Debt Maturity and the Liquidity of Secondary Debt Markets*

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Abstract

We model the debt maturity choice of firms in the presence of fixed issuance costs in the primary market and search frictions in the secondary market for debt. In the secondary market, short maturities improve the bargaining position of sellers, which reduces the required issuance yield. Long maturities reduce reissuance costs. The optimally chosen maturity trades off both considerations. Equilibrium exhibits inefficiently short maturity choices: An individual firm does not internalize that a longer maturity increases expected gains from trade in the secondary market, which attracts more buyers, and hence also facilitates the sale of debt issued by other firms.

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1. Introduction

Some types of corporate debt securities are used to create very substantial maturity mismatch. For example, Schroth et al. (2014) estimate that before the recent financial crisis, asset-backed commercial paper with an average maturity of around 37 days was used to finance assets with an average duration of around 5.8 years. Although the maturity of commercial paper has increased after the crisis, it is still very short. The Federal Reserve, for instance, reported an average maturity of about 55 days for all outstanding commercial paper, on February 23, 2015.[1] A central question is why issuers choose to finance long term assets with such extremely short-term debt securities. In this paper, we explore a theoretical model that provides a possible answer to this question, and in which an externality exacerbates the problem.

An investor who buys a debt security in the primary market might need to convert the security back into cash by selling in the secondary market. In a typical over-the-counter (OTC) market, the investor would first have to locate a potential buyer, and then negotiate over the price. This price depends on how many potential buyers there are in the market, and crucially also on the maturity of the security: a shorter maturity gives the seller a better outside option and thus reduces the value that buyers can extract from sellers.

When an individual issuer chooses its debt maturity, it takes into account that a shorter maturity helps the sellers of its debt securities, and hence (everything else being equal) reduces the cost of its debt. However, a short maturity also hurts buyers, which in turn means that slightly fewer buyers will enter the secondary market. The issuer will not internalize that a short maturity therefore also implies worse prices for sellers of the securities of all other issuers, and more costly debt for those issuers. In consequence, equilibrium maturities are inefficiently short.

A policy conclusion is that regulation that pushes issuers to finance themselves at longer matu-

[1] See `http://www.federalreserve.gov/releases/cp/maturity.htm`.

rities can have the effect of improving the liquidity of secondary markets for debt, and in this way, welfare. Our analysis provides a novel, additional argument in favour of the current regulatory changes that aim to increase the debt maturities at which financial issuers borrow.[2]

While our theory applies broadly to any form of debt security traded in an OTC market, we think that it provides an especially plausible description of commercial paper, which is a very short term-debt security known to have a very illiquid secondary market: Because owners of the paper find it hard to sell in the secondary market, they care a lot about maturity. Issuers therefore find it optimal to choose very short maturities. But this in turn means buyers make low profits in the secondary market for the paper, so there will be few buyers in this market, and it will be very illiquid.

Our continuous-time infinite-horizon model has two types of agents with different time preferences. There are *entrepreneurs* who can each set up a firm to undertake a long-term project generating a perpetual constant cash flow. Entrepreneurs are impatient, i.e. have a high discount rate. There are also *investors*, who are born patient with a low discount rate, but are subject to idiosyncratic preference shocks that make them impatient, i.e. increase their discount rate. We assume that there is a constant and large inflow of new, patient investors.

To exploit the differences in time preferences between entrepreneurs and investors, the firms issue debt to investors, with a maturity chosen by the firm. In the secondary market, investors who become impatient while holding debt will want to sell to patient investors.

We model this secondary market as a search market in which sellers and buyers meet according to a constant-returns-to-scale matching function. The rate at which sellers meet buyers in the secondary market is increasing in the ratio of buyers to sellers. After a match, the transaction price is determined through Nash bargaining, and the price increases with the rate at which sellers

[2]See e.g. the Net Stable Funding Ratio of Basel III (Basel Committee on Banking Supervision, 2010).

can find another buyer (and hence with the ratio of buyers to sellers), because it improves the bargaining position of the seller. At the same time, the price in the secondary market will be decreasing in maturity, because it worsens the bargaining position of the seller. Since investors who buy in the primary market anticipate the effect of maturity, firms obtain lower interest rates in the primary market when issuing at shorter maturities.

In the absence of additional frictions, firms would choose the shortest possible maturity. To obtain an interior solution, we assume that every time firms issue (or reissue) debt in the primary market, they pay a fixed cost. Everything else being equal, firms therefore have an incentive to increase maturity in order to decrease the frequency at which this cost is paid.[3]

The maturity decisions of firms trade off the frictions in the primary and secondary debt markets. When the ratio of buyers to sellers in the secondary market is low, the effect of maturity on price in the secondary market is strong, and hence the effect on interest rates in the primary market is strong. Firms then find it optimal to issue short maturity debt, even if this implies paying the fixed (re-)issuance cost at a higher frequency. Conversely, when the ratio of buyers to sellers is high, the effect of maturity on the price in the secondary market and hence on interest rates in the primary market is weak. Firms then find it optimal to issue long maturity debt, to reduce the frequency at which the fixed (re-)issuance cost is paid.

To close the model, the ratio of buyers to sellers in the secondary market is determined through free entry of buyers. Entry decisions depend on the gains patient buyers expect to realize by trading with impatient sellers. The longer the maturity of debt, the higher these gains, and hence the more entry occurs.

Firms choose maturities as a function of the ratio of buyers to sellers in the secondary market,

[3] Although there are types of debt securities, such as corporate bonds, for which a (small) fixed cost of issuance appears to exist (Altinkiliç and Hansen, 2000), more generally, this assumption can also be interpreted as shorthand for other mechanisms that generate a preference for longer maturities, for instance roll-over risk (see e.g. He and Xiong, 2012b).

and this ratio is determined by free entry as a function of the maturities chosen by firms, so that we can find equilibrium as a fixed point. As described above, our main result is that in equilibrium, firms choose inefficiently short maturities, because they fail to internalize how their maturity choice affects the ratio of buyers to sellers via free entry.

This externality is different from the standard externalities in search models with ex-post bargaining and entry as discussed in the labor literature: Hosios (1990) notes that that there will be too much or too little entry, unless bargaining power parameters take a specific value depending on the elasticities of matching rates. In our context, this standard type of externality relates to how much entry there is for fixed differences in valuation of the debt securities between patient buyers and impatient sellers, whereas our externality relates to maturity choices which affect those differences in valuation, in the presence of entry.

The paper proceeds as follows. Section 2 presents the related literature. Section 3 describes the model. Section 4 discusses the determination of equilibrium. In Section 5, we show that equilibrium maturities are inefficiently short, and discuss the underlying assumptions that generate this result. In Section 6, we illustrate the model with a numerical example. We also briefly describe how the model can be extended to include marketmakers or to consider an increasing-returns-to-scale matching function and comment what additional results can be derived. (Full details on these extensions are available in an Online Appendix.) Finally, we also show that in our context, the type of standard (non-puttable) debt that we consider dominates a form of puttable debt which insures investors against idiosyncratic shocks without the need for a secondary market. Section 7 concludes. All proofs are in the appendix.

2. Related Literature

Our paper relates to the literature that uses search models to describe frictions in OTC secondary markets for securities (see e.g. Duffie et al., 2005). Like He and Milbradt (2014), and following that paper also Chen et al. (2013) and Chen et al. (2015), we relate such frictions to maturity choice. Our paper shares with these papers the insight that a shorter maturity strengthens the bargaining position of sellers. However, while they introduce default to study the dynamic interaction between secondary market illiquidity and default risk, they take the rate at which trades occur in the secondary market as exogenous. In contrast, we abstract from default, but endogenize the entry of buyers and therefore also the rate at which trades occur in the secondary market. This allows us to discuss the relationship between the number of buyers in the market and maturity choice, and the resulting externality, as described above.

Our paper also relates to the literature that discusses other sources of inefficiency in debt maturity choices. For instance, Stein (2012) and Segura and Suarez (2016) find that the interaction between pecuniary externalities in the market for funds during liquidity crises and the financial constraints of banks can lead to excessive short-term debt issuance. In Farhi and Tirole (2012), the collective expectation of a bailout gives incentives to choose maturities that are too short. Finally, the inability of issuers to commit to a maturity structure can lead to inefficiencies: First, it can cause a choice of inefficiently short maturities when existing creditors can be diluted through issuance of new debt with a shorter maturity that is effectively senior (Brunnermeier and Oehmke, 2013). Second, even when covenants prevent such dilution, it can adversely affect the default decision of equityholders (He and Milbradt, forthcoming).

We focus on a particular motive for maturity choice. Others are considered in the corporate finance literature. For example, short-term debt can act as a disciplining device (Calomiris and Kahn, 1991), even though it might produce rollover risk (Cheng and Milbradt, 2012). Short-term debt can be used to signal quality (Diamond, 1991). Shorter maturities can serve to commit equityholders to reducing leverage after poor performance (Dangl and Zechner, 2006), or firms might choose maturities in response to the maturity choices of government, given a fixed demand by investors for certain maturities (Greenwood et al., 2010). Finally, short-term debt with safe harbor protection can avoid the costs of a bankruptcy process, but its issuance might be limited by the availability of liquid collateral (Auh and Sundaresan, 2015).

A key feature of our model, that time-to-maturity matters for liquidity and hence prices, is consistent with findings in the empirical literature: Illiquidity appears to be priced, and time-to-maturity appears to matter for liquidity. For corporate bonds, Edwards et al. (2007) and Bao et al. (2011) find that their preferred measure of illiquidity (the estimated transaction price spread and negative price autocovariance, respectively) increases with time-to-maturity. For commercial paper, Covitz and Downing (2007) find no direct evidence that links a measure of illiquidity to time-to-maturity, but do show that yield spreads increase in time-to-maturity. All of these papers control for credit quality. (Other important characteristics of debt claims that the empirical literature has related to liquidity, and which our model does not shed light on, are age, measured as time since issuance, and credit risk.)

