Herding effects in order driven markets: the rise and fall of gurus

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Abstract
We introduce an order driver market model with heterogeneous traders that imitate each other on a dynamic network structure. The communication structure evolves endogenously via a fitness mechanism based on agents performance. We assess under which assumptions imitation, among noise traders, can give rise to the emergence of gurus and their rise and fall in popularity over time. We study the wealth distribution of gurus, followers and non followers and show that traders have an incentive to imitate and a desire to be imitated since herding turns out to be profitable. The model is then used to study the effect that different competitive strategies (i.e chartist & fundamentalist) have on agents performance. Our findings show that positive intelligence agents can not invade a market populated by noise
traders when herding is high.

**Keywords:** dynamic network, herding, guru, order driver market.

1 Introduction

Mainstream economic theory does not provide a satisfactory explanation for financial market frenzies, crashes and panics. The standard reasoning, dating back to Friedman (1953), is that these phenomena, driven by irrational traders, are irrelevant in the long run since destabilizing speculators would quickly go bankrupt and be eliminated from the market. Thus, according to the mainstream literature, the study of rational speculators is enough to describe the behaviour of stock markets. Nonetheless there is a considerable evidence that investors do not always act rationally and do not follow the economists’ advice. Black (1986) suggests that some traders, when they do not have access to true information, irrationally act on noise, and, following Kyle (1985), calls such investors ”noise traders”.

The presence of noise traders and their impact on prices’ movement is well documented. Some authors (Figlewski (1979), Shiller (1984), Campbell and Kyle (1987), De Long et al. (1990a)) show that if ‘rational agents’ are risk averse, then their ability to take positions against noise traders, who drive prices away from their fundamental value, is limited.

An important mechanism that may explain the deviation of prices from their fundamental value is the formation of expectations. Expectations drive individual behaviours and individual behaviours determine the economic outcome, i.e., prices and trading. ”Therefore, a market, like other social environments, may be viewed as an expectations feedback system” (Heemeijer et al. (2009)). An intuition of how expectations feedback system with ’zero intelligence agents’ works is as follows. If noise traders share pessimistic expectations about an asset, they will sell it frantically, driving down its price. An informed trader who may want to buy the asset will update his expectations recognizing that in the near future noise traders might become even more pessimistic and drive price down even further. The informed trader may eventually conclude that it is not convenient for him to buy now. Conversely, if the informed trader wants to sell an asset about which noise traders have optimistic expectations, that would drive the price up, he may decide not

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1 Our market is populated by agents with naive trading strategies, called noise traders. In this way we are close to the tradition of Zero-Intelligence (ZI) traders as in (Becker (1962), Gode et al. (1993), Gode et al. (1997)).
to sell. Thus convergence to the rational equilibrium price becomes unlikely. In fact, because of the unpredictability of noise traders, prices can fluctuate significantly even when the fundamental price is stable.

It is generally accepted in the economic works on noise traders that “positive feedback” traders prevail in financial markets. Theoretical models and laboratory experiments of positive feedback\(^2\) have been studied in De Long et al. (1990a, 1990b), Marimon and Sunder (1993), Geber et al. (2002), Hommes et al. (2005, 2007, 2008), Heemeijer et al. (2009), Sutan and Willinger (2009) and Adam (2007)). All these works have shown that positive feedback irrational noise traders, if sufficiently aggressive, can destabilize prices and earn larger returns.

An important question is why traders’ expectations are often coordinated. Claude Trichet (2001) remarked: “Some operators have come to the conclusion that it is better to be wrong along with everybody else, rather than take the risk of being right or wrong alone”\(^3\). This “mass-uniform” behaviour was already present in Keynes (1936) who called it “animal spirits”.

Some insights into fluctuations in prices and coordination of expectations have been provided by agent-based models. For example Lux and Marchesi (2000), Iori (2002), Chiarella et al. (2002, 2009), Kirman and Teyssiere (2002), LiCalzi and Pellizzari (2003), Gaumersdorfer et al. (2008), and LeBaron et al. (2007, 2009) have analyzed how the co-ordination of traders’ strategies by market mediated interactions (for example by following chartist trading rules) or mechanisms of behavioural switching can lead to large aggregate fluctuations. In particular Brock and Hommes (1998) consider an asset pricing model with agents switching between fundamentalists and chartists expectations, based upon the recent performance (profits) of these strategies. Chartists survive in the market, because their (short run) profits are higher than those of fundamentalists. In a similar vein, Lux (1998) considers a fundamentalists-chartists model with switching driven by profits and by herding behavior (measured by an opinion index). In the Lux model the performance of a strategy is thus affected by its profitability and by the popularity of the strategy. A good survey of this type of work is Hommes (2006), where these models and how they contradict Friedman’s hypothesis have been extensively discussed.

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\(^2\)Positive feedback in a stock market refer to the situation where positive (negative) expectations about the price do lead to a price increase (decrease)

\(^3\)Claude Trichet, then Governor of the Banque de France.
Collective behaviour nonetheless could reflect the phenomenon known as herding which occurs in situations with information externalities, when agents' private information is swamped by the information derived from directly observing others' actions (Banerjee (1992, 1993); Orléan (1995); Cont and Bouchaud (1999), Stauffer and Sornette (1999), Iori (2002), Markose et al. (2004), LeBaron et al. (2009)). Most of the studies on herding effects have focused on how herding can lead to large price fluctuation but only a few papers have investigated its role on the communication network structure and on traders' wealth, which is the focus of this paper.

