Toll optimisation on river crossings serving large cities

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Abstract

There is renewed interest in the private sector financing and operation of major transportation projects, in which a significant financial contribution comes from toll revenues. Tolling is ideally suited to river crossings, where the tolls are relatively easy to administer and collect. Because of their span, bridges over river estuaries are particularly expensive to build and maintain and so need to be put on a firm financial footing. Toll revenue is therefore a key consideration if such projects are to be financially viable and risk is to be minimised. There may be other issues to do with who benefits from the bridge and whether differential tolls should apply to local residents and non-resident bridge users. In addition, such bridges may be linked to wider economic objectives, such as local development and regeneration. This paper describes a model for estimating optimum bridge tolls, from both a financial and a welfare perspective and provides a case study that illustrates a range of scenarios that are of general interest.

Key words: new bridges, cities, tolls, revenue, congestion

Introduction

River estuaries often make natural harbours, around which many large cities have developed such as Sydney, San Francisco, Lisbon and London. Orbital accessibility is generally provided by a series of bridges or tunnels, mainly upstream from the natural harbour where the estuary is narrow. Over time, lack of downstream orbital access may appear to be constraining the economic development of the entire city. Bridges are expensive to build, requiring long spans as well as navigational clearance for maritime shipping. The option of tunnelling is usually even more expensive than building a bridge, so both the engineering and financing problems are challenging.

The provision of transport infrastructure is traditionally the domain of public authorities. In recent years, there has been a renewed interest in the ability of the private sector to finance and operate major transportation projects, possibly via a partnership with the public authority, but with a major financial contribution coming from toll revenues. Further, tolling is ideally suited to estuarial crossings, where the

[1] Views expressed in this paper are due to the authors and do not necessarily represent Government policy.

tolls are relatively easy to administer and collect. This paper examines alternative tolling strategies and their impact on traffic, revenues and transport user benefits.

Tolling is one form of road pricing which is common in countries like the US (Hoolguin-Verasa et al, 2006). Another form is area based congestion charging, requiring drivers to pay to enter a zone of the city during certain times of the day (see Larson, 1995 or Ieromonachoua et al, 2006 for Norwegian case studies). While the publicised stated objectives may be to reduce traffic congestion and to improve air quality, the use of charging revenues, and the general financial success of the project, are also major factors (Livingstone, 2001; Hyman and Mayhew, 2002). Competing objectives and gaining of public acceptance are two of the reasons for the initially slow uptake of such schemes (Harrington et al, 2001; Eliasson and Mattsson, 2006).

Funding arrangements for tolled bridges are often complex with a mix of public and private finance so that the financial risks are often appreciable. The public sector in turn may involve a mixture of national, city-wide and local forms of administration and there may be interactions with other charging schemes locally. The public sector is more likely to adopt a welfare perspective, but both public and private investors have an interest in knowing which tolling strategies yield the best value for money (for example see Wong et al (2005), De Palma et al (2006), or Rouwendala and Verhoef (2006) for discussions of the principles and issues involved).

It is helpful to distinguish two contrasting situations. Without congestion or financial constraints, a welfare based tolling policy would typically lead to zero toll levels. Without congestion, but under financial constraints, a welfare based tolling would lead to the minimum toll levels that are sufficient to repay any loans for bridge construction. In the presence of congestion, welfare based policies can justify higher toll levels if the alternative of increasing road capacity is available (Romilly, 2004)., A robust public sector case, including the use to which toll revenues are put would be required to make a reasonable decision between these options. In order to inform such discussions, this paper examines bridge toll setting, from both a revenue and welfare perspective.

The lack of cross-river access may appear to be a constraint on the local economy, with communities either side of the river being comparatively disadvantaged. A new bridge could help to stimulate the local economies on both sides of the river by widening the market for locally produced goods and services, generating agglomeration economies. For example, the Standing Committee on Trunk Road Assessment SACTRA (1999) examined the links between transport and the wider economy, and Verhoef (2004) has developed a monocentric urban model to examine the links between road user charges, residential densities and labour supply. A broad review of historical and contemporary topics in urban economics is provided in Mills (2000).

The prospect of wider economic benefits may lead to considerable public interest and could provide part of the rationale for constructing a new bridge. Whilst such a views would appear to be entirely reasonable, the explicit treatment of regeneration benefits is outside the scope of this paper. However, such considerations may be important when assessing selective tolling policies, for example, a policy of setting premium charges for remote traffic, when taken in conjunction with local concerns about congestion resulting from though traffic attracted by the new bridge.

These issues, if improperly understood, may lead to undue tension between local economic objectives on the one hand and fears of excessive congestion from remotely generated traffic on the other. There is a further danger that geographically differentiated tolling could compromise the tolling revenue received on the new bridge, and thereby jeopardise the financial viability of the project. In the illustrative scenarios used in this paper, it will be demonstrated that such a concern appears to be unfounded, but this finding merits further investigation. The development of a rationale for spatially differentiated tolling strategies is resumed towards the end of the paper, under the items for discussion.

Repayment Cost on a Loan for Bridge Construction

What kind of investment and payback is required to finance a new bridge? Suppose that the new bridge cost £500m to construct but the project attracted a (non-repayable) grant of £100m, leaving an outstanding loan requirement of £400m. Over 30 years, at

5% interest p.a., a toll revenue of £68,000/day would be needed to repay the loan, net of the costs of bridge operation.

Now assume that the new bridge attracted 60,000 toll paying vehicles/day. The bridge operator would need to charge an average toll of £1.13/vehicle, plus enough to cover the cost of operation and maintenance of the bridge. If the (non-repayable) grant were increased to £150m, with the same payback period, interest rate and traffic levels, the required daily toll revenue would be £59,000/day the required average toll/vehicle would be 99p, plus the costs of operation and maintenance.

In practice not all types of vehicles would pay identical tolls. Heavy goods vehicles would typically be charged more than cars as they impose much greater road maintenance costs. Sometimes public transport operators may be exempt from charges, although this practice is far from universal (e.g. Golden Gate Bridge).

If the franchise for the new bridge was put out to competitive tender, and the successful operator had to finance the bridge entirely out of toll revenues, under a tendering process that recognised any adverse social impacts of high toll levels, there may be only a small margin between the operators' costs and the toll revenues received. If a tolling strategy other than revenue maximisation was then implemented the private operator may need to be subsidised in order to continue operation. The need for such subsidies depends on just how much revenue is foregone by the alternative tolling strategy and the narrowness of the profit margin faced by the private operator. The methodology contained in this paper can provide a basis for examining questions of this kind.

An example of a proposed new toll bridge

While the methodology presented here is expected to be of general application, when illustrating the methods, it is helpful to have a specific example in mind. The examples given are based, very roughly, on the current geography and road network for the London area. However this example is only mentioned to provide an illustrative focus and is not in any way intended to be a specific subject for investigation in this paper.

In 2003 London introduced central area weekday road charges, covering a three-kilometre radius of the centre of the city. The charging area spans the river Thames with a further westward extension being implemented from February 2007. There is a plan to build a new bridge in East London, in the downstream direction from the central charging zone where there are currently only four existing road crossings. The farthest down the estuary is Dartford, a tolled crossing on the alignment of the M25 outer orbital road. This crossing is 25kms east of the city centre, and consists of a relatively new 6-lane bridge devoted to traffic going north and an older four-lane tunnel for traffic going south. Other Thames downstream un-tolled crossing points include Tower Bridge, adjacent to the central charging zone, and Blackwall, an often congested tunnel. There is also a free ferry with extremely limited capacity, close to the site of the proposed new bridge.

Clearly, the pattern and level of utilisation of the new bridge will depend on the tolling strategy, as well as any associated subsidy policies, or premium charges, for specific classes of user, including charges differentiated by location of residence. For an illustration of discussions of the range of analytical issues that arise, the reader may wish to refer to published documents from the Thames Gateway Board (2004).

Plan of the paper

Our approach entails building a simplified model of traffic flows, showing how demand varies according to the toll levels and how they depend on other factors including prevailing traffic speeds and to the perceived value of travel time savings. Through the inclusion of a demand function and consideration of congestion effects we are able to identify toll levels that would yield the maximum revenue, as well as the revenue implications of alternative tolling strategies in which welfare is an explicit component.

The first part of the paper describes the basic model and spatial metrics used to estimate traffic levels on each river crossing at a sufficient level of detail to be able to be able to estimate the size of route catchments and the market shares at different toll levels, speed and value of time assumptions. The extended theory then deals with the issues of revenue and welfare maximisation. Welfare here represents the sum of transport user benefits (consumer surplus) and transport provider benefits (toll revenue).

