Valuing Voluntary Disclosure with Competitive Interactions using a Real Options Approach

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Abstract

This paper examines the impact of competition, in a duopoly framework, on the voluntary disclosure policy of firms. Each firm is assumed to have invested in a specific product and the manager of each firm must then decide when to optimally disclose its involvement in the product to the market, while taking into consideration the disclosure strategy of the other firm. A preemption, attrition, or synergy equilibrium is found, depending on the trade-off between first and second mover advantages and, also on the advantage from simultaneous disclosure.

Keywords: Voluntary disclosure, Real options, Preemption game, War of attrition, Synergy equilibrium.

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1 Introduction

In Delaney and Thijssen [2], the concept of a disclosure option is proposed, which is a real option available to a firm to voluntarily disclose information to the market. The option to disclose, or withhold, information is a strategic decision on the part of the firm, implying that the manager will only announce the information if he is sufficiently certain that the market response will have a positive impact on the firm’s value. Therefore, a firm with an opportunity to make a disclosure is holding an option which is analogous to a financial option. When the manager discloses some private information to the investors, he exercises his option to disclose. By doing so, he gives up the possibility of waiting for newer information to arrive that might affect the desirability of the firm’s stock, and hence, have a greater positive impact on its profitability. Hence, the option to wait has value.

However, firms may not always have the option to delay or withhold its information disclosure. There can be occasions which make it imperative for a firm to disclose quickly, such as in the face of competition. They must then try to preempt disclosure by competitors, which could have a negative impact on their own profit, relative to the profit of a competing firm. Hence, there is a non-exclusivity feature inherent in a real option which is not associated with its financial counterpart. If, on the other hand, delay is feasible, the risk of disclosure by competing firms, is a cost to delay. The manager of the firm must weigh this cost against the benefit(s) of waiting for new information when deciding on what his optimal disclosure strategy ought to be. In such a setting, one must conduct a game-theoretic analysis of equilibrium disclosure strategies. In this paper I extend the analysis outlined in Delaney and Thijssen [2] to estimate the impact of competition on the timing of corporate voluntary disclosure.

This research primarily touches on two streams of literature; the litera-
ture that deals with voluntary disclosure and the literature that addresses the non-exclusivity feature inherent in a real option, in particular, the issue of imperfect competition. While corporate voluntary disclosure has become an important and topical area of research in recent years, particularly in the accounting literature (see Verrecchia [12] for a detailed discussion), there have been very few real option applications concerned with voluntary disclosure and none, as far as I am aware, concerning competitive interactions between firms in determining equilibrium exercise policies from a real options perspective. Therefore, such an analysis provides an interesting and useful contribution to the literature.

In terms of the competitive aspect of voluntary disclosure, the literature deals with the dilemma with regard to information sharing between firms. According to Bettis [1], “much of the information that would make cash-flow more forecastable for the shareholder is the same information that is competitively valuable. Typical examples include detailed discussions of strategy, new product characteristics, market share objectives, new process innovations and plant costs and capacities”. Furthermore, “the information that investors need to forecast future cash-flow with less uncertainty is the same information a competitor may be able to use to thwart the realisation of this cash-flow. Thus, information disclosure becomes a trade-off with investors and competitors working at cross purposes”.

Models in which competitive issues give rise to a preference for withholding disclosures include Gal-Or [6], Li [7], and Spulber [9]. A key finding in these studies is that the particular form of competition (Bertrand versus Cournot) can have a substantial influence on the firm’s ex ante preference for disclosure.

Dye and Sridhar [4] show that competitive pressures can lead to an increase in disclosures. Roughly stated, if as competitor discloses information, a firm feels pressured to do the same so as to affirm to investors that it too has information worthy of disclosure.
In terms of real option applications concerned with competitive equilibrium in exercise policies, the literature is relatively scant. Furthermore, the application of game theory to continuous-time models is not very well developed, and can be quite difficult to implement. However, from the literature that does exist, the generalisation of the real option approach to include competitive equilibrium exercise strategies appears to provide very different implications from the standard monopoly setting. For example, one of the most well known results in the real options literature is the invalidation of the classical NPV rule of investment. However, the inclusion of competitive access to an investment opportunity leads to a rapid erosion in the value of the option to wait, making the standard NPV rule a much more accurate description of the actual investment threshold.

This typical result is shown clearly in a basic example provided by Dixit and Pindyck [3]. The example they present is based on Smets [8] and essentially it demonstrates the tradeoff between the value of waiting and the fear of preemption by a rival which suggests the need to invest sooner. The parameters of the model determine which of these considerations holds most weight.

This paper most closely resembles Thijssen et al. [11] from the perspective of real options analysis. However, the crucial difference, in terms of technicalities, is that they assume that the value of an unprofitable outcome from option exercise is always zero, while this paper does not make such an assumption and allows for a negative impact from option exercise. By not relaxing the negative impact assumption, the current paper makes a noteworthy contribution in that it shows how a new equilibrium emerges whereby preemption is nonsensical. This so-called “synergistic” equilibrium implies that the optimal strategies of a firm is to never announce, or else to do so only at the same time as its competitor; that is, simultaneous disclosure.

In this paper I consider two firms whose managers each have the opportu-
nity to disclose to the market some private information about the profitability of a product, or technology, in which each firm has invested. Information signals indicating the strength of the product’s profitability are obtained by the managers at random points in time. Hence, disclosing the signals will impact positively or negatively on the value of their respective firms. It is assumed that prior to disclosure, the market is unaware that the firm has invested in the product. Thus, signal disclosure is analogous to disclosing that they have made an investment in the product.

In Delaney and Thijssen [2], a threshold is derived on the probability of a positive shareholder response, above which the manager will opt to make an announcement and otherwise withhold the information from the market. The problem is solved as an optimal stopping problem by examining a number of scenarios whereby the manager has the option to disclose some set of signals to the market. The current paper uses this threshold as a benchmark to examine how the influence of competition (in a duopoly framework) impacts on the disclosure timing decision of a firm. A preemption, attrition, or synergy equilibrium is found, depending on the trade-off between first and second mover advantages and, also on the advantage from simultaneous disclosure.

