



City Research Online

City, University of London Institutional Repository

Citation: Karcianas, N., Leventides, J., Milonidis, E. & Meintanis, I. (2016). Selection of Decentralised Control Structures: Structural Methodologies and Diagnostics. IFAC PapersOnLine, 49(9), pp. 80-85. doi: 10.1016/j.ifacol.2016.07.498

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

Permanent repository link: <https://openaccess.city.ac.uk/id/eprint/15462/>

Link to published version: <https://doi.org/10.1016/j.ifacol.2016.07.498>

Copyright: City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

Reuse: Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

City Research Online:

<http://openaccess.city.ac.uk/>

publications@city.ac.uk

Selection of Decentralised Control Structures: Structural Methodologies and Diagnostics

N. Karcantias * J. Leventides ** E. Milonidis * I. Meintanis *

* *Systems and Control Research Centre
School of Mathematics, Computer Science and Engineering
City University London, Northampton Square, London EC1V 0HB, UK
(e-mail: N.Karcantias@city.ac.uk)*

** *University of Athens, Department of Economics
Section of Mathematics and Informatics, Pezmasoglou 8, Athens,
Greece*

Abstract: The paper aims at formulating an integrated approach for the selection of decentralized control structures using a number of structural criteria aiming at facilitating the design of decentralised control schemes. This requires the selection of decentralisation structure that will allow the generic solvability of a variety of decentralised control problems, such as pole assignment by decentralised output feedback. The approach is based on the use of necessary and sufficient conditions for generic solvability and exact solvability of decentralised control problems. The generic solvability conditions lead to characterisations of inputs and outputs channel partitions. The exact solvability conditions use criteria on avoiding the presence of fixed modes, as well as necessary conditions for pole assignment, expressed in terms of properties of Plücker invariants and Markov type matrices. The structural approach provides a classification of desirable input and output partitions based on structural criteria and it is embedded in an overall framework that may involve aspects related to large scale design.

Keywords: Linear multivariable systems; output feedback control; structural methods; decentralised control; algebraic methods; geometric methods.

1. INTRODUCTION

The selection of a decentralisation scheme is a problem that has not been properly addressed as a design issue and has been handled mostly using process dependent heuristics, conditions derived from the nature and spatial arrangement of sub-process units and the criteria, diagnostics of the interaction analysis. This problem is within the area of control structure selection, or Systems Instrumentation (Karcantias, 1996) and involves steps such as: classification of variables of the model into inputs, outputs and internal variables (Karcantias, 1996); definition of effective sets of inputs, outputs (Karcantias and Vafiadis, 2002); and finally the selection of the feedback coupling of the Control Scheme (Morari and Stephanopoulos, 1980) (Siljak, 1991) etc). These families of problems may be considered within the framework of structural methodologies for linear systems and each one of them involves concrete sub-problems. The design of decentralisation schemes is a problem which may be studied using criteria based on the nature of the process (Morari and Stephanopoulos, 1980), geographical location of subsystems, graph based structural analysis (Siljak, 1991), interaction indicators (Grosdidier and Morari, 1986), (Manousiouthakis et al., 1986) and general coupling diagnostics based on generic properties and invariant based criteria (Leventides and Karcantias, 1996).

We address the problem of design of decentralisation schemes in order to guarantee, or well condition the solvability of families of control problems. As such the overall approach is based on the philosophy of defining schemes which exclude undesirable characteristics, such as fixed modes. Deriving a systematic methodology for synthesis of decentralisation schemes (Karcantias et al., 1997) requires an approach that involves: (a) Classification of variables, definition of system progenitor models and definition of effective sets of inputs, outputs (Karcantias and Vafiadis, 2002) (b) Handling large scale issues using decomposition methodologies (Morari and Stephanopoulos, 1980) and graph methodologies (Siljak, 1991); (c) Developing structural methodologies preconditioning the solvability of control theoretic problems (Karcantias and Leventides, 2005), (Lampakis and Karcantias, 1995); (e) Design indicators and optimization based methodologies enabling the solvability of a number of decentralized control problems (Manousiouthakis et al., 1986). Such an integrated methodology is currently missing and an early form has been introduced in (Karcantias et al., 1997).

