Marking to Market and Inefficient Investment Decisions*

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Abstract

We examine how mark-to-market accounting affects the investment decisions of managers with reputation concerns. Reporting the current market value of a firm's assets can help mitigate agency problems because it provides outsiders (e.g., shareholders) with new information against which the management's decisions can be evaluated. However, the fact that the assets' market value is informative can also have a negative side effect: Managers may shy away from investments that indicate conflicting private information and would damage their reputation. This effect can lead to inefficient investment decisions and make marking to market less desirable when market prices are more informative.

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1 Introduction

Since the recent banking crisis, mark-to-market accounting is again at the center of a policy debate involving banking and accounting regulators, financial institutions and their professional associations, the European Commission, and the U.S. Congress. The supporters point out that current market prices provide more timely and accurate information than book values based on historical costs. Therefore, marking to market can increase transparency, improve decision making, and help monitor a firm's management. The critics emphasize that since prices can diverge from fundamentals, marking to market can lead to excessive fluctuations in valuations as well as to contagion and downward spirals if financial institutions react to their assets' balance sheet values.[1] Both lines of argument suggest that more informative prices make mark-to-market accounting more desirable. We show that this need not be the case: More informative prices can make marking to market less desirable if managers care about their reputation for having accurate private information.

The intuition is as follows. Mark-to-market accounting provides new information to the public that would remain hidden if the reported asset values were based on historical costs. More informative market prices make marking to market more accurate. More accurate public information, however, can induce managers with reputation concerns not to act on conflicting private information. Openly disagreeing with the market would damage the managers' reputation for possessing accurate private information about the assets' fundamental value. As a consequence, mark-to-market accounting can cause managers to rely too much on current market values and thus lead to inefficient decisions – even if the assets' realized payoffs are verifiable ex post. In some cases, this effect can be so strong that marking to market is less desirable when prices are more informative.

Our analysis shows how mark-to-market accounting can interact with agency problems between managers and shareholders and lead to distortions of managerial decisions. Hence, marking to market can be

[1] See Laux and Leuz (2009, 2010) for a detailed discussion of the different arguments.

associated with costs even in normal times (i.e., outside periods of crisis), when prices reflect all available public information, and without exogenous capital requirements that induce firms to react to the balance sheet value of their assets. Our analysis thus helps to broaden the scope of the current policy debate and emphasizes that the accounting rules can have real effects.

To give a concrete example, consider a bank that owns a portfolio of marketable securities. On its balance sheet, the bank reports this position as "financial instruments owned." While the bank may provide a breakdown into several sub-categories (e.g., government obligations, corporate debt, or equities) in the notes to the financial statements, outside investors typically do not know exactly which securities the bank owns. Hence, even if market prices for all securities exist, outside investors cannot produce an accurate valuation of the portfolio on their own. Instead, they must rely on the bank to report this information. Financial statements based on mark-to-market accounting thus provide these investors with new information. Further, the current market value of the bank's assets may be relevant when deciding which new projects the bank should fund. For example, it may be optimal for the bank to take on additional risk only if its existing assets provide a cushion against potential losses.

Suppose now that the market value of the bank's existing assets is high, allowing the bank to make large investments in new, risky projects. Suppose further that the bank management's private information suggests that the assets in place are unlikely to generate high future cash-flows, so that more cautious investments are warranted. Investing cautiously, however, would reveal that the management disagrees with the market. This, in turn, would jeopardize the management's reputation as the private information of skilled managers is more likely to coincide with (rather than diverge from) the information available to the market. Hence, the bank's management may prefer to ignore their private information and fall in line with the market (i.e., to make an inefficient investment decision – taking on, in this case, excessive risk). This effect is well summarized by Charles O. Prince III (at the time the CEO of Citigroup) in his by-now infamous quote "As long as the music is playing, you've got to get up and dance" (Financial Times, 9 July 2007) and consistent with the view that reputation concerns contributed to the herding behavior in the run-up to the recent financial crisis.

The framework we use for our analysis corresponds closely to the above example. Specifically, we consider an agency model with the following features. A firm run by a manager (or insider) on behalf of its shareholders (or outside investors) must make an investment decision. Which decision maximizes expected profits depends on the market value of the firm's existing assets and on the manager's private information about these assets' value. This could be the case, for example, because financing frictions and bankruptcy costs make it optimal to invest more if the value of the assets in place is high. However, there is an agency problem: While the shareholders' objective is to maximize profits, the manager's goal is to maximize his reputation (e.g., because of career concerns).[2] Thus, whenever the reputation maximizing investment differs from the profit maximizing choice, a conflict of interest arises:[3] Rather than profit-maximizing investments, the manager prefers investments that indicate that the market's valuation of the assets and his private information about their value coincide. This can lead to over- or under-investment.

Using this framework, we examine the effect of two accounting rules – marking to market and historical cost accounting – on the manager's investment decision. There is only one difference between the two rules in our setup: Under mark-to-market accounting, the current market value of the firm's assets is reported in the financial statements. Under historical cost accounting, assets on the firm's balance sheet are valued at their historical cost. In that case, the assets' current market value remains hidden from the firm's sharehold-

[2]In an extension in which the manager cares about both his reputation and monetary payoffs, we show that our key findings remain unchanged even in the presence of optimal incentive contracts. See Section 6 for the details.

[3]This is consistent with Holmström and Ricart i Costa (1986), who suggest that an important reason for the misalignment of incentives between managers and shareholders is that the reputation value of an investment can differ from its financial value. Hirshleifer (1993) reviews the research on distortions of investment decisions due to reputation concerns and concludes that "managers have an incentive to use investment choices as a tool for building their personal reputations or the reputation of their firms" (Hirshleifer (1993) p. 146).

ers. The reason is that the shareholders (unlike the manager) do not know exactly which assets the firm owns and thus cannot compute their current market value on their own.

By revealing the current market value of the firm's assets, mark-to-market accounting provides the shareholders with new information. This has two effects. On the one hand, after the assets' market value has been reported to be low (high), the manager can no longer pretend that their market value is high (low). On the other hand, publicly revealing the assets' market value creates incentives for the manager to conceal conflicting private information. The first effect ameliorates the agency problem between the manager and the shareholders. The second effect aggravates the problem. Whether the shareholders are better off when the assets' market value is reported thus depends on which effect dominates.

Two main results emerge from our analysis: First, more informative prices can make the shareholders worse off if the firm's assets are marked to market. The reason is that more informative prices create stronger incentives for inefficient herding. Second, the shareholders may prefer marking to market if prices are less informative but historical cost accounting if prices are more informative. The reason is that, in some cases, the benefits of mark-to-market accounting outweigh the costs (relative to historical cost accounting) if prices are noisy but not if prices are very informative.

We also derive two novel empirical predictions. First, a change from historical cost accounting to mark-to-market accounting (e.g., due to a policy change) leads to an increase in the correlation of investment decisions of firms that own similar assets. The reason is that marking to market increases the decisions' sensitivity to information conveyed by the current market value of the firms' assets. Second, the change in the correlation of the firms' investment decisions is inversely related to the change in their profitability. The reason is that (some of) the increase in the sensitivity to the assets' market value is due to inefficient herding.

Finally, we consider several extensions of our model. First, we show that shareholders prefer their firms to hold more opaque assets under marking to market than under historical cost accounting. Second, we consider voluntary disclosure and fraudulent reporting. Third, we show that our key findings remain unchanged if the manager cares both about his reputation as well as monetary payoffs (even in the presence of optimal incentive contracts). Fourth, we show how the interaction between the informational role of prices and mark-to-market accounting can lead to a reduction in price informativeness. Fifth, we consider the case of a manager that is ex ante better informed than the shareholders about the quality of his private information.

Our paper contributes to the growing literature on the costs and benefits of marking to market.[4] The common view is that mark-to-market accounting is costly during crises. Allen and Carletti (2008) argue that marking to market can lead to contagion across firms during a liquidity crisis. Plantin, Sapra, and Shin (2008) show how marking to market can add endogenous volatility to prices. This, in turn, can lead to financial cycles when financial institutions actively readjust their balance sheets (Adrian and Shin (2009)). Like the aforementioned papers, we show that the accounting rules can have real effects. Unlike these papers, however, we show that mark-to-market accounting can be costly even in normal times, when market prices reflect all available public information, and without exogenous capital requirements due to which firms readjust their balance sheets.[5]

Our paper is also related to the literature on rational herding. In particular, the mechanism at the core of

[4]See Leuz and Wysocki (2008) and Sapra (2010) for detailed surveys. Our work is further related to research showing that more informative disclosure can be detrimental, e.g., Crémer (1995), Goldstein and Sapra (2013), Hermalin and Weisbach (2012), Kanodia, Singh, and Spero (2005), Morris and Shin (2002), Prat (2005), and Vives (2014).

[5]Heaton, Lucas, and McDonald (2010) also focus on financial institutions and argue that marking to market needs to be combined with procyclical capital requirements. O'Hara (1993) shows how marking to market can cause a shift towards short-term lending. Bleck and Liu (2007) argue that marking to market acts as an early warning system, while historical cost accounting allows firms to hide their true economic performance. Ellul, Jotikasthira, Lundblad, and Wang (2014, 2015) find that marking to market can lead to more conservative investments ex ante and that historical cost accounting does not necessarily avoid spillovers and contagion as it can cause gains trading. Plantin and Tirole (2015) derive both marking to market and gains trading as components of an optimal contract and show that individually optimal accounting rules can lead to a socially excessive use of mark-to-market accounting.

our model builds on Scharfstein and Stein (1990), who show how reputation concerns can induce managers to ignore their private information and mimic the investment decisions of others.[6] Unlike Scharfstein and Stein (1990), however, we consider two sources of information that can be of different quality – current market prices (the "market signal") and a manager's private information (the "private signal") – and analyze the costs and benefits of reporting the first signal to the public. Further, we examine how firms react to the disclosure of market prices by changing the composition of their assets, study voluntary disclosure and fraudulent reporting, and consider the effects of optimal incentive contracts. In addition, we investigate the possible interactions between mark-to-market accounting and price informativeness and extend our model to the case of ex ante asymmetric information between the manager and the shareholders regarding the quality of the manager's private information.

The paper proceeds as follows. Section 2 describes our model. Section 3 examines the first-best investment, agency problem, and equilibrium outcomes. Section 4 compares the effects of historical cost and mark-to-market accounting. Section 5 highlights testable predictions. Section 6 presents several extensions, and Section 7 concludes. All proofs are in the supplemental Internet Appendix.

2 Model

In this Section, we present the setup of the model and discuss its main assumptions.

2.1 Setup

Consider a firm that is run by a manager on behalf of its shareholders. Everyone is risk-neutral. There are three dates – $t = 0, 1, 2$ – corresponding to the end of a first fiscal period, an intermediate date during a second fiscal period, and the end date of the second fiscal period.

[6] Also related are Brandenburger and Polak (1996), Gentzkow and Shapiro (2006), Graham (1999), Ottaviani and Sørensen (2006), Prendergast (1993), Prendergast and Stole (1996), and Trueman (1994).

The firm's assets. At $t = 0$, the firm has assets in place that will generate a cash-flow $\pi \in \{0, 1\}$ at $t = 2$. The cash-flow depends on the state of the world $\omega \in \{H, L\}$ as follows:

$$\Pr(\pi = 1|\omega = H) = \Pr(\pi = 0|\omega = L) = p \tag{1}$$

with $p \in (1/2, 1)$. Hence, a high cash-flow is more likely if the state is high ($\omega = H$) than if it is low ($\omega = L$). In the absence of additional information, the two states are equally likely: $\Pr(\omega = L) = \Pr(\omega = H) = 1/2$.

The accounting rules. The value of the assets in place is reported in the firm's financial statements according to the prevailing accounting rules. Under mark-to-market accounting, the assets' book value is equal to their current market value. Under historical cost accounting, the assets' book value is equal to their historical cost (less any accumulated depreciation), which we assume to be uninformative about ω. In the latter case, the assets' current market value remains unknown to the shareholders.

This is the only difference between the two accounting rules in our setup: Under mark-to-market accounting, the financial statements reveal information that the firm's shareholders would not be able to obtain under historical cost accounting. The reason is that the shareholders do not know exactly which assets the firm owns. The idea is that there are many different types of assets in the economy, and the shareholders cannot observe precisely which assets were acquired by the firm. Therefore, they cannot compute the current market value of the firm's assets on their own (even though market prices for these assets exist).[7] Instead, the shareholders must rely on the financial statements, which are released at the end of each fiscal period.

The assets' market price. Assets just like those owned by the firm are traded in a market with three types of investors:[8] a unit mass of atomistic, risk-neutral speculators, noise traders, and competitive, risk-neutral market makers. At $t = 0$, all speculators receive a common signal $\sigma \in \{H, L\}$ about ω. With probability ϕ,

[7] Alternatively, we could assume that learning about the market value of the assets is prohibitively costly to the shareholders if they do so on their own, i.e., without relying on the financial statements.

[8] For example, the assets could be financial securities traded on an exchange.

the signal is informative (i.e., the signal's type is $\theta_M = i$), in which case it perfectly reveals the state:

$$\Pr(\sigma = H|\omega = H, \theta_M = i) = \Pr(\sigma = L|\omega = L, \theta_M = i) = 1. \tag{2}$$

Otherwise, the signal is uninformative ($\theta_M = u$), in which case it is pure noise:

$$\Pr(\sigma = H|\omega = H, \theta_M = u) = \Pr(\sigma = L|\omega = L, \theta_M = u) = \frac{1}{2}. \tag{3}$$

The noise traders' aggregate demand for the assets is $+1$ unit or -1 unit with equal probability, irrespective of the state of the world. The speculators can buy or sell one unit of the asset after observing σ. As in Kyle (1985), the total order flow $f \in \{-2, 0, 2\}$ of speculators and noise traders is absorbed by the market makers at a price such that they break even in expectation, given the information contained in the total order flow.

The firm's manager. The manager's task is to choose an amount $a \in \mathbb{R}_+$ to invest in a new project at $t = 1$. This investment affects the total cash flows of the firm but not the cash flows of the assets in place.

Unlike the shareholders, the manager knows the exact nature of the firm's existing assets and observes their market price irrespective of the accounting rules. In addition, he receives a private signal $s \in \{H, L\}$ about ω that is (fully) informative if the manager is good (type $\theta_A = g$) and pure noise if he is bad (type $\theta_A = b$), i.e.,

$$\Pr(s = H|\omega = H, \theta_A = g) = \Pr(s = L|\omega = L, \theta_A = g) = 1 \tag{4}$$

and

$$\Pr(s = H|\omega = H, \theta_A = b) = \Pr(s = L|\omega = L, \theta_A = b) = \frac{1}{2}. \tag{5}$$

Neither the manager nor the shareholders know the manager's type with certainty. However, their common prior that the manager is good is $\Pr(\theta_A = g) \equiv \delta$.

The final payoffs. At $t = 2$, the firm's final profit $\Pi(\pi, a)$ – net of any costs – is generated by the assets in place and the new project. We assume that $\Pi(\pi, a)$ satisfies

$$\Pi(1,a) > \Pi(0,a) \quad (6)$$

$$\frac{\partial \Pi}{\partial a}(1,a) > \frac{\partial \Pi}{\partial a}(0,a) \quad (7)$$

$$\frac{\partial^2 \Pi}{\partial a^2}(\pi,a) < 0 \quad (8)$$

for all $a \in \mathbb{R}_+$ and $\pi \in \{0,1\}$. That is, we assume that, for any given level of investment in the new project, the firm's profit is increasing in the payoff of the assets in place. Further, the marginal benefit of investing in the new project is increasing in the payoff of the assets in place, and the firm's profit is concave in the level of investment. Finally, we assume that for any given combination of parameters and signal realizations a unique $a^* \in (0, \infty)$ exists that maximizes the firm's expected profit.

Also at $t = 2$, the investment in the new project (a), the payoff of the assets in place (π), and the firm's final profit (Π) are reported in the financial statements. Using this information, the shareholders can update their beliefs regarding the probability that the manager is good. The manager then obtains a benefit $B \cdot \Pr(\theta_A = g | I_2)$, where I_2 is the shareholders' information at $t = 2$, and $B > 0$. That is, we assume that the manager derives utility from being considered good. This utility captures the manager's career concerns.[9]

Figure 1 summarizes the timing of events and decisions.

[Figure 1 about here.]

2.2 Discussion of assumptions

We have made several simplifying assumptions, which we discuss below.

[9] Such career concerns could be modeled explicitly by adding a further period to the setup, during which competition for good managers links compensation to perceived ability. In that case, assuming that legal constraints prevent involuntary servitude (including bonding), the manager's freedom to leave for a better-paid job creates incentives to improve his reputation (e.g., Holmström and Ricart i Costa (1986)).

Role of mark-to-market accounting. The proponents of marking to market often argue that it provides more timely and accurate information than historical cost accounting and that this information can help monitor a firm's management. Our model captures this idea: Marking to market provides new information to the shareholders about the value of the assets in place, allowing them to better assess which investment level is optimal. Note, however, that marking to market does not *create* new information. In this sense, marking to market is different from going public (which can lead to the production of new information).

Relevance of the assets' value for the investment decision. We assume that the marginal benefit of investing in the new project is increasing in the payoff of the assets in place. This assumption captures the notion that information about the market value of the firm's assets is relevant for optimal managerial decisions. This could be the case, for example, because the existing assets and the new project are complements. Alternatively, the expected cost of undertaking the new project could depend on the expected payoff of the assets in place because of financing constraints and bankruptcy costs: Investing a lot in the new project may cause distress or refinancing costs if the project fails and the existing assets do not generate a sufficiently high cash flow to cover the shortfall.

Incentive compensation. The assumption that the manager is motivated only by reputation concerns is for tractability. In Section 6, we show that our key findings remain unchanged (even in the presence of optimal incentive contracts) if the manager cares both about his reputation as well as monetary payoffs.

Observability of prices, payoffs, and the firm's assets. We assume that the market prices and realized payoffs of the different assets in the economy are publicly observable. Thus, our model is most applicable to assets with observable market prices and payoffs – a condition met by many financial securities for which mark-to-market accounting is relevant. Managers, both good and bad, know which assets a firm owns and can compute their current market value irrespective of the accounting rules.[10] Outsiders, however, do not know precisely which assets the firm owns. This assumption corresponds to the fact that while a firm may list a position such as "financial instruments owned" on its balance sheet, the reported information (including any break-down in the footnotes) does typically not reveal exactly which securities the firm owns. Hence, outside investors cannot compute the current market value of the firm's assets on their own, and mark-to-market accounting indeed provides them with new information.

Feedback effects between prices and investment decisions. Several papers (see Bond, Edmans, and Goldstein (2012) for a comprehensive review) examine possible interactions between market prices and investment decisions. In the typical setup in this literature, a manager tries to infer information from the market price of a firm and then uses this information to decide on an action that affects the future cash-flows of the firm. This creates a feedback-loop: The market price of the firm affects the manager's action, and his (expected) action affects the market price of the firm.

In our model, there is no such feedback effect. The reason is that the manager's action does not affect the future cash flows of the assets whose price he uses to decide on the action. The manager infers information from the market price of the assets in place (e.g., financial securities owned by the firm) and then decides how much to invest in a new project. This project, however, does not affect the future cash flows of the assets in place and thus has no effect on their market price.[11]

We believe that this assumption is appropriate when one wants to examine the effect of marking to market, i.e., requiring a firm to report the market value of (some of) its assets. Consider, for example, the case of a bank that owns a portfolio of traded securities. The bank's management may infer information

[10] The only difference between good and bad managers is that good managers receive additional, useful, private information about the expected future payoff of the firm's assets that is not contained in their market price.

[11] Note that this is the price at which assets that the firm owns are traded – not the price of the firm itself. The latter would include the expected value of investing in the new project, but not the former.

from the market price of these securities and then use this information when deciding how much to invest in a new project. Investing in the project affects the future cash flows of the bank but is unlikely to affect the future cash flows of the securities owned by the bank. The price at which these securities are traded in the market – and hence the value that the bank would report under mark-to-market accounting – is thus unlikely to reflect expectations about future investments in new projects that the bank owning the securities may undertake.

Timing of the arrival of private information. We assume that the manager receives his private signal after the financial statements are published, but before he decides how much to invest in the new project. This assumption is motivated by the observation that financial statements are published only periodically, so that it is likely that (additional) private information arrives after their publication (i.e., during the fiscal year). In that case, the manager must take any information that has already been revealed in the financial statements as given and only decides how to respond to his new, private information. However, none of our results change if we assume instead that the private signal is received before the financial statements are released as long as the value of the firm's assets cannot be misreported (conditional on being reported).[12]

Information about the manager's type. We assume that neither the manager nor the shareholders know with certainty whether the manager's private signal is informative. Ex ante, both believe the probability to be δ. This does not mean that the manager cannot have any information regarding the probability that he is good, but merely that there is no asymmetric information ex ante between the manager and the shareholders regarding this probability. We relax this assumption in an extension in Section 6.

State space and signal structure. We assume that there are only two possible realizations of the state of the world, the market signal, and the private signal. In an extension of our model (available upon request)

[12] In that case, the market value of the assets is truthfully reported in the financial statements, and, thereafter, the manager makes his investment decision – exactly as in the case in which the private signal is received after the financial statements are published.

we consider the case of $N > 2$ possible realizations. The driving force behind our findings arises also in this more general setup: More informative market prices increase the manager's incentives to pretend that his private signal coincides with the market signal (relative to revealing a divergence between the signals). This result extends to the limit case of a continuous state and signal space provided there is a non-zero probability that two uninformative signals coincide.

