The Relative Pricing of Sovereign Credit Risk After the Eurozone Crisis

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ABSTRACT

The paper investigates the relative pricing of the sovereign credit risk, for European countries, during and after the sovereign debt crisis of 2010-2012. We investigate empirically the theoretical relationship between CDS spreads and bond yields before and after the announcement of the Outright Monetary Transaction (OMT) Programme, by the European Central Bank, and we show that the relative mispricing of the sovereign credit risk has strongly reduced after the announcement of the OMT. We disentangle the effects of the ECB intervention on the sovereign debt market in different ways, and we provide evidence that the consistent relationship between default risk and bond yields across the Eurozone countries was restored after the ECB intervention. We show that the relative mispricing in the sovereign credit risk has generated arbitrage opportunities before the OMT announcement across all European countries. Nonetheless the arbitrage opportunities were not profitable because of high transaction costs. Following the ECB intervention, instead, we estimate a strong reduction in the transaction costs for the Eurozone countries only; therefore the arbitrage opportunities were cleared, and the equilibrium condition in the Eurozone sovereign debt market was restored.

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1. Introduction

Credit derivatives and debt securities are strictly related, since the pricing of both types of financial assets crucially depends on the risk of default of the reference entity. Credit Default Swaps (CDS) and bonds issued by the CDS reference entity produce similar exposure to the investor in terms of risk and return. The CDS provides protection to the acquirer in case of default of the reference entity, while the bond pays out yields to the bondholder as long as the reference entity is able to comply with its obligations.

In this paper, we study the relationship between sovereign CDS and sovereign bonds, in terms of risk and return, for the European countries, during and after the sovereign debt crisis. Our main finding is the following: after the announcement of the Outright Monetary Transaction (OMT) Programme, by the European Central Bank, the relative mispricing of the sovereign credit risk has strongly reduced for the Eurozone countries. Then, we provide evidence that the consistent relationship between risk and return for the Eurozone sovereign securities is restored after the ECB intervention.

Therefore, we contribute to three strands of academic research. We first offer empirical evidence on the theoretical relationship between CDS premium and bond yields across the European countries. Hull, Predescu, and White (2004) point out that, under a large set of assumptions that ensure absence of frictions in the market, a portfolio including CDS and bond, issued by the reference entity, generates cash flows equal to a riskless bond in all states of the world. The difference between the two portfolios cash-flow is defined as basis, and it is usually adopted as observed measure of mispricing. Hence, the CDS premium should be equal to the excess risky yield over the risk-free rate (zero-basis condition).

Mispricing has been documented for both corporate (Longstaff, Mithal, and Neis (2005), Blanco, Brennen, and Marsh (2005)), and sovereign securities (Palladini and Portes (2011), Arce, Mayor-domo, and Pena (2013), Fontana and Scheicher (2016)). These papers argue that CDS spreads are faster in price discovery, thus reacting quicker to changes in credit condition. As a consequence, the relationship CDS spread - bond spread does not hold in the short-term. However, they show that CDS spreads and bond yields exhibit strong co-movements in a long-term perspective. The
widely used technique of detection of this relationship is the cointegration analysis.

While Palladini and Portes (2011), Arce et al. (2013), and Fontana and Scheicher (2016) provide evidence of the relative pricing of the sovereign credit risk before and during the sovereign crisis, we extend the analysis to the period following the ECB intervention, including also countries outside the Eurozone, with the aim of highlighting the differential effects of the unconventional monetary policy.

We show that the equilibrium condition is violated before the announcement of the OMT, and then restored afterwards, for the Eurozone countries, and in particular for the peripheral countries of the Eurozone. Instead, the deviation from the equilibrium condition is persistent and constant over the entire period for the European countries out of the Eurozone.

Moreover, deviations from the equilibrium condition may generate arbitrage opportunities, that should be unsystematic, and then quickly disappear. We document that these opportunities are large and persistent before the announcement of the OMT, and then almost disappear, for the Eurozone countries. Instead, we do not observe significant changes between before and after the announcement of the OMT for the countries outside the Eurozone. We detail the potential arbitrage strategies implementable by trading sovereign bond yields and CDS, and we show that in the Eurozone the strategies are largely profitable before the launch of the OMT, and then converge towards zero-profits afterwards.

We conjecture that the persistent arbitrage opportunities before the OMT announcement were created by high transaction costs. Therefore, we estimate for each country the threshold below which the arbitrage strategies generate profits that are not even sufficient to cover the costs to be implemented. The idea is that arbitrageurs step into the market only if the arbitrage strategy still generates profits once that the transaction costs have been paid. We show that, before the OMT announcement, the arbitrage opportunities are not cleared because of high transaction costs. Then, we estimate a strong reduction in the transaction costs for the Eurozone countries only, following the ECB intervention. As a consequence, the arbitrage opportunities are cleared, and the equilibrium condition in the Eurozone sovereign debt market is restored. However, we do not estimate a similar reduction in the transaction costs for the No Eurozone countries. Therefore, we observe for those countries a persistent CDS spread - bond yield mispricing even after the OMT
As second contribution, we investigate the consistency of the relationship between risk and return for sovereign securities. The positive relationship between risk and expected return is one of the milestones in financial theory. Investors are willing to buy risky assets only if they are rewarded with a proper expected return. The higher is the risk associated to a given investment, the higher must be the expected return. It turns out that entities marked by higher risk of default should issue more rewarding securities, compared to safer issuers, in order to attract investors. The empirical contradiction of the positive relationship between risk and expected return is known in the financial literature as distress puzzle.

We document that a distress puzzle at the sovereign level emerges during the crisis period for the Eurozone countries, and then is ruled out after the announcement of the OMT programme.

The distress puzzle has been widely investigated in the context of corporate securities, by studying the relationship between the firm’s default risk and the expected return on firm’s equity shares. The empirical evidence is far from being univocal (see, among others, Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2008), Friewald, Wagner, and Zechner (2014)). To the best of our knowledge, however, an analysis of the puzzle at the sovereign level is still missing. As countries do not issue equity, we focus on the debt-related securities.

The intuition is simple. If a country is more likely to default with respect to another country, then the riskier country must issue debt securities that generate higher expected return for the investor, with respect to the safer country. In practice, the riskier country must issue bonds that pay out higher yields. However, it may happen that the riskier country pays out an excess bond yield, with respect to the safer country, that is too low than it should be paid, or that the excess bond yield of the riskier country is too high. Therefore, the monotonic relationship between bond yields and CDS spreads across countries is a necessary but not sufficient condition to rule out the distress puzzle.

To determine the proper distance between bond yields of different countries, we adopt a simple credit risk structural model, in order to obtain a simultaneous relationship between CDS spreads and bond yields for a country. In a structural model, in fact, bond and CDS are implicitly related at
each point in time, as both the securities are derivative contracts on the same underlying quantity, that are the assets and the liabilities of the reference entity. In particular, we adopt a first-passage time model, where the issuer defaults as soon as the value of the assets crosses from above a default boundary, assumed to be deterministic and constant. This framework is an extension of the seminal model of Merton (1974), where the issuer may default only at the maturity of the liability. Gapen, Gray, Lim, and Xiao (2011) introduce contingent claims analysis to study sovereign credit risk, by using a Merton model.

Hence, the default risk of the country is priced in the CDS spread, where the default risk is due to the probability that the leverage of the country, defined as debt-to-asset ratio, reaches a given threshold, to be estimated, that is unsustainable. Then, there is a one-to-one mapping between leverage and CDS spread, where the model provides the specific functional form of the mapping.

We estimate the model with a non-linear Kalman filter in conjunction with maximum likelihood, by using daily data on CDS spreads over three different time horizons, i.e. 1, 5, and 10 years. We reconstruct the dynamics of the market value of the leverage for each country, and we estimate the value of the default boundary. Sovereign assets include current and future surpluses, international reserves, and residual items (see Gapen et al. (2011)). With the estimated parameters, we are then able to compute the bond yields implied by the model estimation using Monte Carlo (MC) simulations. These are the yields implied by the CDS spreads, as we use the observed CDS spreads, and the relationship between default risk and leverage provided by the model, in order to estimate the model parameters, and to reconstruct the dynamics of the country’s leverage. Then, we use the relationship between bond yields and leverage, defined by the model, in order to compute the implied bond yields.

We use the implied bond yields to investigate the monotonicity condition, across countries, in the relationship between CDS spreads and bond yields, for each point in time. We subtract the implied bond yields from the observed bond yields, thus obtaining a net yield for each country, and for each point in time. The idea is the following: if the excess bond yield of the riskier country, with respect to the safer country, is lower than it should be, then the net yield of the safer country is higher than the net yield of the riskier country. The result is a non-monotonic relationship between CDS spreads and net yields.
To investigate the violation of the monotonicity condition over a cross-section of countries, we measure the Spearman’s correlation between CDS spreads and net yields, for each point in time. The Spearman’s correlation evaluates the presence of a monotonic relationship between two variables, regardless the linearity of the relationship. The closer is the correlation to 1, the more consistent are the distances in the bond yields, across countries, with the differences in terms of default risk priced in the CDS.

