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We analyze vertical product differentiation in a model where a good’s quality is unobservable to customers before purchase, a continuum of quality levels is technologically feasible, and minimum quality is supplied by a competitive fringe of firms. After purchase the true quality of the good is revealed. To provide firms with incentives to actually deliver promised quality, prices must exceed unit variable costs. We show that for a large class of customer preferences there is “quality polarization,” that is, only minimum and maximum feasible quality are available in the market. For the case without quality polarization we derive sufficient conditions for the incentive constraints to completely determine equilibrium prices, regardless of demand, for all intermediate quality levels.

1. Introduction

We consider a model of vertical product differentiation in an experience good market with free but costly entry where in each period firms compete simultaneously in quality and price. That is, in contrast to
other papers on vertical product differentiation we study a market for an *experience good*, as introduced by Nelson (1970). In such a market the quality of the respective good is unobservable to buyers at the time of purchase and contracts cannot be conditioned on quality. Only after use, and hence after purchase, buyers detect the true quality of the good and disseminate this information. We are interested in the situation where firms can choose among many alternative quality levels and buyers differ in their preferences regarding quality. For this situation we analyze the (stationary) equilibrium. In particular, we show that for a large class of customer preferences at most two different levels of quality are offered in equilibrium—one of them being the lowest and the other one the highest technologically feasible level. Moreover, if high quality is, in fact, available, then only for a price that is high relative to production cost.

For the binary case, where the quality of the experience good is either good or bad, Klein and Leffler (1981) examined market incentives for the provision of good quality.1 Specifically, Klein and Leffler (1981) show that in order to induce a competitive firm to supply good quality, the product’s price has to be above average production cost and thus includes an informational rent. Delivering bad instead of good quality will drive the respective firm out of the market, and the threat of losing future rents leads to provision of good quality as long as a sufficiently high “quality-assuring price” generates adequate rents.2 However, as Klein and Leffler point out, there is a conflict between positive rents and perfect competition with free entry. Klein and Leffler (1981) eliminate this conflict by postulating sunk costs due to “nonsalvageable” assets, such as, for example, “brand name capital” due to noninformative advertising. The size of these sunk costs, in equilibrium, is implied by the firm’s incentive compatibility constraint to provide good quality. Specifically, it follows from the condition that the firm’s indispensable informational rent is just the normal return on its sunk cost. Thus, excess profits are wiped out by an entry cost equal to the capitalized informational rent.

In our model the cost of market entry is exogenous (rather than being determined by the indispensable informational rent) and determines, together with demand conditions, the number and size of firms active in the market and the range of product qualities available.

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2. This repeat-purchase contract enforcing mechanism is an example of a self-enforcing contract mechanism that plays an important role in contract theory. See, for example, Crawford (1985), Levin (2003), MacLeod (2007).
The range of product qualities that are actually offered in equilibrium is the center stage of our analysis. Obviously, this question cannot be examined in Klein and Leffler’s (1981) model of binary quality. Our main result is that for a large class of customer preferences there is “quality polarization” in equilibrium, that is, only minimum and maximum feasible quality is available in the market. This can explain the casual observation that in some markets for experience goods there is little variety in available levels of quality and, in addition, high quality is quite expensive relative to production cost.

Examples for low quality diversity can be found among markets for “natural food.” In the continuous dimension of “conventionally grown/semi-organic/organic food,” firms can potentially produce food in many different quality levels. However, almost always only two quality levels are actually offered—low quality (conventionally grown) and high quality (organic). Intermediate qualities (semi-organic) are rarely observed. Market surveys indicate that the price premia of organic foods over conventional foods are substantial (see, e.g., Greene et al., 2009 and the references therein). An example in case are organic soybeans in 2006 in the United States: according to Greene et al. (2009) the price premium relative to conventional soybeans exceeded the (generously estimated) cost difference (where even organic transition costs are included) by almost 50%.

Another illustration for quality polarization is the market for catastrophe insurance. In this market customers care about the financial quality and default risk of firms, but cannot observe the complicated system of reinsurance contracts and the respective reinsurers’ financial capabilities, which are decisive for the default risk of a particular insurance company. Zanjani (2002) provides an empirical analysis of this industry and shows that almost all firms (more than 90%, if measured by their market share in the volume of premia—Zanjani 2002 does not contain firm data) deliver top financial quality in the sense that their reserves and reinsurance contracts exceed substantially expected claims and regulatory requirements (see, in particular, Zanjani, 2002, table 1, p. 297).

Although we show that for a well-defined class of customer preferences there will be quality polarization, we do not contend that the majority of experience good markets show low-quality diversity.

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3. “Average costs for producing organic soybeans were as much as $6.20 per bushel higher than conventional production in 2006, after accounting for the influence of other factors on production costs, including organic transition costs […]. The average price premium for organic soybeans was $9.16 per bushel in 2006 […]” (Greene et al., 2009, p. 14).
However, we regard the lack of intermediate qualities as sufficiently widespread to deserve an explanation.

For an intuitive explanation of the polarization result notice that incentive constraints for provision of above minimum quality limit price competition. In particular, for each quality level the respective quality-assuring price (i.e., the lowest incentive compatible price) becomes a price floor. Consequently, Bertrand competition fails to squeeze out profits, enabling multiple firms to cover entry costs in the highest quality segment. Whereas standard economics suggests that firms differentiate to avoid price competition, the quality-assuring price prevents price competition and thus significantly reduces the incentive of firms to differentiate. In the respective analysis we consider customer preferences which imply that at quality-assuring prices (but not necessarily at equilibrium prices) only a corner solution, that is, either the lowest or the highest feasible quality, can be optimal for any customer. For this case we show that it can never be profit-maximizing for a firm to offer intermediate quality, even if all rivals’ prices are above the quality-assuring prices and the respective firm could attract customers for some intermediate quality and make a positive profit. Thus, there cannot be an equilibrium where some intermediate quality is offered. From this the polarization result follows. Because prices are endogenous and can be above quality-assuring prices, quality polarization does not immediately follow from the assumption about customer preferences. For example, it is conceivable and may even seem plausible, that with two firms in the market one offers maximum quality for a high price that well exceeds the respective quality-assuring price and the other firm supplies some intermediate quality at a price at or above the quality-assuring price. Both firms have positive demand and make a positive profit. Our analysis shows that nevertheless this cannot be an equilibrium because the firm offering intermediate quality can do better.

For preferences that do not imply quality polarization an equilibrium in pure strategies may not exist. As in location models of horizontal product differentiation with more than two firms, the reason for nonexistence stems from discontinuities associated with Bertrand competition. Although we do not supply existence conditions for the nonpolarization case, we provide a condition on customer preferences that allows to derive the equilibrium prices for intermediate quality, conditional, of course, on equilibrium existence (in pure or mixed strategies). Specifically, this (sufficient) condition implies that for all intermediate quality levels the equilibrium prices must be the quality-assuring prices. This result follows from the fact that, given the respective assumption on preferences, a firm can always simultaneously raise price and quality in such a way that its profit increases, unless the firm
already charges the quality-assuring price or offers maximum feasible quality.

Our model is one of moral hazard and incentives rather than one of adverse selection and signaling. Because in the recent literature the term “reputation” is used predominantly, though not exclusively, in connection with adverse selection, we avoid that term in this paper. In our analysis the problem is not firms’ reputation with respect to (given) types, but customers’ trust regarding firms’ behavior. Specifically, we assume that customers trust firms whenever conditional on this trust a firm has no incentive to “cheat,” that is, to provide lower quality than promised. This assumption concerns off-equilibrium beliefs, and it excludes equilibria where firms never produce higher than minimum quality because customers stubbornly believe that quality is always at its lowest feasible level. Whenever customers understand firms’ incentives sufficiently well this assumption is justified.

From our theoretical analysis three testable implications follow. (i) As in the Klein–Leffler model, our study implies that price differences for alternative quality levels should significantly exceed the respective differences in production cost. (ii) If for all customers the willingness to pay for quality is convex as a function of quality (measured by production cost), quality polarization should be observed. That is, only extreme quality levels—“high” and “low”—should be available in the market. (iii) If an increase in quality raises all customers’ willingness to pay by more than it raises the respective production cost, prices of alternative quality levels below maximum quality should be a linear function of production cost. Moreover, these prices should be independent of demand shifts (which should affect only the quantities demanded and produced) and of changes in the intensity of competition due to alterations in the firms’ entry cost. The implication’s condition that relative to production cost customers have a sufficiently strong preference for quality, is frequently used in the literature on Bertrand equilibria in markets with vertical product differentiation (e.g., Shaked and Sutton, 1983, 1987; Sutton, 1986, 1991), and it is specified more precisely in Section 6 below. The condition is compatible with convex willingness to pay functions, but in that case intermediate quality levels should not be observed in the market because of implication (ii) above.

4. Examples of papers on reputation that include adverse selection as an essential element are, among others, Holmström (1999), Tadelis (1999), Mailath and Samuelson (2001), Hörner (2002), Cripps et al. (2004). For our analysis of the moral hazard problem we need not be concerned with the questions of how reputation can be acquired, how it is lost, and how it can be used strategically.

5. This distinction coincides with Cabral (2005, p. 3), where “trust” refers to the situation “when agents expect a particular agent to do something,” whereas “reputation” refers to the situation “when agents believe a particular agent to be something.”
Thus, in the case of convex willingness to pay functions implication (iii), though correct, is empty.\(^6\)

As it is the case with other empirical studies, an empirical test of our model would have to overcome several obstacles. First, the product under investigation must be well-defined, that is, different quality levels of the product must unambiguously be distinguished from related but different goods. Second, “quality” must be well-defined and measurable. Specifically, a particular dimension of quality (like durability) or, alternatively, an index of “overall quality” may be used. Third, production cost data for alternative levels of quality, according to the chosen quality criterion, are needed. Finally, customer preferences are not directly observable. However, the willingness to pay functions could, in principle, be detected by questionnaires, experiments or other methods.

