The influence of financial conditions on optimal ordering and payment policies under progressive interest schemes

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Abstract: In many business-to-business transactions, the buyer is not required to pay immediately after the receipt of an order, but is instead allowed to postpone the payment to its suppliers for a certain period. In such a situation, the buyer can either settle the account at the end of the credit period or authorize the payment later, usually at the expense of interest that is charged by the supplier on the outstanding balance. Some payment terms, which are often referred to as trade credit contracts, contain progressive interest charges. In such cases, the supplier offers a sequence of credit periods, where the interest rate that is charged on the outstanding balance usually increases from period to period. If a buyer faces a progressive trade credit scheme, various options for settling the unpaid balance exist, where the financial impact of each option depends on the current credit interest structure and the alternative investment conditions. This paper studies the influence of different financial conditions in terms of alternative investment opportunities and credit interest structure on the optimal ordering and payment policies of a buyer on the condition that the supplier provides a progressive interest scheme. For this purpose, mathematical models are developed and analyzed.

Keywords: Trade credit; progressive interest rates; inventory management; economic order quantity; retail industry

Introduction

The focus of supply chain management has for many years been on the coordination of business functions such as purchasing, production and distribution within and across companies. Although it was stated early by many researchers that the management of supply chains should also include the integration of information and financial flows (cf. Mentzer et al., 2001), the management of financial issues in supply chains has only recently made its way onto research agendas (see, e.g., Pfohl and Gomm, 2009). One financial instrument that has received considerable attention in recent years are trade credits (see Seifert et al., 2013, for a recent review of the literature). Trade credits are short-term debt financing instruments that enable buyers of intermediate goods or services to delay the payment to their suppliers for a predefined credit period, either free of cost or in exchange for a contracted interest rate.

The major advantage of delayed payments is that suppliers provide capital access and thus enable their customers to increase order sizes without approaching a liquidity bottleneck. In addition, they help to improve the competitive position of the suppliers, who can use payment delays instead of price discounts to promote sales and develop their product market position (cf. Summers and Wilson, 2002). Other enablers facilitating the supply of trade credits are differences in the price elasticity between suppliers and buyers, collateral values of goods sold, credit intermediation between buyers and banks as well as the protection of non-salvageable investments in buyers (cf. Seifert et al., 2013). Consequently, in many industries, trade credits have become one of the most important sources of short-term funding. A recent survey of the European Central Bank (2013) showed that access to finance is one of the most pressing problems especially of small- and medium-sized companies in Europe. Trade credits are thus a promising option to get access to short-term finance for companies suffering under a credit crunch. Besides diminishing credit rationing, trade credits may also lead to a reduction of cost by pooling transactions, and they allow more financial flexibility than bank loans in the case of financial distress (Garcia-Teruel and Martinez-Solano, 2010).

Trade credit terms may vary significantly from industry to industry. The simplest way to offer a trade credit is to define a fixed time period in which the buyer is allowed to delay the payment to its supplier. If the buyer fails to settle the account (completely) during this time span, then interest is charged on the outstanding balance. This type of trade credit was first analyzed in the context of an economic order quantity (EOQ) model by Goyal (1985), who showed that the order quantity increases if predefined payment delays are permitted, as compared to the classical EOQ model. Subsequently, Dave (1985) introduced a model that considered different purchasing and selling prices, and Chung (1998) presented a simplified solution procedure for this model. Teng (2002) further extended the model of Goyal (1985) and demonstrated that in certain cases, it is beneficial for the buyer to reduce its order quantity if trade credits are offered, and to benefit from the permissible delay in payments by ordering more frequently. Huang (2007) considered the case of a supplier that specifies a threshold order quantity, where the full trade credit is only granted if the buyer's order quantity exceeds this threshold. If the order quantity is below the predetermined quantity, then only a partial trade credit is offered. Similar works are the ones of Chung et al. (2005) and Yang et al. (2013), which assumed that if the order quantity is smaller than a predetermined quantity, the supplier does not offer a trade credit at all. Taleizadeh et al. (2013) considered a scenario where a fraction of the purchasing cost has to be paid immediately after the order has been received into inventory, and where only the remaining fraction of the purchasing cost is subject to trade credits. A related scenario is the one where the supplier offers the trade credit on a one-time-only basis. Papers that fall into this stream of research assumed that the trade credit is available only for a single order at a pre-specified point in time, which is in contrast to the works discussed above that assumed that the trade credit is available in each order cycle. In case a one-time-only trade credit is offered, the buyer has an incentive to place a special order quantity once to benefit from the trade credit, and to revert to its original order policy after the trade credit option has expired. Works that belong to this stream of research are the ones of Goyal and Chang (2008) and Chung and Lin (2011), among others.

Other authors considered the case where the supplier offers more than a single credit period to the buyer. The general idea of a so-called progressive payment scheme is that no interest is charged in the first credit period, and that the interest rate then increases from credit period to credit period. Goyal et al. (2007) were among the first to consider a progressive payment scheme. The authors studied the case of three different credit periods and analyzed their impact on an EOQ model. This paper was revisited by Chung (2009), who improved the optimization procedure suggested by Goyal et al. (2007). The work of Goyal et al. (2007) has frequently been extended in the past. Some authors, for example, assumed that demand is stock-dependent, which leads to higher customer demand early in the cycle and to lower customer demand at the end of the replenishment cycle (see, e.g., Soni and Shah, 2008; 2009). If such an inventory system is appropriately managed, then higher earnings at the beginning of a cycle enable the buyer to repay the supplier earlier, which leads to a higher profit for the buyer. If demand is stock-dependent, then the profit of the buyer can be increased if inventory is not fully depleted at the end of a cycle, which stimulates additional customer demand (see Teng et al., 2011). Other popular extensions of the work of Goyal et al. (2007) include product deterioration (e.g., Soni et al., 2006b, Teng et al., 2011, Shah et al., 2011), the production of defective items (e.g., Sarker, 2012), the time value of money (e.g., Soni et al., 2006a; 2006b), or limited storage space (e.g., Shah et al., 2011, Teng et al., 2011).

A closer look at the literature reveals that research has frequently relaxed limiting assumptions of earlier works on trade credits to develop more realistic planning models that cover a wide range of practical scenarios. The seminal work of Goyal (1985), for example, assumed that the product is sold to the end customer at the unit purchase price. This assumption was relaxed by Dave (1985), Huang (2002) and Teng et al. (2006), for example, who assumed that the selling price is necessarily higher than the purchase price paid by the buyer. When analyzing the literature, we found that prior research consistently made the assumption that the interest rate charged by the supplier exceeds the credit interest rate of the buyer in all credit periods. The only exception is the work of Cheng et al. (2012), which, however, did not consider a progressive payment scheme and assumed that the buyer settles its open account at the end of the replenishment cycle at the latest, as the supplier is not willing to make a new delivery before receiving the entire purchase price of the previous shipment.

It is clear that in practice, the interest rate charged by the supplier does not always exceed the credit interest rate of the buyer. On the contrary, the credit interest rate of the buyer, which could represent the interest rate the buyer could realize by depositing money in an interest bearing account or by investing it elsewhere, or the interest rate the buyer is charged from its bank (Summers and Wilson, 2002), could exceed the interest charged by the supplier. Several empirical studies revealed that this is especially the case in duopoly industries with a small number of powerful customers (see, e.g., Ng et al., 1999; Klapper et al., 2012). In such a case, it would not be rational from the buyer's point of view to settle the unpaid balance as soon as interest is charged on the outstanding balance, as was assumed in the literature so far. Instead, it would be better to keep the sales revenue invested and to settle the unpaid balance not before the interest charged by the supplier exceeds the incomes from the investment, or just before the next order is issued. Considering such arbitrage gains within the payment policy induces substantial savings and is suited to explain the differences in the working capital structure as can be observed, for example, in the retail sector (cf. Section 2). Another shortcoming we identified is that prior research on inventory models with progressive interest schemes usually assumed that the buyer has the option to settle the outstanding balance only at the end of the credit periods. It is, however, clear that the buyer may benefit from continuously settling the outstanding balance within the credit periods if the interest charged by the supplier exceeds the credit interest rate of the buyer. Finally, we found that compound interest the retailer may realize during the credit periods was neglected in prior trade credit inventory models. Clearly, especially in situations where the credit periods are long and interest rates are high, interest on interest earnings may represent an additional source of profit that should not be neglected.

In light of the research gaps identified above, the purpose of this paper is to generalize the trade credit inventory model with progressive interest scheme by considering a) the case where the credit interest rate of the buyer may (but not necessarily has to) exceed the interest rate charged by the supplier, b) where the buyer has the option to settle the outstanding balance continuously within the credit periods, c) where compound interest accrues at the retailer, and d) bank loans are available as a substitute for the trade credit. In addition, some inaccuracies in earlier formulations of the effective interest cost are corrected. The remainder of the paper is structured as follows: The next section illustrates the role of trade credits and working capital management in the retail sector. Section 3 then outlines assumptions and notations used throughout the paper and develops formal models for determining the optimal order quantity and payment scheme for different interest and payment conditions. Sections 4 and 5 present theoretical findings on the models developed and illustrate their behavior with the help of a benchmark case and an extensive simulation study. Section 6 finally concludes the article.

Trade credits in the retail industry

Although several studies indicate that trade credits are one of the most important means of short term financing for companies (Summers and Wilson (2002), for example, state that more than 80% of business-to-business transactions in the UK include trade credit agreements), the amount of trade credit financing varies significantly from industry to industry (see Ng et al., 1999 or Seifert et al., 2013). The retail sector, which covers all types of companies selling goods or commodities bought from a manufacturer or a wholesaler to the end-user via different distribution channels, is an intensely cash-generating industry that relied extensively on trade credits in the past (see Klapper et al., 2012). Table 1 gives an overview of the operating characteristics of the world's ten largest public-owned non-specialized retailers in terms of total revenues in 2014. In the considered sample, accounts payable reached on average one fifth of the firms' total assets and one third of the firms' total liabilities. At the individual company level, the world's leading retailer, Wal-Mart Stores Inc., already had accounts payable of $38.410 billion in its balance sheet on January 31, 2015. This is about 85% of its total inventories ($45.141 billion). Even higher payables to inventory ratios can be found in the other companies that range from 88% (The Kroger Co. or Target Corporation) to 215% (Carrefour S.A.). Even though the demand for trade credit depends on several factors such as transaction pooling, credit rationing or financial flexibility in vendor-buyer relations (cf. Section 1 and Summers and Wilson, 2002, among others), reduced transaction cost as well as increased demand for liquidity after the economic downturn in the year 2008 seem to be the decisive causes of the high demand for trade credit in the retail sector.

