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Long-run determination of the nominal exchange rate in the presence of national debts: Evidence from the yen-dollar exchange rate

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Long-run determination of the nominal exchange rate in the presence of national debts: Evidence from an intertemporal-modelling framework using the from the yen-dollar exchange rate

This paper develops an intertemporal optimization model to examine the determinants of the nominal exchange rate in the long run. The model is tested empirically using data from the Japan and the USA. The proposed theoretical specification is well supported by the data and shows that relative national debts as well as monetary and financial factors may play a significant role in the determination of the long-run nominal exchange rate between the yen and the dollar.

Keywords: Nominal exchange rate, intertemporal optimization, national debt, asset prices, co-integration.

JEL Classification: F31, G11, G15

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1. Introduction

The construction of appropriate models to understand the long-run determination of the nominal exchange rates remains a major challenge in modern international finance. The original popular models used to determine and forecast the nominal exchange rates include the flexible price monetary models such as Frenkel (1976), Mussa (1976) and Bilson (1978a and 1978b). This models were later followed by the sticky price monetary model of Dornbusch (1976) and the real interest rate differential model of Frankel (1979) who developed a general monetary model that combines elements of both the flexible price and the sticky price monetarist models, as a special cases. The early tests of the monetary models using traditional econometric procedures were not particularly favourable either in terms of significance of the coefficients or in their in-sample or an out-of-sample ability to forecast exchange rates, as shown by Meese and Rogoff (1983a and 1983b) who showed that exchange rate models fail to outperform a simple random walk. However, more recent econometric techniques based on co-integration have produced more favourable results. In addition to the monetary models, the literature on the currency substitution models and the portfolio balance class of models has evolved with mixed empirical support[1].

More recently Duarte and Stockman (2005) have investigated empirically the effects of speculation in an attempt to explore the linkage between exchange rates and asset markets, whereas Dellas and Tavlas (2013) have shown a theoretical and empirical linkage between exchange rate regimes. Exchange rates are generally perceived to be disconnected from macroeconomic fundamentals and as Flood and Rose (1995) report the exchange rate appears to have "a life of its own". Associated with this evidence Bacchetta and Van Wincoup (2004,

[1] For a good exposition on the empirical validity of the various monetary approaches to the exchange rate determination along with the validity of the portfolio balance approach see MacDonald (2007).

2013) have proposed a scapegoat theory in order to interpret the weak link between exchange rates and fundamentals, which is empirically supported by Fratzscher at. (2015).

This paper contributes to the above puzzling fact e existing literature by proposing an alternative approach to the determination of the long-run nominal exchange rate based on micro-foundations. As opposed to the current literature, our proposed theoretical framework contributes toward the portfolio balance approach to the determination of the nominal exchange rate in the long-run by constructing a two country model with optimizing agents where wealth is optimally allocated in an asset choice set that explicitly includes investment in an array of financial assets including domestic and foreign real money balances, domestic and foreign bonds and domestic and foreign stocks. Within this framework the model also contributes to the current literature by looking at the risk of holding relative real money balances in the optimization process, this is done by including the relative government debt to GDP ratio. We argue that the risk associated with increasing national debts can significantly affect the investors' decisions to optimally allocate their wealth among different assets in an open economy setup. The predictions of our theoretical model are tested empirically using data from Japan (treated as the domestic economy) and the USA (treated as the foreign economy). Japan and the USA have high trade and financial relationships with each other and both have high and growing national debt to GDP ratios.

Although Dellas and Tavlas (2013) have shown a theoretical and empirical linkage between exchange rate regimes this differs from our approach which is to show an explicit link between asset prices and the nominal exchange rate.The predictions of our theoretical model are tested empirically using data from Japan (treated as the domestic economy) and the USA (treated as the foreign economy). Japan and the USA have high trade and financial relationships with each other and both have high and growing national debt to GDP ratios.

The model specification that we propose allows for the construction of explicit equations for both domestic and foreign real money balances, which can be utilized in to generate an exchange rate equation based on micro-foundations and optimizing agents. We show that the theoretical model that we derive is empirically well supported by using the Yen-dollar rate in showing that asset prices and returns, along with monetary and real variables, play a significant role in the determination of the nominal exchange rate in the long-run. An important contribution of this paper stems from the strong evidence Our results also indicate that in favour of the relative debt to GDP ratio s as a key variable in to understanding the behaviour of the nominal exchange rate in the long-run. As Aizenman and Marion (2011) indicate the debt variables should be treated as key macroeconomic indicators, dominating current policy debates.

The paper is organised as follows: Section 2 presents the intertemporal optimization model, as a contribution to the understanding of the determination of the nominal exchange rate in the long-run. Section 3 discusses the dataset and the empirical methodology. Section 4 discusses the results from the empirical estimations, Section 5 examines the forecasting ability of the model, Section 6 further explores the statistical performance of the model within different data samples and Section 7 concludes.

2. The theoretical model

An infinitely lived representative agent (individual) is assumed to respond optimally to the economic environment. Utility is assumed to be derived from consumption of domestic and foreign goods, and from holdings of domestic and foreign real money balances[2]. The presence

[2]In Litsios and Pilbeam (2017) the determination of the long-run real exchange rate is investigated based on a similar exposition of the utility function but here we also incorporate a role for national debts and focus on the nominal exchange rate.

of real money balances is intended to represent the role of money used in transactions, without addressing explicitly a formal transaction mechanism. This can distinguish money from other assets like interest bearing bonds or stocks.[3] We extentd Kim's (2000) and Kia's (2006) specification by introducing variable κ_t into the utility function in order to reflect potential risk associated with holding domestic real money balances relative to foreign real money balances. In the current analysis such risk is assumed to be associated with the relative government debt ratio as a percentage of GDP[4]. The representative agent is assumed to maximize the present value of lifetime utility given by:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t{}^{\alpha_1} C_t^*{}^{\alpha_2})^{1-\sigma}}{1-\sigma} + \frac{X}{1-\varepsilon} \left(\left[\frac{M_t}{P_t} [\kappa_t{}^{-1}] \right]^{\eta_1} \left[\frac{M_t^*}{P_t^*} \right]^{\eta_2} [\kappa_t{}^{-1}]^{\eta_3} \right)^{1-\varepsilon} \right] \tag{1}$$

where C_t and C_t^* are single, non-storable, real domestic and foreign consumption goods, $\frac{M_t}{P_t}$ and $\frac{M_t^*}{P_t^*}$ are domestic and foreign real money balances respectively, $0 < \beta < 1$ is the individual's subjective time discount factor, σ, ε, X are assumed to be positive parameters, with $0.5 < \sigma < 1$ and $0.5 < \varepsilon < 1$, and $E_t(\cdot)$ the mathematical conditional expectation at t. For analytical tractability, following Kia's (2006) suggestion, we assume that α_1, α_2, $\eta_1$and η_2 and η_3 are all normalized to unity.