Although overcoming these difficulties may not be easy, they are not fundamentally different from those confronting other empirical studies. In particular, they do not imply that our model lacks testable implications. Rather, we provide three empirical implications, specified above, that can be tested in the market.

The rest of the paper is organized as follows. First, we discuss some related work in Section 2. In Section 3, we present the model. Then, in Section 4, we derive the quality-assuring prices from the incentive compatibility constraints for firms to provide high quality. In Section 5, we analyze the case where customers’ willingness to pay for quality is convex with respect to quality and derive the polarization result. In Section 6, we demonstrate that under a certain condition on customer preferences equilibrium prices for intermediate quality are completely determined by the incentive compatibility constraints alone, whereas customer preferences, the distribution of customer types, and the intensity of competition only determine the quantities demanded and produced, given (predetermined) equilibrium prices. Finally, we conclude in Section 7. All mathematical proofs are relegated to the Appendix.

### 2. Related Work

As pointed out earlier, our analysis is based on Klein and Leffler (1981). In addition, it is also related to the literature on the analysis of Bertrand equilibria in markets with vertical product differentiation (see, e.g.,

\(^6\) However, if customers’ willingness to pay for quality is not convex, existence of an equilibrium is not guaranteed (see Section 6 below). Thus, the usual assumption of empirical research that observations correspond to equilibrium outcomes is perhaps more critical in this case than in other cases.
Gabszewicz and Thise, 1979, 1980; Shaked and Sutton, 1982, 1983). However, it differs from this literature (and the related analysis of endogenous sunk cost and the associated “finiteness result”; see, e.g., Sutton, 1986, 1991; Shaked and Sutton, 1983, 1987; Anderson et al., 1992, 305–313; Berry and Waldfogel, 2009) in three important respects. First, quality cannot be observed by customers before purchase and therefore, due to incentive reasons, high qualities must have high (quality-assuring) prices relative to production costs. That is, costs must also include an incentive component. Second, the quoted literature on Bertrand equilibria in markets with vertical product differentiation assumes basically that the cost increase associated with an increase in quality is negligible in the sense that the cost increase is always below customers’ willingness to pay for that increase in quality. In contrast, we do not use the corresponding assumption in our analysis, which would be that customers’ willingness to pay for higher quality is always larger than the increase in production-cum-incentive cost. Third, while the respective literature concentrates on the case where firms decide first on quality and then, given the (observable) quality choices, on prices, we consider the case where firms decide simultaneously about quality and price (i.e., knowing neither price nor quality chosen by competitors). This approach is natural in our context where quality is unobservable and firms have an incentive to “cheat” customers by producing and selling unobservably low quality for a high price.

Bester (1998) considers a standard Hotelling model of spatial competition for a duopoly where the horizontally differentiated good is an experience rather than an inspection good. The good’s quality is either high or low and, as in our model, the price of high quality includes an incentive component. This puts a floor on the price for high quality that is above the production cost and that the firms cannot undercut. Consequently, price competition is mitigated and because of this the equilibrium outcome may (but need not) be

7. For a Cournot analysis of vertical product differentiation see, for example, Gal-Or (1983). Gabszewicz and Grilo (1992) analyze Bertrand equilibria of a vertically differentiated duopoly where consumers have exogenous beliefs about which of the two firms sells high quality and which sells low quality. Another strain of literature, based on Mussa and Rosen (1978), deals with quality provision by a monopolist (see, e.g., Gabszewicz and Wauthy, 2002 and the references therein). For an analysis of vertical product differentiation with high and low qualities under monopolistic or competitive conditions see Carlton and Dana (2008). However, due to differences in focus the conclusions of this literature are not readily comparable to our results.

8. A fourth respect in which our model differs is our assumption that minimum quality is offered at marginal cost by a competitive fringe. However, although such an assumption would affect the equilibrium prices, the main results of the papers quoted above would remain essentially unchanged because under the assumptions of these models the competitive fringe would have no customers.
“minimum differentiation” rather than “maximum differentiation.” Our model of only vertical product differentiation differs from Bester’s in many respects (in particular, we consider a continuum of qualities, no spatial competition and endogenous entry, whereas Bester considers a continuum of locations, only two quality levels and an exogenously given duopoly) and thus a direct comparison is not possible. However, the two models share the property that asymmetric information about quality mitigates price competition. In fact, our result of low quality diversity may be interpreted as corresponding, in some sense, to Bester’s result of “minimum differentiation,” that is, low spatial diversity.9

Within the literature on vertical product differentiation there is a branch that investigates how customers’ informational differences with respect to different brands may constitute a barrier to entry (see, e.g., Schmalensee, 1982; Bagwell, 1990). The informational differences are in fact differences with respect to supplier reputation. Whereas the incumbent has reputation (the quality of his product is “known”), a new entrant has no reputation (the quality of her product is not “known”). In this literature, firms are of different “types,” e.g., high- and low-quality producers. Thus, customers face an adverse selection problem, and reputation is about firms’ (unalterable) types. In contrast, in our paper firms are not distinguished by types, and consequently, there is no adverse selection. Rather, we analyze the moral hazard problem associated with firms that can choose the quality they provide.

3. The Model

We consider a market for a good that is homogeneous except for quality. That is, there is (potentially) vertical product differentiation but no horizontal product differentiation. Time is measured in discrete periods \( t \in \{1, 2, \ldots \} \). There is a (large) pool of \( N \) firms that are capable to produce each quality \( v \in [0, 1] \) of the good considered at constant marginal cost \( c(v) > 0 \), where the function \( c:[0, 1] \rightarrow \mathbb{R}_{++} \) is strictly increasing. That is, marginal cost is independent of quantity but increasing with respect to quality. Moreover, because \( c(v) \) is strictly increasing, quality can be measured without loss of generality in such a way that cost is linear in

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9. Models of horizontal and vertical differentiation are not as different as frequently perceived. For the case of observable quality Cremer and Thisse (1991) have shown that from the formal point of view the Hotelling model of horizontal differentiation is actually a special case of the standard model of vertical differentiation.
quality with positive slope $\gamma > 0$.

$$c(v) = c(0) + \gamma v.$$  \hspace{1cm} (1)

Put differently, we use production cost to measure quality. In addition to the $N$ firms that are capable to produce each of the technologically feasible quality levels, there are infinitely many firms that are capable of producing minimum quality $v = 0$ at cost $c(0)$. Although a firm may be capable of producing multiple quality levels, we assume, in agreement with most of the literature, that in each period it can produce and offer only one particular level of quality. However, we will show that except for the case of a monopoly the polarization result is independent of this assumption. The number $N$ of firms that can produce all quality levels should be interpreted to be large. Thus, potentially the good can be supplied in many different quality levels.

In equilibrium not all firms will be active in the market and produce. Firms that are active in the market are distinguished between “brand names” and “no names.” No names produce only minimum quality $v = 0$, whereas brand names may produce any quality level $v \in [0, 1]$. Any firm can be active as a no name, and each of the $N$ potential producers of all quality levels can be active as a brand name. Even if a no name is capable of producing positive quality, it will not do so because customers believe that all no names provide only minimum quality (Assumption 3 later). A brand name has to announce the quality of its product in some appropriate way and we call this the “announced quality.” These announcements may take different forms, such as inscriptions on the product or its package, TV or newspaper ads, billboard advertising, promotion campaigns, and so forth. It is also possible that the price itself serves as quality announcement. In fact, we will show below that in a stationary equilibrium quality can be inferred from the price, making explicit quality announcements dispensable.

Actual quality is private information of the firm and may or may not coincide with the announced quality. Between periods brand names can change their announced and actual quality levels, respectively, at no cost. In the model we assume that brand names have to announce the quality in each period, but the absence of a fresh announcement

10. Let $V \in [V_{\min}, V_{\max}]$ be any measure of quality with associated marginal cost $C(V) > 0$ that increases in $V$ but is constant in quantity. Defining $v \equiv [C(V) - C(V_{\min})]/[C(V_{\max}) - C(V_{\min})] \in [0, 1]$ gives the required normalized measurement of quality. For any quality level $\hat{V}$ with normalized equivalent $\hat{v}$ we get the marginal cost $c(\hat{v}) = C(\hat{V}) = C(V_{\min}) + [C(V_{\max}) - C(V_{\min})]\hat{v} = c(0) + \gamma \hat{v}$, where $c(0) \equiv C(V_{\min})$ and $\gamma \equiv C(V_{\max}) - C(V_{\min})$.

may count as implicit renewal of the previous one. We assume that no names have no entry cost and thus form a competitive fringe that sells minimum quality at marginal cost $c(0)$ and has zero payoff. In contrast, each brand name has to incur a positive entry cost, not necessarily identical for all firms, in order to establish the respective brand together with an associated distribution channel. A brand name that voluntarily leaves the market may enter again, either simultaneously or later, but it has to pay the entry cost for each entry. Brand names choose their respective prices, together with their respective quality levels, simultaneously at the beginning of each period. That is, we assume Bertrand competition.

In each period there is an atomless continuum of customers of (Lebesgue) measure 1. Customers live one period and are distinguished by “types” $s \in S = [s_{\min}, s_{\max}] \subset \mathbb{R}$ according to their willingness to pay for quality. Their preferences are specified by the following assumption.

**Assumption 1** (Customer Preferences): Each customer buys at most one unit of the good. The payoff from not buying the good is normalized to zero. For customers of type $s \in S$ the payoff from buying one unit of the good of quality $v$ for the price $p$ is given by $U(v, p, s) = R(v, s) - p$. For all $v \in (0, 1)$ and $s \in S$, the function $R(\cdot, \cdot)$ is twice continuously differentiable, strictly increasing in quality $v$, and $R_{vs}(\cdot, \cdot) > 0$ for $v > 0$, where subscripts of $R(\cdot, \cdot)$ always denote the respective partial derivatives.

