Department of Economics
School of Social Sciences

Trade and Linked Exchange; Price Discrimination Through Transaction Bundling

By Chong Ju Choi, Xeni Dassiou and Daniel Maldoom

Department of Economics
Discussion Paper Series

No. 03/07

Trade and Linked Exchange; Price Discrimination Through Transaction Bundling*

Chong Ju Choi, Xeni Dassiou and Daniel Maldoom

July 2003

Abstract

In this paper we try to explain how price discrimination can cause bilateral trade patterns of the type seen under countertrade agreements. We interpret countertrade as a form of *transaction bundling* which can discriminate between potential trading partners and we combine characteristics from both explanations as to the existence of countertrade. There is both price discrimination through *transaction bundling,* and informational asymmetry in the form of uncertainty in the quality of the goods produced by trading partners in less developed countries (LDCs) leading to a *partner preference* from the side of the Western (DC) firm. Our paper shows that although the ability of firms in LDCs to overcome their creditworthiness constraints by engaging in countertrade arrangements may be restricted by this quality uncertainty as it reduces the willingness of a firm in a DC to exchange, the trade volume prospects of a firm in a LDC can be considerably enhanced if a countertrade transaction does occur.

Our paper goes beyond the case of linked exchange, which is only one of the three cases of transaction bundling examined. The other two cases are that of the Western firm being a monopoly selling a bundle of two goods used as a benchmark case, and the more interesting case of the Western firm being the buyer of two goods and setting both two separate buying prices and a bundling (i.e. package) purchase price. Many procurement decisions are not simply a matter of price, but also the identity and reputation of the supplier matters, especially when the supplier is located in an LDC. We show than when bundling its purchases, the Western firm buyer will be willing to offer a bundled price greater than the sum of the two separate prices, as the option of a bundled purchase would increase its profits even if there are no complementarities between the goods bundled. In our model the argument is that just as it is profitable for a monopolist to offer mixed bundling at a bundled price which is lower than the sum of the individual prices (hence exploiting the average willingness to pay), it is also profitable for a monopsonist to offer a bundled purchase price which is higher that the sum of the individual prices on offer (hence exploiting the average willingness to sell). Equally interestingly, it is found that a LDC can substantially increase its sales of a good with a high degree of quality uncertainty by being offered to bundle it with the sale of a more basic good with a low degree of quality uncertainty.

* Corresponding author: Dr Xeni Dassiou, Dept. of Economics, City University, Northampton Square, London EC1V 0HB, U.K., tel. +44 (0)20 70400206, e-mail: x.dassiou@city.ac.uk

* *The authors wish to thank Martino De Stefano and other conference participants for very useful comments on an earlier version of this paper which was presented at the 1st International Industrial Organization Conference, Boston, USA, April 2003.

Trade and Linked Exchange;
Price Discrimination Through Transaction Bundling

1 Introduction

Countertrade agreements comprise over 20% of the international trade. Numerous examples of countertrade can be found in Hammond (1990). In a series of articles during the 90's countertrade in its various formats (barter, buybacks, and counterpurchase - the latter being the most prevalent form of countertrade) has been examined and a number of alternative explanations offered as to its existence.

A traditional explanation for the existence of countertrade was that of foreign exchange shortages experienced by a firm in a less developed country (LDC). However, as Caves and Marin (1992) have shown empirically the traditional foreign exchange argument does not fully explain the increasing prevalence of countertrade agreements.

The second explanation refers to the moral hazard problems arising from the existence of informational asymmetries. The models by Chan and Hoy (1991) and Choi and Maldoom (1992) analyse buyback contracts as a commitment device which can assist to resolving the problem of technology transfer to developing countries by a Western firm. A buyback contract requires the Western firm to get a specified quantity of future production of the manufactured item to the production of which it has contributed by being the foreign supplier of production technology and equipment. Hence its quality decision has an impact on the quality of the good it is paid by, thus reducing its incentive to cheat by providing low quality production technology. Marin and Schnitzer (1995, 1998) generalised this result to all forms of countertrade, arguing that the tying of two transactions solves a two fold problem: it serves as a collateral and thus reduces the risk of default by the LDC firm, and it also deters cheating on quality by the firm in the developed country (DC) as the value of the collateral depends on its own quality decision (either through the existence of a technological relationship as in the case of buybacks, or by contract design). In a subsequent article on barter, Marin and Schnitzer (2002) re-emphasize the importance of paying an import with export goods as it reduces the anonymity of the medium of exchange and hence improves the LDC trading partner's creditworthiness. In this model it is the Western firm which is unable to determine the quality of the good she is paid with if she has no experience with it, and may thus end up being paid with low quality goods. In other words, since there is uncertainty surrounding the quality of the good provided by the LDC partner, the informational asymmetry is a problem faced by the Western firm. This is less of a problem if the good is not differentiated (more easy to obtain information about its physical and market characteristics), or if the Western firm actively contributes in the good's production by making some costly investment on it (this latter approach re-introduces buyback characteristics into the transaction but this time as a way to resolve an informational asymmetry suffered by the DC partner, not the LDC one).

The third explanation for the existence of countertrade is based on price discrimination. Caves (1974) provides an informal explanation for the existence of countertrade agreements based on price discrimination, while Caves and Marin (1992) have empirically verified this informal explanation; nevertheless they have not explained how it is important in understanding countertrade, nor presented a theoretical model. The main thrust of the argument is that countertrade can be used as a vehicle for price discrimination; the DC exporter can sell its good to LDC partners that will usually have low *a priori* reservation prices. As the good imported from the LDC partner in return is normally not exported and of unknown quality, the effective price paid for the export good is obscure to third parties.

In this paper we try to explain how price discrimination can cause bilateral trade patterns of the type seen under countertrade agreements. Our theoretical model is related to the bundling model of Adams and Yellen (1976), McAffe, McMillan and Whinston (1989, henceforth MMW) and the tying model of Whinston (1990). We interpret countertrade as a form of *transaction bundling* which can discriminate between potential trading partners and we combine characteristics from both the second and the third explanation; in our theoretical framework there is both price discrimination and uncertainty about the quality of the goods produced by LDC partners. This second feature introduces what we refer to in our model as *partner preference* from the side of the Western firm. A firm which is subject to a lemons type problem may be able to signal its type if it is offered the opportunity to engage in a linked auxiliary transaction which will effectively remove the anonymity of the transaction. This requires that the surplus generated by the auxiliary transaction is appropriately related to the type of the firm, thus allowing Spence - signalling to occur. Thus by offering the opportunity of bundling the transactions, the DC firm can effectively enhance its ability to identify desirable trading partners. However, the extent to which the bundling option can serve as a medium of exchange depends on the creditworthiness constraint faced by the DC firm which is the result of the degree of uncertainty in the quality of the good they produce and the extent to which the firm in the developed country can overcome this informational uncertainty. This is a feature our paper shares in common with the Marin and Schnitzer (2002) paper as the latter deals extensively with the issues of anonymity and uncertainty in the quality of the medium of exchange; however it does not contain price discrimination as a feature like our model does. More importantly, our paper presents barter as a special case in exchange bundling and derives the conditions under which this is optimal for the DC partner.

