A joint economic lot size model with financial credits and uncertain investment opportunity

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Abstract: Establishing long-term relationships among the members of a supply chain has become necessary to enhance the supply chain's competitiveness in a globalized environment. Besides coordinating operational decisions, such as how much and when to produce or to order, the members of a supply chain may also share financial resources or act jointly on the capital market. This is important especially when companies have unequal access to capital, for example because they are located in countries with different economic conditions and banking policies and/or ratings. The joint financing of investments across the supply chain may thus ensure the stability of production and of the flow of products to the customers. In addition, it strengthens the established relationships among the supply chain members. This study takes up these issues and presents an integrated inventory model that considers investments jointly financed by the members of a supply chain. In particular, it considers a two-stage supply chain with a single vendor and a single buyer and assumes that the vendor has the option to invest in increasing its production rate. The buyer, however, is assumed to have better access to capital, and therefore has the option to give a credit to the vendor to invest in its productivity, which is beneficial for both parties. To consider uncertainties of the production improvement investments, a success probability for the investment is considered and modeled using a beta distribution.

Keywords: Integrated inventory, supplier credits, variable production rate, uncertain investment, annuity stream approach.

Introduction

Coordinating strategic and operational decisions among the different echelons of a supply chain is a prerequisite for creating and maintaining competitive advantages in today's business environment. Supply chain management therefore encompasses the coordination of material, financial and information flows in supply chains, and has recently received increased attention both in theory and in practice (Mentzer et al., 2001; Li et al., 2006). One stream of supply chain management research that enjoyed increased popularity in recent years studies the coordination of order and production quantities in supply chains. So-called joint economic lot size (JELS) models determine continuous inventory policies from a system's point of view that maximize the profit of the entire supply chain instead of optimizing the positions of individual supply chain members.

Despite the coordination of material movements, supply chain management also focuses on the coordination of financial flows along the stages of a supply chain (Mentzer et al., 2001, Wuttke, et al., 2013). Surprisingly, the focus of prior research on the coordination of supply chains has mainly been on material flows, and only little attention has been paid to the impact a financial coordination may have on the supply chains performance. In a scenario of general economic crisis, however, an innovative opportunity is given by Supply Chain Finance (SCF) services and solutions, which enable a company to improve its financial performance by leveraging on the relationships and dynamics of its supply chain. As it is underlined in Hofmann (2005), SCF focuses on financial aspects and instruments, which might apply to any department of an organization. Moreover, in the above reference, SCF is defined as "an approach for two or more organizations in a supply chain, including external service providers, to jointly create value through means of planning, steering, and controlling the flow of financial resources on an inter-organizational level".

According to Buzacott and Zhang (2004), the main alternatives of financing are: (i) increase payables; (ii) decrease receivables, for example by selling receivables to a third party that pays the firm immediately and collects payments from customers; (iii) increase gross profit; (iv) increase equity; and (v) borrowing. For SMEs, however, it was shown that the best way of financing is borrowing, in particular with asset-based loans (i.e., loans secured on inventories and receivables). The reason is that SMEs lack market power, have limited resources, may not have good credit ratings and, finally, are not always able to compete on price in the market. These results are also supported by a recent survey of the European Central Bank (ECB, 2014) which shows that for many companies and especially for SMEs, getting access to capital is an even more pressing problem than finding customers. Further conclusions that can be drawn from this survey are: (i) SMEs reported a marginal decrease in the perceived availability of bank loans in spite of the fact that their funding highly depends on them, (ii) interest rates offered to SMEs increased over recent years, and (iii) financing conditions for SMEs are in general more difficult than those for larger companies and continue to differ significantly across European countries – for example, in the last years, the average interest rates for SMEs in Italy was much higher than the one in Germany, and the existing gap is still growing.

The reduction in the availability of loans, which is often referred to as 'credit crunch', heavily influences the current world economic scenario. A detailed review of the difficulties SMEs face in financing and on credit crunch issues is presented in Wehinger (2014). A credit crunch may lead to strong asymmetries among the supply chain parties and therefore should be carefully addressed to ensure the competitiveness of the whole supply chain (Hale and Arteta, 2009). For example, financially distressed suppliers, who may end up in bankruptcy, pose significant operational risks to manufacturers. Manufacturers may be able to switch suppliers in the event of a default, but in an environment where many suppliers are financially distressed and where only a few qualified suppliers are available, the number of alternative suppliers decreases significantly. In addition, supplier diversification is expensive especially for non-commodity products. In such a situation, the only remaining option for manufacturers, in order to support the weaker supplier, are subsidies. These subsidies can take the form of cash, agreements for future contracts, or they can be targeted towards supplier development.

To address this issue, this paper considers the case where a manufacturer offers a credit to its supplier to improve the supplier's production performance (cf., for example, Wadecki, et al., 2012). The credit offered by the manufacturer has the purpose to develop the supplier by improving the supplier's capabilities and subsequently its performance (Krause, 1997). One way to improve the performance of a company or the entire supply chain is to invest in the production capacity either by capital-intensive, physical investments (e.g., factories, machines, equipment, etc.), or by investments in intangible assets (e.g., lean production, training, know-how, knowledge etc.). In this paper, we consider an asset-specific investment (i.e. an investment tailored specifically towards the buyer-supplier-relationship), where the invested capital cannot be easily reallocated for another purpose, which increases the investment risk as there are limited recovery values in case of an investment default.

Under these circumstances, lenders pay particular attention to investment performance, and the modeling effort requires considerable accuracy (Borgonovo, et al., 2010). Ojala and Hallikas (2006) provided insights into investment decision-making under uncertainty and risks in buyer-dominated supply networks, where 'risk' refers to the possiblility of a landee not paying its loan on time or not paying it at all. The latter is usually referred to as 'credit default'. The authors also underlined that one key element of an investment is that its outcome is uncertain. Their study leads to the following conclusion (Ojala and Hallikas, 2006, p. 209): "awareness of the potential investment risks and the firm's position and role in the network is essential to be able to make rational investment decisions, also from the point of view of the whole network". Clearly, in a formal model that aims on supporting investment decisions, it is important to model the risk of an investment, as it affects the investment decision. Before the investment is made, the outcome is uncertain: the investment could be a complete success, a complete failure, or it could achieve an intermediate outcome. To model the investment risk, this paper assumes that the investment has a continuous probability of success that follows a beta distribution. The probability distribution represents the stochastic result of the investment and its uncertainty. The beta probability density function has been chosen because it is very flexible and can model a large variety of heterogeneity patterns (Dolgui and Pashkevich, 2008; Ospina and Ferrari, 2012).