The emphasis in our approach is the screening of the bad choices and then leave the final selection to performance dependent criteria. The overall framework developed aims to contribute in answering questions such as selecting between centralised versus decentralised, and if decentralised, then specify the exact nature of decentralisation

that involves the partitioning and pairing for the particular channels. This paper focuses on the development of the structural methods for the selection of decentralization. An integral part of the structural methodologies is the distinction between generic solvability conditions (Leventides and Karcianas, 1996) and parametric invariant dependent conditions, where the model parameter play an important role. We use existing results for generic solvability of decentralized control problems (Wang and Davison, 1973), (Anderson and Clements, 1981), (Karcianas et al., 1988), (Siljak, 1991), (Wang, 1994), (Leventides and Karcianas, 1995), as well as criteria and diagnostics that well condition the solvability of decentralized problems for parameter dependent models. Clearly, these structural results and criteria will have to be used in addition to all other available tools, that are parts of the overall integrated methodology, mentioned above, which will be also summarised in the paper.

The structural approach is based on existing generic solvability conditions and on the Exterior Algebra model based diagnostics (Karcianas and Giannakopoulos, 1984), (Karcianas et al., 1988), (Leventides and Karcianas, 1998b) involving Plücker matrices, decentralized Markov parameters and criteria for avoiding fixed modes depending on parameter model based properties. Such criteria provide the means to characterise the different features of system structure that permit, or prohibit the acceptance of solutions to control problems.

We will use the generic solvability conditions to develop partitions of the input and output sets characterising decentralization, which have good potential and then examine the criteria based on parametric invariants for more detailed evaluation of alternative decentralized control structures. For systems with a given number of inputs, outputs and states but otherwise unstructured models, the problem of selection of decentralisation has to do with the partitioning of inputs outputs into channels, each one characterised by its cardinality and then decide about their coupling, whether feedback control is to be used. Deciding about the nature of cardinality of the particular channel, as well as the number of desirable channels is one of the problem we are concerned with here and it is based on generic properties (Leventides and Karcianas, 1996). The results of this investigation are used prior to the deployment of parameter dependent diagnostics and thus form part of the first screening of the options for selection of decentralisation. We will then consider the significance of the system model parameters and selected decentralization using the results of the exterior algebra framework for the study of Decentralized Determinantal Assignment Problems (Karcianas and Giannakopoulos, 1984; Karcianas et al., 1988).

We aim to eliminate the existence of fixed modes and guarantee full rank properties to the decentralised Plücker matrices (Leventides and Karcianas, 1998b). The paper presents aspects of structural methodologies for design of decentralisation schemes, which are part of an overall integrated philosophy for design of such schemes.

2. OVERALL APPROACH FOR THE SELECTION OF THE DECENTRALISATION STRUCTURE

The selection of the decentralisation structure involves answering questions on whether we have to use centralised, or decentralised schemes; if decentralisation is needed, then we need to decide on the partitioning of the input, output channels, as well as the way we have to couple them in a feedback configuration. An integral part of the design is also the specification of the required order of dynamics of the selected decentralised scheme. Here, however, we will focus on constant feedback design. Our approach for the selection of the decentralisation involves a number of general steps which will be considered in this section.

Step 1. Use knowledge on the process, geographical location of process units and operational requirements, such as the nature of optimisation problem, to define a first appraisal of options as for centralisation versus decentralisation. If decentralisation is needed, then the physical arguments lead to the first structuring of the decentralisation scheme, referred to as feasible decentralisation.

This step aims to take into account the particulars of the application area and nature of the problem. This knowledge is essential and it is part of the overall problem specification. What is expected at this stage is the development of the first structuring of the schemes in terms of super-blocks, which themselves may require some further structuring subsequently. This area of work may be considered as a part of the control structure selection on the entire plant.

Step 2. Use of graph analysis methodology to develop system decompositions (Siljak, 1991), which may indicate structuring of the decentralisation and also use of the concept of structural fixed modes for evaluation of alternatives.