To evaluate the toll that maximises welfare requires the calculation of an equilibrium level of bridge traffic, which recognises the feedback between bridge traffic levels and congestion, at any exogenous level for the bridge toll. The scenarios used to illustrate our results are designed to give a measure of the sensitivity of the model to different assumptions about traffic speeds, the value of time, and congestion effects, all of which may be useful in setting toll levels under operational conditions. Such flexibility is necessary since once the new bridge is built it cannot be guaranteed that the assumptions on which the bridge was planned will still be entirely accurate.

The baseline parameters in scenario A are chosen to broadly correspond with the situation currently pertaining to London. However, any correspondence thereafter is hypothetical. In scenario B we test the robustness of the revenue maximising tolls to changes in the assumed value of travel time savings; in scenario C, the toll at the old Bridge is raised substantially higher; whereas in scenario D, we assume that all traffic passing through the central area of the city would also face a charge.

However, whilst A-D deal with the aggregate effect of bridge usage, they do not discriminate between local users (i.e. locally generated trips) and remote users (i.e. trips generated a long distance from the new bridge). This is the issue of 'spatially differentiated tolling' and concerns the impact of premium charges for remote users of the new bridge and, conversely, the case for toll subsidies to local users.

We deal with this issue by dividing the city into sub-areas, defined in terms of either annuli or radial sectors, and then evaluate the revenue maximising toll for traffic that is generated in each of these sub-areas. Contrary to expectation we find that tolls in the vicinity of the bridge need to be higher but the differences in toll level are small and probably not worth the extra administrative costs of applying differential tolls on traffic using the new bridge.

Finally we reconsider scenario A under congested conditions and estimate what the toll should be under revenue and welfare maximising assumptions, and compare this with the congestion-free revenue maximising toll. The implications of our findings are then discussed in a concluding section.

Basic concepts and assumptions

Our simplified model is based on a circular representation of a city with two major orbital ring roads and a river flowing from west to east, where it widens substantially at it approaches the river estuary. Crossing the river to the west of the centre is assumed to be toll free, whereas on the eastern side, where the river is much wider, a toll is levied at two bridges, an old bridge and a new bridge. Routeing in this idealised city is either along radial arcs through the city centre or by means a combination of orbital or radial routes on either of the two orbital ring roads, which cross the estuary at the two tolled bridges. The outer orbital crosses the estuary at the old tolled bridge and the inner orbital crosses the estuary at the new tolled bridge.

If the model were applied to London the old bridge could be taken to correspond to Dartford, and the new bridge could be taken to correspond to Thames Gateway. London’s central charging zone is small enough for through trips to avoid it by using toll-free bridges near the centre, and this will be assumed in three of the scenarios. With slightly different input assumptions, the model could just as well be applied to other major cities.

There are effectively five routes defined altogether: radial routes via the centre, two orbital routes east (downstream) of the centre and two orbital routes west (upstream) of the centre. The upstream western orbital routes will not incur a toll and the radial routes will normally also be toll-free. This would alter however if there were a central area toll that was unavoidable by traffic using routes passing through the centre. This possibility is considered in the last scenario.

For any origin and destination there will be a preferred or least cost route. We define the area in which it is quicker to reach a given destination by a particular route than via any other route, as the catchment area for that route (Hyman and Mayhew, 2000 and 2001b). In this way the city may be divided up geographically into up to five different route catchments, depending on the location of the fixed destination, the average speeds achievable on each route, the level of the tolls and congestion charge, and on the monetary of travel time.

The five routes that are potentially available are defined as:

- Ring 1 outer orbital route (average radius 25kms)
    - Downstream (tolled)
    - Upstream (un-tolled)
- Ring 2 inner orbital route (average radius 12.5kms)
    - Downstream (tolled)
    - Upstream (un-tolled)
- Double radial (through the city centre): typically un-tolled, except in one scenario where the central charge is applied.

We assume that the traffic on any route will be proportional to the size of its route catchment, but weighted according to the density of trip generations. A route catchment area may therefore be thought of as the 'market area' for any particular facility such as, in this case, a bridge and the traffic flow directed towards the bridge a measure of demand. By calculating the weighted size of each catchment area and summing over all destinations, we can determine how much traffic uses the new tolled bridge, the existing tolled bridge and each of the other available routes. This enables us to undertake an analysis of the resulting traffic flows under a range of charging conditions. Finally, it is important to ensure that the central area toll is modelled on an equivalent (one-way) basis as the bridge tolls. The equivalent one-way toll is simply taken to be the all-day central charge divided by the expected number of daily one-way trips/user.

Route metrics

We assume that travel cost in the idealised city can be represented by an orbital-radial route metric. Such metrics have been extensively studied and their analytical and numerical properties are well understood (Mayhew, 2000; Hyman and Mayhew, 2004) with several published applications **(**Hyman and Mayhew, 2001a, 2001b and 2002**)**. Consider a single circular ring road and a large number of radial routes converging on the centre. When the angular separation between the origin and destination exceeds the *switching angle*, the preferred route is through the centre of the city, whereas for smaller angular separations the ring road is preferred.

Let υ be the average money value of a unit of travel-time savings. The typical travel cost for a trip from i to j, that crosses the river at bridge k is given by

$$C_{ijk} = \upsilon t_{ijk} + \tau_k$$

t_{ijk} is the travel time from i to j using bridge k and τ_k is the toll or charge for bridge k.

Let (r_i,θ_i) denote the polar coordinates of the trip origin and (r_j,θ_j) the polar coordinates of the trip destination (see Figure 1). The five river crossing points are numbered from the east to west, with k=1 to the existing tolled crossing, k=2 to the new tolled bridge, k=3 to the city centre and k=4 and k=5 to the two bridges to the east of the centre. The latter two bridges do not have a toll, so that $\tau_4=\tau_5=0$. Let V_1 denote the orbital spend on the outer ring, of radius R_1, V_2 the orbital speed on the inner ring, of radius R_2 and V_{Radial} the radial speed.

For k=1 and 2 the route uses a downstream bridge and the travel times are:

$$t_{ij1} = \frac{R_1 \,|\, \theta_i - \theta_j \,|}{V_1} + \frac{|\, R_1 - r_i \,| + |\, R_1 - r_j \,|}{V_{Radial}}$$

$$t_{ij2} = \frac{R_2 \,|\, \theta_i - \theta_j \,|}{V_2} + \frac{|\, R_2 - r_i \,| + |\, R_2 - r_j \,|}{V_{Radial}}$$

When k=3 the route is a double radial through the central area and the travel time is given by:

$$t_{ij3} = \frac{r_i + r_j}{V_{Radial}}$$

When k=4 or 5 the route uses an upstream bridge and the travel times are:

$$t_{ij4} = \frac{R_2 (2\pi - |\, \theta_i - \theta_j \,|)}{V_2} + \frac{|\, R_2 - r_i \,| + |\, R_2 - r_j \,|}{V_{Radial}}$$

$$t_{ij5} = \frac{R_1(2\pi - |\theta_i - \theta_j|)}{V_1} + \frac{|R_1 - r_i| + |R_1 - r_j|}{V_{Radial}}$$

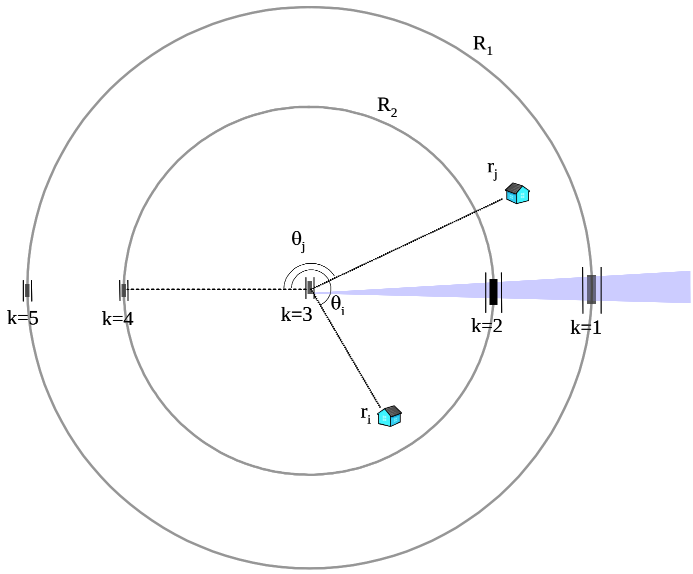

Figure 1: Schematic picture of a city illustrating the polar coordinates for an origin i and destination j and the five river crossing points k. Zero degrees are due west.

