Asian Options with Jumps

A Closed-Form Formula

In this article Marena, Roncoroni, and Fusai derive a closed-form formula for the fair value of call and put options written on the arithmetic average of security prices driven by jump diffusion processes displaying (possibly periodical) trend, time varying volatility, and mean reversion. The model allows one for jointly fitting quoted futures curve and the time structure of spot price volatility. These result extends the no-jump case put forward in [Fusai, G., Marena, M., Roncoroni, A. 2008. Analytical Pricing of Discretely Monitored Asian-Style Options: Theory and Application to Commodity Markets. Journal of Banking and Finance 32 (10), 2033-2045]. A few tests based on commodity price data assess the importance of introducing a jump component on the resulting option prices.

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In Fusai, Marena, and Roncoroni (2008), we put forward a procedure for pricing Asian-style options under the following assumptions:

- Price average is arithmetically computed on market quotes monitored over a finite number of points in time, say $0, \Delta, 2\Delta, ..., n\Delta$: this method is referred to as "*discrete monitoring*".[19]

- Underlying spot price dynamics are driven by continuous diffusion processes, say $(S_t)_{t\geq 0}$, possibly exhibiting mean reversion, time varying trend, and time varying volatility.

- Asian-style options are either puts or calls, including the cases of fixed as well as floating (*i.e.*, depending on the underlying asset quote) strike price.

Under these hypotheses, we devised a new method to calculate the exact analytical expression for the moment generating function of the joint pair consisting of commodity spot price $S_{n\Delta}$ computed at the option's maturity $T := n\Delta$ and the weighed arithmetic average $\sum_{j=0}^{n} \alpha_j S_{j\Delta}$ over the option lifetime. That result allowed us to derive analytical expressions for the relevant transforms[20] of option prices with respect to the strike price. Finally, using the Fourier inversion method, we got to semi-analytical expressions for a large class of Asian-style derivatives.

To the best of our knowledge, the resulting prices are the sole closed-form formulae for options written on arithmetic averages (up to inverse transformation). This comes as opposed to the large number of numerical approximations proposed in the financial literature for those securities.

This article aims at extending our previous result to the case of spot price dynamics driven by a 2-factor jump-diffusion process. We manage to preserve the ability of the model to reproduce mean reversion, time varying trend, as well as time varying volatility. However, we allow for the underlying variable to exhibit discontinuous paths, as is often the case in several financial markets, in particular for energy sources and other commodities.

The rest of the article is organized as follows. The first part states the problem; then there is the setting of the model and the computation of the relevant moment generating function. The result-

[19]Monitoring dates need not be evenly spaced.

[20]Laplace transform for fixed strike options; Fourier transform for floating strike options.

Option	Payoff
Fixed Strike	max $\text{Avg}_n - k, 0$
Floating Strike	max $S_{n\Delta} - \text{Avg}_n - k, 0$
Option	**Underlying Variable**
Standard	$\text{Avg}_n := \sum_{j=0}^{n} \frac{1}{n+1} S_{j\Delta}$
Volume weighed	$\text{Avg}_n := \sum_{j=0}^{n} \frac{V_j}{\sum_i V_i} S_{j\Delta}$

TABLE 1: Payoff functions of Asian-style options under continuous and discrete monitoring.

ing 3-step algorithm producing options prices is illustrated in the section after and its followed by a sensitivity analysis of our formulae; conclusions provide with a few indication about directions for future research work on the subject.

Statement of the Problem

We consider a time horizon $[0, T]$ representing the option's lifetime: 0 is the valuation date, while T is the time of expiration. At time T, the option pays out an amount that is contingent upon the realization of a price average $\text{Avg}_n := \sum_{j=0}^{n} \alpha_j S_{j\Delta}$ (with $\sum \alpha_j = 1$) of discretely monitored spot prices $S_0, S_\Delta, ..., S_{n\Delta}$. Specifically, we consider pay-off structures as reported in Table 1. As an example, we may consider Asian-style options traded in gas markets: at maturity, the position pays out the positive discrepancy, if any, between the last gas quote and the average of daily monitored gas prices (*i.e.*, $\Delta = 1$ day, under the assumed day-count convention), each one being weighed by the proportion of actual delivery V_j over the whole size of physically traded volume $\sum_i V_i$. This case nests our setting provided that $\alpha_j = V_j / \sum_{i=1}^{n} V_i$, where V_i represents delivered volume at time $i\Delta$, for $i = 0, ..., n$.