The assumption $R_{vs}(\cdot, \cdot) > 0$ is a single crossing condition and means that customer types can be defined in such a way that “higher” types have a higher willingness to pay for additional quality, that is, have a higher marginal willingness to pay for quality. Because of this assumption, the difference in willingness to pay between any two quality levels $v'$ and $v'' > v'$, $R(v'', s) - R(v', s) = \int_{v'}^{v''} R_v(v, s)dv$, is strictly increasing in $s$ for any such pair $v', v'' \in [0, 1]$. Specifically, the additional amount of money that a customer of type $s$ is willing to pay when the good is of highest rather than of lowest feasible quality, $r(s) \equiv R(1, s) - R(0, s)$, is strictly increasing. For convenience we henceforth identify customer types by this difference $r(s)$, that is, without loss of generality we assume $s \equiv R(1, s) - R(0, s)$. Notice that the shape of this willingness to pay function $R(\cdot, \cdot)$, in particular, whether or not it is convex, depends on the way quality is measured. The normalization of

12. Except for points of inflection the converse also holds. Whenever for all $v'', v' \in [0, 1]$, $v'' > v'$, the difference $R(v'', s) - R(v', s) = \int_{v'}^{v''} R_v(v, s)dv$, is strictly increasing in $s$ for all $s \in S$, then $R_{vs}(\cdot, \cdot) > 0$ except for isolated points $\tilde{v} \in [0, 1]$. This follows from $\frac{\partial}{\partial s}[R(v'', s) - R(v', s)] = \int_{v'}^{v''} R_{vs}(v, s)dv$. 


(1) implies that we use (constant) marginal cost to measure quality, and the function \( R(\cdot, \cdot) \) is defined using this measurement.

In this study, we consider the case where at the time of purchase a product’s quality is private information of the producer and cannot be observed by other agents; nor can contracts be conditioned on quality. In such a situation, a firm could save on cost by producing lower quality than announced, and in the following we refer to such behavior as “cheating.”

In some cases, the information that a certain product does not have the announced quality may spread slowly and the firm that sells it in the market may be able to exploit the trust among customers it has acquired in the past for a relatively long period. In other cases, this information may become public almost instantaneously and force the firm to close down. The relevant point is not how fast or to what extent the respective firm’s business is diminished, but that a cheating firm risks that it will be punished by a reduction of demand. The expected punishment gives rise to an incentive compatibility constraint that, if satisfied, induces the firm to provide the announced quality. Regardless of the details of the model, this constraint necessarily implies that the price of a good above minimum quality must be sufficiently above its average cost. Otherwise the firm would cheat and produce only minimum quality. Because customers cannot immediately observe quality and contracts cannot be conditioned on quality, a firm that produces high quality must earn an informational rent. The threat of losing this rent provides the incentive for the firm to actually produce the announced quality.

Using the repeat-purchase contract enforcing mechanism, we model the customers’ learning of the respective good’s quality following Klein and Leffler (1981) and the literature on experience goods and assume perfect monitoring, so that at the end of each period the true quality is revealed to the public.13

Assumption 2 (Perfect Quality Monitoring): If in any period \( t \in \{1, 2, \ldots\} \) the true quality of a product is below the quality announced by its producer, the respective brand name has to exit the market at the end of this period and receives a payoff of zero from the next period onwards.

Out-of-equilibrium beliefs may lead to (implausible) equilibria where brand names cannot exist. To avoid such equilibria we assume, like Bester (1998, pp. 833–834), that customers trust firms whenever conditional on this trust the respective firm has no incentive to cheat.

13. Imperfect monitoring, where in each period the true quality is revealed to the public with some probability \( \varphi \in (0, 1] \) and where with complementary probability \( 1 - \varphi \) there is no information about a product’s quality, can easily be accommodated and does not change the results in a substantive way.
Assumption 3 (Customer Beliefs): Customers cannot observe the true quality of a product before purchase. They believe that the true quality is the announced quality unless given these beliefs it is optimal for a firm to provide lower quality. Otherwise, they believe that the true quality is the minimum quality $v = 0$. No names are believed always to provide minimum quality.

Because no names have no entry cost, they always offer quality $v = 0$ for the price $p(0) = c(0)$ under perfectly competitive conditions. Therefore, we restrict the terms “entry” and “incumbent” to brand names. Moreover, because no names do not behave strategically, we do not treat them explicitly as players in the game. In contrast, potential and actual brand names act strategically. In their decisions they take the “competitive fringe” of no names into account, as well as the strategies of all players.

We assume that all firms have the same discount rate $\rho > 0$. The associated discount factor is denoted by $\delta \equiv \frac{1}{1+\rho} \in (0, 1)$. A firm’s payoff is the sum of its discounted profits minus its entry cost, if it has entered the market, and zero otherwise. If more than one brand name offers the same quality $v \in (0, 1]$ for the lowest price in the market, all those brand names share the respective demand equally.

Our analysis is based on the following game with imperfect information. The set of players consists of brand names, customers, and “nature.” Brand names know the distribution of customer preferences, but cannot observe the individual types. Customers cannot observe the quality of the goods offered in the market. The cost function (1) and the rest of the model are common knowledge. In each period $t \in \{1, 2, \ldots\}$, the game proceeds in four phases. In the first phase, at the beginning of the period, brand names decide simultaneously about entry and exit. That is, brand names that are not (or not any more) in the market decide whether to enter, and brand names that are in the market decide whether to exit. In the second phase, still at the beginning of the period, all brand names in the market observe the moves made in the first phase and choose simultaneously announced quality, actual quality, and price (each for the respective period). Customers and firms observe the announced qualities and prices, but actual qualities are private information, unobservable to customers and rival firms. In the third phase, which takes place at the end of the period, customers decide whether and from which supplier to buy one unit of the good. These decisions are executed and the period’s payoffs accrue. Finally, in phase four, each brand name that had provided some quality below the announced quality has to leave the market forever. A brand name that has to leave the market receives no further payoff (but keeps the payoffs received so far).
Because we are not interested in collusion among firms, we want to rule out folk theorem type results. In standard models this can be done by considering only those equilibria of the dynamic game that consist of playing a particular equilibrium of the stage game in every period. In our model the situation is somewhat different because the only reason why firms do not cheat are their future rents, and therefore the game that incumbent firms play in each period is not the relevant “stage game.” In any one-period “stage game” (as well as in any finite version of the dynamic game) incumbents would always cheat, and because customers would anticipate this and thus not buy the respective product, no brand name would enter the market and no positive quality would be available in equilibrium. In our model, the analog of an equilibrium (in pure strategies) of the dynamic game that consists of playing an equilibrium of the stage game in every period, is what we call an equilibrium in stationary strategies or, for short, a stationary equilibrium. We define a brand name’s strategy to be stationary, if it satisfies the following two conditions: (i) the firm either enters the market in the first period \( t = 1 \) or not at all; (ii) if the firm had entered, its announced quality, actual quality, and price are constant in time and independent of the history of actions. Accordingly, an equilibrium (in pure strategies) is stationary, if all brand names’ equilibrium strategies are stationary. The requirement that all brand names’ equilibrium strategies are independent of the history of actions makes collusion unattainable. Throughout the paper we focus on stationary equilibria in pure strategies.

4. Incentive Compatibility Constraints in a Stationary Equilibrium

In this section, we derive the incentive compatibility constraint for a firm to provide a given quality \( \bar{v} \in (0, 1] \) in a stationary equilibrium. Consider a putative stationary equilibrium, where in some period \( t \) a firm offers a product of some announced and actual quality \( \bar{v} \in (0, 1] \) for some price \( \bar{p} \), sells \( \bar{x} \) units in this period, and considers to do the same in every future period. An alternative—deviating—strategy is to cheat and provide in that period \( t \) only minimum quality, which implies involuntary exit immediately after period \( t \).\(^\text{14}\) The firm’s incentive

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\(^\text{14}\) Clearly, it is never optimal to cheat by providing a quality that is below the announced one but above minimum quality. Compared to providing minimum quality, the cost—market exit—is the same whereas the benefit—reduction of production cost—is lower.
constraint to actually provide quality $\bar{v}$ is

$$\frac{\delta}{1 - \delta} [\bar{p} - c(\bar{v})] \bar{x} \geq \delta([\bar{p} - c(0)] \bar{x})$$

where the left hand side is the payoff $\sum_{\tau=1}^{\infty} \delta^\tau [\bar{p} - c(\bar{v})] \bar{x}$ from always producing quality $\bar{v}$ and the right hand side is the payoff from producing quality $v = 0$ followed by market exit. 15 This gives $\frac{1}{\rho} [\bar{p} - c(\bar{v})] \bar{x} \geq \gamma \bar{v} \bar{x}$ or, after re-arranging,

$$\bar{p} \geq c(\bar{v}) + \rho \gamma \bar{v}.$$  (2)

In a stationary equilibrium, this incentive compatibility constraint has to hold for all quality levels $\bar{v} \in (0, 1]$ that are actually produced. It cannot be an equilibrium that some quality $\bar{v}$ is sold at a lower price because at a lower price the respective firm would cheat and in equilibrium this would be anticipated by its customers.

Let $\hat{p}(v)$ denote the minimal incentive compatible price, or “quality-assuring price” (Klein and Leffler, 1981), for quality $v \in [0, 1]$ in a stationary equilibrium, that is,

$$\hat{p}(v) \equiv c(v) + \rho \gamma v,$$  (3)

These prices include the incentive cost $\rho \gamma v$ in addition to the production cost $c(v)$. This incentive cost is proportional to the discount rate $\rho$ and vanishes when $\rho$ converges to 0.16

In any stationary equilibrium the price $p(v)$ must satisfy

$$p(v) \geq \hat{p}(v)$$  (4)

for each quality $v \in [0, 1]$ that is sold in the market. 17

Because of (4), prices exceed marginal costs for positive levels of quality and this has significant consequences for firms’ incentives to differentiate. If quality were immediately observable to customers,

15. Equivalently, the present value of the benefit from cheating, which is $\delta[c(\bar{v}) - c(0)] \bar{x} = \delta \gamma \bar{v} \bar{x}$, must not exceed the present value of the cost of cheating, which is $\delta \frac{\delta}{1 - \delta} [\bar{p} - c(\bar{v})] \bar{x} = \frac{\delta}{\rho} [\bar{p} - c(\bar{v})] \bar{x}$. Both arguments give the incentive constraint $\frac{1}{\rho} [\bar{p} - c(\bar{v})] \bar{x} \geq \gamma \bar{v} \bar{x}$.

