Uncertainty evaluation of trigonometric method for vertical angle calibration of the total station instrument

Lauryna Šiaudinytė [a,*], Kenneth Thomas Victor Grattan [a,b]

[a] Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius, Lithuania
[b] City Graduate School, City University London, Northampton Square, London EC1V0HB, United Kingdom

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A B S T R A C T

The total station is an instrument, widely used in civil and environmental engineering, for flat and vertical angle as well as distance measurements. Typically, routine calibration of such an instrument is obligatory, and depending on the local regulations, is performed typically annually or bi-annually. Analysis of previous research shows that undertaking flat angle calibration under laboratory conditions is more common than calibration of vertical angle measuring systems. This paper deals with a trigonometric vertical angle calibration method and research into determining the main sources of uncertainty is described in the paper. The principle of the method is explained, as well as the uncertainty evaluation based on GUM and the advantages as well as the weaknesses of the setup are discussed.

1. Introduction

Rotary encoders, total stations, laser trackers and other optical-electronic digital instruments are widely used in the fields of robotics, surveying, machine and civil engineering. Circular scales and angular transducers for angle determination in horizontal and vertical planes are commonly the critical components of these optical-electronic geodetic measuring instruments. The accuracy of such instruments depends then directly on the accuracy of the embedded angle measuring systems employed. Measurements using such equipment are specific and require particular arrangements for instrument calibration and especially for measurements in the vertical plane.

The main instruments used in geodetic measurements are total stations, often called tacheometers. Tacheometry (*gr. tacheos* – fast, *metreo* – measure) is the geodetic measurement method for the determination of the Earth's surface point position in three coordinates (x,y,z). During the measurement, readings of both horizontal and vertical angles are recorded to develop a relationship between the points. Total stations (TS) are very useful in survey and civil engineering applications for the angle, distance as well as height difference measurements.

In essence, a total station consists of a theodolite and an electronic distance measurement device (EDM) [2]. Typically, there are two angle measuring systems for both horizontal and vertical angle measurements embedded in a total station. The rotary encoder is the most important component of such angle measuring systems and rotating the telescope of the TS around the vertical axis allows the measurement of the horizontal angle, while rotating it around the horizontal axis measures vertical angle [10,14]. An automatic compensation system for the elimination of temperature deviations as well as terrestrial refraction along with the microprocessor are embedded into a total station. A biaxial electronic compensator integrated in a total station is included to reduce measurement errors. A semiconductor diode (GaAs – gallium and arsenic) is used as the light source in the instrument [1], which is modulated and used for phase difference measurements.

Angles are recorded digitally and the data may be previewed on the screen using image processing (if the total station has an image capture function, the images can be seen on the screen) and pattern recognition methods (when labels are assign to each measured point).

As previous research shows, most of the methods in angle metrology deal with the flat angle calibration [1]. However, calibration of vertical angle measuring systems under the laboratory conditions requires the use of special arrangements.

An implementation of the method as well as an important technique used for vertical angle measuring system calibration under laboratory conditions is analyzed in some detail in this paper.

2. Related research

Although total stations are designed to perform measurements under field conditions, assessments of both horizontal and vertical angle measuring systems are often performed under the laboratory conditions as a means of calibration.

The Theodolite Test Machine (TPM-2: Theodolit-Prufmaschine 2) developed by Leica Geosystems AG is intended for horizontal and vertical angle calibration of theodolites and total stations. During calibration, the readings of the total station under test are compared to the outputs of vertical and horizontal reference encoders embedded into TPM-2 machine. Standard deviations of 0.058″ for the horizontal angles and 0.091″ for the vertical angles can be achieved using this machine [8]. This machine is considered to be an industry standard and is appreciated for its accuracy, although are frequently not available to smaller laboratories due to the high costs involved.