We show that the correlation between CDS spreads and net yields randomly moves around zero for the Eurozone countries before the OMT announcement, then approaching 1 right after the OMT announcement, and remaining stable afterwards. Instead, the countries outside the Eurozone do not show significant change in the cross-sectional correlation between CDS spreads and net yields after the OMT announcement.

Finally, we intervene in the discussion on the effects of the unconventional monetary policies implemented by central banks. Several papers have shown that the ECB intervention in 2012 has significantly lowered the credit spreads of sovereign bond securities, and has also drastically reduced the level of the premium paid on the CDS. Further to the simultaneous reduction of sovereign CDS spreads and bond yields, following the ECB announcement, we document a strong reduction of the distortion in the relative pricing of the sovereign credit risk, which restores the equilibrium conditions, and clears the potential arbitrage opportunities.

Combining this result with our estimation of the transaction costs faced by the traders on the sovereign debt market, before and after the crisis period, we offer the explanation that the disequilibrium at work before the ECB announcement was due to high transaction costs, that did not allow the arbitrageurs to take advantage of the riskless profit opportunities. We show that such barriers disappear after the ECB announcement for the Eurozone countries only, thus creating the condition for the traders to clear the arbitrage opportunities, and finally leading the sovereign debt market back to a consistent relative pricing.

Our paper is organized as follows. We first describe the data in the next section, then we provide empirical evidence on the relationship between CDS spreads and bond yields during and after the OMT announcement, in section 3. In section 4, we detail the underlying credit risk model and our
estimation methodology to compute the implied bond yields. In section 5, we present the empirical investigation. We start the analysis by comparing observed and implied yields, then we proceed with the correlation analysis, and finally we describe potential arbitrage strategies and riskless profits. Moreover, we estimate the transaction costs, before and after the OMT announcement, and we compare such costs with the arbitrage profits. Section 6 concludes the paper.

2. Data

Our main source of data is Thomson Reuter’s DataStream. We download daily data for sovereign CDS spreads and sovereign government bond yields for several European countries, and a sample period going from the 4th January 2010 to the 1st February 2017. Hence, we collect a time series of 1850 daily observations for each country, for both CDS spreads and bond yields, and for three time horizons: 1, 5, and 10 years maturity. Datastream provides reference par yields for sovereign bonds at different maturities. The par yield is the internal rate of return (yield to maturity) of a bond traded at par, and it is expressed as an annualized figure. Instead, the CDS spread is expressed in basis points, and represents the percentage of the CDS notional value that the protection buyer must pay, usually at quarterly frequencies, to the protection seller. CDS spreads are also expressed in annualized terms.

We use all the maturities of the CDS spreads to implement the estimation methodology, however we focus throughout the paper on the 5-years maturity in order to show the results of the empirical analysis. We also collect data on the Euribor to represent the European short term risk-free interest rate curve. At longer maturities we proxy the risk-free rate with the euro area yield curve computed exclusively on AAA-rated central government bonds, and we also use a Nelson-Siegel technique to bootstrap the maturities of the risk-free curve needed to obtain the present values of CDS that we use in the arbitrage strategies.

We apply a filter to the sample, excluding those countries which report an excessive number of missing data on bond yields or CDS spreads -more than 40% of the total observations for at least one maturity- thus dropping from the sample Cyprus, Luxembourg, and Malta. We also exclude Greece that deserves a specific analysis due to the dramatic turbulence experienced during the
sample period. We drop from the sample Estonia, Latvia and Lithuania, as these countries change their status from Non-Eurozone to Eurozone over the sample period. We end up with a final sample of 22 countries. In particular, 12 countries belong to the Eurozone, and 10 are out of the Eurozone. Throughout the analysis, we also divide the sample of the Eurozone countries in two subgroups: core, and periphery. The list of countries is reported in table I.

2.1. Descriptive Statistics

In table I we report data on CDS spreads and bond yields for each single country in the sample. Table I shows that both bond yields and CDS spreads are significantly lower after the announcement of the OMT Programme by ECB governor Mario Draghi on July 26th, 2012. The differences are significant at 5% level (except for the CDS in Slovenia), when considering both mean and median.

In table II we report figures for the time series of mean and median across countries before and after July 2012. We also provide a breakdown of mean and median by different groups of countries. Therefore, we observe that bond yields and CDS spreads are generally lower for the core countries with respect to both the peripheral and the No Eurozone countries, before and after the OMT announcement. Yet, the reduction in both spreads and yields is significant at 5% level even for the core countries.
Table I. Descriptive Statistics by Country

<table>
<thead>
<tr>
<th></th>
<th>Statistics: CDS Spreads Bond Yields</th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Before OMT After OMT Difference Before OMT After OMT Difference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Eurozone</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>78.19</td>
<td>20.22</td>
<td>-57.97*</td>
<td>3.14</td>
<td>1.21</td>
<td>-1.93*</td>
</tr>
<tr>
<td>Belgium</td>
<td>143.11</td>
<td>33.93</td>
<td>-109.19*</td>
<td>3.77</td>
<td>1.44</td>
<td>-2.33*</td>
</tr>
<tr>
<td>Finland</td>
<td>46.50</td>
<td>24.74</td>
<td>-21.76*</td>
<td>2.79</td>
<td>1.14</td>
<td>-1.65*</td>
</tr>
<tr>
<td>France</td>
<td>83.17</td>
<td>31.86</td>
<td>-51.31*</td>
<td>3.13</td>
<td>1.37</td>
<td>-1.76*</td>
</tr>
<tr>
<td>Germany</td>
<td>39.15</td>
<td>12.58</td>
<td>-26.57*</td>
<td>2.48</td>
<td>0.93</td>
<td>-1.55*</td>
</tr>
<tr>
<td>Netherlands</td>
<td>67.26</td>
<td>31.74</td>
<td>-35.53*</td>
<td>7.63</td>
<td>2.31</td>
<td>-5.31*</td>
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<tr>
<td>Peripheral:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>485.07</td>
<td>80.00</td>
<td>-405.07*</td>
<td>4.94</td>
<td>2.81</td>
<td>-2.13*</td>
</tr>
<tr>
<td>Italy</td>
<td>229.15</td>
<td>138.40</td>
<td>-90.75*</td>
<td>2.80</td>
<td>1.15</td>
<td>-1.65*</td>
</tr>
<tr>
<td>Portugal</td>
<td>633.77</td>
<td>247.09</td>
<td>-386.67*</td>
<td>8.85</td>
<td>4.35</td>
<td>-4.49*</td>
</tr>
<tr>
<td>Slovakia</td>
<td>136.00</td>
<td>61.90</td>
<td>-74.10*</td>
<td>4.18</td>
<td>1.89</td>
<td>-2.29*</td>
</tr>
<tr>
<td>Slovenia</td>
<td>164.69</td>
<td>168.27</td>
<td>3.58</td>
<td>5.10</td>
<td>2.90</td>
<td>-2.20*</td>
</tr>
<tr>
<td>Spain</td>
<td>243.27</td>
<td>115.66</td>
<td>-127.62*</td>
<td>5.62</td>
<td>3.07</td>
<td>-2.55*</td>
</tr>
<tr>
<td><strong>No-Eurozone</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td>258.99</td>
<td>130.61</td>
<td>-128.39*</td>
<td>6.53</td>
<td>4.28</td>
<td>-2.25*</td>
</tr>
<tr>
<td>Croatia</td>
<td>316.38</td>
<td>274.95</td>
<td>-41.43*</td>
<td>3.72</td>
<td>1.30</td>
<td>-2.41*</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>98.95</td>
<td>49.67</td>
<td>-49.28*</td>
<td>2.56</td>
<td>1.08</td>
<td>-1.48*</td>
</tr>
<tr>
<td>Denmark</td>
<td>60.44</td>
<td>17.83</td>
<td>-42.61*</td>
<td>7.86</td>
<td>4.66</td>
<td>-3.20*</td>
</tr>
<tr>
<td>Hungary</td>
<td>353.35</td>
<td>191.21</td>
<td>-162.14*</td>
<td>6.75</td>
<td>6.36</td>
<td>-0.39*</td>
</tr>
<tr>
<td>Norway</td>
<td>26.58</td>
<td>16.23</td>
<td>-10.35*</td>
<td>5.17</td>
<td>3.85</td>
<td>-1.32*</td>
</tr>
<tr>
<td>Poland</td>
<td>160.49</td>
<td>71.75</td>
<td>-88.74*</td>
<td>5.51</td>
<td>2.47</td>
<td>-3.04*</td>
</tr>
<tr>
<td>Romania</td>
<td>301.57</td>
<td>145.09</td>
<td>-156.47*</td>
<td>3.01</td>
<td>1.98</td>
<td>-1.02*</td>
</tr>
<tr>
<td>Sweden</td>
<td>36.48</td>
<td>12.96</td>
<td>-23.52*</td>
<td>5.79</td>
<td>3.45</td>
<td>-2.34*</td>
</tr>
<tr>
<td>UK</td>
<td>65.54</td>
<td>27.81</td>
<td>-37.73*</td>
<td>7.51</td>
<td>4.49</td>
<td>-3.02*</td>
</tr>
</tbody>
</table>