16. Thus, $\hat{p}(v)$ approaches zero profit pricing for $\rho \to 0$. This result is not due to the way we model monitoring, but follows from the assumption that the punishment for cheating consists in forced market exit. If in a stationary equilibrium the (constant) profit per period and unit sold is positive, say $\pi > 0$, its capitalized value $\frac{\pi}{\rho}$—and therefore the punishment of forced market exit—goes to infinity as the discount rate $\rho$ goes to zero. This “exploding punishment” explains why the required rent $\pi$ per period, and with it the incentive cost, converges to zero for $\rho \to 0$.

17. There is a related problem in the regulation of banks: deposit-rate ceilings can be used as incentives for banks to invest in safe rather than in inefficiently risky assets (“gambling assets”) because deposit-rate ceilings increase banks’ profits per period and thus their charter values (Hellmann et al., 2000; Repullo, 2004).
Bertrand competition would imply that whenever two or more firms offer the same quality \( v \) the price \( p(v) \) must equal marginal cost \( c(v) \), and consequently profits are zero. Therefore, firms have a strong incentive to differentiate from each other. In contrast, this is not true in our case of unobservable quality and the reason is (4). Because equilibrium prices for positive quality must always strictly exceed marginal costs, Bertrand competition does not any more imply that profits are zero when two or more firms offer the same quality. This reduces, and may even eliminate, the incentive of firms to differentiate from each other. As a consequence, little variety of quality may result.

In the rest of the paper we examine stationary equilibria in pure strategies. In a stationary equilibrium, by definition, equilibrium strategies are constant in time and independent of the history of actions. For our subsequent analysis we define a new game, the restricted game, by restricting the strategy set of each firm to stationary strategies. In this restricted game we have an equilibrium, if all firms’ strategies are best replies in the set of stationary strategies. The following lemma shows that for a stationary equilibrium it is sufficient to consider only stationary strategies.

**Lemma 1:** A strategy profile that constitutes a Nash equilibrium in pure strategies of the restricted game is also a Nash equilibrium in pure strategies of the original game.  

Because of Lemma 1 we can ignore nonstationary strategies. The proof (in the Appendix) demonstrates that whenever a firm has a payoff-increasing nonstationary strategy against the other firms’ stationary strategies, this firm also has a payoff-increasing stationary strategy.

In a stationary equilibrium of our game nothing can be learned from the past and the strategic situation is exactly the same in every period. It follows that equilibrium strategies are sequentially rational and that a stationary equilibrium is a perfect Bayesian equilibrium.  

### 5. Convex Willingness to Pay

The shape of customers’ willingness to pay functions \( R(v, s) \), specifically whether or not they are convex with respect to quality \( v \), has a significant

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18. Obviously, the converse also holds: if a strategy profile of stationary strategies constitutes a Nash equilibrium of the original game, it also constitutes a Nash equilibrium of the restricted game.

19. Because actual qualities are private information, the only proper subgames are the ones starting at the second phase of the first period, after market entry. It is easy to see that all equilibria are subgame perfect.
effect on the equilibrium.\textsuperscript{20} In this section we investigate the case where customers’ willingness to pay \( R(v, s) \) is convex in quality \( v \), that is, \( R_{vv} (\cdot, \cdot) \geq 0 \) for all \( s \in S \).\textsuperscript{21} That includes, in particular, the case where \( R (\cdot, \cdot) \) is linear in \( v \), and specifically the case \( R (v, s) = sv \) that is typically used in models of vertical product differentiation (e.g., Mussa and Rosen, 1978). We show that in the convex case at most the two extreme quality levels, \( v = 0 \) and \( v = 1 \), are available in the market (quality polarization).\textsuperscript{22} If two or more firms can cover their entry cost, all will offer quality \( v = 1 \) for the price \( \hat{p} (1) \).\textsuperscript{23} A plausible assumption is that higher types are willing to pay more for maximum quality, that is, \( R (1, \cdot) \) is increasing in \( s \). In this case, there exists a critical type \( \bar{s} \in S = [s_{\text{min}}, s_{\text{max}}] \) such that all types \( s > \bar{s} \) consume quality \( v = 1 \), whereas all types \( s < \bar{s} \) either abstain from consuming the good or consume quality \( v = 0 \). Whenever \( \bar{s} \in (s_{\text{min}}, s_{\text{max}}) \), customer type \( \bar{s} \) is indifferent between consuming quality \( v = 1 \) and the preferred alternative of either abstention or consumption of quality \( v = 0 \).

**Proposition 1:** Assume that customers’ willingness to pay \( R (\cdot, \cdot) \) is convex in \( v \), i.e., \( R_{vv} (\cdot, \cdot) \geq 0 \). Then in every stationary equilibrium at most the quality levels \( v = 0 \) and \( v = 1 \) are available in the market. If the market can accommodate two or more firms that offer positive quality, the only equilibrium in stationary strategies is that all those firms offer \( v = 1 \) for the price \( p (1) = \hat{p} (1) = c (0) + (1 + \rho)\gamma \). If customers’ willingness to pay for maximum quality \( R (1, \cdot) \) is strictly increasing in type \( s \), there exists a customer type \( \bar{s} \in S \) such that all types \( s < \bar{s} \) either abstain from consuming the good or consume quality \( v = 0 \), and all types \( s > \bar{s} \) consume quality \( v = 1 \). Provided two or more firms

\textsuperscript{20} Some customers will never demand positive quality at prices \( p (v) \geq \hat{p} (v) \) and thus the shape of their willingness to pay functions is irrelevant. Specifically, if for a customer type \( \tilde{s} \) it holds that \( R (v, \tilde{s}) = \hat{p} (v) \leq \max \{ R (0, \tilde{s}) - c (0), 0 \} \) for all \( v \in (0, 1] \), it follows from the incentive constraint (4) that this type will never demand positive quality. Thus we could limit any assumption on the shape of \( R (\cdot, \cdot) \) to “relevant” types.

\textsuperscript{21} Recall that the shape of the function \( R (\cdot, \cdot) \) depends on the way quality is measured. However, if there is some measurement of quality such that, given that measurement, cost is (weakly) concave and willingness to pay is (weakly) convex in that measurement, then the normalization of (1) implies that \( R (\cdot, \cdot) \) is convex in \( v \). To see this, let \( V \in [V_{\text{min}}, V_{\text{max}}] \) be such a measurement of quality. As footnote 10 shows, \( v \equiv \mu + \beta C (V) \), where \( \beta > 0 \) and \( \mu \) are constants, gives (1). Consequently, \( V = C^{-1}(\frac{\mu}{\beta}) \). Defining \( B (v) \equiv C^{-1}(\frac{\mu}{\beta}) \), the willingness to pay of type \( s \) for quality \( V \), denoted by \( \hat{R} (V, s) \), gives \( \hat{R} (v, s) = \hat{R} (V, s) = \hat{R} [B (v), s] \). Therefore, \( \frac{\partial^2 \hat{R} (V, s)}{\partial v^2} \geq \frac{\partial^2 \hat{R} (V, s)}{\partial v^2} [B^2 (v)] \frac{\partial^2 C^2 (V)}{\partial V^2} B^\prime (v) \), where \( \frac{\partial^2 \hat{R} (V, s)}{\partial V^2} > 0 \). Thus, \( C^\prime (v) \leq 0 \) (which implies \( B^\prime (v) \geq 0 \)) and \( \frac{\partial^2 \hat{R} (V, s)}{\partial v^2} \geq 0 \) gives \( \frac{\partial^2 \hat{R} (V, s)}{\partial v^2} \geq 0 \).

\textsuperscript{22}Convexity of the willingness to pay \( R (v, s) \) in \( v \) does not imply that the payoff \( R (v, s) - p (v) \) is convex in \( v \) unless \( p (v) \) is concave. Therefore, extreme qualities are not simply implied by the fact that a convex objective function induces a corner solution. Rather, quality polarization follows from the firms’ optimal choices of quality and price.

\textsuperscript{23} Because we have normalized the population to have measure 1, the entry cost has to be measured relative to the size of the population.
offer positive (and thus maximum) quality, the (indifferent) customer type $s$ decreases strictly with the firms’ discount rate $\rho$ for $s \in (s_{\text{min}}, s_{\text{max}})$.

Intuitively, Proposition 1 follows from two observations. First, as noted above, the limitation of price competition due to the price floor of the quality-assuring price for maximum quality eliminates the need of firms to differentiate themselves from competitors and allows all oligopolistic firms to profitably offer maximum quality. Second, as the proof of Proposition 1 shows, the convexity of customers’ willingness to pay functions has an important implication for the profitability of the firms’ alternative price-quality strategies. If a customer is willing to buy some particular intermediate quality $\tilde{v}$ for some price $p(\tilde{v})$ at or above the respective quality-assuring price $\hat{p}(\tilde{v})$, this customer is also willing to buy any higher quality $v > \tilde{v}$, and in particular maximum quality $v = 1$, at a price that is not below the respective quality-assuring price $\hat{p}(v)$ and, in addition, implies a higher profit per customer. Consequently, a firm’s profit increases when it simultaneously raises price and quality in the right proportion. Because of this, only a strategy where the firm offers maximum quality can be profit-maximizing.