The present value of lifetime utility is assumed to be maximized subject to a sequence of budget constraints given by:

[3] A direct way to model the role of money in facilitating transactions would be to develop a time-shopping model after introducing leisure into the utility function. Another approach, commonly found in the literature, allows money balances to finance certain types of purchases through a cash-in-advance (CIA) modeling. For tractability reasons the specification expressed by Equation (1) is adopted in this paper. See Walsh (2003) for the various approaches in modeling the role of money in the utility function.

[4] Kia (2006) has introduced risk associated to holding domestic money. Such risk can also be associated with the government debt to GDP or the government foreign-financed debt per GDP. In our exposition of the utility function in Equation (1), $\kappa_t{}^{-1}$ reflects the fact that domestic money balances are positively associated with an increase in domestic GDP and an increase in foreign debt and negatively associated with an increase in the domestic debt and an increase in foreign GDP.

$$y_t + \frac{M_{t-1}}{P_t} + \frac{M_{t-1}^*}{e_t P_t} + \frac{B_{t-1}^D(1+i_{t-1}^D)}{P_t} + \frac{B_{t-1}^F(1+i_{t-1}^F)}{e_t P_t} + \frac{S_{t-1}P_t^S}{P_t} + \frac{S_{t-1}^* P_t^{S,*}}{e_t P_t}$$

$$= C_t + C_t^* q_t + \frac{M_t}{P_t} + \frac{M_t^*}{e_t P_t} + \frac{B_t^D}{P_t} + \frac{B_t^F}{e_t P_t} + \frac{S_t P_t^S}{P_t} + \frac{S_t^* P_t^{S,*}}{e_t P_t} \qquad (2)$$

where y_t is current real income, $\frac{M_{t-1}}{P_t}$ and $\frac{M_{t-1}^*}{e_t P_t}$ are real money balances expressed in current domestic unit terms (with M_{t-1} and M_{t-1}^*domestic and foreign nominal money balances respectively carried forward from last period), e_t the nominal exchange rate defined as the amount of foreign currency per unit of domestic currency and P_t the domestic price index. B_{t-1}^D is the amount of domestic currency invested in domestic bonds at $t-1$ and i_{t-1}^D is the nominal rate of return on these domestic bonds. Similarly, $B_{t-1}^F$is the amount of foreign currency invested in foreign bonds at $t-1$ and i_{t-1}^F is the foreign rate of return on these foreign bonds. Both domestic and foreign bonds are assumed to be one period discount bonds paying off one unit of domestic currency next period. S_{t-1} and S_{t-1}^* denote the number of domestic and foreign shares respectively purchased at $t-1$, and $P_t^S, P_t^{S,*}$ denote the domestic and the foreign share prices respectively. q_t denotes the real exchange rate defined as $q_t = \frac{P_t^*}{e_t P_t}$ where P_t^* the foreign price index.

The agent is assumed to observe the total real wealth and then proceed with an optimal consumption and portfolio allocation plan. The right hand side in Equation 2 indicates that total real wealth is allocated at time t among real domestic and foreign consumption $(C_t, C_t^* q_t)$, real domestic and foreign money balances $(\frac{M_t}{P_t}, \frac{M_t^*}{e_t P_t})$, real domestic and foreign bond holdings $(\frac{B_t^D}{P_t}, \frac{B_t^F}{e_t P_t})$, and real domestic and foreign equity holdings $(\frac{S_t P_t^S}{P_t}, \frac{S_t^* P_t^{S,*}}{e_t P_t})$.[5]

[5] All variables are expressed in real domestic terms.

The representative agent is assumed to maximize equation (1) subject to equation (2). To obtain an analytical solution for the intertemporal maximization problem, the Hamiltonian equation is constructed and the following necessary first order conditions are derived:

$$\beta^t U_{c,t} - \lambda_t = 0 \tag{3}$$

$$\beta^t U_{c^*,t} - \lambda_t q_t = 0 \tag{4}$$

$$\beta^t U_{\frac{M}{P},t} \frac{1}{P_t} - \lambda_t \frac{1}{P_t} + E_t\left[\lambda_{t+1}\frac{1}{P_{t+1}}\right] = 0 \tag{5}$$

$$\beta^t U_{\frac{M^*}{P^*},t} \frac{1}{P_t^*} - \lambda_t \frac{1}{e_t P_t} + E_t\left[\lambda_{t+1}\frac{1}{e_{t+1}P_{t+1}}\right] = 0 \tag{6}$$

$$-\lambda_t \frac{1}{P_t} + E_t\left[\lambda_{t+1}\frac{1}{P_{t+1}}(1+i_t^D)\right] = 0 \tag{7}$$

$$-\lambda_t \frac{1}{e_t P_t} + E_t\left[\lambda_{t+1}\frac{1}{e_{t+1}P_{t+1}}(1+i_t^F)\right] = 0 \tag{8}$$

$$-\lambda_t \frac{P_t^S}{P_t} + E_t\left[\lambda_{t+1}\frac{1}{P_{t+1}}P_{t+1}^S\right] = 0 \tag{9}$$

$$-\lambda_t \frac{P_t^{S,*}}{e_t P_t} + E_t\left[\lambda_{t+1}\frac{1}{e_{t+1}P_{t+1}}P_{t+1}^{S,*}\right] = 0 \tag{10}$$

where λ_t the costate variable, $U_{c,t}$, $U_{c^*,t}$ the marginal utilities from domestic and foreign consumption and $U_{\frac{M}{P},t}$, $U_{\frac{M^*}{P^*},t}$ the marginal utilities from domestic and foreign real money balances respectively.

Dividing equation (6) by equation (8) and using equation (4), we obtain equation (11) below:

$$U_{\frac{M^*}{P^*},t} + U_{c^*,t}(1+i_t^F)^{-1} = U_{c^*,t} \tag{11}$$

Equation (11) implies that the expected marginal benefit of holding additional foreign real money balances at time t must equal the marginal utility from consuming foreign goods at time t. Note that the total marginal benefit of holding money at time t is $U_{\frac{M^*}{P^*},t} + U_{c^*,t}$. Equation (11) can be rearranged in order to express the intratemporal marginal rate of substitution of foreign consumption for foreign real money balances as a function of the foreign bond return.