Moreover, our paper goes beyond the case of linked exchange, which is only one of the three cases of transaction bundling examined. The other two cases are that of the Western firm being a monopoly selling a bundle of two goods (in a conventional format of mixed bundling as analysed in MMW) and is as such a benchmark case, and the more interesting case of the Western firm being the buyer of two goods and setting both two separate buying prices and a bundling (i.e. package) purchase price. This latter case is potentially useful to study given the increasing pre-eminence of e-commerce particularly in its B2B format. More specifically, reverse auctions (both off-line and, more frequently, on line) are an area of great recent activity. However, many procurement decisions are not simply a matter of price, but also the identity and reputation of the supplier matters. This is especially true when the supplier is an LDC firm. We show than when bundling its purchases, the Western firm buyer will be willing to offer a bundled price which is higher than the sum of the two separate prices, as the option of a bundle purchase would increase its surplus. This is not as startling as it may initially sound; package bidding in spectrum auctions conducted by the Federal Communications Commission (FCC) results in higher prices being offered for packages as compared to individual licence bidding in the presence of strong complementarities (CRA, 1998; also see Ausubel & Milgrom, 2002 for a theoretical exposition of package bidding involving substitute or complement goods.) This also applies in the retail market; Lucking-Reiley (2000) gives the example of the Winebid online auction of wines, where the high bid for a collection of wines could exceed the sum of the high bids for the individual bottles in the package. However, in our model the demand for the individual goods is independent and there are no complementarities between the goods in the bundle. The argument is that just as it is profitable for a monopolist to offer mixed bundling at a bundle price which is lower than the sum of the individual prices (hence exploiting the average willingness to pay), it is equally profitable for a monopsonist to offer a bundled purchase price which is higher that the sum of the individual prices he is offering (hence exploiting the average willingness to sell). Given that in our paper it is proven that mixed package purchasing is optimal, then if anything the existence of complementarities would reinforce this result by creating additional incentives for bundling. Equally interestingly, it is found that a LDC can substantially increase its sales of a good with a high degree of quality uncertainty by being offered to bundle it with the sale of a more basic good with a low degree of quality uncertainty.

2 The Model

We consider a model which is similar to that of MMW; there are two goods for potential trade, which are traded between a LDC market of firms, called M, and a firm P in a DC, who is either a monopolist or a monopsonist or both. There is a continuum of firms in M, which vary in quality terms and each may choose to transact in neither of the two goods, just one good, or both goods. P is unable to identify the type of a particular firm in M before transacting with it.

The model allows transactions to occur in any direction between M and P. Each firm has a unit demand (supply) for the goods it is buying (selling). (θ_1, θ_2) are the set of valuations placed on the two goods by the firms in M, with θ_1 and θ_2 having independent uniform distributions over $[0, 1]$. Each firm's surplus is equal to

$$S_i(\theta_1, \theta_2) = \theta_i(-\theta_i)$$

If good i is bought (sold) by a firm in M, then θ_i is the value (cost of production) to that firm. Firm P's surplus gross of payments, S_i^p, and for symmetry $\mid S_i^P \mid \in [0, 1]$. S_i^p is equal to the cost of production if P sells good i, i.e. $S_i^p = -c_i$. On the other hand, if P buys the good, $S_i^p = v(\theta_i)$, then we denote by $v(\theta_i)$ the valuation which P assigns to the good. P in his valuation of the good sets a weight of β to that part of the good's quality which depends on the trading partner's choice of quality (and which in turn is a one to one function of its cost of production (θ_i) and hence the type of the trading partner), and a certain component

α [1] of the good' s quality (equal to some minimum quality value that this good would raise if sold in the international market. Hence:

$$v(\theta_i) = \alpha_i + \beta_i \theta_i$$

where $0 \preceq \alpha_i + \beta_i \preceq 1$, if the quality of the goods sold by the firms in M is uncertain. In this case we say that P has a *partner preference*, as its surplus function depends on the quality type of the firm with which it transacts. The term β_i can be thought of as a measure of the intensity of P's *partner preference,* i.e., the importance it assigns to (or the extent of his inability to uncover the features determining) the uncertain quality of the good. If $\alpha_i = 1$, then there is no *partner preference* ($\beta_i = 0$) and P's net surplus function on the good it purchases will definitely be positive.

P announces three prices: p_1, p_2 are the prices for the single transactions in each good, and p_b is the price offered by P for the two transactions bundled together. In other words, this is a model of mixed bundling in which P offers to either transact in each good separately or in both goods bundled together. If P sells (buys) good i then conventionally we will take $p_i > 0$ ($p_i < 0$).

We will assume that it is not possible for P to monitor who it transacts with. An obvious reason for this monitoring problem is that firms in the market can engage a third party to act as a middleman for either of the goods transacted and so hide their identity. The lack of monitoring opportunities means that P faces the constraint that $p_b \preceq p_1 + p_2$ for some firms

[1] α can also be thought as the extent of P's ability to ascertain some of the features of the good which he considers bying.

to wish to engage in the bundled transaction.

There will be four groups of firms in M: those who choose to transact in neither good ($R_0(p)$) at the set of prices $p = (p_1, p_2, p_b)$ announced by P; those who trade in good 1 only ($R_1(p)$); those who trade good 2 only ($R_2(p)$); and those who trade in both goods ($R_{12}(p)$). Those firms who transact in only one good receive a surplus of $S_i(\theta_1, \theta_2) - p_i$, while the firms that transact in both goods get a net surplus of $S_1(\theta_1, \theta_2) + S_2(\theta_1, \theta_2) - p_b$. Accordingly, the four groups are defined as:

$$R_0(p) = \{\theta \in [0,1]^2 : S_1(\theta) < p_1, S_2(\theta) < p_2, S_1(\theta) + S_2(\theta) < p_b\};$$

$$R_1(p) = \{\theta \in [0,1]^2 : S_1(\theta) - p_1 = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b)\};$$

$$R_2(p) = \{\theta \in [0,1]^2 : S_2(\theta) - p_2 = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b)\};$$

$$R_{12}(p) = \{\theta \in [0,1]^2 : S_1(\theta) + S_2(\theta) - p_b = \max(0, S_1(\theta) - p_1, S_2(\theta) - p_2, S_1(\theta) + S_2(\theta) - p_b)\}.$$

These four sets partition the unit square.

The net surplus for P can now be written down. it is equal to:

$$\sum(p) \quad = \quad \int_{R_1(p)}\!\!\int \left(S_1^P + p_1\right) dF(\theta) + \int_{R_2(p)}\!\!\int \left(S_2^P + p_2\right) dF(\theta) + \int_{R_{12}(p)}\!\!\int \left(S_1^P + S_2^P + p_b\right) dF(\theta)$$

The three prices must now be chosen to maximize $\sum(p)$. If the optimal prices are such that $p_b < p_1 + p_2$, then P is offering more favourable terms for the linked transaction than the two separate transactions. We will refer to this as *transaction bundling.* We will proceed to show that transaction (mixed) bundling is optimal not only locally, as in the MMW model for the monopoly case, but globally by checking both the first and the second order conditions in all three cases analysed below.