Proposition 1 implies polarization of quality provision, but does not tell under which conditions both or only one of the extreme quality levels, $v = 0$ and $v = 1$, are available and actually purchased in equilibrium. This depends on the distribution of customer types and, to some extent, on the tie-breaking rules of indifferent agents. In the following brief analysis the tie-breaking rules are as follows (the results for alternative tie-breaking rules are straightforward and analogous): (i) customers choose high quality unless low quality or abstention generates a strictly higher payoff, and choose low quality only if both high quality and abstention generate a strictly lower payoff; (ii) firms enter the market whenever the payoff is non-negative. Because minimum quality $v = 0$ is always available in the market, maximum quality $v = 1$ will be purchased at a given price $p(1)$ only by customers who at that price prefer maximum quality both to minimum quality and to not buying the good at all, that is, by customers who satisfy the condition $R(1, s) - p(1) \geq \max [R(0, s) - c(0), 0]$. Because by definition $s = R(1, s) - R(0, s)$, the condition is equivalent to $s \geq \max [p(1) - c(0), p(1) - R(0, s)]$. For any price $p(1)$, let $\mu(p(1))$ denote the measure (or “quantity”) of customers that satisfy $s \geq \max [p(1) - c(0), p(1) - R(0, s)]$. Consider first the case where only a single firm offers maximum quality and let $p^M(1) \geq \hat{p}(1)$ denote the respective monopoly price. The putative monopolist can cover her entry cost, denoted by $\eta$, if and only if $\eta \leq \frac{1}{\rho} \mu(p^M(1))[p^M(1) - c(1)]$. Let $\eta_1$ and $\eta_2 \geq \eta_1$ denote the lowest and the second lowest entry cost, respectively, in the set $N$.
of potential brand names. If even a monopolist with minimum entry cost cannot cover her entry cost, maximum quality is not available. Consequently, \( \eta_1 \leq \mu(p^M(1))[p^M(1) - c(1)] \) is necessary and sufficient for maximum quality to be available in the market. This condition is satisfied, if (but not only if) the monopolist can cover the entry cost at the price \( \hat{p}(1) \), i.e., if \( \eta_1 \leq \frac{1}{\rho}\mu(\hat{p}(1))[\hat{p}(1) - c(1)] = \gamma\mu(\hat{p}(1)) \), because (3) implies \( \hat{p}(1) - c(1) = \rho \gamma \). Moreover, since with two or more firms offering maximum quality the equilibrium price is \( \hat{p}(1) \), at least two brand names can cover their entry cost if and only if \( \eta_2 \leq \frac{\gamma}{2}\mu(\hat{p}(1)) \).

Finally, minimum quality is always available in the market, but it will actually be purchased only if there are customers who strictly prefer it (a) to maximum quality (if available) and (b) to not buying the good at all. The resulting condition is \( R(0, s) - c(0) > \max[R(1, s) - p(1), 0] \) if maximum quality is available at the price \( p(1) \in \{\hat{p}(1), p^M(1)\} \), and \( R(0, s) - c(0) > 0 \) if maximum quality is not available.

Linearity of marginal cost \( c(v) \) and convexity of willingness to pay \( R(v, s) \) in quality \( v \) imply that the sum of consumer rents and total profits can be maximized with just the two extreme quality levels \( v = 0 \) and \( v = 1 \). Thus, the lack of intermediate quality levels is efficient. It does not follow, however, that the equilibrium outcome is constrained efficient in the sense that a planner who does not have more information than the customers and cannot influence the parameters of the model, could not improve the outcome. Given the inefficiency of intermediate quality levels, the planner would allow at most one brand name to enter, because of the positive entry cost. When setting prices, the planner has to provide incentives for the firms not to cheat and hence is constrained by the incentive constraints (4). Consequently, in the constrained efficient outcome either one brand name offers maximum quality \( v = 1 \) for the price \( \hat{p}(1) \), or—if demand is insufficient—there is no brand name. The market outcome will not be constrained efficient, in general, because it needs at least two brand names, and thus too many, to make sure that the price \( \hat{p}(1) \) is charged. Only if there is no brand name in the market or if there is just one brand name and \( p(1) = \hat{p}(1) \) is nevertheless the payoff-maximizing price, the equilibrium outcome is constrained efficient.

Does it make a difference for the customers whether a given level of unit variable cost consists only of production cost or of production and incentive cost? To answer this question we alter our model in two respects: we assume for the moment that quality is observable (so no incentive costs accrue) and that constant marginal production costs for quality \( v \) are given by \( \tilde{c}(v) = c(0) + (1 + \rho)\gamma v \), \( v \in [0, 1] \), which is identical to the sum of production and incentive costs considered so far. In all other respects, in particular regarding preferences and
entry cost, the model remains the same. At least when customers’ willingness to pay \( R(\cdot, s) \) is strictly convex in \( v \) (i.e., \( R_{vv}(\cdot, s) > 0 \)), the proof of Proposition 1 can be applied to the modified model as well, although with a slight variation.\(^{24}\) It follows that only one brand name can be in the market, because with two or more brand names all will offer \( v=1 \) for the price \( \hat{p}(1) = c(0) + (1+\rho)\gamma \) that equals marginal cost and gives zero profits. Because the brand names could not cover the positive entry cost, only one brand name can be in the market in a stationary equilibrium. The single brand name will offer quality \( v = 1 \) for a price \( p(1) > \hat{p}(1) \). Therefore, customers will be worse off than in the case where the same unit variable costs consist of production costs and incentive costs, provided that the second lowest entry cost \( \eta_2 \) is sufficiently low for two or more brand names to be active when information is asymmetric (i.e., provided \( \eta_2 \leq \frac{1}{2} \mu(\hat{p}(1)); \) see above) and hence \( p(1) = \hat{p}(1) \).

In our analysis we have assumed, that in each period a firm can produce and offer only one particular level of quality. However, except for the case of a monopoly the polarization result is independent of this assumption. This follows because the proof of Proposition 1 implies that every brand name will always offer maximum quality, even when it is allowed to simultaneously offer multiple quality levels. While a monopoly may, in fact, find it profitable to offer intermediate quality in addition to maximum quality, this cannot be the case if two or more brand names are active in the market. Because all brand names offer maximum quality, the equilibrium price must be \( p(1) = \hat{p}(1) = c(0) + (1+\rho)\gamma \). Moreover, minimum quality \( v = 0 \) is available at the price \( p(0) = c(0) \). As a consequence, whenever customers’ willingness to pay is convex but not linear with slope \( (1+\rho)\gamma \), demand is zero for any intermediate quality \( v \in (0, 1) \) at any incentive compatible price \( p(v) \geq \hat{p}(v) = c(0) + (1+\rho)\gamma v \). A customer with a linear willingness to pay is willing to buy some intermediate quality \( \bar{v} \in (0, 1) \) at some incentive compatible price \( p(\bar{v}) \geq \hat{p}(\bar{v}) \) only if \( R(v, s) = R(0, s) + (1+\rho)\gamma v, \) \( R(0, s) \geq c(0), \) and \( p(\bar{v}) = \hat{p}(\bar{v}) = c(0) + (1+\rho)\gamma \bar{v} \).\(^{25}\) In this particular (and rather uninteresting) case the customer’s payoff is \( R(v, s) - p(v) = R(0, s) - c(0) \geq 0 \) for \( v \in \{0, \bar{v}, 1\} \).

\(^{24}\) The only difference is that when quality \( \bar{v} < 1 \) and price \( p(\bar{v}) \) are increased to \( v = 1 \) and \( p(1) = p(\bar{v}) + (1-\bar{v})(1+\rho)\gamma \), respectively, profit per customer \( p(1) - c(0) - (1+\rho)\gamma = p(\bar{v}) + (1-\bar{v})(1+\rho)\gamma - c(0) - (1+\rho)\gamma = p(\bar{v}) - c(0) - (1+\rho)\gamma \bar{v} \) does not increase but remains constant.

\(^{25}\) For a (constant) slope of \( R(v, s) \) below \( (1+\rho)\gamma \) minimum quality \( v = 0 \) at the price \( p(0) = c(0) \) provides a higher payoff, for a (constant) slope greater than \( (1+\rho)\gamma \) maximum quality \( v = 1 \) at the price \( p(1) = \hat{p}(1) \) provides a higher payoff than quality \( \bar{v} \) at price \( \hat{p}(\bar{v}) \). If \( R(v, s) = R(0, s) + (1+\rho)\gamma v \) and \( R(0, s) < c(0) \), the highest payoff is achieved by not buying the good at all.
and thus the same for minimum and maximum quality and intermediate quality \( \bar{v} \). Hence demand for these three quality levels depends on the customer’s tie-breaking rule. Ignoring peculiar tie-breaking rules that favor intermediate quality, firms will maximize their profits by offering only maximum quality at the price \( p(1) = \hat{p}(1) \) because the profit per customer \( \hat{p}(v) - c(v) = \rho \gamma v \) increases in \( v \). This proves the claim that except for the case of a monopoly the polarization result is independent of the assumption that in each period a firm can produce and offer only one particular level of quality.

6. Nonconvex Willingness to Pay

In this section, we consider the situation where customers’ willingness to pay is not convex in quality \( v \). In this case many outcomes, including quality polarization, are possible, depending on customer preferences, the distribution of customer types, and other parameters. Moreover, because of the discontinuity associated with Bertrand competition an equilibrium in pure strategies need not exist, analogous to location models of horizontal product differentiation. Given these difficulties, we focus on the question of prices. Specifically, we derive a sufficient condition that prices for all intermediate quality levels assume their minimum incentive compatible values \( \hat{p}(v) \), regardless of demand and the intensity of competition. In an equilibrium customer preferences and the distribution of customer types only determine the quantities demanded, given (predetermined) prices. Perhaps surprisingly, the relevant condition demands that customers’ willingness to pay for a marginal increase in quality is sufficiently large.

