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D-brane Spectrum and K-theory Constraints of $D = 4$, $\mathcal{N} = 1$ Orientifolds

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Abstract

We study the spectrum of stable BPS and non-BPS D-branes in $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds for all choices of discrete torsion between the orbifold and orientifold generators. We compute the torsion K-theory charges in these $D = 4$, $\mathcal{N} = 1$ orientifold models directly from worldsheet conformal field theory, and compare with the K-theory constraints obtained indirectly using D-brane probes. The K-theory torsion charges derived here provide non-trivial constraints on string model building. We also discuss regions of stability for non-BPS D-branes in these examples.

1 Introduction

Since their discovery more than a decade ago [1], D-branes have been playing a key role in elucidating non-perturbative aspects of string theory. Phenomenologically, they have also become an indispensable tool because D-branes can localize gauge and matter fields and thus stable could in fact be where the Standard Model lives. Therefore if the brane world idea is indeed realized in Nature, it is important to understand given a string compactification what are the allowed stable (BPS or non-BPS) D-branes. While the charges of BPS branes are quite easy to work out, it is in general not a simple task to enumerate the *complete* spectrum of stable D-branes especially the non-BPS ones except for simple backgrounds such as toroidal orbifolds [2–4] or orientifolds [5–8] [1]. As a result, models whose complete D-brane spectrum has been derived so far are those with extended supersymmetry and not much is known about non-BPS branes in the phenomenologically interesting case of $D = 4$, $\mathcal{N} = 1$ supersymmetric backgrounds. In this paper, we shall address this issue by investigating the spectrum of stable D-branes for some prototypical $\mathcal{N} = 1$ examples. In particular, we will focus on the $\mathbf{T^6}/\mathbf{Z_2} \times \mathbf{Z_2}$ orientifolds because in addition to being simple $\mathcal{N} = 1$ compactifications, models with realistic particle physics features have also been constructed in this framework [11]. Moreover, a systematic computer search for the statistics of supersymmetric D-brane models has recently been carried out also for this particular closed string background [12, 13]. Thus, detailed studies of this specific orientifold though undoubtedly limited may serve as a mini-platform for a more ambitious string vacuum project [14].

The stability of a D-brane is typically due to the charges it carries. Although D-branes were originally discovered as objects carrying Ramond-Ramond (RR) charges under p-form supergravity fields, their charges are more properly classified by K-theory [15, 16] instead of cohomology. An important difference between K-theory and cohomology charges arrises when considering discrete *torsion* (e.g., $\mathbf{Z}_2$) valued charges. In fact, the existence of such K-theory torsion charges (sometimes referred to as K-theory charges) is precisely the reason that certain non-BPS D-branes are stable [9]. Due to the K-theoretical nature of D-brane charges, we expect there are in general some additional discrete constraints on string constructions which are invisible in supergravity. Analogously to the usual RR tadpole conditions, the total torsion charges must cancel in a consistent string compactification. However, unlike the usual integral valued RR charges, there are no supergravity fields to which the torsion charged D-branes are coupled. Hence, the discrete constraints on the cancellation of torsion charges are invisible from the usual tadpole conditions obtained by factorization of one-loop open string amplitudes. Nonetheless, these discrete K-theory constraints can be detected in an indirect way by introducing suitable D-brane probes [17]. From a probe brane point of view, a manifestation of these discrete K-theory constraints is the requirement that there should be an even number of Weyl fermions charged under the symplectic gauge group on its worldvolume, for otherwise the worldvolume theory suffers from global anomalies [18]. Moreover, it was recently shown in [19] that for some specific simple examples, these discrete constraints from a probe brane analysis are seen to arise from the standard Dirac quantization conditions of 4-form fluxes when lifted to F-theory. Although the probe brane approach

[1] Reviews and further references of the constructions of these D-brane spectra can be found in [9, 10].

provides a powerful way to determine the K-theory constraints, it is not entirely clear however that all torsion charges can be obtained in this manner.

We are particularly interested in these K-theory constraints because they have proven to provide important non-trivial consistency conditions in building realistic D-brane models from string theory [20,21]. Although the K-theory constraints are automatically satisfied for some simple models [22], it is certainly not the case in general. The K-theory constraints of the $\mathbf{T^6}/\mathbf{Z_2}\times\mathbf{Z_2}$ orientifold that we will analyze in detail in this paper were obtained in [20] using a probe brane approach. Here, we would like to determine these K-theory constraints from a direct conformal field theory (CFT) calculation. It is important to emphasize that these two approaches are to some extent complementary. The stability of D-branes and more importantly their regions of stability are more apparent from the CFT approach, while the relation to anomaly cancellations is more direct from a probe brane perspective. By analyzing the one-loop amplitudes for the open strings that stretch between these branes, as well as imposing consistency conditions for invariance under the orientifold and orbifold generators, we can determine the spectrum of stable BPS and non-BPS branes for a given model. The criteria for a non-BPS brane to be stable is the absence of tachyonic modes in these open string amplitudes.

Our results find agreement with the K-theory constraints derived previously from a probe brane argument [20]. As we shall see, the K-theory constraints in [20] do not constitute the most general set of K-theory charges but nevertheless they are complete for the setup considered in [20]. It will also become clear later from our spectrum of stable non-BPS branes that for a more general setup (e.g., when one considers "oblique" magnetic fluxes on the worldvolumes of D-branes as in [23,24], or D-branes that are stuck at orbifold fixed points, or D-branes that are not space-filling in our four dimensional spacetime, etc), then there are further K-theoretical constraints to be satisfied. In addition to the above subtleties, there are actually 2^4 different types of $\mathbf{Z}_2\times\mathbf{Z}_2$ orientifolds corresponding to different choices of discrete torsion between the orientifold and orbifold generators [25,28]. The orientifold background considered in [20] (which is T-dual to [26] and [27]) is simply one of them. For completeness, we have also enumerated the spectra of torsion and integrally charged D-branes for all other choices of discrete torsion. We expect these results will be useful for future work in building realistic models from more general $\mathbf{Z}_2\times\mathbf{Z}_2$ orientifolds.

Finally, another motivation for this work is to explore the implications of stable D-branes in cosmology. In [29], it was suggested that stable D-branes that are wrapped entirely in the compact directions (and thus appear pointlike to our 4-dimensional universe) might be interesting cold dark matter candidates. In particular, the lightest D-particles (LDPs) are stable because they are the lightest state in the spectrum carrying a specific charge (either an integral or torsion charge). With the specific models at hand, we can investigate in a concrete setting how robust is the existence of such cold dark matter candidates.

This paper is organized as follows. For completeness, we review in Section 2 the boundary state method that we use to compute the spectrum of stable D-brane. Readers who are familiar with this technique can skip directly to the next section. Section 3 gives the specifics of the $\mathbf{Z}_2\times\mathbf{Z}_2$ orientifolds. To be concrete, we will be working in the T-dual frame of Type IIB orientifolds with O3 and O7 planes. A table of orientifold invariant states for two different types of orientifold projections, known as the hyper-multiplet and the tensor-multiplet models, is presented. The effects of discrete torsion are also discussed. To demonstrate explicitly that cancellation of K-theory charges are additional constraints on top of the usual RR tadpole cancellations, we have listed the tadpole conditions in Section 4. For later comparison with results from the worldsheet approach, a probe brane analysis of the K-theory constraints is also included for different choices of discrete torsion. A detailed analysis of the torsion brane and integrally charged brane spectrum is presented in Section 5, along with a discussion of discrete torsion in the model. Some details are relegated to the appendices. Appendix A contains the Klein Bottle, Möbius Strip, and Annulus amplitudes for the models under consideration. Appendix B shows the calculations used to determine the stability of the non-BPS branes. Appendix C discusses the stability regions for the torsion charged branes. Appendix D contains the non-BPS brane spectrum for a $\mathbf{Z}_2$ orientifold as well as that of a T-dual version of the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold under consideration. Finally, Appendix E contains a table of integrally charged D-matter candidates [29] for different choices of discrete torsion.

2 Boundary State Formalism

In this section we give a brief and elementary review of the boundary state techniques[2] used in our calculations. This method will allow us to determine orbifold and orientifold invariant D-branes states (BPS and non-BPS) in the Type IIB model of [20], consisting of O3 and O7 planes, that we want to consider. The results in the T-dual picture with $O5$ and $O9$ planes can be obtained by a simple T-duality.

The initial setup for a boundary state calculation is to have a closed string propagate between two D-brane boundary states, which are evaluated as matrix elements. When evaluated, this expression under open/closed string duality gives the familiar one loop partition function for open strings that end on the D-branes. In addition, one has to require that the boundary state be invariant under the closed string GSO projection ($\frac{1}{4}(1+(-1)^F)(1\pm(-1)^{\tilde{F}})$ where $\mp$ corresponds to IIA/IIB, respectively), as well as the orbifold or orientifold projection. The ψ coordinates (as well as the ∂X coordinates) of the superstring are used to define the boundary state using the condition[3]

$$\begin{aligned} \psi_r^\mu + i\eta\tilde{\psi}_{-r}^{\mu}|\eta\rangle &= 0 \qquad \mu = 1,\dots,p+1 \\ \psi_r^\mu - i\eta\tilde{\psi}_{-r}^{\mu}|\eta\rangle &= 0 \qquad \mu = p+2,\dots,8 \end{aligned} \tag{2.1}$$

where r is half-integer moded in the untwisted NSNS sector, and integer moded in the untwisted RR sector. The boundary state $|\eta\rangle$ has two values, $\eta = \pm 1$, which correspond to the different spin structures. In the sectors where r is integer (such as the untwisted and twisted RR sectors and the $\mathbf{Z}_2$ twisted NSNS sector) the ground state is degenerate, giving rise to additional structure in the boundary state. In this case it is convenient to define

$$\psi_\pm^\mu = \frac{1}{\sqrt{2}}(\psi_0^\mu \pm i\tilde{\psi}_0^\mu)\,, \tag{2.2}$$

[2]See e.g. [3,10,30–33] for a more thorough discussion of the boundary state method for D-branes.

[3]We are using the light cone gauge, with x_0 and x_9 as the light cone coordinates.

which satisfy the usual creation/annihinaltion operator anti-commutation relations,

$$\{\psi^\mu_\pm, \psi^\nu_\pm\} = 0, \quad \{\psi^\mu_+, \psi^\nu_-\} = \delta^{\mu\nu}. \tag{2.3}$$

In terms of the $\psi_\pm$ operators the boundary conditions in the untwisted RR sector give

$$\begin{aligned} \psi^\mu_\eta|\eta\rangle_{R-R} = 0 &\qquad \mu = 1, \ldots, p+1 \\ \psi^\nu_{-\eta}|\eta\rangle_{R-R} = 0 &\qquad \nu = p+2, \ldots, 8 \end{aligned} \tag{2.4}$$

Throughout this paper we will be considering $\mathbf{Z}_2$ orbifold actions, and the corresponding $\mathbf{Z}_2$-twisted sectors. We will take these $\mathbf{Z}_2$ actions to invert n spatial coordinates; the branes we consider will stretch along s of the inverted directions and $r+1$ un-inverted directions, with $r+s=p$. Since the $\mathbf{Z}_2$ twisted NSNS sector is integer moded, we also define the zero mode creation and annihilation operators in this sector, in terms of which the boundary conditions imply

$$\begin{aligned} \psi^\mu_\eta|\eta\rangle_{NS-NS,T} &= 0 \qquad \mu = 9-n, \ldots, 8-n+s \\ \psi^\nu_{-\eta}|\eta\rangle_{NS-NS,T} &= 0 \qquad \nu = 9-n+s, \ldots, 8 \end{aligned} \tag{2.5}$$

where we have assumed the orbifold twist acts on n coordinates, among them s of them are in Neumann directions and $n-s$ are in Dirichlet directions. The coordinates in the twisted RR sector behave in a similar manner, but only in the untwisted directions,

$$\begin{aligned} \psi^\mu_\eta|\eta\rangle_{R-R,T} &= 0 \qquad \mu = 1, \ldots, r+1 \\ \psi^\nu_{-\eta}|\eta\rangle_{R-R,T} &= 0 \qquad \nu = r+2, \ldots, 8-n \end{aligned} \tag{2.6}$$

where we have $r+1$ Neumann directions and $7-n-r$ Dirichlet directions. Each of our operators (GSO, orientifold, orbifold) can be written in terms of the $\psi_\pm$ operators, which then act on the boundary states and impose a set of restrictions for the dimensions of the invariant D-branes.

The action of $\Omega\mathcal{I}_n$ on the fermionic zero modes of a boundary state[4] is given by

$$\Omega\mathcal{I}_n|\eta\rangle = \kappa \prod_1^8 \frac{1-2\psi^i_0\tilde{\psi}^i_0}{\sqrt{2}} \prod_{i=9-n}^8 (\sqrt{2}\psi^i_0) \prod_{i=9-n}^8 (\sqrt{2}\tilde{\psi}^i_0)|\eta\rangle \tag{2.7}$$

The first term on the right hand side ($\prod_1^8 \frac{1-2\psi^i_0\tilde{\psi}^i_0}{\sqrt{2}}$) is the orientifold action acting on the boundary state $|\eta\rangle$, and is written as a condition on the zero modes (i.e. when applied to eqn. (2.4), it takes $\psi_0 \longleftrightarrow \tilde{\psi_0}$). The other two terms come from the $\mathcal{I}_n$ action, and are also conditions on the zero modes. $\kappa = \pm 1$, and is a phase that allows us to keep our choice of states that are even or odd under the projection.

Starting with the untwisted sector, our orientifold action is trivial acting on the NSNS untwisted state, and on the RR untwisted state we have,

$$\Omega\mathcal{I}_n|\eta\rangle_{R-R,U} = \kappa_{R-R,U}\, i^{5-p+2c+n(n+2)}|\eta\rangle \tag{2.8}$$

[4]See the appendix in [8] for a similar treatment of the $\mathbf{Z}_2$ orientifold [34, 35].

p refers to the number of dimensions filled by a D-brane in the theory, and c is the number of coordinates covered by the $\mathcal{I}_n$ action that are also filled by the D-brane. κ is a phase that is either ± 1, and is determined by the action of the orientifold on the untwisted RR sector[5].

