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Merger Clusters during Economic Booms

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Abstract

Merger activity is intense during economic booms and subdued during recessions. This paper provides a non-financial explanation for this observable pattern. We construct a model in which the target—by setting the takeover price—screens the acquirer on his (expected) ability to realize synergy gains when merging. In an economic boom, it is less profitable to sort out relatively “bad fit” acquirers, leading to a hike in merger activity. Although positive economic shocks produce expected gains at the time of merging, these mergers turn out to be less efficient in the long term—a finding that is broadly consistent with the existing empirical evidence. Furthermore, again because of the absence of boom-time screening, the more efficient acquirers earn higher merger profits during “merger waves” than outside of waves, which is also in line with empirical evidence.

JEL Classification Numbers: D21, D80, L11.

Key Words: Mergers, Merger Waves, Screening.


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1 Introduction

The existence of periods of intense merger activity, typically referred to as merger waves, is well documented (Andrade and Stafford, 2004).\(^1\) Merger activity usually heats up in economic booms and slows down in recessions (see e.g. Maksimovic and Philips, 2001). Numerous empirical papers point to various exogenous factors like technological innovation, deregulation, or a demand boom as triggers of merger waves (see e.g. Harford, 2005). There is, however, surprisingly little analysis of how a change in the economic fundamentals can generate a spike in merger activity.

This paper proposes a mechanism that, despite its simplicity, is consistent with a variety of stylized facts about mergers. In our model, the target in a potential merger commits to a takeover price. This enables the target to screen acquirers on the expected synergy gains. Changes in the economic environment that often coincide with a boom induce mergers that would not have occurred otherwise. The reason being that—in an economic boom—merging becomes more profitable and merging with low-synergy types becomes more similar to merging with high-synergy types. This raises the opportunity cost of sorting out relatively inefficient acquirers. The target, therefore, sets a takeover price that is acceptable to both high- and relatively low-type acquirers—leading to a hike in mergers. Thus, an exogenous shift in the economic conditions causes a lack of screening and, as a consequence, a merger cluster or a merger wave.

This lack of screening during booms not only helps explain procyclical spikes in merger activity but also enables the more efficient, high-type acquirers to extract more rents from a merger. This is consistent with recent evidence that in a merger wave, bidders gain on average higher (short term) abnormal returns than bidders outside a wave (Harford, 2003; Gugler et al., 2006; Rosen, 2006).\(^2\) Furthermore, in line with our mechanism, Carow et al. (2004) find that during waves high-type acquirers earn more from a merger than low-type

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\(^1\)Weston et al. (1990) and Martynova and Renneboog (2005) provide excellent reviews of the literature.

\(^2\)Some empirical evidence we will cite in the paper is based on merger waves that coincide with periods of high stock markets rather than economic booms. These stock market booms, however, are (highly) correlated with economic booms as e.g. Jovanovic and Rousseau (2001) indicate.
acquirers.

At the same time, our mechanism can also explain why, on average, mergers that occurred in a wave during an economic boom are less efficient than non-wave mergers. Mergers during a baisse should stay relatively more profitable in the long term, since these are better filtered out by the target. Indeed, in the long term, wave mergers perform on average significantly worse than non-wave mergers, as Gugler et al. (2006), Harford (2003) and Rosen (2006) show. In our model, merged entities are possibly even less efficient than non-merging firms, which is again consistent with empirical evidence of Carow et al. (2004).

Our modeling approach explicitly embodies two related features about mergers that have been prominent in the literature. First, one empirical regularity is that the target extracts considerable rents from bidders in a takeover (see Andrade et al., 2001). Therefore, as Inderst and Wey (2004), we model takeovers as a process in which the target can commit to an optimal reserve price. This is also in line with Cramton (1998), who compares the various tactics employed by the target firm and concludes that these are equivalent to the setting of an (implicit) reserve price. In contrast to Inderst and Wey (2004), our model consists of a single potential acquirer bidding for the target. This allows us to abstract from the free-rider problems between acquirers, emphasized by Inderst and Wey (2004). Below, however, we argue that our results can be extended to the case of multiple bidders.

Second, in the takeover process not only information about the quality of the target is important but also information about the cost synergies that may arise with different acquirers, i.e. the goodness of fit. The distribution of information about potential cost

3Cramton (1998) states that "A target’s board has a great deal of discretion in establishing procedures...This power arises from the target’s prior issuance of a poison pill. In essence, a poison pill makes any acquisition unprofitable...Hence all bids are made conditional on the target board redeeming its pill. At best, poison pills afford the board a (limited) ability to set a reserve price."

4Andrade at al. (2001) do find that the average transaction in the 1990s involved only one bidder; for the period 1973-1998, the number of bidders per deal was on average 1.1. Also, Boone and Mulherin (2006) find strong evidence that the returns to bidders in auctions are similar to the returns to single bidders, suggesting that a bidding war is not responsible for the target’s rent extraction.
synergies across bidders, banks, and targets has played an important role in the analysis of takeover processes. In Scharfstein (1988) and Rhodes-Kropf and Viswanathan (2004), the raider has private information about the synergistic gain or loss of a merger relative to the target. One justification for this assumption, which we also adopt, is that acquirers are on average substantially larger, older, and more experienced in merger activities than targets (see e.g. Rhodes-Kropf et al., 2005). Thus, to get information on synergy gains is substantially easier and relatively cheaper for the acquirer. Also, it seems natural to think that potential acquirers spend considerable time working out a future business plan in the event that the deal is successful.

The target considers the following trade-off in our setting. By requesting a high takeover price, it extracts (all) merger rents whenever the post-merger efficiency gains are high, i.e. whenever the raider is a “good-fit” acquirer (high type). By doing so, however, the target risks that it cannot sell if the acquirer turns out to be a relatively “bad fit” (low type). The target thus compares the gains of setting a high price (and thereby screening the acquirers) to setting a low price and selling for certain (pooling acquirers). An economic boom makes pooling acquirers more attractive as long as (1) merging (with any type of acquirer) becomes more profitable and (2) the benefit of merging with a relatively low-type acquirer approaches that of merging with a high-type acquirer.

