Investigation of optimal parameters for finite element solution of the forward problem in magnetic field tomography based on magnetoencephalography

K Y Aristovich[1], S H Khan[1], and A I Borovkov[2]

[1]School of Engineering and Mathematical Sciences, City University London, Northampton Square, London EC1V 0HB, UK

[2]St Petersburg State Polytechnic University, Polytechnicheskaya Street 29, St Petersburg, 195251, Russia

kirill.aristovich.1@city.ac.uk

Abstract. This paper presents an investigation of optimal parameters for finite element (FE) solution of the forward problem in magnetic field tomography (MFT) brain imaging based on magnetoencephalography (MEG). It highlights detailed analyses of the main parameters involved and evaluates their optimal values for various cases of FE model solutions (e.g., steady-state, transient, etc.). In each case, a detail study of some of the main parameters and their effects on FE solution and its accuracy are carefully tested and evaluated. These parameters include: total number and size of 3D FE elements used, number and size of elements used in surface discretisation (of both white and grey matters of the brain), number and size of elements used for approximation of current sources, number of anisotropic properties used in steady-state and transient solutions, and the time steps used in transient analyses. The optimal values of these parameters in relation to solution accuracy and mesh convergence criteria have been found and presented.

1. Introduction

Magnetic field tomography (MFT) is a relatively new imaging modality which involves localisation and subsequent imaging of active areas in the brain by measuring the extremely weak neuromagnetic fields (10–100 fT) produced by neuronal currents in these areas associated with cognitive processing (magnetoencephalogram). This approach, called the magnetoencephalography (MEG) technique (recording of magnetic fields produced by electrical activity in the brain) is the only truly noninvasive method which could provide information about functional brain activity. Compared to other imaging modalities it is the only imaging modality that combines high temporal with high spatial resolution.

The forward problem in MFT involves the computation of magnetic field distribution from known neuronal current generators (sources) in the brain [1-3]. The inverse problem localizes and images these generators using MEG data measured around the head and the data obtained from the forward solution. An accurate solution of the forward problem is also needed for design, configuration and placement of SQUID sensors, used to measure the neuromagnetic fields around the head, and which constitute the sensing subsystem of the MFT system.

The FE solution of the forward problem involves very accurate modelling of the human brain together with methodologies for accurate evaluation of various solutions. However, these aspects are

rarely covered in the current literature, especially with respect to solution errors and accuracy of FE parameters used for simulations. Human brain is an extremely complex structure for FE modelling both in terms of geometric and material parameters [4, 5]. Although accurate incorporation of these has obvious effects on FE solution accuracy, it needs to be balanced against computational time and complexity involved. Based on various FE analyses, this paper describes optimisation of various parameters highlighting optimal requirements for FE models of the brain and the current sources used.

2. Finite element models and strategy for parameter optimisation

2.1. Model description

The FE models used in this study are based on the methodologies and tools developed by the authors for building realistic models of the human brain [6, 2]. Parametrical solid CAD spline models are used for further FE discretisation which incorporates geometrical accuracy limited only by existing MRI equipment. A typical outline of the solid model used is presented in the Figure 1.

The FE mesh is created, firstly by standard discretisation method involving surface meshing of brain features used and then by an optimised procedure for building 3D volume elements [7-9]. Here the surface mesh, made up of triangular elements is fully closed and the volume is divided into 3D tetrahedral elements. Although the size of these 3D elements can be defined independent of the surface mesh, to ensure the connectivity and transition of regions, the edge-lengths of 3D elements are matched with those of 2D surface elements. Also the 2D mesh is defined to be uniform within 50% variation of the average element size apart from small regions to be occupied by source currents. Therefore in optimisation analyses the overall FE discretisation is almost fully dependent upon initial size of the surface mesh. It is convenient, however to define FE mesh density by the total number of 3D elements in the model [8, 9] which is also the case in this paper.

