The dollar-euro exchange rate and monetary fundamentals

Abstract
This study analyses the relationship between the dollar-euro exchange rate and macroeconomic fundamentals according to the monetary model after 1999. Multivariate and time-varying univariate cointegration techniques are used to test for a long-run equilibrium and changes in the underlying coefficients. Our results provide clear evidence of a long-run relationship between exchange rates and fundamentals. However, we find significant changes in the economic impact of fundamentals on the dollar-euro exchange rate. Both long-run and the short-run coefficients are shown to be strongly time-varying and significantly affected by the financial crisis and the emergence of unconventional monetary policy.

JEL codes: E31, F31

Keywords: cointegration, euro-dollar exchange rate, time-varying coefficient approach

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Introduction

The introduction of the euro brought remarkable changes to the international monetary system. Figure 1 illustrates the path of the dollar-euro exchange rate since 1999. After a significant appreciation during the first three years, the dollar started to rapidly depreciate coinciding with the introduction of the Euro as a currency on the streets in January 2002. The dollar-euro exchange rate moved from $0.83/€1 in January 2002 to $1.58/€1 in July 2007. The dollar strongly appreciated against the euro during the financial crisis in 2008 to $1.20/€1 which might be attributed to the safe haven status of the US dollar in time of financial stress (Tamakoshi and Hamori, 2014). The recent period of the euro area crisis has also been characterized by a strong dollar appreciation.

Figure 1: Dollar-Euro exchange rate after 1999

Note: The graph shows the dollar-euro exchange rates after 1999

Only a few studies have dealt with explaining the path of the dollar-euro exchange rate against the background of traditional exchange rate models; see for example Beckmann et al (2011) and Molodtsova et al (2011). Generally speaking, a researcher faces a difficult task when it comes to explaining exchange rate behavior, even when the focus is on possible long-run linkages between exchange rates and fundamentals. Dollar exchange rates have experienced large fluctuations since the breakdown of Bretton Woods, with the behavior of nominal exchange rates as shown by Engel and Hamilton (1990) being characterized by long swings. The overall evidence as shown by Sarno and Valente (2009) suggests that volatile expectations or departures from rationality are important in the context of those swings.
Despite being unable to capture those features, the traditional monetary exchange rate model first brought forward by Bilson (1978) and Frenkel (1976) remains an important backbone in modeling exchange rate behavior. Even highly sophisticated models, such as the New Open Economy Macroeconomic (NOEM) approach, rely to some extent on the main features of the monetary model, in particular on purchasing power parity (PPP).\(^1\) The fact that the former family of models often does not derive empirical exchange-rate equations is one main reason why the monetary approach is still an important empirical working tool (Sarno, 2002). However, the monetary approach is widely seen as a long-term anchor rather than a short-term driver of exchange rates since the fluctuation of exchange rates is much higher (Sarno and Taylor, 2005). Several studies have therefore turned to analyzing cointegrating relationships and error correction behavior to address the exchange rate disconnect puzzle. The underlying idea is that even long-swings away from fundamentals values mean-revert at some point and to some degree (Mark, 1995; Sarno and Taylor, 2005).

The task for a researcher is further complicated by the issue that the importance of fundamentals varies over time even if they are linked to the exchange rate over the long-run. Empirically what is considered to be the most adequate set of fundamentals varies over time and does not show a recurring pattern as shown in Meese (1990) and Beckmann et al (2011). Recent studies by Rogoff and Stavrakeva (2008) and Inoue and Rossi (2013) both highlight the importance of instabilities when analyzing and forecasting exchange rates. A main problem is that in-sample instabilities do not necessarily transforms into out-of sample predictability since breaks can’t be foreseen (Rossi, 2015). A theoretical explanation for this pattern has been provided by the scapegoat approach of Bacchetta and Wincoop (2004). In a nutshell, the main argument of their approach is that market participants can give “excessive” weight to some (macroeconomic) fundamentals during specific periods, i.e. to so-called “scapegoats”. As a result, the corresponding parameters are subject to instabilities if scapegoat theory holds. A similar explanation of parameter instabilities can be obtained from the imperfect knowledge approach; see for example Goldberg and Frydman (1996 and 2007).\(^2\) Against this background, recent

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\(^1\) See, for example, Sarno and Taylor (2002) for a discussion of such models.

\(^2\) The imperfect knowledge approach is based on the idea that market participants do not know the exact model but use fundamentals to forecast exchange rates in a way consistent with assumed theory. Accordingly, the link
empirical research has incorporated different kinds of nonlinearities when modeling the link between exchange rates and fundamentals.

When analyzing the dollar-euro exchange rate, previous studies have either applied German data or a weighted average of all pre-EMU currencies for Euro exchange rates. Both specifications correspond to alternative characterizations of the Euro. Nautz and Offermanns (2006) compare both approaches up to 2004 on the basis of a monetary exchange rate model and find that the behavior of the Euro cannot be viewed as a simple extrapolation of the German mark.3 By contrast, Beckmann et al (2011) rely on German data for the period prior to the introduction of the euro and find no evidence of a structural break in a time-varying long-run relationship between the exchange rate and monetary fundamentals. Rather than relying on either German or synthetic ECU related data, we begin our analysis with the introduction of the Euro in January 1999. This leaves us with a sufficient span of data for a cointegration analysis and enables us to focus exclusively on actual market exchange rates.