Both conditions are satisfied in our basic market model, where an economic boom is generated by a rise in demand in a Salop-type circular-city model\(^5\) in which mergers involve fixed costs.\(^6\) This is due to two underlying forces. First, since prices are strategic complements, a merger is a commitment to set higher prices—the fat cat strategy in the terminology of Fudenberg and Tirole (1984)—and this commitment is more valuable

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\(^5\)To facilitate technical issues, we follow Cabral (2003) and use quadratic transportation costs to model product differentiation rather than linear transportation costs as Salop (1979) does.

\(^6\)See also Davidson and Deneckere (1985) who argue that merger models based on price competition with differentiated goods are suitable in explaining observed facts of mergers. Furthermore, as argued in Lambrecht (2004), mergers involve significant one-of costs, such as legal fees, fees to investment banks and other merger promoters, and the costs of restructuring and integrating the two companies (see e.g. Houston et al., 2001).
when demand is higher. Second, if demand is higher, merging with the low-type acquirer becomes relatively more attractive as the one-off merging costs become relatively less important. After providing a general expression for conditions (1) and (2) above, we show that our mechanism is also present in mergers across independent markets and in vertically related markets.

Other recent theoretical work has made advances in explaining the procyclicality of merger activity. Lambrecht (2004) shows that when merger synergies are an increasing function of a (stochastic) product market demand, then each firm’s payoff from merging has features similar to call options. Firms therefore have an incentive to merge (exert their option) in periods of economic expansion. Toxvaerd (2004) shows that if an increase in an economic fundamental increases the number of expected future mergers, this in turn can induce preemptive mergers today, leading to cluster effects. Jovanovic and Rousseau (2002) show that bursts in merger activity may follow from technological shocks that lead to a higher dispersion of efficiency in an industry. These shocks lead to a reallocation of physical assets from less efficient targets to more efficient acquirers. Similar with ours, the above mentioned models find pro-cyclical merger clustering. Our screening model, however, differs in that it succeeds to explain long-term less efficient wave-mergers and higher returns for wave-acquirers at the time of merging.

In contrast to our findings, Fauli-Oller (2000) demonstrates that in a Cournot model with constant marginal costs, merger waves are more likely when there is a low realization of demand. The most common interpretation of the “constant-marginal-cost” Cournot merger model is that the merger leads to the closure of the less efficient merger participant (see Perry and Porter, 1985). From this perspective, Fauli-Oller (2000) provides a rationale for elimination of excess capacity in declining industries (see also Dutz, 1989, and Lambrecht and Myers, 2005, for a similar rationale). Our basic market model is a vari-

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7Our conditions are also satisfied in other modelling specifications. For example, in a Cournot set-up with capacity constraints, a merger sometimes conduces little output expansion by outsiders. Higher demand can then lead to higher incentives to merge (see Van Wegberg, 1994).

8A case study of Dutz (1989) and casual evidence in Lambrecht and Myers (2005) indicate that this has occurred in some particular troubled industries. The systematic evidence on merger waves, however,
ant of Salop’s (1979) circular-city model of price competition with differentiated goods. This setup allows the acquirer to benefit from operating the acquired firm—indeed, in non-declining industries acquirers typically do not close the acquired production sites (Lambrecht, 2004).

The paper is structured as follows. In the next section, we introduce the basic model. Section 3 establishes our main result and compares our findings with stylized facts gathered from the empirical merger literature. In Section 4, we outline general conditions for our predictions and provide further examples and extensions. Finally, in Section 5, we conclude. Some proofs are relegated to the Appendix.

2 Basic Model

There are three firms in the market. Absent a merger, all three produce at a marginal (and average) cost $c$.9 A given firm, the acquirer, is potentially interested in buying another firm, the target. If the acquirer is a good fit (“high type”), it can reduce the marginal cost of both units to $c_m = \underline{c} < c$. If less fitting (“low type”), it can only operate the joint entity at marginal cost $c_m = \bar{c} > \underline{c}$.10 The common prior that the post-merger marginal cost is $c_m = \underline{c}$ is $q$; with probability $(1 - q)$ it is $c_m = \bar{c}$.11 Prior to the merger process, however, the acquirer collects information about the profitability of the transaction. Formally, the acquirer receives a perfect private signal about the realized $c_m$. When firms merge, they incur a fixed restructuring cost of merging $R > 0$.

The timing of the merger process is as follows. First, the target sets a reserve price, $r$, clearly indicates that merger activity is on average highly procyclical (see e.g. Gugler et al, 2006).

9The results for the case in which firms have positive instead of zero fixed costs would be exactly the same. Fixed costs may be a reason for which other firms do not enter in the medium term, as we assume throughout the paper.

10Possibly $\bar{c} > c$, which means that a combination of two firms that have no fit may lead to higher marginal costs. The efficiency reduction can happen because of, for example, post-merger integration problems (Banal-Estañol and Selleslachts, 2005).

11All the results can be replicated for a continuous distribution of types. For ease of exposition, we present the two-type model.
at which it is willing to sell the company. Second, the acquirer either accepts or rejects. If he accepts, the merger is carried out—i.e. the target receives a payoff of \( r \) and the acquirer obtains the target’s production facility. Otherwise, the merger is abandoned. Finally, all costs are revealed and the market game specified below is played. Observe that by allowing the target to set a take-it-or-leave-it offer, we assume that the target firm has all the bargaining power in the merger process.

The market game is a variant of Salop’s (1979) circular city model. There are three products located equidistantly from each other on a circle with circumference one. On the same circle, there is a mass \( M \) of consumers with differing tastes, uniformly distributed and indexed by \( x \in [0, 1] \). For any two locations \( x \) and \( y \), let \( d(x, y) \) be their distance on the circle. Utility of consumer \( x \) from buying product \( y \) at price \( p \) is \( v - t \cdot d(x, y)^2 - p \), where \( v - t \cdot d(x, y)^2 \) is her “satisfaction” with the good and \( -p \) is her disutility from paying its price. In other words, \( x \) is the consumer’s “ideal variety”, which yields benefit \( v \). If a person consumes a product \( y \) different from her ideal, she suffers disutility equal to \( t \cdot d(x, y)^2 \), where \( t \) is a measure of the differentiation between varieties of the product—the so-called transportation cost. To avoid unenlightening extra notation, we assume a consumer’s utility from not consuming is negative infinity, so that she always chooses to buy one of the products. In this setup, the mass of consumers \( M \) is a natural measure of the market size.