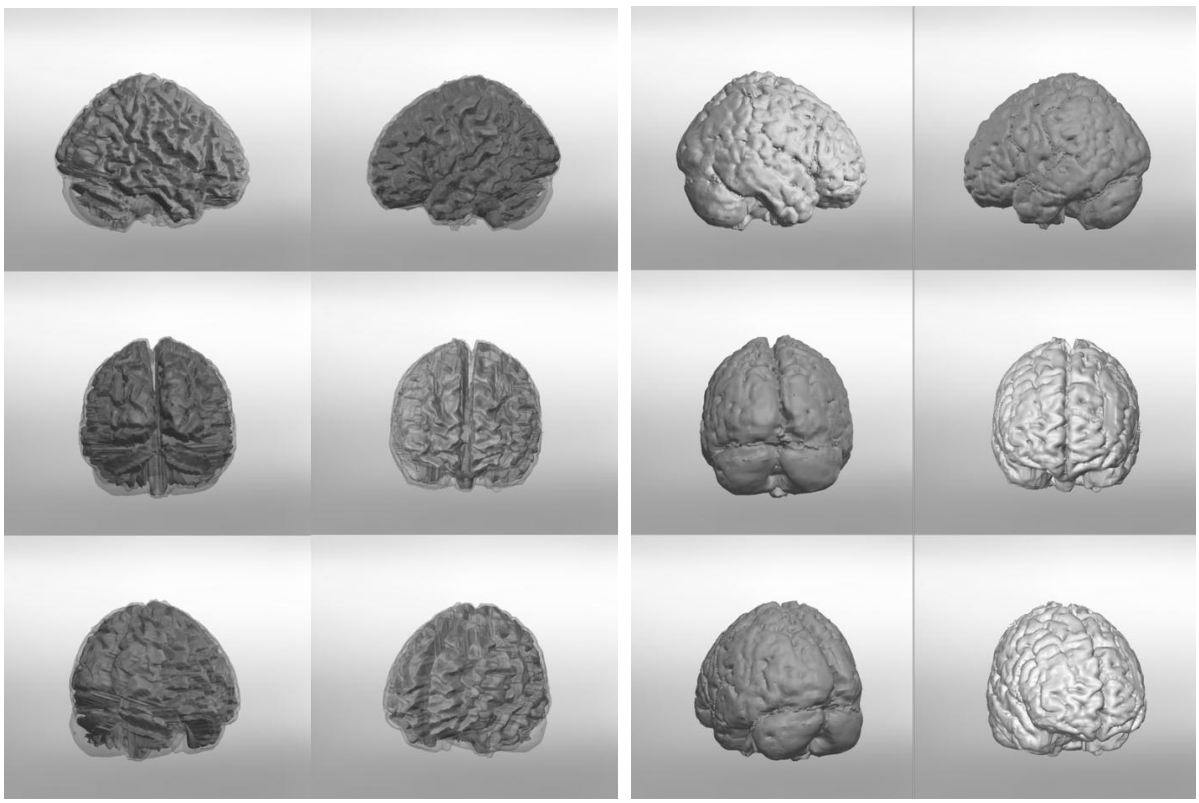

Figure 1. Initial solid model of the human brain: outline of white (left) and grey (right) matters.

2.2. Description of the test problem

As mentioned earlier, the forward problem in MFT based on MEG involves modelling and computation of magnetic fields produced by known neuronal currents on the brain. These currents are modelled as current sources within the FE model domain and can be approximated in a number of different ways [3]. However, they are all based on placing the current source inside the geometric model domain and calculating the resulting magnetic field inside as well as outside the actual brain geometry. This magnetic field is measured on an appropriate simulated detection surface placed outside the head that represents the real measurement surface for sensor arrays (e.g., SQUID sensors) in MFT brain imaging [10].

Because of the linearity of magnetic field problem involved in the forward solution in MFT, superposition principle can be applied to evaluate the effects of multiple current sources based on multiple solutions obtained for a single current source appropriately placed inside the brain geometry. Hence for parameter optimisation of a single current source is used for the test problem (Figure 2).

2.3. Method for the evaluation of optimal parameters

The criterion for mesh convergence was used as a general method for FE optimisation and the refinement of solution accuracy [8]. The method involves successive refinement of FE parameters until the solution achieves a required, predefined accuracy (Figure 3). Thus the outcome of such modelling tests will evaluate the behaviour of model solutions and, hence practical model optimisation can be performed based on required data analyses.