This paper consequently extends the classical JELS model to account for uncertain investment opportunities and joint investment decisions. The remainder of this paper is organized as follows: Section 2 gives an overview of the existing literature, and Section 3 introduces the notations and assumptions used and presents mathematical models for different coordination scenarios. Section 4 provides numerical examples to illustrate the proposed models. Finally, Section 5 concludes the paper by summarizing its main findings and by providing suggestions for future research.

Literature Review

Joint economic lot size models are an important modeling reference for supply chain management and have been studied extensively in the past. Early research in this area focused on the coordination of order and production cycles by assuming that the production and order quantities have to be equal, or by assuming that the supplier may produce an integer number of orders in a single production run (e.g., Goyal, 1977; Banerjee, 1986). Subsequently, researchers focused on managing transportation processes between the supplier and the buyer, for example by assuming that production lots may be split up into equal- and/or unequal-sized shipments, or that some type of constraint on the shipments has to be considered (e.g., Agrawal and Raju, 1996; Hill, 1999; Ertogral et al., 2007; Glock, 2012a). Recent works on the JELS problem focused on the management of containers in supply chains (e.g., Kim et al., 2014; Glock and Kim, 2014), environmental issues (e.g., Jaber et al., 2014), or human learning in production (e.g., Khan et al., 2014). Readers may refer to the works of Glock (2012b) and Glock et al. (2014) for recent reviews.

Many manufacturing firms today recognize the impact of their suppliers' performance on their own. This has made purchasing an active research area, with supplier development being central to improving supply chain performance. Friedl and Wagner (2012), for example, studied a firm's decision of whether to invest in an incumbent supplier or to switch to an

alternative supplier in order to minimize the total costs of purchasing a product. Their work indicated that the development of an incumbent supplier (i.e. a cooperative investment) becomes more attractive when the purchase price of an item on the market is high and uncertain, as compared to what the incumbent supplier can offer. Two other relevant works on supplier development are those of Meisel (2012), who developed a decision support model for supplier selection and supplier development in dynamic markets, and Ries et al. (2013), who studied integrated inventory and financing decisions in supply chains. Talluri et al. (2010) noted that formal decision models have very rarely been developed in this field in the past. The authors provided a decision support system to allocate an investment of a company to its suppliers, such that a target return is achieved at a minimum risk. Recent reviews of works on supplier development are those of Chidambaranathan et al. (2009) and Wagner (2011), who showed that the majority of works on supplier development presented empirical studies based on surveys.
Another stream of research that is relevant for this paper studies investments into process improvements and/or variable production rates. Process improvement investments were first studied by Porteus (1985), who developed an order quantity model where setup costs are variable and dependent on the investment volume. The author showed that investments lead to lower total costs. Investments into the production rate were considered by Buzacott and Ozkarahan (1983), who assumed that production rates in a scheduling problem are variable, and that a changeover cost occurs when varying the production rate. Eiamkanchanalai and Banerjee (1999) developed a model for determining the optimal run time and production rate for a single item. They assumed that the unit production cost is a quadratic function of the production rate. Recently, Glock (2010, 2011) studied the impact of variable production rates on the inventory build-up and the total cost of two- and multi-stage production systems. He showed that considering variable production rates in a production system could reduce excessive inventory in the system. Zanoni et al. (2014) finally considered the case where the production rate influences energy consumption in production, and studied which production rate should be selected to minimize the total costs of the production process.

Finally, as the investments concern a long period of time, it is crucial to consider the temporal allocation of payments, and thus research on the net present value (NPV) approach is of relevance to this paper. Even though the NPV approach is widely accepted as a suitable framework for studying production and inventory control systems, average cost (AC) models have been used more frequently in the literature. Several authors (e.g., Hadley, 1964; Trippi and Lewin, 1974; Klein Haneveld and Teunter, 1998) have argued that for the EOQ model, an AC framework is an approximation of the NPV framework resulting in near-optimal solutions if the following conditions are met: a) products are not moving slowly, b) interest rates are not high, and c) the customer payment structure does not depend on the inventory policy. The main objections against using the AC approach as an approximation of the NPV are: a) the time value of money is not explicitly taken into account, b) there is no distinction between out-of-pocket holding costs and opportunity costs of inventory investment, and other sources of opportunity costs/yields (e.g., fixed ordering costs, product sales) are not taken into account at all, and c) initial conditions are not taken into consideration. However, the NPV approach is often rather complicated, such that an approximation may still be preferred. In the past, it has been shown that a certain transformation of the holding cost parameters in EOQ-type models gives near-optimal results for the NPV approach as well, e.g. the annuity stream approach (AS). Some of the most interesting works on the discounted cash flow approach are those of Grubbström (1980), van der Laan and Teunter (2002) and Beullens and Janssens (2014).

Model development

Problem description

This paper is concerned with the coordination of inventory and financing decisions in a two-stage single-vendor-single-buyer supply chain. As the advantage of centralized decision-making has repeatedly been demonstrated in the literature on the joint economic lot size problem (for a detailed and recent review, see Glock, 2012b), the present work considers only centralized scenarios and assumes that the vendor and the buyer act jointly to maximize the total profit of the supply chain.

Replenishments are assumed to follow an equal-sized shipment policy in this paper, as shown in Figure 1. The buyer orders a lot of size Q at regular time intervals and the vendor manufactures a lot of size nQ at a finite production rate P with a single setup that is delivered to the buyer in n shipments of equal size Q. The vendor incurs a setup cost for each production run, and the buyer incurs an ordering cost for each order placed.

Beside these operational characteristics, it is assumed that the buyer – for different reasons (e.g., larger dimension, a different country of origin, a different financial rating, etc.) – is in a better financial position in terms of liquidity and solvency than the vendor, and thus has access to capital at a lower interest rate than its rather small vendor. As noted by Fazzari et al. (1988), in an environment with information asymmetries, external funds may be more costly than internal capital, and they may thus provide an imperfect substitute for it. A possible way to reduce this cost and to overcome credit crunch is to close agreements with a partner that is in a better financial position, with revenue sharing as an alternative mechanisms (see Lee and Rhee, 2010). As a result, the buyer could engage in supplier development initiatives through direct investments by lending money to the vendor that may be invested to increase its production rate. The increase of the production rate at the vendor is assumed to result in a decrease of the unit production cost. Moreover, the demand at the buyer's side is linked to the selling price, such that there is an indirect relationship between the production rate at the vendor and the demand at the buyer. The investment considered in this paper is assumed to be uncertain, and the probability of success for the investment is modelled using a beta distribution.