Firms’ simultaneously set prices for the products they own, and, upon observing these prices, each consumer purchases the product that maximizes her utility. We solve for perfect Bayesian equilibrium. Furthermore, we restrict attention to interior equilibria in which each product is sold to a positive mass of consumers.\(^{14}\)

\(^{12}\)The original Salop (1979) paper considered linear transportation costs. As Cabral (2003), we assume quadratic transportation costs to ensure the existence of pure strategy equilibria following a merger for a wide range of marginal costs.

\(^{13}\)Our results would be identical if consumers had an option of not buying, but \( v \) was sufficiently high (or costs and product differentiation sufficiently low) so that no consumer would take advantage of this option in equilibrium.

\(^{14}\)As we show in the appendix, a sufficient condition for the existence of such equilibria is that the marginal cost of the merging firm after a merger satisfies \( c - \frac{4}{9}t \leq c_m \leq c + \frac{5}{9}t \).
3 Results

Solving the market game (see the appendix), if no merger takes place the market model yields the following reduced form profits for each firm $i = 1, 2, 3$:

$$\pi^*_i = \frac{Mt}{27}.$$

Suppose that two firms have merged. Then we show in the appendix that the total profits of the merged company are

$$\pi^*_m = \frac{M}{3t} \left( c - c_m + \frac{5t}{9} \right)^2.$$

Then merging gives a relative profit with respect to not-merging of $\Delta \pi^*_m = \pi^*_m - 2\pi^*_i - R$, which, substituting from above, can be rewritten as

$$\Delta \pi^*_m = \frac{M}{3t} \left[ \left( c - c_m + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right] - R.$$

(1)

The acquirer accepts the reserve price $r$ if and only if

$$\pi^*_m(c_m) - r - R \geq \pi^*_i.$$

The optimal reserve price of the target is one of the following three (types of) reserve prices $r$. First, it can set the highest price such that both types of acquirers accept. This “pooling” reserve price $r^p$ is such that the low type is indifferent between accepting and rejecting, $r^p = \pi^*_m(c) - \pi^*_i - R$. Second, it can set the highest price such that only high types accept. At this “separating” reserve price, the high type is indifferent between accepting and rejecting, $r^s = \pi^*_m(c_i) - \pi^*_i - R$. Finally, it can set a reserve price such that no buyer accepts. We denote (any) such prohibitively high reserve price by $r^\infty$, $r^\infty > \pi^*_m(c_l) - \pi^*_i - R$.

We next introduce a condition to ensure that the target might prefer to pool acquirers rather than to screen them. In our model, this corresponds to ensure that pooling is better in the absence of any restructuring cost. The condition requires that low-type acquirers are not too different from high-type acquirers, or that there are enough low types around
in order for the target to be willing to bet on meeting a low type. Using the relative merger profits in equation (1) for both the high and the low type, this translates into the inequality below.

**Assumption 1** The prior \( q \) and the post-merger marginal costs \( \zeta \) and \( \overline{c} \) are such that

\[
\left( c - \overline{c} + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \geq q \left[ \left( c - \zeta + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right].
\]

We are now ready to state our main proposition:

**Proposition 1** If Assumption 1 holds, there exist strictly positive and finite market sizes \( \underline{M} \) and \( \overline{M} \) such that for

- \( M < \underline{M} \), no takeover takes place;
- \( \underline{M} \leq M < \overline{M} \), only high-type takeovers occur;
- \( M \geq \overline{M} \), both high-type and low-type takeovers occur.

**Proof.** We begin by comparing the target’s profit from screening (setting a reserve price \( r^s \)) to setting a prohibitively high reserve price \( r^\infty \). The target strictly prefers the separating over any prohibiting reserve price if and only if

\[
q \cdot r^s + (1 - q) \cdot \pi_i^* > \pi_{i*}^*,
\]

which is equivalent to

\[
\pi_{m*}(c) - 2\pi_i^* - R > 0,
\]

which in turn is equivalent to

\[
\frac{M}{3t} \left[ \left( c - \zeta + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right] - R > 0.
\]

The left-hand side is increasing in \( M \) (given that \( c < c \)), negative at \( M = 0 \) and positive for high enough \( M \). Hence, there exists a unique \( \underline{M}, \overline{M} \equiv \frac{3tR}{(c - \zeta + \frac{5t}{9})^2 - \frac{2t^2}{9}} \), such that if \( M < \underline{M} \), the target strictly prefers \( r^\infty \) and if \( M \geq \underline{M} \), the target prefers \( r^s \). Following
the same arguments (now using \( c < c \) and Assumption 1 to show that the left hand side is increasing), there exists a unique \( \hat{M} \), \( \hat{M} = \frac{3MR}{(c-\bar{e}+\frac{5t}{9})^2-\frac{2t^2}{9}} > M \), such that if \( M < \hat{M} \), the target strictly prefers \( r^\infty \) to the optimal pooling reserve price, \( r^p \), and if \( M \geq \hat{M} \) the target prefers \( r^p \) to \( r^\infty \).

The target strictly prefers setting the optimal pooling reserve price \( r^p \) to \( r^s \) if and only if
\[
r^p > q \cdot r^s + (1-q) \cdot \pi^*_i,
\]
which can be rewritten as
\[
\pi^*_m(\bar{c}) - \pi^*_i - R > q \cdot (\pi^*_m(c) - \pi^*_i - R) + (1-q) \cdot \pi^*_i,
\]
which in turn can be restated as
\[
\frac{M}{3t} \left[ \left( c - \bar{e} + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right] - R - q \left( \frac{M}{3t} \left[ \left( c - \bar{e} + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right] - R \right) > 0.
\]
Given Assumption 1, the left-hand side is increasing in \( M \), negative at \( M = 0 \) and positive for high enough \( M \). Hence, there exists a unique \( \overline{M} \) such that if \( M < \overline{M} \), the target strictly prefers \( r^s \) and if \( M \geq \overline{M} \), the target prefers \( r^p \). Given that this function evaluated at \( \hat{M} \) is negative, one has \( M < \hat{M} < \overline{M} \).

To summarize, if \( M < \overline{M} \) the target sets a prohibitive reserve price. If \( \overline{M} \leq M < \hat{M} \), the target sets the separating reserve price and if \( M \geq \overline{M} \), it sets the pooling reserve price.