Obviously, in a stationary equilibrium brand names have positive profits, do not cheat and do not exit. For each quality \( v \in [0, 1] \) that is available in the market there can be only one price \( p(v) \) and it must hold that \( p(v) \geq \hat{p}(v) \), where \( \hat{p}(v) \) is given by (3). Otherwise the respective firm(s) would cheat. We are interested in the question when the actual price \( p(v) \) is completely determined by the incentive constraint, that is, when \( p(v) = \hat{p}(v) \) holds. For maximum quality \( v = 1 \) the equality \( p(1) = \hat{p}(1) \) is true in two different circumstances. One case occurs when the incentive cost \( \rho \gamma \) is sufficiently large to make the price \( \hat{p}(1) = c(1) + \rho \gamma \) optimal (in the set \( \{ p \mid p \geq \hat{p}(1) \} \) of incentive compatible prices) even for a firm that is the sole brand name in the market. That is, the quality assuring price \( \hat{p}(1) \) exceeds the monopoly price under complete information. The other case, explained next, consists of the situation where two or more brand names offer quality \( v = 1 \).
If at least two brand names offer the same quality \( \bar{v} > 0 \), Bertrand competition will drive the price \( p(\bar{v}) \) to “the lowest possible value.” In the case of observable quality this lowest possible value is the marginal cost \( c(\bar{v}) \). In contrast, when quality is unobservable the lowest possible value, in any stationary equilibrium, is \( \hat{p}(\bar{v}) \) because a brand name that had announced quality \( \bar{v} > 0 \) and charges a price \( p(\bar{v}) < \hat{p}(\bar{v}) \) will cheat. Because of Assumption 3 (Customer Beliefs), customers will buy its product for a price below \( \hat{p}(\bar{v}) \) only if the price is \( c(0) \). Thus, if the firm charges a price \( p(\bar{v}) < \hat{p}(\bar{v}) \), its profit is zero, whereas it is positive if \( p(\bar{v}) = \hat{p}(\bar{v}) \). Consequently, whenever two or more brand names offer the same quality \( \bar{v} \) it must hold that \( p(\bar{v}) = \hat{p}(\bar{v}) \) in any stationary equilibrium.

Although our previous analysis has already shown that two or more brand names may offer maximum quality \( v = 1 \), it is implausible that two or more brand names offer the same intermediate quality \( v \in (0, 1) \). However, even when only one brand name offers some intermediate quality \( \bar{v} \), the lowest possible price, \( \hat{p}(\bar{v}) \), may still be optimal. In fact, we show that \( \hat{p}(\bar{v}) \) is indeed optimal whenever at \( \bar{v} \) all customers’ marginal willingness to pay for quality, \( R_v(\bar{v}, \cdot) \), exceeds the (constant) “marginal cost of quality” \( \gamma \), that is, whenever \( R_v(\bar{v}, \cdot) > \gamma \). Consequently, if \( R_v(v, s) > \gamma \) for all \( v \in (0, 1) \) and all \( s \in S \), then \( p(v) = \hat{p}(v) \) for all \( v \in [0, 1) \), because \( p(0) = c(0) = \hat{p}(0) \) holds trivially. Because of \( R_v(\cdot, \cdot) > 0 \), the condition \( R_v(v, s) > \gamma \) for all \( v \in (0, 1) \) and all \( s \in S \) is equivalent to \( R_v(v, s_{\min}) > \gamma \) for all \( v \in (0, 1) \). Moreover, it is also equivalent to the condition that \( R(v, s) - c(v) \) increases in \( v \) for all \( s \in S \), because \( R(v, s) - c(v) = R(v, s) - c(0) - \gamma v \). As a consequence, at prices that equal unit production costs (but do not include incentive costs), that is, if \( p(v) = c(v) \) for all \( v \in [0, 1) \), all customers’ payoffs \( R(v, s) - p(v) \) are strictly increasing in quality. Exactly this condition is an assumption widely used in the literature on Bertrand equilibria in markets with vertical product differentiation under complete information (e.g., Shaked and Sutton, 1983, 1987; Sutton

26. As Klein and Leffler (1981, p. 625) put it “… the quality-assuring price is, in effect, a minimum price constraint ‘enforced’ by rational consumers.” This requires that consumers either understand the model and know the firms’ cost function for quality or have learned the quality-assuring prices from history.

27. Because brand names have no fixed cost of production, every brand name that has incurred the entry cost can stay in the market and guarantee itself a non-negative profit per period. Consequently, it is impossible that a brand name drives another brand name out of the market by undercutting \( \hat{p}(\bar{v}) \). In the case where (contrary to our model) brand names have a fixed cost of production, a different argument gives the same result. With fixed costs, undercutting will trigger a war of attrition, and in wars of attrition the most plausible equilibria are those in mixed strategies. Because this implies expected payoffs of zero, whereas cheating gives strictly positive payoffs, the respective brand names will cheat. Because customers will recognize this, undercutting is not profitable.
Therefore, the condition used in Proposition 2 below is well in line with the literature.

**Proposition 2:** Assume that \( R_v(\bar{v}, \cdot) > \gamma \) for some intermediate quality level \( \bar{v} \in (0, 1) \), i.e., all customers’ marginal willingness to pay for quality \( \bar{v} \) exceeds the “marginal cost of quality.” Then, if there exists a stationary equilibrium where quality \( \bar{v} \) is offered, its price \( p(\bar{v}) \) is the minimal incentive compatible price, that is, \( p(\bar{v}) = \hat{p}(\bar{v}) \). If the assumption \( R_v(\bar{v}, \cdot) > \gamma \) holds for all intermediate quality levels \( \bar{v} \in (0, 1) \), then this result \( p(\bar{v}) = \hat{p}(\bar{v}) \) holds for each quality level \( \bar{v} \in [0, 1) \) that is below the highest feasible level.

The intuition for Proposition 2 is as follows. Given any intermediate quality \( \bar{v} \) and a price \( p(\bar{v}) > \hat{p}(\bar{v}) \), which is strictly above the associated minimum incentive compatible price \( \hat{p}(\bar{v}) \), the respective firm can always increase its profit by offering a higher quality \( \bar{v} + \epsilon \) (where \( \epsilon \) is positive but sufficiently small) for some higher price \( p(\bar{v} + \epsilon) > p(\bar{v} + \gamma \epsilon) \) that satisfies \( p(\bar{v} + \epsilon) \geq \hat{p}(\bar{v} + \epsilon) \). Because by assumption all its customers are willing to pay more than the cost difference \( \gamma \epsilon \) for the increase in quality, the firm will not loose any customers if \( p(\bar{v} + \epsilon) \) is sufficiently low, and because \( p(\bar{v} + \epsilon) > p(\bar{v}) + \gamma \epsilon \) the firm’s profit per customer will increase. The argument fails when the price is already at the associated minimum incentive compatible price, that is, when \( p(\bar{v}) = \hat{p}(\bar{v}) \), and some or all customers—although willing to pay the higher production cost \( \gamma \epsilon \)—are not willing to pay the higher production plus incentive cost \( \hat{p}(\bar{v} + \epsilon) - \hat{p}(\bar{v}) = (1 + \rho)\gamma \epsilon > \gamma \epsilon \) for the quality increase. Obviously, the argument also fails when quality is already at its maximum \( v = 1 \). Of course, Proposition 2 is only relevant when an equilibrium exists. However, because in an equilibrium with mixed strategies all strategies that are chosen with positive probability must be payoff-maximizing, the proof holds for equilibria in mixed strategies as well.

28. Shaked and Sutton (1983, p. 1472) characterize the respective assumption as follows: “Where that condition is satisfied, all consumers will be agreed in ranking the products in the same strict order, at unit variable cost.” Translated into our setting, this means that \( R(v, s) - c(v) \) is strictly increasing in \( v \), that is, \( R_v(v, s) - c'(v) > 0 \), for all \( s \in S \). Because \( c(v) = c(0) + \gamma v \), it follows that \( R_s(v, s) > \gamma \) for all \( v \in (0, 1) \) and \( s \in S \).

29. For any given \( \bar{v} \in (0, 1) \) it is sufficient that \( R_v(\bar{v}, s) > \gamma \) holds for the (unique) type \( s(\bar{v}) \) that solves \( R(\bar{v}, s) - R(0, s) = (1 + \rho)\gamma \bar{v} \) (thus, \( s(\bar{v}) \) is the “indifferent type” defined by \( R(\bar{v}, s) - \hat{p}(\bar{v}) = R(0, s) - c(0) \)), whenever such a type exists. Because of \( R_v(\bar{v}, s) > 0 \), this implies \( R_v(\bar{v}, s) > \gamma \) for all higher types \( s > s(\bar{v}) \), but for lower types \( s < s(\bar{v}) \) it may hold that \( R_v(\bar{v}, s) \leq \gamma \). If a type \( s(\bar{v}) \) does not exist, either \( R(\bar{v}, s) - R(0, s) < (1 + \rho)\gamma \bar{v} \) for all \( s \in S \) and no type will demand quality \( \bar{v} \) at the price \( \hat{p}(\bar{v}) \), or \( R(\bar{v}, s) - R(0, s) > (1 + \rho)\gamma \bar{v} \) for all \( s \in S \) and only then quality \( \bar{v} \) can be available in the market and the assumption \( R_v(\bar{v}, \cdot) > \gamma \) is needed. If for each \( \bar{v} \in (0, 1) \) the type \( s(\bar{v}) \) exists and satisfies \( R_v(\bar{v}, s) > \gamma \), then \( p(\bar{v}) = \hat{p}(\bar{v}) \) for all \( \bar{v} \in (0, 1) \). Moreover, the proposition can be extended to the case where the strict inequality in \( R_v(\bar{v}, s) > \gamma \) is replaced by the weak inequality.
Proposition 2 shows that if customers have a sufficiently strong preference (relative to production cost) for high quality, then prices for all quality levels \( v \in [0, 1) \) below 1 are determined by the sum of production and incentive cost. The intensity of competition plays no role. Customer preferences, as long as they satisfy \( R_v(\cdot, \cdot) > \gamma \) for \( v > 0 \), and the distribution of customer types determine the quantities demanded but not the equilibrium prices (which follow from the incentive constraints alone). In addition, whenever two or more firms offer the highest feasible quality \( v = 1 \), the price for maximum quality \( v = 1 \) is also determined by the incentive constraint, that is, \( p(1) = \hat{p}(1) \), because of Bertrand competition.

