Debt sustainability, public investment, and natural resources in developing countries: The DIGNAR model

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Abstract

Policymakers in resource-rich developing countries often face complicated fiscal choices to manage natural resource revenues. While investing resource revenues in public capital may promote economic growth, spending without saving or borrowing against future revenues can expose the economy to debt sustainability risks. This paper presents the Debt, Investment, Growth, and Natural Resources (DIGNAR) model for analyzing macroeconomic and debt sustainability effects of scaling up public investment in resource-rich developing countries. It captures pervasive problems of these countries that may be aggravated during scaling-ups, including investment inefficiency and limited absorptive capacity. It also allows for flexible fiscal specifications: investment can be jointly financed by resource revenues and debt; a resource fund may be used as a buffer; and distorting fiscal adjustments are subject to feasibility constraints. The application to an average low-income country shows that, when fiscal adjustment is implementable, a delinked public investment approach combined with the resource fund – such that government spending is a-cyclical with respect to resource revenues – can reduce macroeconomic instability relative to a spend-as-you-go approach. However, even with the fund, ambitious frontloading public investment plans combined with more borrowing can induce debt sustainability risks, especially with declining investment efficiency or when future resource revenues turn out to be lower than expected.

1. Introduction

Public investment scaling-ups offer many opportunities as well as challenges to countries endowed with natural resources. They may raise important concerns, for instance, about their macroeconomic and fiscal implications for the economy, which may be compounded in resource-rich developing countries that also face the challenge of managing their natural resource wealth. To analyze these implications in a coherent framework is not an easy task. This paper constructs a small open economy model, in the tradition of the dynamic stochastic general equilibrium (DSGE) literature, to assess the macroeconomic and fiscal effects of public investment surges in resource-rich developing countries, including the effects on growth and debt sustainability.

In theory, by financing public investments in infrastructure and human capital, natural resource revenues may help foster development and growth in many developing countries. Increases in public capital may raise the productivity of labor and private capital, inducing more accumulation of these productive factors and, therefore, growth – the positive productivity and cost-saving effects described by Agénor (2012). In addition, resource revenues can serve as collateral for borrowing from international markets, making it possible to build up public capital even before these revenues actually arrive. And by providing this external financing, resource revenues may help smooth away the crowding out effects on private consumption and investment that are claimed to be part of public investment increases, particularly when these increases depend somewhat on domestic financing. Through smoothing these crowding out effects, resource revenues then also support the positive public investment growth nexus.

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In practice, natural resource revenues have brought many challenges to developing countries. One of them is the natural resource curse: resource-rich countries often face lower growth rates than those of non-resource-rich counterparts (see, e.g., Sachs and Warner, 1995, 1999; van der Ploeg and Poelhekke, 2009; van der Ploeg, 2011; Satti et al., 2014). As history reveals, several reasons may explain this curse, including the volatility of commodity prices combined with mismanagement of debt and public investment. Manzano and Rigobon (2007) point out that excessive borrowing in the 1970s predicated on the belief of a continuous rising path of oil prices led to inevitable debt crises and lackluster growth in the 1980s, when oil prices plummeted. This was evident in Latin America, where the region witnessed a “lost decade” despite the undertaking of ambitious investment projects (Gelb, 1988; Carrasco, 1999). These effects were probably aggravated by problems of declining public investment efficiency and limited absorptive capacity (van der Ploeg, 2012; Berg et al., 2013). In fact, as suggested by Warner (2014), these problems may be behind the weak empirical link between public investment surges and growth in developing countries.

As more developing countries continue to discover and exploit natural resources and remain committed to scale up public investment to achieve the sustainable development goals, the need to assess the potential macroeconomic effects of public investment surges in resource-rich developing countries has become more prominent for policymakers. Country teams at the International Monetary Fund (IMF), for instance, are frequently asked to provide such macroeconomic assessments, including on debt sustainability. In some cases, resource revenues are expected to come in the future and therefore the ambitious public investment plans involve substantial borrowing in the present. To do these assessments, teams can rely on model-based frameworks such as the Debt, Investment and Growth (DIG) model, developed in Buffie et al. (2012), and the Natural Resource (NR) model, described in Berg et al. (2013). The DIG model makes explicit the public investment-growth nexus and allows for different debt financing schemes. However, it does not have a natural resource sector and models resource revenues as a foreign transfer, such as aid. Meanwhile, the NR model contains a resource sector and features different resource management policies, but it does not allow for borrowing to finance investment spending. For countries that intend to finance investment projects with both resource revenue and debt, neither model seems adequate.

In this paper, we fill the modeling gap by combining the models developed in Buffie et al. (2012) and Berg et al. (2013) into a suitable framework for assessing debt sustainability and growth benefits of public investment surges in resource-rich developing countries. We name it the Debt, Investment, Growth, and Natural Resources (DIGNAR) model. It differs from the DIG model by adding a natural resource sector so it can account for resource GDP and distinguish between the resource sector and the non-resource traded good sector. Also, it differs from the NR model by including a variety of debt instruments — concessional debt, external commercial debt, and domestic debt. Moreover, DIGNAR includes several important economic features of developing countries, namely learning-by-doing externalities in the traded good sector to capture potential Dutch disease effects from spending resource revenues.

Despite the concern of the resource curse, resource revenues have played an important role in supporting public investment spending and economic growth in many developing countries, as documented in Hamdi and Sbia (2013) for Bahrain and Dizaji (2014) for Iran.

The DIG model was developed to address criticisms on the IMF-World Bank debt sustainability framework (DFI, International Monetary Fund and World Bank, 2005). This framework does not make explicit the link between public investment and growth and has often been criticized for its incoherence in making debt projections, which leads to potential biases toward conservative borrowing limits (Eaton, 2002; Hjertholm, 2003).

In addition to micro-founded models, there also exist macroeconomic models without optimizing behaviors for assessing the growth effects of investing resource revenues. See Ali and Harvie (2013) for studying oil revenues and economic development in Libya as an example.

Since the natural resource literature highlights the importance of savings in managing volatile resource revenues (e.g., Collier et al., 2010; van der Ploeg, 2010a; Van den Bremer and van der Ploeg, 2013) and many developing and more developed countries, such as Kazakhstan (Minasyan and Yang, 2013) and Kuwait (Mehrara and Oskou, 2007), have benefited from setting up a stabilization or saving fund, DIGNAR includes a resource fund that serves as a fiscal buffer. For given paths of public investment, aid, resource prices and quantities, a resource fund is drawn down to cover a revenue shortfall or accumulates savings from excessive revenues. In practice, some countries may borrow externally while saving resource revenues at the same time, despite that the interest earned from a resource fund is often lower than the interest cost of borrowing. To accommodate this phenomenon in DIGNAR, the government can borrow before exhausting the resource fund by imposing a minimal saving level in the fund. Public debt accumulation then triggers distorting fiscal adjustments via changes in taxes or in government transfers to households.

After presenting the model, the paper illustrates how DIGNAR can be used to derive policy lessons by relying on some stylized experiments. We calibrate the model to an average low-income developing country and analyze various investment scaling-up paths. Two hypothetical scenarios of resource revenue paths are constructed: one that represents the baseline scenario resembling the qualitative patterns of a country that anticipates a future resource windfall; and the other one, referred to as the adverse scenario, where the baseline is affected by large negative revenue shocks. The investment approaches simulated are (i) the spend-as-you-go (SAYG) approach, which invests all resource windfall each period without saving; and (ii) the delinked approach, which combines investment and saving such that government spending is cyclical with respect to resource revenues.

Several policy lessons are obtained from the simulation results. First, when fiscal adjustment is implementable, the delinked investment approach combined with the resource fund can reduce macroeconomic instability, while the SAYG approach may aggravate it. This holds for both scenarios of resource revenues, including the adverse one. A delinked approach delivers a more resilient and stable growth in non-resource GDP and a less volatile real exchange rate. The novelty of this result lies on showing the key role that pervasive features of developing countries — such as limited absorptive capacity, declining public investment efficiency, and learning-by-doing externalities — play in amplifying the macroeconomic instability effects of the SAYG approach. Under this approach, sudden accelerations in public investment expenditures make the economy more prone to bumping into absorptive capacity constraints, translating into a declining efficiency for public investment. Moreover, the substantial appreciation of the real exchange rate induced by the SAYG approach leads to greater negative learning-by-doing externalities and thus a larger decline in traded output — i.e., more severe Dutch disease effects. With the delinked investment approach combined with the resource fund, these negative effects, however, are contained.

The second lesson is that, when fiscal adjustment is constrained (or cannot be implemented beyond certain magnitudes in tax increases or spending cuts) and borrowing is necessary to fill financing gaps, a front-loaded public investment surge, even if coupled with a resource fund, can induce debt sustainability risks. Simulations for different degrees of investment front-loading, declines in investment efficiency, paths of resource revenues and returns to capital are conducted to show their importance for debt sustainability problems.

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5 This differs from Berg et al. (2013), where the saving rate of a resource windfall into a resource fund is constant, an unrealistic feature compared to the operation of a resource fund in reality.

6 For country applications of DIGNAR, see, for example, Minasyan and Yang (2013), Melina and Xiong (2014), and Deléchat et al. (2015).
traditional concerns about Dutch disease effects from spending resource revenues, we also investigate how various assumptions on the persistence of learning-by-doing externalities affect these effects. Moreover, we explore whether the Marshall–Lerner condition is satisfied in our model by studying how the real appreciation from investing resource revenues affects traded good output and the trade balance.

Overall, this paper contributes to the literature on managing resource revenues for developing countries. This literature has evolved from advising to save most of a resource windfall in a sovereign wealth fund, as suggested by the permanent income hypothesis (e.g., Davis et al., 2001; Barnett and Ossowski, 2003; Bems and de Carvalho Filho, 2011), to recommending to invest the windfall to build productive capital (e.g., van der Ploeg, 2010b; Venables, 2010; van der Ploeg and Venables, 2011; Araujo et al., 2013). To complement this literature, our paper offers a policy tool for assessing the macroeconomic and debt sustainability effects associated with different revenue scenarios and investment trajectories, without necessarily looking at optimal policies. Like other DSGE models developed for policy analysis, DIGNAR is an internally consistent framework that can be used to systematically produce alternative macroeconomic and policy scenarios, making explicit its different assumptions as well and their implications for macroeconomic outcomes (Berg et al., 2015c). In this regard, DIGNAR offers a framework for organizing thinking and informing policy decisions.

2. The DIGNAR model

We first give a non-technical overview of the model and then proceed to provide the full model specification.

2.1. An Overview

DIGNAR is a real model of a small open economy with two types of households and three production sectors. The intertemporal optimizing households have access to capital and financial markets, and the rule-of-thumb households are poor and financially constrained, consuming all the disposable income each period. The three production sectors include a nontraded good sector, a (non-resource) traded good sector, and a natural resource sector. Since resource-rich developing countries tend to export most resource output, we assume that the whole resource output is exported. Also, as most natural resource production is capital intensive, and much of the investment in the resource sector in developing countries is financed by foreign direct investment, natural resource production in the model, both resource quantities and prices are assumed to follow exogenous processes to match the projected resource output and prices.