The contribution of this study is twofold: (i) we analyze whether there is any evidence of a cointegrating relationship between exchange rates and macroeconomic fundamentals for the dollar-euro exchange rate after 1999 and (ii) we focus on the evolution of the corresponding coefficients over time. To the best of our knowledge, this is the first study to tackle both issues simultaneously. Our analysis is based on an evaluation of the in-sample link between exchange rates and fundamentals. To do this we use two different approaches, the first is the multivariate modeling approach in the spirit of Johansen (1988) and Juselius (2006). Such a framework has the advantage of testing the overall adequacy of the monetary model by fully capturing all underlying dynamics rather than focusing exclusively on a reduced form with predetermined causalities with the exchange rate on the left-hand side. The second estimator we apply is a time-varying coefficient approach based on a dynamic OLS estimation. This takes account of the fact that our sample includes several extraordinary events, including the financial crisis which started in 2008.

3 According to their findings, the EU model outperforms the DM model, since the latter is not able to beat a random walk forecast significantly at any forecast horizon.
The paper is structured as follows. Section 2 provides a brief description of theoretical considerations. Section 3 summarizes previous empirical findings. Section 4 describes the data and the empirical methodology. Section 5 reports our empirical findings with regard to long-run equilibrium, adjustment dynamics and time varying impact of fundamentals. We provide evidence for a long-run relationship between the exchange rate and fundamentals and show that the impact of fundamentals is strongly time-varying. Section 6 concludes.

2. The monetary exchange rate model

When considering the monetary approach of exchange rate determination, one should keep in mind that the general idea of the model is that the nominal exchange rate is to some extent driven by relative fundamentals over the long run. On the other hand, we do not expect all features to be empirically identified. However, reconsidering the basic identities is important for us to provide an adequate interpretation of our empirical results.

The key assumption of the monetary exchange rate models is that supply and demand for currencies are a result of transactions on international financial markets. The flexible-price monetary model was developed by Frenkel (1976) and Bilson (1978) and assumes that purchasing power parity holds continuously. It represents a valuable addition to exchange rate theory because it explicitly introduces relative money stocks into the picture as determinants of the relative prices which in turn determine the exchange rate.

We start by assuming that there is a conventional money demand function given by:

\[ m - p = \eta y - \sigma i \]  

(1)

where \( m \) is the log of the domestic money stock, \( p \) is the log of the domestic price level, \( y \) is the log of domestic real income and \( i \) is the nominal domestic interest rate.

Equation (1) states that the demand to hold real money balances is positively related to real domestic income because it leads to an increased transactions demand, and inversely related to the domestic interest rate because an increase in the interest rate makes holding money more expensive. A similar relationship holds for the foreign money demand function which is given by:

\[ m^* - p^* = \eta^* y^* - \sigma^* i^* \]  

(2)

where \( m^* \) is the log of the foreign nominal money stock, \( p^* \) is the log of the foreign price level...
$y^*$ is the log of foreign real income and $r^*$ is the foreign interest rate.

It is assumed that purchasing power parity holds continuously, expressed as:

$$s = p - p^*$$

(3)

where $s$ is the log of the exchange rate defined as domestic currency units per unit of foreign currency.

The monetarist model makes a crucial assumption that domestic and foreign bonds are perfect substitutes. This being the case, the uncovered interest parity condition holds:

$$E_s = i - i^*$$

(4)

Where $E_s$ is the expected rate of change of the spot rate.

Equation (4) says that the expected rate of depreciation of the home currency is equal to the interest rate differential between domestic and foreign interest rates.

We can rearrange equations (1) and (2) to give solutions for the domestic and foreign price levels:

$$p = m - \eta y + \sigma i$$

(5)

$$p^* = m^* - \eta^* y^* + \sigma^* i^*$$

(6)

We then substitute equations (5) and (6) into equation (3) to obtain:

$$s = m - m^* - \eta y + \eta^* y^* + \sigma - \sigma^* i^*$$

(7)

Equation (7) states that the spot exchange rate is driven by economic fundamentals, as given by relative money supplies, relative national incomes and relative interest rates. The interest rates are expressed as a percentage and $\eta$ and $\sigma$ are measures of income and semi-interest elasticity (Frankel, 1979). Taking the difference between both money demand functions and substituting the price differential with the nominal exchange rates according to purchasing power parity gives

$$s_t = \beta_1 m_t - \beta_1^* m_t^* - \beta_2 y_t + \beta_2^* y_t^* + \beta_3 i_t - \beta_3^* i_t^*$$

(8)

where $\beta_2 = \eta$, $\beta_2^* = \eta^*$, $\beta_3 = \sigma$ and $\beta_3^* = \sigma^*$. Foreign variables are denoted by an asterisk. The restriction $\beta_1 = \beta_1^* = 1$ holds in the flexible-price version of the monetary model. See Frenkel (1976), Bilson (1978), Hodrick (1978) and MacDonald and Taylor (1994) for a detailed
discussion. An increase in the domestic money supply or a rise in the relative domestic interest rates (driven by a rise in expected inflation) leads to an increase in domestic prices. Since purchasing power parity (PPP) is assumed to hold, the domestic currency depreciates as a result of the rise in prices. In the case of a domestic income expansion, the demand for real balances increases, domestic prices fall and the currency appreciates (Dornbusch, 1980).