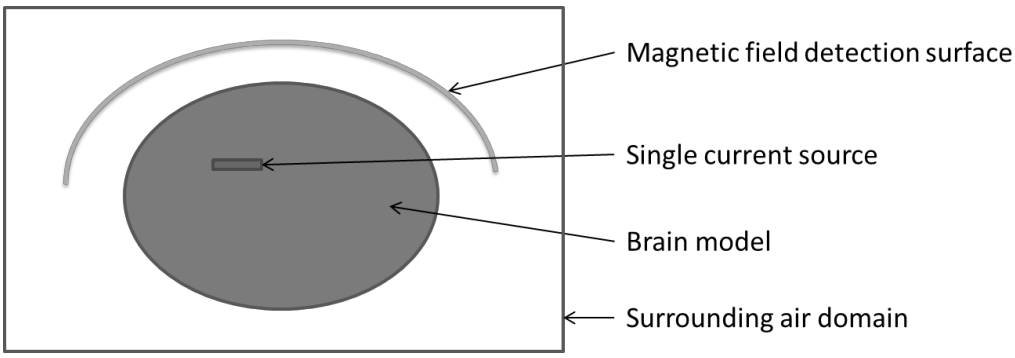

Figure 2. Schematic for the test problem solution

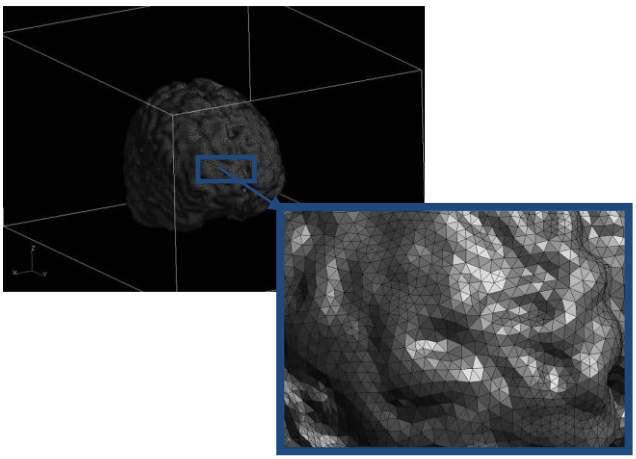

Figure 3. Example of FE surface discretisation.

3. Key parameters for finite element simulations

In FE simulations a number of key parameters are used which require careful analysis and optimisation for accurate and efficient solution. For FE solution of the forward problem presented in this paper the following parameters have been considered.

3.1. Overall mesh density (total number of elements, N and their distribution)

The accuracy of FE solution is inherently dependent upon the total number of FE nodes and hence, FE elements, *N* by which the problem domain is discretised for solution. For a given problem there always exists a maximum threshold number for *N* beyond which any further increase in elements does not result in corresponding increase in solution accuracy. Also the distribution of elements and the uniformity of mesh density in the problem domain play an important role in solution accuracy. A large number of FE elements beyond the above threshold number will lead to unnecessary computational overhead without essentially increasing solution accuracy. In this work the following geometric shapes are considered critical for surface meshing:

– Implemented current source surface
– White matter surface
– Grey matter surface
– Detection electrodes surface
– Surrounding air surface

The critical point of the problem is the variation of mesh size within the problem domain. As mentioned above, the uniformity of mesh density is considered important for FE analysis [11]. Thus the element sizes within the problem domain are kept almost the same except for the elements in and around the current sources where further refinement is made to account for their very small size. This leads to a clear dependence between the number of surface elements and the total number of elements in the model, *N*.

3.2. Mesh size near the current source and its discretisation

As was mentioned earlier, the current source in MFT is relatively small in size (in microns) compared to the total brain model. In some cases submodelling techniques can be used to improve solution accuracy [12]. However, to do this initial courser mesh must satisfy accuracy requirements and mesh refinement is needed within the region occupied by the current source. In this work, the number of elements within the current source region is given by the parameter *M*.

3.3. Number of anisotropic element properties in the model, P

In case of Section 4 below, conductivity of the brain material plays a major role and, therefore has to be carefully considered [13]. Due to high complexity and anisotropy level, each element has to have its own material properties which leads to a complicated structure of matrix for material properties. Reducing the number of material properties (*P*) in the model can result in significant reduction in computational time, especially at the first step of matrix assembly [9].