Each period the government’s total receipts consist of i) taxes, including consumption taxes, labor income taxes, and resource revenues, ii) foreign aid, iii) bond sales, iv) the principal and interest earnings from the resource fund, and v) user fees on infrastructure services. The government’s total expenditures consist of i) government consumption, ii) public investment, iii) transfers to households, iv) debt service payments, and v) savings in the resource fund.

The highlight of the model is that it can further reduce the efficiency of public investment and have negative effects on growth, as suggested by Esfahani and Ramirez (2003).

2.2. Model specification

We denote variables associated with intertemporal optimizing households by the superscript OPT and the rule-of-thumb households by the superscript ROT. Also, we denote variables associated with the traded, nontraded goods, and resource sector by $T$, $N$, and $O$, respectively.

2.2.1. Households

A fraction $\omega$ of the households are intertemporal optimizing and the remaining fraction $1 - \omega$ are rule-of-thumb. Both types of households consume a constant-elasticity-of-substitution (CES) basket ($c^i_t$) of traded goods ($c^i_{T,t}$) and nontraded goods ($c^i_{N,t}$). Thus,

$$c^i_t = \left[\phi^i \left(\frac{c^i_{N,t}}{c^i_{T,t}}\right)^{\frac{1}{\chi}} + (1-\phi)^i \left(\frac{c^i_{N,t}}{c^i_{T,t}}\right)^{\frac{1}{\chi}}\right]^{\frac{1}{1-\phi^i}}$$

for $i = OPT, ROT$. (1)

where $\phi$ indicates the nontraded good bias and $\chi > 0$ is the intratemporal elasticity of substitution. The consumption basket is the numeraire of the economy, with the unit price of this basket corresponding to

$$1 = \left[\phi p^i_{N,t} + (1-\phi) p^i_{T,t}\right]^{\frac{1}{1-\phi^i}}$$

(2)

where $p_{N,t}$ and $s_t$ represent the relative prices of nontraded and traded goods, respectively. Assuming that the law of one price holds for traded goods implies that $s_t$ also corresponds to the real exchange rate, defined as the price of one unit of foreign consumption basket in units of domestic basket.

7 Regarding optimal public investment policies, see Levine et al. (2015).
Minimizing total consumption expenditures subject to the consumption basket Eq. (1) yields the following demand functions for each good:

\[ c_{it} = \psi(p_{it}^t - \bar{X}_{i}^t) \text{ and } \tilde{c}_{it} = (1 - \psi)(s_{i})^{-1} \tilde{X}_{i} \] \quad \forall i = \text{OPT, ROT}. \tag{3}

Both types of households provide labor service (\( L_{it}^t \) and \( L_{i}^{N,t} \), \( i = \text{OPT, ROT} \)) to the traded and nontraded good sectors, denoted by subscript \( T \) and \( N \), respectively. Total labor \( L_{it}^t \) has the following CES specification to capture imperfect substitutability between the two types of labor:

\[ L_{it}^t = \left[ \delta \tilde{L}_{it}^t + (1 - \delta) \tilde{L}_{i}^{N,t} \right]^{\frac{\rho}{\rho - 1}}, \quad \text{for } i = \text{OPT, ROT}. \tag{4} \]

where \( \delta \) is the steady-state share of labor in the nontraded good sector, and \( \rho > 0 \) is the intra-temporal elasticity of substitution. Let \( w_{iT}^t \) and \( w_{i}^{N,t} \) be the real wage rates paid in each sector. The real wage index is

\[ w_{it} = \left[ \delta w_{iT}^{1,ip} + (1 - \delta) w_{i}^{1,ip} \right]^{\frac{1}{\rho}}. \tag{5} \]

A representative optimizing household maximizes the expected discounted value of its utility flows from consuming and working

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_{t}^{\text{OPT}, t}^{\text{OPT}} = E_0 \left[ \sum_{t=0}^{\infty} \left[ \frac{1}{1 - \alpha} \left( c_{it}^{\text{OPT}} \right)^{1 - \alpha} - \frac{\kappa_{i}^{\text{OPT}}}{1 - \psi} \left( l_{it}^{\text{OPT}} \right)^{1 - \psi} \right] \right]^{\frac{1}{1 - \alpha}} \tag{6} \]

subject to the budget constraint:

\[ (1 + \tau_{c}^{t}) c_{it}^{OPT} + b_{it}^{OPT} - \frac{s_{i}^{OPT}}{g} = (1 - \tau_{c}^{t}) w_{iT}^{OPT} + R_{c,t} - \frac{1}{1 - \gamma} k_{it}^{\text{OPT}} - \frac{1}{1 - \gamma} s_{it}^{OPT} + \Omega_{r,t} + \Omega_{N,t} + \delta \mathbb{E}^t (R_{f,t}^{OPT} + k_{N,t}^{K} - 1) + \gamma_{t} r_{m}^{t}; \]

\[ \tau_{c}^{t} - \mu_{c,t}^{OPT} = - \delta \mathbb{E}^t. \tag{7} \]

\( E_0 \) is the expectation operator at time 0; \( \beta \equiv [1 + \rho]^{-1} \) is the subjective discount factor; and \( \rho \) is the pure rate of time preference. \( \alpha \) is the inverse of the inter-temporal elasticity of substitution of consumption, and \( \psi \) is the inverse of the inter-temporal elasticity of substitution of the labor supply. \( \kappa_{i}^{\text{OPT}} \) is the disutility weight of labor, and \( \tau_{c}^{t} \) and \( \tau_{f}^{t} \) are the tax rates on consumption and labor income. The intertemporal optimizing households have access to government bonds \( b_{i}^{OPT} \) that pay a gross real interest rate \( R_{t} \). They can also borrow from abroad \( b_{i}^{OPT} \) at the interest rate \( R_{f,t}^{OPT} \) that pays a constant premium \( u \) over the interest rate that the government pays on external commercial debt \( R_{de,t} \), such that

\[ R_{f,t}^{OPT} = R_{de,t} + u. \tag{8} \]

These households also receive profits, \( \Omega_{r,t} \) and \( \Omega_{N,t} \), from firms in the traded and nontraded good sectors. The term \( \delta \mathbb{E}^t (R_{f,t}^{OPT} + k_{N,t}^{K} - 1) \) is a tax rebate that optimizing households receive on the tax levied on the firms’ capital return.\(^{10} \) \( r_{m}^{t} \)

denotes remittances from abroad, and \( z_{t} \) is government transfers. \( \mu_{c,t}^{OPT} \) is the user fees charged for public capital services, and \( \zeta_{i}^{OPT} \equiv \left( \gamma_{i}^{OPT} - b_{i}^{OPT} \right)^{2} \) is portfolio adjustment costs associated with foreign liabilities, where \( \eta \) controls the degree of capital account openness, and \( b_{i}^{OPT} \) is the initial steady-state value of private foreign debt.\(^{11} \)

Rule-of-thumb households have the same utility function as that of intertemporal optimizing households, so

\[ U_{t}^{\text{ROT}, t}^{\text{ROT}} = \frac{1}{1 - \alpha} \left( c_{it}^{\text{ROT}} \right)^{1 - \alpha} - \frac{\kappa_{i}^{\text{ROT}}}{1 - \psi} \left( l_{it}^{\text{ROT}} \right)^{1 - \psi}. \tag{9} \]

Their consumption is determined by the budget constraint

\[ (1 + \tau_{c}^{t}) c_{it}^{\text{ROT}} + b_{it}^{\text{ROT}} - \frac{s_{i}^{\text{ROT}}}{g} = (1 - \tau_{c}^{t}) w_{iT}^{\text{ROT}} + s_{i} r_{m}^{t} + z_{t} - \mu_{c,t}^{\text{ROT} - 1}. \tag{10} \]

while static maximization of the utility function gives the following labor supply function:

\[ l_{it}^{\text{ROT}} = \left[ \frac{1}{1 - \gamma} \frac{1 - \tau_{c}^{t}}{1 + \tau_{c}^{t}} \left( \frac{c_{it}^{\text{ROT}}}{\eta_{i}^{t} + \tau_{c}^{t}} \right) - \frac{\kappa_{i}^{\text{ROT}}}{1 - \psi} \right]. \tag{11} \]

22.2. Firms

Nontraded good firms produce output \( y_{N,t} \) with the following Cobb–Douglas technology:

\[ y_{N,t} = z_{N} \left( k_{N,t-1} \right)^{1 - \alpha_{N}} \left( k_{G,t-1} \right)^{\alpha_{N}} \left( k_{G,t-1} \right)^{\alpha_{N}}, \tag{12} \]

where \( z_{N} \) is total factor productivity, \( k_{N,t-1} \) and \( k_{G,t-1} \) are private and public capital used at \( t \), \( \alpha_{N} \) is the labor share of sectoral income, and \( \alpha_{N} \) is the output elasticity with respect to public capital.

Private capital installed in the nontraded good sector evolves according to

\[ k_{N,t} = (1 - \delta_{k}) k_{N,t-1} + \frac{1 - \nu_{N}}{2} \left( \frac{b_{N,t}}{b_{N,t-1}} - 1 \right)^{2} k_{N,t}. \tag{13} \]

where \( b_{N,t} \) represents investment expenditure, \( \delta_{k} \) is the capital depreciation rate, \( \nu_{N} \) is the investment adjustment cost parameter, and \( k_{G,t} \) is infrastructure. The investment adjustment costs follow the representation suggested by Christiano et al. (2005).

The representative nontraded good firm maximizes its discounted lifetime profits weighted by the marginal utility of consumption of the intertemporal optimizing households \( \lambda_{r} \). These profits are given by

\[ \Omega_{N,t} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{t} \left( p_{N,t} y_{N,t} - w_{N,t} k_{N,t} - b_{N,t} - \gamma_{t} r_{N}^{K} k_{N,t-1} \right), \tag{14} \]

where \( r_{N}^{K} = (1 - \alpha_{N}) p_{N,t} \frac{y_{N,t}}{k_{N,t}} \) is the (gross) return to capital.

\( ^{10} \) For the sake of simplicity, our model specification assumes that government consumption does not enter the household’s utility function. Instead, it can affect household’s utility indirectly through responses in private consumption and labor due to changes in public investment or government consumption. An alternative specification that allows government spending to generate utility directly is to replace \( c_{it}^{\text{OPT}} \) with \( \bar{X}_{i} \) \( (1 - \alpha) \bar{X}_{i}^{-\alpha} \), where \( \bar{X}_{i} \) is government consumption, \( \alpha \) is the weight of private consumption in utility, and \( \gamma_{t} \) controls the substitutability or complementarity between the two.