Next we will approach the twisted sector, which contains an inherent subtlety that must be explained. When acting on a twisted sector, the $\mathcal{I}_n$ action does not necessarily transform the same coordinates as the twisted boundary state (in our case the twisted sectors created by our orbifold group of 3 $\mathbf{Z}_2$ generators). When this happens the final result depends both on the n of the $\mathcal{I}_n$ and the n' coordinates transformed by the twisted boundary state. We will try to make this distinction clear in our calculations. For the NSNS twisted sector we have,

$$\Omega\mathcal{I}_n|\eta\rangle_{NS-NS,T} = \kappa_{NS-NS,T} i^{2c_c+c_t+n(n+2)+\frac{n'}{2}}|\eta\rangle \tag{2.9}$$

c_c refers to the common filled directions between the n and n' transformed coordinates. c_t is the number of filled directions in the twisted boundary state.

In the twisted RR sector we have

$$\Omega\mathcal{I}_n|\eta\rangle_{R-R,T} = \kappa_{R-R,T} i^{5+r+2c_{nc}+n(n+2)-\frac{n'}{2}}|\eta\rangle \tag{2.10}$$

In this case c_{nc} comes from filled compact directions on the $\mathbf{T^6}$ that are not common between n and n'. r is the number of filled non-compact directions.

We shall now apply these general calculations to a IIB orientifold that contains $D9$ branes, which we will use in Section 3 when we analyze $D9$ branes with magnetic flux. Using the equations above, we take $p = 9$ and so have $c = n$ and $n' = c_t = 4$[6]. The results become

$$\begin{aligned}
\Omega\mathcal{I}_n|\eta\rangle_{R-R,U} &= \kappa_{R-R,U} i^{n(n+4)-4}|\eta\rangle &(2.11)\\
\Omega\mathcal{I}_n|\eta\rangle_{NS-NS,T} &= \kappa_{NS-NS,T} i^{6+2c_c+n(n+2)}|\eta\rangle &(2.12)\\
\Omega\mathcal{I}_n|\eta\rangle_{R-R,T} &= \kappa_{R-R,T} i^{6+2c_{nc}+n(n+2)}|\eta\rangle &(2.13)
\end{aligned}$$

The RR untwisted sector restricts n to be even, which is a result independent of p for p odd. Thus we can have $n = 0, 2, 4, 6$. With n even, the twisted sector equations restrict c_c and c_{nc} to be either even or odd. If c_c is even (odd), then c_{nc} must also be even (odd) relative to the orbifold projections because of the symmetry of the model. Choosing these to be even, for $n = 2$ or 6 we arrive at the orientifold projection in [20]. For $n = 0$ or 4, the orientifold projection would be the same as in [26]. For our model of $D9$ branes with magnetic flux we will use the orientifold projection in [20], with $n = 6$.

Our model will also contain a $\mathbf{Z_2} \times \mathbf{Z_2}$ orbifold with three projections, which we define to cover the coordinates defined by the actions g_1, g_2, and g_3, where the g_i orbifold is orthogonal to the i^{th} $\mathbf{T}^2$:

$$\begin{aligned}
g_1 : (z_1, z_2, z_3) &\rightarrow (z_1, -z_2, -z_3), &(2.14)\\
g_2 : (z_1, z_2, z_3) &\rightarrow (-z_1, z_2, -z_3), &(2.15)\\
g_3 : (z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, z_3) &(2.16)
\end{aligned}$$

[5]In other words, it chooses the orientifold projection on the untwisted RR sector to be symplectic or orthogonal.

[6]$n' = 4$ is due to our $\mathbf{Z}_2$ generators, which are listed at the end of this section.

The complex coordinate z_i defines the complex coordinates[7] on the i^{th} $\mathbf{T}^2$.

3 The Setup

Having shown how we use the boundary state method to determine invariance under the orientifold projection, we will now apply these results to a specific model. After choosing the orientifold projection in the next section, Section 3.2 lists the requirements for invariance in each of the untwisted and twisted NSNS and RR sectors. The section concludes with a brief review of discrete torsion.

3.1 Model Specifics

Now to the specifics of the model. Starting in Type IIA with the orientifold action in [27], we T-dualize along the x^3, x^5, and x^7 directions. The result in Type IIB is the orientifold action ΩR, with R an orbifold projection that inverts all of the coordinates in the $\mathbf{T}^6$. Due to T-duality the orientifold action picks up a factor of $(-1)^{F_L}$ [36]. Pairing our orientifold action with our orbifold generators we obtain 1 O3 and 3 O7 planes, which wrap the coordinates in the internal directions:

$$\begin{aligned} \Omega R(-1)^{F_L}: &\quad (z_1, z_2, z_3) \to (-z_1, -z_2, -z_3), && (3.1)\\ \Omega R g_1(-1)^{F_L}: &\quad (z_1, z_2, z_3) \to (-z_1, z_2, z_3), && (3.2)\\ \Omega R g_2(-1)^{F_L}: &\quad (z_1, z_2, z_3) \to (z_1, -z_2, z_3), && (3.3)\\ \Omega R g_3(-1)^{F_L}: &\quad (z_1, z_2, z_3) \to (z_1, z_2, -z_3) && (3.4) \end{aligned}$$

Finally we come to the branes themselves. The branes fill $r + 1$ coordinates in the uncompactified space. Since each brane will be wrapped on a $\mathbf{T}^2$ in the compactified space, we shall use s_i as the coordinates on the i^{th} $\mathbf{T}^2$. Thus each s_i will be 0, 1, or 2 and a Dp brane will have $p = r + \sum_1^3 s_i$. From now on we will refer to the dimensions that the branes fill using the notation $(r; s_1, s_2, s_3)$.

3.2 Orientifold Invariance

Now that we have chosen the orientifold projection of the model, we can use an adaptation of equations (2.8), (2.9), and (2.10) to determine invariance of the D-brane states under the GSO and orientifold projections. For Type IIB, the GSO projection requires odd values of p for Dp-branes. Using the boundary state method the invariant orientifold states obey the restrictions

$$\begin{aligned} \Omega R(-1)^{F_L}|\eta\rangle_{R-R,U} &= \kappa^{\Omega}_{R-R,U} i^{5-p}|\eta\rangle_{R-R,U} && (3.5)\\ \Omega R(-1)^{F_L}|\eta\rangle_{NS-NS,T_{g_i}} &= \kappa^{\Omega}_{NS-NS,T} i^{s_j+s_k+2}|\eta\rangle_{NS-NS,T_{g_i}} && (3.6)\\ \Omega R(-1)^{F_L}|\eta\rangle_{R-R,T_{g_i}} &= \kappa^{\Omega}_{R-R,T} i^{5+r+s_i-2}|\eta\rangle_{R-R,T_{g_i}} && (3.7) \end{aligned}$$

[7]The complex coordinates (z_1, z_2, z_3) correspond to $z_n = x^{2n+1} + ix^{2n+2}$ where $n = 1, 2, 3$.

Each of the κ is a choice of discrete torsion between the orientifold and the generators of the orbifold group $\{1, g_1, g_2, g_3\}$. We shall discuss discrete torsion more thoroughly in the next subsection, but before we can continue, we need to introduce some terminology. Although our results deal with 4D $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds, we are going to borrow the terms hyper-multiplet and tensor-multiplet, which are used when describing 6D $\mathbf{Z}_2$ orientifolds. These terms refer to different choices of discrete torsion between the orientifold and orbifold projections. In 6D $\mathbf{Z}_2$ orbifolds there are both twisted sector hyper- and tensor-multiplets. One choice of discrete torsion between the orientifold projection and the twisted sector keeps the hyper-multiplets [34, 35], while the other keeps tensor-multiplets [34, 37, 38]. This difference was clarified in [39] using the D-brane language, and in terms of group cohomology in [28]. A similar choice of discrete torsion arises between the orientifold projection and the orbifold generators in the 4D case. Because of this we have retained this terminology and call our respective 4D models with those particular choices of orientifold projection the hyper- and tensor-multiplet models.

Starting with the hyper-multiplet model [34, 35] in the $\mathbf{T^6}/\mathbf{Z_2} \times \mathbf{Z_2}$, the orientifold and orbifold invariant states are

$$
\begin{array}{rl}
|B(r,\mathbf{s})\rangle_{\text{NS-NS}} & \text{for all } r \text{ and } s_i \\
|B(r,\mathbf{s})\rangle_{\text{R-R}} & \left\{\begin{array}{l} \text{for } r=-1,3 \text{ and } s_1=s_2=s_3=0 \text{ or} \\ \text{for } r=-1,3 \text{ and } s_i=0,\ s_j=s_k=2 \text{ or} \\ \text{for } r=1 \text{ and } s_1=s_2=s_3=2 \text{ or} \\ \text{for } r=1 \text{ and } s_i=2,\ s_j=s_k=0 \text{ or} \\ \text{for } r=2 \text{ and } s_1=s_2=s_3=1 \end{array}\right. \\
|B(r,\mathbf{s})\rangle_{\text{NS-NS,T}g_i} & \text{for all } r \text{ and } s_i \text{ and for } s_j=0, s_k=2 \\
|B(r,\mathbf{s})\rangle_{\text{R-R,T}g_i} & \left\{\begin{array}{l} \text{for } r=-1 \text{ and } s_i=2 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=0 \text{ and } s_i=1 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=1 \text{ and } s_i=0 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=3 \text{ and } s_i=2 \text{ and all } s_j,s_k \end{array}\right.
\end{array}
\tag{3.8}
$$

These choices are consistent with the closed string spectrum of the model and can be determined by combining the results in [8] and [4].

The tensor multiplet model has the same orientifold invariant boundary states in the untwisted sectors as the hyper-multiplet model, with the twisted sectors invariant boundary states given by

$$
\begin{array}{rl}
|B(r,\mathbf{s})\rangle_{\text{NS-NS,T}g_i} & \left\{\begin{array}{l} \text{for all } r \text{ and } s_i \text{ and for } s_j=s_k=0 \text{ or} \\ \text{for all } r \text{ and } s_i \text{ and for } s_j=s_k=2 \end{array}\right. \\
|B(r,\mathbf{s})\rangle_{\text{R-R,T}g_i} & \left\{\begin{array}{l} \text{for } r=-1 \text{ and } s_i=0 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=1 \text{ and } s_i=2 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=2 \text{ and } s_i=1 \text{ and all } s_j,s_k \text{ or} \\ \text{for } r=3 \text{ and } s_i=0 \text{ and all } s_j,s_k \end{array}\right.
\end{array}
\tag{3.9}
$$

Given the above restrictions on r and s_i we can construct four different types of integrally charged branes:

- Fractional Branes: These are charged under untwisted and twisted RR forms. There are two types of fractional branes, singly fractional branes coupling to the g_i twisted sector, which are of the form

$$\begin{aligned} |D(r,s)\rangle &= |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{\text{R-R}} \\ &\quad + |B(r,s)>_{NS-NS,T_i} + |B(r,s)>_{R-R,T_i} \end{aligned} \tag{3.10}$$

  or totally fractional branes, which are of the form

$$\begin{aligned} |D(r,s)\rangle &= |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{\text{R-R}} \\ &\quad + \sum_{i=1}^{3} \{|B(r,s)>_{NS-NS,T_i} + |B(r,s)>_{R-R,T_i}\} \end{aligned} \tag{3.11}$$

  The totally fractional branes only exist in the tensor multiplet model, and are the $(-1;0,0,0)$, $(3;0,0,0)$, and $(1;2,2,2)$ branes. Singly fractional branes exist for $(r,s_i) = (-1,2),(1,0),(3,2)$ and $(s_j,s_k) = (2,0),(0,2)$ (hyper), or $(r,s_i) = (-1,0),(1,2),(3,0)$ and $(s_j,s_k) = (0,0),(2,2)$ (tensor). Note that this includes the tadpole cancelling $D7$ branes.

- Bulk Branes: These are charged only under the untwisted RR forms, and are of the form

$$|D(r,s)\rangle = |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{\text{R-R}} \ . \tag{3.12}$$

  These exist for $(r=(-1,3), s=0)$, and $(r,s)=(1,6)$ (hyper) or $(r=1; s_i=0, s_j = s_k = 1)$ and $(r=3; s_i=2, s_j=s_k=1)$ (tensor).

- Truncated Branes: These are charged only under the twisted RR forms. These exist for the invariant states listed in equations (3.8) or (3.9), provided that no fractional branes exist with the same r and s_i. We discuss them in more detail in section (5.2).

- Stuck Branes: These branes are not charged under the twisted RR forms, but are different from bulk branes in that they cannot move from the fixed points. Before orientifolding such branes are a pair of fractional branes with opposite twisted charges; the orientifold projection removes the moduli which allow the brane to move off the fixed points. We will discuss them in Section 5.

With the exception of the truncated branes, the above branes are BPS. The conditions in (3.8) and (3.9) guarantee that the D-branes are both orbifold and orientifold invariant.

3.3 Discrete Torsion

Discrete torsion in orbifolds [40, 41] has been studied extensively, especially in the $\mathbf{Z}_2 \times \mathbf{Z}_2$ case. We will be interested in determining the orbifold invariant boundary states, hence we need to know the effects of discrete torsion on the D-brane sector. Here we will summarize the results[8]. Starting with the projection operator for the orbifold group

$$P = \frac{1}{|\Gamma|} \sum_{g_i \,\epsilon\, \Gamma} g_i \tag{3.13}$$

where Γ is the orbifold group that contains elements g_i, and inserting it into the partition function,

$$Z(q,\overline{q}) = \frac{1}{|\Gamma|} \sum_{g_i,g_j \,\epsilon\, \Gamma} \epsilon(g_i,g_j) Z(q,\overline{q};g_i,g_j) \tag{3.14}$$

The partition function can pick up a phase $\epsilon(g_i,g_j)$ between the elements in the orbifold group. For the $\mathbf{Z}_2$ projections we consider, we have two choices for the phase: the trivial result $\epsilon(g_i,g_j) = 1$ or the non-trivial result $\epsilon(g_i,g_j) = 1$ if $g_i = g_j$, and -1 otherwise. In our case the orbifold group contains 4 elements: $\{1, g_1, g_2, g_3\}$. Choosing discrete torsion between the orbifold generators means that components of our partition function will be modular invariant up to a phase definition.

