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Essays on Multi-Sector Macroeconomic Models for Policy Analysis

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May, 2018
A thesis submitted to
the Academic Faculty
by
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In partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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May, 2018
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5 Thesis conclusions

Bibliography
Abstract

This thesis studies multi-sector macroeconomic models suitable for policy analysis. The first and second chapters use a variety of empirical and theoretical macroeconomic models allowing for the consumption of goods with different durability, and analyze which modeling assumptions and features of the economy are crucial for the conduct of monetary policy. The third chapter focuses on the role of fiscal and monetary policies in the Euro Area, thus providing insights about the joint policy stance that have the potential to inform future policy choices.

In the first chapter, we challenge a crucial assumption made in the literature of Dynamic Stochastic General Equilibrium (DSGE) models with durable and nondurable goods about their relative price stickiness. We start with a thorough empirical analysis by estimating a Structural Vector Autoregressive model of the US economy, in which we find that the response of the relative price of durables to a monetary policy contraction is either flat or mildly positive. It significantly falls only if narrowly defined as the ratio between new-house and nondurables prices. These findings are then rationalized via the estimation of two-sector New-Keynesian (NK) models. Durables prices are estimated to be as sticky as those of nondurables, leading to a flat relative price response to a monetary policy shock. Conversely, house prices are estimated to be almost flexible. Such results survive several robustness checks and a three-sector extension of the NK model. These findings have implications for building NK models with durable and nondurable goods, and for the conduct of monetary policy. This chapter is based on an article co-authored with Dr. Giovanni Melina (International Monetary Fund) and published in the Journal of Economic Dynamics and Control.
The second chapter adds imperfect labor mobility to a two-sector New-Keynesian model with durable and nondurable goods and estimates it with Bayesian methods. We use the model to design optimal monetary policy and find that an inverse relationship between sectoral labor mobility and the optimal weight the central bank should attach to durables inflation arises. Moreover, we show that the combination of nominal wage stickiness and limited labor mobility leads to a nonzero optimal weight for durables inflation even if durables prices were fully flexible. These results survive alternative calibrations and interest-rate rules and point toward a non-negligible role of sectoral labor mobility for the conduct of monetary policy. This chapter is co-authored with Dr. Giovanni Melina (International Monetary Fund).

The third chapter of the thesis focuses on the role of shocks and policies in the Euro Area business cycle. We consider the long-term structure of government debt and introduce a financial sector. These features allow the model to account for both the recent financial and sovereign debt crises, and the effects of the unconventional monetary policy implemented by the European Central Bank. We then determine the joint fiscal and monetary policy stance in the Euro Area and find that it has been expansionary in the aftermath of the financial crisis but has turned to be contractionary after the sovereign debt crisis. The joint effect of the austerity measures taken by governments of European countries and the zero-lower-bound constraint on the monetary policy rate caused the reversion of the policy stance, which was prevented to be even more contractionary only by the quantitative easing implemented by the European Central Bank. This chapter is based on a paper co-authored with Dr. Nicoletta Batini (International Monetary Fund), Dr. Giovanni Melina (International Monetary Fund) and Dr. Stefania Villa (Bank of Italy).
Acknowledgements

This thesis is dedicated to my wife, Valentina, who is my strength, my inspiration, my love. She supported me every and each day of my PhD, she encouraged me to always fight for my objectives and never give up. I owe her all my academic and personal achievements. Without her, not only this PhD thesis but also all the beautiful things that happened in my life would have never been possible.

This thesis is also dedicated to my dear friend Michele. I will keep dedicating you all my achievements.

I am extremely grateful to my supervisors, Prof. Joseph Pearlman and Dr. Giovanni Melina, for their inestimable support and guidance. I would have never been able to complete my PhD without them. They taught me how to approach academic research, they always supported my choices and ideas. I feel I learned from them far more what I could expect.

I also would like to thank City, University of London and the Department of Economics for financing the ideas presented in this thesis and creating the perfect environment for a PhD student.

I thank my parents for their encouragement.

I thank all those I had the privilege to meet during the last four years. Sion, with whom we shared invaluable time at the Pool, and all the other PhD students I met at City.

I owe a lot to Daniela, who, day by day, has become my little sister.
Chapter 1

Introduction

This thesis collects three individual papers studying multi-sector macroeconomic models with the aim of contributing to the current academic debate and informing policy decisions. Since the seminal contributions of Lucas and Prescott (1971), Kydland and Prescott (1982) and Christiano et al. (2005), among many others, DSGE models have been widely used to study the business cycle properties of the economy, to evaluate and design policies and also to forecast macroeconomic variables. In addition to informing the academic debate, DSGE models have become the workhorse macroeconomic models used by central banks and policy institutions throughout the world.

The standard DSGE model, augmented with nominal and real frictions, in the New-Keynesian spirit (see, e.g. Christiano et al. 2005 and Smets and Wouters 2007), often assumes that the economy comprises a single production sector that produces a single homogeneous consumption good purchased by households. However, empirical evidence highlights important sectoral heterogeneity, i.e. that different types of goods react differently to macroeconomic shocks.

In particular, by broadly differentiating goods by their durability, durable goods are estimated to be much more interest rate sensitive than nondurables and services (see Bernanke and Gertler 1995, among others). It follows that a growing literature has developed, along two main strands. Empirical macroeconomic studies have documented a comovement between consump-
tion of durable and nondurable goods in response to a monetary policy shock. However, Barsky et al. (2007) were the first to notice that a standard New-Keynesian model in which agents consume both types of goods, is unable to reproduce such a regularity, hence the so-called comovement puzzle. Several papers then engaged in solving this puzzle but, as noted in the first chapter of this thesis, the reason it arises crucially depends on the assumption about sectoral price stickiness.

From an optimal monetary policy perspective, accounting for durable goods presents policymakers with more severe trade-offs and policy choices. Indeed, central bankers have only one instrument available to stabilize output and inflation in the two sectors. One way to exploit such a trade-off entails creating a suitable measure of aggregate inflation to employ as the target of monetary policy. As a consequence, choosing the appropriate weights to assign to sectoral inflation rates is crucial for the conduct of monetary policy. How the choice of such a weights is influenced by realistic features of multi-sector economies and its consequences for welfare is the focus of the second chapter of this thesis.

Extending the standard New-Keynesian model to include durable goods is not the only dimension explored to build multi-sector models. The global financial crisis erupted in 2008 had consequences not only for the real economy, but also triggered a lively academic and policy debate about the suitability of DSGE models that had hitherto largely neglected any role of the financial sector. As a consequence, DSGE models have been extended to incorporate financial intermediaries with the aim of explaining the financial crisis and policy responses. This is the starting point of the third chapter of this thesis, in which the inclusion of a financial sector is vital for analysis of fiscal and monetary policies in the Euro Area.

Given that accounting for heterogeneity in the economy is of a paramount importance, multi-sector models are the object of this thesis which contributes to the literature along a number of dimensions.

The first chapter, *Monetary Policy and the Relative Price of Durable Goods*, challenges a common assumption made in DSGE models with durable and nondurable goods, namely that prices of durables are substantially more
flexible than those of nondurables. This assumption, mainly made on the basis that house prices are flexible and on the microeconometric evidence reported in Bils and Klenow (2004), prevents a two-sector DSGE model from generating the desired comovement between durable and nondurable consumption in response to a monetary policy shock. In the paper, this assumption is questioned on two grounds. First, more recent microeconometric studies report price stickiness in many varieties of durable goods. Furthermore, durable goods comprise many categories other than houses. As a consequence, we thoroughly assess the definition of the durables sector and estimate the degree of price stickiness in durable goods. We start our analysis by reporting empirical evidence about the relative degree of sectoral price rigidities by estimating a battery of two-sector Structural Vector Autoregressive models and looking at the response of the relative price of durable goods to a monetary policy shocks. We find that the response of the relative price of durables to a monetary policy contraction crucially depends on the definition of the durables sector itself. Indeed, a broad definition of durables, which includes non-housing and housing durables, leads to a flat or non-negative response of the relative price of durables. This result points to a similar degree of price stickiness across sectors. Conversely, a definition of durables that includes only residential investment leads to a fall in the relative price thus suggesting that house prices are significantly more flexible than nondurables prices. To rationalize these results, we construct and estimate a two-sector DSGE model, in which durables are used by credit-constrained households as a collateral to borrow. We estimate the model with Bayesian methods and confirm the results of the empirical model. The results survive also several extensions, in particular the estimation of a three-sector DSGE model in which we treat housing and non-housing durables separately. The results reported in the paper carry two important implications. From a modeling viewpoint, the contribution of the paper implies that a DSGE model with a durables sector has to assume that durables prices are sticky, unless it comprises only housing goods. Moreover, a three-sector model is required to fully capture the heterogeneity of non-housing and housing durables. From a policy viewpoint, we contribute to the debate by inferring that monetary policy
does not create big allocative distortions between non-housing durables and nondurables whereas it might create them between housing and non-housing goods.

The second chapter of the thesis, *Sectoral Labor Mobility and Optimal Monetary Policy*, takes an optimal monetary policy perspective of a two-sector DSGE model. Indeed, when the model comprises two sectors, the central bank has to define an appropriate inflation rate to target. The problem is not trivial as, in a multi-sector context, the central bank has to stabilize sectoral output and inflation with only one instrument, namely the policy rate. In the paper we determine how the central bank optimally assigns weights to sectoral inflations and how such a choice is determined by the degree of labor mobility across sectors. We do so by first estimating a two-sector DSGE model in which households choose in which sector supply their labor services. Crucially we estimate that the degree at which workers can be reallocated across sectors in response to macroeconomic shocks is rather limited. We then use the estimated model to perform optimal monetary policy analysis. We start by finding the first best allocation and then we search for the combination of parameters in the Taylor rule adopted by the central bank that delivers the second best outcome. In doing so, we let the policymaker optimally choose the weight to assign to the sectoral inflation rates. Our novel finding is that such a weight heavily depends on the degree of labor mobility in the economy and we provide intuition for this result. We furthermore determine which interest rate rule entails desirable welfare implications. We thus shed light and contribute to the debate about the inflation measure central banks should target by showing that the degree of segmentation in the labor market is a crucial aspect central bankers should consider in the conduct of monetary policy.

The final chapter of this thesis, *Shocks and Policy Stance in the Euro Area*, focuses on the role of shocks and policies on the Euro Area business cycle. The ultimate aim of the paper is to estimate the Euro Area joint monetary and fiscal policy stance in a model that accounts for crucial features and episodes of the Euro Area. To serve this purpose, the multi-sector dimension of the DSGE model studied in this chapter is achieved by including a financial
sector. This extension is crucial for two reasons. First, it creates a transmission channel of credit shocks, thus allowing the model to account for the recent financial crisis. Moreover, it constitutes also the main channel through which the central bank affects the real economy via unconventional monetary policy. In addition, we model a detailed fiscal sector whereby government debt is long-term and can be purchased by the central bank to implement unconventional monetary policy. The active role of fiscal policy is introduced via distortionary taxes and government expenditures. Introducing long-term government debt allows the model to account both for the sovereign debt crisis and the role of unconventional monetary policy. Such a detailed modeling of monetary and fiscal policies is vital for the analysis of the Euro Area joint policy stance. To reach our conclusions, we bring the DSGE model to the data via Bayesian estimation. We then compute the historical contribution of shocks and policies on the Euro Area output and find interesting insights. First, we detect a crucial role of credit shocks in the build-up and development of the recent financial crisis. Overall, output dynamics of the Euro Area have been mainly shaped by preference and price-markup shocks. As far as policies are concerned, the joint policy stance is estimated to have been countercyclical in the aftermath of the financial crisis. However, as the zero-lower-bound constraint started binding and the sovereign debt crisis forced governments to implement austerity measures, the joint policy stance became contractionary. Unconventional monetary policy helped sustaining output but did not revert the policy stance. We thus find a potential role of fiscal policies in member countries with fiscal space to turn the policy stance to expansionary. Our paper thus adds to the debate about the policy stance in the Euro Area in the aftermath of the financial and sovereign debt crises by assessing the joint role of fiscal, conventional and unconventional monetary policies, which constitutes the main contribution of the paper.
Chapter 2

Monetary Policy and the Relative Price of Durable Goods

2.1 Introduction

Whether monetary policy innovations create distortions in allocations across durable and nondurable goods boils down to the extent to which such shocks change their relative price. The importance of the response of the relative price of durables to monetary policy has been explored in a small number of theoretical contributions, but surprisingly largely neglected in the empirical literature\(^1\).

In the context of optimal policy, Erceg and Levin (2006) show that the relative price of durables affects both the user cost and the demand of durable goods. A stable relative price of durables keeps output close to potential in both sectors and its role for the conduct of monetary policy is therefore non-negligible\(^2\) Petrella and Santoro (2011), in an economy with input-output structure, show that the relative price of services affects sectoral marginal costs and creates a channel through which the comovement between con-

\(^1\) One exception is Reis and Watson (2010) who estimate a dynamic factor model to argue that relative price movements are a main determinant of aggregate inflation.

\(^2\) In a similar model Aoki (2001) reaches the same conclusion.
sumption of the two goods is attained. They claim that their results can be
generalized to any sticky price model with two sectors. In fact, in a similar
model featuring durable and nondurable goods, Sudo (2012) demonstrates
that if the change in the relative price is small, the substitution effect be-
tween durables and nondurables is likewise small and the two goods comove
in response to a monetary policy shock.

The comovement between durables and nondurables in response to mon-
eyary policy has indeed been a popular topic in the literature and has been
documented in a number of papers employing recursive Structural Vector-
Autoregressive (SVAR) models (see Bernanke and Gertler, 1995; Erceg and
Levin 2006; Monacelli 2009; Sterk and Tenreyro 2014; Di Pace and Her-
tweck, 2016, among others). Barsky et al. (2003) confirm this empirical result
using Romer dates. However, Barsky et al. (2003, 2007, BHK henceforth)
were the first to notice that a two-sector New-Keynesian (NK) model fails to
replicate such a comovement, hence the so-called comovement puzzle. Conse-
quently, several extensions of the baseline model have been explored in order
to solve it.

The crucial assumption that prevents the baseline model from generating
the comovement concerns sectoral price stickiness. In fact, BHK assume
that prices of durable goods are flexible whereas prices of nondurables are
sticky. This assumption is made for two reasons. First, durables prices such
as houses are largely negotiated and most homes are priced for the first
time when they are sold. Second, they appeal to microeconometric studies,
such as Bils and Klenow (2004), documenting that durables are more flexible
than nondurables. On these grounds, although durables price stickiness turn
out to play a key role in the comovement issue (see Sterk, 2010), BHK and
most of the subsequent papers assume that durables prices are completely
flexible. In contrast, more recently, Nakamura and Steinsson (2008), Boivin

$^3$Carlstrom and Fuerst (2010), DiCecio (2009) and Iacoviello and Neri (2010) introduce
nominal wage stickiness; Monacelli (2009), Sterk (2010), Chen and Liao (2014) and Tsai
(2016) evaluate the role of credit frictions; Bouakez et al. (2011) and Sudo (2012) study an
economy with input-output interactions; Kim and Katayama (2013) assume non-separable
preferences; finally, Di Pace and Hertweck (2016) introduce search and matching frictions.
For an extensive literature review see Cantelmo and Melina (2015).
et al. (2009), Klenow and Malin (2010) and Petrella and Santoro (2012) report microeconomic evidence of stickiness in many categories of durables other than houses (investment in housing represents about 23% of aggregate durables in US NIPA tables in the post-war period).

The assumption about sectoral price stickiness is closely related to the response of the relative price of durables to a monetary policy shock. In fact, when durables prices are assumed to be flexible, while nondurables prices are sticky, the relative price of durables necessarily falls following a monetary policy tightening, implying that monetary policy creates a distortion in sectoral allocations.

Quite surprisingly, little empirical analysis has focused specifically on this issue. \[4\] Table 2.1 reports unconditional correlations between lags of changes in the federal funds rate (FFR) and changes in key macroeconomics variables over the main sample considered in the paper. The durables sector is defined as the sum of durable goods and residential investment and we report both the relative price of durables and the relative price of houses. \[5\] As expected, changes in the FFR are negatively associated with changes in real GDP, durables, houses and nondurables with some lags. As regards inflation, it takes up to three years to detect a negative (though insignificant) correlation. Changes in the relative price of durables seem to be uncorrelated with changes in the FFR, this being in accordance with overall price stickiness not being dramatically different across the two sectors. The relative price of houses exhibits negative but insignificant correlation.

\[4\] With different objectives in mind, Boivin et al. (2009) estimate a dynamic factor model to show that the price setting behavior of firms changes according to the nature of the shock hitting the economy. Sectoral prices appear to be sticky in response to aggregate shocks such as monetary policy innovations but flexible in response to sector-specific shocks. Makowiak et al. (2009) largely confirm these results and compare a Calvo model with sticky information and rational inattention models to reproduce them. They argue in favor of the latter as the former two models need implausible calibrations to match the distribution of sectoral prices responses to aggregate and sector-specific shocks. However, Beck et al. (2016) challenge these empirical results thus reducing the importance of sector-specific shocks. They conclude that multi-sector, multi-country models are needed be consistent with their empirical findings but a rational inattention model proves to be a good approximation to them.

\[5\] Section 2.2.1 discusses the choice of the sample and the definitions of durable and housing sectors employed.
Table 2.1: Correlations between lags of changes in the Federal funds rate (FFR) and changes in selected macroeconomic variables

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Durables</th>
<th>Houses</th>
<th>Nondurables</th>
<th>Inflation</th>
<th>Rel. Price</th>
<th>Rel. Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Durables</td>
<td>Houses</td>
</tr>
<tr>
<td>FFR (-1)</td>
<td>0.0801</td>
<td>-0.3282*</td>
<td>-0.2534*</td>
<td>-0.3020*</td>
<td>0.1675*</td>
<td>0.0804</td>
<td>-0.0985</td>
</tr>
<tr>
<td>FFR (-4)</td>
<td>-0.1806*</td>
<td>-0.2865*</td>
<td>-0.3081*</td>
<td>-0.2411*</td>
<td>0.2230*</td>
<td>0.1110</td>
<td>-0.0049</td>
</tr>
<tr>
<td>FFR (-8)</td>
<td>-0.1810*</td>
<td>-0.0727</td>
<td>-0.0903</td>
<td>-0.0803</td>
<td>0.1392</td>
<td>0.0438</td>
<td>-0.0497</td>
</tr>
<tr>
<td>FFR (-12)</td>
<td>0.0318</td>
<td>-0.0533</td>
<td>0.1198</td>
<td>0.0634</td>
<td>-0.0070</td>
<td>0.0599</td>
<td>-0.0596</td>
</tr>
</tbody>
</table>

Note: GDP, durables, houses and nondurables are first differences in log real per-capita variables. Inflation is the first difference in the log of the GDP deflator. The relative prices are the first difference of the ratios of the relevant price indices. More data details are available in the Appendix. Frequency: quarterly. Sample: 1969Q2-2007Q4. * denotes significance at a 5 percent level.

Given the important policy and modeling implications, this topic deserves more careful investigation. In the paper we exploit both SVAR and Dynamic Stochastic General Equilibrium (DSGE) models, in order to assess the effects of a monetary policy shock on the relative price of durables and the relative house price. The monetary policy shock in SVAR models is identified through recursive, sign restrictions and narrative approaches. Across subsamples and methodologies, the response of the relative price of durables is either flat or mildly positive, but it never falls, contrary to what most DSGE models imply under the assumption of flexible durables prices. A significant fall is found only if the relative price is narrowly defined as the ratio between house prices and nondurables prices, this being consistent with flexible house prices. The estimation of DSGE models corroborates and helps rationalise the SVAR results. We build a two-sector NK model in which durable goods are used by credit-constrained impatient households as collateral to borrow funds from patient households. The Bayesian estimation unveils that the degree of price stickiness in the sector comprising all durable goods (housing and non-housing) is not significantly different from the nondurables sector. Thus the credible set of impulse responses of the relative price to a monetary policy shock includes zero. In contrast, when durables comprise only housing,
house prices are estimated to be almost flexible whereas nondurables prices are substantially stickier. Only in this case, a monetary policy tightening affects the relative price of durable goods, namely the relative house price.

These results on price stickiness survive also modifications affecting sectoral Phillips curves, i.e. if we allow for imperfect labor mobility across sectors, or if we introduce sectoral price indexation to past inflation, and they hold true also in a generalization to a three-sector model.

Our DSGE analysis is related to recent contributions in the literature. Our results on price stickiness are broadly in line with those of Bouakez et al. (2009) who estimate price stickiness in a six-sector model. In their framework, however, there is full symmetry in modeling the various types of goods, which fully depreciate within one period. In contrast, we capture two important features of durable goods that distinguish them from nondurables. First, they yield utility over time rather than being completely consumed in one use. Second, they serve as collateral for borrowing purposes. In a simpler two-sector model Barsky et al. (2016) show that different degrees of durability have implications also for optimal monetary policy. Normative monetary policy implications in a two-sector model are drawn also in Petrella et al. (2017) who focus on input-output interactions. These last two papers, however, do not estimate price stickiness parameters as we do. Iacoviello and Neri (2010) estimate a two-sector model where durables comprise only housing and house prices are assumed a priori to be fully flexible. In contrast, we consider both housing and non-housing durables and estimate all price stickiness parameters.

Our SVAR and DSGE results have two important implications for modeling and policy. The first is that, when building a two-sector New-Keynesian, model it is desirable to assume that prices of durable goods are somewhat sticky, unless the model’s aim is to focus on the housing sector in isolation from other durables. A three-sector model is needed to fully capture the intrinsic differences between housing and non-housing durables, such as the type of goods that can be used as collateral and their different degree of durability. The second is that overall monetary policy innovations do not foster big distortions in sectoral allocations between durables and nondurables, this
representing a desirable feature of the monetary policy conduct. Conversely, since monetary policy does affect the relative house price, it may potentially create allocative distortions between housing and non-housing goods.

The remainder of the paper is organized as follows. In Section 2.2 we perform the SVAR analysis. Section 2.3 presents the DSGE model, its extensions, and discusses the results of the Bayesian estimation. Section 2.4 concludes. An appendix complements the paper by providing details about the dataset, the theoretical model, and by reporting robustness checks.

2.2 Structural vector-autoregressive models

2.2.1 Methodology

As regards the estimation of the empirical model, we use quarterly, seasonally adjusted US data for the Federal funds rate, real GDP, real durable goods, real nondurable goods and services, the GDP deflator and the relative price of durables. In order to thoroughly investigate the effects of a monetary policy shock on the relative price of durables, we employ two alternative definitions of durables sector. We first follow Erceg and Levin (2006), Mona
celli (2009), Sterk and Tenreyro (2014) and Di Pace and Hertweck (2016) in defining durables as the sum of durable goods consumption and residential investments. Then, we assume that durables comprise only houses. We label the former model baseline SVAR and the latter housing SVAR. Table 2.2 summarizes the various definitions of durables sector and relative prices used throughout the paper. Definitions I and II are used in the main analysis whereas III and IV serve for robustness checks. The algebraic details for the computation of all relative prices are reported in Appendix 2.5.

The main analysis is performed over the sample 1969Q2-2007Q4. This choice is dictated by the availability of the narrative measure of monetary policy shocks constructed by Romer and Romer (2004, RR henceforth) and

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6 A detailed description of the data can be found in Section 2.5 of the Appendix.
7 Erceg and Levin (2006) slightly depart from the other studies by disaggregating GDP into an index of consumer durables and residential investment and an index of all other components of output.
Table 2.2: Definitions of Relative Prices

<table>
<thead>
<tr>
<th>Definition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Relative Price of Durables</td>
</tr>
<tr>
<td>II</td>
<td>Relative House Price</td>
</tr>
<tr>
<td>III</td>
<td>Relative Price of Durables and New Single Family Houses</td>
</tr>
<tr>
<td>IV</td>
<td>Relative Price of Durables and Broad Measure of Houses</td>
</tr>
</tbody>
</table>

The vector of variables employed in the SVAR is the following:

\[ x_t = [GDP_t, D_t, C_t, P_t, Q_t, FFR_t]' \quad (2.1) \]

where \( GDP_t \) denotes gross domestic product; \( D_t \) and \( C_t \) represent consumption of durable and nondurable goods, respectively; \( P_t \) is the GDP deflator; \( Q_t \) is the the relative price of durable goods; and \( FFR_t \) denotes the Federal funds rate, with the ordering used in the estimations being exactly that of equation (2.1). We take the natural logarithm of all variables except for the FFR, which is in levels.

For the sake of robustness, we take three different approaches to the identification of monetary policy shocks:

i) recursive (Cholesky) approach in which we make the standard assumption that the monetary policy variable is ordered last, hence it has no contemporaneous effect on the other variables (see Bernanke and Mihov 1998)

---

8A detailed discussion of the methodologies is in Appendix 2.6.
ii) *sign restrictions* imposed on the impulse responses of the variables and derived from a DSGE model as in Canova (2002), Dedola and Neri (2007), Pappa (2009), and Bermperoglu et al. (2013), among others. Fry and Pagan (2011) critically review the sign restrictions approach arguing that if there is not enough information to discriminate among the various shocks, it may be problematic to correctly identify them. In principle, only if the researcher describes the sign pattern for each shock in the model it is possible to avoid this problem. In order to partially address this identification issue we proceed as follows. Following Peersman (2005), we first determine the sign pattern of two standard supply and demand shocks, and then we identify the monetary policy shock.\(^9\)

Table 2.3 summarizes the set of sign restrictions imposed. A contractionary supply shock curbs output, nondurable and durable consumption, while increasing inflation, which leads the central bank to raise the nominal interest rate. A negative demand shock reduces both all the mentioned real variables and inflation, thus leading the central bank to cut the interest rate. Conversely, the monetary policy shock is characterized by an increase in the nominal interest rate, which leads to a decrease in output, nondurable consumption and inflation. As discussed in Appendix 2.9, notwithstanding the lack of a robust response, in order to correctly identify the monetary policy shock, we assume that the nominal interest rate is positive in the first quarter. We remain agnostic on the response of the relative price and consumption of durables as it is the main objective of our investigation. The different restrictions imposed on the responses of the GDP deflator and the interest rate ensure the orthogonality between the disturbances and the correct identification of the monetary policy shock.

iii) *recursive narrative approach*: we follow Romer and Romer (2004), Coibion (2012) and Cloyne and Hurtgen (2016) and re-estimate the recursive SVAR model by replacing the FFR with the monetary policy shock constructed by RR and extended by Coibion et al. (2012) and Tenreyro and

\(^9\) Robust IRFs for the supply and demand shocks are completely standard and are available upon request.
In the macro-fiscal literature, Mertens and Ravn (2013) and Mertens and Ravn (2014) challenge the use of narrative measures to identify fiscal shocks in SVAR models on the ground that such measures do not represent truly exogenous fiscal shocks. They therefore build on Stock and Watson (2012) and use narrative measures as external instruments for the identification of the structural shocks in the SVAR model (Proxy SVAR henceforth). However, as noted by Cloyne and Hurtgen (2016), while fiscal narrative measures are directly derived from historical sources, thus representing potential noisy proxies for the structural shocks, the monetary policy measure derived by RR is the result of a first-stage regression which yields a direct measure of the structural shock rather than a proxy. It is therefore reasonable to use such a measure directly in the VAR model rather than as an external instrument. In any case, for the sake of robustness, Figure 2.20 in Appendix 2.7.5 reports the impulse responses obtained with a Proxy SVAR model using the RR measure as external instrument to identify the structural shock linked to the monetary policy variable, i.e. the federal funds rate. The relative prices exhibit the same sign pattern to those reported in the main analysis. 

<table>
<thead>
<tr>
<th>Shock</th>
<th>GDP</th>
<th>D</th>
<th>C</th>
<th>P</th>
<th>Q</th>
<th>FFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>none</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Demand</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>none</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>&lt; 0</td>
<td>none</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>none</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

Table 2.3: Sign restrictions

\[\text{Thwaites (2016). In the macro-fiscal literature, Mertens and Ravn (2013) and Mertens and Ravn (2014) challenge the use of narrative measures to identify fiscal shocks in SVAR models on the ground that such measures do not represent truly exogenous fiscal shocks. They therefore build on Stock and Watson (2012) and use narrative measures as external instruments for the identification of the structural shocks in the SVAR model (Proxy SVAR henceforth). However, as noted by Cloyne and Hurtgen (2016), while fiscal narrative measures are directly derived from historical sources, thus representing potential noisy proxies for the structural shocks, the monetary policy measure derived by RR is the result of a first-stage regression which yields a direct measure of the structural shock rather than a proxy. It is therefore reasonable to use such a measure directly in the VAR model rather than as an external instrument. In any case, for the sake of robustness, Figure 2.20 in Appendix 2.7.5 reports the impulse responses obtained with a Proxy SVAR model using the RR measure as external instrument to identify the structural shock linked to the monetary policy variable, i.e. the federal funds rate. The relative prices exhibit the same sign pattern to those reported in the main analysis.}\]
2.2.2 Results

The estimated impulse responses are presented in Figure 2.1. Rows refer to the variables of the model whereas columns refer to the three different identification approaches. The shock is a one standard deviation increase in the monetary policy measure. Solid lines depict the responses for the baseline SVAR model, and the shaded areas are the corresponding one-standard-deviation confidence bands. Dashed lines show the responses for the housing SVAR model, with dotted lines representing the corresponding one-standard-deviation confidence bands. The impulse responses show that results are broadly robust across models and identification approaches, with the exception of the relative price. There is evidence for the comovement between durables and nondurables, and the responses of durables are always larger than those of nondurables, a finding that is consistent with the empirical literature.

Turning to the dynamic behavior of the relative price, the estimated responses to a monetary policy tightening are highly dependent on the definition of the durables sector adopted. If durables account for both consumption goods and residential investment, the response of the relative price is either flat or mildly positive, this being at odds with the assumption of flexible durable prices adopted in most of the theoretical literature. Conversely, a model in which the durables sector coincides exclusively with the housing sector, the relative price falls consistently with the notion of flexible new house prices. These results are confirmed by the responses of the relative price across seven subsamples. In Figure 2.2 rows plot the relative price responses for each subsample, whereas columns represent the three identification approaches. The relative price of durables never falls in the baseline

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footnotes:

13 One-standard-deviation confidence bands in the recursive approaches are computed by Monte Carlo methods based on 2000 draws. In the sign restrictions approach we construct a distribution of impulse responses and we report the median together with the 16th and the 84th percentiles in order to report a comparable confidence band.


15 The size of each subsample is 24 years. See Appendix 2.7.3 for details.
Figure 2.1: impulse responses to a one standard deviation increase in the monetary policy measure. Sample: 1969Q2-2007Q4 (bold lines refer to the model with all durable goods; dashed lines refer to the model with only houses; shaded areas and dotted lines represent one-standard-deviation confidence bands)
Figure 2.2: responses of the relative price to a one standard deviation increase in the monetary policy measure. Rows denote samples, columns denote identification methods (bold lines refer to the model with all durable goods; dashed lines refer to the model with only houses; shaded areas and dotted lines represent one-standard-deviation confidence bands)
SVAR model whereas it significantly decreases in the housing SVAR model, thus confirming the previous results across subsamples and identifications of the monetary policy shock. To sum up, this empirical evidence suggests that the definition of the durables sector is crucial. If durable goods are defined to include both non-housing goods and residential investment, these display dynamics consistent with a non-negligible degree of price stickiness. Conversely, durable goods defined to include only the housing sector exhibit a behavior compatible with flexible prices.

These results survive several robustness checks reported in Appendix 2.7: (i) inclusion of a linear time-trend (2.7.1); (ii) alternative definitions of durables as described in Table 2.2 (see Appendix 2.7.2); (iii) subsample analysis (2.7.3); (iv) sign restrictions imposed for two, four and six quarters (2.7.4); (v) the Proxy SVAR approach (2.7.5); (vi) a three-sector SVAR model in which durables and housing are treated separately in the same model (2.7.6). It is noticeable from Section 2.7.2 that when any measure of house prices is bundled with non-house durables prices, the relative price never falls in response to a monetary policy tightening.

2.3 New-Keynesian model

To rationalize the SVAR estimates, we analyze a two-sector New-Keynesian model in which households consume both durable and nondurable goods. Following Monacelli (2009), Sterk (2010) and Iacoviello and Neri (2010) we assume that impatient households obtain loans from patient ones using their durables stock as collateral, with the amount they can borrow tied to the value of the collateral, thus allowing for a further transmission mechanism of monetary policy beyond the standard interest-rate channel. The economy is characterized by several frictions, the importance of which is empirically assessed. These are price and wage stickiness, investment adjustment costs.

\[\text{16} \] This important transmission mechanism is not considered in related studies, such as Bouakez et al. (2009) and Kim and Katayama (2013). Furthermore, Walentin (2014) estimates the model of Iacoviello and Neri (2010) on the Swedish economy to investigate how macroprudential policies (i.e. changes in the LTV ratio) further alter the effects of monetary policy shocks.
in durable goods (IAC, henceforth) and habit formation in consumption of nondurable goods. Finally, the monetary authority sets the nominal interest rate according to a Taylor-type interest rate rule.

2.3.1 Households

The economy is populated by a continuum of two groups of infinitely-lived households (patient and impatient) each indexed by \( i \in [0, 1] \) in which consumers derive utility from consumption of durable and nondurable goods and get disutility from supplying labor. Impatient households have a lower discount factor than patient ones (\( \beta' < \beta \) ) that is why they borrow in equilibrium. Throughout the paper, variables and parameters with a \( ' \) refer to impatient households.

2.3.1.1 Patient households

Patient household’s lifetime utility is represented by

\[
E_0 \sum_{t=0}^{\infty} e_i^t \beta^t U (X_{i,t}, N_{i,t}),
\]

where \( e_i^t \) is a preference shock, \( X_{i,t} = Z_{i,t}^{1-\alpha} D_{i,t}^{\alpha} \) is a Cobb-Douglas consumption aggregator between nondurable (\( Z_{i,t} \)) and durable goods (\( D_{i,t} \)) with \( \alpha \in [0, 1] \) representing the share of durable consumption on total expenditure, and \( N_{i,t} \) being the household’s labor supply. We assume that nondurable consumption is subject to external habit formation so that

\[
Z_{i,t} = C_{i,t} - \zeta S_{t-1},
\]

\[
S_t = \rho_c S_{t-1} + (1 - \rho_c) C_t,
\]

where \( C_{i,t} \) is the level of the household’s nondurable consumption; \( S_t, \zeta \in (0, 1) \) and \( \rho_c \in (0, 1) \) are the stock, the degree and the persistence of external habit formation, respectively, while \( C_t \) represents average consumption across
Each household monopolistically supplies labor to satisfy the following demand function:

$$N_{i,t} = \left( \frac{w_{i,t}}{w_t} \right)^{-\epsilon^W} N_t,$$

(2.5)

where $w_{i,t}$ is the real wage of each household whereas $w_t$ is the average real wage in the economy. Parameter $\epsilon$ is the intratemporal elasticity of substitution between labor services and $\epsilon^W$ is a wage markup shock. Finally, firms on average demand a quantity $N_t$ of labor services. Nominal wages are subject to quadratic costs of adjustment as in Rotemberg (1982):

$$\phi^W \left( \frac{w_{i,t}}{w_{i,t-1}} \Pi_t^C - \Pi_t^C \right)^2 w_t N_t,$$

where $\phi^W$ is the parameter governing the degree of wage stickiness, $\Pi_t^C$ is the gross rate of inflation in the non-durable sector, and $\Pi_t^C$ is its steady-state level. The stock of durables evolves according to law of motion

$$D_{i,t+1} = (1 - \delta) D_{i,t} + \epsilon^D I_{i,t} \left[ 1 - S \left( \frac{I_{i,t}}{I_{i,t-1}} \right) \right],$$

(2.6)

where $\delta$ is the depreciation rate of durables, $I_{i,t}$ is investment in durable goods that is subject to adjustment costs and $\epsilon^D$ represents an investment-specific shock. The adjustment costs function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $S''(1) > 0$. In addition, each household purchases nominal bonds $B_{i,t}$, receives profits $\Omega_{i,t}$ from firms and pays a lump-sum tax $T_t$ so that the period-
by-period real budget constraint reads as follows:

\[ C_{i,t} + Q_t I_{i,t}^D + \frac{\vartheta W}{2} \left( \frac{w_{i,t}}{w_{i,t-1}} \Pi_t^C - \Pi_t^C \right)^2 w_t N_i + \frac{R_t B_{i,t-1}}{\Pi_t^C} = \frac{B_{i,t}}{P_t^C} + \frac{W_{i,t}}{P_t^C} N_i + \Omega_{i,t} - T_t, \quad (2.7) \]

where \( Q_t \equiv \frac{P_D,t}{P_C,t} \) is the relative price of durables, \( R_t \) is the gross nominal interest rate and \( W_{i,t} \) is the nominal wage. Households choose \( Z_{i,t}, B_{i,t}, D_{i,t+1}, I_{D,i,t}, w_{i,t} \) to maximize (2.2) subject to (2.3), (2.4), (2.5), (2.6) and (2.7). At the symmetric equilibrium, the patient household’s optimality conditions are:

\[ 1 = E_t \left[ \Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right], \quad (2.8) \]

\[ Q_t \psi_t = \frac{U_{D,t}}{U_{Z,t}} + (1 - \delta) E_t [\Lambda_{t,t+1} Q_{t+1} \psi_{t+1}], \quad (2.9) \]

\[ 1 = \psi_t c_{t+1} - S \left( \frac{I_t^D}{I_{t-1}^D} \right) - S' \left( \frac{I_t^D}{I_{t-1}^D} \right) \left( \frac{I_t^D}{I_{t-1}^D} \right)^2 + E_t \left\{ \Lambda_{t,t+1} \psi_{t+1} Q_{t+1} \psi_{t+1} \left[ S' \left( \frac{I_{t+1}^D}{I_t^D} \right) \left( \frac{I_{t+1}^D}{I_t^D} \right)^2 \right] \right\}, \quad (2.10) \]

\[ 0 = [1 - \psi_t] \eta_{t} \mu_t + \frac{\psi_t \eta_{t} \mu_t}{\mu_t} + \vartheta W \left( \Pi_t^W - \Pi_t^C \right) \Pi_t^W + E_t \left[ \Lambda_{t,t+1} \psi_{t+1} \left( \Pi_t^W - \Pi_t^C \right) \Pi_{t+1}^W w_{t+1} N_{t+1} \right]. \quad (2.11) \]

Equation (2.8) is a standard Euler equation with \( \Lambda_{t,t+1} \equiv \beta \frac{U_{Z,t+1}}{U_{Z,t}} \frac{\mu_{t+1}}{\mu_{t}} \) representing the stochastic discount factor and \( U_{Z,t} \) denoting the marginal utility of habit-adjusted consumption of nondurable goods. Equation (2.9) represents the asset price of durables, where \( U_{D,t} \) is the marginal utility of durables consumption and \( \psi_t \) is the Lagrange multiplier attached to constraint (2.6). Equation (2.10) is the optimality condition with respect to investment in durable goods. Finally, equation (2.11) is the wage setting equation in which \( \mu_t \equiv \frac{w_t}{MRS_t} \) is the wage markup, \( MRS_t \equiv \frac{U_{N,t}}{U_{Z,t}} \) is the marginal rate of sub-
stitution between consumption and leisure, $U_{N,t}$ is the marginal disutility of work and $\Pi^C_t = \frac{w_t}{w_{t-1}}\Pi^C$ is the gross wage inflation rate.

2.3.1.2 Impatient households

Impatient households solve a maximization problem analogous to patient households, with the additional assumption that the former are limited in the amount they can borrow from the latter by the value of their durables stock according to the following borrowing constraint:

$$B'_{i,t} \leq mE_t \left( \frac{Q_{t+1}D'_{i,t} \Pi^C_{t+1}}{R_t} \right),$$  

where $m$ represents the loan-to-value (LTV) ratio.\footnote{As noted by Iacoviello and Neri (2010), patient households are subject to a similar constraint that never binds due to $\beta' < \beta$.}

At the symmetric equilibrium, the impatient household’s optimality conditions are:

$$\lambda'_t = e_t^BU'_{Z',t},$$  

$$\lambda'_t = \beta'E_t \left[ \frac{\lambda'_{t+1}}{\Pi^C_{t+1}} \right] + \lambda^{BC}_t R_t,$$

$$Q_t\psi'_t = \frac{U'_{D',t}}{U'_{Z'}} + \beta'(1 - \delta) E_t \left[ \frac{\lambda'_{t+1}}{\lambda'_t} \psi'_t Q_{t+1} \right] +$$

$$+ \lambda^{BC}_t mE_t \left[ Q_{t+1} \Pi^C_{t+1} \right],$$  

$$1 = \psi'_t e'_t \left[ 1 - S' \left( \frac{I'D'}{I'D'} \right) - S' \left( \frac{I'D'}{I'D'} \right) \right] +$$

$$+ \beta'E_t \left\{ \frac{\lambda'_{t+1} Q_{t+1}}{\lambda'_t Q_t} \psi'_t e'_t \left[ S' \left( \frac{I'D'}{I'D'} \right) \right]^2 \right\},$$  

$$0 = \left[ 1 - e'_t W \eta \right] + e'_t W \eta - \partial W \left( \Pi^W - \Pi^C \right) \Pi^W +$$

$$+ \beta'E_t \left[ \frac{\lambda'_{t+1}}{\lambda'_t} \partial W \left( \Pi^W_{t+1} - \Pi^C \right) \Pi^W_{t+1} \frac{w'_{t+1} N'_{t+1}}{w'_{t} N'_t} \right].$$  

Variables $\lambda'_t$ and $\lambda^{BC}_t$ are the Lagrangian multipliers attached to the bud-
get and borrowing constraints, respectively. Notice that (2.14) is a modified version of the typical Euler equation due to the presence of the borrowing constraint. Equations (2.15) and (2.16) show the optimal decisions about the stock and flow of durables whereas (2.17) is the wage equation. Here, \( \Pi_t^{W'} = \frac{w_t'}{w_{t-1}'} \Pi_t^C \) is the gross wage inflation rate of impatient households.