Legend: The table reports the average over the time series, for the CDS spreads and the bond yields, at country level, for the period before the OMT and for the period after the OMT. The third column is the difference between the two subperiods: (After OMT - Before OMT). The * indicates that the difference is significant at the 5% level.
Table II. Descriptive statistics by asset

<table>
<thead>
<tr>
<th></th>
<th>Average of Means</th>
<th></th>
<th>Average of Medians</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before OMT</td>
<td>After OMT</td>
<td>Difference</td>
<td>Before OMT</td>
</tr>
<tr>
<td>Overall:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS</td>
<td>183.10</td>
<td>86.57</td>
<td>-96.53*</td>
<td>125.70</td>
</tr>
<tr>
<td>Yields</td>
<td>4.95</td>
<td>2.66</td>
<td>-2.29*</td>
<td>4.70</td>
</tr>
<tr>
<td>Breakdown by country group:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS</td>
<td>73.23</td>
<td>25.84</td>
<td>-50.39*</td>
<td>70.57</td>
</tr>
<tr>
<td>Yields</td>
<td>3.82</td>
<td>1.40</td>
<td>-2.42*</td>
<td>3.13</td>
</tr>
<tr>
<td>Periphery</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS</td>
<td>315.33</td>
<td>135.22</td>
<td>-180.11*</td>
<td>239.24</td>
</tr>
<tr>
<td>Yields</td>
<td>5.25</td>
<td>2.69</td>
<td>-2.55*</td>
<td>4.84</td>
</tr>
<tr>
<td>Non-Eurozone</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS</td>
<td>167.88</td>
<td>93.81</td>
<td>-74.07*</td>
<td>129.99</td>
</tr>
<tr>
<td>Yields</td>
<td>5.44</td>
<td>3.39</td>
<td>-2.05*</td>
<td>5.84</td>
</tr>
</tbody>
</table>

Legend: The table reports statistics for the time series of CDS spreads and bond yields before and after the OMT announcement date and their difference for the overall sample, and separately for the three different country groups that we identified as: "Core", "Periphery", and "No Eurozone". The "Average of Means" is computed as the average over each subperiod of the average CDS spread and yield across countries at each time $t$. The "Average of Medians" is computed as the average over each subperiod of the meadian CDS spread and yield across countries at each time $t$. The * indicates that the difference is significant at the 5% level.
Legend: The figure reports the dynamics of average and medians of the cross section of countries for CDS spreads and bond yields over the sample time series, at the 5-years maturity, for the three different groups of countries. The blue line represents the dynamic for the core countries of the Eurozone, the green line is for the peripheral countries of the Eurozone, and the yellow line is the average of the cross section of No-Eurozone countries. The red line is the OMT announcement date.
3. The CDS - Bond basis

CDS spreads and yields on a risky bond issued by the reference entity of the CDS contract are strictly related. The CDS provides protection to the acquirer in case of default of the reference entity, while the bond pays out yields to the bondholder as long as the reference entity is able to comply with its obligations. In particular, Hull et al. (2004) have pointed out that, under a large set of assumptions, the $T$-years CDS spread should be equal to the $T$-years excess yield on a risky bond, issued by the reference entity, over the $T$-years riskless bond.

The reason is simple: if the assumptions listed by Hull et al. (2004) hold, a portfolio including a $T$-years CDS and a $T$-years par yield bond, issued by the reference entity, generates cash flows equal to a $T$-years par yield riskless bond in all states of the world, and so

$$s = y - r,$$

where $s$ is the $T$-years CDS spread, $y$ is $T$-years yield on the risky bond, and $r$ is the $T$-years yield on the riskless bond. If this relationship does not hold, then an arbitrage opportunity arises in the market by trading CDS, risky bond, and riskless asset. We will analyze later in the paper the riskless profits generated by the potential arbitrage strategies that exploit the violation of the equation (1).

We now provide empirical evidence on the relationship between CDS spreads and risky bond yields for our sample countries, over the time interval covered by our dataset. We group the countries in three sub-samples: Eurozone-Core (EC), Eurozone-Peripheral (EP), and No-Eurozone (NZ). We define as basis the difference between the $T$-years CDS spread and the $T$-years excess yield on a risky bond, issued by the reference entity, over the $T$-years riskless bond.

Figure 2 shows the dynamics of the basis for each country. The EC countries have basis substantially lower than the EP countries and the NZ countries. More importantly, the basis of both the core and peripheral countries of the Eurozone converge to zero right after the OMT announcement, and then remains around zero over the following years. The NZ countries, instead, do not show the same convergence in terms of basis, and appear to be spread around the zero, before and after the OMT announcement.
Figure 2. CDS spreads - Bond Yields basis

Legend: The figure reports the dynamics of the basis (CDS spread - Bond Yield) for each country over the sample time series, at the 5-years maturity, for the three different groups of countries. The basis is expressed in percentage terms, i.e. basis points divided by 10000. The red line is the OMT announcement date.

This result is also evident looking at the average of the absolute basis across groups of countries. Table III reports that the absolute basis has substantially reduced for the Eurozone countries in the second period of the time series (-65% for the EC, -55% for the PC, respectively), while the decrease is much lower for the NZ countries (-10%).

Table III. Average Absolute Basis (CDS spreads - Bond Yields)

<table>
<thead>
<tr>
<th></th>
<th>Euro - Core</th>
<th>Euro - Periphery</th>
<th>No Eurozone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before OMT</td>
<td>0.0063</td>
<td>0.0078</td>
<td>0.105</td>
</tr>
<tr>
<td>After OMT</td>
<td>0.0022</td>
<td>0.0036</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Legend: The basis is expressed in percentage terms, i.e. basis points divided by 10000. Both CDS spreads and Bond yields are at 5-years maturity.
4. CDS-implied bond yields

In this section, we estimate a credit risk structural model in order to determine the risky bond yield of a country consistent with the country’s default risk priced in the CDS spreads. The procedure that we adopt is the following: first, we reconstruct the unobservable dynamics of the leverage, defined as debt-to-asset ratio, for each country, by performing a non-linear Kalman filter, and using the CDS spreads as observable variables. The Kalman filter enables to retrieve the dynamics of a latent variable, by using an observable variable and the ex-ante known relationship between the two variables. The relationship between the observed and the unobserved variables forms the measurement equation, while the evolution over time of the latent variable is called transition equation. We estimate the model parameters by adopting a quasi-maximum likelihood algorithm, in conjunction with the Kalman filter. Details of the estimation methodology are provided in Appendix A.

Then, we perform Monte Carlo (MC) simulations to compute the implied yields on a risky zero-coupon bond, for each country, over the sample time series. In the MC simulations, for each country, we use the dynamics of the leverage, and the estimates of the model parameters, of the first step. In the next subsection, we describe the underlying model, then we briefly introduce the Kalman filter applied to our estimation problem. In the last subsection, we detail the MC simulations, and we describe the implied yields obtained from the simulations.

4.1. Underlying Model

The asset value of the $i$-th country is described by a geometric Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathcal{P})$:

$$dV_{i,t} = \mu V_i dt + \sigma V_i dW_{i,t},$$

where $\mu_{V_i}$ and $\sigma_{V_i}$ are the $\mathcal{P}$-drift and diffusion constant coefficients, $W_{i,t}$ is a standard Brownian motion under the physical probability measure $\mathcal{P}$.

We define the $i$-th market value of leverage as $L_{i,t} = \ln\left(\frac{F_i V_i}{V_i}\right)$, following an arithmetic Brownian motion,
\[ dL_{i,t} = \mu_{L_i}dt - \sigma_{L_i}dW_{i,t}, \] (2)

where \( \mu_{L_i} = - \left( \mu_{V_i} - \frac{1}{2}\sigma_{V_i}^2 \right) \) is the \( \mathcal{P} \)-leverage drift coefficient, and \( \sigma_{L_i} = \sigma_{V_i} \) is the leverage diffusion component. As result of the inverse relationship between the asset and the leverage values, the minus before the diffusion component stands for the perfect negative correlation between the Brownian motions of the asset value and the leverage dynamics.

In the first-passage time framework, default occurs as soon as the asset value crosses from above a constant and deterministic barrier \( C_i \), that we assume to be below the face value of the debt, at any time \( s \), with \( t \leq s \leq T \), where \( T \) is the outstanding debt maturity. The country’s default risk is priced in the credit default swaps (CDS) issued with different maturity \( \tau_j \), with \( j \) going from 1 to \( J \), where the longest maturity \( \tau_J \) matches the debt maturity \( T \). In a CDS contract, the protection buyer pays a fixed premium each period until either the default event or the contract expiration, and the protection seller is committed to buy back from the buyer the defaulted bond at its par value.