3 Investment strategies

We now examine the effects of the accounting rules on the manager's investment decision. First, we compute the market price of the firm's existing assets at $t = 0$ (which is reported in the firm's financial statements under mark-to-market accounting but not under historical cost accounting). Second, we characterize the first-best investment. Then, we discuss the agency problem in our setup and explore whether the first-best investment is an equilibrium outcome. Finally, we examine which investments can be equilibrium outcomes and characterize the Pareto-dominant equilibria.

3.1 Market prices and information revealed by the reported asset values

The price of the firm's existing assets at $t = 0$ is equal to $E[\pi|f]$. In equilibrium, the speculators submit buy orders after receiving a high signal ($\sigma = H$) and sell orders after a low signal ($\sigma = L$).[13] We thus have

$$E[\pi|f = 2] = \Pr(\omega = H|\sigma = H) \cdot p + \Pr(\omega = L|\sigma = H) \cdot (1 - p) = \frac{1}{2} + \phi\left(p - \frac{1}{2}\right) \tag{9}$$

if both speculators and noise traders buy. If both groups sell, the price is equal to

$$E[\pi|f = -2] = \Pr(\omega = H|\sigma = L) \cdot p + \Pr(\omega = L|\sigma = L) \cdot (1 - p) = \frac{1}{2} - \phi\left(p - \frac{1}{2}\right). \tag{10}$$

Finally, if speculators buy and noise traders sell or vice versa, the price is equal to

$$E[\pi|f = 0] = \Pr(\omega = H) \cdot p + \Pr(\omega = L) \cdot (1 - p) = \frac{1}{2}. \tag{11}$$

[13] The speculators' (ex ante) expected profit from following this strategy is $\phi(p - 1/2)/2$.

The assets' current market value thus reveals the speculators' signal if the price is equal to $E\left[\pi|f=2\right]$, which indicates $\sigma = H$, or if the price is equal to $E\left[\pi|f=-2\right]$, which indicates $\sigma = L$. Mark-to-market accounting makes this information available to the shareholders: They can infer the speculator's signal from the assets' current market value as reported in the firm's financial statements. Henceforth, we therefore treat mark-to-market accounting as directly revealing σ to the shareholders. For brevity, we refer to this information as "the market signal." Under historical cost accounting, instead, the market signal is not revealed by the assets' book value. In that case, the financial statements report the assets' historical cost (less any accumulated depreciation), which is uninformative about the speculators' signal.

The assets' market value is uninformative about the speculators' signal if the price is equal to $E\left[\pi|f=0\right]$. In that case, the distinction between historical cost and mark-to-market accounting is uninteresting. For this reason, we restrict attention to informative prices from now on and suppress the case of an uninformative price realization. Note, however, that doing so does not change any of our results. The reason is that allowing for uninformative price realizations would simply mean that there is always some probability that the firm's financial statements do not reveal the speculators' information even under mark-to-market accounting.[14]

3.2 First-best investment strategy

At $t = 1$, the first-best investment level conditional on σ and s is

$$a^*_{\sigma s} \in \arg\max_{a \geq 0} \Pr\left(\pi = 1|\sigma, s\right) \cdot \Pi\left(1, a\right) + \Pr\left(\pi = 0|\sigma, s\right) \cdot \Pi\left(0, a\right). \tag{12}$$

Given $\Pi\left(\pi, a\right)$'s concavity in a, the unique solution is given by the first order condition

$$\frac{\partial \Pi}{\partial a}\left(0, a^*_{\sigma s}\right) + \Pr\left(\pi = 1|\sigma, s\right) \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a^*_{\sigma s}\right) - \frac{\partial \Pi}{\partial a}\left(0, a^*_{\sigma s}\right)\right] = 0. \tag{13}$$

[14] All results pertaining to the case of an uninformative price can be recovered by setting $\phi = 0$ in our analyses. Moreover, the probability that the price is uninformative could be made arbitrarily small without affecting any of our results by assuming that noise traders buy or sell with probability $\lambda/2$ each and do not trade with probability $1 - \lambda$ for some very small $\lambda > 0$.

The first-best investment level is monotonically increasing in the probability that the assets in place have a high payoff, $\Pr(\pi = 1|\sigma, s)$.[15] Further, this probability is a one-to-one function of the market signal and the private signal (with the exception of the special case $\delta = \phi$). Hence, the four possible combinations of σ and s lead to four distinct first-best investment levels a^*_{HH}, a^*_{HL}, a^*_{LH}, and a^*_{LL}.

3.3 Agency problem

The firm's shareholders must rely on the manager to make the investment decision. However, unlike the shareholders, who care about the firm's final profit, the manager only cares about the benefit he derives from the shareholders' posterior belief regarding his quality.[16] He is motivated only by reputation concerns and thus maximizes the expected posterior belief that he is good, conditional on the information I_2 that the shareholders will infer at $t = 2$. That is, he chooses the investment that maximizes $E\left[\Pr\left(\theta_A = g|I_2\right)|\sigma, s\right]$.

Depending on the accounting rules and the investment decision, the information that the shareholders infer in equilibrium at $t = 2$ is $I_2 \in \{(\pi), (\pi, \sigma), (\pi, s), (\pi, \sigma, s)\}$.[17] They always observe the cash-flow (π) generated by the assets in place. They may also learn the market signal (σ) if there is mark-to-market accounting or if the manager's investment decision reveals σ. If his investment decision reveals the manager's private signal (s), the shareholders can also learn s. Finally, if the manager's investment decision reveals both his private signal and the market signal, the shareholders learn everything: π, σ, and s. This happens, for example, if the manager follows the first-best strategy in equilibrium and invests $a^*_{\sigma s}$.

[15]Specifically, we have $\frac{da^*_{\sigma s}}{d\Pr(\pi=1|\sigma,s)} = -\frac{\frac{\partial \Pi}{\partial a}(1, a^*_{\sigma s}) - \frac{\partial \Pi}{\partial a}(0, a^*_{\sigma s})}{\Pr(\pi=1|\sigma,s)\cdot \frac{\partial^2 \Pi}{\partial a^2}(1, a^*_{\sigma s}) + \Pr(\pi=0|\sigma,s)\cdot \frac{\partial^2 \Pi}{\partial a^2}(0, a^*_{\sigma s})} > 0$.

[16]In an extension in which the manager cares about both his reputation and monetary payoffs, we show that our key findings remain unchanged even in the presence of optimal incentive contracts (Section 6).

[17]The shareholders directly observe (a, π, Π) under historical cost accounting and $(E[\pi|\sigma], a, \pi, \Pi)$ under marking to market. Under marking to market, σ can be inferred from $E[\pi|\sigma]$, the assets' price at $t = 0$. Further, depending on the investment strategy followed in equilibrium, σ and s may be inferred from a. Given that only π, σ, and s are relevant for updating the shareholders' belief regarding the manager's skill, we refer to the relevant information set directly as $I_2 \in \{(\pi), (\pi, \sigma), (\pi, s), (\pi, \sigma, s)\}$.

However, the manager's objective differs from that of the shareholders. Thus, it is not clear that the manager chooses the first-best investment. Further, because the shareholders' information depends on the firm's financial reports, switching from historical cost to mark-to-market accounting or vice versa can ameliorate or aggravate the agency problem.[18]

3.4 The first-best investment is not always an equilibrium outcome

If the manager were to follow the first-best strategy in equilibrium, his investment decision would reveal both the market signal (σ) and his private signal (s). The reason is that the four possible combinations of σ and s lead to four distinct first-best investments. Revealing σ and s, however, is in the manager's interest only if doing so maximizes the expected posterior belief that he is good. We show that this is not always the case.

Proposition 1 *Under historical cost accounting, a perfect Bayesian equilibrium (PBE) in which the manager follows the first-best investment strategy does not exist.*

The intuition is as follows. Under historical cost accounting, the firm's financial statements do not reveal the market price of the assets in place at $t = 0$ to the shareholders. This implies that the shareholders do not directly learn the market signal. Of course, they do not directly observe the manager's private signal either.

Assume now an equilibrium existed in which the manager follows the first-best strategy. The shareholders would learn the market signal (σ), the manager's private signal (s), and the assets' payoff (π). Based on this information, they would then update their beliefs about the manager's quality. For any given private signal, however, the posterior probability that the manager is good is larger if the market signal coincides with the private signal than if the signals diverge. Hence, under historical cost accounting, following the first-

[18] Allowing for sophisticated investors that know the market value of the firm's assets does not change our main results. All that is needed is that there are at least some investors who can learn this information only from the financial statements and that the manager cares about his reputation as perceived by these investors.

best investment strategy is not incentive-compatible: The manager would always prefer to pretend that the market signal coincides with his private signal. He would thus either choose an investment level indicating $\sigma = s = H$ or an investment level indicating $\sigma = s = L$, but never one indicating $\sigma \neq s$.

The situation is different under marking to market. In that case, the shareholders directly learn the market signal from the firm's financial statements. They still do not directly observe the manager's private signal. However, if the manager were to follow the first-best strategy, his investment decision would reveal his private signal, too. We can show that following the first-best investment strategy is indeed an equilibrium outcome under mark-to-market accounting if and only if the quality of the market signal is sufficiently low.

Proposition 2 *Under mark-to-market accounting, a unique $\phi^* \in (0, \delta)$ exists such that a PBE in which the manager follows the first-best investment strategy exists if and only if $\phi \leq \phi^*$.*

The intuition is as follows. In a PBE in which the manager follows the first-best strategy, the shareholders form posterior beliefs about his quality based on π, σ, and s. However, the manager may want to deviate from an investment level that reveals a divergence between the market and his private signal. The difference compared to the case of historical cost accounting is that the manager can only misreport his private signal but not the market signal (which is revealed directly to the shareholders in the financial statements). Thus, the manager cannot choose between investment levels indicating $\sigma = s = H$ or $\sigma = s = L$. Instead, he must take the market signal as given and choose between investment levels indicating $s = \sigma$ or $s \neq \sigma$.

In that situation, the manager may prefer an investment level indicating $s \neq \sigma$ if he is sufficiently more confident about his private signal than the market signal. Hence, whether following the first-best strategy is incentive compatible depends on the quality of the market signal relative to the manager's private information. Revealing a divergence between the signals is less costly when the market signal is less informative. Indeed, it can be shown that a unique threshold $\phi^* < \delta$ exists such that following the first-best strategy is incentive compatible if and only if $\phi \leq \phi^*$. A change from historical cost to mark-to-market accounting can thus lead to the first-best investment if the quality of the market signal is low, but not when the quality of the market signal is high. In that case, the information content of the market signal distorts the manager's decision: Choosing an investment revealing that his private signal differs from the (highly informative) market signal would damage the manager's reputation.

3.5 Equilibrium investment strategies

Given that an equilibrium in which the manager follows the first-best investment strategy does not always exist, we now examine which investment strategies can be followed in equilibrium. For the purpose of this analysis, we focus on PBE in pure strategies. However, we show in the Internet Appendix that also considering equilibria in mixed strategies does not change our main results.

For any combination of parameters and accounting rules, our model has multiple pure-strategy PBE. We characterize all of these equilibria in the Internet Appendix. The manager is ex ante indifferent between the different equilibria because his reputation is a martingale (i.e., $E\left[\Pr\left(\theta_A = g|I_2\right)\right] = \delta$). The shareholders, however, prefer the equilibrium with the highest expected profits for the firm, which is unique in our setup. Given the manager's indifference, this equilibrium Pareto-dominates the other equilibria and thus provides a natural focal point. For this reason, we assume that for any given combination of parameters and accounting rules, the shareholders and the manager coordinate on the unique Pareto-dominant equilibrium.[19]

Importantly, if following the first-best strategy is not an equilibrium outcome, then following any other strategy that reveals both the market signal (σ) and the private signal (s) to the shareholders cannot be an equilibrium outcome either. In that case, one can show that the only strategies that can be followed in equilibrium are such that the manager's investment decision reveals only σ, only s, or neither σ nor s.

The manager is indifferent between all strategies that reveal the same information to the shareholders. The shareholders, however, prefer the strategy that uses the revealed information optimally. That is, among

[19]To be a little more precise, the manager's investment decisions and the shareholders' beliefs on the equilibrium path are unique. The out-of-equilibrium beliefs that can sustain the Pareto-dominant equilibrium are, of course, not unique.

all investment strategies that reveal only s, the shareholders prefer

$$a_s^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1|s) \cdot \Pi(1,a) + \Pr(\pi = 0|s) \cdot \Pi(0,a), \tag{14}$$

given by the first order condition

$$\frac{\partial \Pi}{\partial a}(0, a_s^{**}) + \Pr(\pi = 1|s) \cdot \left[\frac{\partial \Pi}{\partial a}(1, a_s^{**}) - \frac{\partial \Pi}{\partial a}(0, a_s^{**})\right] = 0. \tag{15}$$

We refer to this strategy as the "s-strategy."

Among all investment strategies that reveal only σ, the shareholders prefer

$$a_\sigma^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma) \cdot \Pi(1,a) + \Pr(\pi = 0|\sigma) \cdot \Pi(0,a), \tag{16}$$

given by the first order condition

$$\frac{\partial \Pi}{\partial a}(0, a_\sigma^{**}) + \Pr(\pi = 1|\sigma) \cdot \left[\frac{\partial \Pi}{\partial a}(1, a_\sigma^{**}) - \frac{\partial \Pi}{\partial a}(0, a_\sigma^{**})\right] = 0. \tag{17}$$

We refer to this strategy as the "σ-strategy."

While the manager is ex ante indifferent, the shareholders always prefer the σ-strategy and the s-strategy to any other strategy that reveals neither σ nor s. Among the σ-strategy and the s-strategy, they prefer the σ-strategy if the market signal is more informative than the private signal ($\phi > \delta$). Otherwise, they prefer the s-strategy.[20] We obtain the following results:

Proposition 3 *Under historical cost accounting, in the Pareto-dominant PBE in pure strategies, the manager invests a_s^{**} given by Equation (15) if $\phi \leq \delta$ and otherwise a_σ^{**} given by Equation (17).*

Proposition 4 *Under mark-to-market accounting, in the Pareto-dominant PBE in pure strategies, the manager invests $a_{\sigma s}^{*}$ given by Equation (13) if $\phi \leq \phi^*$ and otherwise a_σ^{**} given by Equation (17).*

If the manager follows the σ-strategy, the shareholders do not update their beliefs about his quality as no new information about the manager's private signal is revealed. As a consequence, the manager is

[20]For $\phi = \delta$, the shareholders are indifferent between the σ-strategy and the s-strategy.

indifferent between the two equilibrium investment levels $a^{**}_{\sigma=H}$ and $a^{**}_{\sigma=L}$. Hence, following the σ-strategy is an equilibrium outcome for any combination of parameters and accounting rules. This equilibrium can be supported, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\tilde{a}) = 0$ for any out-of-equilibrium investment $\tilde{a}$.

If the manager follows the s-strategy, under historical cost accounting, the shareholders form posterior beliefs about his quality based on the manager's private signal and the ex post realized cash-flow (π). It follows that the manager prefers investing $a^{**}_{s=H}$ to $a^{**}_{s=L}$ if he believes that $\pi = 1$ is more likely than $\pi = 0$ (and vice versa). As a consequence, he prefers $a^{**}_{s=H}$ to $a^{**}_{s=L}$ after receiving the private signal $s = H$ (and $a^{**}_{s=L}$ to $a^{**}_{s=H}$ after receiving $s = L$) if and only if $\phi \leq \delta$. To see why, consider the case of $s = H$ and $\sigma = L$: The manager believes that $\pi = 1$ is more likely than $\pi = 0$ and prefers $a^{**}_{s=H}$ only if his private signal is at least as likely to be informative than the market signal. Hence, following the s-strategy is an equilibrium outcome under historical cost accounting if and only if $\phi \leq \delta$. This equilibrium can be supported, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\tilde{a}) = 0$ for any out-of-equilibrium investment $\tilde{a}$.

The situation is different under mark-to-market accounting. In that case, if the manager follows the s-strategy, the shareholders learn the market signal from the financial statements and the private signal from the investment decision. Thus, the shareholders' posterior beliefs regarding the probability that the manager is good and the manager's incentive compatibility constraints are exactly as in the case of the first-best strategy. As a consequence, following the s-strategy is an equilibrium outcome if and only if $\phi \leq \phi^*$.

Figure 2 presents our results in the parameter space (δ, ϕ).

[Figure 2 about here.]

4 Marking to market versus historical cost accounting

We now compare the firm's expected profits under the different accounting rules. As in Section 3, for any given combination of parameters and accounting rules, we focus on the Pareto-dominant PBE in pure strategies. Figure 3 shows the firm's expected profits under historical cost accounting, marking to market, and in the first-best case. It plots the expected profits ($E[\Pi]$) as a function of ϕ under the specific assumptions that $\Pi(\pi, a) = \pi a - a^2$, $p = 0.95$, and $\delta = 0.8$.[21]

[Figure 3 about here.]

When the market signal is not too informative ($\phi \leq \phi^*$), the expected profit under mark-to-market accounting is identical to the first-best case. It increases with ϕ as a more informative market signal improves the manager's investment decision. Under historical cost accounting, the expected profit is both lower and unaffected by the informativeness of the market signal because only the manager's private information is used to choose the level of investment. Hence, if the market price of the assets in place is not too informative ($\phi \leq \phi^*$), the firm's shareholders prefer marking to market to historical cost accounting.

However, as the informativeness of the market signal increases beyond ϕ^* – but still remains below the informativeness of the manager's private signal (δ) – the expected profit under marking to market falls below the expected profit under historical cost accounting.[22] Decisions under mark-to-market accounting are now only based on the market signal, which is less informative than the manager's private signal. This entails a decrease in the firm's expected profits under marking to market. The two main results of our analysis are an

[21]These assumptions are made to facilitate the graphical representation. Similar results obtain for generic parameter combinations.

[22]When the informativeness of the market signal increases beyond the informativeness of the private signal, the investment decisions under historical cost and mark-to-market accounting coincide: Both are based only on the market signal. Thus, there is no difference between marking to market and historical cost accounting when $\phi > \delta$. As ϕ reaches one, the expected profits under both accounting rules converge to the first-best profits because the market signal becomes perfectly informative.

immediate consequence of this effect:

Main Result 1 *More informative prices can make a firm's shareholders worse off if its assets must be marked to market (but not under historical cost accounting).*

Main Result 2 *A firm's shareholders may prefer mark-to-market accounting if prices are less informative but historical cost accounting if prices are more informative.*

The intuition is as follows. Moving from $\phi = \phi'$ to $\phi = \phi''$ with $\phi' < \phi^* < \phi'' < \delta$ increases the informativeness of the market signal by $\Delta\phi = \phi'' - \phi' > 0$. This increase, however, causes a change in the manager's equilibrium behavior under marking to market: He no longer relies on both the market and his private signal. Instead, the manager stops using his private signal and relies exclusively on the information conveyed by the market. The total amount of information that is used to make the investment decision therefore drops by $\delta - \Delta\phi > 0$. As a result, the firm's expected profits under mark-to-market accounting are now lower than before. Under historical cost accounting, instead, the firm's expected profits are not affected by the increase in ϕ because the manager's investment decision is based only on his private signal.

5 Empirical predictions

The informativeness of prices and private signals is typically unobservable and difficult to estimate. As a consequence, several immediate implications of our theory do not lend themselves to simple empirical tests. Thus, we now highlight two predictions based on variables that are more easily observed or estimated.

The first prediction concerns the correlation of the investment decisions of firms that own the same type of assets. If the market price of a firm's assets is relatively uninformative ($\phi \leq \phi^*$), marking to market allows for investment decisions that are based on the market signal in addition to the manager's private signal. If, however, the assets' market price is relatively informative ($\phi > \phi^*$), investment decisions under marking to market are based only on the market signal. Under historical cost accounting, instead, investment decisions are either based only on the private signal (if $\phi \leq \delta$) or only on the market signal (if $\phi > \delta$).

It follows that the average effect (across all values of ϕ) of switching from historical cost to mark-to-market accounting is an increase in the sensitivity of the investment decisions to the market signal. As all firms with the same assets receive the same market signal (while different managers may receive different private signals), changing from historical cost to mark-to-market accounting thus increases the correlation of the firms' investment decisions. Hence, our model predicts that marking to market increases the correlation of the investment decisions of firms that own the same type of assets (relative to historical cost accounting):

Prediction 1 *A change from historical cost accounting to mark-to-market accounting increases the correlation of the investment decisions of firms that own the same type of assets.*

The second prediction concerns the efficiency of investment decisions and thus the effect on firms' values. For a firm whose assets have a relatively uninformative market price ($\phi \leq \phi^*$), changing from historical cost to mark-to-market accounting improves the efficiency of the investment decisions. If, however, the informativeness of the price satisfies $\phi^* < \phi < \delta$, marking to market decreases the efficiency of the investment decisions (relative to historical cost accounting). For $\phi \geq \delta$, a change from historical cost to mark-to-market accounting does not affect the decisions' efficiency.