The intuition is simple. First, when consumer demand \( M \) is low, merger conditions are bad: merger profits are low for a small market size and merging requires a fixed cost \( R \). Hence, it is unprofitable to merge. The target, thus, sets a reservation price too high for any acquirer to accept. When merger conditions become better, high-type mergers become profitable while low-type mergers remain unprofitable. By requesting a high takeover price, the target extracts all post-merger efficiency gains if the acquirer it meets is a high-type acquirer. By doing so, however, the target risks that the acquirer is a “bad” acquirer in which case the takeover will be abandoned. As the market size
increases and merger conditions become even better, mergers involving low-type acquirers become profitable: \( \pi^*_m(c) - 2\pi^*_i - R \geq 0 \). This does not mean, however, that the target sets a reserve price that accommodates low-type acquirers. For if it would do so, it would earn approximately zero benefits from the takeover while if it screens the acquirer it—with a probability \( q \)—gets strictly positive benefits from the takeover. As the market size increases even further, both high- and low-type mergers become more profitable. Setting a price that is acceptable to low-type acquirers becomes more and more attractive as otherwise the target cannot reap the positive merger profits from lower types, which increase relatively faster than those of higher types. Hence, there exists a critical market size at which the target is just indifferent between selling to the high types for a higher price and selling to both types for a lower price. Above this critical market size, the target strictly prefers to set a pooling reserve price.

Observe that above the critical market size at which the target sets the reservation price \( r^p \) such that all acquirers accept, high-type acquirers extract information rents \( (\pi^*_m(c) - \pi^*_i - R) - r^p = \pi^*_m(c) - \pi^*_m(c), \) while below this critical market size they are exactly indifferent between acquiring and abandoning the merger.

**Corollary 1** For market sizes \( M \leq M < M \), the target extracts all the merger rents. For market sizes \( M \geq M \), the high-type acquirer earns rents \( \pi^*_m(c) - \pi^*_m(c) \) and the low-type acquirer earns no rents.

There is evidence that in a merger wave bidders gain, on average, higher abnormal returns than bidders outside waves. In a series of announcement return regressions for bidders, Harford (2003) sets a dummy variable to one for acquisitions made during waves and finds the dummy to be significantly positive in all specifications. Gugler et al. (2006) find that, for tender offers, returns for wave-acquirers in the month of the acquisition are 1% higher than their non-wave counterparts. Finally, Rosen (2006) discovers that bidder stock prices are more likely to increase when a merger is announced in a “hot” merger.
market, i.e. in periods when mergers cluster. These observations are in line with our theoretical prediction.

There is also evidence that during merger waves, high-type acquirers gain more than low-type acquirers at the announcement date.\textsuperscript{15} Carow et al. (2004) distinguish high- and low-type acquirers in a merger wave and find that high types’ abnormal returns are significantly higher for different (short-term) time windows.\textsuperscript{16}

From the proposition, it is also clear that, on average, a merger is technologically less efficient during a merger wave (pooling equilibrium). It may even be that merging firms become less efficient than stand-alone firms, $\bar{c} > c$. These mergers can occur when

$$
\Delta \pi^*_m(\bar{c}) = \frac{M}{3t} \left[ (c - \bar{c} + \frac{5t}{9})^2 - \frac{2t^2}{9} \right] - R \geq 0,
$$

which in the limit becomes

$$
\lim_{M \to \infty} [\Delta \pi^*_m(\bar{c})] \geq 0, \quad (2)
$$

which in turn is equivalent to

$$
\bar{c} \leq c + \frac{(5 - 3\sqrt{2})t}{9}. \quad (3)
$$

Thus, if inequality (3) and Assumption 1 hold, it may be that the merged entity is technologically less efficient.

**Corollary 2** At higher market sizes $M \geq \bar{M}$, technologically less efficient mergers occur. These mergers may even raise the marginal cost of the firms involved.

To relate this corollary with the existing empirical evidence, we provide a simple framework that predicts differences in the long term profitability of mergers as a function of the market size. In particular, we explain why we expect mergers that occur at high values of $M$ (and, thus, are on average technologically less efficient) to perform worse.

\textsuperscript{15} Carow et al. (2004) define types by, among others, the timing of merging in a wave, industry relatedness, and form of payment.

\textsuperscript{16} Measured as industry-adjusted returns over the interval of days $[-1, 1]$ around the announcement of the acquisition, high types earn 4.42% more than low types in the wave and this difference is statistically significant; over the longer interval $[-5, 5]$ the difference is still 4.23%.
in the long run. The simplest justification would be to presume that all (stock) market participants incorrectly presume the current demand conditions $M$ to last forever while in reality boom periods are systematically followed by more normal demand conditions. In this case, since boom mergers have higher marginal cost on average, they perform worse than non-boom mergers in the future. But even if the stock market participants correctly foresee that current boom periods are followed by normal periods in the future, our mechanism survives. For the sake of argument, we distinguish the market size in the “near” and in the “distance” future. Suppose the market game is played twice. First, in the near future with a market size $M$ and, then, in the distant future with (expected) market size $\tilde{M}$. Payoffs of the distant future are discounted. Hence, a takeover is more likely if the market size is large in the near future and small in the distant future than vice versa. Thus, even if the market size switches deterministically between good and bad conditions, we expect more takeover activity when current conditions are good. Somewhat more realistically, suppose that demand can be either high or low and is independently drawn for the near and the distance future. Then—since pooling is more likely if near future demand is high—distant-future profits of mergers that occurred during high demand periods are on average lower because these merged firms have on average higher marginal costs. Thus empirical work that measures profits in the distant future, should indicate that wave mergers perform worse.

Indeed, Gugler et al. (2006) demonstrate that, in the long term, wave mergers perform on average significantly worse than non-wave mergers: The median abnormal return after three years is more than 11% lower for wave-mergers. This is especially true for tender offers, where the difference becomes 34%. Harford (2003) shows the wave-dummy to be significantly negative for different specifications in long-run bidder performance regressions. Also Rosen (2006) finds that long-run returns are significantly lower for mergers announced in periods when the merger market was booming.\footnote{It must be added that Harford (2005) finds a positive effect for wave-mergers on expected long-term earnings. He, however, compares specialists’ forecasts right before and right after the merger. Specialists should reason in the same way as the firms in our model, which should lead to a positive evaluation at the time of merging.} Furthermore, there is a
clear difference between high- and low-type mergers during waves with respect to non-merging firms. Carow et al. (2004) find out that the mean of high-type mergers enjoy abnormal returns of almost 17% after three years with respect to non-merging firms in the same industry. The other (low-type) mergers suffer a relative loss of more than 17%.