2.3.2 Firms

Firms face quadratic costs of changing prices as in Rotemberg (1982):
\[
\Phi_{j\omega,t} = \frac{\vartheta_j}{2} \left( \frac{P_{j\omega,t}}{P_{j\omega,t-1}} - 1 \right)^2 Y_{j\omega,t},
\]
where \( \vartheta_j \) is the parameter of sectoral price stickiness. Each firm produces differentiated goods according to a constant returns to scale production function,
\[
Y_{j\omega,t} = e^{A_t \left( N_{j\omega,t} \right)^\hat{\psi} \left( N_{j\omega,t}'' \right)^{1-\hat{\psi}}},
\]
where \( \omega \in [0, 1] \) and \( j = C, D \) are indices for firms and sectors respectively, \( \hat{\psi} \in [0, 1] \) denotes the share of the patient household and \( e^{A_t} \) is a labor augmenting shock. Firms maximize the present discounted value of profits,
\[
E_t \left\{ \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[ \frac{P_{j\omega,t}}{P_{j,t}} Y_{j\omega,t} - \frac{W_{j\omega,t}}{P_{j,t}} N_{j\omega,t} - \frac{W_{j\omega,t}'}{P_{j,t}'} N_{j\omega,t}' - \Phi_{j\omega,t} \right] \right\},
\]
subject to production function (2.18) and a standard Dixit-Stiglitz demand equation
\[
Y_{j\omega,t} = \left( \frac{P_{j\omega,t}}{P_{j,t}} \right)^{-\epsilon_j} Y_{j,t},
\]
where \( \epsilon_j \) and \( \epsilon_j' \) are the sectoral intratemporal elasticities of substitution across goods and the sectoral price markup shocks, respectively. At the symmetric equilibrium, the price setting equations for

\[\text{We follow Iacoviello and Neri (2010) and assume a Cobb-Douglas production function. Conversely, Monacelli (2009) and Sterk (2010) assume perfect substitutability between labor inputs and use a linear production function. We opted for the former because, given the different saving choices across the two households, they will bargain different wages. The income share of the two households is different and governed by parameter \( \hat{\psi} \). Assuming that workers are perfect substitutes would lead to the same income share across households, thus neglecting the different saving motive across them. It must be said that Iacoviello and Neri (2010) argue that estimating a model in which hours are perfect substitutes doesn’t materially affect their results.}\]
the two sectors read as

\[
(1 - \epsilon_t^C \epsilon_c) + \epsilon_t^C \epsilon_c MC_t^C = \vartheta_c \left( \Pi_t^C - 1 \right) \Pi_t^C - \\
- \vartheta_c E_t \left[ \Lambda_{t,t+1} \frac{Y_{t+1}^C}{Y_t^C} \left( \Pi_{t+1}^C - 1 \right) \Pi_{t+1}^C \right], \quad (2.20)
\]

\[
(1 - \epsilon_t^D \epsilon_d) + \epsilon_t^D \epsilon_d MC_t^D = \vartheta_d \left( \Pi_t^D - 1 \right) \Pi_t^D - \\
- \vartheta_d E_t \left[ \Lambda_{t,t+1} \frac{Q_{t+1}Y_{t+1}^D}{Q_tY_t^D} \left( \Pi_{t+1}^D - 1 \right) \Pi_{t+1}^D \right]. \quad (2.21)
\]

If \( \vartheta_j = 0 \) prices are flexible and are set as constant markups over the marginal costs.

### 2.3.3 Fiscal and monetary policy

Every period, a lump-sum tax equates government spending so that the government budget is balanced. Government spending \( e_t^G \) follows an exogenous process and, as in Erceg and Levin (2006), we assume that the government purchases only nondurable goods and services. Monetary policy is set according to the following Taylor rule:

\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \\
+ (1 - \rho_r) \left[ \rho_w \log \left( \frac{\tilde{\Pi}_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \right] + \epsilon_t^R, \quad (2.22)
\]

where \( \rho_r \) is the interest rate smoothing parameter, \( \rho_w \) and \( \rho_y \) are the monetary policy responses to the deviations of the inflation aggregator and output from their respective steady states, and \( \epsilon_t^R \) represents the exogenous innovation to the monetary policy rule. \( \tilde{\Pi}_t \equiv \left( \Pi_t^C \right)^{1-\tau} \left( \Pi_t^D \right)^{\tau} \) is an aggregator of the gross rates of inflation in the two sectors with \( \tau \in [0, 1] \) representing the weight of durables. Different monetary policy rules have been used in two-sector NK models with no difference in their main implications.\(^{29}\)

\(^{29}\) Barsky et al. (2007); Katayama and Kim (2010); Bouakez et al. (2011); Sudo (2012, 2007) specify a money supply rule. Carlstrom and Fuerst (2010); Monacelli (2009); Sterk (2010).
### 2.3.4 Market clearing and exogenous processes

In equilibrium all markets clear and the model is closed by the following identities:

\[
Y_t = Y_C^t + Q_t Y_D^t + \frac{\vartheta W}{2} (\Pi^W_t - \Pi^C_t)^2 w_t N_t + \frac{\vartheta W}{2} (\Pi^{W'}_t - \Pi^C_t)^2 w'_t N'_t,
\]

\[
Y_C^t = C_t + C'_t + G_t + \frac{\vartheta c}{2} (\Pi^C_t - \Pi^C_t)^2 Y_C^t,
\]

\[
Y_D^t = [D_t - (1 - \delta) D_{t-1}] + [D'_t - (1 - \delta) D'_{t-1}] + \frac{\vartheta d}{2} (\Pi^D_t - \Pi^D_t)^2 Y_D^t,
\]

\[0 = B_t + B'_t,
\]

\[
N_t = N_C^t + N_D^t,
\]

\[
N'_t = N_C^t + N_D^t.
\]

As in Smets and Wouters (2007), the wage markup and the price markup shocks follow ARMA(1,1) processes:

\[
\log \left( \frac{\kappa_t}{\kappa_{t-1}} \right) = \rho_\kappa \log \left( \frac{\kappa_{t-1}}{\kappa_{t-2}} \right) + \epsilon_\kappa^t - \theta_\kappa \epsilon_\kappa^{t-1},
\]

with \( \kappa = [e^W, e^C, e^D] \), \( i = [W, C, D] \), whereas all other shocks follow an AR(1) process:

\[
\log \left( \frac{\kappa_t}{\kappa_{t-1}} \right) = \rho_\kappa \log \left( \frac{\kappa_{t-1}}{\kappa_{t-2}} \right) + \epsilon_i^t,
\]

where \( \kappa = [e^B, e^I, e^R, e^A, e^G] \) is a vector of exogenous variables, \( \rho_\kappa \) and \( \rho_\kappa \) are the autoregressive parameters, \( \theta_\kappa \) are the moving average parameters, \( \epsilon_\kappa^t \) and \( \epsilon_i^t \) are i.i.d shocks with zero mean and standard deviations \( \sigma_\kappa \) and \( \sigma_i \).

---

\( \text{(2010); Chen and Liao (2014) introduce an interest rate rule that responds only to inflation.} \)

Following Junhe (2009); Auray et al. (2013); Kim and Katayama (2013); Katayama and Kim (2013); Di Pace and Hertweck (2016); Tsai (2016), we set a Taylor-type rule featuring interest rate smoothing and responses to inflation and output.

\( ^{21} \) The systems of equations describing the full symmetric equilibrium and the steady state are presented in Sections 2.8.1 and 2.8.2 of the Appendix.
2.3.5 Functional forms

The utility function is additively separable and logarithmic in the consumption aggregator: \( U(X_t, N_t) = \log(X_t) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} \), where \( \nu \) is a scaling parameter for hours worked and \( \varphi \) is the inverse of the Frisch elasticity of labor supply. Following Christiano et al. (2005), we assume quadratic adjustment costs in durables investment: \( S \left( \frac{I_D}{I_{D-1}} \right) = \frac{\phi}{2} \left( \frac{I_D}{I_{D-1}} - 1 \right)^2 \), with \( \phi > 0 \) representing the degree of adjustment costs. The same functional forms are assumed for the impatient households, with the preference and investment adjustment cost parameters specific to them.

2.3.6 Bayesian estimation

The model is estimated with Bayesian methods. The Kalman filter is used to evaluate the likelihood function, which combined with the prior distribution of the parameters yields the posterior distribution. Then, the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm with two parallel chains of 150,000 draws each is used to generate a sample from the posterior distribution in order to perform inference. We estimate the model over the sample 1969Q2-2007Q4, the same as in the SVAR analysis. We use eight observables: GDP, investment in durable goods, consumption of nondurable goods, real wage, hours worked, inflation in the nondurables sector, inflation in the durables sector and the nominal interest rate, using US data. Similarly to the SVAR analysis, we first define the durables sector as the sum of durable goods and residential investments and label this model as the baseline DSGE. Then, we estimate the model by assuming that durables comprise only houses and we will refer to it as the housing DSGE. This model becomes then very close to Iacoviello and Neri (2010) who, however do not estimate the price stickiness parameter in the housing sector and assume that prices are flexible\(^{22}\). The following measurement equations link the data to

\(^{22}\)Another difference between our model and Iacoviello and Neri (2010) is that we assume perfect labor mobility across sectors hence sectoral wages are always equal.
the endogenous variables of the model:

\[
\begin{align*}
\Delta Y^o_t & = \gamma + \hat{Y}_t - \hat{Y}_{t-1}, \\
\Delta I^o_{D,t} & = \gamma + \hat{I}^*_{D,t} - \hat{I}^*_{D,t-1}, \\
\Delta C^o_t & = \gamma + \hat{C}^*_t - \hat{C}^*_{t-1}, \\
\Delta W^o_t & = \gamma + \hat{W}^*_t - \hat{W}^*_{t-1}, \\
N^o_t & = \hat{N}^*_t, \\
\Pi^o_{C,t} & = \bar{\pi}_C + \hat{\Pi}^C_t, \\
\Pi^o_{D,t} & = \bar{\pi}_D + \hat{\Pi}^D_t, \\
R^o_t & = \bar{r} + \hat{R}_t.
\end{align*}
\] (2.31)

Variables with a `\hat{}` are in log-deviations from their own steady state while `\ast` denotes that the variable has been aggregated between the patient and impatient households (i.e. `x^* = x + x'`\(^{23}\)). \(\gamma\) is the common quarterly trend growth rate of GDP, investment of durables, consumption of nondurables and the real wage; \(\bar{\pi}_C\) and \(\bar{\pi}_D\) are the average quarterly inflation rates in nondurable and durable sectors respectively; \(\bar{r}\) is the average quarterly Federal funds rate. Hours worked are demeaned so no constant is required in the corresponding measurement equation (2.35).

### 2.3.6.1 Calibration and priors

The structural parameters and steady state values presented in Table 2.4 are calibrated at a quarterly frequency. As in Iacoviello and Neri (2010), the discount factors \(\beta\) and \(\beta'\) are 0.99 and 0.97, respectively. Following Monacelli (2009), the depreciation rate of durable goods \(\delta\) is calibrated at 0.010 amounting to an annual depreciation of 4\%, and the durables share of total expenditure \(\alpha\) is set at 0.20. The sectoral elasticities of substitution across different varieties \(\epsilon_c\) and \(\epsilon_d\) equal 6 in order to target a steady-state gross mark-up of 1.20. The elasticity of substitution in the labor market \(\eta\) is set equal to 21 as in Zubairy (2014), implying a 5\% steady-state gross wage

\(^{23}\)The aggregation for the real wage is borrowed from Iacoviello and Neri (2010).
mark-up. The preference parameters $\nu$ and $\nu'$ are set to target steady-state hours of work of 0.33 for both households. The government-output ratio $g_y$ is calibrated at 0.20, in line with the data. Finally, we follow Iacoviello and Neri (2010) and set the loan-to-value ratio $m$ to 0.85 and the share of patient households $\tilde{\psi}$ at their estimated value 0.79.

Table 2.4 summarizes the prior and posterior distributions of the parameters and the shocks. The choice of priors correspond to a large extent to those in previous studies of the US economy. We set the prior mean of the inverse Frisch elasticities $\varphi$ and $\varphi'$ to 0.5, in line with Smets and Wouters (2007, SW henceforth) who estimate a Frisch elasticity of 1.92. We also follow SW in setting the prior means of the habit parameter, $\zeta$ and $\zeta'$, to 0.7, the interest rate smoothing parameter, $\rho_r$, to 0.80 and in assuming a stronger response of the central bank to inflation than output. As far as the the constants in the measurement equations are concerned, we set the prior means equal to the average values in the dataset. In general, we use the Beta (B) distribution for all parameters bounded between 0 and 1. We use the Inverse Gamma (IG) distribution for the standard deviation of the shocks for which we set a loose prior with 2 degrees of freedom. Kim and Katayama (2013) are the only authors who jointly estimate the price and wage stickiness parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor patient households</td>
<td>$\beta$ 0.99</td>
</tr>
<tr>
<td>Discount factor impatient households</td>
<td>$\beta'$ 0.97</td>
</tr>
<tr>
<td>Durables depreciation rate</td>
<td>$\delta$ 0.010</td>
</tr>
<tr>
<td>Durables share of total expenditure</td>
<td>$\alpha$ 0.20</td>
</tr>
<tr>
<td>Elasticity of substitution nondurable goods</td>
<td>$\epsilon_c$ 6</td>
</tr>
<tr>
<td>Elasticity of substitution durable goods</td>
<td>$\epsilon_d$ 6</td>
</tr>
<tr>
<td>Elasticity of substitution in labor</td>
<td>$\eta$ 21</td>
</tr>
<tr>
<td>Preference parameters</td>
<td>$\nu, \nu'$ target $N = N' = 0.33$</td>
</tr>
<tr>
<td>Loan-to-value ratio</td>
<td>$m$ 0.85</td>
</tr>
<tr>
<td>Share of patient households</td>
<td>$\tilde{\psi}$ 0.79</td>
</tr>
<tr>
<td>Government share of output</td>
<td>$g_y$ 0.20</td>
</tr>
</tbody>
</table>

Table 2.4: Calibrated parameters

---

24This calibration is also consistent with the findings in Jappelli (1990), who estimates an income share of 80% for savers in the U.S economy.
whereas all the other studies calibrate them such that prices of nondurable goods are sticky whereas prices of durable goods are flexible. However, they define Calvo parameters for prices and a Rotemberg parameter for wages. Our model features Rotemberg parameters for both prices and wages and we choose a Gamma ($G$) distribution, given that these are non-negative. One of our main interests is to assess whether the durables price stickiness parameter is close to zero, or whether it tends towards values closer to those estimated for the nondurables sector. This is crucial in order to assess whether the response of the relative price of durables is significantly different from zero or not. To this aim, we assign a prior whereby durables prices are as sticky as nondurables prices and both degrees of price stickiness are low (corresponding to firms resetting prices around 2.3 quarters on average in a Calvo world).25 Then, we let the data decide whether and to what extent these should depart from one another.

2.3.6.2 Estimation results

Table 2.5 also reports the posterior mean with 90% probability intervals in square brackets of the baseline and the housing DSGE models. The posterior means suggest that various frictions are supported by the data in both models.26 Impatient households display a higher degree of habits in nondurables consumption ($\zeta < \zeta'$), as found by Iacoviello and Neri (2010) but with a lower persistence ($\rho_c > \rho_c'$). In addition, patient households face larger costs of adjusting their durables stock ($\phi > \phi'$). The posterior mean of the inverse Frisch elasticities of labor supply in both models are higher than the prior and are well identified in the data (as can be seen from comparing prior and posterior distributions in Figure 2.25, Appendix 2.11). Estimates of the Taylor rule parameters show a high degree of policy inertia, and a stronger response to inflation than to output, a likely consequence of estimating the

---

25 We follow Woodford (2003) and Monacelli (2009) to convert the Rotemberg to Calvo parameters and obtain the average price duration.

26 In Appendix 2.12 we perform likelihood comparisons and a number of robustness checks and show that the frictions considered are important when the theoretical model is brought to the data. In addition, Appendix 2.12.1 discusses the implications of changing the income share of patient households.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distr.</th>
<th>Prior Mean</th>
<th>Sd/df</th>
<th>Posterior Mean Baseline DSGE</th>
<th>Posterior Mean Housing DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inv. Frisch elasticity patients $\varphi$</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.5504 [0.4010;0.6986]</td>
<td>0.6448 [0.4933;0.7942]</td>
</tr>
<tr>
<td>Inv. Frisch elasticity impatients $\varphi'$</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.6468 [0.4952;0.8028]</td>
<td>0.6860 [0.5300;0.8431]</td>
</tr>
<tr>
<td>Habits patients $\zeta$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.6505 [0.5979;0.7036]</td>
<td>0.6615 [0.6188;0.6965]</td>
</tr>
<tr>
<td>Habits. impatients $\zeta'$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.9336 [0.9240;0.9442]</td>
<td>0.9404 [0.9338;0.9465]</td>
</tr>
<tr>
<td>Habit persist. patients $\rho_c$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.5068 [0.3964;0.6206]</td>
<td>0.6399 [0.5412;0.7436]</td>
</tr>
<tr>
<td>Habit persist. impatients $\rho'_c$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.2195 [0.1564;0.2809]</td>
<td>0.3221 [0.2366;0.4142]</td>
</tr>
<tr>
<td>Price stickiness nondurables $\varphi_W$</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>23.38 [15.82;30.61]</td>
<td>26.06 [18.56;33.99]</td>
</tr>
<tr>
<td>Price stickiness durables $\varphi_D$</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>24.45 [16.09;33.26]</td>
<td>1.79 [1.13;2.46]</td>
</tr>
<tr>
<td>Wage stickiness</td>
<td>G</td>
<td>100.0</td>
<td>10.00</td>
<td>152.39 [136.15;169.71]</td>
<td>168.06 [158.30;177.30]</td>
</tr>
<tr>
<td>IAC durables patients $\phi$</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>3.4738 [2.8002;4.1114]</td>
<td>3.7908 [3.2240;4.4022]</td>
</tr>
<tr>
<td>IAC durables impatients $\phi'$</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>1.9112 [1.2022;2.5902]</td>
<td>1.7710 [1.0228;2.4684]</td>
</tr>
<tr>
<td>Share of durables inflation $\tau$</td>
<td>B</td>
<td>0.20</td>
<td>0.10</td>
<td>0.1440 [0.0519;0.2299]</td>
<td>0.0516 [0.0367;0.0672]</td>
</tr>
<tr>
<td>Inflation -Taylor rule $\rho_H$</td>
<td>N</td>
<td>1.50</td>
<td>0.20</td>
<td>1.4042 [1.2298;1.5702]</td>
<td>1.7285 [1.5062;1.9437]</td>
</tr>
<tr>
<td>Output -Taylor rule $\rho_Y$</td>
<td>G</td>
<td>0.10</td>
<td>0.05</td>
<td>0.0175 [0.0056;0.0291]</td>
<td>0.0221 [0.0059;0.0368]</td>
</tr>
<tr>
<td>Interest rate smoothing $\rho_r$</td>
<td>B</td>
<td>0.80</td>
<td>0.10</td>
<td>0.7088 [0.6657;0.7545]</td>
<td>0.7681 [0.7314;0.8054]</td>
</tr>
<tr>
<td>Averages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trend growth rate $\gamma$</td>
<td>N</td>
<td>0.49</td>
<td>0.10</td>
<td>0.4017 [0.3678;0.4343]</td>
<td>0.4054 [0.3752;0.4356]</td>
</tr>
<tr>
<td>Inflation rate nondurables $\pi_C$</td>
<td>G</td>
<td>1.05</td>
<td>0.10</td>
<td>1.0135 [0.9120;1.1146]</td>
<td>1.0427 [0.9348;1.1438]</td>
</tr>
<tr>
<td>Inflation rate durables $\pi_D$</td>
<td>G</td>
<td>0.37</td>
<td>0.10</td>
<td>0.4324 [0.3200;0.5495]</td>
<td>0.3462 [0.2180;0.4767]</td>
</tr>
<tr>
<td>Interest rate $\bar{r}$</td>
<td>G</td>
<td>1.65</td>
<td>0.10</td>
<td>1.6140 [1.4898;1.7467]</td>
<td>1.6404 [1.4980;1.7574]</td>
</tr>
<tr>
<td><strong>Exogenous processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology $\rho_{eA}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9775 [0.9574;0.9970]</td>
<td>0.9555 [0.9223;0.9903]</td>
</tr>
<tr>
<td>Monetary Policy $\sigma_{eA}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>0.6933 [0.6196;0.7607]</td>
<td>0.7483 [0.6678;0.8308]</td>
</tr>
<tr>
<td>Investment Durables $\rho_{eI}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.4915 [0.3072;0.6710]</td>
<td>0.3925 [0.2872;0.5543]</td>
</tr>
<tr>
<td>Preference $\rho_{eB}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.7465 [0.6805;0.8167]</td>
<td>0.8641 [0.8198;0.9103]</td>
</tr>
<tr>
<td>Price mark-up nondurables $\rho_{eC}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9053 [0.8544;0.9595]</td>
<td>0.9098 [0.8490;0.9728]</td>
</tr>
<tr>
<td>Price mark-up durables $\rho_{eD}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.5643 [0.5060;0.6733]</td>
<td>0.6449 [0.5262;0.7698]</td>
</tr>
<tr>
<td>Price mark-up durables $\rho_{eG}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>2.0840 [1.6533;2.4938]</td>
<td>2.2972 [1.7984;2.7678]</td>
</tr>
<tr>
<td>Price mark-up durables $\rho_{eD}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9867 [0.9768;0.9976]</td>
<td>0.9888 [0.9778;0.9994]</td>
</tr>
<tr>
<td>Wage mark-up $\rho_{eW}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.7037 [0.6025;0.8045]</td>
<td>0.8903 [0.8561;0.9265]</td>
</tr>
<tr>
<td>Government spending $\rho_{eG}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9586 [0.9458;0.9879]</td>
<td>0.9565 [0.9283;0.9857]</td>
</tr>
<tr>
<td>Government spending $\sigma_{eG}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>3.5099 [3.1807;3.8336]</td>
<td>3.3376 [3.0351;3.6504]</td>
</tr>
</tbody>
</table>

Table 2.5: Prior and posterior distributions of estimated parameters (90% confidence bands in square brackets)
model over a sample including the Great Moderation. Overall, estimates from both models are quite close to each other.

As regards price stickiness in the two sectors, when we employ the broad measure of durable goods (baseline DSGE) the posterior means are very similar – with confidence intervals almost entirely overlapping. The point estimates of durables and nondurables price stickiness ($\vartheta_d = 24.45, \vartheta_c = 23.38$) correspond to Calvo probabilities of resetting the price of 35.9% and 36.5% and an average price duration of 2.8 and 2.7 quarters respectively. Conversely, in the housing DSGE the posterior mean of house prices ($\vartheta_d = 1.79$) is dramatically lower than that of nondurables ($\vartheta_c = 26.06$) corresponding to Calvo probabilities of resetting the price of 78.1% and 35% and average price durations of 1.3 and 2.8 quarters respectively. In addition, confidence intervals never overlap. Here, the estimated degree of wage stickiness ($\vartheta^W = 168.06$) guarantees that the comovement between consumption in the two sectors is still attained despite house prices being estimated to be quasi-flexible.\footnote{The importance of wage stickiness for the comovement between the two sectors is also highlighted in a calibrated version of the model with flexible durables prices, see Figure 2.26 in Appendix 2.12.}

Figure 2.3 shows the prior and posterior distributions of the price stickiness parameters in both models. First, we notice that the data is informative as the posterior distributions are rather apart from the prior. In the baseline DSGE (left box), the two distributions almost entirely overlap thus pointing to a negligible difference in the price stickiness across the nondurables and durables sectors. Conversely, in the housing DSGE (right box) the posterior distribution of the housing price stickiness moves towards zero and in opposite direction with respect to the posterior distribution of nondurables prices. Such estimates highlight that when a broad measure of durables is employed, then prices display the same degree of stickiness with respect to nondurables whereas, if the durables sector coincides with the housing sector, then prices are estimated to reset almost every quarter.

This result that durables prices are as sticky as nondurables contrasts with\footnote{Kim and Katayama (2013) who find that prices of durables are substantially more flexible than prices of nondurables, in a model with homogenous}
households and no role for durables as collateral, fewer shocks and different observables. We try and be as close as possible to mainstream estimated models as far as shocks and observables are concerned, with the natural addition of observables related to durables consumption and durables inflation. Moreover, such results are closer to the latest microeconometric evidence. In particular, Nakamura and Steinsson (2008), Klenow and Malin (2010) and Petrella and Santoro (2012) use highly disaggregated data and find no decisive evidence that categories of nondurables are stickier than durables. In addition, Boivin et al. (2009) argue that inflation in sectors with high price stickiness display a high autocorrelation and low volatility. They estimate that durables inflation has higher autocorrelation and lower volatility than nondurables inflation hence it is possible to infer that prices of durables are stickier than nondurables.

Also Bouakez et al. (2009) provide qualitatively similar results to Kim and Katayama (2013) in a larger model estimated using GMM and a different dataset.
2.3.6.3 Impulse response functions

In order to investigate the dynamic properties of the models, Figure 2.4 displays the estimated impulse responses of the variables of interest to a one standard-deviation increase in the nominal interest rate across the baseline and housing DSGE models. As the estimated parameters are very similar across the two models, the mean responses do not show large differences. Taking into account the 68%, 90% and the 95% confidence bands (see Figures 2.23 and 2.24 in Appendix 2.10) further highlights these similarities. An increase in the monetary policy rate leads to an output contraction and a decrease in overall and sectoral inflations. Furthermore, the presence of wage and price stickiness generates the desired comovement between durables and nondurables. The only noticeable difference between the two models concerns the response of the relative price, due to the different degree of

---

29Impulse responses represent percentage deviations from the steady state. Bayesian impulse responses of each model together with confidence bands are reported in Appendix 2.10.
Figure 2.5: Bayesian impulse responses of relative prices to a contractionary monetary policy shock (bold lines are mean responses, dark-shaded areas are 68% confidence bands, medium and lighter shaded areas represent 90% and 95% confidence bands respectively)

estimated price stickiness. In the baseline model, prices of durables and nondurables are equally sticky hence the response of the relative price is flat whereas in the housing model, house prices are almost flexible and prices of nondurables are sticky hence the relative price falls in response to a monetary policy contraction. Figure 2.5 highlights the Bayesian impulse responses of the relative prices in the two models together with the 68%, 90% and the 95% confidence bands. The credible set of estimated impulse responses in the baseline model does not exclude zero at any of the confidence levels considered whereas it is significantly negative in the housing DSGE. Such dynamic properties of the models are consistent with the findings of the SVAR models estimated in Section 2.2 (see Figures 2.1 and 2.2) and represent the main novel contribution of our paper.

To sum up, we have estimated prices of durables (defined as the sum of durable goods and residential investments) to be as sticky as nondurables which is at odds with the assumption made in most two-sectors New-Keynesian models that they are fully flexible. We have then demonstrated that such assumption is consistent only with a narrow definition of durables sector that

\[\text{In general, qualitatively the responses of the estimated DSGE models are consistent with those of the SVAR model. Also from a quantitative perspective, durables turn out to be more volatile than nondurables and output, as in the SVAR results.}\]
coincide with exclusively with residential investments.

2.3.7 Estimated sectoral price stickiness in extended models

The two-sector DSGE model estimated in the previous section builds mainly on Barsky et al. (2007) with the addition of several frictions and of the collateral constraint as in Monacelli (2009), Iacoviello and Neri (2010) and Sterk (2010). In this section we extend the model to account for two additional features affecting sectoral Phillips curves, namely imperfect sectoral labor mobility and price indexation. We re-estimate the DSGE model jointly with the additional parameters. Then, we further generalize our analysis and estimate a three-sector DSGE model in which housing and non-housing durables are treated separately and display heterogeneity in terms of rate of depreciation, adjustment costs and degree of substitutability with nondurable goods. Table 2.6 reports the estimated sectoral price stickiness across the extended models whereas the full set of estimated parameters is in Appendix 2.13.

2.3.7.1 Imperfect sectoral labor mobility

Households in two-sector models are allowed to optimally choose the quantity of labor to supply in each sector according to their preferences. Standard two-sector models typically assume either that labor is perfectly mobile hence sectoral wages are equalized across the two sectors or assume no mobility at all. However, more recent contributions have emphasized the importance of limited labor mobility in accounting for the behavior of the economy in multi-sector models. Indeed, Bouakez et al. (2009) argue that imperfect labor mobility affects the dispersion of hours across sectors whereas Bouakez et al. (2011) show that accounting for limited labor mobility jointly with intersectoral linkages solves the comovement puzzle. Moreover, Iacoviello and

---

Neri (2010) find evidence of limited labor mobility across the consumption and housing sectors.

In the context of our main two-sector model above, perfect labor mobility implies that the production structure and thus marginal costs are always the same across the two sectors. Therefore, it seems sensible to modify it to allow for limited labor mobility, and hence for different dynamics of wages and the marginal costs across the two sectors, and to check the robustness of our results as regards the estimation of the price stickiness parameters. Following the abovementioned contributions, limited labor mobility is introduced by specifying a constant elasticity of substitution (CES) aggregator between sectoral hours for each household:

\[ N_t = \left[ (\chi^C)^{-\frac{1}{\lambda}} \left( N_C^t \right)^{\frac{1+\lambda}{\lambda}} + \left( 1 - \chi^C \right)^{-\frac{1}{\lambda}} \left( N_D^t \right)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{1}{1+\lambda}}, \]  

(2.39)

where the intra-temporal elasticity of substitution \( \lambda \in (0, \infty) \) governs the degree of labor mobility. Note that \( \lambda \to 0 \) denotes the case of labor immobility, while as \( \lambda \to \infty \) labor can be freely reallocated and all workers earn the same wage at the margin. For \( \lambda < \infty \) the economy displays a limited degree of labor mobility and sectoral wages are not equal. Moreover, \( \chi^C \equiv N_C^t/N \) represents the steady-state share of labor supply in the nondurables sector.

We then replace the labor market clearing conditions (2.27) and (2.28) with the CES aggregators and bring the model to the data. Typically, limited labor mobility is calibrated at a value of \( \lambda = 1 \) (see Bouakez et al. 2009, Petrella and Santoro 2011 and Petrella et al. 2017), except Bouakez et al. (2011) who explore values between 0.5 and 1.5 whereas Iacoviello and Neri (2010) estimate values of 1.51 and 1.03 for savers and borrowers, respectively. Accordingly, we set the prior mean of the labor mobility parameters

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32 Imperfect sectoral labor mobility plays a role also for the conduct of optimal monetary policy, as demonstrated by Petrella and Santoro (2011) and Petrella et al. (2017).

33 The same functional form is assumed for impatient households, with variables and parameters specific to them denoted by \( ' \). Details about the symmetric equilibrium are in Appendix 2.8.3.1.

34 Iacoviello and Neri (2010) specify the CES aggregator such that the labor mobility parameter is the inverse of \( \lambda \). They find values of 0.66 and 0.97 for savers and borrowers respectively hence the values of 1/0.66=1.51 and 1/0.97=1.03 we reported.
at the posterior estimates of Iacoviello and Neri (2010) and bring the model to the data. The third and fourth columns of Table 2.6 report the estimated price stickiness in the baseline and housing DSGE models with imperfect sectoral labor mobility and show that the results of our main model (first and second columns of Table 2.6) continue to hold. In the baseline DSGE, price stickiness is similar across the two sectors with 90% confidence intervals widely overlapping. Conversely, in the housing DSGE prices of nondurables are significantly stickier than house prices, which are quasi-flexible. The top panel of Figure 2.6 plots the posterior distribution of the price stickiness parameters in the baseline and housing DSGE models with imperfect sectoral labor mobility. While in the baseline DSGE the two posterior distributions widely overlap, in the housing DSGE the posterior distributions are rather apart from each other thus implying a significant difference between the two sectoral price stickiness parameters. In addition, labor mobility in the housing DSGE is estimated to be somewhat lower than in the baseline DSGE (see Table 2.12, Appendix 2.13).

2.3.7.2 Price indexation

The price setting behavior of firms specified in equations (2.20) and (2.21) yield purely forward-looking sectoral Phillips curves. In this section we introduce a backward-looking component of the Phillips curves by estimating the degree of sectoral indexation to past inflation and verify that the results reported in Section 2.3.6.2 as regards price stickiness are not driven by the absence of indexation. Following Ireland (2007, 2011) and Ascari et al. (2011) we introduce indexation in the Rotemberg price adjustment cost specification, which now read as:

\[
\frac{\varphi_j}{2} \left( \frac{P^{\prime}_{j,t}}{(\Pi^{\prime}_{t-1})^{\gamma_j} P^{\prime}_{j,t-1}} - 1 \right)^2 Y_t^j, \tag{2.40}
\]

\[^{35}\text{Confidence bands of the estimated elasticities do not overlap for patient households, but they overlap for impatient households.}\]
where $\zeta_j \in [0, 1]$ determines the degree of indexation to past inflation and $j = C, D$.

When bringing the extended model to the data, we set the prior mean of the sectoral indexation parameters as in Smets and Wouters (2007) and Lacoviello and Neri (2010) at 0.50 and standard deviation 0.20. Table 2.6 (fifth and sixth columns) shows that the estimated sectoral price stickiness is very similar across sectors in the baseline DSGE, whereas in the housing DSGE nondurables prices are much stickier than house prices, which are virtually flexible. This confirms the results of the main model. Looking at the posterior distributions of the price stickiness parameters in the baseline and housing DSGE models with sectoral price indexation (middle panel of Figure 2.6) leads to the same inference as in the main model. The estimated degrees of price rigidity are not significantly different in the baseline DSGE whereas in the housing DSGE house prices are significantly more flexible than nondurable prices. Finally, we estimate a low degree of sectoral price indexation (see Table 2.13 Appendix 2.13).

2.3.7.3 Three-sector model

We have so far demonstrated that the definition of the durables sector plays a crucial role in the estimation of the sectoral price stickiness. In this section, we verify that our results continue to hold when we generalize the model to a three-sector economy producing nondurables, housing and non-housing durables, where only housing goods serve as collateral. Non-housing and housing durables display several sources of heterogeneity with respect to each other: (i) different depreciation rates, i.e. different degrees of durability; (ii) different adjustment costs in investment in these two goods; (iii) different degree of substitutability between housing and non-housing durables with

\footnote{Appendix 2.8.3.2 provides details about the modified symmetric equilibrium.}

\footnote{Our estimates of the price indexation parameters are in line with Smets and Wouters (2007) and Ascari et al. (2011). In contrast, Benati (2008) and Lacoviello and Neri (2010) report higher values whereas Ireland (2007; 2011) finds evidence of no indexation. However, to the best of our knowledge, this is the first paper that estimates the sectoral degree of price indexation within a two-sector DSGE model with durable and nondurable goods (Lacoviello and Neri (2010) estimate price stickiness and price indexation only for the nondurables sector).}
nondurable goods. In particular, here parameter $\delta$ denotes the depreciation rate only of non-housing durables, while parameter $\delta^H \neq \delta$ denotes the depreciation of housing goods. Similarly, while parameters $\phi$ and $\phi'$ refer to investment adjustment costs of non-housing durables, parameters $\phi^H$ and $\phi'^H$ refer to adjustment costs of housing goods. Finally, accounting for different degrees of substitutability between housing and non-housing durables with nondurable goods requires a generalization of the consumption aggregator, which we specify as a nested constant-elasticity-of-substitution (CES) function of the three goods as follows:

$$X_t = \left[ (1 - \alpha) \tilde{C}_t^{\frac{\rho - 1}{\rho}} + \alpha H_t^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}}, \quad (2.41)$$

$$\tilde{C}_t = \left[ (1 - \tilde{\alpha}) Z_t^{\frac{\tilde{\rho} - 1}{\tilde{\rho}}} + \tilde{\alpha} D_t^{\frac{\tilde{\rho} - 1}{\tilde{\rho}}} \right]^{\frac{\tilde{\rho}}{\tilde{\rho} - 1}}, \quad (2.42)$$

where parameters $\rho, \tilde{\rho} \in (0, \infty)$ represent the elasticities of substitution between non-housing (durables and nondurables) and housing goods and between nondurable and non-housing durable goods, respectively. The resulting degree of substitutability between non-durables and housing and between non-durables and non-housing durables is then a function of these two elasticities and is allowed to be different.\(^{38}\)

Housing goods are used as collateral by impatient households, hence the borrowing constraint (2.12) now reads as:

$$B_t' \leq mE_t \left( \frac{Q_H^t H_t' \Pi_C^t}{R_t} \right), \quad (2.43)$$

with $Q_H^t \equiv \frac{P_H^t}{P_C^t}$ being the relative house price. Firms in the housing sector behave as firms in the nondurables and durables sector, as outlined in Section 2.3.2 they maximize profits and are subject to quadratic costs of adjusting prices, with three different price stickiness parameters: $\vartheta_c, \vartheta_d, \vartheta_h \in [0, \infty)$.

\(^{38}\)The same CES aggregators are used for impatient households, with the corresponding variables denoted by $'$. Full description of the symmetric equilibrium is provided in Appendix 2.8.3.3.
denoting price stickiness in the non-durables, non-housing durables and housing durables sector, respectively. Finally, distinguishing between housing and non-housing investment requires making a distinction between (i) housing and non-housing investment-specific shocks; and (ii) housing and non-housing durables price markup shocks. These are assumed to follow AR(1) and ARMA(1,1) processes, respectively, in line with the analysis above. This means that the extended model features two more shocks relative to that in Section 2.3.

Consistently to this new structure, we bring the model to the data by distinguishing between residential and non-residential investment in durable goods, as well as inflation in the housing and non-housing durables sectors. Since now the observables of durables investment and inflation exclude housing goods, we need to add the following two measurement equations for residential investment and house price inflation respectively:

\[
\Delta I_{H,t} = \gamma + \hat{I}_{H,t} - \hat{I}_{H,t-1},
\]
\[
\Pi_{H,t} = \bar{\pi}_H + \hat{\Pi}_H.
\]

In addition to the parameters calibrated in Table 2.4, we set the elasticity of substitution in the housing sector \( \epsilon_h \) to 6 and the distributional parameters of the CES consumption aggregators \( \alpha, \tilde{\alpha} \in [0, 1] \) to match the sectoral expenditure shares over the sample considered. The calibration of depreciation rates of the non-housing durables \( \delta \) and housing goods \( \delta^H \) deserves more attention. These parameters are crucial for the property of quasi-constancy of the shadow-value of long-lived goods, as demonstrated by Barsky et al. (2007) and Barsky et al. (2016). The literature has used a variety of values, ranging from a quarterly depreciation of 0.01 (see, among others, Monacelli 2009; Sterk 2010; Iacoviello and Neri 2010; Chen and Liao 2014), to 0.025 (see Erceg and Levin 2006; Carlstrom and Fuerst 2010; Petrella and Santoro 2011; Sudo 2012); and 0.0125 (Barsky et al., 2007; Kim and Katayama 2013; Katayama and Kim 2013; Barsky et al., 2016) and 0.0035 (Jeske and Liu 2013). Barsky et al. (2016) explore the implications for optimal monetary policy of several values between 0.01 and 0.20.
In accordance with the microeconometric evidence (see, e.g. Fraumeni, 1997) and the literature just mentioned, we assume that non-housing durables display a higher depreciation rate than housing goods. We thus calibrate $\delta = 0.025$ and $\delta^H = 0.01$. Price stickiness and investment adjustment costs parameters are estimated using the same priors as outlined in Section 2.3.6.1 whereas we set the prior mean of the consumption elasticities of substitution $\rho$ and $\tilde{\rho}$ to 1 (which imply a nested Cobb-Douglas aggregator), and a standard deviation of 0.1.

It turns out that point estimates of investment adjustment cost parameters differ across sectors, and confidence intervals of both consumption elasticities of substitution do not exclude the Cobb-Douglas case (Tables 2.14 and 2.15, Appendix 2.13). The last column of Table 2.6 reports the estimated price stickiness parameters across the three sectors. Although the distance between the point estimates of non-housing durables and nondurables price stickiness is larger than that existing between overall durables and nondurable (see two-sector model estimates, Table 2.6 column 1), their confidence intervals overlap at any conventional confidence level, as it can also be seen by inspecting the posterior distributions (bottom panel of Figure 2.6). This means that they are not significantly different from each other in a statistical sense. It is not surprising that, with respect to our main
Figure 2.6: Posterior distributions of price stickiness parameters in extended models
model, the posterior distribution of the non-housing durable price stickiness moves further to the right, as we have deducted housing from the relevant observables. Indeed, house prices robustly continue to be the most flexible component of durables prices also in the three-sector model. Confidence intervals of house price stickiness still never overlap with the other two, given that its posterior distribution moves rather apart from the others towards zero (see bottom panel of Figure 2.6).

2.4 Concluding remarks

Several papers have engaged in building a two-sector New-Keynesian model able to generate the comovement between durable and nondurable goods following a monetary policy shock, as documented by the SVAR literature. This paper contributes to the existing literature by focusing on a less studied but equally important issue: the effects of a monetary policy innovation on the relative price of durables.

We show that, robustly across identifications and subsamples, in SVAR models the response of the relative price of durables to monetary policy shocks crucially depends on the definition of the durables sector. If durables include both non-housing durable goods and residential investment (as common in the literature), the relative price marginally increases or stays flat in response to a monetary policy contraction. Conversely, employing a narrow measure of durable goods that includes only new houses generates a fall in the relative price.