For systems which have an explicit graph structure, a procedure leading to overall system decomposition, may be used in developing further the possible structures specified in step (1) and then evaluating alternatives based on the exclusion of structural fixed modes based on properties of the system graph.

Step 3. Use results on the generic solvability of decentralised control problems to produce a parameterisation of alternatives based on generic solvability of decentralised control problems.

The study of decentralised control problems has produced some results characterising generic solvability of control problems (Wang and Davison, 1973), (Anderson and Clements, 1981), (Karcianas et al., 1988), (Siljak, 1991), (Wang, 1994), (Leventides and Karcianas, 1995), (Leventides and Karcianas, 1996) which can lead to parameterisation of possible partitions of input, output channels which permit solvability of control problems. These results depend on structural characteristics such as the McMillan degree and the numbers of inputs, outputs.

Step 4. Use results on the solvability of decentralised control problems based on parameter dependent structural invariants to produce a parameterisation of alternatives based on diagnostics for solvability of decentralised control problems.

At this stage we proceed with the evaluation of the available options using linear models and parameter dependent properties such as fixed modes, almost fixed modes (Karcianas et al., 1988), properties of the rank of decentralised Plücker matrices, strong instability and minimum phase phenomena. Tests based on exterior algebra diagnostics include rank of decentralised Markov parameters (Leventides and Karcianas, 1998a). These tests are used to predict formation of undesirable characteristics (fixed, almost fixed modes, loss of rank of Plücker matrices).

Step 5. Use of interaction analysis diagnostics based on steady state models, or simple dynamic models to evaluate the alternatives produced at the previous stage.

Progressing from graph models to steady state, or simple dynamic models, we may use the large number of diagnostics of the RGA, BRGA type (Manousiouthakis et al., 1986), to evaluate further the options specified in the last step. There is a variety of tests for interaction analysis based on simple models that can be used.

Step 6. On a full dynamics linear model, use diagnostics based on performance indicators to evaluate the alternative decentralisation schemes, which have been specified by the previous stage.

Having exhausted all structural methodologies and tests to reduce the set of options we now use computationally intensive methodologies on smaller dimension systems for the further screening of alternatives. This involves indicators such as singular values, structural singular values, properties of cost balanced realisations, energy requirements for coupling, etc. In this paper we focus on issues related to steps (3) and (4).

3. PRELIMINARIES TO THE STRUCTURAL APPROACH

Consider a k -channel linear system $S(A, B, C)$ described by

$$\dot{x} = Ax + \sum_{i=1}^k B_i u_i \quad y_i = C_i x \quad (1)$$

where x, u_i, y_i are n, m_i, p_i vectors, respectively, and u_i, y_i are the input and output of the i -th channel in a decentralisation scheme. We use a right coprime MFD for the transfer function $G(s) = N(s)D(s)^{-1}$. If local feedback laws of the type

$$u_i = K_i y_i + u'_i, \quad K_i \in \mathbb{R}^{p_i \times m_i}, \quad i = 1, 2, \dots, k$$

are applied to each of the k -channels, the closed-loop pole polynomial is expressed as

$$p(s) = \det(sI - A - \sum_{i=1}^k B_i K_i C_i) \\ p(s) = \det \left([I_p, K_{dec}] \begin{bmatrix} D(s) \\ N(s) \end{bmatrix} \right) \quad (2)$$

where $K_{dec} = bl.diag\{K_1, \dots, K_k\}$. The above problem belongs to a more general category of pole placement problems where the multivariable feedback controller is structured. More specifically if we denote the set of all possible pairs i, j corresponding to closing the loop by $\Omega = \{(i, j) : 1 \leq i \leq p \text{ and } 1 \leq j \leq m\}$ where the output j is connected to the input i , then any subset Ω_s of Ω

defines a structured feedback scheme where the permissible loop closures are described by the pairs in Ω_s . In the special case of static decentralised control we have that:

$$\Omega_d = [1, p_1] \times [1, m_1] \cup [p_1 + 1, p_1 + p_2] \times [m_1 + 1, m_1 + m_2] \\ \dots \cup \left[\sum_{i=1}^{k-1} p_i + 1, p \right] \times \left[\sum_{i=1}^{k-1} m_i + 1, m \right]$$