For a fixed $c$, it is also easy to see that the closer $\bar{c}$ and $c$ are, the larger the region where a pooling equilibrium exists.

**Corollary 3** For a fixed $c$, smaller acquirers’ differences, $\bar{c} - c$, implies equal critical mass $M$ but smaller $\bar{M}$.

This result is again intuitive. The more similar (potential) acquirers become, the more costly it is to gamble by setting a high (separating) reserve price. Following the previous explanation, our model implies that smaller differences in long term values should be correlated with periods of merger spikes. We are not aware of any work that tries to test this prediction, which might be difficult to do. Potentially, however, one could use the approach of Rhodes-Kropf et al. (2005) who disentangle firms’ short- and long-term valuations. Using the standard deviations of acquirers’ long term valuations (instead of the means, as they do in their analysis), one could potentially test whether smaller standard deviations correlate with merger clusters.

### 4 Generalization and Extensions

#### 4.1 Market Competition

We briefly consider a more general market model. Let the economic condition be parameterized by a real variable $b \in [b_{\min}, b_{\max}]$ with the interpretation that firms’ profits are higher if $b$ is greater. For a given economic condition, let $\pi^H(b)$ be the post-merger profits if the target is bought by a high-type and let $\pi^L(b)$ be the profits in case the target is bought by a low-type acquirer, with $\pi^H(b)$ and $\pi^L(b)$ being continuously differentiable and $\pi^H(b) > \bar{\pi}^H(b)$ for any $b$. Denote the common prior that the acquirer is of high-type again by $q$. Let $\pi^T(b)$ be the target’s and $\pi^A(b)$ the acquirer’s profit in the
absence of a merger. We denote the net merger gain of a high-type merger by $\Delta \pi^H(b)$, i.e. $\Delta \pi^H(b) = \pi^H(b) - (\pi^T(b) + \pi^H(b))$. Similarly, let $\Delta \pi^L(b)$ be the net merger gain of a low-type merger.

We say that a market model satisfies the regularity conditions if (1) the net-merger gains of a high-type merger, $\Delta \pi^H(b)$, increase in $b$ and if (2) merging with a low type becomes relatively more attractive as $b$ increases, i.e. if $\Delta \pi^L(b)/\Delta \pi^H(b)$ increases in $b$.

The circular-city model introduced above satisfies the regularity conditions. We provide other examples below.

We begin with an example that we view as a stylized model of international mergers. Mergers in the example do not change the intensity of competition in any given market but allow for the exploitation of cost advantages.

**Example 1: Independent Markets**: Initially, both firms (target and acquirer) operate in separate local markets with, say, linear (inverse) demand ($p = b - ax$), where the demand intercept $b > c$ is a measure of the economic condition. A merger leads to efficiency gains (i.e. a reduction in the marginal cost $c$), but requires a fixed up-front restructuring cost $R > 0$. These efficiency gains can be either low leading to post-merger marginal costs $\bar{c} < c$ or high leading to $\underline{c} < \bar{c}$. As we show in the Appendix, it is straightforward to check that this setup satisfies the regularity conditions.

Next, we introduce a simple example in which a vertical merger with unknown synergy gains eliminates the double marginalization problem. A merger here eliminates a stylized contracting problem and may lead to reduction in marginal costs of producing.

**Example 2: Vertical Merger**: Initially, there are two firms—an upstream firm $U$ and a downstream firm $D$. The downstream firm produces the final product for consumers using one unit of the upstream firm’s product to produce one unit of output. For simplicity, we assume that the upstream firm produces its output at constant marginal cost $c$ and that the only costs of the downstream firm are the payments made to the upstream firm.
The upstream firm sets a (linear) price $p_U$ at which it sells its output to the downstream firm. After observing $p_U$, the downstream firm sets a price $p_D$ to final consumers. The demand in the downstream market is $x = b - p_D$, where the demand intercept $b > c$ is a measure of the economic condition.

Suppose the downstream firm is the (potential) target while the upstream firm is the (potential) acquirer. The acquirer can be a high-type acquirer—in which case the post-merger marginal cost are $c$—or a low-type acquirer with marginal cost $\bar{c}$, where we assume that $c \geq \bar{c} > c$. As we show in the Appendix, even absent any restructuring cost, this example also satisfies the regularity conditions.

Obviously, the above examples can be considerably generalized.

**Proposition 2** Suppose the market model satisfies the regularity conditions. Then mergers are more likely to take place during economic booms, i.e. there exist $\hat{b}$ and $\overline{b}$ ($\in [b_{\text{min}}, b_{\text{max}}]$) such that if $b < \hat{b}$ no merger takes place, if $\hat{b} < b < \overline{b}$ only good mergers take place, and if $b > \overline{b}$ all mergers take place.

The proof is analogous to the one for the circular-city model and is included in the Appendix. The intuition is simple. When merger conditions are bad, no merger takes place since it is never profitable to merge. Thus, the target sets a reservation price too high for any acquirer to accept. When merger conditions become better, mergers with high-type and low-type acquirers become profitable. But, the difference between $\pi^H$ and $\pi^L$ is still large. By requesting a high takeover price, the target extracts all rents from the post-merger efficiency gains if the acquirer is a “good” acquirer. By doing so, however, he risks that the acquirer is a “bad” acquirer in which case the takeover will be abandoned. Considering the probabilities and payoffs of these two events, the gains of setting a high price and screening the acquirers are compared to the gains of setting a low price that all acquirers are willing to accept.
4.2 Multiple Bidders

Our model could also be extended to accommodate multiple bidders. As Inderst and Wey (2004), assume that potential acquirers participate in an auction in which the target can commit to an optimal reserve price. For simplicity, suppose a second-price sealed-bid auction is used and that bidders play a (reasonable) equilibrium in which each bidder bids the net-gain from the takeover (defined as the increase in profits in the market game if the bidder succeeds in the takeover versus a situation in which the bidder does not succeed).

