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A Stochastic Dynamics Approach for Seismic Response Spectrum-Based Analysis of Hysteretic MDOF Structures

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Abstract: An efficient nonlinear stochastic dynamics methodology has been developed for estimating the peak inelastic response of hysteretic multi-degree-of-freedom (MDOF) structural systems subject to seismic excitations specified via a given uniform hazard spectrum (UHS), without the need of undertaking computationally demanding non-linear response time-history analysis (NRHA). The proposed methodology initiates by solving a series of inverse stochastic dynamics problems for the determination of input power spectra compatible in a stochastic sense with a given elastic response UHS of specified damping ratio. Relying on statistical linearization and utilizing an efficient decoupling approach the nonlinear *N*-degree-of-freedom system is decoupled and cast into (*N*) effective linear single-degree-of-freedom (SDOF) oscillators with effective linear properties (ELPs): natural frequency and damping ratio. Subsequently, each DOF is subject to a stochastic process compatible with the UHS adjusted to the oscillator effective damping ratio. Next, an efficient iterative scheme is devised achieving convergence of the damping coefficients of all the *N* effective linear SDOF oscillators and the UHS corresponding to each DOF. Finally, peak inelastic responses for all *N* DOFs are estimated through the updated UHS for the *N* different sets of SDOF oscillators ELPs. The proposed approach is numerically illustrated using a yielding 3-storey building exposed to the Eurocode 8 (EC8) UHS following the Bouc-Wen hysteretic model. NRHA involving an ensemble of EC8 compatible accelerograms is conducted to assess the accuracy of the proposed approach in a Monte Carlo-based context.

1 Introduction

Addressing nonlinearities through a design spectrum-based analysis framework usually involves either modification of the elastic design spectrum by applying strength reduction factors [14] or generation of an equivalent elastic SDOF system. Clearly, derivation of an equivalent linear SDOF system (ELS) allows for interpreting the inelastic response spectra as elastic response spectra, utilizing the equivalent properties of the ELS [8]. In this setting, various deterministic and statistical linearization techniques to determine the properties of ELS for various kinds of nonlinearity can be found in the literature [9]. More recently, Giaralis and Spanos [5] proposed a statistical linearization-based framework to estimate the peak inelastic response of an SDOF system exposed to seismic excitations compatible with a given elastic design spectrum.

To circumvent undertaking computationally intensive non-linear response history analysis (NRHA) together with earthquake record selection and scaling [7] this paper puts forth an efficient stochastic dynamics methodology to estimate the peak inelastic response of multi-storey buildings modelled as lumped-mass multi-degree-of-freedom (MDOF) hysteretic systems subject to elastic response UHS. Specifically, the proposed methodology involves the determination of a series of seismically induced stochastic processes characterized by power spectra compatible in a stochastic sense with the assigned elastic response UHS. Relying on statistical linearization and utilizing an efficient decoupling approach the nonlinear *N*-degree-of-freedom system is decoupled and cast into *N* effective linear single-degree-of-freedom (SDOF) oscillators with effective linear properties (ELPs); natural frequency and damping ratio. Particular attention is given to the stochastically derived ELPs for updating appropriately the damping dependent elastic response UHS and the stochastic content of the corresponding input seismic random processes. To this aim, an efficient iterative scheme is devised achieving convergence between the damping ratio coefficient of the effective linear oscillator corresponding to the j-th DOF, and the UHS corresponding to the same j-th DOF and related to the same damping ratio coefficient. In this manner, both the linear oscillator ELPs and the corresponding UHS are iteratively updated, so that the required compatibility between the two is achieved. This significant feature of the proposed scheme is novel in comparison to other alternative treatments in the literature [5], and aims at enhancing the consistency and accuracy of the scheme.