3. The Model

Time is continuous and indexed by $t \geq 0$. There are two types of infinitely-lived and risk-neutral agents: Entrepreneurs and investors. There are many entrepreneurs. Each entrepreneur has a large endowment of funds, and can set up a firm operating one project. The project requires an initial investment of 1 at $t = 0$, and subsequently produces a perpetual cash flow of $x > 0$. Entrepreneurs have discount rate $\rho > 0$.

Investors have large endowments, but are restricted to holding at most one unit of a debt security at any point in time. An investor is either *patient* and has a discount rate of 0, or *impatient* and has a discount rate of ρ. Patient investors are subject to (idiosyncratic) liquidity shocks that arrive at Poisson rate θ and are i.i.d. across investors. Once hit by the shock, a patient investor irreversibly becomes impatient. At every time t there is a large inflow of patient investors into the economy. Investors can consume their endowment, can store it at a net rate of return of zero, or can buy the debt issued by firms, as described below. Without loss of generality, we can assume that investors only consume their funds when they are impatient.

Since entrepreneurs value present consumption more than patient investors do, they may prefer to let the firm finance the investment in the project through debt which is placed with investors. We assume that each firm can have a single debt issue outstanding, with an aggregate face value equal to D, chosen by entrepreneurs. To solve the model, it is not necessary to be very specific about how many investors are required to finance the debt of a single firm. To be concrete, however, we will describe a situation in which the debt of a single firm is held by a continuum of investors of measure D who each hold a debt security with a face value of 1. We assume that any payments promised by the firm cannot exceed project cash flows, so that debt is safe.

We assume that the maturity of debt is stochastic and arrives at Poisson rate $\delta \geq 0$, chosen by the firm at $t = 0$ and held fixed through time. We will refer to δ as the *refinancing frequency*.[4] Since a firm's debt consists of a single debt issue, all of a firm's debt matures at the same time: we are assuming maximum granularity of maturity (or minimum dispersion) in the sense of Choi et al. (2015). At maturity, the repayment of the D units of principal is financed via funds raised from reissuing the debt. Finally, debt also pays a continuous interest rate of r per unit of face value, set as described below.

[4]This assumption of stochastic maturity is for the purpose of analytical tractability, as in Blanchard (1985), Leland (1998), and He and Xiong (2012a). Although a version of our model with deterministic maturity is not very tractable analytically, we have verified that its numerical behavior is very similar. Details are available from the authors upon request.

There is a *primary* and *secondary market* for debt. In the primary market, firms issue debt at $t = 0$ which they then refinance every time it matures. Debt is placed to investors through an auction in which all investors can participate. Investors observe the refinancing frequency δ of a debt issue, and then submit bids of interest rates r at which they are willing to buy a unit of the debt issue at par. Firms incur a cost $\kappa > 0$ each time an auction is held. Because of the stochastic maturity assumption, firms would be exposed to the risk of having to pay κ at random times when they reissue all their debt. This risk would not be present in a model with deterministic maturity. To simplify, we assume that firms can insure against this risk at an actuarially fair rate and cover these costs by paying a flow of $\delta\kappa$ per unit of time, equal to the expected issuance cost. As in Dangl and Zechner (2006), issuance costs generate a preference for issuing debt with longer maturities, as these reduce the frequency at which the cost is incurred.

For convenience, we derive our analytical results under the following condition on parameter values:

Assumption 1. $x > \max(\rho, \theta\kappa)$

This assumption is sufficient (but not necessary) to ensure that the utility that can be obtained from a debt-financed project is positive, and exceeds the utility that can be obtained from a project financed with the entrepreneur's own funds.

A debtholder who becomes impatient attaches a lower value to a debt claim than an investor who is still patient. The gains from trade between these two types of agents can be realized in a secondary market. The debt of all firms trades in the same secondary market. In this market, pairwise random matches occur between investors who own a security and search to sell (the sellers), and investors who do not own a security and search to buy (the buyers). Buyers in this market incur a non-pecuniary flow cost of effort $e_B > 0$ per unit of time while they are searching. For simplicity, we assume that sellers incur no such cost.

We let $\mu(\alpha_t^S, \alpha_t^B)$ denote the aggregate flow of matches between sellers and buyers, where α_t^S, α_t^B are the measures of sellers and buyers, respectively, in the secondary market at time t. These measures will be endogenously determined in equilibrium. The matching function satisfies $\mu(0, \alpha^B) = \mu(\alpha^S, 0) = 0$, is increasing in both arguments, and has continuous derivatives. In order to highlight that the results derived in the paper do not rely on the strong "thick market externalities" inherent in the increasing-returns-to-scale matching function commonly used in the literature on OTC markets (see e.g. Duffie et al., 2005), we assume that the matching function exhibits constant returns to scale, and let μ be homogeneous of degree one in (α^S, α^B). (We discuss how the magnitude of our inefficiency is amplified when using an increasing-returns-to-scale matching function in Section 6.3.) As long as $\alpha^S > 0, \alpha^B > 0$, we can define $\phi := \frac{\alpha^B}{\alpha^S}$, and then define $\mu_S(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^S = \mu(1, \phi)$ as the rate at which sellers find a counterparty, and $\mu_B(\phi) := \mu(\alpha^S, \alpha^B)/\alpha^B = \mu(\phi^{-1}, 1)$ as the rate at which buyers find a counterparty. These rates satisfy the following congestion properties:

$$\begin{aligned} &\lim_{\phi \to 0} \mu_S(\phi) = 0, \quad \lim_{\phi \to \infty} \mu_S(\phi) = \infty, \\ &\lim_{\phi \to 0} \mu_B(\phi) = \infty, \quad \lim_{\phi \to \infty} \mu_B(\phi) = 0. \end{aligned} \tag{1}$$

These equations simply state that when there are more sellers (buyers) in the market it is more difficult for a seller (buyer) to get matched with a buyer (seller).

After a buyer and seller are matched, they engage in Nash bargaining over the price with bargaining power parameters $\beta, 1 - \beta$, respectively, with $\beta \in (0, 1)$.

Summarizing, decisions are as follows: At $t = 0$, each firm i chooses a debt structure (δ_i, D_i) consisting of the refinancing frequency δ_i and the face value D_i of its debt. It takes this decision based on an expectation of the ratio of buyers to sellers $(\phi_t)_{t \geq 0}$, and of the debt structure choices $\{(\delta_j, D_j)\}_{j \neq i}$ of the other firms. Then, for every $t \geq 0$, patient investors decide whether to bid in the primary market auctions of any current debt (re-)issue, whether to search to buy in the secondary

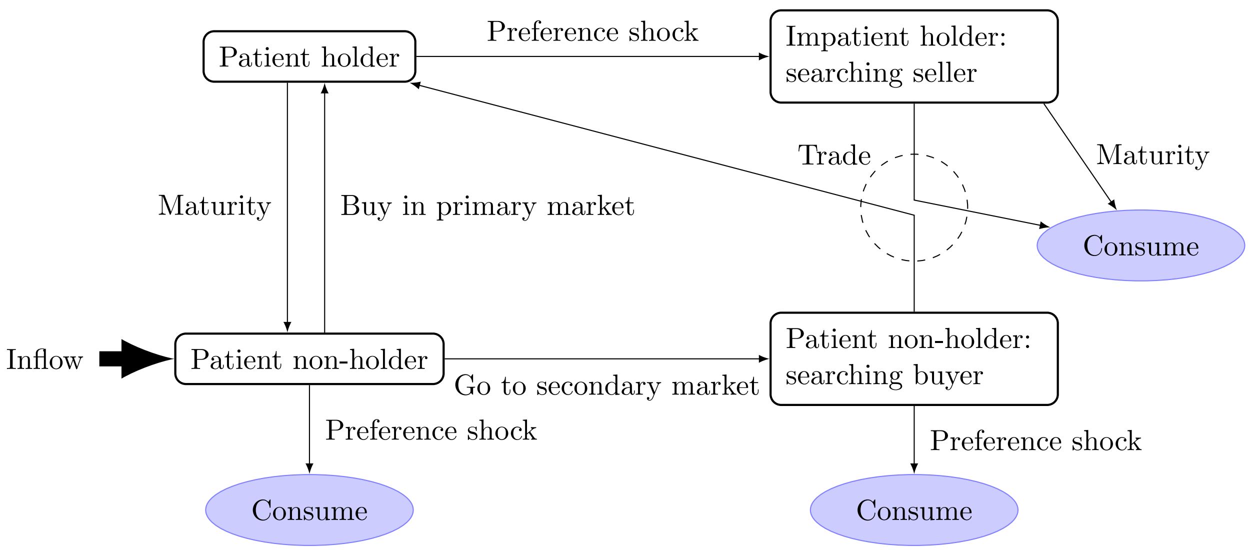

Fig. 1. Flow diagram for investors

This flow diagram illustrates the possible states that investors can transition through in the model.

market, or whether to store their endowment. Impatient investors with funds will consume, and impatient debtholders with funds decide whether to search to sell in the secondary market. These decisions are taken based on the publicly known debt structure choices choices $\{(\delta_i, D_i)\}$ of firms and on an expectation of the ratio of buyers to sellers $(\phi_{t'})_{t' \geq t}$. We illustrate these decisions in Figure 1.

We focus on steady-state equilibria, in which all quantities that are determined in equilibrium are constant through time. This type of equilibrium can be characterized by the set of debt structure choices of firms, $\{(\delta_i^e, D_i^e)\}$, and a ratio of buyers to sellers ϕ^e such that: first, given $(\{(\delta_i^e, D_i^e)\}, \phi^e)$ the debt structure choice (δ_i, D_i) of each firm i is optimal, and second, the free entry decisions of investors into both the primary and secondary market are optimal given $(\{(\delta_i^e, D_i)\}, \phi^e)$, which amounts to the condition that investors obtain no rents in either of these markets.