Dividing equation (6) by equation (10) and using equation (4), we obtain equation (12) below:[6]

$$U_{\frac{M^*}{P^*},t} + U_{c^*,t}\left[\frac{P_{t+1}^{S,*}}{P_t^{S,*}}\right]^{-1} = U_{c^*,t} \tag{12}$$

Equation (12) implies that the expected marginal benefit of holding additional foreign real money balances at time t must equal the marginal utility from consuming foreign goods at time t. Equation (12) can be rearranged to express the intra-temporal marginal rate of substitution of foreign consumption for foreign real money balances as a function of the foreign stock return.

Dividing equation (5) by equation (7) and using equation (3) we obtain equation (13):

$$U_{\frac{M}{P},t} + U_{c,t}(1 + i_t^D)^{-1} = U_{c,t} \tag{13}$$

Equation (13) implies that the expected marginal benefit of holding additional domestic real money balances at time t must equal the marginal utility from consuming domestic goods at time t. Equation (13) can be rearranged to express the intra-temporal marginal rate of substitution of domestic consumption for domestic real money balances as a function of the domestic bond return.

Finally, by dividing equation (5) by equation (9) and using equation (3) we obtain equation (14) below:

$$U_{\frac{M}{P},t} + U_{c,t} = U_{c,t} \tag{14}$$

Equation (14) implies that the expected marginal benefit of holding additional domestic real money balances at time t, must equal the marginal utility from consuming domestic goods at time t. Equation 14 can be rearranged to express the intra-temporal marginal rate of substitution

[6] For notational simplicity we drop the mathematical conditional expectation $E_t(\cdot)$.

of domestic consumption for domestic real money balances as a function of the domestic stock return.

Combining equation (3) and equation (4), we can derive equation (15):

$$\frac{U_{c,t}}{U_{c^*,t}} = \frac{1}{q_t} \tag{15}$$

Equation (15) implies that the marginal rate of substitution of foreign consumption goods for domestic consumption goods is equal to their relative prices.

Using equation (1) the marginal utilities of consumption and real money balances can be derived as follows:

$$U_{c,t} = \beta^t (C_t)^{-\sigma} (C_t^*)^{1-\sigma} \tag{16}$$

$$U_{c^*,t} = \beta^t (C_t)^{1-\sigma} (C_t^*)^{-\sigma} \tag{17}$$

Dividing equation (16) by equation (17) and using equation (15) we derive equation (18):

$$C_t^* = C_t (q_t)^{-1} \tag{18}$$

The marginal utilities for foreign and domestic real money balances are given respectively as:

$$U_{\frac{M^*}{P^*},t} = \beta^t X {\kappa_t}^{\varepsilon-1} \left(\frac{M_t}{P_t}\right)^{1-\varepsilon} \left(\frac{M_t^*}{P_t^*}\right)^{-\varepsilon} \tag{19}$$

$$U_{\frac{M}{P},t} = \beta^t X {\kappa_t}^{\epsilon-1} \left(\frac{M_t^*}{P_t^*}\right)^{1-\varepsilon} \left(\frac{M_t}{P_t}\right)^{-\varepsilon} \tag{20}$$

Equations (11), (17), (18) and (19) imply that:

$$m_t^* = [(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}} \left[(X)^{\frac{1}{\varepsilon}} (m_t)^{\frac{1-\varepsilon}{\epsilon}} \left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}} {\kappa_t}^{\frac{\varepsilon-1}{\varepsilon}} \right] \tag{21}$$

Equations (12), (17), (18) and (19) imply that:

$$m_t^* = [(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}} \left[(X)^{\frac{1}{\varepsilon}} (m_t)^{\frac{1-\varepsilon}{\epsilon}} {\kappa_t}^{\frac{\varepsilon-1}{\varepsilon}} \left[1 - \left(\frac{P_{t+1}^{S,*}}{P_t^{S,*}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}} \right] \tag{22}$$

Equations (13), (16), (18) and (20) imply that:

$$m_t = [(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\epsilon}}\kappa_t^{\frac{\varepsilon-1}{\varepsilon}}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\right] \quad (23)$$

Finally, equations (14), (16), (18) and (20) imply that:

$$m_t = \left[(C_t)^{1-2\sigma}(q_t)^{-[1-\sigma]}\right]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\epsilon}}\kappa_t^{\frac{\varepsilon-1}{\varepsilon}}\left[1-\left(\frac{P_{t+1}^S}{P_t^S}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right] \quad (24)$$

Equations (21) to (24) reflect the demand equations for domestic and foreign real money balances (depicted by m_t and m_t^* respectively) as implied by the economic model. This system of equations can be used in order to solve explicitly for the determinants of the nominal exchange rate. Substituting equation (22) into equation (23) and equation (24) into equation (21), we obtain equation (25):[7]

$$le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 lP_t^S + \delta_4 lP_t^{S,*} + \delta_5 li_t^H + \delta_6 li_t^* + \delta_7 l\kappa_t + \delta_8 ly_t + \delta_9 ly_t^* \quad (25)$$

where:

$\delta_1 = [\frac{-(2\varepsilon-1)+(2\varepsilon-1)(1-\varepsilon)-(1-\varepsilon)}{\varepsilon}]$; $\delta_2 = [2(1-\varepsilon)]$; $\delta_3 = -\left[\frac{(1-\varepsilon)}{\varepsilon}\right]$; $\delta_4 = \left[\frac{(1-\varepsilon)}{\varepsilon}\right]$; $\delta_5 = -1$;

$\delta_6 = \left[\frac{(1-\varepsilon)}{\varepsilon}\right]$; $\delta_7 = \left[\frac{(2\varepsilon-1)(\varepsilon-1)}{\varepsilon}\right]$; $\delta_8 = -\left[\frac{(2\varepsilon-1)(1-\sigma)}{\varepsilon}\right]$; $\delta_9 = \left[\frac{\sigma(2\varepsilon-1)}{\varepsilon}\right]$

The predictions of the model are that:

$\delta_1 < 0$; $\delta_2 > 0$; $\delta_3 < 0$; $\delta_4 > 0$; $\delta_5 = -1$ $\delta_6 > 0$; $\delta_7 < 0$; $\delta_8 < 0$; $\delta_9 > 0$

In addition, the following restrictions (as implied by the economic model) are assumed to hold. These restrictions are imposed on the long-run co-integrating vectors for the real exchange rate as derived in section (3).

$\delta_4 = -\delta_3$; $\delta_4 = \delta_6$

[7] A l before a variable denotes log. See the Appendix for the full derivation of Equation (25) along with the various assumptions employed.