The model was implemented as follows. The urban area was divided into cells representing trip origins and a representative sample of points to indicate destinations. Journey origins were spaced at 1-degree intervals with an incremental radius of 0.5 kilometres such that there were 28,800 within a 40-kilometre radius of the city centre, 18,000 such cells within the 25 km radius of the outer orbital ring; and 9,000 within the inner orbital ring.

The destinations were spaced at intervals of 5, 10, 15, 20 and 30 kilometres and angles of arc at 0, 30, 60, 90, 120, 150, and 175 degrees to give 35 destinations in all. The co-ordinates of the new tolled bridge were assumed to be at a radius of 12.5kms and angle of 180 degrees and the co-ordinates of the old tolled bridge, at a radius of 25kms and angle of 180 degrees. The average trip density in each cell was obtained was obtained by allocating cells to standard traffic areas in the London area and allocating a 'traffic weight' to each cell.

For each destination cell, the cost of on each of travelling between every origin on the opposite side of the river and that destination via each of the five bridges was then calculated. For each origin-destination pair the preferred bridge (giving the minimum cost journey) was identified. The area of each route catchment is numerically estimated by summing the areas of each cell assigned to each route. The results were then plotted in the form of a simplified route catchment map.

To obtain the relative proportion of traffic at each bridge, a weighted sum of the number of cells was calculated, with weights proportional to a measure of the density of trips generated from each trip origin. These weights were based on the National Trip End Model, and provided by the TEMPRO software package, Department for Transport (2002).

Illustrative route catchment maps

To help fix ideas and show the possible range of spatial impacts of traffic on each route, we now consider the examples of catchment maps shown in Figure 2. Each map shows the size and shape of the route catchments pertaining to a fixed location. This is situated half way between the inner and outer ring on the north side of the river at an angle of 150 degrees from due west (the results for the south side of the river are a mirror image of those for the north side). Figure 2 (a) corresponds to revenue maximising scenario A described later in the paper in which the model parameters broadly compare with London. Figure 2 (b) corresponds to revenue maximising scenario D, in which an additional charge of £2.50 is made for crossing the central area[2]. Other toll and parameter values are shown in Table 1.

As is seen the route catchments have distinctive shapes and patterns. As Figure 2(a) shows, in the absence of any central area charge the radial catchment is second in size only to the catchment for the inner ring. However, in Figure 2 (b), it is seen that much of the traffic is diverted onto ring one as a result of the additional central area charge and illustrates clearly the potential of tolls to alter traffic patterns.

As the changes in route catchment areas in Table 1 suggest, both tolled bridges also receive extra traffic. This is also apparent from the changes in the sizes and shapes of

[2] Since this illustration was prepared the congestion charge has increased to a one-way equivalent charge of £4. Additionally there are plans to increase the charge at Dartford, defined in the map as the ‘old bridge’.

the lightly shaded route catchments in the south east quadrant of the maps. Note that the values for catchment areas generated by the model given in the Table 1 include only those areas that fall within the outer ring, which we define as the 'built up area'.

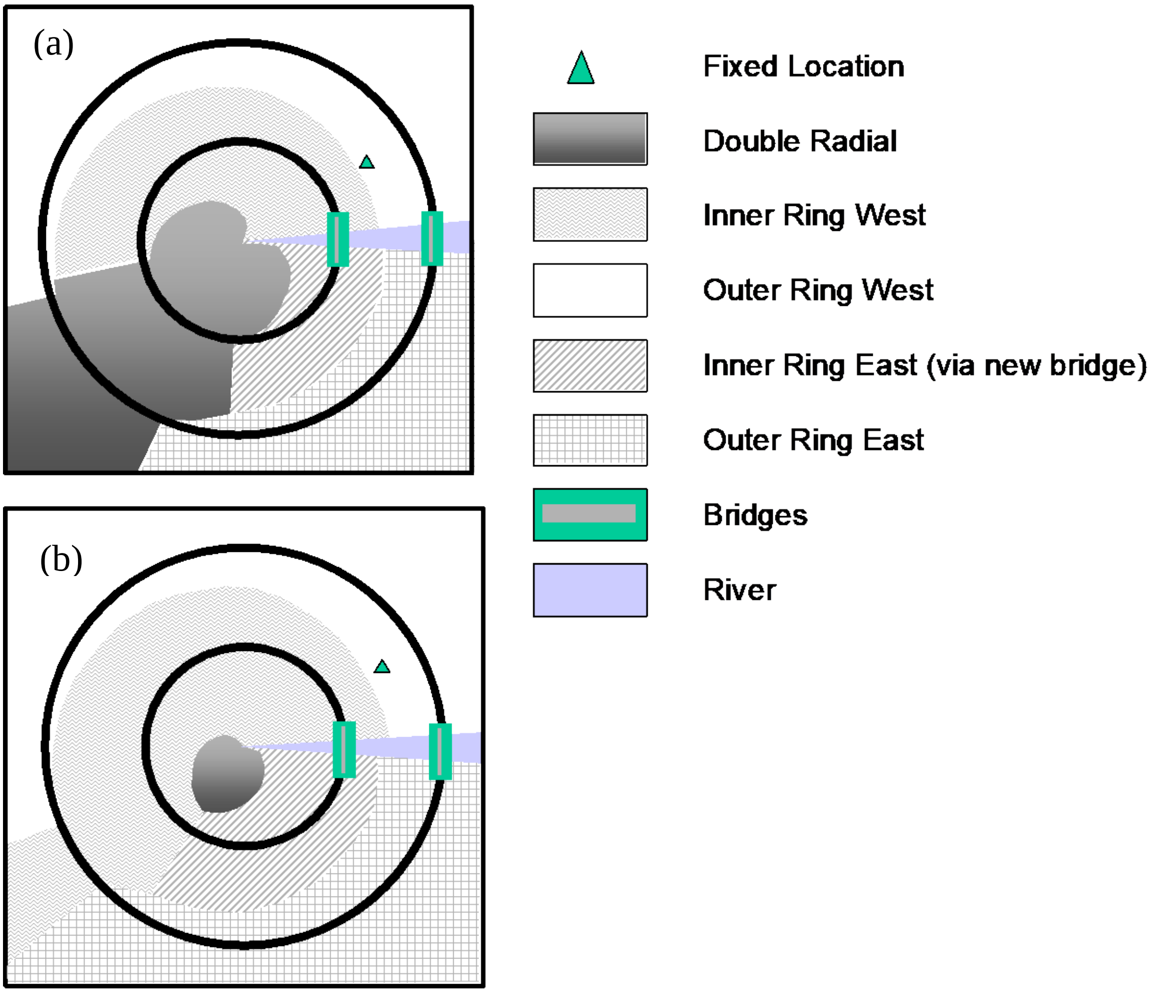

Figure 2: Examples of route catchment maps based on different toll values for a destination 150 degrees from due west between the inner and outer ring road: (a) no central toll; (b) central toll of £2.50. (See Table 1 for catchment area sizes.)

Measuring the spatial impact of the tolled bridges

A different route catchment map is needed for each destination (or origin); this means a more convenient way is needed to evaluate the overall impact of the bridge that does not involve drawing a great many such maps. Using the model, all locations inside the built up area were investigated to find the market shares of each of the five routes. If the market share of any route from any given location is calculated to be say 25%, it means that it will be favoured on cost grounds over all other routes to that location in 25% of journeys from locations within the built up area.

Route catchment area (sq kms)	*(a)*	*(b)*
Double Radial	558	79
Inner Ring West	646	935
Outer Ring West	312	313
New Bridge	275	385
Old Bridge	171	252
Total	*1,963*	*1,963*

Table 1: Route catchment areas (square kilometres) with and without a central toll within the built up area.

Different locations have different route market shares so that contour values at any given location would represent the percentage of traffic using a particular route. The higher the market share the busier the route is expected to be at that location and the greater its influence over traffic at any point; it follows that where the market share extends over a wide area the greater its geographical impact will be in traffic terms. Where market shares rise to 100%, as occurs on double radial routes originating or terminating near the city centre, it means all traffic will opt for this route.

Based on Scenario A the market shares for the new and old bridges were estimated and plotted as contours in Figure 3. The contour values shown are 15%, 20% and 25% of all journeys inside the built up area. The dark shaded area is defined as the impact zone for the new bridge and the light shaded area as the impact zone for the old bridge. Along the boundary between the white area and the dark grey area 15% of cross-river traffic uses the new bridge, and along the boundary between the white and light grey area 15% uses the old bridge. In the interior of the dark grey area more than 15% of cross-river traffic uses the new bridge, and in the interior of the light grey area more than 15% uses the old bridge. The other contours show the 20% and 25% contours for the new and old bridges.