Provided \( R_v(\cdot, \cdot) > \gamma \) for all \( v \in (0, 1) \), Proposition 2 implies that in a stationary equilibrium quality \( v \) is a function of the price \( p \). Specifically, \( v = \frac{1}{(1 + \rho)\gamma}[p - c(0)] \) if \( c(0) \leq p < c(0) + (1 + \rho)\gamma \), and \( v = 1 \) if \( p \geq c(0) + (1 + \rho)\gamma \). If \( p < c(0) \) implies beliefs \( v = 0 \), prices themselves can serve as quality announcements and no explicit quality announcements are needed.

It is useful to compare Proposition 2 with corresponding results of standard oligopoly models of vertical product differentiation, where quality is observable. In these models, it is crucial to distinguish simultaneous price-quality competition (firms observe neither their competitors’ quality choices nor their prices before making their own decisions) and quality-then-price competition (firms decide first about quality and then, after having observed their competitors’ quality choices, about the price). In the latter case of quality-then-price competition (see, e.g., Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983), equilibrium prices will typically be above their competitive levels and firms will differentiate in the sense that no two firms offer the same quality. In the case of simultaneous price-quality competition this is not necessarily so. If for all customers marginal willingness to pay (for quality) exceeds marginal production cost (of quality), only maximum quality is available in equilibrium and prices are given by marginal production costs (Anderson et al., 1992, Section 8.3.2). This replicates the (efficient) perfect competition equilibrium, but it must be assumed that entry is free. If the efficient quality level (the level that maximizes willingness to pay for quality minus production cost of quality) is not the same for all customers and an equilibrium (in pure strategies) exists,

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30. If entry is costly and the entry decisions of rivals can be observed before simultaneous price-quality competition takes place, at most one firm will enter in a pure strategy equilibrium because profits are zero whenever two or more firms produce (but there are also mixed strategy equilibria with stochastic entry).
prices will typically exceed marginal production costs and firms will differentiate.\textsuperscript{31}

While quality-then-price competition depicts probably the more plausible strategic situation when quality is observable, our case of unobservable quality is incompatible with quality-then-price competition. Because firms are by assumption unable to observe their competitors’ quality choices, they are necessarily unaware of these choices when they decide about prices.\textsuperscript{32} Therefore, simultaneous price-quality competition is the appropriate assumption in our study.

7. Conclusions

In this paper, we examined vertical product differentiation in an experience good market with free but costly entry. Customers cannot observe a good’s quality before purchase, but after purchase the true quality of the good is revealed. In our model, a continuum of quality levels is technologically feasible, an endogenous number of oligopolistic firms simultaneously competes in price and quality, and minimum quality is supplied under competitive conditions. For each feasible quality level above the minimum the price must sufficiently exceed average production cost in order to provide incentives for the respective firm not to produce lower quality than announced. That is, prices are bounded from below by “quality-assuring prices” (Klein and Leffler, 1981). In our analysis we examined the effects of this floor of quality-assuring prices on quality diversity and equilibrium prices.

Our study shows that when each customer’s willingness to pay for quality is convex with respect to quality (which includes, in particular, the popular linear case), there is quality polarization: at most two levels of quality, the lowest and the highest, are available in the market. Whereas standard economics suggests that firms differentiate to avoid price competition, the incentive compatibility constraint for quality

\textsuperscript{31} It is easy to see that an equilibrium in pure strategies need not exist. If some firm makes a positive profit in equilibrium, all firms must make the same profit because otherwise a firm with a lower than maximal profit could raise its profit by slightly underbidding a firm with a higher profit. On the other hand, with a finite number of firms an equilibrium where all profits are zero will not exist, provided preferences are sufficiently heterogenous. This follows because if preferences are sufficiently heterogenous, there will always be a price-quality combination with the price exceeding marginal cost that, given the rivals’ price-quality choices, attracts some customers.

\textsuperscript{32} It is conceivable that firms decide first about quality announcements and then, after having observed their competitors’ announcements, about the price. However, because of the incentive constraints (4) these quality announcements have implications for the prices and may even determine them (Proposition 1). If they do, we are back at simultaneous price-quality competition. For these reasons we assume simultaneous price-quality competition in our analysis.
provision keeps the price from falling below the quality-assuring price and thus severely limits price competition. This limitation of price competition eliminates the need of firms to differentiate themselves from competitors and has the effect that all oligopolistic firms offer maximum quality.

If customer’s willingness to pay for quality is not convex with respect to quality, no general results can be achieved. However, we show that if customers have a sufficiently strong preference for high quality, equilibrium prices of all quality levels, except for the maximum, are completely determined by incentive constraints, that is, for all intermediate quality levels equilibrium prices are identical with the respective quality-assuring prices. Neither customer preferences (as long as they satisfy the condition indicated above), nor the distribution of customer types, nor the intensity of competition has any effect on the prices of intermediate quality levels. They only determine the quantities demanded and produced, given (predetermined) equilibrium prices.

**Appendix**

*Proof of Lemma 1.* Because the respective strategy profile is a Nash equilibrium of the restricted game, the incentive constraints for always actually producing the announced quality are satisfied. Otherwise customers would not buy the respective firm’s product for a price above $c(0)$, the firm’s profit would be zero and it would not have entered the market (because it has a positive entry cost). In addition, because all competitors’ actions are constant in time, each firm’s (constant) choice of quality and price maximizes in each individual period its per period profit, given all other incumbents’ qualities and prices. Consequently, no incumbent can improve her payoff by deviating to a different (stationary or non-stationary) strategy with respect to quality and price. Moreover, each incumbent makes in each period a constant positive profit that must exceed the entry cost times the discount rate $\rho$ (otherwise the respective firm would not have entered the market). Therefore, it is optimal for each incumbent to enter in the first period and never to exit.

*Proof of Proposition 1.* First, we show by contradiction that if there is only one brand name, it will offer maximum quality $v = 1$. Clearly, in a stationary equilibrium the respective single brand name will not offer minimum quality $v = 0$ because profits would be zero and the entry cost could not be covered. Assume the respective brand name offers some quality level $\bar{v} \in (0, 1)$ for some price $p(\bar{v}) \geq \hat{p}(\bar{v}) = c(0) + (1 + \rho)\gamma \bar{v}$. For any customer $s$ of the brand name it must hold that she prefers $v = \bar{v}$
for the price \( p(\bar{v}) \) to (i) \( v = 0 \) for the price \( p(0) = c(0) \) and (ii) to not buying at all, that is, it must hold that \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0, s) - c(0), 0] \). If \( R(\bar{v}, s) - p(\bar{v}) \geq R(0, s) - c(0) \geq 0 \), then \( R(\bar{v}, s) - R(0, s) \geq p(\bar{v}) - c(0) \geq \bar{p}(\bar{v}) - c(0) = (1 + \rho)\gamma \bar{v} \). Moreover, if for some \( s \in S \), \( R(\bar{v}, s) - R(0, s) \geq (1 + \rho)\gamma \bar{v} \) for some \( \bar{v} \in (0, 1) \), then \( R(1, s) - R(\bar{v}, s) \geq (1 - \bar{v})(1 + \rho)\gamma \), because from \( R_{vv}(\cdot, s) \geq 0 \) it follows that 

\[
\frac{R(\bar{v}, s) - R(0, s)}{\bar{v}} \geq \frac{R(1, s) - R(\bar{v}, s)}{1 - \bar{v}}.
\]

Thus, \( R(\bar{v}, s) - p(\bar{v}) \geq R(0, s) - c(0) \) implies \( R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] \geq R(\bar{v}, s) - p(\bar{v}) \). In addition, if \( R(\bar{v}, s) - p(\bar{v}) \geq 0 \), then \( R(\bar{v}, s) - R(0, s) > p(\bar{v}) - c(0) \geq \bar{p}(\bar{v}) - c(0) = (1 + \rho)\gamma \bar{v} \) and thus \( R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] > R(\bar{v}, s) - p(\bar{v}) \) because 

\[
\frac{R(1, s) - R(\bar{v}, s)}{\rho} > (1 + \rho)\gamma
\]

gives \( R(1, s) > R(\bar{v}, s) + (1 - \bar{v})(1 + \rho)\gamma \). Consequently, \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0, s) - c(0), 0] \) always implies \( R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] \geq R(\bar{v}, s) - p(\bar{v}) \). Therefore, the respective brand name will not lose demand, if it offers quality \( \bar{v} \in (0, 1) \) for the price \( p(1) = p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma \) instead of quality \( \bar{v} \in (0, 1) \) for the price \( p(\bar{v}) \). In addition, its profit per customer will increase from \( \rho \gamma \bar{v} \) to \( \rho \gamma \). It follows that if there is only one brand name, it will offer maximum quality \( v = 1 \).