Table 1: Top-10 global retailers according to total revenues in 2014

Name of company	Country of origin	Total revenues 2014 [m USD]	Operating income 2014 [m USD]	Total assets 2014 [m USD]	Total liabilities 2014 [m USD]	Accounts payable 2014 [m USD]	Closing date 2014
Wal-Mart	US	485,651	27,147	203,490	122,096	38,410	31/01/2015
Costco	US	112,640	3,220	33,024	20,721	8,491	31/08/2014
The Kroger	US	108,465	3,137	30,497	25,085	5,052	31/01/2015
Tesco	GB	96,532	-6,346	68,218	57,308	7,832	28/02/2015
Carrefour	FR	92,659	2,715	55,592	43,175	16,250	31/12/2014
Metro	DE	80,855	1,461	35,237	28,947	12,495	30/09/2014
Target	US	72,618	4,535	41,404	27,407	7,759	31/01/2015
Aeon	JP	59,354	819	65,905	50,560	7,938	28/02/2015
Group Casino	FR	58,875	2,109	54,974	36,025	10,107	31/12/2014
Seven & I	JP	50,637	2,772	43,893	23,510	3,459	28/02/2015

* other currencies have been converted to USD by the exchange rate at closing date

Due to its practical relevance, trade credits have extensively been studied in the context of economic ordering and payment decisions (cf. Section 1). However, prior research consistently made the assumption that the interest rate charged by the supplier exceeds the interest rate of the buyer. In contrast, empirical studies indicate that even though the average effective interest rate of trade credits is high, the effective interest rates vary from a low of 2% to a high of 100% (Klapper et al., 2012). Consequently, in practice, the credit interest rate of the buyer, which could represent the interest rate the buyer could realize by depositing money in an interest bearing account or by investing it elsewhere, could also exceed the interest rate charged by the supplier. In such a case, it would not be rational from the buyer's point of view to settle the unpaid balance immediately, as was assumed in the literature so far, but instead to keep the sales revenue invested and to settle the unpaid balance not before the interest charged by the supplier exceeds the incomes from the investment, or just before the next order is issued. Consequently, as long as the interest charged by the supplier on the open account is below the internal rate of return or the interest rates on short term deposits, the buyer may realize arbitrage profits from postponing the payment and investing the money in other projects or a bank account. Thus, the buyer has a financial incentive to extend the trade credit period, which also affects the average number of days payables outstanding and finally the effective cash conversion cycles.

Table 2: Days payables outstanding and cash conversion cycles of the retailers

	Year / Company	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**AV**
DPO	**Wal-Mart**	39	40	40	35	37	40	41	40	39	39	**39**
	Costco	33	32	33	30	32	32	31	31	32	32	**32**
	The Kroger	28	28	26	24	24	24	22	21	23	22	**24**
	Tesco	29	31	34	35	37	39	38	38	36	32	**35**
	Carrefour	102	99	96	93	91	96	92	78	78	82	**91**
	Metro	93	101	104	98	103	102	103	96	100	75	**97**
	Target	66	59	57	52	54	48	48	47	54	55	**54**
	Aeon	65	64	65	66	74	73	72	65	69	80	**69**
	Group Casino	84	74	79	70	72	73	70	70	64	66	**72**
	Seven & I	28	22	21	19	34	32	39	39	40	40	**32**
CCC	**Wal-Mart**	14	10	10	10	6	8	9	11	13	12	**10**
	Costco	1	3	2	2	3	1	3	2	3	2	**2**
	The Kroger	12	10	11	11	11	8	8	7	8	7	**9**
	Tesco	-6	-4	-3	-3	-5	-4	0	0	-2	-2	**-3**
	Carrefour	-57	-58	-54	-52	-51	-51	-45	-38	-38	-39	**-48**
	Metro	-38	-44	-47	-46	-48	-46	-45	-44	-36	-27	**-42**
	Target	35	35	47	48	44	46	43	39	37	35	**41**
	Aeon	-9	-3	-4	-9	-16	-12	-8	5	23	19	**-1**
	Group Casino	-17	-13	-11	-8	-9	-7	-6	-5	-10	2	**-8**
	Seven & I	5	4	3	5	-7	-6	0	1	2	1	**1**

Table 2 provides an overview of working capital measures of the retailers listed in Table 1, which gives some insights into the payment behaviors in the retail industry (note that Table 2 displays annual values based on each company`s fiscal year). A common measure for working capital management is the Cash Conversion Cycle (CCC) (cf. Richards and Laughlin, 1980) that measures the length of time (in days) a company's cash is tied up in working capital. CCC is commonly calculated as the days of inventory outstanding (DIO) + the days accounts receivable outstanding (DRO) – the days accounts payable outstanding (DPO). To analyze the working capital management of the retailers, the CCC and its components were calculated for the years 2005 to 2014. The results show that there is a substantial difference between the ten retailers. Whereas some companies such as Wal-Mart Stores Inc. or The Kroger Co. exhibit a positive CCC in the past ten years, it is notably negative for others such as Carrefour S.A. or Metro AG. Regarding CCC's components, the DIOs vary between a low of 19 days and a high of 59 days, while the DROs and DPOs vary between 3 and 36 days as well as 24 and 97 days, respectively. To gain further insights into this aspect, a cluster analysis of the retailers with regard to their DIO, DRO and DPO characteristics was performed with the help of the k-means approach. In the k-means analysis, an input data set is partitioned into k clusters by computing the squared distances between the inputs and the centroids and by assigning these inputs to the nearest centroid to minimize the consequent mean-squared error (cf. Rizman Zalik, 2008). The results show that companies with a positive CCC show a weighted average DIO of 30 days, a weighted average DRO of 5 days and a weighted average DPO of 31 days. In contrast, companies with a negative CCC have extended inventory cycles (DIO 43 days), but also significantly higher payment cycles for inbound and outbound transactions (DRO of 19 days and DPO of 74 days). Obviously, in the second cluster, companies have DPOs of more than two months. As large companies such as the retailers in our sample are supposed to have easy access to other sources of finance, they would not use trade credits extensively if it was as expensive as commonly hypothesized in the literature (cf. Ng et al., 1999, Klapper et al., 2012). Accordingly, beside other causes for the distinct payment behavior discussed in the literature, it is reasonable that due to the large varieties in contract conditions, trade credits appear comparatively cheap for some of the retailers in comparison with the return on alternative investment opportunities (we note that also power relationships cloud play an important role in this context which is, however, not reflected in our sample). This also seems to be supported by the fact that especially larger and investment-grade buyers receive longer net days from their suppliers (cf. Klapper et al., 2012). Hence, with an increasing interest rate, the retailers might realize arbitrage profits from postponing the payment to their suppliers, and therefore they tend to settle their payables outstanding later. To investigate this issue, which has been neglected in the literature so far, in more detail, the influence of financial conditions on the replenishment and payment behavior will be analyzed formally in the following. The results derived from this formal analysis may facilitate further empirical research.

Model development

The problem described in the introduction will subsequently be analyzed under the following conditions:

1. The inventory system involves a single item and has an infinite planning horizon.
2. Shortages are not allowed and the demand rate is constant and deterministic.
3. Lead time is zero and replenishments are made instantaneously.
4. The supplier provides a trade credit with progressive interest rates to the buyer. If the buyer pays before time M, the supplier does not charge any interest, whereas in case the buyer pays between times M and N with $M < N$, the supplier charges interest at the rate of Ic_1. In

case the buyer pays after time *N*, the supplier charges interest at the rate of Ic_2, with $Ic_2 > Ic_1$.
5. Apart from trade credits, the buyer is also assumed to have access to bank loans at the rate of *Ib* that are frequently referred to as substitutes to trade credits and vice versa.
6. The buyer has the option to deposit money in an interest bearing account with a fixed interest rate of *Ie*. Thus, s/he may use sales revenues to earn interest until the account is completely settled. Other investment decisions that are not related to the lot sizing problem are not considered.

In addition, the following terminology is used throughout the paper:

Parameters:

A	cost of placing an order
C	unit purchasing cost with $C < P$
D	demand rate per unit of time
h	physical unit holding cost per unit and unit of time
Ic_1	interest rate per unit of time charged by the supplier between times *M* and *N*
Ic_2	interest rate per unit of time charged by the supplier after time *N*
Ib	interest rate on borrowings at the retailer per unit of time
Ie	interest rate on deposits at the retailer per unit of time
M	permissible delay in payments without any interest charge
N	permissible delay in payments which induces an increase in the interest rate with $N > M$
P	selling price per unit

Decision variables

Q	order quantity of the buyer (can implicitly be derived from *T*)
T	replenishment interval

The buyer faces a constant customer demand rate *D* that leads to a continuous decrease in the inventory level *I*(*t*). Accordingly, the development of the inventory level with respect to time *t* can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -D, \quad 0 \le t \le T \tag{1}$$

with the boundary conditions $I(0) = Q$ and $I(T) = 0$. The solution of this differential equation is:

$$I(t) = D(T - t), \quad 0 \le t \le T \tag{2}$$

which leads to the corresponding order quantity $Q = DT$.

The total relevant costs are given as the sum of ordering, inventory carrying and interest costs, reduced by interest earnings. The cost per unit of time for placing an order at the supplier amounts to:

$$OC = A/T \tag{3}$$

Inventory holding cost is given by:

$$IHC = \frac{h}{T}\int_0^T I(t)dt = hDT/2 \tag{4}$$

Depending on the length of the replenishment cycle, T, the ratio of the interest rates (i.e. the ratio of Ie to Ic_1 and Ic_2) and the lengths of the credit periods, M and N, the buyer may incur interest costs and/or realizes interest earnings.

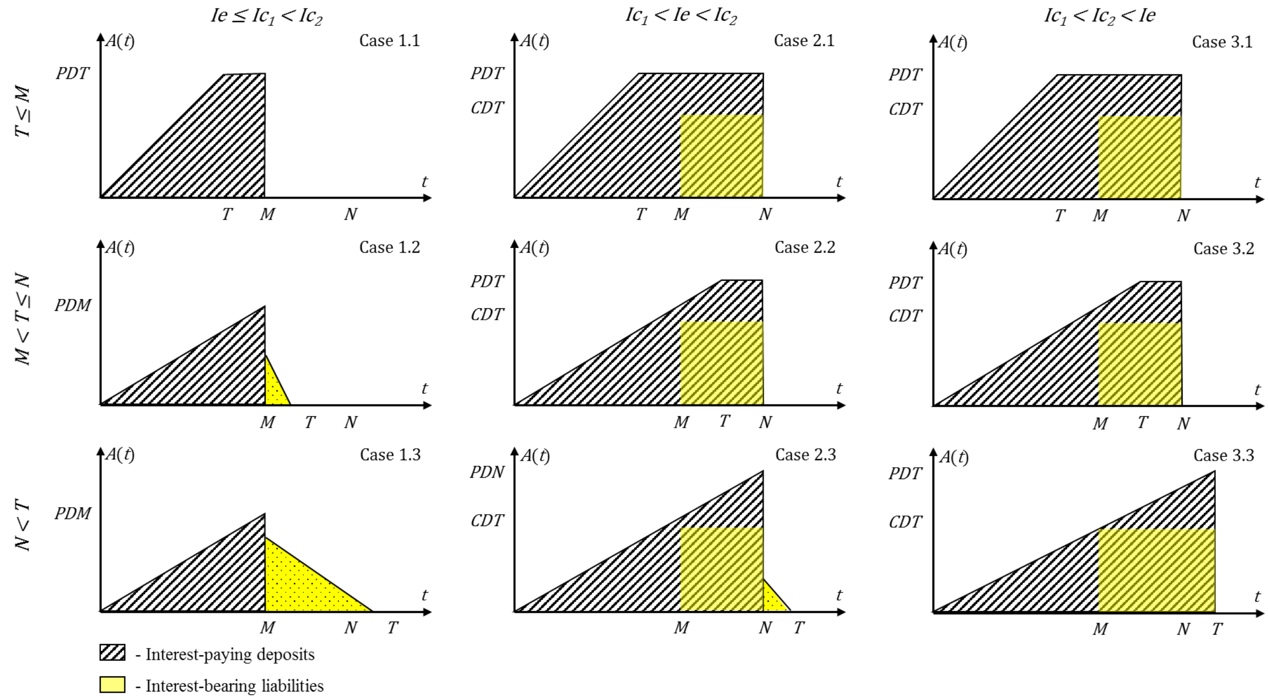

Figure 1: Balance of accounts for deposits and liabilities

Figure 1 exemplarily illustrates the available amount of cash from sales revenues as well as the outstanding debt over time and thus facilitates determining interest cost and/or interest earnings (note that in Cases 1.2, 1.3 and 2.3, the spotted triangles representing continuous settlement of the outstanding debt depend on the ability of the retailer to settle the open account entirely at times M or N). Unless the interest rate on deposits (Ie) exceeds the interest charged by the supplier on the outstanding payments (Ic_1) between times M and N, the buyer settles as much of the account as possible at time M to avoid unnecessary interest cost (cf. left part of Figure 1). Depending on the length of the replenishment interval and the ability to settle the open account, three different cases with respective subcases may arise, namely $T \le M$, $M < T \le N$ and $T > N$ (see also Goyal et al., 2007). However, in case the interest rate on deposits (Ie) exceeds the debit interest rate (Ic_1) between times M and N or even the debit interest (Ic_2) after time N, the buyer realizes an advantage from postponing the settlement of the open account until times N or T (cf. the middle and right parts of Figure 1 and note that it was assume that the retailer settles its account at the latest at time $\max\{N,T\}$, as the supplier would not release further deliveries in the event of delayed or default payment). Again, for both interest conditions depending on the length of the replenishment interval and the ability to settle the open account, three different cases with respective subcases may arise. Each of the arising cases will be discussed in the following with regard to the accruing interest charges and/or earnings and the resulting total cost in order to derive the optimal ordering and payment policy for the retailer.