It is noteworthy that even in the vicinity of the bridges neither bridge achieves market share of greater than 25%. This means that the dominant routes are usually one of the other three; nearer the city centre it is the double radial route, and moving outwards, it is either the upstream, un-tolled inner or outer rings. In the interior of the white area more than 70% use un-tolled routes (i.e. neither the old bridge nor the new bridge). Any location more than three-quarters of the way to the outer ring would typically use the un-tolled inner ring, with remaining traffic using the un-tolled outer ring. At locations a couple of kilometres inside the inner ring the double radial becomes the dominant route.

The major influence of the other routes can also be gauged by the fact that both bridges have negligible market share in the western half of the city, whilst the impact zone for new bridge is largely confined to within the outer ring and tends to straddle the inner ring before tapering away either side of the estuary. On the city centre facing side of the new bridge the contours fall away steeply due to the influence and competition from double radial routes. On the western side the impact zone extends further but is curtailed by the old bridge a little way inside the outer ring.

We conclude therefore that, at the tolls levels set under scenario A, the impact zone for the new bridge would be relatively compact and its market share of all journeys inside the built up area would not rise much above 25% except in the immediate vicinity of the bridge. Further, as is also apparent from Figure 3, it is unlikely that the new bridge would to attract long distance traffic on any scale as compared with other routes. This is because, as the map indicates, there are usually better alternatives, either the old bridge or the un-tolled alternatives.

Different toll combinations yield different impact zones but generally follow the same pattern changing only in size or contour value. For the impact zone of the new bridge to increase in area either the toll would need to be reduced or tolls on other river crossings would have to increase. Figure 2b, for example, shows how extending the tolled area on double radial routes would be one way to increase the impact zone on the new bridge but it is notable that the route catchments for other river crossings would also increase.

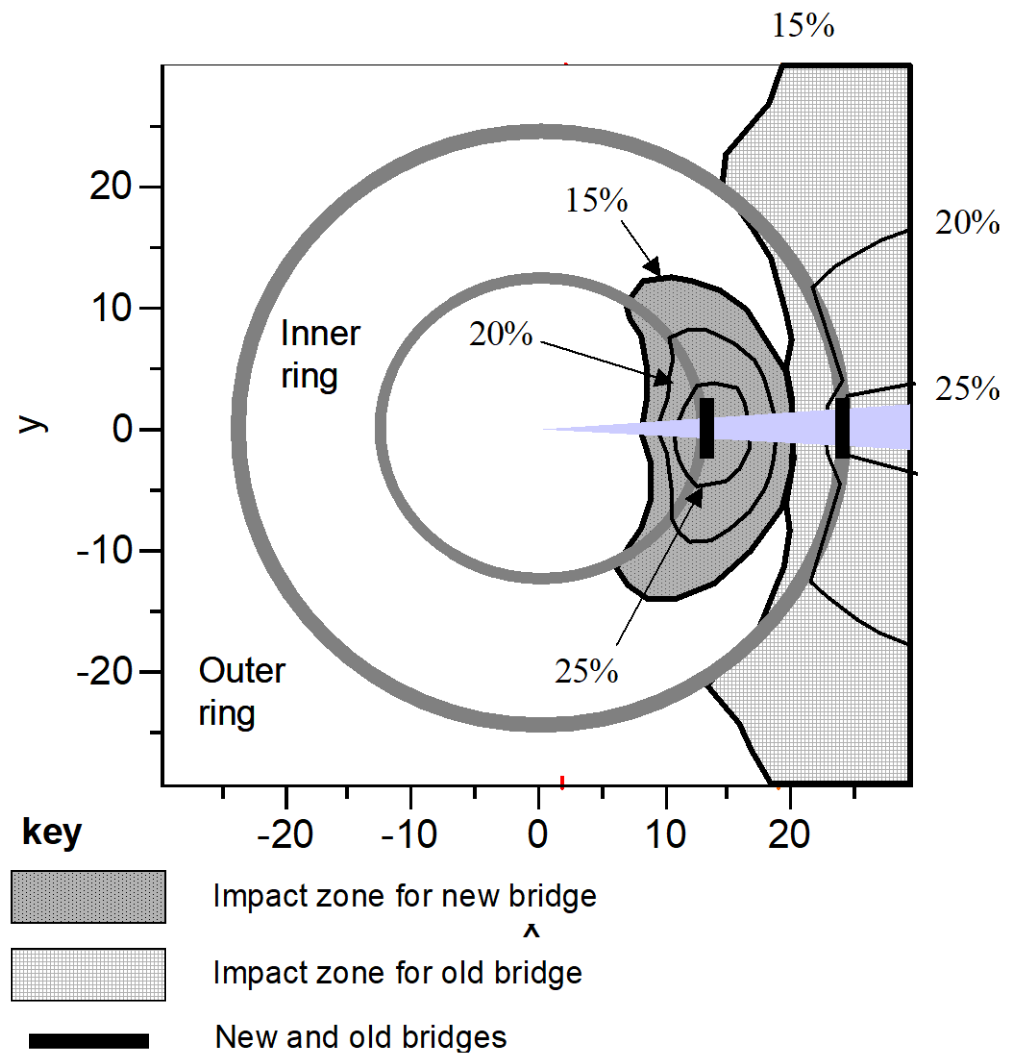

Figure 3: Contour plot showing the destinations in the northeast quadrant for which the new bridge will be the preferred route for the given percentage of the urban area (axes in kilometres)

Traffic Demand

We now describe the demand functions that are used in the analysis. The traffic level Q(τ) at the new tolled bridge is assumed to be a smooth function of the toll τ and is applied over a small range of variation in the toll level. In this region, *Q(τ)* can be approximated by a linear function of the form:

$$Q(\tau) = a - b\tau$$

where a and b are parameters to be estimated. This functional from is applied to both total traffic on the new bridge, and to the components of this traffic that are generated from specific geographical locations.

Tolls which Maximise Revenue

The toll revenue *R* that is derived from this traffic is given by a quadratic function:

$$R = Q\tau = a\tau - b\tau^2$$

The maximum revenue at the new bridge is obtained when the toll level is set to

$$\tau = \frac{a}{2b}$$

It can be noted that the resulting toll levels only depends on the ratio of the parameters a/b.

The old bridge is assumed to have a fixed toll. The traffic on the old bridge is also approximated by a linear function of the toll τ on the new bridge, of the form

$$F(\tau) = c + d\tau$$

Where, again τ is the toll at the new bridge. Let u denote the (fixed) toll on the old bridge. The total toll revenue collected at the old bridge is given by:

$$S = u(c + d\tau)$$

The total revenue from the old and new bridges combined is given by

$$Z = R + S = a\tau - b\tau^2 + u(c + d\tau) = uc + (a + ud)\tau - b\tau^2$$

The combined revenue from the two toll bridges is maximized when the toll at the new bridge is given by

$$\tau = \frac{(a + ud)}{2b}$$

This toll level will be slightly higher than the previous toll, where only new bridge revenues were considered. This difference is given by the term $ud/2b$ and depends on how much traffic the new bridge diverts from the old bridge. This effect will be estimated in the following analysis.

In order to estimate the parameters of the demand functions, we need to convert estimates of catchment area sizes into equivalent traffic volumes. This was implemented by using comparative weights for trips generated in each the three rings: a) $r<R_1$, b) $R_1<r<R_2$ and c) $r>R_2$, where r is the radius of the fixed location, R_1 the radius of the inner ring road and R_2 the radius of the outer ring road. The particular weights used in the examples were a) 8, b) 4 and c) 1, and were derived for London from TEMPRO (see above).

In each set of simulations, we fixed the assumptions about traffic speeds, values of time, central area charging and the toll at the old bridge and varied the toll at the new bridge. We calculated both traffic levels and toll revenues until we were in reasonably close proximity to the maximum revenue. We then assumed that variations in bridge traffic levels were a linear function of variations in the toll: $Q(\tau) = a - b\tau$. To estimate the parameters *a* and *b* we than compared the simulated bridge traffic at two proximate tolls $\tau(1)$ and $\tau(2)$, and used the two resulting traffic levels $Q(\tau(1))$ and $Q(\tau(2))$ to deduce the slope b and intercept a, for each tolled crossing, using the simple formulae:

$$b = -\frac{Q(\tau(2)) - Q(\tau(1))}{\tau(2) - \tau(1)}$$
$$a = Q(\tau(1)) + b\,\tau(2)$$

Tolls which Maximise Welfare

The previous section explained how tolls can be set to maximise toll revenue. This can be contrasted with a welfare based tolling policy. Such a policy is of particular interest when the traffic on the bridge approaches causes congestion, adding to journey times and therefore user costs. A suitably set toll could then increase total welfare by lowering the journey times for a reduced number of travellers. It follows that there is value in exploring the differences in welfare and revenue maximising tolls in order to show how tolls need to be fine-tuned to deal with a range of circumstances, ranging from low to excessive demand.

