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Are Positive Reactions to Bad News Plausible?
The Consideration of Fraud in Audit and Reporting Delays

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Abstract

I formulate a model to emphasize the fraud detection role of auditors in the financial market and relate the role to audit and financial reporting delays. In the model, an auditor considers whether to perform extended audit procedures after observing a red flag generated from regular audit procedures. An audit delay is represented by the event of extending audit procedures and manifested as a financial reporting delay observed by the market. I derive a simple closed-form condition characterizing when a positive market reaction to a delay is possible. The condition provides a theoretical basis for formulating empirically testable hypotheses. I discuss why the fundamental logic behind the counter-intuitive positive-reaction result also applies to other contexts such as internal control weakness disclosure. Documented evidence in the literature suggests that “positive reactions to bad news” (PR2BN) is a general phenomenon. I also discuss other empirical implications of the model, with suggestions for regression equation specifications. (JEL M49/M43/G32/K42)

Keywords: Market reaction, audit delay, financial reporting lag, red flag, fraud detection, SAS 99.

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1 Introduction

On June 4, 2009 came the news of the alleged fraud in Countrywide Financial, the largest mortgage loan provider in the US before the credit crunch hit (Morgenson 2009). The allegation to Countrywide reminds people of a long expected but unfulfilled role of auditors in the financial market: fraud detection (see e.g. Johnson 2010). In this paper, I formulate a model to emphasize this role and relate it to audit and financial reporting delays that have long been studied in the literature.\(^1\)

The Statement on Auditing Standards No. 99 (SAS 99) requires auditors to consider fraud in conducting audits (Ramos 2003).\(^2\) Similarly, the International Auditing and Assurance Standards Board (IAASB) also has a standard on auditors’ responsibilities relating to fraud in audits, namely the International Standard on Auditing 240 (ISA 240). Both standards concern fraud as an intentional act resulting in a misstatement in financial statements and distinguish between two types of fraud: (i) fraudulent financial reporting (e.g., falsification of accounting records); (ii) misappropriation of assets (e.g., false expenditures and theft of cash) (SAS 99, paragraph 6; ISA 240, paragraph 3). The recent literature on fraud predominantly focuses on the first category with little attention given to the second.\(^3\)

Obviously, fraudulent financial reporting is important and deserves a lot of attention. However, attention should also be given to asset misappropriation fraud. According to a 2010 survey by the Association of Certified Fraud Examiners (ACFE), nearly 90% of the reported fraud cases in the US involve some form of asset misappropriation (Association of Certified Fraud Examiners 2010). There is no doubt that in the US the median dollar losses are much higher in financial statement fraud than in asset misappropriation fraud. This however is not necessarily true outside the US (see e.g. KPMG 2010).\(^4\)

In another recent survey with 3,037 respondents from 54 countries, two-thirds of those respondents who have experienced economic crime in the year before the survey suffered asset misappropriation (PricewaterhouseCoopers 2009). Confining to US companies with accounting fraud, the Fraudulent Financial Reporting: 1998-2007 study commissioned by the Committee of Sponsoring Organizations of the Treadway Commission (COSO) finds that 14% of the cases also involve misappropriation of assets (Beasley, Carcello, Hermanson, and Neal 2010, Table 9, page 17). Thus, auditors should not neglect fraud risk involving asset misappropriation in carrying out their duties.

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\(^1\)Prior models of fraud detection such as Matsumura and Tucker (1992) have not related the fraud detection role of auditors to audit and financial reporting delays.

\(^2\)“Auditors will enter a much expanded arena of procedures to detect fraud as they implement SAS no. 99. The new standard aims to have the auditor’s consideration of fraud seamlessly blended into the audit process and continually updated until the audit’s completion.” (Ramos 2003).


\(^4\)For example, 2,035 fraud incidents involving internal perpetrators (management or non-management), with AUD68.7 million of losses, are reported in KPMG’s Fraud and Misconduct Survey 2010, Australia and New Zealand. Among them, 2,001 incidents with AUD57.2 million of losses are in the asset misappropriation categories (e.g., theft of cash, false invoicing, and cheque tampering), whereas 13 with AUD61,800 of losses are in the fraudulent statement categories (e.g., misstated asset/revenue and fraudulent education qualifications) (KPMG 2010, Appendix A, pages 30-31).
The model of this paper discusses diversion of firm resources, which is closer to asset misappropriation fraud as classified by US and international auditing standards. Consistent with SAS 99, the model has an auditor considering whether to perform extended audit procedures, depending on whether a red flag is observed after regular audit procedures. Plenty of examples suggest that financial reporting delays are often the consequence of extending audit procedures to investigate accounting irregularities. A financial reporting delay thus suggests to outside investors that a red flag was observed by the auditor and has triggered extended audit procedures.

The difference between regular and extended audit procedures can be related to the context of the Bagnoli, Kross, and Watts (2002) study. They observe that firms voluntarily disclose in advance the expected earnings announcement dates, and the market reacts adversely to each day of delay. Presumably the extent of regular audit procedures is ascertained during the audit planning phase and should be communicated to a client at the beginning of the whole process. Hence, the extent of regular audit procedures is likely to be a factor affecting the client firm’s expectation on when it can make the earnings announcement. A delay occurs in the model when the auditor extends audit procedures in response to some red flag observed after performing regular audit procedures. Since the delay is triggered by some red flag, it changes the market’s expectation about the existence of fraud in the firm. Consequently, the market might react adversely to the delay.

The analysis of this paper formalizes the intuitions above. Most important, it yields a new result that is not easy to discover without rigorously analyzing a formal model. Specifically, I show that a positive reaction to a delay is possible when it is perceived by market investors as “on the whole good

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5Despite the distinction between fraudulent financial reporting and asset misappropriation, their impacts on a firm can be similar. For example, an executive might prepare fraudulent financial reporting to exaggerate firm performance and thereby obtain a larger bonus, or avoid being fired. In either case, firm resources are lost as a result of the over-compensation to the executive not as competent as believed. More important, the misstated performance might prevent the firm from reacting timely and properly to the business conditions, which may lead to severe losses. On the other hand, fraudsters misappropriating a firm’s assets obviously would not let the company accountant know and state the correct firm value on financial reports. So similar misstatements in financial reporting and losses to a firm occur, regardless of the two types of fraud.

6For instance, “Sensormatic Electronics Corp.’s accounting firm has expanded the scope of its annual audit, forcing the company to delay its annual earnings report and raising the possibility of downward revisions to past earnings” (Wall Street Journal, 1995). Another example is Royal Ahold, a Dutch retail group with subsidiaries such as Stop & Shop in the US and Albert Heijn in the Netherlands. “The audit of Ahold’s consolidated 2002 has been extended because of a number of new internal investigations at US Foodservice (the unit at the heart of the financial scandal) .... Deloitte & Touche, which is carrying out the audit, said that, while important progress had been made in these investigations, various delays in their completion had placed the resumption of important parts of the total audit some four to six weeks behind schedule. ... Based on the information received to date, intentional accounting irregularities involving earnings management and misapplications of generally accepted accounting principles were found ...” (Decision News Media SAS, 2003).

7In a conversation with a recently retired Big4 audit firm partner, he confirmed that red flags observed during audits can trigger additional audit work and result in financial reporting delays of clients. He explained that clients constantly give pressure to urge completing the audits as soon as possible but additional audit work, if deemed necessary, is important to auditors’ risk management.

8Consistent with this feature of the model, Kinney Jr. and McDaniel (1993) wrote that “[p]resence of [client factors such as poor internal controls or intentional violation of the securities acts by client management] is expected to lead to increased year-end audit work and audit/client negotiations about the best disclosure action.” Ettredge, Li, and Sun (2006) also report that audit delays are positively associated with the likelihood of the clients receiving modified audit opinions and their restating financial reports.
news” under certain circumstances. Such circumstances are fully characterized in closed form by a positive-reaction condition. The condition is simple enough for one to see through the underlying economic determinants.

In the model, the auditor takes up the audit engagement of a client firm managed by an entrepreneur referred to as an insider. The insider needs capital to invest in a project. He is interested in raising external capital. Outside investors and the auditor are aware of the possibility that the insider can secretly divert firm resources for his private interest (e.g., pursuing undisclosed investments with negative net present values, private consumption of corporate assets, embezzlement, etc). The outside investors and auditor therefore behave strategically to guard against the potential (but not immediately noticeable) misappropriation, say, by accepting only properly priced equity offering terms and by extending audit procedures adequately.

An audit delay in the model is represented by the event of extending audit procedures. Such a delay might not be immediately observable to outside investors in reality. According to Givoly and Palmon (1982), “[t]he single most important determinant of the timeliness of the earnings announcement is the length of the audit.” I therefore assume an audit delay is manifested as a financial reporting delay, although the latter is not explicitly modeled here.

The first main result of the paper concerns the equilibrium likelihood of an audit and financial reporting delay. I find that the delay likelihood decreases when the reliability of regular and extended audit procedures increases and/or when the prior probability of having a dishonest insider reduces. Intuitively speaking, the more reliable audit procedures are, the lower the chance of observing a false-positive red flag and the higher the chance of catching a dishonest insider. He is thus less aggressive in choosing the extent of diversion. Consequently, the auditor has less incentive to perform extended audit procedures frequently. As a result, a delay occurs less often. If the prior probability of having a dishonest insider is lower, it triggers a red flag less often. A delay is thus less often seen.

Recall that outside investors worry about the insider secretly diverting firm resources for his private interest. Still, they are willing to invest in the firm as long as the shares are fairly priced, i.e., taking into account the expected loss of resources. However, if later the insider could be stopped from diverting resources, the initial share price that reflected the expected loss would be underpriced. For analytical convenience, the real-world benefit from detecting fraud and thereby preventing any future losses is parsimoniously captured in this one-shot model as though the diverted resources were recoverable should the auditor discover the fraud timely. Further explanation on the interpretation of the “recoverable loss” assumption is provided in section 6.2.

9In Table 5 of Bagnoli, Kross, and Watts (2002), they report that the mean and median cumulative market-adjusted returns (CARs) of the entire group of late-announcer firms are negative. However, they also document that the third quartile of CARs is positive and growing as the delay lengthens, suggesting that a delay is not merely bad news.

10Since 2004 the relation between earnings announcement lag and audit report lag has changed somewhat owing to unintended consequences of Public Company Accounting Oversight Board (PCAOB) Auditing Standards No. 2 on internal control and No. 3 on documentation. See Bronson, Hogan, Johnson, and Ramesh (2011) and Krishnan and Yang (2009) for details.
When outside investors see a financial reporting delay, which in this model is due to an audit delay, they know two things have happened. First, the auditor must have observed some red flag of potential fraud; otherwise, extended audit procedures would not have been triggered. So outside investors should revise the expected firm value downward. However, an audit delay also means the auditor is doing extra work, increasing the chance of discovering a fraud and “recovering” the expected losses reflected in the initial share price. For this reason, outside investors should revise the expected firm value upward. Whether the overall market reaction is negative or positive depends on the relative magnitudes of these opposing effects. If the ex ante probability of fraud was high, say, due to weak corporate governance, the discount in the initial share price would have been high. Consequently, the first effect of revising the expected firm value downward would be limited, leaving more room for the second effect to dominate and a greater chance of seeing a positive reaction to a delay.