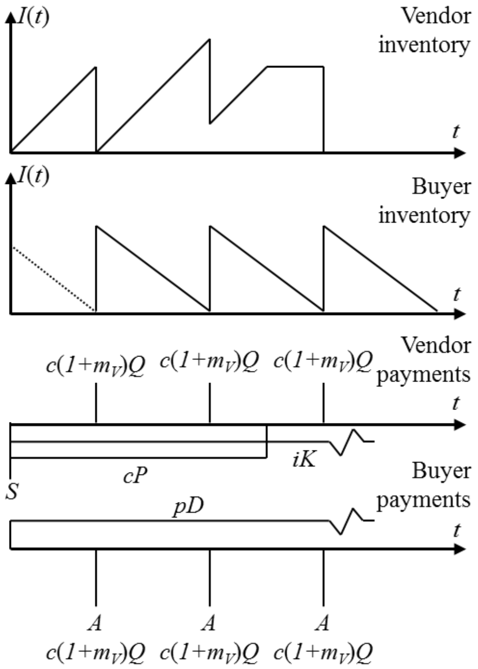

Figure 1: Inventory level and payment structure

While considering investment decisions, the timing of incoming and outgoing payments has a considerable impact on the profitability of the investment and on production decisions. Whereas decisions following from a cost-oriented model are insensitive to the temporal allocation of payments within a period, a cash flow-oriented approach directly considers the impact of the time period on the value of the investment (Hofmann, 1998). Thus, even though the average cost (AC) approach has more frequently been used for studying production and inventory control systems than the superior net present value (NPV) approach, the approximation through the use of the AC model is not correct; for this reason, we use the annuity streams (AS) concept in this paper, i.e. we assume that a constant flow of payments yields the same net present value for a given stream of payments. For an infinite planning horizon, the AS is obtained as a linear transformation of the NPV, which quite generally can be viewed to be the Laplace transformation of a cash-flow function (Grubbström, 1980).

The following section develops formal models to study the problem described above. In developing the models, the following scenarios will be considered:

- Scenario 0, which assumes that no investment opportunity exists. Scenario 0, therefore, is a basic joint economic lot size model that serves as a benchmark case in this paper;
- Scenario 1, which assumes that the vendor has the option to invest in its production rate, with the success of the investment being uncertain;
- Scenario 2, which considers the investment option at the vendor and a settled interest rate for the capital loan between the vendor and the buyer.

The three scenarios outlined above will be discussed in the following.

Notation and assumptions

The following notation will be used throughout the paper:

α parameter of the Beta distribution
A ordering cost per order for the buyer
β parameter of the Beta distribution

b	price-sensitivity of the demand rate
c	unit production cost of the vendor
δ	incremental investment cost for increasing the production rate
d	fixed annual customer demand
γ	incremental unit cost reduction for an increased production rate
i_B	continuous interest rate on loans for the buyer
i_V	continuous interest rate on loans for the vendor
i_S	continuous interest rate settled between the vendor and the buyer
K	capital invested for increasing the production rate
n	number of shipments from the vendor to the buyer
m_V	profit margin of the vendor
m_B	profit margin of the buyer
ϕ	success probability of the investment
p	unit selling price of the buyer
P	production rate of the vendor with $P \in [P_{min}, P_{max}]$
Q	order lot size of the buyer
r	continuous discounting rate
ρ_V	interest rate coefficient for the vendor
ρ_B	interest rate coefficient for the buyer
S	setup cost for the vendor

In addition to the properties already described, the following assumptions are made:

1. The inventory system involves a single item and has an infinite planning horizon.
2. Shortages are not allowed, as both lead time and customer demand are deterministic.
3. The demand rate D is assumed to be a linear function of the selling price, (see Banker et al., 1998; Savaskan et al., 2004; Glock and Kim, 2015), $p=(1+m_V)(1+m_B)c$, i.e. $D(p)=d-bp$. The condition $p<d/b$ has to be satisfied to ensure that demand is always positive. It should be noted that for Scenario 0, where the investment option is not considered, $c = c_0$, $p = p_0$ and $D=D_0$.
4. The vendor's production rate P is finite with $P \geq D$. Due to technological constraints of the production process, it is limited to the interval [P_{min}, P_{max}], but may be increased at an investment $K(P)$. Investments are assumed to follow a logarithmic function $K(P)=\delta \ln(P/P_{\min}) \ \forall \ P \in [P_{\min}, P_{\max}]$ (see, e.g., Porteus, 1985).
5. Beside the continuous capital cost according to the interest on borrowings, the investment in increased production capacities also induces economies of scale. Thus, an increasing production rate leads to reduced unit production cost according to the logarithmic relationship $P-P_{\min}=\gamma \ln(c_0/c)$ (see Christensen et al., 1973). Thus the unit production cost is $c(P)=c_0 e^{-(P-P_{\min})/\gamma}$.
6. In addition, the outcome of potential productivity investments is uncertain, and the realized production rate is affected by the success probability of the investment, ϕ.
7. The interest rates charged by the credit institution to the vendor and the buyer are assumed to depend on their solvency, which is represented by the two coefficients ρ_V and ρ_B that describe the relationship between the average discounting rate r and the respective interest rate on loans, i.e. $i_V=\rho_V r$ and $i_B=\rho_B r$.

Supply chain annuity stream without investment (Scenario 0)

The annuity stream of the vendor is given by the sum of all relevant payments according to their time of occurrence (Grubbström, 1980). In this scenario, which considers a basic joint economic lot size model without investments to increase the production rate, and which serves as a benchmark case, the vendor produces at the lower bound of his/her productivity and, thus, P is constant and equal to P_{min}. The annuity stream in this case only encompasses sales revenues from selling batches to the buyer less the total cost consisting of setup and production costs, and it amounts to:

$$AS_0^V\left(Q,n\right)=\frac{r}{1-e^{-rnQ/D_0}}\left((1+m_V)c_0Q\sum_{j=1}^{n}e^{-r\left(\frac{Q}{P_{\min}}+(j-1)\frac{Q}{D_0}\right)}-S-c_0P_{\min}\frac{1-e^{-rnQ/P_{\min}}}{r}\right) \quad (1)$$

For the buyer, the annuity stream consists of sales revenues reduced by ordering and supply costs:

$$AS_0^B\left(Q,n\right)=\left(1+m_B\right)\left(1+m_V\right)c_0D_0-\frac{r}{1-e^{-rnQ/D_0}}\left(A+\left(1+m_V\right)c_0Q\sum_{j=1}^{n}e^{-r\left(\frac{Q}{P_{\min}}+(j-1)\frac{Q}{D_0}\right)}\right) \quad (2)$$