A further setup for vertical angle calibration of total stations has been proposed by colleagues at KRISS (Korea Research Institute of Standards and Science) [12]. A special piece of apparatus with a tribrach and total station support is mounted on Moore's Special index. The Moore 1440 Precision Index is a serrated-tooth circle divider. During operation, the table is displaced axially to disengage the teeth and radially to the desired angle. The expanded uncertainty of this instrument is $U_{95\%} = 0.1''$ ($k = 2$) at any of the 1440 indexed positions. It is often used in laboratories as a flat angle standard. The total station is fixed to the apparatus in a horizontal position. The mirror is attached to the telescope of a total station, to which an autocollimator is pointed. In this way the setup for flat angle calibration was used to calibrate the vertical angles, with the expanded uncertainty of $U_{95\%} = 0.52''$ ($k = 2$) [12,13].

The trigonometric method for vertical angle calibration was developed, along with the new vertical circle comparator at ESRF, France. For the calibration of the robotic total station, the distance between the vertical circle comparator and the total station can be minimized to 2.5 m which limits vertical angle measurement range to ±23.75°. The vertical angle is measured with respect to the reflector sliding along the vertical circle comparator in the vertical plane. The expanded uncertainty of vertical angle measuring system calibration using this vertical circle comparator is given as $U = 1.65''$ ($k = 2$) [9].

The method proposed in [11] is based on the trigonometric approach of vertical angle determination, achieved by measuring two distances and this can be expressed as follows:

$$\cos(\theta) = \frac{\Delta d\left(\sqrt{\frac{y_0}{y_1}} - 1\right)}{d_0} \tag{1}$$

where Δd – the object displacement distance, d_0 – the effective distance to the measured object before the displacement y_0 and y_1 are the scale readings before and after the displacement and θ – vertical angle [11].

3. Trigonometric vertical angle calibration method

3.1. The principle and instrumentation

In trigonometric methods, an angle is expressed as a function of distance. The method for calibration of vertical angle measuring systems of geodetic instruments was developed at Institute of Geodesy of Vilnius Gediminas Technical University. This method proposes an arrangement to create the reference angle suitable for vertical angle calibration purposes in the laboratory environment [3,4].

Proposed method is suitable for relatively small angle measurements and is based on trigonometric determination of the reference angle using standard means, such as a laser interferometer and a calibrated graduated 1 meter reference scale. Proposed method is based on a comparison of the angle measured by the total station and the reference angle determined by measuring two distances – the horizontal (the distance between the TS vertical axis and the vertically positioned scale) and the vertical (the distance between the grating lines of the vertically positioned reference linear scale). The principle of the method is shown in Fig. 1.

The reference angle based on measuring the horizontal and the vertical distances can be expressed as:

$$\varphi = \arctan\frac{\Delta h}{l} \tag{2}$$

where Δh – the vertical distance determined between the scale grating; and l – the horizontal distance between the axis of the TS and the reference scale.

Using this method, the reference 1 m graduated scale bar was placed vertically using its original mount for stability and leveling on the carriage. The graduated scale must be perpendicular to the optical axis of the total station at initial position and therefore, it was in this way precisely leveled and aligned.

This method has two different approaches – with displaced reference scale and with stationary reference scale, as discussed below.

First, in the approach with the displaced reference scale, the reference angle (φ') is determined according to the horizontal displacement of the scale and the vertical distances between the scale grating, as shown in (3):

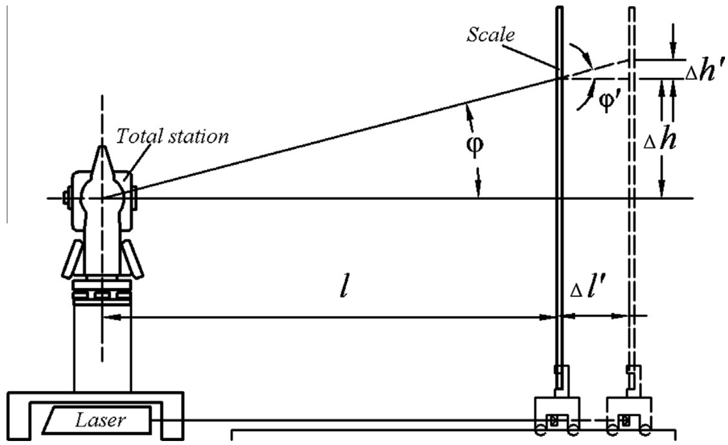

Fig. 1. The principle of the method.