3. Long-Run Empirical Methodology and Results

To empirically test the validity of the economic predictions implied by equation (25) in the long-run, a Vector Error Correction Model (VECM) of the following form is employed[8].

$$\Delta\chi_t = \Gamma_1^m \Delta\chi_{t-1} + \Gamma_2^m \Delta\chi_{t-2} + \cdots + \Gamma_{k-1}^m \Delta\chi_{t-k+1} + \Pi\chi_{t-m} + \varepsilon_t \tag{26}$$

Where $\chi_t = \left(le_t, lM_t, lM_t^*, lP_t^S, lP_t^{S,*}, li_t^H, li_t^*, l\kappa_t, ly_t, ly_t^* \right)$ a $(10x1)$ a vector of variables, m denotes the lag placement of the ECM term[9], Δ denotes the difference, and $\Pi = a\beta'$ with a and β (pxr) matrices with $r < p$, where p the number of variables and r the number of stationary co-integrated relationships.

To test for co-integration among a set of integrated variables the Full Information Maximum Likelihood (FIML) approach is employed as proposed by Johansen (1988, 1991).[10] Having uniquely identified potential co-integrating vectors, stationarity among the variables can be tested, while imposing specific restrictions. The above methodology is applied to test for a potential long-run relationship among the macroeconomic variables depicted by equation (25).

For our empirical test quarterly time series data for Japan and the USA are employed for the period 1983:Q1 to 2015:Q4 for the variables depicted by Equation (25)[11]. le_t is the log of the Japanese bilateral nominal exchange rate defined as dollars per Yen, lM_t is the log of the Japanese nominal money supply $(M3)$, lM_t^* is the log of the USA nominal money supply $(M2)$,

[8] Some of the advantages of the VECM are that it reduces the multicollinearity effect in time series, that the estimated coefficients can be classified into short-run and long-run effects, and that the long-run relationships of the selected macroeconomic series are reflected in the level matrix Π and so can be used for further co-integration analysis. See Juselius (2006).

[9] For an *I(1)* analysis *m* should be equal to 1.

[10] The main advantage of such an approach is that it is asymptotically efficient since the estimates of the parameters of the short-run and long-run relationships are carried out in a single estimation process. In addition, through the FIML procedure potential co-integrating relationships can be derived in an empirical model with more than two variables.

[11] Data are collected from Datastream. Data from the United States are used as a proxy for foreign variables and data from Japan as proxies for domestic variables.

lP_t^S and $lP_t^{S,*}$ are the total return Morgan Stanley Composite Indices for Japan and the USA respectively in the local currency, li_t^h is the log of $\frac{i_t^D}{1+i_t^D}$ where i_t^D is the three month rate on Japanese Treasury securities and li_t^* is the log of $\frac{i_t^F}{1+i_t^F}$ where i_t^F is the three month USA Treasury bill rate, $l\kappa_t$ is the log of the relative government debt as a percentage of GDP between Japan and the USA, and ly_t , ly_t^* are the logs of real output in Japan and the USA respectively.

To proceed with the VECM analysis the time series employed were tested first for stationarity. Evidence suggests that the first differences of the variables appear to be stationary as opposed to their levels. Consequently, the variables can be considered to be integrated of order one, i.e. I (1), and co-integration among the variables is possible.[12]

Before testing for the co-integration rank, the appropriate lag length for the underlying empirical VECM model is identified based on the Lagrangian multiplier (LM) test for serial correlation of the residuals.[13] The Johansen (1995) procedures were then applied to test for the co-integration rank. Following the Trace test and the Max-Eigen test, the rank of the Π-matrix was found to be $r = 3$ implying that statistically a discrimination among three conditionally independent stationary relations is possible. The three unrestricted co-integration relations are uniquely determined but the question remains on whether they can be meaningful for economic interpretation. Consequently, following Johansen and Juselius (1994), identifying restrictions should be imposed to distinguish among the vectors and ensure the uniqueness of the coefficients. By taking linear combination of the unrestricted β vectors, it is always possible to impose $r - 1$ just identifying restrictions and one normalization on each vector without

[12] Evidence is coming from the Augmented Dickey-Fuller (ADF) test and the Phillips Perron (PP) test. For robustness purposes we have also performed the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test with stationarity under the null. The KPSS also suggests that the variables are integrated of order one i.e. I(1). The results are available upon request.

[13] The AIC, SBA, HQ tests are employed for the lag order selection. Beginning with the lowest lag suggested by the tests (based on the SBC criterion) the serial correlation of the residuals is tested using the Lagrangian multiplier (LM) test.

changing the likelihood function. Although the normalization process can be done arbitrarily, it is generally accepted practice to normalize on a variable that is representative of a particular economic relationship. Since the purpose of the paper is to identify the long-run determination of the nominal exchange rate, the first co-integrating vector is normalized with respect to the nominal exchange rate. Two additional restrictions (as implied by the economic model) are also imposed, namely that $\delta_4 = -\delta_3$; $\delta_4 = \delta_6$[14].

Table 1 reports the constrained coefficients from the long-run co-integrating relationship normalized with respect to le_t. All variables are signed in accordance with the predictions of the theoretical model and there is strong evidence for the significance of the coefficients. The To test the stability of the VECM model is tested through the inverse roots of the AR Characteristic Polynomial are reported in **Figure 1** in the Appendix. The analysis confirms that the VECM is stable since all the inverted roots of the model lie inside the unit circle. According to the Chi-squared value (χ^2=5.08) all restrictions are jointly accepted at three degrees of freedom. Consequently, the system is identified and according to theorem 1 of Johansen and Juselius (1994) the rank condition is satisfied. Additional tests related to the statistical viability of the results indicate that there is no serial correlation of the residuals, no evidence of heteroscedasticity and that the residuals are normally distributed[15].

[14] Foreign variables i.e. $M_t^*, lP_t^{S,*}, li_t^*, ly_t^*$ are treated as weakly exogenous variables, thus long run forcing in the co-integrating space. This can be justified under the assumption that Japan is a small open economy, as such domestic policy decisions or more generally domestic economic activity do not have a significant impact on the evolution of foreign variables. Consequently, treating all variables as jointly endogenously determined would lead to inappropriate inference. The co-integrating vectors are linearly independent.

[15] The Breusch-Godfrey serial correlation LM test reports a prob$(\chi)^2 = 0.87$, the Breusch-Pagan-Godfrey for Heteroscedasticity a prob$(\chi)^2 = 0.91$ and the Jarque-Bera Normality test has a probability of 0.39.