Throughout the rest of the paper, we will be presenting graphs that are based on the following parameter values: We interpret a unit of time as one year. We set the project cash flow to

$x = 1\%$, the rate at which investors become impatient to $\theta = 1$, the discount rate of impatient investors to $\rho = 10\%$, the refinancing cost to $\kappa = 3$bp, the bargaining power parameter of sellers to $\beta = 0.5$, the flow cost of searching to buy to $e_B = 2\%$, and use the matching function $\mu(\alpha_S, \alpha_B) = 10(\alpha_S)^{\frac{1}{2}}(\alpha_B)^{\frac{1}{2}}$. (With these parameters, debt issuance will be optimal and entrepreneur utility will be positive, even though the sufficient condition in Assumption 1 is not satisfied.)

4. Equilibrium

We find the equilibrium of the economy by following a sequence of steps: We first work out how free entry of investors into the primary market determines the interest rate r that a firm has to pay on debt as a function of its choice of refinancing frequency δ, taking the ratio of buyers to sellers ϕ and the choices of other firms as given. We then show that in equilibrium, all firms will chose the same refinancing frequency, and find the expression for the optimal refinancing frequency δ as a function of ϕ. Finally, we determine the ratio of buyers to sellers ϕ that is compatible with free entry of investors into the secondary market, for a given refinancing frequency δ chosen by all firms. Taken together, equilibrium is characterized by the intersection of two curves in (ϕ, δ)-space.

4.1. The interest rate in the primary market

In order to compute the interest rate that is determined in the primary market auctions, we first need to consider the value that investors derive from holding debt that pays an interest rate of r and has a refinancing frequency δ. We use $V_0(r, \delta; F, \phi)$ and $V_\rho(r, \delta; F, \phi)$ to denote the value that a patient and an impatient debtholder obtain, respectively, from holding a unit of the debt security (r, δ), when the types and quantities of securities that can be found in the secondary market are described by the distribution $F(r', \delta')$, and given a ratio of buyers to sellers ϕ. We also use $V_B(F; \phi)$ to denote the value that a patient investor attaches to searching to buy in the secondary market.

Below, we will omit the arguments of V_0, V_ρ, and V_B where possible to reduce notational clutter.

Patient debtholders do not search to sell in the secondary market, because buyers do not attach a higher value to holding the debt, and hence there are no potential gains from trade. In contrast, there are gains from trade between impatient debtholders and patient buyers: Suppose that an impatient debtholder is matched with a (patient) buyer, and that trade takes place at price $P = P(r, \delta; F, \phi)$ per unit of face value. Then the surplus that the seller obtains is $P - V_\rho$. The surplus the buyer obtains is $V_0 - P - V_B$. The total gains from trade are therefore $P - V_\rho + V_0 - P - V_B = V_0 - V_\rho - V_B$. We will see later that free entry of investors into the (buy side of the) secondary market implies that in equilibrium, $V_B = 0$ (see Lemma 3), and hence that the total gains from trade in equilibrium are equal to $V_0 - V_\rho$. Due to the higher discount rate of impatient investors, we trivially have that $V_\rho < V_0$, such that the gains from trade are positive, and every match results in a trade. The price P splits the surplus according to Nash bargaining,

$$P = \beta V_0 + (1 - \beta) V_\rho, \tag{2}$$

where β and $1 - \beta$ are the bargaining power parameters of the seller and buyer, respectively.

We can now write a system of recursive flow-value equations that V_0 and V_ρ satisfy in steady state:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0, \tag{3}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) = \rho V_\rho. \tag{4}$$

The first equation states that for a patient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of becoming impatient, just balance the reduction in utility due to discounting at rate 0. The second equation states that for an impatient investor, the utility flow stemming from the continuous interest payments, the possibility of maturity, and the possibility of locating a buyer in the secondary market and selling at price P, just balance the reduction in utility due to discounting at rate ρ.

Obviously, the value of a patient debtholder $V_0(r, \delta; F, \phi)$ is increasing in the interest flow r, and the profits of the firm and hence the utility of the entrepreneur is decreasing in r. There is free entry of patient investors into the primary market auctions, who will compete by bidding successively lower interest rates r, until, in equilibrium, the value to be obtained from buying at par in the primary market is driven to zero:

$$V_0(r, \delta; F, \phi) - 1 = 0. \tag{5}$$

Given the expression for $V_0(r, \delta; F, \phi)$ that can be derived from equations (2),(3), and (4), the condition (5) determines the interest rate $r(\delta; \phi)$ that firms have to pay when issuing debt. This interest rate is a function of the refinancing frequency δ chosen by the firm, and the ratio of buyers to sellers ϕ (and does not depend on F). We summarize this discussion in the following lemma:

Lemma 1. *For a given equilibrium ratio of buyer to sellers ϕ, the interest rate $r(\delta; \phi)$ that is set in the primary market auctions as a function of the firm's refinancing frequency choice δ is given by:*

$$r(\delta; \phi) = \frac{\rho\theta}{\delta + \theta + \rho + \mu_S(\phi)\beta}. \tag{6}$$

The interest rate exceeds 0, the discount rate of patient investors, because bidders require compensation for the utility losses associated with the frictions faced when attempting to sell in the secondary market. They will suffer these losses in case they become impatient before maturity, and need to sell, so that the interest rate can be interpreted as an illiquidity premium. The magnitude of frictions can be indirectly measured via the discount in the secondary market price $1 - P$ that impatient debtholders accept in order to be able to liquidate their position, which can be calculated using equations (2) and (3) as

$$1 - P = (1-\beta)(V_0 - V_\rho) = (1-\beta)\frac{r}{\theta}. \tag{7}$$

The interest rate will just compensate for the expected loss incurred when becoming impatient, which is $\theta(V_0 - V_\rho)$. We can see that the price discount $1 - P$ (which benefits the buyer), is just equal to the buyers share $1 - \beta$ of the gains from trade $V_0 - V_\rho$. The gains from trade, the interest rate, and the price discount are therefore all tightly linked.

As the ratio of buyers to sellers increases, it becomes easier for sellers to find a buyer. The bargaining position of sellers therefore improves, and hence the interest rate decreases. In the limit as $\phi \to \infty$, sellers can find a buyer instantaneously, and the interest rate tends to zero. As the refinancing frequency δ increases, searching sellers are more likely to have their debt mature before they find a buyer, which improves their bargaining position, implying a higher secondary market price, which means that debtholders require less compensation for illiquidity and hence accept a lower interest rate, as shown in Figure 2. As liquidity shocks become more frequent (θ increases) the interest rate that investors demand increases because it is more likely that they become impatient before the debt matures.

4.2. The firm's problem

At $t = 0$, firms choose whether to issue debt and undertake the project. If the project is undertaken and debt is issued, the firm also needs to decide on the refinancing frequency δ and face value D. The firm anticipates that to issue debt at par, it needs to pay an interest flow of $r(\delta;\phi)D$, with the interest rate as given by (6). Debt issuance is feasible as long as the cash flow from the project exceeds the flow cost of debt,

$$x \geq r(\delta;\phi)D + \delta\kappa. \tag{8}$$

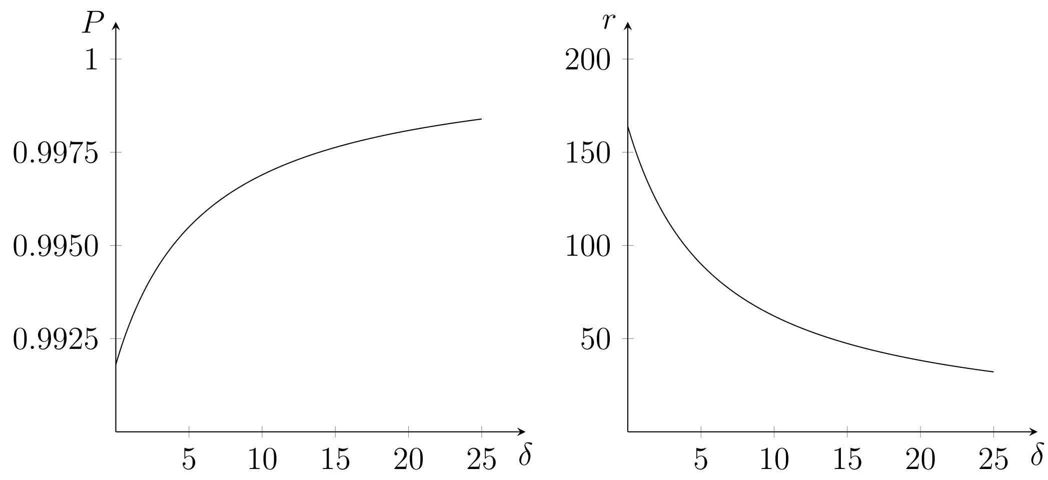

Fig. 2. Price in the secondary market, interest rate in the primary market

Secondary market price P (as fraction of face value), and interest rate r (in bp) that the firm has to pay in the primary market and , both as a function of refinancing frequency δ. (Time is measured in years, such that e.g. $\delta = 10$ means an expected maturity of $\frac{1}{10}$ years or 36.5 days.) The ratio of buyers to sellers is set to $\phi = 1$. All other parameters as in the baseline case, see the end of Section 3.

An entrepreneur consumes the residual cash flows and hence her utility when there is investment and the firm issues debt with refinancing frequency δ and with face value D is:

$$U(\delta, D, r(\delta;\phi)) = -1 - \kappa + D + \int_0^\infty e^{-\rho t}(x - r(\delta;\phi)D - \delta\kappa)dt, \tag{9}$$

$$= -1 - \kappa + D + \frac{x - r(\delta;\phi)D - \delta\kappa}{\rho}, \tag{10}$$

where the first term is the cost of the investment and the second is the cost of the initial debt issuance. The third accounts for the proceeds from debt issuance. The last term accounts for the discounted value of the net excess cash flows that the firm generates.