The structured pole placement map X^s is the function that maps every structured feedback K_{dec} to the n closed loop poles: $X^s : \mathfrak{S}^\mu \rightarrow \mathfrak{S}^n$ where $\mu = |\Omega_s|$ denotes the number of free parameters in Ω_s . The decentralised pole placement map, X^d , is defined this way to be: $X^d : \mathfrak{S}^{\sum m_i p_i} \rightarrow \mathfrak{S}^n$. The pole placement map (Leventides and Karcianas, 1992) carries all the information as far as the pole placement and for the decentralised case can be considered as a restriction of the general pole placement map of the centralised output feedback case and contains a subset of the Markov parameters (Leventides and Karcianas, 1998a). The latter property allows the linking of decentralised Plücker invariants to the state space parameters and thus to system design issues.

We examine next issues linked to the selection of decentralisation schemes: (a) Generic conditions for solvability of decentralisation problems and selection of decentralisation partitioning. (b) Well conditioning of Plücker invariants by selection/redesign of decentralised Markov parameters. These problems are linked to necessary conditions and thus introduce possible solutions for design of decentralisation using the results of the decentralised pole assignment by constant output feedback. The framework can be extended by considering similar decentralised control problems.

4. BACKGROUND TO THE DECENTRALISED DETERMINANTAL ASSIGNMENT PROBLEM

The study of the pole placement map is central in the investigation of solvability conditions of the decentralised pole assignment and provides the required necessary conditions for addressing the design of the decentralisation schemes. In the following, we review two approaches. The first is based on the decentralised Plücker matrix and the second on the differential of the map X^d , which leads to the definition of the decentralised Markov parameters.

Decentralised Plücker Matrices: Using the Binet Cauchy Theorem on 2, we have:

$$p(s) = C_p([I_p, K_{dec}]) \cdot C_p \left(\begin{bmatrix} D(s) \\ N(s) \end{bmatrix} \right) = \underline{k}_{dec}^t \underline{g}(s)$$

where, $C_p(\cdot)$ denotes p -th compound, k_{dec}^t the exterior product of the rows of $[I_p, K_{dec}]$ and $\underline{g}(s)$ the exterior product of the columns of $\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}^t$. We may write $\underline{g}(s) = P \cdot \underline{e}_n(s)$, $\underline{e}_n(s) = [s^n, \dots, s, 1]$, $\underline{e}_n(s) = [s^n, \dots, s, 1]$ is the right *Grassmann-representative*, the right Plücker matrix and they are complete invariants for the $S(A, B, C)$ system (Karcianas and Giannakopoulos, 1984). The vector k_{dec}^t has zeros at certain positions due to the block diagonal structure. If now we cut from $\underline{g}(s)$ those entries corresponding to the fixed zero locations of k_{dec}^t we get a new vector $\underline{g}'(s) = P' \cdot \underline{e}_n(s)$ called the *decentralised Grassmann representative* and P' the *decentralised*

Plücker matrix (Karcaniyas et al., 1988). $g'(s)$ and P' are invariants under the given decentralisation scheme and their significance in the characterisation of fixed modes (Anderson and Clements, 1981) and solvability of decentralised assignment is summarised by the following result (Karcaniyas et al., 1988):

Theorem 1. For a given decentralisation scheme defined on $S(A, B, C)$ by K_{dec} , the following hold true:

- (i) A necessary and sufficient condition for $\lambda \in \mathbb{C}$ to be a decentralised fixed mode is that λ is a zero of $\underline{g}'(s)$, i.e. $\underline{g}'(\lambda) = 0$.
- (ii) A necessary condition for decentralised assignment is that $\text{rank}(P') = n + 1$.
- (iii) If $\text{rank}(P') = n + 1$, then the system $S(A, B, C)$ has no decentralised fixed modes under K_{dec} schemes. □

Remark 2. The presence of fixed modes is the result of the structure of the decentralisation scheme and it is independent of dynamics of decentralisation scheme. The test for the decentralised constant compensation case is thus valid for all decentralised dynamic compensations having a given decentralisation characteristic. □

Such results may be extended to dynamic compensation schemes and can be used to test the properties of the given decentralisation scheme. However, they provide no insight in how we can modify a decentralisation scheme, since the links to the state space parameters are not explicit. Such links are established by examining the differential of the pole assignment map, which leads to the decentralised Markov parameters (Leventides and Karcaniyas, 1998a).