Further, the increase in the sensitivity of the investment decisions to the market signal due to a switch to marking to market is smaller in case of $\phi \leq \phi^*$ than in case of $\phi^* < \phi \leq \delta$. For $\phi > \delta$, the sensitivity to the market signal does not change. Hence, a change from historical cost accounting to marking to market should generate a negative correlation between the change in the efficiency of investment decisions (as measured, for example, by Tobin's Q or return on assets) and the change in their sensitivity to the market signal. Given that a higher sensitivity to the market signal leads to a stronger correlation of the investment decisions of different firms that own the same type of assets, our model generates the following prediction:

Prediction 2 *The change in the correlation of firms' investment decisions (due to a change from historical cost to mark-to-market accounting) is inversely related to the change in their profitability.*

6 Extensions

We now consider five extensions of our model. First, we study how the accounting rules affect the types of assets that firms choose to hold on their balance sheets. Second, we consider voluntary disclosure and fraudulent reporting. Third, we investigate the effect that incentive compensation may have on our findings. Fourth, we discuss possible interactions between marking to market and the informativeness of market prices. Fifth, we examine the case of ex ante asymmetric information between the manager and the shareholders regarding the quality of the manager's private signal.

6.1 Responding to mark-to-market accounting: holding opaque assets

Throughout our analysis, we have treated the informativeness of the market signal as given. A firm, however, may respond to accounting regulation by changing the composition of its assets. In this extension, we assume that the firm's shareholders can give a mandate to the manager to only hold assets that belong to a specific class of assets. Further, we assume that different asset classes are distinguished by the informativeness of their market prices. The manager may thus be mandated to hold transparent assets on the firm's balance sheet, whose market prices are relatively informative. Or he may be mandated to hold opaque assets with less informative market prices.[23] Using this framework, we obtain the following result:

Proposition 5 *Suppose that the firm's shareholders can give a mandate to the manager to only hold assets that belong to a specific class of assets with price informativeness $\phi \leq \bar{\phi}$ for some $\bar{\phi} < 1$. Under historical cost accounting, the shareholders choose to mandate $\phi = \bar{\phi}$ if $\bar{\phi} > \delta$ and are indifferent between all ϕ if*

[23] We maintain the assumption, however, that the shareholders do not know exactly which assets the manager has chosen from the class of assets he is mandated to hold on the firm's balance sheet.

$\bar{\phi} \leq \delta$. Under mark-to-market accounting, a unique $\bar{\delta} \in \left(0, \bar{\phi}\right)$ exists such that for $\delta < \bar{\delta}$ the shareholders mandate $\phi = \bar{\phi}$, and for $\delta \geq \bar{\delta}$ the shareholders mandate $\phi = \min\left\{\phi^*, \bar{\phi}\right\}$.

Intuitively, under historical cost accounting, for $\phi > \delta$, the manager follows the σ-strategy, which depends only on the market signal. In that case, the firm's expected profit is strictly increasing in the market signal's informativeness. Thus, the shareholders choose the highest feasible informativeness $\left(\phi = \bar{\phi}\right)$, provided that $\bar{\phi} > \delta$. For $\phi \leq \delta$, the manager follows the s-strategy, which depends only on his private signal. In that case, the firm's expected profit does not depend on the informativeness of the market signal. Hence, for $\bar{\phi} \leq \delta$, the shareholders are indifferent between all ϕ.

Under marking to market, the firm's expected profit is strictly increasing in the informativeness of the market signal for $\phi \leq \phi^*$ and for $\phi > \phi^*$. However, the firm's expected profit drops at $\phi = \phi^*$. At this point, the manager switches from following the first-best strategy, which is based on both the market signal and the private signal, to the σ-strategy, which is based only on the market signal. Using these results, it can be shown that a unique $\bar{\delta} \in \left(0, \bar{\phi}\right)$ exists such that for $\delta < \bar{\delta}$ the firm's shareholders mandate $\phi = \bar{\phi}$, and for $\delta \geq \bar{\delta}$ the shareholders mandate $\phi = \min\left\{\phi^*, \bar{\phi}\right\}$.

The intuition is as follows. If the manager's private signal is sufficiently uninformative $\left(\delta < \bar{\delta}\right)$, the shareholders are better off if the manager relies only on the market signal. In that case, the most informative market signal $\left(\phi = \bar{\phi}\right)$ is optimal. If, however, the manager's private signal is sufficiently informative $\left(\delta \geq \bar{\delta}\right)$, the shareholders are better off if the manager uses his private signal. In that case, the best the shareholders can do is to choose the highest level of price informativeness that does not prevent the manager from following the first-best investment strategy $\left(\phi = \min\left\{\phi^*, \bar{\phi}\right\}\right)$.

Which types of assets the firm optimally holds on its balance sheet thus depends both on the accounting rules and on the quality of the manager to whom subsequent investments are delegated. Under historical cost accounting, the firm's shareholders prefer the most transparent asset class if its price informativeness exceeds the informativeness of the manager's private information. If not, the shareholders are indifferent among the various asset classes. Under mark-to-market accounting, however, the shareholders either choose the asset class with the highest transparency or a more opaque asset class. If the manager's quality is lower than a given threshold, the most transparent asset class is chosen. If the manager's quality is higher than the threshold, the shareholders prefer more opaque assets. Our analysis thus suggests that if mark-to-market accounting is mandatory, firms – in particular, those with good managers – may optimally respond by holding more opaque assets with less informative market prices on their balance sheets.[24]

6.2 Voluntary disclosure and fraudulent reporting

So far, we did not explicitly consider any type of voluntary disclosure or fraudulent reporting. We assumed that the manager must either report the market value of the firm's assets truthfully under mark-to-market accounting (and thereby reveal the market signal) or not at all under historical cost accounting. In this extension, we assume instead that the manager can voluntarily disclose and/or misreport the market signal.[25] In that case, we obtain the following result:

Proposition 6 *In the Pareto-dominant PBE in pure strategies for $\phi \leq \phi^*$, the manager reveals the market signal truthfully and follows the first-best investment strategy. For $\phi \in (\phi^*, \delta)$, the manager does not reveal the market signal and follows the s-strategy. For $\phi \geq \delta$, the manager reveals the market signal truthfully and follows the σ-strategy.*

The intuition is that the option to voluntarily disclose or misreport effectively allows the firm to choose between historical cost and mark-to-market accounting. For $\phi \leq \phi^*$, marking to market dominates historical cost accounting. Hence, the shareholders prefer the manager to reveal the market signal. For $\phi \in (\phi^*, \delta)$,

[24]Note that the manager is indifferent between the various asset classes: From an ex ante point of view, his expected reputation is a martingale and equal to δ, irrespective of ϕ. As a consequence, in the Pareto-dominant PBE, the manager will follow the shareholders' mandate to choose assets from a specific asset class.

[25]Of course, the manager does not reveal the market signal directly but reports the assets' market value (and chooses the level of investment) which may or may not reveal the market signal to the shareholders in equilibrium.

historical cost accounting dominates, and the shareholders prefer that the market signal is not revealed. For $\phi \geq \delta$, they are indifferent between historical cost accounting and marking to market – but the optimal level of investment (a_{σ}^{**}) will reveal the market signal ex post. The manager is ex ante indifferent between revealing and not revealing the market signal because his expected reputation is equal to the prior about his quality in either case. Hence, the Pareto-dominant equilibrium is the one preferred by the shareholders.

6.3 Incentive compensation

To increase the tractability of our model, we have assumed that the manager is motivated only by his reputation concerns but not by any incentive compensation. We now show that relaxing this assumption and allowing for optimal incentive contracts does not change our main results. For the purpose of this extension, we thus assume that both investment levels and realized profits are verifiable and can be contracted upon. However, as is common in the literature on principal-agent problems, we assume that the manager has zero initial wealth. This assumption rules out the trivial solution of selling the entire firm to the manager.

Our analysis reveals that, under mark-to-market accounting, an increase in the probability that the market signal is informative can make the firm's shareholders worse off – even if incentive contracts can be designed that induce the manager to choose the first-best investment:

Proposition 7 *Under mark-to-market accounting, the firm's shareholders prefer $\phi = \phi'$ to $\phi = \phi''$ with $\phi' < \phi^* < \phi'' < \delta$ for all $B > B^*$ with*

$$B^* = 2 \cdot \frac{\Pi^{FB}(\phi'') - \Pi^{FB}(\phi')}{F(\phi'') \cdot (1 - \delta\phi'')}, \tag{18}$$

where $\Pi^{FB}(\phi'')$ and $\Pi^{FB}(\phi')$ denote the firm's expected profit (gross of any incentive pay) if the manager follows the first-best strategy and the probability that the market signal is informative is equal to ϕ'' and ϕ', respectively. $F(\phi'')$ and ϕ^* are defined as in the proof of Proposition 2.

The intuition is as follows. To achieve the first-best investment, the manager must be willing to reveal his private information. Therefore, any contract that induces the manager to choose the first-best investment must compensate him for the loss of reputation that he suffers from revealing a divergence between the market and his private signal. More informative prices increase this loss. As a consequence, more informative prices can increase the cost of achieving the first-best investment and make the firm's shareholders worse off.

An increase in the informativeness of the market signal can further change the relative desirability of marking to market and historical cost accounting. The shareholders may prefer marking to market for less informative prices but historical cost accounting for more informative prices:

Proposition 8 *For $\phi = \phi' \leq \phi^*$, the firm's shareholders always prefer mark-to-market accounting to historical cost accounting. However, for $\phi = \phi''$ with $\phi^* < \phi'' < \delta$, the firm's shareholders prefer historical cost accounting to mark-to-market accounting for all $B > B^{**}$ with*

$$B^{**} = 2 \cdot \frac{\Pi^{FB}(\phi'') - \Pi^{SB}(\phi'')}{F(\phi'') \cdot (1 - \delta\phi'')}, \tag{19}$$

where $\Pi^{SB}(\phi'')$ denotes the firm's expected final profit (gross of any incentive pay) if the manager follows the s-strategy and the probability that the market signal is informative is equal to ϕ''. $\Pi^{FB}(\phi'')$, $F(\phi'')$, and ϕ^* are defined as in Proposition 7.

The intuition is that more informative prices can make achieving the first-best investment under marking-to-market so costly that the shareholders prefer historical cost accounting.

In summary, our analysis reveals that allowing for optimal incentive contracts that induce the manager to follow the first-best strategy does not change our key findings (Main Results 1 and 2). Without incentive pay, more informative prices can make it impossible to achieve the first-best investment. With incentive pay, more informative prices can make achieving the first-best more costly. In either case, more informative prices can make marking to market less desirable.

6.4 Interactions between marking to market and price informativeness

Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2015) show that inferring information from a price to make a decision that, in turn, affects the price can reduce the informativeness of the price from which one wishes to learn. In our setup, the interaction between the informational role of prices and marking to market can also lead to a reduction in price informativeness.[26] However, the reason is not a feedback effect between actions and prices as in Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2015).

In our model, mark-to-market accounting can induce the manager to ignore his private information when choosing the investment in the new project. In that case, market participants cannot infer the manager's private information from his investment decision. As a consequence, the price of the firm's assets in subsequent periods can be less informative than if the manager were to rely on his private information.

To make the above intuition more precise, consider the following extension of our model: Suppose the speculators described in Section 2 know which assets the firm owns and can observe the manager's investment decision. Suppose further that there is another round of trading after the manager has made the investment in the new project (at $t = 1$). As before, the noise traders' aggregate demand is $+1$ unit or -1 unit with equal probability, the speculators can can buy or sell one unit of the asset, and the total order flow is absorbed by the market makers at a price such that they break even in expectation.

In that case, if the manager's investment in the new project does not reveal his private signal, the price of the assets at $t = 1$ can take three different values: $E[\pi]$, $E[\pi|\sigma = L]$, and $E[\pi|\sigma = H]$. If, instead, the manager's investment decision reveals his private signal, the price can take nine different values: $E[\pi]$, $E[\pi|\sigma = L]$, $E[\pi|\sigma = H]$, $E[\pi|s = L]$, $E[\pi|s = H]$, $E[\pi|\sigma = L, s = L]$, $E[\pi|\sigma = L, s = H]$, $E[\pi|\sigma = H, s = L]$, and $E[\pi|\sigma = H, s = H]$.

[26] This finding is also related to Plantin and Tirole (2015), who show that the individually optimal use of marking to market can result in a socially sub-optimal level of price informativeness as firms fail to internalize the negative externality that their use of mark-to-market accounting imposes on the information available to other firms.

The intuition is as follows. Suppose that the manager's investment in the new project does not reveal his private signal (s). In that case, the market makers can learn at most the speculators' signal (σ). It follows that the market price of the assets at $t = 1$ can at most reflect σ. If, instead, the manager's investment in the new project reveals his private signal, the market makers may learn both the speculators' and the manager's signals. In that case, the assets' market price at $t = 1$ may reflect both signals and hence be more informative. We thus obtain the following result:

Proposition 9 *Mark-to-market accounting can reduce the informativeness of the assets' market price at* $t = 1$ *relative to historical cost accounting if* $\phi \in (\phi^*, \delta)$*. If* $\phi \notin (\phi^*, \delta)$*, mark-to-market and historical cost accounting lead to the same price informativeness at* $t = 1$.

6.5 Ex ante asymmetric information about the quality of the private signal

Throughout our analysis, we have assumed that neither the manager nor the shareholders know with certainty whether the manager is good or bad: Both believe that he is good with probability δ. We now examine whether and how our findings change if we assume instead that the manager is ex ante better informed than the shareholders about the quality of his private information.

For the purpose of this analysis, we assume that with probability μ a good manager learns that he is good before he has to make the investment decision. Specifically, the manager receives a private, unverifiable signal $\tau \in \{g, b\}$ before he has to decide on the investment (e.g., at $t = 0$). It is common knowledge that $\Pr(\tau = g|\theta_A = g) = \mu$ and $\Pr(\tau = g|\theta_A = b) = 0$. Hence, observing $\tau = g$ perfectly reveals the manager's quality: $\Pr(\theta_A = g|\tau = g) = 1$. Observing $\tau = b$, instead, does not perfectly reveal his quality: $\Pr(\theta_A = g|\tau = b) = \delta(1-\mu)/(1-\mu\delta) \equiv \delta_b$.

First-best investment strategy. The first-best investment (conditional on τ, σ, and s) is now

$$a^*_{\tau\sigma s} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\tau, \sigma, s) \cdot \Pi(1, a) + \Pr(\pi = 0|\tau, \sigma, s) \cdot \Pi(0, a). \tag{20}$$

Given $\Pi(\pi, a)$'s concavity in a, the unique solution is given by the first order condition

$$\frac{\partial \Pi}{\partial a}\left(0, a^*_{\tau\sigma s}\right) + \Pr\left(\pi = 1 | \tau, \sigma, s\right) \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a^*_{\tau\sigma s}\right) - \frac{\partial \Pi}{\partial a}\left(0, a^*_{\tau\sigma s}\right)\right] = 0. \tag{21}$$

The first-best investment strategy, however, cannot be an equilibrium outcome (irrespective of the accounting rules). The intuition is that the manager cannot gain by revealing bad news about the quality of his private information ($\tau = b$) and is always better off pretending that he received good news ($\tau = g$). Thus, following the first-best investment strategy is not incentive compatible. For example, in case of $\tau = b$ and $\sigma = s = H$, the manager prefers a^*_{gHH} to a^*_{bHH}.

Equilibrium investment strategies. Given that the manager never follows the first-best strategy in any equilibrium, we now examine which investment strategies can be followed. As before, we restrict attention to pure-strategy PBE and assume that the manager and the shareholders coordinate on the ex ante Pareto-dominant equilibrium.[27] Further, we make the following assumption (stated formally in the Internet Appendix): If two investment levels lead to the same posterior beliefs about the manager's quality – so that his reputation does not depend on the choice between these investment levels – then the manager chooses the investment level that is preferred by the shareholders. This could be motivated, for example, by assuming that the manager has a positive (but arbitrarily small) stake in the firm, so that among two investment levels that lead to the same reputation irrespective of the ex post realized payoff, the manager chooses the investment level that leads to a higher expected profit for the firm. We obtain the following results:

Proposition 10 *Under historical cost accounting, for $\phi \leq \delta(1-\mu)/(1-\mu\delta) \equiv \delta_b \leq \delta$, in the Pareto-dominant PBE in pure strategies, the manager invests a^{**}_s given by*

$$\frac{\partial \Pi}{\partial a}\left(0, a^{**}_s\right) + \Pr\left(\pi = 1 | s\right) \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a^{**}_s\right) - \frac{\partial \Pi}{\partial a}\left(0, a^{**}_s\right)\right] = 0. \tag{22}$$

[27] As in the case of symmetric information about the private signal's quality, there are multiple pure-strategy PBE for any combination of parameters and accounting rules. We characterize them in the Internet Appendix.

For $\phi > \delta_b$, in the Pareto-dominant PBE in pure strategies, the manager invests a_H^{**} given by

$$\frac{\partial \Pi}{\partial a}\left(0, a_H^{**}\right) + \left[\delta p + (1-\delta)\left(\frac{1}{2} + \phi\left(p - \frac{1}{2}\right)\right)\right] \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a_H^{**}\right) - \frac{\partial \Pi}{\partial a}\left(0, a_H^{**}\right)\right] = 0 \tag{23}$$

if $(\tau = g, s = H)$ or $(\tau = b, \sigma = H)$ and otherwise a_L^{**} given by

$$\frac{\partial \Pi}{\partial a}\left(0, a_L^{**}\right) + \left[\delta(1-p) + (1-\delta)\left(\frac{1}{2} - \phi\left(p - \frac{1}{2}\right)\right)\right] \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a_L^{**}\right) - \frac{\partial \Pi}{\partial a}\left(0, a_L^{**}\right)\right] = 0. \tag{24}$$

Proposition 11 *Under mark-to-market accounting, a unique $\phi_b^{**} \in (0, \delta_b)$ exists for any $\mu < 1$ such that for $\phi \leq \phi_b^{**}$, in the Pareto-dominant PBE in pure strategies, the manager invests $a_{\sigma s}^{**}$ given by*

$$\frac{\partial \Pi}{\partial a}(0, a_{\sigma s}^{**}) + \Pr(\pi = 1|\sigma, s) \cdot \left[\frac{\partial \Pi}{\partial a}(1, a_{\sigma s}^{**}) - \frac{\partial \Pi}{\partial a}(0, a_{\sigma s}^{**})\right] = 0. \tag{25}$$

For $\phi > \phi_b^{**}$ as well as for $\mu = 1$, in the Pareto-dominant PBE in pure strategies, the manager invests a_σ^{**} given by

$$\frac{\partial \Pi}{\partial a}(0, a_\sigma^{**}) + \Pr(\pi = 1|\sigma) \cdot \left[\frac{\partial \Pi}{\partial a}(1, a_\sigma^{**}) - \frac{\partial \Pi}{\partial a}(0, a_\sigma^{**})\right] = 0. \tag{26}$$

Corollary 1 *For any $\mu < 1$, a unique $\phi_b^{**} \in (0, \delta_b)$ exists such that the firm's shareholders prefer mark-to-market accounting if $\phi \leq \phi_b^{**}$ and otherwise historical cost accounting. As a consequence, Main Results 1 and 2 (derived in Section 4) hold for any $\mu < 1$. For $\mu = 1$, the firm's shareholders always prefer historical cost accounting to marking to market.*

The intuition for these results is similar to the intuition developed in Section 3. The only difference is that under historical cost accounting, if $\phi > \delta_b$, whether the manager relies on the market or his private signal when making the investment decision depends on his private information about his quality: The manager uses his private signal (s) if he knows that he is good ($\tau = g$). Otherwise, he uses the market signal (σ).

For $\mu < 1$, the shareholders prefer marking to market to historical cost accounting if $\phi \leq \phi_b^{**}$ because, in that case, mark-to-market accounting leads to investment decisions that are based both on the market and the private signal. Historical cost accounting would lead to investment decisions based only on the private signal. If instead $\phi > \phi_b^{**}$, the shareholders prefer historical cost accounting so that the manager relies on his private signal if he knows that it is more likely to be informative than the market signal and otherwise on the market signal. Marking to market would lead to investment decisions based on the market signal irrespective of the manager's knowledge about his private signal's quality.

In the special case of $\mu = 1$ (which implies $\delta_b = 0$), the manager is always perfectly informed about his quality: $\tau = g$ implies that he is good, and $\tau = b$ implies that he is bad. In that case, under mark-to-market accounting, the manager always follows the market signal. This is inefficient if he knows that he is good. For $\mu = 1$, the shareholders thus prefer historical cost accounting, so that the manager makes the investment decision based on his private signal if he knows that he is good.

7 Conclusion

We examine how marking to market affects the investment decisions of a manager with reputation concerns when optimal decisions are based on both information conveyed by market prices and unverifiable private information. As commonly argued by the proponents of marking to market, reporting the market value of a firm's assets plays a disciplinary role in this setting. The information contained in the market prices provides a benchmark against which the manager's decisions can be evaluated. This forces the manager to take the market information into account when choosing the level of investment. However, the fact that market prices are informative about which decision the manager should take has a negative side effect: The manager may prefer to conceal conflicting private information whose use (and thus revelation) would damage his reputation. This effect can render marking to market less desirable when prices are more informative.