$$\varphi' = \arctan\left(\frac{\Delta h'}{\Delta l'}\right) \tag{3}$$

where $\Delta h'$ – the known, calibrated distance between the grating of the scale; and $\Delta l'$ – the distance measured by the laser interferometer between two scale positions. After pointing TS to the line of the reference scale, the scale is displaced until another line matches the reticle central line of the TS. In this approach, the scale displacement distance ($\Delta l'$) is measured by an interferometer and the vertical distance ($\Delta h'$) is determined from the reference scale.

In the alternative approach with the stationary reference scale, the precision reflector was mounted on the scale for determination of horizontal distance between the TS and the reference scale. Since it was not possible to measure directly to the grating surface, the prism constant and the scale depth were taken into consideration and measured separately.

The realization of proposed method was performed at the Korea Research Institute of Standards and Science (KRISS). In collaboration with the Division of Physical Metrology at KRISS, the instrumentation for this experiment was selected to perform measurements at the Center for Length Laboratory for distance measurements.

The Total Station Leica TC 2003, having a focusing range of 1.6 m was used as the instrument under calibration. For the experiment the calibrated reference 1 m H shape invar scale (Gaertner Scientific Corporation Chicago. No. 244 A. U) with a 1 mm grating pitch was chosen with its original mount used for leveling. After leveling the total station, the position of the scale was double-checked and readjusted in order to make the scale grating lines parallel to the horizontal line of the total station reticle.

For the horizontal displacement measurements that were made, a Hewlett Packard Laser System 5519 A with helium neon laser was chosen because it is used as a length standard at KRISS. The system involves a Zeeman-split two-frequency laser output. With the beam diameter of 6 mm, this interferometer can perform 80 m length range distance measurements. The alignment of the devices was adjusted by using a cross-line laser level, which produces beams in two perpendicular planes. In the approach to the stationary reference scale, a Mitutoyo micrometer was used to measure the scale depth, to the surface of the grating.

3.2. Uncertainty evaluation

In order to create the uncertainty evaluation required, it is very important to analyze all the key components influencing measurement accuracy. Error sources are analyzed and the measurement accuracy was calculated and reported in the form of combined and expanded uncertainties as it is required in GUM (Guide to the expression of uncertainty in measurement). During the uncertainty evaluation both type A and type B evaluation methods were used to obtain the results by applying statistical analysis of series of observations as well as other means (i.e. calibration certificates) for the evaluation [6].

Therefore, the correction value (B) can be expressed as follows:

$$B = \arctan\left(\frac{\Delta h'}{\Delta l'}\right) - \theta_{TS} \tag{4}$$

where θ_{TS} – the angle measured by the total station, $\Delta h'$ – the known calibrated distance between the grating of the scale; and $\Delta l'$ – the distance measured by the laser interferometer between two scale positions.