5. GENERIC SOLVABILITY CONDITIONS AND PARAMETERISATION ISSUES

The existing results on the generic solvability of the decentralised pole assignment may be used to provide parameterisations of the decentralised structures with good pole assignment potential. The results below indicate desirable partitions $\sum_{i=1}^k p_i = p$, $\sum_{i=1}^k m_i = m$ for the p -inputs, m -outputs and n states generic system. The fundamental question is under what conditions the Decentralized DAP can be solved for any or almost any $p(s)$ of degree n . Some of these results are stated below:

Theorem 3. (Wang (1994)). The condition $\sum_{i=1}^k m_i p_i \geq n$ implies generic pole assignability, when either the number of all inputs, or the number of all outputs are equal. □

More general sufficient conditions avoiding the equality of dimensions, of input-output channels have been also derived and are summarised below.

Theorem 4. (Leventides and Karcaniyas (1995)). **i)** A necessary condition for arbitrary, pole placement via a k -channel static decentralised output feedback controller is:

$$\sum_{i=1}^k m_i p_i \geq n, \text{rank}(P') = n + 1$$

- ii)** A sufficient condition for arbitrary, pole placement is $\min(m_i)p > n, \text{rank}(P') = n + 1$ □

Theorem 4 gives a simpler test when compared to testing the results in (Wang, 1994) and it is directly related to the decentralisation parameters (m_i, p_i) . A similar family of results is based on the notion of partition of integers and on the evaluation of height function.

Definition 5. A binary partition t of the number n of length k is a sequence of non-negative integers $t(1), t(2), \dots, t(k)$, such that $n = t(1) + t(2) + \dots + t(k)$ and for every j , there is at most one 1 in all the j -th digits of the binary representations of $t(1), t(2), \dots, t(k)$. □

Theorem 6. A sufficient condition for arbitrary pole placement by a real static decentralised output feedback for a proper system with n states and $k(m_i, p_i)$ channels, is that there exists a k -length binary partition of n say $\{t(1), t(2), \dots, t(k)\}$ such that $t(i) \leq h(p_i, m_i)$ for every $i = 1, \dots, k$. □

The computation of $h(p_i, m_i)$ may be achieved as shown in (Leventides and Karcaniyas, 1995):

Lemma 7. If $1 < p \leq m$ and ν is such that $2^\nu \leq m + p - 1 < 2^{\nu+1}$, then the height function $h(p, m)$ is given as

$$h(p, m) = \begin{cases} 2^{\nu+1} - 2 & \text{if } p = 2, 3 \text{ and } m + p = 2^\nu + 1 \\ 2^{\nu+1} - 1 & \text{otherwise} \end{cases}$$
□

6. EXACT SOLVABILITY CONDITIONS AND THE DECENTRALISED MARKOV PARAMETERS

The development of links between structural invariants, the properties of the pole assignment, and the evaluation of decentralisation is through the notion of the decentralised Markov parameters (Leventides and Karcaniyas, 1998a) linked to the properties of the differential of the pole assignment map.

Decentralised Markov Parameters: We consider the decentralised pole placement map X^d that can be factorised as

$$X^d : \mathfrak{S}^{\sum m_i p_i} \xrightarrow{E} \mathfrak{S}^{m \times p} \xrightarrow{X^c} \mathfrak{S}^n$$

where X^c is the centralised pole placement map and $E(\text{row}(K_1), \dots, \text{row}(K_k)) = \text{bl.diag}(K_1, \dots, K_k)$. The calculation of the differential of X^d involves the decomposition $X^d = X^c \circ E$ and this implies

$$D(X^d)_k = D(X^c)_{E(k)} \circ D(E)_k$$

The sets Ω and Ω^d , specify the lower indices of the entries of centralised and decentralised feedback matrices respectively. We consider a basis for $T(\mathfrak{S}^{mp})_k$ the set of all $(\partial/\partial k_a)$, $a \in \Omega$ and for $T(\mathfrak{S}^{\sum m_i p_i})_k$ the set of all $(\partial/\partial k_b)$, $b \in \Omega_d$, where all the indices are lexicographically ordered. Using these bases we have a representation for the differentials (Leventides and Karcaniyas, 1998a):

