Fuzzy-Logic based Inelastic Displacement Ratios of Degrading RC Structures

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ABSTRACT

The existing classical methods for estimating the inelastic displacement ratios of reinforced concrete (RC) structures subjected to seismic excitation are built upon several assumptions that ignore the effect of uncertainties on the concerning phenomenon. Uncertainty techniques are more appropriate to modeling such phenomenon that inherits impreciseness. This research presents a new method predicting the inelastic displacement ratio of moderately degrading RC structures subjected to earthquake loading using expert systems such as fuzzy logic approach.

A well-defined degrading model was used to conduct the dynamic analyses. A total of 300 earthquake motions recorded on firm sites, including recent ones from Japan and New Zealand, with magnitudes greater than 5 and peak ground acceleration (PGA) values greater than 0.08g, were selected. These earthquake records were applied on five RC columns that were chosen among 255 tested columns based on their beam-column element parameters reported by the Pacific Earthquake Engineering Research Center [1]. A total of 96,000 dynamic analyses were conducted. The results from these analyses were used to develop the fuzzy inelastic displacement ratio model inheriting uncertainties in terms of strength reduction factor (R) and period of vibration (T). The performance evaluation of the new fuzzy logic model and four classical methods were investigated using different independent data sets. As a result, more accurate results were predicted using the new fuzzy logic model.

INTRODUCTION

Lately, the performance based design concept has been more and more integrated into seismic design provisions throughout the world. As the life expectancy of structures

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in seismic areas increases, predicting the seismic behavior of systems at different hazard levels becomes more important. Hence, it is essential to predict the seismic demands as accurately as possible.

The existing classical methods predicting the inelastic displacement ratio of SDOF structures are based on several assumptions. The uncertainties that RC columns inherit by nature are simulated using several assumptions which may filter down the effect of vital uncertainties and, therefore, result in estimating the inelastic displacement ratio of SDOF structures less accurately.

In this research, a new method was developed for predicting the inelastic displacement ratio of seismically excited and moderately degrading SDOF RC structures using a Fuzzy Logic approach. A well-defined energy-based degrading model that takes softening of columns into account was used in the analytical studies. The studies were performed on five tested RC columns with similar beam-column element parameters that were proposed in PEER Report 2007/03 [2]. A large earthquake record database consisting of 300 earthquake records measured on firm sites was used in the analyses to increase the statistical significance of the results. Each record was selected to have magnitude greater than 5 and PGA value greater than 0.08g.

Procedures for estimating maximum inelastic displacements of SDOF systems have been developed during the past 50 years. The first research work was conducted by Veletsos and Newmark [3] who investigated the relationship between the maximum inelastic displacements and elastic displacements of SDOF systems. The hysteretic behavior of SDOF systems was assumed to be elasto-plastic and three earthquake records were used. The results of this study have led to the very well-known “equal displacement rule”. Using the equal displacement rule in low frequency regions was also recommended in other studies [4, 5].

Analyses of non-degrading SDOF structures using five different hysteretic models was conducted in [6]. Either bilinear or Clough model [4] were used in their numerical studies and the analyses were performed only using one earthquake record. They concluded that the equal displacement rule applies for periods higher than the characteristic period, which is defined as the period between the constant acceleration
and constant velocity regions of the response spectra, regardless of which hysteretic model is used.

In the beginning of 1990s, Krawinkler and his co-workers [7, 8] investigated SDOF columns using bilinear, Clough or pinching models. They considered either strength degradation or stiffness degradation in their modeling process. Moreover, they derived an equation to estimate the inelastic displacement ratio of SDOF systems [9]. Miranda [10, 11] analyzed the ratio of maximum inelastic displacement to maximum elastic displacement of elasto-plastic SDOF models subjected to 124 earthquake records. He studied the inelastic displacement ratios on three different soil types in short period regions and investigated the limiting period where the equal displacement rule starts to apply. He furthered his study on constant ductility inelastic displacement ratios using 264 earthquake records and developed ratio versus period plots based on different earthquake magnitude, distance to the source, and local soil types [12]. He concluded that neither the earthquake magnitude nor the epicenter distance affects the inelastic displacement ratio under the same constant ductility ratio. He also found that different site conditions do not have a significant effect on the constant inelastic displacement ratio when the average shear wave velocity of the upper 30 m (100 ft) of the sites is higher than 180 m/s (600 ft/s). In addition to his findings, he developed an equation that estimates the inelastic displacement ratio of elasto-plastic SDOF structures. In a later study, he pointed out that maximum inelastic displacements could be related to maximum elastic displacements either through inelastic displacement ratios or strength reduction factors, which are known as direct and indirect methods respectively [13]. He showed that the indirect method underestimates the maximum inelastic displacements compared to the results obtained from the direct method.

Miranda and Ruiz-Garcia [14] evaluated six approximate methods, four of them based on equivalent linearization and two based on multiplying the maximum elastic displacement with a factor, that estimates the maximum inelastic displacement of SDOF systems. They also studied the effects of period of vibration, lateral yielding strength level, site conditions with shear wave velocity higher than 180 m/s (600 ft/s), earthquake magnitude, epicenter distance and strain hardening ratio on inelastic displacement ratio [15]. They derived an equation to predict the inelastic displacement ratios of existing
structures on firm sites restricted to elasto-plastic systems. They also worked on the inelastic displacement ratio of SDOF systems on soft soils [16, 17].

In the late 1990s and early 2000s, several other studies were also conducted. SDOF systems subjected to 15 earthquake records were investigated in [18]. Degradation effect was incorporated in the system using a three-parameter model. Numerical studies on non-degrading Bouc-Wen model [19] subjected to 20 earthquake records were conducted in [20]. Inelastic displacement ratio of structures subjected to 12 ground motions considering strength and stiffness degradation effect only were performed in [21]. In another study, Chopra and Chintanapakdee [22] investigated the inelastic displacement ratio of new and existing structures which were modeled as non-degrading elasto-plastic and bilinear systems subjected to 214 earthquake records. Chenouda and Ayoub [23, 24] and Ayoub and Chenouda [25] developed a new energy-based model, which takes several degradation effects into account, to perform dynamic analysis and predict collapse of structures subjected to seismic excitation. Bilinear and modified Clough models [4] were used in this study. They proposed a new equation, originally based on a study by Krawinkler and Nassar [9], to estimate the maximum inelastic displacement of degrading systems. In addition, they compared their inelastic displacement ratio curves with several other proposed equations.