We introduce a model where agents imitate the expectations of the most successful traders called the Guru$^4$. Price formation is determined by an order-driven market, as in Chiarella et al. (2009). Within their budget constraints, agents can place market orders or limit orders for arbitrary quantities. Limit orders are stored in the book and executed (partially or completely) when they find a matching market order on the opposite side. A market order is filled completely if it finds enough capacity on the book, or partially otherwise. The motivation to use an order-driven market is to avoid the limitations of the market maker approach in which there is no explicit mechanism of trading. In fact, the market maker, typically risk neutral and endowed with unbounded liquidity, absorbs excess demand and makes trading always viable, regardless of its size. In each period, the market maker adjusts the price to reduce the excess demand. Inspired by the metaphor of the Walrasian tâtonnement, this price-adjusting rule fails to recognize that in a real market trade occurs whenever two agents can match their requests at a given price. Because of the simplistic pricing rule adopted by the market maker, herding (that normally leads to a large excess demand) has an obvious and immediate impact on prices.

In an order driven market, where agents imitate the expectations of others and not their action, the role of imitation is less obvious. In fact even if the guru expects a price increase, he himself and/or the agents that imitate his expectations may submit limit orders instead of market orders, and the impact of these trades on the price may be negligible or may be delayed.

Recent models that are related to our are those of Markose et al. (2004), LeBaron et al (2009), and Gerasymchuk et al. (2010). Markose et al. (2004)

$^4$Agent based models have taken many different approaches as to how strategy information could be shared outside price system (see Vriend (2000)). "Obviously, the correct model for information sharing is not identifiable, but it is clear that some imitation must take place in financial markets"-Le Baron (2009).
develop a model where agents make a binary decision, to buy or sell a single unit of an asset, following the average advice of the other agents they are connected to. The interaction network evolves dynamically as agents adaptively modify the weights of their links to their neighbours by reinforcing "good" advisors and breaking away from "bad" advisors. The question Markose et al. address is whether and when the dynamic process of reinforced learning can lead to the creation of small world networks. Nonetheless trading is not explicitly modelled in their model and the wealth of agents is not monitored. LeBaron et al. (2009), instead, develop a dynamic limit order model where traders make weighted forecasts of assets future returns by combining fundamental, chartist and noise rules, following Chiarella et al (2009). As time goes by, agents do trade and look at their own past performances (measured in term of their expected prices versus realized prices) and update the weights of their trading rules via a genetic algorithm that selects those parameters which have performed better. Gerasymchuk et al. (2010) studies the effect of alternative exogenous information network structure in the Brock-Hommes asset pricing model. Agents are located on the nodes of a network and can observe the fitness measure of the strategies employed only by those agents who reside on the nodes directly connected with them. The authors show that the network structure influences the stability and volatility of the asset price dynamics, due to the different speed of information transmission in different networks. In these last two models there is no direct imitation among traders and coordination arises when traders dynamically adopt the same rule.

In our model all agents are uninformed noise traders and directly imitate each other. The originality of this work respect to our previous version (Tedeschi et al. (2009)) is in the communication network. In Tedeschi et al. the guru was fixed exogenously and each agent decided whether to imitate him or not with a given probability. Here we introduce an endogenous mechanism of imitation, by implementing a preferential attachment rule (Barabási and Albert (1999)) such that each trader is imitated by others with a probability proportional to its profit\(^5\). This mechanism of links formation allows

\(^5\)In 1955 Herbert Simon showed that power laws arise when 'the rich get richer', when the amount you get goes up with the amount you already have. In sociology this is referred to as Matthew effect (see Merton (1968)) with reference to the biblical edict. Today, this phenomenon is usually known under the name 'preferential attachment', coined by Barabasi and Albert (1999). Bianconi and Barabasi (2001) have proposed an extension of Barabasi and Albert. In their model each newly appearing vertex \(i\) is given a 'fitness'
us to study under which assumptions a gurus endogenously rise and fall over
time, and how imitation affects the asset price and the distribution of agents'
wealth.

Although agents in our model initially start with the same amount of stock
and cash, when imitation is high, trading generates a fat tail distribution of
individuals' wealth, in accordance with the empirical evidence that market
participants are very heterogeneous in size (see, for example, Pareto (1897),
Zipf (1949), Ijiri and Simon (1977), Axtell (2001), Pushkin and Aref (2004),
Gabaix et al. (2006)). Moreover, in contrast with the prevailing economic
view that informed agents need to hide their private informations in order
to profit from it\textsuperscript{6} (see Benabou and Laroque (1992), Caldentey and Stac-
chetti (2007), Chakraborty and Yilmaz (2008)), our uninformed gurus gain
the highest profits when they reveal their expectations to the highest number
of followers. Furthermore, we will also show that followers, on average, gain
higher profit then non followers, thus providing a justification for herding to
occur in the first place. Since our baseline model deals with zero-intelligence
traders, it seems worthwhile to ask whether more sophisticated agents, such
as Chartists and Fundamentalists, can overperform gurus in a market domi-
nated by herding. Thus, we use the model as a computational laboratory to
run some preliminary experiments on the role of different competitive strate-
gies. Our findings show that while chartist and fundamentalist strategies are
both successful in the absence of herding, they underperform the guru and
his followers when imitation is high.