In reality, many reasons can lead to reporting delays. Examples include financial distress and mergers and acquisitions. The positive-reaction result of this paper is based on audit delays as a reason behind reporting delays. I am not aware of any research documenting the frequencies of different reasons for earnings announcement delays. However, reasons for late 10-K filings can be found in a recent study. Table 6 of Impink, Lubberink, van Praag, and Veenman (2011) reports that in the sample of observations with unique reasons for unambiguously late 10-K filings (i.e., at least two days after the Form 12b-25 filing) and returns data available for analysis, 1,204 of them give an explanation based on accounting/auditing related issues, whereas 145 are due to asset acquisitions/dispositions, with another 112 due to financial distress. In the full sample without returns data requirements and including observations with multiple reasons, the frequencies of the three reasons are 1,721, 426, and 444, respectively (see their Table 1).

The positive-reaction condition can provide a basis for developing hypotheses for empirical testing. Two examples are given below. More discussion on the hypotheses and how the model can guide empirical test designs are provided in appendix A.

**Hypothesis 1.** A positive market reaction to a delay is more likely for a firm with weaker corporate governance and internal control.

**Hypothesis 2.** Negative market reactions to delays are weaker for firms with weaker corporate governance and internal control.12

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11Shareholders appear to be able to recognize the risk of resources diversion and price the shares accordingly. For example, Huang and Zhang (2012) argue that the value of cash reserves can shed light on the extent of the private consumption of corporate assets. They wrote: “As Pinkowitz, Stulz, and Williamson (2006) propose, in the presence of agency costs, the marginal value of cash is significantly below $1, because part of it is reduced by the appropriation of private benefits. In contrast, the value of an extra dollar of cash is less likely to be below its face value when managers utilize liquid assets to maximize shareholder welfare. Specifically, they find that a dollar of cash is worth just $0.33 in countries with poor institutions, whereas it is worth $0.91 in other countries. Similarly, Dittmar and Mahrt-Smith (2007) report that a dollar of cash in a poorly governed firm contributes only $0.42 to shareholder value, as compared to $1.62 in a well-governed firm.”

12These hypotheses use the strength of corporate governance/internal control to proxy for the ex ante probability of fraud. See section 6.3 for a discussion on how the potential endogeneity between ex ante probability of fraud and corporate governance/internal control can be addressed.
The first contribution of the paper is identifying the puzzling phenomenon of positive reactions to bad news (PR2BN) and explaining the insight behind the phenomenon. The phenomenon is not specific to the audit and reporting delay context. It also exists in other interesting contexts such as internal control weakness disclosure (see details in section 6).

The phenomenon of PR2BN is counter-intuitive. It can easily go unnoticed when researchers' attention is on the negative average reaction. Even when PR2BN are noticed, some researchers might not report such distracting findings not central to the main issues of their studies. For reasons like these, it is not easy to find documented examples of the phenomenon. One of the few examples is the positive cumulative abnormal returns (CARs) documented by Palmrose, Richardson, and Scholz (2004). In their Table 2, the third-quartile CARs are positive in all four event windows considered by them. They conjecture on a few reasons for the puzzling positive reactions but do not do any formal analysis to explain the phenomenon.

The clearest and most robust example of the phenomenon that I am aware of is the positive abnormal returns to internal control weakness disclosure reported in Panel B of Table 4 in Beneish, Billings, and Hodder (2008). Although the average reaction is negative, positive abnormal returns consistently constitute over 32% of the uncontaminated sample (i.e., with no other news in the event window), regardless of the choice of the control sample for measuring normal returns.

The second contribution of the paper is arousing the interest of empirical researchers to study the underlying reasons behind the PR2BN phenomenon in different contexts. As explained, instances of the phenomenon are rarely documented in published research. By providing a theoretical explanation to the phenomenon in a particular context and suggesting empirical tests based on the model, this paper serves the purpose of stimulating empirical research to discover and analyze more instances of the phenomenon and thereby deepen the understanding.

The third contribution of the paper is the formulation of a theoretical framework for examining the likelihood of and market reaction to an audit and financial reporting delay. Empirical studies have examined such issues extensively. However, rarely have the studies developed hypotheses for testing using any formal theoretical model. Prior analytical models on financial reporting timeliness follow the voluntary disclosure literature, assuming a financial reporting delay is a voluntary, strategic decision (e.g., Gennette and Trueman 1996). These models are completely different from mine that emphasizes the fraud detection role of auditors in the financial market and the involuntariness of a financial reporting delay arising from an audit delay.

This paper has an empiricist-centric theoretical modeling style. To facilitate follow-up empirical research, I derive theoretical results in closed form, giving clearer guidance on empirical research.

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13 I thank Michelle Hanlon for drawing my attention to this study.
designs. More specific functional forms and exogenous parameters are used in the model to capture the fundamental first-order effects of relevant economic forces. I provide a detailed discussion of the empirical implications of the model, including suggestions for regression equation specifications.

The rest of the paper is organized as follows. The next section briefly reviews the background of SAS 99 that has inspired the model of this paper. The model setup is described in section 3, with different parties’ equilibrium decisions analyzed in section 4. Empirical-oriented readers can skip section 4 and jump directly to section 5, where I discuss the model’s empirical implications. Such readers can go back to the preceding section for a glance of the analysis when doing so is necessary to understand the empirical implications. Section 6 explains the generality of the PR2BN phenomenon beyond the particular context of the model and the nature of the “bad news” in the phenomenon. In addition, the section discusses empirical proxies for a key parameter of the model, namely the ex ante probability of fraud, and relates the paper to several other studies. Concluding remarks are provided in section 7. An illustrative example of an empirical test design is given in appendix A. Technical proof and derivation are relegated to appendix B.

2 Background

The debate on the fraud detection role of auditors in the financial market has a long history. It was initially framed by the accounting profession as an expectation gap issue. Advocates argued that the assumed integrity of the management is a necessary starting point for an audit engagement. This debate cooled down slightly in the 90’s. It has gained attention again following the collapse of Arthur Andersen, one of the biggest five accounting firms in the world. Its collapse was closely tied to the accounting scandals of a number of companies, including the telecommunication giant WorldCom and especially the energy giant Enron.


In October 2002, partly in response to the accounting scandals and the Sarbanes-Oxley Act (SOX) that followed, the AICPA released SAS 99. This statement superseded SAS 82 but with the same title. The updated standard provides more guidelines on how auditors should structure an audit plan to better serve the fraud detection function. According to the standard, auditors are required to “consider whether [misstatements identified by audit test results] may be indicative of fraud. ... If the auditor believes that the misstatement is or may be the results of fraud, and either has determined that the effect could be material to the financial statements or has been unable to
evaluate whether the effect is material, the auditor should ... attempt to obtain additional audit evidence to determine whether material fraud has occurred or is likely to have occurred, and, if so, its effect on the financial statements and the auditor’s report thereon.” (SAS 99, paragraphs 75 and 77; emphasis added.)

In short, this standard requires an auditor to consider whether misstatements identified by audit test results may be indicative of fraud and if so, the auditor should attempt to obtain additional audit evidence to clarify the situation. Considering red flags from audit test results and obtaining additional audit evidence are decisions auditors should think about carefully in assessing fraud risks. They are also two important elements of the model formulated in the next section to understand the fraud detection role of auditors in the financial market.

3 A Model of Extended Audit for Fraud Detection

The model formulated here has a setup similar to the Newman, Patterson, and Smith (2005) (hereafter NPS) model. I depart from their model mainly in the strategic consideration of extending audit procedures contingent on the red flag observed, if any. Patterson and Smith (2007) and Smith, Tiras, and Vichitlekarn (2000) have analyzed models with a “two-round” audit similar to the one examined here. The auditor in their models always performs both rounds of audit. Therefore, their setup is more like a refined modeling of the regular audit procedures of my model, which are performed unconditionally. By contrast, the auditor in my model may or may not perform extended audit procedures, depending on the outcome of regular audit procedures. This distinction from the prior models fits the empirical context, namely the occurrence of a delay, for which I am interested in deriving predictions.15 In the following, I first outline the basic model setup, followed by a description of the sequence of events in the model. Table 1 summarizes the notations used.

The model has an auditor with a client firm managed by an entrepreneur, referred to as an insider. The insider can of two types: honest or dishonest (i.e., opportunistic). His type is determined at random by the nature. An honest insider never diverts the firm’s resources, whereas a dishonest one will make the diversion decision strategically. It is not new in the economics literature to make the parsimonious assumption of two exogenous types of players with one being “irrational.” An example is the widely cited paper by Kreps, Milgrom, Roberts, and Wilson (1982). Another example is Erard and Feinstein (1994), of which the authors assume that there is an exogenous fraction of inherently honest taxpayers. This assumption is made to cope with some perplexing empirical facts often described as the “Why people pay tax” puzzle in the tax compliance literature. This hard-to-explain puzzle suggests that the existence of inherently honest people is a realistic assumption.

15Patterson and Smith (2007) study the effects of SOX on an auditor’s choice of effort in performing control tests (the “first-round” audit) before doing substantive tests (the “second-round” audit). Smith, Tiras, and Vichitlekarn (2000) analyze the interaction between internal control assessments and substantive testing. Their models are more general than mine in the sense that they allow audit choices in both rounds, which lets them address the issues in their papers. In contrast, I simplify the “two-round” audit setup by allowing an audit choice only in the second round. This facilitates deriving closed-form results that are useful for guiding empirical hypotheses for testing.
Accounting studies have also recognized that some people do behave honestly when they should behave strategically given the monetary incentives. A widely cited example is Evans, Hannan, Krishnan, and Moser (2001). Mounting evidence of similar “irrational” behaviors have been documented in the experimental economics literature, with the ultimatum game being an often cited example (see Camerer and Thaler 1995).

In the model, the insider initially owns all the shares of the firm, whose only asset comes from the insider’s endowed wealth $W > 0$. The firm needs an exact amount of external capital, denoted by $K > 0$, to add to $W$ to invest in a project with a known rate of return $g > 0$. Outside investors and the auditor are aware of the possibility that the insider can secretly divert some resources from the firm for his private interest. As a result, the firm value suggested by the firm’s financial statements actually overstates the underlying true value. The outside investors and auditor therefore behave strategically to guard against the potential (but not immediately noticeable) misappropriation, say, by accepting only a properly priced equity offering term and by extending audit procedures adequately. All parties in the model are risk neutral.

Below is the sequence of events in the model, which is summarized with the timeline in Figure 1. The model structure and parameters are common knowledge to the insider, outside investors, and auditor.

1. The nature determines the type of the insider: honest or dishonest. The prior probability of having a dishonest insider is $\theta$, where $0 < \theta < 1$.

2. The insider simultaneously chooses a non-contingent audit fee, $F$, and a fraction of ownership to sell, $\lambda$. For simplicity, I consider only pure strategies in choosing $F$ and $\lambda$ that constitute the pooling equilibrium. The choice of $F$ and $\lambda$ is constrained by competitive audit and financial market conditions. A competitive audit market requires that the audit fee $F$ leaves an auditor zero expected profit. Moreover, a competitive financial market requires that outside investors’ total cost of investment is equal to the value of the investment. Further details of

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16 The intuitions of the model apply to initial public offering (IPO) as well as secondary equity offering (SEO) situations. The assumption of the insider initially owning all the shares is for convenience only. At the expense of more complicated expressions with an additional parameter for the initial ownership level, similar results hold with the assumption relaxed.