Furthermore, there is a relationship between a IIA theory with (without) discrete torsion and a IIB theory without (with) discrete torsion. This effect can be seen by noting that we have two choices for defining the fermionic zero mode operators which make up the orbifold elements g_i,

$$g_i = \prod(\sqrt{2}\psi_0^i) \prod(\sqrt{2}\tilde{\psi}_0^i) \tag{3.15}$$

$$\hat{g}_i = \prod(2\psi_0^i\tilde{\psi}_0^i) \tag{3.16}$$

where we sum over the directions acted on by the $\mathbf{Z}_2$ twists. These two definitions are related by discrete torsion

$$\hat{g}_i|g_j\rangle = \epsilon(g_i,g_j) g_i |g_j\rangle \tag{3.17}$$

where $\epsilon(g_i,g_j)$ has the non-trivial definition. This is equivalent to saying that these two definitions are related by T-duality. The theory that we define to have discrete torsion (g_i or $\hat{g}_i$) is ambiguous, but each theory is unique in that they have different Hodge numbers.

Initially we will look at a model with discrete torsion $g_1|g_2\rangle = +|g_2\rangle$, which corresponds to a model with the Hodge numbers (h_{11}, h_{21}) = (51,3). The model with $g_1|g_2\rangle = -|g_2\rangle$ corresponds to a model with Hodge numbers (h_{11}, h_{21}) = (3,51).

4 RR Tadpole Conditions

In this section, we analyze in detail all the tadpole conditions arising in the Type IIB orientifolds under consideration. The purpose of doing this is to illustrate that the K-theory torsion charges, if uncancelled in a string model, do not show up as the usual tadpole

[8] For more details, e.g. [3, 4, 42–45].

divergences and hence their cancellation impose additional constraints. The reason is that unlike the usual homological RR-charges, there are no supergravity fields to which the K-theory torsion charges are coupled. Therefore, the presence of these K-theory torsion charges does not affect the asymptotics of the Klein bottle, Möbius strip, and Annulus amplitudes in the closed string channel, which correspond to the exchanges of light closed string fields. However, just like the usual RR charges, these K-theory torsion charges need to be cancelled globally in a consistent model. We will derive such K-theory constraints for the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold using a probe brane approach in this section. The corresponding derivation using a CFT approach will be presented in Section 5.

4.1 Homological RR Tadpoles

The tree level interaction for BPS branes on the Annulus, Möbius Strip, and Klein bottle, not considering the momentum and winding sums or Chan-Paton factors, is the same as in the T-dual case [26]. Since these do not change, we have included the full calculations (open and closed string channel) in Appendix A, with eqn.(4.1) below defining the momentum and winding on the Klein Bottle (primed) and Möbius Strip/Annulus (no prime), respectively. The tilded definitions are the momentum and winding after a Poisson resummation.

$$
\begin{aligned}
M'_j &= \textstyle\sum_{n=-\infty}^{\infty} e^{\frac{-\pi t n^2}{R_j^2}}, & W'_j &= \sum_{m=-\infty}^{\infty} e^{-\pi t m^2 R_j^2} \\
\widetilde{M}'_j &= \textstyle\sum_{s=-\infty}^{\infty} e^{\frac{-\pi R_j^2 s^2}{t}}, & \widetilde{W}'_j &= \sum_{r=-\infty}^{\infty} e^{\frac{-\pi r^2}{R_j^2 t}} \\
M_j &= \textstyle\sum_{n=-\infty}^{\infty} e^{\frac{-2\pi t n^2}{R_j^2}}, & W_j &= \sum_{m=-\infty}^{\infty} e^{-2\pi t m^2 R_j^2} \\
\widetilde{M}_j &= \textstyle\sum_{s=-\infty}^{\infty} e^{\frac{-\pi R_j^2 s^2}{2t}}, & \widetilde{W}_j &= \sum_{r=-\infty}^{\infty} e^{\frac{-\pi r^2}{2R_j^2 t}}
\end{aligned}
\tag{4.1}
$$

Calculating the RR tadpole for the BPS branes is similar to the T-dual case. For the purpose of comparison to the K-theory constraints that we will introduce in the next subsection, here we list out the untwisted RR tadpole contribution. The full tadpole, including the twisted contribution and cross terms between different branes is well known in the literature [26], and has also been calculated taking into account factors of discrete torsion [46,47]. For completeness, we summarize these results in Appendix A. The untwisted tadpole is presented below, where the first line is the Klein Bottle contribution, the second line comes from the Möbius Strip, and the final line is from the induced $D3$ and $D7$ brane charge on the Annulus.

$$
\begin{aligned}
&v_4 \int dl \left\{ 32 \left(\frac{1}{v_1 v_2 v_3} + \frac{v_1 v_2}{v_3} + \frac{v_2 v_3}{v_1} + \frac{v_1 v_3}{v_2} \right) \right. \\
&-2 \left(\frac{1}{v_1 v_2 v_3} \mathrm{Tr}\, \gamma_{\Omega 3,3}^{T} \gamma_{\Omega 3,3}^{-1} + \sum_{i=1}^{3} \frac{v_j v_k}{v_i}\ \mathrm{Tr}\, \gamma_{\Omega 7_i,7_i}^{T} \gamma_{\Omega 7_i,7_i}^{-1} \right)
\end{aligned}
$$

$$+\frac{1}{32}\left(\frac{1}{v_1v_2v_3}\ \mathrm{Tr}\ \gamma_{1,3}\gamma_{1,3}^{-1}+\sum_{i=1}^{3}\frac{v_jv_k}{v_i}\ \mathrm{Tr}\ \gamma_{1,7_i}\gamma_{1,7_i}^{-1}\right)=0 \tag{4.2}$$

where $i\neq j\neq k$, and the indices run from 1 to 3. For tadpole cancellation we require

$$\gamma_{\Omega 3,3} = +\gamma_{\Omega 3,3}^T \tag{4.3}$$
$$\gamma_{\Omega 7_1,7_3} = +\gamma_{\Omega 7_1,7_3}^T \tag{4.4}$$
$$\gamma_{\Omega 7_2,7_1} = +\gamma_{\Omega 7_2,7_1}^T \tag{4.5}$$
$$\gamma_{\Omega 7_3,7_2} = +\gamma_{\Omega 7_3,7_2}^T \tag{4.6}$$

The final result is then

$$\frac{1}{32}\int dl\ \left\{\frac{1}{v_1v_2v_3}(32-n_3)^2+\sum_{i=1}^{3}\frac{v_jv_k}{v_i}(32-n_{7_i})^2\right\}=0 \tag{4.7}$$

This result can also be written in terms of magnetic and wrapping numbers[9]. The D-branes we are working with have magnetic flux, which is quantized according to

$$\frac{m_a^i}{2\pi}\int_{\mathbf{T_i^2}}F_a^i=n_a^i \tag{4.8}$$

m_a^i is the number of times a D-brane wraps the i^{th} $\mathbf{T^2}$, and n_a^i is the integer units of flux going through the i^{th} $\mathbf{T^2}$. We can use these numbers to describe our D-branes. For example, a D3 brane ($r=3$, $s=0$) has magnetic and wrapping numbers $[(n_a^1,m_a^1)\times(n_a^2,m_a^2)\times(n_a^3,m_a^3)]=[(1,0)\times(1,0)\times(1,0)]$. A D5 brane that wraps the first $\mathbf{T^2}$ would have numbers $[(n_a^1,m_a^1)\times(1,0)\times(1,0)]$. Our D-branes must be invariant under the orientifold group, which means we also introduce image branes with magnetic and wrapping numbers $(n_a^i,-m_a^i)$. In addition, we shall consider a general setup with K stacks of N_a D-branes. The RR tadpole conditions are now related to cancellation of the O-plane charge by the D-brane and its image, which we will not count separately,

$$\sum_a N_a[\Pi_a]+[\Pi_{O3+O7}]=0 \tag{4.9}$$

where have 64 O3 planes with $-1/2$ D3 brane charge and 4 O7_i branes with -8 D7_i charge that need to be cancelled. The RR tadpole conditions for a general choice of discrete torsion are

$$\sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 n_\alpha^3 = 16\kappa_{\Omega R} \tag{4.10}$$
$$\sum_\alpha N_\alpha n_\alpha^1 m_\alpha^2 m_\alpha^3 = -16\kappa_{\Omega R g_1} \tag{4.11}$$
$$\sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 m_\alpha^3 = -16\kappa_{\Omega R g_2} \tag{4.12}$$
$$\sum_\alpha N_\alpha m_\alpha^1 m_\alpha^2 n_\alpha^3 = -16\kappa_{\Omega R g_3} \tag{4.13}$$

[9]See [49] for a more detailed explanation.

(4.10) refers to the D3 brane, and (4.11), (4.12), (4.13) are the D7_i branes. The κ factors refer to a choice of discrete torsion between the orientifold and the untwisted RR boundary state ($\kappa_{\Omega R}$), and the orientifold and the orbifold generators ($\kappa_{\Omega R g_i}$). The effects of discrete torsion on the D-brane spectrum will be discussed in more detail in Section 5.3.

4.2 K-theoretical RR Tadpoles from Probe Branes

Besides the homological RR tadpole conditions there are the K-theory torsion constraints which can be found using D-brane probes. For example, one can introduce probe $D3$ ($r = 3, s = 0$) and $D7$ ($r = 3, s_i = s_j = 2, s_k = 0$) branes and demand that the number of Weyl fermions on each of the probe brane worldvolume gauge theory to be even for otherwise there are $SU(2)$ $D = 4$ global gauge anomalies [18]. This is the approach adopted in [17, 20, 22]

Though we shall prove our results from a CFT approach in the next section, for the probe brane approach the K-theory constraints in the hyper-multiplet are

$$\sum_\alpha N_\alpha m^1_\alpha m^2_\alpha m^3_\alpha \quad \in \quad 4\mathbb{Z} \tag{4.14}$$

$$\sum_\alpha N_\alpha n^1_\alpha n^2_\alpha m^3_\alpha \quad \in \quad 4\mathbb{Z} \tag{4.15}$$

$$\sum_\alpha N_\alpha n^1_\alpha m^2_\alpha n^3_\alpha \quad \in \quad 4\mathbb{Z} \tag{4.16}$$

$$\sum_\alpha N_\alpha m^1_\alpha n^2_\alpha n^3_\alpha \quad \in \quad 4\mathbb{Z} \tag{4.17}$$

which requires an even number of non-BPS torsion charged $D9$ and $D5$ ($r = 3, s_i = 2, s_j = s_k = 0$) branes. These torsion charged branes are non-BPS D-branes that couple to the NSNS sector (untwisted and/or twisted) but not to RR fields. Nevertheless, they can be stable because of the discrete torsion $\mathbf{Z}_2$ charge they carried [2–10]. See Section 5.1 for a more precise and detailed definition.

For different choices of discrete torsion, the probe brane approach gives the following K-theory constraints for $\kappa = 1$,

$$\sum_\alpha N_\alpha m^1_\alpha m^2_\alpha m^3_\alpha \in 4\mathbb{Z} \qquad \left\{ \begin{array}{c} \text{for } \kappa + \kappa_{\Omega R g_1} + \kappa_{\Omega R g_2} + \kappa_{\Omega R g_3} = \\ 2 \text{ or } 4 \end{array} \right. \tag{4.18}$$

$$\sum_\alpha N_\alpha m^i_\alpha n^j_\alpha n^k_\alpha \in 4\mathbb{Z} \qquad \left\{ \begin{array}{c} \text{for } \kappa + \kappa_{\Omega R} + \kappa_{\Omega R g_j} + \kappa_{\Omega R g_k} = \\ 2 \text{ or } 4, \\ \text{i} \neq \text{j} \neq \text{k} \end{array} \right. \tag{4.19}$$

and for $\kappa = -1$,

$$\sum_\alpha N_\alpha m^1_\alpha m^2_\alpha m^3_\alpha \in 8\mathbb{Z} \qquad \text{for } \kappa + \kappa_{\Omega R g_1} + \kappa_{\Omega R g_2} + \kappa_{\Omega R g_3} = 2 \tag{4.20}$$

$$\sum_\alpha N_\alpha m^i_\alpha n^j_\alpha n^k_\alpha \in 8\mathbb{Z} \qquad \left\{ \begin{array}{c} \text{for } \kappa + \kappa_{\Omega R} + \kappa_{\Omega R g_j} + \kappa_{\Omega R g_k} = 2 \\ \text{i} \neq \text{j} \neq \text{k} \end{array} \right. \tag{4.21}$$

where there are different K-theory constraints for different choices of discrete torsion between the orbifold generators (κ), between the orientifold and the untwisted RR boundary state ($\kappa_{\Omega R}$), and the orientifold and the orbifold generators ($\kappa_{\Omega R g_j}$). The gauge groups for the open string spectrum on the probe branes have been provided for different values of discrete torsion in Table 4. By comparing this result with eqns (4.18), (4.19), we see that the K-theory constraints exist when the probe brane gives USp gauge groups. In terms of the magnetic and wrapping numbers of the branes, we will have a symplectic gauge group on the brane when the charge class $[\mathbf{Q}_a]$ of a D-brane is invariant under the ΩR action. For open strings that begin and end a brane with a symplectic gauge group, the K-theory constraints restrict the number of chiral fermions to be even.

As we shall see in the next section, these are not the entire set of possible K-theory constraints, but the one relevant to our setup. The probe brane approach (at least for the types of probe branes that have been introduced in the literature) gives us the constraints for branes that fill the non-compact space ($r = 3$) and for backgrounds with non-oblique flux. Our results in the next section show that there are additional torsion charged branes that might show up in more general flux backgrounds.