The option can be priced following a 3-step algorithm devised in Fusai, Marena, and Roncoroni (2008), which we now sketch for the reader's convenience:

Step 1. Compute the moment generating function (MGF) of the underlying spot price S at maturity $T = n\Delta$ conditional to $S_0 = s_0$ at current time 0:

$$\gamma \to v_{0,s_0}(\gamma) := \mathbb{E}_0 \left[e^{-(\gamma S_{n\Delta})} \right]$$

Step 2. Using the main theorem in Fusai, Marena, and Roncoroni (2008), calculate the MGF of the pair $\left(S_{n\Delta}, \sum_{j=0}^{n} \alpha_j S_{j\Delta} \right)$ by recursion:

$$(\gamma,\mu) \to v_{0,s_0}^{n,\Delta}(\gamma,\mu) := \mathbb{E}_0 \left[e^{-\left(\gamma S_{n\Delta} + \mu \sum_{j=0}^{n} \alpha_j S_{j\Delta}\right)} \right]$$

Notice that $v_{0,\cdot}(\gamma) = v_{0,\cdot}^{n,\Delta}(\gamma, 0)$.

Step 3. Consider a contingent claim paying off $(Y_T - k)^+$ at time T, where k is the strike and Y is a nonnegative Markovian stochastic process. This form includes plain vanilla calls ($Y_T = S_{n\Delta}$) and standard fixed strike Asian-style options ($Y_T = \sum_{j=0}^{n} \alpha_j S_{j\Delta}$) struck at k. The time 0 arbitrage-free option price seen as a function of the strike price k reads as:

$$k \to C_{0,y_0}^{T}(k) = e^{-rT} \int_k^{+\infty} (y-k) f_{Y_T}(y) dy$$

where f_{Y_T} denotes the risk-neutral probability density of Y_T conditional to $Y_0 = y_0$. Provided that the MGF of Y_T exists, define the Laplace transform $\mathcal{L}$ of the option price $C_{0,y_0}^{T}(k)$ with respect to the strike price k as:

$$\lambda \to \mathcal{L}\left[C_{0,x}^{T}(\cdot)\right](\lambda) := \int_0^{+\infty} e^{-\lambda k} C_{0,x}^{T}(k) dk$$
$$= e^{-rT} \left(\frac{\mathbb{E}_0\left[e^{-\lambda Y_T}\right]}{\lambda^2} + \frac{\mathbb{E}_0(Y_T)}{\lambda} - \frac{1}{\lambda^2} \right)$$

The option price can be written as:

$$C_{0,x}^{T}(k)$$
$$= e^{-rT} \left(\mathcal{L}^{-1} \left[\frac{\mathbb{E}_0\left[e^{-\lambda Y_T}\right]}{\lambda^2} \right](k) + \mathbb{E}_0(Y_T) - k \right)$$

where:

- Expected values $E_0\left[e^{-\lambda Y_T}\right]$ and $E_0(Y_T)$ can be computed based on the output at step 2;
- Transform inversion can be executed using the Fourier inversion method (see Fusai and Roncoroni (2008))

We refer to a table reported in Fusai, Marena, and Roncoroni (2008) for exact expressions across the variety of cases under concern.

An appropriate selection of γ, μ, and α_j allows one to cover the cases of standard European, fixed strike Asian-style, fixed strike volume weighed Asian-style, and floating strike standard Asian-style options.

The goal of this article is to tune this procedure in a way to encompass the case of underlying prices driven by a jump-diffusion process. The key point here is that steps 1 to 3 above stated do occur in automatic cascade, meaning that each step directly follows from the previous one. Consequently, we just need to show that step 1 delivers closed-form output under jump dynamics for the spot price and the rest follows with no particular change.