From Lemma 1, we know that $r(\delta;\phi) < \rho$, so the entrepreneur can borrow at an interest rate that is below her discount rate. This means that U is increasing in D irrespective of the choice of δ and hence the entrepreneur will exhaust debt capacity and always choose the maximum D subject to the constraint (8). So in any optimal debt structure (δ, D), it will always be the case that

$$D = \frac{x - \delta\kappa}{r(\delta;\phi)}. \tag{11}$$

Using this, we can from now on focus only on the optimal choice of the refinancing frequency δ. Substituting the expression (11) into equation (10), the expression for U becomes

$$U(\delta, r(\delta;\phi)) = -1 - k + \frac{x - \delta\kappa}{r(\delta;\phi)}. \tag{12}$$

The firm's program can then be written as

$$\max_{\delta \geq 0} \frac{x - \delta\kappa}{r(\delta;\phi)}, \tag{13}$$

which means that the firm chooses a refinancing frequency to maximize the size of the initial debt issue.

The optimal decision of the firm is described in the following lemma:

Lemma 2. *It is optimal for the firm to undertake the project and to issue debt. In addition, for every ϕ, the firm's problem* (13) *has a unique solution $\delta^*(\phi)$ which is given by:*

$$\delta^*(\phi) = \max\left(\frac{1}{2}\left(\frac{x}{\kappa} - \theta - \rho - \mu_S(\phi)\beta\right), 0\right)$$

We illustrate how the optimal choice of refinancing frequency δ^* varies with the ratio of buyers to sellers ϕ in Figure 3. As buyers become scarce and $\phi \to 0$, the only way in which investors can liquidate their investment is by being repaid at maturity. This makes long maturity debt very expensive for firms and they choose a high refinancing frequency (a short expected maturity). As ϕ increases, the maturity of debt becomes less important to investors, since they can more easily liquidate their investment by selling in secondary markets. Hence firms find it optimal to choose a lower refinancing frequency (that is, to lengthen the expected maturity), to reduce the expected

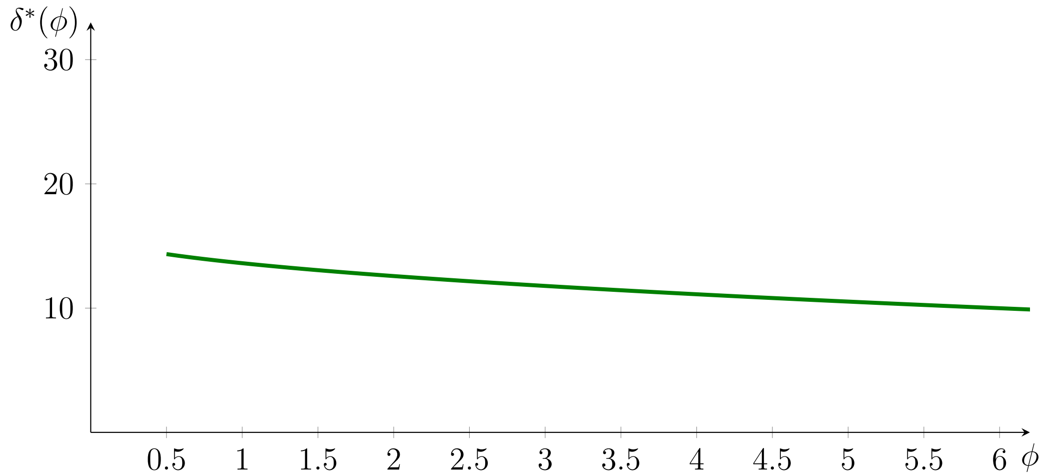

Fig. 3. Optimal refinancing frequency as function of ϕ

Optimal refinancing frequency $\delta^*(\phi)$ chosen by firms, as a function of the ratio of buyers to sellers ϕ. Parameters are as described at the end of Section 3.

issuance costs. When the ratio of buyers to sellers ϕ becomes sufficiently large, the firm eliminates reissuance costs completely by setting $\delta = 0$, that is, by issuing perpetual debt.

All firms are identical and face the same conditions in the secondary market, such that the lemma also implies that:

Corollary 2.1. *In equilibrium, all firms will choose the same refinancing frequency and face value of debt.*

4.3. Entry into the secondary market

We now consider what ratio of buyers to sellers ϕ is consistent with free entry of buyers into the secondary market, given a set of choices of refinancing frequencies by firms. We have that:

Lemma 3. *Free entry ensures that in equilibrium, the value of being a searching buyer satisfies* $V_B(F;\phi) = 0$.

Taking into account that in equilibrium all firms choose the same refinancing frequency δ, the value V_B of being an active buyer satisfies the following flow value equation:

$$-e_B + \mu_B(\phi)(1-\beta)\left(V_0 - V_\rho - V_B\right) - \theta V_B = 0. \tag{14}$$

The first term is the (dis-)utility flow from the effort cost of searching. The second term describes the expected utility flow from the possibility of meeting a seller, all of whom hold identical debt securities. The price is such that the buyer receives a fraction $1-\beta$ of the gains from trade when matched. The third term reflects the possibility of becoming impatient. The equation states that in steady state, all of these flows must balance the reduction in utility due to discounting at rate 0.

After substituting $V_B = 0$ into equation (14) and using equation (3) with condition (5), we obtain the following *free entry condition*, which describes how buyers enter the secondary market:

Lemma 4. *Free entry into the secondary market implies the following* free entry condition*:*

$$e_B = \mu_B(\phi)(1-\beta)\frac{r(\delta;\phi)}{\theta}. \tag{FEC}$$

This equation defines a strictly decreasing function $\phi^{FEC}(\delta)$ which describes the ratio of buyers to sellers that results from free entry of buyers for each possible choice of δ by firms. This function is maximized for $\delta = 0$, when it takes a finite value $\hat{\phi}$, and tends to zero as $\delta \to \infty$.

Figure 4 plots $\phi^{FEC}(\delta)$ (with the axes reversed to facilitate comparison with Figure 3). Using equation (3) we can see that the gains from trade in the secondary market are $V_0 - V_\rho = \frac{r}{\theta}$. At higher refinancing frequencies, the bargaining position of sellers improves, and the interest rate and the gains from trade both decrease. This makes entering the market less attractive for buyers, and reduces the ratio of buyers to sellers ϕ. Such a reduction in ϕ has two effects that lead to the reestablishment of the free entry condition (FEC). First, it increases the matching rate $\mu_B(\phi)$ of buyers. Second, it decreases the matching rate $\mu_S(\phi)$ for sellers, meaning that they are in a worse bargaining position when selling, which increases the interest rate paid on debt and the gains from

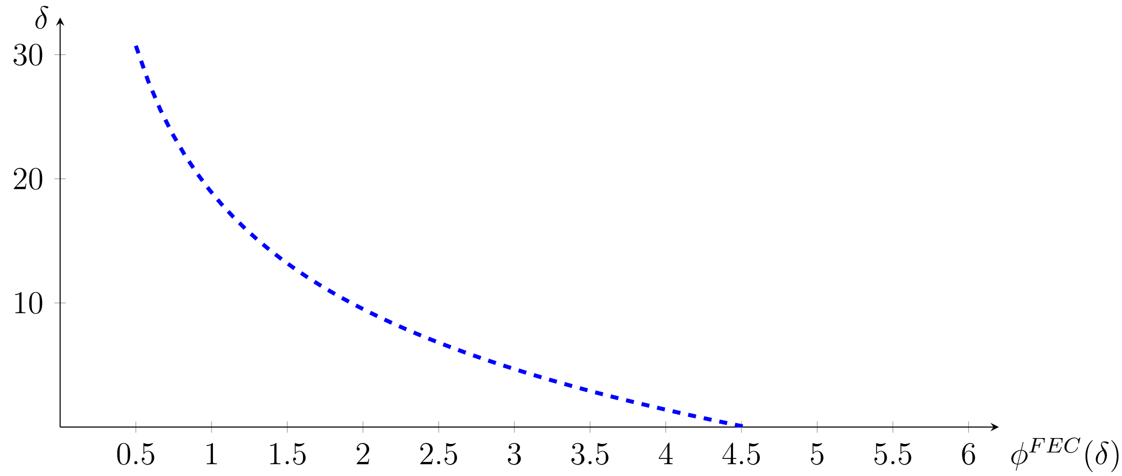

Fig. 4. Free entry, refinancing frequency, and the ratio of buyers to sellers
The ratio of buyers to sellers ϕ produced via free entry of buyers as a function of refinancing frequency δ chosen by firms, as described by $\phi^{FEC}(\delta)$. Note that to facilitate comparison with Figure 3, we have reversed the order of the axes, that is, we are plotting the inverse of $\phi^{FEC}(\delta)$. Parameters are as described at the end of Section 3.

trade in the market. These two effects offset the impact of the increase in refinancing frequency, with the end result that $\phi^{FEC}(\delta)$ is decreasing in δ.

We note that there is a maximum ratio of buyers to sellers of $\phi = \hat{\phi}$ that can be induced via free entry when firms issue perpetual debt ($\delta = 0$). Also, as firms choose refinancing frequencies that tend to infinity, ϕ tends to zero as the gains from trade in the secondary market vanish and buyers choose not to enter.