Empirical studies frequently impose symmetry restrictions such as \( \beta_1 = \beta_1^* \) and \( \beta_2 = \beta_2^* \), as well as \( \beta_3 = \beta_3^* \), with regard to the elasticities of two countries. However, estimation in such a reduced form can result in biased coefficients and there is evidence that the elasticities of both economies are not equal if a formal test is applied, see for example Haynes and Stone (1981), Goldberg (2000) Beckmann et al (2011 and 2013). In the following, we will adopt such a restriction only for the price differential, which we use directly as a regressor while we test for symmetry restrictions for money supply, interest rates and industrial production.

There are three reasons for this procedure: firstly, this restriction has not been rejected by previous studies. In addition, the price differential is considered to be integrated of order one according to unit root tests. Finally, this restriction reduces the complexity of the system under investigation. We will formally test for symmetry restrictions with regard to the other coefficients.

3. Literature review

Owing to the fact that the literature on the monetary approach of exchange rates is voluminous, we will not provide a complete overview which is collectively covered in studies such as Goldberg and Frydman (2007) and Sarno and Taylor (2002). Instead, we briefly summarize

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4 A slightly different version, labelled the ‘Frenkel model’, arises if the interest rate differential is substituted with the inflation differential (Frenkel, 1976; Frankel, 1979). This formulation can be achieved by combining the Cagan-Type money demand function assumed by Frenkel (1976) with PPP. Frankel (1979) labels both formulations as different versions of a Chicago model. The formulation including inflation is often referred to in the literature as the Frenkel-Bilson model.

5 If Equation (3) is combined with uncovered interest rate parity (UIP) and the expected change in the exchange rate is considered stationary, the nominal exchange rate is driven only by relative money and income, that is, by money velocity.

6 Frenkel (1976) analyzes the monetary model for the period of German hyperinflation in the 1920s.
the general arguments and results. Early empirical results, in the seventies, were generally consistent with theoretical predictions when the monetary approach was first brought to bear on the data; see for example, Frenkel (1976); Bilson (1978) and Hodrick, (1978).

However, those studies neglected dynamics which would allow one to distinguish between short-run and long-run effects and possibly also suffered from a spurious regression problems. The empirical findings of studies conducted in the early eighties raised several questions. While the results of Driskell (1981) are mainly in line with the monetary model, most empirical studies from the late seventies and early eighties reported insignificant or false parameters, or, most famously, the failure of traditional exchange rate models to beat a random walk, see Meese and Rogoff (1983).

Once the concept of cointegration and long-run equilibria had been introduced into the literature by Engle and Granger (1987) early studies which adopted their two-step approach struggled to find evidence of a long-run relationship between exchange rates and fundamentals see for example, Meese (1986) and Meese and Rogoff (1988). Empirical studies which adopted the multivariate cointegration estimator proposed by Johansen (1988) turned out in part to be more successful for example, studies by MacDonald and Taylor (1993) and Cushman et al (1996) all provided evidence of cointegration between exchange rates and fundamentals. However, other authors who have applied the same methodology to different samples have not reported significance in cointegration relationships such as Chinn and Meese (1995), Papell (1997) and Goldberg and Frydman (2007).

Around the beginning of the nineties, researchers such as Meese (1990) began to incorporate into their models the fact that the set of fundamentals correlated with the exchange rate can vary over time. Recent empirical research on cointegration and the monetary exchange rate model can, from a methodical point of view, be separated roughly into three different kinds of nonlinear framework: Markov-switching models, smooth transmission models and models with structural breaks or time-varying coefficients. The first two frameworks focus on exchange rate

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7 Further shortcomings include a possible endogeneity bias, since the choice of the nominal exchange rate as the left-hand variable is arbitrary see Sarno and Taylor (2002).

8 For example, the results of Frankel (1982) suggest that an increase in German money supply results in an appreciation of the mark. In addition, several explanations for the rise of the dollar during the early eighties have been raised in the literature.
adjustment to deviations from a fundamental value without modeling a nonlinear long-run structure separately. The difference is that on the one hand Markov-switching models which have for example been applied by Sarno et al. (2004), De Grauwe and Vansteenkiste (2007) and Frömmel et al. (2005a and 2005b) apply a stochastic switching process to the adjustment coefficients, while on the other hand smooth transition models applied, for example, by Taylor and Peel (2000) allow for endogenously determined changes in the adjustment coefficients.

For the following reasons, we do not consider the first two approaches above. Firstly, one of our aims is to analyze changes in the long-run coefficients rather than in the adjustment coefficients although we do analyze time-varying adjustment coefficients in case of the single estimation technique. In addition, Markov-switching models are more appropriate when a longer period sample is under study. One reason for this is that such an approach is particularly useful for studying different regimes. A smooth transition model is also an interesting approach. However, such a strategy is based on transition functions, which in our case would depend on the size of deviations from established long-run relations. A possible caveat for our sample is that such a function may not be adequately defined over the comparable short sample under investigation.