To rationalize the SVAR results, we build a rather canonical two-sector DSGE model in which impatient households borrow from patient households against the value of their durables collateral. We bring the model to the data using Bayesian methods employing, first, the broad definition of durables (non-housing durables and residential investments) and then the narrow measure of durables including only residential investment. Similarly to the most recent microeconomic evidence, we estimate the degree of price stickiness to be almost the same when non-housing durables are bundled with residential investment. It follows that the credible set of responses of the relative price
of durables to a monetary policy shock includes zero. Conversely, durables -defined as including only residential investment- display a much lower stickiness than nondurables hence the credible set of responses of the relative price of durables to a monetary policy shock is significantly negative. Such results not only agree, but also rationalize our SVAR estimates. The results regarding the estimation of price stickiness parameters survive extensions of the DSGE model affecting sectoral Phillips curves and a three-sector generalization. The importance of these findings is twofold. First, when building a two-sector New-Keynesian model it is desirable to assume that prices of durable goods are sticky, unless the aim is modeling the housing sector in isolation from other durables. When this is the case, the comovement puzzle is no longer an issue: if prices are sticky in both sectors, durables and nondurables will move in the same direction in response to monetary innovations. A three-sector model is needed to fully capture the intrinsic differences between housing and non-housing durables, such as the type of goods that can be used as collateral and their different degree of durability. Second, from a policy viewpoint, while the central bank is not likely to create big allocative distortions between the durables and nondurables sector, it may indeed create allocative distortions regarding the housing sector. Whether this has large welfare implications, and the optimal monetary policy design in this context, are beyond the scope of this paper, but these issues should certainly be investigated in future research.
Appendix

2.5 Data: sources and transformations

<table>
<thead>
<tr>
<th>Series</th>
<th>Definition</th>
<th>Source Mnemonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DUR^N$</td>
<td>Nominal Durable Goods</td>
<td>BEA Table 2.3.5 Line 3</td>
</tr>
<tr>
<td>$RI^N$</td>
<td>Nominal Residential Investment</td>
<td>BEA Table 1.1.5 Line 13</td>
</tr>
<tr>
<td>$ND^N$</td>
<td>Nominal Nondurable Goods</td>
<td>BEA Table 2.3.5 Line 8</td>
</tr>
<tr>
<td>$SN^N$</td>
<td>Nominal Services</td>
<td>BEA Table 2.3.5 Line 13</td>
</tr>
<tr>
<td>$P_{DUR}$</td>
<td>Price Deflator, Durable Goods</td>
<td>BEA Table 1.1.9 Line 4</td>
</tr>
<tr>
<td>$P_{RI}$</td>
<td>Price Deflator, Residential Investment</td>
<td>BEA Table 1.1.9 Line 13</td>
</tr>
<tr>
<td>$P_{ND}$</td>
<td>Price Deflator, Nondurable Goods</td>
<td>BEA Table 1.1.9 Line 5</td>
</tr>
<tr>
<td>$P_S$</td>
<td>Price Deflator, Services</td>
<td>BEA Table 1.1.9 Line 6</td>
</tr>
<tr>
<td>$YN$</td>
<td>Nominal GDP</td>
<td>BEA Table 1.1.5 Line 1</td>
</tr>
<tr>
<td>$PY$</td>
<td>Price Deflator, GDP</td>
<td>BEA Table 1.1.9 Line 1</td>
</tr>
<tr>
<td>$FFR$</td>
<td>Effective Federal Funds Rate</td>
<td>FRED FEDFUNDS</td>
</tr>
<tr>
<td>$N$</td>
<td>Nonfarm Business Sector: Average Weekly Hours</td>
<td>FRED PRS85006023</td>
</tr>
<tr>
<td>$W$</td>
<td>Nonfarm Business Sector: Compensation Per Hour</td>
<td>FRED COMPNFB</td>
</tr>
<tr>
<td>$POP$</td>
<td>Civilian Non-institutional Population, over 16</td>
<td>FRED CNP16OV</td>
</tr>
<tr>
<td>$CE$</td>
<td>Civilian Employment, 16 over</td>
<td>FRED CE16OV</td>
</tr>
<tr>
<td>$NH^N$</td>
<td>Nominal New-single family houses</td>
<td>BEA Table 5.3.5 Line 23</td>
</tr>
<tr>
<td>$P_{NH}$</td>
<td>Price Deflator, New-single family houses</td>
<td>BEA Table 5.3.4 Line 23</td>
</tr>
<tr>
<td>$MH^N$</td>
<td>Nominal Multifamily houses</td>
<td>BEA Table 5.3.5 Line 24</td>
</tr>
<tr>
<td>$P_{MH}$</td>
<td>Price Deflator, Multifamily houses</td>
<td>BEA Table 5.3.4 Line 23</td>
</tr>
</tbody>
</table>

Table 2.7: Data Sources

2.5.1 Durables and Residential Investments

1. Sum nominal series: $DUR^N + RI^N = DR^N$
2. Calculate sectoral weights of deflators: $\omega^D = \frac{DUR^N}{DR^N}$; $\omega^{RI} = \frac{RI^N}{DR^N}$
3. Calculate Deflator: $P_D = \omega^D P_{DUR} + \omega^{RI} P_{RI}$
4. Calculate Real Durable Consumption: \( D = \frac{DUR^N + RN}{P_D} \)

### 2.5.2 Nondurables and Services

1. Sum nominal series: \( ND^N + SN = NS^N \)
2. Calculate sectoral weights of deflators: \( \omega^{ND} = \frac{ND^N}{NS^N}; \ \omega^S = \frac{SN}{NS^N} \)
3. Calculate Deflator: \( P_C = \omega^{ND} P_{ND} + \omega^S P_S \)
4. Calculate Real Nondurable Consumption: \( C = \frac{ND^N + SN}{P_C} \)

### 2.5.3 Only broad measure of houses

1. Sum nominal series: \( NH^N + MH^N = DR^N \)
2. Sectoral weights of deflators: \( \omega^{NH} = \frac{NH^N}{DR^N}; \ \omega^{MH} = \frac{MH^N}{DR^N} \)
3. Calculate Deflator: \( P_D = \omega^{NH} P_{NH} + \omega^{MH} P_{MH} \)
4. Calculate Real Durable Consumption: \( D = \frac{NH^N + MH^N}{P_D} \)

### 2.5.4 Durable goods and New-single family houses

1. Sum nominal series: \( DUR^N + NH^N = DR^N \)
2. Calculate sectoral weights of deflators: \( \omega^{D} = \frac{DUR^N}{DR^N}; \ \omega^{NH} = \frac{NH^N}{DR^N} \)
3. Calculate Deflator: \( P_D = \omega^{D} P_{DUR} + \omega^{NH} P_{NH} \)
4. Calculate Real Durable Consumption: \( D = \frac{DUR^N + NH^N}{P_D} \)

### 2.5.5 Durable goods and broad measure of houses

1. Sum nominal series: \( DUR^N + NH^N + MH^N = DR^N \)
2. Sectoral weights of deflators: \( \omega^{D} = \frac{DUR^N}{DR^N}; \ \omega^{NH} = \frac{NH^N}{DR^N}; \ \omega^{MH} = \frac{MH^N}{DR^N} \)
3. Calculate Deflator: \( P_D = \omega^{D} P_{DUR} + \omega^{NH} P_{NH} + \omega^{MH} P_{MH} \)
4. Calculate Real Durable Consumption: \( D = \frac{DUR^N + NH^N + MH^N}{P_D} \)
2.5.6 Data transformation for Bayesian estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$POP_{\text{index}}$</td>
<td>Population index</td>
<td>$\frac{POP_{\text{2009-1}}}{\text{POP}}$</td>
</tr>
<tr>
<td>$CE_{\text{index}}$</td>
<td>Employment index</td>
<td>$\frac{CE_{\text{2009-1}}}{\text{CE}}$</td>
</tr>
<tr>
<td>$Y^{\circ}$</td>
<td>Real per capita GDP</td>
<td>$\ln \left( \frac{Y^{\circ}}{\text{POP}_{\text{index}}} \right) \times 100$</td>
</tr>
<tr>
<td>$I_{D}^{\circ}$</td>
<td>Real per capita investment: durables</td>
<td>$\ln \left( \frac{I_{D}^{\circ}}{\text{POP}_{\text{index}}} \right) \times 100$</td>
</tr>
<tr>
<td>$I_{H}^{\circ}$</td>
<td>Real per capita investment: houses</td>
<td>$\ln \left( \frac{I_{H}^{\circ}}{\text{POP}_{\text{index}}} \right) \times 100$</td>
</tr>
<tr>
<td>$C^{\circ}$</td>
<td>Real per capita consumption: nondurables</td>
<td>$\ln \left( \frac{C^{\circ}}{\text{POP}_{\text{index}}} \right) \times 100$</td>
</tr>
<tr>
<td>$W^{\circ}$</td>
<td>Real wage</td>
<td>$\ln \left( \frac{W^{\circ}}{\text{PY}} \right) \times 100$</td>
</tr>
<tr>
<td>$N^{\circ}$</td>
<td>Hours worked per capita</td>
<td>$\ln \left( \frac{N^{\circ}}{\text{POP}_{\text{index}}} \right) \times 100$</td>
</tr>
<tr>
<td>$\Pi_{C}^{\circ}$</td>
<td>Inflation: nondurables sector</td>
<td>$\Delta \left( \ln P_{C} \right) \times 100$</td>
</tr>
<tr>
<td>$\Pi_{D}^{\circ}$</td>
<td>Inflation: durables sector</td>
<td>$\Delta \left( \ln P_{D} \right) \times 100$</td>
</tr>
<tr>
<td>$\Pi_{H}^{\circ}$</td>
<td>Inflation: housing sector</td>
<td>$\Delta \left( \ln P_{H} \right) \times 100$</td>
</tr>
<tr>
<td>$R^{\circ}$</td>
<td>Quarterly Federal Funds Rate</td>
<td>$\frac{\text{FFR}}{4}$</td>
</tr>
</tbody>
</table>

Table 2.8: Data transformation - Observables

2.6 SVAR methodologies

2.6.1 Recursive approach

Let $\Sigma_{e}$ be the variance-covariance matrix of the reduced-form shocks of the SVAR model. Under the recursive approach, the structural shocks are identified through a Cholesky decomposition of $\Sigma_{e}$. Consequently, the order of the variables in vector $x_{t}$ matters for the identification of the monetary disturbance. Indeed, at time $t$ one variable is affected by the previous but not from those which follow. In our estimation, we make the standard assumption
that the monetary policy variable is ordered last hence it has no contemporaneous effect on the other variables (see Bernanke and Mihov, 1998, among others). Our SVAR model includes a vector of constant terms and four lags, as commonly assumed in the literature for a monetary SVAR with quarterly frequency.

2.6.2 Sign restrictions approach

The second approach we employ is the pure sign restrictions proposed by Uhlig (2005). This method implies that shocks are identified when they follow specific and unique patterns by imposing restrictions on the impulse response functions (IRFs) of the SVAR model. Several orthogonal matrices linking the reduced-form and the structural shocks are drawn, where we retain those generating impulse responses that satisfy the set of restrictions while discarding the others.\footnote{We repeat this process a large number of times until 500 draws are accepted.} We employ the model-based methodology outlined by Canova (2002) and applied in Dedola and Neri (2007), Pappa (2009) and Bermperoglu et al. (2013), among others, according to which the restrictions are extracted from a theoretical model. We can summarize the procedure in three main steps: i) build a nested DSGE model in which nominal and real frictions can be removed via appropriate parametrizations. We do this in Section \ref{sect:2.3}, where our two-sector model encompasses a continuum of models featuring different subsets of frictions; ii) define ranges for the structural parameters, generate thousands of random draws of the parameter values from their support and obtain IRFs for each draw;\footnote{See section \ref{sect:2.9} for a discussion of the choice of ranges and of the impulse responses.} iii) use the robust IRFs to impose sign restrictions on the IRFs of the SVAR model.

2.6.3 Narrative approach

The third approach we employ is based on the contribution of Romer and Romer (2004). RR develop a new measure of U.S. monetary policy shock that is somewhat immune to two problems embedded in monetary policy variables
such as the actual FFR. Indeed, RR argue that such measures suffer from *endogeneity* and *anticipatory movements*. In particular, the former implies that the FFR moves with changes in economic conditions hence not with changes in the conduct of monetary policy. The latter implies that movements in the FFR represent responses to information about future events in the economy. As a result, RR argue that such measures of monetary policy do not really represent exogenous shocks and they derive a new measure that enables the researcher to overcome these shortcomings. The derivation of the alternative monetary policy variable consists of two main steps. RR first derive a series of *intended FFR changes* around meetings of the Federal Open Market Committee (FOMC) of the Federal Reserve (Fed). They rely on a combination of narrative and quantitative evidence in order to retrieve the direction and the magnitude of such intended changes. This step eliminates the endogeneity between the interest rate and economic conditions thus solving the first of the two shortcomings outlined above. The second step consists of controlling for the Fed’s internal forecasts in order to disentangle the effects of information about future economic developments. RR then regress the change in the intended FFR on its level, on the level and the changes of forecasts about GDP growth and the GDP deflator, and forecasts about the unemployment rate. Then they take the residuals of this regression as the new measure of monetary policy shocks. Consequently, the resulting series gains a higher degree of exogeneity with respect to the FFR since it represents movements in the monetary policy measure not stemming from forecasts about inflation, GDP growth and unemployment.
2.7 Robustness checks for the SVAR model

2.7.1 SVAR Models with trend

Figure 2.7: Impulse responses: SVAR models with trend (bold lines = baseline model without trend; dashed lines = baseline model with trend; shaded areas and dotted lines = one-standard-deviation confidence bands)
2.7.2 Alternative definitions of durables

Figure 2.8: SVAR impulse responses. Sample: 1969Q2-2007Q4 (bold lines = baseline model; dashed lines = durables goods and new single family houses; shaded areas and dotted lines = one-standard-deviation confidence bands)

Figure 2.9: SVAR impulse responses. Sample: 1969Q2-2007Q4 (bold lines = baseline model; dashed lines = durables goods and broad measure of houses; shaded areas and dotted lines = one-standard-deviation confidence bands)
Figure 2.10: SVAR impulse responses. Sample: 1969Q2-2007Q4 (bold lines = baseline model; dashed lines = new single family houses; shaded areas and dotted lines represent one-standard-deviation confidence bands)

2.7.3 Subsample analysis

Figure 2.11: SVAR impulse responses. Sample: 1969Q2-1993Q1 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)
Figure 2.12: SVAR impulse responses. Sample: 1971Q4-1995Q3 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)

Figure 2.13: SVAR impulse responses. Sample: 1974Q2-1998Q1 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)
Figure 2.14: SVAR impulse responses. Sample: 1976Q4-2000Q3 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)

Figure 2.15: SVAR impulse responses. Sample: 1979Q2-2003Q1 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)
Figure 2.16: SVAR impulse responses. Sample: 1981Q4-2005Q3 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)

Figure 2.17: SVAR impulse responses. Sample: 1984Q2-2007Q4 (bold lines = all durable goods; dashed lines = only houses; shaded areas and dotted lines = one-standard-deviation confidence bands)
2.7.4 Sign restrictions

Figure 2.18: Sign restrictions imposed for 2, 4 and 6 quarters against 1 quarter, baseline model. Sample: 1969Q2-2007Q4 (bold lines = one quarter; dashed lines = more quarters; shaded areas and dotted lines = one-standard-deviation confidence bands)

Figure 2.19: Sign restrictions imposed for 2, 4 and 6 quarters against 1 quarter, model with broad measure of houses as durables. Sample: 1969Q2-2007Q4 (bold lines = one quarter; dashed lines = more quarters; shaded areas and dotted lines = one-standard-deviation confidence bands)
2.7.5 Proxy SVAR approach

Figure 2.20: Proxy SVAR impulse responses to a one standard deviation increase in the monetary policy measure. Sample: 1969Q2-2007Q4 (bold lines: baseline VAR; dashed lines: housing VAR; shaded areas and dotted lines represent one-standard-deviation confidence bands)

2.7.6 Three-sector SVAR model

Figure 2.21: SVAR impulse responses in a three-sector SVAR model. Sample: 1969Q2-2007Q4 (bold lines = mean response; shaded areas = one-standard-deviation confidence bands)
2.8 The DSGE models

2.8.1 Symmetric equilibrium

Patient households

\[
X_t = Z_t^{1-\alpha} D_t^\alpha \\
Z_t = C_t - \zeta S_{t-1} \\
S_t = \rho_c S_{t-1} + (1 - \rho_c) C_t \\
U(X_t, N_t) = \log(X_t) - \nu N_t^{1+\varphi} / (1 + \varphi) \\
U_{Z,t} = (1 - \alpha) Z_t \\
U_{D,t} = \alpha D_t \\
U_{N,t} = -\nu N_t^\varphi \\
\Lambda_{t,t+1} = \beta \frac{U_{Z,t+1} e_{t+1}^B}{e_t^B} \\
\]

\[
[1 - e_t^W \eta] + \frac{e_t^W \eta}{\mu_t} = \theta^W \left( \Pi_t^W - \Pi_t^C \right) \Pi_t^W + \\
+ E_t \left[ \Lambda_{t,t+1} \theta^W \left( \Pi_{t+1}^W - \Pi_{t+1}^C \right) \Pi_{t+1}^W \frac{w_{t+1} N_{t+1}}{w_t N_t} \right] \\
\mu_t = -\frac{U_{Z,t}}{U_{N,t}} w_t \\
Q_t \psi_t = \frac{U_{D,t}}{U_{Z,t}} + (1 - \delta) E_t [\Lambda_{t,t+1} Q_{t+1} \psi_{t+1}] \\
1 = E_t \left\{ \Lambda_{t,t+1} \psi_{t+1} \frac{Q_{t+1}}{Q_t} e_t^{l_{t+1}} \left[ S' \left( \frac{I_{t+1}^D}{I_{l_{t+1}}^D} \right) \left( \frac{I_{t+1}^D}{I_{l_{t-1}}^D} \right)^2 \right] \right\} + \\
+ \psi_t e_t \left[ 1 - S \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} \right) - S' \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} \right) \frac{I_{l_{t-1}}^D}{I_{l_{t-1}}^D} \right] \\
S \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} \right) = \frac{\phi}{2} \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} - 1 \right)^2 \\
S' \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} \right) = \phi \left( \frac{I_{t}^D}{I_{l_{t-1}}^D} - 1 \right) 
\]
1 = E_t \left[ \Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}^{C}} \right] \quad (2.60)

Impatient households

- \begin{align*}
X'_t &= Z''_t - D''_t \quad (2.61) \\
Z'_t &= C'_t - \zeta S'_{t-1} \quad (2.62) \\
S'_t &= \rho_c S'_{t-1} + (1 - \rho_c') C'_t \quad (2.63) \\
U(X'_t, N'_t) &= \log (X'_t) - \nu' \frac{N'_{t+1} + \varphi'}{1 + \varphi'} \quad (2.64)
\end{align*}

- \begin{align*}
U_{Z',t} &= \frac{1 - \alpha}{Z'_t} \quad (2.65) \\
U_{D',t} &= \frac{\alpha}{D'_t} \quad (2.66) \\
U_{N',t} &= -\nu' (N'_t)^{\varphi'} \quad (2.67) \\
X'_t &= \beta' E_t \left[ \frac{X'_{t+1}}{\Pi_{t+1}^{C}} R_t \right] + \lambda_t^{BC} R_t \quad (2.69)
\end{align*}

- \begin{align*}
Q_t \psi'_t &= \frac{U_{D',t}}{U_{Z',t}} + \beta' (1 - \delta) E_t \left[ \frac{X'_{t+1}}{X'_t} \psi_{t+1} Q_{t+1} \right] \quad (2.70)
\end{align*}

- \begin{align*}
1 &= \beta' E_t \left\{ \frac{X'_{t+1} Q_{t+1}}{\lambda_t} \psi_{t+1} e'_{t+1} \left[ S' \left( \frac{1}{I_{t+1}^{P'}} \right) \left( \frac{I_{t+1}^{P'}}{I_{t+1}^{P}} \right)^2 \right] \right\} + \\
&\quad + \psi_{t} e'_{t} \left[ 1 - S \left( \frac{1}{I_{t+1}^{P'}} \right) - S' \left( \frac{1}{I_{t+1}^{P'}} \right) \left( \frac{I_{t+1}^{P'}}{I_{t+1}^{P}} \right) \right] \quad (2.71)
\end{align*}

- \begin{align*}
S \left( \frac{1}{I_{t+1}^{P'}} \right) &= \frac{\phi'}{2} \left( \frac{I_{t+1}^{P'}}{I_{t+1}^{P}} - 1 \right) \quad (2.72) \\
S' \left( \frac{1}{I_{t+1}^{P'}} \right) &= \phi' \left( \frac{I_{t+1}^{P'}}{I_{t+1}^{P}} - 1 \right) \quad (2.73)
\end{align*}

- \begin{align*}
0 &= \left[ 1 - e'_t \eta \right] + \frac{e'_t \eta}{\mu'_t} - \vartheta W \left( \Pi_{t+1}^{W'} - \Pi_{t}^{W'} \right) \Pi_{t+1}^{W'} + \\
&\quad + \beta' E_t \left[ \frac{X'_{t+1}}{\lambda_t} \vartheta W \left( \Pi_{t+1}^{W'} - \Pi_{t}^{W'} \right) \Pi_{t+1}^{W'} w_{t+1} N_{t+1} \right] \quad (2.74)
\end{align*}

- \begin{align*}
\mu'_t &= -w'_t \frac{U_{Z',t}}{U_{N',t}} \quad (2.75)
\end{align*}
Firms

\[ Y_t^C = e_t^A (N_t^C)^{\tilde{\psi}} \left( N_t^C \right)^{1-\tilde{\psi}} \]  
(2.76)

\[ Y_t^D = e_t^A (N_t^D)^{\tilde{\psi}} \left( N_t^D \right)^{1-\tilde{\psi}} \]  
(2.77)

\[ e_t^C \epsilon_c MC_t^C = (e_t^C \epsilon_c - 1) + \theta_c (\Pi_t^C - 1) \Pi_t^C - \]  
(2.78)

\[- \theta_c E_t \left[ \Lambda_{t+1} \frac{Y_t^C}{Y_t^C} \left( \Pi_{t+1}^C - 1 \right) \Pi_{t+1}^C \right] \]

\[ w_t = MC_t^C \tilde{\psi} \frac{Y_t^C}{N_t^C} \]  
(2.79)

\[ w_t' = MC_t^C \left( 1 - \tilde{\psi} \right) \frac{Y_t^C}{N_t^C} \]  
(2.80)

\[ e_t^D \epsilon_d MC_t^D = (e_t^D \epsilon_d - 1) + \theta_d (\Pi_t^D - 1) \Pi_t^D - \]  
(2.81)

\[- \theta_d E_t \left[ \Lambda_{t+1} \frac{Q_{t+1}}{Q_t} \frac{Y_t^D}{Y_t^D} \left( \Pi_{t+1}^D - 1 \right) \Pi_{t+1}^D \right] \]

\[ w_t = MC_t^D \tilde{\psi} \frac{Q_t Y_t^D}{N_t^D} \]  
(2.82)

\[ w_t' = MC_t^D \left( 1 - \tilde{\psi} \right) \frac{Q_t Y_t^D}{N_t^D} \]  
(2.83)

\[ \tilde{\Pi}_t = (\Pi_t^C)^{1-\tau} (\Pi_t^D)^{\tau} \]  
(2.84)

Monetary policy and market clearing

\[ \log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \]  
(2.85)

\[ + (1 - \rho_r) \left[ \rho_t \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \right] + \epsilon_t^M \]

\[ Y_t = Y_t^C + Q_t Y_t^D + \frac{\theta_W}{2} \left( \Pi_t^W - 1 \right)^2 w_t N_t + \]  
(2.86)

\[ + \frac{\theta_W}{2} \left( \Pi_t^W - 1 \right)^2 w_t' N_t' \]

\[ Y_t^C = C_t + C_t' + G_t + \frac{\theta_c}{2} \left( \Pi_t^C - 1 \right)^2 Y_t^C \]  
(2.87)

\[ Y_t^D = I_t^D + I_t^{D'} + \frac{\theta_d}{2} \left( \Pi_t^D - 1 \right)^2 Y_t^D \]  
(2.88)
0 = B_t + B'_t \quad (2.89)
N_t = N_t^C + N_t^D \quad (2.90)
N'_t = N'_t^C + N'_t^D \quad (2.91)

### 2.8.2 Steady state

In the deterministic steady state all expectation operators are removed and for each variable it holds that \( x_t = x_{t+1} = x \). Moreover, the stochastic shocks are absent. The variables \( U_Z, U'_Z, Q, N_{D}' \) solve equations (2.50), (2.65), (2.82) and (2.91) respectively. In steady state \( N = N' = 0.33 \) and the parameters \( \nu \) and \( \nu' \) are endogenized to match these values. The remaining variables are found recursively as follows:

\[
\begin{align*}
\Lambda &= \beta \\
R &= \frac{1}{\beta} \\
\psi &= 1 \\
\mu &= \frac{\eta}{\eta - 1} \\
\psi' &= 1 \\
\mu' &= \frac{\eta}{\eta - 1} \\
MC^C &= \frac{\epsilon_c - 1}{\epsilon_c} \\
MC^D &= \frac{\epsilon_d - 1}{\epsilon_d} \\
U_D &= Q\psi U_Z \left[ 1 - \beta \left( 1 - \delta \right) \right] \\
D &= \frac{\alpha}{U_D} \\
\lambda' &= U_{Z'} \\
\lambda_{BC} &= \left( 1 - \beta' R \right) \lambda' \\
U_{D'} &= Q\psi' U_{Z'} \left[ 1 - \beta' \left( 1 - \delta \right) \right] - \lambda_{BC} mQ 
\end{align*}
\]
\[ D' = \frac{\alpha}{U_D'} \]  
(2.105)  
\[ B' = \frac{m Q D'}{R} \]  
(2.106)  
\[ Y^D = \delta (D + D') \]  
(2.107)  
\[ w' = MC^D \left( 1 - \frac{1}{\psi} \right) \frac{Q Y^D}{N'} \]  
(2.108)  
\[ N^D = \left[ \frac{Y^D}{N^D} \right]^{\frac{1}{\psi}} N'^D \]  
(2.109)  
\[ N^C = N + N^D \]  
(2.110)  
\[ U_N' = -w' \frac{U_N}{\mu'} \]  
(2.111)  
\[ \nu' = -\frac{U_N}{N' \psi'} \]  
(2.112)  
\[ C' = (1 - R) B' + w' N' - Q \delta D' \]  
(2.113)  
\[ S' = C' \]  
(2.114)  
\[ Z' = C' - \zeta S' \]  
(2.115)  
\[ X' = Z'^{1-\alpha} D'^{\alpha} \]  
(2.116)  
\[ N'^C = \left[ MC^C \left( 1 - \frac{1}{\psi} \right) \frac{N^C}{N'} \right]^{\frac{1}{\psi}} \]  
(2.117)  
\[ Y^C = \left[ N^C \right]^{\frac{1}{\psi}} \left[ N'^C \right]^{1-\frac{1}{\psi}} \]  
(2.118)  
\[ w = MC^C \psi \frac{Y^C}{N^C} \]  
(2.119)  
\[ U_N = -w \frac{U_N}{\mu} \]  
(2.120)  
\[ \nu = -\frac{U_N}{N' \psi} \]  
(2.121)  
\[ Y = Y^C + Q Y^D \]  
(2.122)  
\[ G = g_y Y \]  
(2.123)  
\[ C = Y^C - C' - G \]  
(2.124)  
\[ S = C \]  
(2.125)  
\[ Z = C - \zeta S \]  
(2.126)  
\[ X = Z'^{1-\alpha} D'^{\alpha} \]  
(2.127)
2.8.2.1 Alternative calibration of steady state hours

Throughout the paper, and following the relevant literature, we have defined
\[ U(X_t, N_t) = \log(X_t) - \nu \frac{N_t^{1+\nu}}{1+\varphi} \]
such that hours worked enter the utility function in a separable fashion. We then normalized steady state hours to 1/3 and found the value of the scale parameter \( \nu \) consistent with this normalization.

However, defining hours as \( N_t \) rather than the complement to one of leisure (i.e. \( 1 - L_t \)) implies that hours are dimensionless so targeting a steady state value of 1/3 is arbitrary and the parameter \( \nu \) has merely the effect of scaling the steady state values of the other variables.

Moreover, \( \nu \) does not have any effect in the log-linearized model hence the dynamic properties of the model are not affected by either targeting steady state hours to be 1/3 or by calibrating \( \nu = 1 \) and calculating the resulting steady state value of hours. Indeed, consider the labor supply of the patient households (the same applies to impatients):
\[ \nu N_t^\varphi = U_{z,t} w_t. \]  
(2.128)

Log-linearizing it around the steady-state yields:
\[ \dot{N}_t = \frac{1}{\varphi} \left( \dot{U}_{z,t} + \dot{w}_t \right), \]  
(2.129)
where variables with \( \dot{\cdot} \) are expressed in log-deviations from the steady-state. It is clear that the log-deviations of hours from steady state do not depend on the constant \( \nu \) whose value does have any effect on the dynamics of hours.

2.8.3 Symmetric equilibrium of the extended models

This section reports the changes in the symmetric equilibrium of Appendix 2.8.1 when the model is extended to allow for limited labor mobility, indexation and a three-sector economy as in Sections 2.3.7.1, 2.3.7.2 and 2.3.7.3 respectively.
2.8.3.1 Imperfect Sectoral Labor Mobility

Equations (2.90) and (2.91) are replaced by the CES aggregators of sectoral hours:

\[ N_t = \left( (\chi C)^{-\frac{1}{\lambda}} (N^C_t)^{\frac{1+\lambda}{\lambda}} + (1 - \chi C)^{-\frac{1}{\lambda}} (N^D_t)^{\frac{1+\lambda}{\lambda}} \right)^{\frac{1}{1+\lambda}} \]  
(2.130)

\[ N'_t = \left( (\chi'^C)^{-\frac{1}{\lambda'}} (N'^C_t)^{\frac{1+\lambda'}{\lambda'}} + (1 - \chi'^C)^{-\frac{1}{\lambda'}} (N'^D_t)^{\frac{1+\lambda'}{\lambda'}} \right)^{\frac{1}{1+\lambda'}} \]  
(2.131)

Given these labor aggregators, each households determines their labor supply:

\[ N^j_t = \chi^j \left( \frac{w^j_t}{w_t} \right)^{\lambda} N_t \]  
(2.132)

\[ N'^j_t = \chi'^j \left( \frac{w'^j_t}{w'_t} \right)^{\lambda'} N'_t \]  
(2.133)

with \( j = C, D \). Finally, sectoral wages are different hence equations (2.79), (2.80), (2.82) and (2.83) are replaced by:

\[ w^C_t = MC^C_t \hat{\psi} Y^C_t \frac{N^C_t}{N^C_t} \]  
(2.134)

\[ w'^C_t = MC^C_t \left( 1 - \hat{\psi} \right) Y^C_t \frac{N^C_t}{N^C_t} \]  
(2.135)

\[ w^D_t = MC^D_t \hat{\psi} Q_t Y^D_t \frac{N^D_t}{N^D_t} \]  
(2.136)

\[ w'^D_t = MC^D_t \left( 1 - \hat{\psi} \right) Q_t Y^D_t \frac{N^D_t}{N^D_t} \]  
(2.137)
2.8.3.2 Price Indexation

The sectoral price setting equations (2.78) and (2.81) are amended as follows:

\[
e_C^t e_c MC^C_t = (e_C^t e_c - 1) + \varphi_c \left( \frac{\Pi^C_t}{(\Pi^C_{t-1})^\gamma} - 1 \right) \frac{\Pi^C_t}{(\Pi^C_{t-1})^\gamma}
- \varphi_c E_t \left[ \Lambda_{t,t+1} \frac{Y^C_{t+1}}{Y^C_t} \left( \frac{\Pi^C_{t+1}}{(\Pi^C_t)^\gamma} - 1 \right) \Pi^C_{t+1} \right] \tag{2.138}
\]

\[
e_D^t e_d MC^D_t = (e_D^t e_d - 1) + \varphi_d \left( \frac{\Pi^D_t}{(\Pi^D_{t-1})^\gamma} - 1 \right) \frac{\Pi^D_t}{(\Pi^D_{t-1})^\gamma}
- \varphi_d E_t \left[ \Lambda_{t,t+1} \frac{Q^D_{t+1}}{Q_t} \frac{Y^D_{t+1}}{Y_t} \left( \frac{\Pi^D_{t+1}}{(\Pi^D_t)^\gamma} - 1 \right) \Pi^D_{t+1} \right] \tag{2.139}
\]

Then, since the price adjustment costs enter the sectoral market clearing conditions, equations (2.87) and (2.88) now read as:

\[
Y^C_t = C_t + C_t' + G_t + \frac{\varphi_c}{2} \left( \frac{\Pi^C_t}{(\Pi^C_{t-1})^\gamma} - \Pi^C_t \right)^2 Y^C_t \tag{2.140}
\]

\[
Y^D_t = I_t^P + I_t^D' + \frac{\varphi_d}{2} \left( \frac{\Pi^D_t}{(\Pi^D_{t-1})^\gamma} - \Pi^D_t \right)^2 Y^D_t \tag{2.141}
\]

2.8.3.3 Three-sector model

\[
1 = E_t \left[ \Lambda_{t,t+1} \frac{R_t}{\Pi^C_{t+1}} \right] \tag{2.142}
\]

\[
Q_t \psi_t = \frac{U_{D,t}}{U_{Z,t}} + (1 - \delta) E_t \left[ \Lambda_{t,t+1} Q_{t+1} \psi_{t+1} \right] \tag{2.143}
\]

\[
1 = \psi_t' e_t^l \left[ 1 - S' \left( \frac{I^D_{t-1}}{I^P_{t-1}} \right) - \Pi^D_t \right] \frac{I^D_{t-1}}{I^P_{t-1}}
+ \varphi_t \left\{ \Lambda_{t,t+1} \psi_t' \frac{Q_{t+1}}{Q_t} e_{t+1} \left[ S' \left( \frac{I^D_{t+1}}{I^P_{t+1}} \right) \left( \frac{I^D_{t+1}}{I^P_{t+1}} \right)^2 \right] \right\} \tag{2.144}
\]

\[
Q_t^H \psi_t^H = \frac{U_{H,t}}{U_{Z,t}} + (1 - \delta^H) E_t \left[ \Lambda_{t,t+1} Q_{t+1}^H \psi_{t+1}^H \right] \tag{2.145}
\]
\[ 1 = \psi_t^H e_t^H \left[ 1 - S \left( \frac{I_t^H}{I_{t-1}^H} \right) - S' \left( \frac{I_t^H}{I_{t-1}^H} \right) \right] + \]
\[ + E_t \left\{ \Lambda_{t,t+1} \psi_{t+1}^H Q_{t+1}^H e_{t+1}^H \left[ S' \left( \frac{I_{t+1}^H}{I_t^H} \right) \left( \frac{I_{t+1}^H}{I_t^H} \right)^2 \right] \right\} \] (2.146)

\[ 0 = \left[ 1 - e_t^W \eta \right] + e_t^W \eta \frac{\eta}{\mu_t} - \vartheta^W \left( \Pi_t^W - \Pi_{t+t}^W \right) \Pi_{t+t}^W + \]
\[ + E_t \left\{ \Lambda_{t,t+1} \vartheta^W \left( \Pi_{t+t}^W - \Pi_{t+t}^C \right) \Pi_{t+t}^W \frac{w_{t+1} N_{t+1}}{w_t N_t} \right\} \] (2.147)

\[ D_{t+1} = (1 - \delta) D_t + e_t^I I_t^D \left[ 1 - S \left( \frac{I_t^D}{I_{t-1}^D} \right) \right] \] (2.148)

\[ H_{t+1} = (1 - \delta^H) H_t + e_t^H I_t^H \left[ 1 - S \left( \frac{I_t^H}{I_{t-1}^H} \right) \right] \] (2.149)

\[ X_t = \left[ (1 - \alpha) \tilde{C}_t \frac{\tilde{e}_t}{\tilde{t}} + \alpha H_t \frac{\tilde{e}_t}{\til{t}} \right] \] (2.150)

\[ Z_t = C_t - \varsigma S_{t-1} \] (2.151)

\[ S_t = \rho_c S_{t-1} + (1 - \rho_c) C_t \] (2.152)

\[ \tilde{C}_t = \left[ (1 - \tilde{\alpha}) Z_t \frac{\tilde{e}_t}{\til{t}} + \tilde{\alpha} D_t \frac{\tilde{e}_t}{\til{t}} \right] \] (2.153)

\[ N_t = N_t^C + N_t^D + N_t^H \] (2.154)

\[ \Lambda_{t,t+1} = \beta \frac{U_{Z,t+1} e_{t+1}^B}{U_{Z,t} e_t^B} \] (2.155)
\[\mu_t = -w_t \frac{U_{Z,t}}{U_{N,t}}\]  

(2.156)

\[U_{Z,t} = \frac{(1 - \alpha)(1 - \tilde{\alpha})C_t^\frac{1}{2}}{C_t^\frac{1}{2}X_t^\frac{1}{2}Z_t^\frac{1}{2}}\]  

(2.157)

\[U_{D,t} = \frac{(1 - \alpha)\tilde{\alpha}C_t^\frac{1}{2}}{C_t^\frac{1}{2}X_t^\frac{1}{2}D_t^\frac{1}{2}}\]  

(2.158)

\[U_{H,t} = \frac{1}{H_t^\frac{1}{2}X_t^\frac{1}{2}}\]  

(2.159)

\[U_{N,t} = -\nu N_t^\varphi\]  

(2.160)

\[\lambda_t' = e_t^B U_{Z',t}\]  

(2.161)

\[\lambda_t' = \beta'E_t \left[ \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} \right] + \lambda_{t+1}^b C_t \]  

(2.162)

\[Q_t^H \psi_t' = \frac{U_{D',t}}{U_{Z',t}} + \beta'(1 - \delta) E_t \left[ \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} (1 - \delta^H) \right] + \lambda_{t+1}^b C_t \]  

(2.163)

\[1 = \psi_t'^I \left[ 1 - S \left( \frac{I_{t-1}^D}{I_{t-1}^H} \right) - S' \left( \frac{I_{t-1}^D}{I_{t-1}^H} \right) \right] + \beta'E_t \left[ \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} \right] \]  

(2.164)

\[Q_t^H \psi_t'^H = \frac{U_{H',t}}{U_{Z',t}} + E_t \left[ \beta' \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} (1 - \delta^H) + \lambda_{t+1}^b C_t \right] \]  

(2.165)

\[1 = \psi_t'^H e_t'^H \left[ 1 - S \left( \frac{I_{t-1}^H}{I_{t-1}^H} \right) - S' \left( \frac{I_{t-1}^H}{I_{t-1}^H} \right) \right] + \beta'E_t \left[ \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} \right] \]  

(2.166)

\[0 = \left[ 1 - \epsilon_t^W \eta + \frac{\epsilon_t^W \eta}{\mu_t'} - \varrho^W (\Pi_{t+1}^{W'} - \Pi^C) \Pi_t^{W'} + \beta' \right] E_t \left[ \frac{\lambda_{t+1}I_{t+1}'}{\lambda_t' Q_{t+1}^H} \right] \]  

(2.167)

\[B_t' = mE_t \left( \frac{Q_{t+1}^H (\Pi_{t+1}^{W'} - \Pi^C)}{R_t} \right)\]  

(2.168)
\[ D_{t+1}' = (1 - \delta)D_t' + e_t'I_t^{D'} \left[ 1 - S\left( \frac{I_t^{D'}}{I_{t-1}} \right) \right] \]  

(2.169)

\[ H_{t+1}' = (1 - \delta^H)H_t' + e_t^I'H_t^{H'} \left[ 1 - S\left( \frac{I_t^{H'}}{I_{t-1}} \right) \right] \]  

(2.170)

\[ X_t' = \left[ (1 - \alpha) \tilde{C}_t^{\frac{\rho - 1}{\rho}} + \alpha H_t^{\frac{\rho - 1}{\rho}} \right] \]  

(2.171)

\[ Z_t' = C_t' - \zeta S_{t-1} \]  

(2.172)

\[ S_t' = \rho_c S_{t-1} + (1 - \rho_c)C_t' \]  

(2.173)

\[ \tilde{C}_t' = \left[ (1 - \tilde{\alpha}) Z_t^{\frac{\rho - 1}{\rho}} + \tilde{\alpha} D_t^{\frac{\rho - 1}{\rho}} \right] \]  

(2.174)

\[ N_t' = N_t^{IC} + N_t^{ID} + N_t^{IH} \]  

(2.175)

\[ B_t' = C_t' + Q_t'I_t^{D'} + Q_t^H I_t^{H'} + R_{t-1}B_{t-1} + \]  

(2.176)

\[ + \frac{\delta^W}{2} \left( \frac{w_{t-1}'}{w_{t-1}'} \Pi_C^C - \Pi_C^C \right)^2 w_t'N_t' - w_t'N_t' \]  

(2.177)

\[ \mu_t' = -w_t'U_{Z't'} \]  

(2.178)

\[ U_{Z't'} = \frac{(1 - \alpha)(1 - \tilde{\alpha}) \tilde{C}_t^3}{\tilde{C}_t^{\frac{\rho - 1}{\rho}} X_t^{\frac{\rho - 1}{\rho}} Z_t^{\frac{\rho - 1}{\rho}}} \]  

(2.179)

\[ U_{D't'} = \frac{(1 - \alpha)\tilde{\alpha} C_t^{\frac{\rho - 1}{\rho}}}{\tilde{C}_t^{\frac{\rho - 1}{\rho}} X_t^{\frac{\rho - 1}{\rho}} D_t^{\frac{\rho - 1}{\rho}}} \]  

(2.180)

\[ U_{H't'} = \frac{\alpha}{H_t^{\frac{\rho - 1}{\rho}} X_t^{\frac{\rho - 1}{\rho}}} \]  

(2.181)

\[ U_{N't'} = -\nu'(N_t')^\phi \]  

(2.182)

\[ Y_t^C = A_t \left[ N_t^C \right]^\psi \left[ N_t^C \right]^{1-\psi} \]  

(2.183)

\[ Y_t^D = A_t \left[ N_t^D \right]^\psi \left[ N_t^D \right]^{1-\psi} \]  

(2.184)

\[ Y_t^H = A_t \left[ N_t^H \right]^\psi \left[ N_t^H \right]^{1-\psi} \]  

(2.185)

\[ w_t = MC_t^{\psi \frac{Y_t^C}{N_t^C}} \]  

(2.186)

\[ w_t' = MC_t^{\psi (1 - \psi) \frac{Y_t^C}{N_t^C}} \]  