2.1 The monopoly case

In this case P sells both goods and the market M buys them. The uncertainty parameters are the valuations placed on the two goods by the firms in M. Thus $S_i(\theta_1, \theta_2) = \theta_i$, and P's gross surplus on each transaction is equal to the cost of production of each good, $S_i^P = -c_i$. There is no *partner preference* in this case, since P's gross surplus is independent of the type of the trading partner. This example is encompassed by MMW; however we do not follow the steps of that paper here for two reasons. First, because our purpose is to offer an analytical example in *transaction bundling*, rather than to look at the general framework of seller bundling exclusively as MMW do. Second, because the adoption of a general framework constraints MMW to solve the problem only in terms of local optimisation. Instead, we proceed to solve the problem by global optimisation and in order to do that we need to take a more specific case by assuming independently distributed uniform values for θ_1 and θ_2. We note that a sufficient condition to ensure that there is a local gain from bundling is given by $(p_1^* - c_1)h_1(p_1^*)[1 - H_2(p_2^*)] > 0$ according to MMW, where θ_1 and θ_2 are independently distributed following any h_i and H_i density and distribution functions respectively, and p_1^* and p_2^* are the nonbundling optimal prices.[2]

[2] It is easy to calculate that this equal to $\xi = 0.25(1 - c_1)(1 - c_2)$ in our example.

Theorem 1 *Mixed bundling in the monopoly case is optimal unless the production cost of either good takes its maximum value (one). The monopolist's net surplus from bundling and the bundling discount factor* ϵ $(\epsilon = p_1 + p_2 - p_b)$ *are both decreasing functions of her marginal costs (tables 1 and 2), while the individual prices offered by the monopolist to the trading partner for goods 1 (2) are increasing (decreasing) in* c_1 *and decreasing (increasing) in* c_2 *(table 3).*

Proof. *See the Appendix.* ■

Both $\sum(p)$ and ϵ are decreasing functions of the monopolist's marginal production costs. If either (or both) marginal cost(s) take the upper value of one, then mixed bundling ceases to be surplus enhancing and the optimal value of ϵ is equal to zero. It can be seen that the optimal net surplus achieved by P reaches a peak value of $\Sigma(p) = 0.5492$ when $c_1 = c_2 = 0$, by setting a bundling discount factor of $\epsilon = 0.4714$,while the optimal prices in this case are $p_1^* = p_2^* = \frac{2.}{3}$ (table 3), while the price of the bundle is $p_b = \frac{2}{3} + \frac{2}{3} - 0.4714 = 0.862$.

3 The exchange case

In this case P sells good 1 and buys good 2, whereas a firm in M buys good 1 and sells good 2. The quality of good 2 is uncertain but the quality of good 1 is certain, as the latter is produced by P (a firm in a DC). The first uncertainty parameter θ_1 is the valuation given to good 1 by the firm in M. The second uncertainty parameter θ_2 is the cost to the firm in M of producing good 2. As mentioned earlier P's valuation of good 2 is equal to $\nu(\theta_2) = \alpha + \beta\theta_2$, where $0 \preceq \alpha + \beta \preceq 1$. P produces good 1 at cost c. Thus the gross surplus functions are given by

$$S_1(\theta_1, \theta_2) = \theta_1, S_2(\theta_1, \theta_2) = -\theta_2$$

Theorem 2 *Mixed bundling in the exchange case is optimal for $\alpha > 0$, $c < 1$. If either $\alpha = 0$ or $c = 1$, then non-bundling is optimal for P. Both P's net surplus and the bundling discount factor are decreasing functions of P's production cost (c) and the intensity of partner preference and increasing functions of α. The absolute values of the prices offered to the trading partner for goods 1 and 2 are increasing in α, β and c.*

Proof. *See the Appendix.* ■

It is interesting to note that in the exchange case θ_2 has a dual role: it is both a participation constraint for the firm in M as well as for P, as the latter's valuation of good 2 depends on this variable; hence in the case of no bundling, for example, M will not trade with P in good 2 unless $-p_2 \succeq \theta_2$, and P will not trade with a firm in M unless $-p_2 \preceq \alpha + \beta\theta_2$. This implies a net surplus of $\int\limits_0^{-p_2} (\alpha + \beta\theta_2 + p_2) d\theta_2$, which is maximized for $-p_2^* = \frac{\alpha}{2-\beta}$. In the case of mixed bundling of the transactions of sell and buy, P's surplus maximising values of p_1, p_2 and ϵ are derived by solving the first and second order conditions (see the appendix) and then solved analytically for different values of α, β and $c = 0$ in tables 4-6 and for $c = 0.6$ in tables 7-9. Note that the sufficient condition for ensuring a local gain from bundling in this case written in the terms of MMW specification is $(p_1^* - c)h_1(p_1^*)[H_2(-p_2^*)] > 0$ as here the firm in M sells good 2 rather than buys it[3].

If informational asymmetry is minimal ($\alpha = 1$, $\beta = 0$), we are in the special case of no *partner preference* and θ_2 is no longer a participation determinant for firm P ; in such a case the value of reciprocal exchange bundling will reach a global surplus maximum for $c = 0$ equal to $\Sigma(p) = 0.5492$ (table 5). P offers its own good at a separate price 0.67 which is

[3] This local gain is equal to $\xi = \frac{\alpha(1-c)}{2(2-\beta)}$ in our example and again clearly positive.

double the price it offers for its partner's good but also finds it optimal to offer a substantial enough bundling premium ($\epsilon = 0.47$), while leads to a negative profit maximising bundled price ($p_b = \frac{2}{3} - \frac{1}{3} - 0.4714 = -0.138 < 0$). At this point both p_1 (table 6) and the net surplus are equal to their corresponding peak values in the monopoly case of P selling both goods.

At the other extreme, if either $a = 0$ or $c = 1$, then $\epsilon = 0$ meaning than bundling is no longer optimal if P's marginal cost is equal to 1 or when the certainty component in the quality of the good sold by the partner is zero.

Our model presents countertrade in the form of commodity bundling within a reciprocal exchange, and shows that the existence of such an option in addition to the offering of two separate prices by P is profit enhancing relative to the absence of such a linked exchange option, with only two exceptions as noted in the previous paragraph. Hence, rather than viewing the price discrimination approach to countertrade as an alternative model to the information model of double moral hazard problems relating to the trading partner's credit constraints and the uncertain quality of the good it sells, we combine both by introducing the concepts of *transaction bundling* and *partner preference* (informational asymmetry) respectively. We find support for the view that a countertrade arrangement in B2B trade can produce higher profits than an arrangement in which only separate trading in each good is offered.

Moreover, the volume of trade also increases relative to the case of no bundling. P raises (reduces) the price p_1 ($|p_2|$) of the good it sells (buys) in relation to its unbundled value $p_1^*(|p_2^*|)$, thus decreasing the incentive of a firm in M to participate in $R_1(R_2)$, while offering a premium ϵ for the bundled transaction substantial enough to increase trade in both good 1 ($R_1^B + R_{12}^B$) and good 2 ($R_2^B + R_{12}^B$) relative to its unbundled size[4]. More specifically, the improvement in the trade for good 1 is an increasing function of both α and β, with the former's effect dominating over the latter's in the case of an equal change in the values of both in opposite directions, and a decreasing function of c. The maximum percentage increase achieved as compared to no bundling is equal to 58.1% for $\alpha = 1, \beta = c = 0$. On the other hand, the improvement for good 2 is decreasing in α and c and increasing in β,with β's effect dominating that of α's in the case of an equal change in the values of both in the same direction. It is now the minimum percentage increase in the volume of trade for good 2 which is equal to 20.3% for $\alpha = 1, \beta = c = 0$, while this reaches very high values for low values of α and c and high values of β (for example for $\alpha = 0.2, \beta = 0.8$, $c = 0$ this equals 58.1%), provided that $\alpha > 0$ (as for $a = 0$, bundling is no longer optimal for P) .