17 I allow for an endogenous $K$ in an earlier version of the paper (available upon request), with major results unchanged under mild additional assumptions.

18 A separating equilibrium does not exist in this model. If an honest insider had a way (e.g., by choosing a high reputation auditor) to separate himself from a dishonest insider, the auditor and outside investors would not require “premiums” in the audit fee and equity offering terms to price-protect themselves, and the auditor would never extend audit procedures. Given competitive financial and audit markets, these terms obtained by a honest insider must be more favorable than what can be obtained by a dishonest insider in such a hypothetical separating “equilibrium.” Consequently, a dishonest insider is always better off by mimicking whatever a honest insider does to pool with him. This invalidates the supposition that a separating equilibrium exists in this setup.

19 The literature has discussed the possibility of a firm signaling its accounting quality or the integrity of its management by hiring a high-quality auditor. While this is true to a certain extent, the fact that big 4 auditors are from time to time sanctioned by regulators for wrongdoings related to misreporting or fraud committed by their clients suggest that a separating equilibrium is unlikely to capture the whole truth of the reality. Therefore, examining a pooling equilibrium is interesting and necessary.

Here the “competitive financial market” condition means outside investors are under perfect competition to provide capital to the firm. It does not mean outside investors themselves face a capital supply curve that is perfectly elastic.
this condition will be provided shortly.

3. The insider offers the audit fee $F$, payable after the audit, to an auditor to secure her services for a year-end audit.\textsuperscript{20} The insider also offers the equity offering term $\lambda$ to outside investors. Given competitive audit and financial market conditions and anticipating the insider’s equilibrium choices, the auditor accepts the audit engagement at the fee offered, and outside investors accept the equity offering term.

4. The insider invests $K + W$ in the project, earns the return, and then chooses the diversion rate $\delta$, i.e., the proportion of the year-end (after-audit-fee) firm value $\Pi = (1 + g)(K + W) - F$ to divert.

5. At the end of the year, the auditor follows standard practice to perform regular audit procedures. For simplicity, the cost of these procedures is normalized to zero. The procedures may result in a red flag for potential fraud. The chance of a false positive, or a Type I error, (i.e., red flag on despite no fraud) is $p = \Pr \{ \text{red flag} \mid \text{no fraud} \} > 0$.\textsuperscript{21} For simplicity, the chance of a false negative, or a Type II error, (i.e., red flag off despite fraud) is assumed to be zero. I require that the red flag is an informative signal of potential fraud so that false positives are less likely than true positives, i.e., $p < 1$.\textsuperscript{22}

6. Conditional on whether a red flag is observed, the auditor makes the decision $x$ on extending audit procedures ($x = 1$) or not ($x = 0$). She incurs an extra cost $C$ in extending audit procedures.

7. The auditor cannot give a qualified opinion unless audit evidence supports the opinion. More definite, though not completely conclusive, evidence is available only when extended audit procedures are performed. So the auditor can only issue a qualified opinion if extended audit procedures have been performed.

8. With probability $q > 0$, the evidence obtained from extended audit procedures is a true positive (i.e., proving the existence of fraud). While the evidence can also be a false negative

\textsuperscript{20} That the audit fee is payable after the audit is an inessential difference from NPS’s setup. This change, however, is more consistent with the incentive of the insider to grow the “pie” (firm value) as large as possible, regardless of his incentive to divert resources from the firm.

\textsuperscript{21} The assumption of a non-zero probability of a false positive is consistent with the observation that “while symptoms of fraud (“red flags”) are observed frequently, the presence of such issues is not necessarily indicative of fraud” (see Hogan, Rezaee, Riley, and Velury 2008).

\textsuperscript{22} The results of the paper are essentially unchanged under the more general assumption of $n \equiv \Pr \{ \text{no red flag} \mid \text{fraud} \} \geq 0$ with the false-to-true-positive likelihood ratio $\frac{p}{1-n} < 1$. This likelihood ratio requirement is equivalent to the monotone likelihood ratio property (MLRP) often assumed in moral-hazard agency models. To see this, first note that the relation between the existence of fraud and the observation of a red flag is analogous to that between a binary action and a binary outcome in a moral-hazard setting. Thus, the following is equivalent to the MLRP:

$$\frac{\Pr \{ \text{red flag} \mid \text{fraud} \}}{\Pr \{ \text{red flag} \mid \text{no fraud} \}} > \frac{\Pr \{ \text{no red flag} \mid \text{fraud} \}}{\Pr \{ \text{no red flag} \mid \text{no fraud} \}},$$

which can be written as $\frac{p}{1-n} < 1$. 
(i.e., suggesting no fraud despite fraud) with probability $1 - q$, it can never be a false positive (i.e., showing fraud despite no fraud).

9. The audit opinion is issued based on whether evidence of fraud has been obtained.

10. If fraud is discovered, outsider investors can “recovered” from the initially expected loss of resources. In addition, the insider must bear a penalty equivalent to a monetary fine proportional to the amount of resources diverted, i.e., $b\delta\Pi$, where $b > 0$. The parameter $b$ indicates the severity of the penalty to an insider committing fraud, as in NPS’s model. If fraud exists but is not discovered by the auditor, it will nonetheless have a non-zero chance to be discovered in the future, resulting in some liability cost to the auditor. The present value of the expected liability cost is assumed to be proportional to the diversion rate, i.e., $a\delta$, where $a > 0$. The parameter $a$ is referred to as the penalty multiplier for the auditor, as in NPS’s model.

When the auditor decides whether to take the offer $F$ for the audit engagement and subsequently determines whether to extend audit procedures ($x = 1$) or not ($x = 0$), she knows the insider’s choice of $\lambda$, which is set to ensure outside investors are not worse off by investing in the firm. The auditor does not observe the privately chosen diversion rate $\delta$ but will make decisions based on an equilibrium conjecture about it. The insider chooses $\delta$ after securing the auditor but before she comes in to do the year-end audit. When he makes the diversion decision, he knows everything except for whether the auditor will observe a red flag and her decision on $x$ that follows. Such information about the red flag and extended audit, as well as the diversion rate $\delta$, are not known to outside investors when making the investment decision in response to the insider’s equity offering term. They nonetheless make the decision based on equilibrium conjectures about choices not known to them at that time. Anticipating equilibrium behaviors of other parties and himself, the insider chooses $F$ and $\lambda$ at the very beginning.

To detail the competitive financial market condition, let $R(K) \equiv (1 + r)K$, with $r > 0$, denote outside investors’ total cost of arranging the capital to invest in the firm. For example, if the $K$ dollars are arranged by borrowing from banks, the total cost of arranging the capital would be the principal plus the interest, and $r = \frac{R(K)}{K} - 1$ would be the borrowing interest rate. A competitive

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23 NPS assume the auditor does only a one-round audit with the fraud detection probability $q$ depending on the amount of resources invested in the audit. In contrast, the auditor here always performs regular audit procedures, with the possibility of also performing extended audit procedures. To focus on this unique aspect of the model, I simplify other aspects with assumptions like an exogenous $q$. Closed-form results can also be obtained without affecting the fundamental structure of the equilibrium if alternatively $q$ is proportional to the diversion rate $\delta$.

24 This assumes the auditor follows the GAAS to document her work properly and finds it prohibitively costly to commit a criminal offense by ignoring evidence of fraud.

25 Like NPS, I make the simplifying assumption that the insider’s and auditor’s expected penalties are deadweight losses unrelated to outside investors’ payoff.

26 Whether the audit fee offered to the auditor is observable to outside investors at the moment of making the investment decision is inessential here because even when it is not observable, in equilibrium all players can correctly anticipate the amount as though it was. Consistent with the US regulatory requirement of disclosing the audit fee after the year end, it seems more natural to assume that outside investors cannot observe the year-end audit fee at the moment of making the investment decision, which presumably occurs before the year end, but eventually they can observe it when the fee is disclosed after the year-end audit has been fully completed.
financial market requires that the equity offering term \( \lambda \) leaves outside investors indifferent between investing or not investing in the firm. That is to say, the total cost of investment, \( R(K) \), equals the value of the investment, i.e., the fraction \( \lambda \) of the expected year-end (after-audit-fee) firm value.

To ensure that the insider’s equilibrium choice of \( \lambda \) exists and is an interior solution (and therefore the comparative statics analysis to be carried out later is valid), I assume that the insider has sufficient wealth such that the following inequality holds:\(^{27}\)

\[
R(K) < \mu_1[(1 + g)(K + W) - a\varphi],
\]

where

\[
\mu_1 = \frac{1 + b(1 - \varphi)}{1 + b}
\]

and

\[
\varphi = a^{-1}\left(\frac{C}{q}\right)[p + (1 - p)\theta].
\]

The constant \( \mu_1 \) is the upper bound of a variable \( \mu \), referred to as the effective outside ownership, to be defined shortly. It will be clear that \( \mu \) and \( \lambda \) map to each other uniquely.

It will also be clear that the \( a\varphi \) on the right hand side of (1) is equal to the equilibrium audit fee determined in the model. Therefore,

\[
\Pi(\varphi) = (1 + g)(K + W) - a\varphi
\]

is the equilibrium year-end (after-audit-fee) firm value before any diversion. The constant \( \varphi \) summarizes the effects of several parameters of the model and is larger when (i) the penalty multiplier \( a \) that constitutes the auditor’s expected liability cost is smaller; (ii) the “unit” audit cost \( C/q \) of the extended audit procedures is larger; (iii) the false-positive probability \( p \) that captures the imprecision of the red flag as an early warning is higher; (iv) the prior probability, \( \theta \), of having a dishonest insider is higher. I will come back to the intuitive interpretation of the constant \( \varphi \) later.

For the model to be interesting, the audit procedures have to be sufficiently reliable. Specifically, this means \( q > \frac{1}{1 + \rho} \), which requires that the probability \( q \) of detecting a fraud that exists is sufficiently high. The probability has to be high enough such that given the severity of the penalty to a fraudster, represented by \( b \), the insider would rather not divert any resources if anticipating \( x = 1 \) with certainty. Otherwise, it would be in the auditor’s interest to always extend audit procedures without conditioning on the observation of a red flag. The model would become indistinguishable from the setting already studied by Patterson and Smith (2007).

It will be clear shortly that \( [1 + p(\frac{g}{2} - 1)]^{-1} \) is the posterior probability of the existence of a

---

\(^{27}\)Fully expanding (1) by substituting into it the \( \mu_1, \varphi, \) and \( R(K) \) gives an inequality in terms of \( g, r, a, b, \) and other parameters. The inequality can hold even for \( W = 0 \) if alternatively I assume that the penalty multipliers \( a \) and \( b \) are sufficiently large and \( (g - r)K \), i.e., the net gain from making the investment, is greater than the equilibrium audit fee to be detailed shortly.
fraud after observing a red flag. Given that a fraud exists, the probability of detecting it is \( q \). Note that \( a \) is essentially the auditor’s expected liability cost of not detecting a fraud with a diversion rate of \( \delta = 1 \). Therefore, the condition

\[
C < \frac{aq}{1 + p\left(\frac{1}{\theta} - 1\right)},
\]

referred to as **affordable audit cost**, ensures that after observing a red flag, extending audit procedures reduces the expected liability cost related to a “full-diversion” fraud by an amount that exceeds the extra cost of audit. This condition is sufficient to deter “full diversion,” preventing the uninteresting corner case of \( \delta = 1 \) to constitute an equilibrium. Throughout my analysis, I assume the condition holds.