Our analysis centers on a single aspect of marking to market: Reporting the market value of a firm's assets can be useful because it conveys information about what the firm's management should do next. This information facilitates monitoring by investors – but it also incentivizes managers to conform and conceal conflicting private information. The resulting trade-off is the focus of our paper, and we deliberately abstract away from other aspects of mark-to-market accounting. This is not to say that mark-to-market accounting cannot have other effects or serve additional purposes. However, other benefits (e.g., providing information that is useful to value the firm) and disadvantages (e.g., increasing the volatility of the asset values that are reported on the balance sheet) are unlikely to qualitatively affect our main result: Marking to market can cause distortions in managers' investment decisions precisely because market prices contain relevant information. As a consequence, more informative prices can render mark-to-market accounting less attractive.

Our findings have several important implications. First, neither accounting rule is always optimal. Depending on the relative informativeness of market prices and private information, marking to market or historical cost accounting can be preferable. Hence, one-size-fits-all accounting rules are not optimal. Second, in contrast to the commonly held view that more informative market prices make mark-to-market accounting naturally more appealing, we find that marking to market dominates historical cost accounting only when market prices are not too informative. Third, marking to market can induce firms to rely excessively on public signals. This implies inefficient investment decisions (if there is only one type of investment) and also correlated investment decisions (if firms can choose between different types of investments). Thus, marking to market could lead to less diversification in the economy. Fourth, firms – in particular, those with good managers – may optimally respond to mandatory mark-to-market accounting by holding more opaque assets with less informative market prices on their balance sheets.

Throughout the paper, we focus on the effects of revealing the market value of its assets on a firm's investment decisions. The framework we use for our analysis, however, is not specific to marking to market and investment decisions and could be applied to other settings. For example, our findings suggest that increasing the quality of publicly available credit ratings may adversely affect the quality of financial institutions' internal risk assessment and risk management. If the risk management team is concerned about its reputation, increasing the accuracy of credit ratings provided by rating agencies may increase the team's incentives to confirm the public rating and reduce the incentives for individual research that may uncover additional, conflicting information. We leave such applications for future work.

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Figures

Figure 1: Timing of Events and Decisions

$t = 0$	$t = 1$	$t = 2$
- Firm starts out with assets in place	- Manager receives private signal	- Final net profit is realized
- Manager observes market prices	- Manager chooses investment	- Financial statements are released
- Financial statements are released		- Shareholders update beliefs

Figure 2: Equilibrium Investment Strategies

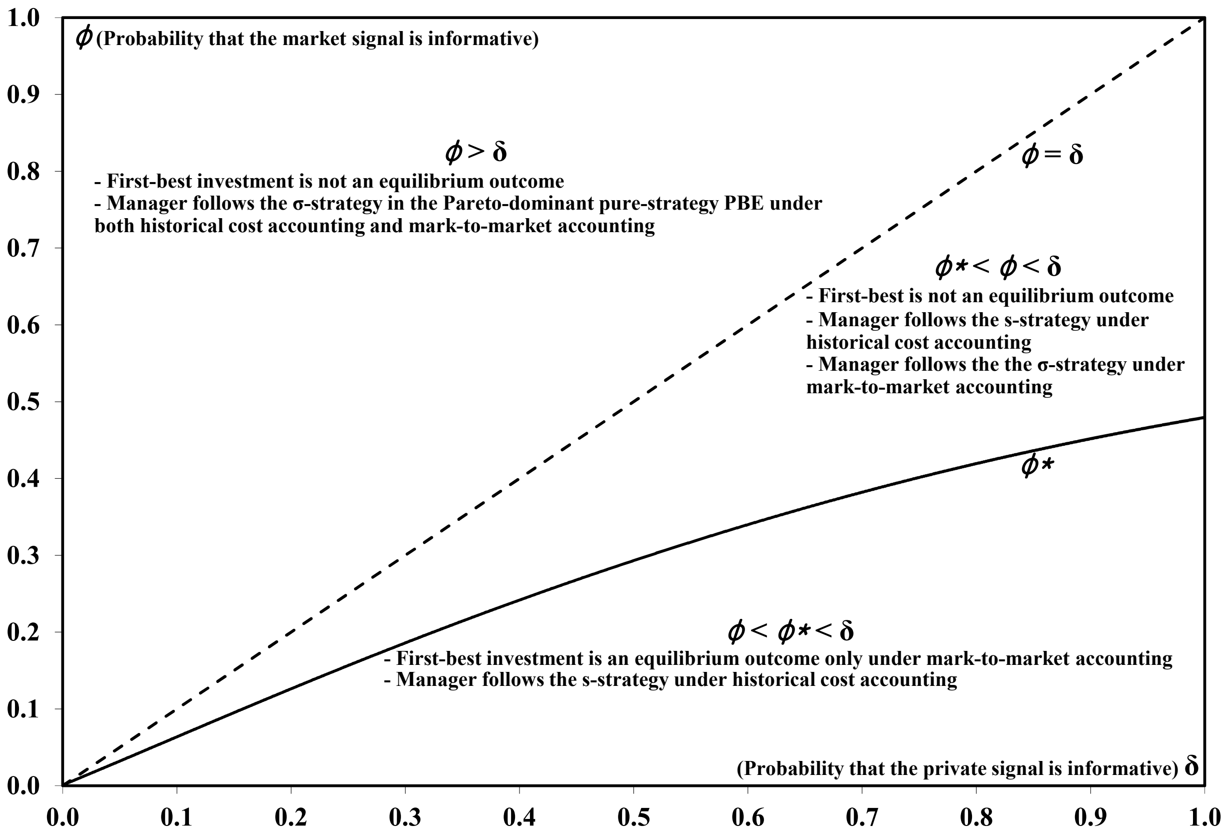
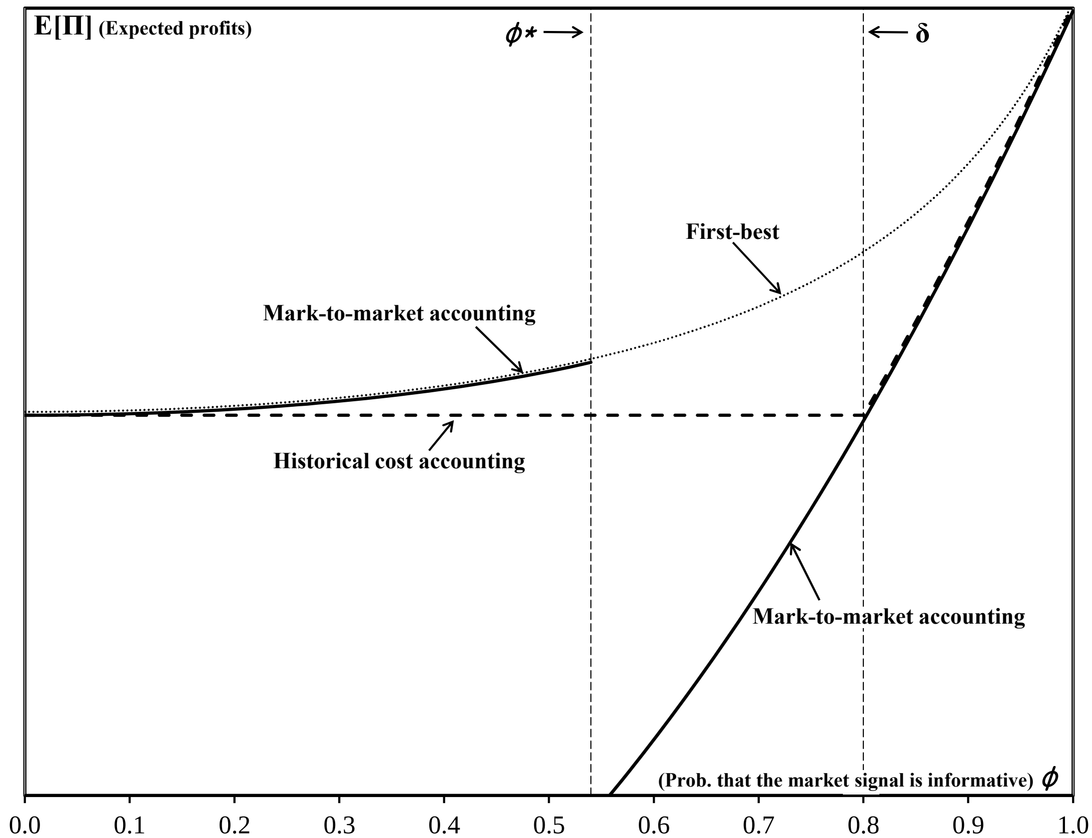


Figure 3: Expected Profits



Supplemental Internet Appendix

Proof of Proposition 1: Assume a PBE in which the manager follows the first-best investment strategy. The shareholders form the following posterior beliefs $\widehat{\delta}(\pi,\sigma,s) \equiv \Pr(\theta_A = g|\pi,\sigma,s)$:

$$\widehat{\delta}(\pi = 1,\sigma = H,s = H) = \widehat{\delta}(\pi = 0,\sigma = L,s = L) = \frac{2\delta p(1+\phi)}{2p(\delta+\phi)+(1-\phi)(1-\delta)} \quad \text{(A1)}$$

$$\widehat{\delta}(\pi = 1,\sigma = L,s = H) = \widehat{\delta}(\pi = 0,\sigma = H,s = L) = \frac{2\delta p(1-\phi)}{2p(\delta-\phi)+(1-\delta)(1+\phi)} \quad \text{(A2)}$$

$$\widehat{\delta}(\pi = 1,\sigma = H,s = L) = \widehat{\delta}(\pi = 0,\sigma = L,s = H) = \frac{2\delta(1-p)(1-\phi)}{2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)} \quad \text{(A3)}$$

$$\widehat{\delta}(\pi = 1,\sigma = L,s = L) = \widehat{\delta}(\pi = 0,\sigma = H,s = H) = \frac{2\delta(1-p)(1+\phi)}{2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)} \quad \text{(A4)}$$

with $\widehat{\delta}(\pi,\sigma = H,s = H) > \widehat{\delta}(\pi,\sigma = L,s = H)$ and $\widehat{\delta}(\pi,\sigma = L,s = L) > \widehat{\delta}(\pi,\sigma = H,s = L)$ for all $\pi \in \{0,1\}$. Hence, the manager always prefers to pretend that σ and s coincide. Thus, the assumed PBE does not satisfy the manager's incentive compatibility constraints under historical cost accounting if σ and s do not coincide – the manager always prefers a^*_{HH} to a^*_{LH} and a^*_{LL} to a^*_{HL}. ■

Proof of Proposition 2: Assume a PBE in which the manager follows the first-best investment strategy, so that the shareholders form posterior beliefs $\widehat{\delta}(\pi,\sigma,s)$ as in the proof of Proposition 1. Under mark-to-market accounting, the shareholders learn σ directly from the firm's financial statements, so that the manager's investment decision can reveal new information about s but not about σ. Regarding the out-of-equilibrium beliefs, we assume that the shareholders believe $\Pr(\theta_A = g|\tilde{a}) = 0$ upon observing an out-of-equilibrium investment $\tilde{a} \notin \{a^*_{HH}, a^*_{HL}\}$ if $\sigma = H$ and $\tilde{a} \notin \{a^*_{LH}, a^*_{LL}\}$ if $\sigma = L$. As a consequence, any out-of-equilibrium investment $\tilde{a}$ is dominated.

Consider the case of $\sigma = L$ and $s = H$. The manager prefers a^*_{LH} to a^*_{LL} if and only if

$$\begin{aligned} F(\phi) \equiv \; & \Pr(\pi = 1|\sigma = L,s = H)\left[\widehat{\delta}(\pi = 1,\sigma = L,s = L) - \widehat{\delta}(\pi = 1,\sigma = L,s = H)\right] \\ & + \Pr(\pi = 0|\sigma = L,s = H)\left[\widehat{\delta}(\pi = 0,\sigma = L,s = L) - \widehat{\delta}(\pi = 0,\sigma = L,s = H)\right] \le 0 \end{aligned} \quad \text{(A5)}$$

$F(\phi)$ is continuous, and we have $F(0) < 0$, $F(\delta) > 0$, and $\partial F/\partial\phi > 0$. Thus, there exists a unique threshold $\phi^* < \delta$ with $F(\phi^*) = 0$, such that the manager prefers a^*_{LH} to a^*_{LL} in case of $\sigma = L$ and $s = H$ if and only if $\phi \leq \phi^*$. An analogous argument can be made for the case of $\sigma = H, s = L$. In case of $\sigma = s$, the manager's incentive compatibility constraint is always satisfied. ■

Proof of Proposition 3: We first characterize all possible pure-strategy PBE and then describe the Pareto-dominant PBE in pure strategies. Under historical cost accounting, the manager's private information (his "type") is $(\sigma, s) \in \{(H,H);(H,L);(L,H);(L,L)\}$. Thus, in any pure-strategy PBE, at most four distinct investment levels can be chosen. There are four possible cases: (1) All types choose the same investment level x. (2) All types choose either an investment level x or another investment level y. (3) All types choose an investment level x, y, or z. (4) Each type chooses a different investment level w, x, y, or z.

Consider case (1). In that case, the shareholders do not update their belief about the manager's quality, i.e., maintain the belief $\Pr(\theta_A = g|\pi, x) = \delta$ for all $\pi \in \{0,1\}$. This equilibrium always exists. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0,1\}$ and any out-of-equilibrium investment $\tilde{x}$.

Consider case (2). There are three possible sub-cases: (i) Types (H,H) and (H,L) choose x, and (L,L) and (L,H) choose y. (ii) Types (H,H) and (L,H) choose x, and (L,L) and (H,L) choose y. (iii) Types (H,H) and (L,L) choose x, and (L,H) and (H,L) choose y. In sub-case (i), σ is revealed to the shareholders (but s is not). As a consequence, the shareholders do not update their belief about the manager's quality, i.e., maintain the belief $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y) = \delta$ for all $\pi \in \{0,1\}$. This equilibrium always exists. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0,1\}$ and any out-of-equilibrium investment $\tilde{x}$. In sub-case (ii), s is revealed to the shareholders (but σ is not). In equilibrium, the shareholders form posterior beliefs $\Pr(\theta_A = g|\pi, x) = \widehat{\delta}(\pi, s = H)$ and $\Pr(\theta_A = g|\pi, y) = \widehat{\delta}(\pi, s = L)$ as follows:

$$\widehat{\delta}(\pi = 1, s = H) = \widehat{\delta}(\pi = 0, s = L) = \frac{p\delta}{p\delta + \frac{1}{2}(1-\delta)} > \delta \tag{A6}$$

$$\widehat{\delta}(\pi = 1, s = L) = \widehat{\delta}(\pi = 0, s = H) = \frac{(1-p)\delta}{(1-p)\delta + \frac{1}{2}(1-\delta)} < \delta. \tag{A7}$$

The manager prefers to make an investment indicating $s = H$ rather than $s = L$ if

$$\begin{array}{ccc} \Pr(\pi = 1|\sigma, s)\,\widehat{\delta}(\pi = 1, s = H) & & \Pr(\pi = 1|\sigma, s)\,\widehat{\delta}(\pi = 1, s = L) \\ & > & \\ +\Pr(\pi = 0|\sigma, s)\,\widehat{\delta}(\pi = 0, s = H) & & +\Pr(\pi = 0|\sigma, s)\,\widehat{\delta}(\pi = 0, s = L) \end{array} \tag{A8}$$

which is satisfied for $s = H$ and violated for $s = L$ for all $\sigma \in \{L, H\}$ if $\delta > \phi$. For $\delta = \phi$, the manager is indifferent. Hence, sub-case (ii) constitutes an equilibrium if and only if $\phi \leq \delta$. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$. Sub-case (iii) cannot constitute an equilibrium because, for all $\pi \in \{0, 1\}$, $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\sigma = s) > \Pr(\theta_A = g|\pi, y) = \Pr(\theta_A = g|\sigma \neq s)$.

Consider case (3). There are six sub-cases: (i) Types (H, H) and (L, H) choose x, (H, L) chooses y, and (L, L) chooses z. (ii) Types (H, H) and (H, L) choose x, (L, H) chooses y, and (L, L) chooses z. (iii) Types (H, H) and (L, L) choose x, (L, H) chooses y, and (H, L) chooses z. (iv) Types (L, H) and (H, L) choose x, (H, H) chooses y, and (L, L) chooses z. (v) Types (L, H) and (L, L) choose x, (H, H) chooses y, and (H, L) chooses z. (vi) Types (H, L) and (L, L) choose x, (H, H) chooses y, and (L, H) chooses z. Sub-cases (i) and (vi) are ruled out as follows:

Lemma 1 *A pure-strategy PBE in which an investment level x is chosen only by type (H, H) and another investment level y is chosen only by type (L, H) does not exist. Similarly, a pure-strategy PBE in which an investment level x is chosen only by type (L, L) and another investment level y is chosen only by type (H, L) does not exist.*

Proof: Assume a PBE in which x is chosen only by type (H, H) and y is chosen only by type (L, H). It follows from the proof of Proposition 1 that we have $\Pr(\theta_A = g|\pi, x) > \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$, so that choosing x dominates choosing y. An analogous argument rules out the case in which x is chosen only by type (L, L) and y is chosen only by type (H, L). ■

In sub-case (ii), we must have $\phi \leq \phi^* < \delta$. The proof of Proposition 2 implies that type (L, H) would otherwise deviate to z. $\phi \leq \phi^* < \delta$, however, implies $\Pr(\pi = 1|\sigma = H, s = H) > \Pr(\pi = 1|\sigma = L, s = H) >$

$\Pr(\pi = 1|\sigma = H, s = L)$. In that case, we can use the following result:

Lemma 2 *In any pure-strategy PBE, if two types* (σ, s) *and* (σ', s') *choose the same investment level* x*, any other type* (σ'', s'') *with* $\Pr(\pi = 1|\sigma, s) < \Pr(\pi = 1|\sigma'', s'') < \Pr(\pi = 1|\sigma', s')$ *or* $\Pr(\pi = 1|\sigma, s) > \Pr(\pi = 1|\sigma'', s'') > \Pr(\pi = 1|\sigma', s')$ *must either choose* x *or another investment level* y *such that for all* $\pi \in \{0, 1\}$ *we have* $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$.

Proof: Define $\Delta x \equiv \Pr(\theta_A = g|\pi = 1, x) - \Pr(\theta_A = g|\pi = 0, x)$ and, analogously, Δy. The following incentive compatibility constraints must be satisfied:

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\sigma, s)\,\Delta x \geq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\sigma, s)\,\Delta y \quad \text{(A9)}$$

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\sigma', s')\,\Delta x \geq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\sigma', s')\,\Delta y \quad \text{(A10)}$$

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\sigma'', s'')\,\Delta x \leq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\sigma'', s'')\,\Delta y \quad \text{(A11)}$$

This implies $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$. ∎

In sub-case (ii), we must therefore have $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$, which in turn would imply $y = x$. An analogous argument rules out sub-case (v). In sub-case (iii), we have $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\sigma = s)$ for all $\pi \in \{0, 1\}$. Type (L, H)'s expected utility from choosing y is $E[\Pr(\theta_A = g|\pi, y)\,|\sigma = L, s = H] = \Pr(\theta_A = g|\sigma \neq s)$. $\Pr(\theta_A = g|\sigma = s) > \Pr(\theta_A = g|\sigma \neq s)$, however, implies that type (L, H) would prefer to deviate to x. In sub-case (iv), type (L, H)'s expected utility from choosing x is $E[\Pr(\theta_A = g|\pi, x)\,|\sigma = L, s = H] = \Pr(\theta_A = g|\sigma = L, s = H)$. However, we have $\Pr(\theta_A = g|\pi, \sigma = H, s = H) > \Pr(\theta_A = g|\pi, \sigma = L, s = H)$ for all $\pi \in \{0, 1\}$, so that type (L, H) would prefer to deviate and choose y. Hence, a pure-strategy PBE in which three distinct investment levels are chosen in equilibrium does not exist.

Finally, consider case (4). Lemma 1 implies that this cannot constitute an equilibrium. The only possible pure-strategy PBE under historical cost accounting are thus as follows:

Lemma 3 *Under historical cost accounting, two types of pure-strategy PBE exist for any ϕ: (1) The manager chooses the same investment level x for all realizations of σ and s. (2) The manager chooses an investment level x if $\sigma = H$ and another investment level y if $\sigma = L$. For $\phi \leq \delta$, a third type of pure-strategy PBE exists: (3) The manager chooses an investment level x if $s = H$ and another investment level y if $s = L$.*

The manager maximizes $E\left[\Pr\left(\theta_A = g|I_2\right)|\sigma, s\right]$ and is thus indifferent between all investment strategies that reveal the same information I_2 to the shareholders. The shareholders, however, prefer the strategy that uses the revealed information optimally. That is, among all investment strategies that reveal only s, the shareholders prefer $a_s^{**} \in \arg\max_{a\geq 0} \Pr\left(\pi = 1|s\right) \cdot \Pi\left(1, a\right) + \Pr\left(\pi = 0|s\right) \cdot \Pi\left(0, a\right)$, to which we refer as the s-strategy. Among all strategies that reveal only σ, they prefer $a_\sigma^{**} \in \arg\max_{a\geq 0} \Pr\left(\pi = 1|\sigma\right) \cdot \Pi\left(1, a\right) + \Pr\left(\pi = 0|\sigma\right) \cdot \Pi\left(0, a\right)$, to which we refer as the σ-strategy. Moreover, the shareholders always prefer the σ-strategy and the s-strategy to any other strategy that reveals neither σ nor s. Among the σ-strategy and the s-strategy, the shareholders prefer the σ-strategy if the market signal is more informative than the private signal ($\phi > \delta$). For $\phi < \delta$, they prefer the s-strategy. For $\phi = \delta$, they are indifferent between the σ-strategy and the s-strategy. Given that, ex ante, the manager's expected reputation in any equilibrium is equal to the prior about his quality (so that he is indifferent between all equilibria), the Pareto-dominant equilibrium is the one preferred by the shareholders. ■

Proof of Proposition 4: Under mark-to-market accounting, the market signal (σ) is common knowledge, and the manager's private information (i.e., his "type") is $s \in \{H, L\}$. Conditional on a realization of σ, there are thus two possible PBE in pure strategies: (1) A pooling equilibrium in which both types $s = H$ and $s = L$ choose the same investment level x. (2) A separating equilibrium in which type $s = H$ invests x, and type $s = L$ invests y.