This paper is arranged as follows; the set-up of the model is described in the next section, while Section 3 outlines some of the equilibrium concepts for timing games that have been developed by Fudenberg and Tirole [5] and outlined further in Thijssen et al. [11]. Section 4 solves for the equilibria of the game. Section 5 provides a numerical example to help better explain the theoretical results. Section 6 finally concludes.
2 Model

2.1 Background and Motivation

Consider two firms, both of which have invested in a new product or technology, and the problem for the manager of each firm is to determine at what point to disclose this information to the market, whilst taking into account the other firm’s potential strategy. I assume that the product is still in the developmental stage and signals regarding the progress of the development, which are indicative of the potential profitability of the product for the firm, are obtained by both managers at random points in time. The uncertainty primarily arises from the managers’ being unsure how the market will respond to such an announcement. The more positive the signals they obtain, the more likely the market will interpret the information favourably. Hence, each time a signal is obtained, the managers update their beliefs as to the likely market response, in a Bayesian way. It is important to assume that each firm can choose to abandon their investment at any point, before they launch product. Thus, by choosing not to make an announcement, if the firm then abandons the investment, the market may never learn that such an investment took place. This implies another option for the managers; namely a divestment option. However, taking account of the value of such an option is beyond the scope of this paper.

I assume that both firms compensate their managers via stock options, and hence, for each manager, the activities of their firm impacts upon their own utility from wealth. Their objectives are, then, to maximise the discounted expected current value of their respective firms.

In terms of the competitive aspect of the problem there are two possibilities; either a Stackelberg competition arises or both firms disclose simultaneously.

After disclosure has taken place by at least one firm, the other firm then
knows how the market interprets the signals or, equivalently, they know the market’s interpretation of such a firm’s prospects given that they have invested in the product. Hence, in the case of Stackelberg competition, there is an information spillover from the leader to the follower, which creates a second mover advantage. The follower then decides immediately on whether to reveal his involvement in the product. It is assumed that this does not take any time (see Thijssen et al. [11]). If one firm discloses at a time $\tau \geq 0$, the follower will either disclose at $\tau$ as well, or not at all. This case is distinguished from the case of simultaneous competition where both firms also disclose simultaneously at $\tau$. However, at the time of disclosure both firms are uncertain as to how the market will respond to the announcement; that is, there is no second mover advantage.

As in Delaney and Thijssen [2], I further assume that all disclosures are (ex post) verifiable; that is, a manager will not issue mis-leading information in an attempt to alter the market’s perception of his firm’s prospects.

### 2.2 Model Set-Up

The managers both hold an option to voluntarily disclose their involvement in the product, via the signals they obtain, to the market and they are uncertain about how this information will be perceived. If the revelation is regarded positively by the market, this will result in an increase in the value of the firm by an amount $V^P$ or, if regarded negatively, a fall in the value by an amount $V^N$, when the announcement is made, such that $V^N < 0 < V^P$. $V^P$ and $V^N$ are the infinitely discounted values resulting from making a disclosure, discounted at a constant rate $r \in (0, 1)$.

In the case that the market response is favourable, the leader’s payoff equals $V^P_L > 0$, whereas if it is not favourable, the payoff is $V^N_L < 0$. If the response is favourable, the follower will immediately disclose and obtain $V^P_F > 0$, but
will not disclose if the response is unfavourable, so $V_N^F = 0$. Without loss of generality, it is assumed $V_L^P > V_F^P > 0$. Hence there is a first mover advantage if the disclosure results in a positive market response, and disclosure is profitable for both firms. In other words, the positive payoff to the manager who is first to disclose is greater than the positive payoff to the manager who follows and discloses in response to the leader’s action.

If the market response is not positive, the payoff is $V_L^N < 0$ (or a first mover disadvantage). If the response is unfavourable, the follower observes this and benefits because he can make his disclosure decision under complete information. This information spillover to the follower when the leader has disclosed earns him a second mover advantage. To ascertain whether the leader or the follower is in the better position the magnitudes of the first and second mover advantages have to be compared. If both firms disclose simultaneously, and the response is positive (negative), both receive $V_M^P > 0$ ($V_M^N < 0$), such that $V_F^P < V_M^P < V_L^P$ and $V_L^N < V_M^N < 0$.\(^1\)

When a firm has the option to disclose its involvement in the product to the market, it is assumed that the manager has some \textit{ex ante} belief about the market reaction to the announcement being either positive or negative. The prior probability of a positive reaction, and therefore, an increase in the firm’s value is given by

$$P(V^P) = p_0,$$

and this is identical for both firms.

I further assume that at some random points in time, both firms obtain imperfect signals, from various sources, indicating whether the profitability of their investment product is positive or negative. A high quality signal occurs with probability $\theta > \frac{1}{2}$. The signals are observed by both firms simultaneously as both have invested in the same product.

\(^1\)Note that the payoffs are regarded as an infinite stream of earnings per share, $\pi_i^j$, discounted at rate $r > 0$; i.e. $V_j^i = \int_0^\infty e^{-rt} \pi_i^j dt = \frac{1}{r} \pi_i^j$, $i = P, N$ and $j = L, M, F$. 
The signal arrivals are modelled via a Poisson process with parameter $\mu > 0$; that is, the probability that the manager obtains a signal is $\mu dt$ and $1 - \mu dt$ if no new signal is obtained.

It is assumed that both firms have an identical belief $p_t \in (0, 1)$ that the market response will be positive at time $t$. Similar to Thijssen et al. [11], I denote by $p_M$ the belief such that the \textit{ex ante} expected payoff for the follower equals the \textit{ex ante} expected payoff of simultaneous disclosure; that is, $p_M$ is such that $F(p_M) = M(p_M)$. When $p_t \geq p_M$, both firms will disclose simultaneously, before the market interpretation is known. On the other hand, if one firm discloses when $p_t < p_M$, it is not optimal for the other firm to do so also at time $t$.