Finally, a value of $p_b = 0$ indicates that the countertrade transaction will take the form of a barter exchange. It can be shown that barter may occur only within a specific range of values: $\alpha \in [0.5, 1]$, $\beta \in [0, 0.5]$, $c \in [0, 0.25]$ resulting into a surplus maximising range of values for ϵ between $\left[\frac{1}{3}, \frac{2}{5}\right]$. More specifically, within this range of values ϵ is a decreasing and convex (concave) function of α (β and c). For $a = 1$ and $c = 0.25$, barter is optimal at $\epsilon = \frac{1}{3}$, while for $a = \beta = 0.5$ and $c = 0$ barter is optimal at $\epsilon = \frac{2}{5}$. Note that barter can not be a profit maximising strategy for P unless the degree of *partner preference* is equal to

[4] It is easy to numerically check that there is always an increase in the size of trade in both goods relative to its unbundled size by substituting for the bundled and unbundled surplus maximising prices and the corresponding optimal value of ϵ and calculating for these values $R_1^B + R_{12}^B - R_1^{UB} - R_{12}^{UB} = p_1^* - p_1 - \epsilon p_2 + \frac{\epsilon^2}{2}$ and $R_2^B + R_{12}^B - R_2^{UB} - R_{12}^{UB} = p_2^* - p_2 + \epsilon(1 - p_1) + \frac{\epsilon^2}{2}$ both of which were found to be always positive for the range of possible values for α, β and c.

or smaller than $\frac{1}{2}$, with marginal costs of production for firm P than do not exceed $\frac{1}{4}$. In other words a necessary condition for a barter contract to be a profitable way to conduct a bundled exchange is that the uncertainty component in the value of the good the firm in M sells does not exceed 50% of its overall value, and also unless P has a low marginal cost in producing good 1.

The above findings support the Marin and Schnitzer 2002 assertion that the extent to which barter (and more generally, in our case, bundled exchange) can be used by developing countries to overcome their creditworthiness constraints and enhance their trade volume is limited by the informational asymmetry that the uncertainty in the quality of good 2 introduces. We can view our α as the equivalent to π in the Marin and Schnitzer article, the anonymity measure which is directly related to the quality of good 2. A case of $\pi = 1$ corresponds to $\alpha = 1$ in our article; in their model it implies that the good produced by the trading partner in a LDC is anonymous and hence its quality is fully known, and in our model that there is no partner preference and hence the type of a trading partner from a developing country does not determine P's willingness to engage in an exchange with him.

On the other hand, provided that P finds it profit maximising to engage into a bundled exchange (for $\alpha \succ 0, c \prec 1$), the prospects of improving the volume of trade for the good sold by the firm in M (good 2) by bundling are more extensive the higher the partner preference for this good is (the greater the extent of quality uncertainty) and the lower the marginal cost of the good that P produces and bundles into the transaction.

3.1 The monopsony case

In this case P is the buyer of both goods from the market in M. The uncertainty parameters θ_1 and θ_2 are the costs of producing the two goods. Hence the gross surplus functions are $S_1(\theta_1, \theta_2) = -\theta_1$, $S_2(\theta_1, \theta_2) = -\theta_2$, $S_1^P = \alpha_1 + \beta_1\theta_1$, $S_2^P = \alpha_2 + \beta_2\theta_2$, where $0 \preceq \alpha_i + \beta_i \preceq 1$. As the surplus of P depends on whom it transacts with, this case also demonstrates *partner preference;* both θ_1 and θ_2 are dual role variables as participation constraints for both M as well as P. In the case of no bundling for example, M will not sell the good i to P unless $-p_i \succeq \theta_i$, and P in turn will not purchase the good unless $-p_i \preceq \alpha_i + \beta_i\theta_i$. This implies a net surplus of $\int_0^{-p_1} (\alpha_1 + \beta_1\theta_2 + p_1)d\theta_1 + \int_0^{-p_2} (\alpha_2 + \beta_2\theta_2 + p_2)d\theta_2$, which is maximized for $-p_1^* = \frac{\alpha_1}{2-\beta_1}$ and $-p_2^* = \frac{\alpha_2}{2-\beta_2}$.

In the case of mixed bundling of the transactions, P's surplus maximising values of p_1, p_2 and ϵ are derived by solving the first and second order conditions (see the appendix) and then solved analytically for different values of α_i and $\beta_i = 1 - \alpha_i$ in tables 10-12. Note that the sufficient condition for ensuring a local gain from bundling in this case written in the terms of MMW specification is $(\alpha_1 + \beta_1\theta_2 + p_1)h_1(-p_1^*)[H_2(-p_2^*)] > 0$ as here the P buys both goods[5].

Theorem 3 *Mixed bundling in the monopsony case is optimal for $\alpha_1, \alpha_2 > 0$. Both P's net surplus and the bundling premium are increasing functions of both α_1 and α_2, the degree to which the qualities of goods 1 and 2 are known to P (tables 10 and 11). The absolute values of the prices offered to the trading partner for goods 1 (2) are increasing (decreasing) in α_1 and decreasing (increasing) in α_2 (table 12).*

Proof. *See the Appendix.* ■

[5] This local gain is equal to $\xi = \frac{\alpha_1\alpha_2}{(2-\beta_1)(2-\beta_2)}$ in our example and again positive as both $\beta_1, \beta_2 > 0$.

Both the maximum net surplus ($\sum(p)$) and the bundling premium (ϵ) are decreasing functions of the monopsonist's degree of *partner preference.* Unless P's ability to ascertain the quality of either (or both) of the goods it considers buying equals zero, purchase bundling is optimal. On the other extreme, it is interesting to note that the optimal net surplus reaches a peak value of 0.5492 for $\alpha_1 = \alpha_2 = 1$ by setting a bundling premium of $\epsilon = 0.4714$ (and prices $p_1^* = p_2^* = -\frac{1}{3}$, $p_b = -\frac{1}{3} - \frac{1}{3} - 0.4714 = -1.138$), is identical to the peak net surplus value and discount factor in the monopoly case when $c_1 = c_2 = 0$ (with prices $p_1^* = p_2^* = \frac{2}{3}$, $p_b = 0.862$). However, for less extreme values of α_1 and α_2, both the net surplus as well as the bundling premium are greater than the corresponding ones in monopoly (translating tables 10 and 11 into 1 and 2 by setting $c_i = 1 - \alpha_i$).