3.4. Time step size, T for transient cases

In case of transient solution, the temporal resolution is important because of integration procedure. Taking into account characteristic speed of signal propagation inside the neurons the optimal time step *T* for numerical integration can be evaluated.

4. Quasistatic magnetic field analysis I

For this analysis in linear FE solution domain, the solution accuracy to a large extent depends on the mesh quality, especially on the overall mesh density. The current source is given in terms of interface/boundary conditions, which does not require special considerations for meshing regions in and around the current source [2]. A typical FE mesh is shown in Figure 4 together with the position

and configuration of the sensor detection surface. The current source was placed approximately in the centre of the brain model.

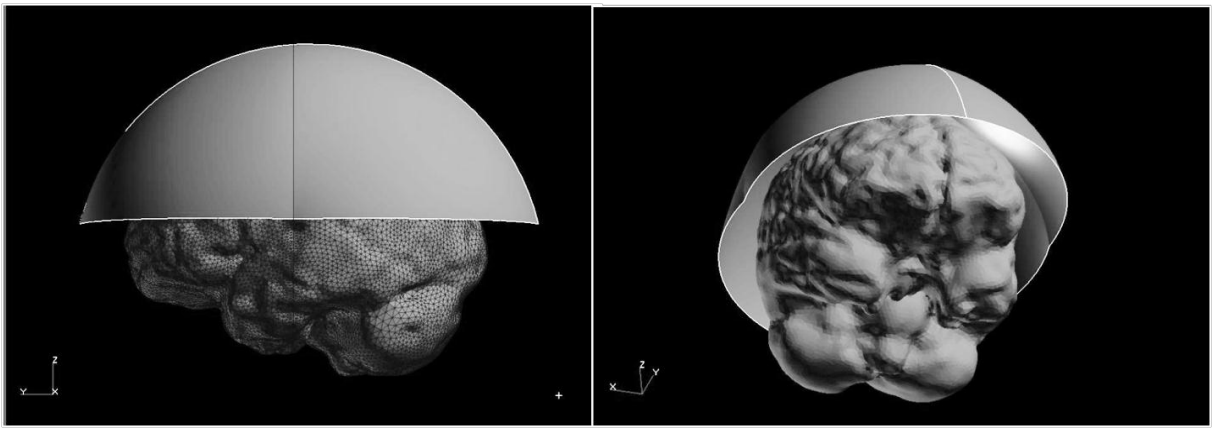

Figure 4. Surface mesh for the brain model and sensor detection surface corresponding to Section 4.

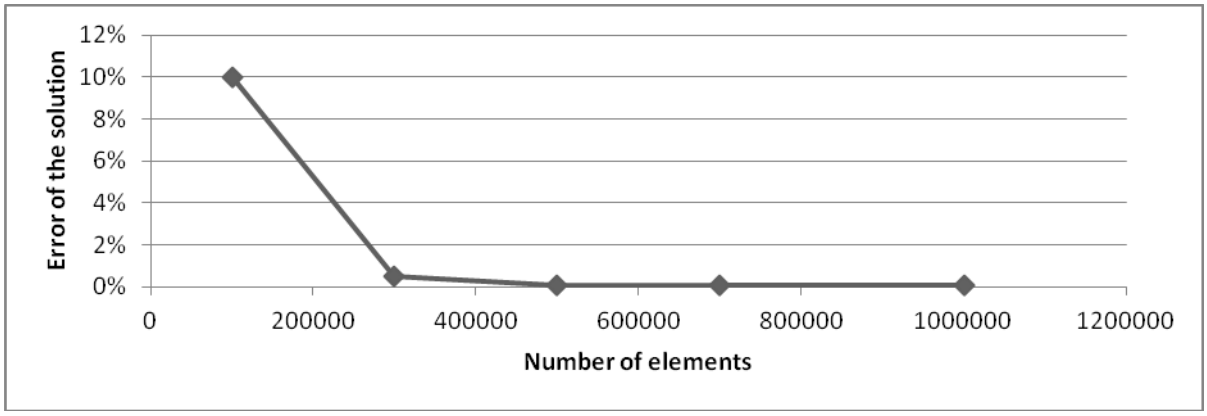

Figure 5. Determination of optimal number of FE elements, *N* for the test problem in Section 4.