Hatzigeorgiou and Beskos [26] investigated the effect of repeated or multiple earthquakes on inelastic displacement ratio of elasto-plastic SDOF systems. They used 112 earthquake records recorded at sites with USGS soil types A, B, C and D in their study. After numerical studies, they proposed an equation not only to estimate the inelastic displacement ratio of SDOF systems subjected to single earthquakes but also subjected to multiple earthquakes. Zhang et al. [27] developed inelastic displacement ratios accounting for shear-flexure interaction behavior of concrete structures. Lately, Erberik et al. [28] used an energy approach to develop degrading models for reinforced concrete columns, and used them to derive new inelastic displacement ratios.

The main purpose of this study is to develop a fuzzy logic model for predicting the inelastic displacement ratio of moderately degrading SDOF RC structures subjected to earthquake loading. The analytical model used takes degradation into consideration. A qualitative and quantitative comparison with the results of existing classical methods is
then performed. A brief description of the earthquake records used and the analytical model adopted is presented first.

EARTHQUAKE RECORDS

A large database of earthquake records was used in this research in order to increase the statistical significance. The database set used in this research consists of 300 earthquake records with PGA values varying between 0.08g and 2.73g. Each record corresponds either to NEHRP soil type C or D (stiff soil or soft rock) based on their shear wave velocities (180 m/s to 760 m/s). Magnitudes in the records were greater than 5 and the distances to the source were greater than 5 km. Most of the earthquake records were selected from the PEER Ground Motion Database [29]. Significant earthquake records that occurred recently were also collected from the Kyoshin Network (K-Net) [30] and GeoNet – Strong Motion Data [31]. The horizontal components with max PGA values of selected earthquake records were used in the analytical modeling. A total of 266 earthquake records with PGA values greater than 0.08g were selected from the PEER Ground Motion Database. A total of 18 earthquake records of Honshu–Japan earthquake (3/11/2011) with PGA value varying between 0.768g and 2.731g were selected from Kyosin Network. The shear wave velocities ($V_s$) of the records were given rather than their soil types. Therefore, the corresponding NEHRP soil types were based on the NEHRP Site Classification (FEMA 450 [32]). A total of 16 earthquake records from the Christchurch-New Zealand earthquake (2/21/2011) with PGA values varying between 0.082g and 0.881g were selected from the GeoNet database. The records correspond to NZS 1170.5:2004 soil types B, C and D [33] which are equivalent to NEHRP soil types C and D.

Scaling of earthquake records for any seismic performance evaluation purpose has been one of the important issues in engineering applications. Huang et al. [34] investigated the nonlinear response histories of SDOF systems using four scaling methods and presented their advantages and disadvantages. The first method was the Geometric-Mean scaling method used by Somerville and his co-workers [35] which is based on amplitude scaling of a pair of ground motion. According to Huang et al.’s findings, it is difficult to select ground motions for this method with median spectrums that closely match the target spectrum of a wide range of periods. The second method
was the spectrum-matching method which is often used for computing the seismic demands in structural framing systems. This particular method was found to underestimate the median peak displacement demand in highly nonlinear SDOF systems. The third method was $S_a(T_1)$ Scaling method which was proposed by Shome et al. [36]. In this method, earthquake records are scaled to match the median elastic spectral acceleration at each period of vibration that is investigated. This method resulted in unbiased median displacement response predictions. The last method that Huang et al. investigated was the Distribution-Scaling method. This method was also found to estimate the median displacement responses with no bias. In this study, the $S_a(T_1)$ Scaling method was adopted due to its efficiency and simplicity. The records used in this study are presented in Table 1. The detailed ground motion record list may be found in Ozkul [37].

VALIDATION OF NUMERICAL MODEL

The material model used in this study follows the degrading modified Clough model (Fig. 1) described in detail in Chenouda and Ayoub [23]. The model uses an eight-parameter energy approach to account for strength, unloading stiffness, accelerated stiffness, and cap degradation under cyclic loads. Collapse of an element is assumed if any of the following two criteria is established: a) the displacement has exceeded the value of that of the intersection point of the softening (cap) slope with the residual strength line, which is referred to as cap failure, or b) the degradation effect results in total loss of strength, which is referred to as cyclic degradation failure. In both cases, the capacity of the element to sustain additional loads is vanished.

In this study, it was necessary to find a group of tested columns that possess particular beam-column element parameters and that have periods of vibration spanning from 0.2 sec to 1.4 sec. The PEER Report 2007/03 study [2] was used as a source for selecting these columns. The report uses a subset of 255 columns from the PEER database that includes results of 430 columns, and provides recommendations for the beam-column element parameters of those columns to be used in nonlinear dynamic analysis. Five of these columns were selected for the study as described below. The geometric and material parameters of all columns are reported in Table 2. The first
column is column B2 tested by Thomsen and Wallace [38]. This column was tested to investigate the suitability of using ductile moment-resisting frames constructed of high-strength concrete in moderate to high seismic areas. The specimen was tested in a cantilever configuration. The second and third columns are columns C2-3 and C3-2 tested by Mo and Wang [39]. These columns were tested to investigate the effect of transverse reinforcement configurations on the seismic behavior of RC columns. The fourth column is column BG-6 tested by Saatcioglu and Grira [40]. This column, representing part of a first story column between the footing and point of inflection, was tested in order to investigate an alternative transverse confinement reinforcement detailing for earthquake resistant construction using welded grids. The fifth column is column 1006015 tested by Legeron and Paultre [41]. This column was tested to investigate the influence of the axial-load level and the volumetric ratio of confinement steel of high strength concrete columns under seismic excitation.