Suppose first that an outsider does not gain from another merger, as would, for example, be the case if bidders and target were operating in different markets (Example 1 above). If one denotes the probability that none of the bidders is of high type as \((1 - q)\), one would obtain the same results as in the single-bidder case. The target would again face a trade-off between setting a pooling reserve price such that a merger occurs for certain and setting a separating reserve price at a risk that none of the bidders is actually of high type. If the outsiders strictly lose from a merger, the reserve prices and, therefore, the thresholds would be quantitatively different. In a preemptive merger equilibrium (see Molnar (2003) and Fridolfsson and Stennek (2005)), reserve prices and thresholds would reflect not the gain from merging but the net gain of being an insider rather than remaining as an outsider. If both insiders and outsiders gain from a merger then—as pointed out by Inderst and Wey (2004) in a setting with symmetric bidders—a free-riding problem between acquirers arises as long as bidders play a symmetric equilibrium.\(^\text{18}\) In this case, Inderst and Wey (2004) show that it is optimal for the target to set a reserve price such that a takeover occurs with probability less than one. As long as this reserve price for the case of high synergies (our “separating” reserve price) is not accepted by the low synergy types, we expect our results to also hold in this setting.

\(^{18}\)There exists an asymmetric equilibrium with \(n\)-bidders in which only one bidder is active. Thus, if independent of the reserve price bidders always play this asymmetric equilibrium, one gets the same results as in our single-bidder model.
5 Conclusions

We constructed a model in which the target screens the acquirer on the effectiveness of realizing synergy gains, by setting the takeover price. As economic conditions become more favorable, screening out relatively “bad” acquirers becomes less desirable. This leads to a hike in merger activity. We, thus, provided a mechanism that can explain how a positive change in economic fundamentals may generate a spike in merger activity, in line with the observed procyclicality of merger waves. When economic conditions are better, efficiency issues become less important in realizing merger profits. But, although positive economic shocks produce gains at the time of merging, these mergers can develop into unprofitable entities in an economic downturn, exactly due to this lack of screening. Furthermore, again because of the lack of screening during economics booms, more efficient acquirers are able to earn (short-term) positive merger profits, which they cannot during economics downturns. We thus offered a simple explanation for why recent empirical evidence finds that (1) wave-mergers are on average less efficient and thus less profitable in the long run and (2) wave-acquirers are able to extract more rents at the time of merging.

Our model is consistent with efficient capital markets and managers maximizing the value of the firm. Authors such as Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) develop alternative models in which merger waves result from managerial reactions to market misvaluations. Interestingly, our theoretical predictions partly coincide with theirs, given that there is a high correlation between economic booms and stock market booms. Rhodes-Kropf et al. (2005) find in an empirical study evidence for the misvaluation theories, but state that: “An alternative explanation [of our empirical findings] is that aggregate merger intensity spikes when short-run growth opportunities are high. However, the long-run growth opportunities go in the opposite direction; they are negatively associated with merger intensities.” This explanation is consistent with our mechanism if one relates short-run growth opportunities with demand and long-term growth opportunities with a firm’s efficiency.

We provide an explanation of merger clustering based on asymmetric information and screening. It reconciles the empirical regularities explained by the “neoclassical
theories”—economic shocks have a determinant impact on merger clustering—with those explained by the “behavioral theories”—wave mergers perform worse in the long term (see Gugler et al., 2006, for a survey of these theories).

As most other models of merger waves, our theory is essentially non-strategic in the sense that it does not explicitly accounts for the mechanism through which one merger is related to others. Rather, our merger wave is characterized by an exogenous shift in the economic environment—an upward shift in the market demand—that simultaneously makes all mergers attractive. In contrast to other explanations, however, we attribute a spike in merger activity mainly to a lack of screening, which has several new implications that match recent empirical evidence. While a dynamic takeover model is beyond the scope of the current paper, strategic elements can be included in our setting. For example, if all mergers take place in the same industry, screening may be less important in subsequent mergers.\textsuperscript{19} We leave a full investigation of this question to future research.

Appendix

Circular City Model

In this appendix, we compute the equilibrium profits for the circular city model. We begin by assuming that, in equilibrium, each product is sold to a positive mass of consumers. Denote as $d_i \equiv d(x, y_i)$ the distance between a consumer with ideal location $x$ and product $i$, which is located at $y_i$. Hence, the “transportation” cost of consumer $x$ if she buys product $i$ is $d_i^2 t$ and her utility is $v - d_i^2 t - p_i$. Since the utility from not buying the good is negative infinity, each consumer solves

$$\max_i \ v - d_i^2 t - p_i.$$

Given a distance of $1/3$ between products, the transportation costs of consumer $x$ to neighboring product $j = i + 1 \mod 3$ (i.e. in case $i = 3$ one has $j = 1$) will be $(\frac{1}{3} - d_i)^2 t$ if

\textsuperscript{19}This effect would, for example, arise if high-type acquirers move early so that the probability of facing a high-type acquirer, and therefore the benefit of screening, is lower in later mergers.
she is located between product $i$ and $j$. Similar, the transportation cost will be $(d_i - \frac{1}{3})^2 t$ if she is located further away. Apart from transportation costs, consumers will take into account the prices charged for products $i$ and $j$. Hence, as long as no product is priced out of the market altogether, the boundary between the two products will be at a location $x$ at which the generalized costs are equal, i.e.

$$p_i + d_i^2 t = p_j + \left(\frac{1}{3} - d_i\right)^2 t.$$  

It is straightforward to solve for boundary distance $d_i$. Since there is a mass $M$ of consumers uniformly distributed on the unit circle, the demand for product $i$ by consumers that are located on the ‘one side’ of product $i$, $d_i M = \left(\frac{3(p_j - p_i)}{2t} + \frac{1}{6}\right) M$. Analogous, we derive demand for the consumers on the other side of product $i$, which is $d_i M = \left(\frac{3(p_k - p_i)}{2t} + \frac{1}{6}\right) M$. Hence, total demand for product $i$ is

$$q_i^c(p_i, p_j, p_k) = \left(\frac{3(p_j + p_k - 2p_i)}{2t} + \frac{1}{3}\right) M. \quad (4)$$