In the next section, I derive the perfect (Bayesian) Nash equilibrium of the model using backward induction. Because separating equilibrium strategies do not exist in this setup (see footnote 18), an honest insider will simply select the audit fee and equity offering term that would be chosen by a dishonest insider. Therefore, I only need to analyze the decisions from the angle of a dishonest insider.

### 4 Equilibrium of the Model

The core structure of the equilibrium is similar to a mixed-strategy equilibrium in the following sense. In equilibrium, a dishonest insider chooses a particular diversion rate making the auditor indifferent between doing or not doing an extended audit. Although the auditor is indifferent among all the probabilities of extending the audit, only one of the probabilities will make the dishonest insider indifferent among all possible diversion rates. This extended audit probability together with the particular diversion rate in concern forms the only equilibrium pair. In the following, I will first determine the pair of equilibrium diversion rate and extended audit probability and then work backward to determine the equilibrium audit fee and outside ownership.

#### 4.1 Diversion and Extended Audit Decisions

After observing a red flag, the auditor weighs the incremental benefit against the incremental cost to decide whether it is gainful to extend audit procedures. Given any conjecture \( \delta \) on a dishonest insider’s diversion decision, the incremental benefit of extending audit procedures is a reduction in expected liability cost equal to \( a\delta q \Pr\{\text{fraud} | \text{red flag}\} \). In equilibrium, this benefit must be equal to the incremental cost \( C \).

The equilibrium diversion rate chosen by a dishonest insider is given by the condition below:

\[
\delta^* = \frac{C}{aq \Pr\{\text{fraud} | \text{red flag}\}}.
\]

Since an honest insider never diverts any resources, the prior probability of fraud is simply the prior probability of having a dishonest insider. By the Bayes’ rule,
\[ \Pr\{\text{fraud|red flag}\} = \frac{1}{1 + p(\frac{1}{\theta} - 1)}. \]  

(7)

Hence,

\[ \delta^* = \frac{C/q}{a \left[ 1 + p(\frac{1}{\theta} - 1) \right]^{-1}}. \]  

(8)

This expression of the equilibrium diversion rate shows clearly its determinants. They are (i) the “unit” audit cost \( C/q \) of the extended audit procedures; (ii) the penalty multiplier \( a \) for the auditor when a fraud was not detected; (iii) the reliability of the red flag as an indicator of fraud, captured by the posterior probability of having a dishonest insider, i.e., \( \left[ 1 + p(\frac{1}{\theta} - 1) \right]^{-1} \).

Given the equilibrium conjecture \( \delta^* \), the auditor is indifferent between choosing \( x = 1 \) or \( x = 0 \). Let \( \omega \) denote her randomized strategy of extending audit procedures after observing a red flag, i.e., \( \omega = \Pr\{x = 1|\text{red flag}\} \).\(^{28}\) The auditor will not extend audit procedures unless a red flag is observed. Thus, the only way to detect a fraud that exists is to follow through the sequence of observing a red flag, extending audit procedures, and finding evidence of fraud. In other words, a dishonest insider knows that the chance of being caught is \( q\omega \).

If a dishonest insider successfully diverts a fraction of the firm’s resources without being detected, his payoff is the sum of (i) diverted resources that he can enjoy privately, \( \delta \Pi \), and (ii) his share of the firm value after the diversion, \( (1 - \lambda)(1 - \delta)\Pi \). However, if the diversion is detected by the auditor, he must return the diverted resources and bear the penalty. Returning the diverted resources \( \delta \Pi \) means that the insider who owns \( 1 - \lambda \) of the firm has a claim worth \( (1 - \lambda)\Pi \). With the penalty \( b\delta\Pi \), his payoff becomes \( (1 - \lambda)\Pi - b\delta\Pi \). Given any conjecture \( \omega \), the expected payoff of a dishonest insider selecting a diversion rate \( \delta \) is as follows:\(^{29}\)

\[ [\delta\Pi + (1 - \lambda)(1 - \delta)\Pi](1 - q\omega) + [(1 - \lambda)\Pi - b\delta\Pi]q\omega. \]

(9)

In order for \( \delta^* \) given by (8) to constitute an equilibrium, the diversion rate must satisfy the first-order condition below:

\[ \lambda(1 - q\omega) = bq\omega. \]

(10)

This condition uniquely determines the auditor’s equilibrium randomized strategy of extending audit procedures, characterized by

\[ \omega^* = \frac{\lambda}{(\lambda + b)q}. \]

(11)

In the following subsections, I continue the analysis by determining the equilibrium audit fee

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\(^{28}\)Modeling this decision of the auditor as a randomized strategy is consistent with the highly stochastic nature of individual-level economic choices now widely recognized in the literatures of psychology, decision theory, experimental economics, and accounting (see e.g. Rieskamp 2008, Wilcox 2011, Loomes 2005, and Fischbacher and Stefani 2007).

\(^{29}\)Like NPS, I assume that if the fraud is not detected by the auditor, the diverted resources will not be recoverable, say, because the dishonest insider will flee from the country.
and fraction of ownership to sell.

4.2 Audit Fee

The auditor’s expected profit prior to performing regular audit procedures is

\[ F - \{ C\omega[\theta + (1 - \theta)p] + a\delta\theta(1 - q\omega)\} \]  \hspace{1cm} (12)

The first term inside the curvy brackets is the expected cost of extending audit procedures. The cost is incurred only when a red flag is observed, which happens with probability \( \theta + (1 - \theta)p \), followed by the outcome \( x = 1 \), which occurs with probability \( \omega \). The second term inside the curvy brackets is the expected liability cost. The auditor will bear the liability cost \( a\delta \) if a fraud exists but is undetected during the audit. This has a chance of \( \theta(1 - q\omega) \) to occur.

Given a competitive audit market and anticipating the insider’s and auditor’s equilibrium choices, the equilibrium audit fee \( F^* \) will be set such that the auditor in equilibrium earns zero expected profit. In other words, the fee is determined by substituting the equilibrium \( \omega^* \) and \( \delta^* \) into (12) and then setting it to zero to solve for the equilibrium \( F^* \). This is the audit fee offered by the insider and accepted by the auditor in equilibrium:

\[ F^* = a\delta^*\theta, \]  \hspace{1cm} (13)

where \( \delta^* \) is given by (8).

The equilibrium audit fee does not depend on \( \omega^* \) because in equilibrium \( \delta^* \) is chosen to make the auditor indifferent between extending audit procedures or not. Since the cost of performing regular audit procedures has been normalized to zero, the audit fee only needs to be high enough to cover the auditor’s expected liability cost, as though she had never considered extending the audit procedures.

Substitute \( \delta^* \) into (13), the equilibrium audit fee becomes

\[ F^* = \left( \frac{C}{q} \right)[p + (1 - p)\theta]. \]  \hspace{1cm} (14)

The false-positive probability \( p \) measures the imprecision of the red flag as an early warning. Ideally one would like regular audit procedures to have a low \( p \) so that false positives occur rarely. In other words, the red flag is more informative when \( p \) is smaller. Consequently, the decision on extending audit procedures can be made more efficiently. The equilibrium audit fee can thus be lower. Similarly, \( F^* \) is lower when the extended audit procedures are more effective in detecting fraud, i.e., \( q \) is higher. In a society with a higher moral standard, people are more likely to be honest, i.e., \( \theta \) is lower, which results in a lower equilibrium audit fee.

It is useful to recognize that \( F^* = a\varphi \) and \( \varphi = \delta^*\theta \), where the constant \( \varphi \) is defined in (3). Note that \( \varphi = F^*/a \) is between 0 and 1. This ratio may be referred to as a liability discount factor. To see why, consider the limiting case where both \( \theta \) and \( \delta^* \) are nearly 1. The auditor in equilibrium is
indifferent between performing and not performing extended audit procedures. She anticipates that her expected payoff would be equivalent to that without extending the audit procedures. Since $\theta$ and $\delta^*$ are nearly 1, it is almost certain that she would need to bear the “full diversion” liability cost, i.e., $a$. Therefore, she would not accept the audit engagement unless the audit fee $F^*$ is as much as $a$. In general, the audit fee is set according to the anticipated fraction of the “full diversion” liability cost to be paid by the auditor. This fraction is lower if the expected liability cost, determined by $\theta$ and $\delta^*$, is lower.

A priori, one might expect that through $\omega^* = \frac{\lambda}{(\lambda+b)q}$, the audit fee $F^*$ should be affected by the ownership sold to outside investors, $\lambda$. This endogenous variable may be seen as a measure of the misalignment of interest between the insider and other shareholders. If $\lambda = 0$, the insider is the only shareholder of the firm and has no incentive to divert resources. The diversion incentive is strongest when $\lambda = 1$. In this model, the strategic interaction between the auditor and her client can result in an equilibrium audit fee independent of $\lambda$. When $\lambda$ is lower, one might think that the fraud risk should be lower and hence the auditor would not extend audit procedures so often. Extra costs would be incurred less frequently, resulting in a lower audit fee. This however is not the complete story. When the auditor worries less because of a lower $\lambda$, she has a weaker incentive to extend audit procedures. Therefore, a dishonest insider worries less about being caught and has a stronger “induced” incentive to divert resources. In the end, the two effects balance out and have no net impact on the equilibrium audit fee.\footnote{30}

I summarize the observations about the equilibrium audit fee as the following proposition.

**Proposition 1 (Effects on equilibrium audit fee).** The equilibrium audit fee $F^*$ decreases when (i) the reliability of regular and/or extended audit procedures improves, i.e., $p$ is lower and/or $q$ is higher, or (ii) the chance of having a dishonest insider reduces, i.e., $\theta$ is lower. Formally,

$$\frac{\partial F^*}{\partial p} > 0, \quad \frac{\partial F^*}{\partial q} < 0, \quad \text{and} \quad \frac{\partial F^*}{\partial \theta} > 0.$$

4.3 Outside Ownership

Given any conjectures about $\omega$, $\delta$, and $F$, outside investors’ expected benefit from accepting the insider’s equity offering term is

$$\lambda(1-\delta)\Pi\theta(1-q\omega) + \lambda\Pi[1-\theta(1-q\omega)], \quad (15)$$

\footnote{30}The discussion here cautious against regression analysis that uses an endogenous variable like $\lambda$ to explain another endogenous variable like $F$. Ideally, equations specified for regression analysis should have only exogenous variables as explanatory variables and endogenous variables as dependent variables. If the model here is used as a framework to interpret empirical findings, any statistical relationship discovered between outsider ownership and audit fee suggests one of the following: (i) the model is inadequate (e.g., fails to capture some important forces affecting the two variables); (ii) the relationship is spurious; (iii) the causality runs from $F$ to $\lambda$. I will in section 5.1 show that the last possibility is consistent with the model.
where \( \Pi = (1 + g)(K + W) - F \) is the year-end (after-audit-fee) firm value.\(^{31}\) The probability \( \theta(1 - q\omega) \) in the first term above is the chance of both having a fraud (i.e., dishonest insider) and not having it detected during the audit. If this event occurs, the value of the shares owned by outside investors is only \( \lambda(1 - \delta)\Pi \). If the insider is honest, or if the fraud of a dishonest insider is detected during the audit, outside investors have a claim on the undiverted firm, which is the \( \lambda\Pi \) in the second term above.