5 Non-BPS Branes

5.1 Torsion Branes in the $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$

The first set of non-BPS branes we are going to analyze in the model are torsion branes. In this section we consider only the hyper-multiplet model, and will consider other models (the T-dual of the $\mathbf{T}^6/\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold as well as a $\mathbf{T}^4/\mathbf{Z}_2$ orientifold) in Appendix D. The effects of discrete torsion will be addressed in Section 5.3. These non-BPS torsion branes do not couple to the untwisted or twisted RR sector, i.e. there are no $(-1)^F$ factors in the corresponding open string projection operators. Torsion branes have boundary states of the form

$$|D(r,s)\rangle = |B(r,s)>_{\text{NS-NS}} \tag{5.1}$$

$$\text{or} \quad |D(r,s)\rangle = |B(r,s)>_{\text{NS-NS}} + \epsilon_i\, |B(r,s)>_{NS-NS,T_i} \qquad i = 1,2,\text{or } 3 \tag{5.2}$$

$$\text{or} \quad |D(r,s)\rangle = |B(r,s)>_{\text{NS-NS}} + \sum_{i=1}^{3} \epsilon_i\, |B(r,s)>_{NS-NS,T_i} \tag{5.3}$$

where each of the twisted boundary states is defined up to a phase $\epsilon_i = \pm 1$ and $\epsilon_3 = \epsilon_1\epsilon_2$. The open strings living on these branes are, respectively, invariant under the following projection operators

$$\left(\frac{1+\Omega R}{2}\right) \tag{5.4}$$

$$\left(\frac{1+\Omega R}{2}\right)\left(\frac{1+g_i}{2}\right) \tag{5.5}$$

$$\left(\frac{1+\Omega R}{2}\right)\left(\frac{1+g_1}{2}\right)\left(\frac{1+g_2}{2}\right) \tag{5.6}$$

Imposing the orientifold projection places restrictions on the allowed r and s_i values for branes in equations (5.2) and (5.3). Specifically we see immediately, from equation (3.8) that no branes of the type given in eqn (5.3) are orientifold invariant. We are now ready to compute the spectrum of stable (i.e. tachyon-free) branes.

The tachyon is extracted from the open string partition function, and an equation is set up to cancel the tachyon between the Möbius strip and Annulus diagrams. Using this technique, there are eight possible contributions to the tachyon: the untwisted Annulus diagram, the twisted Annulus diagram (three contributions), the Möbius strip diagram for a boundary state/$O3$ crosscap interaction, and the Möbius strip diagram for a boundary state/$O7_i$ crosscap interaction (three contributions). See Appendix B, eqns (B.1) - (B.3) for the relevant calculations.

The condition for the tachyon to cancel is

$$\begin{aligned} &2^4\ n^2\ \times (1+\epsilon_{T_1}+\epsilon_{T_2}+\epsilon_{T_3}) \\ &-2n\ \sin\left(\frac{\pi}{4}(r-s+1)\right)-2n\ \sin\left(\frac{\pi}{4}(r+s-2s_3-3)\right) \\ &-2n\ \sin\left(\frac{\pi}{4}(r+s-2s_1-3)\right)-2n\ \sin\left(\frac{\pi}{4}(r+s-2s_2-3)\right)=0 \end{aligned} \tag{5.7}$$

where n is the normalization of the boundary state, and must be solved for when plugging in values of r and s. For stable torsion branes n must be a non-zero positive number. Because we cannot construct torsion branes that couple to all three twisted sectors and are orientifold invariant, we have introduced the parameter ϵ_{T_i} to determine to which of the twisted sectors the brane is coupled. $\epsilon_{T_i}=1$ if the brane couples to the T_i twisted NSNS sector, and 0 otherwise.

It is at this point we would like to emphasize that eqns. (5.7) and (5.16) are conditions for the existence of torsion charged D-branes of certain dimensions (i.e., values of r and s_i) in a background with a specific choice of discrete torsion. The existence of such torsion branes imply the discrete constraints discussed in Subsection 4.2. Hence, for a vaccum configuration (which involves stacks of D-branes) to be consistent, we need to check that the discrete conditions (4.18),(4.19) (for $\kappa=1$) or (4.20), (4.21) (for $\kappa=-1$) are satisfied.

There are two types of torsion branes to consider: branes that couple to twisted NSNS sectors and branes that couple only to the untwisted NSNS sectors. The former is of the form in eqns. (5.2), which corresponds to the open string projection operator (5.5). The latter are of the form in eqns. (5.1), and correspond to the projection operator (5.4).

The branes that couple to twisted NSNS sectors and for which the open string cancels are listed in Table 1. For a torsion brane to be invariant under all orbifold and orientifold projection operators, the branes can only couple to one twisted sector T_i, where i refers to the twisted sector generated by the g_i orbifold action. It is easy to see that the branes listed in Table 1 are orbifold invariant versions of the torsion branes found in [8]. For example the $(3;0,2,0)$-brane coupling to the NSNSTg_i sector is a g_2 invariant combination of $(5,2)$-branes of [8].

Of the three types of branes listed in Table 1, not all are consistent. Indeed, following the discussion in [16] and [6], it was argued in [8] that the $(4,2)$-branes are inconsistent despite being tachyon-free. T-dualising the results in Table 1, it is easy to see that the branes with

Invariant Twisted States T_i	$(r; s_1, s_2, s_3)$	n^2
$i = 1$	$(2;0,2,0)^\dagger$	$\frac{1}{128}$
	$(2;0,0,2)^\dagger$	$\frac{1}{128}$
	$(3;0,2,0)$	$\frac{1}{64}$
	$(3;0,0,2)$	$\frac{1}{64}$
	$(3;1,2,0)^\dagger$	$\frac{1}{128}$
	$(3;1,0,2)^\dagger$	$\frac{1}{128}$
$i = 2$	$(2;0,0,2)^\dagger$	$\frac{1}{128}$
	$(2;2,0,0)^\dagger$	$\frac{1}{128}$
	$(3;0,0,2)$	$\frac{1}{64}$
	$(3;2,0,0)$	$\frac{1}{64}$
	$(3;0,1,2)^\dagger$	$\frac{1}{128}$
	$(3;2,1,0)^\dagger$	$\frac{1}{128}$
$i = 3$	$(2;0,2,0)^\dagger$	$\frac{1}{128}$
	$(2;2,0,0)^\dagger$	$\frac{1}{128}$
	$(3;0,2,0)$	$\frac{1}{64}$
	$(3;2,0,0)$	$\frac{1}{64}$
	$(3;0,2,1)^\dagger$	$\frac{1}{128}$
	$(3;2,0,1)^\dagger$	$\frac{1}{128}$

Table 1: Stable Torsion branes that couple to twisted NSNS sectors. These are torsion branes of the form in eqn. (5.2). Branes that are shown to be inconsistent are marked with a dagger.

$r = 2$ or with $s_i = 1$ for some i, are T-dual to $g_1 \times g_2$ orbifold invariant $(4, 2)$ branes of [8] and hence are inconsistent. Thus the $D5$ branes are the only allowed torsion branes with twisted NSNS coupling. For $i = 1$, for example, these would be the $(3; 0, 2, 0)$ and $(3; 0, 0, 2)$ branes. These can be thought of as g_2 invariant images of the $Z_2 \oplus Z_2$ torsion branes found in [8].

The torsion branes with one twisted coupling in Table 1 can also be thought of as the orientifold invariant bound state of a fractional BPS D-brane and fractional anti-BPS D-brane from the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold. This can be seen from the normalizations of the D-branes. Indeed, the normalization squared of a fractional BPS $D5$ brane is $\frac{1}{256}$. The $D5$ torsion brane has a normalization squared of $2^2 \times \frac{1}{256} = \frac{1}{64}$ confirming that the torsion charged brane can be seen as a superposition of a BPS anti-BPS pair of branes with oppositely charged untwisted and twisted RR sectors.

The second type of torsion brane, branes that only couple to the untwisted NSNS sector, have boundary states of the form in equation (5.1). Our results are presented in Table 2. Of these branes, in IIB the $D4$ and the $(r = 3; s_i = 0, s_j = 1, s_k = 2)$ branes are T-dual to $D6$ branes, and the $(2; 2, 2, 2)$ brane is T-dual to a $(2; 0, 0, 0)$ brane. Therefore these branes are T-dual to branes that can be shown to be inconsistent in Type I.

Note that the $(3; 0, 2, 0)$ branes of the type given in equation (5.1) and listed in Table 2

$(r; s_1, s_2, s_3)$	n^2
$r=2, s_i=2, s_j=s_k=0^{\dagger}$	$\frac{1}{32}$
$r=2, s_1=s_2=s_3=2^{\dagger}$	$\frac{1}{32}$
$r=3, s_i=1, s_j=s_k=0$	$\frac{1}{32}$
$r=3, s_i=s_j=1, s_k=0$	$\frac{1}{16}$
$r=3, s_i=2, s_j=s_k=0$	$\frac{1}{16}$
$r=3, s_1=s_2=s_3=1^{\dagger}$	$\frac{1}{8}$
$r=3, s_i=0, s_j=1, s_k=2^{\dagger}$	$\frac{1}{32}$
$r=3, s_i=2, s_j=s_k=1$	$\frac{1}{16}$
$r=3, s_i=s_j=2, s_k=1$	$\frac{1}{32}$
$r=3, s_1=s_2=s_3=2$	$\frac{1}{16}$

Table 2: Stable Torsion branes that only couple to the untwisted NSNS sectors. These are torsion branes of the form in eqn. (5.1). Branes that are shown to be inconsistent are marked with a dagger.

can be thought of as a pair of torsion branes of the type given in equation (5.2) with opposite twisted torsion charges. This is in fact a g_2-invariant version of the process discussed in [8].

Of the branes in Table 2, the $D9$ and the $(r=3; s_i=2)$ torsion branes correspond to the branes found by a probe brane argument in [20], using $D9$ branes with non-oblique magnetic flux on their world-volumes. The two other odd D-branes, an off-diagonal $D5$ and $D7$ brane, correspond to torsion charged branes in a configuration that includes a more general D-brane background.

The additional branes found in Table 2 do not introduce extra K-theory constraints other than the ones obtained from a probe brane argument reviewed in Section 4.2. This can be seen as follows. One can see that the discrete charges carried by the branes in Table 2 are not independent by showing that they can decay to one another via changing the compactification moduli. The tachyon cancellation equation (5.7) is a condition for the absence of ground state tachyons. However, tachyonic momentum/winding modes can develop as we vary the compactification radii. The stability region of the branes in Table 2 can be found by generalizing eqn. (5.7) to include the contributions from momentum and winding modes. Details are given in Appendix C. For example, one of the stability conditions for a $r=3$, $s=1$ brane that fills x^3 in the compact space is that $R_4 \geq \frac{1}{\sqrt{2}}$. If we consider this $D4$ to be a pair of orientifold invariant truncated branes in the g_2 or g_3 orbifold, for $R_4 \leq \frac{1}{\sqrt{2}}$ each of these branes can decay into a pair of orientifold invariant $r=3$, $s_1=2$ $D5$ branes. Therefore the $D4$ and the $D5$ have the same torsion charge.

As a consistency check one can compute the tree level amplitude between two torsion charged D-branes. If there a tachyon in the spectrum, then it signals that there is a common $\mathbf{Z_2}$ charge between them which causes an instability in the system. We can use this method to find an appropriate basis for the branes in terms of their charges. It can be shown that the branes found in the probe brane argument are a consistent basis for four Z_2 charges, and that all the branes in Table 2 are charged under at least one of these charges.

To enumerate the spectrum of non-BPS torsion charged D-branes, one should analyze the stability regions (as discussed in Appendix C) of the candidate torsion branes in Table 2 and make sure that there are no decay channels by which they can decay to a pathological brane (e.g., those that are T-dual to the $D2$ and $D6$ branes in Type I [6, 16]). Details of such analysis can be found in Appendix C. However, for the purpose of deriving discrete K-theoretical constraints in string model building, this kind of analysis would have to be applied with caution. For example, in [20], the carriers of the discrete K-theory charges are certain BPS bound state of D-branes (which of course must carry also the usual homological RR charges in order for them to be BPS). The D-brane system is BPS only for certain choices of compactification moduli. The analysis of decay channels in Appendix C typically involves decompactifying the theory and would take the D-brane system away from their BPS configuration. It is possible that non-BPS branes carrying a particular discrete K-theory charges cannot be constructed even though BPS branes carrying such charges exist. The stability region of the branes is expanded on in Appendix C, where a similar decay channel analysis for the torsion brane spectrum in the hyper- and tensor-multiplet model is also presented.

5.2 Integrally Charged Branes

In this sub-section we make some comments outside the main focus of this paper by investigating integrally charged D-branes in the $\mathbf{Z_2} \times \mathbf{Z_2}$ orientifolds. In addition to torsion branes that couple only NSNS sectors, in orbifold and orientifold models one often finds so-called truncated branes that couple to the twisted RR sectors. In the present type of models, such integrally charged branes can have boundary states of one of two types. Firstly, they could be of the form

$$|D(r,s)\rangle = |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{R\,-\,R,T_i} \tag{5.8}$$

which corresponds to the open string projection operator

$$\left(\frac{1+\Omega R}{2}\right)\left(\frac{1+g_i(-1)^F}{2}\right) . \tag{5.9}$$

Such branes are just the g_j invariant version of the branes found in [8]. Alternately, the truncated brane boundary states can be of the form

$$\begin{aligned} |D(r,s)\rangle \;\; = \;\; & |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{R\,-\,R,T_i} \\ & + |B(r,s)>_{NS\,-\,NS,T_j} + |B(r,s)>_{R\,-\,R,T_k} \end{aligned} \tag{5.10}$$

which corresponds to the open string projection operator

$$\left(\frac{1+\Omega R}{2}\right)\left(\frac{1+g_i(-1)^F}{2}\right)\left(\frac{1+g_k(-1)^F}{2}\right) \tag{5.11}$$

Using eqns. (3.8) and (3.9) we see that this second type of truncated brane are not possible in the hyper-multiplet model, but are possible in the tensor-multiplet model. [10]

[10] Branes of the form (5.10) are consistent with the orientifold projection in the tensor multiplet model.