(2.187)
\[ e_t^C e_c MC_t^C = (e_t^C e_c - 1) + \vartheta_c (\Pi_t^C - 1) \Pi_t^C - \]

\[ - \vartheta_c E_t \left[ \Lambda_{t,t+1} \frac{Y_{t+1}^C}{Y_t} (\Pi_{t+1}^C - 1) \Pi_{t+1}^C \right] \quad (2.188) \]

\[ w_t = MC_t^D \tilde{Q}_t Y_t^D \]

\[ \tilde{Q}_t = \frac{N_t D_t}{N_t} \]

\[ w_t' = MC_t^D \left( 1 - \tilde{\psi} \right) \frac{Q_t Y_t^D}{N_t^{D_t}} \]

\[ e_t^D e_d MC_t^D = (e_t^D e_d - 1) + \vartheta_d (\Pi_t^D - 1) \Pi_t^D - \]

\[ - \vartheta_d E_t \left[ \Lambda_{t,t+1} \frac{Q_{t+1} Y_{t+1}^D}{Q_t Y_t^D} (\Pi_{t+1}^D - 1) \Pi_{t+1}^D \right] \quad (2.191) \]

\[ w_t = MC_t^H \tilde{Q}_t^H Y_t^H \]

\[ \tilde{Q}_t = \frac{N_t H_t}{N_t^H} \]

\[ w_t' = MC_t^H \left( 1 - \tilde{\psi} \right) \frac{Q_t Y_t^H}{N_t^{H_t}} \]

\[ e_t^H e_h MC_t^H = (e_t^H e_h - 1) + \vartheta_h (\Pi_t^H - 1) \Pi_t^H - \]

\[ - \vartheta_h E_t \left[ \Lambda_{t,t+1} \frac{Q_{t+1} Y_{t+1}^H}{Q_t Y_t^H} (\Pi_{t+1}^H - 1) \Pi_{t+1}^H \right] \quad (2.194) \]

\[ \log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \]

\[ + \left( 1 - \rho_r \right) \left[ \rho_c \log \left( \frac{\tilde{\Pi}_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{\bar{Y}} \right) \right] + \epsilon_t^M \quad (2.195) \]

\[ \tilde{\Pi}_t = \Pi_t^{(1-\tau)} (\Pi_t^D)^{\tau(1-\hat{T})} (\Pi_t^H)^{\hat{T}} \quad (2.196) \]

\[ Y_t = Y_t^C + Q_t Y_t^D + Q_t^{H} Y_t^{H} + \frac{\partial W}{2} (\Pi_t^W - \Pi_t^C)^2 w_t N_t + \]

\[ + \frac{\partial W}{2} \left( \Pi_t^{W'} - \Pi_t^C \right)^2 w_t' N_t' \quad (2.197) \]

\[ Y_t^C = C_t + C_t' + G_t + \frac{\partial c}{2} \left( \frac{\Pi_t^C}{\Pi_{t-1}^C} - 1 \right)^2 Y_t^C \quad (2.198) \]

\[ Y_t^D = [D_t - (1 - \delta) D_{t-1}] + [D_t' - (1 - \delta) D_{t-1}'] + \]

\[ + \frac{\partial d}{2} (\Pi_t^D - 1)^2 Y_t^D \quad (2.199) \]

\[ Y_t^H = [H_t - (1 - \delta^H) H_{t-1}] + [H_t' - (1 - \delta^H) H_{t-1}'] + \]

\[ + \frac{\partial h}{2} (\Pi_t^H - 1)^2 Y_t^H \quad (2.200) \]
2.9 Robust impulse responses

This section describes the methodology employed to impose the sign restrictions in Section 2.2.1. Let \( \theta \) be a \( N \times 1 \) vector of the structural parameters of the model. We assume that each parameter is uniformly distributed over a particular range \( \Theta_i \), that is each parameter \( i \) in \( \theta \) is defined over \( \Theta = \prod_i \Theta_i \). Each interval is set around a value consistent with a quarterly calibration of the U.S. economy and its length is determined both to include reasonable values and to avoid indeterminacy. As a result, some ranges are narrower whereas others are broader, but overall our choices should be uncontroversial. Table 2.9 summarizes the supports of the structural parameters. Consistently with the calibration of the two-sector NK models so far used in the literature, we define the same range for the parameters of price stickiness but we impose the restriction \( \vartheta_c \geq \vartheta_d \) so that prices of nondurables are stickier or at least as sticky as prices of durables. Note that this condition does not prevent us from obtaining a fully-flexible price model whenever a random draw implies that \( \vartheta_c = \vartheta_d = 0 \). We perform our main simulations by randomly drawing the values of the Rotemberg parameter of wage stickiness from the support \([0, 180]\) hence including cases in which wages are completely flexible. However, in order to highlight the crucial role played by wage stickiness in solving the comovement puzzle, we perform another set of simulations with flexible wages while keeping the same ranges for the remaining parameters. \footnote{We calibrate \( \vartheta^W = 2 \). Calibrating \( \vartheta^W < 2 \) leads to severe indeterminacy issues. However, a value of 2 implies almost fully flexible wages. Indeed, in Section 2.3.6 we estimate \( \vartheta^W = 153 \) in the baseline model and \( \vartheta^W = 168 \) in the housing DSGE. These values are extremely larger than 2 that is why we are confident that such value represents a good approximation of the case of fully flexible wages.}

Then we randomly draw the parameter values \( \theta^m_i \), \( i = 1, ..., N; m = 1, ..., 10034 \) from each \( \Theta_i \), where \( m \) is the number of random draws. Two issues are likely to arise when parameter values are randomly drawn from their support. The first is indeterminacy whenever the Blanchard-Kahn conditions are not satisfied. The second consists of violating the condition that we impose on the degree of price stickiness in the two sectors. In order to make our analysis robust, our aim is to generate about 10000 sets of impulse response functions.
## Table 2.9: Parameter ranges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient households’ discount factor $\beta$</td>
<td>[0.985, 0.995]</td>
</tr>
<tr>
<td>Impatient households’ discount factor $\beta'$</td>
<td>[0.96, 0.984]</td>
</tr>
<tr>
<td>Durables depreciation rate $\delta$</td>
<td>[0.0025, 0.025]</td>
</tr>
<tr>
<td>Durables share of total expenditure $\alpha$</td>
<td>[0.05, 0.35]</td>
</tr>
<tr>
<td>Elasticity of substitution in nondurables $\epsilon_c$</td>
<td>[4, 11]</td>
</tr>
<tr>
<td>Elasticity of substitution in durables $\epsilon_d$</td>
<td>[4, 11]</td>
</tr>
<tr>
<td>Elasticity of substitution in labor $\eta$</td>
<td>[4, 25]</td>
</tr>
<tr>
<td>Inverse Frisch elasticities $\varphi, \varphi'$</td>
<td>[0, 3, 3]</td>
</tr>
<tr>
<td>Disutilities of labor $\nu, \nu'$</td>
<td>$N, N' \in [0.2, 0.5]$</td>
</tr>
<tr>
<td>Habits degree parameters $\zeta, \zeta'$</td>
<td>[0, 0.9]</td>
</tr>
<tr>
<td>Habits persistence parameters $\rho_c, \rho'_c$</td>
<td>[0, 0.9]</td>
</tr>
<tr>
<td>Price stickiness in nondurables $\vartheta_c$</td>
<td>[0.58]$^*$</td>
</tr>
<tr>
<td>Price stickiness in durables $\vartheta_d$</td>
<td>[0.58]$^*$</td>
</tr>
<tr>
<td>Nominal wage rigidities $\phi_W$</td>
<td>[0, 180]</td>
</tr>
<tr>
<td>Investment adjustment cost parameters $\phi, \phi'$</td>
<td>[0, 5]</td>
</tr>
<tr>
<td>Loan-to-value ratio $m$</td>
<td>[0.55, 0.95]</td>
</tr>
<tr>
<td>Share of patient households $\tilde{\psi}$</td>
<td>[0.60, 0.90]</td>
</tr>
<tr>
<td>Share of durables inflation in inflation aggregator $\tau$</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Steady state government share of output $g_y$</td>
<td>[0.1, 0.3]</td>
</tr>
<tr>
<td>Monetary policy to inflation $\rho_{\pi}$</td>
<td>[1.05, 5]</td>
</tr>
<tr>
<td>Monetary policy to output gap $\rho_y$</td>
<td>[0, 0.5]</td>
</tr>
<tr>
<td>Interest rate smoothing $\rho_R$</td>
<td>[0, 0.9]</td>
</tr>
<tr>
<td>Persistence of monetary policy shock $\rho_{e,\pi}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of business cycle shock $\rho_{e,c}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of preference shock $\rho_{e,\beta}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of durables investment shock $\rho_{e,I}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of wage markup shock $\rho_{e,W}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of nondurables price markup shock $\rho_{e,C}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of durables price markup shock $\rho_{e,D}$</td>
<td>[0, 0.95]</td>
</tr>
<tr>
<td>Persistence of government consumption shock $\rho_{e,G}$</td>
<td>[0, 0.95]</td>
</tr>
</tbody>
</table>

Note: $^*$ denotes that parameters are subject to the restriction $\vartheta_c \geq \vartheta_d$.

That is why we performed 22000 draws, of which 10034 were accepted. 92% of the discarded draws did not satisfy the restriction on price stickiness and only 7% of them did not satisfy the Blanchard-Kahn conditions. Finally, for each accepted draw, we construct a $K \times 1$ vector of impulse response functions.
Figure 2.22 plots the 68% probability bands of impulse responses to a 1% increase in the nominal interest rate for two sets of simulations. The first leaves the wage stickiness parameter unrestricted (blue dashed lines) whereas, in the second, wages are fully flexible (red dotted lines). Regarding the first set of simulations, on impact, output, nondurable and durable consumption,
and inflation exhibit robust negative responses. In fact, our model features frictions such as wage and price rigidities that solve the comovement puzzle for different combinations of parameter values. In order to be consistent with the literature, we impose that price stickiness of durables can either be lower or equal to price rigidity in the nondurables sector but never higher. Consequently, the response of the relative price of durables is by construction bounded below zero. The response of the nominal interest rate deserves more attention as it is not robust and in some cases at odds with the monetary policy shock being restrictive. However, this is a common issue of two-sector NK models as reported by BHK and Sterk (2010). According to BHK, the counter-intuitive response of the nominal interest rate follows from the near constancy of the shadow value of durables which makes their real rate of return constant thus forcing the nominal interest rate to track expected inflation in the durable goods sector.

We next proceed to discuss the results of the simulations of the model with fully-flexible wages (red dotted lines of Figure 2.22). As expected, nominal wage rigidities play a crucial role in solving the comovement puzzle (see Carlstrom and Fuerst 2006, 2010). Indeed, when wages are kept flexible, there exist combinations of parameter values such that consumption of durables increases in response to a monetary policy tightening. Furthermore, also in this second set of simulations there are cases in which the comovement between durables and nondurables is attained due to specific values of the parameters of price stickiness (see Sterk, 2010). However, the aim of this second set of simulations is to show that when wages are assumed to be flexible there exist fewer combinations of parameter values that generate a comovement between consumption in the two sectors.

2.10 Bayesian impulse responses

In this section we plot the Bayesian impulse responses of the models estimated in Section 2.3.6 together with the 68%, 90% and 95% confidence bands. Figure 2.23 refers to the baseline DSGE whereas Figure 2.24 refers to the housing DSGE.
Figure 2.23: Bayesian impulse responses of relative prices to a contractionary monetary policy shock in the baseline DSGE (bold lines are mean responses, dark-shaded areas are 68% confidence bands, medium and lighter shaded areas represent 90% and 95% confidence bands respectively)

Figure 2.24: Bayesian impulse responses of relative prices to a contractionary monetary policy shock in the housing DSGE (bold lines are mean responses, dark-shaded areas are 68% confidence bands, medium and lighter shaded areas represent 90% and 95% confidence bands respectively)
The same conclusions as in Section 2.3.6.3 can be drawn also when taking into account the different confidence levels. Indeed, in both models the comovement is attained due to the presence of prices and wages stickiness whereas the only noticeable difference concerns the response of the relative prices, as discussed in the main text.

## 2.11 Posterior distributions of Inverse Frisch Elasticities

![Graph](image.png)

Figure 2.25: Prior and posterior densities of Inverse Frisch Elasticities. Left box: baseline DSGE. Right box: housing DSGE (left-scale refers to distribution of housing parameter, right-scale refers to nondurables).

## 2.12 Models comparison

We take two approaches to assess how well our (unrestricted) model’s features help fitting the data. First, we perform a likelihood race between the baseline and five restricted models, in which the DSGE model is estimated with one friction removed at a time. Then, we plot the impulse responses of the

---

43We perform such estimations only for the baseline DSGE model.
Table 2.10: Likelihood comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>Restrictions</th>
<th>Log-marg. likelihood</th>
<th>Kass-Raftery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td></td>
<td>−1472.494</td>
<td></td>
</tr>
<tr>
<td>Flexible Wages</td>
<td>$\vartheta^W = 0$</td>
<td>−1672.300</td>
<td>399.612</td>
</tr>
<tr>
<td>Flexible Durables Prices</td>
<td>$\vartheta_d = 0$</td>
<td>−1538.150</td>
<td>131.312</td>
</tr>
<tr>
<td>No IAC</td>
<td>$\phi = \phi' = 0$</td>
<td>−1970.003</td>
<td>995.018</td>
</tr>
<tr>
<td>No Habit</td>
<td>$\zeta = \zeta' = 0$</td>
<td>−1698.053</td>
<td>451.118</td>
</tr>
<tr>
<td>No Durables Inflation</td>
<td>$\tau = 0$</td>
<td>−1473.396</td>
<td>1.804</td>
</tr>
</tbody>
</table>

baseline and a few restricted models to a contractionary monetary policy shock.

Table 2.10 reports the log-marginal likelihoods of the models, in conjunction with the statistic by [Kass and Raftery (1995) KR henceforth]. The KR statistic decisively favors the baseline model. Indeed, there is slight evidence in favor of this with respect to the model in which the central bank responds only to inflation in nondurables ($\tau = 0$). Furthermore, very strong evidence is found against a model with flexible prices in the durables sector ($\vartheta_d = 0$), a model with flexible wages ($\vartheta^W = 0$), a model without IAC in durable goods ($\phi = \phi' = 0$), and a model without habit formation in consumption of nondurable goods ($\zeta = \zeta' = 0$).

These results suggest that the frictions considered are important when the theoretical model is brought to the data, although the main result about the sectoral price stickiness survives in the restricted models, except for the case of flexible wages. Indeed, Table 2.11 shows that the point estimate of the price stickiness parameter in the durables sector is higher than the price stickiness parameter in the nondurables sector, although by a small margin.

---

$BF_{i/j} = \frac{L(Y|m_i)}{L(Y|m_j)} = \frac{\exp(LL(Y|m_i))}{\exp(LL(Y|m_j))}$

where $L(Y|m_i)$ is the marginal data density of model $i$ for the common dataset $Y$ and $LL$ stands for log-marginal likelihood. Values of the KR statistics above 10 can be considered “very strong” evidence in favor of model $i$ relative to model $j$; between 6 and 10 represent “strong” evidence; between 2 and 6 “positive” evidence; while values below 2 are “not worth more than a bare mention”.

---

44The KR statistic is computed as twice the log of the Bayes Factor (BF), with the BF between the baseline models $m_i$ and the restricted model $m_j$, being
<table>
<thead>
<tr>
<th>Model</th>
<th>Restrictions</th>
<th>Price stickiness nondurables $\vartheta_c$</th>
<th>Price stickiness durables $\vartheta_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td></td>
<td>23.38 [15.82;30.61]</td>
<td>24.45 [16.09;33.26]</td>
</tr>
<tr>
<td>Flexible Wages</td>
<td>$\vartheta^W = 0$</td>
<td>1.2032 [0.5643;1.7338]</td>
<td>2.4006 [1.4801;3.3098]</td>
</tr>
<tr>
<td>No IAC</td>
<td>$\phi = \phi' = 0$</td>
<td>47.135 [32.832;62.022]</td>
<td>51.378 [37.533;65.994]</td>
</tr>
<tr>
<td>No Habit</td>
<td>$\zeta = \zeta' = 0$</td>
<td>27.629 [19.122;35.731]</td>
<td>30.482 [19.540;41.270]</td>
</tr>
<tr>
<td>No Durables Inflation</td>
<td>$\tau = 0$</td>
<td>22.338 [15.209;28.933]</td>
<td>25.961 [16.311;35.075]</td>
</tr>
</tbody>
</table>

Table 2.11: Estimated price stickiness parameters

In all cases, the confidence intervals of the two parameters widely overlap thus pointing to the fact that there is only a negligible difference between the two.

The importance of the real and nominal frictions is further depicted in Figure 2.26. The black-solid line represents the same impulse responses of the baseline model as in Figure 2.4, while the blue-dashed line depicts the dynamic behavior of a model with flexible wages. Thanks to price stickiness in durable goods, the responses are close to the baseline model and the comovement between durables and nondurables is attained. When prices of durables are assumed to be flexible and wages are sticky (red-dotted line), the comovement still survives. The only tangible difference lies in the response of the relative price, which is almost flat in the baseline case, whereas it decreases in the restricted scenario.

Excluding habit formation in consumption of nondurable goods (red-dashed and dotted line) leads to a considerable larger fall in nondurables and output. In particular, we confirm the results of Katayama and Kim (2013) that including this friction is crucial to obtain reasonable sizes in the responses of nondurables consumption and output. Similarly, IACs in durable...

\[45\] We calibrate the parameters with the point estimates of the baseline model and remove a friction at a time. Impulse responses are rescaled to generate a 1% increase in the policy rate. In order to ease the graphical analysis, we do not plot the responses of the model in which the central bank responds only to inflation in nondurables since they overlap with the others.
goods are crucial to account for plausible magnitudes of the responses of durables and output. Indeed, the black-rounded lines show that in the absence of IACs, at the trough, durables fall by almost 7% whereas output falls by about 0.4%. Thus the maximum fall in durables is about 17.5 times larger than the maximum fall of output, an implausible result according to our SVAR estimates.
2.12.1 The importance of the income share of patient households

All the results reported above assume an income share of the patient households of 79%, as estimated by Iacoviello and Neri (2010). This is also in line with estimates by Jappelli (1990), who reports a share of 80% for savers in the U.S. economy. In this section, we use a calibrated version of the baseline DSGE using the posterior mean of all parameters reported in Section 2.3.6 and alternative values for the income share of patient households to assess the importance of this parameter for the dynamic responses of macroeconomic variables to a monetary policy shock. Figure 2.27 shows the impulse responses to an increase in the policy rate for different values of $\tilde{\psi}$. Qualitatively, the dynamic responses of the baseline DSGE are not affected by changes in the income share of the two households. However, a quantitative inspection yields interesting insights. Increasing the share of impatient
households (blue-dashed and red-dotted lines) exacerbates the negative effects of the monetary policy shock. The simple reason is that a higher share of households are credit constrained hence on aggregate, durables investment and nondurables consumption fall more. Here, it is also evident that the transmission channel of monetary policy through the collateral constraint is in fact important and should not be neglected. Conversely, lowering the share of impatient households mitigates the effects of a monetary policy shock.
## 2.13 Posterior estimates of extended models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Sd/df</th>
<th>Posterior Mean Baseline DSGE</th>
<th>Posterior Mean Housing DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inv. Frisch elasticity patients</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.5262 [0.3807;0.6765]</td>
<td>0.4996 [0.4641;0.9352]</td>
</tr>
<tr>
<td>Inv. Frisch elasticity patients</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.6599 [0.5044;0.8172]</td>
<td>0.7803 [0.5300;0.8431]</td>
</tr>
<tr>
<td>Habits</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.6761 [0.6281;0.7238]</td>
<td>0.7022 [0.6532;0.7557]</td>
</tr>
<tr>
<td>Habits. imperfects</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.9346 [0.9259;0.9443]</td>
<td>0.9487 [0.9445;0.9526]</td>
</tr>
<tr>
<td>Habit persist. patients</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.5226 [0.4143;0.6299]</td>
<td>0.5399 [0.4471;0.6339]</td>
</tr>
<tr>
<td>Habit persist. imperfects</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.2135 [0.1500;0.2746]</td>
<td>0.2062 [0.1257;0.2695]</td>
</tr>
<tr>
<td>Labor mobility patients</td>
<td>N</td>
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<td>0.50</td>
<td>2.6036 [1.9903;3.2098]</td>
<td>0.8628 [0.6649;1.0884]</td>
</tr>
<tr>
<td>Labor mobility imperfects</td>
<td>N</td>
<td>1.03</td>
<td>0.50</td>
<td>1.6042 [0.9274;2.2777]</td>
<td>0.7500 [0.1084;1.3567]</td>
</tr>
<tr>
<td>Price stickiness nondurables</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>25.72 [18.07;33.85]</td>
<td>51.08 [45.73;56.06]</td>
</tr>
<tr>
<td>Price stickiness durables</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>27.02 [17.95;35.59]</td>
<td>0.72 [0.56;0.85]</td>
</tr>
<tr>
<td>Wage stickiness</td>
<td>G</td>
<td>100.0</td>
<td>10.00</td>
<td>159.09 [145.64;176.10]</td>
<td>2.5507 [1.9744;3.1002]</td>
</tr>
<tr>
<td>IAC durables patients</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>3.0043 [2.3778;3.6516]</td>
<td>2.5507 [1.9744;3.1002]</td>
</tr>
<tr>
<td>IAC durables imperfects</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>1.6987 [0.9303;2.4394]</td>
<td>1.4886 [0.7305;2.2391]</td>
</tr>
<tr>
<td>Share of durables inflation</td>
<td>B</td>
<td>0.20</td>
<td>0.10</td>
<td>0.2018 [0.1077;0.2887]</td>
<td>0.0534 [0.0359;0.0700]</td>
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<tr>
<td>Inflation -Taylor rule</td>
<td>N</td>
<td>1.50</td>
<td>0.20</td>
<td>1.4099 [1.2523;1.5715]</td>
<td>1.7766 [1.5348;2.0045]</td>
</tr>
<tr>
<td>Output -Taylor rule</td>
<td>N</td>
<td>0.10</td>
<td>0.05</td>
<td>0.0187 [0.0063;0.0308]</td>
<td>0.0230 [0.0061;0.0393]</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>B</td>
<td>0.80</td>
<td>0.10</td>
<td>0.7052 [0.6656;0.7488]</td>
<td>0.7978 [0.7653;0.8323]</td>
</tr>
<tr>
<td><strong>Averages</strong></td>
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</tr>
<tr>
<td>Trend growth rate</td>
<td>N</td>
<td>0.49</td>
<td>0.10</td>
<td>0.3957 [0.3606;0.4315]</td>
<td>0.4127 [0.3881;0.4403]</td>
</tr>
<tr>
<td>Inflation rate nondurables</td>
<td>G</td>
<td>1.05</td>
<td>0.10</td>
<td>0.9872 [0.8885;1.0863]</td>
<td>0.9870 [0.9010;1.0775]</td>
</tr>
<tr>
<td>Inflation rate durables</td>
<td>G</td>
<td>0.37</td>
<td>0.10</td>
<td>0.4598 [0.3549;0.5616]</td>
<td>0.5627 [0.3860;0.7343]</td>
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<tr>
<td>Interest rate</td>
<td>G</td>
<td>1.65</td>
<td>0.10</td>
<td>1.5989 [1.4788;1.7287]</td>
<td>1.6211 [1.5215;1.7221]</td>
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<tr>
<td><strong>Exogenous processes</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Technology</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9792 [0.9603;0.9973]</td>
<td>0.9255 [0.8722;0.9765]</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.7053 [0.6304;0.7743]</td>
<td>0.8059 [0.7240;0.8878]</td>
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<tr>
<td>Investment Durables</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>3.0043 [2.3778;3.6516]</td>
<td>2.5507 [1.9744;3.1002]</td>
</tr>
<tr>
<td>Investment Durables</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>1.6987 [0.9303;2.4394]</td>
<td>1.4886 [0.7305;2.2391]</td>
</tr>
<tr>
<td>Preference</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.7427 [0.6703;0.8182]</td>
<td>0.8251 [0.7745;0.8704]</td>
</tr>
<tr>
<td>Price mark-up nondurables</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9549 [0.9160;0.9930]</td>
<td>0.9324 [0.9874;0.9994]</td>
</tr>
<tr>
<td>Price mark-up durables</td>
<td>N</td>
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<td>2.0</td>
<td>2.6500 [1.7960;2.7354]</td>
<td>3.3818 [2.8323;2.9237]</td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>N</td>
<td>0.50</td>
<td>0.20</td>
<td>0.4908 [0.4293;0.5564]</td>
<td>0.8878 [0.0015;0.1547]</td>
</tr>
<tr>
<td>Government spending</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9657 [0.9445;0.9882]</td>
<td>0.9544 [0.9265;0.9827]</td>
</tr>
</tbody>
</table>

Table 2.12: Prior and posterior distributions of estimated parameters: models with imperfect labor mobility (90% confidence bands in square brackets)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distr.</th>
<th>Prior Mean</th>
<th>Sd/df</th>
<th>Posterior Mean Baseline DSGE</th>
<th>Posterior Mean Housing DSGE</th>
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<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inv. Frisch elasticity patients $\varphi$</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.5442 [0.3963;0.6916]</td>
<td>0.6321 [0.5278;0.8445]</td>
</tr>
<tr>
<td>Inv. Frisch elasticity impatient patients $\varphi'$</td>
<td>N</td>
<td>0.50</td>
<td>0.10</td>
<td>0.6436 [0.4867;0.7980]</td>
<td>0.6853 [0.5300;0.8431]</td>
</tr>
<tr>
<td>Habits patients $\zeta$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.6532 [0.6281;0.7238]</td>
<td>0.6701 [0.6298;0.7050]</td>
</tr>
<tr>
<td>Habits impatient patients $\zeta'$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.9357 [0.9271;0.9452]</td>
<td>0.9411 [0.9333;0.9472]</td>
</tr>
<tr>
<td>Habit persist. patients $\rho_c$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.5102 [0.4013;0.6233]</td>
<td>0.6377 [0.5399;0.7388]</td>
</tr>
<tr>
<td>Habit persist. impatient patients $\rho'_c$</td>
<td>B</td>
<td>0.70</td>
<td>0.10</td>
<td>0.2249 [0.1635;0.2882]</td>
<td>0.3459 [0.2527;0.4370]</td>
</tr>
<tr>
<td>Price stickiness nondurables $\delta_c$</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>20.58 [13.63;27.65]</td>
<td>23.87 [16.55;31.31]</td>
</tr>
<tr>
<td>Price stickiness durables $\delta'_c$</td>
<td>G</td>
<td>15.0</td>
<td>5.00</td>
<td>22.05 [13.77;30.25]</td>
<td>1.26 [0.69;1.79]</td>
</tr>
<tr>
<td>Price indexation nondurables $\varsigma_c$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.2084 [0.0414;0.3636]</td>
<td>0.1439 [0.0263;0.2550]</td>
</tr>
<tr>
<td>Price indexation durables $\varsigma'_c$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.1167 [0.0158;0.2146]</td>
<td>0.2131 [0.0270;0.3994]</td>
</tr>
<tr>
<td>Wage stickiness $\psi^W$</td>
<td>G</td>
<td>100.0</td>
<td>10.00</td>
<td>155.98 [140.08;172.65]</td>
<td>23.87 [16.55;31.31]</td>
</tr>
<tr>
<td>IAC durables patients $\phi$</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>3.5149 [2.8751;4.1782]</td>
<td>3.7729 [3.2011;4.3405]</td>
</tr>
<tr>
<td>IAC durables impatient patients $\phi'$</td>
<td>N</td>
<td>1.5</td>
<td>0.50</td>
<td>1.9209 [1.2333;2.6121]</td>
<td>1.7943 [1.0931;2.4844]</td>
</tr>
<tr>
<td>Share of durables inflation $\tau$</td>
<td>B</td>
<td>0.20</td>
<td>0.10</td>
<td>0.1535 [0.0646;0.2463]</td>
<td>0.0517 [0.0362;0.0671]</td>
</tr>
<tr>
<td>Inflation -Taylor rule $\pi$</td>
<td>N</td>
<td>1.50</td>
<td>0.20</td>
<td>1.3781 [1.2056;1.5351]</td>
<td>1.7361 [1.5127;1.9526]</td>
</tr>
<tr>
<td>Output -Taylor rule $\gamma$</td>
<td>G</td>
<td>0.10</td>
<td>0.05</td>
<td>0.0188 [0.0054;0.0307]</td>
<td>0.0247 [0.0076;0.0411]</td>
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<tr>
<td>Interest rate smoothing $\rho_r$</td>
<td>B</td>
<td>0.80</td>
<td>0.10</td>
<td>0.7100 [0.6676;0.7514]</td>
<td>0.7707 [0.7337;0.8071]</td>
</tr>
<tr>
<td><strong>Averages</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trend growth rate $\gamma$</td>
<td>N</td>
<td>0.49</td>
<td>0.10</td>
<td>0.4075 [0.3749;0.4412]</td>
<td>0.4055 [0.3740;0.4364]</td>
</tr>
<tr>
<td>Inflation rate nondurables $\bar{\pi}_c$</td>
<td>B</td>
<td>1.05</td>
<td>0.10</td>
<td>1.0034 [0.9034;1.1051]</td>
<td>1.0419 [0.9393;1.1453]</td>
</tr>
<tr>
<td>Inflation rate durables $\bar{\pi}_d$</td>
<td>B</td>
<td>0.37</td>
<td>0.10</td>
<td>0.4345 [0.3189;0.5476]</td>
<td>0.3475 [0.2152;0.4830]</td>
</tr>
<tr>
<td>Interest rate $\bar{\gamma}$</td>
<td>G</td>
<td>1.65</td>
<td>0.10</td>
<td>1.6108 [1.4854;1.7334]</td>
<td>1.6450 [1.5027;1.7795]</td>
</tr>
<tr>
<td><strong>Exogenous processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology $\rho_{EA}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9789 [0.9601;0.9977]</td>
<td>0.9590 [0.9290;0.9910]</td>
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<tr>
<td>Monetary Policy $\rho_{M}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.1023 [0.0290;0.1769]</td>
<td>0.0811 [0.0189;0.1380]</td>
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<tr>
<td>Investment Durables $\rho_{I}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>0.6889 [0.6169;0.7574]</td>
<td>0.7369 [0.6553;0.8163]</td>
</tr>
<tr>
<td>Preference $\rho_{P}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.7467 [0.6784;0.8158]</td>
<td>0.8563 [0.8107;0.9026]</td>
</tr>
<tr>
<td>Price mark-up nondurables $\rho_{PC}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9193 [0.8704;0.9734]</td>
<td>0.9161 [0.8539;0.9806]</td>
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<tr>
<td>Price mark-up durables $\rho_{PD}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
<td>0.9867 [0.9771;0.9981]</td>
<td>0.9893 [0.9785;0.9994]</td>
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<tr>
<td>Price mark-up nondurables $\sigma_{PC}$</td>
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<td>0.10</td>
<td>2.0</td>
<td>6.4416 [4.3733;8.5367]</td>
<td>6.1710 [5.4138;8.1980]</td>
</tr>
<tr>
<td>Price mark-up durables $\sigma_{PD}$</td>
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<td>0.10</td>
<td>2.0</td>
<td>2.1614 [1.6924;2.5921]</td>
<td>2.3646 [1.8614;2.8512]</td>
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<tr>
<td>Price mark-up durables $\sigma_{PD}$</td>
<td>IG</td>
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<td>2.0</td>
<td>4.2377 [2.9814;5.4658]</td>
<td>22.506 [18.728;26.259]</td>
</tr>
<tr>
<td>Price mark-up durables $\sigma_{PD}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>4.2377 [2.9814;5.4658]</td>
<td>22.506 [18.728;26.259]</td>
</tr>
<tr>
<td>Wage mark-up $\rho_{W}$</td>
<td>B</td>
<td>0.50</td>
<td>0.20</td>
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<td>0.9533 [0.9310;0.9766]</td>
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<td>0.20</td>
<td>0.5483 [0.4430;0.6580]</td>
<td>0.5650 [0.4652;0.6710]</td>
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<tr>
<td>Government spending $\sigma_{GC}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>5.3348 [4.3919;6.2104]</td>
<td>5.6566 [4.7591;6.5676]</td>
</tr>
<tr>
<td>Government spending $\sigma_{GC}$</td>
<td>IG</td>
<td>0.10</td>
<td>2.0</td>
<td>3.5031 [3.1815;3.8188]</td>
<td>3.3336 [3.0230;3.6309]</td>
</tr>
</tbody>
</table>

Table 2.13: Prior and posterior distributions of estimated parameters: models with price indexation (90% confidence bands in square brackets)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior Mean</th>
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<td>Mean</td>
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<td>φ</td>
<td>N</td>
</tr>
<tr>
<td>Inv. Frisch elasticity impatients</td>
<td>φ'</td>
<td>N</td>
</tr>
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<td>ζ</td>
<td>B</td>
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<td>Habits. impatients</td>
<td>ζ'</td>
<td>B</td>
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<td>ρc</td>
<td>B</td>
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<tr>
<td>Habit persist. impatients</td>
<td>ρ'c</td>
<td>B</td>
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<td>ρ</td>
<td>N</td>
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<tr>
<td>Elast. sub. consumption</td>
<td>ρ'</td>
<td>N</td>
</tr>
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<td>G</td>
</tr>
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<td>Price stickiness durables</td>
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<td>G</td>
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<td>Price stickiness housing</td>
<td>θh</td>
<td>G</td>
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<td>Wage stickiness</td>
<td>θW</td>
<td>G</td>
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<td>φ</td>
<td>N</td>
</tr>
<tr>
<td>IAC housing patients</td>
<td>φH</td>
<td>N</td>
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<tr>
<td>IAC durables impatients</td>
<td>φ'</td>
<td>N</td>
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<tr>
<td>IAC housing impatients</td>
<td>φ'H</td>
<td>N</td>
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<td>Weight in inflation aggregator</td>
<td>τ</td>
<td>B</td>
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<td>Output -Taylor rule</td>
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<td>G</td>
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<td>πD</td>
<td>G</td>
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<tr>
<td>Interest rate</td>
<td>τ</td>
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Table 2.14: Prior and posterior distributions of estimated parameters: three-sector model (90% confidence bands in square brackets)
<table>
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<tr>
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<th>Prior</th>
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<td>$\rho_{eC}$</td>
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<tr>
<td>$\theta_{C}$</td>
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<tr>
<td>$\sigma_{eC}$</td>
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<tr>
<td>Price mark-up durables</td>
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<tr>
<td>$\theta_{D}$</td>
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</tr>
<tr>
<td>$\sigma_{eD}$</td>
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<tr>
<td>Price mark-up housing</td>
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<td>Wage mark-up</td>
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<tr>
<td>$\sigma_{eW}$</td>
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<td>$\rho_{eG}$</td>
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<tr>
<td>$\sigma_{eG}$</td>
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</table>

Table 2.15: Prior and posterior distributions of exogenous processes: three-sector model (90% confidence bands in square brackets)
Chapter 3

Sectoral Labor Mobility and Optimal Monetary Policy

3.1 Introduction

What inflation measure should central banks target? This question naturally arises when a New-Keynesian model is extended to include more than one sector. In fact, with only one instrument available, the central bank has to decide how much weight it has to assign to each sectoral inflation.

In a seminal paper, Aoki (2001) studies a two-sector economy with sticky- and flexible-price sectors and finds that, subject only to a technology shock, the central bank should assign zero weight to the flexible-price sector. A similar result is attained by Benigno (2004) in a two-country New-Keynesian model of a currency union which resembles a two-sector model. Here, more weight is attached to inflation in the region displaying a higher degree of price stickiness. Mankiw and Reis (2003) enrich these results by showing that, in order to construct a price index that—if kept on target—stabilizes economic activity, the sectoral weights should depend on the degree of price stickiness, the responsiveness to business cycles and the tendency to experience idiosyncratic shocks.

The optimal monetary policy literature has hitherto augmented a standard two-sectors New-Keynesian model by either characterizing the sectors
by their durability as in Erceg and Levin (2006), by introducing input-output interactions (I-O, henceforth) between intermediate and final goods firms as in Huang and Liu (2005), or both as in Petrella and Santoro (2011) and Petrella et al. (2017). Abstracting from heterogeneity in price stickiness, Erceg and Levin (2006) show that goods durability plays an important role for the conduct of monetary policy. As durable goods are more sensitive to the interest rate than nondurables, the central bank faces a severe trade-off in stabilizing output and prices across the two sectors. In their setting, with symmetric sectoral price and wage rigidities, the central bank attains the second-best policy by responding to both price and wage inflations. Conversely, I-O interactions imply that the two sectoral inflations reflect the difference between a consumer price index (CPI) and a producer price index (PPI). In such context, Huang and Liu (2005), Gerberding et al. (2012) and Strum (2009) conclude that targeting hybrid measures of inflation delivers desirable welfare results but the weight assigned to each sectoral inflation reflects their size. Similar conclusions are drawn when, neglecting I-O interactions, durable goods are used as collateral by households to borrow (Monacelli, 2008), sectors differ by factor intensities (Jeske and Liu, 2013), or the length of wage contracts differs across sectors (Kara, 2010).

All the above-mentioned contributions look at this important issue from many angles, but overlook the role that the extent to which labor is allowed to reallocate across sectors has for optimal monetary policy. Indeed, Petrella et al. (2017) optimally find the weight attached to durables inflation in an input-output economy with a given limited degree of labor mobility, but do not isolate the impact that labor mobility has on the weight itself. We fill this gap and show that the extent to which labor can freely move across sectors is crucial in the determination of the optimal inflation composite, and it intuitively interacts with price and nominal wage stickiness. In our model the two sectors differ both in goods durability and degree of price stickiness, given that durable goods also display less sticky prices in the data, and this empirical regularity is employed in much of the theoretical literature.

\[1\] Consistently, in their empirical exercise, Mankiw and Reis (2003) conclude that substantial weight should be assigned to the level of nominal wages.
We first use Bayesian methods to estimate the model. This embeds imperfect labor mobility, price and wage stickiness and a set of shocks conventional in the monetary policy literature. Model estimates point at a limited degree of labor mobility across sectors. Then, we use the estimated model to design optimized monetary policy rules and highlight a negative relationship between labor mobility and the weight attached to inflation in the durables sector.

Consistently with the literature, a lower weight is assigned to the sector in which prices are more flexible (durables) but, conditional on the degree of price and wage stickiness, such weight is higher the less mobile labor is across sectors. Intuitively, following a shock, the lower (higher) the friction in the labor market the larger (smaller) is the reallocation of workers across sectors hence the smaller (larger) is the adjustment through prices. It follows that the central bank devotes more (less) attention to the stickier sector.

We furthermore show that wage stickiness plays an important role in setting the optimal weight of sectoral inflations due to second-round effects on marginal costs and explains why even with flexible durable prices, the central bank still assigns a positive weight to inflation in the durables sector. Consistently with Erceg and Levin (2006), we also find that key variables in the durables sector are more volatile than in the nondurables sector, specifically the variability of prices, wages and output is higher in the former than in the latter. Finally, when optimizing the parameters of the interest rate rule, the central bank actually chooses a price level rule to minimize the welfare loss with respect to the first best policy, a result attained by Levine et al. (2008) and Giannoni (2014), among others, in a one-sector economy. Our results carry an important policy implication: the degree of labor mobility between sectors is an aspect of the economy that central banks should not overlook in setting the monetary policy stance.

The remainder of the paper is organized as follows. Section 3.2 presents

\footnote{Estimating the model prior to designing optimized monetary policy rules is crucial since, as noted by Cantore et al. (2012), such rules heavily depend on the persistence and the variance of shocks. Note we use the estimation for the purpose of calibration in the optimal policy analysis while we do not seek a comparison between the estimated and the optimized Taylor rule.}
the two-sector New-Keynesian model. In Section 3.3 we report the results of the Bayesian estimation, whereas Section 3.4 shows the outcomes of the optimal monetary policy exercises. Finally, Section 3.5 concludes. More details about the data, the model’s equilibrium conditions and the steady state are provided in the Appendix.

3.2 Model

We construct a two-sector New-Keynesian model in the spirit of Barsky et al. (2007), where consumers purchase both durable and nondurable goods. In addition, there are several frictions now standard in the New-Keynesian literature, namely price and wage stickiness, investment adjustment costs in durable goods (IAC, henceforth), habit formation in consumption of nondurable goods. We also introduce imperfect labor mobility across sectors and estimate the relevant parameter. Finally, the monetary authority sets the nominal interest rate according to an interest rate rule.

3.2.1 Households

There is a continuum \( i \in [0, 1] \) of identical and infinitely-lived households consuming both durable and nondurable goods and supplying labor, whose lifetime utility is

\[
E_0 \sum_{t=0}^{\infty} e_t^B \beta^t U (X_{i,t}, N_{i,t}),
\]

(3.1)

where \( \beta \in [0, 1] \) is the subjective discount factor, \( e_t^B \) is a preference shock, \( X_{i,t} = Z_{i,t}^{1-\alpha} D_{i,t}^\alpha \) is a Cobb-Douglas consumption aggregator of nondurable \((Z_{i,t})\) and durable goods \((D_{i,t})\) with \( \alpha \in [0, 1] \) representing the share of durables consumption on total expenditure, and \( N_{i,t} \) being the household’s labor supply. Nondurable consumption is subject to external habit formation such that

\[
Z_{i,t} = C_{i,t} - \xi S_{t-1},
\]

(3.2)

\[
S_t = \rho_c S_{t-1} + (1 - \rho_c) C_t,
\]

(3.3)
where $C_{i,t}$ is the level of the household’s nondurable consumption; $S$, $\zeta \in (0, 1)$ and $\rho_c \in (0, 1)$ are the stock, the degree and the persistence of habit formation, respectively, while $C_t$ represents average consumption across households. Members of each household supply labor to firms in both sectors according to:

$$N_{i,t} = \left( (\chi C)^{-\frac{1}{\lambda}} (N_{i,t}^C)^{\frac{1+\lambda}{\lambda}} + (1 - \chi C)^{-\frac{1}{\lambda}} (N_{i,t}^D)^{\frac{1+\lambda}{\lambda}} \right)^{\frac{1}{1+\lambda}}. \quad (3.4)$$

Following Horvath (2000), Petrella and Santoro (2011) and Petrella et al. (2017), this CES specification of aggregate labor captures different degrees of labor mobility across sectors, governed by parameter $\lambda > 0$, i.e. the intra-temporal elasticity of substitution: $\lambda \to 0$ denotes the case of labor immobility, while as $\lambda \to \infty$ labor can be freely reallocated and all workers earn the same wage at the margin. For $\lambda < \infty$ the economy displays a limited degree of labor mobility and sectoral wages are not equal. Moreover, $\chi C \equiv N_C^C / N$ represents the steady-state share of labor supply in the nondurables sector. The stock of durables evolves according to law of motion

$$D_{i,t+1} = (1 - \delta) D_{i,t} + e_l^I I_{i,t}^D \left[ 1 - S \left( \frac{I_{i,t}^D}{I_{i,t-1}} \right) \right], \quad (3.5)$$

---

3 As explained by Fuhrer (2000), the extra source of persistence in superficial habit formation determined by $\rho_c$ implies that the reference level for habit formation can either be only the previous period consumption (for $\rho_c = 0$) or consumption further back in time (for $0 < \rho_c \leq 1$). Fuhrer (2000) then demonstrated that this extra source of persistence is key to fit the model to the data and produce plausible impulse responses of consumption to macroeconomic shocks. The same specification of persistence has been proved to be empirically relevant in estimated models with deep habits in consumption (see Ravn et al., 2006, Cantore et al., 2014a and Zubairy, 2014). Moreover, Cantore et al. (2014b) show that including the extra persistence in superficial habits makes a NK model fit the data as well as a NK model with deep habits. Given that accounting for the plausible response of consumption to a monetary policy shock is crucial in our paper, we included the extra source of persistence in superficial habits.