4.4. Equilibrium

Summarizing the discussion in the previous subsections, a steady-state equilibrium can be characterized by a pair (δ^e, ϕ^e) for which refinancing frequencies are optimal, and for which the free entry condition for buyers into the secondary market is satisfied:

$$\delta^e = \delta^*(\phi^e) \text{ and } \phi^e = \phi^{FEC}(\delta^e).$$

We have that:

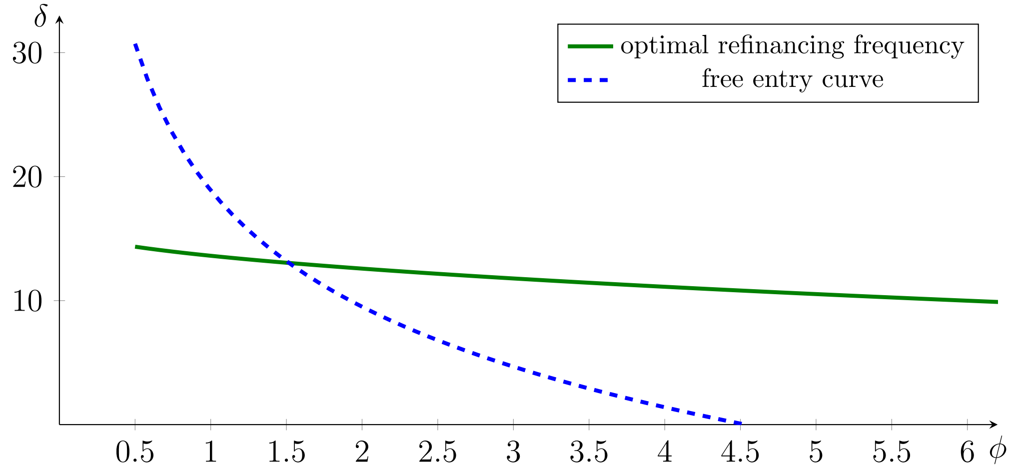

Fig. 5. Equilibrium

The optimal refinancing frequency $\delta^*(\phi)$ (green solid line) and the free entry curve $\delta^{FEC}(\phi)$ (blue dashed line). The unique steady-state equilibrium (δ^e, ϕ^e) occurs at the intersection of the two curves. Parameters are as described at the end of Section 3.

Proposition 1. *There exists a unique steady-state equilibrium* (δ^e, ϕ^e) *in the economy.*

The steady-state equilibrium can be described by the intersection of a *refinancing frequency curve*, and a *free entry curve* as illustrated in Figure 5. Since both curves (seen as functions of ϕ) are decreasing, there could exist multiple intersection points: If firms expect a high ratio of buyers to sellers, they could issue debt with low refinancing frequency which generates important gains from trade in the secondary market. This in turn could attract many buyers, and produce the anticipated high ratio of buyers to sellers. Proposition 1, however, states that this kind of self-fulfilling equilibrium does *not* arise in the model.

5. Inefficiency of equilibrium

In this section we consider the problem of a Social Planner (SP) who chooses the debt structure of firms to maximize surplus in the economy. We show that the SP chooses a higher refinancing frequency than the one which arises in the laissez-faire equilibrium. That is, in the absence of intervention, firms choose maturities which are inefficiently short.

After establishing this normative result, we discuss how it depends on the presence of both primary market frictions and secondary market frictions. We also argue that the source of the inefficiency in our model is not the standard set of entry-related externalities known from the labor literature (Hosios, 1990). Finally, we discuss the importance of our assumption of a single search market.

In our model, the only agents who obtain a surplus are entrepreneurs. Therefore, both the SP as well as any firm will want to maximize entrepreneur utility. Both the SP and all firms will want to maximize the face value of debt for a given maturity choice. However, the programs of the SP and of a firm differ, because a firm takes the ratio of buyers to sellers ϕ as given, while the SP internalizes the effects of maturity choices on ϕ.

More formally, the SP internalizes that a refinancing frequency δ chosen by all firms induces a ratio of buyers to sellers $\phi^{FEC}(\delta)$ in the secondary market via entry. We can write the SP's optimization problem in terms of the expression for the utility of entrepreneurs in equation (12) as follows:

$$\max_{\delta \geq 0} U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))). \tag{15}$$

Now if the competitive equilibrium (δ^e, ϕ^e) has $\delta^e > 0$, then the first order condition for firms implies that at the equilibrium values (δ^e, ϕ^e),

$$\frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\frac{\partial r}{\partial \delta} = 0. \tag{16}$$

At the same time, the full derivative of $U^{SP}(\delta)$ with respect to δ is:

$$\frac{dU^{SP}}{d\delta} = \frac{\partial U}{\partial \delta} + \frac{\partial U}{\partial r}\left(\frac{\partial r}{\partial \delta} + \frac{\partial r}{\partial \phi}\frac{d\phi^{FEC}}{d\delta}\right) \tag{17}$$

We note that $\frac{\partial U}{\partial r} < 0$ or that the utility of entrepreneurs is decreasing in interest rates, that $\frac{\partial r}{\partial \phi} < 0$ or that interest rates are decreasing in the ratio of buyers to sellers, and $\frac{d\phi^{FEC}}{d\delta} < 0$ or that the

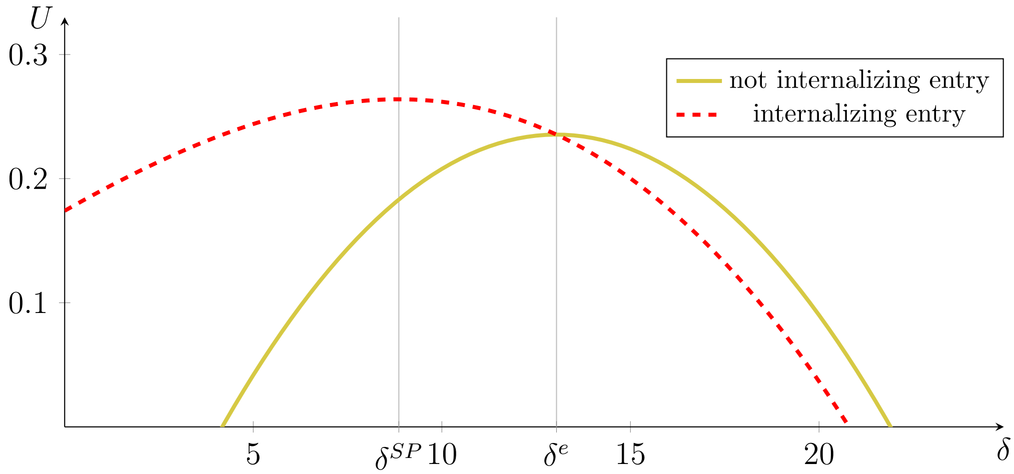

Fig. 6. Firm profit U as function of refinancing frequency δ

Firm profit U as a function of refinancing frequency δ, (a) as perceived by the social planner, internalizing the effect of δ on entry and hence $\phi = \phi^{FEC}(\delta)$ (solid yellow line), and (b) as perceived by an individual firm in equilibrium, not internalizing the effect of δ on entry and hence $\phi = \phi^e$ (red dashed line).

ratio of buyers to sellers induced by free entry is decreasing in refinancing frequency. Together with the first order condition for firms (16), this implies that at the equilibrium values (δ^e, ϕ^e), the objective of the SP, $U^{SP}(\delta)$, is decreasing in δ. Hence the SP can increase entrepreneur utility by reducing δ from its equilibrium value δ^e, as illustrated in Figure 6. The local argument above can be extended to a global result which is the main result of the paper:

Proposition 2. *Let* (δ^e, ϕ^e) *be an equilibrium with* $\delta^e > 0$. *Then the solution to the Social Planner's problem* (15) *satisfies* $\delta^{SP} < \delta^e$, *induces* $\phi^{SP} > \phi^e$, *and Pareto improves on the competitive equilibrium.*

The SP can increase aggregate welfare by reducing the refinancing frequency. The reason is that the SP internalizes that choosing a smaller refinancing frequency for all firms increases the gains from trade in the secondary market, which increases the ratio of buyers to sellers and so makes it easier for sellers to find a buyer. This reduces the interest rates that firms pay, so that they can issue more debt, and therefore increases entrepreneur utility. Since investors always break even, the decrease in refinancing frequency is a Pareto improvement.

This result depends on the existence of frictions in the primary market as well as in the secondary market. If we eliminate the frictions in the primary market by letting the refinancing cost κ tend to zero, it can be seen (in Lemma 2) that the optimal refinancing frequency choice of firms tends to ∞: if there is no cost associated with reissuance, firms can choose debt that matures instantaneously and reissue continuously, which essentially gives investors the option to redeem their investment at any point in time. Since investors then can completely avoid the frictions in the secondary market, the interest rate on debt tends to zero. As this happens, firms can issue a larger and larger amount of debt. This allocation tends towards the first best, and the SP cannot improve on it in the limit.

If instead we eliminate the frictions in the secondary market, by letting the rate at which matches arrive tend to infinity in a suitable manner, we can similarly see that the interest rate would tend to zero for any choice of δ (as investors do not need to be compensated for frictions in the secondary market), such that firms could choose $\delta = 0$ and completely avoid the refinancing cost and hence the friction in the primary market. Again, as the interest rate tends to zero, firms can issue a larger and larger amount of debt. This allocation also tends towards the first best, and the SP cannot improve on it in the limit.

We now compare the source of our inefficiency to the standard entry-related externalities known from the labor literature (Hosios, 1990). To understand the standard externalities, consider our model, and fix the refinancing frequency. Now with our choice of matching function, investors who enter the secondary market in order to buy cause congestion and impose a negative externality on other buyers, by making it more difficult for them to be matched with a seller. This externality could lead to an inefficiently high level of entry. At the same time, buyers do not appropriate the whole surplus from a match, and thus they do not have enough incentives to incur the cost of searching, which might lead to an inefficiently low level of entry. The relative importance of the two opposing forces depends on the bargaining power of buyers: when it is high the first dominates and there is excessive entry, when it is low the second dominates and there is insufficient entry. The amount of entry will therefore only be socially efficient for a particular distribution of the bargaining power which exactly balances the two effects. The relevant condition is sometimes referred to as the "Hosios condition."[5]

The standard externalities also operate in our model, and unless the "Hosios condition" holds, the amount of entry by buyers into the secondary market will be inefficient. However, regardless of whether or not the entry decisions of *buyers* are efficient for a given refinancing frequency, *firms* never internalize how their refinancing frequency choice affects the gains from trade in the secondary market, and hence entry, as the argument for Proposition 2 shows.