A competitive financial market implies that in equilibrium \( \lambda \) will be chosen to equate the expected benefit in (15) with outsider investors’ total cost of arranging the capital, i.e. \( R(K) \). This condition is expressed as the equation below:

\[
R(K) = \mu[(1 + g)(K + W) - F],
\]

where

\[
\mu = \lambda(1 - \Delta)
\]

with \( \Delta = \delta\theta(1 - q\omega) \) referred to as the effective diversion rate. This rate \( \Delta \) differs from the diversion rate \( \delta \) because there is a chance that the insider is honest. Even if he is dishonest, diversion may be detected during the audit and therefore unsuccessful. Given the effective diversion rate \( \Delta \), it is intuitive to refer to \( \mu \) as the effective ownership sold to outside investors (or simply effective outside ownership). From (19) provided below, it will be clear that \( 0 < \mu < \lambda \) for all \( \lambda > 0 \) and \( \mu = 0 \) for \( \lambda = 0 \). When \( \lambda = 1 \), \( \mu \) equals \( \mu_1 = \frac{1+b(1-\varphi)}{1+b} \).

By choosing the effective outside ownership, \( \mu \), a dishonest insider indirectly chooses the fraction of ownership to sell, \( \lambda \). To see how the equilibrium \( \lambda^* \) is determined, consider the equations below, which are (16) and (17) evaluated at the equilibrium \( F^*, \delta^*, \text{ and } \omega^* \):

\[
R(K) = \mu\Pi(\varphi),
\]

\[
\mu = \lambda \left[ 1 - \frac{b\varphi}{(\lambda + b)} \right],
\]

where \( \Pi(\varphi) = (1 + g)(K + W) - a\varphi \) and \( \varphi = a^{-1}\left(\frac{C}{q}\right)[p + (1 - p)\theta] \). The second equation is equivalent to the quadratic equation of \( \lambda \) below:

\[
\lambda^2 + b(1 - \varphi)\lambda = \mu(\lambda + b),
\]

which has only one root between 0 and 1 for any given \( \mu \) between 0 and 1. This solution of the equation, denoted by \( \lambda(\mu) \), defines a one-to-one mapping between \( \mu \) and \( \lambda \). Obviously, the equilibrium \( \mu^* \) is uniquely determined by (18). Accordingly, the equilibrium \( \lambda^* = \lambda(\mu^*) \) is also uniquely determined.

\(^{31}\)I assume that unless a fraud is discovered during the year-end audit, the diverted resources are irrecoverable even though the fraud may be eventually discovered some time in the future. Any future attempt to recover the diverted resources, including filing a lawsuit against the auditor, will end up in a negligible net gain to outside investors. It is thus omitted here.
The following comparative statics result is useful for signing the association between outside ownership and audit fee to be discussed later.

**Proposition 2.** The equilibrium outside ownership \( \lambda^* \) and effective outside ownership \( \mu^* \) are related to the liability discount factor \( \varphi \) as follows:

\[
\left( \frac{d\lambda^*}{d\varphi} \right) = \left[ \left( \frac{d\mu^*}{d\varphi} \right) + \frac{b\lambda^*}{(\lambda^* + b)} \right] \left[ 1 - \frac{b^2 \varphi}{(\lambda^* + b)^2} \right]^{-1} > 0, \tag{21}
\]

\[
\left( \frac{d\mu^*}{d\varphi} \right) = \frac{aR(K)}{\Pi(\varphi)} > 0, \tag{22}
\]

where \( \Pi(\varphi) = (1 + g)(K + W) - a\varphi \) and \( \varphi = a^{-1} \left( \frac{C}{q} \right) [p + (1 - p)\theta] \).

In the next section, I examine some implications of the model, namely how the reliability of regular and extended audit procedures and the ex ante probability of fraud may affect the likelihood of and market reaction to an audit and financial reporting delay.

## 5 Delay Likelihood and Market Reaction

### 5.1 Effects on Delay Likelihood

The empirical literature on audit and financial reporting delays has a long history (see footnote 14). However, rarely have such studies developed empirical tests using a formal theoretical model. In the following I explore some implications of my model, with the intention to develop empirically testable hypotheses not yet investigated in the literature. I focus on two interesting aspects that have been repeatedly examined in the literature, namely the determinants and the effects of audit and financial reporting delays.

Consider first the likelihood of an audit delay, manifested as a financial reporting delay. The delay occurs if and only if the event \( \{x = 1\} \) happens, i.e., the auditor has chosen to extend audit procedures. Conditional on a red flag, the event happens with probability \( \omega^* \). Outside investors understand that a red flag will be observed by the auditor if (i) the insider is dishonest, or (ii) he is honest but the red flag is a false positive. Case (i) has a chance of \( \theta \) to occur, and case (ii) a chance of \( p(1 - \theta) \). Thus the equilibrium likelihood of a delay is

\[
Pr\{x = 1\} = \omega^* [\theta + p(1 - \theta)] = \frac{\lambda^* [p + (1 - p)\theta]}{(\lambda^* + b)q}. \tag{23}
\]

To see clearly what determines the likelihood, it is convenient to look at the log transformation of the relation above:

\[
\log Pr\{x = 1\} = \log \left( \frac{\lambda^*}{\lambda^* + b} \right) + \log [p + (1 - p)\theta] - \log q. \tag{25}
\]
If the equilibrium outside ownership $\lambda^*$ could be held constant,\footnote{The $\lambda^*$ can be held constant if the change in $\theta$, $p$, or $q$ discussed here is balanced out by a covariation in $C$ to fix the value of $\varphi$ that enters the equation system (18) and (19). All these factors affect $\lambda^*$ only through $\varphi$.} then the delay likelihood is higher when the ex ante probability of fraud is higher (i.e., $\theta$ is higher), the imprecision of a red flag is higher (i.e., $p$ is higher), and the effectiveness of extended audit procedures for fraud detection is lower (i.e., $q$ is lower). A higher ex ante probability of fraud triggers a red flag more often. It is thus more likely to see a delay. A red flag appears more often if the red flag is less precise, i.e., false alarms occur more often. A delay thus appears more often. If the effectiveness of extended audit procedures is lower, the chance of seeing a delay is higher because extended audit procedures must be performed more often to provide the same equilibrium level of deterrence to a dishonest insider.

The log-transformed relation can be used as a basis for specifying a regression equation to empirically test the effects of the above-mentioned factors on the delay likelihood. There are two caveats. First, finding adequate proxies for the informativeness of regular audit procedures and effectiveness of extended audit procedures might be difficult. If auditors in a competitive audit market are believed to have similar audit technologies, the $-\log q$ may be treated as the intercept to be estimated. Similarly, with a proxy for $\theta$ included in the regression equation, the estimated coefficient may be interpreted as reflecting the level of $p$.\footnote{The signs of this coefficient and the intercept can be determined by linearizing the term $\log[p + (1 - p)\theta]$ using Taylor’s expansion. If one believes a Big4 audit firm means $q$ is higher and $p$ is lower, a Big4 indicator variable can be included to interact with a proxy for $\theta$ and also as a standalone control to test the effects of $p$ and $q$, whose data usually are difficult to obtain.}

Finding a firm-level proxy for $\theta$ is also challenging. Suppose that the chance of having a dishonest insider is related to the insider’s moral standard. Arguably this may be traced back to his family and education background. Suppose one believes that whether the insider grew up in primary and high schools with strong religious heritages might be relevant. Then observable characteristics like these can be considered proxies for $\theta$. Alternatively, one can argue that $\theta$ is closely related to the inherent risk of a firm. Then variables like industry, capitalization, and beta can also be used as proxies for the ex ante probability of fraud.

The second caveat of specifying a regression equation based on (25) is the endogeneity of the outside ownership $\lambda^*$. Note that $\lambda^*$ correlates with other factors in the relation through $\varphi$ that enters the equation system (18) and (19). Suppose that the random disturbances affecting $\lambda^*$ and the delay likelihood are uncorrelated. Then the estimated coefficient of $\theta$ is unbiased despite direct inclusion of $\lambda^*$ as an explanatory variable in the regression equation. A bias arises if the random disturbances are correlated. To mitigate the bias, a 2SLS regression procedure may be used to first regress $\lambda^*$ on $\theta$ and other exogenous control variables. Then the predicted value of $\lambda^*$ can be included as an explanatory variable in the regression equation specified based on (25). The procedure can control for the bias, provided the control variables for constructing the predicted $\lambda^*$ are exogenous, or proper instruments are used to proxy for the variables.

Alternatively, one can consider a reduced-form regression equation based on (25) that includes the exogenous determinants of the outside ownership but not $\lambda^*$ itself. Because some of these determinants are already in (25), the predicted signs of such factors need to be determined more
carefully than by merely assuming that $\lambda^*$ is being held constant. The next proposition provides the result to sign the effects of those factors. The proposition follows directly from Lemma 2.

**Proposition 3** (Association between outside ownership and audit fee). *For variations in $\phi$ driven by variations in $\theta$, $p$, and/or $q$, the equilibrium outside ownership $\lambda^*$ is positively associated with the equilibrium audit fee $F^*$, i.e.,

$$
\left(\frac{d\lambda^*}{d\phi}\right)\left(\frac{\partial F^*}{\partial \phi}\right) > 0.
$$

Recall that $F^* = a\phi$. Therefore, by Proposition 1, $\partial \phi / \partial \theta > 0$, $\partial \phi / \partial p > 0$, and $\partial \phi / \partial q < 0$. Moreover, $\frac{\lambda^*}{\lambda^* + b}$ is increasing in $\lambda^*$. With $d\lambda^*/d\phi > 0$, the signs of the exogenous factors’ effects on the delay likelihood remain the same, regardless of holding $\lambda^*$ constant or not. This result is formally stated as the proposition below.

**Proposition 4** (Effects on delay likelihood). *The equilibrium delay likelihood decreases when (i) the reliability of the regular and/or extended audit procedures improves, i.e., $p$ is lower and/or $q$ is higher, or (ii) the ex ante probability of fraud reduces, i.e., $\theta$ is lower. Holding constant these factors and the penalty multiplier $b$ for a dishonest insider, the equilibrium delay likelihood is positively associated with the equilibrium outside ownership $\lambda^*$. Formally,

$$
\frac{dPr\{x = 1\}}{dp} > 0, \quad \frac{dPr\{x = 1\}}{dq} < 0,
$$

$$
\frac{dPr\{x = 1\}}{d\theta} > 0, \quad \text{and} \quad \frac{\partial Pr\{x = 1\}}{\partial \lambda^*} > 0.
$$

### 5.2 Market Reaction to Delay

To see how the financial market should react to a delay, I look at the difference between the expected firm values before and after a delay. Note that outside investors’ ownership of the firm will be worth $\lambda^*\Pi(\phi)$ if the insider is honest or dishonest but with the fraud detected timely. In contrast, if the insider is dishonest and the fraud is not detected during the year-end audit, the loss of resources expected by outside investors will not be “recovered” and their shares will be worth $\lambda^*(1 - \delta^*)\Pi(\phi)$ only. Given these, the expected firm value is determined by the probability of undetected fraud. Thus, all that matters to the change in expected firm value is the posterior probability $Pr\{\text{undetected, fraud}|x = 1\}$ conditional on a delay, relative to the unconditional prior probability, i.e.,

$$
\theta(1 - q\omega^*) = \frac{b\theta}{\lambda^* + b}.
$$

(26)

Recall that a delay could not have occurred unless a red flag was observed. Therefore,

$$
Pr\{\text{undetected, fraud}|x = 1\} = \frac{1 - q}{1 + p(\frac{1}{\theta} - 1)}.
$$

(27)

---

$^{34}$When $\theta$, $p$, $q$, and $b$ are held constant, the variations in $\lambda^*$ are due to changes in $C$ and/or $a$.