Case 1: $Ie \leq Ic_1 < Ic_2$

Case 1.1: $T \leq M$

In this case, the buyer sells off the entire batch of $Q = DT$ units at time T, and is able to settle the account completely before the supplier starts charging interest at time M. Between times 0 and T, sales revenues accumulate until the total revenue, PDT, is available at time T. During the period [0,M], the buyer additionally generates interest earnings at the rate Ie by depositing sales revenues in an interest bearing account. The total interest earned per unit of time can be written as:

$$IE_{1.1} = \frac{IeP}{T}\left[\int_0^T Dt\,dt + Q(M-T)\right] = IePD\left(M - \frac{T}{2}\right) \tag{5}$$

To avoid interest payments to the supplier, the buyer settles the balance entirely at time M (i.e. $IC_{1.1} = 0$). Consequently, the total relevant costs amount to:

$$TC_{1.1} = \frac{A}{T} + \frac{hDT}{2} - IePD\left(M - \frac{T}{2}\right) \tag{6}$$

Since the second-order condition of Eq. (6) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 1.1:

$$T^*_{1.1} = \sqrt{\frac{2A}{D(h + Ie\,P)}} \tag{7}$$

Case 1.2: $M < T \leq N$

In the case where $Ie < Ic_1$ and $M < T \leq N$, the buyer tries to settle as much of the unpaid balance as possible at time M to minimize interest payments. In the period [0,M], the buyer sells DM products and generates direct revenues in the amount of PDM dollars. Sales revenues that accumulate over time are deposited in an interest bearing account that earns interest at the rate of Ie per unit of time, which leads to additional earnings of $IePDM^2/2$ (cf. Eq. (8)). Accordingly, at time M, the buyer uses the sum of revenues and interest earnings to settle the open account. The total purchase cost for a lot of size DT amounts to CDT dollars. Depending on the ratio of the total purchase cost to the sum of earnings from sales and interest received at time M, two different subcases may arise that will be discussed in the following:

Case 1.2-1: $CDT \leq PDM(1 + IeM/2)$

In the first subcase, the sum of sales revenues and interest earned at time M is sufficient to settle the unpaid balance completely, i.e. $CDT \leq PDM(1 + IeM/2)$. The interest earnings per unit of time are given as:

$$IE_{1.2-1} = \frac{IeP}{T}\int_0^M Dt\,dt = \frac{IePDM^2}{2T} \tag{8}$$

As the buyer again does not have to pay interest to the supplier in this subcase (i.e. $IC_{2,1} = 0$), the total relevant costs amount to:

$$TC_{1.2-1} = \frac{A}{T} + \frac{hDT}{2} - \frac{IePDM^2}{2T} \tag{9}$$

Since the second-order condition of Eq. (9) is strictly positive (cf. Section 4), the solution can be derived using of the first-order condition, which leads to the optimal value of T for Case 1.2-1:

$$T^*_{1.2-1} = \sqrt{\frac{2A - IePDM^2}{Dh}} \tag{10}$$

Case 1.2-2: $CDT > PDM(1 + IeM/2)$

In contrast to the previous subcase, we now consider the case where the sum of sales revenues and interest earned at time M is not sufficient to settle the balance completely, i.e. $CDT > PDM(1 + IeM/2)$. Thus, the supplier starts charging interest on the unpaid balance at the rate Ic_1 at time M. Interest earned in the period $[0,M]$ is again given as $IePDM^2/2$ (cf. Eq. (8)), which leads to an open account at time M in the amount of $CDT - PDM(1 + IeM/2)$. To minimize interest payments, the buyer transfers each dollar earned after time M directly to the supplier (see Goyal et al., 2007 and Taleizadeh, 2014a; 2014b for a similar assumption in the case of trade credits or prepayments). For the case where the unpaid balance cannot be settled at time M, but before time N, the interest cost can be formulated as follows:

$$IC_{1.2-2} = \frac{Ic_1}{T} \int_M^{M+z_1} \Big(\big(CDT - PDM(1 + IeM/2)\big) - PD(t - M) \Big)\, dt = \frac{Ic_1}{2PDT} \big(CDT - PDM(1 + IeM/2)\big)^2 \tag{11}$$

where $M + z_1$ denotes the point in time when the unpaid balance has been completely settled, with $z_1 = \big(CDT - PDM(1 + IeM/2)\big)/PD$. In case of $Ib < Ic_1 < Ic_2$, the retailer may benefit from bridgeover finance by bank loans that reduce the effective interest rate (note that in this case in Eq. (11), Ic_1 needs to be replaced by Ib; everything else would remain unchanged). Thus, the total costs for this case amount to:

$$TC_{1.2-2} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{2PDT} \big(CDT - PDM(1 + IeM/2)\big)^2 - \frac{IePDM^2}{2T} \tag{12}$$

Since the second-order condition of Eq. (12) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 1.2-1:

$$T^*_{1.2-2} = \sqrt{\frac{2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)}{D(h + Ic_1 C^2/P)}} \tag{13}$$

Case 1.3: $N < T$

The case where $Ie < Ic_1$ and $T > N$ is similar to Case 1.2. Again, the buyer uses the revenues and interest earned to pay the supplier. To minimize interest payments, he/she settles as much of the outstanding balance as possible at time M and afterwards reduces the outstanding amount continuously by transferring each dollar of the sales revenues to the supplier's account. This helps to avoid unnecessary interest costs as compared to prior works in this area assuming that partial payments are made at times M and N only. In addition, interest charges that accrue between times M and N will be considered, which leads to an unsettled balance at time N in the amount of $CDT - PDM(1 + IeM/2) - PD(N - M) + Ic_1 \int_M^N \Big(\big(CDT - PDM(1 + IeM/2)\big) - PD(t - M) \Big)\, dt$. Integrating this term over the limits and rearranging the resulting

expression leads to $(CDT - PDM(1 + IeM/2))(1 + Ic_1(N - M)) - PD(N - M)(1 + Ic_1(N - M)/2)$. The first part of this expression describes the outstanding balance at time *M*, which includes interest incurred at the rate Ic_1. The second part of the expression, in contrast, represents the amount of repayment during times *M* and *N*, which also considers the interest effect due to the continuous refund.
In contrast to prior works on trade credits with progressive credit periods, these modifications allow us to consider the interest charges that accumulate between times *M* and *N* as well as the payments the buyer makes between times *M* and *N* to reduce the accruing interest. According to the ratio of the total purchasing cost to the sum of sales and interest earnings, three possible subcases may arise that can be distinguished based on the balance of the buyer's account at times *M* and *N*, respectively.

Case 1.3-1: $CDT \leq PDM(1 + IeM/2)$
The case where $Ie < Ic_1$ and $T > N$ is identical to Subcase 1.2-1. Thus, the buyer settles the account completely at time *M* without paying any interest charges to the supplier.

Case 1.3-2: $CDT > PDM(1 + IeM/2)$ and $(CDT - PDM(1 + IeM/2))(1 + Ic_1(N - M)) \leq PD(N - M)(1 + Ic_1(N - M)/2)$
The case where $Ie < Ic_1$ and $T > N$ is identical to Subcase 1.2-2. Thus, the buyer is not able to pay off the entire purchase cost at time *M*, but settles as much of the account as possible at time *M*. Afterwards, he/she continuously reduces the open account by transferring sales revenues to the supplier. The account is settled at time $M + z_1$ with $M + z_1 < N$, and the supplier charges an interest on the unpaid balance between times *M* and $M + z_1$.

Case 1.3-3: $CDT > PDM(1 + IeM/2)$ and $(CDT - PDM(1 + IeM/2))(1 + Ic_1(N - M)) > PD(N - M)(1 + Ic_1(N - M)/2)$
In the case where $Ie < Ic_1$ and $T > N$, the buyer is not able to pay off the total purchase cost at times *M* or *N*. Thus, he/she settles as much of the unpaid balance as possible at time *M*. Afterwards, the open account is continuously reduced by transferring each dollar earned from sales to the supplier. As the buyer has no incentive to authorize any payments before time *M*, he/she realizes interest earnings in the period [0,*M*] (cf. Eq. (8)). The supplier, in turn, charges interest on the gradually decreasing unpaid balance at the rate Ic_1 between times *M* and *N* and at the rate Ic_2 after time *N*.
Hence, the overall interest cost per unit of time for this case, $IC_{1.3\text{-}3}$, amounts to:

$$IC_{1.3-3} = \frac{Ic_1}{T}\int_M^N \left(\left(CDT - PDM\left(1 + \frac{IeM}{2}\right)\right) - PD(t - M)\right)dt + \frac{Ic_2}{T}\int_N^{N+z_2}\left(\left(CDT - PDM\left(1 + \frac{IeM}{2}\right)\right)(1 + Ic_1(N - M)) - PD(N - M)\left(1 + \frac{Ic_1(N-M)}{2}\right) - PD(t - N)\right) dt = \frac{Ic_1(N-M)}{T}\left(CDT - PDM\left(1 + \frac{IeM}{2}\right) - \frac{PD(N-M)}{2}\right) + \frac{Ic_2}{2PDT}\left(\left(CDT - PDM\left(1 + \frac{IeM}{2}\right)\right)(1 + Ic_1(N - M)) - PD(N - M)\left(1 + \frac{Ic_1(N-M)}{2}\right)\right)^2 \quad (14)$$

where $N + z_2$ denotes the point in time when the unpaid balance has been completely settled, with $z_2 = \big((CDT - PDM(1 + IeM/2) - PD(N - M))(1 + Ic_1(N - M)) +$