2 Derivation of Design Spectrum Compatible Power Spectra

Contemporary codes for the aseismic design of structures allow seismic motion representation by means of artificial accelerograms generated as parts of finite duration T_s of samples of a stationary stochastic process which is characterized by a power spectrum compatible in a stochastic sense with a given elastic response uniform hazard spectrum (UHS). In this regard, the following nonlinear equation consists the basis for relating a pseudo-acceleration response spectrum $S_a(\omega_i, \zeta_o)$ to an one-sided power spectrum corresponding to a Gaussian stationary stochastic process $X_i(t)$ [1]; that is,

$$S_a(\omega_i, \zeta_o) = \eta_{X_i} \omega_i^2 \sqrt{\lambda_{0,X_i}} \tag{1}$$

where η_{X_i} and λ_{0,X_i} stand for the peak factor and the variance of the stationary stochastic response process $X_i(t)$ of an elastic oscillator of natural frequency ω_i and damping ratio ζ_o. Further, the spectral moment of zero order of the stationary response process that appears in Eq.(1), reads for the general case of n order

$$\lambda_{n,X_i} = \int_0^\infty \omega^n \frac{1}{(\omega_i^2 - \omega^2)^2 + (2\zeta_o \omega_i \omega)^2} G_{X_i}^{\zeta_o}(\omega) d\omega. \tag{2}$$

The evaluation of the stochastically compatible power spectrum $G_{X_i}^{\zeta_o}(\omega)$, which does not appear explicitly in Eq.(1), necessitates a careful handling of the inverse stochastic dynamics problem. In this setting, several methods for generating a consistent power spectrum can be found in the literature [1,4]. Following the hypothesis of a barrier outcrossing in clumps, the peak factor η_{X_i} which is related with the first-passage problem is determined as

$$\eta_{X_i}(T_s,p) = \sqrt{2\,\ln\{2\,v_{X_i}[1-\exp[-\delta_{X_i}^{1.2}\sqrt{\pi\,\ln(2\,v_{X_i})}]]\}} \tag{3}$$

where the mean zero crossing rate v_{X_i} and the spread factor δ_{X_i} of the stochastic response process $X_i(t)$ are defined as

$$v_{X_i} = \frac{T_s}{2\pi}\sqrt{\frac{\lambda_{2,X_i}}{\lambda_{0,X_i}}}(-\ln p)^{-1} \tag{4}$$

and

$$\delta_{X_i} = \sqrt{1-\frac{\lambda_{1,X_i}^2}{\lambda_{0,X_i}\,\lambda_{2,X_i}}}. \tag{5}$$

respectively. The peak factor η_{X_i} consists the critical factor by which the standard deviation of the considered elastic oscillator response should be multiplied to predict a level S_a below which the peak response will remain, with probability p (see Eq.(4)). This probability p is reasonably set to be equal to 0.5. Besides this selection, there is the underlying compatibility criterion which prescribes that from an ensemble of stationary samples of duration T_s of the stochastic process $X_i(t)$, half of the associated response spectra lie below the assigned pseudo-acceleration response spectrum S_a [5]. For the purposes of this study, the following approximate formula for obtaining a reliable estimation of the variance of the response process $X_i(t)$ of an oscillator of natural frequency ω_i and damping ratio ζ_o is used [18], i.e.,

$$\lambda_{0,X_i} = \frac{G_{X_i}^{\zeta_o}(\omega_i)}{\omega_i^3}\left(\frac{\pi}{4\zeta_o}-1\right)+\frac{1}{\omega_i^4}\int_0^{\omega_i} G_{X_i}^{\zeta_o}(\omega)d\omega. \tag{6}$$

Considering Eq.(6) and manipulating appropriately Eq.(1) yields

$$S_\alpha^2(\omega_i,\zeta_o) = \eta_{X_i}^2\omega_i G_{X_i}^{\zeta_o}(\omega_i)\left(\frac{\pi-4\zeta_o}{4\zeta_o}\right)+\eta_{X_i}^2\int_0^{\omega_i} G_{X_i}^{\zeta_o}(\omega)d\omega \tag{7}$$

which is further simplified by substituting the integral in Eq.(7) by a discrete summation. In this setting, the stochastically compatible power spectrum $G_{X_i}^{\zeta_o}(\omega)$ can be derived as

$$G_{X_i}^{\zeta_o}(\omega_i) = \begin{cases} \dfrac{4\zeta_o}{\omega_i\pi-4\zeta_o\omega_{i-1}}\left(\dfrac{S_\alpha^2(\omega_i,\zeta_o)}{\eta_{X_i}^2}-\Delta\omega\displaystyle\sum_{q=1}^{i-1}G_{X_i}^{\zeta_o}(\omega_q)\right), & \omega_i > \omega_b^l \\ 0, & \omega_i \le \omega_b^l \end{cases} \tag{8}$$