Model Setting

We consider price dynamics under a risk-neutral probability $\mathbb{P}^*$. They are assumed to exhibit mean reversion to (possibly) time varying trend, time varying volatility, and a jump component of Poisson type, leading to a general expression:

$$dS_t = \beta\left(\eta_t - S_t\right)dt + \sigma_t\sqrt{S_t}dW_t + d\bar{J}_t, \tag{8}$$

where:

- β is a mean reversion constant frequency expressed in $1/\mathit{time}$ units;
- $(\eta_t)_{t\geq 0}$ is a deterministic time-varying price trend which spot quotes revert to;
- $(\sigma_t)_{t\geq 0}$ is a deterministic time-varying spot price volatility parameters: squares σ_t^2 represents the time t variance of instantaneous price variations per unit of price value S_t and is expressed in $1/\mathit{time}$ units;
- $(W_t)_{t\geq 0}$ is a standard Brownian motion;
- $(J_t)_{t\geq 0}$ is a compound Poisson process $\sum_{i=1}^{N_t} Y_i$ with the following properties:
  - N_t, Y_i's, and W_t are all mutually stochastically independent;
  - Jump intensity λ_t is deterministic, time varying, and bounded by a constant from above;
  - Jump magnitudes Y_i are i.i.d. copies of an exponential variable Y with mean $\xi > 0$;
- $(\bar{J}_t)_{t\geq 0}$ is the compensated martingale process defined as: $d\bar{J}_t := dJ_t - \xi\lambda_t dt$.

Notice that drift components β and η are meant under (a) risk-neutral probability: in principle they cannot be statistically estimated on time series of observed spot prices, but they ought to be calibrated on plain vanilla option quotes via pricing formulae as those we have described in the previous section. In particular, drift term η_t can be selected in a way that model (8) fits an observed forward price curve $(F_{0,T}, T \geq 0)$ quoted in the market, *i.e.*,

$$\mathbb{E}_0^*\left(S_T\right) = F_{0,T}, \tag{9}$$

where the * superscript indicate that expectation is computed under $\mathbb{P}^*$. By inserting the integral version of dynamics (8) into this formula, we come up to identifying the risk-neutral trend function:

$$\eta_T = F_{0,T} + \frac{1}{\beta}\partial_T F_{0,T} \tag{10}$$

ensuring the claimed fitting of observed forward curve.

Spot Price MGF

We now compute an analytical expression for the MGF of the underlying spot price $S_{t+\Delta}$, conditional to the market information available at time t, which is formally represented by the σ-algebra $\mathcal{F}_t^S$ generated by the price process $(S_t)_{t\geq 0}$ up to time $t \in [0, T]$.

Under spot price dynamics (8), the MGF of the spot price $S_{t+\Delta}$ given $S_t = x$ is:

$$\nu_{t,x}\left(\gamma\right) = e^{-\left[A_t(\Delta;\gamma)x + B_t(\Delta;\gamma)\right]}$$

where:

$$A_t\left(\Delta;\gamma\right) = \frac{\gamma e^{-\beta\Delta}}{1 + \frac{\gamma}{2}\int_t^{t+\Delta}\sigma_u^2 e^{-\beta(t+\Delta-u)}du}, \tag{11}$$

$$\begin{aligned} B_t\left(\Delta;\gamma\right) = {} & \gamma F_{0,t+\Delta} - F_{0,t}A_t\left(\Delta;\gamma\right) + \\ & -\frac{1}{2}\int_t^{t+\Delta} F_{0,u}\sigma_u^2 A_u\left(\Delta;\gamma\right)^2 du + \\ & -\xi^2\int_t^{t+\Delta}\lambda_u \frac{A_u\left(\Delta;\gamma\right)^2}{1+\xi A_u\left(\Delta;\gamma\right)}du \end{aligned} \tag{12}$$

Proof. Consider the MGF $\nu_{t,x}(\gamma)$ as a function $v(t,x)$ for fixed Δ and γ. Similarly, define $A(t)$ and $B(t)$ as $A_t(\Delta;\gamma)$ and, respectively, $B_t(\Delta;\gamma)$. Function v solves the partial integro-differential equation:

$$\begin{cases} \partial_t v\left(u,x\right) + \left[\beta\left(\eta_u - x\right) - \lambda_u\xi\right]\partial_y v\left(u,x\right) \\ \qquad + \frac{1}{2}\sigma_u^2 y\partial_{yy}v\left(u,x\right) + \\ \qquad + \lambda_u E_u\left[v\left(u,x+Y\right) - v\left(u,x\right)\right] = 0 \\ \qquad \text{v(t+}\Delta, x) = e^{-\gamma x} \end{cases}$$

on $[t, t+\Delta] \times \mathbb{R}$.

