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1 **Determination of the aerodynamic droplet breakup boundaries based**
2 **on a total force approach**

3

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23

24 **Abstract**

25 The determination of the critical We_g number separating the different breakup regimes
26 has been extensively studied in several experimental and numerical works, while
27 empirical and semi-analytical approaches have been proposed to relate the critical
28 We_g number with the Oh_l number. Nevertheless, under certain conditions, the Re_g
29 number and the density ratio ε may become important. The present work provides a
30 simple but reliable enough methodology to determine the critical We_g number as a
31 function of the aforementioned parameters in an effort to fill this gap in knowledge. It
32 considers the main forces acting on the droplet (aerodynamic, surface tension and
33 viscous) and provides a general criterion for breakup to occur but also for the
34 transition among the different breakup regimes. In this light, the present work
35 proposes the introduction of a new set of parameters named as $We_{g,eff}$ and Ca_l
36 monitored in a new breakup plane. This plane provides a direct relation between gas
37 inertia and liquid viscosity forces, while the secondary effects of Re_g number and
38 density ratio have been embedded inside the effective We_g number ($We_{g,eff}$)

39 **Keywords:** droplet breakup; critical We number; VOF simulations

40

41 **1 Introduction**

42 The aerodynamic droplet breakup has been extensively studied in experimental and
43 numerical works due to its importance in spray systems. Depending on the relative
44 strength of the main forces acting on the droplet (aerodynamic, surface tension and
45 viscous forces), different breakup types can be observed such as the bag breakup, the
46 transitional breakup (including several sub-types), the sheet-thinning breakup and the
47 catastrophic breakup. A complete description of these breakup modes can be found in
48 the review article of (Guildenbecher et al., 2009) among others.

49 Increasing the gas phase inertia results in the successive transition between the
50 aforementioned breakup regimes. The parameters affecting droplet breakup are
51 grouped into dimensionless numbers, such as the We_g , the Oh_l and the Re_g numbers,
52 but also the density and viscosity ratios of the liquid/gas phase (ε and N respectively);
53 see section 2.1 for a complete description of these numbers. Among them, the We_g
54 number is the most influential, while the liquid viscous damping becomes important
55 only when $Oh_l > 0.1$; see for example the breakup map of (Hsiang and Faeth, 1995).

56 The We_g number leading to droplet breakup (or generally separating different breakup
57 regimes) is called critical We number ($We_{g,cr}$) and in the limit of negligible liquid
58 viscosity (i.e. low Oh_l), we call it in the present work as $We_{g,cr,0}$ (the subscript 0
59 denotes negligible viscosity). Having also in mind that the experimental data are
60 characterized by high Re_g numbers, the $We_{g,cr,0}$ generally represents negligible
61 viscosity effects both in the gas and liquid phases. In the following paragraphs, the
62 various approaches found in literature to relate $We_{g,cr}$ with $We_{g,cr,0}$ will be presented.

63 In (Guildenbecher et al., 2009) it is stated that breakup is observed for $We_{g,cr,0}=11\pm 2$,
 64 indicating that there is a scatter in the results of experimental works; in (Hanson et al.,
 65 1963) an even lower value of ~ 7 is reported. Regarding the dependency between the
 66 $We_{g,cr}$ and Oh_l numbers (the two most influential), this is generally expressed through
 67 the empirical equation 1, where C and n are fitting coefficients:

$$\frac{We_{g,cr}}{We_{g,cr,0}} = 1 + C \cdot Oh_l^n \quad (1)$$

68 A list of the coefficients C , n which were determined in past works is given in Table
 69 1. (Brodkey, 1967) and (Gelfand, 1996) obtained these coefficients by fitting
 70 experimental data, while (Cohen, 1994) assumed that the energy required for breakup,
 71 is that of an inviscid droplet plus the energy required to overcome the viscous
 72 dissipation (see details in section 6.3); this resulted in $n=1$, while the coefficient C
 73 was determined by fitting experimental data.

74

75 Table 1: List of the coefficients C , n of eq. 1 proposed by different sources for the bag
 76 breakup regime.

source	coeff. C	coeff. n	derivation	comments
(Brodkey, 1967)	1.077	1.6	Empir.	$Oh_l < 10$
(Cohen, 1994)	1-1.8	1	Semi-Anal.	$10 < We_{g,cr,0} < 100$
(Gelfand, 1996)	1.5	0.74	Empir.	$Oh_l < 4$

77

78 In (Hsiang and Faeth, 1995) the droplet momentum equation was used and adopting
 79 the viscous timescale of (Hinze, 1949) (eq. 14 in section 2.1), they derived equation 2.
 80 Assuming an average value of the drag coefficient $\overline{C_D}$, they determined the coefficient
 81 C (without mentioning its value) by comparing against experimental data and the
 82 model performance was very good.

$$\frac{We_{g,cr}}{We_{g,cr,0}} = \frac{1}{4} \left(1 + C \cdot \frac{\overline{C_D}}{\sqrt{\varepsilon \cdot We_{g,cr,0}}} Oh_l \right)^2 \quad (2)$$

83

84 Another approach for the estimation of the critical We_g number, is to assume that the
 85 breakup is ought to Rayleigh-Taylor (R-T) instabilities as in (Zhao et al., 2011) and
 86 (Yang et al., 2017). According to this model, when the droplet deformation (usually
 87 the cross-stream diameter) exceeds the critical wavelength of the R-T instability
 88 (which depends on liquid properties and droplet acceleration), then breakup occurs.
 89 The resulting equation (e.g. in (Zhao et al., 2011)) has the form of eq. 3, where C is an
 90 adjustable coefficient, in the range 1.18-1.48.

$$\left(\frac{We_{g,cr,0}}{We_{g,cr}} \right)^{1/2} + C \left(\frac{Oh_l^2}{We_{g,cr}} \right)^{1/3} = 1 \quad (3)$$

91 The concept of R-T instabilities has been considered as the main mechanism for
 92 breakup in other works as in (Joseph et al., 1999),(Theofanous and Li, 2008),
 93 (Theofanous et al., 2012). The group of Prof. Theofanous considered also a different
 94 characterization of breakup, with Rayleigh-Taylor piercing (RTP) happening at lower
 95 We_g numbers and shear-induced entrainment (SIE) above a transition We_g . Generally,

96 the aforementioned correlations are in qualitative agreement between them, but they
97 do not give insight into the effects of Re_g and ε numbers

98 Turning now to the effect of the Re_g number and density ratio ε , this has not been in
99 detail examined in experimental works due to technical limitations in obtaining low
100 Re_g and ε numbers. On the other hand, their effect has been examined in a few
101 numerical works but without providing correlations similar to the aforementioned for
102 the Oh_l number (e.g. as in eq. 1). As a general remark, they have all concluded that
103 the critical We_g number increases for low Re_g and ε numbers. More specifically,
104 (Aalburg, 2002) found that there is no effect on breakup for $Re_g > 100$ and $\varepsilon > 32$.
105 Nevertheless, their numerical model could not predict the actual breakup and they
106 assumed that breakup happens when the cross-stream deformation exceeds 60%;
107 despite this limitation, they were able to reproduce the breakup map of (Hsiang and
108 Faeth, 1995). In (Han and Tryggvason, 2001) the authors examined low density ratios
109 ($\varepsilon < 10$) and found that the Re_g effect is minimal for $Re_g > 200$, while decreasing the Re_g
110 and keeping the other parameters constant can lead to different breakup modes. A
111 similar conclusion was also drawn when the density ratio decreases and approaches
112 unity. In (Jing and Xu, 2010) it is stated that shear breakup is observed only for
113 $\varepsilon > 100$, while for Re_g numbers in the range 10^2 up to 10^6 there are slight differences in
114 the topology of the bag and the rim. Regarding the effect of density ratio they found
115 different breakup modes for $\varepsilon = 10$ and 1000 (forward bag and sheet-thinning
116 respectively for $We_g = 27.5$) and also significantly lower droplet acceleration and
117 displacement as the density ratio increases. Recently, (Yang et al., 2016) used a 3D
118 model to study breakup at highly unstable conditions ($Re_g \sim 10^4$) and found that
119 breakup is affected even for $\varepsilon > 32$ (the limit proposed by (Aalburg, 2002)), and a

120 lower density ratio results in a higher deformation rate but less intensive
121 fragmentation. Finally, (Kékesi et al., 2014) examined various combinations of Re_g
122 and ε numbers (generally low values) and identified new breakup regimes that have
123 not been observed in experiments.

