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Correction: Exchange Option Under Jump-Diffusion Dynamics

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Abstract In this note, we provide the correct formula for the price of the European exchange option given in Cheang and Chiarella (2011) in a bi-dimensional jump diffusion model.

Key Words: exchange option, jump-diffusion.

Theorem 5.1 in Cheang and Chiarella (2011), page 259, gives a formula for the price of a European exchange option under jump diffusion dynamics. The formula is based on a wrong application of the change of numéraire from the risk-neutral to the spot measure. We amend the proof and provide the correct pricing formula for the exchange option.

Theorem 1: Suppose the asset prices follow the dynamics in formula (38) of Cheang and Chiarella (2011), and the continuous dividend rate for each asset is $\xi_i$. Then when $S_{1,t} = s_1$ and $S_{2,t} = s_2$, the European exchange option price is

$$C^E_{s_1,s_2} = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} e^{-(\lambda_1 + \lambda_2 + \lambda)(T-t)} \left( \frac{(\lambda_1(T-t))^k}{k!} \right) \left( \frac{(\lambda_2(T-t))^m}{m!} \right) \left( \frac{(\lambda(T-t))^n}{n!} \right) \times 
\{ 
\begin{align*}
    s_1 e^{-(\xi_1 + \hat{\lambda}_1 Z_1 + \hat{\lambda}_2 Z_2)(T-t) + k\hat{\mu}_1 + m\hat{\mu}_2 + n\hat{\mu}_3 + \sigma_1^2 \sqrt{T-t}} \Phi(d_1) \\
    -s_2 e^{-(\xi_2 + \hat{\lambda}_2 Z_2 + \hat{\lambda}_1 Z_1)(T-t) + m\hat{\mu}_2 + n\hat{\mu}_3 + \sigma_2^2 \sqrt{T-t}} \Phi(d_2)
\end{align*}
\}$$

where

$$d_{1,t,k,m,n} = \frac{\ln \left( \frac{s_1}{s_2} \right) + (\xi_2 - \xi_1 - \hat{\lambda}_1 - \hat{\lambda}_2)(T-t) + \tilde{\lambda}_1 Z_1 + \tilde{\lambda}_2 Z_2) + \mu_{k,m,n} + \frac{\sigma_{k,m,n}^2}{2}(T-t)}{\sigma_{k,m,n} \sqrt{T-t}},$$

$$d_{2,t,k,m,n} = d_{1,t,k,m,n} - \sigma_{k,m,n} \sqrt{T-t};$$

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with

\[ \mu_{k,m,n} = k(\bar{\alpha}_{11} + \delta_{11}^2/2) - m(\bar{\alpha}_{22} + \delta_{22}^2/2) + n(\bar{\alpha}_1 - \bar{\alpha}_2 + \delta_1^2/2 - \delta_2^2/2), \]

and

\[ \sigma_{k,m,n}^2 = \sigma^2 + \frac{k\delta_{11}^2}{T-t} + \frac{m\delta_{22}^2}{T-t} + \frac{n(\delta_1^2 + \delta_2^2 - 2\rho\gamma_1\delta_2)}{T-t}, \quad \sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2, \]

where \( \Phi \) is the standard normal probability distribution function.

**Proof:** Without loss of generality, we derive a formula for the exchange option price at time \( t = 0 \). The option price at \( t = 0 \) is then

\[ C_0^E(S_{1,0}, S_{2,0}) = \mathbb{E}_Q \left\{ \frac{(S_{1,T} - S_{2,T})^+}{e^{rT}} \right\} = \]

\[ S_{1,0}e^{-\xi_1T}\mathbb{E}_Q \left\{ \exp \left[ -\frac{\sigma_1^2}{2}T + \sigma_1W_{1,T} - \tilde{\lambda}_1\tilde{k}_1T + \sum_{n=0}^{N_T}Y_{1,n} - \tilde{\lambda}_1\tilde{k}_Z_1T + \sum_{i=1}^{N_{1,T}}Z_{1,i} \right] \cdot 1\{S_{1,T}\leq S_{2,T}\} \right\} \]

\[ -S_{2,0}e^{-\xi_2T}\mathbb{E}_Q \left\{ \exp \left[ -\frac{\sigma_2^2}{2}T + \sigma_2W_{2,T} - \tilde{\lambda}_2\tilde{k}_2T + \sum_{n=1}^{N_T}Y_{2,n} - \tilde{\lambda}_2\tilde{k}_Z_2T + \sum_{l=1}^{N_{2,T}}Z_{2,l} \right] \cdot 1\{S_{1,T}\leq S_{2,T}\} \right\}. \]