In this case, the maximum value of the magnetic field flux density on the detection surface was taken as the solution parameter for testing modelling accuracy as a function of solution errors for successive solutions for various mesh densities. For comparison, precise analytical solutions were used to calculate FE solution error for a given mesh. The mesh convergence analysis shows a stable convergence as seen in Figure 5. The solution error was found to be minimum and stable at around 0.05% for the number of FE elements corresponding to about *N*=500k elements.

5. Quasistatic magnetic field analysis II

5.1. Description of the test problem

In this analysis the current source was approximated by a straight short conductor of quadratic cross-sectional area (Figure 6). The conductivity values of the current source were varied starting from the conductivity of the copper (6×10^7 S/m) in order to give high conductivity difference between the source and the surrounding brain matter. The conductivity value was then uniformly decreased to the realistic one (100 S/m, 10 S/m and finally 0.33 S/m). The current source was placed approximately at the centre of the brain model (the same as in the case of Section 4 above). In this case the brain model incorporates complex anisotropic conductivity tensor for brain material properties [6].

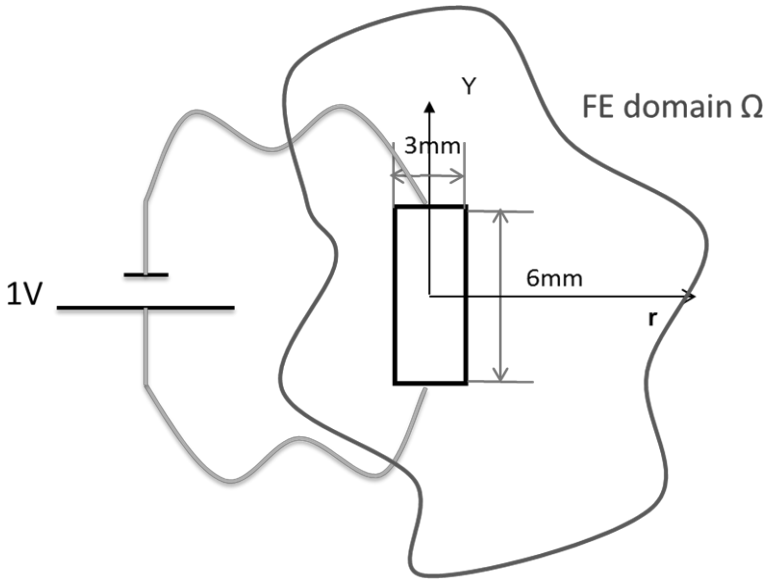

Figure 6. Current source approximation corresponding to FE analysis in Section 5.1.

5.2. Results of computer simulation for number of FE elements, N=500k

Initial simulations were performed which established the optimal number of elements for FE solution as N=500k. The characteristic number for source mesh density, M was taken as the number of elements along the longest edge of the current source; initially M=4. Results have been plotted as the variation of current density along the r-axis (Figure 6) of the current source (Figure 7). The corresponding analytical value for each solution was also calculated in form of total current flowing through the current source, and the solution error was calculated as the difference between analytical and numerical solutions (Figure 8).

The results show good agreement between analytical and numerical results for high conductivity ratios. However, in cases where conductivity of the current source was comparable to the average conductivity of surrounding media, FE results give unrealistic trend (Figure 7, bottom right graph). Here the current density peak is associated with inappropriate finite element mesh. Also this corresponds to a total current density difference of 500% between analytical and numerical solutions. These results show that both the total number of elements, N and the number of elements in the current source, M need to be increased together to improve solution accuracy.