The combined uncertainty of the correction value may be expressed as follows:

$$u_c^2(B) = c_{\Delta h'}^2 u^2(\Delta h') + c_{\Delta l'}^2 u^2(\Delta l') + c_{\theta_{TS}}^2 u^2(\theta_{TS}) \tag{5}$$

where c – the different sensitivity coefficients above; u – the standard uncertainties arising from the vertical distance ($\Delta h'$), the horizontal distance ($\Delta l'$) and the angle measured with the TS (θ_{TS}). The uncertainty due to vertical distance is dependent on the accuracy, tilt, thermal expansion and compression of the reference scale and thus may be expressed as:

$$u_{\Delta h'}^2 = c_{\Delta h'}^2 \left\{ \begin{array}{c} u^2(\Delta h'_{Scal}) + u^2(\Delta h'_{tilt}) + u^2(\Delta h'_{therm}) + \\ + u^2(\Delta h'_{comp}) + u^2(\Delta h'_{point}) \end{array} \right\} \tag{6}$$

where $u^2(\Delta h'_{Scal})$ – the standard uncertainty due to the reference scale; $u^2(\Delta h'_{tilt})$ – the standard uncertainty due to the tilt of the scale; $u^2(\Delta h'_{therm})$ – the standard uncertainty due to thermal expansion; $u^2(\Delta h'_{comp})$ – the standard uncertainty due to compression of the scale; and $u^2(\Delta h'_{point})$ – the standard uncertainty due to pointing to the center of the scale line. The uncertainty due to the pointing of the instrument must be evaluated with reference to the different widths of the cross line of the telescope and the reference scale.

The uncertainty due to thermal expansion can be evaluated with reference to the thermal expansion coefficient. The uncertainty due to the thermal expansion of the scale $u(\Delta h'_{therm})$ can be determined for every measured pitch and for the total measured length of the scale (Δh_{therm}) as a result of linear thermal expansion:

$$\Delta h_{therm} = \alpha \Delta T h \tag{7}$$

where α – the thermal expansion coefficient of invar ($\alpha = 1 \cdot 10^{-6}$); ΔT – the temperature deviation from 20 °C ($\Delta T = 0.5$ °C); h – the initial length of the scale at temperature of 20 °C ($h = 1.0$ m). Therefore, $u(\Delta h'_{therm})$ – the uncertainty due to the thermal expansion of the scale was determined as $\Delta h'_{therm} = 0.5$ µm/m over the 1 meter length of the scale.

The reference scale is 1 m long and is used in the vertical orientation and therefore, the correction for the compression due to the effect of gravity has to be evaluated [7]. The expression for the correction due to the compression in this case can be assumed to be the uncertainty due to compression of the scale, arising from the standard uncertainties of the expression shown in Eq. (8)as the specific components are unknown. Therefore, the uncertainty due to compression of the scale can be evaluated, as follows (8):

$$u(\Delta h_{comp}) = 500 \frac{\rho g}{E} (L)^2 \tag{8}$$

where ρ – the density of the gauge block material; E – Young's modulus of elasticity of the gauge block material; g – the acceleration of gravity; and L – the length of the gauge block. The uncertainty due to the compression of the reference scale was evaluated as $u(\Delta h'_{comp}) = 7.1$ nm.

Since the reference scale was thus calibrated, its uncertainty can be evaluated using the type B evaluation method. The uncertainty due to the effect of pointing $u(\Delta h'_{point})$ was analyzed in greater depth. Since the widths of graduation lines of the reference scale and the reticle crosshair of TS telescope do not match exactly, they have to be measured separately [5]. As is shown in Fig. 2, the center of the TS reticle was pointed along the line center of the reference scale. The vertical angle was measured between the two line centers of the reference scale with the vertical distance (Δh) between them. The zoom inset shown in Fig. 2 illustrates the fact that width of the reference scale line (W_S) and the cross line of the TS reticle (W_{TS}) differ. Therefore, this uncertainty has to be evaluated.

The uncertainty due to the pointing to the line center of the reference scale $u(h_{point})$ varies in a way that is dependent on the distance between the scale and the telescope. It can be expressed below, this being based on a triangular distribution as follows:

$$u(\Delta h'_{point}) = \frac{W_S - W_{TS}}{\sqrt{6}} \tag{9}$$

where $u(\Delta h'_{point})$ – the uncertainty due to pointing to the line center of the reference scale; W_S – the line half-width of the reference scale in the image plane, this depending on the distance between the device and the reference scale; and W_{TS} – the constant half-width of the reticle line of the TS telescope.