Finally, a key assumption necessary for generating our externality is that debt claims with different maturities are all traded in a *single* secondary search market, in the sense that if there are claims with different maturities being sold in the market, buyers cannot search to be matched only with specific maturities. This means that a single firm that deviates from the equilibrium refinancing frequency and chooses $\delta \neq \delta^e$ knows that this deviation will not affect the distribution of maturities available in the market, hence knows that this will not affect entry, and hence will correctly anticipate that this will not affect the ratio of buyers to sellers ϕ^e.

Conversely, consider a situation in which debt claims with different maturities are all traded in different sub-markets, and that buyers can decide in which sub-market they search, and hence can search to be matched only with a specific maturity. In this case, we would have a free entry condition for each sub-market j, and a corresponding ratio of buyers to sellers ϕ_j^e. Suppose that

[5] Given a fixed refinancing frequency in our model, the first order condition for maximization of welfare with respect to β holds when $\beta = -\phi\mu_B'(\phi)/\mu_B(\phi)$, i.e. when β is equal to the elasticity of $\mu_B(\phi)$ with respect to ϕ. See Pissarides (1990, chapter 7), or Hosios (1990) for a more general discussion of this type of condition.

firms who deviate and offer a maturity not yet traded in the market know that this creates a new sub-market. This means that even a single firm which deviates from the equilibrium refinancing frequency knows that the ratio of buyers to sellers will be determined by its maturity choice. In this situation firms would internalize the effect of maturity on entry, and on the ratio of buyers to sellers in sub-markets, ϕ_j^e. As a consequence, the maximization problem of the SP would coincide with the one of firms and the laissez-faire equilibrium would exhibit efficient maturity choice.

We note that in a *competitive search* model (or *directed search* model) (Moen, 1997) neither the standard externalities, nor our externality would exist. In such a model, there exist sub-markets, and prices are competitive in the sense that they equalize marginal rates of substitution between market tightness and prices across buyers and sellers in each sub-market. The key feature in that type of model that would eliminate our externality is the existence of sub-markets, not the competitive pricing.

This raises the question as to what extent the assumption of a single search market is empirically plausible. It is clear that the externality that we describe cannot operate across debt markets that are clearly distinct. For instance, in practice, maturity decisions on corporate bonds are unlikely to affect the ratio of buyers to sellers in the market for commercial paper or syndicated loans, and hence the maturity decisions on commercial paper and syndicated loans. Also, to the extent that some participants in one of these markets specializes in trading a subset of maturities only, the externality will be confined to operating within these subsets of maturities, and not across the subsets.

From interviews with market practitioners we learnt that the degree of maturity specialization varies across markets. To our knowledge, there is not much maturity specialization of market participants within the commercial paper market (neither on the side of dealers, nor of investors), and we therefore believe the metaphor of a single search market to be a plausible description of the commercial paper market in isolation.

The corporate bond market is more complicated. On the one hand, on a typical corporate bond trading desk there will be a single trader assigned to a set of issuers, trading in bonds of all maturities of these issuers. Investors who contact traders therefore cannot know ex-ante what specific maturities the traders will be interested in trading.[6] On the other hand, many funds that invest in corporate bonds have mandates that restrict the maturities that they can invest in (probably due to agency issues between the fund managers and the investors in the funds). These mandates are known to other market participants. It is therefore possible that a more segmented form of the inefficiency operates in corporate bond markets.

6. Discussion

In this section we illustrate the results in the paper with a numerical example, discuss versions of the model with marketmakers and a different matching function, and argue that in the context of our model, puttable debt would not be preferable to the standard (non-puttable) debt that we have described so far.

6.1. Numerical illustration

To illustrate our results numerically, we use the parameter values introduced at the end of Section 3. These lead to short maturities, as observed in practice for commercial paper. In particular, we have an equilibrium refinancing frequency of $\delta^e \approx 13.04$, which implies an expected maturity of debt securities of about $1/\delta^e \approx 28$ days. The ratio of buyers to sellers is $\phi^e \approx 1.52$, implying more

[6]This contrasts with sovereign bonds, where there are typically several traders assigned to a single large sovereign, with each trader specializing in bonds in a certain maturity subset. Investors who contact a trader will know ex-ante what range of maturities a trader will trade. We believe that the underlying friction that prevents maturity specialization of traders in the case of corporate bonds is one of economies of scale: The much larger volume on a large sovereign can make having several traders that specialize in different maturities for that sovereign viable.

sellers than buyers and an expected time for a seller to contact and trade with a buyer of about 30 days, while the expected time for a buyer to contact and trade with a seller is about 45 days. (We believe that these numbers are very large, and that the numbers produced with an extended model that includes marketmakers who speed up trading are more plausible, see Subsection 6.2.) The interest rate / illiquidity premium r that firms have to pay at this maturity of 28 days is equal to about 49bp. Entrepreneur utility is equal to $U \approx 0.235$.

To understand prices, first note that the value that an impatient investor attaches to the debt security is $V_\rho \approx 0.9951$. The value that a patient investor attaches to that security is $V_0 = 1$, so that the gains from trade are about 49bp. The price at which investors agree to trade debt in the secondary market is equal to $P \approx 0.9975$, indicating that because of the equal bargaining power, they divide the gains from trade equally (slightly more than 24bp per party).

In this situation, a social planner would choose a refinancing frequency of $\delta^{SP} \approx 8.86$, implying an expected maturity of $1/\delta^{SP} \approx 41$ days, as compared to the 28 days in the laissez-faire equilibrium. This increases gains from trade in the secondary market from about 49bp to about 58bp, which induces more entry of buyers leading to a ratio of buyers to sellers of $\phi \approx 2.11$. The expected time for a seller to contact a buyer then is reduced to about 25 days.

The interest rate / illiquidity premium r that firms have to pay at this new maturity of 41 days is equal to 58bp. At the same time, an individual firm that considers deviating from the laissez-faire equilibrium would perceive the interest rate required for issuing debt at a maturity of 41 days to be 62bp. The difference (62bp versus 58bp) arises because a coordinated increase in maturity choice increases gains from trade in the secondary market and encourages the entry of buyers and marketmakers into this market.

Finally, even though the interest rate at the social planner's maturity is higher than in the laissez-faire economy, entrepreneurs benefit because the longer maturity allows them to save on the refinancing cost and lower the overall flow cost of debt. This increases the amount of debt issued, and leads to a higher entrepreneur utility of $U \approx 0.264$, representing a 12% increase over the laissez-faire equilibrium.

6.2. *Marketmakers*

Most trade between final sellers and buyers in secondary OTC debt markets is intermediated by marketmakers. Following the approach in Duffie et al. (2005), we extend the model to incorporate this new class of agents. Marketmakers have access to an additional matching technology with which they get matched with buyers and sellers, and also have access to an inter-dealer market in which they can instantly offset any position that they have entered into with an investor. They profit from the difference between the bid (ask) price at which they buy from sellers (sell to buyers) and the price in the inter-dealer market at which they close out their positions. As in the baseline model, we allow for free entry of marketmakers to close the model, and we are able to show uniqueness of equilibrium and the presence of the same inefficiency that renders laissez-faire maturities excessively short. A complete analytical solution is in the Online Appendix.

This extension produces two additional implications. First, the presence of marketmakers generates a model-implied bid-ask spread, and we show that it is increasing in maturity. This implication is consistent with Edwards et al.'s (2007) evidence on estimated corporate bond transaction price spreads.

Second, in the presence of marketmakers investors are able to trade faster. Moreover, if marketmakers have a high bargaining power, the quantitative importance of the externality is preserved. Extending the numerical illustration of the baseline model with marketmakers, who have a Nash bargaining power parameter of $\gamma = .95$ and a matching technology which is ten times faster than which that matches buying and selling investors directly, the expected time for a selling investor to trade is reduced from the arguably implausible 30 days of the baseline model to a more realistic four days. At the same time, issuers still choose a relatively short maturity of around 33 days in laissez-faire equilibrium. Given that marketmakers speed up trading and improve the secondary market in this way, a social planner would find it optimal to choose a much longer maturity than in the baseline case, of around 109 days. This would increase entrepreneur utility by about 16%. We describe this numerical illustration in more detail in the Online Appendix.

6.3. Increasing-returns-to-scale matching function

Many other papers that apply the search approach to modeling OTC markets differ from ours in that they use an increasing-returns-to-scale (IRS) matching function, with the functional form $\mu^{IRS}(\alpha^S, \alpha^B) = \lambda \alpha^S \alpha^B$ for some $\lambda > 0$ (Duffie et al., 2005, Vayanos and Wang, 2007, Vayanos and Weill, 2008, Afonso, 2011, and others). Loosely speaking, with this IRS matching function, an additional seller entering the market will make it more attractive for buyers to enter, without making it harder for other sellers to find buyers, i.e. there are no congestion effects. This means that strong "thick market externalities" are assumed as part of the technology. To emphasize that our main result does not rely on these thick market externalities in the matching function, we have instead used a constant-returns-to-scale (CRS) matching function. In this sense, our assumptions are closer to those of Weill (2008) and Lagos and Rocheteau (2007), who also consider alternatives to the IRS matching function.

As stated above, our main result can also be derived in a version of the model that uses the IRS matching function. The formal analysis, which we conduct in the Online Appendix of the paper, is slightly more complicated because equilibrium cannot be characterized in terms of the steady-state ratio of buyers to sellers, but must instead be characterized in terms of both the steady-state measure of buyers and the steady-state measure of sellers. However, as in the baseline model, we can prove that the maturity is inefficiently short in laissez-faire equilibrium. The basic intuition for this result is the same as in the baseline model: An individual issuer does not internalize that raising maturity increases the gains from trade in the secondary market, which attracts more buyers, which reduces the interest rates paid by other issuers. However, with an IRS matching function, entering buyers do not produce any form of congestion, so more buyers can enter than in the CRS case, and the inefficiency is amplified.