In the pooling equilibrium, the manager's private information (s) is not revealed to the shareholders. As a consequence, the shareholders do not update their belief about the manager's quality, i.e., maintain the belief $\Pr\left(\theta_A = g|\pi, x\right) = \delta$ for all $\pi \in \{0, 1\}$. This equilibrium always exists. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

In the separating equilibrium, s is revealed to the shareholders (who also learn σ from the firm's financial statements). The proof of Proposition 2 implies that this equilibrium exists if and only if $\phi \leq \phi^*$. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

Ex ante, the possible pure-strategy PBE under marking to market are thus as follows:

Lemma 4 *Under mark-to-market accounting, two types of pure-strategy PBE exist for any ϕ: (1) The manager chooses the same investment level x for all realizations of σ and s. (2) The manager chooses an investment level x if $\sigma = H$ and another investment level y if $\sigma = L$. Further, a unique $\phi^* \in (0, \delta)$ exists such that four additional types of pure-strategy PBE exist for $\phi \leq \phi^*$: (3) The manager chooses an investment level x if $s = H$ and an investment level y if $s = L$. (4) The manager chooses an investment level x if $s = H$ and $\sigma = H$, y if $s = L$ and $\sigma = H$, and z if $\sigma = L$. (5) The manager chooses an investment level x if $s = H$ and $\sigma = L$, y if $s = L$ and $\sigma = L$, and z if $\sigma = H$. (6) The manager chooses an investment level w if $s = H$ and $\sigma = H$, x if $s = H$ and $\sigma = L$, y if $s = L$ and $\sigma = H$, and z if $s = L$ and $\sigma = L$.*

The manager is indifferent ex ante between all equilibria because his expected reputation is always equal to the prior belief about his quality. The shareholders, however, prefer the manager to rely on more (rather than less) information when choosing the level of investment and to use this information optimally. That is, if $\phi \leq \phi^*$, the shareholders prefer the manager to follow the first-best investment strategy and invest $a^*_{\sigma s} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma, s) \cdot \Pi(1, a) + \Pr(\pi = 0|\sigma, s) \cdot \Pi(0, a)$. This corresponds to case (6) in Lemma 4, where w, x, y and z are chosen optimally. Otherwise, the shareholders prefer the manager to follow the σ-strategy and invest $a^{**}_{\sigma} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma) \cdot \Pi(1, a) + \Pr(\pi = 0|\sigma) \cdot \Pi(0, a)$. This corresponds to case (2) in Lemma 4, where x and y are chosen optimally. Given the manager's indifference ex ante between all possible equilibria, the Pareto-dominant equilibrium is the one preferred by the shareholders. ■

Proof of Proposition 5: Under historical cost accounting, for $\phi > \delta$, the investment strategy followed in the Pareto-dominant PBE in pure strategies depends only on σ, and the shareholders' expected utility ex ante is

$$\begin{aligned} EU &= \Pr(\sigma = H)\left\{\Pi\left(0, a^{**}_{\sigma=H}\right) + \Pr(\pi = 1|\sigma = H)\left[\Pi\left(1, a^{**}_{\sigma=H}\right) - \Pi\left(0, a^{**}_{\sigma=H}\right)\right]\right\} \\ &\quad + \Pr(\sigma = L)\left\{\Pi\left(0, a^{**}_{\sigma=L}\right) + \Pr(\pi = 1|\sigma = L)\left[\Pi\left(1, a^{**}_{\sigma=L}\right) - \Pi\left(0, a^{**}_{\sigma=L}\right)\right]\right\}. \end{aligned} \tag{A12}$$

Using the envelope theorem, we obtain

$$\frac{\partial EU}{\partial \phi} = \frac{1}{2}\left(p - \frac{1}{2}\right)\left[\Pi\left(1, a^{**}_{\sigma=H}\right) - \Pi\left(1, a^{**}_{\sigma=L}\right) + \Pi\left(0, a^{**}_{\sigma=L}\right) - \Pi\left(0, a^{**}_{\sigma=H}\right)\right] > 0. \tag{A13}$$

Thus, if the shareholders can mandate any $\phi \leq \bar{\phi}$ with $\bar{\phi} > \delta$, they mandate $\phi = \bar{\phi}$. For $\phi \leq \delta$, the investment strategy followed in the Pareto-dominant PBE in pure strategies under historical cost accounting depends only on s, and the shareholders' expected utility is independent of ϕ. Thus, for $\bar{\phi} \leq \delta$, the shareholders are indifferent between all $\phi \leq \bar{\phi}$.

For the case of mark-to-market accounting, we first establish the following Lemma:

Lemma 5 *Under mark-to-market accounting, the shareholders' expected utility is increasing in ϕ for $\phi \leq \phi^*$ and for $\phi > \phi^*$.*

Proof: Under mark-to-market accounting, if $\phi > \phi^*$, the investment strategy followed in the Pareto-dominant PBE in pure strategies depends only on σ. In that case, we know from equation (A13) that the shareholders' expected utility is increasing in ϕ.

In case of $\phi \leq \phi^*$, the investment strategy followed in the Pareto-dominant PBE in pure strategies is the first-best strategy (which depends on both σ and s). For ease of exposition, we define $p_{\sigma s} \equiv \Pr(\pi = 1|\sigma, s)$ and $q_{\sigma s} \equiv \Pr(\sigma = \zeta, s = \xi)$ for $(\zeta, \xi) \in \{H, L\} \times \{H, L\}$. Conditional on a signal pair $(\sigma, s) \in \{H, L\} \times \{H, L\}$, the shareholders' expected utility under the first-best strategy is $g(p_{\sigma s}) \equiv \Pi(0, a^*_{\sigma s}) + p_{\sigma s} \cdot [\Pi(1, a^*_{\sigma s}) - \Pi(0, a^*_{\sigma s})]$. Before σ and s are realized, the ex ante expected utility is $E[g(p_{\sigma s})] = q_{HH} \cdot g(p_{HH}) + q_{LH} \cdot g(p_{LH}) + q_{HL} \cdot g(p_{HL}) + q_{LL} \cdot g(p_{LL})$. We have $\partial g(p_{\sigma s}) / \partial p_{\sigma s} > 0$

and $\partial^2 g(p_{\sigma s})/\partial p_{\sigma s}^2 > 0$. Thus, the function $g(\cdot)$ is increasing and convex in $p_{\sigma s}$. This implies that the shareholders' ex ante expected utility increases if the informativeness of the market signal increases from ϕ to $\phi' \in (\phi, \phi^*]$. The shareholders' expected utility is thus increasing in ϕ for both $\phi > \phi^*$ and $\phi \leq \phi^*$. ■

We can now show that a $\bar{\delta} \in (0, \bar{\phi})$ exists such that for $\delta < \bar{\delta}$ the shareholders mandate $\phi = \bar{\phi}$ and for $\delta > \bar{\delta}$, the shareholders mandate $\phi = \min\{\phi^*, \bar{\phi}\}$. First, note that it follows from Lemma 5 that the shareholders mandate either $\phi = \bar{\phi}$ or $\phi = \phi^*$. Second, note that for $\delta = 0$ we obtain $\phi^* = 0$, so that the shareholders' expected utility is maximized for $\phi = \bar{\phi}$. Third, for $\delta = \bar{\phi}$, the shareholders' expected utility is maximized for $\phi = \phi^*$. To see this, note that $\delta = \bar{\phi}$ implies $\bar{\phi} > \phi^*$. Hence, if the shareholders mandate $\phi = \bar{\phi}$, the investment strategy followed in the Pareto-dominant PBE in pure strategies is based only on σ. This, however, would lead to the same expected utility as choosing $\phi = 0$, so that the investment strategy followed in the Pareto-dominant PBE in pure strategies would be based only on s. Consider now the shareholders' expected utility after mandating $\phi = \phi^*$. In that case, the investment strategy followed in the Pareto-dominant PBE in pure strategies is based not only on s but also on σ. We know that, in that case, the shareholders' expected utility is increasing in ϕ. Thus, mandating $\phi = \phi^*$ must dominate choosing $\phi = \bar{\phi}$. Finally, note that, for $\phi = \phi^*$, the shareholders' expected utility is increasing in δ because we have $\partial EU/\partial\delta > 0$ and $\partial\phi^*/\partial\delta > 0$. It follows that a unique $\bar{\delta} \in (0, \bar{\phi})$ exists such that for $\delta < \bar{\delta}$ the shareholders mandate $\phi = \bar{\phi}$ and for $\delta > \bar{\delta}$ the shareholders mandate $\phi = \min\{\phi^*, \bar{\phi}\}$. ■

Proof of Proposition 6: At $t = 0$, the manager's private information (i.e., his "type") is $\sigma \in \{H, L\}$. Thus, there are two types of pure-strategy PBE: (1) Both types report the same asset value, say v. (2) The two types report different values, say v and w. (Note that both v and w could also be "no report," so that not reporting any value is in the manager's choice set.)

Suppose first that both types report v at $t = 0$, so that no information about the true market signal (σ) is revealed to the shareholders. The sub-game at $t = 1$ is then the same as under historical cost accounting, and Proposition 3 implies that the manager follows the s-strategy if $\phi \leq \delta$ and the σ-strategy otherwise.

This equilibrium can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\tilde{v}) = 0$ for any out-of-equilibrium report $\tilde{v}$.

Suppose now that one type reports v and the other type reports w. Without loss of generality, assume type $\sigma = H$ reports v and type $\sigma = L$ reports w. In that case, the shareholders learn σ. The sub-game at $t = 1$ is then the same as under mark-to-market accounting, and Proposition 4 implies that the manager follows the first-best strategy if $\phi \leq \phi^*$ and the s-strategy otherwise. What remains to be verified is that type $\sigma = H$ ($\sigma = L$) indeed prefers to report v (w). For notational simplicity, denote the market signal implied by the manager's report with $\widehat{\sigma}$ and consider the case of $\sigma = H$. (The proof for the case of $\sigma = L$ is symmetric.)

Assume first that $\phi \leq \phi^*$. It follows from Proposition 2 that after reporting $\widehat{\sigma} = H$, the manager prefers investing a^*_{HH} to a^*_{HL} if his private signal is $s = H$ and a^*_{HL} to a^*_{HH} if $s = L$. Hence, if the manager truthfully reports $\widehat{\sigma} = H$, his expected reputation is

$$\Pr(s = H|\sigma = H)\begin{bmatrix} \Pr(\pi = 1|\sigma = H, s = H)\,\widehat{\delta}(\pi = 1, \widehat{\sigma} = H, s = H) \\ + \Pr(\pi = 0|\sigma = H, s = H)\,\widehat{\delta}(\pi = 0, \widehat{\sigma} = H, s = H) \end{bmatrix} \tag{A14}$$
$$+ \Pr(s = L|\sigma = H)\begin{bmatrix} \Pr(\pi = 1|\sigma = H, s = L)\,\widehat{\delta}(\pi = 1, \widehat{\sigma} = H, s = L) \\ + \Pr(\pi = 0|\sigma = H, s = L)\,\widehat{\delta}(\pi = 0, \widehat{\sigma} = H, s = L) \end{bmatrix}.$$

Suppose now the manager were to deviate and report $\widehat{\sigma} = L$ instead. We know from Proposition 2 that, for $\phi \leq \phi^*$, the manager prefers investing a^*_{LH} to a^*_{LL} if $\sigma = L$ and $s = H$. Further, the manager's expected reputation after investing a_{LH}^* is higher in case of $\sigma = H$ and $s = H$ than in case of $\sigma = L$ and $s = H$:

$$\begin{aligned}
&\Pr(\pi = 1|\sigma = H, s = H) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H) \\
&+ \Pr(\pi = 0|\sigma = H, s = H) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H) \\
&> \\
&\Pr(\pi = 1|\sigma = L, s = H) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H) \\
&+ \Pr(\pi = 0|\sigma = L, s = H) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H).
\end{aligned} \tag{A15}$$

Moreover, the manager's expected reputation after investing a_{LL}^* is lower in case of $\sigma = H$ and $s = H$ than in case of $\sigma = L$ and $s = H$:

$$\begin{aligned}
&\Pr(\pi = 1|\sigma = H, s = H) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = L) \\
&+ \Pr(\pi = 0|\sigma = H, s = H) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = L) \\
&< \\
&\Pr(\pi = 1|\sigma = L, s = H) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = L) \\
&+ \Pr(\pi = 0|\sigma = L, s = H) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = L).
\end{aligned} \tag{A16}$$

Thus, if the true market signal is $\sigma = H$, after reporting $\widehat{\sigma} = L$, the manager prefers a_{LH}^* to a_{LL}^* if $s = H$.

We know from Proposition 1 that if $\sigma = H$ and $s = L$, the manager prefers a_{LL}^* to a_{HL}^*. In that case, the manager also prefers a_{HL}^* to a_{LH}^*:

$$\begin{aligned}
&\Pr(\pi = 1|\sigma = H, s = L) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = H, s = L) \\
&+ \Pr(\pi = 0|\sigma = H, s = L) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = H, s = L) \\
&> \\
&\Pr(\pi = 1|\sigma = H, s = L) \cdot \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H) \\
&+ \Pr(\pi = 0|\sigma = H, s = L) \cdot \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H).
\end{aligned} \tag{A17}$$

Thus, if the true market signal is $\sigma = H$, after reporting $\widehat{\sigma} = L$, the manager prefers a_{LL}^* to a_{LH}^* if $s = L$.

Hence, if the manager deviates from the proposed equilibrium and untruthfully reports $\widehat{\sigma} = L$, his expected reputation is

$$\Pr(s = H|\sigma = H)\begin{bmatrix} \Pr(\pi = 1|\sigma = H, s = H)\,\widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H) \\ + \Pr(\pi = 0|\sigma = H, s = H)\,\widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H) \end{bmatrix} \tag{A18}$$
$$+ \Pr(s = L|\sigma = H)\begin{bmatrix} \Pr(\pi = 1|\sigma = H, s = L)\,\widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = L) \\ + \Pr(\pi = 0|\sigma = H, s = L)\,\widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = L) \end{bmatrix}.$$

Comparing (A14) and (A18), it can be shown that the manager prefers to report truthfully if

$$\begin{gathered} \left[\frac{1}{4}(1 + \phi\delta) + \frac{1}{2}\left(p - \frac{1}{2}\right)(\delta + \phi)\right] \cdot \left[\widehat{\delta}(\pi = 1, \widehat{\sigma} = H, s = H) - \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H)\right] \\ + \left[\frac{1}{4}(1 + \phi\delta) - \frac{1}{2}\left(p - \frac{1}{2}\right)(\delta + \phi)\right] \cdot \left[\widehat{\delta}(\pi = 0, \widehat{\sigma} = H, s = H) - \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H)\right] \\ \geq \\ \left[\frac{1}{4}(1 - \phi\delta) + \frac{1}{2}\left(p - \frac{1}{2}\right)(\delta - \phi)\right] \cdot \left[\widehat{\delta}(\pi = 1, \widehat{\sigma} = H, s = H) - \widehat{\delta}(\pi = 1, \widehat{\sigma} = L, s = H)\right] \\ + \left[\frac{1}{4}(1 - \phi\delta) - \frac{1}{2}\left(p - \frac{1}{2}\right)(\delta - \phi)\right] \cdot \left[\widehat{\delta}(\pi = 0, \widehat{\sigma} = H, s = H) - \widehat{\delta}(\pi = 0, \widehat{\sigma} = L, s = H)\right] \end{gathered} \tag{A19}$$

which is satisfied for any $\phi\delta > 0$. Hence, the manager indeed prefers to report the market signal truthfully. This equilibrium can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\tilde{v}) = 0$ for any out-of-equilibrium report $\tilde{v}$.

Assume now $\phi > \phi^*$. In that case, if the manager follows the σ-strategy at $t = 1$, there is no updating regarding his quality. As a consequence, the manager is indifferent between reporting v and w at $t = 0$ and has no incentive to deviate from the proposed equilibrium. This equilibrium can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\tilde{v}) = 0$ for any out-of-equilibrium report $\tilde{v}$.

Finally, note that the manager is ex ante indifferent between all possible equilibria: His expected reputation is a martingale and equal to the prior about his quality in any equilibrium. The shareholders, however, prefer the equilibrium that leads to the highest expected profits for the firm. Hence, in the Pareto-dominant PBE in pure strategies, the manager truthfully reveals σ and follows the first-best investment strategy if

$\phi \leq \phi^*$, does not reveal σ and follows the s-strategy for $\phi \in (\phi^*, \delta)$, and reveals σ and follows the σ-strategy for $\phi \geq \delta$. ■

Proof of Proposition 7: To simplify notation, define $\Pr(ijk) \equiv \Pr(\pi = i|\sigma = j, s = k)$ and $\widehat{\delta}(ijk) \equiv \widehat{\delta}(\pi = i, \sigma = j, s = k)$ for $i, j, k \in \{1,0\} \times \{H,L\} \times \{H,L\}$. Assume $\phi > \phi^*$ and consider first the case of $\sigma = L$. Define $b^{\pi}_{\sigma s}$ as the incentive payment that the manager receives if he has invested $a_{\sigma s}$ and the assets in place generate cash-flow π. Under mark-to-market accounting, any contract that induces the manager to choose the first-best investment must satisfy[28]

$$B \cdot \left[\Pr(1LH) \cdot \widehat{\delta}(1LH) + \Pr(0LH) \cdot \widehat{\delta}(0LH)\right] + E\left[b^{\pi}_{LH}|\sigma = L, s = H\right]$$
$$\geq \tag{A20}$$
$$B \cdot \left[\Pr(1LH) \cdot \widehat{\delta}(1LL) + \Pr(0LH) \cdot \widehat{\delta}(0LL)\right] + E\left[b^{\pi}_{LL}|\sigma = L, s = H\right]$$

and

$$B \cdot \left[\Pr(1LL) \cdot \widehat{\delta}(1LH) + \Pr(0LL) \cdot \widehat{\delta}(0LH)\right] + E\left[b^{\pi}_{LH}|\sigma = L, s = L\right]$$
$$\leq \tag{A21}$$
$$B \cdot \left[\Pr(1LL) \cdot \widehat{\delta}(1LL) + \Pr(0LL) \cdot \widehat{\delta}(0LL)\right] + E\left[b^{\pi}_{LL}|\sigma = L, s = L\right].$$

Further, because the manager has zero initial wealth, we must have

$$b^{\pi}_{\sigma s} \geq 0 \quad \text{for} \quad \pi, \sigma, s \in \{1,0\} \times \{H,L\} \times \{H,L\}. \tag{A22}$$

Consider now the following contract:

$$b^{\pi}_{\sigma s} = \begin{cases} B \cdot F(\phi) & \text{if } a = a^*_{LH} \\ 0 & \text{otherwise} \end{cases} \tag{A23}$$

[28]Note that under mark-to-market accounting, the shareholders learn the market signal directly from the firm's financial statements. Hence, while the level of investment in the new project can reveal information about the manager's private signal, it cannot reveal any new information about the market signal. The manager's choice is thus whether to make an investment that indicates $s = H$ or to make an investment that indicates $s = L$.

with $F(\phi) \equiv \Pr(1LH) \cdot \left[\widehat{\delta}(1LL) - \widehat{\delta}(1LH)\right] + \Pr(0LH) \cdot \left[\widehat{\delta}(0LL) - \widehat{\delta}(0LH)\right]$. The proposed contract satisfies (A20) by construction, (A21) because $\Pr(0LH) \leq \Pr(0LL)$, and (A22) because $F(\phi) > 0$ for $\phi > \phi^*$. Further, note that any other incentive compatible contract cannot have a lower expected cost for the shareholders because the manager has zero initial wealth and $E\left[b^{\pi}_{LH}|\sigma = L, s = H\right] = B \cdot F(\phi)$ is the minimum expected incentive payment that induces the manager to choose the first-best investment if $s = H$.

The case in which $\phi > \phi^*$ and $\sigma = H$ is symmetric. It follows that, for $\phi > \phi^*$, all optimal contracts that induce the first-best investment under mark-to-market accounting are such that, in equilibrium, $E\left[b^{\pi}_{\sigma s}|\sigma \neq s\right] = B \cdot F(\phi)$ and $E\left[b^{\pi}_{\sigma s}|\sigma = s\right] = 0$. The expected cost of inducing the first-best investment is therefore $B \cdot F(\phi) \cdot \Pr(\sigma \neq s) = B \cdot F(\phi) \cdot (1 - \delta\phi)/2$. The proof of Proposition 2 shows that no incentive pay is necessary to induce the first-best investment under mark-to-market accounting if $\phi \leq \phi^*$.