\( ^{11} \) According to Schindler (2009), measures of de jure restrictions on cross-border financial transactions suggest that the private capital account for the median sub-Saharan African country – a typical low-income country – is relatively closed. Therefore, to capture this, we assume that intertemporally optimizing households face portfolio adjustment costs associated with foreign assets/liabilities. These adjustment costs also ensure stationarity in this small open economy model, as discussed in Schmitt-Grohe and Uribe (2003).

Note that a variable without a time subscript refers to the steady-state value of such variable.
Analogously to the nontraded good sector, firms in the traded good sector produce traded output with the following technology

\[ y^T_t = z^T_t \left( k^T_{t-1} \right)^{1-\alpha_L} \left( l^T_t \right)^{\alpha_L} \left( c^T_{t-1} \right)^{\theta}. \]  

(15)

To capture the common Dutch disease effects associated with spending resource revenues, we assume that the total factor productivity in this sector, \( z^T_t \), is subject to learning-by-doing externalities:

\[ \frac{z^T_t}{z^T_1} = \left( \frac{z^T_t}{z^T_1} \right)^{\rho^T} \left( \frac{y^T_t}{y^T_1} \right)^{\rho^T}. \]  

(16)

where \( \rho^T, \rho^T \in [0, 1] \) control the severity of Dutch disease. This specification is a variation of the one in Krugman (1987), Matsuyama (1992), Torvik (2001), and Adam and Bevan (2006).\(^{12}\) When \( \rho^T \) or \( \rho^T \approx 1 \), there is no permanent effect of learning by doing on productivity or output, but deviations of traded sector output from the trend can have some persistent productivity effects.

Private capital in the traded sectors is accumulated according to

\[ k^T_t = (1 - \delta) k^T_{t-1} + \left[ 1 - \frac{\nu^T}{2} \left( \frac{y^T_t}{k^T_{t-1}} - 1 \right) \right]^2 l^T_t. \]  

(17)

Like nontraded good firms, a representative traded good firm maximizes the following discounted lifetime profits:

\[ \Omega_{2} = E_{0} \sum_{t=0}^{\infty} \beta^{t} \lambda \left( s_{t} y_{t} - w_{t} L_{t} - i_{t} - \tau_{t}^{R} \theta^{L} k_{t-1} \right). \]  

(18)

Resource production and prices follow exogenous processes.\(^{13}\) The model can incorporate any exogenous path for these production and prices, but also allows for a simple parametric representation of the following types. For resource production, we assume

\[ \hat{Y}^{p}_{t} = \frac{Y^{p}_{t-1}}{Y^{p}_{0}} \exp(\varepsilon^{p}_{p}). \]  

(19)

where \( \varepsilon^{p}_{p} \sim \text{iid } N(0, \sigma^{p}_{p}) \) is a production shock. For the international commodity price (relative to the foreign consumption basket), we assume

\[ \frac{P^{O}_{t}}{P^{O}_{0}} = \left( \frac{P^{O}_{t}}{P^{O}_{0}} \right)^{\theta} \exp(\varepsilon^{p}_{p}). \]  

(20)

where \( \varepsilon^{p}_{p} \sim \text{iid } N(0, \sigma^{p}_{p}) \) is a price shock. We assume that resource production is small relative to world production; hence, the country cannot control \( P^{O}_{t} \).

\(^{12}\) Aside from assuming learning-by-doing with respect to total factor productivity, the literature has alternative approaches to modeling learning-by-doing. Chang et al. (2002) assume that labor skill depends on the hours worked last period. Cooper and Johri (2002) and Johri and Lahiri (2008) assume that organization capital (proxied by experience) depends on production levels of the same and similar goods. Also, Stokke (2008) models learning-by-doing through sectoral labor shares in both traded and nontraded goods sectors.

\(^{13}\) Resource production in reality is not exogenous to countries' authorities' decisions; we abstract from modeling them. This is not very restrictive in the case of LDCs. In fact, in these countries, these decisions typically happen via negotiations between governments and foreign multinational corporations. As such, one could think of foreign direct investment (FDI) as the outcome of these negotiations. Then FDI is accumulated to create capital \( K^T \), which in turn is used for resource production \( y^T = f(K^T) \). From this perspective, there is no substantial value added from explicitly modeling this mechanism — one could assume that either FDI or \( Y^T \) is exogenous with no repercussions. For introducing endogenous resource production, which seems more relevant for advanced economies, see, e.g., Ferrero and Seneca (2015) where the representative resource producer uses a domestic intermediate good.

Resource GDP in units of the domestic consumption basket corresponds to

\[ Y^{O}_{t} = s_{t} P^{O}_{t} Y^{O}_{t}. \]  

(21)

The total real GDP \( y_t \) in this economy is defined as

\[ y_t = p_{t} \nu_{t} + s_{t} y^{T}_{t} + Y^{O}_{t}. \]  

(22)

22.2. The government

The government flow budget constraint is given by

\[ \tau^{G}_{t} C_t + \tau^{R}_{t} W_t L_t + (1 - \theta^{L})^{\theta^{K}} \left( e^{K}_{t} k^{K}_{t-1} + r^{K} k^{K}_{t-1} \right) + s_{t} \alpha^{G}_{t} \]  

\[ + \mu_{t} \delta_{t-1} + e^{C}_{t} + s_{t} d_{t} + s_{t} d_{t+1} + \theta^{R} \delta^{F}_{t} f_{t-1}^{C} = \tau^{G}_{t} \left( g^{T}_{t} + g^{T}_{t} \right) + z_{t} + R_{t-1} b_{t-1} + s_{t} d_{t-1} + s_{t} R^{d}_{t-1} d_{t-1} + s_{t} f_{t}. \]  

(23)

Besides the tax revenues from consumption, labor income and capital income — \( \tau^{G}_{t} C_t, \tau^{R}_{t} W_t L_t, and \sum_{t-1}^{t+1} (1 - \theta^{L})^{\theta^{K}} \tau^{R} \delta^{F}_{t} f_{t} \) — the government also receives international grants, \( g^{G}_{t} \), user fees, \( \mu_{t} \delta_{t-1} \), and resource-related royalties, \( g^{T}_{t} \). As in Buffie et al. (2012), the user fee charged on public capital is computed as a fraction \( f \) of recurrent costs, \( \mu_{p} g^{T}_{t} \). The resource revenues collected each period are computed as

\[ t^{G}_{t} = t^{G}_{t} s_{t} P^{O}_{t} Y^{O}_{t}. \]  

(24)

where \( t^{G}_{t} \) is a constant royalty rate that can be made time-varying, if necessary. The government has three debt instruments: external concessional debt, \( d_{t} \), external commercial debt, \( d_{t} \), and domestic debt, \( b_{t} \). Concessional loans extended by official creditors are taken as exogenous in the model and charge a constant (gross) real interest rate \( R_{t} \). The gross real interest rates paid on external commercial debt, on the other hand, incorporates a risk premium depending on the deviations of total external public debt to GDP ratio from its initial steady state. That is

\[ R^{d}_{t} = R^{d} + \nu_{d} \exp \left[ \eta_{d} \frac{s_{t} (d_{t} + d_{t+1})}{y_{t}} - s (d + d_{t+1}) \right]. \]  

(25)

where \( R^{d} \) is a (constant) risk-free world interest rate, and \( \nu_{d} \) and \( \eta_{d} \) are structural parameters. We now proceed to describe the government spending variables and the resource fund in (23).

Government purchases comprise government consumption \( (g^{G}_{t}) \) and public investment \( (g^{P}_{t}) \). Like private consumption, government expenditure, \( g^{G}_{t} = g^{T}_{t} + g^{C}_{t} \), is also a CES aggregate of domestic traded goods, \( g^{T}_{t} \), and domestic nontraded goods, \( g^{C}_{t} \). Thus, \( g^{T}_{t} \) is the weight given to nontraded goods in government purchases. We assume that government purchases have the same intra-temporal elasticity of substitution \( \chi > 0 \) as that of private consumption, but different degrees of home bias (\( \nu_{t} \neq \varphi \) in Eq. (1)).

\(^{14}\) Expenditures on government consumption implicitly include wage spending on public employment, which generally accounts for half of government consumption expenditures. To formally model the effects of government expenditures on public employment, one should introduce public-sector wages and labor, and their interaction with private-sector wages and labor. Since our focus is on the use of a resource windfall for scaling up public investment, we abstract from the details of modeling public employment and its effects.
Minimizing total government expenditures $p_f^e G_t = p_n^e G_{nt} + s_t G_{ct}$, subject to the government consumption basket (26), yields the following public demand functions for each good:

$$g_{NA} = v_1 \left( \frac{p_{NA}}{p_f^e} \right)^{-x} g_t$$ and $$g_{NA} = (1 - v_1) \left( \frac{s_1}{p_f^e} \right)^{-x} g_t,$$

where $p_f^e$ is the government consumption price index in terms of units of the consumption basket:

$$p_f^e = \left[ v_1 p_n^e + (1 - v_1) s_1 \right]^{\frac{1}{x}}.$$

Note that $v_1$ is time-varying. As we focus on the effects of additional government spending in the form of public investment, the weight given to nontraded goods for the additional government spending, $v_1 e$, can differ from its steady-state value, $v$. Thus,

$$v_1 = \left( \frac{p_f^e g_t}{p_f^e g_t} \right) v + (p_f^e g_t - p_f^e g_t) v_1 e.$$

To reflect public inefficiencies and absorptive capacity constraints, we assume that effective investment $g_d^i$ is a function of the proportional deviation of public investment from its steady-state value, $\gamma_t^d = \frac{g_t^d - 1}{g_t^d}$, and a threshold $\gamma_t^d$. Specifically,

$$g_d^i = \begin{cases} \gamma_t^d, & \text{if } \gamma_t^d \leq \gamma_t^d, \\ (1 + \gamma_t^d) g_t^i + \epsilon (\gamma_t^d - \gamma_t^d) g_t^i, & \text{if } \gamma_t^d > \gamma_t^d, \end{cases}$$

where $\epsilon \in [0, 1]$ represents steady-state efficiency and $\epsilon (\gamma_t^d - \gamma_t^d)$ governs the efficiency of the portion of public investment exceeding the threshold $\gamma_t^d$, following:

$$\epsilon (\gamma_t^d) = \exp \left[ -\frac{\gamma_t^d}{\gamma_t^d - \gamma_t^d} \right].$$

This captures the fact that, because of absorptive capacity constraints, the efficiency of that part of public investment exceeding the threshold drops proportionally to the magnitude of the scaling-up. In simple words, if the government invests too fast, it may face substantial declines in efficiency because of the limited absorptive capacity as discussed in policy circles. The severity of these constraints, and thus the extent of the drop, is governed by parameter $\gamma_t^d \in [0, \infty)$.