Note that it is theoretically possible for welfare maximising tolls to exceed revenue-maximising tolls. Effects of this kind merit detailed empirical investigation, but do not arise under the assumptions made in this paper.

Firstly, we define welfare as the sum of transport user benefits (consumer surplus) and transport provider benefits (toll revenues). We assume that the new bridge is already in operation, allowing us to confine our analysis to the benefits arising from alternative tolling strategies, rather than from the provision of the new bridge itself.

The theory is developed for a restricted case, where the congestion associated with the new bridge is considered in isolation, i.e. assuming that, as tolls on the new bridge are varied, any user benefit *arising from changes in congestion on other crossing points* are negligible.

For clarity of exposition, the theory is developed in two stages. In stage 1, we estimate user benefits of alternative toll levels on the new bridge, without including congestion effects. Here the maximum welfare occurs when the bridge toll is set to the lowest level that is consistent with any prevailing financial constraints. In stage 2, we include the effect of congestion on the new bridge and on its approach roads. Stage 2 requires the calculation of an equilibrium level of bridge traffic, which recognises the feedback between bridge traffic levels and congestion, at any exogenous level for the bridge toll. Now the perceived cost of travel includes both congestion, which varies according to traffic levels, and tolls.

We then examine the corresponding revenue maximising policy on comparable assumptions to stage 2 (congestion effects associated only with the new bridge) and calculate the ratio of the welfare-maximising tolls to the revenue-maximising tolls. From the point of view of the provider, only the toll revenues from the new bridge are considered. From the point of view of the users, we assume that the transport user benefits *arising from the removal of congestion on other crossing points* are negligible, so congestion modelling can be confined to the new bridge. We returned to this issue later, in the subsequent discussion.

Stage 1) Welfare analysis without any congestion effects

The total welfare benefit of the new bridge is given by

$$W = UB + R$$

In annex A, we show that this expression, the sum of user benefit and toll revenue, is equivalent to the difference between willingness to pay and social cost and therefore consistent with normally accepted approaches for evaluating transport projects (Verhoef and Small, 2003; Department for Transport, 2003; Hau, 1992; Ministry of Transport, 1964). Since there is no congestion, the only variable user cost is the bridge toll itself τ and the *user benefit* of the new bridge is given by the consumer surplus:

$$UB = -\int_0^{\tau} Q(\tau)\,d\tau$$

The *provider benefit* of the new bridge was given earlier by the toll revenue:

$$R = \tau Q(\tau)$$

The traffic demand for the new bridge is given by:

$$Q(\tau) = a - b\tau$$

Evaluating the integral and adding toll revenues, it can be verified that W is given by:

$$W = -\tfrac{1}{2} b \tau^2$$

Hence the maximum welfare value occurs when $\tau=0$, i.e. when the bridge toll is set to zero. In general, the best policy would therefore be to set the toll at the lowest affordable level that is consistent with the repayment conditions for the bridge loan. Once the loan has been paid for, the toll is normally removed. This is the usual basis for setting tolls on bridges and other estuarial crossings in the absence of congestion.

Stage 2) Congestion associated with the new bridge

2(a) Traffic Equilibrium

When the new bridges, or its approach roads, are congested the level of the bridge toll will influence the journey times of bridge users. For any given toll level, the utilisation of the new bridge needs to be consistent with levels of congestion resulting from that level of utilisation. This entails the calculation of an equilibrium between bridge demand and bridge congestion levels.

We assume that, for each bridge user, congested travel times are given by a linear function:

$$t(\tau) = \alpha + \beta Q(\tau)$$

where α is the free-flow travel time, β is the additional time resulting from a unit increase in bridge traffic and τ is the level of the bridge toll. This is the upward sloping solid line shown in Figure 4. The horizontal dashed line represents the un-congested case.

Recalling that ν is the value of travel time and since congestion adds a cost of $\upsilon(t-\alpha)$ to the cost of travel, the traffic demand for the new bridge is now given by:

$$Q(\tau) = a - b(\tau + \upsilon(t - \alpha))$$

In Figure 4, two downward sloping demand schedules are shown, the rightmost line the one where the toll is zero, the leftmost line, when the toll is τ.

Substituting for the congested times *t* gives:

$$Q(\tau) = a - b(\tau + \upsilon\beta Q(\tau))$$

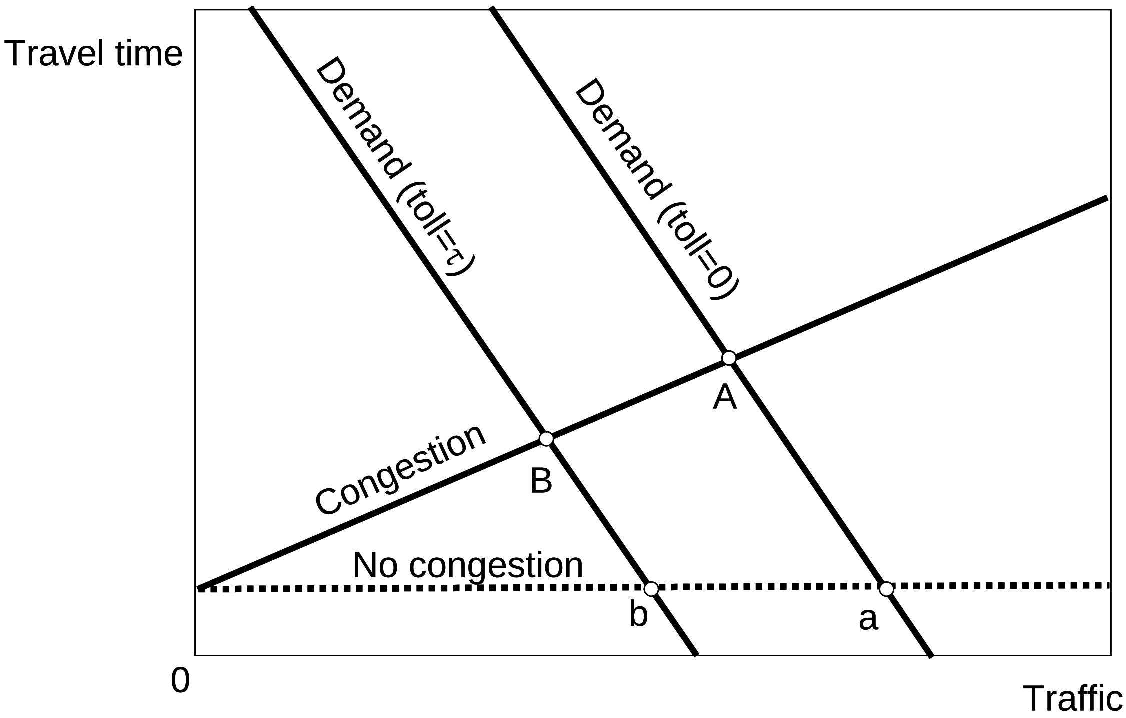

Figure 4: The effect of a toll on traffic, with and without congestion

Solving for the equilibrium flow *Q(τ)* gives:

$$Q(\tau) = Q(0) - B\tau$$

where

$$Q(0) = \frac{a}{1 + \nu\beta b}$$

is the equilibrium level of bridge traffic when the bridge toll is zero, and

$$B = \frac{b}{1+\nu\beta b}$$

is an adjusted slope parameter. The slope B modifies the demand function to take account of the equilibrium level of congestion. The calculation of the equilibrium traffic level, for both the congested and un-congested cases is illustrated in Figure 4 by the intersection points A and B of the congestion line and the demand schedules. The point A is the congested equilibrium with no tolls, the point B is the congested equilibrium in the presence of a toll. The points 'a' and 'b' represent the corresponding equilibria in the absence of congestion.