4 According to Horvath (2000), the CES aggregator allows us to capture labor market mobility without deviating from the representative agent assumption. Di Pace and Hertweck (2016) show that search and matching frictions improve the fit of a two-sector model to the data but, although desirable, capturing sectoral labor market mobility in a search and matching framework is not straightforward and would inevitably complicate our analysis (e.g. it would require using sectoral and aggregate matching functions).
where $\delta$ is the depreciation rate, $I_{t,t}^D$ is investment in durable goods that is subject to adjustment costs, and $e^I_t$ represents an investment-specific shock. The adjustment costs function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $S''(1) > 0$. Each household consumes $C_{i,t}$, purchases nominal bonds $B_{i,t}$, receives profits $\Omega_t$ from firms and pays a lump-sum tax $T_t$. Finally, $Q_t \equiv \frac{P_{D,t}}{P_{C,t}}$ denotes the relative price of durables so that the period-by-period real budget constraint reads as follows:

$$C_{i,t} + Q_t I_{i,t}^D + \Phi_t + \frac{B_{i,t}}{P_{C,t}} N_{i,t} + R_{t-1} \frac{B_{i,t-1}}{P_{C,t}} + \Omega_t - T_t,$$

(3.6)

where the term $\Phi_t = \frac{dW}{2} \left( \frac{w_{i,t}}{w_{i,t-1}} \right)^2 w_t N_t$ accounts for nominal wage adjustment costs as in Rotemberg (1982). Households choose $Z_{i,t}$, $B_{i,t}$, $D_{i,t} + 1$, $I_{i,t}^D$, $w_{i,t}$, $N_{C,t}^C$, $N_{D,t}^D$ to maximize (3.1) subject to (3.2), (3.3), (3.4), (3.5) and (3.6).

At the symmetric equilibrium, the household’s optimality conditions are:

$$1 = E_t \left[ \Lambda_{t,t+1} \frac{R_t}{I_{i,t}^{C+1}} \right],$$

(3.7)

$$Q_t \psi_t = U_{D,t} U_{Z,t} + (1 - \delta) E_t \left[ \Lambda_{t+1, t+1} Q_{t+1} \psi_{t+1} \right],$$

(3.8)

$$1 = \psi_t e^I_t \left[ 1 - S \left( \frac{I_{t}^D}{I_{t-1}^D} \right) - S' \left( \frac{I_{t}^D}{I_{t-1}^D} \right) \frac{I_{t}^D}{I_{t-1}^D} \right] +$$

$$+ E_t \left\{ \Lambda_{t,t+1} \psi_{t+1} Q_{t+1} e^I_{t+1} \left[ S' \left( \frac{I_{t+1}^D}{I_{t}^D} \right) \frac{I_{t+1}^D}{I_{t}^D} \right]^2 \right\},$$

(3.9)

$$N_{C,t}^C = \chi^C \left( \frac{w_{C,t}}{w_t} \right)^{\lambda} N_t,$$

(3.10)

$$N_{D,t}^D = \chi^D \left( \frac{w_{D,t}}{w_t} \right)^{\lambda} N_t,$$

(3.11)

$$0 = \left[ 1 - e^W \eta_t \right] + \frac{e^W}{\mu_t} \eta_t - \theta^W \left( \Pi_{t+1}^W - \Pi_{C}^W \right) \Pi_{t+1}^W +$$

$$+ E_t \left[ \Lambda_{t,t+1} \theta^W \left( \Pi_{t+1}^W - \Pi_{C}^W \right) \Pi_{t+1}^W \frac{w_{t+1} N_{t+1}}{w_t N_t} \right].$$

(3.12)
Equation (3.7) is a standard Euler equation with $\Lambda_{t,t+1} \equiv \beta \frac{U_{Z,t+1}}{U_{Z,t}} e_{B,t+1}$ representing the stochastic discount factor and $U_{Z,t}$ denoting the marginal utility of habit-adjusted consumption of nondurable goods. Equation (3.8) represents the asset price of durables, where $U_{D,t}$ is the marginal utility of durables consumption and $\psi_t$ is the Lagrange multiplier attached to constraint (3.5). Equation (3.9) is the optimality condition w.r.t. investment in durable goods. Equations (3.10) and (3.11) are the household’s labor supplies in each sector, whereas equation (3.12) is the wage setting equation in which $\hat{\mu}_t \equiv \frac{w_t}{MRS_t}$ is the wage markup, $MRS_t \equiv -\frac{U_{N,t}}{U_{Z,t}}$ is the marginal rate of substitution between consumption and leisure, $U_{N,t}$ is the marginal disutility of work and $\Pi_t^W$ is the gross wage inflation rate.

### 3.2.2 Firms

A continuum $\omega \in [0, 1]$ of firms in each sector $j = C, D$ operates in monopolistic competition and face quadratic costs of changing prices $\vartheta_j \left( \frac{P_{j,t+1}}{P_{j,t-1}} - 1 \right)^2 Y^{j}_{t}$, where $\vartheta_j$ is the parameter of sectoral price stickiness. Each firm produces differentiated goods according to a linear production function,

$$Y^{j}_{\omega,t} = e^{A}_{t} N^{j}_{\omega,t}, \quad (3.13)$$

where $e^{A}_{t}$ is a labor-augmenting productivity shock. Firms maximize the present discounted value of profits,

$$E_t \left\{ \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[ \frac{P^j_{\omega,t}}{P^j_t} Y^{j}_{\omega,t} - \frac{W_{\omega,t}}{P^j_t} N^{j}_{\omega,t} - \frac{\vartheta_j}{2} \left( \frac{P^j_{\omega,t}}{P^j_{\omega,t-1}} - 1 \right)^2 Y^{j}_{t} \right] \right\}, \quad (3.14)$$

subject to production function (3.13) and a standard Dixit-Stiglitz demand equation $Y^{j}_{\omega,t} = \left( \frac{P^j_{\omega,t}}{P^j_t} \right)^{-\epsilon_{j,t}} Y^{j}_{t}$, where $\epsilon_j$ and $e^j_{t}$ are the sectoral intratemporal elasticity of substitution across goods and the sectoral price markup shock, respectively. At the symmetric equilibrium, the price setting equations for
the two sectors read as

\[
(1 - e_t^C e_c) + e_t^C e_c MC_t^C = \vartheta_c \left( \Pi_t^C - \Pi_t^C \right) - \vartheta_e E_t \left[ \Lambda_{t,t+1} \frac{Y_{t+1}^C}{Y_t^C} (\Pi_{t+1}^C - \Pi_t^C) \Pi_{t+1}^C \right],
\]

(3.15)

\[
(1 - e_t^D e_d) + e_t^D e_d MC_t^D = \vartheta_d \left( \Pi_t^D - \Pi_t^D \right) - \vartheta_d E_t \left[ \Lambda_{t,t+1} \frac{Q_{t+1} Y_{t+1}^D}{Q_t Y_t^D} (\Pi_{t+1}^D - \Pi_t^D) \Pi_{t+1}^D \right],
\]

(3.16)

where \(MC_t^C = \frac{\omega_t}{e_t^C}\) and \(MC_t^D = \frac{\omega_t}{e_t^D Q_t}\). When \(\vartheta_j = 0\) prices are fully flexible and are set as constant markups over the marginal costs.

### 3.2.3 Fiscal and monetary policy

The government purchases nondurable goods as in Erceg and Levin (2006) and runs a balanced budget by levying lump-sum taxes. Monetary policy is conducted by an independent central bank via the following interest rate rule:

\[
\log \left( \frac{R_t}{\bar{R}} \right) = \rho_r \log \left( \frac{R_{t-1}}{\bar{R}} \right) + (1 - \rho_r) \left\{ \rho_r \log \left( \frac{\Pi_t}{\bar{\Pi}} \right) + \rho_y \log \left( \frac{Y_t}{Y_t^f} \right) \right. \\
+ \rho_{\Delta y} \left[ \log \left( \frac{Y_t}{Y_t^f} \right) - \log \left( \frac{Y_{t-1}}{Y_{t-1}^f} \right) \right] \right\} + e_t^R.
\]

Equation (3.17) is that employed by Smets and Wouters (2007) and implies that the central bank reacts to inflation, the output gap and the output gap growth to an extent determined by parameters \(\rho_r, \rho_y\) and \(\rho_{\Delta y}\), respectively. The output gap is defined as the deviation of output from the level that would prevail with flexible prices and wages, \(Y_t^f\), and \(\rho_r\) is the degree of interest rate smoothing. The aggregator of the gross rates of sectoral inflations is

\[
\bar{\Pi}_t \equiv \left( \Pi_t^C \right)^{1-\tau} \left( \Pi_t^D \right)^{\tau},
\]

(3.18)

where \(\bar{\Pi}\) is its steady-state value, and \(\tau \in [0, 1]\) represents the weight assigned by the central bank to durables inflation.
3.2.4 Market clearing conditions and exogenous processes

In equilibrium all markets clear and the model is closed by the following identities:

\[
Y_t^C = C_t + e_t^C + \frac{\partial_c}{2} (\Pi_t^C - \Pi^C)^2 Y_t^C, \tag{3.19}
\]

\[
Y_t^D = [D_t - (1 - \delta) D_{t-1}] + \frac{\partial_d}{2} (\Pi_t^D - \Pi^D)^2 Y_t^D, \tag{3.20}
\]

\[
Y_t = Y_t^C + Q_t Y_t^D + \frac{\partial_w}{2} \left( \frac{w_t}{w_{t-1}} \Pi_t^C - \Pi^C \right)^2 w_t N_t. \tag{3.21}
\]

As in Smets and Wouters (2007), the wage markup and the price markup shocks follow ARMA (1,1) processes:

\[
\log \left( \frac{\kappa_t}{\kappa} \right) = \rho_{\kappa} \log \left( \frac{\kappa_{t-1}}{\kappa} \right) + \epsilon_t^\kappa - \theta_i \epsilon_{t-1}^\kappa, \tag{3.22}
\]

with \( \kappa = [e^W, e^C, e^D] \), \( i = [W, C, D] \), whereas all other shocks follow an AR (1) process:

\[
\log \left( \frac{\kappa_t}{\kappa} \right) = \rho_{\kappa} \log \left( \frac{\kappa_{t-1}}{\kappa} \right) + \epsilon_t^\kappa, \tag{3.23}
\]

where \( \kappa = [e^B, e^I, e^R, e^A, e^G] \) is a vector of exogenous variables, \( \rho_{\kappa} \) and \( \rho_{\kappa} \) are the autoregressive parameters, \( \theta_i \) are the moving average parameters, \( \epsilon_t^\kappa \) and \( \epsilon_t^\kappa \) are i.i.d shocks with zero mean and standard deviations \( \sigma_{\kappa} \) and \( \sigma_{\kappa} \).

3.2.5 Functional forms

The utility function is additively separable and logarithmic in consumption: \( U(X_t, N_t) = \log(X_t) - \nu N_t^{1+\varphi}, \) where \( \nu \) is a scaling parameter for hours worked and \( \varphi \) is the inverse of the Frisch elasticity of labor supply. Adjustment costs in durables investment are quadratic: \( S \left( \frac{I_t^D}{I_{t-1}^D} \right) = \frac{\phi}{2} \left( \frac{I_t^D}{I_{t-1}^D} - 1 \right)^2, \) \( \phi > 0 \) as in Christiano et al. (2005).
### 3.3 Bayesian estimation

The model is estimated with Bayesian methods. The Kalman filter is used to evaluate the likelihood function that, combined with the prior distribution of the parameters, yields the posterior distribution. Then, the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm with two parallel chains of 150,000 draws each is used to generate a sample from the posterior distribution in order to perform inference. We estimate the model over the sample 1969Q2-2007Q4 by using US data on: GDP, consumption of durable goods, consumption of nondurable goods, real wage, hours worked, inflation in the nondurables sector, inflation in the durables sector and the nominal interest rate. The following measurement equations link the data to the endogenous variables of the model:

\[
\begin{align*}
\Delta Y_{t}^o &= \gamma + \hat{Y}_{t} - \hat{Y}_{t-1} , \\
\Delta I_{D,t}^o &= \gamma + \hat{I}_{D,t} - \hat{I}_{D,t-1} , \\
\Delta C_{t}^o &= \gamma + \hat{C}_{t} - \hat{C}_{t-1} , \\
\Delta W_{t}^o &= \gamma + \hat{W}_{t} - \hat{W}_{t-1} , \\
N_{t}^o &= \hat{N}_{t} , \\
\Pi_{C,t}^o &= \bar{\pi}_C + \hat{\Pi}_{t}^C , \\
\Pi_{D,t}^o &= \bar{\pi}_D + \hat{\Pi}_{t}^D , \\
R_{t}^o &= \bar{r} + \hat{R}_{t} ,
\end{align*}
\]

where \( \gamma \) is the common quarterly trend growth rate of GDP, consumption of durables, consumption of nondurables and the real wage; \( \bar{\pi}_C \) and \( \bar{\pi}_D \) are the average quarterly inflation rates in nondurable and durable sectors respectively; \( \bar{r} \) is the average quarterly Federal funds rate. Hours worked are demeaned so no constant is required in the corresponding measurement equation (3.28). Variables with a \( \hat{\cdot} \) are in log-deviations from their own steady state.
### Table 3.1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.99</td>
</tr>
<tr>
<td>Durables depreciation rate</td>
<td>$\delta$ 0.010</td>
</tr>
<tr>
<td>Durables share of total expenditure</td>
<td>$\alpha$ 0.20</td>
</tr>
<tr>
<td>Elasticity of substitution nondurable goods</td>
<td>$\epsilon_c$ 6</td>
</tr>
<tr>
<td>Elasticity of substitution durable goods</td>
<td>$\epsilon_d$ 6</td>
</tr>
<tr>
<td>Elasticity of substitution in labor</td>
<td>$\eta$ 21</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$\nu$ target $\bar{N} = 0.33$</td>
</tr>
<tr>
<td>Government share of output</td>
<td>$g_y$ 0.20</td>
</tr>
</tbody>
</table>

### 3.3.1 Calibration and priors

Table 3.1 presents the structural parameters calibrated at a quarterly frequency. The discount factor $\beta$ is equal to the conventional value of 0.99, implying an annual steady-state gross interest rate of 4%. Following [Mona- celli (2009)](Mona-cell), we calibrate the depreciation rate of durable goods $\delta$ at 0.010 amounting to an annual depreciation of 4%, and the durables share of total expenditure $\alpha$ is set at 0.20. The sectoral elasticities of substitution across different varieties $\epsilon_c$ and $\epsilon_d$ equal 6 in order to target a steady-state gross mark-up of 1.20 in both sectors. We target a 5% steady-state gross wage mark-up hence we set the elasticity of substitution in the labor market $\eta$ equal to 21 as in [Zubairy (2014)](Zubairy). The preference parameter $\nu$ is set to target steady-state total hours of work of 0.33. The government-output ratio $g_y$ is calibrated at 0.20, in line with the data.

Prior and posterior distributions of the parameters and the shocks are reported in Table 3.2. We set the prior mean of the inverse Frisch elasticity $\varphi$ to 0.5, broadly in line with [Smets and Wouters (2007)](Smets and Wouters) who estimate a Frisch elasticity of 1.92. We also follow SW in setting the prior means of the habit parameter, $\zeta$, to 0.7, the interest rate smoothing parameter, $\rho_r$, to 0.80 and in assuming a stronger response of the central bank to inflation than output. We set the prior means of the constants in the measurement equations equal to the average values in the dataset. In general, we use the Beta distribution for all parameters bounded between 0 and 1. We use the Inverse Gamma (IG) distribution for the standard
deviation of the shocks for which we set a loose prior with 2 degrees of freedom. We choose a Gamma distribution for the Rotemberg parameters for both prices and wages, given that these are non-negative. The price stickiness parameters are assigned the same prior distribution corresponding to firms resetting prices around 1.5 quarters on average in a Calvo world. Finally, we follow Iacoviello and Neri (2010) who choose a Normal distribution for the intra-temporal elasticity of substitution in labor supply $\lambda$, with a prior mean of 1 which implies a limited degree of labor mobility.

### 3.3.2 Estimation results

We report the posterior mean of the parameters together with the 90% probability intervals in square brackets in Table 3.2. We first devote our attention to the parameters that will be crucial for remainder of the analysis. In line with the literature, the labor mobility parameter $\lambda$ is estimated to be 1.1769 implying a non-negligible degree of friction in the labor market. Indeed, Horvath (2000) estimates a regression equation to find a value of 0.999 whereas Iacoviello and Neri (2010) estimate values of 1.51 and 1.03 for savers and borrowers, respectively.

Typically, limited labor mobility is calibrated at a value of $\lambda = 1$ (see Bouakez et al. 2009, Petrella and Santoro 2011 and Petrella et al. 2017) except Bouakez et al. (2011) who explore values between 0.5 and 1.5. Figure 3.1 displays the prior and posterior distribution of $\lambda$ to confirm that the parameter is correctly identified as the posterior distribution is rather apart from the prior and that it includes the above-mentioned values used in the literature.

Prices are estimated to be stickier in the nondurables sector, with a non-negligible degree of stickiness in durable goods. In the literature there is no decisive evidence that prices of nondurable goods are much stickier than those of many durables (see Bils and Klenow 2004 and Nakamura and Steinsson).

---

5Iacoviello and Neri (2010) specify the CES aggregator such that the labor mobility parameter is the inverse of $\lambda$. They find values of 0.66 and 0.97 for savers and borrowers respectively hence the values of $1/0.66=1.51$ and $1/0.97=1.03$ we reported to ease the comparison.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distrib.</th>
<th>Prior Mean</th>
<th>Std/df</th>
<th>Posterior Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor mobility</td>
<td>$\lambda$</td>
<td>Normal</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\varphi$</td>
<td>Normal</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Habit in nondurables consumption</td>
<td>$\zeta$</td>
<td>Beta</td>
<td>0.70</td>
<td>0.10</td>
</tr>
<tr>
<td>Habit persist. nondurables consumption</td>
<td>$\rho_c$</td>
<td>Beta</td>
<td>0.70</td>
<td>0.10</td>
</tr>
<tr>
<td>Price stickiness nondurables</td>
<td>$\delta_e$</td>
<td>Gamma</td>
<td>15.0</td>
<td>5.00</td>
</tr>
<tr>
<td>Price stickiness durables</td>
<td>$\delta_d$</td>
<td>Gamma</td>
<td>15.0</td>
<td>5.00</td>
</tr>
<tr>
<td>Wage stickiness</td>
<td>$\sigma_W$</td>
<td>Gamma</td>
<td>100.0</td>
<td>10.00</td>
</tr>
<tr>
<td>Invest. adjust. costs durable goods</td>
<td>$\phi$</td>
<td>Normal</td>
<td>1.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Share of durables inflation in aggregator</td>
<td>$\tau$</td>
<td>Beta</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Inflation -Taylor rule</td>
<td>$\rho_n$</td>
<td>Normal</td>
<td>1.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Output -Taylor rule</td>
<td>$\rho_y$</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>Output growth -Taylor rule</td>
<td>$\rho_{\Delta y}$</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho_r$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Averages</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trend growth rate</td>
<td>$\gamma$</td>
<td>Normal</td>
<td>0.49</td>
<td>0.10</td>
</tr>
<tr>
<td>Inflation rate nondurables</td>
<td>$\pi_C$</td>
<td>Gamma</td>
<td>1.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Inflation rate durables</td>
<td>$\pi_D$</td>
<td>Gamma</td>
<td>0.55</td>
<td>0.10</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\bar{\rho}$</td>
<td>Beta</td>
<td>1.65</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Exogenous processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_{eA}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>$\rho_{eR}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Investment Durables</td>
<td>$\rho_{eI}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Preference</td>
<td>$\rho_{eB}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Price mark-up nondurables</td>
<td>$\rho_{eC}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Price mark-up durables</td>
<td>$\rho_{eD}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>$\rho_{eW}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\rho_{eG}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Log-marginal likelihood</td>
<td></td>
<td></td>
<td></td>
<td>-1378.927</td>
</tr>
</tbody>
</table>

Table 3.2: Prior and posterior distributions of estimated parameters (90% confidence bands in square brackets)
However, prices of new houses are generally rather flexible. Therefore, given the importance of the degree of price stickiness of durable goods for our results, in the remainder of the paper we use both the estimated value in the baseline model and alternative calibrations. Similarly, wages are found to be sticky but we also explore the effects of flexible wages.

The remaining parameters are broadly in line with the literature and suggest a relevance of the real frictions (IAC in durable goods and habits in consumption of nondurables) and a stronger response of monetary policy to inflation with respect to output with a high degree of policy inertia. Overall, our estimation delivers results consistent with both standard and two-sectors New-Keynesian models estimated with Bayesian methods and serves as the starting point for our analysis of optimal monetary policy.

### 3.4 Optimal monetary policy

#### 3.4.1 The monetary policy rule

To design the optimal monetary policy, we follow Schmitt-Grohe and Uribe (2007) and write the interest rate rule (3.17) as

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Bouakez et al. (2014), Cantelmo and Melina (2018) and the references therein provide a more detailed discussion about the macro and micro evidence of sectoral price stickiness.
\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \alpha_\pi \log \left( \frac{\tilde{\Pi}_t}{\Pi} \right) + \\
+ \alpha_y \log \left( \frac{Y_t}{Y_t^f} \right) + \alpha_{\Delta y} \left[ \log \left( \frac{Y_t}{Y_t^f} \right) - \log \left( \frac{Y_{t-1}}{Y_{t-1}^f} \right) \right],
\]

(3.32)

to include the case of price-level rules in the spirit of Woodford (2003), Giannoni (2014), and Cantore et al. (2012) when \( \rho_r = 1 \). In Section 3.4.4 we assess also the robustness of our results to a superinertial rule in which \( \rho_r > 1 \) and to an alternative rule that responds only to inflation and deviations of output from its steady state.

### 3.4.2 Welfare measure

The optimal monetary policy analysis serves two purposes: (i) determining the optimal weights the central bank should assign to sectoral inflations subject to given degrees of labor mobility, and (ii) seeking parameter values for interest rate rule (3.32) to mimic the first best allocation, i.e. that minimize the welfare loss with respect to the Ramsey policy. The social planner maximizes the present value of households’ utility,

\[
\Upsilon_t = E_t \left[ \sum_{s=0}^{\infty} e^{R_s} \beta^s U(X_{t+s}, N_{t+s}) - w_r(R_{t+s} - R)^2 \right],
\]

(3.33)

subject to the equilibrium conditions of the model. As established by Schmitt-Grohe and Uribe (2007), while more stylized models allow for a first-order approximation to the equilibrium conditions to be sufficient to accurately approximate welfare up to a second order, the presence of numerous frictions requires taking a second-order approximation both of the mean of \( \Upsilon_t \) and of the model’s equilibrium conditions around the deterministic steady state. In particular, we take the approximation around the steady state of the Ramsey equilibrium. Similarly to many other NK models in the literature

\[\text{For the remainder of the optimal monetary policy analysis we shut down stochastic monetary policy innovations, that is why no random term is included in (3.32).}\]
the steady-state value of the gross inflation rate in the Ramsey equilibrium turns out to be very close to unity, which implies an almost zero-inflation steady state. Since it is not straightforward to account for the zero-lower-bound (ZLB, henceforth) on the nominal interest rate when using perturbation methods, we follow Schmitt-Grohe and Uribe (2007) and Levine et al. (2008) and introduce a term in (3.33) that penalizes large deviations of the nominal interest rate from its steady state. Hence, the imposition of this approximate ZLB constraint translates into appropriately choosing the weight \( w_r \) to achieve an arbitrarily low per-period probability of hitting the ZLB, \( Pr(ZLB) \equiv Pr(R^u_t \leq 1) \), which we set at less than 0.01 for each calibration, and corresponds to a value of the penalty parameter \( w_r = 80 \). We optimize the interest rate rule (3.32) by numerically searching for the combination of the policy parameters and the weight on durables inflation \( \tau \in [0, 1] \) that maximizes the present value of households’ utility (3.33). In doing so, the support of \( \rho_r \) is \([0, 1]\) whereas the support of \( \alpha_{\pi}, \alpha_y \) and \( \alpha_\Delta y \) is \([0, 5]\). Then, we assess the role of labor mobility by considering three cases: (i) \( \lambda = 0.25 \) represents the case of quasi labor immobility, (ii) \( \lambda = \infty \) the case of perfect labor mobility and (iii) \( \lambda = 1.1769 \), the imperfect labor mobility estimated in Section 3.3. Then we compare the welfare losses in terms of steady-state consumption-equivalent, \( \omega \), with respect to the Ramsey policy, as in Schmitt-Grohe and Uribe (2007). In particular, for a regime associated to a given Taylor-type interest rate rule \( A \), the welfare loss is implicitly defined as

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ U \left( (1 - \omega) X_t^R, N_t^R \right) \right] \right\} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ U \left( X_t^A, N_t^A \right) \right] \right\}, \quad (3.34)
\]
where $\omega \times 100$ represents the percent permanent loss in consumption that should occur in the Ramsey regime (R) in order for agents to be as well off in regime $R$ as they are in regime $A$.

### 3.4.3 Results

#### 3.4.3.1 Impulse responses to an inflation shock: Ramsey policy

We first graphically explore how different degrees of labor mobility alter the optimal responses to structural shocks. Figure 3.2 shows the responses to an inflation shock (namely a shock to the price mark-up) under the Ramsey policy and sticky durables prices. Higher mark-ups in the durables sector increase inflation and decrease investment and employment in durables for any degree of labor mobility. However, when labor is prevented from moving across sectors, output and employment fall also in nondurables. Indeed, although households would substitute durables with nondurables, firms in the nondurables sector are not able to hire workers from the durables sector and increase production. It follows that output in both sectors decreases and aggregate output is persistently below the steady-state.

Conversely, when labor is mobile, consumption in nondurables increases since labor can flow from the durables sector. Output in the two sectors displays a negative comovement and the response of durables is more gradual. This makes aggregate output increase in the first seven quarters, i.e. when the positive effect on nondurables output outweighs the fall in durables. Later on, the stronger decline of durables takes over and aggregate output falls. Moreover, the optimal response of the nominal interest rate is significantly different when labor is perfectly mobile: i.e. it accommodates the increase in inflation, while it is almost constant for lower degrees of labor mobility.\footnote{The procyclical behavior of the nominal interest rate will be important in our analysis as explained in the next section.}

The intuition is that a fall in the nominal interest rate stimulates consumption in nondurables and, since more workers can flow from the durable to the nondurable sector, supply will be able to catch up with demand and avoid a recession, at least in the first seven quarters after the shock. This
Figure 3.2: Responses to a durables price markup shock: Ramsey policy suggests that labor mobility has important implications for the transmission mechanism of structural shocks and optimal monetary policy.

3.4.3.2 Optimized interest-rate rules

Turning to the welfare properties of the interest rate rule (3.32), Table 3.3 reports its optimized parameters together with the associated welfare costs $\omega$. The top panel displays the optimized parameters in the baseline model in which all the parameters (except $\lambda$ and those that are optimally set), are calibrated at the posterior means reported in Section 3.3.

We first notice that regardless of the degree of labor mobility, the central bank response to the output gap and output gap growth is absent whereas a stronger reaction is devoted to inflation, a result in line with the findings of Schmitt-Grohe and Uribe (2007) in a one-sector model. Moreover, the interest rate smoothing parameter systematically hits the upper bound of one,
Table 3.3: Optimized monetary policy rule: Sticky vs Flexible Durables Prices

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_y$</th>
<th>$\alpha_{\Delta y}$</th>
<th>$\tau$</th>
<th>$100 \times \omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1</td>
<td>0.0633</td>
<td>0.0000</td>
<td>0.0109</td>
<td>0.4831</td>
<td>0.0716</td>
</tr>
<tr>
<td>1.1769</td>
<td>1</td>
<td>0.1104</td>
<td>0.0000</td>
<td>0.0191</td>
<td>0.1612</td>
<td>0.0846</td>
</tr>
<tr>
<td>$\infty$</td>
<td>1</td>
<td>0.4611</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.5947</td>
</tr>
</tbody>
</table>

$\textit{Sticky durables prices}$

| 0.25      | 1        | 0.0404       | 0.0000     | 0.0135              | 0.1581| 0.0791          |
| 1.1769    | 1        | 0.1000       | 0.0000     | 0.0187              | 0.0000| 0.0920          |
| $\infty$  | 1        | 0.5055       | 0.0000     | 0.0000              | 0.0000| 0.9572          |

$\textit{Flexible durables prices}$

thus characterizing (3.32) as a price-level rule. As discussed by Giannoni (2014), price-level rules deliver better welfare results than Taylor-type rules by introducing a sufficient amount of history dependence in an otherwise entirely forward-looking behavior of price setters, thus reducing the volatility of inflation. It is therefore not surprising that the optimal value of $\rho_r$ equals 1.\footnote{Similar results hold in other contexts, such as the New-Keynesian model with financial frictions studied by Melina and Villa (2018).} Moreover, the coefficient attached to inflation ($\alpha_\pi$) in the interest-rate rule is smaller than one for any degree of labor mobility. However, indeterminacy is not an issue since, as demonstrated by Giannoni (2014), any price level rule with positive coefficients yields a determinate equilibrium.\footnote{Giannoni (2014) further demonstrates that price level rules are somewhat insensitive to the persistence of the shocks.}

Along these results, here the novel finding concerns the inverse relationship that arises between the optimal weight placed on durables inflation $\tau$ and sectoral labor mobility. Uniformly, the higher weight in the inflation composite is assigned to nondurables ($\tau < 0.5$), as this is the sector with the higher price stickiness. However, an inverse relationship between sectoral labor mobility and the optimal weight placed on durables inflation arises. As
labor becomes more mobile (i.e. $\lambda$ increases) the central bank finds it optimal to place even less weight on durables inflation (i.e. optimal $\tau$ decreases). Indeed, when labor is perfectly mobile ($\lambda = \infty$), no weight is assigned at all. The intuition is that, with more mobile labor, adjustments to shocks easily occur through quantities (via the reallocation of labor itself) rather than prices, and the central bank finds it optimal to focus more on the sector with the higher price stickiness.

The lower panel of Table 3.3 shows that these results hold also when prices of durables are assumed to be flexible ($\vartheta_d = 0$). Interestingly, for a sufficiently limited degree of labor mobility, $\tau$ is still nonzero. This result is driven by nominal wage stickiness. In fact, wage stickiness affects firms’ marginal costs and their price setting behavior. The pass-through of sticky wages to the durables sector marginal cost induces the central bank to place some weight on inflation in this sector despite price flexibility. Figure 3.3 shows the relationship between the optimal $\tau$ and durables price stickiness, $\vartheta_d$, under sticky and flexible wages.\footnote{Labor mobility is limited at the estimated value of $\lambda = 1.1769$.} We notice that $\tau$ is a strictly increasing function of $\vartheta_d$ when wages are sticky. Even when durables prices are very flexible ($\vartheta_d$ approaches zero), the optimal weight on durables inflation is positive.\footnote{As sectoral labor mobility decreases, this result is exacerbated. Indeed, the lower panel of Table 3.3 shows that with low mobility and fully flexible durables prices the optimal weight on durables inflation is substantially positive.}
Conversely, with flexible wages a sufficiently high degree of durables price stickiness is needed for the central bank to place a positive weight on durables inflation.

Comparing the welfare losses with respect to the Ramsey policy (Table 3.3), we notice that when labor becomes perfectly mobile, the welfare loss is higher than in the cases of limited labor mobility. This is due to the procyclical optimal behavior of the nominal interest rate under the Ramsey policy and perfect labor mobility, which by construction cannot be replicated by the simple interest rate rules. Indeed, Figure 3.4 shows the impulse responses to a durables inflation shock under the optimized interest-rate rule.

Indeed, Figure 3.2 is an example of the optimal procyclicality of the nominal interest rate. Under perfect labor mobility, the social planner decreases the nominal interest rate despite the increase in inflation. Such behavior holds under the other structural shocks of the model and impulse responses are available upon request.

Figure 3.4: Responses to a durables price markup shock: optimized interest rate rule
By comparing them with those under the Ramsey policy (Figure 3.2) it is noticeable that, in the free mobility case, the responses of the nominal interest rate are dramatically different, thus yielding large welfare losses. In addition, with limited labor mobility, the response of the nominal interest rate under the Ramsey policy is almost constant, whereas it is positive under the optimized simple rule in order to bring inflation back to the target.

Implied relative volatilities in these exercises confirm existing findings in the literature and add further insights. In particular, Table 3.4 reports the standard deviation of durables inflation, wages and output relative to nondurables in the baseline model and under different degrees of labor mobility. In all cases the volatility of sectoral prices, wages and output in the durables sector is larger than in nondurables, which extends the results that Erceg and Levin (2006) obtain in a model with no sectoral labor mobility to a model with limited or perfect mobility. Obviously, under perfect labor mobility wages are the same in the two sectors, hence there is no difference in their volatility. In contrast, under limited labor mobility, the standard deviation of wages in durables is always larger than nondurables (about 4.7 times under the estimated degree of labor mobility) with this difference increasing as labor mobility decreases and adjustments to shocks occur predominantly through wages rather than the reallocation of labor. Conversely, the relative volatilities of prices and output in the two sectors are increasing functions of the degree of labor mobility (variables in the durables sector become much more volatile than in nondurables).

All in all, our results reveal that the degree at which workers are able to reallocate across sectors is crucial for the optimal design of monetary policy,
and an inverse relationship exists between sectoral labor mobility and the importance of the sector which displays relatively more flexible prices. Wage stickiness, via its substantial pass-through onto marginal costs, leads to a non-zero optimal weight on durables inflation even if durables prices were fully flexible.

3.4.4 Robustness to alternative calibrations and interest rate rules

In this section we perform two types of robustness checks. We first look at the implications of two alternative calibrations of price and wage stickiness. Then, we replace the monetary policy rule (3.32) with two alternative rules and compare the results with the baseline model.

The top panel of Table 3.5 shows the case of flexible wages ($\vartheta^w = 0$) whereas in the lower panel both durables prices and wages are flexible ($\vartheta_d = \vartheta^w = 0$). As shown in Figure 3.3 at the estimated limited degree of labor mobility the optimal weight on durables inflation drops as wages become flexible ($\tau$ falls from 0.1612 to 0.0214) and becomes zero as both frictions are removed. The main conclusions drawn in the previous section are carried over these two alternative calibrations: i) $\tau$ and $\lambda$ are negatively related hence a lower weight is assigned to durables inflation as labor becomes mobile; ii) the central bank finds it optimal to implement a price-level rule ($\rho_r = 1$).

As the optimal degree of interest rate inertia systematically hits the upper bound, it is natural to ask what would happen should we allow $\rho_r$ to be larger than 1. Interest rate rules as equation (3.32) with $\rho_r > 1$ are usually referred to as superinertial rules and have the desirable feature of adding further history dependence in policy as explained by Giannoni and Woodford (2003) and Giannoni (2014). The top panel of table 3.6 reports the optimized parameters of the baseline model when rule (3.32) can be superinertial, with $\rho_r \in [0, 5]$. It is noteworthy that for any degree of labor mobility: i) the central bank optimally chooses a superinertial rule, with $\rho_r$ hitting the upper bound as labor becomes more mobile; ii) the welfare losses with respect to the Ramsey policy are smaller with respect to the price-level rules; iii) crucially,
the negative relationship between labor mobility and the optimal weight on durables inflation survives. We then replace rule (3.32) with an interest rate rule that responds only to inflation and the deviation of output from its steady state. Following Schmitt-Grohe and Uribe (2007), this type of interest rate rule is typically labeled implementable rule and, after the appropriate reparametrization, reads as follows:

$$\log \left( \frac{R_t}{\bar{R}} \right) = \rho_r \log \left( \frac{R_{t-1}}{\bar{R}} \right) + \alpha_\pi \log \left( \frac{\bar{\Pi}_t}{\bar{\Pi}} \right) + \alpha_y \log \left( \frac{\bar{Y}_t}{\bar{Y}} \right). \quad (3.35)$$

The lower panel of Table 3.6 demonstrates that despite these modifications, the choice towards a price-level rule and the inverse relationship between labor mobility and the optimal weight on durables inflation still hold true. In addition, the implied welfare losses are similar to the baseline model.

### 3.5 Conclusion

As the New-Keynesian literature on two-sector models has demonstrated, setting the appropriate weights to sectoral inflations is a crucial task for a central bank in order to achieve its objectives. We look at this issue from

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_y$</th>
<th>$\alpha_{\Delta y}$</th>
<th>$\tau$</th>
<th>$100 \times \omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1</td>
<td>0.1570</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.1765</td>
<td>0.0274</td>
</tr>
<tr>
<td>1.1769</td>
<td>1</td>
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Table 3.5: Optimized monetary policy rules: robustness to alternative calibrations

Flexible wages

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<tr>
<th>$\lambda$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_y$</th>
<th>$\alpha_{\Delta y}$</th>
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</table>

Flexible durables prices and wages
Table 3.6: Robustness to alternative optimized monetary policy rule

<table>
<thead>
<tr>
<th></th>
<th>λ</th>
<th>ρ_τ</th>
<th>α_τ</th>
<th>α_γ</th>
<th>α_Δγ</th>
<th>τ</th>
<th>100 × ω</th>
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<td><strong>Superinertial rule</strong></td>
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<th>α_τ</th>
<th>α_γ</th>
<th>α_Δγ</th>
<th>τ</th>
<th>100 × ω</th>
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<td><strong>Implementable rule</strong></td>
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an angle the literature has so far overlooked. In particular, we study the relation between the degree of sectoral labor mobility and the optimal weight the central bank should assign to inflation in the sector with relatively more flexible prices (durables). We first estimate the model with Bayesian methods and find evidence of a limited sectoral labor mobility. Then, we exploit the estimated model to perform optimal monetary policy analysis. Under the Ramsey policy, the optimal responses to structural shocks are significantly altered by different degrees of sectoral labor mobility. Preventing labor from moving freely between sectors dramatically changes the optimal path of the policy rate, thus leading to different effects on sectoral and aggregate variables, as well as on welfare. We then let the central bank optimize the parameters of a simple monetary policy rule along with the weight on inflation in the durables sector for different degrees of labor mobility. Our main result is that conditional on the intensity of price stickiness in the durables sector, an inverse relationship between labor mobility and the optimal weight on the sector with relatively more flexible prices arises: a lower weight is assigned to durables inflation as the degree of labor mobility increases. Intuitively, with more mobile labor, adjustments to shocks easily occur through quantities (via the reallocation of labor itself) rather than prices, and the the central bank finds it optimal to focus more on the sector with the higher price stickiness. Wage stickiness also plays an important role on the optimal
weight of durables inflation. Via the pass-through on marginal costs it always implies a higher weight on durables inflation with respect to the case of flexible wages. In the design of optimal monetary policy, we also find that the central bank chooses to implement a price-level rule by introducing desirable history dependence in the model to reduce the volatility of prices, thus confirming results the literature has found in one-sector models. These results are confirmed by various robustness checks and point to a non-negligible role of sectoral labor mobility for the conduct of monetary policy.

Appendix

3.6 Data

We define the durables sector as the a composite of durable goods and residential investments whereas the nondurables sector comprises nondurables goods and services.