The law of motion of public capital is described as $k_{ct} = (1 - \delta_{ct}) k_{ct-1} + \xi_{ct}$,

where $\delta_{ct}$ is a time-varying depreciation rate of public capital in the spirit of Rioja (2003). Since insufficient maintenance can shorten the life of existing capital, we assume that the depreciation rate increases proportionally to the extent to which effective investment fails to maintain existing capital. Specifically,

$$\delta_{ct} = \begin{cases} \phi \delta \delta_{ct-1}, & \text{if } \xi_{ct} < \delta_{ct-1} k_{ct-1} \\ \rho \delta \delta_{ct-1} + (1 - \rho) \delta_{ct}, & \text{if } \xi_{ct} > \delta_{ct-1} k_{ct-1} \\ \end{cases},$$

where $\delta_{ct}$ is the steady-state depreciation rate, $\phi > 0$ determines the extent to which poor maintenance produces additional depreciation, and $\rho \in [0, 1]$ controls the persistence.

We introduce a resource fund in the model along the lines of Berg et al. (2013). A resource windfall is defined as resource revenues that are above their initial steady-state level, i.e., $f^r_t = f^r_t - f^r_t$. Let $f^r_t$ be the foreign financial asset in a resource fund. Each period, the resource fund earns interest income $s_t(R_t^f - 1)f^r_t$, with a constant gross real interest rate $R_t^f$. The resource fund evolves by the process

$$f_t^r - f_t^r = \max \left\{ -f_{out}^r, (f_t^r - f_t^r) + \frac{f_{in}^r - f_{out}^r}{s_t R_t^f} \right\},$$

where $f_{in}$ and $f_{out}$ represent the total fiscal inflow and outflow that we define below. $f_{out} < 0$ is a lower bound for the fund that the government chooses to maintain. If no minimum savings are required in a resource fund, the lower bound can be set at zero. At each period, if the fiscal inflow exceeds the fiscal outflow, the value of the resource fund increases. Instead, if the resource fund is above $f_{out}$, any fiscal outflow that exceeds the fiscal inflow is absorbed by a withdrawal from the fund. As we discuss below, whenever the floor of a resource fund binds, potential fiscal gaps can be covered via borrowing and/or fiscal adjustment. This adjustment, in turn, is achieved by increasing taxes (on consumption and factor incomes) or by cutting government non-capital expenditures (government consumption and transfers).

One of the purposes of the model is to analyze the effects of investing a resource windfall. The simulations presented in this paper focus on two investing approaches: the spend-as-you-go approach and the delinked investing approach. These approaches are formulated as follows.

- **Spend-as-you-go approach (SAYG).** With spend-as-you-go, the resource fund stays at its initial steady-state level ($f^r_t = f^r_t$, $\forall t$), and the entire windfall is spent on public investment projects:

$$p_f^e g_t^i - p_f^e g_t^i = \frac{f_{out}^r - f_{out}^r}{f_{out}^r - f_{out}^r}.$$

- **A delinked investment approach.** With delinked investing, a scaling-up path of public investment is specified as a second-order delay function,

$$g_t^i = 1 + \exp(-k_{t-1} t) - 2 \exp(-k_t t) g_{inss},$$

where $g_{inss}$ is the scaling-up investment target expressed as a percentage deviation from the initial steady state, $k > 0$ represents the speed of adjustment of public investment to the new level, and $k_{t-1}$ represents the degree of investment frontloading. In particular, if $k_{t-1} = k_t = 0$, public investment stays at its original steady-state level, i.e., $g_t^i = g_t^i, \forall t$. If instead $k_{t-1} = \infty$, public investment jumps to the new steady-state level immediately. Lastly, if $k_{t-1} = k_t$, public investment increases gradually and is not frontloaded. The mechanics of this functional form on public investment trajectories are illustrated in Figs. 1 and 2.

We borrow the structure of the fiscal gap and the mechanisms to cover it, from Buffie et al. (2012), but expand the number of fiscal instruments and take into account the dynamics of the fund. Given the paths of public investment, concessional borrowing, and foreign grants, algebraic manipulation of the budget constraint of the government Eq. (23) allows us to rewrite it as follows:

$$gap = f_{out} - f_{in} + s_t (f_t^r - f_{t-1}^r).$$

---

16. To guarantee that the resource fund does not follow an explosive process, we assume that in the very long run, a small autoregressive coefficient $p_t \in (0, 1)$ is attached to $(f^r_t - f^r_t)$. The model is typically solved at a yearly frequency for a 100-period horizon. The coefficient $p_t$ is activated after the first 100 years of simulations.

17. In addition to the two approaches simulated here, the model allows for analyzing an exogenously-specified public investment path proposed by the user of the model.
and

gap_i = \Delta b_i + s_i \Delta d_{i-1} + \left( t_{i}^c - \tau_i \right) c_{i-1} + \left( t_{i}^c - \tau_i \right) \omega_i L_{i-1} - p_i^G (g^c - g^c) - (z_i - 1),

\tag{38}

\begin{align*}
\frac{f_{i,N}}{t_i} & = t_i^c c_i + t_i^c W_i^c L_i + \left( 1 - \phi \right) i \left( \frac{\bar{k}_i}{t_i^c} k_{i,i-1} + (\frac{\bar{k}_i}{t_i^c} k_{i,i-1} - 1) + t_i^c + t_i^c + \mu k_{i,i-1} + s_i g_{i}^c + s_i \left( \frac{\bar{k}_i}{t_i^c} - 1 \right) f_{i-1} - s_i \Delta d_i, \\
\tag{39}
\end{align*}

and

\begin{align*}
\frac{f_{i,N}}{t_i} & = p_i^G g_i^c + p_i^G g_i^c + (1 - \phi) i \left( \frac{\bar{k}_i}{t_i^c} k_{i,i-1} + (\frac{\bar{k}_i}{t_i^c} k_{i,i-1} - 1) + t_i^c + t_i^c + \mu k_{i,i-1} + s_i g_{i}^c + s_i \left( \frac{\bar{k}_i}{t_i^c} - 1 \right) f_{i-1} - s_i \Delta d_i, \\
\tag{40}
\end{align*}

Eq. (38) says that covering the fiscal gap entails domestic and/or external commercial borrowing or adjustments in various fiscal instruments. By combining Eqs. (34) and (37), we can see that if $f_i^c > f_i^floor$, then gap_i = 0; i.e., the resource fund absorbs any fiscal gap and no fiscal policy adjustments are needed. On the other hand, when $f_i^c = f_i^floor$, the gap satisfies gap_i > 0 and it needs to be covered by more borrowing and/or by fiscal adjustments, as we proceed to explain.

The split of government borrowing between domestic and external commercial debt, to help cover the gap, is determined based on the simple rule:

$$\kappa \Delta b_i = (1 - \kappa) s_i \Delta d_i, \tag{41}$$

where $\kappa \in [0, 1]$. This rule accommodates the limiting cases of supplementing concessional loans and grants with only domestic borrowing ($\kappa = 0$) or with only external commercial borrowing ($\kappa = 1$).

Debt sustainability requires that eventually revenues have to increase and/or expenditures have to be cut to cover the gap. The debt stabilizing target values of (i) the consumption tax rate, (ii) the labor income tax rate, (iii) government consumption, and (iv) transfers are determined by:

$$\tau^e_{target,i} = \tau^e + \lambda_1 \frac{gap_i}{c_i}, \tag{42}$$

$$\tau^d_{target,i} = \tau^d + \lambda_2 \frac{gap_i}{W_i L_i}, \tag{43}$$

$$g^c_{target,i} = g + \lambda_3 \frac{gap_i}{p_i^G}, \tag{44}$$

and

$$z_{target,i} = z + \lambda_4 gap_i, \tag{45}$$

where $\lambda_i, i = 1, \ldots, 4$ split the fiscal burden across the different fiscal instruments, satisfying $\sum_{i = 1}^{4} \lambda_i = 1$. Tax rates and expenditure items are then determined according to the policy reaction functions

$$\tau^e_i = \min \left\{ \tau^e_{rule,i}, \tau^e_{ceiling} \right\}, \tag{46}$$

$$\tau^d_i = \min \left\{ \tau^d_{rule,i}, \tau^d_{ceiling} \right\}, \tag{47}$$

$$g^c_i = \max \left\{ \frac{g^c_{rule,i}}{g_i^c}, g^c_{floor} \right\}, \tag{48}$$

and

$$z_i = \max \left\{ \frac{z_{rule,i}}{z_i}, z_{floor} \right\}, \tag{49}$$

where $\tau^e_{ceiling}$ and $\tau^d_{ceiling}$ are the maximum levels of the tax rates that can be implemented, and $g^c_{floor}$ and $z_{floor}$ are minimum deviations of government consumption and transfer from their initial steady-state values. All these ceilings and floors are set exogenously and reflect policy adjustment constraints that governments may face. Finally, $\tau^e_{rule,i}$, $\tau^d_{rule,i}$, $g^c_{rule,i}$, and $z_{rule,i}$ follow the linear rules.

$$\tau^e_{rule,i} = \tau^e_{i-1} + \zeta_1 \left( \tau^e_{target,i} - \tau^e_{i-1} \right) + \zeta_2 (x_{i-1} - x), \tag{50}$$

$$\tau^d_{rule,i} = \tau^d_{i-1} + \zeta_3 \left( \tau^d_{target,i} - \tau^d_{i-1} \right) + \zeta_4 (x_{i-1} - x), \tag{51}$$

$$g^c_{rule,i} = g^c_i + \zeta_5 \left( g^c_{target,i} - g^c_i \right) - \zeta_6 (x_{i-1} - x), \tag{52}$$

$$z_{rule,i} = z_i + \zeta_7 (z_{target,i} - x_{i-1} - x), \tag{53}$$
and

\[
\frac{z_{\text{rule},t}}{z} = \frac{z_{t-1}}{z} + \zeta' \left( \frac{z_{\text{target},t} - z_{t-1}}{z} \right) - \zeta_8 (z_{t-1} - x), \quad \text{with } \zeta', \zeta_8 > 0. \tag{53}
\]

where \(\zeta', \zeta_8\) control the speed of fiscal adjustments, and \(x = \frac{b_{\text{share}} + d_{\text{share}}}{R_f - 1}\) is the sum of domestic and external commercial debt as a share of GDP.

2.2.4. Identities and market clearing conditions

To close the model, the goods market clearing condition and the balance of payment conditions are imposed. The market clearing condition for nontraded goods is

\[
y_{N,t} = \psi y_{ST,t} (c_t + i_{t,t} + f_{t,t}) + v_t \left( \frac{p_{N,t}}{p_c} \right)^{-y} g_t,
\]

while the balance of payment condition corresponds to

\[
c a^{d,t}_t = \rho \left( 1 - \delta \right)^t - \Delta f_t + \Delta d_t + \Delta L_t - (1 - t^0) \delta t_0,
\]

where \(ca^{d,t}_t\) is the current account deficit defined as

\[
ca^{d,t}_t = c_t + i_{t,t} + f_{t,t} + \psi^{OP}_1 \cdot y_{t} - \delta t_0 + \left( R_d - 1 \right) s_t d_t - (1 - t^0) y_{t_0}.
\]

Given model complexity, Table 1 gives a list of variable definitions.