$Ic_1PD(N-M)^2/2\Big)/PD$. In case of $Ib < Ic_1 < Ic_2$ or $Ic_1 < Ib < Ic_2$, the retailer may benefit from bridgeover finance by bank loans that reduce the effective interest rate (note that in this case in Eq.(14) Ic_1 and /or Ic_2 need to be replaced by *Ib*; everything else would remain unchanged). The total cost for this case amounts to:

$$TC_{1.3-3} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1(N-M)}{T}\left(CDT - PDM\left(1+\frac{IeM}{2}\right) - \frac{PD(N-M)}{2}\right) + \frac{Ic_2}{2PDT}\left(\left(CDT - PDM\left(1+\frac{IeM}{2}\right)\right)\left(1+Ic_1(N-M)\right) - PD(N-M)\left(1+\frac{Ic_1(N-M)}{2}\right)\right)^2 - \frac{IePDM^2}{2T} \quad (15)$$

Since the second-order condition of Eq. (15) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of *T* for Case 1.3-3:

$$T^*_{1.3-3} = \sqrt{\frac{2A+Ic_2PD\left(M\left(1+Ic_1(N-M)\right)\left(1+\frac{Ic_1(N-M)}{2}\right)+(N-M)\left(1+\frac{IeM}{2}\right)\right)^2-2Ic_1(N-M)\left(PDM\left(1+\frac{IeM}{2}\right)+\frac{PD(N-M)}{2}\right)-IePDM^2}{D\left(h+Ic_2\frac{C^2}{P}\left(1+Ic_1(N-M)\right)^2\right)}} \quad (16)$$

After some algebraic manipulations, the buyer's total cost function for the interest structure $Ie < Ic_1 < Ic_2$ (Case 1) may be summarized as follows:

$$TC_1 = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{2PDT}\left[\left(max\left\{CDT - PDM\left(1+\frac{IeM}{2}\right), 0\right\}\right)^2 - \left(max\left\{CDT - PDM\left(1+\frac{IeM}{2}\right) - PD(N-M), 0\right\}\right)^2\right] + \frac{Ic_2}{2PDT}\left[\left(max\left\{\left(CDT - PDM\left(1+\frac{IeM}{2}\right)\right)\left(1+Ic_1(N-M)\right) - PD(N-M)\left(1+\frac{Ic_1(N-M)}{2}\right), 0\right\}\right)^2\right] - \frac{IePD}{2T}\left[M^2 - (max\{M-T, 0\})^2\right] \quad (17)$$

Case 2: $Ic_1 < Ie \le Ic_2$

Case 2.1: $T \le M$

For *Ie* > Ic_1 and $T \le M$, the buyer may benefit from keeping the sales revenue in an interest bearing account until time *N* and from settling the account after this point in time. Between times *M* and *N*, he/she has to pay interest to the supplier. However, since *Ie* > Ic_1, the interest earned exceeds the interest paid during this period. Similar to Subcase 1.1 (cf. Eq. (5)), the interest earned per unit of time can be calculated as:

$$IE_{2.1} = \frac{IeP}{T}\left[\int_0^T Dt\,dt + Q(N-T)\right] = IePD\left(N - \frac{T}{2}\right) \quad (18)$$

The overall interest cost between times *M* and *N* amounts to:

$$IC_{2.1} = \frac{Ic_1}{T}CQ(N-M) \quad (19)$$

The total costs are thus calculated as:

$$TC_{2.1} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N - M) - IePD\left(N - \frac{T}{2}\right) \quad (20)$$

Since the second-order condition of Eq. (21) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 2.1:

$$T^*_{2.1} = \sqrt{\frac{2A}{D(h+IeP)}} \quad (21)$$

Case 2.2: $M < T \leq N$

The case where $M < T < N$ and $Ic_1 < Ie$ is identical to Case 2.1. In this case, the buyer accepts interest charges between times M and N caused by the postponement of the payment and realizes interest earnings by depositing sales revenues in an interest bearing account. The account is completely settled at time N.

Case 2.3: $N < T$

In the case where $Ic_1 < Ie$ and $N < T$, as much of the unpaid balance is settled at time N as possible to minimize interest payments. In the period $[0,N]$, the buyer sells a total quantity of DN units and generates direct sales revenues in the amount of PDN dollars. Sales revenues that accumulate over time are deposited in an interest bearing account that earns interest at the rate of Ie per unit of time, which leads to additional earnings of $IePDN^2/2$ (cf. Eq. (23)). Thus, at time N, the buyer uses the sum of sales revenues and interest earnings to settle the open account. The total purchase cost for a lot of size DT again amounts to CDT dollars. According to the ratio of the total purchase cost and the accruing interest to the total sales revenues and interest earnings at time N, two possible subcases may arise that will be discussed in the following:

Case 2.3-1: $CDT\big(1 + Ic_1(N - M)\big) \leq PDN(1 + IeN/2)$

If the interest rate of the buyer, Ie, exceeds the interest charges of the supplier for the first credit period, Ic_1, he/she will again not settle the account before time N. Instead, the buyer keeps the sales revenues earned in period $[M,N]$ in an interest bearing account. As the account is completely settled at time N, the interest earned per unit of time is given as:

$$IE_{2.3-1} = \frac{IeP}{T} \int_0^N Dt\, dt = \frac{IePDN^2}{2T} \quad (22)$$

The interest charges in this case are the same as those derived for Case 2.1 (cf. Eq. (20)). Thus, the total costs can be formulated as:

$$TC_{2.3-1} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N - M) - \frac{IePDN^2}{2T} \quad (23)$$

Since the second-order condition of Eq. (24) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 2.3-1:

$$T^*_{2.3-1} = \sqrt{\frac{2A - IePDN^2}{hD}} \quad (24)$$

Case 2.3-2: $CDT\big(1 + Ic_1(N - M)\big) > PDN(1 + IeN/2)$

For the case where $Ie > Ic_1$ and where the buyer is unable to pay off the total purchase cost at time N, the account is partially settled at time N. Afterwards, the unpaid balance is continuously reduced by transferring each dollar earned to the supplier's account until the balance has been completely settled. The interest earned in the period $[0,N]$ is given as $IePDN^2/2$ (cf. Eq. (23)), which again considers the interest advantage that results from accruing interest earnings due to the delayed payment. The supplier, however, charges interest on the gradually decreasing unpaid balance at the rate Ic_2 after time N which is given as:

$$IC_{2.3-2} = \frac{Ic_1}{T} CQ(N-M) + \frac{Ic_2}{T} \int_N^{N+z_3} \left(\left(CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2) \right) - PD(t-N) \right) dt = \frac{Ic_1}{T} CQ(N-M) + \frac{Ic_2}{2TPD} \left[CQ\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2) \right]^2 \quad (25)$$

where $N+z_3$ denotes the point in time when the unpaid balance has been completely settled, with $z_3 = \left(CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2) \right) / PD$. In case of $Ic_1 < Ib < Ic_2$, the retailer may benefit from bridgeover finance by bank loans that reduce the effective interest rate (note that in this case in Eq.(25) Ic_2 needs to be replaced by Ib; everything else would remain unchanged). The total costs for this case amount to:

$$TC_{2.3-2} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N-M) + \frac{Ic_2}{2PDT} \left(CDT\big(1 + Ic_1(N-M)\big) - PDN(1 + IeN/2) \right)^2 - \frac{IePDN^2}{2T} \quad (26)$$

Since the second-order condition of Eq. (27) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 2.3-1:

$$T^*_{2.3-2} = \sqrt{\frac{2A + PDN^2\left(Ic_2(1+IeN/2)^2 - Ie\right)}{D/_P\left(hP + Ic_2 C^2\big(1 + Ic_1(N-M)\big)^2\right)}} \quad (27)$$

After some algebraic manipulations, the buyer's total cost function for the interest structure $Ic_1 < Ie < Ic_2$ (Case 2) can be summarized as follows:

$$TC_2 = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N-M) + \frac{Ic_2}{2PDT} \left[\left(max\left\{ \left(CDT\big(1 + Ic_1(N-M)\big) - (PDN + IePDN^2/2) \right), 0 \right\} \right)^2 \right] - \frac{IePD}{2T} \left[N^2 - (max\{N-T, 0\})^2 \right] \quad (28)$$

Case 3: $Ic_1 < Ic_2 < Ie$

Case 3.1: $T \leq M$

Since $Ie > Ic_2$, the retailer again has an incentive to postpone the payment or even to never pay back the trade credit to the supplier. However, as the supplier will not provide infinite advance financing and would not be willing to release further deliveries in the event of a delayed or default credit, it is reasonable to assume that the buyer has to settle the open account at the end of the last credit period. Thus, for $Ie > Ic_2$ and $T \leq M$, the buyer may again benefit from keeping the sales revenue in an interest bearing account until time N. Between times M and N, he/she has to pay interest to the supplier that will, however, be offset by the interest earnings in this period. As for Subcase 2.1 (cf. Eq. (20)), the total costs are given as:

$$TC_{3.1} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N - M) - IePD\left(N - \frac{T}{2}\right) \tag{29}$$

Since the second-order condition of Eq. (29) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 3.1:

$$T_{3.1}^{*} = \sqrt{\frac{2A}{D(h+IeP)}} \tag{30}$$

Case 3.2: $M < T \leq N$

The case where $M < T < N$ and $Ic_2 < Ie$ is identical to Case 3.1. Again, the buyer accepts interest charges between times M and N that result from postponing the payment to the supplier, and he/she thus realizes interest earnings by depositing sales revenues in an interest bearing account. The account is completely settled at time N.

Case 3.3: $N < T$

In the case where $Ic_2 < Ie$ and $N < T$, the buyer may benefit from postponing the payment as long as possible and could also benefit from never settling the open account. We may, however, assume that the supplier would not be willing to offer a credit with infinite duration. As the trade credit serves the purpose to influence the buyer's ordering behavior on a per-order bases, we may assume that the buyer has to settle the open account at time T, right before the next order is issued (and right before the next trade credit is granted). In the period $[0,T]$, the buyer sells off the entire lot of DT units and generates direct sales revenues in the amount of PDT dollars. In addition, sales revenues that accumulate over time are deposited in an interest bearing account that earns interest at the rate of Ie per unit of time, which leads to additional earnings of:

$$IE_{3.3} = \frac{IeP}{T}\int_{0}^{T} Dt\, dt = \frac{IePDT}{2} \tag{31}$$

At time T, the buyer uses sales revenues and interest earnings to settle the total purchase cost of CDT dollars as well as the accruing interest charges between times M and T. The interest cost amounts to:

$$IC_{3.3} = \frac{Ic_1}{T} CQ(N - M) + \frac{Ic_2}{T} CQ(T - N)\big(1 + Ic_1(N - M)\big) \tag{32}$$

Consequently, the total costs can be formulated as:

$$TC_{3.3} = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T} CQ(N - M) + \frac{Ic_2}{T} CQ(T - N)\big(1 + Ic_1(N - M)\big) - \frac{IePDT}{2} \tag{33}$$

Since the second-order condition of Eq. (33) is strictly positive (cf. Section 4), the solution can be derived using the first-order condition, which leads to the optimal value of T for Case 3.3:

$$T_{3.3}^{*} = \sqrt{\frac{2A}{D\left(h+2Ic_2C\left(1+Ic_1(N-M)\right)-IeP\right)}} \tag{34}$$

After some algebraic manipulations, the buyer's total cost function for the interest structure $Ic_1 < Ic_2 < Ie$ (Case 3) can be summarized as follows:

$$TC_3 = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{T}CDT(N-M) + \frac{Ic_2}{T}CDT\left(1+Ic_1(N-M)\right)max\{T-N,0\} - IePD\left(\frac{T}{2} + max\{N-T,0\}\right) \tag{35}$$

A summery table with closed form solutions as well as the corresponding optimality conditions for all relevant cases can be found in the online supplement.