The experimental cyclic force-displacement responses of the columns taken from the PEER Structural Performance Database were compared with the modified Clough model simulations. The first set of cyclic loading simulations was conducted using the beam-column element parameters recommended in [2] (Table 3). The second set of simulations was conducted using newly proposed constant element parameters for all columns as described in Table 3. These values were selected in order to parameterize the problem at hand. Reasonable agreement between the two approaches was observed as shown in the results below.

The cyclic load-displacement and force-pseudo time figures of Column B2 [38], Columns C2-3 and C3-2 [39], Column BG-6 [40], and Column 1006015 [41] are shown in Fig. 2(a) to 2(k) respectively. Overall a good correlation is observed between the experimental results and the analytical simulations.

**DISPLACEMENT ESTIMATES OF RC STRUCTURAL COLUMNS**

The five columns described in the previous sections with the same model parameters reported in Table 3 were used to simulate the deformation behavior of a large set of RC columns by generating 40 different periods of vibration ranging from 0.2 sec to 1.4 sec. Seven to nine different axial loads were applied on the column specimens that
varied between 5 to 30% of their axial loading capacities. The axial loads applied on each column did not exceed ±10% of its original tested axial loading (Table 2). The range of periods covered by each column is represented in Fig. 3.

The scaled earthquake records presented earlier were used to conduct the analytical study using the modified Clough model. Moderately degraded SDOF systems (γ = 100) with several strength reduction factors (R = 1.5, 2, 3, 4, 5, 6, 7 and 8) were evaluated. The model parameters (αs = 6%, αc = -6%, γ = 100) described in Table 3 were adopted in the study.

The ratio of maximum inelastic displacement to maximum elastic displacement (inelastic displacement ratio) generated for 40 different period values was plotted for each strength reduction factor. Collapse was defined when the SDOF columns subjected to earthquake records failed under more than 50% of the records. The period before collapse occurred in the system is indicated with a “*” symbol in the plots. It was observed that the SDOF RC columns collapsed at every period smaller than the period indicated with “*”. Therefore, the inelastic displacement ratio estimations for those periods were not plotted. The collapse period for the different strength reduction factors are shown in Table 4.

The numerically-evaluated inelastic displacement ratios were compared with four classical equations used to estimate the inelastic displacement ratios of SDOF degrading systems. The first equation is the modified Krawinkler and Nassar equation for modified Clough systems, and is proposed by Chenouda and Ayoub [23]:

$$\delta_{inelastic} \over \delta_{elastic} = \frac{1}{R} \left[ 1 + \frac{Rc - 1}{c} \right] \quad (2)$$

where \( a = 0.7 \) and

$$b = 0.39 - \frac{R}{50} + \frac{0.033R^2}{\sqrt{\gamma}} \quad (3)$$

The constant values of coefficients \( a \) and \( b \) depend on the strain hardening ratio (\( \alpha_s \)), which was considered as 3%, the period of the structure (T), and the strength reduction...
factor (R). The equation is only valid for systems with period values higher than the expected collapse period.

The second equation is proposed by Chopra and Chintanapakdee [22]:

\[ L_R = \frac{1}{R} \left( \frac{R - 1}{\alpha} + 1 \right) \]  \hfill (4)

\[
\frac{\delta_{\text{inelastic}}}{\delta_{\text{elastic}}} = 1 + \left[ (L_R - 1)^{-1} + \left( \frac{a}{R^b} + c \right) \left( \frac{T}{T_c} \right)^{d-1} \right]^{-1}
\]  \hfill (5)

where \( T_c \) is the period separating the acceleration and velocity sensitive regions and equal to 0.33 for NEHRP soil type C and \( \alpha \) is the strain hardening ratio of the degrading SDOF system. Coefficients of the equation were proposed by the authors to be: \( a = 61 \), \( b = 2.4 \), \( c = 1.5 \) and \( d = 2.4 \). Chenouda and Ayoub [23] recalibrated the coefficient of \( c \) to simulate modified Clough systems and found the value to be equal to 0.5.

The third equation is proposed by Ruiz-Garcia and Miranda [15]:

\[
\frac{\delta_{\text{inelastic}}}{\delta_{\text{elastic}}} = 1 + \left[ \frac{1}{a(T/T_s)^b} + \frac{1}{c} \right] (R - 1)
\]  \hfill (6)

where \( T_s \) is the characteristic period at the site which is assumed to equal 0.85 for NEHRP soil type C. Constants of the equations, \( a \), \( b \) and \( d \) are also site dependent and equal to 48, 1.8 and 50 respectively. The constant \( b \) was recalibrated later by Chenouda and Ayoub [23] to fit modified Clough systems and was found to be equal to 2.2.

The fourth equation is proposed by Hatzigeorgiou and Beskos [26] as follow:

\[
\frac{\delta_{\text{inelastic}}}{\delta_{\text{elastic}}} = 1 = a \left( \frac{R - 1}{R} \right) \left( T^b + R^c + d \right)
\]  \hfill (7)

\[ a(\xi, H) = a_1 + a_2\xi + a_3H + a_4H^2 \]  \hfill (8)

\[ b(\xi, H) = b_1 + b_2\xi + b_3H + b_4H^2 \]  \hfill (9)

\[ c(\xi, H) = c_1 + c_2\xi + c_3H + c_4H^2 \]  \hfill (10)

\[ d(\xi, H) = d_1 + d_2\xi + d_3H + d_4H^2 \]  \hfill (11)
The viscous damping ratio is denoted by $\xi$ and H refers to the post-yield stiffness ratio. The coefficients $a_i$, $b_i$, $c_i$, and $d_i$ for NEHRP soil type C are given in Table 5.