$^{35}$The derivation of the posterior probability is provided in the appendix.
Intuitively, this means conditional on a delay, the posterior probability of undetected fraud is lower when the extended audit procedures are more effective in detecting fraud, i.e., $q$ is higher. Note that $[1 + p(\frac{1}{\theta} - 1)]^{-1}$ is the posterior probability of having a dishonest insider after observing a red flag, i.e., $Pr\{\text{fraud|red flag}\}$. Suppose the regular audit procedures that generate a red flag are more informative, i.e., $p$ is lower, or the ex ante probability of fraud is higher, i.e., $\theta$ is higher. Then $Pr\{\text{fraud|red flag}\}$ is higher and consequently the posterior probability of undetected fraud after observing a delay is also higher.

Compared below are the unconditional and conditional probabilities of undetected fraud. The comparison highlights the key intuition behind the condition on whether a positive reaction to a delay is possible:

\[
\Pr\{\text{undetected, fraud}\} > \Pr\{\text{undetected, fraud|}x = 1\}
\]

\[
\iff \frac{1}{\theta} > \frac{1}{\theta + p(1-\theta)}
\]

This inequality is equivalent to the positive-reaction condition to be stated shortly. Component-by-component comparisons indicate that $Pr\{\text{undetected|fraud}\} = 1 - q\omega$ is higher but $Pr\{\text{fraud}\} = \theta < Pr\{\text{fraud|red flag}\} = \frac{[1 + p(\frac{1}{\theta} - 1)]^{-1}}{\theta + p(1-\theta)}$. When the ex ante probability of fraud, $\theta$, is sufficiently high, the difference between $\theta$ and $\frac{\theta}{\theta + p(1-\theta)}$ is small compared to the difference between $1 - q\omega$ and $1 - q$. Consequently, the first component of the probability on each side of the inequality will dominate, allowing the condition to hold. This intuition about the effect of $\theta$ provides the basis for developing Hypotheses 1 and 2 stated in the introduction.

Let $V_b$ and $V_a$ denote respectively the expected market values of the shares owned by outside investors before and after observing a delay. The market reaction to a delay can be represented by the percentage deviation of $V_a$ from $V_b$, denoted by $v$:

\[
v \equiv \frac{V_a - V_b}{V_b} = \frac{\lambda^*\Pi(\varphi)[1 - \delta^*Pr\{\text{undetected, fraud|}x = 1\}] - 1}{\lambda^*\Pi(\varphi)[1 - \delta^*Pr\{\text{undetected, fraud}\}]}
\]

\[
= \frac{\delta^* \left[ \frac{b\theta}{\lambda^* + b} - \frac{1 - q}{1 + p(\frac{1}{\theta} - 1)} \right]}{1 - \frac{b\varphi}{\lambda^* + b}}
\]

\[
= \varphi \frac{[1 - (1 - \theta) (1 - p) - (1 - q) (1 + \frac{\lambda^*}{b})]}{(1 - \varphi + \frac{\lambda^*}{b}) [p + (1 - p)\theta]}.
\]

Since $0 < \varphi < 1$, the denominator of the expression for $v$ above is positive. Below is the last result of the paper.

**Proposition 5 (Market reaction to delay).** The market reaction to a delay, measured by the percentage deviation of the post-delay market value of the outside ownership from the pre-delay market value, is equal to

\[
v = \varphi \frac{[1 - (1 - \theta) (1 - p) - (1 - q) (1 + \frac{\lambda^*}{b})]}{(1 - \varphi + \frac{\lambda^*}{b}) [p + (1 - p)\theta]}.
\]
which is positive if and only if

\[(1 - \theta)(1 - p) + (1 - q)\left(1 + \frac{\lambda^*}{b}\right) < 1.\]  

The inequality above provides a necessary and sufficient condition for observing a positive market reaction to a delay. If such a market reaction is observed, a delay is perceived by outside investors as on the whole good news. When will this happen?

If the red flag generated from regular audit procedures is imprecise, the delay so triggered may well be just a false alarm to outside investors. When the ex ante probability of fraud is high, observing a red flag by the auditor can actually be good to outside investors because the red flag may trigger an extended audit. This is especially good when the extended audit is very effective in detecting fraud, leaving little chance for a fraud to sneak through. Thus a delay can be on the whole good news to outside investors when the ex ante probability of fraud, the imprecision of a red flag, and the effectiveness of extended audit procedures for detecting fraud are all high.\(^\text{36}\)

The condition in the proposition above can serve as a basis for specifying a logistic regression equation. The dependent variable of the regression is binary with the value equal to 1 for a positive market reaction observed following a delay. The condition suggests that if the equilibrium outside ownership \(\lambda^*\) could be held constant, the odd of seeing a positive market reaction increases with \(\theta\), \(p\), and \(q\). On the other hand, holding constant \(\theta\), \(p\), \(q\), and \(b\), an increase in the equilibrium outside ownership \(\lambda^*\) (e.g., driven by variations in \(C\), \(a\), \(g\), and/or \(W\)) makes the positive reaction condition harder to be met. If taking into account the effects of \(\theta\) and \(p\) on the equilibrium outside ownership, only \(q\) has an unambiguously positive effect on the odd of seeing a positive market reaction to a delay.

Even for \(q\), its effect on the magnitude of the market reaction, \(|v|\), cannot be unambiguously determined. This ambiguity is due to the appearance of \(\lambda^*\) both in the top and bottom components of \(v\). However, if \(b\) is very high in a country, given that \(\lambda^*\) cannot exceed 1, one would expect the changes in \(\frac{\lambda^*}{b}\) as a result of the changes in other factors could be negligible. In such circumstances, a higher \(q\) would result in a lower \(\varphi\) and accordingly could lead to a smaller top component and a larger bottom component of the expression of \(|v|\), assuming that \(v\) is negative. That is to say, when the market reaction to a delay is negative, outside investors perceive a delay as less of bad news if extended audit procedures are more effective in detecting fraud, giving outside investors stronger protection.

\(^{36}\)It might appear that if a financial reporting delay can result in a positive market response, then the firm would always delay financial reporting even when there is no audit delay, seemingly invalidating the result that a financial reporting delay can be good news to market investors. This argument, however, assumes the firm has no cost in adopting such a strategy, which is not true. After the annual report is released, the date of the audit report would suggest whether the firm has deliberately delaying financial reporting. Anticipating that the initial positive market response to a financial report delay would reverse, the firm has little incentive to mimic a delay.
6 Discussion

6.1 Positive Reactions to Bad News: Plausible?

I have provided an intuitive explanation to the seemingly counter-intuitive result of a positive reaction to an audit and reporting delay. Despite the specific context of the model, the logic of the reasoning holds generally: If there is potentially good news wrapped inside bad news, PR2BN are plausible. The generality of the logic implies that PR2BN should occur in other contexts. However, for reasons explained in the introduction, instances of PR2BN are rarely documented in the academic literature. Provided below are a few examples that I am aware of.

Like fraud discovery, internal control weakness (ICW) disclosure has potentially good news wrapped inside bad news. Obviously, the shareholders of a company would like the company to have no ICW. However, if such weaknesses exist anyway whether they are brought to light or not, it is better to have them disclosed and thereby raise the chance of remediating them than leave the weaknesses unaddressed.

Because of the similarity between fraud discovery and ICW disclosure, I expect a significant proportion of the share price reactions to ICW disclosure is positive. Results documented in Table 4 of Beneish, Billings, and Hodder (2008) are broadly consistent with this prediction. In Panel A of the table, 40% of the 3-day price reactions in their full sample are positive, although the mean reaction is negative. The evidence remains strong when observations potentially contaminated by concurrent news are removed: 36% of the reactions are positive. The findings are robust to alternative choices of the control sample for measuring normal returns. Using control firms matched by industry and size, industry and performance, or industry and earnings quality, they find that the proportion of positive reactions is in the range of 32% to nearly 39% for the uncontaminated samples and 44% to 50% for the potentially contaminated samples (see Panel B of their Table 4).

Beneish, Billings, and Hodder (2008) also report other findings that are consistent with the logic behind PR2BN. For example, for firms with Big 4 auditors, about 45% of the price reactions are positive. The proportion drops to only 22% for firms with small auditors. This finding is consistent with the view that as higher-quality gatekeepers, Big 4 auditors can facilitate the remediation of discovered ICW. In line with this, Gupta and Nayar (2007) report in their Table 5 higher stock price reactions (i.e., either less negative or maybe more positive) to ICW disclosure for firms with Big 4 auditors.

Another example of PR2BN is about dividend changes. Because managers are reluctant to cut dividends unless earnings decreases are persistent (Koch and Sun 2004), reductions in dividends are regarded as bad news. Many studies have shown that average share price reactions to dividend cuts or omissions are negative (e.g., Woolridge 1983, Dhillon and Johnson 1994, and Christie 1994). However, reasons have been proposed to argue that there might be potentially good news wrapped inside the bad news of dividend reductions. For example, reducing cash dividends could be a way to facilitate internally financed growth (Ghosh and Woolridge 1989). Interestingly, Hallock and Mashayekhi (2003) document that during 1970-2000, the mean 3-day cumulative excess returns to
cash dividend decrease announcements are positive for six of the years for the smallest 20% of firms. When confining to the largest 20% of firms, positive average reactions occurred even more often: about eleven years during the 31-year period. For the full sample of firms, the average reactions to cash dividend decreases are positive for at least four of the years (see Figures 6A, 6B, and 6C of the study).

Table 5 of Bagnoli, Kross, and Watts (2002) also demonstrates that PR2BN are possible (see my footnote 9). However, this example of PR2BN does not constitute supporting evidence for the explanation advanced here (based on audit delays) because annual and interim earnings announcements are pooled together in their analysis. In contrast, Table 4 of Begley and Fischer (1998) separately reports the cumulative market adjusted pre-announcement returns of late announcing firms for annual and interim announcements. The 3-day returns are significantly negative for interim announcements but not significantly different from zero for annual announcements. While this is not a clean test for the explanation advanced here, the findings are consistent with the argument for potential benefits resulting from audit delays.

6.2 Crisis as Opportunity Lying in the Middle of Danger

For analytical convenience, the benefit of a timely discovery of fraud is captured in the model as the diverted resources recovered. This needs not be taken literally. Alternatively, a two-period model could have been formulated to allow a fraudster to steal from the firm without any possibility of recovery and to continue to do so as long as the fraud is not revealed in the first period. In such an alternative formulation, the benefit of an early discovery of fraud is to stop further irrecoverable losses of firm resources. Despite this change, the intuition of the fundamental logic remains the same but the technical analysis is more tedious.