The trade volume increases in the monopsony case relative to the case of no bundling. P reduces in absolute value both of the separate prices it offers to its trading partner ($|p_1|, |p_2|$) relative to their unbundled values ($|p_1^*|, |p_2^*|$), thus reducing the incentive for a firm in M to participate in R_1 and R_2, while offering a premium ϵ for the bundled transaction substantial enough to increase trade in both good 1 ($R_1^B + R_{12}^B$) and good 2 ($R_2^B + R_{12}^B$) relative to its unbundled size[6]. More specifically, the improvement in the trade for good 1 is more substantial the higher α_2 and the lower α_1 is, while the reverse is true for good 2. This is intuitive, as it implies that there is more room for an increase in the volume of trade through bundling , the greater the intensity of P's *partner preference* for the good in question is. For

[6] It is easy to numerically check that there is always an increase in the size of trade in both goods relative to its unbundled size by substituting for the bundled and unbundled surplus maximising prices and the corresponding optimal value of ϵ and calculating for these values $R_1^B + R_{12}^B - R_1^{UB} - R_{12}^{UB} = p_1^* - p_1 - \epsilon p_2 + \frac{\epsilon^2}{2}$ and $R_2^B + R_{12}^B - R_2^{UB} - R_{12}^{UB} = p_2^* - p_2 - \epsilon p_1 + \frac{\epsilon^2}{2}$ both of which were found to be always positive for the range of possible values for α_1 and α_2.

example, the percentage increase in the trade volume of good 1 (good 2) resulting from the introduction of bundling is equal to 58.1%[7] (4.4%) for $\alpha_1 = 0.2, \alpha_2 = 1$, and equal to 4.4% (58.1%) for $\alpha_1 = 1$, $\alpha_2 = 0.2$.

4 Conclusions

The purpose of this paper was to try to provide a general theoretical framework for explaining the continued prevalence of countertrade transactions. The cases of selling, exchanging or buying goods in a mixed bundling format were all proved to be optimal (profit maximising) strategies for the western firm, except if marginal costs of production were extremely high (the monopoly case), or the ability to assess the quality of the goods sold by LDC partners extremely low (the monopsony case), or both (the exchange case).

In the exchange case, the volume of trade for a firm in a developing country can be considerably increased by the introduction of bundling (by up to 58%) as compared to having the transactions conducted separately. Obviously, there is more room for an increase in the volume of trade for the good sold by a firm in a LDC (relatively to selling it separately) the larger the quality uncertainty component of the good it trades is, and the smaller the marginal cost of the good that the firm in the developed country bundles into the transaction is. Since the uncertainty in the value of the good traded by the firm in a LDC acts as a participation constraint for the firm in the developing country, it implies that the willingness of the latter to bundle will increase as the degree of partner preference, as well as its own marginal cost

[7] Equal to the improvement in good 1's volume of trade in the bundled exchange case for $\alpha = 1$ and $\beta = c = 0$.

for the good it sells, both decrease. The existence of such participation constraints means that barter can only emerge as an optimal strategy for a certain range of values of marginal cost (below 0.25) and the degree of ability to assess quality (above 50%). This suggests that barter is limited in its ability to help firms in LDCs overcome problems of creditworthiness by the existence of informational asymmetry the uncertainty in the quality of the goods sold by such partners introduces; however as our model shows it is only but a special case of bundled exchange.

The third and final part of our paper introduces the more unconventional idea of mixed bundling in purchases (monopsony) with the use of a bundling premium. It is shown that a monopsonist can profitably exploit the average willingness to sell by offering such a premium, even when there are no complementarities between the goods in the bundle. Our result is therefore important in explaining what format may be profitable for designing such "reverse" packaged transactions. Moreover, in terms of the volume of trade for a good with high quality uncertainty, this can be significantly increased if the transaction is bundled with another good sold by the LDC which has a low degree of quality uncertainty (a more basic good) acting as a collateral to the uncertainty of the other good purchased by the firm in a DC.

5 References

Adams, W.J. and Yellen, J.L. (1976). 'Commodity bundling and the burden of monopoly', *Quarterly Journal of Economics*, vol. 90, pp. 475-498

Ausubel, L.M. and Milgrom, P.R. (2002). 'Ascending auctions with package bidding', *Frontiers of Theoretical Economics,* vol. 1, pp. 1-42.

Caves, R.E. (1974). 'The economics of reciprocity; theory and evidence in bilateral trading arrangements', in W. Sellekaerts, eds., *International Trade and Finance: Essays in Honour of Jan Tinbergen*, London: Macmillan.

Caves, R.E. and Marin, D. (1992). 'Countertrade transactions: theory and evidence', *The Economic Journal,* vol. 102, pp. 1171-1183.

Chan, R. and Hoy, M. (1991). 'East-West joint ventures and buyback contracts', *Journal of International Economics*, vol. 30, pp. 331-343.

Charles Rivers Associates Incorporated & Market Design, Inc. (1998) 'Report 2: Simultaneous ascending auctions with package bidding', *submitted to Federal Communications Commission*, CRA no. 1351-00.

Choi, C.J. and Maldoom, D. (1992). 'A simple model of buybacks', *Economics Letters*, vol. 40, pp. 77-82.

Hammond, G.T. (1990). *Countertrade, Offsets and Barter in International Political Economy*, London: Pinter.

Lucking-Reiley, D. (2000). 'Auctions on the internet: What's being auctioned, and how?', *Journal of Industrial Economics*, vol. 48, pp.227-252.

McAfee, R.P., McMillan, J. and Whinston, M.D. (1989). 'Multiproduct monopoly, commodity bundling, and correlation of values', *Quarterly Journal of Economics*, vol. 114, pp. 371-383.

Marin, D. and Schnitzer, M. (1995). 'Tying trade flows: A theory of countertrade with evidence', *The American Economic Review,* vol. 85, pp. 1047-1064.

Marin, D. and Schnitzer, M. (1998). 'Economic incentives and international trade', *European Economic Review*, vol. 42, pp. 705-716.

Marin, D. and Schnitzer, M. (2002). 'The economic institution of international barter', *The Economic Journal*, vol. 112, pp. 293-316.

Whinston, M. (1990). 'Tying, foreclosure and exclusion' *American Economic Review,* vol. 80, pp. 837-857.

6 APPENDIX

6.1 Proof of Theorem 1

The first step is to identify the sets of inequalities defining regions R_0, R_1, R_2, R_{12}:

$$R_0(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : p_1 > \theta_1, p_2 > \theta_2, p_b > \theta_1 + \theta_2\};$$

$$R_1(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > p_1, p_b - p_1 > \theta_2\};$$

$$R_2(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_2 > p_2, p_b - p_2 > \theta_1\};$$

$$R_{12}(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > p_b - p_2, \theta_2 > p_b - p_1, \theta_1 + \theta_2 > p_b\}.$$

Based on these, the net surplus to P is given by:

$$\begin{aligned}
\sum &= \int_0^{p_b - p_1} \int_{p_1}^{1} (p_1 - c_1)\, d\theta_1 d\theta_2 + \int_0^{p_b - p_2} \int_{p_2}^{1} (p_2 - c_2)\, d\theta_2 d\theta_1 + \\
&\quad \int_{p_b - p_1}^{1} \int_{p_1}^{1} (p_b - c_1 - c_2)\, d\theta_1 d\theta_2 + \int_{p_b - p_2}^{p_1} \int_{p_b - \theta_1}^{1} (p_b - c_1 - c_2)\, d\theta_2 d\theta_1 \\
&= (p_1 - c_1)(1 - p_1)(p_b - p_1) + (p_2 - c_2)(1 - p_2)(p_b - p_2) + \\
&\quad (p_b - c_1 - c_2)\left((1 - p_b + p_1)(1 - p_b + p_2) - \frac{1}{2}(p_1 + p_2 - p_b)^2\right)
\end{aligned}$$