If the leader discloses at a point where the belief in a positive response is $p_t$, the leader’s \textit{ex ante} expected payoff can be written as

$$L(p_t) = \begin{cases} p_t V_P^L + (1 - p_t) V_N^L & \text{if } p_t < p_M \\ p_t V_P^L + (1 - p_t) V_N^L & \text{if } p_t \geq p_M. \end{cases}$$

(2)

The follower only discloses in the case of a positive market response. Hence, the \textit{ex ante} expected payoff for the follower, if the leader discloses when the belief in a positive response is $p_t$, is given by

$$F(p_t) = \begin{cases} p_t V_P^F & \text{if } p_t < p_M \\ p_t V_P^F + (1 - p_t) V_N^F & \text{if } p_t \geq p_M. \end{cases}$$

(3)

Finally, in the case of simultaneous disclosure at belief $p_t$, each firm has an \textit{ex ante} expected payoff given by

$$M(p_t) = p_t V_P^M + (1 - p_t) V_N^M.$$  

(4)

The preemption belief, denoted $p_P$, is defined as being the belief at which the managers are indifferent between being the leader or the follower; that is, $L(p_P) = F(p_P)$. Equating equations (2) and (3) gives the preemption belief

$$p_P = \frac{V_N^L}{V_N^M - V_P^L + V_F^P}.$$  

(5)
which is well defined for $V^N_L < 0$.

Furthermore, since $F(p_M) = M(p_M)$,

$$p_M = \frac{V^N_M}{V^N_M - V^P_M + V^P_F},$$

(6)

which is the belief threshold above which both firms find it optimal to disclose simultaneously.

A graphical depiction of the situation is given by Figure 1 for a specific numerical parameterisation defined in Section 5.

It must be noted that a knife-edge result on whether the preemption belief threshold is below the threshold where both managers disclose simultaneously is not possible to obtain. However, $p_P < p_M$ for

$$V^P_L - V^P_F > (V^P_M - V^P_F) \frac{V^N_L}{V^N_M}. \quad (7)$$

Indeed, if $p_P > p_M$ preemption is nonsensical since a point above which both firms try preempting each other cannot intuitively exist after both firms have disclosed simultaneously. This point is particularly important because it is the main technical difference between the equilibrium derived by Thijssen et al. [11]. They assume that $V^N_j = 0$ for $j = L, M, F$, and hence, they find that it is always the case that $p_P < p_M$. However, in this paper, $V^N_L$ and $V^N_M$ are assumed to be non-zero, and consequently the inequality $p_P < p_M$ does not always hold.

The intuition governing equation (7) is the following; the simultaneous disclosure effect outweighs the information spillover; i.e. $p_P < p_M$, when the magnitude of the first mover advantage, $V^P_L - V^P_F$, is greater than a multiple $V^N_L/V^N_M (> 1)$ of the cost to the follower from not making a simultaneous disclosure with the leader, $V^P_M - V^P_F$.

To compute the managers’ belief that there will be a positive response to the announcement, $p_t := p(s_t)$, one must apply Bayes’ rule because their
beliefs as to the profitability of the product are updated each time a new signal arrives. This is given by (cf. Thijssen et al. [10])

\[ p(s_t) = \frac{\theta^{s_t}}{\theta^{s_t} + \zeta(1 - \theta)^{s_t}}, \tag{8} \]

where \( \zeta = \frac{1-p_0}{p_0} \) denotes the unconditional odds of a negative shareholder response and \( s_t \) the number of positive signals in excess of negative signals obtained by the managers at time \( t \).

Furthermore, from equation (8), a solution for \( s_t \) can be obtained and is given by

\[ s_t = \frac{\log \left( \frac{1-p_t}{p_t} \right) - \log \zeta}{\log \left( \frac{1-\theta}{\theta} \right)}. \tag{9} \]

### 3 Equilibrium Concepts for Timing Games

In this section I outline a formalisation of strategy spaces and payoffs for continuous-time games. This formalisation is developed in Fudenberg and Tirole [5] and Thijssen et al. [11].

Let \( (\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq \infty}, \mathbb{P}) \) be a filtered probability space such that \( \mathcal{F}_0 \) contains all the \( \mathbb{P} \)-null sets of \( \mathcal{F} \) and the filtration \( (\mathcal{F}_t)_{0 \leq t \leq \infty} \) is right-continuous. It is assumed that the stochastic process \( (Y_t)_{t \geq 0} \) captures all of the uncertainty on the filtered probability space.

In the current paper, a player’s only decision is to choose a (single) time to disclose information signals to the market. The starting point of the game is \( t = 0 \). A strategy consists of both a distribution function and an intensity function. All the definitions below are defined for each path \( (Y_t(\omega))_{t \geq 0} \), resulting from \( \omega \in \Omega \).

**Definition 1.** A simple strategy for player \( n \in \{i, j\} \) in the subgame starting at \( t_0 \in [0, \infty) \) is given by a tuple of real-valued functions \( (G_n^{t_0}, \alpha_n^{t_0}) : [t_0, \infty) \times \Omega \to [0, 1] \times [0, 1] \), such that for all \( \omega \in \Omega \)
1. $G^t_n(\cdot, \omega)$ is non-decreasing and right continuous with left limits;

2. $\alpha^t_n(\cdot, \omega)$ is right continuous with left limits;

3. if $\alpha(\cdot, \omega) = 0$ and $t = \inf\{u \geq t_0 | \alpha^t_n(u, \omega) > 0\}$, then the right-derivative of $\alpha^t_n(u, \omega)$ exists and is positive.

Denote the strategy set of simple strategies of player $n$ in the subgame starting at $t_0$ by $S^*_{n}(t_0, \omega)$. Furthermore, define the strategy space by $S^*_{n}(t_0, \omega) = \Pi_{n=i,j} S^*_{i}(t_0, \omega)$ and denote the strategy at $t \in [t_0, \infty)$ by $s^i(t, \omega) = (G^t_{n_i}(t, \omega), \alpha^t_n(t, \omega))_{n=i,j}.$

Simple strategies allow for several disclosure strategies. $G^t_n$ is interpreted as the cumulative probability that one player has disclosed before, or at, time $t$. Additionally, it allows for continuous disclosure strategies (used in war of attrition models) and single jumps. $\alpha^t_n$ is a measure for the “intensity” of the atoms in the interval $[t, t + dt]$. This intensity function allows for coordination between firms in cases where disclosure by one firm is optimal, but simultaneous disclosure is not. At such points in time, solely using distribution functions $G_i$ leads to inefficient outcomes. Therefore, we allow for a randomisation device which is provided by the intensity function. The intensity function is included in the strategy space to replicate the discrete time results that are lost by modelling in continuous time (see Fudenberg and Tirole [5] for a more in-depth explanation). As soon as the intensity function is non-zero a game is played where both managers disclose with probabilities $\alpha_i$ and $\alpha_j$, respectively. The game is repeated until at least one of the firms has disclosed. Playing the game is assumed to take no time, so that the stochastic process $Y_t$ remains constant during this repetition process. The third condition is imposed for technical convenience.