The rest of the paper is organized as follows. In section 2 we describe the
model; in section 3 we present the results of the baseline model simulations;
in section 4 we present results when including competing chartists and fund-
damentalists strategies, and section 5 concludes.

that represents its attractiveness and thus its propensity to gain more links. When one
considers a fitness algorithm it is true that the larger the fitness the larger the degree but
a large degree is a consequence of some intrinsic quality, not the cause of the improvement
of site connectivity. In our model this intrinsic quality is, precisely, the agents' profit.

\textsuperscript{6}In fact revealing private intentions, specially for large agents, could decrease their
fitness. For this reason large investors refrain from revealing their demand, supply or their
expectation (see Vaglica et al. (2008)).
2 The model

2.1 The network

We start the description of the model by explaining formation and evolution of the communication network. In our network, nodes represent agents and edges are the connective links between them. Links are directional, they are created and deleted by agents who look for advice and point to the agent that provides advice. Information travels in the opposite direction.

In general local interaction models agent interacts directly with a finite number of others in the population. The set of nodes with whom a node is linked is referred to as its neighbourhoods. In our model the number of outgoing links is constrained to be one, thus agents can only get advice from one neighbor. The reason being that in a highly connected random network synchronisation could be achieved via indirect links. The effects of direct imitation are easier to be tested in a diluted network where indirect synchronisation is less likely to arise.

We implement an endogenous mechanism of preferential attachment based on a fitness parameter given by agent’s wealth. Agents start with the same amount of cash $C_{t=0}$ and stocks $S_{t=0}$, so that all agents have the same initial wealth $W_{t=0} = C_{t=0} + p_{t=0}S_{t=0}$. As time goes by, some traders may become richer than others. As a measure of agents’ success we define their fitness at time $t$ as their wealth relative to the wealth $W_{t}^\text{max}$ of the richest agent $i_{\text{max}}$:

$$f_i^t = \frac{W_i^t}{W_{t}^\text{max}}. \quad (1)$$

Each agent $i$ starts with one outgoing link with a random agent $j$, and possibly with some incoming links from other agents. Links are rewinded at the beginning of each period, in the following way: each agent $i$ cuts its outgoing link, with agent $k$, and forms a new link, with a randomly chosen agent $j$, with a probability

$$p_i^t = \frac{1}{1 + e^{-\beta(f_j^t - f_i^t)}},$$

or keep its existing link with probability $1 - p_i^t$. The rewind algorithm is designed so that successful traders, here called gurus, gain a higher number of incoming links and thus have a higher probability of being imitated. Nonetheless the algorithm introduces a certain amount of randomness, and links with more successful agent have a finite probability to be cut in favour
of links with less successful agents. In this way we model imperfect information and bounded rationality of agents. The randomness also helps unlocking the system from the situation where all agents link to the same guru.

2.2 The expectation formation mechanism

Trading happens over a number of periods $t_k$, with $k = 1, \cdots T$. At the beginning of each trading period $t_k$, agents make idiosyncratic expectations about the spot return, $\hat{r}_{i, t_k + \tau}^i$ in the interval $(t_k, t_k + \tau)$. We assume that agents are not informed and have random expectation about future returns. We also assume that agents are heterogeneous in that they have different forecasts of the returns’ volatility, $\sigma^i_{t_k}$. Idiosyncratic expected returns are thus given by

$$\hat{r}_{i, t_k + \tau}^i = \sigma^i_{t_k} \epsilon^i_{t_k},$$

where $\sigma^i_{t_k}$ is a positive, agent specific, constant and $\epsilon^i_{t} \sim N(0, 1)$ is a normal noise.

After individual expectation are generated, a consultation round starts during which agents sequentially, and in a random order, revise their expectations. The revised expected return is obtained by weighing agent $i$’s own expectation with that of agent $j$ to which $i$ is linked to

$$r_{i, t_k + \tau}^i = w \hat{r}_{i, t_k + \tau}^i + (1 - w) \hat{r}_{j, t_k + \tau}^j,$$

where $w$ measures the impact that agent $j$’s expectation has on the agent $i$’s expectation. When $w$ is equal to zero, $i$ trusts completely the opinions of $j$, while, when $w$ is equal to one, $i$ considers exclusively his own opinion and agents decisions are fully independent from each other. At the end of each period $t_k$, after trading has taken place, agents expectations are reset to random values. We stress that in the model imitation is purely expectation based, and agents do not observe the actions of others. This choice is motivated by the fact that in a real market the order book is not normally fully visible to traders, and that the order submission is anonymous.

While our agents do not have a priori information about price movements, they do acquire some information via the imitation mechanism. An agent who realizes to have several incoming links knows (when $w$ is small) he will be able to influence the decisions of several others, and can better predict future demand. Highly popular agents thus become overoptimistic or over-pessimistic. This effect is incorporated in the model by assuming that the
volatility of forecasted returns is proportional to the number of incoming links and to the weights $w$, such that

$$\sigma^i_{t_k} = \sigma^i_0 (1 + l^i_{t_k} (1 - w)),$$

(4)

where $l^i_{t_k}$ is the percentage of existing links that point to agent $i$ at time $t_k$. The values of $\sigma^i_0$ are chosen, with uniform probability, in the interval $(0, \sigma_0)$.  