Assuming that the vendor and the buyer cooperate by determining a jointly optimal inventory policy, the annuity stream of the supply chain is given by the sum of Eqs. (1) and (2):

$$AS_0^S\left(Q,n\right)=\left(1+m_B\right)\left(1+m_V\right)c_0D_0-\frac{r}{1-e^{-rnQ/D_0}}\left(A\sum_{j=1}^{n}e^{-r\left(\frac{Q}{P_{\min}}+(j-1)\frac{Q}{D_0}\right)}+S+c_0P_{\min}\frac{1-e^{-rnQ/P_{\min}}}{r}\right) \quad (3)$$

In this setting, all decision variables are determined jointly to maximize the annuity stream of the supply chain. Using for illustrative purpose a first order Maclaurin expansion of the exponential components in Eq. (3), truncated at the second term, the annuity stream of the supply chain may be approximated (cf. Grubbström, 1980) by:

$$AS_0^S\left(Q,n\right)\approx AS_{\tilde{0}}^S\left(Q,n\right)=\left(1+m_B\right)\left(1+m_V\right)c_0D_0-\left(\frac{D_0}{nQ}+\frac{r}{2}\right)\left(nA\left(1+rQ\left(\frac{(1-n)}{2D_0}-\frac{1}{P_{\min}}\right)\right)+S+c_0nQ\right) \quad (4)$$

The approximated formulation in Eq. (4) represents a useful reference to solve the presented model.

Supply chain annuity stream with investment (Scenario 1 and 2)

In the other scenarios, the annuity stream of the vendor consists of the revenues from selling the product to the buyer, reduced by the total cost consisting of setup, production and investment costs. In contrast to the previous model, the production rate of the vendor may now be influenced by equipment investments that enable the vendor to reduce the unit production cost. Moreover, it is assumed that investments in an increased production rate are uncertain and may not necessarily lead to the expected outcome. Accordingly, the success probability of the investment ϕ is a random variable that follows a beta distribution with mean $\mu=a/(a+b)$ and variance $\sigma=ab/(a+b)^2(a+b+1)$. As the success of the investment is uncertain, the increase in productivity and thus the realized production rate after the investment becomes uncertain as well. In consequence, the expected production rate is given as:

$$E[P] = P_{\min} + (P - P_{\min})E[f] = P_{\min} + \alpha(P - P_{\min})/(\alpha + \beta) \tag{5}$$

Note that in case of a fully successful investment, $E[P] = P$, while in the case of a complete failure of the investment, $E[P] = P_{\min}$.

Thus, in Scenario 1, considering the opportunity of an uncertain investment in an increased production rate, the annuity streams of the vendor and the buyer are given as:

$$AS_1^V(Q,n,P) = \frac{r}{1-e^{-rnQ/D(E[P])}}\left((1+m_V)c(E[P])Q\sum_{j=1}^{n} e^{-r\left(\frac{Q}{E[P]}+(j-1)\frac{Q}{D(E[P])}\right)} - S - c(E[P])\frac{1-e^{-rnQ/E[P]}}{r}\right) - i_V K(P) \tag{6}$$

$$AS_1^B(Q,n,P) = (1+m_B)(1+m_V)c(E[P])D(E[P]) - \frac{r}{1-e^{-rnQ/D(E[P])}}\left(A + (1+m_V)c(E[P])Q\sum_{j=1}^{n} e^{-r\left(\frac{Q}{E[P]}+(j-1)\frac{Q}{D(E[P])}\right)}\right) \tag{7}$$

Where $D(E[P])$ denotes the price-dependent demand based on the unit production cost $c(E[P]) = c_0 e^{-(E[P]-Pmin)/\gamma}$ and the profit margin, and $K(P)$ the vendor's investment function.

Assuming that the vendor and the buyer cooperate by determining a jointly optimal inventory policy, the annuity stream of the supply chain is given by the sum of Eqs. (6) and (7):

$$AS_1^S(Q,n,P) = (1+m_B)(1+m_V)c(E[P])D(E[P]) - \frac{r}{1-e^{-rnQ/D(E[P])}}\left(A\sum_{j=1}^{n} e^{-r\left(\frac{Q}{E[P]}+(j-1)\frac{Q}{D(E[P])}\right)} + S + c(E[P])E[P]\frac{1-e^{-rnQ/E[P]}}{r}\right) - i_V K(P) \tag{8}$$

Using for illustrative purposes a first order Maclaurin expansion of the exponential components in Eq. (8), truncated at the second term, the annuity stream of the supply chain for Scenario 1 can be approximated by:

$$\begin{aligned} & AS_1^S(Q,n,P) \approx AS_1^{\bar{S}}(Q,n,P) \\ & = (1+m_B)(1+m_V)c(E[P])D(E[P]) - \left(\frac{D(E[P])}{nQ}+\frac{r}{2}\right)\left(nA\left(1+rQ\left(\frac{(1-n)}{2D(E[P])}-\frac{(\alpha+\beta)}{\alpha P+\beta P_{\min}}\right)\right)+S+c(E[P])nQ\right) - i_V K(P) \end{aligned} \tag{9}$$

Scenario 2 assumes that the buyer provides the necessary capital to the vendor at an interest rate i_S that is in-between the two individual interest rates i_V and i_B, with the aim of sharing the potential advantageous productivity investment at the vendor side. As the buyer has better access to capital from financial institutions than the vendor, he/she is able to realize a profit of $(i_S - i_B)K(P)$ as long as the interest rate he/she has to pay is smaller than the credit conditions he/she offers to the vendor. In this case, the vendor's capital expenses reduce to $i_S K(P)$. Assuming that the vendor and the buyer jointly determine the optimal inventory policy, the overall capital expense is equal to $i_B K(P)$, and the supply chain annuity stream becomes:

$$AS_2^S(Q,n,P) = (1+m_B)(1+m_V)c(E[P])D(E[P]) - \frac{r}{1-e^{-rnQ/D(E[P])}}\left(A\sum_{j=1}^{n} e^{-r\left(\frac{Q}{E[P]}+(j-1)\frac{Q}{D(E[P])}\right)} + S + c(E[P])E[P]\frac{1-e^{-rnQ/E[P]}}{r}\right) - i_B K(P) \quad (10)$$

Using for illustrative purposes a first order Maclaurin expansion of the exponential components in Eq. (10), truncated at the second term, the annuity stream of the supply chain for Scenario 2 can be approximated by:

$$AS_2^S(Q,n,P) \approx AS_2^S(Q,n,P)$$
$$= (1+m_B)(1+m_V)c(E[P])D(E[P]) - \left(\frac{D(E[P])}{nQ}+\frac{r}{2}\right)\left(nA\left(1+rQ\left(\frac{(1-n)}{2D(E[P])}-\frac{(\alpha+\beta)}{\alpha P+\beta P_{\min}}\right)\right)+S+c(E[P])nQ\right) - i_B K(P) \quad (11)$$

In the given scenarios, all decision variables are determined jointly to maximize the annuity stream of the supply chain. Since the objective functions (3), (8) and (10) are too complex to derive closed-form expressions for any of the decision variables, numerical procedures are required to derive a solution for the given setting.