The definition of simple strategies does not a priori exclude the possibility that both firms choose an intensity function that turns out to be inconsistent with $G^t_n$. To see this, suppose that for some $t \geq 0$ it holds that $\alpha_i(t) = 0.5$
and $\alpha_j = 0$. Then firm $i$ invests with probability 1 at time $t$ (see $P(i, \neg j)$). This should be reflected in the distribution function $G$; i.e., $G_i(t) = 1$. Any other value for $G_i(t)$ would be inconsistent with $(\alpha_i(t), \alpha_j(t))$. In equilibrium it should naturally be the case that such inconsistencies do not occur. Therefore, the notion of $\alpha$-consistency is introduced.

**Definition 2.** A tuple of simple strategies $((G^n_0, \alpha^n_0))_{n=i,j}$ for the subgame starting at $t_0 \geq 0$ is $\alpha$-consistent if for all $\omega \in \Omega$ and $t \geq t_0$,

$$\alpha^n_0(t, \omega) - \alpha^n_0(t, \omega) \neq 0 \Rightarrow G^n_0(t, \omega) - G^n_0(t, \omega) = \frac{(1 - G^n_0(t, \omega)) \alpha^n_0(t, \omega)}{\alpha^n_0(t, \omega) + \alpha^n_0(t, \omega) - \alpha^n_0(t, \omega) \alpha^n_0(t, \omega)}.$$ (10)

Definition 2 requires that if for either firm there is a jump in the intensity function, then the jump in the cumulative distribution function of both firms should equal the probability that the firm discloses by playing the game described above. Note that if $\alpha^n_0(t, \omega) - \alpha^n_0(t, \omega) \neq 0$ and $\alpha^n_0(t, \omega) = 1$, then $\alpha$-consistency implies that $G^n_0(t, \omega) = 1$.

Let the payoff function for firm $n \in \{i,j\}$ in the subgame starting at $t_0$ be given by $V_n : [0, \infty) \times S^s(t_0, \omega) \to \mathbb{R}$. An $\alpha$-equilibrium for the subgame starting at $t_0 \geq 0$ is then defined as follows:

**Definition 3.** A tuple of simple strategies $s^* = (s^*(\omega))_{\omega \in \Omega}$, $s^*(\omega) \in S^s(t_0, \omega)$, for all $\omega \in \Omega$, is an $\alpha$-equilibrium for the subgame starting at $t_0$ if for all $\omega \in \Omega$, $s^*(\omega)$ is $\alpha$-consistent and

$$\forall n \in \{1,2\} \forall s_n \in S^s_n(t_0, \omega) : V_n(t_0, s^*(\omega)) \geq V_n(t_0, s_n, s^*_{-n}(\omega)).$$ (11)

A caveat with $\alpha$-equilibrium is that it does not exclude time inconsistent strategies. Hence, we need a family of strategies $(G^n(t, \omega))$, otherwise known as a closed loop. This closed loop is necessary, because to test for perfectness, strategies conditional on zero-probability events must be defined; in other words it is needed to define a subgame perfect equilibrium. Furthermore, let for all $\omega \in \Omega$ and $t_0 \geq 0$, $\tau$ be defined as $\tau = \min_{n=i,j} \{ \inf\{ t \geq t_0 | \alpha^n_0(t, \omega) > 0 \} \}$. 13
**Definition 4.** A closed loop strategy for player \( n \in \{i, j\} \) is for all \( \omega \in \Omega \) a collection of simple strategies \( (G^n_t(\cdot, \omega), \alpha^n_t(\cdot, \omega))_{0 \leq t < \infty} \), with \( (G^n_t(\cdot, \omega), \alpha^n_t(\cdot, \omega)) \in S^s_n(t, \omega) \) for all \( t \geq 0 \) that satisfies the following intertemporal consistency condition for all \( \omega \in \Omega \):

\[
\forall 0 \leq t \leq u \leq v < \infty : v = \inf \{ \tau > t | Y_\tau = Y_v \} \Rightarrow G^n_t(v, \omega) = G^n_u(v, \omega)
\]

and

\[
\alpha^n_t(v, \omega) = \alpha^n_u(v, \omega).
\]

The set of closed loop strategies for player \( n \in \{i, j\} \) is denoted by \( S^{cl}_n(\omega) \).

As before, the strategy space is defined by \( S^{cl}(\omega) = \Pi_{n=i,j} S^{cl}_n(\omega) \).

A consistent \( \alpha \)-equilibrium is defined as follows:

**Definition 5.** A tuple of closed loop strategies \( \bar{s} = (\bar{s}(\omega))_{\omega \in \Omega} \), \( \bar{s}(\omega) \in S^{cl}(\omega) \) all \( \omega \in \Omega \) is a consistent \( \alpha \)-equilibrium if for all \( t \in [0, \infty) \), the corresponding tuple of simple strategies \( \left((G^n_t, \alpha^n_t), (G^j_t, \alpha^j_t)\right) \) is an \( \alpha \)-equilibrium for the subgame starting at \( t \).

For the remainder of the analysis, let \( \omega \in \Omega \) be fixed. For notational convenience, \( \omega \) will be dropped as an argument.