124 The aim of the present work is to provide a simple but reliable methodology to relate
125 the critical We_g number with all the actual dimensionless numbers affecting droplet
126 breakup, as there is a lack of such a model in literature. In the text follows there is a
127 description of the methodology and then the model results are presented. In the
128 appendix, the derivation of correction factors for the effect of Re_g number and density
129 ratio is presented along with a correlation to predict the breakup initiation time.
130 Finally in the appendix, the present methodology is related to a modified version of
131 the energy approach of (Cohen, 1994), showing that both concepts are equivalent.

132

133 **2 Methodology**

134 **2.1 Forces and dimensionless numbers**

135 Before proceeding to the presentation of the methodology adopted in the present
136 work, it is essential to discuss the forces acting on the droplet and the dimensionless
137 numbers describing droplet breakup.

138 The main forces controlling droplet breakup are the aerodynamic forces induced from
139 the gas phase ($\sim \rho_g U_g^2 D^2$), the surface tension forces ($\sim \sigma D$), the gas viscosity forces
140 ($\sim \mu_g U_g D$) and the liquid viscosity forces ($\sim \mu_l U_l D$). In the latter case, the liquid

141 velocity U_l appearing can be estimated from (Hinze, 1955), (Hsiang and Faeth, 1992)

142 as:

$$U_l = \sqrt{\frac{\tau}{\rho_l}} = \sqrt{\frac{\rho_g U_g^2}{\rho_l}} = \frac{U_g}{\sqrt{\varepsilon}} \quad (4)$$

143 , where $\varepsilon = \rho_l/\rho_g$ is the density ratio and τ denotes stress. This equation implies that

144 gas and liquid inertia forces are essentially equal to one another ($\rho_l U_l^2 = \rho_g U_g^2$).

145 Turning now to the definition of the dimensionless numbers, these are derived by

146 combing different types of forces:

$$We_g \sim \frac{\text{gas inertia forces}}{\text{surface tension forces}} = \frac{\rho_g U_g^2 D^2}{\sigma D} = \frac{\rho_g U_g^2 D}{\sigma} \quad (5)$$

$$Oh_l \sim \frac{\text{liquid viscous forces}}{\sqrt{(\text{inertia} \cdot \text{surface tension}) \text{ forces}}} = \frac{\mu_l U_l D}{\sqrt{\rho_l U_l^2 D^2 \cdot \sigma D}} \quad (6)$$
$$= \frac{\mu_l}{\sqrt{\sigma D \rho_l}}$$

$$Re_g \sim \frac{\text{gas inertia forces}}{\text{gas viscous forces}} = \frac{\rho_g U_g^2 D^2}{\mu_g U_g D} = \frac{\rho_g U_g D}{\mu_g} \quad (7)$$

147 Among them, the We_g and the Oh_l numbers are the most influential in droplet

148 breakup, while one can notice that the Oh_l number despite its wide use and

149 importance, it has a rather strange physical meaning by relating the liquid viscous

150 forces with the square root of inertia times surface tension forces.

151 Using different combinations of the aforementioned forces, one can define additional

152 dimensionless numbers, such as the liquid and gas Capillary numbers (Ca_l and Ca_g),

153 the liquid Re number (Re_l) and the gas-liquid Re number (Re_{gl}):

$$Ca_l \sim \frac{\text{liquid viscous forces}}{\text{surface tension forces}} = \frac{\mu_l U_l D}{\sigma D} = \frac{\mu_l U_g}{\sigma \sqrt{\varepsilon}} \quad (8)$$

$$Ca_g \sim \frac{\text{gas viscous forces}}{\text{surface tension forces}} = \frac{\mu_g U_g D}{\sigma D} = \frac{\mu_g U_g}{\sigma} \quad (9)$$

$$Re_l \sim \frac{\text{liquid inertia forces}}{\text{liquid viscous forces}} = \frac{\rho_l U_l^2 D^2}{\mu_l U_l D} = \frac{\rho_l U_g D}{\mu_l \sqrt{\varepsilon}} = Re_g \frac{\sqrt{\varepsilon}}{N} = \frac{\sqrt{We_g}}{Oh_l} \quad (10)$$

$$Re_{g/l} \sim \frac{\text{gas inertia forces}}{\text{liquid viscous forces}} = \frac{\rho_g U_g^2 D^2}{\mu_l U_l D} = \frac{\sqrt{\rho_g \rho_l} U_g D}{\mu_l} = Re_g \frac{\sqrt{\varepsilon}}{N} \quad (11)$$

154 The gas-liquid Re number ($Re_{g/l}$) and the liquid Re number (Re_l) is proved to represent
155 the same quantity. Both of them are equal to $Re_g \sqrt{\varepsilon}/N$ or $\sqrt{We_g}/Oh_l$; the latter was
156 used in (Aalburg, 2002) to develop a new breakup map as it was proved to dominate
157 the breakup at large Oh_l numbers. The gas-liquid Re number ($Re_{g/l}$) has also appeared
158 in the work of (Schmehl, 2002), named there as “deformation” Re number. Finally,
159 the term $N/\sqrt{\varepsilon}$ in equations 10 and 11 depends on the physical properties and has
160 appeared in (Gelfand, 1996), (Aalburg, 2002), while in (Kékesi et al., 2014) it was
161 also used to develop a new breakup map along with the Re_g number. Among the
162 aforementioned new dimensionless numbers, the Ca_l number will be proved in the
163 subsequent sections to be the most valuable one and it is related to the other numbers
164 with the following equation:

$$Ca_l = \frac{\mu_l U_g}{\sigma \sqrt{\varepsilon}} = Oh_l \sqrt{We_g} = \frac{We_g}{Re_l} \quad (12)$$

165

166 Finally, for the non-dimensionalization of time, the shear breakup timescale of
 167 (Nicholls and Ranger, 1969) is widely used (eq. 13), which in fact represents the
 168 liquid convection timescale. For large Oh_l numbers, the viscous timescale of (Hinze,
 169 1949) has also been used (eq. 14)

$$t_{sh} = \frac{D}{U_l} = \frac{D\sqrt{\varepsilon}}{U_g} \quad (13)$$

$$t_{vis} = \frac{\mu_l}{\rho_g U_g^2} = t_{sh}/Re_l \quad (14)$$

170

171 **2.2 Total Force approach**

172 As mentioned in the previous section 2.1, there are various types of forces acting on
 173 the droplet. One can group them into forces that tend to deform the droplet (F_{DEF}) and
 174 forces tending to restore the droplet (F_{RES}) in its original shape, or equivalently into
 175 forces from the gas side (F_{gas}) and forces from the liquid side (F_{liq}). These two types
 176 of forces are overall calculated as in equations 15 and 16,

$$F_{DEF/gas} = \rho_g U_g^2 D^2 - C_{vis,g} \mu_g U_g D \quad (15)$$

$$F_{RES/liq} = \sigma D + C_{vis,l} \mu_l U_l D \quad (16)$$

177 , where the terms $C_{vis,g}$ and $C_{vis,l}$ are adjustment factors aiming to reveal the
 178 contribution of the gas and liquid viscous forces correspondingly on the evolution of
 179 the phenomenon. For convenience here, the gas viscous forces (which are restorative)
 180 appear in the deformation forces but with a negative sign. Thus, the term $F_{DEF/gas}$
 181 represents the net deformation forces from the gas side.