However, by using the approximated annuity stream functions, it can be shown that the annuity stream of the supply chain given in Eq. (4) is concave in Q for fixed values of n and P. Given the condition that $c_0 > rA((n-1)/2D + 1/P_{min})$, it is possible to derive a closed-form expression for the optimal lot size:

$$Q_0^* = \sqrt{\frac{2(A+S/n)D_0}{rn\left(c_0 - rA\left(\frac{(n-1)}{2D_0}+\frac{1}{P_{\min}}\right)\right)}} \quad (12)$$

Similarly, by using the approximated annuity stream functions, it can be shown that the annuity stream of the supply chain given in Eq. (9) or Eq. (11) is concave in Q for fixed values of n and P. Given the condition that $c(E[P]) > rA((n-1)/2D(E[P]) + (\alpha+\beta)/(\alpha P + \beta P_{min}))$, it is possible to derive a closed-form expression for the optimal lot size:

$$Q_{1/2}^* = \sqrt{\frac{2(A+S/n)D(E[P])}{rn\left(c(E[P]) - rA\left(\frac{(n-1)}{2D(E[P])}+\frac{\alpha+\beta}{\alpha P+\beta P_{\min}}\right)\right)}} \quad (13)$$

To derive good, but not necessarily optimal, solutions, these expressions can be used to develop two-dimensional line search algorithms for finding appropriate values of *n* and *P* (cf. Glock, 2010). In the numerical examples, we compared the results from exact and approximated annuity stream functions and performed a sensitivity analyses with the exact functions to minimize deviations from the optimal solution. The presented problem was solved in Microsoft Excel enhanced with Visual Basic Codes.

Numerical Analysis

This section presents numerical examples to illustrate the behavior of the models developed above. We first introduce a set of parameters to define a base scenario, and then vary important model parameters to study how a) the opportunity to invest influences the annuity stream of the supply chain, b) the interest rate the vendor and the buyer agree upon influences the allocation of the supply chain's annuity stream among both actors, c) the investment uncertainty influences the performance of the supply chain, and d) the discount rate impacts the investment behavior of the supply chain.

Consider a vendor-buyer system with the following parameters: A = €100/order, b=10, c_0 = €30/unit, γ = 8000, δ = 2000, d = 1000 units/year, i_S= 0.14, m_v = 0.5, m_B = 0.5, P_{min} = 1000 units/year, P_{max} = 6000 units/year, ρ_v = 2.5, ρ_b = 1, r = 0.08 and S = €400/setup. Moreover, for the investment opportunity, we assumed that the probability of success of the investment is modeled using a Beta distribution with parameters $(\alpha, \beta) = (8, 2)$.

Considering the inventory and investment decisions for the given scenarios shown in the previous section, Table 1 illustrates the numerical comparison between the exact equations and the approximated ones. As can be seen, the deviation regarding the value of the supply chain annuity stream is acceptable and the highest deviation of about 1.42% was observed in Scenario 0. However, in every scenario, the approximations lead to systematically lower order quantities especially in the case without investments.

	Scenario 0		Scenario 1		Scenario 2	
[units]	$Q_0^* = 445.05$	$Q_{\tilde{0}}^* = 368.04$	$Q_1^* = 546.53$	$Q_{\tilde{1}}^* = 509.67$	$Q_2^* = 558.48$	$Q_{\tilde{2}}^* = 522.29$
[shipment]	$n_{\mathbf{0}} = 1$	$n_{\tilde{\mathbf{0}}} = 1$	$n_1 = 1$	$n_{\tilde{1}} = 1$	$n_2 = 1$	$n_{\tilde{2}} = 1$
[€/unit]	$c_{\mathbf{0}} = 30$	$c_{\tilde{\mathbf{0}}} = 30$	$c_1 = 23.21$	$c_{\tilde{1}} = 23.10$	$c_2 = 22.66$	$c_{\tilde{2}} = 22.56$
[€/unit]	$p_{\mathbf{0}} = 67.50$	$p_{\tilde{\mathbf{0}}} = 67.50$	$p_1 = 52.22$	$p_{\tilde{1}} = 51.99$	$p_2 = 50.97$	$p_{\tilde{2}} = 50.77$
[unit]	$D_{\mathbf{0}} = 325$	$D_{\tilde{\mathbf{0}}} = 325$	$D_1 = 477.78$	$D_{\tilde{1}} = 480.11$	$D_2 = 490.25$	$D_{\tilde{2}} = 492.34$
[units]	$P_{\min} = 1000$	$P_{\min} = 1000$	$P_1 = 3566.15$	$P_{\tilde{1}} = 3611.05$	$P_2 = 3807.97$	$P_{\tilde{2}} = 3848.99$
[€]	$K(P)_0 = -$	$K(P)_{\tilde{0}} = -$	$K(P)_1 = 2{,}542.97$	$K(P)_{\tilde{1}} = 2{,}567.99$	$K(P)_2 = 2{,}674.19$	$K(P)_{\tilde{2}} = 2695.62$
[€]	$AS_0^S = 11{,}441.99$	$AS_{\tilde{0}}^S = 11{,}280.07$	$AS_1^S = 12{,}464.31$	$AS_{\tilde{1}}^S = 12{,}392.56$	$AS_2^S = 12{,}777.52$	$AS_{\tilde{2}}^S = 12{,}708.30$

Table 1. Comparison between exact (results in the left column for each scenario) and approximated formulation (results in the right column for each scenario

As can be seen, the opportunity to invest in increasing the production rate leads to a significant increase in the annuity stream for the supply chain (cf. a shift from Scenario 0 to Scenario 1). Moreover, considering the opportunity of a contractually agreed interest rate between the actors of the supply chain (cf. a shift from Scenario 1 to Scenario 2), the supply chain benefits from an additional increase in the annuity stream due to the reduced interest expenses.

It should be noted that the agreed interest rate i_S is not necessarily fixed; instead, depending on the bargaining power of the two players and on the willingness of the stronger to help the weaker, it would adopt a value between the two individual interest rates i_V and i_B. As can be seen in Figure 2, higher interest rates settled by the actors increase the vendor's total cost and decrease its annuity stream in Scenario 2, while it has no effect in Scenario 1. Thus, the increase in the annuity stream that occurs when shifting from Scenario 1 to Scenario 2, for higher i_S, is reduced. For the buyer, in contrast, the opposite effect can be observed: higher settled interest rates decrease its total cost and increase its annuity stream in Scenario 2. As the total cost and total annuity stream are independent of the agreed-upon interest rate in the integrated scenario, varying interest rates merely affect the allocation of the cooperation gain.