Table 1 Long-run co-integration relationship (constrained coefficients)

$$le_t = -10.91(lM_t) + 3.57(lM_t^*) - 1.33(lP_t^S) + 1.33\left(lP_t^{S,*}\right) - 1.05(li_t^H) + 1.33(li_t^*) - 4.73(l\kappa_t) - 6.63ly_t + 2.36(ly_t^*)$$

(- 4.428) (1.986) (-1.943) (1.943) (-3.291) (-1.977) (1.943) (-1.483) (-2.152)

Note: t statistics in parentheses.
All constrained coefficients are correctly signed in accordance with the predictions of the model.

4. Economic Interpretation of Results

Having established that the VECM is stable, the identified long-run co-integrating relationship, normalized on the nominal exchange rate, can be interpreted.

4.1 Nominal money supply

The economic model as reflected by equation (25) predicts that an expansionary monetary policy in Japan leads to a depreciation of the Yen i.e. $\delta_1 < 0$. The estimated coefficient for the domestic (Japanese) nominal money supply lM_t is negative and significant, thus supporting the prediction of the model. This reflects the fact that as the money supply increases the price level rises in the domestic economy leading to a depreciation of the nominal exchange rate via the purchasing power parity (PPP). Given that the Uncovered Interest Rate Parity (UIRP) also holds expectations for a future nominal depreciation are incorporated[16].

In a similar manner, the data supports the prediction of equation (25) related to the foreign nominal money supply lM_t^* ($\delta_2 > 0$). The coefficient is positive and significant, implying that an expansionary monetary policy in the USA will cause the yen to appreciate as predicted by the model.

[16] The PPP is assumed to hold as depicted by real money balances in equations (1) and (2). The validity of PPP in the long run is validated by authors such as Hall *et al* (2013).

4.2 Share price indices

The model predicts that as the Japanese share price index lP_t^S increases the yen depreciates i.e. $\delta_3 < 0$. The estimated coefficient for lP_t^S is negative and significant, thus supporting the prediction of the model. A possible explanation is that as the price of equities increases, equities become more attractive to investors causing a substitution effect (which dominates the wealth or income effect) from money and other risk free assets towards equities. The demand of less risky assets relative to equities will decrease, implying a fall in their price and an increase in the interest rate. This increase in the interest rate will induce a further decrease in the demand for real balances. The price level will adjust to equilibrate the money market. Inflationary expectations will be revised upwards (given that the expected return on risky assets increases) which will induce a depreciation of the nominal exchange rate. Using analogous reasoning, in accordance with the prediction of the model ($\delta_4 > 0$), the coefficient for the USA stock price index $lP_t^{S,*}$ is positive and significant, implying an appreciation of the yen.

4.3 Interest rates

As the model predicts the estimated coefficient for the domestic interest rate li_t^H is negative implying that as the domestic nominal interest rate increases the yen depreciates i.e. $(\delta_5 = -1)$. An explanation is that an increase in the domestic interest rate reflects rising inflation expectations and hence a depreciation of the yen against the dollar. It is worth noting that the estimated coefficient for the Japanese interest rate is equal to -1.05, which is very close to the theoretical prediction of the model.

A similar reasoning applies for the increase in the US interest rate, which induces a depreciation of the dollar and an appreciation of the yen, hence the positive and significant coefficient for li_t^* in Table 1, this result is also consistent with the prediction of the model i.e. $\delta_6 > 0$.

4.4 Relative debt to GDP ratio

The relative debt to GDP ratio is of a particular interest as a potential determinant of the nominal exchange rate in the long-run since governments in both Japan and the USA are highly indebted and they have both experienced large increases in their debt to GDP ratios over time. We use the relative debt to GDP ratio as a proxy for the risk associated with holding domestic currency relative to foreign currency. Based on the theoretical predictions of the model an increase in relative debt to GDP ratio in Japan will lead to a depreciation of the yen against the dollar i.e. $\delta_7 < 0$. The estimated coefficient for $l\kappa_t$ is negative and significant supporting this prediction of the model. A possible explanation is that as the government debt to GDP increases in Japan, as compared with the USA, the riskier the environment is perceived by economic agents who become reluctant to invest in Japan and to hold the Japanese currency. It may also mean that economic agents have a greater fear of monetization of the national debt in the future or of higher future taxes. Consequently the yen depreciates against the dollar.

To further highlight the importance of the relative government debt to GDP ratio for the determination of the yen-dollar long run nominal exchange rate, we re-estimate equation (25) by dropping the $l\kappa_t$ variable, implying that the risk associated with holding domestic and foreign real money balances is not considered as a major factor affecting the nominal exchange rate in the two economies. After excluding the relative debt to GDP ratio from the theoretical setup, but following the same analytical procedures, the model has the following predictions:

$\delta_1 < 0$; $\delta_2 > 0$; $\delta_3 < 0$; $\delta_4 > 0$; $\delta_5 < 0$; $\delta_6 > 0$; $\delta_8 < 0$; $\delta_9 > 0$

In addition the following restrictions are assumed to hold:

$\delta_2 = -\delta_1$; $\delta_4 = -\delta_3$; $\delta_6 = -\delta_5$; $\delta_9 = -\delta_8$

$\delta_1 = \delta_5$; $\delta_2 = \delta_6$; $\delta_3 = \delta_8$; $\delta_4 = \delta_9$

Table 2 reports the constrained coefficients from the long-run co-integrating relationship normalized with respect to le_t. Evidence suggests that all coefficients have the right sign, but only 4 coefficients appear to be significant. In addition, only the stock market and the real output coefficients are both right signed and significant. Furthermore, according to the Chi-squared value (χ^2=52) the restrictions imposed are not jointly accepted at 12 degrees of freedom[17]. Consequently, it can be inferred that the presence of the $l\kappa_t$ variable in the analysis significantly improves the empirical validity of the theoretical model implying that the relative government debt to GDP is potentially an important factor that should be considered when trying to understand the long-run determination of the yen-dollar nominal exchange rate.

Table 2 Long-run co-integration relationship after dropping $l\kappa_t$ (constrained coefficients)

$$le_t = -0.06(lM_t) + 0.06(lM_t^*) - 0.39(lP_t^S) + 0.39\left(lP_t^{S,*}\right) - 0.06(li_t^H) + 0.06(li_t^*) - 0.39ly_t + 0.39(ly_t^*)$$

[17] The rank of the Π-matrix was found to be $r = 2$. The Breusch-Godfrey serial correlation LM test reports a prob$(\chi)^2 = 0.40$, the Breusch-Pagan-Godfrey for Heteroscedasticity a prob$(\chi)^2 = 0.79$ and the Jarque-Bera Normality test a probability of 0.003 implying that residuals are not normally distributed.