Therefore, the price of the CDS, i.e. the premium (the spread) paid by the insurance buyer, is defined at the inception date of the contract in order to equate the expected value of the two contractual legs. Then, by assuming the existence of a default-free money market account appreciating at a constant continuous interest rate \( r \), and \( M \) periodical payments occurring during one year, the CDS spread \( \gamma \) with time-to-maturity \( \tau_j \), priced at \( t = 0 \), solves the following equation:

\[
\sum_{m=1}^{M} T \frac{\gamma}{M} \exp \left( -r \frac{m}{M} \right) \mathcal{E}_0^Q \left[ 1_{t^* > \frac{m}{M}} \right] = \mathcal{E}_0^Q \left[ \exp(-rt^*)\alpha 1_{t^* < \tau_j} \right],
\]

where \( t^* \) stands for the time of default, \( \alpha \) is the amount paid by the protection seller to the protection buyer in case of default, and \( \mathcal{E}_0^Q \) indicates that the expectation is taken under the risk-neutral measure \( \mathcal{Q} \). Therefore, \( \mathcal{E}_0^Q \left[ 1_{t^* < \tau_j} \right] \) is the probability that the country defaults at any time before \( \tau_j \), that is the probability that the asset value crosses from above the barrier \( C_i \). At \( t \), this probability is equal to:
\[ PD_{i,t}^Q(\tau_j) = \Phi \left( \frac{K_i + L_{i,t} - \left( r - \frac{1}{2} \sigma_{Li}^2 \right) (\tau_j - t)}{\sigma_{Li} \sqrt{(\tau_j - t)}} \right) + \exp \left( (K_i + L_{i,t}) \left( \frac{2r}{\sigma_{Li}^2} - 1 \right) \right) \Phi \left( \frac{(K_i + L_{i,t}) + \left( r - \frac{1}{2} \sigma_{Li}^2 \right) (\tau_j - t)}{\sigma_{Li} \sqrt{(\tau_j - t)}} \right) \] , \quad (3) 

if \( \tau_j < T \), otherwise

\[ PD_{i,t}^Q(\tau_J) = 1 - \Phi \left( \frac{-L_{i,t} + \left( r - \frac{1}{2} \sigma_{Li}^2 \right) (\tau_J - t)}{\sigma_{Li} \sqrt{(\tau_J - t)}} \right) + \exp \left( (K_i + L_{i,t}) \left( \frac{2r}{\sigma_{Li}^2} - 1 \right) \right) \Phi \left( \frac{(2K_i + L_{i,t}) + \left( r - \frac{1}{2} \sigma_{Li}^2 \right) (\tau_J - t)}{\sigma_{Li} \sqrt{(\tau_J - t)}} \right) \] , \quad (4)

as \( \tau_J = T \), and we have to consider not only the early bankruptcy risk as in the equation (2), but also the probability of the country not being able to pay back the outstanding debt \( F_i \) at time \( T \), even though the asset value never crossed the default boundary.

\( \Phi \) stands for the cumulative distribution function of a standard normal variable, and \( K_i = \ln \left( \frac{C_i}{F_i} \right) \). As the default barrier is below the face value of the debt, \( K_i \) assumes only negative values. The larger is the magnitude of the absolute value of \( K_i \), the larger is the distance between the face value of the debt \( F_i \) and the default barrier \( C_i \).

4.2. Model Estimation

We formulate our problem in a state-space model, where the measurement equations come from (3)-(4). The noise terms associated with the CDS implied-default probability for different time to maturities \( \tau_j \) are assumed to be uncorrelated, and with equal variance.

\[ PD_{i,t}^Q(\tau_j) = g(L_{i,t}; K_i, \sigma_{Li}) + \epsilon_{i,t}(\tau_j), [j = 1, 5, 10] \]

where the time to maturity is expressed in years, and \( j = 10 \) stands for the maturity \( T \) of the outstanding debt \( F_i \) (i.e., 10 years). The function \( g \) defines the non-linear relationships between the observable and the latent variable, and \( \epsilon_{i,t}(\tau_j) \) is the measurement noise associated with the
CDS implied-default probability equation and the time horizon \( j \). These four measurement noises, for each country \( i \), are assumed to follow a multivariate normal distribution, with zero mean, and diagonal covariance matrix \( R_i \). We assume a homoscedastic covariance matrix, which is country-varying.

On the other side, the transition equation describes the evolution of the country’s leverage. It follows from the discretization of the stochastic process defined in (2):

\[
L_{i,t+\delta t} = L_{i,t} + \mu_{L_i} \delta t + \eta_{i,t+\delta t},
\]

where \( \eta_{i,t+\delta t} = \sigma_{L_i}(W_{i,t} - W_{i,t+\delta t}) \sim N(0, Q_i) \) is the transition error, and \( Q_i = \sigma_{L_i}^2 \delta t \).

The dynamics of \( L_{i,t} \), and the parameters of the model, such as the parameters of the leverage dynamics \( (\mu_{L_i}, \sigma_{L_i}) \) and \( K_i \), are then estimated by performing a non-linear Kalman filter in conjunction with quasi-maximum likelihood estimation. For parsimony, the steps to implement the non-linear Kalman filter, and the construction of the likelihood function, are described in details in the Appendix A.

Figure 3 provides an idea of the estimation results, thus comparing the reconstructed dynamics of the leverage, for the European countries, over the sample time series, against the observed dynamics of the 5-years CDS spreads and the 5-years observed bond yields. The dynamics of both CDS spreads and bond yields is in line with the dynamics of the country’s leverage. When the CDS spreads and the bond yields approach to very low values, in particular in the last part of the time series, then we estimate a leverage that moves far away from zero, towards negative values.
Figure 3. Leverage, CDS spreads and Bond Yields. Eurozone Countries

Legend: The figure shows the dynamics of the leverage of the country (blue line), as defined in the equation (2), reconstructed for each country by using the Kalman filter, the 5-years CDS spreads (dashed line) and the 5-years bond yields (red line), both expressed in percentage terms, i.e. basis points divided by 10000.

4.3. Monte Carlo simulations

The implied risky yields, for each point in time, and for each country, are obtained as average over 10000 simulations. In particular, for each point in time $t$, and each country, we simulate the dynamics of the leverage for a time interval going from $t$ to $t + M \times 360$, where $M$ is the maturity of the bond expressed in years.

The leverage of a country is simulated by using the equation (2), where $dt$ is a one-day step. The parameters of the stochastic process are the estimates obtained in the previous step, and we use the estimated leverage for the time $t$ as starting point of the simulated dynamics. We generate $M \times 360$ normally distributed random numbers for each country to simulate the daily increment of the Brownian motion, thus finally obtaining a simulated dynamics of the leverage of length $M \times 360$.

Then, we use the condition of default implied by the model. The country defaults if $V_{i,t} < C$, that corresponds to $L_{i,t} > (-K_i)$. Therefore, if the simulated leverage of the country is above $-K_i$, at least for one point in time over the simulation time horizon, then we impose that the bond
defaults and the $t$-value of the bond is zero. Otherwise, the $t$-value of the bond is equal to the risk-free discount factor, by using the risk-free rate at time $t$.

We then compute the bond price for each time $t$ averaging across the 10000 simulations, and the corresponding yield by simple inversion. Let define $B$ the price of the bond obtained with MC simulations, then the implied yield $Y$ is equal to

$$Y = \log\left(\frac{1}{B}\right) \frac{1}{M \times 360}$$

5. Empirical Analysis

We now implement our empirical analysis by combining the information on the CDS spreads and the observed bond yields with the estimation of the model-implied bond yields. We disentangle the main question of the paper from three different points of view. First, we study the distance between observed and implied bond yields for each country. Then, we study the correlation between CDS spreads and bond yields, by using both observed and implied yields. Finally, we test the consistency of the bond yields in terms of default risk priced in the CDS spreads, by constructing riskless arbitrage strategies, and we verify whether the strategies are profitable.

5.1. Implied and Observed Bond Yields

The difference between observed and implied yields should be zero for each country, and each point in time, if the observed risky yields of a country are consistent with the default risk priced in the CDS spreads. Indeed, the maintained assumption behind this statement is that the model-implied yields are well estimated, and the model is able to fully capture whatever drives the relationship between default risk and bond prices. With these caveats in mind, we compare observed and implied yields for each country, over the sample time-series.

Figures 4-5 show that the implied yields are generally closer to the observed yields for the Eurozone countries with respect to the No Eurozone countries. Within the Eurozone group (figure 4), we obtain implied yields that are very close to the observed yields for the core countries in the second part of the time series. At the opposite, the NZ countries show a persistent distance between implied and observed yields, over the all time series.
Figure 4. Implied versus Observed Yields. Eurozone

Legend: The figure shows the observed (blue line) and the implied (red line) yields, at 5-years maturity, for each country in the Eurozone group, over the sample time series. The implied yields are obtained by implementing the steps of the estimation methodology described in section 4.
Figure 5. Implied versus Observed Yields. No Eurozone

Legend: The figure shows the observed (blue line) and the implied (red line) yields, at 5-years maturity, for each country in the No Eurozone group, over the sample time series. The implied yields are obtained by implementing the steps of the estimation methodology described in section 4.