The uncertainty due to horizontal distance in the approach of the displaced target technique depends on the laser interferometer measurements and this can be expressed as follows:

$$u^2(\Delta l') = c_l^2 \left\{ u^2(\Delta l'_{LIcal}) + u^2\left(\Delta l'_{LIrep}\right) + u^2(\Delta l'_{LIres}) \right\} \tag{10}$$

where $u^2(\Delta l'_{LIcal})$ – the standard uncertainty due to the laser interferometer; $u^2(\Delta l'_{LIrep})$ – the standard uncertainty due to repeatability of the laser interferometer; and $u^2(\Delta l'_{LIres})$ – the standard uncertainty due to limited display resolution of the laser interferometer.

The uncertainty due to the total station angle measurements (11) contains the effect of the uncertainty due to the limited display resolution of the device $u(\theta_{TSres})$ and the uncertainty due to the repeatability $u(\theta_{TSrep})$:

$$u^2(\theta_{TS}) = c_{\theta_{TS}}^2 \{ u^2(\theta_{TSres}) + u^2(\theta_{TSrep}) \} \tag{11}$$

The uncertainties due to the limited display resolution of the devices can be evaluated as shown below in (12):

$$u(\theta_{TSres}) = \frac{R}{2\sqrt{3}} \tag{12}$$

where R – the display resolution of the total station. Eq. (12) can be used for the uncertainty, arising due to the limited resolution determination of any instrumentation used for measurement. The uncertainties due to the repeatability can be evaluated by determining the standard uncertainties of the measurement sets and then calculating the pooled standard deviation. The uncertainty due to the tilt of the reference scale may be expressed as:

$$u(h_{tilt}) = \frac{W_{LB0.5}}{\sqrt{3}} \tag{13}$$

where $W_{LB0.5}$ – the half width of the laser beam reflection.

In the approach where the reference scale remains stationary, the uncertainty due to the vertical distance has the same components. However, the uncertainty due to the horizontal distance (l) can be expressed as follows:

$$u^2(l) = c_l^2 \{ u^2(l_{TS}) + u^2(l_p) + u^2(l_M) + u^2(l_S) \} \tag{14}$$

where $u^2(l_{TS})$ – the standard uncertainty due to the TS measurements of the distance between the TS and the prism; $u^2(l_p)$ – the standard uncertainty due to the prism measurements of the prism constant determination; $u^2(l_M)$ – the standard uncertainty due to the measurements to the mirror for prism constant determination; and $u^2(l_S)$ – the standard uncertainty due to the reference scale depth measurements.

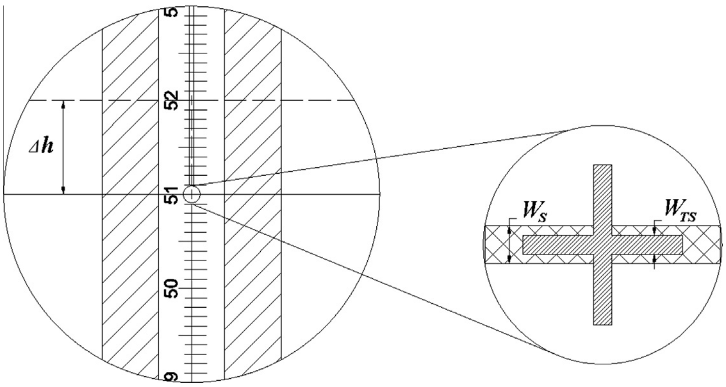

Fig. 2. General and zoomed – in views of the TS telescope pointed to the reference scale.