Comparison of the maximum inelastic displacement ratio curves obtained is calculated for $R$ equal to 1.5, 2, 3, 4, 5, 6, 7, 8. The results for $R = 3, 4, 6, \text{ and } 8$ are shown in Fig. 4(a) to (d) respectively. All equations provided conservative estimates in general for the inelastic displacement ratios. The Modified Krawinkler and Nassar equation was the closest equation to the data obtained from the analytical model. It has the tendency of slightly underestimating the inelastic displacement ratios for $R$ equal to 3 and 4 systems at very short periods. The Chopra-Chintanapakdee equation gives more or less the same results as the Modified Krawinkler and Nassar equation for the systems with $R$ up to 3. It starts to give more conservative results compared to the Modified Krawinkler and Nassar equation for $R$ values larger than 3. It has the same tendency of underestimating the inelastic displacement ratios for $R$ equal to 3 and 4 systems at very short periods. The Ruiz-Garcia and Miranda equation gives more conservative results compared to the two equations mentioned above. As the period of the structure and the strength reduction factor increases, the overestimation of the inelastic displacement ratio increases compared to the data obtained from the analytical model. The equation proposed by Hatzigeorgiou and Beskos is very conservative for structures with any strength reduction level. It produces high overestimations within the period range of 0.2 sec to 1.0 sec.

**Fuzzy Logic-Based Inelastic Displacement Ratios**

The Fuzzy logic approach was originally introduced by Zadeh [42] and has been adopted in various engineering problems since. Mamdani and his co-workers [43, 44], and Takagi and Sugeno [45], for example, applied this approach in electronic engineering. Applications on various hydraulic and hydrology problems were introduced in [46-48]. Incorporation of fuzzy logic in earthquake engineering applications was conducted in [49-52].

The use of a fuzzy logic approach to estimate seismic demands was not performed before. This study aims at adopting a fuzzy logic approach to estimate the inelastic displacement ratios of SDOF structures.
Classical logic (crisp) sets and fuzzy logic sets consist of elements with some common features or properties. However, the boundaries of the sets are defined differently. Classic sets have precise boundaries meaning that an element is either a member of a set or not. On the other hand, fuzzy sets have imprecise boundaries letting an element to be partially a member of one or more fuzzy sets. This ability of describing the uncertainties of input variables by partial involvement to fuzzy sets generates the basis of fuzzy logic approach. Membership functions are used to define the membership degree of an element in a fuzzy set. In other words, the partial involvement of an element to a fuzzy set is defined by membership functions. These functions have great importance in fuzzy logic approach and, therefore, they need to be carefully assigned. Gaussian-shaped, bell-shaped and trapezoidal shaped are the most commonly used membership functions in the literature. The membership functions are named using linguistic fuzzy words such as low, medium and high fuzzy sets.

Membership degree values of each fuzzy set range between 0 and 1. If the membership degree equals to zero, this indicates the element in consideration is not a member of that particular set. On the contrary, if the membership degree equals to 1, that indicates the element belongs completely to that set. If the membership degree is between 0 and 1, however, the element partially belongs to that fuzzy set.

Fuzzy logic methods are constructed for defining a relationship between inputs and outputs using fuzzy rules. The method lets the user take the uncertainties of the input data into consideration rather than making assumptions. For a successful fuzzy logic modeling, four interdependent steps are necessary to be followed:

**Step 1 - Fuzzification:** The input values are converted to membership functions (fuzzy sets) which are interfering with each other. In this study, a Gaussian-shaped function was adopted for both the strength reduction factor (R) and the fundamental period of vibration (T) (Figs. 6 and 7). Linguistic fuzzy terms such as low, medium and high are used for naming the fuzzy sets.

**Step 2 - Constructing Fuzzy Rules:** After the fuzzification process, the relationships between the combined linguistic input fuzzy sets and output fuzzy sets are built using a series of IF-THEN rules. “IF” statements are referred to as the “antecedent” part of the rules and combine the linguistic input fuzzy sets. “THEN” statements coming
after “IF” statements are referred to as the “consequent” part of the rules which include the convenient output fuzzy sets based on the antecedent part. Fuzzy rules necessitate expert knowledge and/or synthetic data to be constructed. In this study, nine rules were developed for prediction of seismic inelastic displacement ratios (Table 6).

**Step 3 - Implication:** The consequent part (output) is shaped based on the antecedent part on this step.

**Step 4 - Defuzzification:** The outputs that are obtained as fuzzy sets are reduced to scalar values. Engineers use those scalar results rather than the fuzzy sets in their further studies, in this case to represent inelastic displacement ratios. Therefore, this step is required for engineers. The commonly used defuzzification procedures are the centroid and weighted abscissa methods [53].

Two commonly used fuzzy logic approaches exist in the literature applications, which differ in dealing with the consequent part of the fuzzy rules. The first one is the Mamdani approach [43] and the second approach is the Takagi-Sugeno approach [45].

The Takagi-Sugeno fuzzy logic approach was used in this study to develop a new model for estimating the inelastic displacement ratio of moderately degrading SDOF RC structures. The analytical model results obtained in the previous section were used for this new development purpose. Consideration of eight strength reduction factors (R) and 40 periods of vibration (T) resulted in 320 inelastic displacement ratio points in total. These data points were divided into training and testing data. The training (calibration) data consisted of 70% of the data, which were randomly selected, and was used to establish the fuzzy logic model; whereas, the testing (prediction) data consisted of the remaining 30% of the data, and was used to validate the model.

The general Takagi-Sugeno Fuzzy Logic model input-output diagram is as shown in Fig. 5. The training data was used to optimize the fuzzy sets shown in the figure and the fuzzy rules relating the fuzzy input sets to the output.

The strength reduction factor (R) and period of vibration (T) were considered as the fundamental fuzzy input variables having uncertain boundaries in this study. At the end of the training process, three fuzzy sets were defined for each of the input variable qualifying their uncertainties with linguistic expressions such as low, medium and high. Gaussian membership functions were used to establish those fuzzy sets for R and T.
respectively (Figs. 6 and 7). Nine fuzzy rules with three fuzzy sets for each input variable were then optimized as depicted in Table 6.