In the Online Appendix, we construct a numerical example to illustrate this: we consider an economy differing from that in the numerical illustration of the baseline model only in the matching function, which is assumed to be of the IRS type. In order to make the two economies comparable, we chose the multiplicative parameter $\lambda > 0$ to produce a laissez-faire equilibrium in which all the endogenous equilibrium variables coincide with those in Section 6.1. We find that in such an economy a SP would choose an optimal refinancing frequency of $\delta_{IRS}^{SP} = 0$, that is, the social planner would require firms to issue perpetual debt (compared to the maturity of around 41 days that the SP would choose in the CRS economy), which would increase entrepreneur utility by 420% (compared to 12% in the CRS economy).

6.4. Puttable debt

In our baseline model, investors can redeem their security for face value when it matures. When the refinancing frequency δ tends to infinity, debt is maturing instantaneously, and it becomes puttable in the sense that investors can redeem their security for face value at any point in time.[7] The advantage of puttable debt would be that it could be issued at an interest rate of zero, since investors do not need to be compensated for frictions in the secondary market.

In the baseline model, however, we assumed that maturing debt is immediately reissued, so

[7] Alternatively, this type of debt could be described as a demand deposit, from which the deposited amount can be withdrawn at any point in time.

that if debt matures instantaneously, then it is also reissued continuously. There is a fixed cost of κ associated with each reissuance, so that if debt is reissued continuously, then the flow of reissuance costs goes to infinity. This cannot be optimal. However, a firm could issue puttable debt, but instead of reissuing continuously, it could let patient investors roll over their debt, finance redemptions by impatient investors out of project cash flows, and reissue only periodically to take the amount of debt back to its original level.

In other models in which firms place debt with investors who face liquidity shocks, e.g. in the canonical model of Diamond and Dybvig (1983), it is optimal for a firm (or bank) to insure risk-averse investors via puttable debt. So a natural question is whether this is also the case in our model, and hence whether it is internally consistent for us to focus on standard debt.

In our model, investors are risk-neutral and hence there is no need to insure them via puttable debt. However, debt type has an effect on debt capacity, which implies that standard debt dominates: When firms finance redemptions out of project cash flows, this puts a very tight constraint on debt capacity, especially just after issuance, when the flow of redemptions will be highest. In contrast, when firms issue standard debt, they have to finance interest payments (which compensates for secondary market frictions) out of project cash flows, but no redemptions. We can show that interest on standard debt is low in comparison to redemptions on puttable debt just after issuance, which implies that debt capacity and hence entrepreneur utility is higher with standard debt (see Appendix B for details).

7. Conclusion

Debt holders who need to sell in an OTC secondary market are in a worse bargaining position the longer they are locked-in into their contracts, i.e. the longer the time-to-maturity of their debt is. This worse bargaining position implies a larger discount when selling. Firms anticipate that they need to offer higher yields on debt with longer maturities, and especially so if the secondary market is very illiquid, in the sense that the ratio of buyers to sellers is low. But the entry of buyers into the secondary market and hence its liquidity is a function of the profits that buyers can obtain in this market, which decreases in the bargaining position of the sellers. We present a model in which the liquidity of secondary markets for corporate debt, and maturities, are jointly determined in equilibrium, on the basis of this mechanism.

Our main result is that in equilibrium, maturities chosen by firms are inefficiently short. This is because firms do not internalize the effect of their maturity decisions on the gains from trade in the secondary market and hence on the incentives for buyers to enter this market. When an individual firm increases the maturity of its debt, this worsens the bargaining positions of the holders of this debt who need to sell. But this also increases the gains from trade in the secondary market. The latter attracts more buyers into the secondary market in search of more profitable deals, which increases liquidity and reduces the interest rates demanded by investors on the debt of all firms, at all possible maturities.

From a practical perspective, this might explain why prior to the crisis, financial institutions relied on extremely short term asset-backed commercial paper to fund long term assets, while at the same time the secondary market for commercial paper was so illiquid as to be almost non-existent. Our model highlights that if issuers were forced to sell longer maturity paper, this would attract more buyers to the secondary market for commercial paper, making it easier to sell, and hence decreasing the need to issue such short maturity paper.

There are some avenues for future research that could be pursued using the type of model that we describe. For example, one could examine the consequences of a financial transaction tax, as currently being considered by the European Commission.[8] It can be shown that when

[8] See “Proposal for a council directive implementing enhanced cooperation in the area of financial transaction tax,” http://ec.europa.eu/taxation_customs/resources/documents/taxation/com_2013_71_en.pdf.

such a tax is introduced into our model (e.g. modeled as a cash amount to be paid by the buyer every time a transaction occurs), the equilibrium refinancing frequency increases (the equilibrium maturity decreases), indicating that a financial transactions tax might interfere with regulatory objectives such as the reduction in the maturity mismatch produced by financial intermediaries. More generally, it would be interesting to evaluate how this type of tax affects the externality that we have identified (as well as the standard congestion externality present in search models with ex-post bargaining and free entry), and hence its impact on welfare.

Appendix

Appendix A. Proofs

Proof of Lemma 1: Substitute (2) into (4), and solve the resulting equation together with (3) for V_0. Apply condition (5) and solve for r to obtain the result. ■

Proof of Lemma 2: Assumption 1 (i) guarantees that $\frac{x}{\rho} - 1 > 0$ and hence that undertaking the project with own funds produces positive entrepreneur utility, and (ii) also guarantees that $U(0, r(0;0)) = -1 - \kappa + \frac{x}{r(0;0)} > \frac{x}{\rho} - 1$. Since $r(0;\phi) \leq r(0;0)$ which implies $U(0, r(0,\phi)) \geq U(0, r(0,0))$, and $U(\delta^*, r(\delta^*;\phi)) \geq U(0, r(0;\phi))$, we have that $U(\delta^*, r(\delta^*;\phi)) > \frac{x}{\rho} - 1$ (> 0), so that the entrepreneur will find it optimal to issue debt to undertake the project.

We now characterize the optimal refinancing frequency decisions for given ϕ. Substituting the expression for $r(\delta;\phi)$ in equation (6) into the program (13), we obtain

$$U(\delta) = \frac{(\delta + \theta + \rho + \mu_S(\phi)\beta)(x - \delta\kappa)}{\rho\theta} \tag{18}$$

$U(\delta)$ is concave and has a unique maximum, although the δ that achieves this maximum might be negative. The δ that maximizes $U(\delta)$ under the constraint $\delta \geq 0$ is therefore given by

$$\delta^*(\phi) = \max\left(\frac{1}{2}\left(\frac{x}{k} - \theta - \rho - \mu_S(\phi)\beta\right), 0\right), \tag{19}$$

which concludes the proof. ■

Proof of Lemma 3: After a match between a buyer and a seller with contract (r, δ) there is trade at the price $P = P(r, \delta; F, \phi)$. The surpluses of the buyer and seller are $V_0 - P - V_B$ and $P - V_\rho$, respectively. Both surpluses have to be non-negative for the agents to agree to trade, which implies the following necessary condition for matches to result in trade:

$$V_0 - V_B \geq V_\rho. \tag{20}$$

With these surpluses, Nash bargaining results in the price

$$P = \beta(V_0 - V_B) + (1-\beta)V_\rho. \tag{21}$$

(This differs from the expression for the price in Section 4, because here, we cannot set $V_B(F,\phi) = 0$, since this is what we want to prove.)

Following a match, a buyer is paired with a random seller, and trade results at a price given by (21) if condition (20) is satisfied. The flow-value equation for $V_B(F;\phi)$ is then:

$$-e_B + \mu_B(\phi)(1-\beta)\int_{V_0 - V_B \geq V_\rho} (V_0 - V_\rho - V_B)\, dF(r,\delta) - \theta V_B = 0. \tag{22}$$

(This differs from the corresponding flow-value equation in Section 4 because here, we do not yet impose that in equilibrium only a single contract (r,δ) is traded, and because potentially not all matches result in trade.) We now prove the lemma by contradiction.

Suppose that in equilibrium $V_B(F;\phi) < 0$. Buyers strictly prefer not to search in the secondary market. Therefore it has to be the case that $\alpha_B = 0$ and hence that $\phi = 0$. Moreover, since we trivially have that $V_0(r,\delta;F,\phi) > V_\rho(r,\delta;F,\phi), \forall (r,\delta)$, we also have that $V_0 - V_\rho - V_B > -V_B > 0, \forall(r,\delta)$. It follows that $\lim_{\phi\to 0}\mu_B(\phi)(1-\beta)\int_{V_0-V_B\geq V_\rho}(V_0 - V_\rho - V_B)\, dF(r,\delta) = +\infty$, and hence the flow-value equation (22) is not satisfied for $V_B(F;\phi) < 0$.

Suppose that in equilibrium $V_B(F;\phi) > 0$. Due to the assumption of a large inflow of patient investors, this would imply $\alpha^B \to \infty$ and hence $\phi \to \infty$. It suffices to prove that there exists C such that for all $V_0(r,\delta;F,\phi) - V_\rho(r,\delta;F,\phi) - V_B(F;\phi) < C$ for all (r,δ) and ϕ, since this implies that $\lim_{\phi\to\infty}\int_{V_0-V_B\geq V_\rho}(V_0 - V_\rho - V_B)dF < +\infty$, and hence that $\lim_{\phi\to\infty}\mu_B(\phi)(1-\beta)\int_{V_0-V_B\geq V_\rho}(V_0 - V_\rho - V_B)dF \leq 0$, so that the flow-value equation (22) is not satisfied for $V_B(F;\phi) > 0$.