### 4 Equilibria of the Game

Suppose, for now, that one firm, say firm \( i \), has been preassigned the leader’s role and firm \( j \) can only disclose once the leader has done so. In this case, there exists a \( p_t \in (0, p_M) \) such that the *ex ante* expected payoff for the leader is greater than the *ex ante* expected payoff from simultaneous disclosure; i.e. \( L(p_t) > M(p_t) \). The intuition is that for such a belief, \( p_t \), the leader’s decision has no effect on the optimal response of the follower. Thus, the leader acts as
if there is no follower and becomes a monopolist. From Delaney and Thijssen [2], it is optimal for the leader to disclose when \( p_t \) hits the threshold

\[
p^*_L = \left[ 1 - \frac{V_P}{V_N} \Pi \right]^{-1},
\]

where

\[
\Pi = \frac{(\beta_1(r + \mu) - \mu \theta(1 - \theta)) \left( \frac{r}{\mu} + 1 - \theta \right) - \mu \theta(1 - \theta) \beta_1}{\beta_1(r + \mu) - \mu \theta(1 - \theta) \left( \frac{r}{\mu} + \theta \right) - \mu \theta(1 - \theta) \beta_1}
\] (14)

and

\[
\beta_1 = \frac{r + \mu}{2\mu} + \frac{1}{2} \sqrt{\left( \frac{r}{\mu} + 1 \right)^2 - 4\theta (1 - \theta) > \theta}.
\]

(15)

From equation (9), it is easy to verify that \( s_t \) is increasing in \( p_t \). Then \( s_L := s(p_L) > s_P := s(p_P) \) when \( p_L > p_P \). This is true for

\[
\frac{V_L^P - V_L^F}{V_P^P} > \Pi.
\]

(16)

The left-hand side (LHS) of equation (16) is the cost to the follower for waiting to obtain the information spillover relative to the leader’s payoff. Comparative statics show that this relative cost is increasing in \( V_P^P \). The greater the payoff to the leader, the more the follower “suffers” as a result of not having been the first to disclose. Conversely, this cost is decreasing in \( V_L^F \), which intuitively makes sense.

Comparative statics show that the right-hand side (RHS) of equation (16) is decreasing in signal quality, \( \theta \), and in signal quantity, \( \mu \). Intuitively, the value of the information spillover to the follower is greater when the quality and quantity of the information signals are low; that is for lower values of \( \theta \) and \( \mu \). Hence, if a manager becomes the disclosure leader, he provides relatively more information to his competitor when the quality and quantity of signals are low, and thus, for the RHS of (16) relatively high, compared with when the RHS of (16) is low. This shows that (16) is essentially a relative comparison between the first and second mover advantages.
I further note that $p_P > p_{N_{PV}}^L$. This implies that preemption, in a real options framework, still asserts later disclosure than the traditional net present value (NPV) rule would suggest. However, it is not possible to obtain an unambiguous relationship between $p_{N_{PV}}^L$ and $p_M$; that is, between the classical NPV rule and the point at which firms will disclose simultaneously. This is due to the fact that the condition $p_M > p_P$ only holds for certain values of $V_j^i$ ($i \in \{P, N\}$ and $j \in \{L, M, F\}$).

### 4.1 Preemption

If $p_L > p_P$, the leader advantage outweighs the information spillover. This implies that the firm who first discloses that it has invested in this new product will benefit, through an increase in outside investment, more than the firm who waits to ascertain how the market will react to the information. This is because the signals regarding the product’s development are sufficiently good that each firm wants to be the first to disclose that it has undertaken this investment, and thereby, attain a greater positive impact on its value than the impact it would obtain from being the follower. The likely reaction to the firm that is the follower, while it will be positive, will be more muted than the reaction to the leader’s disclosure, simply because the revelation is less novel from the follower. Additionally, in this instance, the signals are sufficiently strong that neither firm feels the need to wait for their competitor to disclose its decision to invest in the product so that they may observe the market’s reaction to this information. This type of scenario will be covered in the following subsection.

For $p_P \leq p_L < p_M$, the optimal $\alpha^i_n(\cdot)$ function is obtained through maximising firm $i$’s payoff in a competitive game such that if neither firm discloses, the game is repeated, and can be repeated infinitely many times.
I denote firm $i$'s payoff by $V_i$. Then

$$V_i(t_0, s_i, s_j) = \alpha_i \alpha_j M + \alpha_i (1 - \alpha_j)L + (1 - \alpha_i)\alpha_j F + \ldots$$

$$+ \alpha_i \alpha_j (1 - \alpha_i)^{T-1}(1 - \alpha_j)^{T-1}M$$

$$+ \alpha_i (1 - \alpha_i)^{T-1}(1 - \alpha_j)^{T}L$$

$$+ \alpha_j (1 - \alpha_i)^{T}(1 - \alpha_j)^{T-1}F$$

$$= (\alpha_i \alpha_j M + \alpha_i (1 - \alpha_j)L + \alpha_j (1 - \alpha_i)F) \times$$

$$\sum_{t=0}^{T-1} [(1 - \alpha_i)(1 - \alpha_j)]^t.$$  \hspace{1cm} (17)

If $dt$ is the size of one time period and if $T_{\Delta} := Tdt$, then $T - 1 \equiv \frac{T_{\Delta}}{dt} - 1$. Hence, letting $dt \downarrow 0$, the summation over $t$ is from 0 to $\infty$, implying (17) is the infinite sum of a geometric series with common ratio $(1 - \alpha_i)(1 - \alpha_j) < 1$.

Therefore

$$V_i = \frac{\alpha_i \alpha_j M + \alpha_i (1 - \alpha_j)L + \alpha_j (1 - \alpha_i)F}{1 - (1 - \alpha_i)(1 - \alpha_j)}.$$ \hspace{1cm} (18)

Maximising this expression with respect to $\alpha_i$ (and noting that only symmetrical strategies are considered) gives

$$\alpha_j = \frac{L(p_t) - F(p_t)}{L(p_t) - M(p_t)}$$

$$= \frac{p_t (V^P_L - V^F_P) + (1 - p_t) V^N_L}{p_t (V^P_L - V^F_P) + (1 - p_t) (V^N_L - V^N_M)}$$

$$= \alpha_i.$$ \hspace{1cm} (19)

Let $P(i, \neg j|\tau)$ denote the probability that firm $i$ is the only firm that discloses at time $\tau$. By a similar limiting argument to that already outlined in equation (17),

$$P(i, \neg j|\tau) = \frac{\alpha_i (1 - \alpha_j)}{\alpha_i + \alpha_j - \alpha_i \alpha_j}.$$ \hspace{1cm} (20)

If $P(i, j|\tau)$ denotes the probability that both firms disclose simultaneously at $\tau$,

$$P(i, j|\tau) = \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j - \alpha_i \alpha_j}.$$ \hspace{1cm} (21)

To analyse the equilibrium outcome in the preemption game, I consider three separate regions: (i) $p_t < p_P$, (ii) $p_P \leq p_t < p_M$, and (iii) $p_t \geq p_M$. 