We consider a solution with exponential affine structure:

$$v(t,x) = e^{-A(t)x-B(t)}$$

which would lead to the following ODE system for functions $A(t)$ and $B(t)$:

$$\begin{cases} -A'(t) + \beta A(t) + \frac{1}{2}\sigma_t^2 A(t)^2 = 0 \\ -B'(t) - [\beta\eta_t - \xi\lambda_t] A(t) - \lambda_t \frac{\xi A(t)}{1+\xi A(t)} = 0 \end{cases}$$

with boundary conditions $A(t+\Delta) = \gamma$ and $B(t+\Delta) = 0$.

Let $C(t)$ be defined by:

$$A(t) = e^{\beta t} C(t) \tag{13}$$

Plugging this expression into the relevant equation, we have:

$$-\left(\beta e^{\beta t} C(t) + e^{\beta t}\partial_t C(t)\right) +$$
$$+\beta e^{\beta t} C(t) + \frac{1}{2}\sigma_t^2 e^{2\beta t} C(t)^2 = 0$$

$$C(t+\Delta) = \gamma e^{-\beta(t+\Delta)}$$

By separating variables, we get to:

$$C(t) = \frac{\gamma e^{-\beta(t+\Delta)}}{1 + \frac{\gamma}{2}\int_t^T \sigma_u^2 e^{-\beta(T-u)} du}$$

which, combined with assumption (13), leads to expression (11).

From the second ODE, we have

$$B(t) = \beta \int_t^{t+\Delta} \eta_u A(u)\, du +$$
$$+\xi \left(-\int_t^{t+\Delta} \lambda_u A(u)\, du + \int_t^{t+\Delta} \lambda_u \frac{A(u)}{1+\xi A(u)} du \right)$$

By using (10), we get to expression (12).

Remark. In absence of jumps, the stated Proposition matches Lemma 5 in Fusai, Marena, and Roncoroni (2008).

Pricing Algorithm

Combining the result obtained in the previous section with the procedure described earlier, we come up to the following algorithm for pricing Asian-style call options:

Algorithm

- **Step 0**: Assume:
  - A time interval $[0,T]$, which refines into n monitoring lags of length Δ, and a strike index k;
  - A continuously compounded rate of interest r;
  - Risk-neutral spot dynamics:

$$\left.\begin{array}{r} \text{mean reversion freq. } \beta \\ \text{fwd curve } (F_{0,s})_{0\le s\le T} \\ \text{volatility } (\sigma_t)_{0\le t\le T} \\ \text{jump freq. } (\lambda_t)_{0\le t\le T} \\ \text{average size of jump } \xi \\ \text{starting price } s_0 \end{array}\right\}$$

$$\rightarrow \begin{cases} dS_t = \beta(\eta_t - S_t)dt + \sigma_t\sqrt{S_t}dW_t + d\tilde{J}_t \\ J_t = \sum_{i=1}^{N_t} Y_i, \text{ with } Y_i \overset{i.i.d.}{\sim} \mathcal{E}xp\left(\xi^{-1}\right) \\ \mathbb{E}(dN_t) = \lambda_t dt \\ S(0) = s_0 \end{cases}$$

- **Step 1**. Compute the MGF of $S_{(j+1)\Delta}$ conditional to $S_{j\Delta} = x$, for $j = n-1, n-2, ..., 0$, using formula:

$$\gamma \to v_{j\Delta,x}(\gamma) = e^{-\left[A_{j\Delta}(\Delta;\gamma)x + B_{j\Delta}(\Delta;\gamma)\right]}$$

with:

$$A_{j\Delta}(\Delta;\gamma) = \frac{\gamma e^{-\beta\Delta}}{1 + \frac{\gamma}{2}\int_{j\Delta}^{(j+1)\Delta} \sigma_s^2 e^{-\beta(t+\Delta-s)} ds}$$

$$\begin{aligned} B_{j\Delta}(\Delta;\gamma) =& \gamma F_{0,(j+1)\Delta} - F_{0,j\Delta} A_{j\Delta}(\Delta;\gamma) + \\ & -\tfrac{1}{2}\int_{j\Delta}^{(j+1)\Delta} F_{0,u}\sigma_u^2 A_u(\Delta;\gamma)^2 du \\ & -\xi^2 \int_{j\Delta}^{(j+1)\Delta} \lambda_u \frac{A_u(\Delta;\gamma)^2}{1+\xi A_u(\Delta;\gamma)} du \end{aligned}$$