Suppose now the probability that the market signal is informative increases from $\phi = \phi'$ to $\phi = \phi''$ with $\phi' < \phi^* < \phi'' < \delta$. Under mark-to-market accounting, the firm's shareholders prefer $\phi = \phi'$ to $\phi = \phi''$ if $\Pi^{FB}(\phi') > \Pi^{FB}(\phi'') - B \cdot F(\phi'') \cdot (1 - \delta\phi'')/2$, where $\Pi^{FB}(\phi'')$ and $\Pi^{FB}(\phi')$ denote the firm's expected profit (gross of any incentive pay) if the manager follows the first-best strategy and the probability that the market signal is informative is equal to ϕ'' and ϕ', respectively. $F(\phi'')$ and ϕ^* are defined as in the proof of Proposition 2. Hence, under mark-to-market accounting, the firm's shareholders prefer $\phi = \phi'$ to $\phi = \phi''$ for all $B > B^*$ with $B^* = 2\left[\Pi^{FB}(\phi'') - \Pi^{FB}(\phi')\right] / \left[F(\phi'') \cdot (1 - \delta\phi'')\right]$.[29] ■

Proof of Proposition 8: Propositions 1 and 2 imply that the firm's shareholders prefer marking to market to historical cost accounting for $\phi \leq \phi^*$ because mark-to-market accounting leads to the first-best investment without any incentive pay. However, the shareholders prefer historical cost accounting (which leads to

[29] Note that under marking to market, for $\phi \in (\phi^*, \delta)$, inducing the manager to follow the s-strategy has the same expected cost as inducing him to follow the first-best strategy because, in either case, the shareholders learn both σ and s. For this reason, under mark-to-market accounting, inducing the first-best investment strategy always dominates inducing the s-strategy. Further, the first-best strategy in case of $\phi = \phi'$ always dominates the σ-strategy in case of $\phi = \phi''$ because $\phi'' < \delta$.

the σ-strategy without incentive pay) to marking to market for $\phi = \phi''$ with $\phi^* < \phi'' < \delta$ if $\Pi^{SB}(\phi'') > \Pi^{FB}(\phi'') - B \cdot F(\phi'') \cdot [1 - \delta\phi'']/2$, i.e., for all $B > B^{**}$ with $B^{**} = 2\left[\Pi^{FB}(\phi'') - \Pi^{SB}(\phi'')\right] / \left[F(\phi'') \cdot (1 - \delta\phi'')\right]$, where $\Pi^{SB}(\phi'')$ denotes the firm's expected final profit (gross of any incentive pay) if the manager follows the s-strategy and the probability that the market signal is informative is equal to ϕ''. $\Pi^{FB}(\phi'')$, $F(\phi'')$, and ϕ^* are defined as in Proposition 7.[30] ■

Proof of Proposition 9: At $t = 1$, the market makers set the price equal to $E[\pi | f_0, f_1]$, where f_0 and f_1 are the total order flows at $t = 0$ and $t = 1$, respectively. Suppose first that the manager's investment in the new project does not reveal his private signal (s). In that case, marker makers can learn at most the speculators' signal (σ).

If $f_0 = -2$ or $f_0 = 2$, the market makers have learned σ at $t = 0$. The price at $t = 1$ will then be equal to either $E[\pi | f_0 = -2, f_1] = E[\pi | f_0 = -2] = E[\pi | \sigma = L] = 1/2 - \phi(p - 1/2)$ or $E[\pi | f_0 = 2, f_1] = E[\pi | f_0 = 2] = E[\pi | \sigma = H] = 1/2 + \phi(p - 1/2)$ irrespective of the total order flow at $t = 1$.

If $f_0 = 0$, the market makers have not learned the speculators' signal at $t = 0$. The speculators submit buy orders at $t = 1$ if $\sigma = H$ and sell orders if $\sigma = L$. The market price at $t = 1$ is thus equal to $E[\pi | f_0 = 0, f_1 = -2] = E[\pi | \sigma = L] = 1/2 - \phi(p - 1/2)$ if both speculators and noise traders sell at $t = 1$. The price is equal to $E[\pi | f_0 = 0, f_1 = 2] = E[\pi | \sigma = L] = 1/2 + \phi(p - 1/2)$ if both speculators and noise traders buy at $t = 1$. The price is equal to $E[\pi | f_0 = 0, f_1 = 0] = E[\pi] = 1/2$ if at $t = 1$ speculators buy and noise traders sell or vice versa. Thus, if the manager's investment in the new project does not reveal his private signal, the price of the assets at $t = 1$ can take three different values: $E[\pi]$, $E[\pi | \sigma = L]$, and $E[\pi | \sigma = H]$.

Suppose now that that the manager's investment in the new project reveals his private signal to the speculators. The market makers may now learn both the speculators' signal (σ) and the manager's signal (s).

[30] Note that the expected cost of inducing the first-best strategy is never lower under historical cost accounting than under marking to market because, in either case, the shareholders learn both σ and s.

If $f_0 = -2$, the market makers have already learned the speculators' signal ($\sigma = L$ in this case) at $t = 0$. At $t = 1$, the speculators then submit buy orders if $s = H$ and sell orders if $s = L$. The possible price realizations are therefore $E[\pi | f_0 = -2, f_1 = -2] = E[\pi | \sigma = L, s = L] = 1/2 - (p - 1/2)(\delta + \phi)/(1 + \phi\delta)$, $E[\pi | f_0 = -2, f_1 = 0] = E[\pi | \sigma = L] = 1/2 - \phi(p - 1/2)$, $E[\pi | f_0 = -2, f_1 = 2] = E[\pi | \sigma = L, s = H] = 1/2 + (p - 1/2)(\delta - \phi)/(1 - \phi\delta)$.

Similarly, if $f_0 = 2$, the market makers have already learned $\sigma = H$ at $t = 0$, and the possible price realizations are $E[\pi | f_0 = 2, f_1 = -2] = E[\pi | \sigma = H, s = L] = 1/2 - (p - 1/2)(\delta - \phi)/(1 - \phi\delta)$, $E[\pi | f_0 = 2, f_1 = 0] = E[\pi | \sigma = H] = 1/2 + \phi(p - 1/2)$, and $E[\pi | f_0 = 2, f_1 = 2] = E[\pi | \sigma = H, s = H] = 1/2 + (p - 1/2)(\delta + \phi)/(1 + \phi\delta)$.

If $f_0 = 0$, the market makers have not learned the speculators' signal at $t = 0$. In equilibrium, the speculators then submit buy orders at $t = 1$ if $E[\pi | \sigma, s] > 1/2$ and sell orders if $E[\pi | \sigma, s] < 1/2$. If $E[\pi | \sigma, s] = 1/2$, the speculators are indifferent between buying and selling, and we assume that they do not trade in this case (e.g., because of a small trading cost).

Suppose first that $\phi > \delta$. In that case we have $E[\pi | \sigma = H, s = H] > E[\pi | \sigma = H, s = L] > 1/2 > E[\pi | \sigma = L, s = H] > E[\pi | \sigma = L, s = L]$. If $\phi < \delta$, we have $E[\pi | \sigma = H, s = H] > E[\pi | \sigma = L, s = H] > 1/2 > E[\pi | \sigma = H, s = L] > E[\pi | \sigma = L, s = L]$. Finally, for $\sigma = \delta$, we have $E[\pi | \sigma = H, s = H] > E[\pi | \sigma = L, s = H] = E[\pi | \sigma = H, s = L] = 1/2 > E[\pi | \sigma = L, s = L]$.

If $\phi > \delta$, the possible price realizations at $t = 1$ are thus $E[\pi | f_0 = 0, f_1 = -2] = E[\pi | \sigma = L] = 1/2 - \phi(p - 1/2)$, $E[\pi | f_0 = 0, f_1 = 2] = E[\pi | \sigma = H] = 1/2 + \phi(p - 1/2)$, and $E[\pi | f_0 = 0, f_1 = 0] = E[\pi] = 1/2$. If $\phi < \delta$, the possible price realizations are $E[\pi | f_0 = 0, f_1 = -2] = E[\pi | s = L] = 1/2 - \delta(p - 1/2)$, $E[\pi | f_0 = 0, f_1 = 2] = E[\pi | s = H] = 1/2 + \delta(p - 1/2)$, and $E[\pi | f_0 = 0, f_1 = 0] = E[\pi] = 1/2$.

Finally, if $\phi = \delta$, the speculators do not trade at $t = 1$ if $\sigma \neq s$. In that case, only the noise traders submit orders and $f_1 \in \{-1, 1\}$. The price at $t = 1$ in that case is $E[\pi | f_0 = 0, f_1 = -1] = E[\pi | f_0 = 0, f_1 = 1] = E[\pi] = 1/2$. The speculators sell if $\sigma = s = L$ and buy if $\sigma = s = H$, and the possible price realizations are

$E[\pi|f_0 = 0, f_1 = -2] = E[\pi|\sigma = L, s = L] = 1/2 - (p - 1/2)(\delta + \phi)/(1 + \phi\delta)$, $E[\pi|f_0 = 0, f_1 = 0] = E[\pi] = 1/2$, and $E[\pi|f_0 = 0, f_1 = 2] = E[\pi|\sigma = H, s = H] = 1/2 + (p - 1/2)(\delta + \phi)/(1 + \phi\delta)$.

Thus, if the manager's investment decision reveals his private signal, the price of the assets at $t = 1$ can take nine different values: $E[\pi]$, $E[\pi|\sigma = L]$, $E[\pi|\sigma = H]$, $E[\pi|s = L]$, $E[\pi|s = H]$, $E[\pi|\sigma = L, s = L]$, $E[\pi|\sigma = L, s = H]$, $E[\pi|\sigma = H, s = L]$, and $E[\pi|\sigma = H, s = H]$.

Proposition 3 implies that the manager's investment decision reveals his private signal under historical cost accounting if and only if $\phi \leq \delta$. Proposition 4 implies that the manager's investment decision reveals his private signal under mark-to-market accounting if and only if $\phi \leq \phi^*$. Hence, mark-to-market accounting can reduce the informativeness of the assets' market price at $t = 1$ relative to historical cost accounting if and only if $\phi \in (\phi^*, \delta)$. ■

Proof of Proposition 10: Under historical cost accounting, the manager's private information (i.e., his "type") is $(\tau, \sigma, s) \in \{(g, H, H); (g, H, L); (g, L, H); (g, L, L); (b, H, H); (b, H, L); (b, L, H); (b, L, L)\}$. Before we proceed, we formally state our assumption regarding the manager's behavior in case two investment levels lead to the same posterior beliefs and establish three useful Lemmas.

Assumption 1 *Denote with* $\Pr(\theta_A = g|\pi, x)$ *the shareholders' posterior belief about the probability that the manager is good after observing payoff* π *and investment level* x*. If for two investment levels* x *and* y *we have* $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ *for all* $\pi \in \{0, 1\}$*, then type* (τ, σ, s) *chooses* x *if* $E[\Pi(\pi, x)|\tau, \sigma, s] > E[\Pi(\pi, y)|\tau, \sigma, s]$ *and* y *if* $E[\Pi(\pi, x)|\tau, \sigma, s] < E[\Pi(\pi, y)|\tau, \sigma, s]$*. Otherwise, type* (τ, σ, s) *is indifferent between* x *and* y*.*

Lemma 6 *In any PBE, any investment level* x *chosen in equilibrium by a type that has received the private signal* $\tau = g$ *must also be chosen by (at least) one type that has received the private signal* $\tau = b$*.*

Proof: Suppose not. In that case, choosing the investment level x perfectly reveals $\theta_A = g$ and thus dominates choosing any other investment level (that does not perfectly reveal $\theta_A = g$). ■

Lemma 7 *In any PBE, if in equilibrium two types* (τ, σ, s) *and* (τ', σ', s') *choose the same investment level* x*, any other type* (τ'', σ'', s'') *with* $\Pr(\pi = 1|\tau, \sigma, s) < \Pr(\pi = 1|\tau'', \sigma'', s'') < \Pr(\pi = 1|\tau', \sigma', s')$ *or* $\Pr(\pi = 1|\tau, \sigma, s) > \Pr(\pi = 1|\tau'', \sigma'', s'') > \Pr(\pi = 1|\tau', \sigma', s')$ *must either choose* x *or another investment level* y *such that for all* $\pi \in \{0, 1\}$ *we have* $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$.

Proof: Define $\Delta x \equiv \Pr(\theta_A = g|\pi = 1, x) - \Pr(\theta_A = g|\pi = 0, x)$ and, analogously, Δy. The following incentive compatibility constraints must be satisfied:

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\tau, \sigma, s)\, \Delta x \geq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\tau, \sigma, s)\, \Delta y \tag{A24}$$

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\tau', \sigma', s')\, \Delta x \geq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\tau, \sigma', s')\, \Delta y \tag{A25}$$

$$\Pr(\theta_A = g|\pi = 0, x) + \Pr(\pi = 1|\tau'', \sigma'', s'')\, \Delta x \leq \Pr(\theta_A = g|\pi = 0, y) + \Pr(\pi = 1|\tau'', \sigma'', s'')\, \Delta y \tag{A26}$$

This implies $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$. ■

Lemma 8 *Consider a PBE in which types* (τ, σ, s) *and* (τ', σ', s') *invest* x*, and type* (τ'', σ'', s'') *invests* y*. If either* $\Pr(\pi = 1|\tau, \sigma, s) < \Pr(\pi = 1|\tau'', \sigma'', s'') < \Pr(\pi = 1|\tau', \sigma', s')$ *or* $\Pr(\pi = 1|\tau, \sigma, s) > \Pr(\pi = 1|\tau'', \sigma'', s'') > \Pr(\pi = 1|\tau', \sigma', s')$*, then* $x = y$.

Proof: Suppose $\Pr(\pi = 1|\tau, \sigma, s) < \Pr(\pi = 1|\tau'', \sigma'', s'') < \Pr(\pi = 1|\tau', \sigma', s')$. (The proof in case of $\Pr(\pi = 1|\tau, \sigma, s) > \Pr(\pi = 1|\tau'', \sigma'', s'') > \Pr(\pi = 1|\tau', \sigma', s')$ is identical). Lemma 7 implies that $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$. Define $\Delta\Pi(x) \equiv \Pi(1, x) - \Pi(0, x)$ and $\Delta\Pi(y) \equiv \Pi(1, y) - \Pi(0, y)$. Assumption 1 implies that in equilibrium we must have

$$\Pi(0, x) + \Pr(\pi = 1|\tau, \sigma, s)\, \Delta\Pi(x) \geq \Pi(0, y) + \Pr(\pi = 1|\tau, \sigma, s)\, \Delta\Pi(y) \tag{A27}$$

$$\Pi(0, x) + \Pr(\pi = 1|\tau', \sigma', s')\, \Delta\Pi(x) \geq \Pi(0, y) + \Pr(\pi = 1|\tau', \sigma', s')\, \Delta\Pi(y) \tag{A28}$$

$$\Pi(0, x) + \Pr(\pi = 1|\tau'', \sigma'', s'')\, \Delta\Pi(x) \leq \Pi(0, y) + \Pr(\pi = 1|\tau'', \sigma'', s'')\, \Delta\Pi(y) \tag{A29}$$

This implies $\Pi(\pi, x) = \Pi(\pi, y)$ for all $\pi \in \{0, 1\}$, which in turn implies $x = y$. ■

We now characterize all possible pure-strategy PBE. For ease of exposition, we define

$$\Pr(\pi = 1|ijk) \equiv \Pr(\pi = 1|\tau = i, \sigma = j, s = k) \quad \text{for} \quad i, j, k \in \{g, b\} \times \{H, L\} \times \{H, L\}. \tag{A30}$$

Suppose type (g, H, H) chooses investment level x. In that case, Lemma 6 implies that x must also be chosen by (at least) one of the types (b, H, H), (b, H, L), (b, L, H), and (b, L, L). Assume type (b, H, H) chooses a different investment level y, so that (at least) one of the types (b, H, L), (b, L, H), and (b, L, L) must choose x. In that case, Lemma 7 implies that we must have $\Pr(\theta_A = g|\pi, x) = \Pr(\theta_A = g|\pi, y)$ for all $\pi \in \{0, 1\}$. Lemma 8 then implies $x = y$. An analogous argument applies for the investment levels chosen by types (g, L, H) and (b, H, H), and types (g, L, L), (b, H, L), and (b, L, L). Hence, in any pure-strategy PBE, types (g, H, H), $(g, L, H,)$ and (b, H, H) must chose the same level of investment, and types (g, L, L), $(g, H, L,)$ and (b, L, L) must chose the same level of investment.

Suppose first that types (g, H, H), $(g, L, H,)$ and (b, H, H) and types (g, L, L), $(g, H, L,)$ and (b, L, L) choose the same investment level x. In that case, Lemma 8 implies that all types, including (b, H, L) and (b, L, H), choose the same investment level x.

Now suppose types (g, H, H), $(g, L, H,)$ and (b, H, H) choose investment level x, and types (g, L, L), $(g, H, L,)$ and (b, L, L) choose investment level y. There are ten possible sub-cases:

(i) Type (b, L, H) chooses x, and type (b, H, L) chooses x.

This cannot constitute an equilibrium if $\Pr(\pi = 1|bHL) \leq 1/2 \leq \Pr(\pi = 1|bLH)$ because type (b, H, L) would deviate to y. To see why, suppose x were chosen only by types (g, H, H), (g, L, H), and (b, H, H), and y were chosen only by types (g, L, L), (g, H, L), and (b, L, L). In that case, type (b, H, L) would prefer y over x. If x is chosen also by types (b, L, H) and (b, H, L), the expected reputation after choosing x is even lower. Hence, type (b, H, L) would deviate to y. Similarly, the assumed constellation cannot be an equilibrium if $\Pr(\pi = 1|bHL) > 1/2 > \Pr(\pi = 1|bLH)$ because type (b, L, H) would deviate to y.

(ii) Type (b, L, H) chooses x, and type (b, H, L) chooses y.

(iii) Type (b, L, H) chooses x, and type (b, H, L) chooses z.

We have $\Pr(\theta_A = g|\pi, \tau = g) > \Pr(\theta_A = g|\pi, bLL) > \Pr(\theta_A = g|\pi, bHL)$ for all $\pi \in \{0,1\}$, so that type (b, H, L) would prefer y over z. Hence, the proposed constellation cannot constitute an equilibrium.

(iv) Type (b, L, H) chooses y, and type (b, H, L) chooses x.

(v) Type (b, L, H) chooses y, and type (b, H, L) chooses y.

This constellation cannot constitute an equilibrium for the same reason as in sub-case (i). If $\Pr(\pi = 1|bHL) > 1/2 > \Pr(\pi = 1|bLH)$, then type (b, H, L) would deviate to x. If $\Pr(\pi = 1|bHL) \leq 1/2 \leq \Pr(\pi = 1|bLH)$, then type (b, L, H) would deviate to x.

(vi) Type (b, L, H) chooses y, and type (b, H, L) chooses z.

In that case, we must have $\Pr(\pi = 1|bHL) \geq 1/2 \geq \Pr(\pi = 1|bLH)$. Otherwise, Lemma 8 would imply $z = y$. However, $\Pr(\pi = 1|bHL) \geq 1/2$ implies that type (b, H, L) would prefer x over z. Hence, the proposed constellation cannot constitute an equilibrium.

(vii) Type (b, L, H) chooses z, and type (b, H, L) chooses x.

In that case, we must have $\Pr(\pi = 1|bHL) \leq 1/2 \leq \Pr(\pi = 1|bLH)$. Otherwise, Lemma 8 would imply $z = x$. However, $\Pr(\pi = 1|bHL) \leq 1/2$ implies that type (b, L, H) would prefer y over z. Hence, the proposed constellation cannot constitute an equilibrium.

(viii) Type (b, L, H) chooses z, and type (b, H, L) chooses y.

We have $\Pr(\theta_A = g|\pi, \tau = g) > \Pr(\theta_A = g|\pi, bHH) > \Pr(\theta_A = g|\pi, bLH)$ for all $\pi \in \{0,1\}$, which implies that type (b, L, H) would prefer x over z. Hence, the proposed constellation cannot constitute an equilibrium.

(ix) Type (b, L, H) chooses z, and type (b, H, L) chooses z.

This constellation cannot constitute an equilibrium because type (b, L, H) would deviate to x if $\Pr(\pi = 1|bLH) \geq 1/2 \geq \Pr(\pi = 1|bHL)$, and type (b, H, L) would deviate to y if $\Pr(\pi = 1|bLH) \leq 1/2 \leq \Pr(\pi = 1|bHL)$.

(x) Type (b, L, H) chooses z, and type (b, H, L) chooses w.

We have $\Pr(\theta_A = g|\pi, \tau = g) > \Pr(\theta_A = g|\pi, bHH) > \Pr(\theta_A = g|\pi, bLH)$ for all $\pi \in \{0, 1\}$, which implies that type (b, L, H) would prefer x over z. Hence, the proposed constellation cannot constitute an equilibrium.

It follows that either type (b, L, H) chooses x and type (b, H, L) chooses y as in sub-case (ii), or type (b, H, L) chooses x and type (b, L, H) chooses y as in sub-case (iv).

Hence, there are three types of potential pure-strategy PBE: (1) All types choose the same investment level x. (2) All types with $s = H$ choose the same investment level x, and all types with $s = L$ choose the same investment level y. (3) All types with $(\tau = g, s = H)$ or $(\tau = b, \sigma = H)$ choose the same investment level x, and all types with $(\tau = g, s = L)$ or $(\tau = b, \sigma = L)$ choose the same investment level y.