In this study we stick with a time-varying coefficient approach. The main advantage is that we allow for continuously changes in both long-run and short-run coefficients. Early empirical investigations which adopted time-varying coefficient models when forecasting exchange rates without relying on cointegration were provided by Schinasi and Swamy (1989) and Wolff (1987). Recent examples include Beckmann et al (2011) and Goldberg and Frydman (2001 and 2007), who apply a piecewise linear in-sample relationship to the monthly dollar-euro (deutschmark) exchange rate to account for changes in the parameters. Both studies show that structural breaks also occur with regard to the long-run coefficients. Heimonen (2007) examines changes in the impact of economic fundamentals on the dollar-euro exchange rate between 1987 and 2001. However, he adopts a time-varying parameter approach to analyze the adjustment dynamics rather than the direct long-run estimates.
4. Data, empirical methodology and specification tests

4.1 Data

Our sample contains monthly data running from January 1999 to July 2015. We use the aggregate M1 for money supply, industrial production as a proxy for real income, consumer prices for the price differential and money market rates with a maturity of three months as the short-term interest rates. Three month interest rates are a benchmark choice to assure comparison with previous studies. M1 is used since it is better controlled by central banks compared to M3. Exchange rates (Dollar/Euro), money supply and industrial production are expressed in logarithms. All series are taken from International Financial Statistics or the OECD and can be approximated as integrated of order one according to unit root tests.

4.2 Framework and specification tests for linear long-run and short-run dynamics

The cointegrated VAR approach proposed by Juselius (2006) has as its main advantage the fact that the analysis is carried out without pre-assuming a specific causal structure for long-run relationships. According to Hoover et al. (2007) the philosophy of the approach is to let the data speak freely. Sticking with early work carried out by Haavelmo (1944), the empirical evidence is viewed as the driving force while the economic theory is adjusted to it. In short, the basic model draws upon the following vector autoregression representation (VAR):

\[ \Delta Z_t = \Pi Z_{t-1} + \Gamma(L) \Delta Z_{t-l} + \Phi D_t + \epsilon_t, \quad t = 1, \ldots, T. \]  \hspace{1cm} (9)

The vector \( Z_t = [s_t, m_t, m_t^*, y_t^*, i_t^*, i_t^*, p_t^d] \) contains the exchange rate and fundamentals as outlined in Section 2 with \( p_t^d \) denoting the price differential, i.e. \( (P_t - P_t^*) \). The term \( \Gamma(L) \Delta Z_{t-l} \) describes the short-run dynamics of the model using \( p \) equations between current variables, \( L \)-lagged variables and equilibrium errors; see Juselius (2006). The deterministic components are given by the \( (z \times 1) \) vector \( \Phi D_t \), while \( \epsilon_t \) describes an independent and identically distributed error term. The non-stationary behavior is accounted for by a reduced rank \( (r < z) \) restriction of the long-run level matrices \( \Pi \), which can be fragmented into two matrices \( \alpha \) and \( \beta' \) (\( \Pi = \alpha \beta' \)). The \( r \times z \) matric \( \beta' \) gives the coefficients of the variables for the \( r \) long-run relations.

\(^9\) Estimations with alternative configurations leave our main results unchanged and are available upon request.
while the $z \times r$ matrix $\alpha$ contains the adjustment coefficients describing the reaction of each variable to disequilibria from the $r$ long-run relations $\beta'Z_{t-1}$.

Having provided a general description of our approach, we now turn to preliminary analysis which is necessary before final estimates to be achieved. Owing to the fact that our sample period includes several extraordinary events, such as the recent financial crisis, dummy variables have been introduced to account for resulting outliers in the data, following the methodology described by Juselius (2006). Generally speaking, the fairly short sample under investigation plus those events make an adequate model specification even more important. Table 1 reports skewness, kurtosis and a test for normality for each coefficient. For all quantities normality is not rejected at the 1% significance level except for interest rates, where the rejection of normality is due to excess kurtosis, so that our results are still reliable.\(^{10}\) Table 2 details the tests for autocorrelation and ARCH-effects. The results show that autocorrelation is rejected three out of four cases. According to Rahbek \textit{et al} (2002), the results we obtain in the following are still robust under the remaining ARCH-effects.

\begin{table}[h]
\centering
\caption{Tests for normality and descriptive statistics}
\begin{tabular}{lccc}
\hline
 & Normality & Skewness & Kurtosis \\
\hline
$s_t$ & 1.666 & [0.645] & 0.084 & 3.807 \\
$M_t$ & 0.462 & [0.927] & 0.024 & 3.765 \\
$M_t^\ast$ & 0.695 & [0.874] & 0.221 & 3.953 \\
$Y_t$ & 14.072 & [0.003] & 0.271 & 3.379 \\
$Y_t^\ast$ & 5.959 & [0.114] & -0.051 & 3.790 \\
$i_t$ & 8.464 & [0.037] & -0.137 & 5.857 \\
$i_t^\ast$ & 0.336 & [0.953] & -0.695 & 6.089 \\
$p_t^d$ & 6.087 & [0.107] & -0.420 & 4.363 \\
\hline
\end{tabular}
\end{table}

Note: The table provides tests for normalities as well as skewness and kurtosis for each quantity with p-values in parentheses for the normality test.

