Abstract. The Shepard tabletop illusion, consisting of different perspective embeddings of two identical parallelograms as tabletops, affords a profound difference in their perceived surface shapes. My analysis reveals three further paradoxical aspects of this illusion, in addition to its susceptibility to the ‘inverse perspective illusion’ of the implied orthographic perspective of the table images. These novel aspects of the illusion are: a paradoxical slant of the tabletops, a paradoxical lack of perceived depth, and a paradoxical distortion of the length of the rear legs. The construction of the illusion resembles scenes found in ancient Chinese scroll paintings, and an analysis of the source of the third effect shows that the interpretation in terms of surfaces can account for the difference in treatment of the filled-in versus open forms in the Chinese painting from more than 1000 years ago.

1 Introduction

One of the most profound visual illusions was discovered not in the 19th but in the late-20th century by Roger Shepard. It is the Shepard tabletop illusion, in which the perspective view of two identical parallelograms as tabletops at different orientations gives a completely different sense of the aspect ratio of the implied rectangles in the two cases (Shepard 1990). In my version of the illusion (figure 1) each tabletop is a 45 deg. parallelogram of equal height and width, as indicated by the rectangular boxes aligned with the corresponding sides; the boxes are all of the same size and shape. A parallel wood grain has been added to the two surfaces, further enhancing the perceived discrepancy between the two shapes. “The image on the right was produced by a 45 deg clockwise rotation of the tabletop and the two boxes at left—but without rotating either the legs or the grain. The illusion is so strong that it is scarcely credible that the two tabletops are the same shape.
2 Analysis

Note that an incidental aspect of the Shepard tabletops is that the parallelogram construction makes them subject to the well-known ‘inverse perspective illusion’ of orthographic perspective; although the oblique receding lines for the two sides are parallel, they appear to diverge into the distance, giving the impression that the tabletops are getting wider as they recede in depth. This widening occurs because the perspective context of the rest of the table gives the impression that the tabletops are, indeed, horizontal receding surfaces, which generates the perceptual expectation that the sides should be converging to a single vanishing point. The deviation from this expectation thus produces the impression of divergence, even though the sides are actually parallel.

Inspection of figure 1 reveals three paradoxical properties of this illusion that have been unremarked both by Shepard and subsequent purveyors of the illusion.

- Paradoxical slant of the two tabletops.
- Paradoxical lack of perceived depth.
- Paradoxical distortion of the length of the rear legs.

The first paradox is that the two tabletops do not appear to be parallel horizontal surfaces. The left-hand tabletop appears to be sloping down to the left, while the right-hand one appears to be sloping down somewhat to the right (especially when one focuses attention between the two tables). Thus, both tabletops seem to slope away from a high point of the center of the picture. Paradoxically, this effect is in the opposite direction from the classic orientation induction of acute angle enlargement (Zöllner 1860; Blakemore et al 1970), in which the oblique edges would induce the horizontals to appear to slope up from the center.

One explanation for the primary illusion may be Gregory’s (1963, 1990) concept of Misapplied Depth Constancy, which would take the form of the tabletop surfaces appearing longer than their retinal projections because they are interpreted as surfaces receding in depth, which would require that they were proportionately deeper in order to project to the physical length of the obliques (which is $\sqrt{2}$ longer and $1/\sqrt{2}$ shorter than the front for the left and right images, respectively). Since the right-hand table appears to be approximately square with a width of, say, 2 cm, the implication is that the depth angle of the table surface required to account for the illusory distortion is a rotation of about 45 deg. into the page, which in turn would imply that it has a perceived depth of $2 \times \sqrt{2} = 2.828$ cm. For the same depth angle, the left-hand table would have a perceived depth of $2 \times \sqrt{2} \times \sqrt{2} = 4$ cm.

The second paradoxical aspect of the illusion is the apparent flatness of the appearance of the depictions. Although the table surfaces are seen as receding in depth, they do not do so to the logical extent of rectangular objects (whose physical depths would be about 4 and 3 cm, respectively, for the left and right tables; see below). Instead, the perceived depth of these images in real object terms is much flatter than this, and indeed may appear to be perfectly flat on the page to many observers. It seems curious, therefore, that the perspective distortion seems so strong in the face of only a weak illusion of perceived depth.

Thus, the fundamental paradox of the perceived depth, and indeed of the Misapplied Depth Constancy theory in general, is that the illusory distortions seem to operate in a brain module independent of the one that delivers the sense of perceived depth. In many cases the illusions explained by this theory have been susceptible to other explanations, with this depth paradox weakening the explanatory power of the theory. In this case, however, the identity of the surface shapes makes it difficult to argue for any other theory, so it seems that we need to address the paradox head on.

There are several factors that might arguably mitigate the degree of depth perceived from the perspective implied by the oblique sides of the parallelograms, including the lack of convergence in their orthographic form of perspective, the invariance of the line
thickness, the uniformity of the overlaid texture, the flatness of the stereoscopic depth cues, the knowledge that it is a printed image on a page, and so on. In the face of these cues, therefore, it is more understandable that the perceived depth should be weak to nonexistent. The paradox is that despite all these cues to flatness, the illusory distortion corresponds to a depth angle as steep as 45°, even without the corresponding depth percept.