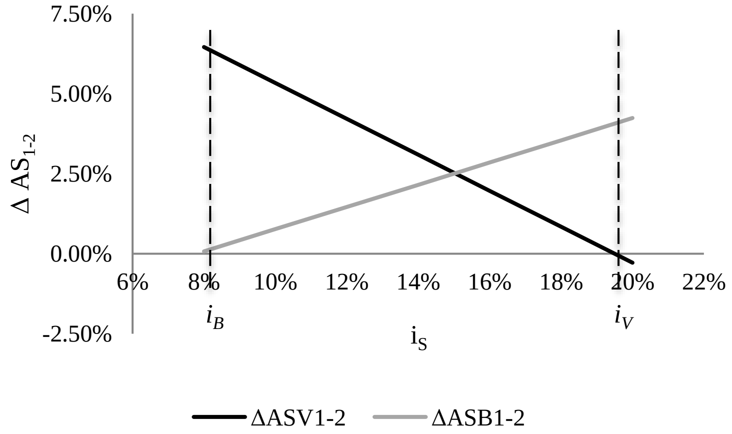

Figure 2 – Variation of the annuity streams of the actors for varying interest rates from Scenario 1 to Scenario 2

Furthermore, the value of the annuity stream considered in Scenarios 1 and 2 is an expected value, which depends on the Beta distribution. Thus, it is necessary to analyze the probability distribution of the supply chain's annuity stream in order to evaluate the dispersion of the results around the mean value. Figure 3 illustrates the probability distribution of the supply chain's annuity stream in Scenario 2. In this paper, uncertainty, represented by the probability distribution of the investment success, is modelled using a Monte Carlo simulation implemented as an add-in module in Microsoft Excel @Risk.

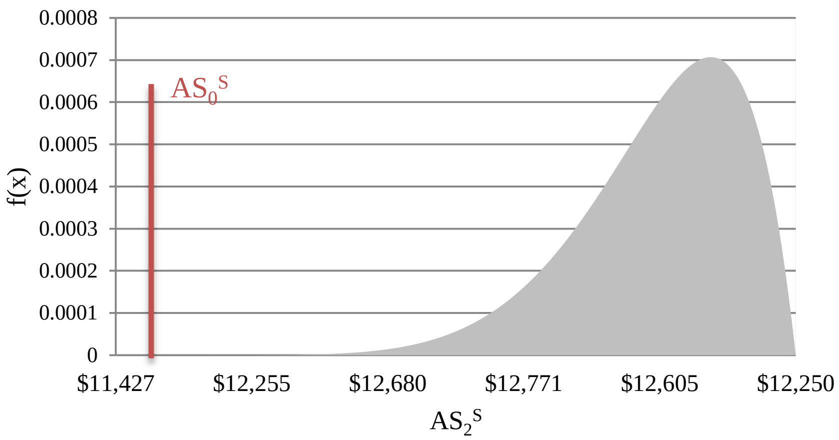

Figure 3 - Probability density function of the supply chain's annuity stream in Scenario 2

An interesting indicator for the prospects of success of the risky investment is the probability that the annuity stream of Scenario 2 exceeds the one of Scenario 0, i.e. the part of the area under the probability density function representing the distribution of AS_2^S for values larger than AS_0 (according to Figure 3): $P(AS_2^S > AS_0^S) = P(AS_2^S > 11,441.99€)$. In the given example, it is possible to observe that the annuity stream in Scenario 2 is larger than the one of Scenario 0 with a probability close to 100% (see Figure 3). Thus, the investment with a settled interest rate between the vendor and the buyer will lead to better results than the benchmark case without investment at a very high probability.

To observe the behavior of the supply chain's annuity stream for all scenarios and for varying discount rates, a sensitivity analysis was performed, and the results are reported in Figure 4 (parts a to d). The main results show that a higher discount rate *r* increases the total costs of both actors and decreases their annuity stream and the investments made (Figure 4a-b). Moreover, it is possible to observe that Scenario 2 leads to a higher annuity stream and a higher capital invested than the other two scenarios as the settled interest rate leads to a more efficient capital expenditure in this case. Figure 4c shows the effect on the annuity stream of the supply chain that result from a shift from Scenario 0 to Scenario 1 ($\mathbf{\Delta AS^S_{1\text{-}0}}$) and from Scenario 1 to Scenario 2 ($\mathbf{\Delta AS^S_{2\text{-}1}}$): for higher values of *r*, the change of the supply chain annuity stream introduced with the investment, in a situation of individual decision-making, is less significant, while the settled interest rate between the vendor and the buyer assumes greater relevance. Figure 4d illustrates the variation of the annuity stream for a shift from Scenario 0 to 1 and from Scenario 1 to 2 for higher opportunity cost for buyer and vendor. The increase of the discount rate for the buyer decreases its annuity stream, while considering the option of an investment (i.e. shifting from Scenario 0 to 1) and when the settled interest rate is considered (i.e., from Scenario 1 to 2). For the vendor, the behavior is different: while the discount rate increases, its annuity stream decreases while considering the option of investment (i.e. from Scenario 0 to 1); in contrast, it increases if additionally the settled interest rate is considered as an additional option (i.e. from Scenario 1 to 2).