(1.157)	(-1.157)	(6.180)	(-6.180)	(1.157)	(-1.157)	(6.180)	(-6.180)

Note: t statistics in brackets

4.2 Real income

The economic model predicts that a higher real income in Japan will lead to a depreciation of the yen i.e. $\delta_8 < 0$. **Table 1** shows that the estimated coefficient for the domestic (Japanese) real income ly_t is negative. The evidence supports the prediction of the economic model, which is consistent with a mechanism that links income to imports and thereby to the exchange rate[18]. The implication of such a mechanism is that higher income results in a higher demand for imports and a depreciation of the domestic currency. In this case, the evidence is relatively weak as the coefficient although right signed is not statistically significant.

On similar grounds the coefficient for the foreign (USA) real income ly_t^* comes with a positive sign, which suggests, as the model predicts, that an increase in the foreign real income will lead to an appreciation of the yen i.e. $\delta_9 > 0$

5. Statistical performance of the Exchange rate Predictability

Given our relatively successful empirical estimates of the model coefficients we also conduct both an in-sample and out-of-sample forecasting analysis to see if the model has any useful forecasting ability in comparison to the Random Walk forecast made famous by Meese and Rogoff (1983). To do this we look at 4 possible models encompassed by equation (25):

1st Model: $le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 li_t^H + \delta_4 i_t^* + \delta_5 ly_t + \delta_6 ly_t^*$
2nd Model: $le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 lP_t^S + \delta_4 lP_t^{S,*} + 5 li_t^H + \delta_4 i_t^* + \delta_5 ly_t + \delta_6 ly_t^*$
3rd Model: $le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 lP_t^S + \delta_4 lP_t^{S,*} + \delta_5 li_t^H + \delta_6 li_t^* + \delta_7 l\kappa_t + \delta_8 ly_t + \delta_9 ly_t^*$
4th Model: $le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_5 li_t^H + \delta_6 l\kappa_t + \delta_7 i_t^* + \delta_8 ly_t + \delta_9 ly_t^*$

[18] The results in the literature related to the way that domestic real income affects the nominal exchange rate over the long-run are somewhat mixed. See Morley (2007) and Wilson (2009)

Table 3. Statistical Performance of the Exchange Rate Predictability

	In-sample	Out-of-sample (2007:Q1-2009:Q4)	Out-of-sample (2007:Q1-2012:Q4)	Out-of-sample (2007:Q1-2015:Q4)
RMSE	RW: 0.049	RW: 0.051	RW: 0.040	RW: 0.044
RMSE	1st Model: 0.047	1st Model: 0.064	1st Model: 0.164	1st Model:0.156
RMSE	2nd Model: 0.045	2nd Model: 0.119	2nd Model: 0.113	2nd Model: 0.320
RMSE	3rd Model: 0.040	3rd Model: 0.099	3rd Model: 0.080	3rd Model: 0.350
RMSE	4th Model: 0.044	4th Model: 0.071	4th Model: 0.093	4th Model: 0.206

The first model resembles the conventional Monetary Approach to the Exchange Rate determination (MAER) in the presence of domestic and foreign money supplies, domestic and foreign interest rates and domestic and foreign real incomes. The second model is an augmented version after introducing the log of the real level of stock market indices for both Japan and the USA. Model 3 is the one reflected by equation (25) incorporating the relative government debt to GDP ratio i.e. $l\kappa_t$. Finally, model 4 is an augmented version of model 1 incorporating the relative government debt to GDP ratio $l\kappa_t$.

Table 3 reports the Root Mean Squared Errors (RMSE) for all models including the random walk forecast. It is apparent from the results that all 4 versions of the model depicted by equation (25) have a superior forecasting power than the simple random walk model for the in-sample forecasting exercise. In addition, after comparing Model 1 to Model 4 and Model 2 to Model 3 it can be inferred that the predictive power improves in the presence of the relative government debt to GDP ratio, which further highlights the importance of this variable for the determination of the nominal exchange rate.

Although in the out-of-sample exercise the constructed models do not perform better than the random walk forecast, it seems that the forecasting ability of Model 3 is better as compared with the other models when the out-of-sample performance is compared within a 5 years window.

6. The statistical performance of the model within different data samples

To further explore the statistical performance of the theoretical model implied by equation (25) we split the whole sample into 2 sub-periods. The first sub-period spans from 1983:Q1 to 1999:Q4 when the Japanese economy suffered from a recession, which has been followed by a financial crisis, and the second sub-period spans from 2000:Q1-2015:Q4. It is worth noting that since the early 2000s the Japanese economy has entered a state of expansion accompanied by deflation and a significant increasing in its government debt to GDP ratio. The Japanese government debt to GDP ratio increased from 63.26% in 1983:Q1 to 142.37% in 2000:Q4 and to 254.29% in 2015:Q4. The USA government debt to GDP was much lower starting from 47.86%in 1983:Q1 and gradually increasing to 53.53% in 2000:Q4 and to 105.48% by 2015:Q4. **Table 4** reports the constrained coefficients from the long-run co-integrating relationship normalized with respect to le_t for the second sub-period. The empirical results are quite supportive of the predictions of the theoretical model since all coefficients are coming with predicted sign and they all appear to be statistically significant[19]. In addition, according to the Chi-squared value (χ^2=5.98) the restrictions imposed are jointly accepted at 3 degrees of freedom[20]. It is therefore apparent that during the second sub-period, which is characterized by particularly high debts to GDP ratios the model performs very well with the coefficients of the $l\kappa_t$ variable exhibiting the highest level of significance implying that relative government debts to GDP should be considered as a potential determinant of the long-run nominal exchange rate between the yen and the dollar.

[19] The magnitude of the coefficient for the domestic interest rate is not close to unity as predicted by the theoretical model. The empirical support of the model coming from the first sub-period (1983Q1-1999Q4) is rather limited. The results are available upon request.

[20] The rank of the Π-matrix was found to be $r = 4$. The Breusch-Godfrey serial correlation LM test reports a $\text{prob}(\chi)^2 = 0.17$, the Breusch-Pagan-Godfrey for Heteroscedasticity a $\text{prob}(\chi)^2 = 0.77$ and the Jarque-Bera Normality test a probability of 0.03.

Table 4 Long-run co-integration relationship (constrained coefficients) 2000:Q1-2015:Q4

$$le_t = -5.76(lM_t) + 3.46(lM_t^*) - 0.12(lP_t^S) + 0.12\left(lP_t^{S,*}\right) - 0.03(li_t^H) + 0.12(li_t^*) - 2.53(l\kappa_t) - 0.92 ly_t + 1.05(ly_t^*)$$

(- 6.753) (5.817) (-3.733) (3.733) (-2.305) (-13.110) (3.733) (-3.630) (-2.239)

Note: t statistics in brackets.