where the discretization scheme $\omega_i = \omega_b^l + (i-0.5)\Delta\omega$ is employed. The value of ω_b^l is related with the lowest bound of the frequency domain of Eq.(3). Obviously, a preselection of an input power spectrum shape has to be preceded for deriving a stochastically compatible spectrum, according to the numerical scheme of Eq.(8). In the herein study the utilization of a more elaborate input power spectrum shape is deemed necessary in achieving better matching between the target design spectrum and the compatibly generated design spectrum $S_a(\omega_i,\zeta_o)$ calculated by Eq.(1) (see Figure 1). In this regard, the widely used Kanai-Tajimi spectrum appropriately modified by Clough and Penzien (CP) is considered herein [11]. Further, the values of ω_i range in the closed frequency interval $[\omega_b^l, \omega_b^u]$. Numerical experimentation conducted in [5], indicates that for the case of a CP input spectrum, the value of the lowest bound ω_b^l is equal to zero. An upper cut-off frequency bound ω_b^u is defined according to

$$\int_0^{\omega_b^u} G_{X_i}^{\zeta_o}(\omega)d\omega = (1-\varepsilon)\int_0^{\infty} G_{X_i}^{\zeta_o}(\omega)d\omega \tag{9}$$

where $\varepsilon \ll 1$ (e.g. ε is equal to 10^{-3}). In Figure 1a the target EC8 design spectrum is compared with the solution of the direct formulation of Eq.(1) utilizing the stochastically compatible power spectra $G_{X_i}^{\zeta_o}(\omega)$ for the cases of CP and white-noise (WN) input spectrum shape. Further, in Figure 1b the compatible power spectra for various spectral shapes are presented.

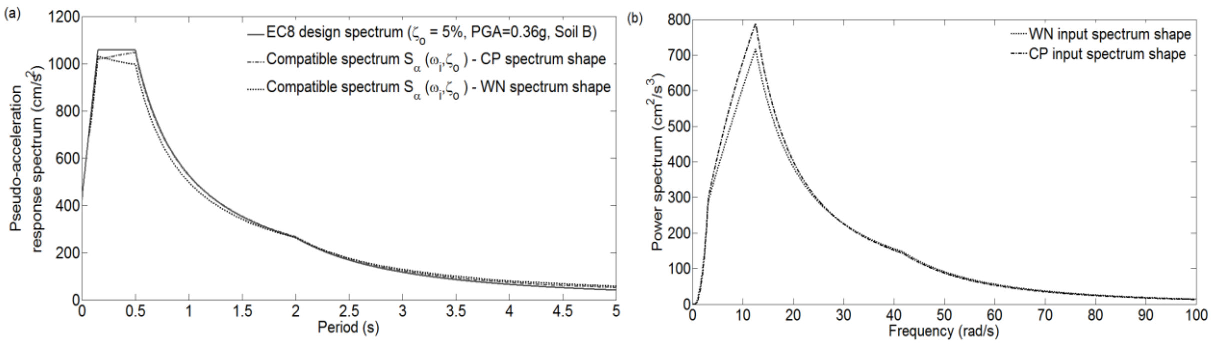

Figure 1: (a) EC 8 design spectrum and the solution to the direct formulation of Eq.(1) $S_a(\omega_i, \zeta_o)$ for various spectral shapes (ζ_o = 5%, PGA= 0.36g, Soil Conditions B). (b) Compatible design spectrum power spectra.

3 Inelastic Stochastic Design Spectrum Analysis

In this section a novel stochastic dynamics methodology for conducting efficiently inelastic design spectrum-based analysis of MDOF nonlinear structural systems is developed.

3.1 Statistical Linearization

Consider the N-DOF structural system governed by the nonlinear differential equation

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{g}[\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)] = -\boldsymbol{F}(t), \tag{10}$$