In order to evaluate the inelastic displacement ratio (IDR), first the weight of each IDR function was calculated and then the weighted average of the nine IDR functions was evaluated using the following equations:

\[ r_r = (m_T^r * m_R^r) \] and
\[ \text{IDR} = \frac{\sum_{r=1}^{9}(r_r \text{ IDR}_r)}{\sum_{r=1}^{9}(r_r)} \]

where \( m_T^r \) and \( m_R^r \) represent respectively the degrees of membership of the period and strength reduction factor fuzzy sets of the \( r \) rule.

An example of applying the fuzzy logic-based model developed in this study for estimating the IDR is shown in Appendix A.

After the new model that estimates the inelastic displacement ratio of moderately degrading RC SDOF structures was established using the training data, the testing data was used for validating the method. The graphical representations of fuzzy logic model predictions using testing data are shown in 8(a) to (h). It is noteworthy to mention that all of the graphs result in one conclusion: inelastic displacement ratio predictions of the fuzzy logic-based model show remarkably good agreement with the synthetic data (Model Data).

**Evaluation of Fuzzy Logic-Based and Classical Methods**

The quantitative performance evaluation of the fuzzy logic-based method and the existing classical methods mentioned in a previous section are conducted in terms of statistical criteria, such as the mean square error (MSE) and the coefficient of efficiency (CE). These expressions can be written as follow:

\[ \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (\text{IDR}_{pi} - \text{IDR}_{oi})^2 \]

\[ \text{CE} = 1 - \frac{1}{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{IDR}_{oi} - \text{IDR}_{mean})^2}} \]
where IDR\textsubscript{pi} and IDR\textsubscript{oi} are predicted and synthetic values of the inelastic displacement ratio at the \textit{i}^\text{th} observation and IDR\textsubscript{mean} is the mean value of the synthetic values respectively. \(N\) represents the total number of observations.

The models considered are: the proposed Fuzzy Logic model, Modified Krawinkler and Nassar (MK-N), Chopra and Chintanapakdee (C-C), Ruiz-Garcia and Miranda (RG-M), and Hatzigeorgiou and Beskos (H-B) methods respectively. The CE values of Fuzzy Logic model, MK-N, C-C, RG-M and H-B methods are calculated as 0.91, 0.76, 0.6, 0.11 and -7.53 respectively. It can be concluded that the fuzzy logic model is more accurate than the other classical methods when compared to the analytical model data.

The performance evaluation of the Fuzzy Logic model is shown in Fig. 9(a) which depicts the relationship between the synthetic data and the data predicted using the proposed model for all \(R\) values considered. Most of the predicted values are around a 45° diagonal line, with a CE value equal to 0.91. It can be concluded that the proposed fuzzy logic model is a viable approach that estimates the inelastic displacement ratio of moderately degrading SDOF RC structures with high accuracy, which makes it a good potential alternative to existing design guidelines.

Fig. 9(b) shows the performance evaluation of MK-N method. Most of the predicted values are above the 45° diagonal line, which means that the results of this method overestimate the inelastic displacement ratio of moderately degrading SDOF RC structures. However, with a CE value of 0.76, this method is still good for design purposes.

The C-C method shows a similar performance trend as the M-K method. As seen in Fig. 9(c), the CE value equals 0.6. The method is still acceptable for design purposes.

Fig. 9(d) and Fig. 9(e) show the performance evaluations for RG-M and H-B methods. All of the predicted values fall above the 45° diagonal line. The CE values for these methods are calculated as 0.11 and -7.53 respectively meaning that the predictions are conservative and may result in inefficient design.

Finally, a sensitivity study was conducted in order to examine the applicability and accuracy of the proposed Fuzzy-Logic method. In this case, the percent of training
data was varied from 30% to 80%, with the remaining data used for testing purposes. Figures 10(a) and 10(b) show the performance evaluation for the cases of 30% training data and 70% testing data; and 80% training data and 20% testing data respectively. In the former case, the CE value was 0.73, slightly lower than the MK-N method; while in the latter case the CE value improved to 0.95. However, this improvement was based on prediction of a much fewer number of data points. This concludes that the originally proposed model based on 70% training data and 30% testing data, which resulted in a CE of 0.91, strikes a good balance between accuracy and confidence in results.

**SUMMARY AND CONCLUSIONS**

The use of fuzzy logic techniques has become popular in today’s world and fuzzy logic has been widely used in several engineering problems. This study presents a newly developed model using a fuzzy logic approach for estimating the maximum inelastic displacements of moderately degrading SDOF RC columns under seismic excitation. For this purpose, dynamic analyses of RC columns subjected to 300 earthquake records were evaluated using an eight-parameter modified Clough degrading model. Five tested RC columns provided in PEER Report 2007/03 [2] were used to acquire a large range of vibration periods reflecting real cases. A total of 96,000 dynamic analyses of SDOF systems were performed to accurately evaluate inelastic displacement ratios. The data obtained from these dynamic analyses were then used for the development of new inelastic ratio functions. The accuracy of the new method and four existing classical methods were evaluated. Several conclusions were drawn from this study:

1) The predicted inelastic displacement ratio values obtained from the fuzzy logic model matches the experimental data with great accuracy.

2) The comparison of experimental data with four existing classical methods showed that all methods estimated the inelastic displacement ratios of moderately degrading SDOF RC columns conservatively.

3) The Modified Krawinkler and Nassar method [23] is the most accurate equation compared to the other classical methods. It slightly underestimates the inelastic displacement ratios of SDOF systems with strength reduction factors (R) equal to 3, 4 and 5 at very short periods.
4) The Chopra and Chintanapakdee method [22] gives almost the same results as the Modified Krawinkler and Nassar method for the systems with R less than 3. It gives more conservative results when R is greater than 3. It has the same tendency of underestimating the inelastic displacement ratios of SDOF systems for R equal to 3, 4, and 5 at very short periods.