2.3 The market

This section describes the order placement mechanism which is based on the order-driven market used in Chiarella et al. (2009). Our $N$ traders can either place market orders, which are immediately executed at the current best listed price, or they can place limit orders. Limit orders are stored in the exchange’s book and executed using time priority at a given price and price priority across prices. A transaction occurs when a market order hits a quote on the opposite side of the market.

At the beginning of each period, traders make expectations about the price at the end of a given time horizon $\tau$ (that we take to be the same for all traders). The future price expected at time $t_k + \tau$ by agent $i$ is given by

$$\hat{p}^i_{t_k,t_k+\tau} = p_{t_k} e^{r^i_{t_k,t_k+\tau} + \sqrt{\tau}},$$

(5)

where $r^i_{t_k,t_k+\tau}$ is the agent’s expectation on the spot return given by Eq. (3) and $p_{t_k}$ is the reference price observed by all agents at the beginning of each period. The square root term indicates that if agents are purely noise traders (as when in the absence of imitation) they expect prices to be geometric random walks\(^7\). After expectations are made, agents enter the market, sequentially and in a random order and place a buy or a sell order of a certain size. Orders that are not executed after a period $\tau$ are removed from the book.

The number of stocks an agent is willing to hold in its portfolio at a given price level $p$ depends on the choice of the utility function. Our agents are modelled as risk averse and maximize an exponential CARA utility function

$$U(W^i_{t_k}, \alpha) = -e^{-\alpha W^i_{t_k}},$$

(6)

\(^7\)The square root term in the geometric Brownian motion is introduced for analogy with continuous time models. The square term nonetheless could be replaced with a linear term without considerable implications for the dynamic itself.
where the coefficient \( \alpha \) measures the risk aversion of traders. The portfolio wealth of each agent is given by

\[
W^i_{tk} = S^i_{tk}p_{tk} + C^i_{tk},
\]

where \( S^i_{tk} \geq 0 \) and \( C^i_{tk} \geq 0 \) are respectively the stock and cash position of agent \( i \) at time \( t_k \). The optimal composition of the agent’s portfolio is determined in the usual way by trading-off expected return against expected risk. However here the agents are not allowed to engage in short-selling. When agents place a market order, their cash and stocks positions are updated accordingly. When agents place a limit order, the cash they commit to buy and the stocks they commit to sell are tentatively removed from their portfolios (even if a limit order does not comport an immediate transaction). In this way agents can not spend money or sell stocks that have already been committed in the book. If an order is cancelled, the stocks and cash that were tied down in the order are returned to the trader who had submitted it.

For the CARA utility function assumed here the optimal composition of the portfolio, that is the number of stocks the agent wishes to hold at any given price is given by

\[
\pi^i(p) = \frac{\ln(\hat{p}^i_{tk} + \tau/p)}{\alpha V^i_{tk}p},
\]

where \( V^i_{tk} \) is the risk perceived by agent \( i \), normally taken as the unconditional variance of returns. We assume that agents’ risk assessment depend on their connectivity. In particular, we assume that those agents who are highly imitated, and consequently, as we will see later, are more successful, flattered by their numerous followers become more confident about their forecasts. To these popular agents the risky asset appears as less risky, because they are good at forecasting it. Thus their assessment of risk is far below the unconditional variance of the risky asset returns. This effect is captured by setting

\[
V^i_{tk} = V_{tk}(1 - (1 - w)l^\%_{i,t_k}),
\]

where \( l^\%_{i,t_k} \) is the percentage of existing links that point to agent \( i \) at time \( t_k \). Note that this implies that highly connected agents are willing to hold larger amounts of stocks in their portfolio.

Combining equations (5, 8, 9) together, we obtain

\[
\pi^i(p) = \frac{\ln(p_{tk}/p) + (w\sigma^i_0(1 + l^\%_{i,t_k}(1 - w))\epsilon_{tk} + (1 - w)\sigma^0_0(1 + l^\%_{j,t_k}(1 - w))\epsilon_{tk})\sqrt{\tau}}{\alpha(1 - (1 - w)l^\%_{i,t_k})V_{tk}p}.
\]

\(^8\) Eq. (8) can be derived on the basis of mean-variance one-period portfolio optimization.
The unconditional variance $V_{t_k}$ is estimated as

$$V_{t_k} = \frac{1}{\tau} \sum_{j=1}^{\tau} [r_{t_k-j} - \bar{r}_{t_k}]^2,$$

where the average spot return $\bar{r}_t$ is given by

$$\bar{r}_t = \frac{1}{\tau} \sum_{j=1}^{\tau} r_{t_k-j} = \frac{1}{\tau} \sum_{j=1}^{\tau} \ln \frac{p_{t_k-j}}{p_{t_k-j-1}}.$$  

(12)

If the amount $\pi^i(p)$ is larger (smaller) than the number of stocks already in the portfolio of agent $i$ then the agent decides to buy (sell). In order to determine the buy/sell price range of a typical agent, we first estimate numerically the price level $p^*$ at which agents are satisfied with the composition of their current portfolio, which is determined by

$$\pi^i(p^*) = \frac{\ln(\hat{p}_{t_k+\tau}/p^*)}{\alpha V_{t_k} p^*} = S_{t_k}^i.$$  