\(*<p=0.05; **<p=0.01; ***<p=0.001\)

\(^{10}\) Since excess kurtosis does not introduce a significant bias to the estimated cointegration vectors, the findings are more sensitive to excess skewness (Juselius, 2006).
Regarding the deterministic components, Johansen (1991) distinguishes between five different configurations of a cointegrated VAR model. The inclusion of a deterministic trend in the long-run relationship having been rejected, we include only a constant in the cointegrating space. Then we analyze the number of stationary long-run relationships. This is a crucial step in our analysis, since the results of restriction and validity tests, as well as the reliability of the estimation, depend on the right choice of rank \((r)\). To tackle this issue, we rely on the trace test developed by Johansen (1988). Starting with the hypothesis of full rank, the rank is determined using a top-bottom procedure until the null cannot be rejected (Juselius, 2006). As a check of robustness we have also simulated the asymptotic critical values for the rank test based on random walks with lengths of 400 and 2,500 replications.

For both simulated and standard critical values, we additionally consider the Bartlett-corrected test statistic. The latter version implements a small sample size correction, which might be reasonable considering the available span of data. The results are given in Table 3.

### Table 3: Rank tests results

<table>
<thead>
<tr>
<th>(p)</th>
<th>(r)</th>
<th>Elig. Value</th>
<th>Trace</th>
<th>Trace*</th>
<th>Frac95</th>
<th>p-Value</th>
<th>p-Value*</th>
<th>Frac95*</th>
<th>P-Value</th>
<th>P-Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
<td>0.465</td>
<td>395.740</td>
<td>336.339</td>
<td>169.405</td>
<td>0.000</td>
<td>0.000</td>
<td>160.931</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.358</td>
<td>270.072</td>
<td>115.013</td>
<td>134.543</td>
<td>0.000</td>
<td>0.397</td>
<td>128.707</td>
<td>0.000</td>
<td>0.250</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.315</td>
<td>180.850</td>
<td>73.938</td>
<td>103.679</td>
<td>0.000</td>
<td>0.810</td>
<td>99.737</td>
<td>0.000</td>
<td>0.684</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.196</td>
<td>104.691</td>
<td>45.081</td>
<td>76.813</td>
<td>0.000</td>
<td>0.950</td>
<td>73.617</td>
<td>0.000</td>
<td>0.908</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.130</td>
<td>60.737</td>
<td>28.205</td>
<td>53.945</td>
<td>0.010</td>
<td>0.945</td>
<td>52.103</td>
<td>0.006</td>
<td>0.911</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.079</td>
<td>32.670</td>
<td>14.511</td>
<td>35.070</td>
<td>0.091</td>
<td>0.957</td>
<td>34.268</td>
<td>0.067</td>
<td>0.929</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.051</td>
<td>16.027</td>
<td>8.021</td>
<td>20.164</td>
<td>0.176</td>
<td>0.820</td>
<td>19.505</td>
<td>0.156</td>
<td>0.777</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>0.027</td>
<td>5.432</td>
<td>1.332</td>
<td>9.142</td>
<td>0.248</td>
<td>0.889</td>
<td>9.315</td>
<td>0.232</td>
<td>0.851</td>
</tr>
</tbody>
</table>

Note: The table shows Johansen’s (1988) cointegration test. \(r\) denotes the cointegration rank. The p-values denoted * correspond to a simulation with \(T = 400\) and 2500 replications. The asterisk refers to Bartlett corrected values.

\(*<p=0.05; **<p=0.01; ***<p=0.001\)
For both simulated and standard critical values, the Bartlett corrected test statistic suggests a rank of one, while the non-corrected values suggest four long-run relationships. A consideration of the recursive graph of the trace test and an inspection of the alpha coefficients belonging to the second, third and fourth relations suggest that a rank of two or three seems reasonable, since the fourth long-run relationship only gains significance at the end of the sample. To check the robustness of our findings, in the following we estimate the model with a rank of both two and three.

In cases of a rank larger than one, it is necessary to impose merely identifying restrictions on \( \beta \) to achieve interpretable maximum likelihood estimates of the cointegrating relations \( \beta' X_{t-1} \). To identify the cointegration vector. By having an economic model such as the monetary approach at hand, further restrictions can also be implemented, so that the model is over-identified. Hypothesis testing on cointegration vectors is done by specifying the \( s_i \)-free varying parameters in each \( \beta \) vector, according to the term

\[
\beta = (H_1, \ldots, H_r)
\]

where \( \beta \) as \((z \times r)\) coefficient matrix. \( H_i = \begin{pmatrix} p_{l1} s_1 \\ p_{lj} s_j \\ p_{l2} s_z \end{pmatrix} \). The term \( p_{ij} \) corresponds to the coefficient on the long-run relationship \( i \) for variable \( j \) and \( s_j \) being 0 or 1. If \( s_j \) is 0, the coefficient of variable \( j \) in the long-run relationship \( i \) is restricted to zero. If \( s_j \) is 1, variable \( j \) is included in long-run relationship \( i \). In the section which describes our results, we base the tests of our hypotheses on a likelihood ratio procedure as described in Juselius (2006).

4.3 Framework for time-varying long-run and short-run dynamics

Adopting a single equation approach rather than the multivariate estimator also provides a robustness check and is justified by the fact that a long-run relationship was not detected without import prices. Hargreaves (1994) has shown that single equation estimators provide efficient estimates even in case of more than one long-run relationship.