2(b) Welfare Analysis

We now look at the welfare effects, taking account of local congestion effects. We assume that user benefits are given by:

$$UB = -\int QdC$$

where C is the cost of travel (*including tolls)*, given by:

$$C = \upsilon t + \tau$$

where we have now included a term representing the varying cost of congestion. The incremental change in the cost of travel is therefore:

$$dC = \upsilon dt + d\tau$$

Substituting this in the user benefit integral adding toll revenues (and allowing a slight abuse of notation for the toll variable in second integral) gives:

$$W = -\nu\int Qdt - \int Qd\tau + \tau Q(\tau)$$

Now, noting that travel times are given by:

$$t(\tau) = \alpha + \beta Q(\tau)$$

we can evaluate the first integral in the expression for welfare to obtain:

$$W = Const - \tfrac{1}{2}\nu\beta Q^2 - \int Q d\tau + \tau Q(\tau)$$

Also, noting that the equilibrium traffic using the new bridge is given by:

$$Q(\tau) = Q(0) - B\tau$$

We can evaluate the remaining integral to obtain the simple expression:

$$W = Const - \tfrac{1}{2}(\nu\beta Q^2 + B\tau^2)$$

It is now straightforward to verify that the toll that maximises welfare satisfies:

$$\tau^W = \upsilon\beta Q^W = \frac{\nu\beta Q(0)}{1+\nu\beta B}$$

Where the superscript *W* indicates that the bridge toll and traffic level are evaluated at the welfare optimum. We showed earlier that the equilibrium slope parameter *B* is given by:

$$B = \frac{b}{1+\nu\beta b}$$

Substituting for *B* gives the toll that maximises welfare in the form:

$$\tau^W = \nu\beta Q^W = \frac{\nu\beta Q(0)(1+\nu\beta b)}{1+2\nu\beta b}$$

The resulting equilibrium traffic level on the new bridge is therefore:

$$Q^W = \frac{Q(0)(1+\nu\beta b)}{1+2\nu\beta b}$$

2(c) Revenue and welfare policies compared

We now compare welfare and revenue maximising policies, assuming local congestion. It is straightforward to verify that the revenue-maximising toll is:

$$\tau^R = \frac{Q(0)}{2B} = \frac{Q(0)(1+\upsilon\beta b)}{2b}$$

and the resulting equilibrium level of bridge traffic is:

$$Q^R = Q(0)/2$$

The ratio of the welfare-maximising toll to the revenue-maximising toll is given by:

$$r_g = \frac{\tau^W}{\tau^R} = \frac{2\nu\beta b}{1+2\nu\beta b}$$

Note that the welfare-based toll is less than the revenue-based toll. To understand why this occurs, we need to return to the welfare function and look at the user benefit term explicitly. It can be verified that this is given by:

$$UB = Const - \tfrac{1}{2}\nu\beta Q^2 + \tfrac{1}{2}B\tau^2 - Q(0)\tau$$

Differentiating UB with respect to the bridge toll gives:

$$\frac{\partial UB}{\partial \tau} = \nu\beta BQ(\tau) + B\tau - Q(0) = Q(\tau)(\upsilon\beta B - 1) = -\frac{Q(\tau)}{1+\upsilon\beta b}$$

This expression is not positive at any level of the bridge toll. This derivative represents the difference between the welfare derivative and the revenue derivative. So, as tolls rise, welfare must rise at rate that is not greater than the rate at which revenue rises. Hence welfare cannot reach its maximum value at a higher toll than the toll that maximises revenue.

It should be noted at this point that this finding is a consequence of the assumptions made in deriving the model used here, and different assumptions can, in theory, yield a different conclusion. Determination of the correct set of assumptions is a question that requires detailed empirical investigation.

The ratio of the welfare-maximising bridge traffic to revenue-maximising bridge traffic is given by:

$$Q^W = \frac{Q(0)(1+\nu\beta b)}{1+2\nu\beta b}$$

$$Q^R = \tfrac{1}{2}Q(0)$$

$$r_q = \frac{Q^W}{Q^R} = \frac{2+2\upsilon\beta b}{1+2\upsilon\beta b}$$

Note that this fraction cannot be less than unity, so welfare based bridge traffic cannot be less than the revenue based bridge traffic. This is a direct consequence of the welfare-maximising toll being less than the revenue-maximising toll.

Assume, for illustrative purposes, that the welfare policy eliminates 20% of the traffic that would have used the bridge if there had been no toll. Then

$$\frac{Q^W}{Q(0)} = \frac{1+\nu\beta b}{1+2\nu\beta b} = 0.8$$

Then the value of $\nu\beta b$ would be equal to 1/3. So we obtain:

$$r_g = \frac{\tau^W}{\tau^R} = \frac{2\nu\beta b}{1+2\nu\beta b} = \frac{2}{5} = 0.4$$

The welfare-maximising toll would be equal to 40% of the revenue-maximising toll.

We need to ensure that tolling policies are compared on equivalent assumptions about congestion. In the presence of congestion associated with the new bridge, the equilibrium traffic level is less sensitive to the toll, by a factor B/b, where:

$$B = \frac{b}{1 + \nu\beta b}$$

So, with *νβb=1/3*, the effect of congestion is to increases the revenue-maximising toll by a factor of 4/3.

For a more extended comparison, we recall that the welfare maximising toll is:

$$\tau^W = \frac{\nu\beta Q(0)(1 + \nu\beta b)}{1 + 2\nu\beta b}$$

In the absence of congestion, the revenue maximising toll is:

$$\tau = \frac{a}{2b} = \frac{Q(0)}{2b}$$

So the ratio of *congested welfare*-maximising toll to the *congestion-free revenue* maximising toll is given by:

$$\frac{2\upsilon\beta b(1 + \upsilon\beta b)}{1 + 2\upsilon\beta b} = \frac{8}{15}$$

on comparable numerical assumptions.

Scenarios

Following a large number of simulations using the model, we reduced these to four key scenarios, A-D. Scenario A is a baseline scenario with comparable parameters to London. Scenarios B is designed to test the robustness of the revenue maximising tolls to changes in the assumptions of the value of time by raising it from £10 per hour to £20 per hour. Our motivation here is that if the value of time were to differ significantly once the bridge was in operation then this may affect the estimates for the revenue maximising toll, so this test was required

Scenario C is designed to test the effect of changes in the toll on the old bridge. In this scenario, the toll at the old bridge was raised from £1 to £2. Scenario D is designed to test the effect of a central area toll. In this scenario, the central area charge is set at £2.50. In the case of London, such a scenario might arise if, at some time in the future, there were an eastward extension of the current congestion charging zone to incorporate existing un-tolled crossing points both at, and downstream from Tower Bridge.

Table 2 show our main assumptions and Table 3 our main results. These are the cases based on maximising revenue on the new bridge, maximising revenue on the new bridge and old bridge together, and the toll on the new bridge maximising welfare. As is apparent from Table 3 revenues are maximised over a relative small range of toll values, and all results are in the range £1 to £2. The higher revenue maximising tolls only arise when either the value of time is doubled (scenario B) or if the current congestion-charging zone is extended eastwards (scenario D).

It is noteworthy that raising the toll at the old bridge makes virtually no difference to the optimal toll level at the new bridge. The value of time makes only a modest difference to the findings, but the implications for cost recovery might prove to be critical in some financial scenarios. The central area charge (extension) has a more substantial impact. It is also seen that the resulting tolls levels on the new bridge when revenues on the new and old bridge are jointly maximised would be about 10% higher than the corresponding results for the new bridge alone.

For the welfare maximising case we used the 8/15 ratio derived earlier to obtain welfare toll estimates from equivalent revenue based estimates. This has been done for each of the basic scenarios A, B, C and D. For scenario A with a congestion-free revenue maximising toll of £1.17 (scenario A), this would give a revenue maximising toll of £1.56 in the presence of congestion. The corresponding welfare maximising toll would be 62p.

Scenario	*Value of time*	*Assumed Toll on Old Bridge*	*Assumed Toll in Central Area*
A	£10/hr	£1	0
B	£20/hr	£1	0
C	£10/hr	£2	0
D	£10/hr	£1	£2.5

Table 2 Parameter assumptions used for scenarios A to D.

Scenario	*Toll on New Bridge giving Max Revenue on New Bridge*	*Toll on New Bridge giving Max Revenue on New+Old Bridge*	*Toll on New Bridge giving Max Welfare*
A	£1.17	£1.30	£0.62
B	£1.36	£1.52	£0.73
C	£1.17	£1.27	£0.62
D	£1.74	£1.92	£0.93

Table 3: Optimal toll results for scenarios A-D

Figure 5 is a plot of the predicted daily revenues on both bridges as a function of the toll on the new bridge, using the baseline scenario A. Tolls are in pounds sterling/vehicle and revenues are in pounds per day. For a toll of £1 on both new and old bridges, these results correspond to traffic levels of just under 60,000 vehicles/day on the new bridge and just over 120,000/day on the old bridge. The results indicate that, on these assumptions about traffic levels, the new toll bridge revenues are close to being sufficient to repay the illustrative loans assumed above, but that between 20% and 30% of these revenues represent transfers from the old tolled bridge. This fraction represents the interest that the recipient of the revenues of the old bridge has in the proceeds of the new bridge.