Theorem 8. For a given decentralised feedback gain K and a system $S(A, B, C)$, a matrix representation of the differential of the decentralised pole placement map $D(X^d)_k$,

with respect to the bases previously defined is denoted by $R(X^d)_k$ and it is given by

$$R(X^d)_k = R(X^c)_{E(k)} \circ R(E)_k$$

$$R(X^c)_k = Q^t [\text{col}CB, \text{col}CHB, \dots, \text{col}CH^{n-1}B]^t$$

which is an $n \times mp$ matrix, col maps an $m \times p$ matrix to $mp \times 1$ matrix formed by superimposing its columns, $H = A + BKC$ and Q is associated with the coefficients of the closed loop polynomial and $R(E)$ is an $mp \times \sum m_i p_i$ matrix such that for $\forall \alpha, \beta \in \Omega$

$$Q = \begin{bmatrix} 1 & p_n & \dots & p_2 \\ 0 & 1 & \dots & p_3 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & 1 \end{bmatrix} R(E)_{\alpha\beta} = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{otherwise} \end{cases}$$

□

Remark 9. Note that $R(X^d)_k$ is obtained by $R(X^c)_{E(k)}$ by keeping only those rows of $R(X^c)_{E(k)}$ which correspond to Ω_d set of indices. The column representation of the Markov parameters of $S(A, B, C)$ is obtained by computing the differential of X^c at $k = 0$. Similarly, we may define the decentralised Markov parameters by using the differential of X^d at $K_{dec} = 0$. In fact, if $H_i = CA^i B$, then for $K_{dec} = 0$ we have

$$R(X^d)_0 = Q^t [\text{col}\hat{H}_0, \text{col}\hat{H}_1, \dots, \text{col}\hat{H}_{n-1}]^t = Q^t M_d$$

where $\text{col}\hat{H}_i$ denotes the reduced column obtained from $\text{col}H_i$ after eliminating all the entries that do not correspond to the indices Ω_d .

□

The matrix M_d is known as decentralised Markov matrix and its properties are summarised below (Karcianas et al., 1988):

Theorem 10. For a system $S(A, B, C)$ and a decentralisation scheme defined by Ω_d set the following properties hold true:

- i) $\text{rank}(M_d) \leq \dim(\text{Im } X^d) \leq n$.
- ii) For a generic system $S(A, B, C)$ such that $\sum m_i p_i \geq n$, then $\text{rank}(M_d) = \dim(\text{Im } X^d) = n$. Furthermore, under these conditions the arbitrary pole assignment via complex output feedback is solvable.

□

The link between the decentralised Plücker and the decentralised Markov matrices is expressed as follows (Leventides and Karcianas, 1998a):

Proposition 11. For the system described by $S(A, B, C)$, let $P_s = [\underline{e}_1, P_s']$ be its Plücker matrix. We may compute $M = [\text{col}CB, \dots, \text{col}CA^{n-1}B]$ from P_s' as follows:

- (a) Select the rows of P_s' that correspond to the set of indices $(1, 2, \dots, i-1, i+1, \dots, p, p+j)$ for $i : 1 \leq i \leq p$ and $j : 1 \leq j \leq m$. This set of rows is multiplied by $(-1)^{i-1}$.
- (b) Repeat step (a) for all i and j and form the $pm \times n$ matrix, which is then post-multiplied by Q^{-1} , where Q is the $n \times n$ matrix defined above and which corresponds to the *open loop pole polynomial*.

□

The above indicates that the Markov matrix M is modulo $Q \in \mathbb{R}^{n \times n}$, $\det(Q) \neq 0$, a sub-matrix of P_s and thus the full rank of M implies full rank of P_s , but not vice versa. This explicit relationship can thus be used as an indicator for designing systems (by selection of C , or B) such that the corresponding Plücker matrix has full rank.