(13)

Eq. (13) admits a unique solution with $0 < p^* \leq \hat{p}_{t_k+\tau}$ since $S_{t_k}^i \geq 0$ (short selling is not allowed). Agents are willing to buy at any price $p < p^*$ since in this price range their demand is greater than their holding, and are willing to sell at any price $p > p^*$ since in this case their demand is less than their holding. Note that agents may thus wish to sell even if they expect a future price increase. In order to impose budget constraints we need to restrict to values of $p \leq \hat{p}_{t_k+\tau} = p_{M}^i$ to ensure $\pi(p) \geq 0$ and so rule out short selling. Furthermore to ensure that an agent $i$ has sufficient cash to purchase the desired stocks, the smallest value of $p$ we can allow, $p_m^i$, is determined by its cash position (see Eq. (7)), and is given by the condition

$$p_m^i \left( \pi^i(p_m^i) - S_{t_k}^i \right) = C_{t_k}^i.$$  

(14)

Again one can easily show that this equation also admits a unique solution with $0 < p_m^i \leq \hat{p}_{t_k+\tau}$ since $S_{t_k}^i, C_{t_k}^i \geq 0$. Indeed, comparing Eqs. (13) and (14) it can be easily proven that $0 < p_m^i \leq p^* \leq \hat{p}_{t_k+\tau}$. The price at which the agent is willing to trade is finally chosen randomly, with uniform probability, in the interval $[p_m^i, p_M^i]$. Suppose now that the agent chooses to trade at a price $p < p^*$, then it submits a limit order to buy an amount

$$s^i = \pi^i(p) - S_{t_k}^i,$$

while if $p > p^*$ it submits a limit order to sell an amount

$$s^i = S_{t_k}^i - \pi^i(p).$$
Position | Type of order | Volume
---|---|---
\( p_m^i < p < a^i_q \) | BUY | Limit order \( s_i = \pi^i(p) - S^i_t \)
\( a^i_q \leq p < p^* \) | BUY | Market order \( s_i = \pi^i(a^i_q) - S^i_t \)
p = p^* | | No order placement
\( p^* < p \leq b^i_q \) | SELL | Market order \( s_i = S^i_t - \pi^i(b^i_q) \)
\( b^i_q < p \leq p^*_M \) | SELL | Limit order \( s_i = S^i_t - \pi^i(p) \)

Table 1: Summary of the trading mechanism of a typical trader \( i \) with a random price level \( p \) limited between the value \( p^*_m \) given by Eq. (14) and the value \( p^*_M = \tilde{p}^i_{t_k + \tau} \). The current quoted best ask and best bid are \( a^i_q \) and \( b^i_q \) respectively.

However if \( p < p^* \) and \( p > a^i_q \) (the best ask) the buy order can be executed immediately at the ask. An agent in this case would submit a market order to buy an amount

\[
s^i = \pi^i(a^i_q) - S^i_{t_k}.
\]

Similarly if \( p > p^* \) and \( p < b^i_q \) (the best bid) the agent would submit a market order to sell an amount

\[
s^i = S^i_{t_k} - \pi^i(b^i_q).
\]

If the depth at the bid (ask) is not enough to fully satisfy the order, the remaining volume is executed against limit orders in the book. The agent thus takes the next best buy (sell) order and repeats this operation as many times as necessary until the order is fully executed. This mechanism applies under the condition that sufficient quotes of these orders are above (below) price \( p \). Otherwise, the remaining volume is converted into a limit order at price \( p \). If the limit order is still unmatched at time \( t_k + \tau \) it is removed from the book.

The essential details of the trading mechanism are summarized in Table 1, showing how it depends on the price level \( p \), the “satisfaction level” \( p^* \), the best ask \( a^i_q \) and the best bid \( b^i_q \).

3 Simulations and results

The model is studied numerically for different values of the parameter \( w \). In the first part we focus the analysis on some properties of the network such as the in-degree and fitness distribution. Then we analyze the probability
distribution of wealth and stocks and the positive feedback on prices.
In the simulations the number of traders is set at $N = 150$. Each agent is
initially given the same amount of stock $S_0 = 100$ and cash $C_0 = 100$. The
initial stock price is chosen at $p_0 = 1000$. We fix $\tau = 200, \alpha = 0.01$, and $\beta^i$
uniformly distributed in the interval $[5, 45]$ The results reported here are the
outcome of simulations of $T = 1000$ periods and $N_t = 300$ trades per period.
Simulations are repeated $M = 100$ times with a different random seed$^9$.