182 The ratio of these forces is called here TFR (Total Force Ratio) and it is shown in eq.
 183 17, in which the liquid velocity U_l has been replaced with the corresponding gas terms
 184 according to eq. 4:

$$TFR = \frac{F_{DEF/gas}}{F_{RES/liq}} = \frac{\rho_g U_g^2 D^2 - C_{vis,g} \mu_g U_g D}{\sigma D + C_{vis,l} \mu_l \frac{U_g}{\sqrt{\epsilon}} D} \quad (17)$$

185 Diving both forces with σD (the surface tension forces) and after some manipulation
 186 of eq. 17 we can reach the following equation 18, where $f_{vis,g}$ represents a correction
 187 factor for the gas viscosity effects.

$$TFR = \frac{We_g}{f_{vis,g} (1 + C_{vis,l} \cdot Ca_l)} \quad (18)$$

$$f_{vis,g} = \frac{1}{1 - C_{vis,g} (1/Re_g)} \quad (19)$$

188

189 As seen, the TFR is in fact the We_g number divided/corrected by two terms (both
 190 higher than unity) to account for viscosity effects. In the limit of inviscid flow, TFR is
 191 simply the We_g number and a close physical approximation of this situation
 192 corresponds to conditions characterized by low Oh_l and high Re_g numbers.

193 The model proposed in this study, assumes that there is a critical TFR value (TFR_{cr}),
 194 above which, breakup occurs (or there is generally transition among the different
 195 breakup modes). It is further assumed that the critical TFR value depends only on the
 196 breakup mode (bag, sheet-thinning etc) and not on other dimensionless numbers (e.g.
 197 the Oh_l number as in the case of $We_{g,cr}$), since by definition TFR accounts for all types

198 of forces. This critical value pertains to the $We_{g,cr,0}$ value, which depends only on the
 199 breakup mode and it is generally known from experiments ($We_{g,cr,0}$ was defined in the
 200 introduction for low Oh_l and high Re_g numbers).

201 Nevertheless, there is also one parameter that has not been yet included in the present
 202 analysis. This is the density ratio ε . Past works (presented in the introduction) have
 203 shown that the critical We_g number increases for low ε values, thus the $We_{g,cr,0}$ has to
 204 be multiplied by a correction factor f_ε ($f_\varepsilon > 1$) to account for low density ratio effects.
 205 Thus the final criterion for breakup should be defined as $TFR_{cr} = We_{g,cr,0} \cdot f_\varepsilon$.

206 Replacing the TFR_{cr} from eq. 18, the following equations 20-23 describe the relation
 207 among $We_{g,cr}$ and the rest dimensionless numbers.

$$\frac{We_{g,cr}}{We_{g,cr,0}} = f_\varepsilon \cdot f_{vis,g} \cdot f_{vis,l} \quad (20)$$

$$f_\varepsilon = 1 + C_\varepsilon \frac{1}{\varepsilon} \quad (21)$$

$$f_{vis,g} = \frac{1}{1 - C_{vis,g} (Re_g)^{-ng}} \quad (22)$$

$$f_{vis,l} = 1 + C_{vis,l} \cdot (Ca_l)^{nl} \quad (23)$$

208 The density correction factor f_ε is given by equation 21 with $C_\varepsilon=3$ (see section 6.1.2
 209 for details), while the gas and liquid viscosity correction factors are re-written in
 210 equations 22 and 23 in a more generic way (using the exponents ng , nl) to account for
 211 deviations from the preceding theoretic analysis (theoretically it is $ng=nl=1$).

212 Regarding the adjustable coefficients $C_{vis,g}$ and ng for the effect of gas viscosity in eq.
 213 22, these were determined by performing numerical simulations (see section 6.1.1 for
 214 details) and found to be equal to $C_{vis,g}=55$ and $ng=1.1$, following a best fitting
 215 algorithm, which is close to the estimated value of 1. Regarding the liquid viscosity
 216 coefficients, these were depending on the breakup mode and were estimated to be in
 217 the range $C_{vis,l}=0.06 - 0.26$ and $nl=0.9 - 1.0$ (close to the theoretic value of 1);
 218 nevertheless, a value of $nl=1$ was used throughout this study for all breakup modes.
 219 These were initially determined based on the breakup boundaries of (Hsiang and
 220 Faeth, 1995) and then fine-tuned using the experimental and numerical data shown in
 221 sections 3.2.2 and 3.2.3.

222 The values used for the adjustable coefficients $C_{vis,g}$, $C_{vis,l}$, ng , nl and C_ϵ are given in
 223 Table 2, as well as the $We_{g,cr,0}$ value; the catastrophic breakup regime is also included,
 224 but it has been estimated without having a sufficient amount of data. All these
 225 coefficients are assumed, for the current status of work, to be constant numbers but it
 226 is likely, that they are functions of additional numbers (e.g the density ratio), or there
 227 are interdependencies between them. It has also to be noted, that all coefficients were
 228 assumed to be unaffected by the breakup mode, except of the $C_{vis,l}$ coefficient which is
 229 the most influential.

230

231 Table 2: Values of the adjustable coefficients $C_{vis,g}$, ng , $C_{vis,l}$, nl , C_ϵ used in equations
 232 21-23. The $We_{g,cr,0}$ value is also shown.

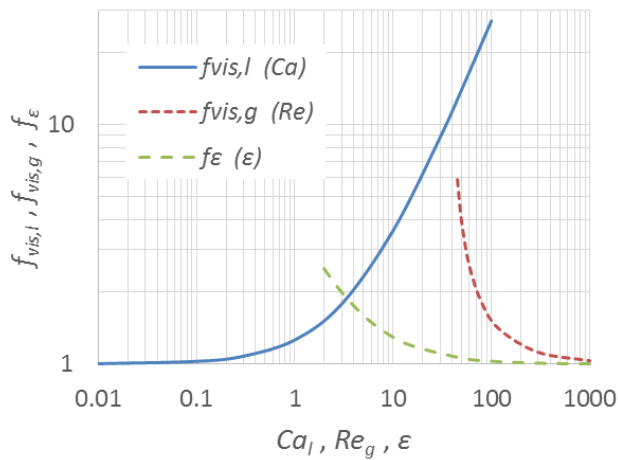
Breakup mode	$We_{g,cr,0}$	$C_{vis,g}$	ng	$C_{vis,l}$	nl	C_ϵ
---------------------	---------------------------------	-------------------------------	------------------------	-------------------------------	------------------------	--------------------------------

bag	10	55	1.1	0.26	1.0	3
transitional	16	55	1.1	0.20	1.0	3
Sheet-thinning	63	55	1.1	0.06	1.0	3
catastrophic	350	55	1.1	0.01	1.0	3

233

234 A graphical representation of the aforementioned correction factors is shown in Fig.1,
 235 according to which liquid viscosity effects become important for $Ca_l > 0.5$, gas
 236 viscosity effects for $Re_g < 300$ and density ratio effects for $\epsilon < 20$.