All constrained coefficients are statistically significant and correctly signed in accordance with the predictions of the model.

7. Conclusions

This paper contributes towards the theoretical determination of the long-run nominal exchange rate by constructing an intertemporal optimization model, which incorporates investment in an array of different assets including domestic and foreign bonds, domestic and foreign stocks, and domestic and foreign real money balances. In addition, special consideration has been given to relative government debt to GDP ratio as potential explanatory variable for determining the nominal exchange. The importance of relative government debt to GDP ratio as a key determinant of the long-run nominal exchange rate has been somewhat neglected in the current literature, which is heavily oriented towards various versions of the conventional flexible price or sticky price monetary approaches.

The model has been tested on the two highly indebted economies of Japan and the USA although it could be applied more broadly. The predictions of the model are borne out empirically suggesting that asset prices and returns, along with monetary and real variables, play an important role in the determination of the long-run nominal exchange rate and its evolution. More specifically, the model suggests that an increase in the domestic (Japanese) money supply, an increase in the domestic economy's stock market, an increase in the domestic bond returns and an increase in real income lead to a depreciation of the yen against the dollar in the long-run, while increases in the corresponding foreign (USA) variables lead to a nominal yen appreciation in the long-run. Of a particular interest the results suggest that an increase in the relative debt to GDP ratio between the Japan and the USA, which is a proxy for the relative risk associated with holding the corresponding national currencies, leads to a depreciation of the yen against the dollar. Our empirical results, clearly highlight the significance of the relative debt to GDP ratio as an important variable in determining the long-run exchange rate between the two economies. In addition, our in-sample forecasting exercise shows the statistical forecasting ability of the model is superior to the simple random walk model and better than other models that incorporate micro-foundations but lack the relative debt to GDP ratio.

Given the importance of the role of the nominal exchange rate for policy makers and for the functioning of open economies our contribution provides an alternative framework to much of the existing literature. Our results suggest that future research would benefit from incorporating a range of asset prices and consideration of the relative government debt to GDP ratio when considering the determination of the nominal exchange rate. There is also scope for future research to consider how mispricing of financial assets may have feedback effects on the nominal exchange rate and hence on the real economy.

APPENDIX

The derivation of the nominal exchange rate equation

Substituting equation (22) into equation (23) and equation (24) into equation (21) in the text the following equation is derived:

$$\frac{m_t}{m_t^*}=\frac{[(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}\left\{[(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}(m_t)^{\frac{1-\varepsilon}{\varepsilon}}\left[1-\left(\frac{P_{t+1}^{S,*}}{P_t^{S,*}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{[(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}\left\{[(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\varepsilon}}\left[1-\left(\frac{P_{t+1}^{S}}{P_t^{S}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}$$

which simplifies to:

$$\frac{m_t}{m_t^*}=\left(\frac{{q_t}^{\sigma-1}}{{q_t}^{\sigma}}\right)^{-\frac{1}{\varepsilon}}\left[\left(\frac{{q_t}^{\sigma}}{{q_t}^{\sigma-1}}\right)^{-\frac{1}{\varepsilon}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{{m_t}^{\frac{1-\varepsilon}{\varepsilon}}}{{m_t^*}^{\frac{1-\varepsilon}{\varepsilon}}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\frac{\left\{\left[\frac{P_t^{S,*}-[P_{t+1}^{S,*}]}{P_{t+1}^{S,*}}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}{\left\{\left[\frac{P_t^{S}-[P_{t+1}^{S}]}{P_{t+1}^{S}}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}\frac{\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}\qquad (A.1)$$

Dividing equation (7) by equation (9) yields: $\frac{1}{P_t^S}=\frac{1+i_t^D}{P_{t+1}^S}$, which implies that:

$$P_t^S-[P_{t+1}^S]=-[P_{t+1}^S]\frac{i_t^D}{1+i_t^D}\qquad (A.2)$$

In a similar manner, dividing equation (8) by equation (10) implies that:

$$P_t^{S,*}-[P_{t+1}^{S,*}]=-[P_{t+1}^{S,*}]\frac{i_t^F}{1+i_t^F}\qquad (A.3)$$

Using equations (A.2) and (A.3), equation (A.1) simplifies to:

$$\frac{m_t}{m_t^*}=[{q_t}^{\sigma-1}{q_t}^{-\sigma}]^{-\frac{1}{\varepsilon}}\left\{[{q_t}^{\sigma}{q_t}^{1-\sigma}]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[{m_t}^{\frac{1-\varepsilon}{\epsilon}}{m_t^*}^{-[\frac{1-\varepsilon}{\varepsilon}]}\right]^{\frac{1-\varepsilon}{\varepsilon}}\frac{\left\{\frac{\left[-[P_{t+1}^{S,*}]\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}{[P_{t+1}^{S,*}]^{-\frac{1}{\varepsilon}}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}{\left\{\frac{\left[-[P_{t+1}^{S}]\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{[P_{t+1}^{S}]^{-\frac{1}{\varepsilon}}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}\frac{\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}$$

$$\frac{m_t}{m_t^*}=[{q_t}^{\sigma-1}{q_t}^{-\sigma}]^{-\frac{1}{\varepsilon}}\left\{[{q_t}^{\sigma}{q_t}^{1-\sigma}]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[{m_t}^{\frac{1-\varepsilon}{\epsilon}}{m_t^*}^{-[\frac{1-\varepsilon}{\epsilon}]}\right]^{\frac{1-\varepsilon}{\varepsilon}}$$

$$\left[P_{t+1}^{S,*}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\frac{\left[P_{t+1}^{S}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}{\left[P_{t+1}^{S,*}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}\left[P_{t+1}^{S}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{\frac{1}{\varepsilon}} \qquad (A.4)$$