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11 Both adjustment coefficients and recursive estimations are available upon request.
The framework we use is a state space model which combines the Kalman filter with a regression based on dynamic OLS. More precisely, the basic equations have the following form:

\[ s_t = F_t\theta_t + \epsilon_t \quad \epsilon_t \sim N(0,H_t) \]  
\[ \theta_t = \theta_{t-1} + \eta_t \quad \eta_t \sim N(0,Q_t) \]  

Equation (11) is the observation equation and Equation (12) is the state equation. \( F_t \) includes all fundamentals: \( F_t = [m_t, m_t^*, y_t, y_t^*, i_t, i_t^*, p_t^d] \). We also consider a second setting where we use cross-country differences not only for prices but for all fundamentals. The latter is motivated by the fact that the multivariate results presented in the next section suggest that symmetry restrictions are not rejected by the data. Estimating both settings provides another implicit robustness test for our results. \( \theta_t \) provides separate time-varying coefficients for each fundamental. In the long-run setting, those coefficients are the time varying but inverse equivalents to \( \beta' \) in Equation (9) if normalization is carried out on the exchange rate. Time varying adjustment dynamics are obtained in a similar fashion if \( s_t \) in Equation (11) is replaced by \( \Delta s_t \) while \( \hat{\epsilon}_t \) replaces \( \epsilon_t \) so that the change of the exchange rate is linked to the estimation error of the previous period. The interpretation then is equivalent to an adjustment coefficient provided by \( \alpha \) in Equation 9.

The matrix \( Q_t \) corresponds to the variances and covariances of the states and determines changes in the coefficients. The errors in the observation and the state equation covariances are assumed to be mutually independent at all leads and lags.

At each point in time Kalman-filtering begins with a prediction of both equations based on an optimization of the projected error covariances. After making a new observation estimates are corrected based on the Kalman gain or the blending factor which minimizes the posterior error covariances. Since we deal with a model based on nonstationary I(1) variables, the error term may be serially and contemporaneously correlated with the regressors. These assumptions govern the Kalman filter for this model. The DOLS estimator introduced by Stock and Watson (1988) corrects traditional OLS with regard to endogeneity and serial correlation by including
leads and lags for the first differences of the regressors, that is, the fundamentals, on the right hand side of the equation. This is the approach we adopt in the following.

5. Empirical Results

As a first step, we compare the full-sample estimates of both approaches. The results of the multivariate approach are provided in Table 4, those of the time varying DOLS approach in Table 5. The signs of the coefficients of all fundamentals have been reversed in Table 4. This allows for a direct comparison of the estimates with the coefficients of Table 5 and the theoretical considerations.

Table 4: Results of multivariate cointegration analysis

4a): Results for three long-run relationships

<table>
<thead>
<tr>
<th></th>
<th>(s_t)</th>
<th>(m_t)</th>
<th>(m_t^*)</th>
<th>(Y_t)</th>
<th>(Y_t^*)</th>
<th>(i_t)</th>
<th>(i_t^*)</th>
<th>(p_t^d)</th>
<th>(constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td>1.000</td>
<td>1.741</td>
<td>-1.741</td>
<td>-7.427</td>
<td>7.427</td>
<td>-0.239</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.714</td>
</tr>
<tr>
<td></td>
<td>(10.582)</td>
<td>(-10.582)</td>
<td>(-8.205)</td>
<td>(8.205)</td>
<td>(-8.314)</td>
<td>(NA)</td>
<td>(NA)</td>
<td>(NA)</td>
<td>(8.137)</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>0.000</td>
<td>1.000</td>
<td>-0.482</td>
<td>0.000</td>
<td>1.562</td>
<td>-0.287</td>
<td>-0.017</td>
<td>-10.210</td>
<td>0.574</td>
</tr>
<tr>
<td></td>
<td>(NA)</td>
<td>(NA)</td>
<td>(-4.113)</td>
<td>(NA)</td>
<td>(4.607)</td>
<td>(-8.929)</td>
<td>(-1.176)</td>
<td>(-6.722)</td>
<td>(0.457)</td>
</tr>
<tr>
<td>(\beta_3)</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.397</td>
<td>0.039</td>
<td>0.039</td>
<td>-25.144</td>
<td>-1.902</td>
</tr>
<tr>
<td></td>
<td>(NA)</td>
<td>(NA)</td>
<td>(NA)</td>
<td>(NA)</td>
<td>(6.032)</td>
<td>(1.278)</td>
<td>(1.278)</td>
<td>(-7.544)</td>
<td>(-10.439)</td>
</tr>
</tbody>
</table>

Panel (b): Test of restricted model: \(\chi^2(6)=6.645\ [0.355]\)

<table>
<thead>
<tr>
<th></th>
<th>(ds_t)</th>
<th>(dm_t)</th>
<th>(m_t^*)</th>
<th>(dY_t)</th>
<th>(dY_t^*)</th>
<th>(di_t)</th>
<th>(di_t^*)</th>
<th>(dp_t^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>-0.027</td>
<td>0.010</td>
<td>-0.021</td>
<td>0.016</td>
<td>-0.007</td>
<td>0.018</td>
<td>-0.063</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(-1.861)</td>
<td>(2.508)</td>
<td>(-5.194)</td>
<td>(3.122)</td>
<td>(-2.272)</td>
<td>(0.368)</td>
<td>(-0.932)</td>
<td>(-3.766)</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.054</td>
<td>0.020</td>
<td>-0.009</td>
<td>0.010</td>
<td>0.002</td>
<td>0.196</td>
<td>0.131</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(2.880)</td>
<td>(3.635)</td>
<td>(-1.738)</td>
<td>(1.479)</td>
<td>(0.458)</td>
<td>(3.050)</td>
<td>(1.493)</td>
<td>(8.061)</td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>0.032</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.070</td>
<td>0.071</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(3.336)</td>
<td>(-0.483)</td>
<td>(0.387)</td>
<td>(-1.369)</td>
<td>(1.149)</td>
<td>(2.094)</td>
<td>(1.594)</td>
<td>(8.057)</td>
</tr>
</tbody>
</table>