Solution approach

For convenience, we assume that all $TC_i(T)$ with i representing the respective case ($i = \{1.1, 1.2-1, \ldots\}$) are defined on $T > 0$. To find the optimal solution for the problem presented above, all $TC_i(T)$ are minimized separately and compared with regard to their range of validity to obtain an optimal value of T.

In Case 1.1, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{1.1}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{\mathrm{Ie}PD}{2} = 0 \tag{36}$$

Theorem 1. Let $T = T_{1.1}^{*}$ be the solution of (36).

(a) Eq. (36) has a unique solution.
(b) If $T_{1.1} \leq M$, then $T_{1.1}^{*}$ is the global minimum of $TC_{1.1}$.

Proof: See Appendix A. □

In Case 1.2-1, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{1.2-1}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePDM^2}{2T^2} = 0 \tag{37}$$

Theorem 2. Let $T = T_{1.2-1}^{*}$ be the solution of (37).

(a) Eq. (37) has a unique solution.
(b) If $M < T_{1.2-1} \leq N$ and $CDT_{1.2-1} \leq PDM(1 + IeM/2)$, then $T_{1.2-1}^{*}$ is the global minimum of $TC_{1.2-1}$.

Proof: See Appendix B. □

In Case 1.2-2, the first-order condition for a minimum is:

$$\frac{dTC_{1.2-2}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{Ic_1DC^2}{2P} - \frac{PDM^2}{2T^2}\left(Ic_1(1 + IeM/2)^2 - Ie\right) = 0 \tag{38}$$

Theorem 3. Let $T = T_{1.2-2}^{*}$ be the solution of (38).

(a) Eq. (38) has a unique solution.
(b) If $M < T_{1.2-2} \leq N$ and $CDT_{1.2-2} > PDM(1 + IeM/2)$, then $T^*_{1.2-2}$ is the global minimum of $TC_{1.2-2}$.

Proof: See Appendix C. □

The total cost function in Case 1.3-1 is identical to Subcase 1.2-1. Accordingly, the theoretical analysis for this case is consequently the same than the one presented for Theorem 2.

Corollary 1. If $N < T_{1.3-1}$ and $CDT_{1.3-1} \leq PDM(1 + IeM/2)$, then $T^*_{1.3-1}$ is the global minimum of $TC_{1.3-1}$.

Proof:
If $T = T^*_{1.3-1}$ is the solution to $dTC_{1.3-1}/dT = 0$, the second-order derivative of $TC_{1.3-1}$ at this point is:

$$\left.\frac{d^2TC_{1.3-1}(T)}{dT^2}\right|_{T^*_{1.3-1}} = \sqrt{(hD)^3/2A - IePDM^2} > 0.$$

Hence, $T^*_{1.3-1}$ is the global minimum of $TC_{1.3-1}$. Additionally, by substituting ${T_{1.3-1}}^*$ into $N < T$ and $CDT \leq PDM(1 + IeM/2)$, we know that if and only if $IePDM^2 + hDN^2 < 2A \leq DM^2(IeP + h(P/C)^2(1 + IeM/2)^2)$, then $N < {T_{1.3-1}}^*$ and $C D {T_{1.3-1}}^* \leq PDM(1 + IeM/2)$. □

The total cost function in Case 1.3-2 is identical to Subcase 1.2-2. Accordingly, the theoretical analysis for this case is consequently the same than the one presented for Theorem 3.

Corollary 2. If $N < T_{1.3-2}$, $CDT_{1.3-2} > PDM(1 + IeM/2)$ and $\big(CDT_{1.3-2} - PDM(1 + IeM/2)\big)\big(1 + Ic_1(N - M)\big) \leq PD/2\,(2(N - M) - Ic_1(N - M)^2)$, then $T^*_{1.3-2}$ is the global minimum of $TC_{1.3-2}$.

Proof:
If $T = T^*_{1.3-2}$ is the solution to $dTC_{1.3-2}/dT = 0$, the second-order derivative of $TC_{1.3-2}$ at this point is:

$$\left.\frac{d^2TC_{1.3-2}(T)}{dT^2}\right|_{T^*_{1.3-2}} = \sqrt{\big(D(h + Ic_1C^2/P)\big)^3/\big(2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)\big)} > 0.$$

Hence, $T^*_{1.3-2}$ is the global minimum of $\mathrm{TC}_{1.3-2}$. Additionally, by substituting $T^*_{1.3-2}$ into $N < T$ and $CDT \leq PDM(1 + IeM/2)$, we know that if and only if $(h + Ic_1C^2/P)D{\Delta_2}^2 - PDM^2(Ic_1(1 + IeM/2)^2 - Ie) < 2A \leq (h + Ic_1C^2/P)D{\Delta_3}^2 - PDM^2(Ic_1(1 + IeM/2)^2 - Ie)$ with $\Delta_2 = max\{N, M(P/C)(1 + IeM/2)\}$ and $\Delta_3 = \big(P(2(N - M) - Ic_1(N - M)^2)/2C\big(1 + Ic_1(N - M)\big) + MP(1 + IeM/2)/C\big)$, then $N < T^*_{1.3-2}$, $CDT^*_{1.3-2} \leq PDM(1 + IeM/2)$ and $\big(CDT^*_{1.3-2} - PDM(1 + IeM/2)\big)\big(1 + Ic_1(N - M)\big) \leq PD/2\,(2(N - M) - Ic_1(N - M)^2)$. □

In Case 1.3-3, the first-order condition for a minimum is:

$$\frac{dTC_{1.3-3}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{Ic_1(N-M)}{T^2}\left(PDM\left(1+\frac{IeM}{2}\right) + \frac{PD(N-M)}{2}\right) + \frac{Ic_2DC^2}{2P}\left(1+Ic_1(N-M)\right)^2 - \frac{Ic_2PD}{2T^2}\left(M\left(1+Ic_1(N-M)\right)\left(1+\frac{Ic_1(N-M)}{2}\right) + (N-M)\left(1+\frac{IeM}{2}\right)\right)^2 + \frac{IePDM^2}{2T^2} = 0 \quad (39)$$

Theorem 4. Let $T = T^*_{1.3-3}$ be the solution of (39).

(a) Eq. (39) has a unique solution.
(b) If $N < T_{1.3-3}$, $CDT_{1.3-3} > PDM(1+IeM/2)$ and $\left(CDT_{1.3-3} - PDM(1+IeM/2)\right)\left(1+Ic_1(N-M)\right) > PD(N-M)(1+Ic_1(N-M)/2)$, then $T^*_{1.3-3}$ is the global minimum of $TC_{1.3-3}$.

Proof: See Appendix D. □

In Case 2.1, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{2.1}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePD}{2} = 0 \quad (40)$$

Theorem 5. Let $T = T^*_{2.1}$ be the solution of (40).

(a) Eq. (40) has a unique solution.
(b) If $T_{2.1} \le M$, then $T^*_{2.1}$ is the global minimum of $TC_{2.1}$.

Proof: The proof of this theorem is similar to the proof of Theorem 1 since Eq. (36) and Eq. (40) are identical. □

The total cost function in Case 2.2 is identical to Case 2.1. The theoretical analysis for this case is consequently the same as the one presented for Theorem 1.

Corollary 3. If $M < T_{2.2} \le N$, then $T^*_{2.2}$ is the global minimum of $TC_{2.2}$.

Proof:
If $T = T^*_{2.2}$ is the solution to $dTC_{2.2}/dT = 0$, the second-order derivative of $TC_{2.2}$ at this point is:

$$\left.\frac{d^2TC_{2.2}(T)}{dT^2}\right|_{T^*_{2.2}} = \sqrt{(D(h+PIe)^3/2A} > 0.$$

Hence, $T^*_{2.2}$ is the global minimum of $TC_{2.2}$. Additionally, by substituting $T^*_{2.2}$ into $M < T \le N$, we know that if and only if $(h+IeP)DM^2 < 2A \le (h+IeP)DN^2$, then $M < T^*_{2.2} \le N$. □

In Case 2.3-1, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{2.3-1}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePDN^2}{2T^2} = 0 \quad (41)$$

Theorem 6. Let $T = T^*_{2.3-1}$ be the solution of (41).

(a) Eq. (41) has a unique solution.

(b) If $N < T_{2.3-1}$ and $CDT_{2.3-1}\big(1 + Ic_1(N - M)\big) \le PDN(1 + IeN/2)$, then $T^*_{2.3-1}$ is the global minimum of $TC_{2.3-1}$.

Proof: See Appendix E. □

In Case 2.3-2, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{2.3-2}}{dT} = -\frac{A}{T^2} - \frac{PDN^2}{2T^2}\big(Ic_2(1 + IeN/2)^2 - Ie\big) + \frac{D}{2}\Big(h + \frac{C^2}{P}Ic_2\big(1 + Ic_1(N - M)\big)^2\Big) = 0 \quad (42)$$

Theorem 7. Let $T = T^*_{2.3-2}$ be the solution of (42).

(a) Eq. (42) has a unique solution.
(b) If $N < T_{2.3-2}$ and $CDT_{2.3-2}\big(1 + Ic_1(N - M)\big) > PDN(1 + IeN/2)$, then $T^*_{2.3-2}$ is the global minimum of $TC_{2.3-2}$.

Proof: See Appendix F. □

In Case 3.1, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{3.1}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePD}{2} = 0 \quad (43)$$

Theorem 8. Let $T = T^*_{3.1}$ be the solution of (43).

(a) Eq. (43) has a unique solution.
(b) If $T_{3.1} \le M$, then $T^*_{3.1}$ is the global minimum of $TC_{3.1}$.

Proof: The proof of this theorem is similar to the proof of Theorem 1 since Eq. (36) and Eq. (43) are identical. □

The total cost function in Case 3.2 is identical to Case 3.1. The theoretical analysis for this case is consequently the same as the one presented for Theorem 1.

Corollary 4. If $M < T_{3.2} \le N$, then $T^*_{3.2}$ is the global minimum of $TC_{3.2}$.

Proof:
If $T = T^*_{3.2}$ is the solution to $dTC_{3.2}/dT = 0$, the second-order derivative of $TC_{3.2}$ at this point is:

$$\left.\frac{d^2TC_{3.2}(T)}{dT^2}\right|_{T^*_{3.2}} = \sqrt{(D(h + PIe)^3/2A} > 0.$$

Hence, $T^*_{3.2}$ is the global minimum of $TC_{3.2}$. Additionally, by substituting $T^*_{3.2}$ into $M < T \le N$, we know that if and only if $(h + IeP)DM^2 < 2A \le (h + IeP)DN^2$, then $M < T^*_{3.2} \le N$. □

In Case 3.3, the first-order condition for a minimum is:

$$\frac{d\mathrm{TC}_{3.3}}{dT} = -\frac{A}{T^2} + \frac{hD}{2} + Ic_1Ic_2CD(N - M) - \frac{IePD}{2} = 0 \quad (44)$$

Theorem 9. Let $T = T^*_{3.3}$ be the solution of (44).

(a) Eq. (44) has a unique solution.
(b) If $N < T_{3.3}$, then $T^*_{3.3}$ is the global minimum of $TC_{3.3}$.

Proof: See Appendix G. □

Numerical studies

To illustrate some properties of the models developed in Section 3, we first perform a benchmark case study (Section 5.1) and then report the results of an extensive numerical experiment (Section 5.2).

5.1 Benchmark case study

In the following, the proposed approach has been applied to a case study considering the ordering and payment behavior of large diversified retailers (cf. Section 2) in order to exemplify the applicability and implications of the models developed above.