The diversion of resources committed by the dishonest insider is closer to asset misappropriation fraud. But the fundamental logic behind PR2BN is unchanged as long as (i) any problematic issues (e.g., fraudulent reporting or questionable accounting choices) of which the concern triggers an audit delay (e.g., due to extended audit procedures or simply lengthier auditor-client negotiations) are costly to the shareholders, and (ii) when the issues indeed exist, the auditor’s timely response helps stopping such issues or preventing/limiting them in the future. Whether the revelation of the problematic issues is potentially good news or purely bad news depends on whether the news is a crisis, defined as an opportunity for improvement lying in the middle of a danger.

What opportunities for improvement can the revelation of fraud lead to? Some studies find that following the revelation of fraud, there are improvements in corporate governance/internal control attributes and operating/stock price performance. For example, Marciukaityte, Szewczyk, Uzun, and Varma (2006) have examined a sample of firms charged with financial reporting, government, or stakeholder fraud or regulatory violation in the US during 1978-2001. They find that after the accusation of fraud, the firms increase the proportion of outsider directors on their boards and subcommittees. Compared to a no-fraud control sample, firms charged with fraud show similar subsequent long-run stock price and operating performance. The results suggest that improvements
in corporate governance/internal control after fraud accusations help restoring investor confidence and firm performance.

Kedia and Philippon (2009) also find improvement in firm performance, in terms of resource allocation efficiency and productivity, after misreporting is detected. They study a sample of firms identified by the US General Accounting Office (GAO) as involving accounting irregularities leading to materially misstated financial results. During periods of suspicious accounting, the firms over-employ in labor and over-invest in capital. Following fraud detection, they reduce employment and investment and improve in productivity.

The research by Farber (2005) is another study documenting potential benefits from fraud discovery. He investigates a sample of firms identified by the US Securities and Exchange Commission (SEC) as involving fraudulent financial reporting. Like prior research, he finds that relative to a control sample, firms with accounting fraud have poorer corporate governance in the year before fraud detection (e.g., fewer audit committee meetings, fewer financial experts on the audit committee, a smaller percentage of Big 4 auditing firms, and a greater likelihood of CEO-chairman duality). Following the revelation of fraud, the firms take actions to improve their governance; three years after, they have governance characteristics similar or even superior to the control firms. He also finds that investors appear to value governance improvements: the firms taking actions to improve governance have better stock price performance, even after controlling for earnings performance.

The existence of fraud could be due to abuses by entrenched executives or could be facilitated by incompetent executives/outside directors. The managerial power theory (Bebchuk and Fried 2003 and 2004) provides an explanation to why despite a great deal of suspicion (namely a very high \( \theta \)), the board of directors and audit committee do not take investigative or remedial actions until after fraud is revealed by some third party. Additional pressure from outside following fraud discovery/allegation increases the chance of improving the firm value by ousting entrenched executives or incompetent directors. For this and other reasons, Agrawal, Jaffe, and Karpoff (1999) argue that the revelation of fraud can increase the likelihood of management turnover.

In line with this, a recent study by Krishna-Moorthy (2011) shows that CEO and CFO turnover is higher following the settlement of fraud allegations. Similar results are obtained by Fich and Shivdasani (2007) for tainted directors (i.e., outside directors also sitting on the boards of other firms involved in fraud). Moreover, they find that tainted directors’ departure from non-fraudulent firms is associated with increases in firm valuation.

In other contexts where PR2BN are expected, opportunities for improvement also exist. For instance, ICW disclosure and fraud discovery are material negative events with similarities. Johnstone, Li, and Rupley (2011) argue that during ordinary times, costs and difficulties associated with governance changes might prevent improvements from happening. However, following material negative events, there are stronger incentives to make changes to improve reputational capital or bring in new directors with experience to remediate negative events. Supporting this argument, they find that the turnover of audit committee members is higher following ICW disclosure. Moreover, remediation of ICW is positively associated with the turnover of audit committee members. Additionally,
remediation is positively associated with changes in important corporate governance characteristics. They include an increase in the proportion of independent directors on the board, having an audit committee member chairing the board, and improvements in audit committee member financial expertise.

6.3 Empirical Proxies for Ex Ante Probability of Fraud

Hypotheses 1 and 2 stated in the introduction assume that the strength of corporate governance and internal control can be used to proxy for the ex ante probability of fraud (i.e., $\theta$). In testing the hypotheses, the potential endogeneity between ex ante probability of fraud and corporate governance/internal control needs to be addressed. A procedure similar to the one adopted by Fich and Shivdasani (2007) may be used to mitigate the issue.

First of all, they notice that corporate governance attributes such as CEO-chairman duality and the Gompers, Ishii, and Metrick's (2003) governance index vary systematically by industry. Therefore, they follow Barber and Lyon (1996) to construct an industry-matched control sample for their analysis. Additionally, they use a simultaneous-equations framework to account for the endogeneity between tainted directors and fraud lawsuits of a firm. The framework addresses the concern about what tainted directors represent: (i) they might be viewed as weaker gatekeepers and hence tend to allow fraud to occur in other firms where they are on the boards; (ii) they might be viewed as someone with expertise dealing with fraud lawsuits and are therefore sought for service by firms at greater risk of fraud.

Applying the framework to here, one needs to recognize that although stronger corporate governance/internal control tends to prevent fraud from occurring in the first place, firms with a higher ex ante probability of fraud might be self-selected to adopt stronger corporate governance/internal control. Therefore, separate regression equations need to be specified to account for the endogenous relationship between the strength of corporate governance/internal control and the ex ante probability of fraud. Such a simultaneous-equations framework can mitigate the endogeneity issue, provided that variables required for identification of the simultaneous-equations system are selected properly. Additionally, detection controlled estimation (Li 2010) is required to correct for the unobservability of undetected fraud in estimating the ex ante probability of fraud.

To the extent that risk factors underneath fraud and accounting misstatements are correlated, one can consider using the F-Score proposed by DeChow, Ge, Larson, and Sloan (2011) or the commercially developed Accounting and Governance Risk (AGR) and Accounting Risk (AR) measures (Price, Sharp, and Wood 2011) as proxies for $\theta$. These proxies, however, might also involve endogeneity issues. In contrast, cultural-based proxies for $\theta$ are likely to be exogenously determined.

Prior research has shown that cultural factors can affect economic decisions like financial contracting (Giannetti and Yafeh 2011). Hilary and Hui (2009) find that firms located in US counties with higher levels of religiosity display lower degrees of risk exposure, in terms of variances in equity returns or returns on assets. Moreover, these firms generate a stronger positive stock price reaction

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37I thank Michelle Hanlon for suggesting the F-Score.
when announcing new investments. They also document that CEOs are more likely to join a firm with a similar religious environment as in their prior employers.

Several recent studies have demonstrated that religiosity influences unethical corporate behavior and financial reporting irregularities. Grullon, Kanatas, and Weston (2009) show that firms with headquarters in highly religious counties are less likely to backdate options and grant excessive compensation packages to executives. Dyreng, Mayew, and Williams (2009) document that companies operating in counties with higher levels of religious adherence have lower risk of fraudulent accounting and are less likely to restate financial results. Similarly, McGuire, Omer, and Sharp (2010) find a negative association between religiosity and abnormal accruals, suggesting that firms headquartered in areas with stronger religious social norms have higher accounting quality. Based on these studies, it seems reasonable to use religiosity as an exogenous empirical proxy for the ex ante probability of fraud.

6.4 Other Related Studies

The two-round audit process modeled in the paper is probably the simplest way to capture the considerations prescribed in SAS 99 (see the discussion in section 2). The formulation provides a parsimonious structure to relate the fraud detection role of auditors to audit and financial reporting delays, serving the purpose of obtaining closed-form results to guide empirical research.

Compared to NPS’s continuous audit effort formulation, the binary audit choice model here has some dissimilar results. For example, in this model a greater incentive to extend audit procedures owing to a higher penalty multiplier \( a \) is neutralized by a new equilibrium diversion rate that continues to make the auditor indifferent between extending or not extending the audit. In the end, the equilibrium audit fee does not depend on \( a \), unlike the case in the NPS model. For a similar reason, outside ownership \( \lambda \) affects the audit fee in their model but not here.

To clearly differentiate my contributions from NPS’s, I have chosen not to include any analysis about the effects of the penalty multipliers, \( a \) and \( b \), on the equilibrium of my model. While mine is similar to NPS’s model, the strategic consideration of extending audit procedures is unique. Recognizing this key difference between the two models, I believe it should be interesting to examine in future research the effects of the penalty multipliers on the equilibrium.

The Knechel and Payne (2001) study reports that “audit report lag is decreased by the potential synergistic relationship between MAS [(i.e., management advisory services)] and audit services.” The model of this paper provides a theoretical foundation to interpreting the empirical finding in this way. Specifically, I have shown that \( d \Pr\{x = 1\}/dp > 0 \) and \( d \Pr\{x = 1\}/dq < 0 \). If indeed a synergistic relationship between MAS and audit services results in a less imprecise red flag generated by regular audit procedures and more effective extended audit procedures for detecting fraud, it will reduce the likelihood of a delay. Intuitively speaking, this is consistent with a reduction in the audit report lag.\(^3\)

\(^3\)In its present form, the model cannot directly talk about audit report lag because the delay is modeled as a 0-1 event with a fixed delay duration. A modification to it using some hazard-rate function modeling technique would
result from a closer business relationship with an audit client purchasing also MAS. Extending the model in this direction provides another avenue for future research.

Ettredge, Li, and Sun (2006), p. 5, argue that “[a]uditors ... need to extend their scope of work and perform additional substantive tests to compensate for the control weakness. ... The extended audit effort due to control weakness should lead to longer audit delay.” However, I believe that such a delay needs not be significant if the auditors have prepared enough manpower to do the additional tests concurrently. Presumably, the control weaknesses are identified mainly in the planning phase. It can be well ahead of the peak period around the fiscal year end when the auditors are completely tied up and unable to easily find more new hires to provide the extra help. Such a control weakness driven delay might not be as unanticipated as a delay due to contingent extended audit procedures discussed in this paper.\(^\text{39}\)

7 Concluding Remarks

This paper contributes to the literature by formalizing intuitions and deriving predictions consistent with findings in the literature (e.g., the positive association between audit fee and outside ownership). Most important, on top of this, it adds an interesting new insight that is hard to discover without the help of such a model (i.e., the possibility of a positive market reaction to a delay). The positive-reaction result illustrates the fundamental logic behind the general phenomenon of PR2BN whenever the bad news is a crisis (i.e., an opportunity for improvement lying in the middle of a danger). The logic is consistent with several examples of the PR2BN phenomenon documented in the literature.

The paper also reaches out to an empirical-oriented audience by discussing the model’s empirical implications in detail, including specifics about how the model can provide suggestions for regression equation specifications. Any empirical test based on the model is a joint test of the model predictions and assumptions. Keeping in mind this caveat can avoid mistaking spurious findings as evidence for supporting the model. For example, the model assumes fraud is the main issue behind a delay. In reality, there are other reasons leading to delays. In constructing the sample for an empirical test, it is important to take into consideration such reasons. I elaborate on this point in appendix A where an illustrative example of an empirical test design is discussed.