where $\ddot{\boldsymbol{x}}(t)$ denotes the response acceleration vector, $\dot{\boldsymbol{x}}(t)$ is the response velocity vector and $\boldsymbol{x}(t)$ is the response displacement vector. For the sake of clarity, a distinction should be made between inter-story drifts vector $\boldsymbol{y}(t)$ and the normalized by the nominal yielding displacement $\mathrm{x_y}$ inter-story drifts vector $\mathbf{x}(\mathrm{t})$ that appears in Eq.(10); namely $\mathbf{x}(\mathrm{t}) = \mathbf{y}(\mathrm{t})\, x_y^{-1}$. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denote the $(N \times N)$ mass, damping and stiffness matrices, respectively; $\mathbf{g}[\mathbf{x}(\mathrm{t}), \dot{\mathbf{x}}(\mathrm{t})]$ is an arbitrary nonlinear $(N \times 1)$ vector function of the variables $\mathbf{x}(\mathrm{t})$ and $\dot{\mathbf{x}}(\mathrm{t})$. $\mathbf{F}(\mathrm{t})^{\mathrm{T}} = [\mathrm{f_1(t), f_2(t), \dots, f}_N\mathrm{(t)}]$ is a $(N \times 1)$ zero mean, stationary random vector process defined as $\mathbf{F}(\mathrm{t}) = \bar{\mathbf{M}}\boldsymbol{\gamma} x_y^{-1}\ddot{\alpha}_{\mathrm{g}}(\mathrm{t})$, where $\boldsymbol{\gamma}$ is the unit column vector, $\ddot{\alpha}_{\mathrm{g}}(\mathrm{t})$ is a stochastic stationary seismic excitation process characterized by a power spectrum $G_{X_i}^{\zeta_o}(\omega)$ and $\bar{\mathbf{M}}$ stands for the $(N \times N)$ mass matrix defined in absolute coordinates. In this regard, $\mathbf{F}(\mathrm{t})$ possesses the spectral density matrix of the diagonal form

$$\boldsymbol{S_{FF}}(\omega) = m_j^2 \mathrm{x_y}^{-2} G_{X_i}^{\zeta_o}(\omega)\, \mathbf{I}_{NxN}, \qquad j = 1, 2\dots, N \tag{11}$$

In the following, a statistical linearization approach [11,15] is employed for determining the response power spectrum matrix $\boldsymbol{S_{xx}}(\omega)$. A linearized version of Eq.(10) is given in the form

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \left(\boldsymbol{C} + \boldsymbol{C_{eq}}\right)\dot{\boldsymbol{x}}(t) + \left(\boldsymbol{K} + \boldsymbol{K_{eq}}\right)\boldsymbol{x}(t) = -\,\boldsymbol{F}(t). \tag{12}$$

Next, relying on the standard assumption that the response processes are Gaussian, the elements of the equivalent linear matrices $\boldsymbol{C_{eq}}$ and $\boldsymbol{K_{eq}}$ are given by the expressions

$$c_{j,l}^{eq} = E\left[\frac{\partial g_j}{\partial \dot{x}_l}\right], \tag{13}$$

and

$$k_{j,l}^{eq} = E\left[\frac{\partial g_j}{\partial x_l}\right]. \tag{14}$$

Further, the fourier transform of the response cross-correlations matrix defined by convoluting the impulse response function matrix with the vector of the applied stochastic seismic loads leads for the general case of a *N*-degree-of-freedom system in the celebrated frequency domain relation [11,15]

$$\boldsymbol{S_{xx}}(\omega) = \boldsymbol{H_x}(i\omega)\boldsymbol{S_{FF}}(\omega)\boldsymbol{H_x}^{T^*}(i\omega), \tag{15}$$

where the superscripts (T) and (*) denote matrix transposition and complex conjugation, respectively, and the non-symmetric frequency response function (FRF) matrix is defined as

$$\boldsymbol{H_x}(i\omega) = \left[\left[\left(\boldsymbol{K} + \boldsymbol{K_{eq}}\right) + \boldsymbol{M}(i\omega)^2\right] + i\omega\left(\boldsymbol{C} + \boldsymbol{C_{eq}}\right)\right]^{-1}. \tag{16}$$

Furthermore, the cross–variance of the response due to a vector of stochastic excitation processes characterized by power spectra of the form $G_{X_i}^{\zeta_o}(\omega)$ can be evaluated as

$$E\left[x_j(t)x_l(t)\right] = \int_{-\infty}^{\infty} S_{x_j x_l}(\omega)\, d\omega \tag{17}$$

where $S_{x_j x_l}(\omega)$ is the $(j,l)^{th}$ element of the response power spectrum matrix $\mathbf{S_{xx}}(\omega)$. It can be readily seen that Eqs.(12-17) constitute a coupled nonlinear system of algebraic equations to be solved iteratively for the system response covariance matrix. Further for the j^{th} degree of freedom, using Eq.(8), Eq.(11), Eq.(15) and Eq.(17) yields