The additional straightforward consequence of a perfect equality between observed and implied yields is that the differences in the observed yields across countries are perfectly consistent with the differences in the default risk priced in the CDS spreads, under the assumption that the differences across countries in terms of default risk are well reflected by the model on the differences across bond yields.

Indeed, this assumption is very strong and not strictly necessary for the purpose of our analysis. What we actually aim to investigate is whether the differences in the observed yields across countries are in line with the differences in the yields derived by the model estimates, thus implied by the CDS spreads.

Therefore, we define the observed bond yield as the sum of the unobservable true yield and the mispricing currently arising in the market. We define true yield as the yield that should be paid by the risky bond in absence of any market distortion and friction, thus being perfectly consistent with the default risk of the country.
\[ \hat{Y}_{i,t} = Y_{i,t} + \varepsilon_{i,t}, \]

where \( \hat{Y}_{i,t} \) is the observed yield, \( Y_{i,t} \) is the true yield, and \( \varepsilon_{i,t} \) is the market mispricing, for each country \( i \), and each point in time \( t \). The true yield is indeed unobservable, therefore we assume that the true yield is the sum of an observable proxy and an error:

\[ Y_{i,t} = \tilde{Y}_{i,t} + \eta_{i,t}, \]

where the error term \( \eta_{i,t} \) is proportional to the current level of the true yield proxy, for a given constant \( k \) to be estimated. Thus, we have:

\[ \hat{Y}_{i,t} = (1 + k) \tilde{Y}_{i,t} + \varepsilon_{i,t} = \beta \tilde{Y}_{i,t} + \varepsilon_{i,t}, \]  

(5)

where \( k \) is assumed to be constant across countries and time. Therefore, we can estimate the equation (5) with a panel regression, where \( i \) goes from 1 to 22, and \( t \) goes from 1 to \( T \), where \( T \) is the length of the sample time series (i.e., 1850 daily observations).

We adopt two specifications for the true yield proxy. First, we use the theoretical true yield given by the zero-basis condition described in equation (1). The second proxy is instead the yield implied by the model estimation and generated by MC simulations. The corresponding error terms are then easy to interpret. As for the first proxy, the error is given by the strong set of assumption at the base of the zero-basis condition, while the error in the second proxy is the result of the model assumptions and the estimation error.

Two additional consequences of (5) are straightforward. First, the closer is the regression \( \beta \) to 1, the closer is the error term of the proxy to zero. Moreover, the bond yield market mispricing is simply measured by the regression residuals.

Table IV reports a value of the coefficient close to 1 for both the true yield proxies. Then, we generate the estimation residuals for the two regressions, and we compare them in the next plot.
Table IV. Panel Regression - Observed and True Yield

<table>
<thead>
<tr>
<th></th>
<th>Obs Yield</th>
<th>Obs Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Yield</td>
<td>0.882***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td></td>
</tr>
<tr>
<td>Basis Yield</td>
<td>1.052***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>40656</td>
<td>40656</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.73</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Legend: The table reports the results of the panel data regression of the observed risky bond yields against the proxy of the unobservable risky bond true yields. The Model Yield is the result of the MC simulations using the model estimates, and the Basis Yield is the theoretical true yield given by the zero-basis condition. The stars over the coefficient stand for a 1% significance level, and we report in brackets the standard errors.

Figure 6. Regression Residuals and Basis. Eurozone

Legend: The figure shows the residuals of the panel regression (5) by using the two different true yield proxies (blue line for the model yields regression residuals, and red line for the observed yields regression residuals), and the CDS spreads - Bond Yield basis (yellow line), for the Eurozone countries, over the sample time series

Figure 6 shows that the residuals of both the regressions are very close to the CDS spread - Bond Yield basis, for all the countries, and over the whole time series, thus supporting the interpretation
of the regressions $\beta$ close to 1. Moreover, measuring the market mispricing by using either the observed basis or the regression residuals does not lead to great differences. We report here only the results for the Eurozone, however equivalent results hold for the No Eurozone countries.

5.2. Correlations Analysis

If the distance in terms of default risk across countries is consistently reflected on the distance in terms of risky yields across countries, then the cross-sectional correlation between CDS spreads and risky bond yields should be close to 1. When the CDS spread of the country $A$ is higher than the CDS spread of the country $B$, then the yield on a bond issued by $A$ must be higher than the yield on a bond issued by $B$. Such a relationship should hold across the whole set of countries, therefore the cross-sectional correlation between CDS spreads and risky bond yields should be close to 1, for each point in time.

In fact, if we compute the cross-sectional correlation between CDS spreads and risky bond yields by using the theoretical yield implied by the zero-basis condition, this correlation is always equal to 1, at each point in time.

However, computing only the correlation between CDS spreads and observed bond yields is not enough to rule out the distress puzzle. In particular, the monotonicity of bond yields is a necessary but not sufficient condition to rule out the distress puzzle. We require, in fact, that the relationship between CDS spreads and observed bond yields across countries is not only monotonic, but also that the size of the differences in terms of default risk across countries is reflected in the size of the differences in the bond yields. The rationale behind this condition is that a country might be paying a disproportionately high or low yield compared to what the default risk priced in the CDS would imply, without violating the monotonicity condition.

Therefore, we proceed as follows. First, we deduct the implied yields from the actual yields for each country, thus obtaining a net yield. Once the observed yields have been adjusted by deducting the corresponding implied yields, we can evaluate whether the monotonicity condition still holds, by computing the Spearman’s correlation between CDS spreads and the net yields across countries, at each point in time. As result, we generate a series of cross-sectional correlations over time, between CDS spreads and net yields. The closer is the correlation to one, then the closer is the market
to ruling out the distress puzzle. We adopt the Spearman’s index of correlation as it fits much better the goal of our analysis, by evaluating the monotonic relationships between two variables, regardless the linearity of the relationship.

The intuition behind this approach is simple. If the excess bond yield of the riskier country, with respect to the safer country, is too low then the net yield of the safer country is higher than the net yield of the riskier country. The result is a non-monotonic relationship between CDS spreads and net yields. However, if the observed distance between the bond yields of two countries is too high, then the monotonic relationship between CDS spreads and net yields still holds. Hence, we say that the correlation analysis is able to detect only if there is a sufficient distance between bond yields, across countries. However, as a consequence, if the distance between the bond yields of two countries is too high, then it is likely that the distance between the yield of one of the two countries and a third country’s bond yield is too low, thus returning at the end a lower value of the correlation coefficient.

The next figure represents graphically the main result of the paper. Figure 7 shows the dynamics of the cross-sectional correlations between the 5-years CDS spreads and the implied bond yields (blue line), the observed bond yields (red line), and the net yields (red line), for the Eurozone and the No-Eurozone countries, respectively. Moreover, the bottom plots report the corresponding p-values associated to the test on the statistical significance of the correlation.
The top plots show that the correlation of the CDS spreads with both observed and implied yields is close to 1, over the whole time series, and for both groups of countries. This result is natural for the implied yields, that are estimated by using the CDS spreads. Though, the correlation is not perfectly equal to 1, as the model is subject to an error, and because the yields are then generated by MC simulations still subject to an error. On the other hand, this result documents that also the relationship between CDS spreads and actual yields is monotonically positive, as it should be. This means that riskier countries issue bonds with higher yields.

However, this result does not imply that the distress puzzle is ruled out. What really matters is the dynamics of the red line, where we analyse the presence of a monotonic relationship between CDS spreads and bond yields, only after adjusting the observed yields by using the implied yields.

Indeed, the key result arises when we focus on the correlation between CDS spreads and net yields. This correlation, in fact, randomly moves around zero for the Eurozone countries before the OMT announcement, and approaches 1 right after the OMT announcement, thus remaining stable.
afterwards. It turns out that, before the OMT announcement, the cross-sectional differences across the sovereign bond yields of the Eurozone countries are not consistent with the cross-sectional differences in terms of default risk, and that right after the announcement this consistency is restored.

This result is even more interesting and stronger if we compare Eurozone and No Eurozone countries. In fact, the NZ countries do not show any change in the cross-sectional correlation between CDS spreads and net yields. The correlation, in fact, is quite stable over the whole time series, however never approaching 1. Moreover, the jump to 1 of the cross-sectional correlation across the Eurozone countries is also highlighted by the jump to zero of the corresponding p-value. Therefore, after the OMT announcement, the correlation between CDS spreads and net yields is always significantly different from zero, whereas before the OMT announcement we observe large and very volatile p-values.

Table V. Correlation CDS spreads - Bond Yields

<table>
<thead>
<tr>
<th></th>
<th>Eurozone</th>
<th>No Eurozone</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs Yields</td>
<td>Imp Yields</td>
</tr>
<tr>
<td>Before OMT</td>
<td>0.883</td>
<td>0.938</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>After OMT</td>
<td>0.951</td>
<td>0.927</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Legend: The table reports the average cross-sectional correlation of the CDS spreads and Observed Yields, Implied Yields, and Net Yields (Observed Yields - Implied Yields) for the 5-years time horizon, across Eurozone and No Eurozone countries, and within the pre and the post OMT announcement. We first compute the series of the cross-sectional correlations over the sample period for each group of countries, and then we compute the average within each of the two time intervals (before/after OMT announcement). The same method is applied to compute the p-values, that we report in parentheses.