Although the variation of trade credit terms within a certain industry is often low as compared to variation across industries (cf. Ng et al., 1999), the degree of within-variation may differ significantly. In some industries, companies even seem to vary credit terms from customer to customer (cf. Wilson and Summers, 2002), which often leads to a myriad of purchasing contracts with distinct trade credit conditions for the buying company (Seifert and Seifert, 2011). To take account of this heterogeneity, all trade credit parameters were derived based on extensive firm-level data containing information of about 17,000 transactions between 26 large retail companies and their suppliers, which allows estimating the range of existing credit conditions (note that this information on credit conditions is part of the descriptive analysis performed by Klapper et al., 2012, pp. 842-851). Trade credit contracts in this sample generally have a very long duration compared to other industries. About 75% of the considered transactions have net days longer than 30 days. However, variation in net days differs significantly between the different groups. Diversified retailers, for example, tend to be offered much longer net payment durations than grocery or soft good retailers. About 20% of the contracts also offer early payment discounts, which seems to be more common in hard good retailing and grocery. In contrast to net days, discount periods are rather short and contain less than 30 days in about 75% of the transactions. Of contracts with discounts, 32% have a discount equal to 1% or less, 61% have a discount between 1% and 2%, and the remaining 7% have a discount greater than 2%. The effective interest rate of considered credits, defined as the implied interest rate if the buyer does not utilize the discount and pays on the due date, calculated as $(1/(1 - discount\ rate))^{360/(net\ days - discount\ days)} - 1$, varies from a low of 2% to a high of 100% (cf. Klapper et al., 2012; note that due to high interest rates arising from a low spread between discount and net days, effective interest rate was truncated at 100%). According to these empirical findings, we exemplarily assumed a progressive interest scheme with the following conditions: If the retailer settles its balance before time M = 30/365, the supplier charges no interest to the retailer, whereas in case when the retailer settles its balance after time M, but before time N = 80/365, the supplier charges an interest rate Ic_1 = 5% on the outstanding balance. In the case when the retailer pays after time N, the supplier charges an interest rate Ic_2 = 12%.

Revenues the retailer receives from sales may be deposited in an interest bearing account until the account is settled entirely. Again, comparing the ten highest-grossing retailers' average RoE within the last ten years, values from a low of 4.34% up to a high of 22.64% were observed (cf. Table 3). Accordingly, interest earned was assumed at the rate of Ie = 6% (note that to ensure comparability, Examples 1 and 2 require a different interest structure with $Ie < Ic_1$; therefore it is assumed that Ie = 5% and Ic_1 = 6% in this case).

Depending on the specific segment, average gross margins in the retail sector vary from a low of 6% to a high of 42% (cf. Gaur et al., 2005). Considering the presented sample of the ten largest diversified retailers (cf. Section 2), average gross margins within the last ten years range from a low of 6.41% to a high of 38.67% (cf. Table 3). Thus, we assume an average gross margin of 25% based on a unit purchase cost of C = 15, which leads to a sales price per unit of P = 20.

Table 3: Ten year averages for RoE and gross margins of the retailers

	Company	10y-AV		Company	10AV
RoE	**Wal-Mart**	20.93%	Gross margin	**Wal-Mart**	26.71%
	Costco	13.50%		**Costco**	13.45%
	The Kroger	22.64%		**The Kroger**	21.16%
	Tesco	4.34%		**Tesco**	6.41%
	Carrefour	13.05%		**Carrefour**	22.34%
	Metro	7.80%		**Metro**	24.39%
	Target	14.17%		**Target**	29.39%
	Aeon	4.47%		**Aeon**	38.67%
	Group Casino	8.25%		**Group Casino**	19.66%
	Seven & I	6.31%		**Seven & I**	38.15%

Beside sectoral differences, holding costs strongly depend on the warehouse system used. Nevertheless, recent studies revealed that the average unitary holding cost rate of an item in a manually operated warehouse is around 25% of the inventory value (Azzi et al., 2014). Considering the average unit cost of the items of C = 15, the annual holding cost per item becomes h = 3.75, whereas ordering cost related to fixed freight fees as well as internal documentation and administration expenses for containerized overseas supply amounts to A = 200 (cf. Arikan et al., 2014). Standardized annual demand was assumed as D = 1000. To illustrate some properties of the models developed in Section 3 regarding compound interest, continuous settlement and different interest relations, we performed numerical experiments, whose results are reported in the following.

Example 1. This example illustrates that considering compound interest that accrues at the retailer influences the ordering decision of the retailer and its total cost. Compound interest arises when the buyer is not able to settle the open account at times M or N. In this case, the interest cost accruing until time N increases the principal of the loan and thus induces further interest cost in the next credit period. Considering the compound interest as described in Section 3, we obtain the following computational results for the cycle time and the total costs:

$T^* = 0.307467$ and $TC(T^*) = 1295.71$

In contrast, if we assume that no compound interest accrues at the retailer, then we obtain the following values for the cycle time and the total costs:

$T^* = 0.307025$ and $TC(T^*) = 1295.82$

As can be inferred from the example, considering compound interest leads to slightly reduced replenishment intervals and smaller order sizes. The total costs, however, remain nearly at the same level regardless of whether the effective interest cost is considered or not. Thus, the approximation proposed in previous approaches achieves good results for the given example. However, the effect of compound interest is stronger when the interest rate Ic_1 and/or the distance between the two settling times increase.

Example 2. This example reveals some shortcomings of previous approaches in developing efficient payment structures for the case that the cycle time exceeds both credit periods, i.e. for the case where $N < T$. In case the interest costs for the first credit period exceed interest earnings, the open account should be reduced gradually during the period [M,N], instead of making partial payments during this period. In previous inventory models with progressive payment schemes, it was merely assumed that in case the buyer is not able to settle the account entirely at times M or N, s/he partially settles the account in M and N and afterwards gradually reduces the remaining balance. This assumption, however, leads to unnecessary interest expenditures. Consequently, the strategy of settling the open account continuously after time M leads to substantial savings. Likewise, we obtain the following computational results for the cycle time and the total costs:

$T^* = 0.299812$ and $TC(T^*) = 1258.69$

If we assume that between times M and N no payment is made, we again obtain the following values for the cycle time and the total costs:

$T^* = 0.307025$ and $TC(T^*) = 1295.82$

Again, it can be seen that settling the account earlier reduces the length of the replenishment intervals. These findings are illustrated in Figure 2, where the left part shows the buyer's account for the traditional payment policy and the right part shows the buyer's account for the payment policy proposed in this paper. It is obvious that in case the interest rate for the first credit period exceeds interest earnings (e.g. $Ic_1 > Ie$), the open account should be reduced continuously between times M and N instead of making partial payments at times M and N, which induces lower effective interest cost.

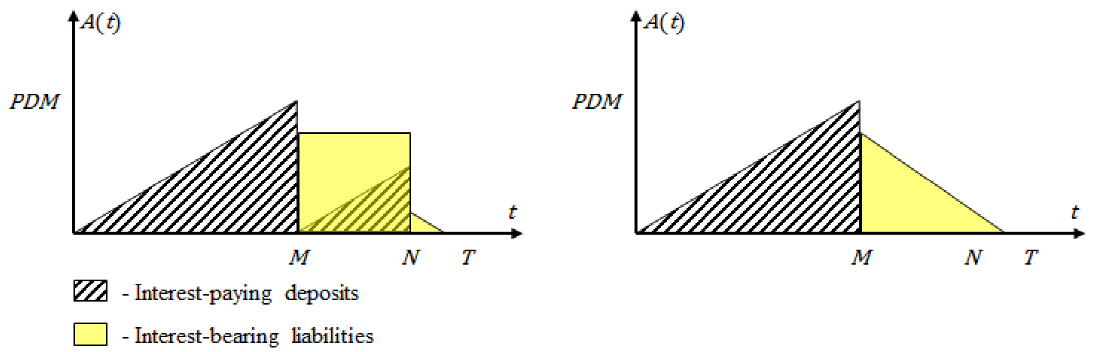

Figure 2: Balance of accounts for the different payment policies

In fact, comparing the total cost expressions with (see Eq. (14)) and without (see Eq. (13) in Goyal et al. (2007) and note that for comparison reasons, the compound interest has to be added in the model) continuous redemption of the debt between times M and N, it can be shown that the proposed strategy leads to lower total costs for all parameter settings (cf. Proposition 1).

Proposition 1. If $Ie < Ic_1$ and $CDT > PDM(1 + IeM/2)$, then continuously reducing the open account between times *M* and *N* induces lower total cost than partial payments in *M* and N.

Proof:
Subtracting the total cost that consider continuous redemption of the debt between M and N from the total cost with partial payments in M and N only, we get: $\Delta TC_{1.3-3} = (Ic_1 - Ie)\left(\frac{PD(N-M)^2}{2T} + \frac{Ic_2(N-M)^2}{2T}\left(\left(1 + Ic_1(N-M)\right)\left(CDT - PDM\left(1 + \frac{IeM}{2}\right)\right) + (N-M)\left(1 + \frac{(Ic_1 - Ie)(N-M)}{4}\right)\right)\right) > 0 \; \forall \, T > 0$. Hence, earlier payments are always profitable in this case. □

Example 3. The final example demonstrates that the interest structure has only a minor influence on the replenishment interval and the size of the order. However, it is shown that the interest structure strongly influences the optimal time for settling the account, which may lead to substantial savings if the appropriate payment policy is applied. Assuming that the interest rate on deposits at the buyer exceeds the interest rate charged by the supplier between times *M* and *N*, the buyer may benefit from postponing the settlement of the account. By comparing the results on the appropriate order quantities obtained from the cost functions in Cases 1 and 2 (cf. Section 3), it can be shown that the interest structure has only a minor influence on the order quantity decision. Thus, in both cases described in this paper ($Ie \leq Ic_1 < Ic_2$ and $Ic_1 < Ie < Ic_2$), the order quantity is almost identical. On the other hand, comparing the total relevant costs for the different cases, it can be shown that the interest structure has a strong influence on the payment decision, which leads to substantial savings of more than 20% in case the appropriate payment policy is used and the settlement is postponed to time *N* (cf. Table 4).

Table 4: Order quantities and total cost for different order cost levels

	Case 1: With early settlement			Case 2: With late settlement			Comparative statics	
A	T_1	Q_1	TC_1	T_2	Q_2	TC_2	$(Q_1\text{-}Q_2)/Q_1$	$(TC_1\text{-}TC_2)/TC_1$
15	0.07785	77.850	286.73	0.07785	77.850	225.08	0.00000	0.21499
30	0.11665	116.651	441.26	0.11010	110.096	384.70	0.05619	0.12818
50	0.15127	151.271	590.56	0.14213	142.134	543.29	0.06040	0.08004
100	0.21464	214.642	863.85	0.20101	201.008	834.71	0.06352	0.03373
150	0.26317	263.172	1073.13	0.25422	254.219	1056.06	0.03402	0.01591
200	0.30210	302.103	1249.61	0.29951	299.511	1235.97	0.00858	0.01092
250	0.33287	332.871	1407.10	0.33052	330.520	1394.69	0.00706	0.00882
400	0.41160	411.599	1810.07	0.40970	409.699	1799.97	0.00462	0.00558
500	0.45660	456.604	2040.43	0.45489	454.892	2031.30	0.00375	0.00447
600	0.49755	497.554	2250.04	0.49598	495.984	2241.63	0.00316	0.00374

5.2 Numerical experiment

To gain further insights into the optimal ordering and payment behavior, this section analyzes 10,000 randomly generated problem instances in a numerical experiment (note that these instances are nearly equally distributed among the two different interest structures). Parameters were taken randomly from the ranges obtained by the real system as described in the previous section assuming an equal distribution (cf. Table 5).