$$E\left[x_j^2(t)\right] = \int_{-\infty}^{\infty} \left(\left|\mathrm{H}_{x_{j1}}(i\omega)\right|^2 \mathrm{m}_1^2 + \left|\mathrm{H}_{x_{j2}}(i\omega)\right|^2 \mathrm{m}_2^2 + \cdots + \left|\mathrm{H}_{x_{jd}}(i\omega)\right|^2 \mathrm{m}_d^2\right) \mathrm{x}_y^{-2}\, G_{X_i}^{\zeta_o}(\omega)\mathrm{d}\omega, \tag{18}$$

and

$$E\left[\dot{x}_j^2(t)\right] = \int_{-\infty}^{\infty} \omega^2 \left(\left|\mathrm{H}_{x_{j1}}(i\omega)\right|^2 \mathrm{m}_1^2 + \left|\mathrm{H}_{x_{j2}}(i\omega)\right|^2 \mathrm{m}_2^2 + \cdots + \left|\mathrm{H}_{x_{jd}}(i\omega)\right|^2 \mathrm{m}_d^2\right) \mathrm{x}_y^{-2}\, G_{X_i}^{\zeta_o}(\omega)\mathrm{d}\omega. \tag{19}$$

To this end, Eqs.(18) and (19) provide with estimates of the normalized response displacement and velocity variance corresponding to each and every DOF of the nonlinear MDOF structural system subject to a vector of stochastic seismic excitation processes characterized by a power spectrum compatible in the stochastic sense delineated in section 2 with an assigned pseudo-acceleration design/response spectrum $\mathrm{S}_\alpha(\omega_i, \zeta_o)$.

3.2 Derivation of Effective Linear SDOF Oscillators Properties

Modal analysis is typically employed for decoupling the coupled linear/linearized differential equation of motion, however the requirement of a special form of damping (e.g., Rayleigh) imposes limitations to the method. In this section, an efficient decoupling approach, which can readily address arbitrary forms of damping matrices, for determining a stochastically equivalent linear SDOF system for each DOF is outlined [10]. In the herein study an auxiliary effective linear oscillator corresponding to the j^{th} degree of freedom is defined as

$$\ddot{x}_j(t) + 2\zeta_{ef_j}\omega_{ef_j}\dot{x}_j(t) + \omega_{ef_j}^2 x_j(t) = -\ddot{\alpha}_g(t), \tag{20}$$

where the variables ω_{ef_j} and ζ_{ef_j} are the effective natural frequency and damping ratio, respectively. In this regard, by equating the expressions for the variances of the response displacement and velocity of the auxiliary effective linear oscillator, expressed utilizing the FRF corresponding to Eq.(20), with the corresponding ones determined via Eqs.(18-19) yields

$$E\left[x_j^2(t)\right] = \int_{-\infty}^{\infty} \left|H_{x_{ef_j}}(i\omega)\right|^2 x_y^{-2}\, G_{X_i}^{\zeta_o}(\omega)d\omega, \tag{21}$$

$$E\left[\dot{x}_j^2(t)\right] = \int_{-\infty}^{\infty} \omega^2 \left|H_{x_{ef_j}}(i\omega)\right|^2 x_y^{-2}\, G_{X_i}^{\zeta_o}(\omega)d\omega. \tag{22}$$

where

$$H_{x_{ef_j}}(i\omega) = \left[(i\omega)^2 + i2\zeta_{ef_j}\omega_{ef_j}\omega + \omega_{ef_j}^2\right]^{-1} \tag{23}$$

Clearly, Eqs.(21) and (22) in conjunction with Eqs.(18) and (19) constitute a nonlinear system of two algebraic equations to be solved for the evaluation of the linear oscillator effective natural frequency ω_{ef_j} and damping ratio ζ_{ef_j} coefficients. In this setting, determining the effective natural frequency ω_{ef_j} is especially important for a number of reasons such as tracking and avoiding moving resonance phenomena [10,12], or developing efficient approximate techniques for determining nonlinear system survival probability and first-passage PDF [13,17].