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Region 1: If $p_t < p_P$, the payoff to the follower from disclosing at $p_t$ is greater than the payoff to the leader; i.e. $F(p_t) > L(p_t)$. Therefore, neither firm wants to be the first to disclose and both will abstain from disclosing until the excess number of positive over negative signals, $s_P$, has been reached. Intuitively, the excess number of positive signals is insufficient for the manager to be confident of a positive market response to the revelation that they have invested in such a product. Therefore, each firm would prefer to wait until the other firm has disclosed so as to obtain the information spillover before deciding whether to make an announcement or not.

In equilibrium there are two possible outcomes. In the first outcome, firm $i$ is the leader and discloses when the belief is $p_P$ and firm $j$ is the follower and discloses at $p_M$. The second outcome is the symmetric counterpart.

Region 2: If $p_P \leq p_t < p_M$ is the starting point of the game, both firms try to preempt each other to obtain a first mover advantage since $L(p_t) > M(p_t)$. However $p_t < p_M$ implies that the belief in a positive response is not strong enough such that simultaneous disclosure is optimal.

If $p_t = p_P$, recall that $F(p_P) = L(p_P)$, and thus, from equation (19), $\alpha_i = 0$. The probability that $i$ is the only firm that discloses is zero, from (20). Similarly, the probability that both firms disclose simultaneously is zero, from (21). However, firm $j$ invests with probability one because $\mathbb{P}(-i,j|\tau) = \frac{(1-\alpha_i)\alpha_j}{\alpha_i + \alpha_j - \alpha_i \alpha_j} = 1$. Thus, the expected disclosure payoff for firm $i$ is zero and for firm $j$ is $\mathbb{P}(-i,j|\tau)F(p_P) = F(p_P)$.

However, if $p_t < p_P$, $L(p_t) > F(p_t)$ which implies $\alpha_i(p_t) > 0$. The probability that firm $i$ discloses at $p_t$ and firm $j$ at $p_M$ is given by (20). Both firms disclose simultaneously at $p_t$ with probability given by (21), leaving both with
a low payoff $M(p_t)(< F(p_t))$. The expected payoff to each firm is then

$$P(i, ¬j|τ)L(p_t)+P(¬i, j|τ)F(p_t)+P(i, j|τ)M(p_t)$$

$$=\frac{α_i L + α_j F - α_i α_j (L+F-M)}{α_i + α_j - α_i α_j}$$

$$=\frac{F(F-2M+L)}{L-2M+F} ⇔ F(p_t),$$

by substituting for $α_i$ and $α_j$ using equation (19).

Region 3: If $p_t ≥ p_M$, $M(p_t) = F(p_t)$. Therefore, both firms will disclose simultaneously, each getting $F(p_t)$.

Therefore, the overall equilibrium strategy of firm $n ∈ \{i,j\}$ for $p_L > p_P$ is as follows:

$$G^t_n = \begin{cases} 0 & \text{if } p_t < p_P \\ \frac{p_t(V^P_L - V^P_M) + (1-p_t)(V^N_L - V^N_M)}{p_t(V^P_L - 2V^P_M + V^P_F) + (1-p_t)(V^N_L - 2V^N_M)} & \text{if } p_P ≤ p_t < p_M \\ 1 & \text{if } p_t ≥ p_M, \end{cases}$$

and

$$α^t_n = \begin{cases} 0 & \text{if } p_t < p_P \\ \frac{p_t(V^P_L - V^P_M) + (1-p_t)V^N_L}{p_t(V^P_L - V^P_M) + (1-p_t)(V^N_L - V^N_M)} & \text{if } p_P ≤ p_t < p_M \\ 1 & \text{if } p_t ≥ p_M. \end{cases}$$

4.2 War of Attrition

On the other hand, if $p_P > p_L$ the information spillover outweighs the leader advantage. This implies that signals are less strong (in terms of content rather than quality) than in the preemption case, and that both firms are less convinced that the likely market reaction to the news that they have invested in this new product will be positive. For example, if the shareholders learn of the investment the manager has undertaken, they may regard such an investment as too risky a venture and that the sunk investment costs the firm may have incurred are not likely to be recouped. Therefore, it is optimal for the manager of each firm to wait until his competitor has disclosed so that they
can make their own decision over whether to also disclose, or to wait for more signals to arrive, under complete information. That is, he will only disclose if he knows for sure that the market will respond positively to the information. Both firms wish to be the follower so as to obtain the information spillover and protect themselves against a negative response. Hence, a war of attrition arises between the two firms.

For \( p_t > p_P \), the game is exactly the same as the preemption game already discussed. However, if the excess number of signals is such that \( p_t \in [p_L, p_P) \), a war of attrition arises since both firms would prefer to be the follower. The game ends once \( p_P \) is reached. In a war of attrition, two asymmetric equilibria arise trivially; either firm \( i \) discloses with probability one and firm \( j \) with probability zero, or vice versa.

To find a symmetric equilibrium, I argue in line with Thijssen et al. [11] that for each point in time during a war of attrition the expected payoff from disclosing immediately exactly equals the payoff from waiting a small period of time \( dt \) and disclosing when a new signal arrives. The probability that the other firm discloses at belief \( p_t \) is denoted by \( \gamma(s_t) \),\(^2\) and following the analysis outlined in Thijssen et al. [11], \( \gamma(\cdot) \) is given by:

\[
\gamma(s_t) = \frac{1 - \gamma(s_t)}{F(s_t) - M(s_t)} \left[ L(s_t) - \frac{\mu}{r + \mu} \frac{\theta^{s_t+1} + \zeta(1 - \theta)^{s_t+1}}{\theta^{s_t} + \zeta(1 - \theta)^{s_t}} \times \left( \gamma(s_t + 1) \left( M(s_t + 1) - L(s_t + 1) \right) + L(s_t + 1) \right) 
+ \frac{\mu}{r + \mu} \frac{\theta(1 - \theta)(\theta^{s_t-1} + \zeta(1 - \theta)^{s_t-1})}{\theta^{s_t} + \zeta(1 - \theta)^{s_t}} \times \left( \gamma(s_t - 1) \left( M(s_t - 1) - L(s_t - 1) \right) + L(s_t - 1) \right) \right].
\]

To solve for \( \gamma(\cdot) \), note that if \( p_t < p_L \), neither firm will disclose, since the option value of waiting is higher than the expected payoff from disclosing. Therefore \( \gamma(s_L) = 0 \). On the other hand, if \( p_t > p_P \), the firms enter a preemption game.