5) The Ruiz-Garcia and Miranda method [15] gives more conservative results than the other methods. Overestimation of the inelastic displacement ratio increases as the period of the structure and the strength reduction factor increases.

6) The Hatzigeorgiou and Beskos method [26] is extremely conservative for structures with any strength reduction level within vibration periods ranging from 0.2 sec to 1.0 sec.

7) In terms of the coefficient of efficiency, the methods from high to low accuracy are lined up as follow: Fuzzy Logic model, Modified Krawinkler and Nassar method, Chopra and Chintanapakdee method, Ruiz-Garcia and Miranda method, and Hatzigeorgiou and Beskos method.

8) The conducted sensitivity study confirms that the selected percent of training and testing data provides a good balance between accuracy and practicality.

REFERENCES


APPENDIX A

In this appendix, the use of the newly-developed fuzzy logic technique for estimating inelastic displacement ratios is clarified. Consider a system with period of vibration $T=0.5$ sec, and a strength reduction factor $R=3$. When $T$ is equal to 0.5 and $R$ is equal to 3, all of the rules of Table 6 are triggered, as evidenced in Figs. 6 and 7. The degree of memberships of $T$ and $R$ from Figs. 6 and 7 are found as shown in column 2 of Table 7 below. The weights of the rules $r_i$ are calculated in column 3. Each rule has different weight contributions to the result as seen in this column. The output IDR values for each triggered rule are calculated based on the IDR output functions (Table 6) and are provided in column 4 below. The last column gives the multiplication of the weight of the rules and the results of the output fuzzy functions. The final output is computed by calculating the weighted average of all triggered output functions. The inelastic displacement ratio when $T=0.5$ sec and $R=3$ is predicted as 0.75.
Table 1  Earthquake Ground Motions Used in This Study

<table>
<thead>
<tr>
<th>Earthquake Name</th>
<th>Date</th>
<th>Mag (M)</th>
<th>PGA Range (g)</th>
<th>NEHRP Site Class</th>
<th># of Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max.</td>
<td>Min.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.731</td>
<td>0.768</td>
<td>C</td>
</tr>
<tr>
<td>Honshu, Japan</td>
<td>3/11/11</td>
<td>9.00</td>
<td>0.881</td>
<td>0.881</td>
<td>C</td>
</tr>
<tr>
<td>Christchurch, N. Zealand</td>
<td>2/21/11</td>
<td>6.30</td>
<td>1.153</td>
<td>0.086</td>
<td>C</td>
</tr>
<tr>
<td>Chi-Chi, Taiwan</td>
<td>9/20/99</td>
<td>7.62</td>
<td>0.951</td>
<td>0.087</td>
<td>C</td>
</tr>
<tr>
<td>Chi-Chi, Taiwan-02</td>
<td>9/20/99</td>
<td>5.90</td>
<td>0.347</td>
<td>0.092</td>
<td>C</td>
</tr>
<tr>
<td>Chi-Chi, Taiwan-03</td>
<td>9/20/99</td>
<td>6.20</td>
<td>0.520</td>
<td>0.085</td>
<td>C</td>
</tr>
<tr>
<td>Chi-Chi, Taiwan-05</td>
<td>9/22/99</td>
<td>6.20</td>
<td>0.374</td>
<td>0.093</td>
<td>C</td>
</tr>
<tr>
<td>Hector Mine</td>
<td>10/16/99</td>
<td>7.13</td>
<td>0.337</td>
<td>0.081</td>
<td>C</td>
</tr>
<tr>
<td>Kocaeli, Turkey</td>
<td>8/17/99</td>
<td>7.51</td>
<td>1.024</td>
<td>0.087</td>
<td>C</td>
</tr>
<tr>
<td>Northridge-01</td>
<td>1/17/94</td>
<td>6.69</td>
<td>0.184</td>
<td>0.184</td>
<td>C</td>
</tr>
<tr>
<td>Northridge-04</td>
<td>1/17/94</td>
<td>5.93</td>
<td>0.105</td>
<td>0.105</td>
<td>C</td>
</tr>
<tr>
<td>Northridge-05</td>
<td>1/17/94</td>
<td>5.13</td>
<td>0.228</td>
<td>0.088</td>
<td>C</td>
</tr>
<tr>
<td>Northridge-06</td>
<td>3/20/94</td>
<td>5.28</td>
<td>0.457</td>
<td>0.089</td>
<td>C</td>
</tr>
<tr>
<td>Whittier Narrows-01</td>
<td>10/1/87</td>
<td>5.99</td>
<td>0.262</td>
<td>0.178</td>
<td>C</td>
</tr>
<tr>
<td>Whittier Narrows-02</td>
<td>10/4/87</td>
<td>5.27</td>
<td>0.457</td>
<td>0.089</td>
<td>C</td>
</tr>
<tr>
<td>Honshu, Japan</td>
<td>3/11/11</td>
<td>9.00</td>
<td>1.630</td>
<td>0.820</td>
<td>D</td>
</tr>
<tr>
<td>Christchurch, N. Zealand</td>
<td>2/21/11</td>
<td>6.30</td>
<td>0.718</td>
<td>0.082</td>
<td>D</td>
</tr>
<tr>
<td>Dinar, Turkey</td>
<td>10/1/87</td>
<td>6.40</td>
<td>0.352</td>
<td>0.352</td>
<td>D</td>
</tr>
<tr>
<td>Duzce, Turkey</td>
<td>11/12/99</td>
<td>7.14</td>
<td>0.822</td>
<td>0.535</td>
<td>D</td>
</tr>
<tr>
<td>Erzincan, Turkey</td>
<td>3/13/92</td>
<td>6.69</td>
<td>0.515</td>
<td>0.515</td>
<td>D</td>
</tr>
<tr>
<td>Kocaeli, Turkey</td>
<td>8/17/99</td>
<td>7.51</td>
<td>0.358</td>
<td>0.103</td>
<td>D</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