Remark 12. If $M = [\text{col}CB, \dots, \text{col}CA^{n-1}B]$ has full rank, then the Plücker matrix has also full rank. Furthermore, the matrix $M_d Q$ is a full row submatrix of the decentralised Plücker matrix P_s .

□

Corollary 13. If $\sum m_i p_i \leq n$ and $\text{rank}(M_d) = n$, then the system has no fixed modes. The latter condition is necessary for solvability of the Static Decentralised Output Feedback problem.

□

7. A STRUCTURAL PROCEDURE FOR THE SELECTION OF THE DECENTRALISATION SCHEME

The design of the decentralisation scheme has as starting point the *physical decentralisation set* produced by the analysis in Steps (1) and (2). This set corresponds to pairs of indices i, j , $i \in \{1, 2, \dots, p\}$, $j \in \{1, 2, \dots, m\}$ expressing permissible loop closures from the j -th output to the i -th input. The feasible sets are subsets of the physical decentralisation set with cardinality greater or equal to n . The main steps of the procedure are:

Procedure for selection of decentralisation based on constant output feedback:

Step (1): Define the physical decentralisation set and from this all feasible sets corresponding to all possible cardinalities of the partition k .

Step (2): Test whether the necessary conditions for centralised output feedback $mp > n$ and $\text{rank}(P) = n + 1$ are satisfied. If yes, then proceed to next step; otherwise, alternative schemes based on dynamic compensation have to be used.

Step (3): For every element of the feasible set produced in (1) and for all possible cardinality partitions $\{(m_i, p_i), i = 1, \dots, k\}$ select those partitions for which the condition $\sum_{i=1}^k m_i p_i > n$ is satisfied.

Step (4): Compute the Markov parameters set H_0, H_1, \dots, H_{n-1} and the corresponding matrix

$$M = [\text{col}H_0, \text{col}H_1, \dots, \text{col}H_{n-1}]$$

If $\text{rank}(M) = n$ then proceed to next step; otherwise, alternative schemes based on dynamic compensation have to be used, or work with the corresponding Plücker matrices.

Step (5): For every partition produced by (3) define the corresponding decentralised Markov matrix $M_d = [\text{col}\hat{H}_0, \text{col}\hat{H}_1, \dots, \text{col}\hat{H}_{n-1}]$ and test $\text{rank}(M_d) = n$. If the condition is not satisfied, then test the condition on the decentralised Plücker matrix; otherwise, use dynamic schemes.

Step (6): For every element of the feasible set coming from the previous step, use sufficient conditions for generic

assignability. If such conditions are not valid, then use alternative means based on dynamics.

The procedure produces schemes with no fixed modes and which satisfy the necessary conditions. If the generic sufficient conditions are not satisfied, then tests based on non-generic cases, or dynamic schemes have to be used. The selection of the decentralisation involves the selection of full rank Markov matrices for both centralised and decentralised as elaborated in (Karcianas and Leventides, 2005).

8. CONCLUSION

A structural approach for designing the decentralisation scheme for a system has been introduced using as criteria the conditions for solvability of the constant output feedback problem. The overall philosophy has been based on satisfying the necessary conditions which guarantee absence of fixed modes and existence of at least complex solutions for the generic solvability of the decentralised pole assignment. Further screening of candidate partitions may be achieved using sufficient conditions guaranteeing solvability of the generic problem. Results linking the parameterisation of decentralisation schemes with the existence of degenerate solutions of the dynamic assignment problem (Karcianas et al., 2014) may be also used to extend the set of criteria on which selection of decentralisation is made. The advantage of the latter schemes is that they also provide means for computing the decentralised solutions, using the notion of Global Linearization Leventides and Karcianas (2007). The decentralised Markov matrices provides an explicit link between the necessary conditions and the state space parameters and this enables system redesign by modification of matrices. The conditions characterising the desirable partitions are a mixed set of equalities and inequalities and their solution may lead to parameterisations of desirable sets and it is a topic under investigation. Additional structural results have to be integrated that will enable the emergence of a powerful methodology for selection of the decentralisation.