### 3. Calibration

The model is calibrated, at the annual frequency, to an average low-income developing country (LIDC) that just starts exploitation of liquefied natural gas (LNG). Other types of commodities and other stages of exploitations can be accommodated by imposing an exogenous path of resource quantities and prices. Table 2 summarizes the baseline calibration, explained below.

- **National accounting.** Our calibration largely reflects LIDC averages of the last decade in the IMF World Economic Outlook database. The trade balance is set at 6% of GDP, government consumption and public investment are set at 14 and 6% of GDP, respectively, and private investment is set at 15% of GDP. We choose the shares of traded goods to be 50% in private consumption and 40% in government purchases, as government consumption typically has a larger component of nontraded goods than private consumption. Since the economy is at the early stages of exploitation, the share of natural resources is assumed to be only 1% of GDP at the initial steady state.

- **Assets, debt and grants.** We assume that government savings are small initially, only 1% of GDP (\(RF_{\text{share}} = 0.01\)). For government domestic debt, concessional debt and grants, we rely on LIDC averages of the last decade as in Buffie et al. (2012). This implies \(b_{\text{share}} = 0.20, d_{\text{share}} = 0.50, \text{ and } g_{f_{\text{share}}} = 0.04\). To highlight the financial constraints faced by LIDCs in international capital markets, we set \(b_{\text{share}} = 0 \text{ and } d_{\text{share}} = 0\).

- **Interest rates.** We set the subjective discount rate \(\rho\) such that the real annual interest rate on domestic debt \((R - 1)\) is 10%. Consistent with stylized facts, domestic debt is assumed to be more costly than external commercial debt. We fix the real annual risk-free interest rate \((R^f - 1)\) at 4%. The premium parameter \(v_f\) is chosen such that the real interest rate on external commercial debt \((R^c - 1)\) is 6%, and the real interest rate paid on concessional loans \((R_d - 1)\) is 0%, as in Buffie et al. (2012). We assume no additional risk premium in the baseline calibration, implying \(\eta_{\text{de}} = 0\). The parameter \(v\) is chosen to have \(R = R^c\) in the steady state, required by Eqs. (A.4) and (A.5). Based on the average real return of the Norwegian Government Pension Fund from 1997 to 2011 (Gros and Mayer, 2012), the annual real return on international financial assets in the resource fund \((R^{RF} - 1)\) is set at 2.7%.

### Table 1: Variables in the model.

<table>
<thead>
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<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_t)</td>
<td>Total consumption by household’s type (i)</td>
<td>(y_t)</td>
<td>Total output</td>
</tr>
<tr>
<td>(c_{ij})</td>
<td>Consumption of good (j) by household’s type (i)</td>
<td>(g_{t,j})</td>
<td>Foreign grants</td>
</tr>
<tr>
<td>(p_{N,t})</td>
<td>Relative price of nontradables</td>
<td>(\Delta )</td>
<td>Value of national resource revenue</td>
</tr>
<tr>
<td>(\bar{v}_t)</td>
<td>Real exchange rate</td>
<td>(b_t)</td>
<td>Domestic government debt</td>
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<tr>
<td>(L_{ij,t})</td>
<td>Labor supply by household’s type (i) to sector (j)</td>
<td>(d_t)</td>
<td>Concessional government debt</td>
</tr>
<tr>
<td>(G_{ij,t})</td>
<td>Total labor supply by household’s type (i)</td>
<td>(d_{t,ij})</td>
<td>Commercial foreign government debt</td>
</tr>
<tr>
<td>(w_t)</td>
<td>Average real wage</td>
<td>(g_{t,j})</td>
<td>Government consumption</td>
</tr>
<tr>
<td>(w_{t,ij})</td>
<td>Real wage paid in sector (j)</td>
<td>(g_{t,ij})</td>
<td>Government investment</td>
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<tr>
<td>(\rho^{OPT}_{ij})</td>
<td>Government bonds (held by optimizers)</td>
<td>(g_{t,ij})</td>
<td>Total government expenditures</td>
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<tr>
<td>(\rho^{OPT}_{ij})</td>
<td>Foreign domestic debt (held by optimizers)</td>
<td>(g_{t,ij})</td>
<td>Government expenditures in sector (j)</td>
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<td>(\tau_{ij}^{T})</td>
<td>Consumer tax rate</td>
<td>(p_t^{c})</td>
<td>Relative price of government expenditures</td>
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<td>(\tau_{ij}^{L})</td>
<td>Labor income tax rate</td>
<td>(v_{i,j})</td>
<td>Share of tradables in government expenditures</td>
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<tr>
<td>(\bar{v}_t)</td>
<td>Domestic real interest rate</td>
<td>(\bar{v}_t)</td>
<td>Effective government investment</td>
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<td>(\bar{y}_{ij})</td>
<td>Foreign real interest rate</td>
<td>(\bar{y}_{ij})</td>
<td>Growth rate of government investment</td>
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<tr>
<td>(R_{cij})</td>
<td>Concessional real interest rate</td>
<td>(R_{cij})</td>
<td>Depreciation rate of public capital</td>
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<td>(\Omega_{ij})</td>
<td>Profits in sector (j)</td>
<td>(\Omega_{ij})</td>
<td>Resource fund</td>
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<td>(\Omega_{ij})</td>
<td>Real return of capital in sector (j)</td>
<td>(I_{t,ij})</td>
<td>Inflows from the resource fund</td>
</tr>
<tr>
<td>(k_{ij})</td>
<td>Private capital in sector (j)</td>
<td>(I_{t,ij})</td>
<td>Outflows from the resource fund</td>
</tr>
<tr>
<td>(\delta_{ij})</td>
<td>Remittances</td>
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<td>Fiscal gap</td>
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<td>Government transfers</td>
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<td>Target consumption tax rate</td>
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<td>Target labor income tax rate</td>
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<td>Portfolio adjustment costs</td>
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<td>Target government consumption</td>
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<td>(\delta_{ij})</td>
<td>Output in sector (j)</td>
<td>(\delta_{ij})</td>
<td>Target government transfers</td>
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<td>Labor in sector (j)</td>
<td>(\delta_{ij})</td>
<td>Rule-based consumption tax rate</td>
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<tr>
<td>(\delta_{ij})</td>
<td>Private investment in sector (j)</td>
<td>(\delta_{ij})</td>
<td>Rule-based labor income tax rate</td>
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<td>(\delta_{ij})</td>
<td>TFP in tradables</td>
<td>(\delta_{ij})</td>
<td>Rule-based government consumption</td>
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<tr>
<td>(\delta_{ij})</td>
<td>Value of resource production</td>
<td>(\delta_{ij})</td>
<td>Rule-based government transfers</td>
</tr>
<tr>
<td>(\delta_{ij})</td>
<td>Relative price of natural resources</td>
<td>(\delta_{ij})</td>
<td>Total government debt to GDP ratio</td>
</tr>
<tr>
<td>(\delta_{ij})</td>
<td>Production of natural resources</td>
<td>(\delta_{ij})</td>
<td>Current account deficit</td>
</tr>
</tbody>
</table>

\(i = \text{OPT, ROT}, \quad j = T, N.\)
**Private production.** Consistent with the evidence on Sub-Saharan Africa (SSA) surveyed in Buffie et al. (2012), the labor income shares in the nontraded and traded good sectors correspond to $\omega_T = 0.45$ and $\omega_N = 0.60$, respectively. In both sectors private capital depreciates at an annual rate of 10% ($\delta_T = \delta_N = 0.10$). Following Berg et al. (2010), we assume a minor degree of learning-by-doing externality in the traded good sector ($\phi_T = \phi_T = 0.10$). Also as in Berg et al. (2010), investment adjustment costs are set to $\gamma_T = \gamma_N = 25$.

**Households preferences.** The coefficient of risk aversion $\sigma = 2.94$ implies an inter-temporal elasticity of substitution of 0.34, the average LIDC estimate according to Ogaki et al. (1996). We assume a low Frisch labor elasticity of 0.10 ($\psi = 10$), similar to the estimate of wage elasticity of working in rural Malawi (Goldberg, forthcoming). The labor mobility parameter $\rho$ is set to 1 (Horvath, 2000), and the elasticity of substitution between traded and nontraded goods is $\chi = 0.44$, following Stockman and Tesar (1995). To capture limited access to international capital markets, we set $\eta = 1$ as in Buffie et al. (2012).^{18}

**Measure of intertemporal optimizing households.** Since a large proportion of households in LIDCs are liquidity constrained, we pick $\omega = 0.40$, implying that 60% of households are rule-of-thumb. Depending on the degree of financial development of a country, the measure of intertemporal optimizing households can be lower than 40% in some SSA countries. Based on data collected in 2011, Demirguc-

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**Table 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>g$_{share}$</td>
<td>0.14</td>
<td>Govt. consumption to GDP</td>
</tr>
<tr>
<td>b$_{share}$</td>
<td>0.20</td>
<td>Govt. domestic debt to GDP</td>
</tr>
<tr>
<td>d$_{share}$</td>
<td>0.50</td>
<td>Concessional debt to GDP</td>
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<tr>
<td>g$_{share}$</td>
<td>0.04</td>
<td>Grants to GDP</td>
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<tr>
<td>(K − 1)</td>
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<td>Domestic net real int. rate</td>
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<tr>
<td>(K$^{pr}$ − 1)</td>
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<td>Foreign net real int. rate on savings</td>
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<tr>
<td>(R$_{g}$ − 1)</td>
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<td>Net real int. rate on concessional debt</td>
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<tr>
<td>(N − 1)</td>
<td>0.04</td>
<td>Net real risk-free rate</td>
</tr>
<tr>
<td>(R$_{g,2}$ − 1)</td>
<td>0.06</td>
<td>Net real int. rate on external commercial debt</td>
</tr>
<tr>
<td>$\eta_lk$</td>
<td>0.45</td>
<td>Elast. of sovereign risk</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>0.60</td>
<td>Labor income share in nontraded sector</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>0.10</td>
<td>Depreciation rate of $k_{st}$</td>
</tr>
<tr>
<td>$\rho_{lt}$</td>
<td>0.10</td>
<td>Learning by doing in traded sector</td>
</tr>
<tr>
<td>$\rho_{lt}$</td>
<td>0.10</td>
<td>Persist. in TFP in traded sector</td>
</tr>
<tr>
<td>$\kappa_{lt}$</td>
<td>25</td>
<td>Investment adjust. cost, nontraded sector</td>
</tr>
<tr>
<td>$\kappa_{lt}$</td>
<td>25</td>
<td>Investment adjust. cost, traded sector</td>
</tr>
<tr>
<td>$\psi$</td>
<td>10</td>
<td>Inverse of Frisch labor elast.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>Inverse of intertemporal elast. of substitution</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.40</td>
<td>Measure of optimizers in the economy</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.44</td>
<td>Substitution elast. b/w traded/nontraded goods</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Elast. of portfolio adjust. costs</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.65</td>
<td>Royalty tax rate on natural resources</td>
</tr>
<tr>
<td>$\rho_{ma}$</td>
<td>1</td>
<td>Persist. of the commodity price shock</td>
</tr>
</tbody>
</table>

---

18 This implies that perfect arbitrage that equalizes the returns on foreign and domestic assets breaks down, or no perfect substitutability between foreign and domestic assets.