3.1 The network

In figure (1) we plot one shot of the configuration of the endogenous network
for $w = 0.1$, $w = 0.5$ and $w = 1.0$. The graphs show that few gurus could
co-exist and compete for popularity. As $w$ increases the network becomes
less and less centralized with a higher number of smaller gurus. We can
immediately notice how the network structure depends on the imitation level
$w$. Links are formed according to preferential attachment and relative wealth
in Eq. (1), but these are independent of $w$. The weight $w$, however, affects
the profits of the guru and his followers and, thus, the network formation.
Moreover the topology of the network is different from that of the random

\[\text{Figure 1: Network configuration for } w = 0.1 \text{ (the guru is agent 108) (left side), for } w = 0.5 \text{ (the guru is agent 78) (centre) and for } w = 1 \text{ (the guru is agent 6) (right side).}\]

$^9$We have tested the stability of our results and verified that the model shows a qualitatively similar behaviour for a range of values of the parameters.
graph studied extensively by Erdos and Renyi (1960). While in an Erdos-Renyi random graph the in-degree\textsuperscript{10} has a Binomial (or Poisson) distribution, in real world networks some agents are found to have a disproportionately large number of incoming links while others have very few. In figure (2) we plot the complementary cumulative distribution (CCD) of the normalized in-degree (left side) and the complementary cumulative distribution (CCD) of the normalized fitness (right side) for $w = 0.1$ (black line), $w = 0.5$ (red line) and $w = 1.0$ (green line). Colors are available on the web site version.

The distribution of in degree in our model, when imitation is large, is in keeping with that of scale-free networks and displays a ’fat tail’. In Table (2) we plot the index of the current guru (black solid line), the percentage of incoming links to the current guru (red dotted line) and the fitness of the current guru (green dashed line), for different $w$, as function of the time. The figure shows that agents alternate as the guru during the simulation (black solid line). In fact, as the guru acquires an increasing num-

\textsuperscript{10}In directed graphs, there is the in-degree, number of edges pointed to it, and out-degree, number of edges pointing away from it. Note, the out-degree of an agent defined by those edges starting from $i$ gives the number of his first order neighbours that, in our model, are constrained to be one.
ber of links (red dotted line), one or more of his followers may become richer than the guru himself, as signalled by the fact that the fitness (green dashed line) of the guru becomes, at times, smaller than 1. As other agents become rich they start to be imitated more and more and eventually one of them becomes the new guru.

The stability (or average life) of the guru becomes longer as imitation increases (i.e $w$ decreases) as shown in table (2) (bottom right side).

The evolution of our interaction network is in line with other works (Brock and Hommes (1998), Lux (1998), LeBaron et al (2009), and Gerasymchuk et al. (2010)), showing that a switching, driven by profits, creates interesting dynamics.

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<thead>
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<th>Table 2: The index of current guru (black solid line), the percentage of incoming link to current guru (red dotted line) and fitness of current guru (green dashed line) for $w = 0.1$ (top left side), $w = 0.5$ (top right side) and $w = 1$ (bottom left side). Average Guru’s live as a function of $w$ (bottom right side). Colors are available on the web side version.</th>
</tr>
</thead>
</table>

3.2 Wealth analysis

In table. (3) we compare the different performances, in terms of wealth, of the guru (black line), his direct followers (red line) and the rest of the traders (green line) for the same parameters as in table (2). Comparing table (3)
and table (2) we observe that the wealth of the guru increases with the imitation (i.e. decreasing $w$) and that the gap between the wealth of the guru and the wealth of the rest of the system (both followers and non followers) widens with the level of imitation. This result is better quantified by table (3) (bottom right side) that shows the average wealth, over all times and all simulations, of the gurus (black line), followers (red line) and rest of the system (green line) as a function of $w$.

Our findings are in line with theoretical models and laboratory experiments on noise traders (see Hommes (2006) for a survey of the relevant literature). All these works have proved that aggressive zero-intelligence agents gain higher profits thanks to herding behaviors.

In accordance with the empirical evidence (see, for example, Pareto (1897), Gabaix et al. (2006)), figure (3) shows how, raising imitation, the model generates heterogeneity, as indicated by the fat tail distribution of agents’ wealth and stock.
Figure 3: The decumulative distribution function (DDF) of the wealth (left side) and the decumulative distribution function (DDF) of the stocks (right side) for $w = 0.1$ (black line), $w = 0.5$ (red line) and $w = 1.0$ (green line). Colors are available on the web side version.

3.3 Price Analysis

Figure. (4) shows prices (black lines) and average expected prices (red lines) for $w = 0.1$, $w = 0.5$ and $w = 1.0$. The trading constrains of the model determine, in the absence of imitation, a sort of equilibrium level around which the price mean reverts. As imitation increases prices show wilder deviation from the equilibrium level, and large price jumps\textsuperscript{11}. This result is in line with other agent-based models (see, for example, Lux and Marchesi (2000), Iori (2002), Chiarella et al. (2002, 2009), Gaunersdorfer et al. (2008), and LeBaron et al. (2007, 2009)), all of which show that coordination of traders' expectations can generate large price fluctuations. For very low level of $w$ prices do not converge anymore to the equilibrium level (or at least do not do so on short time scales).