Using twice the change of numéraire from the risk neutral measure \( Q \) to the spot measures \( Q_1 \) (stock \( S_1 \) is taken as numéraire) and \( Q_2 \) (stock \( S_2 \) is taken as numéraire), and conditioning on the number of idiosyncratic and common jumps the pricing formula of the exchange option requires the computation of \( Q_1(A|N_{1,T} = k, N_{2,T} = m, N_T = n) \) and \( Q_2(A|N_{1,T} = k, N_{2,T} = m, N_T = n) \), where \( A|N_{1,T}=k, N_{2,T}=m, N_T=n \) is the set defined as

\[ \{\Xi_{k,m,n} > \ln \left( \frac{S_{2,0}}{S_{1,0}} \right) - \left( \xi_2 - \xi_1 - \frac{\sigma_1^2}{2} + \frac{\sigma_2^2}{2} - \tilde{\lambda}(\tilde{k}_1 - \tilde{k}_2) - \tilde{\lambda}_1\tilde{k}_Z_1 + \tilde{\lambda}_2\tilde{k}_Z_2 \right)T \} \]

and

\[ \Xi_{k,m,n} = \sigma_1W_{1,T} - \sigma_2W_{2,T} + \sum_{i=0}^{k} Z_{1,i} - \sum_{l=0}^{m} Z_{2,l} + \sum_{j=0}^{n} (Y_{1,j} - Y_{2,j}). \]

The proof in Cheang and Chiarella (2011) has to be corrected in the specification of the distribution of \( \Xi_{k,m,n} \) under \( Q_1 \) and \( Q_2 \). In particular to compute the distribution of \( Y \) under \( Q_1 \) and \( Q_2 \), we have to apply Theorem 3.1 of Cheang and Chiarella (2011), according to the following Radon–Nikodym derivatives

\[
\frac{dQ_1}{dQ} \bigg|_T = \exp \left[ -\frac{\sigma_1^2}{2}T + \sigma_1W_{1,T} - \tilde{\lambda}_1\tilde{k}_1T + \sum_{n=1}^{N_T}Y_{1,n} - \tilde{\lambda}_1\tilde{k}_Z_1T + \sum_{i=1}^{N_{1,T}}Z_{1,i} \right],
\]

and

\[
\frac{dQ_2}{dQ} \bigg|_T = \exp \left[ -\frac{\sigma_2^2}{2}T + \sigma_2W_{2,T} - \tilde{\lambda}_2\tilde{k}_2T + \sum_{n=1}^{N_T}Y_{2,n} - \tilde{\lambda}_2\tilde{k}_Z_2T + \sum_{i=1}^{N_{2,T}}Z_{2,i} \right].
\]

The parameter \( \gamma \) defined in Theorem 3.1 determines the distribution of the jump component \( Y \) through the following relation on the moment-generating function

\[
M_{Q,Y}(u) = \frac{M_{Q,Y(u + \gamma)}}{M_{Q,Y}(\gamma)}, \quad i = 1, 2.
\]
Setting $\gamma = [1, 0]^T$, Theorem 3.1 implies that the Wiener and the jump components, conditioned on the event $N_{1,T} = k$, $N_{2,T} = m$, $N_T = n$, are normally distributed as

$$
\Xi_{k,m,n} \sim N((\sigma_1^2 - \rho \sigma_1 \sigma_2)T + n(\alpha_1 - \tilde{\alpha}_2 + \delta_1^2 - \rho \gamma \delta_2) + k(\tilde{\alpha}_{11} + \delta_{11}^2) - m\tilde{\alpha}_{22}, \sigma_{k,m,n}^2(T)).
$$

The Poisson process $N_T$ has arrival intensity $\lambda_1 = \tilde{\lambda}(1 + \kappa_1)$ and the Poisson process $N_{1,T}$ has arrival intensity $\tilde{\lambda}_{Z_1} = \lambda_1(1 + \kappa_{Z_1})$ under $Q_{11}$, with the intensity of $N_{2,T}$ unchanged.

Similarly setting $\gamma = [0, 1]^T$, it follows that the random variable $\Xi_{T,k,m,n}$ is therefore normally distributed as

$$
\Xi_{k,m,n} \sim N((\rho \sigma_1 \sigma_2 - \sigma_2^2)T + n(\alpha_1 - \alpha_2 + \rho \gamma \delta_1 \delta_2 - \delta_2^2) + k\tilde{\alpha}_{11} - m(\tilde{\alpha}_{22} + \delta_{22}^2), \sigma_{k,m,n}^2(T)).
$$

The Poisson process $N_T$ has arrival intensity $\tilde{\lambda}_2 = \tilde{\lambda}(1 + \kappa_2)$ and the Poisson process $N_{2,T}$ has arrival intensity $\tilde{\lambda}_{Z_2} = \lambda_2(1 + \kappa_{Z_2})$ under $Q_{22}$, and the intensity of $N_{1,T}$ unchanged.

Straightforward computations as in Cheang and Chiarella (2011) conclude the proof. □

Table 1 provides numerical results. We consider nine different parameter scenarios (we also set $\xi_1 = \xi_2 = 0$, $r = 0$, $T = 1$). Formula (1), row $C^E_t$, has been computed truncating the triple sum to $n = m = k = 25$. We also provide the Monte Carlo estimate, row MC, obtained with $N_{MC} = 10^5$ random trials, implemented using a control variate method as described in Caldana and Fusai (2013). The row labeled C.I.L. gives the length of the 95% mean-centered Monte Carlo confidence interval. In all cases $C^E_t$ matches the Monte Carlo solution up to the sixth digit.

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<td>4.785 x 10^{-7}</td>
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Table 1. Exchange option values are computed for nine different scenarios. $C^E_t$ prices the exchange option according to the analytical formula (1). MC displays the Monte Carlo estimate and C.I.L. gives the length of the 95% mean-centered Monte Carlo confidence interval.

References