To evaluate the uncertainty due to the TS distance measurements $u(l_{TS})$, the uncertainty due to the limited display resolution of the distance measurement readings $u(l_{TSres})$ and uncertainty due to the TS distance measurement repeatability $u(l_{TSrep})$ have to be taken into consideration. This is done for three separate distance measurements as well as for the micrometer used for the depth measurements parameters. Therefore, the uncertainty due to the horizontal distance measurements with the TS can be determined as shown in Eq. (15):

$$u^2(l) = c_l^2 \left\{ \begin{array}{c} u^2(l_{TSres1}) + u^2(l_{TSrep1}) + u^2(l_{TSres2}) + \\ +u^2(l_{TSrep2}) + u^2(l_{TSres3}) + u^2(l_{TSrep3}) + \\ +u^2(l_{\mu mcal}) + u^2(l_{\mu mrep}) + u^2(l_{\mu mres}) \end{array} \right\} \tag{15}$$

where $u(l_{Tsres1})$, $u(l_{Tsres2})$, $u(l_{Tsres3})$ – the standard uncertainties due to the limited display resolution of the TS for distance measurement; $u(l_{Tsrep1})$ – the standard uncertainty due to the repeatability of the distance measurements between the TS and the prism mounted on the scale; $u(l_{Tsrep2})$ – the standard uncertainty due to the repeatability of the distance measurements between the TS and the prism mounted on the mirror (the prism constant determination); $u(l_{Tsrep3})$ – the standard uncertainty due to the repeatability of the distance measurements between the TS and the mirror (the prism constant determination); $u(l_{\mu mcal})$ – the standard uncertainty due to the depth micrometer; $u(l_{\mu mrep})$ – the standard uncertainty due to repeatability of the depth micrometer; $u(l_{\mu mres})$ – the standard uncertainty due to the limited display resolution of the depth micrometer.

According to the expression for the measurement function, in the approach using the displaced scale the sensitivity coefficients for the uncertainty due to the horizontal distance measurements can be expressed as:

$$c_{li} = \left(\frac{\partial b}{\partial \Delta l_i}\right) = -\frac{\Delta h}{\Delta l_i^2 + \Delta h^2} \tag{16}$$

where Δl_i – the average of horizontal distances measured by interferometer, and Δh – the vertical distance between the two lines of the reference scale.

Sensitivity coefficients for the uncertainty due to vertical distance determination can be expressed:

$$c_{hi} = \left(\frac{\partial b}{\partial \Delta h}\right) = \frac{\Delta l_i}{\Delta l_i^2 + \Delta h^2} \tag{17}$$

The sensitivity coefficient for angle measurements by the TS can be expressed as:

$$c_{\theta TS} = \left(\frac{\partial b}{\partial \theta_{TS}}\right) = -1 \tag{18}$$

The uncertainty evaluation was performed according to industry standards and based on the principles provided in JCGM 100:2008 [6].

4. Experimental results

The correction value was evaluated taking into consideration all the instrumentation used for the measurements described above and as shown in (5). The uncertainty components obtained are given in Tables 1 and 2.

In the approach undertaken with the moving scale, eight displacements of the reference scale were measured by using the laser interferometer. Therefore, there were eight sensitivity coefficients which were determined as well as eight combined uncertainties expressed for the correction values. Since the sensitivity coefficients vary, an expression is then given in Table 1.

As can be seen from Table 1, in the displaced scale approach, the repeatability of the laser interferometer influences the horizontal distance measurements most significantly. For the vertical distance measurements, the tilt of the reference scale is crucial. Overall, taking into account the variable sensitivity coefficients, the combined uncertainty of the correction value is determined $u_c(B_i) = 0.294''$ and the expanded uncertainty of this approach is given by $U_{95\%} = 0.59''$ ($k = 2$).

The uncertainty due to the tilt of the scale also plays a major role in the stationary scale approach. It is also clear from Table 2 that repeatability of both the total station distance and the angle measurements has a significant impact on the measurement results. In the stationary scale approach the uncertainty of the correction value was determined to be $u_c(B) = 0.10''$ and the expanded uncertainty of this setup $U_{95\%} = 0.24''$ ($k = 2.447$).