3.3 Efficient Stochastic Iterative Scheme for Updated Design Power Spectra

The proposed methodology incorporates an efficient iterative scheme which includes successive solution of an inverse stochastic dynamics problem for the determination of a series of seismically induced stochastic processes characterized by power spectra compatible in a stochastic sense with the assigned elastic response UHS. Relying on statistical linearization and utilizing the efficient decoupling approach, the nonlinear *N*-degree-of-freedom system is decoupled and cast into (*N*) effective linear SDOF oscillators with effective natural frequency ω_{ef_j} and damping ratio ζ_{ef_j}. Next, the derived effective damping coefficients ζ_{ef_j} redefine the damping ratios of the updated input elastic response UHS which in turn define stochastically compatible design spectrum power spectra. The aforementioned procedure establishes a cyclic relationship between the stochastically equivalent damping coefficients of the effective linear SDOF oscillators $\zeta_{ef_j}^{out}$ and the damping ratios of the input elastic response UHS $\zeta_{ef_j}^{in}$. Lastly, the values of the derived effective linear coefficients attained at the last iteration when convergence has been achieved between $\zeta_{ef_j}^{in}$ and $\zeta_{ef_j}^{out}$, are used in conjunction with the corresponding UHS to approximate the peak inelastic system response of every DOF of the system.

Concisely, the developed stochastic dynamics framework includes the following steps:

a) Determination of a series of seismically induced stochastic processes characterized by power spectra compatible in a stochastic sense with the assigned elastic response UHS.
b) Statistical linearization treatment of the nonlinear MDOF system subject to a vector of stochastic seismic processes characterized by power spectra defined in the first step.
c) Efficient decoupling approach for the determination of (*N*) effective linear SDOF oscillators with effective natural frequency ω_{ef_j} and damping ratio ζ_{ef_j} properties.
d) Redefinition of a series of stochastic seismic processes compatible with the UHS adjusted to the oscillator stochastically derived effective damping ratios.
e) An efficient iterative scheme is devised achieving convergence between the damping ratio coefficient of the effective linear oscillator corresponding to the j-th DOF, and the UHS corresponding to the same j-th DOF and related to the same damping ratio coefficient.

In this regard, the analyst/engineer can readily resort to the updated elastic design/response UHS for reading spectral ordinates without the need for utilizing any additional reduction factors for considering the underlying nonlinearity of the system. Moreover, the problem of utilizing an arbitrary damping ratio of questionable value for the initial input elastic design UHS is addressed as the proposed iteration scheme provides with informed values of the effective damping ratios which are used in a straightforward manner for the redefinition of the most appropriate elastic response UHS and its successive spectral ordinates reading.

The developed stochastic dynamics technique exhibits a number of noteworthy and novel characteristics such as: (i) it accounts for nonlinear and MDOF structural systems, (ii) it provides with efficient inelastic peak response estimates by avoiding NRHA, (iii) it exhibits enhanced accuracy as compared to [5] due to the novel feature of updating the response UHS over the iterative process; the UHS is treated as a unknown variable of the system of nonlinear equations rather than a constant parameter, (iv) the latter point provides with updated values for the elastic response UHS. Note that the attribute of identifying the most appropriate UHS to be used by the analyst/designer constitutes an additional advantage of the technique.

4 Numerical Application

One of the widely used models in earthquake engineering is the Bouc-Wen model [11] that allows to include hysteretic phenomena. Having adopted basic shear-beam idealizations, the j^{th} inter-story restoring force can be given as the composition of an elastic and a hysteretic part

$$\Phi_{s_j}(t) = \alpha k_j y_j(t) x_y^{-1} + (1-\alpha) k_j z_j(t), \tag{24}$$

where the parameter α stands for the rigidity ratio, k_j is the initial elastic stiffness, $y_j(\mathrm{t})$ is the inter-story drift and x_y is the yielding displacement. The additional hysteretic variable $\mathrm{z}_j(\mathrm{t})$ is a state variable so that each story of the structure is now described by a triplet of state variables, i.e. inter-story drift, velocity and hysteretic variable. The constitutive law is introduced by

$$\dot{\mathrm{z}}_{\mathrm{j}}(\mathrm{t}) = \mathrm{x}_{\mathrm{y}}^{-1}\left\{\mathrm{A}\dot{\mathrm{y}}_{\mathrm{j}}(\mathrm{t}) - \beta\dot{\mathrm{y}}_{\mathrm{j}}(t)\left|\mathrm{z}_{\mathrm{j}}(t)\right|^{\mathrm{n}} - \gamma\left|\dot{\mathrm{y}}_{\mathrm{j}}(t)\right|\mathrm{z}_{\mathrm{j}}(t)\left|\mathrm{z}_{\mathrm{j}}(t)\right|^{\mathrm{n}-1}\right\}. \tag{25}$$