Dividing equation (9) by equation (10) and using equations (16), (17) and (18) implies

that: $\frac{P_t^S}{P_t^{S,*}} = \frac{e_{t+1}}{e_t}\frac{P_{t+1}^S}{P_{t+1}^{S,*}}$ which can be used to substitute for: $\frac{\left[P_{t+1}^{S}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}{\left[P_{t+1}^{S,*}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}$ in equation (A.4):

$$\frac{m_t}{m_t^*} = \left[{q_t}^{\sigma-1}{q_t}^{-\sigma}\right]^{-\frac{1}{\varepsilon}}\left\{\left[{q_t}^{\sigma}{q_t}^{1-\sigma}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[{m_t}^{\frac{1-\varepsilon}{\epsilon}}{m_t^*}^{-\left[\frac{1-\varepsilon}{\epsilon}\right]}\right]^{\frac{1-\varepsilon}{\varepsilon}}$$

$$\left[P_{t+1}^{S,*}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_t}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^S}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_{t+1}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^{S,*}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}$$

$$\left[P_{t+1}^{S}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{\frac{1}{\varepsilon}} \qquad (A.5)$$

which further implies that:

$$m_t {m_t^*}^{-1} = {q_t}^{\left[\frac{2\varepsilon-1}{\varepsilon^2}\right]}{m_t}^{\left[\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]}{m_t^*}^{-\left[\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]}{P_{t+1}^S}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^*}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_t}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^S}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_{t+1}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^{S,*}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[P_{t+1}^S\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^H}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^H}^{-\left[\frac{1}{\varepsilon}\right]}{i_t^*}^{\left[\frac{1}{\varepsilon}\right]}$$

(A.6)

where $i_t^* = \frac{i_t^F}{1+i_t^F}$ and $i_t^H = \frac{i_t^D}{1+i_t^D}$

Taking logs of equation (A.6) yields:[21]

$$le_t = -\left[\frac{\varepsilon^2-(1-\varepsilon)^2}{1-\varepsilon}\right]lm_t + \left[\frac{\varepsilon^2-(1-\varepsilon)^2}{1-\varepsilon}\right]lm_t^* + \left[\frac{2\varepsilon-1}{1-\varepsilon}\right]lq_t - l\left[P_{t+1}^{S,*}\right] + \left[\frac{2\varepsilon-1}{1-\varepsilon}\right]li_t^* - lP_t^S$$

$$+\, le_{t+1} + lP_t^{S,*} + l\left[P_{t+1}^S\right] + \left[\frac{1-2\varepsilon}{1-\varepsilon}\right]li_t^H \qquad (A.7)$$

Using the fact that $m_t = \frac{M_t}{P_t}$, $m_t^* = \frac{M_t^*}{P_t^*}$ and $q_t = \frac{P_t^*}{e_t P_t}$ Equation A.7 becomes:

$$le_t = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t + \left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t^* - \left[\frac{1-\varepsilon}{\varepsilon}\right]l\left[P_{t+1}^{S,*}\right] + \left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^{S,*} - \left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^S$$

$$+\left[\frac{1-\varepsilon}{\varepsilon}\right]l\left[P_{t+1}^S\right] + \left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^* - \left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^H$$

$$+\left[\frac{1-\varepsilon}{\varepsilon}\right]le_{t+1} \qquad (A.8)$$

[21] A l before a variable denotes log.

Following the fact that $\frac{P_t^S}{P_t^{S,*}} = \frac{e_{t+1}}{e_t}\frac{P_{t+1}^S}{P_{t+1}^{S,*}}$ and assuming that capital and consumption are homogeneous goods equation (A.8) becomes:

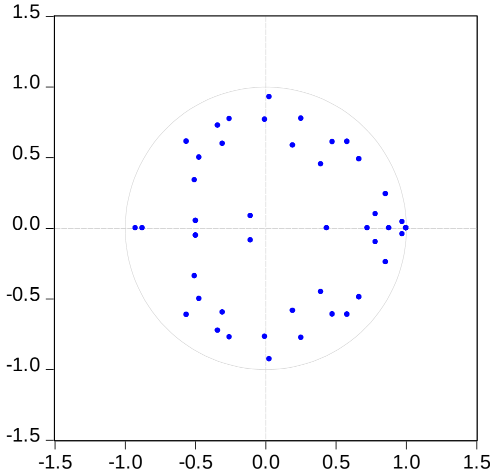
$$le_t = -\left[\frac{2\varepsilon - 1}{\varepsilon}\right] lM_t + \left[\frac{2\varepsilon - 1}{\varepsilon}\right] lM_t^* + \left[\frac{2\varepsilon - 1}{\varepsilon}\right] li_t^* - \left[\frac{2\varepsilon - 1}{\varepsilon}\right] li_t^H - \left[\frac{1 - \varepsilon}{\varepsilon}\right] lP_t^S + \left[\frac{1 - \varepsilon}{\varepsilon}\right] lP_t^{S,*} - \left[\frac{1 - \varepsilon}{\varepsilon}\right] q_t \quad (A.9)$$

Given the fact that $\frac{U_{\frac{M^*}{P^*},t}}{U_{c^*,t}} = i_t^*$ that $\frac{U_{c,t}}{U_{c^*,t}} = \frac{U_{\frac{M}{P},t}/i_t}{U_{\frac{M^*}{P^*},t}/i_t^*} = \frac{1}{q_t}$ and following Kia's (2006) assumption that domestic and foreign real consumption (C_t, C_t^*) are a constant proportion ω of the domestic and foreign real income, equation (A.10) is derived:

$$le_t = \Omega + \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 lP_t^S + \delta_4 lP_t^{S,*} + \delta_5 li_t^H + \delta_6 i_t^* + \delta_7 l\kappa_t + \delta_8 ly_t + \delta_9 ly_t^* \quad \text{(A.10)}$$

Equation (A.10) corresponds to equation (25) in the text.

Figure 1: Inverse Roots of AR Characteristic Polynomial



EExplanation of the variables employed

Variable	Explanation
C_t	Real consumption of a composite bundle of goods
$m_t = \frac{M_t}{P_t}$	Domestic real money balances, with M_t domestic nominal money balances and P_t the domestic price index.
$m_t^* = \frac{M_t^*}{P_t^*}$	Foreign real money balances, with M_t^* foreign nominal money balances and P_t^* the foreign price index.
y_t	Domestic real income
y_t^*	Foreign real income
κ_t	Relative debt to GDP ratio
e_t	Nominal exchange rate (amount of foreign currency per unit of domestic currency)
B_t^D	Amount of domestic currency invested in domestic bonds
B_t^F	Amount of foreign currency invested in foreign bonds
i_t^D	Nominal rate of return on domestic bonds
i_t^F	Nominal rate of return on foreign bonds
S_t	Number of domestic shares
S_t^*	Number of foreign shares
P_t^S	Domestic share price
$P_t^{S,*}$	Foreign share price
$U_{c,t}$	Marginal utility from consumption
$U_{\frac{M}{P},t}$	Marginal utility from domestic real money balances
$U_{\frac{M^*}{P^*},t}$	Marginal utility from foreign real money balances
q_t	Real exchange rate
i_t^h	$\left[\frac{i_t^D}{1 + i_t^D}\right]$
i_t^*	$\left[\frac{i_t^F}{1 + i_t^F}\right]$

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