1 Earthquake Records collected from Kyosin Network (K-Net)
2 Earthquake Records collected from NZSEE Database
Table 2  Miscellaneous Design Properties of the Tested Columns

<table>
<thead>
<tr>
<th>Columns</th>
<th>Geometric properties</th>
<th>Material Properties</th>
<th>Reinf. Ratio</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b (mm)</td>
<td>h (mm)</td>
<td>Aspect ratio (L/h)</td>
<td>A_g (mm^2)</td>
</tr>
<tr>
<td>B2</td>
<td>152.4</td>
<td>152.4</td>
<td>3.9</td>
<td>23226</td>
</tr>
<tr>
<td>C2-3</td>
<td>400.0</td>
<td>400.0</td>
<td>3.5</td>
<td>160000</td>
</tr>
<tr>
<td>C3-2</td>
<td>400.0</td>
<td>400.0</td>
<td>3.5</td>
<td>160000</td>
</tr>
<tr>
<td>BG-6</td>
<td>350.0</td>
<td>350.0</td>
<td>4.7</td>
<td>122500</td>
</tr>
<tr>
<td>1006015</td>
<td>305.0</td>
<td>305.0</td>
<td>6.6</td>
<td>93025</td>
</tr>
</tbody>
</table>

Table 3  Hazelton et al. [2] Parameters and Proposed Parameters

<table>
<thead>
<tr>
<th>Columns</th>
<th>Hazelton et al. Parameters</th>
<th>Proposed Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hardening Slope (a_s)</td>
<td>Cap Slope (a_c)</td>
</tr>
<tr>
<td>B2</td>
<td>0.070</td>
<td>-0.05</td>
</tr>
<tr>
<td>C2-3</td>
<td>0.070</td>
<td>-0.07</td>
</tr>
<tr>
<td>C3-2</td>
<td>0.060</td>
<td>-0.05</td>
</tr>
<tr>
<td>BG-6</td>
<td>0.045</td>
<td>0.07</td>
</tr>
<tr>
<td>1006015</td>
<td>0.001</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Table 4  Collapse Periods for moderately degrading SDOF RC Structures

<table>
<thead>
<tr>
<th>R=1.5</th>
<th>R=2</th>
<th>R=3</th>
<th>R=4</th>
<th>R=5</th>
<th>R=6</th>
<th>R=7</th>
<th>R=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collapse Periods (sec)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.242</td>
<td>0.279</td>
<td>0.315</td>
<td>0.406</td>
</tr>
</tbody>
</table>

Table 5  Coefficients of a_i, b_i, c_i, and d_i for NEHRP soil type C (Hatzigeorgiou and Beskos [26])

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.488390</td>
<td>0.330289</td>
<td>-9.61847</td>
<td>142.252</td>
</tr>
<tr>
<td>b</td>
<td>-1.24221</td>
<td>-0.547800</td>
<td>-5.51635</td>
<td>-19.4654</td>
</tr>
<tr>
<td>c</td>
<td>0.472032</td>
<td>-0.440450</td>
<td>-2.15621</td>
<td>4.98701</td>
</tr>
<tr>
<td>d</td>
<td>-2.49009</td>
<td>4.81703</td>
<td>-2.89469</td>
<td>67.5202</td>
</tr>
<tr>
<td>RULE</td>
<td>DESCRIPTION</td>
<td>THEN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₁</td>
<td>T is Low and R is Low</td>
<td>IDR₁ = -5.39 + 0.1014 T + 1.898 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>T is Low and R is Medium</td>
<td>IDR₂ = -34.39 + 0.1594 T + 13.26 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>T is Low and R is High</td>
<td>IDR₃ = 21.81 - 0.8248 T + 4.139 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₄</td>
<td>T is Medium and R is Low</td>
<td>IDR₄ = -1.812 + 0.0177 T + 1.442 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₅</td>
<td>T is Medium and R is Medium</td>
<td>IDR₅ = -3.131 + 0.09987 T + 1.482 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₆</td>
<td>T is Medium and R is High</td>
<td>IDR₆ = -5.499 + 0.209 T + 1.141 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₇</td>
<td>T is High and R is Low</td>
<td>IDR₇ = -0.2643 - 0.159 T + 2.533 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₈</td>
<td>T is High and R is Medium</td>
<td>IDR₈ = -5.557 - 0.1276 T + 3.765 R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₉</td>
<td>T is High and R is High</td>
<td>IDR₉ = -1.049 - 0.189 T + 6.607 R</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7 Example of using Fuzzy Logic Based Method

<table>
<thead>
<tr>
<th>Rules</th>
<th>Degree of memberships (mT and mR)</th>
<th>r_r (mT*mR)</th>
<th>IDR_r</th>
<th>r_r * IDR_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Low 0.0034</td>
<td>Low 0.4417</td>
<td>0.0015</td>
<td>-0.4928</td>
</tr>
<tr>
<td>R2</td>
<td>Low 0.0034</td>
<td>Medium 0.1523</td>
<td>0.0005</td>
<td>-4.4132</td>
</tr>
<tr>
<td>R3</td>
<td>Low 0.0034</td>
<td>High 0.0193</td>
<td>0.0000</td>
<td>12.5696</td>
</tr>
<tr>
<td>R4</td>
<td>Medium 0.8748</td>
<td>Low 0.4417</td>
<td>0.3855</td>
<td>0.5891</td>
</tr>
<tr>
<td>R5</td>
<td>Medium 0.8748</td>
<td>Medium 0.1523</td>
<td>0.1329</td>
<td>0.21611</td>
</tr>
<tr>
<td>R6</td>
<td>Medium 0.8748</td>
<td>High 0.0193</td>
<td>0.0168</td>
<td>-0.9815</td>
</tr>
<tr>
<td>R7</td>
<td>High 0.2785</td>
<td>Low 0.4417</td>
<td>0.1227</td>
<td>1.92385</td>
</tr>
<tr>
<td>R8</td>
<td>High 0.2785</td>
<td>Medium 0.1523</td>
<td>0.0423</td>
<td>0.6037</td>
</tr>
<tr>
<td>R9</td>
<td>High 0.2785</td>
<td>High 0.0193</td>
<td>0.0053</td>
<td>5.5155</td>
</tr>
</tbody>
</table>