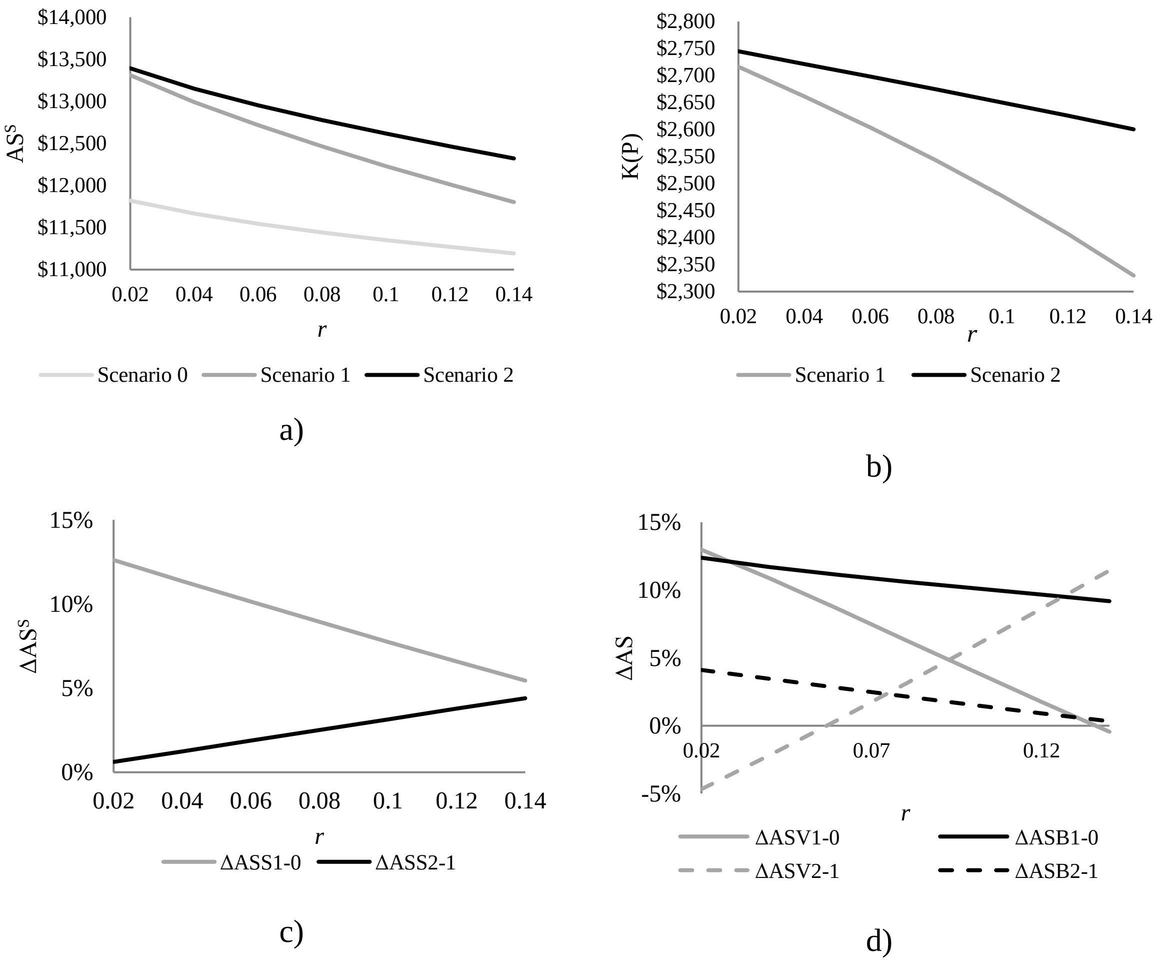

Figure 4(a-d)- Influence of the discount rate *r* on the annuity streams of the vendor and the buyer

Another sensitivity analysis was performed for the expected success probability of the investment (note that this was only performed for the scenarios with investment, i.e.

Scenarios 1 and 2). Figure 5 illustrates the results for different α-parameter value and, thus, expected success probabilities (i.e. higher α-parameter corresponds to higher success probability) and shows that a greater probability of success for the investment in an increased production rate leads to a higher supply chain annuity stream (Figure 5a), less capital invested in both scenarios (Figure 5b), and a higher variation of the supply chain's annuity stream generated by the investment option. In addition, the profit introduced by the settled interest rate slightly decreases (Figure 5c). Moreover, the scenario with the settled interest rate leads to a higher supply chain annuity stream and to a higher capital investment.

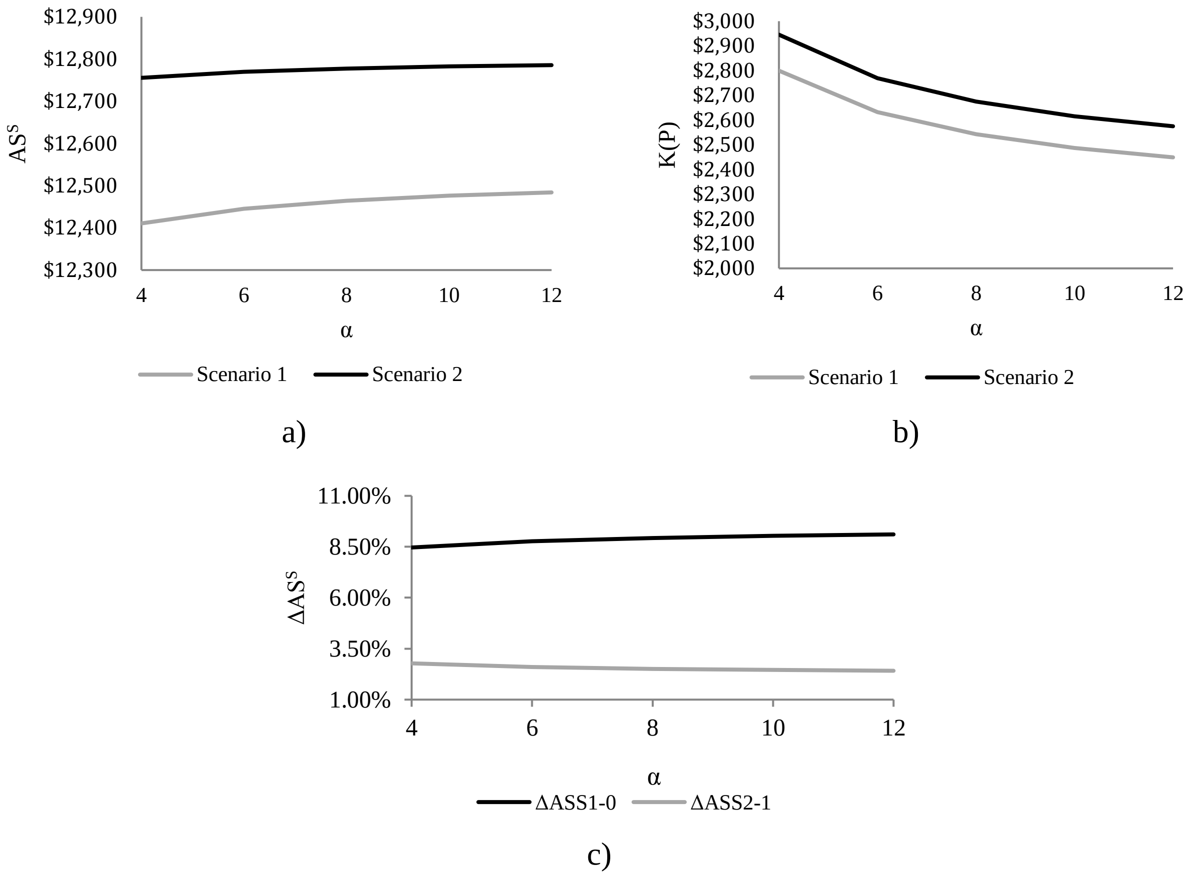

Figure 5(a-c)- Influence of the α-parameter of the beta distribution on the annuity streams and the investment

Subsequently, additional sensitivity analyses were performed to evaluate the changes in the numerical solutions for alternative ratios of the interest rate coefficients of the two actors, which leads to the following results: a higher ratio of the interest rate coefficient of the buyer to the one of the vendor, which implies a lower difference between the capital loan conditions of the two actors, leads to a reduction in the annuity stream of the buyer and the entire supply chain, as the investment becomes less attractive for the buyer. For the vendor, the annuity stream increases (Figure 6).