Calculating the spectrum of integrally charged branes is similar to finding torsion branes. To determine the integrally charged branes, the tachyon cancellation condition

$$\begin{aligned}
&2^4\ n^2\ \times (1+\epsilon_{T_1}+\epsilon_{T_2}+\epsilon_{T_3})\\
&-2n\ \sin\left(\frac{\pi}{4}(r-s+1)\right) - 2n\ \sin\left(\frac{\pi}{4}(r+s-2s_3-3)\right)\\
&-2n\ \sin\left(\frac{\pi}{4}(r+s-2s_1-3)\right) - 2n\ \sin\left(\frac{\pi}{4}(r+s-2s_2-3)\right) = 0
\end{aligned}$$

is still useful, but now the twisted sector parameter $\epsilon_{T_i} = -1$ if the brane couples to the respective twisted RR sector, and 0 otherwise.

A further condition which restricts the allowed values of r and s_i for these types of non-BPS D-branes is that there cannot be any fractional branes with the same values for r and s_i. Indeed if fractional branes exist for a given value of r and s_i we may always consider a pair of them with opposite bulk RR charge and suitable Wilson lines, in analogy to [52]. Such a combination also carries the required twisted RR charges. For completeness we list such pairs of fractional branes in Table 3.

Returning to the tachyon cancelling condition (5.12) it is easy to see that in the case of the hyper-multiplet model the tachyon only cancels for those values of r and s_i for which fractional branes exist. Since such pairs of fractional branes are unstable in certain regimes of moduli space, there will have to be other D-branes into which these fractional branes decay. Such new D-branes will have boundary states different from the ones in equations (5.8) and (5.10) and we hope to investigate them in the future.

In general non-BPS configurations of branes with integral charges have a possible cosmological application as candidates for cold dark matter [29]. Integrally charged branes with $r = 0$ are interesting because they would appear point-like to a 4D observer, but could fill some of the compactified dimensions. We have listed some of the branes for different values of discrete torsion in Appendix E.

Besides the non-BPS branes we have been considering, there are also the BPS stuck branes, which were mentioned briefly at the end of Section 3.2. The stuck branes can be divided into those that are stuck at all the fixed points of the orbifold generators, or only at one set of fixed points. For the first group we have the branes $(3; 0, 0, 0)$, $(1; 2, 2, 2)$, and $(-1; 0, 0, 0)$. These branes do not couple to the twisted sectors, and are located at the fixed points of the g_1 and g_2 generators. An example of the second type of stuck brane is the $(1; 0, 0, 2)$ brane. This brane is stuck under the g_3 orbifold generator but not the g_1 or g_2. Thus the model contains three types of $(1; 0, 0, 2)$ branes: 1) Singly fractional branes with g_1 twisted couplings (sitting at the g_1 fixed points), 2) Singly fractional branes with g_2 twisted couplings (sitting at the g_2 fixed points), and 3) Stuck branes sitting at the g_3 fixed points.

Finally, we would also like to point out that the $(2; 1, 1, 1)$ brane, which might seem to be a bulk brane, is actually formed from a pair of branes of the form

$$\begin{aligned}
|D_1(r,s)\rangle &= |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{\text{R-R}}\\
&\quad + |B(r,s)>_{\text{NS-NS,T}} + |B(r,s)>_{\text{R-R,T}}
\end{aligned} \tag{5.12}$$

and

$$\begin{aligned}
|D_2(r,s)\rangle &= |B(r,s)>_{\text{NS-NS}} + |B(r,s)>_{\text{R-R}}\\
&\quad - |B(r,s)>_{\text{NS-NS,T}} - |B(r,s)>_{\text{R-R,T}}
\end{aligned}$$

Invariant Twisted States T_i	$(r; s_1, s_2, s_3)$
$i = 1$	$(-1; 2, 0, 2)$
	$(-1; 2, 2, 0)$
	$(1; 0, 0, 2)$
	$(1; 0, 2, 0)$
	$(3; 2, 0, 2)$
	$(3; 2, 2, 0)$
$i = 2$	$(-1; 0, 2, 2)$
	$(-1; 2, 2, 0)$
	$(1; 0, 0, 2)$
	$(1; 2, 0, 0)$
	$(3; 0, 2, 2)$
	$(3; 2, 2, 0)$
$i = 3$	$(-1; 0, 2, 2)$
	$(-1; 2, 0, 2)$
	$(1; 0, 2, 0)$
	$(1; 2, 0, 0)$
	$(3; 0, 2, 2)$
	$(3; 2, 0, 2)$

Table 3: Stable brane–anti-brane pairs carrying twisted RR charges

For a singly fractional $(2; 1, 1, 1)$ brane that couples to the g_1 twisted sector, we could place $|D_1(r, s)\rangle$ at position x^4 and $|D_2(r, s)\rangle$ at position $-x^4$, where neither fractional brane is orientifold or orbifold invariant by itself, but is invariant as a pair. This brane was discussed in [42–44].

5.3 Discrete Torsion Revisited

In this section we consider discrete torsion in the model and the extra quantum numbers it introduces:

- We have to choose the discrete torsion between the orbifold generators $g_1|g_2\rangle$. Calling this choice of phase κ, the choice $\kappa = -1$ has Hodge numbers $(h_{11}, h_{21}) = (3, 51)$, and is the model with discrete torsion, and $\epsilon = 1$ has $(h_{11}, h_{21}) = (51, 3)$, and is the model without discrete torsion.

- We have factors of discrete torsion coming from the orientifold and the orbifold group, $\kappa_{\Omega R g_i}$. There are four choices to be made: $\Omega R(-1)^{F_L}$ acting on the untwisted R-R boundary state[11], and acting on each of the $|g_i\rangle$ boundary states. See equations (3.8) and (3.9) in Section 3.2 for the invariant states.

[11] i.e., defining the O3 projection to be orthogonal or symplectic.

Though these choices of discrete torsion might seem independent, they are actually related to each other [46, 47], through the equation [50],

$$\kappa_{\Omega R}\kappa_{\Omega Rg_1}\kappa_{\Omega Rg_2}\kappa_{\Omega Rg_3} = \kappa \tag{5.13}$$

In other words there are 2^4 choices of discrete torsion that are allowed in these orientifolds. These choices correspond to the orbifold discrete torsion, the signs of the RR charge of the O3-plane, and two of the three O7-planes. These sixteen choices have also been derived in [28] using group cohomology techniques. In [28], the idea of orbifold discrete torsion [40], was generalised to orientifolds. In particular the allowed orientifolds for a given orientifold were shown to be classified by a generalised group cohomology with local coefficients. For the models we have been considering in this paper the orientifold group is $G = \Omega R(-1)^{F_l} \times g_1 \times g_2$, and the relevant group cohomology was found to be [28]

$$H^2(G, \tilde{U}(1)) = \mathbf{Z_2}^{\oplus 4}\,, \tag{5.14}$$

which gives exactly the 16 choices discussed above. The elements of this group cohomology, $[H] \in H^2(G, \tilde{U}(1))$ can also be used to define twisted K-theories [28]

$$K_G^{[H]}(X)\,. \tag{5.15}$$

which classify the allowed D-brane charges for a brane transverse to X.

We may easily extend our calculations from this section to the remaining 15 orientifolds in this family. In particular, rewriting equation (5.7) for the torsion branes charged under the twisted NS-NS sector and including factors of discrete torsion we have

$$\begin{aligned} &2^4\ n^2 \times (1 + \epsilon_{T_1} + \epsilon_{T_2} + \epsilon_{T_3}) \\ &-2\kappa_{\Omega R}\ n\ \sin(\frac{\pi}{4}(r - s + 1)) - 2\kappa_{\Omega R,g_1}\ n\ \sin(\frac{\pi}{4}(r + s - 2s_1 - 3)) \\ &-2\kappa_{\Omega R,g_2}\ n\ \sin(\frac{\pi}{4}(r + s - 2s_2 - 3)) - 2\kappa_{\Omega R,g_3}\ n\ \sin(\frac{\pi}{4}(r + s - 2s_3 - 3)) = 0 \end{aligned} \tag{5.16}$$

Our results in the previous sections dealt with $(\kappa, \kappa_{\Omega R}, \kappa_{\Omega Rg_1}, \kappa_{\Omega Rg_2}, \kappa_{\Omega Rg_3}) = (+, +, +, +, +)$ for the hyper model while for the tensor model the choice is $(-, +, -, -, -)$. We present an analysis of the tensor-multiplet torsion brane spectrum in Appendix C.3.

When calculating the torsion brane spectrum, some of the allowed branes do not fill the non-compact space (i.e. $r \neq 3$). This means that the simple probe branes introduced in [20], which fill the non-compact space, will not detect these extra torsion charges branes, which might lead to extra K-theory constraints. For the original case we considered (i.e. the hyper-multiplet model), our torsion brane spectrum only included $r = 3$ branes, so our results matched with the probe brane argument.

To see the relation between the observed torsion charges from the probe brane approach and the gauge groups on the $D3$ and $D7_i$ branes, we have included a chart of the gauge groups in Table 4. Comparing these gauge groups to the K-theory constraints in eqns (4.18) - (4.21), we see that the discrete constraint on $D9$-branes is due to a symplectic group on the probe $D3$-brane, whereas the discrete constraints on $D5_i$-branes are the results of symplectic groups on the probe $D7_i$ branes.

6 Discussion

In this paper, we have investigated D-branes in $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds. We are particularly interested in torsion charged D-branes because they cannot be detected from the usual (homological) tadpole conditions. Nevertheless, their charges need to be cancelled in consistent string vacua and hence their existence imposes non-trivial constraints on model building. Because of the generality of the results, we expect the constraints derived here will be useful for future work in building realistic D-brane models from more general orientifolds. The search for realistic intersecting/magnetized D-brane models within the framework of $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds has so far been focused mainly on a particular choice[12] of discrete torsion (i.e., the hyper-multiplet model whose non-BPS brane spectrum was discussed in detail in Section 5) and moreover with limited types of branes. For example, the set of branes considered in [20] included only bulk branes whose D-brane charges are induced by turning on "non-oblique" (in the sense of [23, 24]) magnetic fluxes on the worldvolume of D9-branes. As the search continues into different choices of discrete torsion or when more general branes are included, we would have to go beyond the K-theory constraints in eqns (4.14) - (4.17) (and analogously eqns. (4.18) - (4.21) for other choices of discrete torsion). The CFT approach adopted here not only reproduces the probe brane results in [20], but finds additional torsion branes that arise from probe branes that are different from the simple probes usually considered [20]. For these more general cases, one would have to check that the discrete K-theory constraints are indeed satisfied or else the models are inconsistent.

So far, the derivations of K-theory constraints have been done in a case by case basis. It would be useful to have a more general or perhaps an alternative understanding of how these discrete constraints arise. The probe brane approach introduced in [17] provides a powerful way to derive some (and in some cases all) of these discrete constraints. Having checked the torsion brane spectrum in the $\mathbf{Z_2} \times \mathbf{Z_2}$ orientifold for all cases of discrete torsion, we have explicitly shown that the set of bulk[13] probe branes with symplectic gauge groups that one can introduce is in one-to-one correspondence with the set of bulk torsion branes which are space-filling in the four non-compact dimensions. Recent work [19] has suggested yet another interesting way to understand the K-theory constraints in orientifold constructions by uplifting to F-theory. A non-vanishing magnetic flux on the world-volume of D-branes in Type IIB can be encoded by the 4-form flux G_4 in F-theory. The K-theory constraints can then be seen to follow from the standard Dirac quantization conditions on G_4. It would be interesting to see if a similar analysis can be done for $D = 4$, $\mathcal{N} = 1$ orientifold backgrounds such as the ones considered here.

The effects of K-theory constraints have recently been explored in the statistical studies of string vacua [12, 21]. The significance of these discrete constraints in reducing the string landscape is somewhat model dependent. The authors of [12] investigated an ensemble of (homological) tadpole cancelling intersecting D-brane models in the $\mathbf{Z_2} \times \mathbf{Z_2}$ orientifold, and found that considering K-theory constraints reduced the possible brane configurations by a factor of five. This is in contrast to [21] which carried out a similar analysis for an ensemble

[12]See, however, [50], for model building from $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold with a different choice of discrete torsion.

[13]By bulk branes, we mean branes that are not stuck at fixed points and so they cannot carry twisted NSNS or RR charges.

of Rational Conformal Field Theory (RCFT) orientifolds with qualitative features of the Standard Model, and found that the additional K-theory constraints did not significantly reduce the number of solutions. It would be interesting to revisit [12] for other choices of discrete torsion using the K-theory constraints derived here.

Finally, our results suggest that the spectrum of stable non-BPS D-branes for $D = 4$, $\mathcal{N} = 1$ orientifolds can be quite rich (e.g. Table 5 for integrally charged non-BPS branes). As discussed in [29], stable non-BPS branes that are point-like in our four non-compact dimensions can be interesting candidates for cold dark matter. An interesting direction for future investigation would be to compute their scattering cross sections and see if they can give rise to sharp signatures that could distinguish them from other cold dark matter candidates.

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Appendices

A Some details about the amplitudes

In this appendix, we summarize the results of the full Klein bottle, Möbius strip, and Annulus amplitudes [26,46,47] for the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifolds[14]. The expressions are more complete than that in Section 4.1 in that we also list the twisted sector contribution to the partition function. We have also included factors of discrete torsion, see Section 5.3 for more details. Each of these amplitudes is calculated first in the open channel and then converted into the closed channel after a Poisson resummation and a modular transformation. The winding and momentum terms are written out explicitly in eqn (4.1). For the modular transformations, the Klein Bottle has $t = 1/4l$; for the Möbius Strip, $t = 1/8l$; and for the Annulus, $t = 1/2l$, where t is the modulus for the open string one loop amplitude and l is the modulus for the closed string vacuum amplitude.