237



238

239 Fig.1: Correction factors for the effect of Ca_l (for bag breakup), Re_g and density ratio
 240 ϵ .

241

242 A final comment has to be made as concerns the methodology described in this
 243 section. It represents an extension of the experimental observations for the dominant

244 role that We_g number plays on distinguishing and controlling breakup regimes and it
245 is not based on a physical principle, such as the momentum conservation equation or
246 the deformation of the droplet beyond a threshold (e.g. the R-T wavelength).
247 Nevertheless, in the appendix (section 6.3) it is proved that the model equations are
248 equivalent to those obtained by using a modified version of the energy approach by
249 (Cohen, 1994). As it will be shown in the following sections, the present model
250 provides with sufficient accuracy a unified criterion to predict the breakup outcome
251 for any combination of We_g , Ca_l (or Oh_l), Re_g and ε numbers.

252

253 **3 Results and discussion**

254 **3.1 Qualitative model performance**

255 In this section the qualitative model performance is examined by using equations 20-
256 23 and adopting some reference values for the Re_g number and density ratio ε (equal
257 to 1000 for both) to calculate the correction factors $f_{vis,g}$ and f_ε . Except of the classical
258 $We_g - Oh_l$ breakup map, alternative breakup maps in the $We_g - Re_{l/g}$ and $We_g - Ca_l$
259 planes are also presented.

260 **3.1.1 The $We_g - Oh_l$ plane**

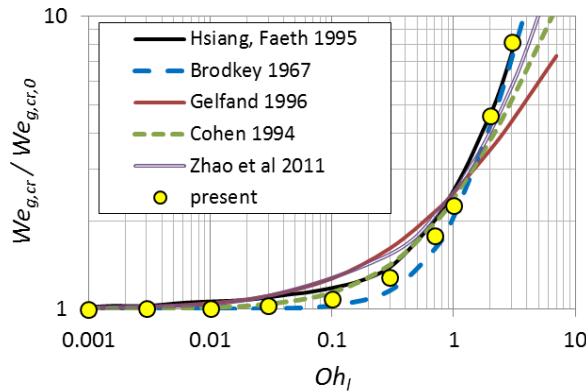
261 In order to reproduce the $We_g - Oh_l$ plane, the Ca_l number in eq. 23 is replaced with
262 $Ca_l = Oh_l \sqrt{We_g}$ (see eq. 12). Using these modifications and setting $nl=1$, eq 20 is
263 transformed into:

$$\frac{We_{g,cr}}{We_{g,cr,0}} = f_\varepsilon \cdot f_{vis,g} \cdot \left(1 + C_{vis,l} \sqrt{We_g \cdot Oh_l}\right) \quad (24)$$

264 This is a quadratic equation in respect to $\sqrt{We_g}$ having two roots. Keeping only the
 265 positive one, the final expression for the dependency of We_g versus Oh_l is given in
 266 equation 25.

$$We_{g,cr} = \frac{1}{4} \left[(C_{vis,l} \cdot We_{g,cr,0} \cdot f_\varepsilon \cdot f_{vis,g} \cdot Oh_l) \right. \\ \left. + \sqrt{(C_{vis,l} \cdot We_{g,cr,0} \cdot f_\varepsilon \cdot f_{vis,g} \cdot Oh_l)^2 + 4We_{g,cr,0} \cdot f_\varepsilon \cdot f_{vis,g}} \right]^2 \quad (25)$$

267 The model results for the bag breakup mode ($We_{g,cr,0} = 10$) are shown in Fig.2 along
 268 with the corresponding correlations from similar referenced works. As it can be seen,
 269 the present model can capture the qualitative behaviour of the dependency between
 270 $We_{g,cr}$ and Oh_l , while it is very close to the results of (Hsiang and Faeth, 1995).
 271 Similar agreement has been achieved for the transitional and the sheet-thinning
 272 breakup regimes, using the adequate coefficients $C_{vis,l}$.

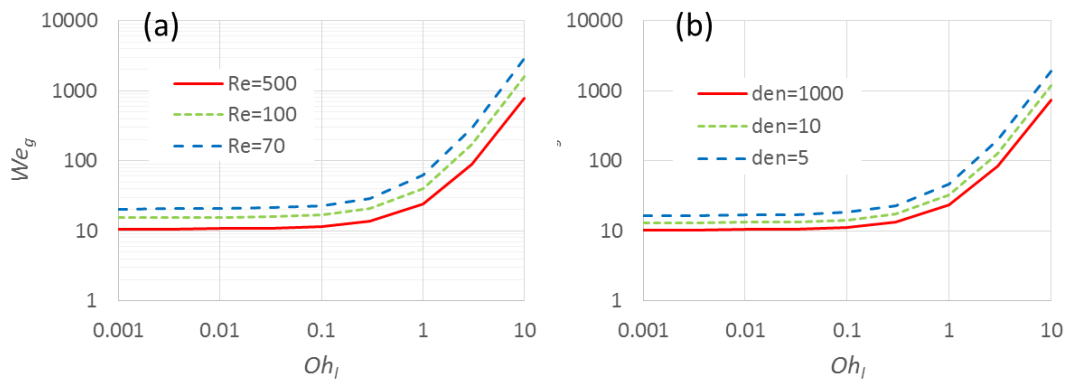


273

274 Fig.2: Results of the present model for the bag breakup regime in the $We_g - Oh_l$ plane.
 275 The corresponding results from other correlations are also shown.

276

277 Regarding the effect of Re_g number and density ratio on the critical We_g number, this
278 is shown in Fig.3 for the bag breakup case ($We_{g,cr,0}=10$), which is representative for all
279 breakup modes. As seen, decreasing the Re_g and ε numbers results in a slight increase
280 of the critical We_g number. This is in accordance with the findings of past works
281 presented in the introduction.



282

283 Fig.3: (a) effect of Re number, (b) effect of density ratio on the bag breakup
284 boundary.

285

286 Regarding the asymptotic behavior of eq. 24 for large Oh numbers, the product
287 $C_{vis,l}\sqrt{We_g} \cdot Oh_l$ is much higher than unity, thus equation 24 becomes $\sqrt{We_g}/Oh_l =$
288 $f_\varepsilon \cdot f_{vis,g} \cdot C_{vis,l} \cdot We_{g,cr,0}$, which is a constant number. This is in accordance with the
289 findings of (Aalburg, 2002) and (Zhao et al., 2011); the first one proved this by
290 performing numerical simulations and the second one by using the R-T instabilities
291 theory.