where the parameters $\mathrm{A}, \beta, \gamma$ and n are capable of representing a wide range of hysteresis loops (in the herein study $\mathrm{A} = 1$, $\beta = \gamma = 0.5$, $\mathrm{n} = 1$ and $\alpha = 0.15$). In this section, the proposed methodology is numerically illustrated using a yielding three-storey building structure which is modeled as a nonlinear/hysteretic three-DOF structural system subject to stochastic seismic excitations defined by the Eurocode 8 (EC8). All floors are assumed to be rigid and have a constant height equal to 3 m. The masses of the plates are lumped at the floor levels and a value of $\mathrm{m_{plate}} = 9.5 \times 10^4$kg is considered herein. The Young's modulus and the mass density are taken equal to 30×10^9Pa and 2.5×10^3 kg/m^3 respectively. The yielding displacement x_y is equal to $5cm$. Further, columns cross-section dimensions for a given floor are assumed to be equal, and thus, a design vector $\mathbf{r}$ can be defined, having one component for every story, i.e. the width of the square cross-sections. Considering the normalized inter-story drifts $\mathrm{x_j}$ as well as the additional states $\mathrm{z_j}$, the three-DOF hysteretic structural system is governed by Eq.(10), where the nonlinear vector function is defined as

$$\boldsymbol{g}[\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)] = [\mathbf{0}_{\mathbf{1}\times \boldsymbol{N}}\ \dot{\boldsymbol{z}}_{\mathbf{1}\times \boldsymbol{N}}]^{\,T}. \tag{26}$$

Next, a pseudo-acceleration design/response spectrum prescribed by EC8 for soil conditions B, damping ratio $\zeta_o = 5\%$, and peak ground acceleration (PGA) equal to $0.36g$ is initially considered (Figure 1). In the ensuing analysis the duration T_s is taken equal to $20s$, whereas the discetization step is set to $\Delta\omega = 0.1\, rads^{-1}$. The parameters values of the CP input shape power spectrum are $\xi_g = 0.78$, $\omega_g = 10.78\, rads^{-1}$, $\xi_f = 0.92$ and $\omega_f = 2.28\, rads^{-1}$. For illustration purposes a multi-storey building structure characterized by the following design vector $\mathbf{r} = [25, 25, 25\,]^T\ cm$ is considered. Note that the convergence rate is reasonably fast, and the stabilization of the estimates of the effective damping ratio ζ_{ef_j} elements, which are function of $\zeta_{\mathrm{ef}_j}^{in}$ is achieved after six iterations. The convergence process is terminated when successive values of the estimated effective damping ratio display difference lower than a threshold β (e.g. $\beta = 10^{-4}$). Lastly, the values of the stochastically derived ELPs attained at the last iteration when convergence has been achieved between $\zeta_{\mathrm{ef}_j}^{in}$ and $\zeta_{\mathrm{ef}_j}(\zeta_{\mathrm{ef}_j}^{in})$ are plotted on the updated inelastic response spectrum of S_{a-d} format presented in Figure 2.

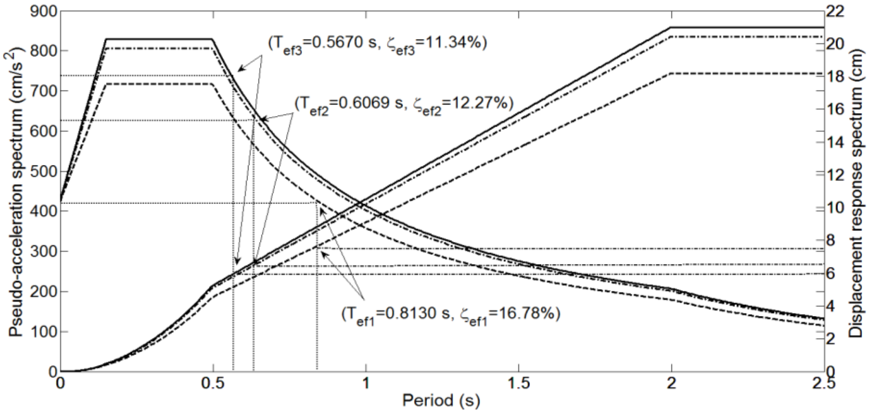

Figure 2: Inelastic design/response spectrum in $S_{a-d}\left(T_{ef_j}, \zeta_{ef_j}\right)$ format for PGA= $0.36g$ and soil conditions B.