\(^2\)Of course, this is also the probability that the manager’s own firm has disclosed since the equilibria are symmetric.
It is also possible that $p_P = p_M$, and then the game proceeds directly from a war of attrition into a game where simultaneous disclosure is optimal. Thus, for other values of $p_t$; that is, for $p_t \in [p_L, p_P)$, it is necessary to solve a system of equations where the $p_t$-th entry is given by (25). A system such as this cannot be solved analytically but for any specific set of parameter values a numerical solution may be determined. Thijssen et al. [11] prove that a solution to a system of equations given by (25) always exists, and furthermore, $\gamma \in [0, 1]$.

Defining the time at which preemption occurs by $T_P^{t_0} := \inf\{t \geq t_0|p_t \geq p_P\}$, and the number of signals that has arrived up until time $t$ by $k_t := \sup\{k|T_k^{t_0} \leq t\}$, the symmetric ($\alpha$-consistent) equilibrium is given by

$$G_n^t = \begin{cases} 0 & \text{if } p_t \leq p_L \\ \sum_k \frac{\gamma(s_k)}{1-\gamma(s_k)} \Pi^k_{s_k = k_t} (1-\gamma(s_k)) & \text{if } p_L < p_t < p_P \\ \left(1 - G_n^t(T_P^{t_0} - t)\right) \times \frac{p_t(V_{P}^L - V_{F}^P) + (1-p_t)(V_{N}^L - V_{N}^M)}{p_t(V_{P}^L - 2V_{M}^P + V_{F}^P) + (1-p_t)(V_{N}^L - 2V_{M}^N)} & \text{if } p_P \leq p_t < p_M \\ 1 & \text{if } p_t \geq p_M, \end{cases}$$

(26)

and

$$\alpha_n^t = \begin{cases} 0 & \text{if } p_t < p_P \\ \frac{p_t(V_{P}^L - V_{F}^P) + (1-p_t)V_{N}^M}{p_t(V_{P}^L - V_{F}^P) + (1-p_t)(V_{N}^L - V_{N}^M)} & \text{if } p_P \leq p_t < p_M \\ 1 & \text{if } p_t \geq p_M, \end{cases}$$

(27)

The technicalities of this result are not new to this paper, and thus, the reader is referred to Thijssen et al. [11] for further details.

### 4.3 Synergy

The condition given by equation (7) is necessary for $p_P < p_M$ to be true. However, if the values of $V_i^j$ ($i = P, N$ and $j = L, M, F$) are such that this condition does not hold, then intuitively, there cannot be a preemption point. This produces a type of “synergistic effect” in that the simultaneous revelation that both firms have invested in the product is expected to generate a stronger
positive market response (and thus a higher payoff from disclosure) than stand-alone disclosure by either firm at $p_P$ would generate.

This scenario occurs if the managers both believe that by disclosing with its competitor that it has invested in this new product, the market will react more favourably (or less negatively) than if the firm was to disclose as the leader. In other words, if the market learns that two very similar firms have chosen to undertake an investment in the same product, investors are more reassured of the product’s potential success than if they believed only one such firm had chosen to undertake the investment.

This new type of equilibrium that arises is driven by the assumption in my paper that $V_j^N \neq 0$ for $j = \{L, M\}$. If there was no direct negative impact on the firms’ value through making a disclosure, then an attempt by the managers to temper the extent to which the investors will sell off their firm’s stock would not be an issue. If this was simply an investment problem whereby the sunk costs incurred are the main loss to the firm, then it would be plausible to let $V_j^N = 0$, for all $j$. Indeed, this is the problem examined by Thijsen et al. [11]. However, with respect to disclosure, if the investors do not like what they learn, then they may sell their stock which lowers the firm’s value. If such a negative response were to ensue under the condition that $p_P > p_M$, then revelation that a similar firm has also chosen to invest in this new product will serve to reassure the market of the product’s potential success, and thereby temper the extent of the market sell-off. Conversely, if a positive response were to ensue, simultaneous disclosure would boost the extent of the market’s investment in the firms through a firmer confidence in the product’s success.

Technically, the inequality given by (7) is reversed when the first mover advantage is less than a multiple, $V_L^N / V_M^N$, of the difference between the positive payoff from simultaneous disclosure and the positive payoff obtained by a firm that is the follower. The situation occurs if the negative impact to the leader from disclosing, $V_L^N$, is very strong relative to the negative impact obtained
from disclosing simultaneously with the other firm, $V^N_M$. To see this more clearly, if $V^N_L \to -\infty$, then the RHS of (7) becomes infinitely large, and the condition $p_P < p_M$ no longer holds. Similarly, if the negative payoff obtained from simultaneous disclosure is not particularly low, that is, if $V^N_M \uparrow 0$ then once again, the RHS becomes infinitely large and the condition breaks down.

If the leader effect outweighs the synergy effect, $p_M < p_L$, neither firm will disclose until $p_t \geq p_M$ is reached, and then both will disclose simultaneously each getting the payoff $F(p_M)$. It is never optimal for one firm to disclose on its own and information spillover has no value. This contrasts with the case when $p_P < p_M < p_L$. In this case, the market will learn sooner about the investment since one of the firms will disclose once $p_P$ is reached. The other firm then decides whether to reveal its involvement in the investment or not, and hence, the ensuing market reaction to the two firms’ actions is likely to have different impacts than if they were to only ever disclose together or not at all.