Now consider first the case in which there are three firms each owning one product. Restricting attention to the case in which each product is sold to a positive mass of consumers, profits are

$$\Pi_i = (p_i - c) \left(\frac{3(p_j + p_k - 2p_i)}{2t} + \frac{1}{3}\right) M. \quad (5)$$

The (candidate) optimal price is then found by letting $\frac{\partial \Pi}{\partial p_i} = 0$. By symmetry, $p_i = p_j = p_k = p^*$, and

$$p^* = c + \frac{t}{9}. \quad (6)$$

Observe that a firm would have to charge a price at or below marginal cost to price its neighbor out the market, which is clearly an unprofitable deviation. Thus, non-local

\footnote{Notice that the indifferent consumer between product $i$ and $j$ must be located at a distance less than $1/2$ from product $i$ because if the consumer located at a distance $1/2$ from product $i$ prefers product $i$ to $j$ then all consumers on the circle prefer $i$ to $j$. Furthermore, under the assumption that all products are sold, such an indifferent consumer must exist.}
deviations are unprofitable and \( p^* \) is the equilibrium price. Substituting the equilibrium prices into the demand function (4) yields equilibrium demand

\[
q^* = \frac{M}{3}. \tag{7}
\]

Substituting equilibrium prices (6) and quantities (7) in the profit function (5) yields

\[
\pi_i^* = \pi_j^* = \pi_k^* = \frac{Mt}{27}.
\]

The next step is to determine profits after the merger under the maintained assumption that each product is sold to some consumers. Let \( \Pi_m \) denote the post-merger profit of the insiders \( i \) and \( j \), which produces both products at constant marginal costs \( c_i = c_j = c_m \).

The merged firm’s profit function is given by

\[
\Pi_m = (p_i - c_m) \left( \frac{3(p_j + p_k - 2p_i)}{2t} + \frac{1}{3} \right) M + (p_j - c_m) \left( \frac{3(p_i + p_k - 2p_j)}{2t} + \frac{1}{3} \right) M, \tag{8}
\]

and the first order condition with respect to prices of product \( i \) is \( \frac{\partial \Pi_m}{\partial p_i} = 0 \). The outsider \( o \)’s first order condition is as in the single-product case. Simplifying, we obtain the candidate post-merger equilibrium price of the outsider \( o \),

\[
p_o^* = \frac{1}{27}(9c_m + 18c + 4t),
\]

and the (identical) candidate post-merger price for each of the two insiders’ products,

\[
p_m^* = \frac{1}{27}(18c_m + 9c + 5t).
\]

Using the demand function (4) and the candidate equilibrium prices, we determine total quantity of the insiders,

\[
q_m^* = 2 \left( \frac{3(p_o^* - p_m^*)}{2t} + \frac{1}{3} \right) M = M \left( \frac{c - c_m}{t} + \frac{5}{9} \right),
\]

and the total quantity of the outsider,

\[
q_o^* = M - M \left( \frac{c - c_m}{t} + \frac{5}{9} \right) = M \left( \frac{4}{9} - \frac{c - c_m}{t} \right).
\]
Substituting candidate equilibrium quantities and prices into the profit function (8) yields

$$\pi^*_m = (p^*_m - c_m) q^*_m = \frac{Mt}{3} \left( \frac{c - c_m}{t} + \frac{5}{9} \right)^2,$$

and

$$\pi^*_o = (p^*_o - c) q^*_o = \frac{Mt}{3} \left( \frac{4}{9} - \frac{c - c_m}{t} \right)^2.$$

We now check whether our candidate equilibrium satisfies the assumption that all products are sold to a positive mass of consumers. The merged firm sells a positive quantity of each product if

$$p^*_o + \left(\frac{1}{2} - \frac{1}{3}\right)t > p^*_m + \left(\frac{1}{2} - \frac{1}{3}\right)t,$$

and therefore if

$$p^*_o > p^*_m - \frac{2t}{9},$$

which, substituting, is satisfied when

$$c_m - c < \frac{5}{9}t.$$

Similarly, the outsider sells a positive quantity when

$$p^*_m + \left(\frac{1}{2} - \frac{1}{3}\right)t > p^*_o,$$

and therefore when

$$p^*_m > p^*_o - \frac{t}{9},$$

which, substituting, is equal to

$$c - c_m < \frac{4}{9}t.$$

Summarizing, each product is sold to a positive mass of consumers as long as

$$-\frac{4}{9}t < c_m - c < \frac{5}{9}t.$$

To prove that the candidate equilibrium is an equilibrium for the above range of cost differences, we need to show that it is unprofitable to deviate to a situation in which not all products are sold. Observe that it is never a best response for the merged firm to sell only one of its products. This cannot be a best response because by lowering the price for the product it does not sell to the same level as for the other product, the merged firm would gain market share while selling at the (same) price above marginal cost. Furthermore, situations in which a firm (just) prices its competitor out of the market are limiting cases of the above and, hence, it is unprofitable to do so.

The net gains from merging for insiders are then given by

$$\Delta \pi^*_m = \pi^*_m - (\pi^*_i + \pi^*_j)$$

or

$$\Delta \pi^*_m = \frac{M}{3t} \left[ \left( c - c_m + \frac{5t}{9} \right)^2 - \frac{2t^2}{9} \right],$$

whereas the outsider’s change in profits are given by

$$\Delta \pi^*_o = \pi^*_o - \pi^*_k$$

or

$$\Delta \pi^*_o = \frac{M}{3t} \left[ \left( \frac{4t}{9} - (c - c_m) \right)^2 - \frac{t^2}{9} \right].$$
Proof of the Claim in Example 1

The net profits from merging are given by

\[ \Delta \pi_m^* (c_m) = 2 \left( \frac{(b-c_m)^2}{4a} - \frac{(b-c)^2}{4a} \right) - R. \]

Taking the first derivative with respect to \( b \) and simplifying,

\[ \frac{\partial}{\partial b} \left( \Delta \pi_m^* (c) \right) = \frac{c - \xi}{4a} > 0, \]

since \( b > c > \bar{\xi} > \xi \). Similarly,

\[ \Delta \pi_m^* (\bar{\xi}) = \frac{(b-\bar{\xi})^2 - (b-c)^2 - 2aR}{(b-\xi)^2 - (b-c)^2 - 2aR}, \]

and therefore,

\[ \frac{\partial}{\partial b} \left( \frac{\Delta \pi_m^* (\bar{\xi})}{\Delta \pi_m^* (\xi)} \right) = \frac{2(\bar{\xi} - \xi) \left[ (c - \bar{\xi})(c - \xi) + 2aR \right]}{\left[ (b - \xi)^2 - (b - c)^2 - 2aR \right]^2} > 0. \]