Further, proposed methodology based data are compared with pertinent Monte Carlo simulation data utilizing an ensemble of 1,000 realizations. Specifically, non-stationary time-histories stochastically compatible with the design spectrum consistent power spectrum of Eq.(8) [6], for the case of CP input spectral shape (see Figure 1) are generated based on the spectral representation technique in [16]. Next, the nonlinear differential equation of motion (Eq.(10)) is numerically integrated via a standard fourth order Runge-Kutta scheme, and finally, system response statistics are obtained based on the ensemble of the response realizations. Relevant results are presented in Table 1, illustrating a satisfactory achieved degree of accuracy. The error in the estimation of the peak inelastic displacement (PID) response is defined as

$$error_{j,\kappa} = \frac{\left|PID_{j,k}^{\boldsymbol{Analytical}} - PID_j^{\boldsymbol{MCS}}\right|}{PID_j^{\boldsymbol{MCS}}}, \tag{27}$$

where k stands for the iteration index.

Table 1: Error (%) in the estimation of the peak inelastic displacement responses through the iterations of the proposed methodology. Comparison with MCS-based results

Iteration	1st	2nd	3rd	4th	5th	6th	MCS (cm)
1st DOF	*6.52*	*7.37*	*4.25*	*4.82*	*4.67*	*4.67*	*7.06*
2nd DOF	*7.57*	*0.16*	*0.48*	*0.16*	*0.16*	*0.16*	*6.21*
3rd DOF	*7.97*	*0.16*	*1.33*	*1.16*	*1.16*	*1.16*	*6.02*

For the cases of a relatively weak non-linearity (e.g. 2nd and 3rd DOF) it is seen from Table 1 that the proposed methodology is particularly accurate. As the degree of non-linearity increases (e.g. 1st DOF) the achieved degree of accuracy will tend to decrease, remaining however, within reasonable levels.

The proposed methodology can be seen as a system identification procedure which redefines/updates incrementally the elastic design/response UHS, considering appropriately the underlying nonlinearity of the system. In this regard, the analyst/engineer can readily resort to the standard graphical spectral ordinates reading [3], without the need of utilizing any additional reduction factors. Note in passing that the proposed scheme can be applied in a straightforward manner to address cases of nonlinear structural systems having a large number of DOFs. Further, the proposed methodology is characterized by considerable versatility since it is liberated from the dependency on the damping ratio of the imposed elastic design/response spectrum, as well as the form of damping itself.

5 Concluding Remarks

An efficient nonlinear stochastic dynamics methodology has been developed for estimating the peak inelastic response of hysteretic multi-degree-of-freedom (MDOF) structural systems subject to seismic excitations specified via a given uniform hazard spectrum (UHS), without the need of undertaking computationally demanding non-linear response history analysis (NRHA). The proposed methodology initiates by solving a series of inverse stochastic dynamics problems for the determination of input power spectra compatible in a stochastic sense with a given elastic response UHS of specified damping ratio. Relying on statistical linearization and utilizing an efficient decoupling approach the nonlinear N-degree-of-freedom system is decoupled and cast into (N) effective linear single-degree-of-freedom (SDOF) oscillators with effective linear properties (ELPs): natural frequency and damping ratio. Subsequently, each DOF is subject to a stochastic process compatible with the UHS adjusted to the oscillator effective damping ratio. Next, an efficient iterative scheme is devised achieving convergence of the damping coefficients of all the N effective linear SDOF oscillators and the UHS corresponding to each DOF. Finally, peak inelastic responses for all N DOFs are estimated through the UHS for the N different sets of SDOF oscillators ELPs.

The accuracy of the developed approach is numerically demonstrated using a yielding 3-storey building exposed to the Eurocode 8 (EC8) UHS following the Bouc-Wen hysteretic model.

NRHA involving an ensemble of EC8 non-stationary compatible accelerograms is conducted to assess the accuracy of the proposed approach in a Monte Carlo-based context.

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