To facilitate deriving closed-form results useful for guiding empirical hypothesis testing, simplifying assumptions have been made to keep the model tractable. For example, unlike prior models with a “two-round” audit, I do not allow the auditor to make continuous audit choices. Allowing continuous choices potentially could make the model useful for addressing a number of additional questions: e.g., When is it more efficient for the auditor to focus on the first-round audit and when is it more efficient to focus on the second? How does the auditor respond to an increase in the strictness of the legal environment when there are two rounds of audit instead of one? Are the

\(^39\)For control weaknesses discovered during the audit after the planning phrase, they are equivalent to the red flag discussed in this paper and can lead to a delay not fully anticipated.
effort levels in the two rounds substitutes or complements? While these questions are interesting, they are not the focus of this paper and are left for future research.

I have also assumed that honesty is exogenously determined in the model. Recent development in economic theory has begun analyzing endogenous honesty behavior using “lying cost” models (e.g., Kartik 2009). Advocates argue that business ethics education in MBA programs should be strengthened to reduce unethical behaviors of future business leaders. Suppose business ethics education is effective. The model here then suggests if such education reduces \( \theta \) (i.e., the prior probability of having a dishonest insider) from 5\% to 4\% (a one-percent difference), it will save society in audit cost by a percentage approximately equal to \( \frac{(1 - p)}{[1 - 0.95 (1 - p)]} \).\(^{40}\) Investigation along this line provides an avenue for quantitative business ethics research.

Suppose a government has one extra dollar to be used on reducing fraud. Should it be spent on reducing the imprecision of a red flag, i.e., \( p \), or on enhancing the effectiveness of extended audit procedures for detecting fraud, i.e., \( q \)? Or would it be even better to spend the extra dollar on decreasing the chance of having a dishonest insider, i.e., \( \theta \), say, by providing better business ethics education that works? These are interesting questions unanswered here. However, the model of this paper can provide a vehicle for future research to examine these and some other interesting questions.

\[^{40}\text{Formally,} \]

\[
\frac{\partial \log F^*}{\partial \theta} \bigg|_{\theta=0.05} = \frac{1 - p}{1 - 0.95 (1 - p)}. 
\]
References


Appendix

A Illustrative Example of an Empirical Test Design

The purpose of this appendix is to illustrate how a theoretical result of the paper can provide an easier starting point for formulating an empirical test design. It is not intended to be a detailed cookbook for one to follow step by step to conduct an empirical study.

The positive-reaction condition (32) predicts that with everything else being equal, if a positive reaction to a delay occurs to a firm with an ex ante probability of fraud \( \theta \), it should also occur to firms with higher \( \theta \)'s. To construct a sample for testing this hypothesis, it is important to exclude observations with known reasons for delays unrelated to fraud concerns, e.g., due to mergers and acquisitions.

In the model, the two types of insider, namely dishonest and honest, are identical in productivity (i.e., the rate of return \( g \) on investment does not depend on the insider type). Consequently, a dishonest insider is clearly undesirable. In reality, investors might want a dishonest CEO as long as he provides the highest net value added to the firm (after deducting for misappropriation) when compared to what could be provided by a replacement CEO. For such a high-productivity, dishonest insider, an audit delay is unlikely to be on the whole good news to investors. Considerations like this need to be taken into account in testing the model’s predictions. For example, control variables for holding constant the capital, labor, and productivity (Kedia and Philippon 2009), as well as the executive compensation, might help reducing the influence by such differences.

Holding constant \( b \), i.e., the penalty multiplier for a fraudster, should not be a problem if the sample observations are under a single jurisdiction. This is true for many countries but not necessarily for the US because the states can differ in their choices of \( b \). Because the outside ownership \( \lambda \) is observable, an attempt to hold it constant can be made by ranking firms by decile so that the test can be done based on observations in each decile.

Holding constant the imprecision \( p \) and the effectiveness \( q \) of the two-round audit process is a challenge because they are not directly observable. Suppose both \( p \) and \( q \) are highly correlated with a single observable quality of auditors. Then ranking auditors by decile based on the quality and testing the hypothesis using observations in each decile can provide a way to reduce the variations in the unobservable \( p \) and \( q \). Besides the Big4 versus non-Big4 classification, the accuracy rate of an auditor’s going-concern opinions can be another candidate to proxy for the observable quality.

Finding proxies for \( \theta \) is less difficult. One can twist the interpretation of the model by arguing that insiders are always dishonest. However, whether they have the opportunity to divert firm resources for private benefit depends crucially on the corporate governance and internal control environments. With this alternative interpretation, \( \theta \) can be proxied by a variety of corporate governance variables like CEO-chairman duality, the number of independent directors, and the size of the board. It can also be proxied by data on internal control weaknesses. (See also the discussion in section 6.3 for issues such as endogeneity.)

With a proxy for \( \theta \), one can test a weak version of the hypothesis by arguing as follows: A
positive reaction to a delay might be rare. However, even when the usual negative reaction to a
delay is observed, the response should be weaker if the news is less bad. The positive-reaction
condition provides a way to classify good and bad news. If $\theta$ is high, the news of a delay is not
as bad as when $\theta$ is low because the market is “psychologically prepared” and not shocked by the
delay. Based on this reasoning, one can partition the sample by the median value of a proxy for $\theta$.
A weaker negative market reaction is thus expected for the above-median subsample.

B Proof and Derivation

Proof. (Proposition 2) Substituting the optimal $\lambda^*$ and $\mu^*$ into (18) and (19), I obtain the following
identities:

$$R(K) = \mu^*[(1 + g)(K + W) - a\varphi];$$

$$\mu^* = \lambda^* \left[1 - \frac{b\varphi}{(\lambda^* + b)}\right].$$

Totally differentiating both sides of the identities with respect to $\varphi$ yields the following identities:

$$0 = \left(\frac{d\mu^*}{d\varphi}\right) \Pi(\varphi) - \mu^* a;$$

$$\left(\frac{d\mu^*}{d\varphi}\right) = \frac{d\lambda^*}{d\varphi} \left[1 - \frac{b\varphi}{(\lambda^* + b)}\right] - b\lambda^* \left[\frac{1}{(\lambda^* + b)} - \frac{d\lambda^*}{d\varphi} \varphi\right],$$

where $\Pi(\varphi) = (1 + g)(K + W) - a\varphi$. Rearranging the terms and recognizing that $\mu^* = R(K)/\Pi(\varphi)$,
the identities become

$$\left(\frac{d\mu^*}{d\varphi}\right) = \frac{aR(K)}{\Pi(\varphi)^2};$$

$$\left(\frac{d\lambda^*}{d\varphi}\right) \left[1 - \frac{b^2\varphi}{(\lambda^* + b)^2}\right] = \left(\frac{d\mu^*}{d\varphi}\right) + \frac{b\lambda^*}{(\lambda^* + b)}.\quad (38)$$

Clearly, $d\mu^*/d\varphi > 0$. Recall that $\varphi = \delta^*\theta < 1$. Thus, $d\lambda^*/d\varphi > 0$.

Derivation. (Posterior probability of undetected fraud conditional on a delay) Because a
delay necessarily means a red flag has been observed,

\[
\Pr\{\text{undetected, fraud}\mid x = 1\} = \Pr\{\text{undetected}\mid \text{fraud}, x = 1\} \Pr\{\text{fraud}\mid x = 1\} = (1 - q) \Pr\{\text{fraud}\mid x = 1\}
\]

\[
= (1 - q) \left( \frac{\Pr\{x = 1\mid \text{fraud}\} \Pr\{\text{fraud}\}}{\Pr\{x = 1\}} \right)
\]

\[
= (1 - q) \left( \frac{\Pr\{x = 1\mid \text{fraud}\} \theta}{\omega^*[\theta + p(1 - \theta)]} \right)
\]

\[
= (1 - q) \left( \frac{\omega^* \theta}{\omega^*[\theta + p(1 - \theta)]} \right)
\]

\[
= \frac{1 - q}{1 + p(\frac{1}{\theta} - 1)}.
\]

\[\square\]
# Table 1: List of parameters and variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>prior probability of having a dishonest insider</td>
</tr>
<tr>
<td>( p )</td>
<td>chance of observing a false-positive red flag (i.e., red flag on despite no fraud)</td>
</tr>
<tr>
<td>( q )</td>
<td>probability of detecting fraud that exists</td>
</tr>
<tr>
<td>( C )</td>
<td>extra cost incurred as a result of extending audit procedures</td>
</tr>
<tr>
<td>( x )</td>
<td>decision on extending audit procedures (( x = 1 )) or not (( x = 0 ))</td>
</tr>
<tr>
<td>( \omega )</td>
<td>probability of the auditor’s randomized choice of ( x ), conditional on observing a red flag</td>
</tr>
<tr>
<td>( \delta )</td>
<td>diversion rate, i.e., the proportion of the firm’s resources to divert</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>( \delta \theta (1 - q \omega) ), referred to as effective diversion rate</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>fraction of ownership to sell (also referred to as outside ownership)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( \lambda (1 - \Delta) ), referred to as effective outside ownership</td>
</tr>
<tr>
<td>( K )</td>
<td>amount of external capital to raise</td>
</tr>
<tr>
<td>( R(K) )</td>
<td>outside investors’ total cost of arranging ( K ) dollars of capital to invest in the firm; ( r = \frac{R(K)}{K} - 1 = “cost of capital” for raising ( K ) dollars</td>
</tr>
<tr>
<td>( g )</td>
<td>constant rate of return on any amount of capital invested in the firm’s project</td>
</tr>
<tr>
<td>( W )</td>
<td>endowed wealth of the insider</td>
</tr>
<tr>
<td>( F )</td>
<td>non-contingent audit fee, payable after the audit</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>( (1 + g)(K + W) - F ), i.e., year-end (after-audit-fee) firm value</td>
</tr>
<tr>
<td>( b )</td>
<td>penalty multiplier for an insider committing fraud</td>
</tr>
<tr>
<td>( a )</td>
<td>penalty multiplier for an auditor failing to discover fraud during an audit</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>( a^{-1} \left( \frac{C}{q} \right) [p + (1 - p) \theta] ), referred to as liability discount factor.</td>
</tr>
<tr>
<td>( V_a )</td>
<td>after-delay market value of the shares owned by outside investors</td>
</tr>
<tr>
<td>( V_b )</td>
<td>before-delay market value of the shares owned by outside investors</td>
</tr>
<tr>
<td>( v )</td>
<td>( \frac{V_a - V_b}{V_b} ), i.e., the percentage deviation of the after-delay market value of the shares owned by outside investors from its before-delay market value</td>
</tr>
</tbody>
</table>
The nature determines the insider type: being dishonest with probability $\theta$ and honest with $1-\theta$.

The insider chooses the audit fee $F$ and the fraction of ownership to sell $\lambda$. Outside investors accept the equity offering term; the auditor accepts the audit fee offered.

The insider invests in the project and earns the return; a dishonest insider chooses the rate $\delta$ of the year-end firm value $\Pi$ to divert. (An honest insider always chooses $\delta = 0$.)

The auditor performs regular audit procedures; a red flag is observed with probability 1 or $p$, depending on whether fraud exists or not, respectively.

The auditor decides whether to extend audit procedures ($x = 1$), by incurring an extra cost $C$, or not ($x = 0$).

The evidence obtained from extended audit procedures can be a false negative with probability $1-q$ but can never be a false positive.

The audit opinion is issued based on whether evidence of fraud has been obtained.

If fraud is discovered, the insider must return the diverted resources and also bear a penalty of $b\delta \Pi$. If fraud exists but is not discovered by the auditor, she has an expected liability cost equal to $a\delta$. 

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