The third paradoxical aspect of the Shepard illusion is that the rear legs of the tables appear to be shorter than the front legs. This effect seems to derive from a contrast with the ‘inverse perspective illusion’ of the parallelogram shapes of the tabletops discussed above, making the tabletops appear wider at the back than at the front. Conversely, the rear legs of the tables appear shorter than the front legs, despite the fact that they are physically the same length (and that they define similar parallelograms alongside those of the tabletops). This apparent shortening of the rear legs seems to be induced in contrast with the apparent widening of the back of the tabletop, despite the fact that the back is at the same perceived distance as the rear legs. It is not obvious that this contrast should occur, since the bottoms of the legs are one edge of the vertical parallelogram defining the plane of the sides of the tables, which might be expected to show the same illusory expansion as the tabletop. One explanation for the different perceptual treatment of the top and the sides may be that the illusory expansion depends on the presence of complete boundaries around the surfaces so that the vertical parallelogram loses its effectiveness as a depth structure because the legs are seen as individual objects rather than components of a rectangular figure.

Figure 2. Han Xi-zai Gives a Banquet (retouched sector of extended scroll); Gu Hong-zhong (950). Solid white construction lines reveal the strong dominance of parallelity within surfaces (orthographic perspective). Dashed white lines complete the implied surface where the chair and table legs meet the floor. Though parallel between the two pairs of table legs, they show a pronounced lengthening of the rear legs relative to the front legs in each case, violating the parallel requirement of orthographic perspective.

As mentioned, the parallel construction of Shepard’s two tabletops implies that they are both rendered in orthogonal (‘Chinese’) perspective, which has been the dominant form of perspective representation in China (Osborne 1970), so we may expect to find similar illusions in Chinese paintings. A good example of the use of patches of oblique parallel perspective with angles is seen in the scroll Han Xi-zai Gives a Banquet (顧關夜宴圖) by Gu Hong-zhong (顧関中), (Five Dynasties period, 五代 ~ 950). One section of the scroll (which has been retouched for clarity in this reproduction) is shown in figure 2. The entire booth with two seated personages and the long food table in front of it conform accurately to one set of parallels with no convergence. In fact, within any given surface, all the perspective
lines in this painting conform to the principle of parallelity (solid white lines), including the implied surface rectangles where the legs of tables or chairs meet the floor (dashed white lines).

Remarkably, although the two sides of the foreground tabletop to the left are parallel, we find that the legs are painted with a strong divergence of their parallels with respect to the ones of the tabletop; the back legs are substantially longer than the front legs (as indicated by the dashed white lines, which should be parallel to their counterparts in the corresponding tabletops). This is not just a lack of accuracy on the part of the painter, as the same effect is seen for the legs of both the chair in the foreground and the long table to the right. In each case the back legs of the pair extend down further in the back than in the front, diverging from the angle of the tabletop. Somehow, Gu Hong-zhong must have seen this divergence between the leg angles and the top surface angle as the proper way to draw the perspective of a table (although I have not found any previous report of this effect in Chinese painting; Tyler 2011).

Notice, however, that there is neither convergence nor divergence between the left and right pairs of legs of each table—these lateral obliques are perfectly parallel with each other despite the divergence of the vertical angles. This distortion may be explained by the third unremarked aspect of the Shepard illusion—the induced shortening of the rear legs in opposition to the induced lengthening of the tabletop. Gu Hong-zhong’s tendency to paint the rear legs of his tables longer than the front suggests that he is either intuitively painting them to look right visually without being aware of the rear-leg-shortening illusion or consciously lengthening them to counteract the illusion that he knows his viewers will perceive if all the legs are painted the same length.

### 3 Predictive Test

As with many illusory figures, this detailed analysis of the properties of the illusion reveals a number of unremarked aspects of the perceived configuration, whose interpretation is subject to further evaluation. The interpretation of the perceived shortening of the rear legs, and its (over)correction in the Chinese painting, for example, may be tested by replacing the legs with solid sides. The prediction is that the solid sides should be subject to the same perceived expansion as the tabletops once the sides are completed into full parallelograms. This prediction is tested in the depiction of the same table shapes from figure 1 as marble blocks in figure 3. The distortion effect of the horizontal surfaces remains just as strong in this marble-block configuration, but it is immediately evident that the sides now also appear wider at the back than the front (the opposite effect from the shortening of the rear verticals when depicted as legs). Thus, we should expect that solid rectangular objects in Chinese paintings are not subject to the correction effect seen for the three tables, as is indeed the case for the seating booth at the right in figure 2.

Figure 3 also includes a verification that the primary illusion is not due to the orientation parallelogram of the top surfaces. The upper figure is a block-rotated version of the right-hand block with its surface parallelogram aligned with that of the left-hand block, showing that the illusion persists even in the aligned orientation. This result verifies that the illusion is due to the perspective context of the other two sides.

### 4 Conclusion

In summary, the interaction between the filled-in versus open forms of the parallelograms and the rear expansion of the orthographic perspective in the two versions of the Shepard illusion can explain the difference in treatment of the filled-in versus open forms in a Chinese painting from more than 1000 years ago. These results imply that the diverse aspects of the
Paradoxes in the Shepard Illusion

Figure 3. This marble block version of the Shepard illusion is similarly subject to the perceived difference between the horizontal surface shapes (compare left and right blocks) but reverses the shortening of the rear sides, which appear expanded at the rear once they are filled in as completed rectangles (especially when viewed close up). The upper figure is a block-rotated version of the right-hand block (red arrow) with its surface parallelogram aligned with that of the left-hand block (green arrow), showing that the illusion persists even in the aligned orientation.

illusion are not just attributable to the orientation relationships of the component lines but are significantly impacted by the degree to which the lines are incorporated into filled-in surfaces.

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References

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