Table 1
Uncertainty budget for the calibration method with the displaced reference scale.

Source of uncertainty	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Uncertainty contribution $\|c_i\| \cdot u(x_i)$	Probability distribution
Combined uncertainty $u(\Delta l')$	$1.102 \cdot 10^{-4}$ m	$-\frac{\Delta h}{\Delta l_i^2+\Delta h^2}$	$\left(-\frac{\Delta h}{\Delta l_i^2+\Delta h^2}\right) \cdot 1.102 \cdot 10^{-4}$	t-(Student's)
Uncertainty due to the laser interferometer $u(\Delta l'_{LIcal})$	$1.0 \cdot 10^{-6}$ m			Normal
Uncertainty due to repeatability of the laser interferometer $u(\Delta l'_{LIrep})$	$1.1 \cdot 10^{-4}$ m			t-(Student's)
Uncertainty due to limited display resolution of the laser interferometer $u(\Delta l'_{LIres})$	$2.89 \cdot 10^{-5}$ m			Rectangular
Combined uncertainty $u(\Delta h')$	$4.398 \cdot 10^{-4}$ m	$\frac{\Delta l_i}{\Delta l_i^2+\Delta h^2}$	$\left(\frac{\Delta l_i}{\Delta l_i^2+\Delta h^2}\right) \cdot 4.398 \cdot 10^{-4}$	Rectangular
Uncertainty due to reference scale $u(\Delta h'_{Scal})$	$7.7 \cdot 10^{-5}$ m			Normal
Uncertainty due to thermal expansion of the reference scale $u(\Delta h'_{Stherm})$	$5.0 \cdot 10^{-7}$ m			Rectangular
Uncertainty due to compression of the reference scale $u(\Delta h'_{Scomp})$	$7.1 \cdot 10^{-9}$ m			Rectangular
Uncertainty due to pointing $u(\Delta h'_{point})$	$5.5 \cdot 10^{-7}$ m			Triangle
Uncertainty due to tilt of the scale $u(\Delta h'_{tilt})$	$4.33 \cdot 10^{-4}$ m			Rectangular
Combined uncertainty $u(\theta_{TS})$	0.294″	-1	0.294″	t-(Student's)
Uncertainty due to limited display resolution of angle readings of the TS $u(\theta_{TSres})$	0.029″			Rectangular
Uncertainty due to repeatability of TS (angle measurements) $u(\theta_{TSrep})$	0.293″			t-(Student's)
Uncertainty of the correction value $u_c(B)$			0.294″	t-(Student's)
Expanded uncertainty $U_{95\%}$ ($k = 2$)			0.59″	

Table 2
Uncertainty budget for the calibration method with the stationary reference scale.