An evaluation of the results shows that the presented approach induces an average reduction in total cost of 7.6%. In contrast to previous models (cf., for example, Goyal et al. (2007)), the inventory replenishment cycle decreases slightly by 3.4% on average, whereas the payment interval increases significantly by 26.2% on average. Thus, carefully considering the prevailing interest structure that governs the optimal payment policy may lead to significant cost reductions with only minor changes in the inventory policy. Additionally, taking into account arbitrage gains arising from different interest structures, the cash conversion cycles tend to decrease as DIOs decrease, whereas DPOs increase. Subsequently, we analyzed the influence of the problem parameters on the optimal ordering and payment behavior as well as the effective total cost of the proposed approach. For this purpose, we conducted several multivariate regression analyses between the problem parameters and different performance measures whose results are given in Tables 6 and 7.

Table 5: Parameter ranges for simulation data sets

D	$\in$	[500,1500]	demand in units per year
A	$\in$	[15,600]	ordering cost per order
C	$\in$	[10,40]	unit purchase cost
P	$\in$	[max{C,30},50]	unit selling price
h	$\in$	[2,8]	inventory holding cost per unit and year
Ic_1	$\in$	[0.005,0.08]	interest rate per year for the first credit period
Ic_2	$\in$	[0.08,0.16]	interest rate per year for the second credit period
Ie	$\in$	[0.005,0.08]	interest rate on deposits for the retailer
M	$\in$	[10;60]	first permissible credit period
N	$\in$	[max{M,40},90]	second permissible credit period

Table 6: Results of the regression analysis for the ratio of payment intervals (left part of Table 5; R^2 = 0.388) and the ratio total costs (right part of Table 5; R^2 = 0.166)

Model parameter	Standardised Beta	t	Sig.
D	-0.113	-14.466	0.000
A	0.317	40.495	0.000
C	0.293	36.581	0.000
P	-0.124	-15.486	0.000
h	-0.118	-15.095	0.000

Model parameter	Standardised Beta	t	Sig.
D	0.048	4.823	0.000
A	-0.101	-10.250	0.000
C	-0.025	-2.428	0.015
P	0.017	1.720	0.086
h	-0.024	-2.413	0.016

Ic_1	0.214	27.304	0.000	Ic_1	-0.050	-5.054	0.000
Ic_2	0.002	0.239	0.811	Ic_2	0.001	0.136	0.892
Ie	-0.263	-33.613	0.000	Ie	0.084	8.504	0.000
M	0.108	13.493	0.000	M	-0.031	-3.103	0.002
N	-0.220	-27.650	0.000	N	0.059	5.902	0.000

Table 7: Results of the regression analysis for the cash conversion cycle (left part of Table 6; $R^2 = 0.740$) and the total cost (right part of Table 6; $R^2 = 0.933$)

Model parameter	Standardised Beta	t	Sig.	Model parameter	Standardised Beta	t	Sig.
D	-0.233	-45.560	0.000	D	0.290	112.176	0.000
A	0.541	105.966	0.000	A	0.831	321.514	0.000
C	-0.560	-107.192	0.000	C	0.083	31.296	0.000
P	0.195	37.395	0.000	P	-0.034	-12.878	0.000
h	-0.241	-47.165	0.000	h	0.361	139.749	0.000
Ic_1	0.077	15.017	0.000	Ic_1	0.049	19.020	0.000
Ic_2	-0.006	-1.187	0.235	Ic_2	0.010	3.708	0.000
Ie	-0.131	-25.611	0.000	Ie	-0.064	-24.861	0.000
M	-0.017	-3.311	0.001	M	-0.053	-20.007	0.000
N	-0.099	-19.099	0.000	N	-0.038	-14.547	0.000

At first, we compared the payment intervals and the consequent total cost to those obtained by previous approaches (see, for example, Goyal et al. (2007)). The results of the regression analysis with the problem parameters as independent variables and the ratio of the payment intervals as well as the ratio of expected total costs as the dependent variables are shown in Table 6. As can be seen, a statistically relevant relationship (with Sig. < 0.05) could be found between all problem parameters and the ratio of the payment intervals, with the exception of Ic_2 (note that this can be explained by the assumption that Ic_2 always exceeds Ic_1 and Ie). An increase in the time span between M and N as well as an increase in Ie or a decrease in Ic_1 leads to a postponement of the point in time when the balance is settled entirely, which is a reaction of the buyer to realize arbitrage gains. The higher the arbitrage gains the buyer can realize, the more our model outperforms the previous models, which explains the results mentioned above. Accordingly, the ratio of the total costs is influenced by the model parameters Ic_1, Ie, M and N in the exact opposite way than the ratio of the payment intervals: The longer the time span between M and N (i.e., the longer the time span the buyer could realize arbitrage gains in), the more beneficial it is to use the model developed in this paper. The same effect can be observed for high values of Ie and low values of Ic_1.

Finally, to assess the effects on the optimal working capital structure and the consequent total cost, we conducted further regression analyses with the problem parameters as independent variables and the cash conversion cycles as well as the total cost as the dependent variables. The results are presented in Table 7. Regarding the cash conversion cycle, again a statistically relevant relationship (with Sig. < 0.05) could be found between for problem parameters with the exception of Ic_2. It can be seen that the CCC is reduced as Ic_1 decreases or as Ie, M and N increase. This can again be explained by the potential arbitrage gains of the buyer. When Ic_1 or

Ie increase, the DIOs adopt lower values whereas the DPOs adopt higher values. Regarding the permissible delays in payment, an increase in M or N reduces the DIOs as well as the DPOs. However, as the effect is much stronger for the DPOs, the CCC tends to decrease. Obviously, lower values for Ic_1 and higher values for Ie, M and N lead to higher arbitrage gains the buyer can realize by postponing the payment to his/her supplier. Thus, it can also be inferred that the differences in the working capital structure of large retailers may also be influenced by their distinct cash investment opportunities. Provided that some of the retailers are able to realize comparably higher interest rates than others, these retailers aim at earning profits from capital investments by postponing the payments to their suppliers, whereas the other retailers tend to pay their suppliers earlier to avoid interest cost. In addition, an increase in the interest rates Ic_1 or Ic_2 leads to an increase in the expected total cost of our model. This effect may, however, be moderated to some degree by the borrowing rate as in case of $Ib < Ic_i$, the retailer benefits from bridgeover finance by bank loans that reduce the effective interest rate. Higher values for *Ie* and longer payment intervals M and N, in turn, reduce the accruing total costs.

Conclusion

The purpose of this paper was to generalize the trade credit inventory model with progressive interest scheme by considering the case where a) the credit interest rate of the buyer may exceed the interest rate charged by the supplier, b) the buyer has the option to settle the outstanding balance continuously during the credit periods, c) compound interest accrues at the retailer, and d) bank loans are available as a substitute for the trade credit. This paper provided the necessary and sufficient conditions for the optimal solution and derived explicit closed-form solutions for the optimal replenishment interval in the generalized setting. In addition, numerical studies illustrated the behavior of the model and showed that the optimal payment policy, which depends on the current interest structure, may lead to lower cost and slightly smaller order sizes.

From a managerial point of view, considering the prevailing interest structure that governs the payment policy is indispensable for minimizing total cost. Neglecting characteristics of financial conditions in calculating order sizes may lead to inferior order and payment policies, which unnecessarily increases the total costs of the buyer. This is especially the case when the interest rate charged by the supplier exceeds the deposit rate of the retailer, which leads to a situation where a continuous settlement policy possibly supported by electronic payment solutions is beneficial. In scenarios where the deposit rate exceeds the liability rate, deferring the settlement, in turn, may lead to additional interest earnings and lower total cost. The results of this paper also illustrate the close linkage between operational and financial aspects in supply chain management, which should be considered by employing integrated planning approaches. In addition, assuming an optimized replenishment and payment policy, the results also indicate that the differences in the working capital structure of large retailers are caused by their distinct cash investment opportunities. Considering incentives caused by different interest conditions, cash conversion cycles tend to decrease as DIOs decrease and DPOs increase, which also is observable in the retail sector where many companies aimed at reducing money tied up in stocks over the past years.

The proposed approach can be extended in several ways. Future research could study how the length of the credit periods influences total cost and treat the lengths of the credit periods as decision variables. There is a stream of research that studies the design of credit term conditions from the supplier's point of view (e.g., Sarmah et al., 2007, 2008), and linking this research stream to the study conducted in this paper would further our understanding of how trade credits may influence the coordination of supply chains. In addition, alternative demand functions could be studied, for example functions that assume that demand is dependent on the inventory level on hand. Earlier research has shown that in the presence of stock-dependent demand, orders should be placed earlier, such that a positive inventory level occurs at the end of each cycle (e.g., Teng et al., 2011), which could be analyzed in the presented scenario as well. Finally, further analysis of alternative sources of financing the retailer could utilize (in addition to or as a substitute of trade credits) seems promising (see also Moussawi-Haidar and Jaber, 2013).

Appendix A: Proof of Theorem 1.

(a) By rearranging (29), we get $F(T) = A/T$ and $G(T) = T/2\,(hD + IePD)$. If there is a unique solution to $F(T) = G(T)$ with $T > 0$, then (29) has a unique solution. Since $F'(T) = -A/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = 1/2\,(hD + IePD) > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique T such that $F(T) = G(T)$, which implies that $dTC_{1.1}/dT = 0$ has a unique solution.

(b) If $T = T_{1.1}^*$ is the solution to $dTC_{1.1}/dT = 0$, the second-order derivative of $TC_{1.1}$ at this point is:

$$\left.\frac{d^2TC_{1.1}(T)}{dT^2}\right|_{T_{1.1}^*} = \sqrt{(D(h + PIe))^3/2A} > 0.$$

Hence, $T_{1.1}^*$ is the global minimum of $TC_{1.1}$. Additionally, by substituting ${T_{1.1}}^*$ into $T \leq M$, we know that if and only if $2A \leq DM^2(h + P\mathrm{Ie})$, then ${T_{1.1}}^* \leq M$. □

Appendix B: Proof of Theorem 2.

(a) By rearranging (30), we get $F(T) = (2A - IePDM^2)/T$ and $G(T) = hDT$. If there is a unique solution to $F(T) = G(T)$ with $T > 0$, then (30) has a unique solution. Since $F'(T) = -(2A - IePDM^2)/T^2 < 0$ (note that the necessary condition $2A > IePDM^2$ already needs to be satisfied for the presence of this subcase), $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = hD > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique T such that $F(T) = G(T)$, which implies that $dTC_{1.2-1}/dT = 0$ has a unique solution.

(b) If $T = T_{1.2-1}^*$ is the solution to $dTC_{1.2-1}/dT = 0$, the second-order derivative of $TC_{1.2-1}$ at this point is:

$$\left.\frac{d^2TC_{1.2-1}(T)}{dT^2}\right|_{T_{1.2-1}^*} = \sqrt{(hD)^3/2A - IePDM^2} > 0.$$

Hence, $T_{1.2-1}^*$ is the global minimum of $TC_{1.2-1}$. Additionally, by substituting ${T_{1.2-1}}^*$ into $M < T \leq N$ and $CDT \leq PDM(1 + IeM/2)$, we know that if and only if $DM^2(h + IeP) < 2A \leq IePDM^2 + hD{\Delta_1}^2$ with $\Delta_1 = min\{N, M(P/C)(1 + IeM/2)\}$, then $M < {T_{1.2-1}}^* \leq N$ and $C D {T_{1.2-1}}^* \leq PDM(1 + IeM/2)$.