Proof of the Claim in Example 2

The net profits from merging are given by

\[ \Delta \pi_m^* (c_m) = \frac{(b-c_m)^2}{4} - \frac{(b-c)^2}{8} - \frac{(b-c)^2}{16}. \]

Deriving and simplifying,

\[ \frac{\partial}{\partial b} \left( \Delta \pi_m^* (\xi) \right) = \frac{b + 3c - 4\xi}{8} > 0, \]

since \( b > c > \bar{\xi} > \xi \). Similarly,

\[ \Delta \pi_m^* (\bar{\xi}) = \frac{(b-\bar{\xi})^2 - (b-c)^2 - (b-c)^2}{4} - \frac{(b-c)^2}{8} - \frac{(b-c)^2}{16}. \]

Rewriting and differentiating yields

\[ \frac{\partial}{\partial b} \left( \frac{\Delta \pi_m^* (\bar{\xi})}{\Delta \pi_m^* (\xi)} \right) = \frac{8(\bar{\xi} - \xi) \left[ b^2 + 3c^2 + 4\xi \xi - (b + 3c)(\xi + \xi) \right]}{\left[ b^2 + 6bc - 3c^2 - 8b\xi + 4\xi^2 \right]^2}. \]

Therefore, this derivative has the same sign as the numerator. Since \( \bar{\xi} > \xi \) it suffices to establish that \( f(\bar{\xi}) \equiv b^2 + 3c^2 + 4\xi \xi - (b + 3c)(\bar{\xi} + \xi) > 0 \). We have that \( f'(\bar{\xi}) = 4\xi - (b + 3c) < 0 \) and \( \bar{\xi} < c \) and \( f(\bar{\xi}) > f(c) = (b - c)(b - c) > 0 \). Hence, the derivative is positive.
Proof of Proposition 2

As we look for optimal reserve prices for the target, we can restrict attention to the reserve prices such that either the low-cost type is indifferent between accepting and rejecting, \( r^p = \pi^L(b) - \pi^A(b) \), or such that the high-cost type is indifferent between accepting and rejecting, \( r^*(b) = \pi^H(b) - \pi^A(b) \), or such that no buyer accepts, \( r^\infty > \pi^H(b) - \pi^A(b) \).

The target will strictly prefer the separating over the prohibiting reserve price if and only if \( qr^*(b) + (1 - q) \pi^T(b) > \pi^T(b) \), which is equivalent to \( \Delta \pi^H(b) > 0 \). By the regularity condition, there exists a unique \( b^* \in [b_{\min}, b_{\max}] \) such that if \( b < b^* \) the target strictly prefers \( r^\infty(b) \) and if \( b > b^* \) the target strictly prefers \( r^*(b) \).

The target will strictly prefer the pooling over the prohibiting reserve price if and only if \( r^P(b) > \pi^T(b) \), which is equivalent to \( \Delta \pi^L(b) > 0 \). Since, trivially, \( \Delta \pi^H(b) > \Delta \pi^L(b) \) the optimal reserve price is prohibitive if \( b < b^* \) and if \( b > b^* \) the optimal reserve price is either separating or pooling. Notice that if \( b = b_{\max} \) the target always prefer the prohibiting reserve price and therefore \( b = b_{\max} \). Suppose from now on that \( b < b_{\max} \).

The target strictly prefers the pooling over the separating reserve price if and only if \( r^P(b) > qr^*(b) + (1 - q) \pi^T(b) \), which is equivalent to \( \Delta \pi^L(b) - q \Delta \pi^H(b) > 0 \). We are now going to show that the regularity condition implies that there exists a unique \( b^* \in [b_{\min}, b_{\max}] \) such that if \( b < b^* \) the target strictly prefers \( r^P(b) \) and if \( b > b^* \) the target strictly prefers \( r^P(b) \).

Let \( f(b) \equiv \Delta \pi^L(b) - q \Delta \pi^H(b) \). We will establish that for any \( b^* \) such that \( f(b^*) = 0 \), one has \( f'(b^*) > 0 \). Since \( f(\cdot) \) is continuously differentiable, this establishes that there exists at most one \( b^* \in [b_{\min}, b_{\max}] \). This together with the fact that \( f'(b^*) > 0 \) establishes the existence of a unique \( b^* \in [b_{\min}, b_{\max}] \) such that if \( b < b^* \) then \( f(b) < 0 \) and if \( b > b^* \) then \( f(b) > 0 \).

Consider any \( b^* \in [b_{\min}, b_{\max}] \) for which \( f(b^*) = 0 \). Since \( b^* \geq b \), \( \Delta \pi^H(b^*) \geq 0 \). Furthermore, since \( \Delta \pi^H(b^*) > \Delta \pi^L(b^*) \), \( \Delta \pi^H(b^*) > 0 \). The regularity condition (i.e. that \( \frac{\partial}{\partial b} \left[ \frac{\Delta \pi^L(b)}{\Delta \pi^H(b)} \right] > 0 \)) implies that \( \frac{\partial \Delta \pi^L(b)}{\partial b} \Delta \pi^H(b) - \frac{\partial \Delta \pi^H(b)}{\partial b} \Delta \pi^L(b) > 0 \) for all \( b \). By definition of \( b^* \), \( \Delta \pi^L(b^*) = q \Delta \pi^H(b^*) \). Thus, \( \frac{\partial \Delta \pi^L(b^*)}{\partial b} \Delta \pi^H(b^*) - \frac{\partial \Delta \pi^H(b^*)}{\partial b} q \Delta \pi^H(b^*) > 0 \). Given that \( \Delta \pi^H(b^*) > 0 \), it follows that \( \frac{\partial \Delta \pi^L(b^*)}{\partial b} - \frac{\partial \Delta \pi^H(b^*)}{\partial b} q > 0 \). Finally, since \( f'(b^*) = \)
\[
\frac{\partial \pi^t(b^*)}{\partial b} - q \frac{\partial \pi^D(b^*)}{\partial b}
\] we have that \( f'(b^*) > 0. \)

References