ACKNOWLEDGEMENTS

The last author was supported by a grant of the Greek State Scholarship Foundation - I.K.Y. which support is gratefully acknowledged.

REFERENCES

- Anderson, B. and Clements, D. (1981). Algebraic characterisation of fixed modes in decentralised control. *Automatica*, 17, 703–712.
- Grosdidier, P. and Morari, M. (1986). Interaction measures for systems under decentralised control. *Automatica*, 22, 309–319.
- Karcianas, N. (1996). Control problems in global process instrumentation: A structural approach. *Comp. and Chem. Engineering, Part B*, 1101–1106.
- Karcianas, N. and Giannakopoulos, C. (1984). Grassmann Invariants, Almost Zeros and the Determinantal Pole-zero Assignment Problems of Linear Systems. *Int. Journal of Control*, 40, 673–698.
- Karcianas, N., Laios, B., and Giannakopoulos, C. (1988). The Decentralised Determinantal Assignment Problem: Fixed and Almost Fixed Modes and Zeros. *International Journal of Control*, 48, 129–147.
- Karcianas, N. and Leventides, J. (2005). Design of decentralised control schemes: An algebraic approach. In *16th IFAC World Congress*. Prague.
- Karcianas, N., Leventides, J., and Milonidis, E. (1997). An integrated framework for input, output and control structure selection: Advanced control diagnostics. In *5th IEEE Medit. Conference*. Cyprus.
- Karcianas, N., Meintanis, I., and Leventides, J. (2014). System Degeneracy and Decentralised Output Feedback: Parametrization of Families of Decentralised Degenerate Gains. preprint S&C/CRC/12-2014/NK-IM-JL 2/2, City University London.
- Karcianas, N. and Vafiadis, K. (2002). Derivation of effective transfer function models by input-output variables selection. *KYBERNETICA*, 38, 657–683.
- Lampakis, E. and Karcianas, N. (1995). Parametrisation issues of the general decentralised stabilization problem. In *ECC '95 Conf. in Rome*, volume 4, 2928–2933.
- Leventides, J. and Karcianas, N. (1992). The pole assignment map, its properties and relationships to systems invariants. *IEEE Trans. Aut. Control*, 38, 1266–1270.
- Leventides, J. and Karcianas, N. (1995). Global Asymptotic Linearisation of the Pole Placement Map: A Closed Form Solution of the Output Feedback Problem. *Automatica*, 31, 1303–1309.
- Leventides, J. and Karcianas, N. (1996). Genericity, generic system properties and generic values of invariants. SESDIP Report SDCU058, Control Engineering Centre, City University.
- Leventides, J. and Karcianas, N. (1998a). The decentralised markov parameters and the selection of control structures. *Int. Journal Control*, 70(70), 815–830.
- Leventides, J. and Karcianas, N. (1998b). Dynamic Pole Assignment Using Global Blow-up Linearisation: Low Complexity Solution. *Journal of Optimization Theory and Applications*, 96, 57–86.
- Leventides, J. and Karcianas, N. (2007). Decentralised dynamic pole assignment with low order compensators. *IMA Journal of Mathematical Control Theory and Information*, 24, 395–410.
- Manousiouthakis, V., Savage, R., and Arkun, Y. (1986). Synthesis of decentralised process control structures using the concept of block relative gain. *AIChE J.*, 32(6), 991.
- Morari, M. and Stephanopoulos, G. (1980). Studies in the synthesis of control structures for chemical processes: Part ii: Structural aspects and the synthesis of alternative feasible control schemes. *AIChE Journal*, 26, 232–246.
- Siljak, D.D. (1991). *Decentralised Control of Complex Systems*. Academic Press, Boston, MA.
- Wang, S. and Davison (1973). On the stabilisation of decentralised control systems. *IEEE Trans. Aut Control*, 18, 473–479.
- Wang, X. (1994). Decentralised Pole Assignment and product Grassmannians. *SIAM J. Control Optim.*, 32, 855–875.