- **Step 2**. Compute the MGF of the pair $\left(S_{n\Delta}, \sum_{j=0}^{n} \alpha_j S_{j\Delta}\right)$ conditional to $S(0) = s_0$:

$$(\gamma,\mu) \to v_{0,s_0}^{n,\Delta}(\gamma,\mu) = e^{-\Lambda_0(\Delta;\gamma,\mu)s_0 - \sum_{j=0}^{n-1} B_{j\Delta}\left(\Delta;\Lambda_{j+1}(\Delta;\gamma,\mu)\right)}$$

where the function $\Lambda_j(\Delta;\gamma,\mu)$ satisfies the recursive equation:

$$\Lambda_j(\Delta;\gamma,\mu) = A_{j\Delta}\left(\Delta;\Lambda_{j+1}(\Delta;\gamma,\mu)\right) + \mu\alpha_j$$

for $j = n-1, n-2, ..., 0$, starting with:

$$\Lambda_n(\Delta,\gamma,\mu) = \gamma + \mu\alpha_n$$

Here, $A_{j\Delta}$ and $B_{j\Delta}$ are as in Step 1.

- **Step 3**. The fixed-strike Asian-style call option price can be represented as:

$$C_{0,s_0}^T(k) = e^{-rT}\left(\mathcal{L}^{-1}\left[\frac{v_{0,s_0}^{n,\Delta}(0,\mu)}{\mu^2}\right](k) + \sum_{j=0}^{n} \alpha_j F_{0,j\Delta} - k\right) \tag{14}$$

Whenever analytical inverse of transform $\mathcal{L}$ is not available, numerical evaluation is required. For instance, the Fourier-Euler algorithm proposed in Abate and Whitt (1992) leads to pricing formula (14) with:

$$\mathcal{L}^{-1}\left[\frac{v_{0,s_0}^{n,\Delta}(0,\mu)}{\mu^2}\right](k) \approx \sum_{m=0}^{M} \binom{M}{m} 2^{-m} d_{N+m}(k)$$

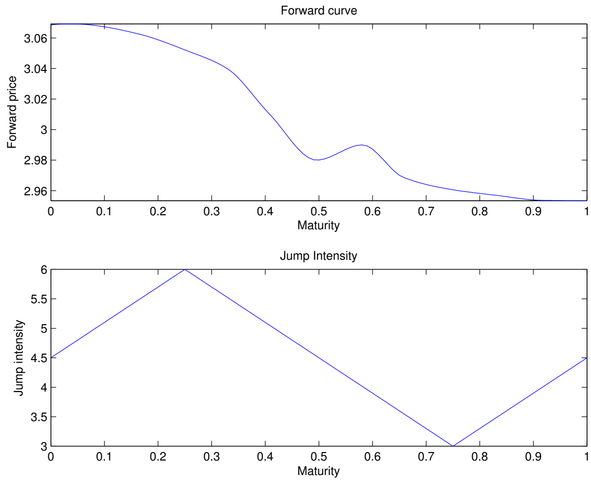

FIGURE 2: Heating Oil futures price curve fitting October 31, 2010, market quotes (panel 1); price jump frequency function (panel 2).

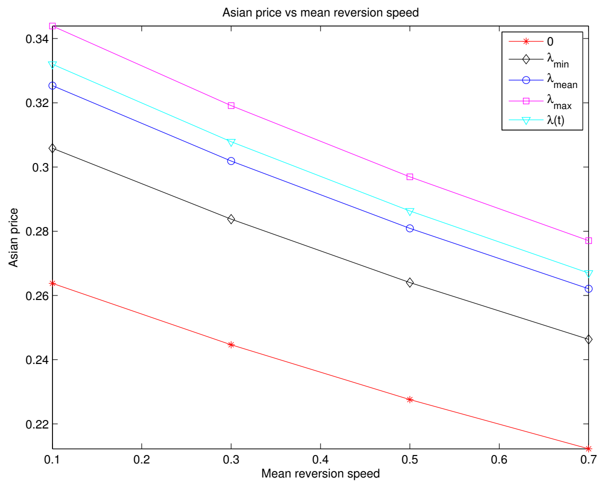

FIGURE 3: Asian-style options price against mean reversion frequency level and varying assumptions about the jump frequency function.