Period = 0.5  R = 3  

\[
\sum(r_r) = 0.71  \quad \sum(r_r \times IDR_r) = 0.53
\]

\[
IDR = \frac{\sum(r_r \times IDR_r)}{\sum(r_r)} = \frac{0.53}{0.71} = 0.75
\]
Fig. 1. Modified Clough Model

Fig. 2. Simulation Results of Columns (a) Force-Displacement of Column B2 using PEER Report 2007/03 recommended beam-column element parameters

- \( \alpha_s = 0.07 \)
- \( \alpha_c = -0.05 \)
- \( \gamma = 78 \)
Fig. 2. Simulation Results of Columns (b) Force-Displacement of Column B2 using model parameters

\[ \alpha_s = 0.06 \]
\[ \alpha_c = -0.06 \]
\[ \gamma = 100 \]
Fig. 2. Simulation Results of Columns (c) Force-Pseudo Time of Column B2 using model parameters

Fig. 2. Simulation Results of Columns (d) Force-Displacement of Column C2-3 using model parameters

α_s = 0.06
α_c = -0.06
γ = 100
Fig. 2. Simulation Results of Columns (e) Force-Pseudo Time of Column C2-3 using model parameters

- $\alpha_s = 0.06$
- $\alpha_c = -0.06$
- $\gamma = 100$

Fig. 2. Simulation Results of Columns (e) Force-Pseudo Time of Column C2-3 using model parameters

- $\alpha_s = 0.06$
- $\alpha_c = -0.06$
- $\gamma = 100$
Fig. 2. Simulation Results of Columns (f) Force-Displacement of Column C3-2 using model parameters

Fig. 2. Simulation Results of Columns (g) Force-Pseudo Time of Column C3-2 using model parameters
Fig. 2. Simulation Results of Columns (h) Force-Displacement of Column BG-6 using model parameters

\[ \alpha_s = 0.06 \]
\[ \alpha_c = -0.06 \]
\[ \gamma = 100 \]

\[ \text{Column BG-6 - Experiment} \quad \text{Column BG-6 - Model} \]

Fig. 2. Simulation Results of Columns (i) Force-Pseudo Time of Column BG-6 using model parameters

\[ \alpha_s = 0.06 \]
\[ \alpha_c = -0.06 \]
\[ \gamma = 100 \]

\[ \text{Column BG-6 - Experiment} \quad \text{Column BG-6 - Model} \]
Fig. 2. Simulation Results of Columns (j) Force-Displacement of Column 1006015 using model parameters

Top displacement (mm)

- Column 1006015 - Experiment
- Column 1006015 - Model

\( \alpha_s = 0.06 \)
\( \alpha_c = -0.06 \)
\( \gamma = 100 \)
Fig. 2. Simulation Results of Columns (k) Force-Pseudo Time of Column 1006015 using model parameters

\[ \alpha_s = 0.06 \]
\[ \alpha_c = -0.06 \]
\[ \gamma = 100 \]

Fig. 2. Simulation Results of Columns (k) Force-Pseudo Time of Column 1006015 using model parameters

Fig. 3. Period of Vibration Range of Each Selected Column
Fig. 4. Inelastic Displacement Ratio for Varying $R$ values (a) $R = 3$

Fig. 4. Inelastic Displacement Ratio for Varying $R$ values (b) $R = 4$
Fig. 4. Inelastic Displacement Ratio for Varying R values (b) R = 4

Fig. 4. Inelastic Displacement Ratio for Varying R values (c) R = 6
Fig. 4. Inelastic Displacement Ratio for Varying R values (d) R = 8

Fig. 5. The input-output diagram of Takagi-Sugeno Fuzzy Logic model
Fig. 6. Fuzzy Sets and Membership Functions of Strength Reduction Factor (R)

Fig. 7. Fuzzy Sets and Membership Functions of Period of Vibration (T)
Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (a) R = 1.5

Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (b) R = 2
Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (c) $R = 3$

Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (d) $R = 4$
Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (e) $R = 5$

Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (f) $R = 6$
Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (g) R = 7

Fig. 8. Inelastic Displacement Ratio versus Period for Varying R Values (h) R = 8
Fig. 9. Model Data (Synthetic) versus Varying Methods (a) Fuzzy Logic Method

**Fuzzy Logic Method**

Coef. of Efficiency = 0.91

Fig. 9. Model Data (Synthetic) versus Varying Methods (b) MK-N Method

**M. Krawinkler & Nassar Method**

Coef. of Efficiency = 0.76

Fig. 9. Model Data (Synthetic) versus Varying Methods
Fig. 9. Model Data (Synthetic) versus Varying Methods (c) C-C Method

δ\textsubscript{inelastic}/δ\textsubscript{elastic} - Synthetic

Chopra & Chintanapakdee Method
Coef. of Efficiency = 0.60

Fig. 9. Model Data (Synthetic) versus Varying Methods (d) RG-M Method

δ\textsubscript{inelastic}/δ\textsubscript{elastic} - Synthetic

Ruiz-Garcia & Miranda Method
Coef. of Efficiency = 0.11
Fig. 9. Model Data (Synthetic) versus Varying Methods (e) H-B Method

Hatzigeorgiou and Beskos Method

Coef. of Efficiency = -7.53

Fig. 10(a). Model Data (Synthetic) versus Fuzzy Logic Method
(30% Training, 70% Testing)
Fig. 10(b). Model Data (Synthetic) versus Fuzzy Logic Method
(80% Training, 20% Testing)

Fuzzy Logic Method
Coeff. of Efficiency = 0.95