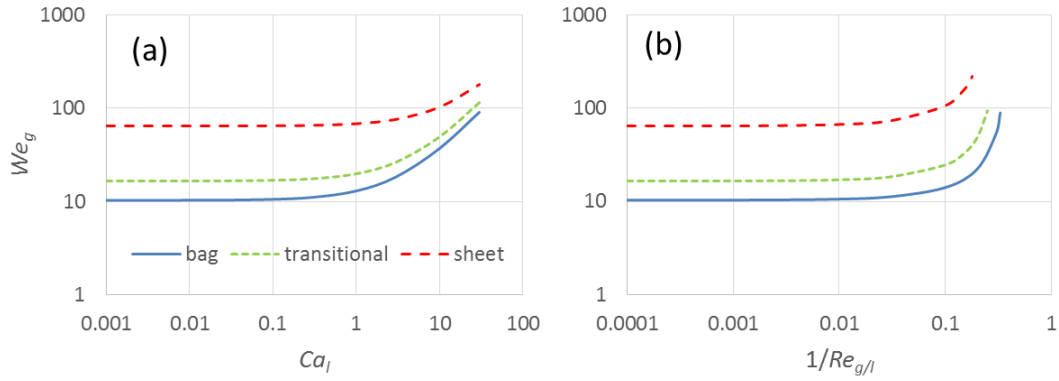
292

293 **3.1.2 Alternative breakup planes**

294 Alternative breakup planes can be developed by using directly eq. 23 (for the $We_g -$
 295 Ca_l plane) or by replacing the Ca_l number in eq. 23 with $Ca_l = We_g/Re_{g/l}$ (from eq.
 296 12) in order to develop the $We_g - 1/Re_{g/l}$ plane; after some manipulation, the resulting
 297 equation is eq. 26. As in the previous case, the coefficient nl in eq. 23 has been set
 298 equal to 1.

$$\frac{We_{g,cr}}{We_{g,cr,0}} = \frac{f_\varepsilon \cdot f_{vis,g}}{1 - f_\varepsilon \cdot f_{vis,g} \cdot C_{vis,l} \frac{1}{Re_{g/l}} We_{g,cr,0}} \quad (26)$$

299 The planes $We_g - Ca_l$ and $We_g - 1/Re_{g/l}$ are plotted in Fig.4a, b respectively by using
 300 the same coefficients as in section 3.1.1 for the $We_g - Oh_l$ plane. All planes presented
 301 so far look similar to one another, but there is a substantial difference in the $We_g -$
 302 $1/Re_{g/l}$ plane. In this plane based on eq. 26 there is a critical condition for breakup, i.e
 303 $1/Re_{g/l} < 1/(f_\varepsilon \cdot f_{vis,g} \cdot C_{vis,l} \cdot We_{g,cr,0})$. This means that breakup is not always
 304 observed for high $1/Re_{g/l}$ numbers. This contradicts the results deduced from the We_g
 305 $- Oh_l$ and $We_g - Ca_l$ planes in which there is no limitation for breakup. For the time
 306 being, there are no experimental data examining extremely high values of $\sqrt{We_g}/Oh_l$
 307 (or We_g/Ca_l , or $1/Re_{g/l}$), thus a clear suggestion for the most appropriate breakup
 308 plane, cannot be given.



309

310 Fig.4: Results of the present model in (a) the $We_g - Ca_l$ plane and (b) the $We_g - 1/Re_{g/l}$
 311 plane.

312

313 Among the breakup planes presented so far, our opinion is that the most suitable is the
 314 $We_g - Ca_l$ plane, since its axes represent gas inertia versus liquid viscous forces, both
 315 non-dimensionalised with the same quantity, i.e the liquid surface tension forces,
 316 while the asymptotic behavior at large We_g/Ca_l values agrees with the one predicted
 317 by (Aalburg, 2002) and (Zhao et al., 2011). Furthermore, there is an explicit relation
 318 between We_g and Ca_l for any value of the coefficient nl , while for the other numbers
 319 (Oh_l or $Re_{g/l}$) there is an implicit relation with the We_g number when nl is not unity.
 320 For the aforementioned reasons, the $We_g - Ca_l$ plane is further used in this work.

321

322 3.2 Comparison against experimental and numerical data

323 In this section, a large amount of experimental and numerical data are superimposed
 324 in the proposed $We_g - Ca_l$ plane to reveal the model capabilities compared to other
 325 works. It is of importance to highlight that the Re_g number and the density ratio are

326 not predefined by assuming reference values as in section 3.1, but they are explicitly
 327 calculated. The results presented here have been grouped according to the breakup
 328 outcome into a) non-breakup, b) bag breakup, c) transitional, d) sheet-thinning and
 329 finally e) catastrophic regimes. The transitional breakup regime includes the
 330 intermediate regimes bag-stamen, dual-bag, multi-bag and plume-shear for reasons of
 331 simplicity and clearness. For reasons of distinctness and readability, the experimental
 332 data and the numerical data are discussed in separate sections. Prior to the
 333 presentation of the results, the concept of the effective We_g number, is introduced.

334

335 **3.2.1 The effective We number**

336 In order to avoid using multi-dimensional graphs (or 2-dimensional planes with
 337 parametric curves as in Fig.3) for the cases in which different Re_g numbers or density
 338 ratios are examined, equation 20 is rearranged and the We_g number is replaced with an
 339 effective We_g number ($We_{g,eff}$), which takes into consideration the secondary effects of
 340 Re_g number and density ratio on breakup outcomes. This is numerically represented in
 341 eq. 27:

$$\frac{We_{g,eff}}{We_{g,cr,0}} = 1 + C_{vis,l} \cdot (Ca_l)^{nl}, \quad We_{g,eff} = \frac{We_g}{f_\varepsilon \cdot f_{vis,g}} \quad (27)$$

342 For example, in a case with $We_g=13$, $Re_g=70$ and a density ratio equal to 10, the
 343 effective We_g number is:

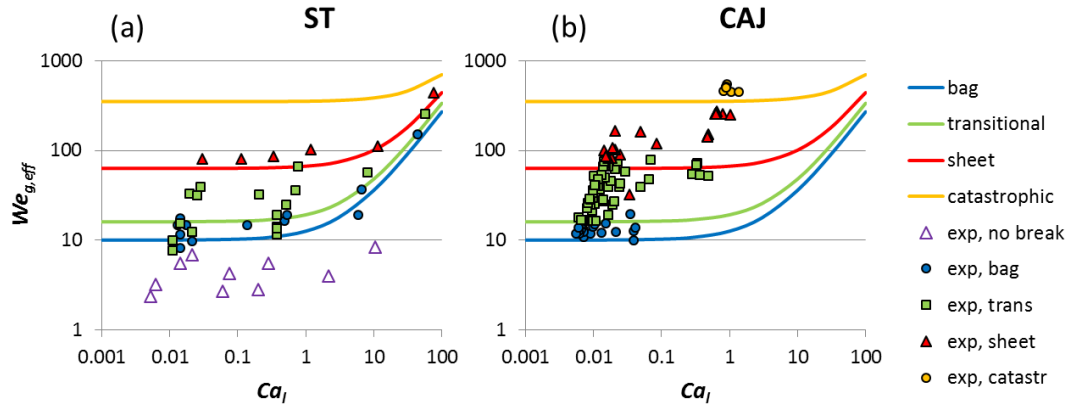
$$We_{g,eff} = \frac{13}{(1 + 3/10) \cdot (1 - 55 \cdot 70^{-1.1})^{-1}} = \frac{13}{1.33 * 2.04} = 4.79$$

344 and probably the droplet will not breakup, since it is much lower than $We_{g,cr,0}=10$. On
345 the other hand, in a case with large Re_g and density ratio (e.g 8000 and 1000
346 respectively), $We_{g,eff}$ is equal to 12.93, close to the normal We_g number of 13.
347 Therefore, it is better and more straightforward for droplet breakup characterization,
348 the $We_{g,eff}$ instead of the We_g number to be used. In the appendix (section 6.2), the
349 effective We_g number is also used to predict the breakup initiation time.