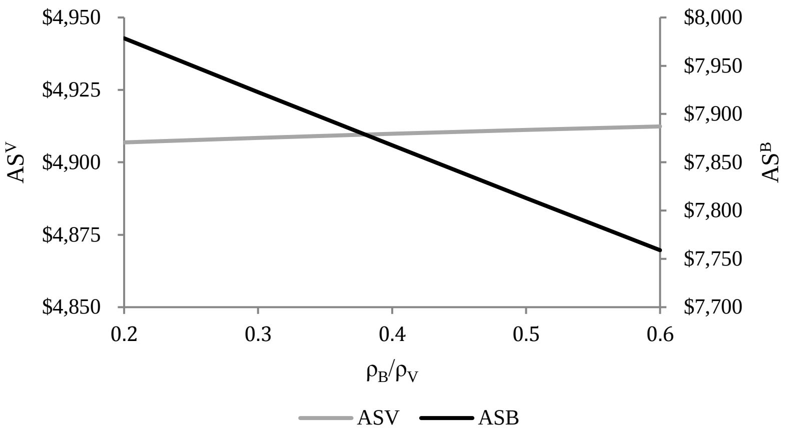

Figure 6 - Sensitivity analysis for the annuity streams of both actors in the ratio of the capital cost coefficient of the two actors (Scenario 2)

The models proposed in this paper can also be used to estimate the investment required to achieve a given increase in the annuity stream, *x*. For this purpose, we consider an integrated decision with financing agreement (Scenario 2), and as a base case the scenario with integrated decisions without investment and financing agreement (Scenario 0). Thus, the variation of the annuity stream is given by $\Delta AS = \left(E\left[AS_2^S\right] - AS_0^S\right) / AS_0^S$. This can be achieved by considering a further constraint in the numerical optimization problem as follows:

$$\Delta AS \geq x \qquad (14)$$

It should be noted that in case the targeted increase in the annuity stream cannot be reached by the investment, the problem may become infeasible.

For illustrative purposes, starting from the numerical example reported at the beginning of this section, and imposing an expected annuity stream increase of $x = 5\%$, we obtain the solution reported in Table 2:

	Scenario 0	Scenario 2
[units]	$Q_0^* = 445.05$	$Q_2^* = 445.28$
[shipment]	$n_0 = 1$	$n_2 = 1$
[€/unit]	$c_0 = 30$	$c_2 = 28.13$
[€/unit]	$p_0 = 67.50$	$p_2 = 63.29$
[unit]	$D_0 = 325$	$D_2 = 367.04$
[units]	$P_{min} = 1000$	$P_2 = 1643.02$
[€]	$K(P)_0 = -$	$K(P)_2 = 993.07$
[€]	$AS_0^V = 4{,}479.53$	$AS_2^V = 4{,}680.88$
[%]	-	$\Delta AS_{0-2}^V = 4.50\%$
[€]	$AS_0^B = 6{,}962.46$	$AS_2^B = 7{,}333.21$
[%]	-	$\Delta AS_{0-2}^B = 5.32\%$
[€]	$AS_0^S = 11{,}441.99$	$AS_2^S = 12{,}014.10$
[%]	-	$\Delta AS_{0-2}^S = 5\%$

Table 2 - Numerical results for the models developed in this paper for the case of a minimum growth target for the annuity stream

As illustrated in Table 2, it is possible to reach an increment of 5% of the annuity stream of the supply chain with respect to the benchmark scenario (Scenario 0), with a low investment in the production rate, while the other decision variables are affected only slightly.

Conclusions

The key contribution of the present work is the evaluation of an uncertain investment opportunity. This paper presented a supply chain inventory model where the buyer, for different reasons (e.g. dimension, location, bank agreements, etc.), has access to capital at a lower interest rate than its vendor. Therefore, the buyer has the option to invest in improving its vendor's productivity either directly or indirectly by lending him/her capital. The investment aimed at increasing the production rate of the vendor was assumed uncertain in this paper, such that the probability of a successful investment can be smaller than 1. The probability of success has been modeled using a beta distribution.

The models developed in this paper considers different scenarios, namely integrated decisions without investments, integrated decisions with investment option at the vendor side, and integrated decisions with the investment option and a settled interest rate for the capital loan between the vendor and the buyer. These models aim at evaluating the efficiency of collaboration between the members of the supply chain, and at determining optimal investment levels for different success probabilities of the investment. The present study illustrates that in case investment opportunities exist, closely coordinating the investment and inventory replenishment activities, and considering the option to share the cost of the investment, help to improve the performance of the entire supply chain.

The paper showed that limited access to credit and the linked worst credit condition (i.e. higher interest rate for bank loans for the investments) at the vendor's side induce strong asymmetry in the supply chain. Thus, considering a centralized scenario with joint decision-making and a settled interest rate for the capital loan between the vendor and the buyer, it is possible to obtain a benefit for the entire chain. Based on the results of our numerical analysis, we conclude that a partnership agreement on sharing financial resources between supply chain members may help to overcome the potential skepticism of uncertain investments.

A possible extension of the present work is to consider efficient profit sharing mechanisms that induce cooperation between different members of the supply chain even under decentralized decision making. In a decentralized setting, it is necessary to establish incentives for cooperation, for example when one of the actors of the supply chain undergoes a reduction in profits. As was shown by Arshinder et al. (2008), the dependencies between supply chain members can be managed with different means and mechanisms of coordination and by utilizing these mechanisms, the performance of the whole supply chain can be improved. Considering such incentive mechanisms might be a valuable extension of the presented approach. The supply chain members could, for example, adjust their terms of trade via a contract that establishes a transfer payment scheme, e.g. in a profit or revenue sharing contract (Chen, 2012; Cachon, 2003). By using such a contract, none of the participants would have an incentive to deviate from the optimal supply chain decisions and actions (Govindan, et al., 2013). Some examples of profit sharing mechanisms are found in Govindan et al. (2013), Cachon (2003), Jaber and Osman (2006), and Jaber et al. (2006). Apart from inventive mechanisms, it would be interesting to analyze alternative methods of investment financing (e.g. loan, leasing, mortgage, etc.). Finally, developing an integrated model that considers different investment types simultaneously would also be interesting.

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