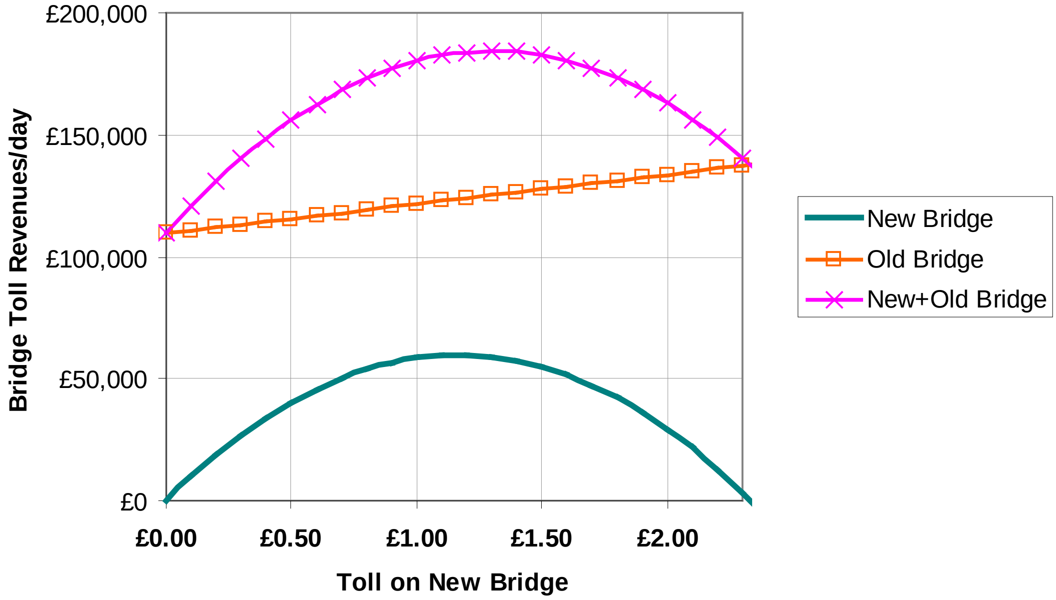

Figure 5: The revenue curves for scenario A

In order to assess the impact of charging policies that differentiate tolls on the new bridge by place of residence, we estimated location-specific traffic demand functions The locations were specified so that one end of the trip was in a particular angular sector or was in a particular radial annulus. Table 4 show the results for five different radii, with destinations (5, 10, 15, 20 and 30 kms from the city centre. and four different angles, (175, 150, 120 and 30 degrees from due west). The 175-degree sector corresponds to the sector immediately adjacent to the new bridge. (Note that zero-degrees points due west).

The results shown in Table 4 indicate that the revenue maximising toll would need to be highest in the two sectors closest to the bridge and lowest in the sector farthest from the bridge. The charges range from 75 pence to £1.28 pence to maximise revenues from the new bridge and from 80 pence to £1.42 to maximise revenues from both new and old bridges It is of interest to note that the spatially differentiated tolls are largest in sector 175, which is the closest one to the new bridge.

Sector (degrees from due west)	*Toll on New Bridge giving Max Revenue on New Bridge*	*Toll on New Bridge giving Max Revenue on New+Old Bridge*
175	1.23	1.42
150	1.28	1.41
120	1.11	1.22
90	0.75	0.80

Table 4: Results for different sectors under scenario A

Annulus (km from city centre)	*Toll on New Bridge giving Max Revenue on New Bridge*	*Toll on New Bridge giving Max Revenue on New+Old Bridge*
5	0.73	0.73
10	1.28	1.34
15	1.23	1.37
20	1.19	1.33
30	0.92	1.09

Table 5: Results for different annuli under scenario A

When the same procedure is carried out for different annuli we found that the revenue maximising toll is lowest in the 5km annulus nearest the city centre rising to £1.28 at 10kms before falling to 92 pence at 30 kms, which covers an area outside the R2 (see Table 5). The results are similar whether the new bridge is considered singly or jointly with the old bridge. Note that the new bridge is positioned 12.5kms form the centre in the model, close to the largest spatially differentiated tolls. The largest spatially differentiated tolls are obtained for the locations closest to the new bridge.

Combining the sector results with the annuli results, we conclude that revenues would tend to be greatest by charging lower tolls to remote distance traffic and higher tolls to local traffic. It is somewhat paradoxical that this is the exact reverse of the pattern that might be desired on the basis of local economic development policies i.e. that such policies might have a price-tag in terms of toll revenues. The plot of toll revenues given earlier suggests that this price tag would be small as long as toll variations are less than 30%. Further any additional administrative costs of implementing spatially differentiated toll, of any from, would need to be offset against the increased, or reduced, revenues.

Discussion

The impact of tolls on the new bridge on the congestion levels of other crossing points has not, so far been discussed, but it may also influence welfare based tolling policies that distinguish between local and remote users. To the extent that other crossing points lie on alternative, (parallel) routes to the new bridge, taking account of wider congestion impacts would tend to result in *lower* welfare based tolls on the new bridge. When the crossing points are some distance apart, as in an estuary, these benefits would tend to be particularly perceived by the more remote prospective users. This suggests that, like revenue based tolling, welfare based tolling would also tend to be lower for remote users than for local users.

Where congestion on remote links that are *en route* (in series with) the new tolled bridge is larger than congestion on alternative crossing points, a modification to this conclusion becomes a possibility. In such cases, the welfare- based tolls for remote users could, in theory, turn out to be higher than for local users. However, as we demonstrated above, the contribution that remote users make to traffic on the new bridge appears to be fairly small, so this issue may prove to be quite difficult to resolve. To assess its likelihood, accurate empirical measurements would be required of the sensitivity of congestion levels at remote locations to bridge toll levels.

Some local districts close to the core of the bridge impact zone might decide that a welfare-based tolling policy is a good means of promoting local economic development but for various reasons it is operationally difficult to implement differential tolls. These districts may then seek to implement a policy of reimbursement to local residents and/or local businesses. The method of reimbursement may take the form of a combination of bulk discounts on advance purchases of toll ‘tickets’, credit for discounted fares on local public transport and discounted access to local attractions. Measures of this kind, if administered well, may help to promote local use of the tolled bridge and to stimulate the local economy.

In contrast, districts outside, or on the periphery of the impact zone are much less likely to perceive benefits from promotions of this kind. The net result of such decentralised arrangements would be that some local residents, and some local business would end up paying less to use the toll bridge than remote users. It is quite possible that the resulting ‘spatially differentiated tolls’ represent good value for money for the local districts concerned but a business case would need to be established. At the time of writing, there are some particularly encouraging prospects for UK local authorities to establish business cases for transport funding innovations.

Summary and conclusions

River crossings are major public investments and normally require the payment of a toll to be financially viable. Bridges can take years to plan and be costly to construct and so it is important that the tolling assumptions are robust. This paper has described a model for assessing tolling strategies for evaluating the revenues and traffic flows associated with a new river crossing in a large urban area. To facilitate the exposition and motivate the illustrative scenarios used in the paper, the model parameters and urban dimensions adopted broadly corresponded to the London area, which itself is currently contemplating the construction of a major new crossing point to the east of the city centre.

Routing in the model was based on a radial-orbital metric with tolls applicable on each route as required. Five river crossing points were assumed, two upstream stream of the centre, the centre itself and two downstream. Although parameterised for London, the model is general in its application if necessary using other metrics (Hyman and Mayhew, 2004). Because there may be a range of planning objectives, revenue maximisation and welfare maximisation are extreme cases of a spectrum of possibilities. These depend on the level of public sector involvement, on the need for private finance, and on the importance of wider local economic and social objectives.

Mathematical expressions for determining tolls that maximised revenue and welfare were derived. Subsequent scenarios showed that revenue maximising tolls are fairly robust to changes in tolls at downstream crossing points and to changes in the value of time. In contrast, these toll levels were significantly more dependent on central area congestion charging policies. Before drawing any policy conclusions from this observation, there are wider factors to be considered, including the implications on local government and private finance, on local economic development etc.