**Knut and Klapper (2012)** report that on average only 24% of the adults in SSA countries have an account in a formal financial institution.

**Mining.** Resource production shocks are assumed to be persistent with $\rho_{ys} = 0.90$. Based on Hamilton’s (2009) estimates, we assume resource prices follow a random walk so $\rho_{pt} = 1$. The royalty tax rate $\tau^0$ is set such that the ratio of natural resource revenue to total revenue at the peak of natural resource production is substantial, almost 50% of total revenues. In this case $\tau^0 = 0.65$. When applying the model to individual countries, the resource tax rate should be calibrated to match the share of resource revenue in total revenues in the data.

**Tax rates and user fees.** The steady-state taxes on consumption, labor and capital are calibrated as $\tau = 0.10$, $\tau = 0.15$, and $\tau = 0.20$, consistent with data collected by the International Bureau of Fiscal Documentation in 2005–06. This combination of tax rates and the implied inefficiency in revenue mobilization implies a non-resource revenue of about 18% of GDP at the initial steady state. Following Briceño Garmendia et al. (2008), we set $f = 0.5$ in the baseline calibration, which implies that half of the recurrent cost of public capital is covered by user fees.

**Fiscal rules.** We impose a non-negativity constraint for the resource fund by setting $\delta_{floor} = 0$. In the baseline calibration, fiscal instruments do not have floors or ceilings. This translates in setting, for instance, $\delta_{floor} = \delta_{floor} = -100, 000$ and $\tau{ceil} = \tau{ceil} = 100, 000$ (or some arbitrarily large numbers in absolute values). The baseline calibration also implies that the whole fiscal adjustment takes place through changes in external commercial borrowing and consumption taxes. This is achieved by setting $\kappa = \kappa = 1$, $\lambda_2 = \lambda_3 = \lambda_4 = 0$, $\delta_3 = \delta_5 = \delta_7 = 1$, and $\delta_4 = \delta_6 = \delta_8 = 0$ in the...
fiscal rules. To smooth tax changes, we choose an intermediate adjustment of the consumption tax rate relative to its target (ζ1 = 0.5) and a low responsiveness of the consumption tax rate to the debt-to-GDP ratio (ζ2 = 0.001). The selection of values for these policy parameters should be guided by the policy scenario that the user wants to simulate as well as by what she considers as a feasible fiscal adjustment.

- **Public investment.** Public investment efficiency is set to 50% (ε = 0.5), which is in line with Arestoff and Hurlin’s Arestoff and Hurlin’s (2006) estimates for developing countries.19 The annual depreciation rate for public capital is 7% (δ = 0.07). The home biases for government purchases v and for investment spending above the initial steady-state level k are 0.6 and 0.4, respectively. The smaller degree of home bias in additional spending reflects that most of the investment goods are imported in LDCs. The output elasticity to public capital αe is set at 0.15, implying a marginal net return of public capital of 28% at the initial steady state. This is in the high end of the range of returns reported by Buffle et al. (2012). The severity of public capital depreciation corresponds to φ = 1 and the change in the depreciation rate of public capital is assumed to be persistent by setting ρδ = 0.8. In the baseline, absorptive capacity constraints start binding when public investment rises above 75% from its initial steady state (δf2 = 0.75). The calibration of absorptive capacity constraints with c1 = 25 implies that the average investment efficiency approximately halves to around 25% when public investment spikes to around 200% from its initial steady state. For illustrative purposes, in the delinked investment approach, we set the planned long-term scaling up of investment such that public investment at the new steady state is 80% higher than at the initial steady state (gE = 0.8).

### 4. Scaling up public investment with a resource windfall

The hypothetical scenarios we analyze assume that the economy discovers a sizable reserve of natural gas, and that production will reach full capacity several years later. When the shock about the current or future increases in resource production hits, the responses of the private sector begin and continue until the system settles in the new steady state. The model dynamics are governed by resource shocks, as well as by exogenously imposed fiscal policy paths. Since households in the model are aware that there are no additional shocks hit in our simulation, the solutions are equivalent to the perfect-foresight solution.

With formidable development needs, the government plans to start investment before resource exploitation is fully in place. To do this, we assume the government uses the prospected natural resource revenues as a collateral to borrow commercially, creating challenges to ensure fiscal sustainability and macroeconomic stability.

In the baseline scenario, the production of LNG increases gradually, reaches full capacity by 2021 and then starts to decline after 2035. At peak, we assume a production of about 1500 millions of cubic feet per year. For the initial years of simulations, we use the oil price forecast per barrel available in the World Economic Outlook of the IMF, multiplied by the conversion factor for full oil parity (0.1724), which yields the price in dollars per million of British Thermal Units (BTUs). The projection of the LNG price in the baseline scenario assumes a non-volatility path, fluctuating around the mean price. The adverse scenario assumes that from 2025 onwards, the resource revenue quickly declines, due to both reduced production quantity and large negative shocks to LNG prices.

### 4.1. The spend-as-you-go approach versus the delinked investment approach

We begin the analysis of policy scenarios by considering two investment approaches and assuming there is no commercial or domestic borrowing to finance public investment increases. With the spend-as-you-go (SAYG) approach, the government spends all of its resource windfall in public investment each period and the resource fund remains at its initial steady state, as analyzed in Richmond et al. (2015) for Angola. Policymakers faced with impoverished population and urgent infrastructure needs could easily find the SAYG approach appealing because of its immediate increase in investment and output. With the delinked investment approach, the government combines investment spending with savings in a resource fund, consistent with the sustainable investing approach analyzed in Berg et al. (2013). We assume both approaches resort only to the consumption tax rate to close any fiscal gap by setting k1 = 1, k2 = 0, ζ1 = ζ2 = ζ3 = ζ4 = 1, and ζ5 = ζ6 = ζ7 = 0 in the fiscal rules.

Figs. 3 and 4 compare the two investment approaches under two resource revenue scenarios: the dotted–dashed lines refer to the SAYG approach and the solid lines correspond to the delinked investment approach.20 With SAYG, public investment does not increase much because of the initial low LNG production. With the delinked approach, public investment scales up gradually with no overshooting (k1 = 0.20, k2 = 0.20). Since the scaling-up is deliberately chosen to be commensurate with the magnitudes of resource revenues, the investment path does not require a large increase in tax rates.

The main difference between the two investment approaches is that the SAYG approach results in a volatile path for public investment, mirroring the volatility of resource revenue flows. Fiscal volatility is translated into macroeconomic instability as shown by fluctuations in macro variables. In contrast, the delinked approach can build up a fiscal buffer and maintain a stable spending path without major fiscal adjustments. Comparing the two scenarios of resource revenues, the economy can build a bigger stabilization fund (of around 150% of GDP) under the baseline scenario than under the adverse scenario of rapidly declining resource revenues — it only peaks at around 25% of GDP.

Another concern with the SAYG approach is the reduced public investment efficiency during the years when resource revenue flows accelerate. Sudden accelerations in public investment expenditures make the economy more prone to bumping into absorptive capacity constraints, translating into lower efficiency. As shown in Fig. 3, with the SAYG approach, public investment accelerates to an extent that average investment efficiency drops from a baseline value of 50% down to almost 25%. Also, when public investment significantly drops (due to a sharp decline in the natural resource revenue), failure to maintain public capital leads to a higher depreciation rate than the steady-state level.

In the baseline scenario without negative shocks, SAYG can perform reasonably well as it leads to a higher accumulation of public capital than the delinked approach. As a result, non-resource output, private consumption, and investment may reach a higher level than that with a delinked approach. However, in the presence of negative shocks to the resource revenue, as captured under the adverse scenario, the delinked approach performs much better, leading to overall more public capital, real non-resource output, private consumption, and investment. Moreover, in both scenarios, a delinked approach delivers a more resilient and stable growth in non-resource GDP and a less volatile real exchange rate. The greater real exchange appreciation induced by SAYG (in periods of particularly high resource revenue) leads to greater

---

19 Other papers such as van der Ploeg (2012) have used the Public Investment Management Index (PIMI) of Dabla-Norris et al. (2012) to calibrate these inefficiencies.

20 The numerical simulations were generated by a set of programs written in Matlab and Dynare (see http://www.ceremaip.cnr.fr/dynare). In all scenarios, we assume perfect foresight and the simulations track the global nonlinear saddle path. Therefore, the model is not log-linearized. Depending on the experiment, the economy may converge to a different steady state from the initial one.
negative learning-by-doing externalities and thus a larger decline in traded output, implying more severe Dutch disease.

Lastly, the two revenue scenarios assume that the reserve of natural gas will deplete after 2040, and this has important consequences for public capital under SAYG. If the public investment level cannot be maintained (like with SAYG), public capital built with the resource windfall eventually declines back to the initial steady-state level. Consequently, the growth benefits of more public capital also diminish. Thus, when determining a scaling-up magnitude, financing needs to sustain capital should be accounted for to ensure long-lasting growth benefits from a resource windfall.

4.2. Front loading public investment with commercial borrowing

Under the constraints of no additional borrowing, any front-loading of public investment is not fiscally feasible unless the government chooses to sharply increase taxes (or significantly cut non-capital expenditures). In this section, we analyze the effects of a front-loaded investment path financed jointly by resource revenues and commercial borrowing.

Figs. 5 to 7 compare the public investment effects under different degrees of frontloading. All three investment paths eventually reach a long-run investment level 80% higher than the level in the initial steady state \((k_1 = 0.20)\). The dotted lines represent a conservative path \((k_2 = 0.10)\), in which public investment is scaled up slowly enough, so it does not require significant debt accumulation when LNG production is low initially. The solid lines represent a gradual path \((k_2 = 0.20)\) with a small degree of frontloading. The dashed lines correspond to an aggressive path \((k_2 = 0.70)\), which generates a pronounced overshooting of public investment. During the two peak years, public investment is around 100% from the initial steady-state level.

In terms of fiscal adjustment, we assume that the government makes use of external commercial borrowing \((\kappa = 1)\) to close the fiscal gap when the resource fund reaches its lower bound. Also, the consumption tax rate is used as the adjustment instrument that stabilizes debt in the long run \((\lambda_1 = 1)\).

Since tax collection in LIDCs is generally weak, we assume there exists a ceiling for the consumption tax rate at 12.5% and it is difficult to increase the tax rate by more that 2.5 percentage points in the short run.