Note: Panel (a) shows the estimates of the cointegration vector with t-statistics in parenthesis. Panel (b) shows the test for over-identifying restrictions, which is an LR-test [p-value]. Panel (c) gives the adjustment coefficients towards the long-run equilibrium.

\[12\] From a theoretical point of view, the OLS estimator is superconsistent in case of cointegration.
When considering the multivariate estimates, we focus on the exchange rate equation given by the first long-run relationship. The reason for this is that we analyze two different configurations with two and three long-run relationships respectively and are interested mainly in the long-run link that arises between the exchange rate and fundamentals.

For both multivariate settings, the over-identifying restrictions are not rejected at the 10% level. The corresponding estimates differ only marginally, with money supply, industrial production and either EMU interest rates (three long-run relationships) or both interest rates (two long-run relationships) entering. Symmetry restrictions are not rejected for the money supply and industrial production in both settings. Interestingly, the signs for industrial production and money supply are reversed compared to those in the monetary approach: an increase in the euro area relative money supply results in an appreciation of the domestic currency, while a relative...
increase in industrial production in the euro area results in a depreciation of the euro. An increase in euro area interest rates leads to an appreciation of the euro. A similar pattern is observed for U.S. interest rates in the case of two long-run relationships.

The results of the time-varying DOLS estimates (Table 5) are very close to those obtained from the multivariate cointegration in terms of the signs of the coefficients. However, the magnitudes of the coefficients for industrial production and money supply are significantly smaller compared to the multivariate estimates. The interest rate differential is again insignificant. The price differential is only significant in the first setting, where cross-country differences are used. Overall, the direction of the causalities remains the same, since a relative increase in domestic money supply results in an appreciation of the domestic currency, while the opposite holds for a relative increase in domestic industrial production. Table 5 presents results with one lead and one lag; increasing this number to two or three does not change the results to a significant degree. As another robustness check, we have also followed a general-to-specific methodology by excluding insignificant quantities step by step. The results again did not change to a significant degree. In both cases, the results are available upon request.

For the multivariate framework, a theory-conform adjustment of the nominal exchange rate to deviations from the first long-run relation is only observed in the first configuration with three long-run relationships. However, this finding may be traced back to nonlinearities in the spirit of smooth transition or Markov-switching models, as described in Section 2. The time-varying adjustment pattern to deviations from the DOLS estimation is analyzed in the next section.

Overall, we find robust evidence that the dollar-euro exchange rate is cointegrated with monetary fundamentals over the long run. In both frameworks, an exchange rate equation as suggested by the monetary approach can be identified. However, the observed coefficients over the full sample are not always in line with the suggestions of the monetary approach. From a multivariate perspective, this finding may be explained by the fact that some of the underlying key equations, such as money demand functions (Equation 2) or purchasing power parity (Equation 1) do not hold as stable long-run relation. This is not surprising, considering that we analyze a comparably small sample which also includes the financial crisis and previous findings which have shown that several kinds of nonlinearities may be responsible for this finding. A detailed multivariate analysis of the monetary approach based on subsystems is beyond the scope of this paper. LaCour and MacDonald (2000) have carried out such an
exercise for the dollar/ECU exchange rate. Against this background, in our next step we turn to time variation in the coefficients for our single-equation framework. 13

Evolution of time-varying long-run and adjustment dynamics

Next, we analyze the evolution of long-run coefficients over time. To disentangle possible effects from both countries, we report the results for the configuration without symmetry restrictions. Figures 2-6 provide the corresponding graphs.

Figure 2: Time-varying coefficient for relative consumer prices

Note: The graph shows the evolution of the coefficient for consumer prices as described in Equations (11) and (12)

13 As a robustness check, we have also carried out recursive maximum likelihood estimations of the cointegrating vectors in the multivariate framework. The results, which are available upon request, show a related pattern of higher volatility compared to the estimates reported in the next section.
Figure 3: Time-varying coefficient for relative industrial production

Note: The graph shows the evolution of the coefficient for industrial production as described in Equations (11) and (12)

Figure 4: Time-varying coefficient for relative interest rates

Note: The graph shows the evolution of the coefficient for interest rates as described in Equations (11) and (12)
Figure 5: Time-varying coefficient for relative money supplies

![Image](image1.png)

Note: The graph shows the evolution of the coefficient for money supply as described in Equations (11) and (12)

Figure 6: Time-varying adjustment coefficient of the exchange rate to long-run disequilibria

![Image](image2.png)

Note: The graph shows the evolution of the exchange rates adjustment coefficient disequilibria from Equations (11 and 12)