350

351 **3.2.2 Comparison against experimental data**

352 The experimental data used to assess the model performance are presented separately
353 according to the experimental technique used, i.e the shock tube and the continuous
354 air jet. For the shock tube experiments (denoted as ST), there are 46 experimental
355 points obtained from the works of (Hanson et al., 1963), (Hirahara and Kawahashi,
356 1992), (Hsiang and Faeth, 1995), (Dai and Faeth, 2001), while for the continuous air
357 jet experiments (CAJ), there are 101 experimental points obtained from the works of
358 (Krzeczkowski, 1980),(Arcoumanis et al., 1994), (Liu and Reitz, 1997), (Lee and
359 Reitz, 2000), (Zhao et al., 2010), (Opfer et al., 2012), (Flock et al., 2012), (Jain et al.,
360 2015). The results of the present model are shown in Fig.5(a) and (b) for both the
361 experimental techniques (ST and CAJ respectively).



362

363 Fig.5: Results of the present model in the $We_{g,eff} - Ca_l$ plane for (a) Shock Tube and
 364 (b) Continuous Air Jet experiments.

365

366 As seen, the fitting of the experimental data is generally good, especially for the CAJ
 367 experiments with an exception for one experimental point of (Flock et al., 2012) in the
 368 sheet-thinning regime, which was observed at a small We_g number of 32, rather
 369 corresponding to the transitional regime; nevertheless, in the experimental photos of
 370 this case, the sheet formation was clear. Also the experiments by Reitz and co-
 371 workers (Liu and Reitz, 1997), (Lee and Reitz, 2000) at $We_g=54$ were considered as
 372 “bag” in the relevant paper, but from their experimental photos it seems rather to
 373 undergo a multi-bag breakup; thus in the present paper they were included in the
 374 “transitional” regime. Regarding the ST experiments, there is a scattering in the
 375 transitional breakup regime in which there are some cases (5 from (Hanson et al.,
 376 1963) and 1 from (Hirahara and Kawahashi, 1992)) characterized by a relatively low
 377 We number of 7-8 which exhibit bag-stamen breakup regime. Nevertheless, it is not
 378 always clear what someone considers as bag or bag-stamen, while other parameters
 379 such as the Mach number and turbulence levels may affect the breakup outcome. Such

380 secondary controlling physical parameters have not been considered in the present
381 model.

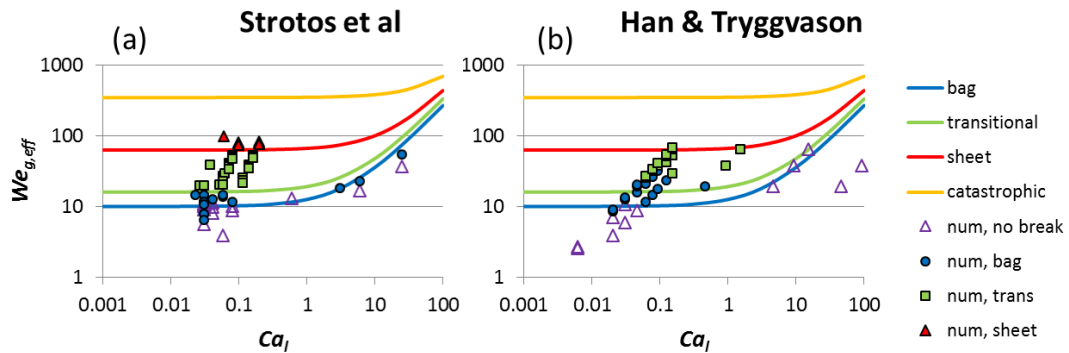
382 In relevance to the breakup map of (Hsiang and Faeth, 1995), the boundary of
383 transitional breakup appears for lower values of We_g number, and includes the bag-
384 stamen, dual-bag, bag-plume, shear plume breakup regimes. The determination of this
385 regime was mainly based on the experiments of (Zhao et al., 2010), which are more
386 recent than the experiments of (Hsiang and Faeth, 1995). One should recall also, that
387 these two works use a different experimental technique.

388

389 **3.2.3 Comparison against numerical data**

390 In this section the model performance is compared against numerical 2D
391 axisymmetric simulations. The numerical data used to assess the model performance
392 are 66 simulations performed in the past from the authors' group in (Strotos et al.,
393 2016a, b; Strotos et al., 2016c), as well as simulations performed in the current work
394 (see appendix), and 43 simulations from (Han and Tryggvason, 2001). For reasons of
395 readability these are presented in different graphs, i.e. in Fig.6 (a) and (b)
396 respectively. The “forward bag” observed in (Han and Tryggvason, 2001) was
397 included in the transitional regime here. In these simulations the Re_g number was in
398 the range 50 – 4000 and the density ratio in the range 5 – 800, which means that there
399 are cases (those with low Re_g and ε) in which the effective $We_{g,eff}$ number differs
400 significantly among them compared to using the classical We_g number. This is an
401 additional reason, why the use of the effective We_g number is proposed as more

402 representative for such types of droplet breakup characterization, compared to the
 403 standard one.



404

405 Fig.6: Results of the present model in the $We_{g,eff} - Ca_l$ plane for (a) simulations of the
 406 authors' group and (b) simulations of (Han and Tryggvason, 2001).

407

408 Based on the graphs, the overall model performance towards separating the various
 409 breakup regimes is good and only a few exceptions seem to deviate from the proposed
 410 breakup boundaries. In the authors' group simulations, there are two cases which
 411 breakup but appear in the non-breakup region of the map, whilst in the (Han and
 412 Tryggvason, 2001) simulations, there are 5 bag breakup cases which appear in the
 413 transitional region of the map. Nevertheless, in all these cases (characterized by low
 414 Re_g numbers and density ratios) the breakup modes differ from those observed in the
 415 experiments and it is a matter of convention what someone considers as transitional
 416 breakup. Furthermore, the breakup phenomenon is a continuous process (as stated in
 417 (Guildenbecher et al., 2009)) and there is not yet a deterministic single criterion for
 418 the transition among different breakup regimes. Based on their recommendation a
 419 zone rather than a single line should be used to separate the breakup regimes.

420 However, the present work offers the introduction of an alternative set of parameters
421 for visualizing the transition of droplet breakup mechanisms, which seems to be more
422 representative and close to reality compared to previous work and in that respect
423 should be considered as a step-forward towards understanding the underlying physics
424 represented by more correct variables.

425

426 **4 Conclusions**

427 In the present work, a new proposed total force approach has been used to determine
428 the dependency of the critical We_g number ($We_{g,cr}$) separating different breakup
429 regimes on all other non-dimensional numbers (Re_g , ε and Oh_l or Ca_l). According to
430 this approach, the breakup phenomenon is controlled by the ratio of the sum of the
431 deformation versus the sum of restorative forces; for negligible viscosities, this ratio
432 reduces to the classical We_g number. Breakup (or generally transition between
433 breakup regimes) occurs when this ratio exceeds a critical value; the latter is equal to
434 the critical We_g number corresponding to low Oh_l numbers (termed here as $We_{g,cr,0}$)
435 and it is known from experimental data.

436 The proposed model includes adjustable coefficients, which were determined by
437 performing numerical simulations and comparing against a large amount of
438 experimental and numerical data found in literature. Overall, a good qualitative and
439 quantitative agreement has been achieved. To unify cases with different conditions
440 (namely Re_g and ε numbers) an effective We_g number ($We_{g,eff}$) was proposed. This is
441 essentially the classical We_g number, corrected by two factors which account for the
442 secondary effects of Re_g and ε numbers. The model results were presented in a new

443 breakup map, the $We_{g,eff} - Ca_l$ plane; using the Ca_l number instead of the Oh_l ,
444 corresponds directly to the relation of gas inertia versus liquid viscosity when both are
445 non-dimensionalised using the same quantity (the surface tension forces), while using
446 the $We_{g,eff}$ instead of the We_g number, enables the inclusion of additional parameters in
447 the same plane. The effective $We_{g,eff}$ number was also used to predict the breakup
448 initiation time, shown in the appendix

449 The present methodology is not derived from physical principles, such as the
450 momentum equation or the Rayleigh-Taylor instabilities. It is rather an extension of
451 experimental observations and numerical data towards including in a unified way all
452 possible interdependencies among the forces acting on a droplet. Nevertheless, it is
453 shown in the appendix that this model is fully compatible with an energy approach
454 relating the required kinetic energy for breakup with that of an inviscid droplet. The
455 methodology proposed applies for Newtonian fluids, in laminar, isothermal and
456 incompressible flow conditions.