Klein Bottle

$$
\begin{aligned}
\mathcal{K} \;=\; & \frac{1}{16}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t^3}\Big(\frac{{f_3}^8(e^{-2\pi t})}{{f_1}^8(e^{-2\pi t})} - \frac{{f_4}^8(e^{-2\pi t})}{{f_1}^8(e^{-2\pi t})} - \frac{{f_2}^8(e^{-2\pi t})}{{f_1}^8(e^{-2\pi t})}\Big) \qquad \text{(A.1)}\\
& \times\Big(\prod_{i=3}^{8} W'_i + \prod_{i=3}^{6} M'_i \prod_{j=7,8} W'_j + \prod_{i=5}^{8} M'_i \prod_{j=3,4} W'_j + \prod_{i=3,4,7,8} M'_i \prod_{j=5,6} W'_j\Big)\\
& +16\times\frac{1}{8}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t^3}\Big(\frac{{f_3}^4(e^{-2\pi t}){f_2}^4(e^{-2\pi t})}{{f_1}^4(e^{-2\pi t}){f_4}^4(e^{-2\pi t})} - \frac{{f_2}^4(e^{-2\pi t}){f_3}^4(e^{-2\pi t})}{{f_1}^4(e^{-2\pi t}){f_4}^4(e^{-2\pi t})}\Big)\\
& \times\Bigg(\kappa_{\Omega R}\kappa_{\Omega R g_1}\Big(\kappa\prod_{i=3,4} W'_i + \prod_{i=3,4} M'_i\Big) + \kappa_{\Omega R}\kappa_{\Omega R g_2}\Big(\kappa\prod_{i=5,6} W'_i + \prod_{i=5,6} M'_i\Big)\\
& +\kappa_{\Omega R}\kappa_{\Omega R g_3}\Big(\kappa\prod_{i=7,8} W'_i + \prod_{i=7,8} M'_i\Big)\Bigg)\\
& = 2\frac{V_4}{(2\pi)^4}\int_0^\infty dl\;\Big(\frac{{f_3}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})} - \frac{{f_2}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})} - \frac{{f_4}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})}\Big)\\
& \times\Big(\prod_{i=3}^{8}\frac{1}{R_i}\tilde{W}'_i + \prod_{i=3}^{6} R_i\tilde{M}'_i \prod_{j=7,8}\frac{1}{R_j}\tilde{W}'_j + \prod_{i=5}^{8} R_i\tilde{M}'_i \prod_{j=3,4}\frac{1}{R_j}\tilde{W}'_j + \prod_{i=3,4,7,8} R_i\tilde{M}'_i \prod_{j=5,6}\frac{1}{R_j}\tilde{W}'_j\Big)\\
& +16\frac{V_4}{(2\pi)^4}\int_0^\infty dl\;\Big(\frac{{f_3}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})} - \frac{{f_4}^4(e^{-2\pi l}){f_3}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}\Big)\\
& \times\Bigg(\kappa_{\Omega R}\kappa_{\Omega R g_1}\Big(\kappa\prod_{j=3,4}\frac{1}{R_j}\tilde{W}'_j + \prod_{i=3,4} R_i\tilde{M}'_i\Big) + \kappa_{\Omega R}\kappa_{\Omega R g_2}\Big(\kappa\prod_{j=5,6}\frac{1}{R_j}\tilde{W}'_j + \prod_{i=5,6} R_i\tilde{M}'_i\Big)\\
& +\kappa_{\Omega R}\kappa_{\Omega R g_3}\Big(\kappa\prod_{j=7,8}\frac{1}{R_j}\tilde{W}'_j + \prod_{i=7,8} R_i\tilde{M}'_i\Big)\Bigg)
\end{aligned}
$$

[14] See, also [48] for this and other $\mathbf{Z}_M \times \mathbf{Z}_N$ orientifolds.

Möbius Strip

The following are the results on the Möbius Strip, written as interaction between a crosscap state and a boundary state for a $(r; s_1, s_2, s_3)$ brane. Note that when switching from open to closed channel, the $f_3(iq)$ and $f_4(iq)$ functions pick up a factor of $e^{i\pi}$. This is due to the transformation of the functions with imaginary arguments (see, e.g., [6]). We mention this because the partition function will gain a factor of sine, and this term will show up when discussing torsion branes (see Appendix B for more details). The NS sector to the Möbius Strip contribution is

$$
\begin{aligned}
\mathcal{M} \;=\;& \frac{1}{16}\frac{V_{r+1}}{(2\pi)^{r+1}}\kappa_{\Omega R}\int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2} \\
&\times\left(\frac{f_4^{5+r-s}(ie^{-\pi t})f_3^{3+s-r}(ie^{-\pi t}) - f_3^{5+r-s}(ie^{-\pi t})f_4^{3+s-r}(ie^{-\pi t})}{f_1^{5+r-s}(ie^{-\pi t})f_2^{3+s-r}(ie^{-\pi t})2^{-(3+s-r)/2}}\right) \\
&\times\left(\mathrm{Tr}\ \gamma^T_{\Omega 3,D_p}\gamma^{-1}_{\Omega 3,D_p}\prod_{i=6-s} W_i\right) \\
&+\sum_{i=1}^{3}\frac{1}{16}\frac{V_{r+1}}{(2\pi)^{r+1}}\kappa_{\Omega R g_i}\int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2}\frac{1}{2^{-(7-r-s+2s_i)/2}} \\
&\times\left(\frac{f_4^{1+r+s-2s_i}(ie^{-\pi t})f_3^{7-r-s+2s_i}(ie^{-\pi t}) - f_3^{1+r+s-2s_i}(ie^{-\pi t})f_4^{7-r-s+2s_i}(ie^{-\pi t})}{f_1^{1+r+s-2s_i}(ie^{-\pi t})f_2^{7-r-s+2s_i}(ie^{-\pi t})}\right) \\
&\times\left(\mathrm{Tr}\ \gamma^T_{\Omega 7_i,D_p}\gamma^{-1}_{\Omega 7_i,D_p}\prod_{j=s-s_i} M_j\prod_{k=2-s_i} W_k\right) \\
=\;& \frac{1}{8}\frac{V_{r+1}}{(2\pi)^{r+1}}\kappa_{\Omega R}\int_0^\infty dl\left(\frac{e^{i(\pi/4)(s-r-1)}f_3^{5+r-s}(ie^{-2\pi l})f_4^{3+s-r}(ie^{-2\pi l})}{f_1^{5+r-s}(ie^{-2\pi l})f_2^{3+s-r}(ie^{-2\pi l})2^{-(3+s-r)/2}}\right. \\
&\left.-\frac{e^{-i(\pi/4)(s-r-1)}f_4^{5+r-s}(ie^{-2\pi l})f_3^{3+s-r}(ie^{-2\pi l})}{f_1^{5+r-s}(ie^{-2\pi l})f_2^{3+s-r}(ie^{-2\pi l})2^{-(3+s-r)/2}}\right) \\
&\times\left(\mathrm{Tr}\ \gamma^T_{\Omega 3,D_p}\gamma^{-1}_{\Omega 3,D_p}\prod_{i=6-s}\frac{1}{R_i}\tilde{W}_i\right) \\
&+\sum_{i=1}^{3}\frac{1}{8}\frac{V_{r+1}}{(2\pi)^{r+1}}\kappa_{\Omega R g_i}\int_0^\infty dl\ \frac{1}{2^{-(7-r-s+2s_i)/2}} \\
&\left(\frac{e^{i(\pi/4)(3-r-s+2s_i)}f_3^{1+r+s-2s_i}(ie^{-2\pi l})f_4^{7-r-s+2s_i}(ie^{-2\pi l})}{f_1^{1+r+s-2s_i}(ie^{-2\pi l})f_2^{7-r-s+2s_i}(ie^{-2\pi l})}\right. \\
&\left.-\frac{e^{-i(\pi/4)(3-r-s+2s_i)}f_4^{1+r+s-2s_i}(ie^{-2\pi l})f_3^{7-r-s+2s_i}(ie^{-2\pi l})}{f_1^{1+r+s-2s_i}(ie^{-2\pi l})f_2^{7-r-s+2s_i}(ie^{-2\pi l})}\right) \\
&\times\left(\mathrm{Tr}\ \gamma^T_{\Omega 7_i,D_p}\gamma^{-1}_{\Omega 7_i,D_p}\prod_{j=s-s_i} R_j\tilde{M}_j\prod_{k=2-s_i}\frac{1}{R_k}\tilde{W}_k\right)
\end{aligned}
\qquad (A.2)
$$

Annulus

Below is the Annulus contribution to the partition function for strings stretched between the $D3$ and $D7$ branes, including contributions from twisted sectors and cross terms.

$$
\begin{aligned}
\mathcal{C} &= \frac{1}{16}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_3}^8(e^{-\pi t})}{{f_1}^8(e^{-\pi t})} - \frac{{f_4}^8(e^{-\pi t})}{{f_1}^8(e^{-\pi t})} - \frac{{f_2}^8(e^{-\pi t})}{f_1(e^{-\pi t})}\right) \qquad \text{(A.3)}\\
&\times\left(\mathrm{Tr}\ \gamma_{1,D_p}\gamma^{-1}_{1,D_p}\prod_s M_i \prod_{6-s} W_j\right)\\
&+\sum_{i=1}^3\frac{1}{4}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_3}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})} - \frac{{f_4}^4(e^{-\pi t}){f_3}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})}\right)\\
&\times\left(\mathrm{Tr}\ \gamma_{g_i,D_p}\gamma^{-1}_{g_i,D_p}\prod_{s_i} M_j \prod_{k=2-s_i} W_k\right)\\
&+\frac{1}{8}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_3}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})} - \frac{{f_3}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})}\right)\\
&\times\left(\sum_{i=1}^3 \mathrm{Tr}\ \gamma_{1,D3}\gamma^{-1}_{1,D7_i}\prod_{s_i} W_i\right)\\
&+\frac{1}{8}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_3}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})} - \frac{{f_3}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})}\right)\\
&\times\left(\sum_{i=1,i\neq j\neq k}^3 \mathrm{Tr}\ \gamma_{1,D7_i}\gamma^{-1}_{1,D7_j}\prod_{s_k} M_k\right)\\
&-\frac{1}{8}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_2}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_3}^4(e^{-\pi t})}\right)\times\left(\sum_{i=1}^3 \mathrm{Tr}\ \gamma_{g_i,D3}\gamma^{-1}_{g_i,D7_i}\prod_{s_i} W_i\right)\\
&-\frac{1}{8}\frac{V_4}{(2\pi)^4}\int_0^\infty \frac{dt}{2t}(2t)^{-2}\left(\frac{{f_2}^4(e^{-\pi t}){f_4}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_3}^4(e^{-\pi t})}\right)\times\left(\sum_{i=1,i\neq j\neq k}^3 \mathrm{Tr}\ \gamma_{g_k,D7_i}\gamma^{-1}_{g_k,D7_j}\prod_{s_k} M_k\right)\\
&= \frac{1}{512}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_3}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})} - \frac{{f_4}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})} - \frac{{f_2}^8(e^{-2\pi l})}{{f_1}^8(e^{-2\pi l})}\right)\\
&\times\left(\mathrm{Tr}\ \gamma_{1,D_p}\gamma^{-1}_{1,D_p}\prod_s R_i\tilde{M}_i \prod_{6-s}\frac{1}{R_j}\tilde{W}_j\right)\\
&+\sum_{i=1}^3\frac{1}{32}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_3}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})} - \frac{{f_2}^4(e^{-2\pi l}){f_3}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}\right)\\
&\times\left(\mathrm{Tr}\ \gamma_{g_i,D_p}\gamma^{-1}_{g_i,D_p}\prod_{j=s_i} R_j\tilde{M}_j \prod_{k=2-s_i}\frac{1}{R_k}\tilde{W}_k\right)
\end{aligned}
$$

$$+\frac{1}{64}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_3}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}-\frac{{f_3}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}\right)$$
$$\times\left(\sum_{i=1}^3 \mathrm{Tr}\ \gamma_{1,D3}\gamma_{1,D7_i}^{-1}\prod_{s_i}\frac{1}{R_i}\tilde{W}_i\right)$$
$$+\frac{1}{64}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_3}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}-\frac{{f_3}^4(e^{-2\pi l}){f_4}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}\right)$$
$$\times\left(\sum_{i=1,i\neq j\neq k}^3 \mathrm{Tr}\ \gamma_{1,D7_i}\gamma_{1,D7_j}^{-1}\prod_{s_k}R_k\tilde{M}_k\right)$$
$$-\frac{1}{64}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_4}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_3}^4(e^{-2\pi l})}\right)\times\left(\sum_{i=1}^3 \mathrm{Tr}\ \gamma_{g_i,D3}\gamma_{g_i,D7_i}^{-1}\prod_{s_i}\frac{1}{R_i}\tilde{W}_i\right)$$
$$-\frac{1}{64}\frac{V_4}{(2\pi)^4}\int_0^\infty dl\ \left(\frac{{f_4}^4(e^{-2\pi l}){f_2}^4(e^{-2\pi l})}{{f_1}^4(e^{-2\pi l}){f_3}^4(e^{-2\pi l})}\right)\times\left(\sum_{i=1,i\neq j\neq k}^3 \mathrm{Tr}\ \gamma_{g_k,D7_i}\gamma_{g_k,D7_j}^{-1}\prod_{s_k}R_k\tilde{M}_k\right)$$

B Finding the tachyon

As mentioned in Section 5.1, there are eight possible contribution to the tachyon: the untwisted Annulus diagram, the twisted Annulus diagram (three contributions), the Möbius strip diagram for a string exchange between the Dp and the $O3$ plane, and the Möbius strip diagram for a string exchange between the Dp and the $O7$ plane (three contributions). Each of these contributions is listed below, and was originally calculated in the closed string channel using the boundary state method. The open string expression was obtained after a Poisson resummation and a modular transformation.

Since the orbifold action produces NSNS and RR twisted sectors that give equal and opposite contributions to the amplitudes, the parameter ϵ_i has been introduced. $\epsilon_i = 1$ if the brane couples to the T_i twisted NSNS sector, and $\epsilon_i = -1$ if the brane couples to the T_i twisted RR sector. $\epsilon_i = 0$ if the brane does not couple to the T_i twisted sector.