21 The model allows for a flexible arrangement of using various fiscal instruments – the consumption and labor tax rates, government consumption, and transfers to households – to maintain debt sustainability; see Eqs. (41)–(53). The analysis presented here uses only the consumption tax rate as an example.
In both resource revenue scenarios (baseline and adverse), front-loading investment results in no savings in the resource fund and rising public debt. As expected, the debt increase is most pronounced with the aggressive investment path under the adverse scenario. In contrast, with either the conservative or gradual path, public debt as a share of GDP does not increase significantly. Also, the increase in the consumption tax rate is smaller than that with the aggressive path. Moreover, the conservative path is able to accumulate some savings in the resource fund even under the adverse scenario.

When the economy can resort to external commercial borrowing, front-loading public investment can advance the benefits of expected resource windfalls, relative to the case with no borrowing. If the degree of front-loading is not excessive and/or the economy does not experience particularly bad shocks, public debt can be stabilized despite taxing constraints. In this respect, the model can serve as a tool to determine a proper front-loading degree under various assumptions on the rate of return to public capital, fiscal policy, and projections of resource revenues. Among the three investment paths analyzed here, the aggressive path signals a likely infeasible path because government debt appears to be on an explosive path under the adverse scenario.

4.3. Welfare analysis

In this subsection, we report the intertemporal welfare associated with the various public investment policies across the baseline and the adverse scenario of natural resource production and prices. The intertemporal welfare of the two types of consumers at time \( t = 0 \) can be calculated as

\[
W_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, l_t^i) \right\}, \quad i = \text{OPT, ROT};
\]

Fig. 4. Spend-as-you-go vs. delinked investment approach (continued): no additional commercial borrowing. X-axis is in years.
while the aggregate welfare is computed using the shares of the two types of agents as follows:

\[ W_0 = \omega W_0^{OPT} + (1 - \omega) W_0^{ROT}. \]

Table 3 reports the welfare measure \( W_0 \). Also, in order to quantitatively compare the various welfare outcomes, we compute the consumption-equivalent (C.E.) welfare change, \( \lambda \), between, say, policy (A) and policy (B), implicitly defined as:

\[
W_0^A = \omega \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t U\left( c_t^{OPT, A} + \frac{1}{1 + \rho} r_t^{OPT, A} \right) \right\} + (1 - \omega) \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t U\left( c_t^{ROT, A} + \frac{1}{1 + \rho} r_t^{ROT, A} \right) \right\}
\]

\[
= \omega \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t U\left( c_t^{OPT, B} + \frac{1}{1 + \rho} r_t^{OPT, B} \right) \right\} + (1 - \omega) \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t U\left( c_t^{ROT, B} + \frac{1}{1 + \rho} r_t^{ROT, B} \right) \right\}
\]

where \( (\lambda - 1) \times 100 \) represents the permanent percent change that should occur to consumption in order for the representative consumer to be as well off under policy (B) as she is under policy (A).

Our calculations show that, in the case of no additional commercial borrowing, the delinked approach delivers a welfare loss under the baseline scenario, but implies a welfare gain (equivalent to a permanent increase in consumption of more than half a percent) under the adverse scenario, relative to the SAYG approach. When the government can resort to external commercial borrowing, a gradual approach – a moderate frontloading of public investment – yields a substantial welfare gain (above 1% in C.E. terms) relative to a conservative approach.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Baseline scenario</th>
<th>Adverse scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>C.E. % change</td>
<td>Welfare</td>
</tr>
<tr>
<td>SAYG (A)</td>
<td>-17.5884</td>
<td>-17.9600</td>
</tr>
<tr>
<td>Delinked (B)</td>
<td>-17.6663 -0.2672 w.r.t. (A)</td>
<td>-17.7887 0.5739 w.r.t. (A)</td>
</tr>
<tr>
<td>Conservative (C)</td>
<td>-18.0521</td>
<td>-18.1253</td>
</tr>
<tr>
<td>Gradual (D)</td>
<td>-17.6629 1.2953 w.r.t. (C)</td>
<td>-17.7830 1.1349 w.r.t. (C)</td>
</tr>
<tr>
<td>Aggressive (E)</td>
<td>-17.6587 0.0149 w.r.t. (D)</td>
<td>-17.7983 -0.0512 w.r.t. (D)</td>
</tr>
</tbody>
</table>

Fig. 5. Various degrees of investment frontloading: external commercial borrowing. X-axis is in years.
which awaits natural resource revenues to materialize. Under the baseline scenario, the aggressive approach – a substantial public investment frontloading – yields a very small welfare gain (0.01% in C.E. terms) relative to the gradual approach. In the adverse scenario, however, the aggressive investment plan causes a welfare loss. This result is driven by the anticipation of a higher government debt overhang and the consequent increase in taxes that would occur with an aggressive public investment policy in the adverse scenario.

4.4. Domestic versus external commercial borrowing

Instead of using external commercial borrowing, the government can use domestic borrowing to finance an investment scaling-up. Fig. 8 compares the macroeconomic effects of the two borrowing strategies (domestic versus external commercial borrowing). The solid lines refer to domestic borrowing ($x = 0$), and the dotted-dashed lines reflect external borrowing ($x = 1$). The public investment path is the same as the aggressive frontloading path depicted in Fig. 5.

The most important difference is that external borrowing brings in additional financial resources, while domestic borrowing shifts domestic resources away from the private sector into the public sector. Because of this and the fact that the real interest rate rises more with domestic borrowing, private investment is crowded out more with domestic borrowing. And since the amount of domestic borrowing is higher under the adverse scenario, the crowding-out effect is more pronounced than that under the baseline scenario. The higher interest rate associated with domestic borrowing also feeds into higher interest payments, more accumulation of public debt, and on average higher consumption tax rates to stabilize debt. This has important consequences for debt sustainability under the adverse scenario: With external commercial borrowing public debt remains stable, while with domestic borrowing public debt becomes unsustainable.

4.5. The role of limited capital account mobility

In this model, optimizing households can save in both domestic government debt and foreign assets. Due to limited capital account mobility, the two types of assets, however, are not perfect substitutes, as shown by the modified interest parity of the model:

$$E_{t}(\lambda_{t+1}R_t) = E_{t}\left[\frac{\lambda_{t-1}S_t}{S_t-\eta(B^t_{OPT}+B^t_{OPT})}\right].$$  

Since the baseline calibration sets $\eta = 1$, the perfect arbitrage between the two assets breaks down. While households are aware of a certain investment path and resource revenue at the beginning, private investment and consumption do not rise immediately to the new steady-state level because households cannot smooth perfectly in an

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22 Our simulation results appear to favor external commercial borrowing to domestic borrowing, mainly due to the reduced crowding-out effect with external borrowing. Since the model only accounts for shocks to resource prices and quantity, it does not capture the increased vulnerability from a higher stock of external debt resulting from other economic shocks. For example, an unexpected shock that depreciates the real exchange rate would expand the size of foreign liabilities, threatening debt sustainability, as the negative terms-of-trade shock analyzed in Buffie et al. (2012).

23 To derive this, we combine the first order conditions (A.4) and (A.5) in Appendix A.
environment with restricted access to financial markets, and productive capital does not build up at the beginning to raise productivity of private production factor immediately.

5. Sensitivity analysis

Among various aspects of the model, public investment efficiency and the return to public capital are particularly important in shaping the macroeconomic effects of public investment. In addition, the degree of learning-by-doing externalities and its persistence play a crucial role on Dutch disease or later Dutch vigor. In this section, we conduct some sensitivity analysis with respect to these aspects. Also, we investigate whether the Marshall–Lerner condition is satisfied by examining the relationship among the real exchange rate, traded output, and trade balance.

5.1. Public investment efficiency, return to public capital, and debt sustainability

To see how efficiency and the return to public capital affect debt sustainability, Fig. 9 compares three different assumptions. The solid lines reflect the baseline calibration ($\tau = 0.5$ and $\alpha_c = 0.15$); the dotted-dashed lines assume efficiency increases from 0.5 to 0.7 over time; and the dotted lines correspond to improving efficiency together with $\alpha_c = 0.18$. The figure is depicted for the case of external commercial borrowing and the adverse natural resource scenario.

As shown in Fig. 9, improving efficiency and/or raising the return to public capital deliver better macroeconomic outcomes than those from the baseline calibration. Higher efficiency generates more public capital for a given investment level, which then helps produce more non-resource output, leading to higher income and, therefore, more consumption. If public capital also becomes more productive, these positive macroeconomic effects are further amplified. In the example provided for the combined changes (dotted lines), the additional growth rate in non-resource GDP is doubled in the long run.

On the fiscal side, government debt is on an explosive path with the baseline calibration, given the adverse natural resource path. However, with the efficiency improvement, the same investment path turns out to be fiscally sustainable as the additional positive effect on non-resource GDP growth is capable of generating enough non-resource revenues that close the fiscal gap. The simulations presented here demonstrate that for the same resource revenue flows and same investment paths, different investment efficiencies and returns to public capital can easily change the outlook of debt sustainability.

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24 Berg et al. (2010) coined the term “Dutch vigor,” as the learning-by-doing externalities cut both ways: increases of traded output relative to its trend can also generate productivity gains in this sector and, therefore, amplify the positive effects of resource revenues, especially over longer horizons.

25 Because public investment efficiency in DIGNAR is time-varying, efficiency matters for growth outcomes. See Berg et al. (2015a) for some misconceptions about efficiency and its implications for growth.
5.2. Learning-by-doing externalities

The simulations so far assume minor learning-by-doing externalities ($\rho zT = \rho yT = 0.1$). Studies on learning-by-doing for developed countries, however, assume a relatively high degree of externalities. Cooper and Johri (2002), using data of the U.S. manufacturing sector, estimate that the persistence parameter (similar to our $\rho zT$) is 0.63 and the learning-by-doing parameter (similar to our $\rho yT$) is 0.37.26 Chang et al. (2002) allow for permanent learning-by-doing effects via a permanent shock to technology. Fig. 10 compares the baseline with two alternative calibrations on $\rho zT$ and $\rho yT$. The case simulated is for external commercial borrowing under the baseline resource revenue scenario with the delinked investment path as shown by the solid lines in Fig. 5.

Relative to the baseline (solid lines), stronger leaning-by-doing ($\rho zT = 0.1$ and $\rho yT = 0.4$, dotted-dashed lines) worsens the initial Dutch disease slightly but amplifies Dutch vigor substantially later on. While the TFP of the traded good sector falls slightly, it increases more than 4% above the trend growth path by 2040, compared to less than 1% with the baseline calibration. Although the TFP of nontraded production is unaffected, higher traded good production increases income and also the demand for nontraded output. Overall, with stronger learning-by-doing externalities the non-resource GDP is about 2% higher relative to the trend-growth path than with the baseline calibration.