By introducing the bundle discount ϵ, $\epsilon = p_1 + p_2 - p_b$, P's net surplus $\sum$ can be expressed in terms of p_1, p_2 and ϵ as

$$\begin{aligned}\sum &= [(p_1 - c_1)(1 - p_1) + (p_2 - c_2)(1 - p_2)] + \\ &[(p_2 - c_2)(1 - p_1) + (p_1 - c_1)(1 - p_2) - (1 - p_1)(1 - p_2)]\,\epsilon + \\ &\frac{1}{2}\left[3(p_1 + p_2) - (c_1 + c_2) - 4\right]\epsilon^2 - \frac{1}{2}\epsilon^3\end{aligned}$$

The next step is to calculate P's optimum prices without bundling, i.e. set $\epsilon = 0$. The net surplus without bundling is given by:

$$\textstyle\sum_0 = (1 - p_1)(p_1 - c_1) + (1 - p_2)(p_2 - c_2)$$

The optimal prices p_1^*, p_2^* for the two goods without bundling are given by the first order conditions $\frac{\partial \sum_0}{\partial p_1} = \frac{\partial \sum_0}{\partial p_2} = 0$ (while it is easy to check that the second order conditions are also satisfied):

$$p_i^* = \frac{1}{2}(1 + c_i) \quad i = 1, 2$$

In order to derive the net surplus maximising level of bundling, we need to set

$$\frac{\partial \sum}{\partial p_1} = \frac{\partial \sum}{\partial p_2} = \frac{\partial \sum}{\partial \epsilon} = 0$$

for deriving the three first order conditions, while the three second order conditions require that:

$$-2 < 0$$

$$\det\begin{bmatrix} -2 & -3\epsilon \\ -3\epsilon & -2 \end{bmatrix} = 4 - 9\epsilon^2 > 0 \Longrightarrow \epsilon < \frac{2}{3}$$

$$\det\begin{bmatrix} -2 & -3\epsilon & -3(p_2-\epsilon)+c_2+2 \\ -3\epsilon & -2 & -3(p_1-\epsilon)+c_1+2 \\ -3(p_2-\epsilon)+c_2+2 & -3(p_1-\epsilon)+c_1+2 & 3(p_1+p_2-\epsilon)-c_1-c_2-4 \end{bmatrix} < 0 \Longleftrightarrow$$

$$(9\epsilon^2-4)(3(p_1+p_2-\epsilon)-(c_1+c_2))+6\epsilon(3p_1c_2+3p_2c_1-9p_1p_2-c_2c_1)+$$

$$+(\sqrt{18}p_1-\sqrt{2}c_1)^2+(\sqrt{18}p_2-\sqrt{2}c_2)^2<0$$

Using the DERIVE mathematical package we have calculated for different values of c_1 and c_2 the net surplus maximising value of ϵ (table 1), the maximum net surplus value (table 2) and the optimal prices (p_1, p_2) (table 3). It can be seen that the optimal net surplus reaches a peak value of 0.5492 for $c_1 = c_2 = 0$ by setting a bundling discount factor of $\epsilon = 0.4714$,while the optimal prices are $p_1 = p_2 = \frac{2}{3}$. Both $\sum(p)$ and ϵ are decreasing functions of the monopolist's marginal production costs. If either (or both) marginal cost takes its upper value of one, then mixed bundling ceases to be surplus enhancing and the optimal value of ϵ is equal to zero.

6.2 Proof of Theorem 2

The first step is to identify the sets of inequalities defining regions R_0, R_1, R_2, R_{12}:

$$R_0(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : p_1 > \theta_1, \theta_2 > -p_2, \theta_2 - \theta_1 > -p_b\};$$

$$R_1(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > p_1, \theta_2 > p_1 - p_b\};$$

$$R_2(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : -p_2 > \theta_2, p_b - p_2 > \theta_1\};$$

$$R_{12}(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : p_b - p_2 < \theta_1, p_1 - p_b > \theta_2, \theta_2 - \theta_1 < -p_b\}.$$

Based on these, the net surplus to P is given by:

$$\begin{aligned}\sum \;=\; & \int_{p_1-p_b}^{1}\int_{p_1}^{1} (p_1 - c_1)\, d\theta_1 d\theta_2 + \int_{0}^{-p_2}\int_{0}^{p_b-p_2} (\alpha + \beta\theta_2 + p_2)\, d\theta_1 d\theta_2 + \\ & \int_{0}^{p_1-p_b}\int_{p_1}^{1} (\alpha + \beta\theta_2 - c + p_b)\, d\theta_1 d\theta_2 + \int_{p_b-p_2}^{p_1}\int_{0}^{\theta_1-p_b} (\alpha + \beta\theta_2 - c + p_b)\, d\theta_2 d\theta_1\end{aligned}$$

Substituting $p_b = p_1 + p_2 - \epsilon$ and evaluating the integrals in P's net surplus gives:

$$\begin{aligned}\sum \;=\; & \left[(p_1 - c_1)(1 - p_1) - \alpha p_2 + (0.5\beta - 1)p_2^2\right] + \\ & \left[\alpha(1 - p_1) + (2 - \beta + c)p_2 + (\beta - 3)p_1 p_2\right]\epsilon + \\ & \frac{1}{2}\left[\alpha + \beta - c - 2 + (3 - \beta)(p_1 + p_2)\right]\epsilon^2 + \frac{1}{6}(\beta - 3)\epsilon^3\end{aligned}$$

Partially differentiating this with respect to α and β it is easy to show that $\sum$ is an increasing function of both parameters. The net surplus with no bundling, that is when $\epsilon = 0$, is given by

$$\sum_0 = (1-p_1)(p_1-c_1) - \alpha p_2 + (0.5\beta - 1)p_2^2$$

Maximising this with respect to p_1 and p_2 $\left(\frac{\partial \sum_0}{\partial p_1} = \frac{\partial \sum_0}{\partial p_2} = 0\right)$ gives the optimal unbundled prices p_1^*, p_2^* for the two goods (while it is easy to check that the second order conditions are also satisfied):

$$p_1^* = \frac{1}{2}(1+c) \quad p_2^* = \frac{\alpha}{\beta - 2}$$

In order to derive the net surplus maximising level in bundled exchange, we need to set

$$\frac{\partial \sum}{\partial p_1} = \frac{\partial \sum}{\partial p_2} = \frac{\partial \sum}{\partial \epsilon} = 0,$$

for deriving the three first order conditions, while the three second order conditions require that:

$$-2 < 0$$

$$\begin{aligned} \det \begin{bmatrix} -2 & \epsilon(\beta-3) \\ \epsilon(\beta-3) & \beta-2 \end{bmatrix} &> 0 \Longleftrightarrow \\ -(\beta-3)^2\epsilon^2 - 2\beta + 4 &> 0 \Longrightarrow \epsilon < \frac{\sqrt{2}\sqrt{2-\beta}}{3-\beta} \end{aligned}$$

Hence, for $\beta = 1 \Rightarrow \epsilon < 0.7071$ and for $\beta = 0 \Longrightarrow \epsilon < \frac{2}{3}$. Finally, the determinant of the 3x3 matrix below should be negative:

$$\begin{array}{ccc} -2 & \epsilon(\beta-3) & (\beta-3)(p_2-\epsilon)-\alpha \\ \epsilon(\beta-3) & \beta-2 & (\beta-3)(p_1-\epsilon)-\beta+c+2 \\ (\beta-3)(p_2-\epsilon)-\alpha & (\beta-3)(p_1-\epsilon)-\beta+c+2 & (3-\beta)(p_1+p_2-\epsilon)+\alpha+\beta-c-2 \end{array}$$

Using the DERIVE mathematical package we have calculated for $c = 0$ and different values of α and β the net surplus maximising value of ϵ (table 4), the maximum net surplus value (table 5) and the optimal prices (p_1, p_2) (table 6). It can be seen that the optimal net surplus reaches a peak value of 0.5492 for $\alpha = 1$ and $\beta = 0$, which is identical to the peak value in the monopoly case. We have also done the same for $c = 0.6$ (tables 7-9) resulting in substantially smaller values for ϵ and P's net surplus.