Table V reports the average correlation, for the different measures of bond yields, across countries in each group, and within each time interval (before/after the OMT announcement). The average correlation between CDS spreads and both actual and implied yields is very close to 1 for both groups, and in each period. Instead, the average correlation across Eurozone countries
between CDS spreads and net yields is more than double in the second period with respect to the first period, thus approaching 1. On the other side, this correlation is very similar across the two periods for the NZ countries, and is even lower after the OMT announcement. Moreover, the corresponding average p-value is large for the Eurozone countries before the OMT announcement, and approaches zero after the OMT announcement.

5.3. Arbitrage Strategies

In this section, we examine two potential arbitrage strategies that exploit riskless profit opportunities. We show that the announcement of the ECB drastically reduces these opportunities for the Eurozone countries. We compare the arbitrage profits across Eurozone and No Eurozone countries, and we show that for the second set of countries, instead, the OMT announcement does not generate any difference in the potential arbitrage profits over the sample time series.

Before looking at the strategies, we recall the definition of the no-arbitrage condition, obtained from the definition of the basis that we used in the previous section of the paper.

\[ s = y - r, \]  

(6)

where \( s \) is the \( T \)-years CDS spread, \( y \) is \( T \)-years yield on the risky bond, and \( r \) is the \( T \)-years yield on the riskless bond. If this relationship does not hold, then an arbitrage opportunity arises in the market by trading CDS, risky bond, and the riskless asset, under a set of assumptions exhaustively explained in Hull et al. (2004). Here, we report only the most relevant assumptions that support the flow of our argument.

1. Market participants can short sovereign bonds
2. Market participants can short the risk-free bond (they can borrow money at the risk-free rate)
3. The ”cheapest-to-deliver bond” option is ruled out, so that the profit is not affected by the ability of the protection seller to find a cheaper bond to deliver in case of default
4. The recovery rate of the bond in case of default is equal to zero

In order to compute the profits, we express all the variables in monetary terms, thus com-
puting the present value of the CDS, the risk-free bond and the risky bond by using continuous compounding, such that the no-arbitrage condition can be rewritten as follows

\[ P_{CDS} = P_{BY} - P_{RF}, \]

where \( P_{CDS}, P_{BY}, P_{RF} \) denote the present value of the CDS, the risky bond, and the riskless bond respectively, and we omit the subscripts \( i \) and \( t \) to save in notation.

**Strategy 1:** The first arbitrage strategy is based on the CDS spread-bond yield basis. Suppose that for the \( i \)-th country, at time \( t \),

\[ P_{CDS} > P_{BY} - P_{RF} \]

then the arbitrageur can sell the risk-free asset, and purchase the CDS and the risky bond issued by the CDS reference entity. The mispricing of the bond generates a positive difference that is exactly the risk-free arbitrage profit. Conversely, if

\[ P_{CDS} < P_{BY} - P_{RF} \]

the arbitrageur obtains the same arbitrage profit by reversing the strategy. In practice, the arbitrageur purchases the risk free asset, and sells the mispriced risky bond and the CDS to obtain the risk-free profit.

Figure 8 shows the arbitrage profits potentially obtained on a portfolio in which each country has equal weight. The panel on the left shows the profits that an arbitrageur can obtain by trading assets of the Eurozone countries, while the panel on the right shows the potential profits by trading assets of the No Eurozone countries. The profits are large and volatile before the OMT Programme announcement in both the Eurozone and No Eurozone areas. After the announcement, however, the profits drop immediately, and start to converge towards zero, for the Eurozone countries. Instead, the riskless profits remain positive and volatile for the countries outside the Eurozone.
Legend: The figure shows the arbitrage profits that could be made on an equally weighted portfolio of sovereign CDS and bonds using strategy 1 described in the paper, over the sample time series. The profits are expressed in monetary terms assuming nominal value of 1 for the bonds, and where the CDS price is computed as present value of the CDS spreads expressed in percentage terms.

Strategy 2: The second strategy exploits the deviation of the observed yields from the yields implied by the model estimates, that are consequently consistent with the default risk priced in the CDS spreads, used to estimate the model. We compute the difference between observed and implied bond yields, at each time $t$, for each country $i$, and we calculate the unconditional mean of those differences for each country, which we consider the benchmark to which the difference should tend to.

Then, at time $t$, for the country $i$, if the difference between observed and implied yield is above the $i$-th country’s unconditional mean, we say that the $i$-th bond is undervalued at $t$, whereas if the difference between observed and implied yield is below the $i$-th country’s unconditional mean, we say that the $i$-th bond is overvalued at $t$.

If the $i$-th country is undervalued, the arbitrageur can sell the risk-free asset, and purchase
the CDS and the risky bond issued by the CDS reference entity. Otherwise, if the $i$-th country is overvalued, the arbitrageur purchases the risk free asset, and sells the mispriced risky bond and the CDS to obtain the risk-free profit.

The implementation of the strategy 2, then, works exactly as for the strategy 1. The difference between the two strategies is only given by which is the signal of an opening of a riskless profit opportunity. While in strategy 1 the signal is the zero-basis condition, in strategy 2 the signal is given by the distance between observed and implied yield.

In figure 9, we compare the potential profits obtained with the second strategy by trading on Eurozone and No Eurozone countries, respectively, with an equally weighted portfolio across countries. The profits plotted in figure 9 are very similar to those presented in figure 8, for both sets of countries. Therefore, the second arbitrage strategy supports our interpretation of the outcome generated by the OMT programme announcement in terms of sovereign bonds market mispricing for the Eurozone countries.

Finally, table VI and table VII report the mean and the standard deviation of the potential profits obtained with the two arbitrage strategies, before and after the OMT announcement, and for the Eurozone and the No Eurozone countries, respectively. Table VI reports the results for the Eurozone countries, and shows a pronounced difference in the average profits between the two subperiods. Further, the standard deviation drops sensibly after the announcement. Such numbers indicate that after the OMT announcement the arbitrage opportunities are approximately zero, or immediately cleared. Instead, for the No Eurozone area, table VII reports similar figures for mean and standard deviation, across the periods before and after the OMT announcement. All the differences reported, in fact, are not statistically different from zero.
Figure 9. Arbitrage Profits - Strategy 2

Legend: The figure shows the arbitrage profits that could be made on an equally weighted portfolio of sovereign CDS and bonds using strategy 2 described in the paper, over the sample time series. The profits are expressed in monetary terms assuming nominal value of 1 for the bonds, and where the CDS price is computed as present value of the CDS spreads expressed in percentage terms.

Table VI. Arbitrage Profits. Eurozone

<table>
<thead>
<tr>
<th>Statistic:</th>
<th>Before OMT</th>
<th>After OMT</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.034</td>
<td>0.014</td>
<td>-0.020*</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.012</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td><strong>Strategy 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.029</td>
<td>0.003</td>
<td>-0.027*</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.012</td>
<td>0.005</td>
<td></td>
</tr>
</tbody>
</table>

Legend: The table reports the mean and the standard deviation of the profits due to the arbitrage strategy applied to the Eurozone countries before and after the OMT announcement date. In the last column the difference between the two subsamples statistic is reported (After OMT-Before OMT). The * indicates that the difference is significant at 5% level.
Table VII. Arbitrage Profits. No Eurozone

<table>
<thead>
<tr>
<th>Statistic:</th>
<th>Before OMT</th>
<th>After OMT</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.036</td>
<td>0.036</td>
<td>-0.000</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.006</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td><strong>Strategy 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.020</td>
<td>0.012</td>
<td>-0.008</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.013</td>
<td>0.017</td>
<td></td>
</tr>
</tbody>
</table>

Legend: The table reports the mean and the standard deviation of the profits due to the arbitrage strategy applied to the No Eurozone countries before and after the OMT announcement date. In the last column the difference between the two subsamples statistic is reported (After OMT-Before OMT). The * indicates that the difference is significant at 5% level.

5.4. Transaction Costs

Arbitrage opportunities can persist in the market if the riskless profits are not sufficient to cover the costs to implement the arbitrage strategy. The idea is that arbitrageurs step into the market only if the arbitrage strategy still generates profits once that the transaction costs have been paid. Therefore, we control for transaction costs by estimating the threshold beyond which the riskless trading gains become sufficiently profitable.

We estimate a vector error correction model (VECM) that includes the CDS spreads and the bond yields in excess of the risk-free rate (excess risky bond yields), for each country, adjusted for the nonlinearity due to the transaction costs threshold (TVECM). In a linear VECM, any deviation from the long-run equilibrium (zero-basis condition) would trigger trades leading the market back to the equilibrium. It turns out that in absence of frictions, such as transactions costs, we should observe a CDS-bond yield basis moving around zero. Instead, when frictions arise in the market, we expect to observe a persistent deviation from the equilibrium. In particular, with non-zero transaction costs, the deviation should persist as long as the magnitude of the deviation is below a given threshold, which introduces the nonlinearity in the error correction model.