457

458 **5 Acknowledgement**

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461

462 6 Appendix

463 6.1 Derivation of correction factors

464 In this section, the 2D axisymmetric simulations performed for the determination of
465 the correction factors $f_{vis,g}$ and f_ε are presented. To determine these factors, all
466 parameters were kept constant and only one was changing each time to reveal the
467 effect of Re_g number and density ratio; the reference settings used were $Ca_l=0.03$,
468 $Re_g=400$ and $\varepsilon=800$. The cases examined were 55 and 31 of them are presented in
469 detail in Table 3. The VOF methodology (Hirt and Nichols, 1981) has been used and
470 implemented in ANSYS FLUENT v16.1 (ANSYS®FLUENT, 2015); details on the
471 methodology used can be found in earlier authors' work mentioned in section 3.2.3.
472 Note also that here, only the breakup outcome is presented, grouped into “breakup”,
473 “no breakup” and “marginal”, with the latter representing cases with an unclear
474 breakup outcome; more details on the droplet shapes and physical mechanisms are
475 going to be published in a separate article.

476

477 Table 3: List of the physical parameters of selected cases for the determination of the
478 correction factors.

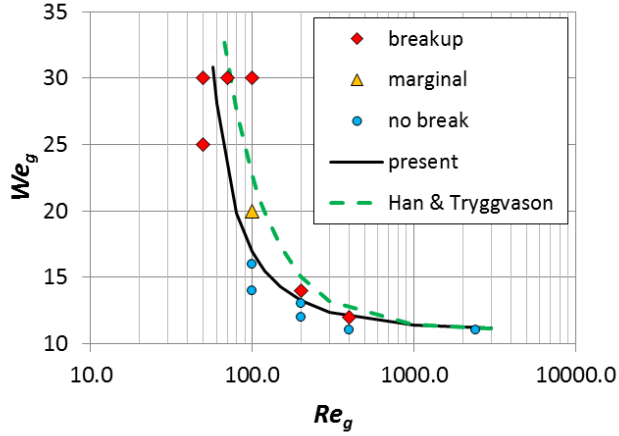
We_g	Re_g	Oh_l	Ca_l	ε	N
25	50.0	0.006	0.03	800	1.70
30	50.0	0.005	0.03	800	1.41
30	70.0	0.005	0.03	800	1.98
14	100.0	0.008	0.03	800	6.06
16	100.0	0.008	0.03	800	5.30
20	100.0	0.007	0.03	800	4.24
30	100.0	0.005	0.03	800	2.83
12	200.0	0.009	0.03	800	14.14

We_g	Re_g	Oh_l	Ca_l	ε	N
13	200.0	0.008	0.03	800	13.05
14	200.0	0.008	0.03	800	12.12
11	400.0	0.009	0.03	800	30.86
11	400.0	0.009	0.03	800	30.86
11	400.0	0.009	0.03	800	30.86
12	400.0	0.009	0.03	800	28.28
12	400.0	0.009	0.03	800	28.28
12	400.0	0.009	0.03	800	28.28
11	2400.0	0.009	0.03	800	185.13
12	400.0	0.009	0.03	100	10.00
13	400.0	0.008	0.03	20	4.13
14	400.0	0.008	0.03	20	3.83
16	400.0	0.008	0.03	10	2.37
16	400.0	0.008	0.03	5	1.68
20	400.0	0.007	0.03	5	1.34
20	400.0	0.007	0.03	3	1.04
20	400.0	0.007	0.03	2	0.85
14	400.0	0.160	0.6	800	484.87
20	400.0	0.671	3	800	1697.06
18	400.0	1.414	6	800	3771.24
25	400.0	1.200	6	800	2715.29
40	400.0	3.953	25	800	7071.07
60	400.0	3.227	25	800	4714.05

479

480 **6.1.1 Effect of gas viscosity**

481 The effect of gas viscosity (i.e the Re_g number) on droplet breakup for $Ca_l=0.03$,
482 $\varepsilon=800$ is presented in Fig.7 in a $We_g - Re_g$ plane. The curve representing the limiting
483 condition for breakup is $11.1 \cdot (1 - 55 \cdot Re^{-1.1})^{-1}$, while the corresponding curve
484 representing the data of (Han and Tryggvason, 2001) for $\varepsilon=10$ is also shown. As seen,
485 the present results are in qualitative agreement with those of (Han and Tryggvason,
486 2001) despite the fact that the density ratios are different; additional simulations are
487 required to investigate possible dependency on the density ratio and the Ca_l (or Oh_l)
488 number.



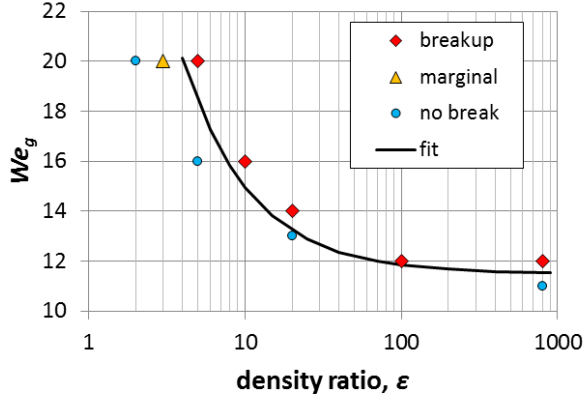
489

490 Fig.7: Effect of gas viscosity (Re_g number) on droplet breakup for $Ca_l=0.03$, $\varepsilon=800$. A
 491 curve representing the corresponding results of (Han and Tryggvason, 2001) for $\varepsilon=10$,
 492 is also shown.

493

494 6.1.2 Effect of density ratio

495 The effect of density ratio on droplet breakup for $Ca_l=0.03$, $Re_g=400$ is shown in
 496 Fig.8 in a $We_g - \varepsilon$ plane. The curve representing the limiting condition for breakup is
 497 $11.5(1+3 \cdot 1/\varepsilon)$ and it is in close agreement with the one representing the data of
 498 (Aalburg, 2002) for $Re_g=50$ and $Oh_l=0.001$ in which the correction factor was
 499 estimated (by the authors of the present work) to be $\exp(2.68/\varepsilon)$; possible dependency
 500 on the Ca_l (or the Oh_l) number was not examined and additional simulations are
 501 required for that. A final comment has to be made for the correction factor
 502 $f_\varepsilon=1+C_\varepsilon \cdot 1/\varepsilon$ used to account for the effect of density ratio. Since the density ratio ε
 503 not appearing in the TFR number, it was “manually” included in the present analysis.
 504 Nevertheless, its form is not arbitrary. It was inspired by the work of (Jalaal and
 505 Mehravaran, 2014) who found that the interfacial instabilities on the droplet’s surface
 506 begin at a We_g number which is analogous to $(1+ 1/\varepsilon)$.