We have also calculated the conditions under which the optimal discount factor gives a bundled price of 0, i.e. the cases where barter is optimal. A barter optimal value of ϵ is an inverse and convex (concave) function of α (β and c) and takes a maximum value $\epsilon = 0.4$ for $\alpha = \beta = 0.5$ and $c = 0$, and a minimum value of $\epsilon = \frac{1}{3}$ for $\alpha = 1$, $\beta = 0$ and $c = 0.25$.

6.3 Proof of Theorem 3

As previously, the first step is to identify the sets of inequalities defining regions R_0, R_1, R_2, R_{12}:

$$R_0(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > -p_1, \theta_2 > -p_2, \theta_1 + \theta_2 > -p_b\};$$

$$R_1(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 < -p_1, \theta_2 > p_1 - p_b\};$$

$$R_2(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 > p_2 - p_b, \theta_2 < -p_2\};$$

$$R_{12}(p) = \{(\theta_1, \theta_2) \in [0,1]^2 : \theta_1 < p_2 - p_b, \theta_2 < p_1 - p_b, \theta_1 + \theta_2 < -p_b\}.$$

By identifying the above sets, the net surplus to P is given by:

$$\begin{aligned}
\sum &= \int_{p_1-p_b}^{1} \int_{0}^{-p_1} (\alpha_1 + \beta_1\theta_1 + p_1)\, d\theta_1 d\theta_2 + \int_{p_2-p_b}^{1} \int_{0}^{-p_2} (\alpha_2 + \beta_2\theta_2 + p_2)\, d\theta_2 d\theta_1 + \\
&\int_{0}^{p_1-p_b} \int_{0}^{-p_1} (\alpha_1 + \beta_1\theta_1 + \alpha_2 + \beta_2\theta_2 + p_b)\, d\theta_1 d\theta_2 \\
&+ \int_{p_1}^{p_2-p_b} \int_{0}^{-p_b-\theta_1} (\alpha_1 + \beta_1\theta_1 + \alpha_2 + \beta_2\theta_2 + p_b)\, d\theta_2 d\theta_1
\end{aligned}$$

Substituting $p_b = p_1 + p_2 - \epsilon$ and evaluating the integrals in P's net surplus gives:

$$\begin{aligned}
\sum &= -0.5\left[p_1^2(2-\beta_1) + p_2^2(2-\beta_2) + 2\alpha_1 p_1 + 2\alpha_2 p_2\right] - \\
&[(3-\beta_1-\beta_2)p_1p_2 + \alpha_1 p_2 + \alpha_2 p_1]\,\epsilon + \\
&\frac{1}{2}\left[\alpha_1 + \alpha_2 + (3-\beta_1-\beta_2)(p_1+p_2)\right]\epsilon^2 - \frac{1}{6}(3-\beta_1-\beta_2)\epsilon^3
\end{aligned}$$

The net surplus with no bundling, that is when $\epsilon = 0$, is given by

$$\textstyle\sum_0 = -0.5\left[p_1^2(2-\beta_1) + p_2^2(2-\beta_2) + 2\alpha_1 p_1 + 2\alpha_2 p_2\right]$$

Maximising this with respect to p_1 and p_2 $\left(\frac{\partial \sum_0}{\partial p_1} = \frac{\partial \sum_0}{\partial p_2} = 0\right)$ gives the optimal unbundled prices p_1^*, p_2^* for the two goods (while it is easy to check that the second order conditions are also satisfied):

$$p_1^* = \quad -\frac{\alpha_1}{2-\beta_1} \qquad p_2^* = -\frac{\alpha_2}{2-\beta_2}$$

In order to derive the net surplus maximising level in bundled exchange, we need to set

$$\frac{\partial \sum}{\partial p_1} = \frac{\partial \sum}{\partial p_2} = \frac{\partial \sum}{\partial \epsilon} = 0,$$

for deriving the three first order conditions, while the three second order conditions require that:

$$\beta_1 - 2 < 0$$

$$\det \begin{bmatrix} \beta_1 - 2 & -\epsilon(3-\beta_1-\beta_2) \\ -\epsilon(3-\beta_1-\beta_2) & \beta_2 - 2 \end{bmatrix} > \ 0 \Longleftrightarrow$$

$$(\beta_1 - 2)(\beta_2 - 2) - \epsilon^2(\beta_1 + \beta_2 - 3)^2 > 0$$

, which implies that for $\beta_1 = \beta_2 = 0 \Rightarrow \epsilon < 0.67$, for $\beta_1 = 1$, $\beta_2 = 0$ (or vice versa) $\Longrightarrow$ $\epsilon < 0.7071$, and for $\beta_1 = \beta_2 = 0 \Longrightarrow \epsilon < 1$. Finally, the determinant of the 3x3 matrix below should be negative:

$$\begin{matrix} \beta_1 - 2 & -(3-\beta_1-\beta_2)\epsilon & (\epsilon - p_2)(3-\beta_1-\beta_2) - \alpha_1 \\ -(\alpha_1+\alpha_2+1)\epsilon & \beta_2 - 2 & (\epsilon - p_1)(3-\beta_1-\beta_2) - \alpha_2 \\ (\epsilon - p_2)(3-\beta_1-\beta_2) - \alpha_1 & (\epsilon - p_1)(3-\beta_1-\beta_2) - \alpha_2 & (3-\beta_1-\beta_2)(p_1+p_2-\epsilon) + \alpha_1 + \alpha_2 \end{matrix}$$

Setting $\beta_i = 1 - \alpha_i$, we have calculated for different values of α_1 and α_2, the net surplus maximising value of ϵ (table 10), the maximum net surplus value (table 11) and the optimal prices (p_1, p_2) (table 12). It can be seen that the optimal net surplus reaches a peak value of 0.5492 for $\alpha_1 = \alpha_2 = 1$, by setting a bundling premium of $\epsilon = 0.4714$, which is identical to the peak value in the monopoly case when $c_1 = c_2 = 0$,although in that case ϵ plays the role of a bundling discount factor. All other things being equal, the optimal price p_1 (p_2) is increasing (decreasing) in α_1, and decreasing (increasing) in α_2 as seen in table 12. For $\alpha_1 = \alpha_2 = 1$, the optimal prices are $p_1 = p_2 = -\frac{1}{3}$. Both $\sum(p)$ and ϵ are decreasing functions of the monopsonist's degree of *partner preference* as measured by $(1 - \alpha_1)$ and $(1 - \alpha_2)$. If either (or both) of these take the upper value of one, then mixed bundling ceases to be surplus enhancing and the optimal value of ϵ is equal to zero.