Hence the equilibrium strategy is given by

$$G^t_n = \begin{cases} 
0 & \text{if } p_t < p_M \\
1 & \text{if } p_t \geq p_M,
\end{cases} \quad (28)$$

and

$$\alpha^t_n = \begin{cases} 
0 & \text{if } p_t < p_M \\
1 & \text{if } p_t \geq p_M.
\end{cases} \quad (29)$$

If the synergy effect outweighs the leader effect; i.e. $p_L < p_M$, a war of attrition arises in the region $[p_L, p_M)$ because both firms would prefer to wait and disclose simultaneously rather than be the leader. The analysis is similar to that which yields the equilibrium strategy given by equations (26) and (27),
so it suffices to state that the equilibrium strategy for this scenario is given by

\[
G^t_n = \begin{cases}
0 & \text{if } p_t \leq p_L \\
\left(1 - G^t_n(T^t_L \ominus)\right) \times \frac{p_t(V^P_L - V^F_P) + (1 - p_t)(V^N_L - V^N_M)}{p_t(V^L_P - 2V^F_M + V^F_P) + (1 - p_t)(V^L_L - 2V^F_M)} & \text{if } p_L < p_t < p_M \\
1 & \text{if } p_t \geq p_M, 
\end{cases}
\]

and

\[
\alpha^t_n = \begin{cases}
0 & \text{if } p_t \leq p_L \\
\frac{p_t(V^P_L - V^F_P) + (1 - p_t)V^N_L}{p_t(V^P_L - V^F_M) + (1 - p_t)(V^N_L - V^N_M)} & \text{if } p_L < p_t < p_M \\
1 & \text{if } p_t \geq p_M, 
\end{cases}
\]

where \(T^t_L := \inf\{t \geq t_0 | p_t \geq p_L\}\).

A graphical depiction of this equilibrium, for a specific parameterisation defined in Section 5, is given in Figure 2.

### 5 Numerical Example

My aim in this section is to provide an insight into the magnitude of some of the effects which I discuss in previous sections. For the parameterisation given in Table 1; \(s_L = 6, s_M = 4\) and \(s_P = 1\). The corresponding belief probabilities are \(p_L \approx 93\%, p_M \approx 83\%\) and \(p_P \approx 60\%.\) Since \(s_P < s_M < s_L\) the leader advantage outweighs the information spillover. Both firms will try

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>(V^P_L)</td>
<td>20</td>
</tr>
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</tr>
<tr>
<td>(V^P_F)</td>
<td>10</td>
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<tr>
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</tr>
<tr>
<td>(r)</td>
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</tr>
<tr>
<td>(\theta)</td>
<td>0.6</td>
</tr>
<tr>
<td>(\mu)</td>
<td>4</td>
</tr>
</tbody>
</table>
to preempt each other when they have one extra positive signal regarding the product’s profitability, and if the excess of positive over negative signals is four or more, it is optimal for both firms to disclose simultaneously, before the market response is known. The probability of simultaneous disclosure can be found using equations (24) and (21).

Figure 1 depicts this situation graphically. The leader’s payoff, \( L(p_t) \), intersects with the follower’s payoff, \( F(p_t) \), at \( p_P = 0.6 \). As shown, for all \( p_t < 0.6 \), the leader curve lies below the follower’s curve; i.e. \( L(p_t) < F(p_t) \) and neither firm wants to be the first to disclose. \( F(p_t) \) intersects with the payoff curve from simultaneous disclosure, \( M(p_t) \), at \( p_M \approx 0.83 \), and for \( p_t \geq 0.83 \) it is clear that \( F(p_t) = M(p_t) \). However, for \( p_t < p_M \), \( M(p_t) \) lies below \( F(p_t) \) implying that the manager would prefer to wait and obtain the information spillover from the leader rather than disclosing simultaneously before the market response is known. The final intersection point is for \( L(p_t) = M(p_t) \) at \( p_t \approx 0.38 \). However, since this plot depicts the situation whereby the leader advantage outweighs the information spillover; i.e. \( p_L > p_P \) (recall \( p_L \approx 0.93 \)), no action will be taken by either firm for \( p_t < 0.6 \). Hence, \( p_t = 0.38 \) is not a point that needs to be discussed.

Consider, however, if the parameterisation is such that \( V_P = 15 \) and \( V_N = -25 \). The condition (7) no longer holds; that is \( p_M > p_P \), and a synergy equi-
librium emerges. The situation is depicted graphically in Figure 2. The payoff functions \( M(p_t) \) and \( F(p_t) \) intersect at \( p_M \approx 0.83 \). It appears from the figure that \( L(p_t) \) also intersects them at \( p_M \approx 0.83 \), but it actually lies slightly below the intersection point at \( p_M \) since \( F(p_M) = M(p_M) \approx 4.17 \) and \( L(p_M) \approx 3.33 \).

For all values of \( p_t \leq p_M \), the leader’s payoff function lies below the follower’s implying that the information spillover outweighs the leader advantage, and thus, neither manager wants to be the first to disclose. Thus, for all \( p_t \leq p_M \), there is no point, \( p_P \), such that the leader and follower payoffs are equal. Similarly, for all \( p_t < p_M \), \( F(p_t) > M(p_t) \) implying that the information spillover also outweighs the synergy effect, and thus, it is not optimal for either manager to disclose simultaneously with his competitor. Hence, no disclosure will be taken by either firm until \( p_M \) is reached. Once \( p_t \geq p_M \), it is optimal for both firms to disclose simultaneously, and the game ends.

![Figure 2: Synergistic equilibrium.](image)

6 Conclusion

In this paper I examine the impact of competition, in a duopoly framework, on the voluntary disclosure policy of firms. Each firm is assumed to have invested in a specific product and the manager of each firm must then decide when to optimally disclose its involvement in the product to the market, while taking
into consideration the disclosure strategy of the other firm. A preemption, attrition, or synergy equilibrium is found, depending on the trade-off between first and second mover advantages and, also on the advantage from disclosing simultaneously with the competing firm.

I do not make the assumption that the negative impact of disclosure on firm value is zero, and therefore, a new equilibrium emerges whereby preemption is nonsensical. This so-called “synergistic” equilibrium implies that the optimal strategy of a firm is to never announce, or else to do so only at the same time as its competitor. The intuition behind this equilibrium result is that the investors’ conviction that their firm has invested in a profitable venture is strengthened by the fact that another similar firm has also chosen to undertake the same investment. This will then temper any sell-off in shares, if the market were to respond in a negative way to the information, than if only one firm were to disclose, or conversely, amplify the effect of a positive market response.

References