When these terms are expanded, combining the tachyonic modes (q^{-1}) from the Annulus and Möbius Strip diagrams produces the constraint equation (5.7). After each term we provide the expansion of the tachyonic and massless modes.

$$\mathcal{A} = 2^4 N_{(r,s),U}^2\ \frac{\prod_{6-s}R_j}{\prod_s R_i}\ \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2}\left([\frac{f_3(e^{-\pi t})}{f_1(e^{-\pi t})}]^8-[\frac{f_2(e^{-\pi t})}{f_1(e^{-\pi t})}]^8\right) \tag{B.1}$$
$$\times\left(\prod_s M_i\prod_{6-s}W_j\right)$$
$$+\sum_{i=1}^3 2^{2+s-s_i}N_{(r,s),T_i}^2\ \frac{\prod_{2-s_i}R_j}{\prod_{s_i}R_k}\ \epsilon_i\ \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2}\frac{{f_4}^4(e^{-\pi t}){f_3}^4(e^{-\pi t})}{{f_1}^4(e^{-\pi t}){f_2}^4(e^{-\pi t})})$$
$$\times\left(\prod_{s_i}M_k\prod_{2-s_i}W_j\right)$$

$$
\begin{aligned}
&= 2^4 N^2_{(r,s),U} \frac{\prod_{6-s} R_j}{\prod_s R_i} \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2}(q^{-1} - 8q^0 + \ldots) \times \prod_s M_i \prod_{6-s} W_j \\
&+ \sum_{i=1}^{3} 2^{2+s-s_i} N^2_{(r,s),T_i} \frac{\prod_{2-s_i} R_j}{\prod_{s_i} R_k} \epsilon_i \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2}((4q)^{-1} + \ldots) \times \prod_{s_i} M_k \prod_{2-s_i} W_j
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_3 &= 32 N_{(r,s),U} N_{O3} \prod_{6-s} R_j \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2} \\
&\quad \frac{e^{-i(\pi/4)(s-r-1)} f_4^{5+r-s}(ie^{-\pi t}) f_3^{3+s-r}(ie^{-\pi t}) - e^{i(\pi/4)(s-r-1)} f_3^{5+r-s}(ie^{-\pi t}) f_4^{3+s-r}(ie^{-\pi t})}{f_1^{5+r-s}(ie^{-\pi t}) f_2^{3+s-r}(ie^{-\pi t})} \\
&\quad \times \frac{1}{2^{-(3+s-r)/2}} \Big(\prod_{i=6-s} W_i \Big) \\
&= 32 N_{(r,s),U} N_{O3} \prod_{6-s} R_j \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2} \Big(2 \sin\frac{\pi}{4}(r-s+1)\Big)(q^{-1} + \ldots) \times \Big(\prod_{i=6-s} W_i \Big)
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
\mathcal{M}_{7_i} &= 4 N_{(r,s),U} N_{O7_i} \frac{\prod_{2-s_i} R_k}{\prod_{s-s_i} R_j} \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2} \\
&\quad \Big(\frac{e^{-i(\pi/4)(3-r-s+2s_i)} f_4^{1+r+s-2s_i}(ie^{-\pi t}) f_3^{7-r-s+2s_i}(ie^{-\pi t})}{f_1^{1+r+s-2s_i}(ie^{-\pi t}) f_2^{7-r-s+2s_i}(ie^{-\pi t})} \\
&\quad - \frac{e^{i(\pi/4)(3-r-s+2s_i)} f_3^{1+r+s-2s_i}(ie^{-\pi t}) f_4^{7-r-s+2s_i}(ie^{-\pi t})}{f_1^{1+r+s-2s_i}(ie^{-\pi t}) f_2^{7-r-s+2s_i}(ie^{-\pi t})} \Big) \\
&\quad \times \frac{1}{2^{-(7-r-s+2s_i)/2}} \Big(\prod_{i=s-s_i} M_k \prod_{2-s_i} W_j \Big) \\
&= 4 N_{(r,s),U} N_{O7_i} \frac{\prod_{2-s_i} R_k}{\prod_{s-s_i} R_j} \int_0^\infty \frac{dt}{2t}(2t)^{-(r+1)/2} \Big(2 \sin\frac{\pi}{4}(r+s-2s_i-3)\Big)(q^{-1} + \ldots) \\
&\quad \times \Big(\prod_{i=s-s_i} M_k \prod_{2-s_i} W_j \Big)
\end{aligned}
\tag{B.3}
$$

where the normalizations for the Annulus contribution are matched to the open string one loop diagram,

$$
N^2_{(r,s),U} = \frac{V_{r+1}}{(2\pi)^{r+1}} \, n^2 \, \frac{\prod_s R_i}{\prod_{6-s} R_j} \tag{B.4}
$$

$$
N^2_{(r,s),T_k} = 2^{4-s+s_i} \frac{V_{r+1}}{(2\pi)^{r+1}} \, n^2 \, \frac{\prod_{s_k} R_i}{\prod_{2-s_k} R_j} \tag{B.5}
$$

and the normalization for the Möbius strip contribution comes from the normalizations of the crosscap states and are defined

$$
N^2_{O3} = \frac{1}{1024} \frac{V_4}{(2\pi)^4} \prod_s \frac{1}{R_j} \tag{B.6}
$$

$$
N^2_{O7_i} = \frac{1}{16} \frac{V_4}{(2\pi)^4} \frac{\prod_{4-s+s_i} R_i}{\prod_{s_i} R_j} \tag{B.7}
$$

n is a constant that is determined for different values of (r,s). For BPS D-branes that couple to all three twisted sectors, $n^2 = \frac{1}{256}$.

C Stability Region of the Torsion Branes

C.1 Higher Winding and Momentum Modes

In this section we will calculate the stability region of the torsion branes in Section 5.1. In that section the branes were considered stable if the ground state tachyonic mode vanished. To determine the stability region of these branes, we must analyze the higher winding and momentum modes, and require that these not become tachyonic.

For a general (r,s) torsion brane with possible twisted couplings, the potentially tachyonic modes, with higher winding and momentum included, are

$$\begin{aligned}
&2^4\, n^2\, q^{-1} \;\times\; \prod_{s} M_i \prod_{6-s} W_j + 2^4\, n^2 \sum_{k=1}^{3} \epsilon_k\, q^{-1} \;\times\; \prod_{s_k} M_i \prod_{2-s_k} W_j \\
&-2n\kappa_{\Omega R}\, q^{-1}\, \sin\left(\frac{\pi}{4}(r-s+1)\right) \times \prod_{6-s} W_j \\
&-2n\kappa_{\Omega R g_1}\, q^{-1}\, \sin\left(\frac{\pi}{4}(r+s-2s_1-3)\right) \times \prod_{s_2+s_3} M_i \prod_{2-s_1} W_j \\
&-2n\kappa_{\Omega R g_2}\, q^{-1}\, \sin\left(\frac{\pi}{4}(r+s-2s_2-3)\right) \times \prod_{s_1+s_3} M_i \prod_{2-s_2} W_j \\
&-2n\kappa_{\Omega R g_3}\, q^{-1}\, \sin\left(\frac{\pi}{4}(r+s-2s_3-3)\right) \times \prod_{s_1+s_2} M_i \prod_{2-s_3} W_j
\end{aligned} \tag{C.1}$$

where $\epsilon_k = 1$ if the brane couples to the g_k NSNS twisted sector, and 0 otherwise. By expanding the winding and momentum terms, in addition to having the vacuum state cancel (eqn. (5.7)), we must restrict the size of the compact directions in the expression above such that the higher modes are not tachyonic. For example, the $(3;0,2,0)$ brane that couples to the g_1 twisted NSNS sector, along the g_1 twisted directions (x^5,x^6,x^7,x^8) the brane is stable for

$$\frac{2}{R_{5,6}^2} - 1 \;\geq\; 0 \tag{C.2}$$

$$2R_{7,8}^2 - 1 \;\geq\; 0\,. \tag{C.3}$$

A $(3;0,2,0)$ torsion brane without any twisted couplings, whose stability region in any of the compact directions

$$\frac{2}{R_i^2} + 2R_j^2 - 1 \geq 0\,, \qquad i = 5,6 \qquad j = 3,4,7,8\,. \tag{C.4}$$

C.2 Torsion branes in the hyper-multiplet model

In Section 5 we have identified stable, torsion charged D-branes in the hyper-multiplet model, and have presented in equation (5.16) a general condition for the absence of an open string tachyon on a D-brane for the different allowed choices of discrete torsion in these models. Enumerating such tachyon free torsion-charged D-branes is then straightforward. Not all tachyon free branes however lead to allowed branes in string theory. In the case of branes that couple to twisted NSNS sectors, one needs to ensure that the corresponding boundary states are invariant with respect to the GSO and orientifold projections. Further, as in the case of Type I SO and Sp theories we need to ensure that our D-branes do not suffer from the pathologies associated to the $D6$ and $D2$ branes in those theories [6, 16]. It is also possible for apparently consistent D-branes to decay into inconsistent D-branes. Such decays signal the need to exclude the former branes [8]. In this sub-section we present the allowed torsion charged D-branes in the hyper-multiplet model as an example.

For the hyper-multiplet model the choice of discrete torsion is

$$\kappa = 1\,, \qquad \kappa_{\Omega R} = 1\,, \qquad \kappa_{\Omega R g_i} = 1\,, \tag{C.5}$$

and equation (5.16) together with the conditions (3.8) imply that tachyon-free torsion D-branes only exist in the form given in eqns (5.2) and (5.1). Torsion branes of the form given in equation (5.2) with coupling to the NSNSTg_1 twisted sector need to have $(r; s_1, s_2, s_3)$ of the form

$$(2;0,2,0)\,, \quad (2;0,0,2)\,, \quad (3;0,2,0)\,, \quad (3;0,0,2)\,, \quad (3;1,2,0)\,, \quad (3;1,0,2)\,. \tag{C.6}$$

Tachyon-free torsion branes of the form given in equation (5.1) need to have $(r; s_1, s_2, s_3)$ of the form

$$\begin{array}{lllll}(2;2,0,0)\,, & (2;2,2,2)\,, & (3;1,0,0)\,, & (3;1,1,0)\,, & (3;2,0,0)\,, \\ (3;1,1,1)\,, & (3;0,1,2)\,, & (3;2,1,1)\,, & (3;2,2,1)\,, & (3;2,2,2)\,. \end{array} \tag{C.7}$$

and all permutations of the s_i.

For the torsion branes with one twisted coupling given by equation (5.2), the $(2;0,2,0)$, $(2;0,0,2)$, $(3;1,2,0)$, and $(3;1,0,2)$ branes are g_3 images of the $\mathbf{Z_2} \oplus \mathbf{Z_2}$ $(4,2)$-branes which were found to be inconsisntent in [8]. Therefore the only consistent brane with one twisted coupling is the $(3;2,0,0)$ brane and all permutations of the s_i.

When considering possible decay channels, they suggest either one of two potential decays: a) Dp brane $\rightarrow D(p\pm1)$ brane or b) Dp brane $\rightarrow D(p\pm2)$ brane. General transitions of the form a) have been beautifully analysed by [52]. At a critical compactification radius the two CFTs that correspond to the two D-branes are equivalent, allowing a Dp brane to decay into a $D(p\pm1)$ brane. Decays of the form b) are in general more complicated, and it is not clear what are the decay channels allowed by matching the whole CFTs. For example, while a CFT matching exists for certain freely acting orbifolds [53], in more complicated settings such as [4, 8] no such matching exists and it is in general not known what branes decay into. Therefore we will only consider decays of the form a) to exclude inconsistent torsion branes.

Since the torsion branes that couple only to the NSNSU sector (of the form in equation (5.1)) are pairs of torsion branes that couple to the NSNSTg_i sector (of the form in equation (5.2)) with opposite twisted torsion charge, then we must exclude branes that have the same values of $(r; s_1, s_2, s_3)$. This means that for branes that couple only to the NSNSU sector, the $(2; 2, 0, 0)$ and $(3; 0, 1, 2)$ are excluded. The $(2; 2, 2, 2)$ and $(3; 1, 1, 1)$ are T-dual to a $D2$ and $D6$ brane in Type I, and are also excluded. In addition, for certain values of the compactification radii there are the allowed decays

$$(3; 1, 1, 0) \to (3; 1, 2, 0)\,, \tag{C.8}$$

and

$$(3; 1, 2, 1) \to (3; 1, 2, 0)\,. \tag{C.9}$$

Therefore the consistent stable torsion branes that have no twisted couplings are

$$(3; 1, 0, 0)\,, \qquad (3; 2, 0, 0)\,, \qquad (3; 2, 2, 1)\,, \qquad (3; 2, 2, 2)\,. \tag{C.10}$$

and all permutations of the s_i.

Considering the remaining torsion branes above, the $(3; 1, 0, 0)$ and $(3; 2, 0, 0)$ branes are related by decay, and the $(3; 2, 2, 1)$ and $(3; 2, 2, 2)$ branes are also related by decay. Since the $(3; 2, 0, 0)$, $(3; 0, 2, 0)$, $(3; 0, 0, 2)$, and $(3; 2, 2, 2)$ branes are not related by decay, then these branes form an independent basis of K-theory torsion charge, and match with the probe brane results in eqns. (4.14) - (4.17).

C.3 Torsion branes in the tensor-multiplet model

In this subsection we continue our analysis to the torsion branes in the tensor-multiplet model. For the tensor-multiplet model the choice of discrete torsion is

$$\kappa = -1\,, \qquad \kappa_{\Omega R} = 1\,, \qquad \kappa_{\Omega R g_i} = -1\,, \tag{C.11}$$

and equation (5.16) together with the conditions (3.9) imply that tachyon-free torsion D-branes of the type given in equation (5.3) need to have $(r; s_1, s_2, s_3)$ of the form

$$(-1; 2, 2, 2)\,, \qquad (0; 2, 2, 2)\,, \qquad (0; 0, 0, 0)\,, \qquad (1; 0, 0, 0)\,, \qquad (2; 0, 0, 0)\,. \tag{C.12}$$

Tachyon-free torsion D-branes of the type given in equation (5.2), with couplings to the NSNSTg_1 sector, need to have $(r; s_1, s_2, s_3)$ of the form

$$\begin{array}{lllll} (-1; 2, 2, 2)\,, & (-1; 1, 2, 2)\,, & (0; 2, 2, 2)\,, & (0; 1, 2, 2)\,, & (0; 0, 0, 0)\,, \\ (0; 1, 0, 0)\,, & (1; 0, 0, 0)\,, & (1; 1, 0, 0)\,, & (2; 0, 0, 0)\,, & (2; 1, 0, 0)\,. \end{array} \tag{C.13}$$

Finally, tachyon-free torsion charged D-branes of the form given in equation (5.1) need to have $(r; s_1, s_2, s_3)$ of the form

$$\begin{array}{lllll} (-1; 2, 2, 2)\,, & (-1; 1, 2, 2)\,, & (-1; 1, 1, 2)\,, & (-1; 1, 1, 1)\,, & (0; 2, 2, 2)\,, \\ (0; 1, 2, 2)\,, & (0; 1, 1, 2)\,, & (0; 1, 1, 1)\,, & (0; 0, 1, 1)\,, & (0; 0, 0, 1)\,, \\ (0; 0, 0, 0)\,, & (1; 0, 0, 0)\,, & (1; 1, 0, 0)\,, & (1; 1, 1, 0)\,, & (1; 1, 1, 1)\,, \\ (2; 0, 0, 0)\,, & (2; 1, 0, 0)\,, & (2; 1, 1, 0)\,. & & \end{array} \tag{C.14}$$

and all permutations of the s_i.