Detailed investigation of the geographical pattern of congestion is required to determine the applicability of such findings to welfare based tolling policies, or to hybrid policies that give a greater or lesser weight to financial and welfare objectives. Our results show that a simple welfare based tolling strategy would yield lower toll revenues than a strategy based on financial objectives. If the bridge were only just breaking even when tolls are set to maximise revenue, the revenue shortfall would need to be made up from other sources, such as a public subsidy. If a primary reason for subsidies were to increase economic development in the areas either side of the bridge, it would then be appropriate to establish robust evidence that this aim would be achieved. Alternative methods of achieving welfare objectives, via local user subsidies have been discussed.

To summarise our findings are of a qualitative nature, but suggest that:

A (i) Under un-congested conditions, welfare maximising toll levels on the new bridge would typically be appreciably lower than revenue maximising toll levels

A (ii) Under congested conditions, welfare maximising toll levels on the new bridge would still be substantially lower than revenue maximising toll levels

B (i) Raising the toll at the old bridge makes virtually no difference to the revenue-maximising toll level at the new bridge.

B (ii) Raising the toll at the old bridge also makes virtually no difference to the welfare-maximising toll level at the new bridge.

C (i) If local users were directly subsidised, the cost of the subsidy may fall on local districts that are least able to afford it, compromising their expenditure on other local economic development projects. These issues may require new funding innovations.

C (ii) If higher tolls than those yielding optimum revenues were levied on remote users this is likely to increase the revenue shortfall for the new bridge. As remote users tend to be in the minority, the magnitude of this impact appears to be comparatively small.

D (i) It has been demonstrated that the optimum bridge toll and bridge traffic levels are particularly sensitive to the level of a wide area central congestion charge. This is because the central area provides the principle alternative crossing point to the new bridge. The policy issues arising from changes and extensions of central area charging are particularly complex matters that merit a much wider investigation.

There are also several further details would require analysis and refinement. For example, we have only considered tolls for a single type of vehicle and for a single time period. We have not examined the effects of differential tolls for commercial vehicles, or different toll for peak period, night time or weekend use. The effect of tolls on commercial vehicle travel behaviour particularly merits detailed empirical study. Public transport and public service vehicles also compete for road space, and so would need to be included in a more substantive in-depth analysis. Related to these issues are the implications of diversity in the ability to pay toll charges on the distributional effects of tolling policies.

Notwithstanding these detailed issues, the general methodology presented in this paper is likely to be applicable to many cities built around major rivers. The geometrical representation of the network has some analytical advantages over more traditional network approaches: clarity of exposition of the issues and low requirements on network and traffic information. One of the innovative features introduced in this paper is the handling of competition between alternative tolled crossings, including the definition of an ‘impact zone’ for the new bridge. This has considerably improved our understanding of the geographical sensitivity of travel demand to bridge toll levels. Such insights would not have been available from the one-dimensional geometry that are often adopted in urban economic models, although even such very simple geometric models can also have useful applications.

Finally, we have discussed some of the implications of our findings for transport appraisal, particularly in terms of reconciling financial and welfare objectives. It is hoped that these will help to stimulate further examination of the implications of innovative approaches to the funding of major transport projects.

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Annex A: Equivalence of Two Welfare Measures

This annex demonstrates the equivalence of two measure of welfare: a) the difference between willingness to pay and social cost and b) the sum of user benefit and toll revenue. Let q denote the flow on an isolated new river crossing, c the average variable cost of using the crossing, t the average travel time (subject to congestion) and τ the toll on the crossing. The tolling literature (e.g. Verhoef and Small, 2003) adopts a welfare measure of the form:

$$W = WTP - SC \tag{1}$$

WTP represents the total users' willingness to pay for using the crossing point and SC the total social cost that they impose on each other through network congestion. It is assumed that all users of the crossing have identical subjective values of travel time. The social cost is given by:

$$SC(\tau) = \upsilon t(\tau) q(\tau) \tag{2}$$

where υ is the value of travel time savings. Both flows and travel times depend on the toll level. The dependency of travel times on the toll arises indirectly because the toll influences traffic flows, and therefore congestion on the crossing. The total willingness to pay to use the new river crossing (sometimes call its 'worth') is:

$$WTP(\tau) = \int_{0}^{q(\tau)} c\,dq \tag{3}$$

For historical interest, WTP was described as the 'gross benefit to all road users' (Ministry of Transport, 1964). The 'theoretically most efficient' use of the network is obtained when the toll is selected to maximize W, subject to any problem-specific constraints.

Note: in this annex all integrals are specified as definite integrals, and correspond to boundary conditions for which demand on the bridge vanishes. This contrasts with the main text where both welfare and revenue integrals are specified in indefinite form *and therefore contain constants of integration. The indefinite forms are used exclusively for calculating optimum tolls. In such applications the value of the constants of integration are expressed in terms of the toll-free bridge traffic levels. The choice of boundary conditions has no influence on the optimum toll values.*

The average variable price of the crossing is given by:

$$c(\tau) = \upsilon t(\tau) + \tau \qquad (4)$$

(Note: we have omitted any other terms apart from travel time and tolls, such as vehicle operating costs).

UK Transport appraisal practice (Department for Transport, 2003) recognizes benefits to both transport users and also to the providers of transport services. Hua (1992) describes this practice as the ‘Change in Total Benefit and Total Cost Approach’ or the ‘British’ approach. The user benefit of the new crossing is the consumer surplus:

$$UB(\tau) = \int_{c(\tau)}^{c^*} q\,dc \qquad (5)$$

where c* is the *extinction price* i.e. the price at which demand for the crossing would just vanish. Integration by parts of WTP yields the identity:

$$WTP(\tau) = c(\tau)q(\tau) + UB(\tau) \qquad (6)$$

Equation (6) simply states that users of the new crossing receive a benefit equal to the difference between what they are willing to pay and what they actually pay to use it. Using (2) and (4) in (6) yields:

$$WTP(\tau) = SC(\tau) + UB(\tau) + \tau q(\tau) \qquad (8)$$

The last term in (8) is the toll revenue collected at the crossing. Equation (8) is equivalent to the identity:

$$W(\tau) = WTP(\tau) - SC(\tau) = UB(\tau) + \tau q(\tau) \qquad (9)$$

This is the required result.

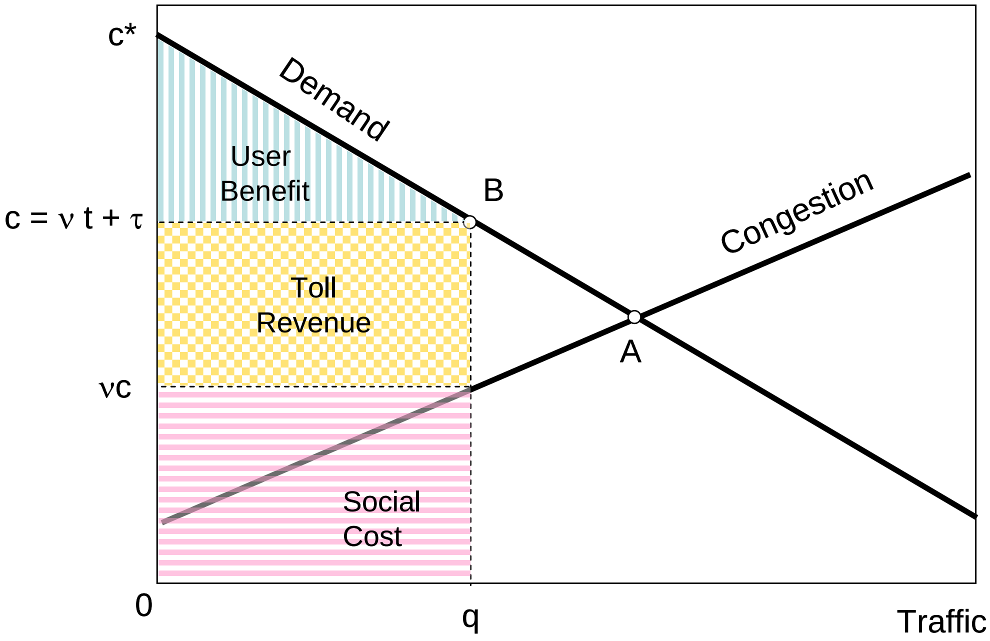

Figure A.1: Decomposition of WTP into user benefits, social costs and toll revenues.

Equation (8) is illustrated in Figure A.1. The point A denotes the equilibrium traffic on the crossing when the toll is zero. The point B is the equilibrium when the toll on the crossing is τ. The trapezium 0C*Bq is the total willingness to pay for using the crossing at this value for the toll. The identity (8) can be verified visually.