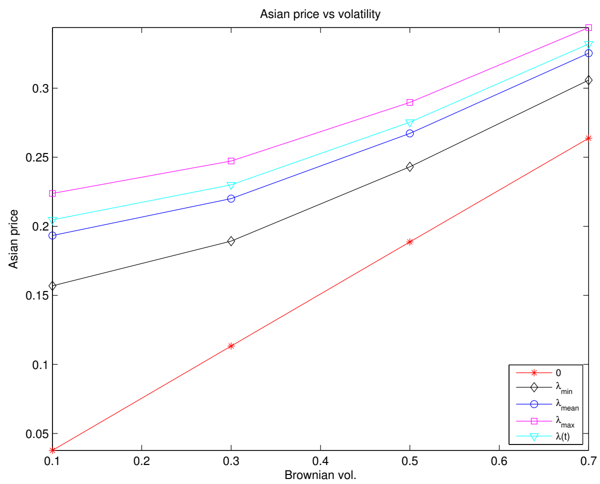

FIGURE 4: Asian-style options price against spot price volatility parameter and varying assumptions about the jump frequency function.

Maturity	# of mon. dates	Avg_n	Jump mean size
3m	3	3.0628	0.3063
6m	6	3.0396	0.3040
9m	9	3.0195	0.3019
12m	12	3.0045	0.3005

TABLE 2: Parametric settings across varying times-to-maturity.

with:

$$d_P(k) = \frac{e^{a_l/2}}{2k} Re\left(\frac{v_{0,s_0}^{n,\Delta}\left(0,\frac{a_l}{2k}\right)}{\mu^2}\right) + $$
$$+\frac{e^{a_l/2}}{k}\sum_{j=1}^{P}(-1)^j Re\left(\frac{v_{0,s_0}^{n,\Delta}\left(0,\frac{a_l+2j\pi i}{2k}\right)}{\mu^2}\right)$$

N and M are suitable constants, and a_l is located to the right-hand side of the real part of the largest singularity of the Laplace transform, *i.e.*, $a_l > 0$. We suggest to adopt the following parametric setting: $a_l = 18.4, M = 25, N = 15$ (see Fusai and Roncoroni (2008) for details).

Pricing Analysis

We used our formula to evaluate Asian-style call options written on Heating Oil price averages. Our goal is to assess option price sensitivity to key input parameters and data including time-to-maturity, market forward curve, and jump frequency.

We begin by defining values for each of the input quantities indicated on step 0 of the pricing algorithm stated earlier. Our base case assume that:

- Current time is $0 :=$ October 31, 2012.
- Options mature within $T = 3, 6, 9$, and 12 months.
- Averages are computed based on monthly monitoring, *i.e.*, $\Delta = 1/12$ years.
- Strike index k is assumed to match the ATM level defined as:

$$\overline{Avg}_{0,n} := \mathbb{E}_0^*\left(\text{Avg}_n\right) = \frac{1}{n+1}\sum_{j=0}^{n} F_{0,j\Delta},$$

  where $n = T/\Delta$. Table 2 provides these values for the cases under consideration.
- For each maturity, interest rate r is bootstrapped from LIBOR quotes on spot date 0.
- Mean reversion frequency $\beta = 0.1$ *per annum*.
- Heating Oil standing forward curve is reported in Table 4: a continuous curve $F_{0,\cdot}$ obtains through interpolation using using a cubic spline over the set of quoted values; this procedure results into the path shown in Figure 2 (panel 1).
- Spot price volatility parameter is constant $\sigma = 0.7$ *per annum*.
- Jump frequency is indicated in Table 3, where February experiences the highest value of the calendar year: a continuous curve $\lambda_\cdot$ obtains through linear interpolation over the set of assigned values; this procedure results into the path shown in Figure 2 (panel 2).
- Average size of jump $\xi = 0.1 \times \overline{Avg}_{0,n}$, that is 10% of the average expected spot price. Table 2 provides these parameters across varying maturities.
- Current spot price is set equal to $s_0 := F_{0,0}$.

Time	Jump Intensity
1m	5.00
2m	5.50
3m	6.00
4m	5.50
5m	5.00
6m	4.50
7m	4.00
8m	3.50
9m	3.00
10m	3.50
11m	4.00
12m	4.50

TABLE 3: Time varying jump frequency.