<table>
<thead>
<tr>
<th>Series</th>
<th>Definition</th>
<th>Source Mnemonic</th>
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<tr>
<td>$DUR^N$</td>
<td>Nominal Durable Goods</td>
<td>BEA Table 2.3.5 Line 3</td>
</tr>
<tr>
<td>$RI^N$</td>
<td>Nominal Residential Investment</td>
<td>BEA Table 1.1.5 Line 13</td>
</tr>
<tr>
<td>$ND^N$</td>
<td>Nominal Nondurable Goods</td>
<td>BEA Table 2.3.5 Line 8</td>
</tr>
<tr>
<td>$SN$</td>
<td>Nominal Services</td>
<td>BEA Table 2.3.5 Line 13</td>
</tr>
<tr>
<td>$P_{DUR}$</td>
<td>Price Deflator, Durable Goods</td>
<td>BEA Table 1.1.9 Line 4</td>
</tr>
<tr>
<td>$P_{RI}$</td>
<td>Price Deflator, Residential Investment</td>
<td>BEA Table 1.1.9 Line 13</td>
</tr>
<tr>
<td>$P_{ND}$</td>
<td>Price Deflator, Nondurable Goods</td>
<td>BEA Table 1.1.9 Line 5</td>
</tr>
<tr>
<td>$P_S$</td>
<td>Price Deflator, Services</td>
<td>BEA Table 1.1.9 Line 6</td>
</tr>
<tr>
<td>$Y^N$</td>
<td>Nominal GDP</td>
<td>BEA Table 1.1.5 Line 1</td>
</tr>
<tr>
<td>$P_Y$</td>
<td>Price Deflator, GDP</td>
<td>BEA Table 1.1.9 Line 1</td>
</tr>
<tr>
<td>$FFR$</td>
<td>Effective Federal Funds Rate</td>
<td>FRED FEDFUNDS</td>
</tr>
<tr>
<td>$N$</td>
<td>Nonfarm Business Sector: Average Weekly Hours</td>
<td>FRED PRS85006023</td>
</tr>
<tr>
<td>$W$</td>
<td>Nonfarm Business Sector: Compensation Per Hour</td>
<td>FRED COMPNFN</td>
</tr>
<tr>
<td>POP</td>
<td>Civilian Non-institutional Population, over 16</td>
<td>FRED CNP16OV</td>
</tr>
<tr>
<td>CE</td>
<td>Civilian Employment, 16 over</td>
<td>FRED CE16OV</td>
</tr>
</tbody>
</table>

Table 3.7: Data Sources

112
3.6.1 Durables and Residential Investments

1. Sum nominal series: $DUR^N + RI^N = DR^N$

2. Calculate sectoral weights of deflators: $\omega^D = \frac{DUR^N}{DR^N}; \omega^R = \frac{RI^N}{DR^N}$

3. Calculate Deflator: $P_D = \omega^D P_{DUR} + \omega^R P_{RI}$

4. Calculate Real Durable Consumption: $D = \frac{DUR^N + RI^N}{P_D}$

3.6.2 Nondurables and Services

1. Sum nominal series: $ND^N + S^N = NS^N$

2. Calculate sectoral weights of deflators: $\omega^{ND} = \frac{ND^N}{NS^N}; \omega^{S} = \frac{S^N}{NS^N}$

3. Calculate Deflator: $P_C = \omega^{ND} P_{ND} + \omega^{S} P_{S}$

4. Calculate Real Nondurable Consumption: $C = \frac{ND^N + S^N}{P_C}$

3.6.3 Data transformation for Bayesian estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$POP_{index}$</td>
<td>Population index</td>
<td>$POP_{index}$</td>
</tr>
<tr>
<td>$CE_{index}$</td>
<td>Employment index</td>
<td>$CE_{index}$</td>
</tr>
<tr>
<td>$Y^o$</td>
<td>Real per capita GDP</td>
<td>$\ln \left( \frac{Y^N}{\ln P_{index}} \right)$</td>
</tr>
<tr>
<td>$I_D^o$</td>
<td>Real per capita consumption: durables</td>
<td>$\ln \left( \frac{I_D^N}{\ln P_{index}} \right)$</td>
</tr>
<tr>
<td>$C^o$</td>
<td>Real per capita consumption: nondurables</td>
<td>$\ln \left( \frac{C^N}{\ln P_{index}} \right)$</td>
</tr>
<tr>
<td>$W^o$</td>
<td>Real wage</td>
<td>$\ln \left( \frac{W}{\ln P_{index}} \right)$</td>
</tr>
<tr>
<td>$N^o$</td>
<td>Hours worked per capita</td>
<td>$\ln \left( \frac{H \times CE_{index}}{\ln P_{index}} \right)$</td>
</tr>
<tr>
<td>$\Pi_C^o$</td>
<td>Inflation: nondurables sector</td>
<td>$\Delta \left( \ln P_C \right)$</td>
</tr>
<tr>
<td>$\Pi_D^o$</td>
<td>Inflation: durables sector</td>
<td>$\Delta \left( \ln P_D \right)$</td>
</tr>
<tr>
<td>$R^o$</td>
<td>Quarterly Federal Funds Rate</td>
<td>$FFR_4$</td>
</tr>
</tbody>
</table>

Table 3.8: Data transformation - Observables
3.7 Symmetric equilibrium

\[ X_t = Z_t^{1-\alpha} D_t^\alpha \]  
\[ Z_t = C_t - \zeta S_{t-1} \]  
\[ S_t = \rho_c S_{t-1} + (1 - \rho_c) C_t \]  
\[ U (X_t, N_t) = \log (X_t) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} \]  
\[ U_{Z,t} = (1 - \alpha) \]  
\[ U_{D,t} = \alpha D_t \]  
\[ U_{N,t} = -\nu N_t^\varphi \]  
\[ N_t = \left[ (\chi^C)^{-\frac{1}{\lambda}} (N_t^C)^{\frac{1+\lambda}{\lambda}} + (1 - \chi^C)^{-\frac{1}{\lambda}} (N_t^D)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{1}{1+\lambda}} \]  
\[ N_t^C = \chi^C \left( \frac{w_t^C}{w_t} \right)^\lambda N_t \]  
\[ N_t^D = \chi^D \left( \frac{w_t^D}{w_t} \right)^\lambda N_t \]  
\[ \Lambda_{t,t+1} \equiv \beta \frac{U_{Z,t+1} e_{t+1}^B}{U_{Z,t} e_t^B} \]  
\[ \frac{e_t^W}{\hat{\mu}_t} = [e_t^W \eta - 1] + \vartheta^W (\Pi_t^W - \Pi_t^C) \Pi_t^W + \]  
\[ + E_t \left[ \Lambda_{t,t+1} \vartheta^W (\Pi_{t+1}^W - \Pi_t^C) \Pi_{t+1}^W \frac{w_{t+1}N_{t+1}}{w_t N_t} \right] \]  
\[ \hat{\mu}_t = -\frac{U_{Z,t}}{U_{N,t}} \]  
\[ Q_t \psi_t = \frac{U_{D,t}}{U_{Z,t}} + (1 - \delta) E_t [\Lambda_{t,t+1} Q_{t+1} \psi_{t+1}] \]  
\[ 1 = E_t \left\{ \Lambda_{t,t+1} \psi_{t+1} \frac{Q_{t+1}}{Q_t} e_{t+1}^I \left[ S' \left( \frac{I_{t+1}^D}{I_t^D} \right) \left( \frac{I_{t+1}^D}{I_{t-1}^D} \right)^2 \right] \right\} + \]  
\[ + \psi e_t^I \left[ 1 - S \left( \frac{I_{t}^D}{I_{t-1}^D} \right) - S' \left( \frac{I_{t}^D}{I_{t-1}^D} \right) \frac{I_{t}^D}{I_{t-1}^D} \right] \]
\begin{align*}
S \left( \frac{I_{t}^D}{I_{t-1}^D} \right) &= \frac{\phi}{2} \left( \frac{I_{t}^D}{I_{t-1}^D} - 1 \right)^2 \quad (3.51) \\
S' \left( \frac{I_{t}^D}{I_{t-1}^D} \right) &= \phi \left( \frac{I_{t}^D}{I_{t-1}^D} - 1 \right) \quad (3.52) \\
1 &= E_t \left[ \Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right] \quad (3.53) \\
\Pi_t^D &= \Pi_t^C \frac{Q_t}{Q_{t-1}} \quad (3.54) \\
\Pi_t^W &= \frac{w_t}{w_{t-1}} \tilde{\Pi}_t \quad (3.55) \\
Y_t^C &= e_t^A N_t^C \quad (3.56) \\
Y_t^D &= e_t^A N_t^D \quad (3.57) \\
e_t^C \epsilon_c MC_t^C &= (e_t^C \epsilon_c - 1) + \vartheta_c (\Pi_t^C - \Pi^C) \Pi_t^C - \vartheta_c E_t \left[ \Lambda_{t,t+1} \frac{Y_t^C}{Y_{t+1}^C} \left( \Pi_{t+1}^C - \Pi^C \right) \Pi_{t+1}^C \right] \quad (3.58) \\
MC_t^C &= \frac{w_t}{e_t^A} \quad (3.59) \\
e_t^D \epsilon_d MC_t^D &= (e_t^D \epsilon_d - 1) + \vartheta_d (\Pi_t^D - \Pi^D) \Pi_t^D - \vartheta_d E_t \left[ \Lambda_{t,t+1} \frac{Y_t^D}{Y_{t+1}^D} \left( \Pi_{t+1}^D - \Pi^D \right) \Pi_{t+1}^D \right] \quad (3.60) \\
MC_t^D &= \frac{w_t}{e_t^A Q_t} \quad (3.61) \\
\tilde{\Pi}_t &= (\Pi_t^C)^{1-\tau} (\Pi_t^D)^{\tau} \quad (3.62) \\
\log \left( \frac{R_t}{\bar{R}} \right) &= \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \alpha_x \log \left( \frac{\tilde{\Pi}_t}{\Pi} \right) + \alpha_y \log \left( \frac{Y_t}{Y_{t-1}^f} \right) + \alpha_{\Delta y} \left[ \log \left( \frac{Y_t}{Y_{t-1}^f} \right) - \log \left( \frac{Y_{t-1}}{Y_{t-1}^f} \right) \right], \quad (3.63) \\
Y_t^C &= C_t + e_t^C + \frac{\vartheta_c}{2} \left( \Pi_t^C - \Pi^C \right)^2 Y_t^C \quad (3.64) \\
Y_t^D &= I_t^D + \frac{\vartheta_d}{2} \left( \Pi_t^D - \Pi^D \right)^2 Y_t^D \quad (3.65) \\
Y_t &= Y_t^C + Q_t Y_t^D + \frac{\vartheta_w}{2} \left( \Pi_t^W - \Pi^C \right)^2 w_t N_t \quad (3.66)
\end{align*}
### 3.8 Steady state

In the deterministic steady state all expectation operators are removed and for each variable it holds that \( x_t = x_{t+1} = x \). Moreover, the stochastic shocks are absent. The steady-state inflation rate in the nondurables sector is the optimal under the Ramsey policy and is denoted by \( \Pi_{opt}^C \). \( C \) solves equation (3.64) whereas all other variables can be found recursively from the following relationships:

\[
\Pi^D = \Pi_{opt}^C \quad (3.67) \\
\Pi' = \Pi_{opt}^C \quad (3.68) \\
\Pi^W = \Pi_{opt}^C \quad (3.69) \\
\Lambda = \beta \quad (3.70) \\
R = \frac{1}{\beta} \quad (3.71) \\
MC_t^C = \frac{\epsilon_c - 1}{\epsilon_c} \quad (3.72) \\
w^C = MC^C e^A \quad (3.73) \\
w^D = w^C \quad (3.74) \\
w = w^D \quad (3.75) \\
MC_t^D = \frac{\epsilon_d - 1}{\epsilon_d} \quad (3.76) \\
Q = \frac{w^D}{MC^D e^A} \quad (3.77) \\
S = 0 \quad (3.78) \\
S' = 0 \quad (3.79) \\
\psi = 1 \quad (3.80) \\
\bar{\mu} = \frac{\eta}{\eta - 1} \quad (3.81) \\
S = C \quad (3.82) \\
Z = (1 - \zeta) C \quad (3.83) \\
U_Z = \frac{(1 - \alpha)}{Z} \quad (3.84) \\
U_D = U_Z Q \psi [1 - (1 - \delta) \beta] \quad (3.85)
\]
\[ D = \frac{\alpha}{U_D} \]  
\[ U_N = - \frac{U_z \nu}{\mu} \]  
\[ N = - \left( \frac{U_N}{\nu} \right)^{\frac{1}{\nu}} \]  
\[ Y^D = \delta D \]  
\[ N^D = Y^D \]  
\[ N^C = N - N^D \]  
\[ \chi^C = \frac{N^C}{N} \]  
\[ e^G = g_y Y \]  
\[ Y^C = N^C \]  
\[ X = Z^{1-a} D^a \]

### 3.8.1 Alternative calibration of steady state hours

Throughout the paper, and following the relevant literature, we have defined
\[ U(X_t, N_t) = \log(X_t) - \nu \frac{N_t^{1+\varphi}}{1+\varphi} \] such that hours worked enter the utility function in a separable fashion. We then normalized steady state hours to 1/3 and found the value of the scale parameter \( \nu \) consistent with this normalization. However, defining hours as \( N_t \) rather than the complement to one of leisure (i.e. \( 1 - L_t \)) implies that hours are dimensionless so targeting a steady state value of 1/3 is arbitrary and the parameter \( \nu \) has merely the effect of scaling the steady state values of the other variables. Moreover, \( \nu \) does not have any effect in the log-linearized model hence the dynamic properties of the model are not affected by either targeting steady state hours to be 1/3 or by calibrating \( \nu = 1 \) and calculating the resulting steady state value of hours. Indeed, consider the labor supply of the patient households (the same applies to impatients):
\[ \nu N_t^\varphi = U_{z,t} \nu_t. \]  
Log-linearizing it around the steady-state yields:
\[ \dot{N}_t = \frac{1}{\varphi} \left( \dot{U}_{z,t} + \dot{\nu}_t \right), \]
where variables with ` are expressed in log-deviations from the steady-state. It is clear that the log-deviations of hours from steady state do not depend on the constant $\nu$ whose value does have any effect on the dynamics of hours.

### 3.9 Estimation diagnostics

In this section we report some estimation diagnostics to verify that the model is correctly estimated. Figure 3.5 plots the prior and posterior distributions of all the estimated parameters. Overall, the posterior distributions are quite apart from the prior thus implying that the data is informative to identify the parameters. Then, we verify that the two parallel chains of the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm have actually converged. Figure 3.6 plots the multivariate convergence diagnostic for the Brooks and Gelman (1998) diagnostics (upper panel) and for the second and third central moments (middle and lower panels, respectively). The fact that the red and blue lines are very close to each other implies that the two chains have almost certainly converged. To sum up, estimation diagnostics reveal that the model is correctly estimated.
Figure 3.5: Prior and posterior distributions of estimated parameters. Prior: black-dashed line; Posterior: blue-solid line.
Figure 3.6: Prior and posterior distributions of estimated parameters. Prior: black-dashed line; Posterior: blue-solid line.
Chapter 4

Shocks and Policy Stance in the Euro Area

4.1 Introduction

Since its establishment in 1999, the Euro Area (EA) experienced two large crises. Like the US and all major economies in the world, the region was hit first by the 2008-2009 financial crisis but, differently from the US, it soon afterwards experienced a sovereign debt crisis that challenged the solvency of some of its member countries. Both crises required bold policy responses and triggered a lively debate both in policymaking and academia on the design and effectiveness of fiscal and monetary policies in the eurozone.

On the monetary policy side, following the financial crisis, the European Central Bank (ECB) reacted by lowering the nominal interest rate to provide stimulus and fight the recession. In contrast, many fiscal policymakers faced a trade-off between providing fiscal stimulus and dealing with the public debt overhang that manifested itself in the years 2009-2011, starting with the Greek crisis and then spreading to the rest of the EA, in particular to Ireland, Italy, Spain and Portugal.

The situation became even more challenging in 2012 when the ECB’s policy rate reached its effective zero-lower-bound (ZLB), leaving the conventional monetary policy without firepower to stimulate the economy and fight
deflation. Thus, the ECB started to implement a series of unconventional monetary policy measures, often labeled as Quantitative Easing (QE), that became part of the so-called Asset Purchase Programme (APP). The APP expanded the ECB’s balance sheet, via the purchase of both public and private securities the former of which fell under the Public Sector Purchase Programme (PSPP). This represented the largest portion of the program.\footnote{ECB’s holdings of public securities account, on average, for 76% of the total asset holdings under the APP since March 2015 (source: ECB’s History of cumulative purchase breakdowns under the APP, see https://www.ecb.europa.eu/mopo/implement/omt/html/index.en.html).}

In the meantime, in response to the sovereign debt crisis, several countries engaged in austerity measures aimed at keeping public debt under control and restoring confidence in financial markets.

This picture suggests that in addition to the usual macroeconomic and financial shocks, contrasting types of policy shocks—expansionary and contractionary—hit the eurozone in its recent past. Disentangling exogenous shocks, notably credit and demand shocks, from policy shocks due to fiscal and monetary actions is thus key to understand recent cyclical developments in the area, and can shed light on the appropriateness of the current policy stance in the area relative to the state of the credit cycle and economic confidence.

To this end, this paper attempts to shed light precisely on which shocks are the main drivers of the EA business cycle since its establishment, and what role fiscal and monetary policies have had in shaping economic outcomes. The paper also seeks to measure the current level of policy stance, and discuss its role in supporting the strength of the euro area and global recovery, and in helping recover stronger inflation dynamics in line with the area’s definition of mid-term price stability.

Our analytical tool is an estimated dynamic stochastic general equilibrium (DSGE) model of the EA augmented with specific features crucial for the analysis. First, we account for households’ limited asset market participation, as in related studies on the EA (see, Forni et al. 2009, Ratto et al. 2009, Coenen et al. 2012, 2013 and Albonico et al. 2016 among others). This feature is key to introduce New-Keynesian effects of fiscal poli-
cies, which would otherwise be absent, as explained by Mankiw (2000) and Gali et al. (2007). Second, we introduce a financial sector in which financial intermediaries (FIs, henceforth) purchase long-term private and government bonds, as in Carlstrom et al. (2017). This modeling block is important for two reasons. On one hand, FIs provide funds for private investment financed by collecting short-term deposits and accumulating net worth. This maturity transformation performed by FIs introduces a transmission channel for credit shocks, which had a prominent role during the Global Financial Crisis (GFC). On the other hand, the fact that FIs hold long-term government debt implies that QE, by impacting the banks’ balance sheets, has real effect on the economy. FIs have been introduced in DSGE models of the EA also by Gerali et al. (2010) and Kollmann et al. (2013). These papers, however, do not allow FIs to hold government debt, hence they do not take this important transmission channel of unconventional monetary policy into account. Third, we specify a detailed government sector whereby government debt is long-term and, as already anticipated, can also be purchased by the central bank. The fiscal authority levies distortionary taxes to finance expenditures and stabilize government debt, via fiscal rules that allow also for automatic stabilizers. Fourth, the central bank conducts both conventional and unconventional monetary policy by setting the short-term interest rate and purchasing long-term government debt, respectively, in the spirit of Chen et al. (2012) and Gertler and Karadi (2013), among others.

Our estimates suggest that preference and price markup shocks have been important drivers of the EA business cycle. Interestingly, in accordance with the findings of Gerali et al. (2010) and Kollmann et al. (2013), we find evidence for a substantial contribution of expansionary credit shocks to the cyclical component of EA’s output before the financial crisis, followed by a large negative contribution after its outburst.

Gerali et al. (2010) highlight the role of credit supply factors in the build-up of the financial crisis whereas Kollmann et al. (2013) conclude that government support for banks dampened the effects of the financial crisis on the EA economy.

The same modeling of QE has been employed by Carlstrom et al. (2017) and Hohberger et al. (2017). Kollmann et al. (2016) also make the same argument in a model without financial intermediaries, where the financial shock is proxied by a shock to the discount factor.
As far as monetary and fiscal policies are concerned, we focus on their historical joint and individual contributions on EA output. In general, we refer to policies as contractionary (expansionary) whenever their contribution to output is negative (positive). Moreover, we label policies as countercyclical (procyclical) whenever their contribution has the opposite (same) sign as the output deviation from trend.

Our main findings regarding the role of monetary and fiscal shocks are as follows. First, we find that fiscal policy has generally played a countercyclical role during the sample of estimation. One exception is represented by the last part of our sample, when monetary policy was constrained by the ZLB, in which we estimate that discretionary fiscal shocks had a negative contribution to output fluctuations despite output being below trend. Here, the austerity measures implemented in the EA to fight the sovereign debt crisis are estimated to have caused the negative contribution of fiscal policy on output. Second, we estimate a larger contribution of monetary policy in the overall discretionary policy stance. This was expansionary until the ZLB started binding in 2012, while it was contractionary afterwards. Third, with the interest rate stuck at the ZLB and a tight fiscal policy, the implementation of QE since 2015 has helped stabilize EA GDP although it has not been countercyclical enough to undo the procyclical impact of EA fiscal policy over that period. Taken together, these results lead to the conclusion that a more expansionary fiscal policy in countries with fiscal space could have facilitated the recovery of the EA from the financial and sovereign debt crises.

Recent studies have assessed the role of fiscal policy on the EA. Forni et al. (2009) make the argument that tax cuts are more expansionary than expenditure increases. Along these lines, Coenen et al. (2013) conclude that the fiscal stimulus package implemented in the EA, known as the European Economic Recovery Plan, generated a fiscal multiplier smaller than one since it comprised both revenues and expenditure measures. Moreover, while Ratto of intermediate firms which affects their investment decisions. However, by specifying a financial sector, as we do, it is possible to disentangle the role of investment-specific shocks from that of credit shocks.
et al. (2009) find a general countercyclical role of fiscal policy before the financial crisis. Kollmann et al. (2016) argue that austerity measures weighed on the EA recovery until the end of 2014, whereas Albonico et al. (2016) find evidence of muted fiscal policy.

Our results add to this literature along two dimensions. First, we assess the joint contribution of both fiscal and monetary policies to determine the overall fiscal policy stance of the EA. Second, we bring unconventional monetary policy into the picture.

The remainder of the paper is organized as follows. Section 4.2 describes the DSGE model. Section 4.3 presents the Bayesian estimation and the results while Section 4.4 concludes. The Appendix provides details about the dataset, the theoretical model, and complementary results.

### 4.2 Model

The core of the model is a quite standard New-Keynesian setting with the usual nominal and real frictions, as in Smets and Wouters (2007). These are price and wage stickiness, investment adjustment costs (IAC, henceforth) and habit formation in consumption. The model is further augmented with (i) two types of households—optimizers (or Ricardian) and rule-of-thumbers (or non-ricardian)—as in Galí et al. (2007); (ii) financial intermediaries accumulating net worth and short-term liabilities to finance the purchase of long-term private investment and government bonds as in Carlstrom et al. (2017); (iii) a central bank conducting quantitative easing beside a conventional interest rate policy; (iv) a detailed fiscal policy whereby the government purchases goods and services, provides transfers and finances the budget with a mixture of debt of different maturities and distortionary taxes.

This section outlines the details of the model. The full set of equilibrium conditions and the deterministic steady state are reported in appendices 4.6 and 4.7, respectively.

Hohberger et al. (2017) instead focus only on the effects of QE on the EA GDP growth neglecting any role for fiscal policies.
4.2.1 Households

The economy is populated by a continuum \( i \in [0, 1] \) of households, a fraction \( \omega \) of which is optimizer (\( o \)), while the remaining fraction \( 1 - \omega \) is rule-of-thumb (\( r \)). Optimizing households have access to financial markets hence they smooth consumption via the purchase of short-term deposits whereas rule-of-thumb households do not have access to saving or borrowing, hence each period they consume their entire disposable income. As common in the literature, preferences are assumed to be identical across the two types of households.

4.2.1.1 Optimizing households

Optimizing households derive utility from consumption, \( C^o_t \) and disutility from providing labor services, \( H^o_t \), in a monopolistically competitive labor market. The intertemporal utility function is given by

\[
E_t \left\{ \sum_{s=0}^{\infty} e_t^h \beta^{t+s} \left[ \ln \left( C_{t+s}^o - hC_{t+s-1}^o \right) - \frac{B}{1 + \eta} \left( H_{t+s}^o \right)^{1+\eta} \right] \right\},
\]

where \( E_t \) is the expectation operator at time \( t \), \( e_t^h \) is a preference shock to the discount factor \( \beta \in (0, 1) \), \( h \in (0, 1) \) is the internal habit formation parameter, \( B > 0 \) is the disutility weight of labor, \( \eta > 0 \) is the inverse of the intertemporal elasticity of substitution of the labor supply. Optimizing households have access to two assets: short-term deposits \( D^o_t \) in financial intermediaries (FIs henceforth) and physical capital \( K^o_t \). They may also hold short-term government bonds (T-bills) and short-term debt issued by the central bank to finance its QE programme, but since these are perfect substitute with deposits and move endogenously to hit the central bank’s short term interest rate target, \( D^o_t \) can be treated as the households’ net resource flow into FIs. The need for financial intermediation arises because all the investment, \( I^o_t \), needs to be financed by issuing long-term bonds purchased by FIs \( ^6 \). As in \( \text{Carlstrom et al.} \ (2017) \), these are assumed to be perpetual

\( ^6 \)This assumption is needed to make FIs relevant in the model.
bonds with cash flows 1, \( \kappa \), \( \kappa^2 \), ... (see, e.g. Woodford [2001]). In other words, if \( Q_t \) is the time-\( t \) price of a new issue, \( \kappa Q_t \) is the time-\( t \) price of the perpetuity issued in period \( t - 1 \) and so on. The duration of these bonds is defined by \((1 - \kappa)^{-1}\) and their gross yield to maturity by \( Q^{-1} + \kappa \). Define \( CI_t \) as the number of perpetuities issued at time \( t \) to finance investment, then the representative household’s overall nominal liability is:

\[
F^o_t = CI_t + \kappa CI_{t-1} + \kappa^2 CI_{t-2} + \ldots, \tag{4.2}
\]

with the time-\( t \) new issue of perpetuities defined as

\[
CI_t = F^o_t - \kappa F^o_{t-1}. \tag{4.3}
\]

In maximizing life-time utility (4.1), optimizing households face three constraints:

\[
(1 + \tau^c_t) C^o_t + \frac{D^o_t}{P_t} + P^k_t I^o_t + \frac{F^o_{t-1}}{P_t} \leq (1 - \tau^w_t) W^o_t H^o_t + \frac{R_{t-1} D^o_{t-1}}{P_t} + (1 - \tau^k_t) R^o_t K^o_t + \frac{Q_t (F^o_t - \kappa F^o_{t-1})}{P_t} + \tau^l_t - \Phi_t + \mathcal{P}_t, \tag{4.4}
\]

\[
K^o_{t+1} \leq (1 - \delta) K^o_t + I^o_t, \tag{4.5}
\]

\[
P^k_t I^o_t \leq \frac{Q_t (F^o_t - \kappa F^o_{t-1})}{P_t} = Q_t CI_t. \tag{4.6}
\]

The consumption good is the numeraire of the economy hence \( P_t \) is the price level. \( P^k_t \) is the real price of capital while \( R^k_t \) is its real rental rate; \( R_{t-1} \) is the gross nominal interest rate on deposits; \( W_t \) is the real wage; \( \delta \in (0, 1) \) is the capital depreciation rate; \( \tau^c_t, \tau^w_t \) and \( \tau^k_t \) are distortionary tax rates on consumption, labor income and the return on capital, respectively; \( \delta P^k_t \tau^k_t K^o_t \) is a depreciation allowance for tax purposes; \( \tau^l_t \) is a lump-sum transfers; \( \Phi_t \) is a labor union membership fee; and \( \mathcal{P}_t \) are profits from all financial and non-financial firms. Equation (4.4) is the households’ budget constraint, equation (4.5) represents the capital accumulation equation, while equation (4.6) is a “loan-in-advance” constraint, which increases the cost of purchasing investment goods, i.e. all investment projects must be financed by issuing
perpetuities purchased by FIs, therefore the total expenditure on investment cannot exceed the total value of perpetuities.

Assuming that constraints hold with equality, substituting for (4.5) into (4.4) and (4.6), and taking first-order conditions with respect to $C^o_t, D^o_t, K^o_{t+1}$ and $F^o_t$ yields:

\[
(1 + \tau^c_t) \Lambda^o_t = \frac{e^b_t}{C^o_t - hC^o_{t-1}} - h\beta E_t \left[ \frac{e^b_{t+1}}{C^o_{t+1} - hC^o_t} \right], \quad (4.7)
\]

\[
\Lambda^o_t = \beta E_t \left[ \frac{\Lambda^o_{t+1} \frac{R_t}{\Pi_{t+1}}}{\Pi_{t+1}} \right], \quad (4.8)
\]

\[
\Lambda^o_t P^k_t M_t = \beta E_t \left\{ \Lambda^o_{t+1} \left[ (1 - \tau^k_{t+1}) R^k_{t+1} + \right. \right.
\]
\[
\left. \left. + \delta P^k_{t+1} \tau^k_{t+1} + (1 - \delta) P^k_{t+1} M_{t+1} \right] \right\}, \quad (4.9)
\]

\[
\Lambda^o_t Q_t M_t = \beta E_t \left[ \Lambda^o_{t+1} \frac{1 + \kappa Q_{t+1} M_{t+1}}{\Pi_{t+1}} \right], \quad (4.10)
\]

where $\Lambda^o_t$ is the Lagrange multiplier associated to the budget constraint (4.4), $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the gross inflation rate, and $M_t \equiv 1 + \frac{\vartheta_t}{\Lambda_t}$ is a dynamic distortion caused by market segmentation, where $\vartheta_t$ is in turn the multiplier associated to the loan-in-advance constraint (4.6). As shown by Carlstrom et al. (2017), the distortion $M_t$ arises because of financial market segmentation (to be explained in further detail below) and is approximated by the discounted sum of the spread between the one-period loan and the deposit rates. Equation (4.7) is the marginal utility of consumption which, together with equation (4.8), determines the usual Euler equation of consumption. Equations (4.9) and (4.10) are asset price equations for capital and investment bonds, respectively.

### 4.2.1.2 Rule-of-thumb households

Rule-of-thumb households have the same instantaneous utility function as that of intertemporal optimizing consumers:

\[
E_t \left\{ \sum_{s=0}^{\infty} \epsilon^b_t \beta^{t+s} \left[ \ln \left( C^r_{t+s} - hC^r_{t+s-1} \right) - \frac{B}{1 + \eta} \left( H^r_{t+s} \right)^{1+\eta} \right] \right\}. \quad (4.11)
\]
They do not have access to financial markets hence they cannot smooth consumption by saving and borrowing. It follows that their consumption is entirely determined by their budget constraint:

$$(1 + \tau^c_t) C^r_t = (1 - \tau^w_t) W_t H^r_t + \tau^l_t - \Phi_t,$$  \hspace{1cm} (4.12)

while their marginal utility of consumption, useful to derive the wage-setting equation in Section 4.2.1.3 is:

$$(1 + \tau^c_t) \Lambda^r_t = \frac{e^b_t}{C^r_t - h C^r_{t-1}} - h \beta_t E_t \left[ \frac{e^b_{t+1}}{C^r_{t+1} - h C^r_t} \right].$$  \hspace{1cm} (4.13)

The presence of rule-of-thumb households helps capturing Keynesian effects of fiscal policy as the economy is thus populated also by agents for which the Ricardian equivalence does not hold. Intuitively, the Keynesian effect of fiscal policy is larger the larger the share of rule-of-thumbers in the economy, i.e. the larger $1 - \omega$.

4.2.1.3 Wage setting

Each type of household provides labor to a continuum of labor unions $z \in [0, 1]$. Each union sets the wage rate for its members and aggregates labor services according to a Dixit-Stiglitz aggregator such that labor demanded by firms from each union is $H^z_t = \left( \frac{W^z_t}{W_t} \right)^{1-e^w_t} H_t$, where $e^w_t$ is the intratemporal elasticity of substitution between labor services and $e^w_t$ is a wage markup shock. As in Colciago (2011) and Furlanetto and Seneca (2012), among others, each period the union $z$ chooses the wage rate $W^z_t$ to maximize a weighted average utility of its members:

$$\max_{W^z_t} E_t \sum_{k=0}^{\infty} \beta^{t+k} [\omega U^o_{t+k} + (1 - \omega) U^r_{t+k}],$$  \hspace{1cm} (4.14)

subject to the labor demand functions and the households’ budget constraints (4.4) and (4.12). Each union is subject to quadratic adjustment costs of wages as in Rotemberg (1982), which are ultimately paid by its members through
a membership fee,

\[ \Phi_t = \frac{\theta_w}{2} \left[ \frac{W_t^2}{\Pi_{t-1}^w W_{t-1}^z} \Pi_t - \Pi_t^{1-\epsilon_w} \right]^2 W_t H_t, \]  

(4.15)

where \( \theta_w \) governs the degree of nominal wages stickiness while \( \epsilon_w \in [0, 1] \) denotes the degree of wage indexation to past inflation.\(^7\) Firms do not discriminate between optimizing and rule-of-thumb households, hence labor supply is identical across households, that is \( H_t^o = H_t^r = H_t \).\(^8\) The wage schedule thus reads as

\[
0 = \left\{ (1 - \pi_t^w) (1 - e_t^w \epsilon_t^w) - \theta_w \left[ \frac{\Pi_t^w}{\Pi_{t-1}^w} - \Pi_t^{1-\epsilon_w} \right] \frac{\Pi_t^w}{\Pi_{t-1}^w} \right\} + \frac{B(H_t)^n}{\Lambda_t W_t} e_t^w \epsilon_t^w \\
+ \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \theta_w \left[ \frac{\Pi_{t+1}^w}{\Pi_{t+1}^{\epsilon_w}} - \Pi_{t+1}^{1-\epsilon_w} \right] \frac{\Pi_{t+1}^w W_{t+1} H_{t+1}}{\Pi_{t+1}^{1-\epsilon_w} W_{t+1} H_{t+1}} \right\},
\]

(4.16)

where \( \Pi_t^w = \frac{W_t}{W_{t-1}} \Pi_t \) denotes the nominal wage inflation, \( \bar{\Lambda}_t = \omega \Lambda_t^o + (1 - \omega) \Lambda_t^r \) represents a weighted average of the marginal utilities of consumption across types of households, and the term \( \frac{B(H_t)^n}{\Lambda_t W_t} \) is the (inverse of the) wage markup, that is the marginal rate of substitution between consumption and leisure divided by the nominal wage.\(^9\) Setting \( \theta_w = 0 \) implies that nominal wages are flexible and set as a constant markup of the marginal rate of substitution between consumption and leisure.

### 4.2.1.4 Aggregation

Aggregate consumption is given by a weighted average of consumption of each type of consumers. Rule-of-thumb consumers do not hold any assets or liabilities, therefore aggregate deposits, liabilities, investment and capital

7Steady-state inflation \( \Pi \) is raised to the power \( 1 - \epsilon_w \) to ensure that wage adjustment costs are zero in steady-state. A similar specification applies to price adjustment costs, see Section 4.2.3.

8This is a standard assumption in models with rule-of-thumbers and nominal wage stickiness, see e.g. Colciago (2011), Furlanetto and Seneca (2012), Coenen et al. (2012) and Albonico et al. (2017), among many others.

9The detailed derivation of the wage setting equation is in Appendix 4.8.
reflect this feature:

\[ C_t = \omega C_t^o + (1 - \omega) C_t^o, \]  
\[ D_t = \omega D_t^o, \]  
\[ F_t = \omega F_t^o, \]  
\[ I_t = \omega I_t^o, \]  
\[ K_t = \omega K_t^o. \]  

(4.17) \hspace{1cm} (4.18) \hspace{1cm} (4.19) \hspace{1cm} (4.20) \hspace{1cm} (4.21)

### 4.2.2 Financial intermediaries

Financial intermediaries are modeled as in Carlstrom et al. (2017). They are the sole buyers of investment bonds \( F_t \) and long-term government bonds \( B_{t}^{\text{FI}} \), which are perfect substitutes, and hence are sold at the same price of a time-\( t \) issue \( Q_t \). The FI’s portfolio is financed by collecting deposits \( D_t \) from optimizing households and by accumulating net worth \( N_t \). Let \( \bar{F}_t = \frac{F_t}{P_t} Q_t \) and \( \bar{B}_{t}^{\text{FI}} = \frac{B_{t}^{\text{FI}}}{P_t} Q_t \) denote the real values of investment and long-term government bonds, respectively. Then, the FI’s balance sheet reads as:

\[ \bar{B}_{t}^{\text{FI}} + \bar{F}_t = \frac{D_t}{P_t} + N_t = L_t N_t, \]  

(4.22)

where \( L_t \) is leverage which is assumed to be taken as given by FIs while long-term investment and government bonds constitute the asset side of the FI’s balance sheet (4.22). Each period, FIs raise profits determined by the spread between the lending and borrowing rates. In particular, let \( \text{prof}_t \equiv (\bar{B}_{t}^{\text{FI}} + \bar{F}_t) R_t^L - \frac{D_t}{P_t} R_{t-1} \), then use (4.22) to define profits as:

\[ \text{prof}_t \equiv \left[ (R_t^L - R_{t-1}) L_{t-1} + R_t \right] \frac{N_{t-1}}{\Pi_t}, \]  

(4.23)

where \( R_t^L \equiv \left( \frac{1 + \kappa Q_t}{Q_{t-1}} \right) \) denotes the return on FIs’ assets.\(^{11}\) A share of the profits are then distributed to households as dividends \((\text{div}_t)\), while the rest

---

\(^{10}\)Long-term government bonds have exactly the same structure as investment bonds, hence they are modeled as perpetuities with maturity \((1 - \kappa)^{-1}\).

\(^{11}\)The interest rate paid on deposits, \( R_t^d \), equals the risk free (policy) rate hence we directly use \( R_t \) in the FIs’ profit function.
is retained as net worth. It follows that each FI chooses dividends and net worth to solve:

$$V_t \equiv \max_{N_t, div_t} E_t \sum_{j=0}^{\infty} (\beta \zeta)^j \Lambda_t^0 \Lambda_{t+j} \Lambda_{t+j}^{\theta} div_{t+j},$$

subject to the following budget constraint

$$div_t + N_t \left[1 + f(N_t)\right] \leq prof_t,$$

which states that dividends are limited by the amount of profits not devoted to net worth. The discount factor $\beta \zeta < \beta < 1$ implies that FIs discount future profits at a lower rate than optimizing households discount future utility, i.e. the former are more impatient than the latter. The portfolio adjustment cost function $f(N_t) = \psi_n \left(\frac{N_t - \bar{N}}{N_t}\right)^2$ prevents the FI from fully adjusting its assets side of the balance sheet in response to shocks, as governed by parameter $\psi_n \in [0, \infty]$. Financial frictions are introduced via a simple hold-up problem. Each period, before aggregate shocks realize, FIs can default on their debt towards depositors and retain a fraction $\Theta$ of the assets. It follows that, in order for optimizing households to be willing to lend to FIs, the following incentive compatibility constraint (ICC) must hold:

$$E_t V_{t+1} \geq \Theta_t L_t N_t E_t \Lambda_t^{\theta} \Pi_{t+1} \frac{R_{t+1}^L}{R_t},$$

according to which net worth limits the amount FIs can borrow from optimizing households, i.e. the expected value of the FI, $V_{t+1}$, needs to be at least as great as the amount it can divert. Variable $\Theta_t$ determines the extent of

---

12 As noted by Carlstrom et al. (2017), this assumption differs from Gertler and Karadi (2011; 2013), who assume that dividends are paid only upon the FIs’ exogenous death. However, this mechanism delivers the same implication as the one designed by Gertler and Karadi (2011; 2013), namely that FIs are not able to infinitely accumulate net worth hence they have to borrow in order to purchase investment and government bonds.

13 This assumption is necessary to induce FIs to borrow in equilibrium from households and is in the same spirit of the one employed by Iacoviello (2005) between patient and impatient households.

14 Note that $V_t$ is increasing in the spread between the return on assets $R_{t+1}^L$ and the interest rate on deposits $R_t$, while it is decreasing in $f(N_t)$. 

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the financial friction and depends negatively on $N_t$ and positively on an exogenous credit shock $e_t^\phi$. Unexpected increases in $e_t^\phi$ exacerbate the financial friction thus lowering real activity, with larger effects the larger the portfolio adjustment costs. Assuming that the ICC is binding, equation (4.26) can be explicitly defined as

$$L_t = \frac{E_t \left( \frac{N_{t+1}}{\Pi_{t+1}} \right)}{E_t \left( \frac{N_{t+1}}{\Pi_{t+1}} \right) + (e_t^\phi - 1) E_t \left( \frac{N_{t+1}}{\Pi_{t+1}} \right) \frac{R_{L_{t+1}}}{R_t}}.$$  (4.27)

It is evident that leverage depends only on aggregate variables and not on each FI’s net worth, hence only aggregate net worth is required to analyze the model’s dynamics. Aggregate net worth is chosen to maximize the representative FI’s value (4.24) subject to (4.25) and (4.27) thus yielding the following optimal accumulation of net worth:

$$\Lambda_t^* \left[ 1 + N_t f' (N_t) + f (N_t) \right] = E_t \left( \frac{N_{t+1}}{\Pi_{t+1}} \right) \left[ R_t + L_t (R_{L_{t+1}} - R_t) \right].$$  (4.28)

The main channel through which the financial friction affects real activity is a limit to arbitrage between the return on long-term bonds $R_{L_{t+1}}$ and the deposit rate $R_t$. The leverage constraint (4.27) poses a limit on the ability of the FI to collect deposits, which can be alleviated by a higher net worth. However, adjustments in net worth are lumpy thus limiting arbitrage. Indeed, increases in net worth allow the FI to collect deposits at a lower rate and exploit arbitrage opportunities with respect to the lending rate. A slow increase in net worth due to adjustment costs prevents the FI from taking advantage of these arbitrage opportunities. Given that investments are feasible only through financial intermediation, the FI’s inability to quickly adjust its net worth implies that central bank purchases of long-term government bonds alter the supply of those bonds, hence the composition of the FI’s portfolio and, ultimately, affect the real economy. Indeed, an increase in the central bank holdings of long-term government bonds decreases the amount held by FIs, which utilize the spare net worth to purchase investment bonds

\footnote{This is the reason why the single FI takes leverage as given in maximizing its value.}
causing an increase in private investment.