Source of uncertainty	Standard uncertainty	Sensitivity coefficient	Uncertainty contribution $\|c_i\| \cdot u(x_i)$	Probability distribution
Combined uncertainty $u(\Delta h)$	$4.398 \cdot 10^{-4}$ m	$-1.068 \cdot 10^{-5}$	$2.03 \cdot 10^{-7}$ rad	Rectangular
Uncertainty due to the reference scale $u(\Delta h_{Scal})$	$7.7 \cdot 10^{-5}$ m			Normal
Uncertainty due to thermal expansion of the reference scale $u(\Delta h_{Stherm})$	$5.0 \cdot 10^{-7}$ m			Rectangular
Uncertainty due to compression effect of the reference scale $u(\Delta h_{Scomp})$	$7.1 \cdot 10^{-9}$ m			Rectangular
Uncertainty due to pointing $u(\Delta h_{point})$	$5.5 \cdot 10^{-7}$ m			Triangular
Uncertainty due to the tilt of the scale $u(\Delta h_{tilt})$	$4.3 \cdot 10^{-4}$ m			Rectangular
Combined uncertainty $u(l)$	$6.738 \cdot 10^{-5}$ m	$4.620 \cdot 10^{-4}$	$7.2 \cdot 10^{-10}$ rad	Normal
Uncertainty due to limited display resolution of the TS (distance measurements between TS and the prism) $u(l_{TSres1})$	$2.8 \cdot 10^{-6}$ m			Rectangular
Uncertainty due to repeatability of TS (distance measurements between TS and the prism) $u(l_{TSrep1})$	$5.0 \cdot 10^{-5}$ m			t (Student's)
Uncertainty due to limited display resolution of the TS (distance between TS and the prism measurements – prism constant determination) $u(l_{TSres2})$	$2.8 \cdot 10^{-6}$ m			Rectangular
Uncertainty due to repeatability of TS (distance between TS and the prism measurements – prism constant determination) $u(l_{TSrep2})$	$2.0 \cdot 10^{-5}$ m			t (Student's)
Uncertainty due to limited display resolution of the TS (prism constant determination) $u(l_{TSres3})$	$2.89 \cdot 10^{-6}$ m			Rectangular
Uncertainty due to repeatability of TS (distance between TS and the mirror measurements – prism constant determination) $u(l_{TSrep3})$	$4.0 \cdot 10^{-5}$ m			t (Student's)
Uncertainty due to micrometer $u(l_{\mu mcal})$	$3.0 \cdot 10^{-6}$ m			Normal
Uncertainty due to repeatability of the micrometer $u(l_{\mu mrep})$	$2.25 \cdot 10^{-6}$ m			t-(Student's)
Uncertainty due to limited display resolution of the micrometer $u(l_{\mu mres})$	$8.66 \cdot 10^{-7}$ m			Rectangular
Combined uncertainty $u(\theta_{TS})$	0.099″	−1	0.099″	t-(Student's)
Uncertainty due to limited display resolution of the TS (angle measurements) $u(\theta_{TSres})$	0.029″			Rectangular
Uncertainty due to repeatability of TS (angle measurements) $u(\theta_{TSrep})$	0.095″			t-(Student's)
Uncertainty of the correction value $u_c(B)$			0.10″	t-(Student's)
Expanded uncertainty $U_{95\%}$ ($k = 2.447$)			0.24″	

To sum up, the horizontal distance measurements have the most combined uncertainty components in the method with the stationary reference. However, this provides a smaller level of uncertainty of the correction value. The uncertainties due to the repeatability and the resolution of the total station (TS) have the highest impact on the measurement results, as well as the uncertainty due to the tilt of the reference scale. In the method with the displaced reference scale, the uncertainty due to the horizontal distance measurements is very small compared to the use of an alternative approach. However, the expanded uncertainty was most significantly influenced by the repeatability of the TS measurements, which have a greater standard deviation, this most likely arising due to the motion of the reference scale.

5. Conclusions

A novel trigonometric setup for the calibration of the vertical angle measuring systems has been proposed in the paper. In the work done, it was determined that the expanded uncertainty for the method using the displaced reference scale was $U_{95\%} = 0.59''$ ($k = 2$) and using the stationary reference scale, $U_{95\%} = 0.24''$ ($k = 2.447$). It was determined that the most significant uncertainty sources seen are the repeatability and the resolution of the total station (TS), as well as the tilt of the reference scale. Therefore, this leads to the conclusion that the motion of the scale increases the uncertainty by a factor of 2.5. The angle measurement pitch can be controlled by adjusting the distance between the device and the reference scale. Although, the measurement range is limited to $90° \pm 17°$, which is smaller in comparison with method described in [12], it can be expanded by adapting a longer calibration scale. Moreover, by using proposed method, some very small angles could be measured by controlling both horizontal and vertical distances. However, the smallest measurement pitch mentioned in [12] is dependent on the indexing table used and often limited to 15″ when using Moore 1440 Precision Index.

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