To assess the reciprocal influences among agents and the coordinations of their strategies, we compute the herding coefficient. Since herding is the consequence of mimetic responses by agents interacting on a communication network, for each time $t_k$, the number of agents $T_{r,s,t_k}$ taking the same decision to sell on market is measured. So the herding phenomena can be

\textsuperscript{11}Prices do not explode because as agents accumulate a larger fraction of their wealth in stocks they become more likely to sell and vice-versa, as can be easily seen from equations (13-14).
captured at each $t_k$ by a simple time varying herding function.

$$H_{t_k} = \frac{Tr_{s,t_k}}{Tr_{t_k}} \in [0, 1],$$

where $Tr_{s,t_k}$ is the number of sellers at time $t_k$ on the market and $Tr_{t_k}$ is the total number of active agents per time\(^{12}\). When this function is close to a half, the market show no herding since traders play without no coordination. When $H_{t_k}$ is close to zero, the market is herding in the direction of buying and vice versa when $H_{t_k}=1$. Figure (5) (left side) exhibits the average herding coefficient over time and the number of simulations as a function of $w$. Decreasing $w$, our model exhibits a significant coordination among agents, as reflected by the high standard deviation of the herding coefficient.

Further, we observe that prices (black line) and expected prices (red line) follow each other closely when $w$ is small\(^{13}\).

Figure 4: Prices (black line) and average expected prices (red line) with $w=0.1$ (left), $w=0.5$ (center) and $w=1.0$ (right). Colors are available on the web side version.

To better quantify this observation we calculate the mean price deviation between realized prices and expected prices defined as $|\frac{p_{t_k} - \hat{p}_{t_k}}{p_{t_k}}|$, and average it over time and the number of simulations. As figure (5) (right side) shows this deviation is smaller when imitation is high and price expectations become self-fulfilling. As shown in the same figure, in the case of no imitation

\(^{12}\)Note that $Tr_{t_k}=N_t=300$.

\(^{13}\)According to the literature (see De Long et al. (1990), Hommes et al. (2005), Heemeejer et al. (2009)), when in a stock market higher average price forecast produces higher realized market price we say that there is a positive expectations feedback.
expected prices become more volatile then realized prices. In this case we lose both the correlation between expected and realized prices and their large excursions.

Figure 5: Average herding coefficient over time and the number of simulations as a function of $w$ (left side). Deviation between realized prices and expected prices as a function of $w$ (right side).

3.4 Discussion

To explain the above results we first need to show that the imitation of expectations translates into imitation of trading actions. An expected price increase (decrease) in our model does not necessarily lead to a decision to buy and, even if so, buy order could be submitted as limit orders. Market orders are more likely to be submitted when agents are very optimistic or pessimistic. In fact in this case the interval $[p_m, p_M]$ over which orders can be placed is wider and it becomes more likely that a price level is chosen such that the order can be immediately executed. In our model it is the agents with many incoming links who forecast a high volatility $\sigma^2_i$ (via equation 4) and are more likely to submit market orders. In addition, if a popular agent has enough connections it can influence several others to overestimate price changes and submit market orders in turns.

In figure (6) we plot the average fraction of the volume of market orders to buy or sell, over the total volume of orders in the same direction. The average is taken over each trading period and plotted for different values of $w$. The result shows, as anticipated, that, when increasing imitation, a higher
fraction of market orders is submitted. Thus the coordination of expectation leads to a coordination of actions and the model generates a positive expectations feedback system.

In turn, a series of market orders in the same direction can generate considerable price changes, as shown by figure (7) where we observe an increase of the realized variance of returns for low values of $w$.

Thus, the forecasts from highly connected agents of an overall high volatility are self-fulfilling, providing an ex-post justification for equation (4).

Next we explain the distribution of agents’ wealth and stocks holdings. First of all, as long as the guru is not the last to trade (we assume a random entrance to the market for all agents including the guru) he will consistently gain on the trades that follow, in the same direction, his trade. Furthermore, while agents are risk averse, highly connected agents underestimate risk, according to equation (9). Consequently these traders, when $w$ is small and their percentage of incoming links, $l^F$, is high, invest more (on average) in the risky asset than others, as confirmed by figure (8). Followers in turn invest on average more than non followers because they, like the guru, overestimate returns. By investing more, gurus and followers earn, on average, higher profits than no followers (as was shown in table (3)). These results are in line with other studies on noise traders risk with positive feedback in financial markets. Particularly, De Long et al. (1990a) show that
noise traders can earn higher returns solely by bearing more of the risk that they themselves create. Our results are in line with previous studies (see Haltiwanger and Waldman (1985), Heemeijer et al. (2009)) and confirm that traders have an incentive to imitate and a desire to be imitated, since predicting a price close to the predictions of other players turns out to be most profitable.

4 Competitive strategies

In this section, our goal is to understand the impact of more ‘rational strategies’ on agents’ performances. In order to do so, we introduce in the baseline model two groups of agents able to counteract the guru. Specifically, we focus our attention on chartist and fundamentalist strategies. In financial literature, in fact, Chartists mirror myopic strategies, while Fundamentalists represent some sort of ‘full rationality’.