Following Gyntelberg, Hördahl, Tersand, and Urban (2017), we model CDS spread and excess risky bond yields as follows, in vector form:
\[ \Delta y_t = [\lambda^L ec_{t-1} + \Gamma^L(\ell) \Delta y_t]d_{Lt}(\beta, \theta) + [\lambda^U ec_{t-1} + \Gamma^U(\ell) \Delta y_t]d_{Ut}(\beta, \theta) + \epsilon_t, \]

where \( ec_{t-1} = CDS_{t-1} - \beta_0 - \beta_1 ER_{t-1} \) is the error correction term, with \( ER \) standing for the excess risky bond yield, \( \Gamma(\ell) \Delta y_t \) is the VAR term of order \( \ell \), and \( \epsilon_t \) are white noise shocks. Moreover, \( d_{Lt} \) and \( d_{Ut} \) are defined as follows:

\[ d_{Lt} = I(ec_{t-1} \leq \theta) \]
\[ d_{Ut} = I(ec_{t-1} > \theta), \]

where \( I \) is an indicator function, and \( \theta \) is the threshold to be estimated. We force \( \beta_1 \) equal to 1, and we estimate \( \beta_0 \). An estimate of \( \beta_0 \) different from zero stands for a persistent non-zero CDS-bond yield basis. Therefore, the average transactions costs faced by the arbitrageurs are given by \( \theta + \beta_0 \). We estimate the model following the approach of Hansen and Seo (2002), who estimate a two-regimes TVECM by using a maximum likelihood algorithm. Also, we estimate the model for the two time sub-samples, such as before and after the OMT announcement. As result, we obtain an estimate of the average transaction costs for each country, and for each time period.\(^1\)

\(^1\)The statistical significance of the thresholds is evaluated following the approach of Hansen and Seo (2002), who calculate standard errors by mean of both parametric and non-parametric bootstrap analysis. Gyntelberg et al. (2017) provide a short description of the two alternative bootstrap procedures, and the decision criterion for the threshold statistical significance.
### Table VIII. Average Transaction Costs

<table>
<thead>
<tr>
<th>Countries</th>
<th>Before OMT</th>
<th>After OMT</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.0152</td>
<td>0.0025</td>
<td>-83</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.0468</td>
<td>0.0073</td>
<td>-84</td>
</tr>
<tr>
<td>Finland</td>
<td>0.0105</td>
<td>0.0036</td>
<td>-66</td>
</tr>
<tr>
<td>France</td>
<td>0.0131</td>
<td>0.0060</td>
<td>-54</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0133</td>
<td>0.0013</td>
<td>-90</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.0098</td>
<td>0.0059</td>
<td>-40</td>
</tr>
<tr>
<td>Average Core</td>
<td>0.0181</td>
<td>0.0044</td>
<td>-75</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.2466</td>
<td>0.0312</td>
<td>-87</td>
</tr>
<tr>
<td>Italy</td>
<td>0.1255</td>
<td>0.0508</td>
<td>-59</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.4179</td>
<td>0.0897</td>
<td>-78</td>
</tr>
<tr>
<td>Slovakia</td>
<td>0.0723</td>
<td>0.0131</td>
<td>-81</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.0321</td>
<td>0.0368</td>
<td>+14</td>
</tr>
<tr>
<td>Spain</td>
<td>0.1123</td>
<td>0.0479</td>
<td>-57</td>
</tr>
<tr>
<td>Average Peripheral</td>
<td>0.1678</td>
<td>0.0449</td>
<td>-73</td>
</tr>
<tr>
<td>Average Eurozone</td>
<td>0.0930</td>
<td>0.0247</td>
<td>-73</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>0.1132</td>
<td>0.0542</td>
<td>-52</td>
</tr>
<tr>
<td>Croatia</td>
<td>0.1958</td>
<td>0.1161</td>
<td>-40</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.0303</td>
<td>0.0084</td>
<td>-72</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.0152</td>
<td>0.0044</td>
<td>-71</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.2156</td>
<td>0.0975</td>
<td>-54</td>
</tr>
<tr>
<td>Norway</td>
<td>0.0111</td>
<td>0.0407</td>
<td>+268</td>
</tr>
<tr>
<td>Poland</td>
<td>0.1170</td>
<td>0.0789</td>
<td>-32</td>
</tr>
<tr>
<td>Romania</td>
<td>0.2012</td>
<td>0.1034</td>
<td>-48</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.0070</td>
<td>0.0109</td>
<td>+56</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.0057</td>
<td>0.0340</td>
<td>+495</td>
</tr>
<tr>
<td>Average No Eurozone</td>
<td>0.0912</td>
<td>0.0548</td>
<td>-39</td>
</tr>
</tbody>
</table>

**Legend:** The table reports the average transaction costs ($\theta + \beta_0$), for each country, before and after the OMT announcement date. The average transaction costs are expressed in percentage terms (basis points/10000). The last column reports the variation in percentage terms across the two time periods, for each country. We also report the mean across group of countries (Eurozone Core, Eurozone Peripheral, No Eurozone)
We find that, in general, the key threshold is substantially higher before the OMT announcement (the average transaction costs across countries is 922bp) with respect to the second period (384bp). This result is consistent with the findings of Gyntelberg et al. (2017), who estimate a threshold during the Eurozone sovereign debt crisis period more than twice higher with respect to the pre-crisis period. We also find that the core Eurozone countries have much lower average transaction costs in both the time periods (181bp before the OMT announcement, and 44bp after the OMT announcement). Moreover, we find that the drop in the average transaction costs across the two time periods is much more pronounced for the Eurozone countries (from 930bp to 247bp), and in particular for the peripheral countries (from 1678bp to 448bp), with respect to the No Eurozone countries (from 912bp to 548bp).

Next, we compare the estimated transaction costs in the two periods with the potential arbitrage profits generated by the arbitrage strategies described above. Consistently with the approach followed in the analysis of the arbitrage strategies, we compare the riskless profits over the full time series with the estimated transaction costs across groups of countries, by splitting our sample in two groups (Eurozone, No Eurozone). Note that the key insights hold if we split again the Eurozone countries in two sub-samples (Core, Peripheral). The plot in figure 10 offers a straightforward interpretation of our results.

Before the OMT announcement, we estimate average transaction costs similar across groups of countries, and that are above the arbitrage profits for all groups of countries. Therefore, the arbitrageurs do not have incentive to intervene and clear the arbitrage opportunities, as the riskless profits are even not sufficient to cover the costs to implement the strategy. As a consequence, over this period, there is a persistent deviation from the zero-basis equilibrium condition.

After the OMT announcement, instead, we estimate a pronounced reduction of the average transaction costs for the Eurozone countries. Hence, the arbitrageurs find profitable to step into the market and take advantage of the deviation from the equilibrium condition. The riskless profits, then, quickly converge to zero, and the same happens for the CDS spread - bond yield basis. In other words, the lower transaction costs have created the condition for the traders to profit from the arbitrage opportunities generated by the relative CDS spread - bond yield mispricing, then leading the sovereign debt market back to the equilibrium (zero basis).
**Legend:** The figure compares the profits generated by the arbitrage strategy 1 for the Eurozone countries (blue line) and the No Eurozone countries (yellow line) against the average transaction costs across the Eurozone countries (green dotted line) and the No Eurozone countries (purple dotted line). The red line stands for the OMT announcement date.

On the other hand, this condition does not occur for the No Eurozone countries. The estimated key threshold, in fact, is lower in the second time period with respect to the pre-announcement period, however the threshold reduction is not enough to create the condition for the traders to clear the arbitrage opportunities. Therefore, we observe a persistent CDS spread - bond yield mispricing even after the OMT announcement, with a persistent deviation from the zero-basis condition.

We conjecture two different explanations, though perhaps simultaneously at work, for the reduction in the Eurozone market transaction costs after the OMT announcement, and the consequent alignment of the relative CDS spread - bond yield pricing to the equilibrium condition. First, the announcement of the ECB has stimulated a pronounced inflow of liquidity in the market, thus reducing the trading costs (by reducing, for instance, the bid-ask spread). Moreover, the intervention of the ECB has generated a risk-reduction effect on the sovereign debt market, by reducing the debt securities volatility, and consequently reducing the risk-premium required by the arbitrageurs.
as compensation to trade. In this case, we can interpret the transaction costs as a lower bound for the risk-premium sought by the traders to step into the market.

6. Conclusion

In the paper, we conduct an empirical investigation of the relationship between sovereign CDS spreads and sovereign bond yields. In a nutshell, we document that, after the announcement of the OMT programme by the ECB, the consistent relationship between CDS spreads and bond yields across Eurozone countries is restored, differently from the No Eurozone countries, which instead show a persistent deviation from the theoretical equilibrium relationship over the all sample period.

We shed light on the effects of the unconventional monetary policy of the ECB on the CDS-bond relationship, and more in general on the consistent risk-return relationship in the sovereign context, with different approaches, that produce a unified and homogenous evidence on the behaviour of the sovereign credit risk market prior and following the announcement of the programme, and across groups of countries.