Appendix C: Proof of Theorem 3.

(a) By rearranging (31), we get $F(T)=\left(2A+PDM^2(Ic_1(1+IeM/2)^2-Ie)\right)/T$ and $G(T)=TD(hP+Ic_1C^2)/P$. If there is a unique solution to $F(T)=G(T)$ with $T>0$, then (31) has a unique solution. Since $F'(T)=-\left(2A+PDM^2(Ic_1(1+IeM/2)^2-Ie)\right)/T^2<0$ (note that the necessary condition $(Ic_1(1+IeM/2)^2>Ie)$ already needs to be satisfied for the presence of this subcase with $Ic_1>Ie$), $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T)=D(hP+Ic_1C^2)/P>0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0)>G(0)$ whereas $F(\infty)<G(\infty)$. Consequently, there is a unique T such that $F(T)=G(T)$, which implies that $dTC_{1.2-2}/dT=0$ has a unique solution.

(b) If $T=T^*_{1.2-2}$ is the solution to $dTC_{1.2-2}/dT=0$, the second-order derivative of $TC_{1.2-2}$ at this point is:

$$\left.\frac{d^2TC_{1.2-2}(T)}{dT^2}\right|_{T^*_{1.2-2}}=\sqrt{\left(D(h+Ic_1C^2/P)\right)^3/\left(2A+PDM^2(Ic_1(1+IeM/2)^2-Ie)\right)}>0.$$

Hence, $T^*_{1.2-2}$ is the global minimum of $TC_{1.2-2}$. Additionally, by substituting ${T_{1.2-2}}^*$ into $M<T\le N$ and $CDT>PDM(1+IeM/2)$, we know that if and only if $DM^2(IeP+h(P/C)^2(1+IeM/2)^2)<2A\le DN^2(h+Ic_1C^2/P)-PDM^2(Ic_1(1+IeM/2)^2-Ie)$, then $M<{T_{1.2-2}}^*\le N$ and $CD{T_{1.2-2}}^*>PDM(1+IeM/2)$.

Appendix D: Proof of Theorem 4.

(a) By rearranging (32), we get $F(T)=\left(2A+Ic_2PD\left(M\left(1+Ic_1(N-M)\right)(1+Ic_1(N-M)/2)+(N-M)(1+IeM/2)\right)^2-Ic_1(N-M)\left(PDM(1+IeM)+PD(N-M)\right)-IePDM^2\right)/T$ and $G(T)=TD\left(hP+Ic_2\left(C+CIc_1(N-M)\right)^2\right)/P$. If there is a unique solution to $F(T)=G(T)$ with $T>0$, then (32) has a unique solution. Since $F'(T)=-\left(2A+Ic_2PD\left(M\left(1+Ic_1(N-M)\right)(1+Ic_1(N-M)/2)+(N-M)(1+IeM/2)\right)^2-Ic_1(N-M)\left(PDM(1+IeM)+PD(N-M)\right)-IePDM^2\right)/T^2$ <0 (note that the necessary condition already needs to be satisfied for the presence of this subcase with $Ic_1>Ie$), $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T)=D\left(hP+Ic_2\left(C+CIc_1(N-M)\right)^2\right)/P>0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0)>G(0)$, whereas $F(\infty)<G(\infty)$. Consequently, there is a unique T such that $F(T)=G(T)$ which implies that $dTC_{1.3-3}/dT=0$ has a unique solution.

(b) If $T=T^*_{1.3-3}$ is the solution to $dTC_{1.3-3}/dT=0$, the second-order derivative of $TC_{1.3-3}$ at this point is:

$$\left.\frac{d^2TC_{1.3-3}(T)}{dT^2}\right|_{T^*_{1.3-3}} = \sqrt{\frac{\left(Dh+Ic_2D\frac{C^2}{P}\left(1+Ic_1(N-M)\right)^2\right)^3}{2A+Ic_2PD\left(M\left(1+Ic_1(N-M)\right)\left(1+\frac{Ic_1(N-M)}{2}\right)+(N-M)\left(1+\frac{IeM}{2}\right)\right)^2-2Ic_1(N-M)\left(PDM\left(1+\frac{IeM}{2}\right)+\frac{PD(N-M)}{2}\right)-IePDM^2}} > 0.$$

Hence, $T^*_{1.3-3}$ is the global minimum of $TC_{1.3-3}$. Additionally, by substituting $T^*_{1.3-3}$ into $N < T$ and $\left(CDT - PDM(1+IeM/2)\right)\left(1+Ic_1(N-M)\right) > PD(N-M)(1+Ic_1(N-M)/2)$, we know that if and only if $IePDM^2 + D\left(h + Ic_2\frac{C^2}{P}\left(1+Ic_1(N-M)\right)^2\right){\Delta_4}^2 - Ic_2PD\left(M\left(1+Ic_1(N-M)\right)\left(1+\frac{Ic_1(N-M)}{2}\right)+(N-M)\left(1+\frac{IeM}{2}\right)\right)^2 + 2Ic_1(N-M)\left(PDM\left(1+\frac{IeM}{2}\right)+\frac{PD(N-M)}{2}\right) < 2A$ with $\Delta_4 = \max\left\{N; \frac{P\left(2(N-M)+Ic_1(N-M)^2\right)}{2C\left(1+Ic_1(N-M)\right)} + \frac{PM(1+IeM/2)}{C}\right\}$, then $N < T^*_{1.3-3}$ and $\left(CDT^*_{1.3-3} - PDM(1+IeM/2)\right)\left(1+Ic_1(N-M)\right) > PD(N-M)(1+Ic_1(N-M)/2)$.

Appendix E: Proof of Theorem 6.

(a) By rearranging (34), we get $F(T) = (2A - IePDN^2)/T$ and $G(T) = hDT$. If there is a unique solution to $F(T) = G(T)$ with $T > 0$, then (34) has a unique solution. Since $F'(T) = -(2A - IePDN^2)/T^2 < 0$ (note that the necessary condition $2A > IePDN^2$ already needs to be satisfied for the presence of this subcase), $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = hD > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique T such that $F(T) = G(T)$ which implies that $dTC_{2.3-1}/dT = 0$ has a unique solution.

(b) If $T = T^*_{2.3-1}$ is the solution to $dTC_{2.3-1}/dT = 0$, the second-order derivative of $TC_{2.3-1}$ at this point is:

$$\left.\frac{d^2TC_{2.3-1}(T)}{dT^2}\right|_{T^*_{2.3-1}} = \sqrt{(hD)^3/2A - IePDN^2} > 0.$$

Hence, $T^*_{2.3-1}$ is the global minimum of $TC_{2.3-1}$. Additionally, by substituting $T^*_{2.3-1}$ into $N < T$ and $CDT\left(1+Ic_1(N-M)\right) \le PDN(1+IeN/2)$, we know that if and only if $DN^2(IeP+h) < 2A \le DN^2\left(IeP + h{\Delta_4}^2\right)$ with $\Delta_4 = \left(P(1+IeN/2)/C\left(1+Ic_1(N-M)\right)\right)$, then $N < T^*_{2.3-1}$ and $CDT^*_{2.3-1}\left(1+Ic_1(N-M)\right) \le PDN(1+IeN/2)$.

Appendix F: Proof of Theorem 7.

(a) By rearranging (35), we get $F(T) = \left(2A + PDN^2(Ic_2(1+IeN/2)^2 - Ie)\right)/T$ and $G(T) = TD\left(h + C^2Ic_2\left(1+Ic_1(N-M)\right)^2/P\right)$. If there is a unique solution to $F(T) = G(T)$ with $T > 0$, then (35) has a unique solution. Since $F'(T) = -\left(2A + PDN^2(Ic_2(1+IeN/2)^2 - Ie)\right)/T^2 < 0$ (note that the necessary condition $(Ic_2(1+IeN/2)^2 > Ie)$ already needs to be satisfied for the presence of this subcase with $Ic_2 > Ie$), $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = D\left(h + C^2 Ic_2\big(1 + Ic_1(N-M)\big)^2/P\right)$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique T such that $F(T) = G(T)$ which implies that $dTC_{2.3-2}/dT = 0$ has a unique solution.

(b) If $T = T^*_{2.3-2}$ is the solution to $dTC_{2.3-2}/dT = 0$, the second-order derivative of $TC_{2.3-2}$ at this point is:

$$\left.\frac{d^2TC_{2.3-2}(T)}{dT^2}\right|_{T^*_{2.3-2}} = \sqrt{\left(D\left(h + Ic_2\frac{C^2}{P}\big(1 + Ic_1(N-M)\big)^2\right)\right)^3 \Big/ 2A + PDN^2(Ic_2(1+IeN/2)^2 - Ie)} > 0.$$

Hence, $T^*_{2.3-2}$ is the global minimum of $TC_{2.3-2}$. Additionally, by substituting $T^*_{2.3-2}$ into $N < T$ and $CDT\big(1 + Ic_1(N-M)\big) > PDN(1 + IeN/2)$, we know that if and only if $2A > DN^2\left(h + Ic_2\frac{C^2}{P}\big(1 + Ic_1(N-M)\big)^2\right){\Delta_5}^2 - PDN^2(Ic_2(1+IeN/2)^2 - Ie)$ with $\Delta_5 = \max\left\{1, \left(\frac{P(1+IeN/2)}{C(1+Ic_1(N-M))}\right)\right\}$, then $N < T^*_{2.3-2}$ and $CDT^*_{2.3-2}\big(1 + Ic_1(N-M)\big) > PDN(1 + IeN/2)$.

Appendix G: Proof of Theorem 9.

(a) By rearranging (44), we get $F(T) = 2A/T$ and $G(T) = D\big(h + 2Ic_2C\big(1 + Ic_1(N-M)\big) - IeP\big)T$. If there is a unique solution to $F(T) = G(T)$ with $T > 0$, then (44) has a unique solution. Since $F'(T) = -2A/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = D\big(h + 2Ic_2C\big(1 + Ic_1(N-M)\big) - IeP\big) > 0$ (note that the necessary condition $(h + 2Ic_2C\big(1 + Ic_1(N-M)\big) > IeP)$ needs to be satisfied to avoid situations in which retailer has an incentive to never pay back the trade credit to the supplier), $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique T such that $F(T) = G(T)$ which implies that $dTC_{3.3}/dT = 0$ has a unique solution.

(b) If $T = T^*_{3.3}$ is the solution to $dTC_{3.3}/dT = 0$, the second-order derivative of $TC_{3.3}$ at this point is:

$$\left.\frac{d^2TC_{3.3}(T)}{dT^2}\right|_{T^*_{3.3}} = \sqrt{\Big(D\big(h + 2Ic_2C\big(1 + Ic_1(N-M)\big) - IeP\big)\Big)^3/2A} > 0.$$

Hence, given the condition that $h + 2Ic_2C\big(1 + Ic_1(N-M)\big) > IeP$, $T^*_{3.3}$ is the global minimum of $TC_{3.3}$. Additionally, by substituting $T^*_{3.3}$ into $N < T$, we know that if and only if $DN^2\big(h + 2Ic_2C\big(1 + Ic_1(N-M)\big) - IeP\big) < 2A$, then $N < T^*_{3.3}$.

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