5.3. Mesh convergence results

Full mesh convergence analyses were performed by varying the total number of elements N=500k-1600k and M=4-25. FE computations were performed for conductivity ratios from $1\text{-}10^8$. For each of the conductivity ratios the optimal number of elements, N together with characteristic current source mesh density, M were obtained which corresponded to a minimum mesh convergence ratio of 0.1%. These results are presented in Figure 9. It shows that for conductivity ratios near 1 the total number of elements required for accurate solution will exceed 1.5m. The current source in this case will be optimally divided into 20 elements along the longest edge. This corresponds to an approximate element size of 0.06 mm. This result essentially shows that the above FE approach for solving this particular problem is associated with high computational overhead in order to achieve the required accuracy of solution. However, as shown in [12] the same accuracy can be obtained by using submodelling approach which significantly decreases computational overhead.

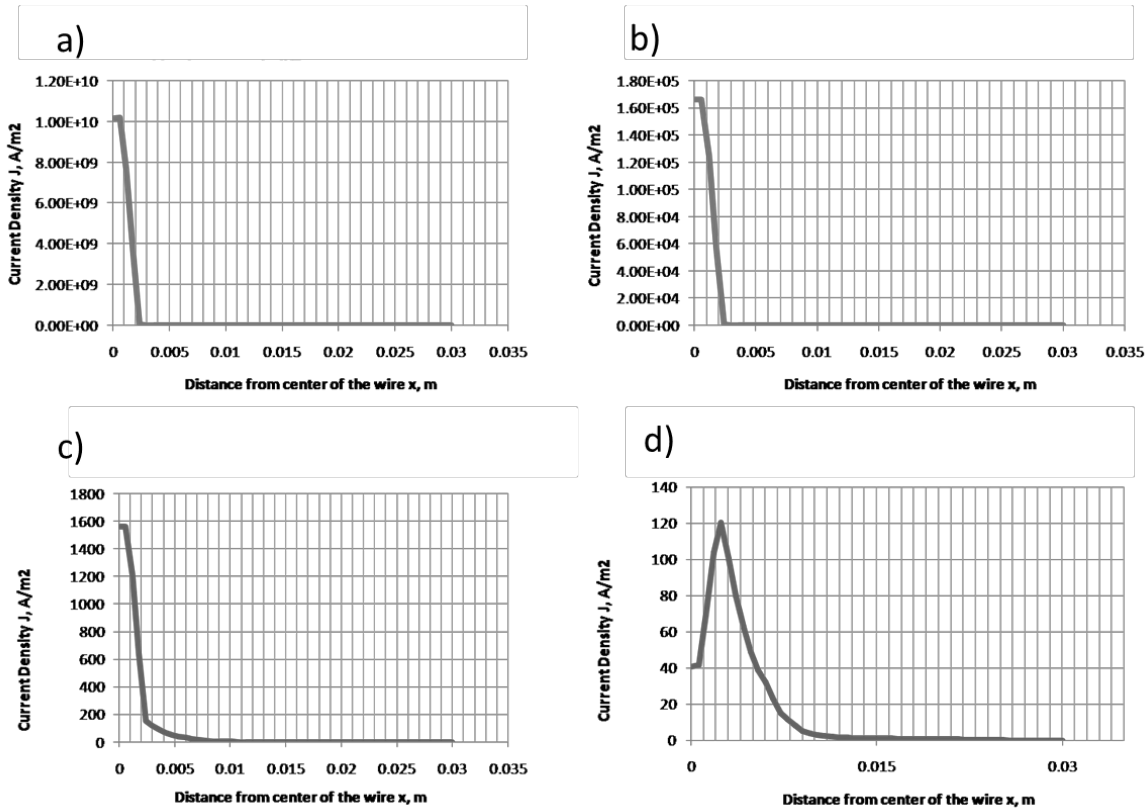

Figure 7. Variation of current density with distance for various conductivity values: a) Electric conductivity σ=6·10^7 S/m, resulting current I=8.15·10^4 A; b) σ=10^3 S/m, I=1.34 A; c) σ=10 S/m, I=2.45·10^{-2} A; d) σ=0.33 S/m, I=1.11·10^{-2} A.