Further investigation should focus on the big challenge of isolating the long term effects of the OMT programme on the relative pricing of the sovereign credit securities, in order to prove and identify a robust causal relationship. The main issue in a sovereign analysis is created by the unavoidable interaction between external and internal factors simultaneously at work. With this paper, we want to highlight a crucial evidence for the analysis of the risk-return relationship, linking this cornerstone of the financial theory with macro-economic and monetary events, then awaiting for further and deeper research.
Appendix A. Kalman filter and Quasi-Maximum Likelihood Estimation

In a general formulation, with a non-linear relationship between the measurement and the state variables, the state-space model is defined by two sets of equations, the transition and the measurement equation, respectively:

\[
X_{i,t+\delta t} = X_{i,t} + c_i + \epsilon_{i,t+\delta t},
\]

\[
Y_{i,t+\delta t} = \psi(X_{i,t+\delta t}) + u_{i,t+\delta t},
\]

where \(X_{i,t+\delta t}\) is the \(i\)-th observation of the state variable at time \(t + \delta t\), \(c_i\) is the time-invariant component driving the evolution of the state variable, \(\epsilon_{i,t+\delta t}\) is the transition error on the \(i\)-th observation of the state variable at time \(t + \delta t\). On the other hand, \(Y_{i,t+\delta t}\) is the \(i\)-th observation of the measurement variable at time \(t + \delta t\), \(\psi\) is the measurement function which links the observable and the latent variable, and \(u_{i,t+\delta t}\) is the measurement error.

For a Gaussian state-space model, under standard assumptions, the discrete Kalman filter is proved to be the minimum mean squared error estimator. However, in the case of non-linear relation between the measurement and the state variable, the classic linear Kalman filter is not longer optimal. One possible solution is to linearize the estimation around the current estimate by using the partial derivatives of the process and measurement functions. To linearize the measurement process, we need to compute the derivatives of \(\psi\) with respect to

(a) the state variable: \(H_{i,j} = \frac{\partial \psi_i}{\partial X_j}(\tilde{X}_t, 0)\),

where \(H\) is the Jacobian matrix of partial derivatives of the generic measurement function \(\psi(\cdot)\) with respect to the state variable \(X\), and \(\tilde{X}_t\) is the current estimate of the state.

(b) the measurement noise: \(\tilde{H}_{i,j} = \frac{\partial \psi_i}{\partial \nu_j}(\tilde{X}_t, 0)\),

where \(\tilde{H}\) is the Jacobian matrix of partial derivatives of \(\psi(\cdot)\) with respect to the noise term \(\nu\).
Once the linearization has been completed, we can implement the discrete Kalman filter in the usual steps. First, we need to set the initial conditions:

\[ \lambda_{i,0} \quad P_{i,0}, \]

where \( P_{i,t} := \text{var}[X_{i,t} - \lambda_{i,t}] \) is the variance of the estimation error, and \( \lambda_{i,t} \) is the estimate of the state at time \( t \) based on the information available up to time \( t \). Then, the filter implementation is based upon two sets of equations, the predicting equations, and the updating equations, that must be repeated for each time step in the data sample.

- **State Prediction**

\[ \lambda_{i,t+\delta t/t} = \lambda_{i,t} + c, \]

and

\[ P_{i,t+\delta t/t} = P_{i,t} + Q_{i}, \]

where \( \lambda_{i,t+\delta t/t} \) is the estimate of the state at time \( t + \delta t \) based on the information available up to time \( t \), and \( Q_{i} \) is the covariance of the transition noise.

- **Measurement Update**

\[ \lambda_{i,t+\delta t} = \lambda_{i,t+\delta t/t} + P_{i,t+\delta t/t} H'_{i,t+\delta t} Z_{i,t+\delta t}^{-1} (Y_{i,t+\delta t} - \psi(\lambda_{i,t+\delta t/t})) \]

\[ P_{i,t+\delta t} = P_{i,t+\delta t/t} - P_{i,t+\delta t/t} H'_{i,t+\delta t} Z_{i,t+\delta t}^{-1} H_{i,t+\delta t} P_{i,t+\delta t/t} \]

\[ Z_{i,t+\delta t} = H_{i,t+\delta t} P_{i,t+\delta t/t} H'_{i,t+\delta t} + R_{i}, \]

where \( H \) stands for the Jacobian matrix of partial derivatives of the generic measurement function \( \psi \) with respect to the state variable \( X \), \( Z_{i,t+\delta t} \) is the covariance matrix of the prediction errors at
time $t + \delta t$. The prediction errors are defined as $v_{i,t+\delta t} = Y_{i,t+\delta t} - \psi(\lambda_{i,t+\delta t/\delta t})$, where $Y_{i,t+\delta t}$ is the observation of the measurement variable at time $t + \delta t$.

The parameters that describe the dynamics of the transition and the measurement equations (i.e., hyperparameters) are unknown, and need to be estimated.

Let rewrite the state-space model as follows:

\[
(y_{t+\delta t}, x_{t+\delta t}) = (x_t, \{\theta\}), \quad \{\theta\} = \{\theta^{(f)}; \theta^{(g)}\}
\]

where $y_{t+\delta t}$ is the observable variable at time $t + \delta t$, $x_{t+\delta t}$ is the state variable at time $t + \delta t$, $\{\theta^{(f)}\}$ is the set of unknown parameters in the transition equation, and $\{\theta^{(g)}\}$ is the set of unknown parameters in the measurement equation. The measurement and transition equations of the system are:

\[
g(y_{t+\delta t}, \alpha) = \varphi(x_{t+\delta t}, \beta) + \epsilon_{t+\delta t}, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_\epsilon)
\]

\[
x_{t+\delta t} = f(x_t, \gamma) + \eta_{t+\delta t}, \quad \eta_t \sim \mathcal{N}(0, \sigma^2_\eta)
\]

Then,

\[
\{\theta^{(f)}\} = \{\gamma, \sigma^2_\eta\}
\]

\[
\{\theta^{(g)}\} = \{\alpha, \beta, \sigma^2_\epsilon\}
\]

We assume that the nonlinear regression disturbance, $\epsilon_t$, is normally distributed:

\[
f(\epsilon_t) = \frac{1}{\sqrt{2\pi\sigma^2_\epsilon}} \exp\left[-\frac{\epsilon_t^2}{2\sigma^2_\epsilon}\right]
\]

By transformation of variable, the density of $y_t$ is given by

\[
f(y_t) = f(\epsilon_t) \left| \frac{\partial \epsilon_t}{\partial y_t} \right| = \frac{\partial g(y_t, \alpha)}{\partial y_t} \frac{\partial \epsilon_t}{\partial y_t}
\]
Then, the density of $y_t$ is

$$f(y_t) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left[ -\frac{(g(y_t, \alpha) - \varphi(x_t, \beta))^2}{2\sigma^2} \right] \left| \frac{\partial g(y_t, \alpha)}{\partial y_t} \right|$$

The log-likelihood function for observation $t$ is

$$\ln \Omega_t (y_t; \theta) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma^2) - \frac{(g(y_t, \alpha) - \varphi(x_t, \beta))^2}{2\sigma^2} + \ln \left| \frac{\partial g(y_t, \alpha)}{\partial y_t} \right|,$$

and the log-likelihood function for $t = 1, 2, ..., T$ observations (i.e., $\delta t = 1$) is

$$\ln \Omega = \sum_{t=1}^{T} \ln \Omega_t (y_t; \theta) = -\frac{T}{2} \ln(2\pi) - \frac{T}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} (g(y_t, \alpha) - \varphi(x_t, \beta))^2$$

$$+ \sum_{t=1}^{T} \ln \left| \frac{\partial g(y_t, \alpha)}{\partial y_t} \right|$$

As long as $g(y_t, \alpha) = y_t$, then

$$f(y_t) = f(\epsilon_t) \Rightarrow \ln \Omega_t (y_t; \theta) = \ln \Omega_t (\epsilon_t; \theta)$$

The last term in the log-likelihood function is equal to zero, and the space of the hyperparameters to be estimated is reduced to:

$$\{\theta^{(f)}\} = \{\gamma, \sigma^2\}$$

$$\{\theta^{(g)}\} = \{\beta, \sigma^2\}$$

In practice, the iteration of the filter generates a measurement-system prediction error, and a prediction error variance at each step. Under the assumption that measurement-system prediction errors are Gaussian, we can construct the log-likelihood function as follows:

$$\ln \Omega(y_t; \theta) = \ln \prod_{t=0}^{T-\delta t} p(y_{t+\delta t/t}) = \sum_{t=0}^{T-\delta t} \ln p(y_{t+\delta t/t}) =$$
\[
= -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=0}^{T-\delta t} \ln |Z_{t+\delta t}| - \frac{1}{2} \sum_{t=0}^{T-\delta t} v_{t+\delta t}'Z_{t+\delta t}^{-1}v_{t+\delta t},
\]

where \( N \) is the number of time steps in the data sample. Finally, this function is maximized with respect to the unknown parameters vector \( \{\theta\} \). This is known as the Quasi-Maximum Likelihood estimation, in conjunction with the non-linear Kalman filter.
REFERENCES