The graphs clearly demonstrate the usefulness of our approach. All coefficients are subject to changes over time. Starting with the coefficient for the price differential (Figure 2), the coefficient changes to a significant degree but is positive after 2008. For industrial production (Figure 3), the coefficient is negative for the eurozone and positive for the US, implying that an increase in relative real output in the eurozone depreciates the euro and vice versa which might be explained by trade effects. However, output in the Eurozone is partly insignificant while us production in the US has a stronger and continuously significant effect. An ongoing decrease is observed at the beginning of the financial crisis between 2007 and 2008.
The coefficients for money supplies (Figure 4) and relative rates (Figure 5) show an interesting pattern: in both cases, coefficients changes around 2007. Both interest rates become insignificant after the initial drop in interest rates as a response to the financial crisis. US interest rates remain insignificant afterwards while Eurozone interest rates display a small negative coefficient. The coefficients of money supply also change significantly around 2010 when unconventional monetary policy emerged. Prior to this date, the coefficients for both money supplies are mostly in line with the suggestions of the monetary model, with a relative increase in the relative euro area money supply resulting in a depreciation of the euro. Although the reversed pattern observed afterwards cannot easily be explained, one possible reason is that an increase in domestic money supply in the context of quantitative easing is considered to be an adequate response to financial turbulence and therefore stabilizes the domestic currency. Since we are analyzing the dollar, safe haven aspects may also play an important role.

Finally, we put the time-varying adjustment pattern to deviations from our long-run relations displayed in Figure 6 under closer scrutiny. The path of the adjustment coefficient again shows great variability but turns out to be negative and therefore in line with theory for most of the time. Altogether, the results are compatible with the concept of regime-sensitive cointegration introduced by Siklos and Granger (1997) since the long-run coefficients and the adjustment dynamics both change over time.

Having established time variation in coefficients, an obvious question corresponds to reasons and implications of those changes. Taking the recent changes in the stance of monetary policy into account, it is not surprising that the effects of monetary policy change over time. The long-run effects of unconventional monetary policy on exchange rates have yet be determined but an important transmission channel corresponds to announcement and expectation effects such as highlighted during the famous “whatever it takes” speech by Mario Draghi in July 2012 which triggered a short-run appreciation of the euro. At the same point in time, unconventional monetary policy has resulted in a substantial increase in money supply.

Our findings are also related to the out-of-sample evidence on exchange rates. Several studies have shown that the monetary exchange rate model is unable to outperform random walk benchmarks, a finding first established by the study of Meese and Rogoff (1983). Recent studies by Engel et al. (2008) and Ince, Molodtsova, and Papell (2015) rely on the monetary approach as a benchmark when evaluating the out-of-sample performance of fundamental exchange rate
models. Our results are also consistent with the findings of Molodtsova and Papell (2013) who evaluate the Euro/U.S. dollar rate out-of-sample during the financial crisis using a range of different models. The ongoing evaluation of the long-run link between fundamentals and exchange rates we find presents a major burden for exchange rate forecasting. A major issue is that it is essentially impossible to accommodate expectation effects resulting from fundamentals changes in the present framework.

5. Conclusion

This paper has analyzed the path of dollar-euro exchange rates since 1999 against the background of the monetary model. Our findings show that a long-run relationship is detected on a basis of different configurations of the fundamentals and estimation techniques. However, in line with previous studies, the underlying coefficients are subject to substantial instabilities. This suggests that the empirical results crucially depend on the sample size under investigation. In this sense, a researcher dealing with long-run exchange rate modeling should not pay too much attention to the specific magnitudes of the coefficients. In the present study, the full-sample estimates for money and income are significant, while their signs are not in line with the strict version of the monetary model. However, an inspection of the time variation in the coefficients shows that this finding may be driven by the specific economic environment prevalent in the midst of the financial crises. The coefficients for both money supply and the interest rate differential change their signs around 2007 and after the emergence of unconventional monetary policy, with the former coefficients being in line with theory prior to that date.

Those findings show that a piece-wise linear relationship adopted by Goldberg and Frydman (1996) and Beckmann et al (2011) is not necessarily capable of tracking coefficient changes and accounting for all instabilities. Even in short subsamples of a few years, coefficients potentially change quite abruptly. One important question should be further investigated: What drives the evolution of the coefficients over time? Considering the recent changes in the stance of monetary policy, expectation effects obviously play quite an important role. Overall, the continuous instabilities in the link between exchange rates and fundamentals represents a major task for policymakers and investors. Even if policy instruments affect fundamentals, the effect on the exchange rate is not-predictable since even the direction of causality partly changes over
time. In this regard, another observation is that discretionary policies affect the link between exchange rates and fundamentals. In the case of unconventional monetary policy, the importance of money supply increases while interest rates become less important. The scapegoat approach of Bachetta and Wincoop (2004) may provide an explanation for this finding in the sense that the observed changes reflect higher weights attached to a “scapegoat” fundamental, in this case money supply which deviates from expectations.

Another important area for further research is the application of Bayesian estimation techniques, which allow one to tackle the caveat of model and parameter uncertainty in a more sophisticated way. By analyzing posterior probabilities rather than focusing on point estimates and allowing for different numbers of long-run relationships, as recently proposed by Jochmann and Koop (2014), the scope of exchange rate economics in terms of cointegration might be greatly extended.

Bibliography


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