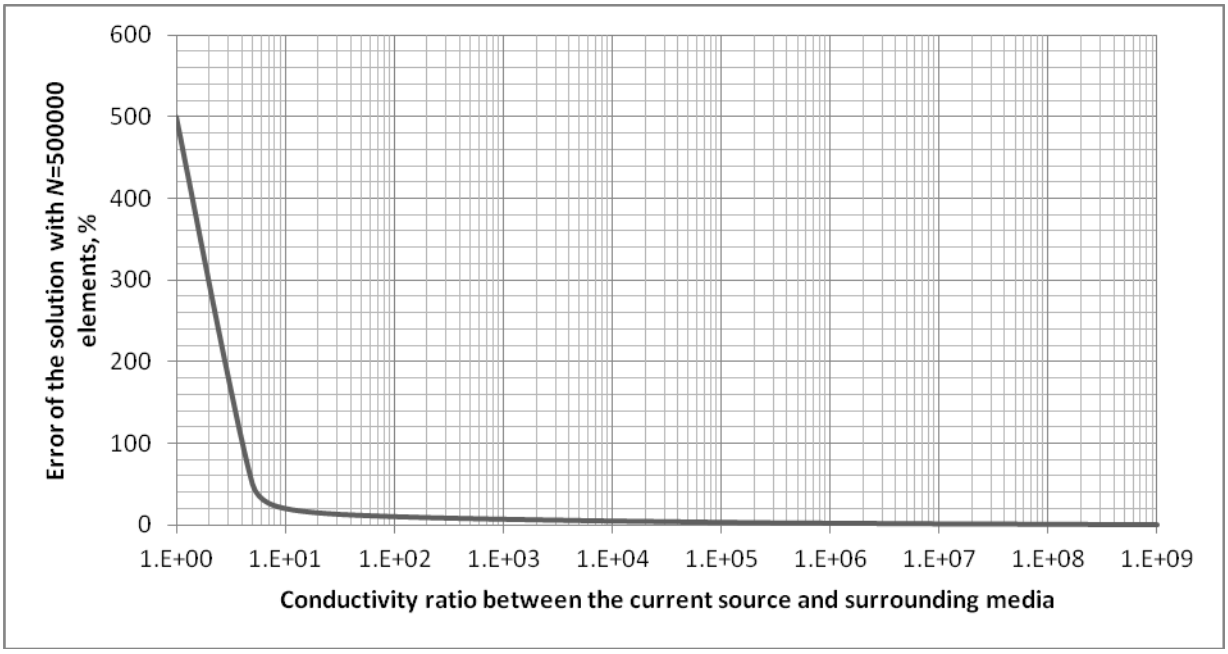

Figure 8. Variation of FE solution error with conductivity ratio for N=500k.