In the first equilibrium, the manager's private information (τ, σ, s) is not revealed to the shareholders. As a consequence, the shareholders do not update their beliefs about the manager's quality, i.e., maintain the belief $\Pr(\theta_A = g|\pi, x) = \delta$ for all $\pi \in \{0, 1\}$. This pooling equilibrium always exists. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

In the second equilibrium, s is revealed to the shareholders (but τ and σ are not). Proposition 3 implies that revealing s to the shareholders is incentive compatible for all types if and only if ϕ is (weakly) smaller than δ_b. Hence, the second equilibrium exists if and only if $\phi \leq \delta_b$. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

In the third equilibrium, types (g, H, H), (g, L, H), and (b, H, H) always prefer x to y, and types (g, L, L),

(g, H, L), and (b, L, L) always prefer y to x. Type (b, H, L) prefers x to y, and type (b, L, H) prefers y to x if $\phi > \delta_b$. Hence, the third equilibrium exists if and only if $\phi > \delta_b$. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

We now describe the Pareto-dominant PBE in pure strategies. First, consider all possible equilibria in which all types choose the same level of investment x. The manager is indifferent between all these equilibria. The shareholders prefer the equilibrium in which the optimal level of investment is chosen, i.e., the equilibrium in which

$$x = a^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1) \cdot \Pi(1, a) + \Pr(\pi = 0) \cdot \Pi(0, a). \tag{A31}$$

Second, consider all possible equilibria in which all types with $s = H$ choose the same investment level x, and all types with $s = L$ choose the same investment level y. The manager is indifferent between all these equilibria. The shareholders prefer the equilibrium in which the optimal level of investment (conditional on s) is chosen, i.e., the equilibrium in which

$$x = a^{**}_{s=H} \in \arg\max_{a \geq 0} \Pr(\pi = 1|s = H) \cdot \Pi(1, a) + \Pr(\pi = 0|s = H) \cdot \Pi(0, a) \tag{A32}$$

and

$$y = a^{**}_{s=L} \in \arg\max_{a \geq 0} \Pr(\pi = 1|s = L) \cdot \Pi(1, a) + \Pr(\pi = 0|s = L) \cdot \Pi(0, a). \tag{A33}$$

Finally, consider all possible equilibria in which all types with $(\tau = g, s = H)$ or $(\tau = g, \sigma = H)$ choose the same investment level x, and all types with $(\tau = g, s = L)$ or $(\tau = b, \sigma = L)$ choose the same investment level y. The manager is indifferent between all these equilibria. The shareholders prefer the equilibrium in which the optimal level of investment is chosen, i.e., the equilibrium in which $x = a^{**}_H$ is given by

$$\frac{\partial \Pi}{\partial a}\left(0, a^{**}_H\right) + \left[\delta p + (1 - \delta)\left(\frac{1}{2} + \phi\left(p - \frac{1}{2}\right)\right)\right] \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a^{**}_H\right) - \frac{\partial \Pi}{\partial a}\left(0, a^{**}_H\right)\right] = 0. \tag{A34}$$

and $y = a^{**}_L$ is given by is given by

$$\frac{\partial \Pi}{\partial a}\left(0, a^{**}_L\right) + \left[\delta(1 - p) + (1 - \delta)\left(\frac{1}{2} - \phi\left(p - \frac{1}{2}\right)\right)\right] \cdot \left[\frac{\partial \Pi}{\partial a}\left(1, a^{**}_L\right) - \frac{\partial \Pi}{\partial a}\left(0, a^{**}_L\right)\right] = 0. \tag{A35}$$

Ex-ante, the manager is indifferent between all possible equilibria (because his expected reputation is equal to the prior belief about his quality in all equilibria). The shareholders prefer the second and the third equilibrium (with two equilibrium investment levels x and y) to the first equilibrium (with a single investment level x). Hence, the Pareto-dominant PBE in pure strategies is the second equilibrium if and only if $\phi \leq \delta_b$ and otherwise the third equilibrium. ■

Proof of Proposition 11: Under marking to market, the market signal is common knowledge, and the manager's private information (i.e., his "type") is $(\tau, s) \in \{(g, H) ; (g, L) ; (b, H) ; (b, L)\}$. Consider the case of $\sigma = H$. (The case of $\sigma = L$ is symmetric.) Suppose type (g, H) chooses investment level x. By Lemma 6, x must also be chosen by type (b, H), type (b, L), or both. Suppose type (b, L) chooses x, and type (b, H) chooses another investment level y. In that case, Lemma 8 implies $x = y$. Hence, if type (g, H) chooses x, type (b, H) also chooses x. An analogous argument implies that types (g, L) and (b, L) must choose the same investment level.

Hence, there are two potential pure-strategy PBE: (1) All types choose the same level of investment x. (2) Types (g, H) and (b, H) choose the same level of investment x, and types (g, L) and (b, L) choose the same level of investment y.

In the first equilibrium, the manager's private information (τ, s) is not revealed to the shareholders. As a consequence, the shareholders do not update their belief about the manager's quality, i.e., maintain the belief $\Pr(\theta_A = g|\pi, x) = \delta$ for all $\pi \in \{0, 1\}$. This pooling equilibrium always exists. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

In the second equilibrium, s is revealed to the shareholders (but τ is not). Proposition 4 implies that revealing s is incentive compatible for all types only if ϕ is (weakly) smaller than some cut-off ϕ_b^{**}, where $\phi_b^{**} < \delta_b$ is determined analogously to ϕ^* in the proof of Proposition 4 (and considering the incentive compatibility constraint in case of $\tau = b$ and $\sigma \neq s$). Hence, the second equilibrium exists if and only if

$\phi \leq \phi_b^{**}$. It can be sustained, for example, by the out-of-equilibrium belief $\Pr(\theta_A = g|\pi, \tilde{x}) = 0$ for all $\pi \in \{0, 1\}$ and any out-of-equilibrium investment $\tilde{x}$.

We now describe the Pareto-dominant PBE in pure strategies. First, consider all possible pure-strategy PBE (conditional on a realization of σ) in which all four types choose the same level of investment x. The manager is indifferent between all these equilibria. The shareholders prefer the equilibrium in which the optimal level of investment (conditional on σ) is chosen, i.e., the equilibrium in which

$$x = a_\sigma^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma) \cdot \Pi(1, a) + \Pr(\pi = 0|\sigma) \cdot \Pi(0, a). \tag{A36}$$

Second, consider all possible pure-strategy PBE in which types (g, H) and (b, H) choose investment level x, and types (g, L) and (b, L) choose y. The manager is indifferent between all these equilibria. The shareholders prefer the equilibrium in which the optimal level of investment (conditional on σ and s) is chosen, i.e., the equilibrium in which

$$x = a_{\sigma,s=H}^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma, s = H) \cdot \Pi(1, a) + \Pr(\pi = 0|\sigma, s = H) \cdot \Pi(0, a) \tag{A37}$$

and

$$y = a_{\sigma,s=L}^{**} \in \arg\max_{a \geq 0} \Pr(\pi = 1|\sigma, s = L) \cdot \Pi(1, a) + \Pr(\pi = 0|\sigma, s = L) \cdot \Pi(0, a). \tag{A38}$$

Ex-ante, the manager is indifferent between all possible equilibria because his expected reputation is equal to the prior belief about his quality in any equilibrium. The shareholders prefer the equilibrium in which types (g, H) and (b, H) choose investment level $x = a_{\sigma,s=H}^{**}$, and types (g, L) and (b, L) choose $y = a_{\sigma,s=L}^{**}$. Given the manager's indifference, the Pareto-dominant equilibrium is the one preferred by the shareholders. ■

Considering PBE in mixed strategies does not change our main results

We now show that considering PBE in mixed strategies does not change our main results. The intuition is as follows. Propositions 1 to 4 imply: (1) For $\phi \leq \phi^*$, the manager follows the first-best investment strategy in the Pareto-dominant PBE under marking to market but not under historical cost accounting. Hence, for $\phi \leq \phi^*$, the shareholders are better off under marking to market whether we restrict attention to pure-strategy PBE or not. (2) For $\phi > \phi^*$, if we restrict attention to pure-strategy PBE, the shareholders are better off under historical cost accounting than under marking to market.

We now show the following result: (3) For $\phi > \phi^*$, the Pareto-dominant PBE under marking to market is a pure-strategy PBE. It follows from (1), (2), and (3) that considering also PBE in mixed strategies does not change Results 1 and 2 (derived in Section 4).

Under mark-to-market accounting, the market signal (σ) is common knowledge, and the manager's private information (i.e., his "type") is $s \in \{H, L\}$. Assume $\sigma = H$. (The proof for the case of $\sigma = L$ is symmetric.) Suppose type s follows a mixed strategy. There must be (at least) two distinct investment levels, say x and y, that type s chooses with positive probability. This leads to three possible sub-cases: (i) x and y are chosen only by type s (but not by type s'). (ii) x and y are chosen by both types s and s'. (iii) One investment level, say x, is chosen by both types s and s', but y is chosen only by one of the two types.

The following two Lemmas imply that sub-cases (i) and (ii) cannot be the Pareto-dominant PBE. To simplify the notation, we define $\Pr(jk) \equiv \Pr(\sigma = j, s = k)$ for $j, k \in \{H, L\} \times \{H, L\}$ as well as $\Pr(ijk) \equiv \Pr(\pi = i | \sigma = j, s = k)$ and $\widehat{\delta}(ijk) \equiv \Pr(\theta_A = g | \pi = i, \sigma = j, s = k)$ for $i, j, k \in \{1, 0\} \times \{H, L\} \times \{H, L\}$.

Lemma 9 *Suppose a PBE E exists in which type s chooses investment level x with probability $\alpha \in (0, 1)$ and investment level $y \neq x$ with probability $\beta \in (0, 1)$, and both x and y are chosen by type s' with probability zero. Then another PBE E^* exists that Pareto-dominates E.*

Proof: Without loss of generality, assume $x < y$. It follows from $\Pi(\pi, a)$'s continuity and concavity in a that an investment level $z \in [x, y]$ exists such that $E[\Pi(\pi, z)|\sigma, s] > \alpha \cdot E[\Pi(\pi, x)|\sigma, s] + \beta \cdot E[\Pi(\pi, y)|\sigma, s]$ and z is chosen by types s and s' with probability zero in E. Consider now the following PBE E^*: Type s' follows the same strategy as in E. Type s invests z whenever type s invests x or y in E and otherwise follows the same strategy as in E. Define $\widehat{\delta_E} \equiv \Pr(\theta_A = g|\pi, \sigma, x) = \Pr(\theta_A = g|\pi, \sigma, y)$ in E, and $\widehat{\delta_{E^*}} \equiv \Pr(\theta_A = g|\pi, \sigma, z)$ in E^*. We have $\widehat{\delta_E} = \widehat{\delta_{E^*}}$ for all $\pi \in \{0, 1\}$, so that both type's (s and s') incentive compatibility constraints are satisfied in E^* if they are satisfied in E. It follows that E^* exists if E exists. Further, E^* Pareto-dominates E because the firm's expected profits are higher in E^* than in E. ■

Lemma 10 *Suppose a PBE E exists in which type s chooses investment level x with probability* $\alpha \in (0, 1)$ *and investment level* $y \neq x$ *with probability* $\beta \in (0, 1)$*, and type* s' *chooses x with probability* $\alpha' \in (0, 1)$ *and y with probability* $\beta' \in (0, 1)$*. Then another PBE* E^* *exists that Pareto-dominates E.*

Proof: Without loss of generality, assume $x < y$. Define $\Delta x \equiv \Pr(\theta_A = g|\pi = 1, \sigma, x) - \Pr(\theta_A = g|\pi = 0, \sigma, x)$ and $\Delta y \equiv \Pr(\theta_A = g|\pi = 1, \sigma, y) - \Pr(\theta_A = g|\pi = 0, \sigma, y)$. In equilibrium, the following incentive compatibility constraints must be satisfied simultaneously:

$$\Pr(\theta_A = g|\pi = 0, \sigma = H, x) + \Pr(1HH)\,\Delta x = \Pr(\theta_A = g|\pi = 0, \sigma = H, y) + \Pr(1HH)\,\Delta y \tag{A39}$$

and

$$\Pr(\theta_A = g|\pi = 0, \sigma = H, x) + \Pr(1HL)\,\Delta x = \Pr(\theta_A = g|\pi = 0, \sigma = H, y) + \Pr(1HL)\,\Delta y. \tag{A40}$$

This implies $\Pr(\theta_A = g|\pi, \sigma = H, x) = \Pr(\theta_A = g|\pi, \sigma = H, y)$ for all $\pi \in \{0, 1\}$, $\Pr(s = H|\pi, \sigma = H, x) = \Pr(s = L|\pi, \sigma = H, y)$, and $\Pr(s = H|\sigma = H, x) = \Pr(s = H|\sigma = H, y) = \Pr(s = H|\sigma = H)$. Hence, in the assumed equilibrium E, the manager's choice of x or y does not provide any new information to the shareholders about the manager's private signal. The profit maximizing x and y would thus be $x = y = a^{**}_{\sigma=H}$.

It thus follows from $\Pi(\pi, a)$'s continuity and concavity in a that an investment level $z \in [x, y]$ exists such that the firm's expected profits would be higher if both types s and s' would choose z instead of x and

y, and z is chosen by types s and s' with probability zero in E. Consider now the following PBE E^*: Both types s and s' invest z whenever s and s' invest x or y in E and otherwise follow the same strategies as in E. Define $\widehat{\delta}_E \equiv \Pr(\theta_A = g|\pi,\sigma,x) = \Pr(\theta_A = g|\pi,\sigma,y)$ in E, and $\widehat{\delta}_{E^*} \equiv \Pr(\theta_A = g|\pi,\sigma,z)$ in E^*. We have $\widehat{\delta}_E = \widehat{\delta}_{E^*}$ for all $\pi \in \{0,1\}$, so that both type's (s and s') incentive compatibility constraints are satisfied in E^* if they are satisfied in E. It follows that E^* exists if E exists. Further, E^* Pareto-dominates E because the firm's expected profits are higher in E^* than in E. ■

Consider now sub-case (iii). Assume first that type $s = L$ invests x with probability $\alpha \in (0,1)$ and y with probability $\beta \in (0,1)$, while type $s = H$ invests x with probability $\gamma > 0$ and y with probability zero.

Note that investing y perfectly reveals $s = L$, so that

$$E\left[\widehat{\delta}(\pi,\sigma = H,y)\,|\sigma = H,s = L\right] = \Pr(1HL)\,\widehat{\delta}(1HL) + \Pr(0HL)\,\widehat{\delta}(0HL)\,. \tag{A41}$$

We further have

$$\begin{aligned} E\left[\widehat{\delta}(\pi,\sigma = H,x)\,|\sigma = H,s = L\right] &= \Pr(1HL)\Pr(\theta_A = g|\pi = 1,\sigma = H,x) \\ &\quad + \Pr(0HL)\Pr(\theta_A = g|\pi = 0,\sigma = H,x)\,, \end{aligned} \tag{A42}$$

It follows that type $s = L$ is indifferent between investing x and y if and only if

$$\alpha = \frac{\Pr(0HH)\Pr(HH)\left\{\frac{\Pr(0HL)\left[\widehat{\delta}(0HL)-\widehat{\delta}(0HH)\right]}{\Pr(1HL)\left[\widehat{\delta}(1HH)-\widehat{\delta}(1HL)\right]} - 1\right\}}{\Pr(0HL)\Pr(HL)\left\{1 - \frac{\Pr(0HH)\left[\widehat{\delta}(0HL)-\widehat{\delta}(0HH)\right]}{\Pr(1HH)\left[\widehat{\delta}(1HH)-\widehat{\delta}(1HL)\right]}\right\}} \tag{A43}$$

Note, however, that $\phi > \phi^*$ implies $\Pr(0HL)\left[\widehat{\delta}(0HL) - \widehat{\delta}(0HH)\right] < \Pr(1HL)\left[\widehat{\delta}(1HH) - \widehat{\delta}(1HL)\right]$. Thus, for $\phi > \phi^*$, no $\alpha \in (0,1)$ exists for which type $s = L$ is indifferent between investing x and y.

An analogous argument rules out the case in which type $s = H$ invests x with probability $\alpha \in (0,1)$ and y with probability $\beta \in (0,1)$, while type $s = L$ invests x with probability $\gamma > 0$ and y with probability zero. Hence, a PBE in which one investment level x is chosen by both types s and s', and another investment level y is chosen only by one of the two types does not exist if $\phi > \phi^*$. It follows that the Pareto-dominant PBE under mark-to-market accounting must be a PBE in pure strategies.

Derivations – Not for Publication

Probabilities:

$$\Pr(s=H) = \Pr(\sigma=H) = \frac{1}{2} \quad \text{(A44)}$$

$$\Pr(\sigma=H,s=H) = \Pr(\sigma=L,s=L) = \frac{1}{4}(1+\delta\phi) \quad \text{(A45)}$$

$$\Pr(\sigma=H,s=L) = \Pr(\sigma=L,s=H) = \frac{1}{4}(1-\delta\phi) \quad \text{(A46)}$$

$$\Pr(\pi=1|s=H) = \Pr(\pi=0|s=L) = \frac{1}{2}+\delta\left(p-\frac{1}{2}\right) \quad \text{(A47)}$$

$$\Pr(\pi=1|\sigma=H) = \Pr(\pi=0|\sigma=L) = \frac{1}{2}+\phi\left(p-\frac{1}{2}\right) \quad \text{(A48)}$$

$$\Pr(\pi=1|\sigma=H,s=H) = \frac{1}{2}+\frac{\left(p-\frac{1}{2}\right)(\delta+\phi)}{1+\phi\delta} \quad \text{(A49)}$$

$$\Pr(\pi=1|\sigma=L,s=H) = \frac{1}{2}+\frac{\left(p-\frac{1}{2}\right)(\delta-\phi)}{1-\phi\delta} \quad \text{(A50)}$$

$$\Pr(\pi=1|\sigma=H,s=L) = \frac{1}{2}-\frac{\left(p-\frac{1}{2}\right)(\delta-\phi)}{1-\phi\delta} \quad \text{(A51)}$$

$$\Pr(\pi=1|\sigma=L,s=L) = \frac{1}{2}-\frac{\left(p-\frac{1}{2}\right)(\delta+\phi)}{1+\phi\delta} \quad \text{(A52)}$$

Posterior beliefs about the manager's type:

$$\widehat{\delta}(\pi=1,\sigma=H,s=H) = \frac{\Pr(\pi=1,\sigma=H,s=H|\theta_A=g)\cdot\Pr(\theta_A=g)}{\Pr(\pi=1,\sigma=H,s=H|\theta_A=g)\cdot\Pr(\theta_A=g)+\Pr(\pi=1,\sigma=H,s=H|\theta_A=b)\cdot\Pr(\theta_A=b)} \quad \text{(A53)}$$

with

$$\begin{aligned}\Pr(\pi=1,\sigma=H,s=H|\theta_A=g) &= \Pr(\pi=1,\sigma=H,s=H|\theta_M=i,\theta_A=g)\cdot\Pr(\theta_M=i)\\ &\quad+\Pr(\pi=1,\sigma=H,s=H|\theta_M=u,\theta_A=g)\cdot\Pr(\theta_M=u)\\ &= \tfrac{1}{2}\cdot p\cdot\phi+\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot p\cdot(1-\phi)\\ &= \tfrac{1}{4}p(1+\phi)\end{aligned} \quad \text{(A54)}$$

and

$$\begin{aligned}\Pr(\pi=1,\sigma=H,s=H|\theta_A=b) &= \Pr(\pi=1,\sigma=H,s=H|\theta_M=i,\theta_A=b)\cdot\Pr(\theta_M=i)\\ &\quad+\Pr(\pi=1,\sigma=H,s=H|\theta_M=u,\theta_A=b)\cdot\Pr(\theta_M=u)\\ &= \tfrac{1}{2}\cdot p\cdot\tfrac{1}{2}\cdot\phi+\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot(1-\phi)\\ &= \tfrac{1}{4}\left[\tfrac{1}{2}+\phi\left(p-\tfrac{1}{2}\right)\right]\end{aligned} \quad \text{(A55)}$$

Hence:

$$\widehat{\delta}(\pi=1,\sigma=H,s=H) = \frac{\frac{1}{4}p(1+\phi)\cdot\delta}{\frac{1}{4}p(1+\phi)\cdot\delta+\frac{1}{4}\left[\frac{1}{2}+\phi\left(p-\frac{1}{2}\right)\right]\cdot(1-\delta)} = \frac{2\delta p(1+\phi)}{2p(\delta+\phi)+(1-\phi)(1-\delta)} \quad \text{(A56)}$$