We now build five alternative assessment of the jump frequency:

- $\lambda_0 := 0$, which corresponds to continuous price paths;

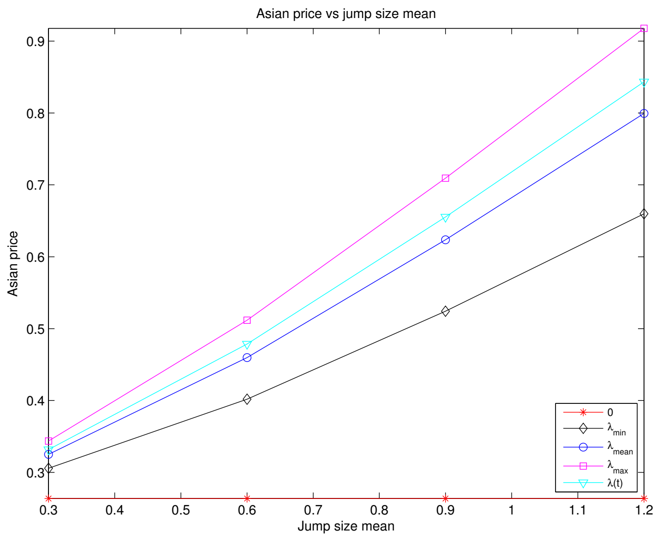

FIGURE 5: Asian-style options price against jump size mean and varying assumptions about the jump frequency function.

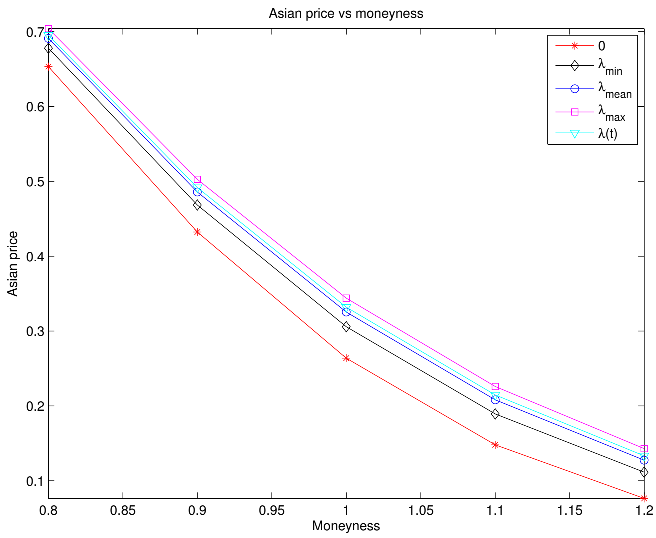

FIGURE 6: Asian-style options price against moneyness and varying assumptions about the jump frequency function.

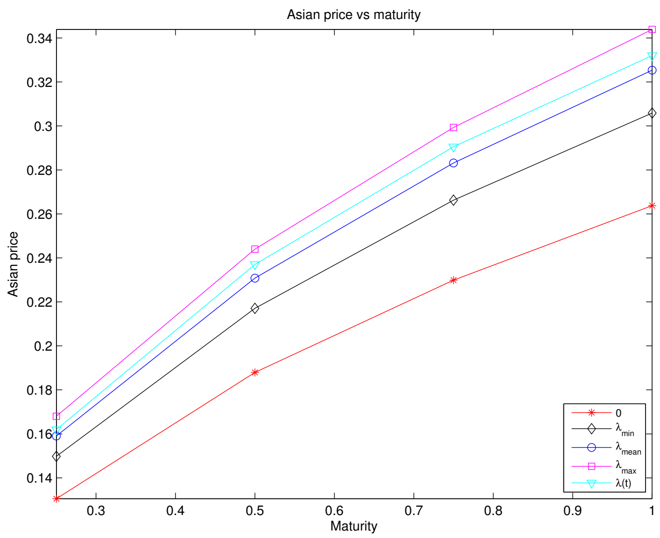

FIGURE 7: Asian-style options price against maturity and varying assumptions about jthe ump frequency function.