14 An example of this phenomenon, known under the name of market manipulation, is the ‘pool’ in RCA stock operated by Michael Meehan between March 7 and March 22, 1929. (see De Long et al. (1990b))
As a first exercise, we add to the model 10 chartists, $c$. These agents do not imitate and are not imitated by any one else. Chartists expected returns in the interval $(t_k, t_k + \tau)$ are given by

$$\hat{r}_{t_k, t_k + \tau} = \bar{r}_{t_k}$$

(15)

where $\bar{r}_{t_k}$ is the chartists’ expected trend based on the observations of the average spot returns over last $\tau$ time steps, as defined in Eq. (12). The following results reproduce the outcome of 10 simulation runs of 10000 steps for different levels of imitation $w$. In table (4) (first raw) we report the average time at which gurus wealth dominates the wealth of the chartists. For high levels of imitation, $w = 0.1$, the guru\textsuperscript{15} performs quickly better than chartists. In an intermediate scenario of imitation, $w = 0.5$, competition between guru and chartists becomes tighter and the two strategies initially appear to be equivalent. Also in this case, however, the guru eventually prevails over Chartists. Only in the absence of herding, $w = 1$, myopic strategies are the most successful over the ten full simulation runs. This is due to the fact that, with no imitation, prices have a mean-reverting behaviour and chartist strategies succeed at forecasting future price movements\textsuperscript{15}

\textsuperscript{15}While not reported in the table for this level of $w$ also the guru’ followers perform better than the chartists.
from past trends. Nonetheless, as imitation increases, with the guru randomly choosing the direction of his trades at each time step, price fluctuations become uncorrelated over time and past trends cannot predict anymore future price movements. Overall the average percentage of guru’s incoming links is lower than in the baseline model and the average life of the guru shorter, as shown in figure (9). A simple myopic strategy, therefore, is not sufficient to destroy the impact of herding, but reduces the guru influence on the market.

In a second exercise, we add to the model 10 fundamentalists, \( f \). Such as chartists, fundamentalists do not imitate and are not imitated by other agents. They form an expectation about the spot return, \( \hat{r}_{t_k,t_k+\tau}^f \), in the interval \((t_k, t_k + \tau)\), as follow

\[
\hat{r}_{t_k,t_k+\tau}^f = \frac{1}{\tau} \ln(p^f/p_t)
\]  

where \( p^f \) is the fundamental value and \( \tau \) is the time scale at which fundamentalists expect the price to mean revert to the fundamental level. The equilibrium level is calculated separately for each simulation run, by pre-running each simulation for 1000 steps in the absence of herding and of fundamentalist strategies. Once the equilibrium level has been determined, we assume that the fundamentalists know it and take it is the fundamental
price. In table (4) (second row) we report the average time at which gurus wealth dominates the wealth of the fundamentalist. When imitation is very high, the guru and the fundamentalists compete among themselves for long time periods, until the guru manages to attract many followers, and gains over the rest of the system. Increasing $w$, however, fundamentalists predominate over the system. Even more than in the case of chartists, the presence of fundamentalist reduces the influence of the guru on the system, the guru does not perform sufficiently well to attract a large number of followers and his life time becomes shorter than in the baseline model (as shown in figure (9)).

<table>
<thead>
<tr>
<th>Imitation level</th>
<th>$w = 0.1$</th>
<th>$w = 0.5$</th>
<th>$w = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^g_i &lt; W^g_t$</td>
<td>Ave $\bar{t}$: 400, st.dev: 230</td>
<td>Ave $\bar{t}$: 2507, st.dev: 1022</td>
<td>never</td>
</tr>
<tr>
<td>$W^f_i &lt; W^g_t$</td>
<td>Ave $\bar{t}$: 800, st.dev 122</td>
<td>never</td>
<td>never</td>
</tr>
</tbody>
</table>

Table 4: Average time ($\bar{t}$) at which the guru’s wealth ($W^g$) dominates the two competitive strategies (Chartists ($W^c$) & Fundamentalists ($W^f$)) across 10 Monte Carlo simulations of $T = 10000$ periods at different level of $w$.

To summarize, our results show that, when imitation is high, the guru overperforms both chartist and fundamentalist strategies. Chartistist strategies are dominated by herding in a shorter period than the fundamentalist ones. Nevertheless, decreasing the imitation level, the guru needs a longer time to undermine the chartists, and definitely loses his advantage over fundamentalists.

In line with other works (see Hommes (2006) for a survey of the relevant literature), we can conclude that, risk averse 'rational agents' are not able to counteract noise traders when herding occurs, contradicting the Friedman's hypothesis.

5 Conclusions

Our results allow us to conclude that profit is a good mechanism of links formation, capable of generating the famous Matthew effect. The endogenous attachment mechanism introduced in our model allows a guru to emerge spontaneously in the system, rise and fall in popularity over time, and possibly be replaced by a new guru. A few gurus could also co-exist and compete among themselves for popularity. Our endogenous attachment mechanism
succeed at creating, sustaining and destroying a guru, because agents benefit both from imitating and being imitated. In fact, if an agent profits from being imitated, he becomes richer, which induces an even larger fraction of agents to follow him. Nonetheless, if only the agents who are imitated benefit from imitation, once an agent becomes the guru he would remain the guru for ever. On the other hand, if followers also profit from imitating the guru, they could eventually outperform the guru and become guru in turn.

The fact that our unsophisticated investors, trivially driven by imitative behaviour, can earn very high profits implies that Friedman’s hypothesis is inadequate. The assumption that noise traders quickly go bankrupt and are eliminated from the market is unrealistic in presence of herding and positive feedback. In fact we have shown that more sophisticated strategies, i.e chartist and fundamentalist, underperform the guru and his followers when imitation is high. These results should not be underestimated, particularly in those situations when market prices exhibit large fluctuation. In these cases in fact is unlikely that prices incorporate true information and the idea of full rationality is implausible.

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