4.2.3 Non-financial firms

A continuum of monopolistically competitive firms indexed by \(i \in [0, 1]\) buys capital, \(K_{it-1}\), and hires labor, \(H_{it}\), to produce differentiated goods, \(Y_{it}\), with convex technology \(F(H_{it}, K_{it-1})\), sold at price \(P_{it}\), and faces a Dixit-Stiglitz firm-specific demand:

\[
Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon_t^p p} Y_t,
\]

(4.29)

where \(\epsilon^p\) is the elasticity of substitution across goods varieties and \(\epsilon_t^p\) is a price mark-up shock. At the end of period \(t-1\) firms acquire capital from capital producers for use in production in period \(t\). Firms also face quadratic price adjustment costs \(\theta_p^2 \left(\frac{P_{it}}{\Pi_{it-1}^p} - \Pi^{1-tp}\right)^2 Y_t\), as in Rotemberg (1982) – where parameters \(\theta_p \in [0, \infty]\) and \(t_p \in [0, 1]\) measure the degree of price stickiness and price indexation, respectively – and maximize the following flow of discounted profits:

\[
J_{it} = E_t \left\{ \sum_{s=0}^{\infty} \beta^{t+s} \frac{P_{it+s}^{\alpha}}{\Pi_{it+s-1}^p} Y_{it+s} - P_{it+s} R_{it+s} K_{it-1+s} - w_{it+s} H_{it+s} - \frac{\theta_p}{2} \left(\frac{P_{it+s}}{\Pi_{it+s-1}^p} - \Pi^{1-tp}\right)^2 Y_t \right\},
\]

(4.30)

with respect to \(K_{it+s}, H_{it+s},\) and \(P_{it+s}\), subject to the firm’s resource constraint

\[
Y_{it} = F(e_t^p, H_{it}, K_{it-1}),
\]

(4.31)

where \(F(e_t^p, H_{it}, K_{it-1}) = e_t^p K_{it-1}^{\alpha} H_{it}^{1-\alpha}\), with \(\alpha\) being the labor share of income and \(e_t^p\) being a total factor productivity shock. The corresponding first-order conditions for this problem are

\[
R_{it}^k = MC_t MPP_K t,
\]

(4.32)

\[
W_t = MC_t MPL_t,
\]

(4.33)
\[ 0 = 1 + e_t^p \varepsilon^p (MC_t - 1) - \theta_p \left( \frac{\Pi_t}{\Pi_{t-1}^{\varepsilon_p}} - \Pi^{1-\varepsilon_p} \right) \frac{\Pi_t}{\Pi_{t-1}^{\varepsilon_p}} + \theta_p E_t \left[ \beta \frac{\Lambda_{t+1}^\omega}{\Lambda_t^\omega} \left( \frac{\Pi_{t+1}}{\Pi_t^{\varepsilon_p}} - \Pi^{1-\varepsilon_p} \right) \frac{\Pi_{t+1}}{\Pi_t^{\varepsilon_p}} \frac{Y_{t+1}}{Y_t} \right], \] (4.34)

where \( MC_t \) is the Lagrange multiplier associated with resource constraint (4.31). In particular, \( MC_t \) is the shadow value of output and represents the firm’s real marginal cost, while \( MPK_t = \alpha e_t^a H_t^{1-\alpha} K_{t-1}^{\alpha-1} \), and \( MPL_t = (1-\alpha) e_t^a K_{t-1}^{\alpha} H_t^{-\alpha} \) are the marginal products of capital and labor, respectively. When prices are flexible (\( \theta_p = 0 \)), equation (4.34) implies that prices are set as a constant markup over the marginal cost.

### 4.2.4 New capital producers

New capital is produced by firms that take investment goods \( I_t \) and converts them into \( P_t^k e_t^\mu \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \) units of new capital goods. These firms are owned by optimizing households and maximize the following profit function:

\[ P_t^k e_t^\mu \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t - I_t, \] (4.35)

where \( S \left( \frac{I_t}{I_{t-1}} \right) \equiv \frac{\psi_t}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \) represents the investment adjustment costs as governed by parameter \( \psi_t \), while \( e_t^\mu \) is an investment-specific shock\(^{16}\). Maximizing (4.35) with respect to \( I_t \) yields the following asset price equation for investment:

\[ P_t^k e_t^\mu \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] = 1 - E_t \left[ \beta \frac{\Lambda_{t+1}^\omega}{\Lambda_t^\omega} e_{t+1}^\mu P_{t+1}^k S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right]. \] (4.36)

\(^{16}\)As in Christiano et al. (2005), the adjustment costs function \( S(\cdot) \) satisfies \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \).
4.2.5 Policymakers

4.2.5.1 Central bank

The central bank conducts both conventional and unconventional monetary policies. Conventional monetary policy is set according to the following Taylor-type interest-rate rule, which determines the nominal interest rate:

$$\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) +$$

$$+ (1 - \rho_r) \left\{ \rho_r \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \right\} + \epsilon_t^n,$$

(4.37)

where $\rho_r$ is the interest rate smoothing parameter, $\rho_\pi$ and $\rho_y$ are the monetary responses to inflation and output, and $\epsilon_t^n$ is a AR(1) monetary policy shock.

Unconventional monetary policy (or QE) refers to the purchase of long-term government bonds at the market price to alleviate periods of economic distress, financed by issuance of riskless short-term debt $\bar{B}_S$ at the interest rate $R_t$, as in Gertler and Karadi (2013). This implies that the central bank borrows from households at the riskless rate and lends to the government at the rate $R_{t+1}$ thus earning profits from the spread between the two. Let $\bar{B}_t \equiv \frac{Q_t B_t}{P_t}$ be the real value of long-term government debt. The central bank holds a fraction $\bar{B}^{CB}_t = \psi_t \bar{B}_t$, where $\psi_t \in [0, 1)$, while the remaining fraction $\bar{B}^{FI}_t = (1 - \psi_t) \bar{B}_t$ is held by FIs. Following Gertler and Karadi (2011, 2013), Chen et al. (2012) and Carlstrom et al. (2017), among others, the central bank sets unconventional monetary policy by varying $\psi_t$, which follows an AR(1) process. Each period, the central bank’s balance sheet implies that assets $\bar{B}^{CB}_t$ are fully covered by liabilities $B^*_t$ and profits from QE are transferred to the government.

The introduction of long-term bonds entails the presence of a term pre-

\[\text{[17] We follow Chen et al. (2012), Gertler and Karadi (2013), Quint and Rabanal (2017), Carlstrom et al. (2017) and Hobberger et al. (2017) in setting an exogenous process for the central bank’s asset purchases because we want it to be active only in those quarters when purchases by the ECB took place. Conversely, Gertler and Karadi (2011) and Ellison and Tischbirek (2014) employ feedback rules.} \]

\[\text{[18] See Section 4.2.5.2 and Appendix 4.9 for details about the consolidated government budget constraint and its derivation, which mimics Gertler and Karadi (2013).}\]
mium in the economy. Consider a ten-year bond, then the term premium is defined as the difference between the observed yield on the this bond and the corresponding yield calculated by applying the expectation hypothesis (EH) of the term structure to the series of short rates (see Carlstrom et al., 2017). Let the yield on the ten-year bond under EH be

\[ R_{10,EH}^t = \kappa + \frac{1}{Q_{EH}^t}, \]

with its price satisfying

\[ R_t = 1 + \kappa Q_{EH}^t. \]

Then, in gross terms, the term premium is defined as

\[ TP_t = 1 + R_{10}^t - R_{10,EH}^t. \]

A QE policy shock alters the supply of long-term government bonds, hence their price \( Q_t \), their yield \( R_{10}^t \), and ultimately the term premium. An increase in the fraction of long-term government bonds held by the central bank will reduce their supply, increasing the price \( Q_t \), lowering the yield \( R_{10}^t \) and the term-premium. The joint effects of asset purchases on the term premium and the FI’s balance sheet stimulates investment and real activity.

4.2.5.2 Fiscal authority

The government finances government spending, \( G_t \) and lump-sum transfers \( \tau^l_t \), via long-term debt and a mix of distortionary taxes \( T_t \). In addition, it receives profits from the central bank’s asset purchases hence the consolidated budget constraint reads as:

\[ \bar{B}_t = \frac{R^L_t}{\Pi_t} \bar{B}_{t-1} + \bar{G}_t - T_t - \frac{R^L_t - R_{t-1}}{\Pi_t} \bar{B}^{CB}_{t-1}, \]

where \( \bar{G}_t = G_t + \tau^l_t \) denotes total government expenditure while the last term on the right-hand side are the central bank’s profits which depend on the spread between the long-term and short-term rates. Total real tax revenues,
\[ T_t \text{ are given by:} \]
\[ T_t = \tau_t^C C_t + \tau_t^W W_t H_t + \tau_t^k \left( P_t^k - \delta P_t^h \right) K_t. \] (4.42)

In order to reduce the number of tax instruments to two, we follow Cantore et al. (2017) and assume that distortionary taxes \( \tau_t^C, \tau_t^W \) and \( \tau_t^k \) as well as the two types of government expenditure deviate from their respective steady state by the same proportion, i.e. \( \tau_t^C = \tau_t^C, \tau_t^W = \tau_t^W, \tau_t^k = \tau_t^k \), \( G_t = g_t G \), and \( \tau_t^l = g_t \tau_t^l \). The government uses the following fiscal rules to stabilize debt and react to deviations of output from steady state:

\[
\log \left( \frac{\tau_t}{\tau} \right) = \rho_\tau \log \left( \frac{\tau_{t-1}}{\tau} \right) + \rho_{\tau b} \log \left( \frac{\bar{B}_{t-1}}{B} \right) + \rho_{\tau y} \log \left( \frac{Y_t}{Y} \right) + \\
+ \vartheta_\tau \epsilon_{\tau t} + (1 - \vartheta_\tau) \epsilon_{\tau t-1},
\] (4.43)

\[
\log \left( \frac{g_t}{g} \right) = \rho_g \log \left( \frac{g_{t-1}}{g} \right) - \rho_{gb} \log \left( \frac{\bar{B}_{t-1}}{B} \right) - \rho_{gy} \log \left( \frac{Y_t}{Y} \right) + \\
+ \vartheta_g \epsilon_{g t} + (1 - \vartheta_g) \epsilon_{g t-1},
\] (4.44)

where \( \rho_g \) and \( \rho_\tau \) govern the persistence of the fiscal policy instruments, while \( \rho_{\tau b} \) and \( \rho_{gb} \) define their responsiveness to deviations of government debt from steady state, and \( \rho_{\tau y} \) and \( \rho_{gy} \) determine their reaction to deviations of output from steady state, to introduce an automatic stabilizer component. Finally, \( \epsilon_{\tau t} \) and \( \epsilon_{g t} \) are i.i.d. government spending and tax shocks, respectively. Following Leeper et al. (2013), we allow for pre-announcement effects of fiscal policy via the parameters \( \vartheta_\tau, \vartheta_g \in [0, 1] \). Such a specification of the fiscal rules is important to account for anticipated effects of discretionary fiscal policies and avoid biased estimates. If \( \vartheta_i = 0 \), with \( i \in \{\tau, g\} \), then agents have perfect foresight of discretionary innovations in fiscal policies as, at time \( t \), they can perfectly observe \( g_{t+1} \) and \( \tau_{t+1} \). Conversely, if \( \vartheta_i = 1 \) agents have no foresight of the discretionary component of fiscal policies and receive news only about contemporaneous government spending and tax rates, in addition to those determined by their endogenous responses to debt and output deviations from steady state. Finally, values of \( \vartheta_i \) between 0 and 1 imply a limited degree of discretionary fiscal foresight by private agents.
4.2.6 Equilibrium and exogenous processes

In equilibrium all markets clear and the model is closed by the resource constraint:

\[ Y_t = C_t + I_t + G_t + \frac{\theta_p}{2} \left( \frac{\Pi_t}{\Pi_{t-1}^p} - \Pi_1^{1-\eta_p} \right)^2 Y_t + \frac{\theta_w}{2} \left[ \frac{\Pi_t^w}{\Pi_{t-1}^w} - \Pi_1^{1-\eta_w} \right]^2 W_t H_t. \]

(4.45)

As in Smets and Wouters (2007), the wage markup and the price markup shocks follow ARMA(1,1) processes:

\[ \log \left( \frac{e^{\kappa_t}}{e^{\kappa_{t-1}}} \right) = \rho_{\kappa} \log \left( \frac{e^{\kappa_{t-1}}}{e^{\kappa_{t-2}}} \right) + \epsilon^{\kappa}_t + \vartheta^{\kappa} e^{\kappa}_{t-1}, \]

(4.46)

with \( \kappa = [p, w] \), whereas all other shocks follow an AR(1) process:

\[ \log \left( \frac{e^{\kappa}}{e^{\kappa_{t-1}}} \right) = \rho_{\kappa} \log \left( \frac{e^{\kappa_{t-1}}}{e^{\kappa_{t-2}}} \right) + \epsilon^{\kappa}_t \]

(4.47)

where \( \kappa = [a, m, \psi, \phi, \mu, b, \eta] \); \( \rho_{\kappa} \) and \( \rho_{\kappa} \) are autoregressive parameters; \( \vartheta_{\kappa} \) are the moving average parameters; \( \epsilon^{\kappa}_t \) and \( \epsilon^{\eta}_t \) are i.i.d shocks with zero mean and standard deviations \( \sigma_{\kappa} \) and \( \sigma_{\kappa} \).

4.3 Bayesian estimation

The model is estimated with Bayesian methods. The Kalman filter is used to evaluate the likelihood function, which combined with the prior distribution of the parameters yields the posterior distribution. Then, the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm with two parallel chains of 200,000 draws each is used to generate a sample from the posterior distribution in order to perform inference. We estimate the model on the EA starting from the introduction of the currency union in 1999 until 2017. In particular, our sample is 1999Q1-2017Q2. We therefore account for crucial episodes of the EA business cycle, namely the Great Recession, the Sovereign Debt Crisis and the implementation of the PSPP started by the ECB in March 2015. The model features 10 structural shocks, hence
we use 10 observables: real GDP, real private consumption, real private investment, real wage, real government spending (which includes government consumption, investment and transfers), real tax revenues, inflation, nominal interest rate, term premium and the purchase of public sector securities by the ECB. We use data on eleven countries of the EA, namely Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain; and aggregate them weighting by nominal GDP. This choice is dictated by the fact that these countries are the founding members of the currency union hence data is available for the entire sample selected. Finally, we follow related studies on the EA by detrending the real variables before the estimation with a linear trend, and demeaning inflation and interest rates (see, Gerali et al., 2010, and Coenen et al. 2012, 2013). The following measurement equations link the data to the endogenous variables of the model:

\[
\begin{bmatrix}
\Delta Y_t^o \\
\Delta C_t^o \\
\Delta I_t^o \\
\Delta W_t^o \\
\Delta G_t^o \\
\Delta T_t^o \\
\bar{\Delta}_t^{CB,o} \\
\Pi_t^o \\
TP_t^o \\
R_t^o
\end{bmatrix}
= 
\begin{bmatrix}
\dot{Y}_t \\
\dot{C}_t \\
\dot{I}_t \\
\dot{W}_t \\
\dot{G}_t \\
\dot{T}_t \\
\dot{\bar{b}}_t^{CB} - \dot{\bar{b}}_{t-1}^{CB} \\
\dot{\Pi}_t \\
T\dot{P}_t^o \\
\dot{R}_t
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Variables with a \(\hat{\cdot}\) are in log-deviations from their own steady state, while \(\bar{b}_t^{CB} = \frac{B_t^{CB}}{Y_t}\) denotes the stock of government debt held by the central bank as a fraction of GDP. Our set of observables include the real variables for each term of the resource constraint (4.45), thus posing an issue of stochastic singularity. Indeed, the resource constraint imposes an exact linear restrict-

\[\text{We excluded Greece because it adopted the euro in 2001 and due to the incomplete availability of quarterly fiscal data.}\]

\[\text{Details about data sources and transformations are provided in Appendix 4.5.}\]
tion between the observables that has a zero probability of being observed which prevents the estimation. We therefore follow Schmitt-Grohe and Uribe (2012) and overcome this issue by introducing a i.i.d. measurement error $\epsilon_{t}^{me}$ in one of the observables.\footnote{Schmitt-Grohe and Uribe (2012) add the measurement error to output while we add it to consumption. This choice has no material implications for the estimation but in this way we avoid a potential bias in the historical decomposition of output which is one of the main focus of the paper. The results are robust to alternatively adding the measurement error to output or investment, see Appendix 4.13.2.}

Our sample includes a period in which the Euro Interbank Offered Rate (EONIA), which we use as a proxy for the ECB policy rate, approached the ZLB (2012Q1) and then turned negative (2014Q4), thus posing potential issues in the estimation of the model.

One way to deal with the ZLB is to estimate the model up to period before it started binding and then use non-linear techniques to simulate it with a binding ZLB. However, there is little agreement about which non-linear method is more appropriate. Moreover, Kollmann et al. (2016) estimate a DSGE model of the Euro Area up to 2016Q4 without accounting for the ZLB and then, as a robustness, re-estimate the model using the method developed by Guerrieri and Iacoviello (2015). They find only marginal changes in their results and argue that the ZLB was not a significant constraint on monetary policy, in line with the conclusions of Fratto and Uhlig (2014) and Linde et al. (2016).

Alternatively, one could estimate the DSGE model replacing the policy rate with a shadow rate, as Mouabbi and Sahuc (2017) do for the Euro Area. However, using the shadow rate comes at the cost of shutting down the explicit unconventional monetary policy channel in our model. Indeed, UMP is captured by the shadow rate and we will not be able to disentangle the effects of conventional vs unconventional monetary policies (see Section 4.13.1 of the Appendix for a more detailed discussion on the issues posed by the ZLB and the robustness of our results to using the shadow rate).

We therefore follow Kollmann et al. (2016) in estimating the model without explicitly modeling the ZLB. I Section 4.13.1 of the Appendix we then verify that our results are robust to the use of the shadow rate as the ob-

\[ \text{\footnote{\text{Schmitt-Grohe and Uribe (2012) add the measurement error to output while we add it to consumption. This choice has no material implications for the estimation but in this way we avoid a potential bias in the historical decomposition of output which is one of the main focus of the paper. The results are robust to alternatively adding the measurement error to output or investment, see Appendix 4.13.2.}}} \]
servable for the monetary policy rate.

4.3.1 Calibration and priors

Structural parameters and steady state values presented in Table 4.1 are calibrated at a quarterly frequency. The calibration of the households’ discount factor ($\beta = 0.99$, which yields an annual steady state interest rate of 4%), the capital depreciation rate ($\delta = 0.025$, which implies a 10% annual capital depreciation rate) and the capital share of income $\alpha = 0.33$ are standard in quarterly DSGE models and have been used in other studies on the EA (see, Smets and Wouters 2003, 2005, among others). The elasticities of substitution in goods and labor markets $\varepsilon^p$ and $\varepsilon^w$ equal 6 in order to target a steady-state gross mark-up of 20%, as in Gerali et al. (2010). The ratios of government spending and government debt to GDP are set to 23% and 68%, respectively, in line with the data. Steady state tax rates are borrowed from Forni et al. (2009).

At the steady state, the stock of government bonds held by the ECB is calibrated at 0.7% to match the ratio of general government debt denominated in euros in the asset side of the consolidated balance sheet of the Eurosystem to the EA GDP over the period 1999-2014 (before the start of the PSPP). We calibrate $\kappa$ such that the duration of the long-term bonds is set to 10 years. The scale parameter of the disutility of labor $B$ is set to match steady state hours equal to 1, whereas the additional discount factor of the FIs is set to match a steady state leverage of 6, as in Villa (2016).

Tables 4.2 and 4.3 summarize the prior and posterior distributions of the parameters and the shocks, respectively. The choice of priors corresponds to a large extent to those in previous studies of the EA. We generally follow Smets and Wouters (2003, 2005) in choosing the prior distribution of the structural parameters and the parameters governing the shock processes. We set the prior mean of the inverse Frisch elasticity $\eta$ to 0.5. Estimated DSGE models of the EA largely agree in setting the prior mean of the habit

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22 Christiano et al. (2010) set the same steady-state tax rate on labor income at 0.45, while the steady-state tax rates on consumption and capital are very similar to ours and set at 0.20 and 0.28, respectively.
parameter $h$ to 0.70. Turning to the share of optimizing households, we start from a prior whereby their share equals that of rule-of-thumbers, as common in the literature. We follow Gerali et al. (2010) by setting the prior distributions of the IAC $\psi_i$ and price and wages stickiness parameters, including indexation. In particular, prices are a priori assumed to last 3.7 quarters while wages last 2.5 quarters. Given the lack of previous estimates for the parameter of FI:s’ net worth adjustment for the EA, we center our prior on the estimate of Carlstrom et al. (2017) for the US economy. In particular, we assume that $\psi_n$ takes a Normal prior distribution with mean 0.785 and standard deviation 0.10. This prior distribution is sufficiently loose to include cases of low and high degrees of financial frictions. The priors of the tax rules coefficients are taken from Zubairy (2014) and are broadly consistent with EA studies, e.g. Forni et al. (2009) and Kollmann et al. (2013). It is important to note that we remain agnostic about the countercyclicality of government spending. Indeed, we set a Normal prior distribution for $\rho_{gy}$ with mean 0.10 and standard deviation 0.10 thus not excluding the case of procyclical government spending should the parameter take negative values. Finally, the prior distributions of the parameters of the Taylor rule are fairly standard, with the interest rate smoothing parameter, $\rho_r$, set to have a prior mean 0.80 and with a stronger response of the central bank to inflation than output. In general, we use the Beta (B) distribution for all parameters bounded between 0 and 1. We use the Inverse Gamma (IG) distribution for the standard deviation of the shocks for which we set a loose prior with 2 degrees of freedom.

### 4.3.2 Posterior estimates

There is evidence of preference parameters $\eta$ and $h$ in line with values common to the literature, while the fraction of optimizing households is rather high–0.90. This implies that about 10% of EA households do not have access to financial markets. The posterior mean of $\omega$ is relatively close to Coenen et al. (2012, 2013), who estimate a fraction of rule-of-thumbers of 18%, but farther from Forni et al. (2009), Ratto et al. (2009) and Albonico et al.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Capital share of income</td>
<td>$\alpha = 0.33$</td>
</tr>
<tr>
<td>Elasticity of substitution goods</td>
<td>$\varepsilon^p = 6$</td>
</tr>
<tr>
<td>Elasticity of substitution labor</td>
<td>$\varepsilon^w = 6$</td>
</tr>
<tr>
<td>Government spending to GDP</td>
<td>$g_y = 0.23$</td>
</tr>
<tr>
<td>Government debt to GDP</td>
<td>$b_y = 0.68$</td>
</tr>
<tr>
<td>Steady state Tax rate consumption</td>
<td>$\tau^c = 0.16$</td>
</tr>
<tr>
<td>Steady state Tax rate capital</td>
<td>$\tau^k = 0.19$</td>
</tr>
<tr>
<td>Steady state Tax rate labor income</td>
<td>$\tau^w = 0.45$</td>
</tr>
<tr>
<td>Steady state CB holdings to gov. debt</td>
<td>$\psi = 0.007$</td>
</tr>
<tr>
<td>Duration of long-term bonds</td>
<td>$(1 - \kappa)^{-1} = 40$</td>
</tr>
<tr>
<td>Disutility of labor</td>
<td>$B = H = 1$</td>
</tr>
<tr>
<td>FI additional discount</td>
<td>$\zeta = L = 6$</td>
</tr>
</tbody>
</table>

Table 4.1: Calibrated parameters

(2016), who estimate in different models and with different datasets that around 35% of EA households are completely prevented from participating in financial markets. We estimate sizable investment adjustment costs with $\psi_i = 5.52$, a value close to e.g. Smets and Wouters (2005), Forni et al. (2009) and Villa (2016). Turning to the posterior estimate of $\psi_n$, we find a value of 0.63 which implies a non-negligible degree of financial frictions, lower than the US estimate of 0.79 by Carlstrom et al. (2017). Consistently with Villa (2016), the US estimated degree of financial frictions is larger than that in the EA. In line with EA studies and differently from the US economy, prices are stickier than wages. Indeed, prices are estimated to be reset every 6.5 quarters ($\theta_p = 173$ which corresponds to a 15.25% probability of resetting the price in a Calvo world), while wages last almost 4 quarters ($\theta_w = 109$ which implies a probability of 29.15% of resetting wages in a Calvo world). Moreover, wages display a higher degree of indexation than prices ($\iota_w = 0.33$, $\iota_p = 0.15$). Estimates of the fiscal rules reveal slightly stronger responses of government spending than taxes to government debt and output deviations.

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23Smets and Wouters (2003, 2005), Forni et al. (2009), Coenen et al. (2012, 2013), Kollmann et al. (2016), and Villa (2016), all estimate that in the EA prices are stickier than wages.
Table 4.2: Prior and posterior distributions of estimated structural parameters (90% confidence bands in square brackets).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distr.</th>
<th>Mean</th>
<th>Sd/df</th>
<th>Posterior Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inv. Frisch elasticity</td>
<td>η</td>
<td>N</td>
<td>0.50</td>
<td>0.10 0.2795</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.1044;0.4380]</td>
</tr>
<tr>
<td>Habits in consumption</td>
<td>h</td>
<td>B</td>
<td>0.70</td>
<td>0.10 0.8551</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.8045;0.9021]</td>
</tr>
<tr>
<td>Fraction of optimizing households</td>
<td>ω</td>
<td>B</td>
<td>0.50</td>
<td>0.10 0.9021</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.8559;0.9473]</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>ψ_i</td>
<td>N</td>
<td>2.50</td>
<td>1.00 5.5202</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[4.2326;6.9571]</td>
</tr>
<tr>
<td>Net worth adjustment costs</td>
<td>ψ_n</td>
<td>N</td>
<td>0.785</td>
<td>0.10 0.6362</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.4104;0.8575]</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>θ_p</td>
<td>G</td>
<td>50</td>
<td>20.0 172.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[137.19;199.99]</td>
</tr>
<tr>
<td>Price indexation</td>
<td>ι_p</td>
<td>B</td>
<td>0.50</td>
<td>0.15 0.1535</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0366;0.2774]</td>
</tr>
<tr>
<td>Wage stickiness</td>
<td>θ_w</td>
<td>G</td>
<td>50</td>
<td>20.0 108.98</td>
</tr>
<tr>
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<td>[66.861;154.22]</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>ι_w</td>
<td>B</td>
<td>0.50</td>
<td>0.15 0.3253</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.1102;0.5573]</td>
</tr>
<tr>
<td>Tax smoothing</td>
<td>ρ_r</td>
<td>B</td>
<td>0.70</td>
<td>0.20 0.8649</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.8012;0.9280]</td>
</tr>
<tr>
<td>Tax reaction to debt</td>
<td>ρ_gb</td>
<td>G</td>
<td>0.50</td>
<td>0.25 0.0849</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0536;0.1180]</td>
</tr>
<tr>
<td>Tax reaction to output</td>
<td>ρ_γ</td>
<td>G</td>
<td>0.15</td>
<td>0.10 0.0454</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0046;0.0941]</td>
</tr>
<tr>
<td>Government spending smoothing</td>
<td>ρ_g</td>
<td>B</td>
<td>0.70</td>
<td>0.20 0.9191</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.8962;0.9418]</td>
</tr>
<tr>
<td>Government spending reaction to debt</td>
<td>ρ_gb</td>
<td>G</td>
<td>0.50</td>
<td>0.25 0.1063</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0852;0.1291]</td>
</tr>
<tr>
<td>Government spending reaction to output</td>
<td>ρ_γγ</td>
<td>N</td>
<td>0.10</td>
<td>0.10 0.1057</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0583;0.1551]</td>
</tr>
<tr>
<td>Inflation -Taylor rule</td>
<td>ρ_π</td>
<td>N</td>
<td>1.70</td>
<td>0.10 1.7243</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[1.5324;1.9277]</td>
</tr>
<tr>
<td>Output -Taylor rule</td>
<td>ρ_γ</td>
<td>G</td>
<td>0.125</td>
<td>0.05 0.0839</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0263;0.1496]</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>ρ_r</td>
<td>B</td>
<td>0.80</td>
<td>0.10 0.9615</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.9480;0.9739]</td>
</tr>
</tbody>
</table>

from steady state. Government spending is estimated to be countercyclical given the positive value of $\rho_{\gamma\gamma}$, while both tax rules display a high degree of inertia. Estimates of the parameters governing the degree of fiscal foresight reveal that agent foresee part of these shocks, with stronger pre-announcement effects of taxes ($\vartheta_r = 0.49$) than government spending ($\vartheta_y = 0.29$). Finally, the Taylor rule parameters and the parameters of the shock processes take standard values. The QE shock is very persistent ($\rho_\psi = 0.99$), which is consistent with the ongoing PSPP programme by the ECB.

24Indeed, on October 26th 2017, the ECB announced “…to continue the Asset Purchase Programme (APP) at a pace of 60 billion until the end of December 2017. From January 2018 the net asset purchases are intended to continue at a monthly pace of 30 billion until the end of September 2018, or beyond, if necessary...”.
### Table 4.3: Prior and posterior distributions of estimated shock processes (90% confidence bands in square brackets).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Posterior Mean</th>
<th>Sd/df</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exogenous processes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$</td>
<td>B 0.50</td>
<td>0.9897</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_a$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>$\rho_m$</td>
<td>B 0.50</td>
<td>0.4271</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_m$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Unconventional Monetary Policy</td>
<td>$\rho_\psi$</td>
<td>B 0.50</td>
<td>0.9883</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_\psi$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Preference</td>
<td>$\rho_t$</td>
<td>B 0.50</td>
<td>0.8855</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_b$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Investment specific</td>
<td>$\rho_\mu$</td>
<td>B 0.50</td>
<td>0.9326</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_\mu$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\rho_p$</td>
<td>B 0.50</td>
<td>0.3529</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_p$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>$\rho_w$</td>
<td>B 0.50</td>
<td>0.2945</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_w$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Government spending</td>
<td>$\phi_g$</td>
<td>B 0.50</td>
<td>0.4933</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_g$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Tax</td>
<td>$\phi_t$</td>
<td>B 0.50</td>
<td>0.9690</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_\tau$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Measurement error</td>
<td>$\sigma_{me}$</td>
<td>IG 0.10</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

#### 4.3.3 Dynamic properties of the estimated model

Before studying the role of the structural shocks, we assess the dynamic properties of the estimated model by plotting the Bayesian impulse responses (at the posterior mean) of the main macroeconomic variables to the ten shocks that perturb the economy. All shocks are expansionary, i.e. they generate a positive response of output. Overall, the selected macroeconomic variables display standard responses to each shock.

Figure [4.1](#) reports the response to monetary policy, government spending
and tax shocks. An expansionary monetary policy (first column of Figure 4.1) increases output, investment, consumption and inflation. Government spending is muted in the first five quarters due to the opposite effects of higher output and lower public debt. Then, the latter outweighs the former and government spending increases. In response to lower debt and higher output, tax rates are reduces. Finally, the term premium decreases thus stimulating investment.

An increase in government spending (second column of Figure 4.1) stimulates output via its positive effect on consumption. Indeed, the presence of rule-of-thumb consumers helps avoiding a crowding out of private consumption. Nevertheless, private investment is crowded out while the nominal interest rate reacts to the increase in inflation. Tax rates increase in reaction to the higher public debt while the term premium decreases.

A fiscal stimulus in the form of lower tax rates (third column of Figure 4.1) causes an initial rise in output. Higher government debt reduces the fiscal space so that the government cuts spending and output is lower than the steady state after four quarters. Note also that pre-announcement effects mitigate the impact responses of the real variables which reach their peak only in the following periods.

When the economy is perturbed with standard supply and demand shocks (technology, price and wage markups, preference and investment shocks, respectively), variables follow a path consistent with standard DSGE models (see Figures 4.9 and 4.10 in Appendix 4.10).

Two shocks are peculiar to the model. A QE shock entails an increase in the stock of long-term government debt held by the central bank financed by issuance of short-term liabilities. The first column of Figure 4.2 shows that this shock has clearly expansionary effects on the economy. The shock is transmitted through the economy, via investment, by reducing the available quantity of long-term government bonds thus driving up their price and reducing their yield and the term premium. Investment is therefore stimulated, hence its expansionary effect on output. Inflation exceeds its steady state, which is why the central bank reacts by increasing the nominal interest rate.25

25In the historical decomposition of the business cycle presented in the following section,
Figure 4.1: Mean Bayesian impulse responses of selected macroeconomic variables to standard policy shocks. Rows are variables, columns are shocks. All shocks are expansionary.
Figure 4.2: Mean Bayesian impulse responses of selected macroeconomic variables to QE and credit shocks. Rows are variables, columns are shocks. All shocks are expansionary.
Moreover, profits earned by the central bank during the QE programme are transferred to the government, therefore they reduce government debt which allows higher government spending and lower taxes, which in turn further increases output.

The other shock peculiar in our model is the credit shock (second column of Figure 4.2). An expansionary shock perturbs the economy via an exogenous decrease in the term premium, as a result of which the hold-up problem becomes less severe. FIs then increase their leverage because they can adjust their net worth only gradually. Investment is thus stimulated hence causing an expansion. The reduction in public debt allows more government spending and lower tax rates. A contractionary version of this shock (in which impulse responses have the reversed sign) proxies the financial crisis erupted in 2008 and will prove important for the historical analysis of the EA business cycle.

4.3.4 The role of non-policy structural shocks

In this section we assess the role of all non-policy structural shocks of the model in explaining the dynamics of output and inflation in the EA, while we assess the role of monetary and fiscal policy shocks in the next section. During this period of time, the EA has been impacted by two major events, namely the financial crisis of 2008 and the subsequent sovereign debt crisis that hit Greece in 2009 and then propagated to other European countries in the following years.\textsuperscript{26} The historical decomposition of output (Figure 4.3) shows a large contribution of the preference shock over the sample, with remarkable negative effects during financial and sovereign debt crises. Interestingly, the years before the 2008 financial crisis, the credit shock had a positive effect on output, which reveals an important role of leverage build-up leading to the crisis itself. After the crisis erupted, FIs engaged in substantial deleveraging which is captured by the the negative contributions of the credit shock

\begin{footnotesize}
\textsuperscript{26}Recall that our set of countries does not include Greece. However, we do include countries that experienced sovereign debt distress starting from 2011.
\end{footnotesize}
during the years 2009-2010. The missed recovery has been driven mainly by weak demand, with the large negative contribution of the preference shock lasting until 2015. At the same time, FIs engaged in rebuilding their balance sheets hence the positive effect of the credit shock on output. Supply shocks, especially total factor productivity and price markup shocks, play a role but display larger effects on the dynamics of inflation. Indeed, Figure 4.4 reveals larger effects of wage and price markup shocks although the decomposition is similar to output. In particular, the wage markup shock seems to explain the bulk of the missed inflation from 2014, a period in which the ECB has increasingly loosened its policy. Overall, structural shocks other than policy shocks, explain a substantial portion of the dynamics of EA output and inflation. Moreover, they help us identify key episodes of the EA business cycle.

4.3.5 Policy stance in the Euro Area

We now turn to the analysis of the role of discretionary policy shocks in shaping the EA business cycle since the currency union. Given that in our model policymakers have four policy instruments available, our ultimate goal is to infer whether the EA policy stance, defined as the joint effect of all policies, has been contractionary or expansive, especially in the aftermath of the financial and sovereign debt crises. Moreover, we want to determine which policy, if any, contributes the most to the overall stance and what is the role of the remaining policies. To do so, we proceed in two steps. First, we look at the joint contribution of all policies and determine the EA policy stance. Then we disentangle the role of each policy by studying the historical decompositions of output and inflation when each policy is treated separately.

---

27 The forecast error variance decomposition reported in Appendix 4.11 largely confirms these results at different horizons.
28 Recall that we design the unconventional monetary policy shock such that this policy is active only from the third quarter of 2015, when the ECB started the purchase of long-term government bonds under the PSPP.
29 Sections 4.13.6 and 4.13.10 of the appendix reports also the historical decompositions of output and inflation to the monetary and fiscal policies stances, i.e. when we treat
Figures 4.3 and 4.6 report the historical decomposition of output and inflation to the joint effect of all the available policy instruments whereas Figures 4.7 and 4.8 plot the historical contribution of the four policy instruments to the evolution of output and inflation, respectively.\textsuperscript{30}

Key findings emerge. First, with the exception of the period 2001-2003 and up to 2013, the overall policy stance has been generally countercyclical. For most of the EA existence, policies display negative (positive) contributions to output when its deviations from trend are positive (negative).

Figure 4.7 sheds more light on the 2001-2003 period: there was a positive contribution of government spending and monetary policy, whereas tax policy had a muted effect. Conversely, the historical decomposition of inflation

\textsuperscript{30}In Figures 4.5\textsuperscript{4.8} the gaps between the bars and the actual values of the variables arise because we only plot the policy shocks.
Figure 4.4: Historical decomposition of inflation to all shocks. Bold line represents the data, bars represent the contribution of each shock.

(Figure 4.6) shows that the policy stance counteracts inflation, where the lion’s share belongs of course to monetary policy (see Figure 4.8).

Turning our attention to the build up, explosion and aftermath of the financial crisis, we notice that the years between 2004 and 2008 saw a large expansion of output while the policy stance was overall contractionary, mainly due to the monetary policy stance. The joint policy stance witnessed a negative effect on inflation which was prevented from further increasing. However, as financial turmoils hit the EA in 2008, GDP and inflation contracted sharply, therefore the policy stance became expansionary to fight the recession. Looking more in detail, the major role was played by the ECB, which immediately reacted by lowering the interest rate. The two fiscal policies show opposite contributions, thus adding very little to output and inflation. However, the overall fiscal stance remained expansionary during the 2009-2010 drop in output as found by Coenen et al. (2012).14

14Kollmann et al. (2013) argue that the non-systematic components of government ex-
Figure 4.5: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks. Bold line represents the data, bars represent the contribution of each shock.

However, after an initial sharp reaction, the EA policy stance has gradually decreased its expansionary role and, from mid 2013, it turned contractionary on output and less powerful on inflation. Two crucial factors might have contributed to this outcome. First, EA was hit by the sovereign debt crisis which exerted most of its effects from 2011 onward. In addition, in 2012 monetary policy started to be constrained by the zero-lower-bound on the policy rate. While monetary policy started losing power (see the smaller contributions to output in Figure 4.7), fiscal adjustments entailed opposite effects of discretionary government spending and taxes which almost penditures rose sharply during the crisis thus concluding that the EA fiscal stance has clearly been expansionary. However, they keep all tax rates fixed during the estimation, thus ruling out the role of taxes in determining the fiscal stance. In addition, in their environment public debt is only short term.

32The Euro Interbank Offered Rate (EONIA), which we use as a proxy for the ECB policy rate in our analysis, approached the ZLB in June 2012. At the ZLB the policy rate is higher than it should be and this effect shows up as a negative contribution of monetary policy on output in the historical decomposition.
neutralized each other.

At the end of 2014, all policies started being contractionary on output due to the joint effect of fiscal adjustments and the zero-lower-bound. Absent strong non-policy structural shocks, this is the main likely cause of the sluggish EA recovery after 2014.

In March 2015, the ECB started implementing its QE programme to stimulate the economy, as evident from the historical decomposition of output in Figure 4.7. QE displayed an increasingly larger positive effect on output and inflation, being the only expansionary policy on the former. QE helped counteract, but was never strong enough to undo the other contractionary policies, due to the joint drag caused by fiscal policies and the zero-lower-bound.
Figure 4.7: Historical decomposition of cyclical deviations of output from trend to policy shocks. Bold line represents the data, bars represent the contribution of each shock.

4.4 Concluding remarks

The Global Financial Crisis and the subsequent Sovereign Debt Crisis triggered a lively debate about the appropriate policy reactions policymakers should take in the EA. To accomplish this difficult task, on one hand, it is crucial to understand what drives the EA business cycle and which shocks contributed the most to its dynamics. On the other hand, it is vital to assess the role of fiscal and monetary policies in affecting economic fluctuations.

Our analysis contributes to the current debate and the literature along a number of dimensions. First, we estimate a crucial role of credit and demand shocks in generating the deep recession experienced in 2008-2009. Second, we disentangle the role of the overall policy stance in the EA, thus accounting for the joint effects of fiscal and monetary policies. We find that the overall EA policy stance was expansionary after the onset of the financial crisis mainly due to a loosening of monetary policy. However, it started loosing
its power due to the almost contemporaneous effects of the ZLB and the Sovereign Debt Crisis. The former made monetary policy unable to provide further stimulus via conventional measures, while the latter implied a muted, if not contractionary, fiscal policy. The QE program that the ECB started in March 2015 is estimated to have positively contributed to the EA policy stance which overall has remained contractionary, due to the joint negative contributions of fiscal policies and the zero-lower-bound constraint.

Our results carry the policy implication that, having the ECB exploited most of its instruments to stimulate the economy, a more expansionary fiscal policy in countries with fiscal space could boost the EA economic activity and also accelerate public debt consolidation.
Appendix

4.5 Data

We collect data on the following countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. For each country we collect data from Eurostat and IMF International Financial Statistics (IFS). Nominal data are then transformed to real series by dividing for the respective country’s GDP deflator. All series are seasonally adjusted. The EONIA rate is taken from the ECB Statistical Data Warehouse. Quarterly 10-years government bond yields are downloaded from IFS. The Term premium is constructed as the spread between the 10-years government bond yields and the EONIA rate. Euro-Area variables are created by aggregating countries series weighted by countries’ Nominal GDP. Inflation and interest rates are demeaned before the estimation while all other variables are detrended using a linear trend.