Delivery Month	Maturity	Settlement Price (USD)
December	30-Nov-2012	3.0682
January	31-Dec-2012	3.0623
February	31-Jan-2013	3.0519
March	28-Feb-2013	3.0400
April	29-Mar-2013	3.0100
May	30-Apr-2013	2.9800
June	31-May-2013	2.9900
July	28-Jun-2013	2.9687
August	31-Jul-2013	2.9607
September	30-Aug-2013	2.9570
October	30-Sep-2013	2.9537
November	31-Oct-2013	2.9535

TABLE 4: Heating Oil Futures prices quoted on October 31, 2012 at ICE.

	3m	6m	9m	12m
Fwd min	0.155	0.226	0.278	0.320
Fwd mean	0.157	0.228	0.281	0.324
Fwd max	0.159	0.232	0.285	0.329
Fwd curve	0.159	0.231	0.283	0.325

TABLE 5: Asian-stype option prices across maturities and varying assumptions about standing forward curve. (Jump frequency is assumed to be flat at level λ_{mean}.)

	3m	6m	9m	12m
λ_0	0.129	0.186	0.228	0.262
λ_{min}	0.148	0.215	0.264	0.304
λ_{mean}	0.157	0.228	0.281	0.324
λ_{max}	0.165	0.241	0.297	0.342
$\lambda(t)$	0.160	0.234	0.288	0.330

TABLE 6: Asian prices across maturities and varying assumptions about jump frequency functions. Forward curve is assumed to be flat at the level "medium flat" defined as $\frac{1}{12}\sum_{i=1}^{12} F_{0,i\Delta} = 2.9962$.

- Time-varying intensity $\lambda(t)$.
- Lowest bound flat: $\lambda_{\min} := \min_{1\le i\le 12} \lambda(i\Delta) = 3$.
- Medium level flat: $\lambda_{mean} := \frac{1}{12}\sum_{i=1}^{12} \lambda(i\Delta) = 4.5$.
- Greatest bound flat: $\lambda_{\max} := \max_{1\le i\le 12} \lambda(i\Delta) = 6$.

For each of these assessments, we calculate option prices and plot the corresponding values against:

- Mean reversion frequency β: Figure 3 shows that option prices decrease with an increase in the speed at which prices tend to revert back to their long-term trend. In fact, higher mean reversion leads to smoothing jumps, a fact that reduces underlying price dispersions, so reducing the likelihood of ending up ITM.
- Brownian volatility σ: Figure 4 shows that option prices increase with an increase in the spot index volatility.
- Jump size mean ξ: Figure 5 shows that option prices increase with an increase in the jump size mean.
- Option moneyness $k/\overline{Avg}_{0,n}$: Figure 6 shows that option prices decrease with an increase in the option moneyness.
- Time-to-maturity T: : Figure 7 shows that option prices increase with an increase in the option lifetime.

Clearly, the greater the jump frequency, the higher the corresponding values for call options. Clearly, a zero jump intensity makes option prices independent of jump related parameters. This is the case reported with a red path in Figure 5.

We finally build four alternative assessment of the input forward curve:

- Market $\{F_{0,i\Delta}\}_{1\le i\le 12}$, as we indicated earlier;
- Lowest flat: $\min_{1\le i\le 12} F_{0,i\Delta} = 2.9535$;
- Medium flat: $\frac{1}{12}\sum_{i=1}^{12} F_{0,i\Delta} = 2.9962$;
- Greatest flat: $\max_{1\le i\le 12} F_{0,i\Delta} = 3.0682$.

For each case, we report option prices against pairs of (maturity, input forward curve) and (maturity, input jump frequency function). Results are indicated in Tables 5 and 6, respectively.

Conclusions

We extend the semi-analytical price formula for Asian-style options derived in Fusai, Marena, and Roncoroni (2008) to the case of underlying spot prices driven by jump-diffusion processes. The key result is the calculation of the MGF for the spot price under these assumptions. Experiments conducted on market price data show that jumps may have a serious impact on the assessment of option prices despite the smoothing effect introduced by arithmetic averaging.

Future investigation might focus of the following spot price dynamics:

- Bivariate processes driven by a stochastic convenience yield;
- Multivariate processes with stochastic volatility;
- Jump-diffusions with random frequency of jumps.

Further extensions might encompass:

- Asian-style options written on a basket of prices;
- Convergence to continuous monitoring;
- Implied calibration on plain vanilla quotes.

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