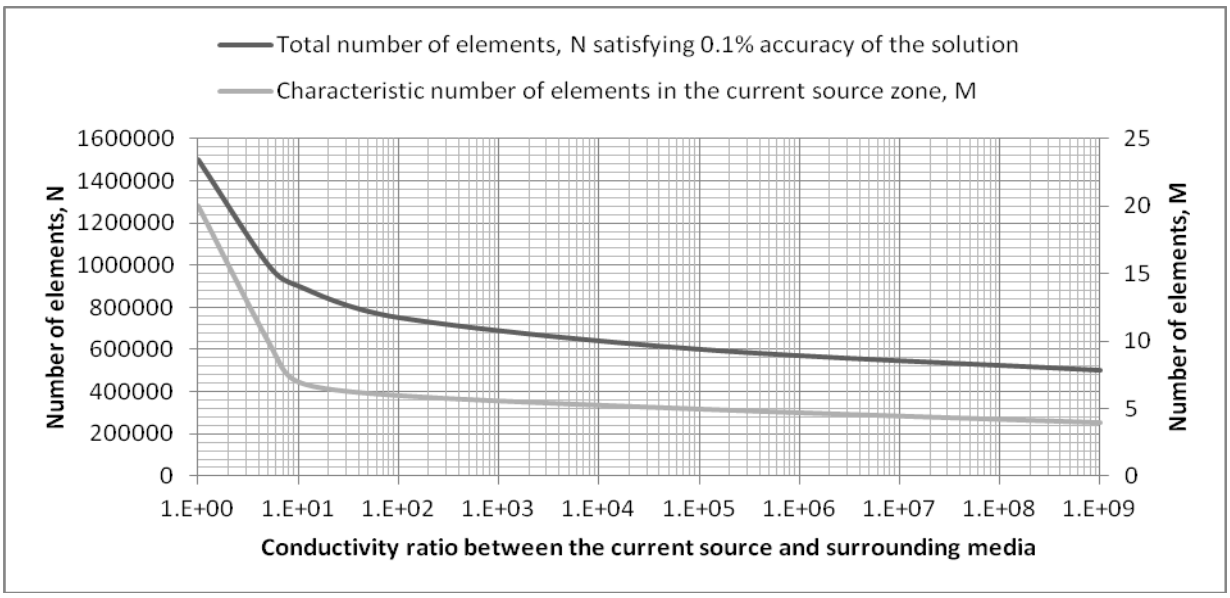

Figure 9. Optimal number of elements in the FE model *N* and characteristic number of elements in the current source *M* as a function of conductivity ratio.

5.4. Optimal number of material properties

The number of material properties *P* was also studied during mesh convergence analysis. The result is independent of the conductivity ratio. However, the number of common properties was found increased with increasing *N*. The variation of *P* as a function of *N* is shown in Figure 10.

6. Transient analysis

The transient analysis together with other parameters involves temporal resolution for numerical integration to be performed. The problem for transient cases was chosen exactly the same as in steady-state case, with the only difference in the application of voltage for source approximation as opposed to current elements. The voltage was applied to approximate the neuronal signal transmission with time along a given pathway. The exact parameters of this function can be found in [5]. The results obtained are presented in Figure 11.

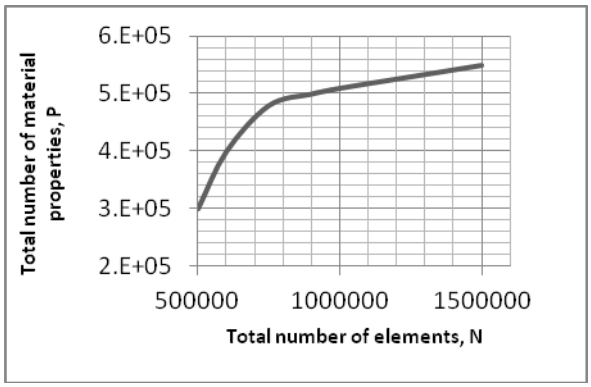

Figure 10. Optimal number of material properties in the model *P* as a function of total number of FE elements *N*.

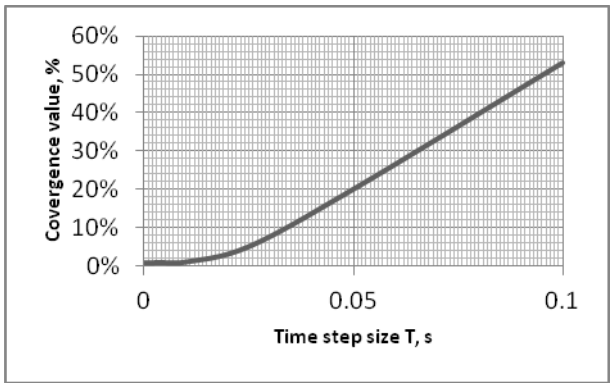

Figure 11. Convergence value as a function of the size of time step *T* in transient solutions.

7. Summary of optimal parameters and conclusions

The optimal parameters for FE solution of the forward problem in MFT based on MEG have been investigated in this paper. The summary of optimal parameters for each particular case is presented in Table 1. This can be used as a guide for forward solutions in MFT based on accurate FE modelling of the brain. In addition, parameters such as the number of surface elements and the average size of elements are also given to inform any follow-up simulations.

Table 1. Summary of optimal parameters.

Solution case	Total number of FE elements, *N*	Average size of elements, (mm)	Number of elements in the current source region, *M*	Average size of elements in the current source region (mm)	Average number of surface elements in the grey matter	Average number of surface elements in the white	Time step, *T*
Quasistatic field analysis I	500000	3	n/a	n/a	10000	40000	n/a
Quasistatic field analysis II	1500000	0.5	20	0.06	50000	130000	n/a
Transient electromagnetic	1500000	0.5	20	0.06	50000	130000	0.01s

8. References

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