Consider now the case of at least two brand names. By the same argument as before, at least one firm offers quality \( v = 1 \). If two or more firms offer quality \( v = 1 \), \( p(1) = \bar{p}(1) \) because of Bertrand competition. In this case, any third firm that offers some quality \( \bar{v} \in (0, 1) \) for some price \( p(\bar{v}) \geq \bar{p}(\bar{v}) \) will attract customer type \( s \) only if \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(1, s) - \bar{p}(1), R(0, s) - c(0), 0] \). However, we have already shown that \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0, s) - c(0), 0] \) implies \( R(\bar{v}, s) - p(\bar{v}) \leq R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] \leq R(1, s) - [\bar{p}(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] = R(1, s) - \bar{p}(1) \). That is, whenever some type \( s \) prefers \( v = \bar{v} \) for the price \( p(\bar{v}) \) to \( v = 0 \) for the price \( p(0) = c(0) \) and to not buying at all, that type prefers, at least weakly, \( v = 1 \) for the price \( \bar{p}(1) \) to \( v = \bar{v} \) for the price \( p(\bar{v}) \). Consequently, any firm that offers some quality \( \bar{v} \in (0, 1) \) for some price \( p(\bar{v}) \geq \bar{p}(\bar{v}) \) has demand of measure zero, and thus zero profits, and cannot cover its entry cost. Hence whenever at least two firms offer \( v = 1 \) no intermediate quality \( v \in (0, 1) \) will be available in the market. The same conclusion follows if only one firm offers \( v = 1 \) for the price \( p(1) = \bar{p}(1) \). The remaining case is the one where \( v = 1 \) is being offered for some price \( p(1) > \bar{p}(1) \) by a single firm and at least one other firm offers \( \bar{v} \in (0, 1) \) for some price \( p(\bar{v}) \geq \bar{p}(\bar{v}) \). We have already shown that \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0, s) - c(0), 0] \) implies \( R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] \geq R(\bar{v}, s) - p(\bar{v}) \). Therefore, the firm offering \( v = \bar{v} \) can have customers only if \( p(1) \geq p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma \) because otherwise \( R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0, s) - c(0), 0] \) implies \( R(\bar{v}, s) - p(\bar{v}) \leq R(1, s) - [p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma] < R(1, s) - p(1) \), that
is, rather than buying $v = \bar{v}$ for the price $p(\bar{v})$ each type $s \in S$ prefers either not to buy at all, to buy $v = 0$ or to buy $v = 1$. If equality $p(1) = p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma$ holds, the set of customers is of measure zero, thus positive profits of the firm offering $v = \bar{v}$ imply $p(1) > p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma$. If the respective firm offers instead $v = 1$ for the price $\hat{p}(1) = p(\bar{v}) + (1 - \bar{v})(1 + \rho)\gamma < p(1)$, it will not lose any customers (because, as shown, $R(1, s) - \hat{p}(1) \geq R(\bar{v}, s) - p(\bar{v})$ for each customer $s$) and take all customers from the rival firm that offers $v = 1$ for some price $p(1) > \hat{p}(1)$. In addition, its profit per customer and hence its payoff increases. Thus, also in this last case it cannot be an equilibrium that some firm offers some intermediate quality $\bar{v} \in (0, 1)$. Consequently, there is no case where a brand name offers an intermediate quality $v \in (0, 1)$. In contrast, it is an equilibrium that all brand names offer quality $v = 1$ for the price $\bar{p}(1)$, provided the market can accommodate at least two firms (i.e., if two firms offer quality $v = 1$ for the price $\hat{p}(1)$, each firm’s discounted stream of profits covers the entry cost). If the market accommodates at least one firm (i.e., if for a single firm that offers quality $v = 1$ for the profit-maximizing price $p(1) \geq \bar{p}(1)$ the discounted stream of profits covers the entry cost), quality $v = 1$ will be available in the market for some price $p(1) \geq \hat{p}(1)$.

For the proof of the second part of the proposition, which assumes $R_s(1, \cdot) > 0$, consider first the case where quality $v = 1$ is available for some price $p(1) \geq \hat{p}(1)$ and is demanded by some but not all customers, that is, $R(1, s_{\text{max}}) - p(1) \geq \max [R(0, s_{\text{max}}) - c(0), 0]$ and $R(1, s_{\text{min}}) - p(1) \leq \max [R(0, s_{\text{min}}) - c(0), 0]$. Then there exists a unique type $s \in S$ such that $R(1, s) - p(1) = \max [R(0, s) - c(0), 0]$. Uniqueness follows because $R(1, \cdot)$ and $s = R(1, \cdot) - R(0, \cdot)$ are both strictly increasing in $s$ and because of the following contradiction: if $R(1, \bar{s}) - p(1) = R(0, \bar{s}) - c(0) > 0$ and $R(1, s') - p(1) = 0 > R(0, s') - c(0)$ for some $s \in S$ and $s' \in S$, then $R(1, s) - p(1) > 0 = R(1, s') - p(1)$ implies $\bar{s} > s'$ because $R_s(1, \cdot) > 0$, whereas $R(1, \bar{s}) - p(1) = R(0, \bar{s}) - c(0)$ and $R(1, s') - p(1) > R(0, s') - c(0)$ imply $\bar{s} = R(1, \bar{s}) = R(0, \bar{s}) = p(1) - c(0) < R(1, s') - R(0, s') = s'$ and thus $s' > \bar{s}$. Moreover, because $R(1, \cdot)$ and $s = R(1, \cdot) - R(0, \cdot)$ are both strictly increasing in $s$, all types $s \in [s_{\text{min}}, s_{\text{max}}]$ either consume quality $v = 0$ or abstain from consuming the good, and all types $s \in (s_{\text{min}}, s_{\text{max}}]$ consume quality $v = 1$. Because $R(1, s) - \hat{p}(1) = R(1, s) - c(0) - (1 + \rho)\gamma$ decreases strictly with $\rho$, the solution $\bar{s}$ of $R(1, s) - \hat{p}(1) = \max [R(0, s) - c(0), 0]$ must also strictly decrease with $\rho$ for $s \in (s_{\text{min}}, s_{\text{max}})$. Finally, if all types $s \in [s_{\text{min}}, s_{\text{max}}]$ demand $v = 1$, $\bar{s} = s_{\text{min}}$; and if no type $s \in [s_{\text{min}}, s_{\text{max}}]$ demands $v = 1$, $\bar{s} = s_{\text{max}}$. □
Proof of Proposition 2. The proof is by contradiction. Consider some firm that in a stationary equilibrium offers quality $\bar{v} \in (0, 1)$ for the price $p(\bar{v}) > \hat{\beta}(\bar{v})$. Assume the firm increases quality $\bar{v}$ to $\bar{v} + \varepsilon > \bar{v}$, where $\varepsilon \in (0, 1 - \bar{v}]$. Because by assumption $R_v(\bar{v}, s) > \gamma$ for all $s \in S$, it follows that $R(\bar{v} + \varepsilon, s) = R(\bar{v}, s) + \int_{\bar{v}}^{\bar{v} + \varepsilon} R_v(v, s) dv > R(\bar{v}, s) + \gamma \varepsilon$. This implies $R(\bar{v} + \varepsilon, s) - R(\bar{v}, s) + p(\bar{v}) > p(\bar{v}) + \gamma \varepsilon$, and thus there exists a price $p(\bar{v} + \varepsilon)$ such that $R(\bar{v} + \varepsilon, s) - R(\bar{v}, s) + p(\bar{v} + \varepsilon) > p(\bar{v} + \varepsilon) + \gamma \varepsilon$. Consequently, $R(\bar{v} + \varepsilon, s) - p(\bar{v} + \varepsilon) > R(\bar{v}, s) - p(\bar{v})$ holds for all $s \in S$ for a price $p(\bar{v} + \varepsilon) > p(\bar{v}) + \gamma \varepsilon$. Moreover, if a sufficiently small $\varepsilon > 0$ is chosen, $p(\bar{v}) + \gamma \varepsilon \geq \hat{\beta}(\bar{v}) + (1 + \rho) \gamma \varepsilon = \hat{\beta}(\bar{v} + \varepsilon)$ because by assumption $p(\bar{v}) > \hat{\beta}(\bar{v})$. Hence there exists an $\varepsilon > 0$ such that for a price $p(\bar{v} + \varepsilon) > p(\bar{v}) + \gamma \varepsilon$, that is, for a price that exceeds $p(\bar{v}) + \gamma \varepsilon$ and satisfies the incentive compatibility constraint (4), it holds for all $s \in S$ that $R(\bar{v} + \varepsilon, s) - p(\bar{v} + \varepsilon) > R(\bar{v}, s) - p(\bar{v})$. The latter inequality implies that every customer who in the assumed stationary equilibrium buys quality $\bar{v}$ at the price $p(\bar{v})$ would buy quality $\bar{v} + \varepsilon$ at the price $p(\bar{v} + \varepsilon)$, if available. Consequently, if the firm that offers quality $\bar{v} \in (0, 1)$ for the price $p(\bar{v}) > \hat{\beta}(\bar{v})$ deviates and chooses the alternative strategy to announce and produce quality $\bar{v} + \varepsilon$ and charge the price $p(\bar{v} + \varepsilon)$, the firm will have at least as many customers as under its original strategy. In addition, because $p(\bar{v} + \varepsilon) > p(\bar{v}) + \gamma \varepsilon$ and hence $p(\bar{v} + \varepsilon) - c(\bar{v} + \varepsilon) > p(\bar{v}) + \gamma \varepsilon - c(\bar{v}) - \gamma \varepsilon = p(\bar{v}) - c(\bar{v})$, the firm’s profit per customer will increase. Hence the deviation will increase the firm’s payoff, and thus the strategy to offer quality $\bar{v} \in (0, 1)$ for the price $p(\bar{v}) > \hat{\beta}(\bar{v})$ cannot be optimal. This contradicts the assumption that it is an equilibrium strategy and proves that under the assumptions of the proposition it cannot hold that $p(\bar{v}) > \hat{\beta}(\bar{v})$ in a stationary equilibrium. Together with the incentive compatibility constraint (4) this implies $p(\bar{v}) = \hat{\beta}(\bar{v})$. If $R_v(v, s) > \gamma$ for all $v \in (0, 1)$ and all $s \in S$, then the same argument shows that it cannot be optimal to offer any quality $v \in (0, 1)$ for a price $p(v) > \hat{\beta}(v)$, and thus $p(v) = \hat{\beta}(v)$. Finally, $p(0) = c(0) = \hat{\beta}(0)$ holds trivially. □

References