To explore the effect of permanent externalities, the dashed lines represent the case with $\rho zT = 1$ and $\rho yT = 0.4$. With permanent learning-by-doing externalities, the initial fall in the productivity of the traded good sector triggers a snowball effect that suppresses the productivity below the trend-growth path throughout the simulation horizon. As a result, traded output turns slightly above trend only almost 30 years after, because the productivity gain from more productive capital is largely offset by the TFP decline due to permanent learning-by-doing effects.

Fig. 10 demonstrates that the intensity of learning-by-doing plays an important role in the macroeconomic effects of public investment. Lack of empirical estimates on the degree of learning-by-doing in developing countries suggests that sensitivity analysis should be performed. In particular, our specification is symmetric in terms of output changes that can deteriorate as well as enhance TFP. An overly strong learning-by-doing assumption (especially $\rho yT$) may depict a very rosy picture about non-resource output and hence tax revenues, which can potentially underestimate debt sustainability risks.

26 These estimates are obtained under the assumption that the accumulation of organization capital and output production have constant returns to scale. Other assumptions, such as increasing returns to scale in organization capital, are also imposed for estimation. Overall, the persistent parameter is around 0.5.
Fig. 9. Sensitivity on public investment efficiency and return to public capital: external commercial borrowing and adverse natural resource scenario. X-axis is in years.

Fig. 10. Sensitivity on learning-by-doing externality: external commercial borrowing and baseline natural resource scenario. X-axis is in years.
5.3. The Marshall–Lerner condition

In addition to learning-by-doing externalities, another dimension related to the Dutch disease is the extent to which the trade balance (or traded output) worsens in response to a real appreciation from investing resource revenues. Under the baseline calibration, the simulation results exhibit a positive correlation between the real exchange rate and trade balance – real appreciations are accompanied by a deterioration of the trade balance – suggesting the validity of the Marshall–Lerner condition. Taking a step further, this section investigates how robust this correlation is in our model with respect to a few key parameters.

In a small, open, New Keynesian model, Gali and Monacelli (2005) find that parameters important for the net export responses to the real exchange rate include the intratemporal elasticity of substitution between home and foreign goods and the inverse of intertemporal elasticity of substitution. Thus, Fig. 11 investigates how trade balance and traded output change when the intratemporal elasticity becomes larger ($\chi = 0.44 \rightarrow 1000$, top row) and when the inverse of intertemporal elasticity of consumption becomes smaller ($\sigma = 2.94 \rightarrow 1$, bottom row).

In our model, trade balance dynamics are largely driven by exogenous components, i.e., resource prices, quantities, and the degree of home bias in public investment (which use both nontraded and traded goods). Thus, we focus on the endogenous responses of the trade balance by looking at the differences between the responses of the trade balance across two parameter settings in each case.

The top row of Fig. 11 shows that, as the degree of intratemporal elasticity between traded and nontraded goods becomes higher, the real exchange rate appreciates less. In an extreme case as represented by dashed lines ($\chi = 1000$, nontraded and traded goods are nearly perfect substitutes), an initial real appreciation (driven by spending resource revenues on public investment) induces households to substitute away from nontraded to traded goods: the demand pressure on nontraded goods from higher public investment is largely offset by the reduced private demand for nontraded goods. In equilibrium, the increase in the trade balance (mainly due to higher resource output and public investment) matches the increase in the financial account balance (mainly due to higher savings in the resource fund and remittance of resource dividends to foreign investors). Thus, the real exchange rate when $\chi = 1000$ stays roughly on the steady-state trend growth path. A higher $\chi$ is associated with a smaller real exchange rate appreciation and an improvement in the trade balance.

The bottom row shows that as the intertemporal elasticity of substitution for consumption increases ($\sigma$ goes from 2.94 to 1), the degree of real appreciation is reduced and trade balance improves. A higher intertemporal elasticity means that private consumption is more sensitive to real interest rate movements. Anticipating future resource output and higher productive public capital implies that current savings are expected to be more productive in the future; the real interest rate must rise to reduce current consumption. The magnitude of such a decline is bigger when $\sigma$ is smaller (higher elasticity). Since private consumption is composed of both nontraded and traded goods, the decrease in nontraded good demand of private consumption reduces the demand pressure of nontraded goods relative to the case of $\sigma = 2.94$. Thus, the real exchange rate appreciates less, leading to better performance in traded output. Later as public capital is gradually built up, the better performance in traded output results in a faster improvement in the TFP of the traded good sector (due to Dutch vigor) and higher non-resource output. Without more domestic production, the appreciation pressure is further reduced relative to the baseline case, generating further improvements in trade balance, as shown in Fig. 11.

In sum, our baseline simulation results, as well as the sensitivity analysis on intra- and inter-temporal elasticity of consumption, suggest that the result that real appreciation leads to a deterioration in the trade balance is generally robust in our model.

6. Conclusion

This paper presents the DIGNAR model, which can be used to assess debt sustainability and growth effects of public investment scaling-ups in resource-abundant developing countries. The model has most of the relevant developing country features of the frameworks developed in Buffie et al. (2012) and Berg et al. (2013), including public investment inefficiencies, absorptive capacity constraints, and learning-by-doing externalities that can deliver Dutch disease effects. It also introduces novel features especially in the fiscal policy structure. DIGNAR can accommodate flexible fiscal arrangements, with domestic and external

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27 There are other features that would be interesting to include such as labor informality. We leave this for future research.
commercial borrowing as options to close the fiscal gap in the short-to-medium run, and several fiscal instruments (taxes and expenditures rules) to maintain debt sustainability in the long run. The model also has a resource fund that can be used as a fiscal buffer as well as a saving device, subject to a minimal saving level that a government intends to maintain.

To illustrate how to use DIGNAR in policy analysis, the paper calibrates the model to an average low-income developing country and constructs some hypothetical and stylized public investment scaling up and resource revenue scenarios. It then discusses the macroeconomic effects of different investment approaches – spend-as-you-go and delinked investment – under different fiscal adjustment and borrowing schemes as well as the implications of different degrees of public investment frontloading. The stylized scenarios show that, when fiscal adjustment is unconstrained, a delinked public investment approach combined with the resource fund can reduce macroeconomic instability relative to the spend-as-you-go approach. However, even with the fund, ambitious frontloading public investment plans combined with more borrowing can induce debt sustainability risks, especially with declining investment efficacy or when future resource revenues turn out to be lower than expected.

The analysis reveals the importance of considering country-specific information that can be mapped into parameter values of the model. When this is not possible for some parameters, sensitivity analysis can be conducted for these parameters, as the paper shows for the public investment efficiency and the return to public capital under a negative resource revenue scenario. Also, the analysis only focuses on two resource revenue scenarios, but in reality the degree of resource revenue uncertainty can be greater than what is depicted here. One way to address this issue is to conduct simulations under a wide range of resource revenue scenarios that account for the historical resource price volatility and likely production profiles. The probability of an unfavorable outcome associated with an investment path can then serve as an indicator of whether a proposed investment path is overly aggressive (see the analysis for Angola in Richmond et al., 2015).28

DIGNAR is an integrated model-based macroeconomic framework that may be useful in constructing the scenarios necessary for debt sustainability analysis of resource-rich developing countries that intend to scale up public investment. Judgment is still critical to calibrate, construct, and interpret these scenarios. However, DIGNAR can help make the assumptions underlying the projections explicit, organize policy discussions based on different simulated scenarios, apply empirical information, and allow more systematic risk assessments in natural resource-rich developing countries.

Appendix A. The First order conditions

This appendix consists of the first order conditions to the optimization problems in the model.

Maximizing the household’s total labor income \((w_i L_i^t = w_i L_{i,t} + w_{0k} L_{0,k})\) subject to labor (4) yields the following labor supply schedules for each sector:

\[
L_{i,h,t}^t = \delta \left( \frac{w_{i,k}}{w_i} \right) x_i \quad \text{and} \quad L_{i,t}^t = \left(1 - \delta \right) \left( \frac{w_{i,k}}{w_i} \right)^{\sigma} x_i, \text{ for } i = \text{OPT, ROT},
\]  

The first-order conditions with respect to \(c_1^\text{OPT}, L_1^\text{OPT}, t_1^\text{OPT}, \) and \(b_0^\text{OPT} \) are

\[
\lambda_t (1 + t_1^t) = (c_1^\text{OPT})^{-\sigma},
\]

where \(\lambda_t \) is the Lagrange multiplier associated with the budget constraint (7).

Let \(\lambda_t q_{N,t} \) be the Lagrange multiplier associated with the law of motion of capital, where \(q_{N} \) is the sectoral Tobin’s q. Then, the first-order conditions with respect to \(L_{N,t}, k_{N,t} \), and \(b_{0,t} \) are given by

\[
w_{N,t} = \alpha_{N} q_{N,t} \frac{y_{N,t}}{L_{N,t}},
\]

\[
q_{N,t} = E_t \left[ \beta \frac{LN_{t+1} - LN_{t} + (1 - \tau_N)E_t (1 - \alpha|q_{N,t+1}|) q_{N,t+1} y_{N,t+1}}{k_{N,t}} \right],
\]

and

\[
1 = \beta E_t \left[ \left(1 - \frac{\alpha}{N} \right) q_{N,t+1} \left(1 - \tau_N \right) \left(1 - \alpha N \right) \frac{y_{N,t+1}}{k_{N,t+1}} \right] + E_t \left[ \beta \frac{LN_{t+1} - LN_{t} + (1 - \tau_N)E_t (1 - \alpha|q_{N,t+1}|) q_{N,t+1} y_{N,t+1}}{k_{N,t} + 1} \right].
\]

The first-order conditions with respect to \(L_{T,t}, k_{T,t} \), and \(d_{0,t} \) are given by

\[
w_{T,t} = \alpha_{T} q_{T,t} \frac{y_{T,t}}{L_{T,t}},
\]

\[
q_{T,t} = E_t \left[ \beta \frac{LN_{t+1} - LN_{t} + (1 - \tau_N)E_t (1 - \alpha|q_{T,t+1}|) q_{T,t+1} y_{T,t+1}}{k_{T,t}} \right],
\]

and

\[
1 = \beta E_t \left[ \left(1 - \frac{\alpha}{N} \right) q_{T,t+1} \left(1 - \tau_N \right) \left(1 - \alpha N \right) \frac{y_{T,t+1}}{k_{T,t+1}} \right] + E_t \left[ \beta \frac{LN_{t+1} - LN_{t} + (1 - \tau_N)E_t (1 - \alpha|q_{T,t+1}|) q_{T,t+1} y_{T,t+1}}{k_{T,t} + 1} \right].
\]

References


Bems, R., de Carvalho Filho, I., 2011. The current account and precautionary savings for exporters of exhaustible resources. J. Int. Econ. 84, 48–64.


28 Berg et al. (2015b) incorporate uncertainty about shocks and parameters more systematically in a debt sustainability model, while maintaining the non-linear structure, to construct confidence bands around debt trajectories. A similar approach could in principle be taken for DIGNAR.