It is easy to see that the $(0;0,0,0)$-brane and the $(1;1,0,0)$-brane of the type (5.2) are g_3 images of the $\mathbf{Z_2} \oplus \mathbf{Z_2}$ $(2,4)$-branes found in [8][15]. These branes were shown not to be consistent [8], and so we exclude them here as well. For branes of the type (5.1), the $(-1;1,1,1)$, $(0;1,1,2)$, and $(2;1,1,0)$ branes are also inconsistent and must be removed. Since it is possible to form branes of the type (5.2) from branes of the type (5.3), and branes of the type (5.1) from branes of the type (5.2), we need to exclude all torsion branes with the above values of $(r;s_1,s_2,s_3)$.

We may next consider torsion branes of the type (5.1). It is possible to show that the following decay processes between torsion-charged branes of this type can occur for suitable values of radii

$$(0;1,1,1) \to (0;0,1,1) \to (0;0,0,1) \to (0;0,0,0)\,. \tag{C.15}$$

as well as

$$(1;1,1,1) \to (1;1,1,0) \to (1;1,0,0)\,. \tag{C.16}$$

Since the end of each of these decays is an inconsistent D-brane, all of the D-branes in the above decays are also inconsistent.

To summarize, the tensor-multiplet model includes torsion D-branes of the type given in equation (5.3) with $(r;s_1,s_2,s_3)$ of the form

$$(-1;2,2,2)\,, \qquad (0;2,2,2)\,, \qquad (1;0,0,0)\,, \qquad (2;0,0,0)\,. \tag{C.17}$$

The tensor model has torsion D-branes of the type given in equation (5.2), with couplings to the NSNSTg_1 sector, with $(r;s_1,s_2,s_3)$ of the form

$$\begin{array}{llll} (-1;2,2,2)\,, & (-1;1,2,2)\,, & (0;2,2,2)\,, & (0;1,2,2)\,, \\ (1;0,0,0)\,, & (2;0,0,0)\,, & (2;1,0,0)\,. & \end{array} \tag{C.18}$$

Similar results hold for branes of the type given in equation (5.2) with couplings to the other NSNSTg_i sectors. Finally, the tensor model has torsion D-branes of the form given in equation (5.1) for $(r;s_1,s_2,s_3)$ of the form

$$\begin{array}{llll} (-1;2,2,2)\,, & (-1;1,2,2)\,, & (-1;1,1,2)\,, & (0;2,2,2)\,, \\ (0;1,2,2)\,, & (1;0,0,0)\,, & (2;0,0,0)\,, & (2;1,0,0)\,. \end{array} \tag{C.19}$$

and all permutations of the s_i.

C.4 A comment on other choices of discrete torsion

Naively, there are many torsion charged branes that are a solution to eqn (5.16). Though in our analysis for the hyper-multiplet model we have excluded some of the branes in Tables 1 and 2 due to their relation to inconsistent branes, this *does not* mean that for other choices

[15] The $(2,4)$-brane was not explicitly mentioned as an inconsistent brane in [8]. See Section D for a brief review of the 6D tensor multiplet, including the equations needed to calculate the spectrum of stable D-branes.

of discrete torsion that the discrete K-theory charges found using the probe brane approach in equations (4.18) - (4.21) are also excluded if branes that could carry those charges are excluded. These discrete K-theory charges can be carried by BPS branes even when non-BPS branes that also carry the same charges do not exist. The probe brane approach in the hyper-multiplet uses a configuration of BPS $D3$ and $D7$ branes to detect these discrete charges, and the same charges can also be carried by non-BPS torsion charged $D9$ and $D5$ branes respectively. This method is independent of whether the non-BPS torsion charged $D9$ and $D5$ branes exist. In addition, the discussion above is based upon changing the compactification radii of the torsion branes along certain directions so that they can decay. By doing this we are moving the model away from its BPS configuration, and thus must treat this analysis with caution.

There are interesting features for the torsion brane spectrum for other choices of discrete torsion. Some of the other 14 cases contain branes that do not fill the non-compact directions (i.e. $r \neq 3$), and can possibly carry torsion charge not carried by the $r = 3$ branes. In addition, these models can contain $r = 3$ branes with oblique flux on their worldvolume, and might be useful for future model building. To determine the torsion brane spectrum, a case by case analysis must be done to determine whether the branes are stable, orientifold and orbifold invariant, and consistent. A full study of the remaining cases would be useful both to string phenomenology and to the study of twisted K-theory.

D Some Known Examples

Here, we will examine a previously worked example of a $\mathbf{T^4}/\mathbf{Z_2}$ and a T-dual version of the $\mathbf{T^6}/\mathbf{Z_2} \times \mathbf{Z_2}$ considered in this paper. The former corresponds to the hyper-multiplet model of a $\mathbf{T^4}/\mathbf{Z_2}$ orientifold [34,35] analyzed in [8] where the underlying K-theory group structure was also discussed. The model contains 1 O5 and 1 O9 plane whose charge is cancelled by introducing D5 and D9 branes. The RR tadpole conditions are

$$\begin{aligned} &\frac{v_6 v_4}{16}\{32^2 - 64\mathrm{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^{T}) + (\mathrm{Tr}(\gamma_{1,9}))^2\} \\ +&\frac{v_6}{16 v_4}\{32^2 - 64\mathrm{Tr}(\gamma_{\Omega R,5}^{-1}\gamma_{\Omega R,5}^{T}) + (\mathrm{Tr}(\gamma_{1,5}))^2\} \\ &+\frac{v_6}{64}\sum_{I=1}^{16}\{\mathrm{Tr}(\gamma_{R,9}) - 4\mathrm{Tr}(\gamma_{R,I})\}^2 = 0 \end{aligned} \tag{D.1}$$

The open string projection operator is

$$(\frac{1+\Omega}{2})(\frac{1+I_4}{2})(\frac{1+(-1)^F}{2})$$

Adapting the results from [8], the tachyon cancellation condition[16,17] is

$$2^4 n^2(1+\epsilon) - 2\sqrt{2}\ n\ \sin(\frac{\pi}{4}(r+s-5)) - 2\sqrt{2}\ n\ \sin(\frac{\pi}{4}(r-s-1)) = 0 \tag{D.2}$$

[16]In the $\mathbf{T^4}$, r $\leq$ 5 and s $\leq$ 4.

[17]See Table I in [8] for more details on the value of n for different values of r and s.

For $\epsilon = -1$ this corresponds to integrally charged non-BPS D-branes that couple to the twisted R-R sector, and reproduces the result s=0,4 for all r and $r = -1, 3$ for all s. The other option of $\epsilon = 1$ gives us torsion branes that couple to the twisted NSNS sector, and are stable for r=5, s=2.

In [8] the tensor-multiplet model was also investigated. As mentioned previously, the tensor-multiplet differs from the hyper-multiplet by a choice of discrete torsion. Changing the discrete torsion between the orientifold and $\mathcal{I}_4$, is equivalent to changing the sign in the Möbius strip diagram in the Dp - $O5$ amplitude. The new tachyon cancellation condition for torsion branes in the tensor-multiplet is

$$2^4 n^2(1+\epsilon) - 2\sqrt{2}\ n\ \sin(\frac{\pi}{4}(r+s-5)) + 2\sqrt{2}\ n\ \sin(\frac{\pi}{4}(r-s-1)) = 0 \tag{D.3}$$

The BPS fractional branes have r, s = (1,0), (1,4), (5,0), and (5,4). The new non-BPS branes from eqn. (D.3) are r=1,5, s=1,2,3 for the integrally charged branes, and for the torsion branes r=-1,0 and s=0 or r=3,4 and s=4.

Let us now consider the $\mathbf{T^6/Z_2 \times Z_2}$ orientifold in the T-dual frame where there are O9-planes and 3 types of O5 planes [26]. The RR tadpole conditions are

$$\begin{aligned} &\{v_1 v_2 v_3 (32^2 - 64\mathrm{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^{T}) + (\mathrm{Tr}(\gamma_{1,9}))^2) \\ \sum_i v_i \prod_{j\neq i} \frac{1}{v_j} &(32^2 - 64\mathrm{Tr}(\gamma_{\Omega R_i,5_i}^{-1}\gamma_{\Omega R_i,5_i}^{T}) + (\mathrm{Tr}(\gamma_{1,5_i}))^2\} \end{aligned} \tag{D.4}$$

The open string projection operator is

$$(\frac{1+\Omega}{2})(\frac{1+g_1}{2})(\frac{1+g_2}{2}) \tag{D.5}$$

which lead to the following tachyon cancellation condition

$$\begin{aligned} 2^4\ n^2 \times (1 + \epsilon_{T_1} + \epsilon_{T_2} + \epsilon_{T_3}) \\ -2n\ \sin(\frac{\pi}{4}(r+s-5)) \\ -2n\ \sin(\frac{\pi}{4}(r-s+2s_3-1)) \\ -2n\ \sin(\frac{\pi}{4}(r-s+2s_1-1)) \\ -2n\ \sin(\frac{\pi}{4}(r-s+2s_2-1)) = 0 \end{aligned} \tag{D.6}$$

All of the branes we find are T-dual to the branes in Section 5, and are related under the transformation $s_i \longrightarrow 2 - s_i$.

E Other Choices of Discrete Torsion

Table 4 contains the gauge group for the open strings in the 33 and 7_i7_i sectors. Only 8 of the possible 16 choices of discrete torsion have been listed; the other 8 cases can be obtained from a permutation of our results. In addition, we have included tables of non-BPS integrally charged branes with $r = 0$, since these are possible D-matter candidates [29] in addition to the torsion branes. The integrally charged brane spectrum for different choices of discrete torsion is listed in Table 5. As noted in Section 5.1, a consistent open string projection requires each of these integrally charged branes to couple to only one twisted sector. Our results are only stated for 8 of the possible 16 choices of discrete torsion. The spectrum for the other cases can be obtained through a simple permutation of the branes in the tables. Branes that are shown to be inconsistent are marked with a dagger.

κ	$\kappa_{\Omega R}$	$\kappa_{\Omega R g_1}$	$\kappa_{\Omega R g_2}$	$\kappa_{\Omega R g_3}$	$D3$	$D7_1$	$D7_2$	$D7_3$
$+$	$+$	$+$	$+$	$+$	$USp(N_\alpha)$	$USp(N_\alpha)$	$USp(N_\alpha)$	$USp(N_\alpha)$
$+$	$+$	$+$	$-$	$-$	$SO(N_\alpha)$	$SO(N_\alpha)$	$USp(N_\alpha)$	$USp(N_\alpha)$
$+$	$-$	$+$	$+$	$-$	$USp(N_\alpha)$	$SO(N_\alpha)$	$SO(N_\alpha)$	$USp(N_\alpha)$
$+$	$-$	$-$	$-$	$-$	$SO(N_\alpha)$	$SO(N_\alpha)$	$SO(N_\alpha)$	$SO(N_\alpha)$
$-$	$-$	$+$	$+$	$+$	$USp(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$
$-$	$-$	$+$	$-$	$-$	$U(N_\alpha/2)$	$SO(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$
$-$	$+$	$+$	$+$	$-$	$U(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$	$USp(N_\alpha/2)$
$-$	$+$	$-$	$-$	$-$	$SO(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$	$U(N_\alpha/2)$

Table 4: The gauge group for the open strings in the 33 and $7_i 7_i$ sectors. The gauge groups for the other eight choices of discrete torsion can be obtained by a simple permutation of the results above.

κ	$\kappa_{\Omega R}$	$\kappa_{\Omega R g_1}$	$\kappa_{\Omega R g_2}$	$\kappa_{\Omega R g_3}$	Integrally Charged Branes $(r = 0, s)$
$+$	$+$	$+$	$+$	$+$	None
$+$	$+$	$+$	$-$	$-$	$s_1 = 1, s_j = 1, s_k = 0$ $s_1 = s_2 = s_3 = 1$ $s_1 = 1, s_j = 1, s_k = 2^{\dagger}$
$+$	$-$	$+$	$+$	$-$	$s_1 = 1$ $s_2 = 1$ $s_1 = 1, s_j = 1, s_k = 0$ $s_2 = 1, s_j = 1, s_k = 0$ $s_1 = s_2 = s_3 = 1$ $s_1 = 1, s_j = 0, s_k = 2$ $s_2 = 1, s_j = 0, s_k = 2$ $s_1 = 1, s_j = 1, s_k = 2^{\dagger}$ $s_1 = 2, s_2 = s_3 = 1^{\dagger}$ $s_2 = s_3 = 2, s_1 = 1$ $s_1 = s_3 = 2, s_2 = 1$
$+$	$-$	$-$	$-$	$-$	None
$-$	$-$	$+$	$+$	$+$	$s_i = 0, s_j = 1, s_k = 2$
$-$	$-$	$+$	$-$	$-$	$s_1 = 1, s_j = 0, s_k = 2$
$-$	$+$	$+$	$+$	$-$	$s_1 = 1$ $s_2 = 1$ $s_2 = s_3 = 2, s_1 = 1$ $s_1 = s_3 = 2, s_2 = 1$
$-$	$+$	$-$	$-$	$-$	None

Table 5: D-matter candidates for different choices of discrete torsion. Branes that are shown to be inconsistent are marked with a dagger.

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