Let (r,δ) be a debt contract. We must have $r < \rho$ because otherwise from equation (12) in the main text we can easily check that the issuing firm would find more profitable to invest in the project out of the entrepreneur's wealth (which would generate utility $-1+\frac{x}{\rho}$). Using the flow-value equation (3) we have that: $r = \theta(V_0 - V_\rho)$, and then using that $r < \rho$ we obtain

$$V_0 - V_\rho - V_B < V_0 - V_\rho < \frac{\rho}{\theta}, \tag{23}$$

so that $V_0 - V_\rho - V_B$ is bounded above, as required. ■

Proof of Lemma 4: In order to prove that (FEC) defines a function $\phi^{FEC}(\delta)$, we substitute the expression for $r(\delta;\phi)$ in (6) into the free entry condition in (FEC) and obtain

$$\mu_S(\phi)\beta + \rho + \theta + \delta = \mu_B(\phi)\frac{1-\beta}{e_B}\rho. \tag{24}$$

We note that as $\phi \downarrow 0$ the left hand side tends to a positive constant which is a function of δ, whereas the right hand side tends to ∞. As $\phi \uparrow \infty$, the left hand side tends to infinity, whereas the right hand side tends to 0. Furthermore, from the properties of the matching function, we know that $\mu_S(\phi)$ is continuous and strictly increasing in ϕ (and hence so is the left hand side), and that $\mu_B(\phi)$ is continuous and strictly decreasing in ϕ (and hence so is the right hand side). It therefore follows that for each $\delta \in [0, \infty)$, there exists a unique ϕ that satisfies (24). We denote the function that describes this mapping as $\phi^{FEC}(\delta)$. Its domain is $[0, \infty)$. Using the implicit function theorem, it can be seen that the function is strictly decreasing, implying that it is maximized at $\hat{\phi} := \phi^{FEC}(0)$. We note that since $\lim_{\delta\to\infty} \phi^{FEC}(\delta) = 0$, the function $\phi^{FEC}(\delta)$ has as its image the interval $(0, \hat{\phi}]$. Since $\phi^{FEC}(\delta)$ is strictly decreasing its inverse function $\delta^{FEC}(\phi)$ is well defined, its domain is the interval $(0, \hat{\phi}]$, its image is the interval $[0, \infty)$ and it is strictly decreasing. ■

Proof of Proposition 1: We first consider existence, and distinguish between two cases.

First, let us suppose that $\delta^*(\hat{\phi}) = 0$. By definition $\delta^{FEC}(\hat{\phi}) = 0$. Then trivially $\delta^e = 0, \phi^e = \hat{\phi}$ is an equilibrium. Second, suppose the converse, that $\delta^*(\hat{\phi}) > 0$. By definition, $\delta^{FEC}(\hat{\phi}) = 0$, and hence $\delta^{FEC}(\hat{\phi}) < \delta^*(\hat{\phi})$. At the same time, $\lim_{\phi\to 0} \delta^{FEC}(\phi) = \infty$, while $\delta^*(0)$ is finite, implying that $\delta^{FEC}(\phi) > \delta^*(\phi)$ for ϕ sufficiently close to zero. By continuity of the two functions $\delta^{FEC}(\phi), \delta^*(\phi)$, there must then exist a pair (δ^e, ϕ^e) such that $\delta^{FEC}(\phi^e) = \delta^*(\phi^e) = \delta^e$. This pair is an equilibrium.

We now prove uniqueness. In order to do so it suffices to prove that

$$\frac{d\delta^{FEC}(\phi)}{d\phi} < \frac{d\delta^*(\phi)}{d\phi} \text{ for all } \phi \in (0, \hat{\phi}]. \tag{25}$$

From the expression for $\delta^*(\phi)$ in Lemma 2, it can be seen that

$$\frac{d\delta^*(\phi)}{d\phi} \geq -\frac{1}{2}\beta\frac{d\mu_S(\phi)}{d\phi} > -\beta\frac{d\mu_S(\phi)}{d\phi}, \quad \forall\phi, \tag{26}$$

since $\frac{d\mu_S(\phi)}{d\phi} > 0$. From (24), we obtain

$$\frac{d\delta^{FEC}(\phi)}{d\phi} = -\beta\frac{d\mu_S(\phi)}{d\phi} + \frac{d\mu_B(\phi)}{d\phi}\frac{1-\beta}{e_B}\rho, \quad \forall\phi \in (0, \hat{\phi}]. \tag{27}$$

Since $d\mu_S(\phi)/d\phi > 0$ and $\mu_B(\phi)/d\phi < 0$, a direct comparison between equations (26) and (27) leads to the inequality (25).

■

Proof of Proposition 2: For all $\delta > \delta^e$ we have $\phi^{FEC}(\delta) < \phi^{FEC}(\delta^e)$. It follows that for $\delta > \delta^e$,

$$U^{SP}(\delta) = U(\delta, r(\delta; \phi^{FEC}(\delta))) < U(\delta, r(\delta; \phi^{FEC}(\delta^e))) \leq U(\delta^e, r(\delta^e; \phi^{FEC}(\delta^e))) = U^{SP}(\delta^e)$$

where in the first inequality we have used that $U(\delta, r)$ is decreasing in r, which in turn is decreasing in ϕ, and in the second that by the definition of equilibrium, δ^e maximizes firms' objective function for liquidity $\phi^{FEC}(\delta^e)$. Using the inequality $\frac{dU^{SP}(\delta^e)}{d\delta} < 0$ which has been proved in the main text we can write

$$\arg\max_{\delta\geq 0} U^{SP}(\delta) < \delta^e.$$

■

Appendix B. Puttable debt

In our baseline model, investors can redeem their security for face value when it matures. When $\delta \to \infty$, debt is maturing instantaneously, and it becomes puttable in the sense that investors can redeem their security for face value at any point in time. There is no need to trade such debt in the secondary market, and the interest rate on it is zero. However, in the baseline model, debt is reissued as it matures. When $\delta \to \infty$, this implies that the fixed cost of reissuance κ is paid continuously, such that the flow of reissuance costs goes to infinity. This type of puttable debt cannot be optimal.

In this appendix, we consider whether it might be optimal for a firm to issue puttable debt, but to avoid continuous reissuance. In particular, a firm might let patient investors roll over their debt, finance redemptions by impatient investors out of project cash flows, and reissue only periodically. We show that this type of arrangement is in fact not optimal.

We maintain the assumption of a stationary debt structure. We assume that puttable debt has deterministic maturity T, meaning that the firm initially issues debt with a face value of D, and reissues debt with face value of D every T units of time. D and T are chosen to maximize the utility of the entrepreneur. We let $t \in (0, T]$ denote the time since the last issuance.

Let $D_t \leq D$ be the aggregate face value of remaining (unredeemed) debt at time t. For $t \in (0, T)$, D_t satisfies the laws of motion $\frac{dD_t}{dt} = -\theta D_t$, implying

$$D_t = e^{-\theta t} D, \tag{28}$$

The flow of redemptions is maximized at $t = 0$, when it takes the value θD. For debt issuance to be feasible, we therefore need

$$x \geq \theta D. \tag{29}$$

This constraint corresponds to the constraint (8) in the baseline case.

The entrepreneur consumes any issuance proceeds and project cash flows net of redemptions and issuance costs. When financing the project with puttable debt with face value D and maturity T, the utility of the entrepreneur therefore is

$$\begin{aligned} U_{PD}(D,T) = & -1 - \kappa + D + \frac{1}{1-e^{-\rho T}} \int_0^T e^{-\rho t}(x - \theta D_t)dt \\ & + \frac{1}{1-e^{-\rho T}} e^{-\rho T} \left(D - D_T - \kappa \right). \end{aligned}$$

The first and second term represent the cost of the investment and the issuance cost at $t = 0$, respectively. The third term accounts for the funds obtained by issuing puttable debt at $t = 0$. The fourth term is the expected value of the continuous flow of project cash flows net of redemptions, paid by the firm to the entrepreneur. The last term is the expected value of the left over cash after reissuing, which is paid as a discrete dividend. (T needs to be sufficiently large, such that this term is non-negative.)

Using equation (28) and integrating, we obtain

$$U_{PD}(D,T) = -1 - \kappa + \frac{x}{\rho} + \underbrace{D \frac{\rho}{\rho+\theta} \frac{1-e^{-(\theta+\rho)T}}{1-e^{-\rho T}}}_{A} - \underbrace{\kappa \frac{e^{-\rho T}}{1-e^{-\rho T}}}_{B}$$

We can see that T affects U_{PD} through two terms. The term labelled A is decreasing in T, reflecting that more gains from trade between the entrepreneur and patient investors can be realized if the face value of debt is frequently reset, to raise the average face value of unredeemed debt. The term labelled B is also decreasing in T, reflecting that refinancing costs can be reduced by lengthening the maturity.

As in the baseline model, the utility of the entrepreneur is increasing in D, so the entrepreneur will choose the largest D consistent with the constraint (29), i.e. will choose $D = \frac{x}{\theta}$. Since the term A is maximized at value $D = \frac{x}{\theta}$ when $T \to 0$, and the term B is minimized at value 0 when

$T \to \infty$, we have that for all T,

$$U_{PD}(D,T) < -1 - \kappa + \frac{x}{\rho} + \frac{x}{\theta} \tag{30}$$

We can compare this to the utility that an entrepreneur can obtain from issuing our traded plain-vanilla debt, with $\delta = 0$. Since $r(\delta, \phi)$ is decreasing in ϕ, $r(0,\phi) < r(0,0) = \frac{\rho\theta}{\rho+\theta}$, and hence

$$U(0, r(0,\phi)) = -1 - \kappa + \frac{x}{r(0,\phi)} > -1 - \kappa + \frac{x}{r(0,0)} = -1 - k + \frac{x}{\rho} + \frac{x}{\theta} \tag{31}$$

Taken together, (30) and (31) imply that $U_{PD}(D,T) < U(0, r(0,\phi))$, and hence since $U(0, r(0,\phi)) \leq U(\delta, r(\delta,\phi))$, this means that non-traded puttable debt produces lower utility for the entrepreneur than the traded standard debt described in the main text.

The reason is that when firms finance redemptions out of project cash flows, this severely reduces debt capacity. With traded, standard debt, firms pay interest which compensates for secondary market frictions out of project cash flows. The argument here shows that such interest is low in comparison to redemptions on non-traded puttable debt just after issuance, which implies that debt capacity and entrepreneur utility is higher with standard debt.

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