Monopolist's optimal discount factor values

c_2/c_1	0	0.2	0.4	0.6	0.8	1
0	0.47	0.36	0.27	0.19	0.10	0
0.2	0.36	0.28	0.22	0.16	0.09	0
0.4	0.27	0.22	0.17	0.13	0.08	0
0.6	0.19	0.16	0.13	0.10	0.06	0
0.8	0.10	0.09	0.08	0.06	0.04	0
1	0	0	0	0	0	0

Table 1

Monopolist's maximum net surplus

c_2/c_1	0	0.2	0.4	0.6	0.8	1
0	0.55	0.44	0.36	0.30	0.26	0.25
0.2	0.44	0.34	0.26	0.21	0.17	0.16
0.4	0.36	0.26	0.19	0.13	0.10	0.09
0.6	0.30	0.21	0.13	0.08	0.05	0.04
0.8	0.26	0.17	0.08	0.05	0.02	0.0
1	0.25	0.16	0.09	0.02	0.01	0

Table 2

Prices (p_1, p_2) charged by the monopolist

c_2/c_1	0	0.2	0.4	0.6	0.8	1
0	(0.67,0.67)	(0.73,0.59)	(0.80,0.55)	(0.87,0.53)	(0.93,0.51)	(1,0.5)
0.2	(0.59,0.73)	(0.68,0.68)	(0.76,0.65)	(0.84,0.62)	(0.92,0.61)	(1,0.6)
0.4	(0.55,0.8)	(0.65,0.76)	(0.74,0.74)	(0.83,0.72)	(0.91,0.71)	(1,0.7)
0.6	(0.53,0.87)	(0.62,0.84)	(0.72,0.83)	(0.81,0.81)	(0.91.0.80)	(1,0.8)
0.8	(0.51,0.93)	(0.61,0.92)	(0.71,0.91)	(0.80,0.91)	(0.90,0.90)	(1,0.9)
1	(0.5,1)	(0.6,1)	(0.7,1)	(0.8,1)	(0.9,1)	(1,1)

Table 3

P's optimal discount factor values in exchange for c=0

β/α	0	0.2	0.4	0.6	0.8	1
0	0	0.10	0.19	0.27	0.36	0.47
0.2	0	0.13	0.23	0.33	0.45	
0.4	0	0.16	0.29	0.42		
0.6	0	0.21	0.37			
0.8	0	0.29				
1	0					

Table 4

P's maximum net surplus in exchange for c=0

β/α	0	0.2	0.4	0.6	0.8	1
0	0.25	0.26	0.30	0.36	0.44	0.55
0.2	0.25	0.26	0.31	0.37	0.47	
0.4	0.25	0.27	0.32	0.40		
0.6	0.25	0.27	0.33			
0.8	0.25	0.28				
1	0.25					

Table 5

Prices (p_1, p_2) charged by P in exchange for c=0

β/α	0	0.2	0.4	0.6	0.8	1
0	$(0.5, 0)$	$(0.51, -0.07)$	$(0.53, -0.13)$	$(0.55, -0.20)$	$(0.60, -0.27)$	$(0.67, -0.33)$
0.2	$(0.5, 0)$	$(0.51, -0.07)$	$(0.54, -0.14)$	$(0.58, -0.21)$	$(0.64, -0.29)$	
0.4	$(0.5, 0)$	$(0.52, -0.08)$	$(0.55, -0.15)$	$(0.61, -0.23)$		
0.6	$(0.5, 0)$	$(0.53, -0.08)$	$(0.58, -0.17)$			
0.8	$(0.5, 0)$	$(0.54, -0.09)$				
1	$(0.5, 0)$					

Table 6

P's optimal discount factor values in exchange for c=0.6

β/α	0	0.2	0.4	0.6	0.8	1
0	0	0.06	0.10	0.13	0.16	0.19
0.2	0	0.07	0.11	0.15	0.18	
0.4	0	0.09	0.13	0.17		
0.6	0	0.11	0.16			
0.8	0	0.13				
1	0					

Table 7

P's maximum net surplus in exchange for c=0.6

β/α	0	0.2	0.4	0.6	0.8	1
0	0.04	0.05	0.08	0.13	0.21	0.30
0.2	0.04	0.05	0.09	0.14	0.23	
0.4	0.04	0.05	0.09	0.16		
0.6	0.04	0.06	0.10			
0.8	0.04	0.06				
1	0.04					

Table 8

Prices (p_1, p_2) charged by P in exchange for c=0.6

β/α	0	0.2	0.4	0.6	0.8	1
0	$(0.8, 0)$	$(0.80, -0.09)$	$(0.81, -0.18)$	$(0.83, -0.28)$	$(0.84, -0.38)$	$(0.87, -0.47)$
0.2	$(0.8, 0)$	$(0.81, -0.10)$	$(0.82, -0.20)$	$(0.83, -0.31)$	$(0.86, -0.42)$	
0.4	$(0.8, 0)$	$(0.81, -0.11)$	$(0.82, -0.23)$	$(0.85, -0.35)$		
0.6	$(0.8, 0)$	$(0.81, -0.13)$	$(0.83, -0.26)$			
0.8	$(0.8, 0)$	$(0.82, -0.15)$				
1	$(0.8, 0)$					

Table 9

Monopsonist's optimal bundling premium values for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

α_2/α_1	0	0.2	0.4	0.6	0.8	1
0	0	0	0	0	0	0
0.2	0	0.20	0.25	0.27	0.28	0.29
0.4	0	0.25	0.31	0.35	0.36	0.37
0.6	0	0.27	0.35	0.39	0.41	0.42
0.8	0	0.28	0.36	0.41	0.44	0.45
1	0	0.29	0.37	0.42	0.45	0.47

Table 10

Monopsonist's maximum net surplus for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

α_2/α_1	0	0.2	0.4	0.6	0.8	1
0	0	0.02	0.06	0.11	0.18	0.25
0.2	0.02	0.04	0.08	0.14	0.20	0.28
0.4	0.06	0.08	0.13	0.19	0.26	0.33
0.6	0.11	0.14	0.19	0.25	0.32	0.40
0.8	0.18	0.20	0.26	0.32	0.39	0.47
1	0.25	0.28	0.33	0.40	0.47	0.55

Table 11

Prices (p_1, p_2) charged by the monopsonist for $\beta_1 = 1 - \alpha_1$ and $\beta_2 = 1 - \alpha_2$

α_2/α_1	0	.2	.4	.6	.8	1
0	$(0, 0)$	$(-.17, 0)$	$(-.29, 0)$	$(-.38, 0)$	$(-.44, 0)$	$(-.50, 0)$
.2	$(0, -.17)$	$(-.14, -.14)$	$(-.25, -.13)$	$(-.33, -.11)$	$(-.40, -.10)$	$(-.45, -.09)$
.4	$(0, -.29)$	$(-.13, -.25)$	$(-.22, -.22)$	$(-.30, -.20)$	$(-.36, -.18)$	$(-.42, -.17)$
.6	$(0, -.38)$	$(-.11, -.33)$	$(-.20, -.30)$	$(-.27, -.27)$	$(-.33, -.25)$	$(-.38, -.23)$
.8	$(0, -.44)$	$(-.10, -.40)$	$(-.18, -.36)$	$(-.25, -.33)$	$(-.31, -.31)$	$(-.36, -.29)$
1	$(0, -.50)$	$(-.09, -.45)$	$(-.17, -.42)$	$(-.23, -.38)$	$(-.29, -.36)$	$(-.33, -.33)$

Table 12