By analogy:

$$\widehat{\delta}(\pi=0,\sigma=L,s=L) = \frac{2\delta p(1+\phi)}{2p(\delta+\phi)+(1-\phi)(1-\delta)} \quad \text{(A57)}$$

$$\widehat{\delta}(\pi=1,\sigma=L,s=H) = \frac{\Pr(\pi=1,\sigma=L,s=H|\theta_A=g)\cdot\Pr(\theta_A=g)}{\Pr(\pi=1,\sigma=L,s=H|\theta_A=g)\cdot\Pr(\theta_A=g)+\Pr(\pi=1,\sigma=L,s=H|\theta_A=b)\cdot\Pr(\theta_A=b)} \quad \text{(A58)}$$

with

$$\begin{aligned}\Pr(\pi=1,\sigma=L,s=H|\theta_A=g) &= \Pr(\pi=1,\sigma=L,s=H|\theta_M=i,\theta_A=g)\cdot\Pr(\theta_M=i) \\ &\quad +\Pr(\pi=1,\sigma=L,s=H|\theta_M=u,\theta_A=g)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot 0\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot p\cdot(1-\phi) \\ &= \frac{1}{4}p(1-\phi)\end{aligned} \quad \text{(A59)}$$

and

$$\begin{aligned}\Pr(\pi=1,\sigma=L,s=H|\theta_A=b) &= \Pr(\pi=1,\sigma=L,s=H|\theta_M=i,\theta_A=b)\cdot\Pr(\theta_M=i) \\ &\quad +\Pr(\pi=1,\sigma=L,s=H|\theta_M=u,\theta_A=b)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot(1-\phi) \\ &= \frac{1}{4}\left[\frac{1}{2}-\phi\left(p-\frac{1}{2}\right)\right]\end{aligned} \quad \text{(A60)}$$

Hence:

$$\widehat{\delta}(\pi=1,\sigma=L,s=H) = \frac{\frac{1}{4}p(1-\phi)\cdot\delta}{\frac{1}{4}p(1-\phi)\cdot\delta+\frac{1}{4}\left[\frac{1}{2}-\phi\left(p-\frac{1}{2}\right)\right]\cdot(1-\delta)} = \frac{2\delta p(1-\phi)}{2p(\delta-\phi)+(1-\delta)(1+\phi)} \quad \text{(A61)}$$

By analogy:

$$\widehat{\delta}(\pi=0,\sigma=H,s=L) = \frac{2\delta p(1-\phi)}{2p(\delta-\phi)+(1-\delta)(1+\phi)} \quad \text{(A62)}$$

$$\widehat{\delta}(\pi=1,\sigma=H,s=L) = \frac{\Pr(\pi=1,\sigma=H,s=L|\theta_A=g)\cdot\Pr(\theta_A=g)}{\Pr(\pi=1,\sigma=H,s=L|\theta_A=g)\cdot\Pr(\theta_A=g)+\Pr(\pi=1,\sigma=H,s=L|\theta_A=b)\cdot\Pr(\theta_A=b)} \quad \text{(A63)}$$

with

$$\begin{aligned}\Pr(\pi=1,\sigma=H,s=L|\theta_A=g) &= \Pr(\pi=1,\sigma=H,s=L|\theta_M=i,\theta_A=g)\cdot\Pr(\theta_M=i) \\ &\quad +\Pr(\pi=1,\sigma=H,s=L|\theta_M=u,\theta_A=g)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot 0\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot(1-p)\cdot(1-\phi) \\ &= \frac{1}{4}(1-p)(1-\phi)\end{aligned} \quad \text{(A64)}$$

and

$$\begin{aligned}\Pr(\pi=1,\sigma=H,s=L|\theta_A=b) &= \Pr(\pi=1,\sigma=H,s=L|\theta_M=i,\theta_A=b)\cdot\Pr(\theta_M=i) \\ &\quad +\Pr(\pi=1,\sigma=H,s=L|\theta_M=u,\theta_A=b)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot p\cdot\frac{1}{2}\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot(1-\phi) \\ &= \frac{1}{4}\left[\frac{1}{2}+\phi\left(p-\frac{1}{2}\right)\right]\end{aligned} \quad \text{(A65)}$$

Hence:

$$\begin{aligned}\widehat{\delta}(\pi=1,\sigma=H,s=L) &= \frac{\frac{1}{4}(1-p)(1-\phi)\cdot\delta}{\frac{1}{4}(1-p)(1-\phi)\cdot\delta+\frac{1}{4}\left[\frac{1}{2}+\phi\left(p-\frac{1}{2}\right)\right]\cdot(1-\delta)} \\ &= \frac{2\delta(1-p)(1-\phi)}{2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)} \end{aligned} \tag{A66}$$

By analogy:

$$\widehat{\delta}(\pi=0,\sigma=L,s=H)=\frac{2\delta(1-p)(1-\phi)}{2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)} \tag{A67}$$

$$\widehat{\delta}(\pi=1,\sigma=L,s=L)=\frac{\Pr(\pi=1,\sigma=L,s=L|\theta_A=g)\cdot\Pr(\theta_A=g)}{\Pr(\pi=1,\sigma=L,s=L|\theta_A=g)\cdot\Pr(\theta_A=g)+\Pr(\pi=1,\sigma=L,s=L|\theta_A=b)\cdot\Pr(\theta_A=b)} \tag{A68}$$

with

$$\begin{aligned}\Pr(\pi=1,\sigma=L,s=L|\theta_A=g) &= \Pr(\pi=1,\sigma=L,s=L|\theta_M=i,\theta_A=g)\cdot\Pr(\theta_M=i) \\ &\quad+\Pr(\pi=1,\sigma=L,s=L|\theta_M=u,\theta_A=g)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot(1-p)\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot(1-p)\cdot(1-\phi) \\ &= \frac{1}{4}(1-p)(1+\phi)\end{aligned} \tag{A69}$$

and

$$\begin{aligned}\Pr(\pi=1,\sigma=L,s=L|\theta_A=b) &= \Pr(\pi=1,\sigma=L,s=L|\theta_M=i,\theta_A=b)\cdot\Pr(\theta_M=i) \\ &\quad+\Pr(\pi=1,\sigma=L,s=L|\theta_M=u,\theta_A=b)\cdot\Pr(\theta_M=u) \\ &= \frac{1}{2}\cdot(1-p)\cdot\frac{1}{2}\cdot\phi+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot(1-\phi) \\ &= \frac{1}{4}\left[\frac{1}{2}-\phi\left(p-\frac{1}{2}\right)\right]\end{aligned} \tag{A70}$$

Hence:

$$\begin{aligned}\widehat{\delta}(\pi=1,\sigma=L,s=L) &= \frac{\frac{1}{4}(1-p)(1+\phi)\cdot\delta}{\frac{1}{4}(1-p)(1+\phi)\cdot\delta+\frac{1}{4}\left[\frac{1}{2}-\phi\left(p-\frac{1}{2}\right)\right]\cdot(1-\delta)} \\ &= \frac{2\delta(1-p)(1+\phi)}{2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)}\end{aligned} \tag{A71}$$

By analogy;

$$\widehat{\delta}(\pi=0,\sigma=H,s=H)=\frac{2\delta(1-p)(1+\phi)}{2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)} \tag{A72}$$

Comparison of posterior beliefs about the manager's type:

$$\widehat{\delta}(\pi=1,\sigma=H,s=H)>\widehat{\delta}(\pi=1,\sigma=L,s=H) \tag{A73}$$

and

$$\widehat{\delta}(\pi=0,\sigma=L,s=L)>\widehat{\delta}(\pi=0,\sigma=H,s=L) \tag{A74}$$

if

$$\begin{aligned}\frac{2\delta p(1+\phi)}{2p(\delta+\phi)+(1-\phi)(1-\delta)} &> \frac{2\delta p(1-\phi)}{2p(\delta-\phi)+(1-\delta)(1+\phi)} \\ (1+\phi)\left[2p(\delta-\phi)+(1-\delta)(1+\phi)\right] &> (1-\phi)\left[2p(\delta+\phi)+(1-\phi)(1-\delta)\right] \\ 1 &> p\end{aligned} \tag{A75}$$

$$\widehat{\delta}(\pi = 0, \sigma = H, s = H) > \widehat{\delta}(\pi = 0, \sigma = L, s = H) \tag{A76}$$

and

$$\widehat{\delta}(\pi = 1, \sigma = L, s = L) > \widehat{\delta}(\pi = 1, \sigma = H, s = L) \tag{A77}$$

if

$$\begin{aligned}
\frac{2\delta(1-p)(1+\phi)}{2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)} &> \frac{2\delta(1-p)(1-\phi)}{2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)} \\
(1+\phi)\left[2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)\right] &> (1-\phi)\left[2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)\right] \\
1 &> 1-p \\
p &> 0
\end{aligned} \tag{A78}$$

$F(\phi)$ and ϕ^*:

$$\begin{aligned}
F(\phi) \equiv\ & \Pr(\pi = 1|\sigma = L, s = H)\left[\widehat{\delta}(\pi = 1, \sigma = L, s = L) - \widehat{\delta}(\pi = 1, \sigma = L, s = H)\right] \\
& + \Pr(\pi = 0|\sigma = L, s = H)\left[\widehat{\delta}(\pi = 0, \sigma = L, s = L) - \widehat{\delta}(\pi = 0, \sigma = L, s = H)\right]
\end{aligned} \tag{A79}$$

$$\Pr(\pi = 1|\sigma = L, s = H) = \frac{\Pr(\sigma=L,s=H|\pi=1)\cdot\Pr(\pi=1)}{\Pr(\sigma=L,s=H|\pi=1)\cdot\Pr(\pi=1)+\Pr(\sigma=L,s=H|\pi=0)\cdot\Pr(\pi=0)} \tag{A80}$$

with

$$\begin{aligned}
\Pr(\sigma = L, s = H|\pi = 1) =\ & \Pr(\sigma = L, s = H|\pi = 1, \theta_M = i, \theta_A = g)\cdot\Pr(\theta_M = i, \theta_A = g) \\
& + \Pr(\sigma = L, s = H|\pi = 1, \theta_M = i, \theta_A = b)\cdot\Pr(\theta_M = i, \theta_A = b) \\
& + \Pr(\sigma = L, s = H|\pi = 1, \theta_M = u, \theta_A = g)\cdot\Pr(\theta_M = u, \theta_A = g) \\
& + \Pr(\sigma = L, s = H|\pi = 1, \theta_M = u, \theta_A = b)\cdot\Pr(\theta_M = u, \theta_A = b) \\
=\ & 0\cdot\phi\cdot\delta + (1-p)\cdot\frac{1}{2}\cdot\phi\cdot(1-\delta) \\
& + \frac{1}{2}\cdot p\cdot(1-\phi)\cdot\delta + \frac{1}{2}\cdot\frac{1}{2}\cdot(1-\phi)\cdot(1-\delta) \\
=\ & \frac{1}{2}\left[\frac{1}{2}(1-\phi\delta) - \left(p-\frac{1}{2}\right)(\phi-\delta)\right]
\end{aligned} \tag{A81}$$

and

$$\begin{aligned}
\Pr(\sigma = L, s = H|\pi = 0) =\ & \Pr(\sigma = L, s = H|\pi = 0, \theta_M = i, \theta_A = g)\cdot\Pr(\theta_M = i, \theta_A = g) \\
& + \Pr(\sigma = L, s = H|\pi = 0, \theta_M = i, \theta_A = b)\cdot\Pr(\theta_M = i, \theta_A = b) \\
& + \Pr(\sigma = L, s = H|\pi = 0, \theta_M = u, \theta_A = g)\cdot\Pr(\theta_M = u, \theta_A = g) \\
& + \Pr(\sigma = L, s = H|\pi = 0, \theta_M = u, \theta_A = b)\cdot\Pr(\theta_M = u, \theta_A = b) \\
=\ & 0\cdot\phi\cdot\delta + p\cdot\frac{1}{2}\cdot\phi\cdot(1-\delta) \\
& + \frac{1}{2}\cdot(1-p)\cdot(1-\phi)\cdot\delta + \frac{1}{2}\cdot\frac{1}{2}\cdot(1-\phi)\cdot(1-\delta) \\
=\ & \frac{1}{2}\left[\frac{1}{2}(1-\delta\phi) + \left(p-\frac{1}{2}\right)(\phi-\delta)\right]
\end{aligned} \tag{A82}$$

Hence:

$$\Pr(\pi = 1|\sigma = L, s = H) = \frac{1}{2} + \frac{\left(p-\frac{1}{2}\right)(\delta-\phi)}{1-\delta\phi} \tag{A83}$$

and

$$
\begin{aligned}
\Pr(\pi=0|\sigma=L,s=H) &= 1-\Pr(\pi=1|\sigma=L,s=H) \\
&= \frac{1}{2}-\frac{\left(p-\frac{1}{2}\right)(\delta-\phi)}{1-\delta\phi} \qquad \text{(A84)}
\end{aligned}
$$

If $\phi=0$:

$$
\begin{aligned}
\Pr(\pi=1|\sigma=L,s=H) &= \frac{1}{2}+\delta\left(p-\frac{1}{2}\right) \qquad \text{(A85)}\\
\Pr(\pi=0|\sigma=L,s=H) &= \frac{1}{2}-\delta\left(p-\frac{1}{2}\right) \qquad \text{(A86)}\\
\widehat{\delta}(\pi=1,\sigma=L,s=L) &= \widehat{\delta}(\pi=0,\sigma=L,s=H)=\frac{2\delta(1-p)}{2(1-p)\delta+(1-\delta)} \qquad \text{(A87)}\\
\widehat{\delta}(\pi=1,\sigma=L,s=H) &= \widehat{\delta}(\pi=0,\sigma=L,s=L)=\frac{2\delta p}{2p\delta+(1-\delta)} \qquad \text{(A88)}
\end{aligned}
$$

and hence

$$
\begin{aligned}
F(\phi=0) &= \left[\frac{1}{2}+\delta\left(p-\frac{1}{2}\right)\right]\left[\frac{2\delta(1-p)}{2(1-p)\delta+(1-\delta)}-\frac{2\delta p}{2p\delta+(1-\delta)}\right] \\
&\quad+\left[\frac{1}{2}-\delta\left(p-\frac{1}{2}\right)\right]\left[\frac{2\delta p}{2p\delta+(1-\delta)}-\frac{2\delta(1-p)}{2(1-p)\delta+(1-\delta)}\right] \\
&= -\frac{8\delta^2(1-\delta)\left(p-\frac{1}{2}\right)^2}{[2(1-p)\delta+(1-\delta)]\cdot[2p\delta+(1-\delta)]} \\
&< 0 \qquad \text{(A89)}
\end{aligned}
$$

If $\phi=\delta$:

$$
\begin{aligned}
\Pr(\pi=1|\sigma=L,s=H) &= \Pr(\pi=0|\sigma=L,s=H)=\frac{1}{2} \qquad \text{(A90)}\\
\widehat{\delta}(\pi=1,\sigma=L,s=L) &= \frac{2\delta(1-p)(1+\delta)}{4(1-p)\delta+(1-\delta)^2} \qquad \text{(A91)}\\
\widehat{\delta}(\pi=1,\sigma=L,s=H) &= \frac{2\delta p}{1+\delta} \qquad \text{(A92)}\\
\widehat{\delta}(\pi=0,\sigma=L,s=L) &= \frac{2\delta p(1+\delta)}{4p\delta+(1-\delta)^2} \qquad \text{(A93)}\\
\widehat{\delta}(\pi=0,\sigma=L,s=H) &= \frac{2\delta(1-p)}{1+\delta} \qquad \text{(A94)}
\end{aligned}
$$

and hence

$$
F(\phi=\delta)=\frac{1}{2}\left[\frac{2\delta p(1+\delta)}{4p\delta+(1-\delta)^2}-\frac{2\delta p}{1+\delta}+\frac{2\delta(1-p)(1+\delta)}{4(1-p)\delta+(1-\delta)^2}-\frac{2\delta(1-p)}{1+\delta}\right] \qquad \text{(A95)}
$$

Furthermore,

$$\begin{aligned}
\frac{2\delta p\,(1+\delta)}{4p\delta+(1-\delta)^2}-\frac{2\delta p}{1+\delta} &= 2\delta p\left\{\frac{(1+\delta)}{4p\delta+(1-\delta)^2}-\frac{1}{1+\delta}\right\}\\
&= 2\delta p\left\{\frac{(1+\delta)^2}{\left[4p\delta+(1-\delta)^2\right][1+\delta]}-\frac{4p\delta+(1-\delta)^2}{\left[4p\delta+(1-\delta)^2\right][1+\delta]}\right\}\\
&= \frac{8\delta^2 p\,(1-p)}{\left[4p\delta+(1-\delta)^2\right](1+\delta)}\\
&> 0 \qquad \text{(A96)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{2\delta\,(1-p)\,(1+\delta)}{4\,(1-p)\,\delta+(1-\delta)^2}-\frac{2\delta\,(1-p)}{1+\delta} &= \frac{8\delta^2 p\,(1-p)}{\left[4\,(1-p)\,\delta+(1-\delta)^2\right](1+\delta)}\\
&> 0 \qquad \text{(A97)}
\end{aligned}$$

imply

$$F\,(\phi=\delta)>0 \qquad \text{(A98)}$$

$$\begin{aligned}
\frac{\partial F}{\partial\phi} = {} & \Pr(\pi=1|\sigma=L,s=H)\left[\frac{\partial\widehat{\delta}(\pi=1,\sigma=L,s=L)}{\partial\phi}-\frac{\partial\widehat{\delta}(\pi=1,\sigma=L,s=H)}{\partial\phi}\right]\\
&+\Pr(\pi=0|\sigma=L,s=H)\left[\frac{\widehat{\delta}(\pi=0,\sigma=L,s=L)}{\partial\phi}-\frac{\partial\widehat{\delta}(\pi=0,\sigma=L,s=H)}{\partial\phi}\right]\\
&+\frac{\partial\Pr(\pi=1|\sigma=L,s=H)}{\partial\phi}\left[\widehat{\delta}\,(\pi=1,\sigma=L,s=L)-\widehat{\delta}\,(\pi=1,\sigma=L,s=H)\right]\\
&+\frac{\partial\Pr(\pi=0|\sigma=L,s=H)}{\partial\phi}\left[\widehat{\delta}\,(\pi=0,\sigma=L,s=L)-\widehat{\delta}\,(\pi=0,\sigma=L,s=H)\right]
\end{aligned} \qquad \text{(A99)}$$

with

$$\frac{\partial\widehat{\delta}(\pi=0,\sigma=H,s=L)}{\partial\phi}=\frac{\partial\widehat{\delta}(\pi=1,\sigma=L,s=H)}{\partial\phi}=\frac{-4p(1-p)\delta(1-\delta)}{[2p(\delta-\phi)+(1-\delta)(1+\phi)]^2}<0 \qquad \text{(A100)}$$

$$\frac{\partial\widehat{\delta}(\pi=0,\sigma=L,s=H)}{\partial\phi}=\frac{\partial\widehat{\delta}(\pi=1,\sigma=H,s=L)}{\partial\phi}=\frac{-4p(1-p)\delta(1-\delta)}{[2(1-p)(\delta-\phi)+(1-\delta)(1+\phi)]^2}<0 \qquad \text{(A101)}$$

$$\frac{\partial\widehat{\delta}(\pi=1,\sigma=L,s=L)}{\partial\phi}=\frac{\partial\widehat{\delta}(\pi=0,\sigma=H,s=H)}{\partial\phi}=\frac{4p(1-p)\delta(1-\delta)}{[2(1-p)(\delta+\phi)+(1-\phi)(1-\delta)]^2}>0 \qquad \text{(A102)}$$

$$\frac{\widehat{\delta}(\pi=0,\sigma=L,s=L)}{\partial\phi}=\frac{\widehat{\delta}(\pi=1,\sigma=H,s=H)}{\partial\phi}=\frac{4p(1-p)\delta(1-\delta)}{[2p(\delta+\phi)+(1-\phi)(1-\delta)]^2}>0 \qquad \text{(A103)}$$

and

$$\frac{\partial\Pr(\pi=1|\sigma=L,s=H)}{\partial\phi} = -\frac{\left(p-\frac{1}{2}\right)\left(1-\delta^2\right)}{(1-\phi\delta)^2}<0 \qquad \text{(A104)}$$

$$\frac{\partial\Pr(\pi=0|\sigma=L,s=H)}{\partial\phi} = -\frac{\partial\Pr(\pi=1|\sigma=L,s=H)}{\partial\phi} \qquad \text{(A105)}$$

Hence:

$$
\begin{aligned}
\frac{\partial F}{\partial \phi} &= \Pr(\pi = 1|\sigma = L, s = H)\left[\underbrace{\frac{\partial\widehat{\delta}(\pi = 1, \sigma = L, s = L)}{\partial \phi}}_{+} - \underbrace{\frac{\partial\widehat{\delta}(\pi = 1, \sigma = L, s = H)}{\partial \phi}}_{-}\right] \\
&\quad + \Pr(\pi = 0|\sigma = L, s = H)\left[\underbrace{\frac{\widehat{\delta}(\pi = 0, \sigma = L, s = L)}{\partial \phi}}_{+} - \underbrace{\frac{\partial\widehat{\delta}(\pi = 0, \sigma = L, s = H)}{\partial \phi}}_{-}\right] \\
&\quad + \underbrace{\frac{\partial \Pr(\pi = 1|\sigma = L, s = H)}{\partial \phi}}_{-}\left[\begin{array}{c} \underbrace{\widehat{\delta}(\pi = 1, \sigma = L, s = L) - \widehat{\delta}(\pi = 0, \sigma = L, s = L)}_{-} \\ + \underbrace{\widehat{\delta}(\pi = 0, \sigma = L, s = H) - \widehat{\delta}(\pi = 1, \sigma = L, s = H)}_{-} \end{array}\right] \\
&> 0 \qquad \text{(A106)}
\end{aligned}
$$