4.6 Equilibrium conditions

Optimizing households

\[
(1 + \tau^e_t) \Lambda^o_t = \frac{e^b_t}{C^o_t - hC^o_{t-1}} - h\beta E_t \left[ \frac{e^b_{t+1}}{C^o_{t+1} - hC^o_t} \right]
\]
\[
\Lambda^o_t = \beta E_t \left[ \Lambda^o_{t+1} \frac{R_t}{\Pi_{t+1}} \right]
\]

\[
\Lambda^o_t P^k_t M_t = \beta E_t \left\{ \Lambda^o_{t+1} \left[ (1 - \tau^k_{t+1}) R^k_{t+1} + \delta P^k_{t+1} \tau^k_{t+1} + (1 - \delta) P^k_{t+1} M_{t+1} \right] \right\}
\]
\[
\Lambda^o_t Q_t M_t = \beta E_t \left[ \Lambda^o_{t+1} \frac{1 + \kappa Q_{t+1} M_{t+1}}{\Pi_{t+1}} \right]
\]
Rule-Of-Thumb households

\[(1 + \tau^c_t) C^r_t = (1 - \tau^w_t) W_t H_t + \tau^l_t \]

\[(1 + \tau^c_t) \Lambda^r_t = \frac{e^b_t}{C^r_t - hC^r_{t-1}} - h\beta E_t \left[ \frac{e^b_{t+1}}{C^r_{t+1} - hC^r_t} \right] \]

Wage setting

\[0 = \left\{ (1 - \tau^w_t) (1 - \epsilon^w_t \xi^w_t) - \theta_w \left[ \frac{\Pi^w_t}{\Pi^w_{t-1}} - \Pi^{1-\epsilon_w} \right] \frac{\Pi^w_{t-1}}{\Pi^w_{t-1} - \Pi^{1-\epsilon_w}} \right\} + \frac{B (H_t)^\eta}{\Lambda_t W_t} \epsilon^w_t \epsilon^w_t \xi^w_t + \beta E_t \left\{ \bar{\Lambda}_{t+1} \theta_w \left[ \frac{\Pi^w_{t+1}}{\Pi^w_t} - \Pi^{1-\epsilon_w} \right] \frac{\Pi^w_{t+1} W_{t+1} H_{t+1}}{\Pi^w_t W_t H_t} \right\} , \]

\[\bar{\Lambda}_t = \omega \Lambda^o_t + (1 - \omega) \Lambda^r_t \]

\[\Pi^w_t = \frac{W_t}{W_{t-1}} \Pi_t \]

Aggregation

\[C_t = \omega C^o_t + (1 - \omega) C^r_t \]

\[\bar{F}_t = \omega \bar{F}^o_t \]

\[I_t = \omega I^o_t \]

\[K_t = \omega K^o_t \]
Financial intermediaries

\[ \bar{B}_t^{FI} + \bar{F}_t = N_t L_t \quad (4.62) \]

\[ L_t = \frac{\Lambda_t^{o+1}}{\Pi_{t+1}} + \left( e_t^\phi - 1 \right) E_t \left( \frac{\Lambda_t^{o+1}}{\Pi_{t+1}} \right) \frac{R_{t+1}^L}{R_t} \quad (4.63) \]

\[ P_t^k I_t = \bar{F}_t - \frac{k \bar{F}_{t-1}}{Q_t} \frac{Q_t}{Q_{t-1}} \quad (4.64) \]

\[ \Lambda_t^o \left[ 1 + N_t f' (N_t) + f (N_t) \right] = \frac{\Lambda_t^{o+1} \beta_c}{\Pi_{t+1}} \left[ R_t + L_t (R_{t+1}^L - R_t) \right] \quad (4.65) \]

\[ f (N_t) = \frac{\psi_n}{2} \left( \frac{N_t - \bar{N}}{N} \right)^2 \quad (4.66) \]

\[ f' (N_t) = \psi_n \left( N_t - \bar{N} \right) \left( \frac{1}{N} \right)^2 \quad (4.67) \]

\[ R_{t+1}^L = \frac{1 + \kappa Q_{t+1}}{Q_t} \quad (4.68) \]

\[ R_t^{10} = \kappa + \frac{1}{Q_t} \quad (4.69) \]

\[ R_t = \frac{1 + \kappa Q_{t+1}^{EH}}{Q_t^{EH}} \quad (4.70) \]

\[ R_t^{10, EH} = \kappa + \frac{1}{Q_t^{EH}} \quad (4.71) \]

\[ TP_t = 1 + R_t^{10} - R_t^{10, EH} \quad (4.72) \]
Intermediate goods producers

\[ Y_t = e_t^K a K_{t-1} H_t^{1-\alpha} \tag{4.73} \]
\[ MPK_t = \alpha e_t^K a H_t^{1-\alpha} K_{t-1}^{\alpha - 1} \tag{4.74} \]
\[ MPL_t = (1 - \alpha) e_t^K a K_{t-1} H_t^{-\alpha} \tag{4.75} \]
\[ R_t^k = MC_t MPK_t \tag{4.76} \]
\[ W_t = MC_t MPL_t \tag{4.77} \]
\[ e_t^P (1 - MC_t) - 1 = -\theta_p \left( \frac{\Pi_t}{\Pi_t^{1-\varphi}} - \Pi_t^{1-\varphi} \right) \frac{\Pi_t}{\Pi_t^{1-\varphi}} + \theta_p E_t \left[ \beta \frac{\Lambda_{t+1}^P}{\Lambda_t^P} \left( \frac{\Pi_{t+1}}{\Pi_t^P} - \Pi_t^{1-\varphi} \right) \frac{\Pi_{t+1} Y_{t+1}}{\Pi_t^Y} \right] \tag{4.78} \]

Capital producers

\[ P_t^k e_t^\mu \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] = 1 - E_t \left[ \beta \frac{\Lambda_{t+1}^\phi}{\Lambda_t^\phi} e_t^\mu P_{t+1}^k S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \tag{4.79} \]
\[ K_t = (1 - \delta) K_{t-1} + I_t e_t^\phi \left[ 1 - S_t \left( \frac{I_t}{I_{t-1}} \right) \right] \tag{4.80} \]
\[ S_t = \frac{\psi_1}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{4.81} \]
\[ S_t' = \psi_1 \left( \frac{I_t}{I_{t-1}} - 1 \right) \tag{4.82} \]

Government

\[ \bar{B}_t = \frac{R_t^L}{\Pi_t} \bar{B}_{t-1} + \bar{G}_t - T_t - \frac{R_t^L - R_{t-1}}{\Pi_t} \bar{B}_{t-1}^{CB} \tag{4.83} \]
\[ T_t = \tau^C_t C_t + \tau^W_t W_t + \tau^l_t \left[ (R_t^k - \delta P_t^k) K_t \right] \tag{4.84} \]
\[ \bar{G}_t = G_t + \tau^l_t \tag{4.85} \]
\[ \log \left( \frac{\tau_t}{\tau} \right) = \rho_t \log \left( \frac{\tau_{t-1}}{\tau} \right) + \rho_{\tau^b} \log \left( \frac{\bar{B}_{t-1}}{B} \right) + \rho_{\tau^r} \log \left( \frac{Y_t}{Y} \right) + \vartheta_t e_t^\tau + (1 - \vartheta_t) e_{t-1}^\tau \tag{4.86} \]
\[
\log \left( \frac{g_t}{g} \right) = \rho_g \log \left( \frac{g_{t-1}}{g} \right) - \rho_{gy} \log \left( \frac{\bar{B}_{t-1}}{\bar{B}} \right) - \\
- \rho_{gy} \log \left( \frac{Y_t}{Y} \right) + \vartheta_g \epsilon^g_t + (1 - \vartheta_g) \epsilon^g_t
\]  
(4.88)

\[
\tau^c_t = \tau_t^c
\]  
(4.89)

\[
\tau^k_t = \tau_t^k
\]  
(4.90)

\[
\tau^w_t = \tau_t^w
\]  
(4.91)

\[
G_t = g_t G
\]  
(4.92)

\[
\tau^l_t = g_t \tau^l
\]  
(4.93)

Central bank

\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \\
+ (1 - \rho_r) \left\{ \rho_p \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y^u_t}{Y^u} \right) \right\} + \epsilon^m_t,
\]  
(4.94)

\[
\bar{B}^C = \bar{B}^C_t
\]  
(4.95)

\[
\bar{B}^F = (1 - \psi_t) \bar{B}_t
\]  
(4.96)

Equilibrium

\[
Y_t = C_t + I_t + G_t + \frac{\theta_p}{2} \left( \frac{\Pi_t}{\Pi^p_{t-1}} - \Pi_1^{1-\tau_p} \right)^2 Y_t + \frac{\theta_w}{2} \left[ \frac{\Pi^w_{t-1}}{\Pi^w_{t-1}} - \Pi_1^{1-\tau_w} \right]^2 W_t H_t
\]  
(4.97)

Shocks processes

\[
\log \left( \frac{\epsilon^p_t}{\epsilon^p} \right) = \rho_p \log \left( \frac{\epsilon^p_{t-1}}{\epsilon^p} \right) + \epsilon^p_t - \vartheta_p \epsilon^p_{t-1}, \quad \kappa = [p, w]
\]  
(4.98)

\[
\log \left( \frac{\epsilon^w_t}{\epsilon^w} \right) = \rho_w \log \left( \frac{\epsilon^w_{t-1}}{\epsilon^w} \right) + \epsilon^w_t, \quad \kappa = [a, m, \psi, \phi, \mu, b] \]  
(4.99)
4.7 Steady state

In the deterministic steady state all expectation operators are removed and for each variable it holds that \( x_t = x_{t+1} = x \). Moreover, the stochastic shocks are absent. The variables \( \Lambda^o \) and \( C^r \) solve equations (4.49) and (4.53) respectively. The remaining variables are found recursively as follows:

\[
R = \frac{1}{\beta} \quad (4.100)
\]
\[
P_t^k = 1 \quad (4.101)
\]
\[
S = 0 \quad (4.102)
\]
\[
S' = 0 \quad (4.103)
\]
\[
R^L = R + \varsigma \quad (4.104)
\]
\[
R^{10} = R^L \quad (4.105)
\]
\[
Q = (R^L - \kappa)^{-1} \quad (4.106)
\]
\[
M = \beta [Q (1 - \beta\kappa)]^{-1} \quad (4.107)
\]
\[
R^k = \frac{P^k M [1 - \beta (1 - \delta)] - \beta\delta P^k \tau^k}{\beta (1 - \tau^k)} \quad (4.108)
\]
\[
MC = \frac{\varepsilon_p - 1}{\varepsilon_p} \quad (4.109)
\]
\[
\Pi^w = \Pi \quad (4.110)
\]
\[
K = \alpha \frac{MC}{R^k} \quad (4.111)
\]
\[
K = \left( \frac{K}{\bar{Y}} \right)^{1-\alpha} \quad (4.112)
\]
\[
Y = K^\alpha \quad (4.113)
\]
\[
I = \delta K \quad (4.114)
\]
\[
G = g_y Y \quad (4.115)
\]
\[
C = Y - I - G \quad (4.116)
\]
\[
MPK = \alpha H^{1-\alpha} \frac{K^{\alpha-1}}{\gamma} \quad (4.117)
\]
\[
MPL = (1 - \alpha) \frac{K^\alpha}{\gamma} H^{-\alpha} \quad (4.118)
\]
\[ W = M C M P L \]  
\[ \Lambda^r = \frac{[(1 - h) C^r]^{-1} - h \beta [(1 - h) C^r]^{-1}}{(1 + \tau^c)} \]  
\[ B = \left\{ [\omega \Lambda^o + (1 - \omega) \Lambda^r] (1 - \tau^w) \left( \frac{\varepsilon^w - 1}{\varepsilon^w} \right) \frac{W}{H^q} \right\} \]  
\[ C^o = \frac{C - (1 - \omega) C^r}{\omega} \]  
\[ I^o = \frac{I}{\omega} \]  
\[ K^o = \frac{K}{\omega} \]  
\[ F = P^k I (1 - \kappa)^{-1} \]  
\[ F^o = \frac{F}{\omega} \]  
\[ B = b_Y A \gamma Y \]  
\[ \bar{B}^{CB} = \psi \bar{B} \]  
\[ \bar{B}^{FI} = (1 - \psi) \bar{B} \]  
\[ L = l \]  
\[ e^\phi = 1 + (1 - L) \left( \frac{R^L}{R} L \right)^{-1} \]  
\[ N = \frac{\bar{B}^{FI} + \bar{F}}{L} \]  
\[ Q^{EH} = (R - \kappa)^{-1} \]  
\[ R^{10,EH} = \kappa + \frac{1}{Q^{EH}} \]  
\[ TP = 1 + R^{10} - R^{10,EH} \]  
\[ f(N) = 0 \]  
\[ f'(N) = 0 \]  
\[ \zeta = \frac{1}{\beta} [R + L (R^L - R)]^{-1} \]  
\[ T = \tau^c C + \tau^w W H + \tau^k \left( (R^k - \delta P^k) K \right) \]  
\[ \tau^L = T - G + (1 - R^L) \bar{B} + (R^L - R) \bar{B}^{CB} \]  
\[ \bar{G} = G + \tau^L \]
4.8 Detailed derivation of the wage setting equation

Remember that the union objective is to:

$$\max_{W_t^z} \sum_{k=0}^{\infty} \beta^{t+k} \left[ \omega U_{t+k}^o + (1 - \omega) U_{t+k}^r \right]$$

subject to the labor demand functions $H_t^z = \left[ \frac{W_t^z}{W_t} \right]^{-\epsilon_t^w \epsilon_t^w} H_t$ and the budget constraints (4.4) and (4.12). Notice also that $U_t^i = f(C_t^i, N_t^z)$ and $C_t^i = g(W_t^z, N_t^z)$, with $i = o, r$. Then the first-order condition of the union with respect to $W_t^z$ reads as:

$$0 = \omega \frac{\partial U_o^t}{\partial C_t^o} \frac{\partial C_t^o}{\partial W_t^z} + (1 - \omega) \frac{\partial U_r^t}{\partial C_t^r} \frac{\partial C_t^r}{\partial W_t^z} + \omega \frac{\partial U_o^t}{\partial H_t^z} \frac{\partial H_t^z}{\partial W_t^z} + (1 - \omega) \frac{\partial U_r^t}{\partial H_t^z} \frac{\partial H_t^z}{\partial W_t^z} + \beta E_t \left[ \omega \frac{\partial U_o^t}{\partial C_t^o} \frac{\partial C_t^o}{\partial W_t^z} + (1 - \omega) \frac{\partial U_r^t}{\partial C_t^r} \frac{\partial C_t^r}{\partial W_t^z} \right]$$

(4.142)

Given the demand function $H_t^z = \left[ \frac{W_t^z}{W_t} \right]^{-\epsilon_t^w \epsilon_t^w} H_t$, we have

$$\frac{\partial U_o^t}{\partial H_t^z} = -B(H_t^z)^\eta,$$

$$\frac{\partial U_r^t}{\partial H_t^z} = -B(H_t^z)^\eta,$$

$$\frac{\partial H_z^t}{\partial W_t^z} = -\epsilon_t^w \epsilon_t^w \frac{H_t^z}{W_t^z}.$$

The derivatives from the households budget constraints read as

$$\frac{\partial C_t^i}{\partial W_t^z} = \frac{\partial \{ (1 - \tau_t^w) W_t^z N_t^z \} }{\partial W_t^z} - \frac{\partial \Phi_t}{\partial W_t^z},$$

$$\frac{\partial C_t^{i+1}}{\partial W_t^z} = \frac{\partial \Phi_t^{i+1}}{\partial W_t^z},$$

with
\[
\frac{\partial \{W_t^z N_t^z\}}{\partial W_t^z} = \frac{\partial}{\partial W_t^z} \left( (1 - \tau_t^w) W_t^z \frac{W_t^z}{W_t} \left( -e_t^w \right) H_t \right)
\]

\[
= (1 - \tau_t^w) W_t^z \frac{W_t^z}{W_t} \left( -e_t^w \right) H_t - (1 - \tau_t^w) e_t^w \frac{W_t^z}{W_t} H_t
\]

\[
= (1 - \tau_t^w) (1 - e_t^w) \frac{W_t^z}{W_t} \left( -e_t^w \right) H_t
\]

\[
= (1 - \tau_t^w) (1 - e_t^w) H_t^z.
\]

and

\[
\frac{\partial \Phi_t}{\partial W_t^z} = \theta_w \left( \frac{W_t^z}{\Pi_{t-1}^w W_{t-1}^z} \Pi_t - \Pi_t^{1-t_w} \right) \frac{W_t^z}{\Pi_{t-1}^w W_{t-1}^z} \Pi_t H_t,
\]

\[
\frac{\partial \Phi_{t+1}}{\partial W_t^z} = \theta_w \left( \frac{W_{t+1}^z}{\Pi_{t+1}^w W_{t+1}^z} \Pi_{t+1} - \Pi_t^{1-t_w} \right) \frac{W_t^z}{\Pi_{t+1}^w (W_{t+1}^z)^2} \Pi_{t+1} W_{t+1}^z H_{t+1}.
\]

Finally, remember that \( \frac{\partial U_t^o}{\partial C_t^o} = \Lambda_t^o \) and \( \frac{\partial U_t^r}{\partial C_t^r} = \Lambda_t^r \). Then, substituting all the derivatives into [4.142] and assuming symmetry so that \( W_t^z = W_t \) and \( H_t^z = H_t \), yields

\[
0 = \omega \Delta_t^o \left( (1 - \tau_t^w) (1 - e_t^w) - \theta_w \left( \frac{W_t}{\Pi_w^w W_{t-1}^z - \Pi_t^{1-t_w} \Pi_t} \right) \right) H_t
\]

\[
+ (1 - \omega) \Delta_t^r \left( (1 - \tau_t^w) (1 - e_t^w) - \theta_w \left( \frac{W_t}{\Pi_w^w W_{t-1}^z - \Pi_t^{1-t_w} \Pi_t} \right) \right) H_t
\]

\[
+ B \left( H_t \right)^n e_t^w \frac{H_t}{W_t}
\]

\[
+ \beta E_t \left( \omega \Delta_t^{o+1} \theta_w \left( \frac{W_{t+1}}{\Pi_{t+1} W_{t+1}^z} \Pi_{t+1}^{1-t_w} \Pi_{t+1} H_{t+1} \right) \right)
\]

\[
+ (1 - \omega) \Delta_t^{r+1} \theta_w \left( \frac{W_{t+1}}{\Pi_{t+1} W_{t+1}^z} \Pi_{t+1}^{1-t_w} \Pi_{t+1} H_{t+1} \right).
\]
Multiply by $\frac{W_t}{H_t}$ and define the nominal wage inflation as $\Pi_t^w = \frac{W_t}{W_{t-1}}$: 

$$0 = \omega \Lambda_t^o \left\{ (1 - \tau_t^w) (1 - e_t^w \varepsilon_t^w) - \theta_w \left[ \frac{\Pi_t^w}{\Pi_{t-1}^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^w}{\Pi_{t-1}^w} \right\} W_t$$

$$+ (1 - \omega) \Lambda_t^r \left\{ (1 - \tau_t^w) (1 - e_t^w \varepsilon_t^w) - \theta_w \left[ \frac{\Pi_t^w}{\Pi_{t-1}^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^w}{\Pi_{t-1}^w} \right\} W_t$$

$$+ B (H_t)^n e_t^w \varepsilon_t^w$$

$$+ \beta E_t \left\{ \omega \Lambda_{t+1}^o \theta_w \left[ \frac{\Pi_{t+1}^w}{\Pi_t^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^{1-t_w} H_{t+1}}{\Pi_t^w H_t} W_{t+1} \right\}$$

$$+ (1 - \omega) \Lambda_{t+1}^r \theta_w \left[ \frac{\Pi_{t+1}^w}{\Pi_t^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^{1-t_w} H_{t+1} W_{t+1}}{\Pi_t^w H_t} \right\}.$$

Factorizing the terms in the curly brackets yields:

$$0 = \left[ \omega \Lambda_t^o + (1 - \omega) \Lambda_t^r \right] \left\{ (1 - \tau_t^w) (1 - e_t^w \varepsilon_t^w) - \theta_w \left[ \frac{\Pi_t^w}{\Pi_{t-1}^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^w}{\Pi_{t-1}^w} \right\} W_t$$

$$+ B (H_t)^n e_t^w \varepsilon_t^w$$

$$+ \beta E_t \left\{ \left[ \omega \Lambda_{t+1}^o + (1 - \omega) \Lambda_{t+1}^r \right] \theta_w \left[ \frac{\Pi_{t+1}^w}{\Pi_t^w} - \Pi_t^{1-t_w} \right] \frac{\Pi_t^{1-t_w} H_{t+1} W_{t+1}}{\Pi_t^w H_t} \right\}.$$

Finally, dividing by $\left[ \omega \Lambda_t^o + (1 - \omega) \Lambda_t^r \right] W_t$ and defining $\bar{\Lambda}_t = \omega \Lambda_t^o + (1 - \omega) \Lambda_t^r$ yields the wage schedule (4.16).

### 4.9 The consolidated government budget constraint

Following Gertler and Karadi (2013), in order to conduct its asset purchase programme, the central bank issues short-term debt to households at the riskless interest rate. In particular, let $\bar{B}_t^{CB} = \frac{Q_t B_t^{CB}}{P_t}$ be the real value of long-term government debt purchased by the central bank and let $\bar{B}_t^S = \frac{B_t^S}{P_t}$ be the real value of short term debt issued by the central bank. The central
bank balance sheet reads as:

\[ \bar{B}^{CB}_t = \bar{B}^S_t, \]  

(4.143)

which states that the purchases of long-term debt (assets of the CB) are financed by short-term liabilities. The central bank lends at the long-term rate \( R_{t+1}^L \) and borrows at the riskless rate \( R_t \). At time \( t \), the central bank earns the return from the stock of government bonds carried from the previous period and pays the interest on the short-term liabilities issued in \( t - 1 \). It follows that the central bank’s profits are determined by:

\[ \Xi_t = \left( \frac{R_t^L}{\Pi_t} - 1 \right) \bar{B}^{CB}_{t-1} - \left( \frac{R_{t-1}^L}{\Pi_t} - 1 \right) \bar{B}^S_{t-1}. \]  

(4.144)

Substituting the CB’s balance sheet (4.143) into it’s profits function (4.144) yields:

\[ \Xi_t = \left( \frac{R_t^L}{\Pi_t} - 1 \right) \bar{B}^{CB}_{t-1} - \left( \frac{R_{t-1}^L}{\Pi_t} - 1 \right) \bar{B}^{CB}_{t-1} \]

\[ = \frac{R_t^L - R_{t-1}}{\Pi_t} \bar{B}^{CB}_{t-1}; \]  

(4.145)

which enters the consolidated government budget constraint (4.41) since we assume that profits from QE are returned to the Treasury.
4.10 Bayesian impulse response to standard shocks

Figure 4.9: Mean Bayesian impulse responses of selected macroeconomic variables to standard supply shocks. Rows are variables, columns are shocks. All shocks are expansionary.
Figure 4.10: Mean Bayesian impulse responses of selected macroeconomic variables to standard demand shocks. Rows are variables, columns are shocks. All shocks are expansionary.
### Variance decomposition of the estimated model

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<th>TFP specific</th>
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Table 4.4: Variance decomposition
4.12 Estimation diagnostics

In this section we report some estimation diagnostics to verify that the model is correctly estimated. Figure 4.11 plots the prior and posterior distributions of all the estimated parameters. Overall, the posterior distributions are quite apart from the prior thus implying that the data is informative to identify the parameters. Then, we verify that the two parallel chains of the Monte-Carlo-Markov-Chain Metropolis-Hastings (MCMC-MH) algorithm have actually converged. Figure 4.12 plots the multivariate convergence diagnostic for the Brooks and Gelman (1998) diagnostics (upper panel) and for the second and third central moments (middle and lower panels, respectively). The fact that the red and blue lines are very close to each other implies that the two chains have almost certainly converged. To sum up, estimation diagnostics reveal that the model is correctly estimated.

4.13 Bayesian estimation and historical decompositions: robustness checks

In this section we assess the robustness of the Bayesian estimation and the following historical decompositions of output and inflation. We first check whether our results are affected by the ZLB constraint on the policy rate. Then, we assess the role of imposing the measurement error on the measurement equation of consumption.

We report the posterior estimates of the alternative models along with the estimates of the baseline model estimated in Section 4.3 in Tables 4.5 and 4.6.

We furthermore plot the historical decompositions of output and inflation, along with those of the baseline model, in Sections 4.13.3-4.13.10. All the main results survive the robustness checks and the conclusions about the policy stance in the Euro Area carry over the alternative models.
Figure 4.11: Prior and posterior distributions of estimated parameters. Prior: black-dashed line; Posterior: blue-solid line.
4.13.1 The role of the ZLB

In the main text we estimate the model including a period in which the EONIA rate hit the ZLB (2012Q1) and then turned negative (2014Q4). However, using standard log-linear approximation methods to estimate the DSGE model does not allow for the imposition of the ZLB. It follows that the difference between the observed variable and the corresponding variable in the model is picked up by the exogenous shock to the interest rate whenever the ZLB constraint binds (see also footnotes 25 and 32).

One way to circumvent this issue is to estimate the model up to period before the ZLB started binding and then use non-linear techniques to simulate it with the ZLB, see i.e. Chen et al. (2012), Drautzburg and Uhlig (2015), Del Negro et al. (2015), Hirose and Inoue (2016), Linde et al. (2016), Anzoategui et al. (2016) and Gust et al. (2017). However, these papers have
to rely on different methods to simulate the model, i.e. either by using a perfect foresight solution and/or by using a non-linear solution only for the ZLB constraint or for the entire system of equations, with little agreement about which method is actually more appropriate.

Moreover, Kollmann et al. (2016) estimate a DSGE model of the Euro Area up to 2016Q4 without accounting for the ZLB and then, as a robustness, re-estimate the model using the method developed by Guerrieri and Iacoviello (2015). They find only marginal changes in their results and argue that the ZLB was not a significant constraint on monetary policy, in line with the conclusions of Fratto and Uhlig (2014) and Linde et al. (2016).

Alternatively, one could estimate the DSGE model replacing the policy rate with a shadow rate, as Mouabbi and Sahuc (2017) do for the Euro Area. The shadow rate is a counterfactual policy rate that takes into account the effects of all the unconventional monetary policies implemented by the central bank and is free to move in negative territory, thus circumventing the estimation issues posed by the ZLB.

To check the robustness of our results we will opt for the latter since it is more directly comparable to our main analysis. We therefore re-estimate the model using the Eonia shadow rate constructed by Wu and Xia (2017).

Figure 4.13 plots the actual Eonia rate against the shadow Eonia rate computed by Wu and Xia (2017) from 2004Q4 to 2017Q2. By construction the shadow rate equals the actual rate until unconventional policies are implemented at the ZLB. Indeed, the two rates are very close to each other (with a correlation of 98% from 2004Q4 to 2008Q4, when they start diverging) until 2011Q4 when the Eonia rate approached the ZLB and unconventional measures started to be implemented by the ECB. Even if the Eonia was not actually prevented from turning negative from 2014Q4, the shadow rate fell more due to the implementation of unconventional monetary policies (still

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33 Studies that estimate DSGE models of the US economy during the ZLB period using standard methods are Albonico et al. (2017) and Quint and Rabanal (2017).

34 Mouabbi and Sahuc (2017) construct their own shadow rate which is similar to the one constructed by Wu and Xia (2017) but it is not publicly available. The main difference between the two is that the latter allows for a time-varying lower bound of the interest rate.

35 Interest rates are expressed at quarterly frequency.
the two rates display a correlation of 90% throughout the entire sample).

It must be said that using the shadow rate in place of the Eonia rate comes at the cost of shutting down the explicit unconventional monetary policy channel in our model. Indeed, UMP is captured by the shadow rate and we will not be able to disentangle the effects of conventional vs unconventional monetary policies.\(^{36}\)

To estimate the model, we first eliminate the ECB government bonds purchase from our observables and the corresponding measurement equation. We then assume that the central bank does not hold any government debt in steady state and prevent it from buying newly issued debt, which results in shutting down the UMP channel by eliminating the exogenous shock \(\epsilon_t^y\). Finally, we replace the Eonia rate with the shadow rate as the observable

\(^{36}\)This shortcoming actually dictated the choice of using the Eonia rate despite the ZLB in the main analysis.
of the policy rate. The series constructed by Wu and Xia (2017) starts in 2004Q4 hence we extend it back to 1999Q1 using the Eonia rate given that the two coincide in normal times and the high correlation displayed from 2004Q4 onwards.

The fourth column of Tables 4.5 and 4.6 report the posterior means of the parameters estimated using the shadow rate as the observable for the policy rate. The estimated values are virtually unchanged with respect to the baseline model estimated using the Eonia rate (third column of Tables 4.5 and 4.6).

The historical decompositions of output and inflation are likewise extremely similar with respect to the baseline model with a slight exception for inflation (compare the first two figures of Sections 4.13.3 to 4.13.6 for output, and the first two figures of Sections 4.13.7 to 4.13.10 for inflation). Indeed, in the last part of the sample, the contribution of monetary policy, which embeds both conventional and unconventional measures, is overall negative on inflation while it is positive in the baseline model. Therefore, not explicitly modeling the UMP channel might bias the results for inflation. Overall, the ZLB constraint does not seem to be a binding constraint on the Eonia rate, as argued by Kollmann et al. (2016).

It must be said that the majority of papers studying the issues that the ZLB poses on the estimation of DSGE models (and cited above) focus on the US economy. However, the Federal Funds (FFR) and the Eonia rates display different behaviors. First, the FFR reached the ZLB much earlier (2008Q1) than the Eonia (2012Q1). Then, the FFR was effectively prevented from turning negative while the Eonia rate became negative in 2014Q4, although with a much higher inertia.

We therefore conclude that the path followed by the Eonia rate implied a binding ZLB constraint only to some extent, thus resulting in very similar results when estimating the model using the actual Eonia or the shadow rate.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Mean</th>
<th>90% Confidence Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Shadow rate</td>
<td>ME</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>ME</td>
<td></td>
</tr>
<tr>
<td>Structural Inv. Frisch elasticity</td>
<td>$\eta$</td>
<td>$0.2795 \ [0.1044;0.4380]$</td>
</tr>
<tr>
<td>Habits in consumption</td>
<td>$h$</td>
<td>$0.8551 \ [0.8045;0.9021]$</td>
</tr>
<tr>
<td>Fraction of optimizing households</td>
<td>$\omega$</td>
<td>$0.9021 \ [0.8559;0.9473]$</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\psi_i$</td>
<td>$5.5202 \ [4.2326;6.9571]$</td>
</tr>
<tr>
<td>Net worth adjustment costs</td>
<td>$\psi_n$</td>
<td>$0.6362 \ [0.4104;0.8575]$</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\theta_p$</td>
<td>$172.92 \ [137.19;199.99]$</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\iota_p$</td>
<td>$0.1535 \ [0.0366;0.2774]$</td>
</tr>
<tr>
<td>Wage stickiness</td>
<td>$\theta_w$</td>
<td>$108.98 \ [66.861;154.22]$</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>$\iota_w$</td>
<td>$0.3253 \ [0.1102;0.5573]$</td>
</tr>
<tr>
<td>Tax smoothing</td>
<td>$\rho_{\tau}$</td>
<td>$0.8649 \ [0.8012;0.9280]$</td>
</tr>
<tr>
<td>Tax reaction to debt</td>
<td>$\rho_{\tau b}$</td>
<td>$0.0849 \ [0.0536;0.1180]$</td>
</tr>
<tr>
<td>Tax reaction to output</td>
<td>$\rho_{\tau y}$</td>
<td>$0.0454 \ [0.0046;0.0941]$</td>
</tr>
<tr>
<td>Government spending smoothing</td>
<td>$\rho_g$</td>
<td>$0.9191 \ [0.8962;0.9418]$</td>
</tr>
<tr>
<td>Government spending reaction to debt</td>
<td>$\rho_{gb}$</td>
<td>$0.1063 \ [0.0852;0.1291]$</td>
</tr>
<tr>
<td>Government spending reaction to output</td>
<td>$\rho_{gy}$</td>
<td>$0.1057 \ [0.0583;0.1551]$</td>
</tr>
<tr>
<td>Inflation - Taylor rule</td>
<td>$\rho_{\pi}$</td>
<td>$1.7243 \ [1.5324;1.9277]$</td>
</tr>
<tr>
<td>Output - Taylor rule</td>
<td>$\rho_{y}$</td>
<td>$0.0839 \ [0.0263;0.1496]$</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho_r$</td>
<td>$0.9615 \ [0.9480;0.9739]$</td>
</tr>
</tbody>
</table>

Table 4.5: Prior and posterior distributions of estimated structural parameters: baseline vs alternative models (90% confidence bands in square brackets).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Shadow rate</th>
<th>ME Output</th>
<th>ME investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous processes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$</td>
<td>0.9897 [0.9752;0.9992]</td>
<td>0.9901 [0.9814;0.9986]</td>
<td>0.9890 [0.9794;0.9981]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_a$</td>
<td>1.2241 [0.9757;1.4902]</td>
<td>1.2231 [1.0192;1.4201]</td>
<td>1.1799 [0.8967;1.4496]</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>$\rho_m$</td>
<td>0.4271 [0.2901;0.5606]</td>
<td>0.4270 [0.3158;0.5418]</td>
<td>0.4369 [0.3220;0.5573]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_m$</td>
<td>0.0779 [0.0649;0.0918]</td>
<td>0.0774 [0.0663;0.0883]</td>
<td>0.0786 [0.0671;0.0900]</td>
</tr>
<tr>
<td>Unconventional Monetary Policy</td>
<td>$\rho_\psi$</td>
<td>0.9883 [0.9747;0.9990]</td>
<td>N.A.</td>
<td>0.9888 [0.9798;0.9987]</td>
</tr>
<tr>
<td>Preference</td>
<td>$\rho_b$</td>
<td>0.8855 [0.8380;0.9319]</td>
<td>0.8878 [0.8487;0.9262]</td>
<td>0.9249 [0.8900;0.9604]</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\rho_\sigma$</td>
<td>0.9730 [0.2737;0.8760]</td>
<td>0.5741 [0.3217;0.8316]</td>
<td>0.4990 [0.2052;0.8130]</td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>$\rho_\omega$</td>
<td>0.8781 [0.8254;0.9277]</td>
<td>0.8785 [0.8355;0.9218]</td>
<td>0.8650 [0.8167;0.9131]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\omega$</td>
<td>0.2072 [0.0279;0.3968]</td>
<td>0.2111 [0.0467;0.3675]</td>
<td>0.1863 [0.0355;0.3216]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_g$</td>
<td>0.2945 [0.1982;0.3851]</td>
<td>0.2981 [0.2244;0.3763]</td>
<td>0.2615 [0.1835;0.3412]</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\tau$</td>
<td>2.4933 [1.3260;1.6651]</td>
<td>2.4909 [1.3533;1.6326]</td>
<td>2.4905 [1.3641;1.6128]</td>
</tr>
<tr>
<td>Credit</td>
<td>$\rho_\phi$</td>
<td>2.8638 [2.2236;3.5395]</td>
<td>2.8619 [2.2899;3.4011]</td>
<td>2.2347 [1.7399;2.6905]</td>
</tr>
<tr>
<td>Measurement error</td>
<td>$\sigma_{me}$</td>
<td>2.2781 [1.9745;2.5868]</td>
<td>2.2767 [1.9744;2.5860]</td>
<td>1.2432 [1.0812;1.4064]</td>
</tr>
</tbody>
</table>

Table 4.6: Prior and posterior distributions of estimated shock processes: baseline vs alternative models (90% confidence bands in square brackets).
4.13.2 The role of the measurement error

In estimating the main model, we follow Schmitt-Grohe and Uribe (2012) by introducing a measurement error in one of the measurement equations since we observe all the variables in the resource constraint. Schmitt-Grohe and Uribe (2012) assume that the measurement error applies to output while we added it to consumption. Our choice is dictated by the fact that output is our main variable of interest hence adding the measurement error to it might lead to a potential bias in the historical decomposition.

Nevertheless, in this section we check whether our results are robust to adding the measurement error first to output and then to investment instead of adding it to consumption.

The fifth and sixth columns of Tables 4.5 and 4.6 report the posterior means of the parameters for the models estimated with the measurement error on output and investment, respectively. The posterior estimates are very similar across the three specifications, with the exception for the fraction of optimizing households in the model with the measurement error on investment, which declines from 90% to 76%.

Historical decompositions of output and inflation are very similar across the three specifications (compare first, third and fourth figures of Sections 4.13.3 to 4.13.10), with a slightly stronger effect of fiscal policies in the model with the measurement error on investment due to the higher estimated fraction of rule-of-thumbers. Overall, the results are robust to adding the measurement error to one of the main three observables of the resource constraint.

37 We also tried to estimate the model by removing consumption from our observables and the measurement error. However, the estimation was severely impaired since we have not accounted for the main component of GDP.
4.13.3 Output historical decompositions: Policy stance

Figure 4.14: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.15: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.16: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.17: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.4 Output historical decompositions: all shocks

Figure 4.18: Historical decomposition of cyclical deviations of output from trend to all shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.19: Historical decomposition of cyclical deviations of output from trend to all shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.20: Historical decomposition of cyclical deviations of output from trend to all shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.21: Historical decomposition of cyclical deviations of output from trend to all shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.5 Output historical decompositions: all policies

Figure 4.22: Historical decomposition of cyclical deviations of output from trend to policy shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.23: Historical decomposition of cyclical deviations of output from trend to policy shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.24: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.25: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.6 Output historical decompositions: Monetary vs Fiscal policies

Figure 4.26: Historical decomposition of cyclical deviations of output from trend to monetary and fiscal policies shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.27: Historical decomposition of cyclical deviations of output from trend to monetary and fiscal policies shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.28: Historical decomposition of cyclical deviations of output from trend to monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.29: Historical decomposition of cyclical deviations of output from trend to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.7 Inflation historical decompositions: Policy stance

Figure 4.30: Historical decomposition of inflation to joint monetary and fiscal policies shocks. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.31: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.32: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.33: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.8 Inflation historical decompositions: all shocks

Figure 4.34: Historical decomposition of inflation to all shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.35: Historical decomposition of inflation to all shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.36: Historical decomposition of inflation to all shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.37: Historical decomposition of inflation to all shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.9 Inflation historical decompositions: all policies

Figure 4.38: Historical decomposition of inflation to policy shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.39: Historical decomposition of inflation to policy shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.40: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.41: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
4.13.10 Inflation historical decompositions: Monetary vs Fiscal policies

Figure 4.42: Historical decomposition of inflation to monetary and fiscal policies shocks: baseline model. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.43: Historical decomposition of inflation to monetary and fiscal policies shocks: model with shadow Eonia. Bold line represents the data, bars represent the contribution of each shock.
Figure 4.44: Historical decomposition of inflation to monetary and fiscal policies shocks: model with measurement error on output. Bold line represents the data, bars represent the contribution of each shock.

Figure 4.45: Historical decomposition of inflation to joint monetary and fiscal policies shocks: model with measurement error on investment. Bold line represents the data, bars represent the contribution of each shock.
Chapter 5

Thesis conclusions

This doctoral thesis is a collection of three papers in macroeconomics studying multi-sector models suitable for policy analysis. It sheds light on the importance of accounting for a multi-sector environment and draws conclusions about the conduct of macroeconomic policies.

The first chapter of the thesis builds on the literature of two-sector New-Keynesian models with a distinction between durable and nondurable goods. The aim of the paper is to challenge a typical assumption made in the literature about the degree of price stickiness in the durables sector. While prices of durables have been usually assumed to be flexible, the paper shows that this assumption is consistent with a definition of the durables sector comprising only housing durables. Conversely, when the durables sector includes both housing and non-housing goods, prices of durables are estimated to be as sticky as nondurables. To reach these conclusions, the paper estimates both Structural Vector Autoregressive and New-Keynesian two-sector models. Moreover, we further extend the analysis of the New-Keynesian model to a three-sector economy, thus accounting for realistic features of housing and non-housing durable goods. We confirm that prices of non-housing durables and nondurables display a similar degree of price stickiness whereas prices of housing durables are dramatically more flexible. These results carry the important implications that when building a multi-sector New-Keynesian model, prices of non-housing durables should be assumed to be sticky while
housing durables are consistent with a notion of flexible prices. From a policy
point-of-view, we use both models to estimate the responses of the relative
price of durables to a monetary contraction and find that monetary pol-
cy is non-distortive in the allocation between nondurables and non-housing
durables, whereas it can potentially create allocative distortions between
nondurables and housing durables.

The second chapter of this thesis performs optimal monetary policy anal-
ysis in a two-sector New-Keynesian model, similar to the one studied in the
first chapter. The ultimate aim is to determine how the central bank assigns
weights to sectoral inflation rates in constructing the inflation measure to
target, and how such a choice is affected by the degree of labor mobility
across sectors. Indeed, we allow workers to be reallocated across sectors and
estimate the model together with the parameter governing the degree of sec-
toral labor mobility. We estimate a rather limited degree of labor mobility
and show that this feature is crucial for the conduct of monetary policy. Our
novel finding is that an inverse relationship between sectoral labor mobility
and the optimal weight the central bank should attach to durables inflation
arises. The paper finally argues that the degree of segmentation in the labor
market is a crucial aspect central bankers should consider in the conduct of
monetary policy.

The final chapter of the thesis focuses on the role of shocks and poli-
cies on the Euro Area business cycle. A standard New-Keynesian model is
enriched with three crucial features, namely a financial sector, long-term gov-
ernment debt and unconventional monetary policy. The model studied thus
accounts for the recent global financial crisis, the subsequent sovereign debt
crisis experienced in the eurozone, and the unconventional monetary policy
implemented by the European Central Bank. The ultimate goal of this chap-
ter and its novel contribution is to assess the joint monetary and fiscal policy
stance in the eurozone. As far as the importance of the shocks are concerned,
the model estimates a crucial role of the credit shock in generating the 2008
financial crisis. The joint policy stance in the Euro Area is estimated to have
been expansionary in the aftermath of the crisis. However, as countries in
the eurozone experienced a sovereign debt distress that forced governments
to implement austerity measures, and conventional monetary policy started being constrained by the zero-lower-bound, the policy stance turned to be contractionary on the Euro Area output. The implementation of unconventional monetary policy by the European Central Bank helped counteract the negative contribution of the other policies but the overall stance remained contractionary, due to the joint drag caused by fiscal policies and the zero-lower-bound. Our results carry the important policy implication that more expansionary fiscal policies could be designed to help the Euro Area recover faster from the financial and sovereign debt crises.

To conclude, this thesis contributes to the macroeconomic literature along several dimensions. First, it demonstrates that multi-sector macroeconomic models uncover features of the economy and relative policy implications that are overlooked in standard one-sector DSGE models. By questioning one of the typical assumption made in the literature of two-sector New-Keynesian models with durable goods, it provides future research with a better understanding of sectoral heterogeneity and establishes which are the crucial features such a models should include. This thesis further sheds light on the importance of sectoral labor mobility for the optimal conduct of monetary policy. Central banks should consider the degree at which workers can be reallocated across sectors when constructing the inflation measure to target. Finally, the thesis demonstrates that it is vital to account for a financial sector when studying the Euro Area business cycle. By using a multi-sector model the thesis provides a thorough assessment of the Euro Area policy stance that highlights crucial interactions between monetary and fiscal policies.

Overall, the thesis provides a thorough assessment of multi-sector macroeconomic models that contributes to both academic and policy debates.


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