507

508 Fig.8: Effect of density ratio on droplet breakup for $Ca_l=0.03$, $Re=400$.

509

510 6.2 Estimation of the breakup initiation time

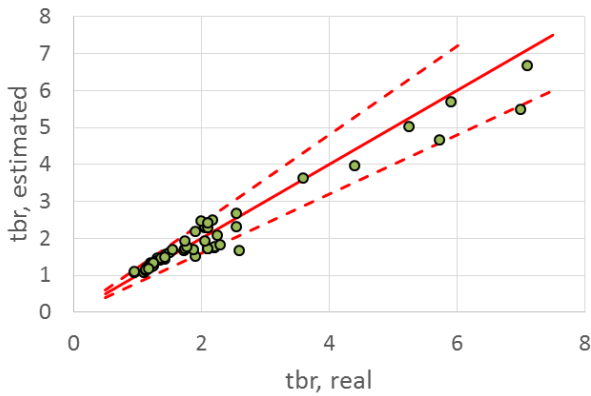
511 The effective We_g number ($We_{g,eff}$) defined in section 3.2.1 can be used to include the
512 effects of Re_g number and density ratio on the breakup initiation time. Generally,
513 there is not a clear definition what is meant by the term “initiation” time. Here,
514 breakup initiation time is the instant at which there is droplet detachment or the
515 instant at which the droplet interface is corrupted and holes are created. Analyzing the
516 numerical data used in section 3.2.3, the following equation 28 can be used to predict
517 the breakup initiation time, with less than 20% error (see Fig.9):

$$\frac{t_{break}}{t_{sh}} = 2.87 (We_{g,eff} - 8)^{-0.26} (1 + 2.56 Oh_l^{0.63}) \quad (28)$$

518 This equation has the same form as the one proposed by (Pilch and Erdman, 1987),
519 but additionally predicts that the breakup time increases with decreasing Re_g and ϵ
520 numbers. In (Pilch and Erdman, 1987) the dependency of break initiation time versus
521 the We_g number was analogous to $(We_g - 12)^{-0.25}$ and in (Reinecke and Waldman,

522 1975) proportional to $(We_g - 8)^{-0.25}$. Here an exponent of -0.26 has been estimated,
 523 which is close to the aforementioned values.

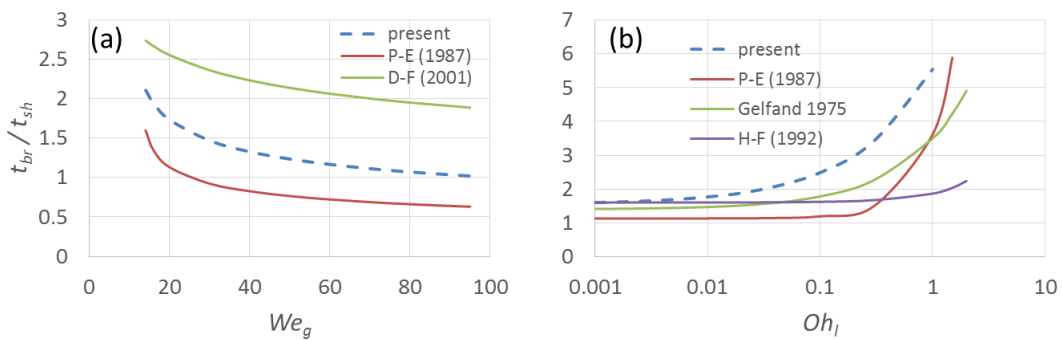
524 Regarding the overall behavior of eq. 28 in relevance to other correlations based on
 525 experimental data, the predicted breakup time is in-between the one predicted by the
 526 correlations of (Pilch and Erdman, 1987) and (Dai and Faeth, 2001) for a wide range
 527 of We_g numbers (all others parameters regarded constant), while it predicts a similar
 528 effect of Oh_l number as the one predicted by the correlations of (Pilch and Erdman,
 529 1987),(Hsiang and Faeth, 1992),(Gelfand, 1996). These trends are shown in Fig.10.



530

531 Fig.9: Prediction of breakup initiation time with eq. 28. The error lines of $\pm 20\%$ are
 532 also shown.

533



534

535 Fig.10: Prediction of breakup initiation time with eq. 28. (a) effect of We_g , (b) effect
 536 of Oh_l number. Correlations from other researchers are also shown.

537

538 **6.3 Relation to Cohen's approach**

539 In (Cohen, 1994) it was assumed that the kinetic energy required for breakup is that
 540 of an inviscid droplet plus the energy required to overcome the energy dissipated by
 541 the liquid viscosity; this is shown mathematically in eq. 29 including also the
 542 dissipation in the gas phase.

$$\frac{1}{2} \frac{\pi D^3}{6} \rho_g U_g^2 = \left(\frac{1}{2} \frac{\pi D^3}{6} \rho_g U_g^2 \right)_{vis=0} + C_{vis,l} (\mu_l U_l D^2) + C_{vis,g} (\mu_g U_g D^2) \quad (29)$$

543 In relevance to Cohen's approach, in the liquid dissipation term he assumed that there
 544 is a "mixing velocity" U_{mix} , which was determined by comparing against experimental
 545 data; instead of that here, the liquid phase velocity U_l is used, along with the
 546 adjustable coefficient $C_{vis,l}$. Non-dimensionalising eq 29 with the surface energy of the
 547 spherical droplet $\sigma\pi D^2$, and substituting the liquid velocity from eq. 4, we get equation
 548 30 in which the We_g number in the limit of inviscid flow ($We_{g,cr,0}$) has appeared in the
 549 RHS of the equation.

$$We_g = We_{g,cr,0} + 12 \frac{C_{vis,l}}{\pi} Ca_l + 12 \frac{C_{vis,g}}{\pi} \frac{We_g}{Re_g} \quad (30)$$

550 Rearranging eq. 30 and dividing by $We_{g,cr,0}$ we get eq. 31 which is identical to the
 551 equation derived with the total force approach in section 2.2 (with ng , nl , $f\varepsilon$ equal to
 552 unity and including all constants inside the terms $C_{vis,g}$ and $C_{vis,l}$)

$$\frac{We_g}{We_{g,cr,0}} \left(1 - \left(12 \frac{C_{vis,g}}{\pi} \right) \frac{1}{Re_g} \right) = 1 + \left(12 \frac{C_{vis,l}}{\pi We_{g,cr,0}} \right) Ca_l \quad (31)$$

553

554 7 Nomenclature

555

Roman symbols

Symbol	Description	Units
C	Adjustable coefficient	-
Ca	Capillary number $Ca = \mu U / \sigma$	-
D	diameter	m
f	Correction factor	-
F	force	N
n, ng, nl	Adjustable exponent	-
Oh	Ohnesorge number $Oh = \mu / \sqrt{\rho \sigma D}$	-
Re	Reynolds number $Re = \rho U D / \mu$	-
t	time	s
U	reference velocity	m/s
We	Weber number $We = \rho U^2 D / \sigma$	-

556

557

Greek symbols

Symbol	Description	Units
ε	density ratio $\varepsilon = \rho_l / \rho_g$	-
μ	viscosity	kg/ms
N	Viscosity ratio $N = \mu_l / \mu_g$	-
ρ	density	kg/m ³
σ	surface tension coefficient	N/m

558

Subscripts

Symbol	Description
0	Reference value
br	breakup
cr	critical
DEF	deformation
eff	effective
g or gas	gas
l or liq	liquid
RES	restore
vis	viscosity

559

Abbreviations

Symbol	Description
CAJ	Continuous Air Jet
R-T	Rayleigh-Taylor
ST	Shock tube
TFR	Total Force Ratio
VOF	Volume of Fluid

560

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