A SENSITIVITY ANALYSIS OF THE PARAMETERS USED IN A PHI MULTIPLE STATE MODEL

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Abstract

This paper presents a multi-state model for Permanent Health Insurance. It includes more states than are normally found in such a model to act as a proxy to duration spent in a particular state. This enables a Markov approach to be adopted. Lapses are incorporated within the model and the net premium for a particular policy is tested for sensitivity to the various parameters used, including the lapse rate. The most significant conclusion is that the net premium is insensitive to changes in the lapse rate.

Key words

PHI benefits; Markov chain; lapses.

"A sensitivity analysis of the parameters used in a PHI multiple state model"

1. Introduction

When Permanent Health Insurance (PHI) is modelled using a multi-state process, the approach is usually to consider only 3 states: healthy, sick and dead.

Furthermore, in order to make the calculations tractable, the transition forces are usually assumed to be constant, at least over each year of age of the policyholder.

The problem with this approach is that the model is over simplified since, in practice, the probability that a policyholder moves out of the sick state (into the dead state or back into the healthy state) will depend on the time spent in the sick state. In other words, the transition probabilities depend on duration in a particular state as well as the age of the policyholder.

It would be possible to incorporate within the model the duration-dependence aspect described above by adopting a semi-Markov, as opposed to Markov, model. However, it is generally accepted (Jones 1994) that semi-Markov models are complicated and do not lead to convenient expressions for the transition probabilities required to obtain actuarial values.

In this paper, we adopt an alternative approach suggested by Jones (1994) whereby we introduce more states into the model as a means of dealing with duration-dependence whilst maintaining the Markov as opposed to semi-Markov property of the model. Increasing the number of states makes the state space more complicated but maintaining the Markov process keeps the calculations tractable.

Having introduced our model, including a status for lives who have lapsed their policies, we use the data contained within Continuous Mortality Investigation Report No 12, published by the Institute and Faculty of Actuaries, to test the sensitivity of the parameters involved.

2. The Model

We set out below a diagrammatic representation of the multi-state model adopted.

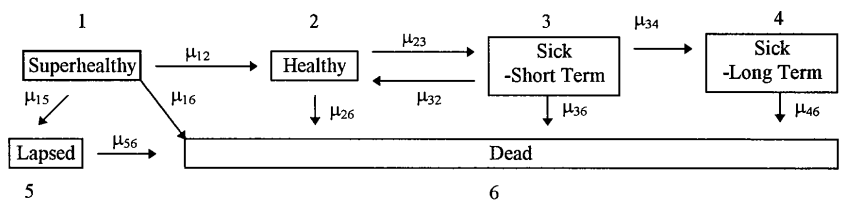

Figure 1. Outline of PHI model

The model has 6 states. State 1 is the state in which policyholders enter the model when their policy commences. Since they will have needed to provide satisfactory medical evidence, they

are deemed to be select lives and therefore healthier than other insured lives of the same age. We describe these lives as "superhealthy". It is likely that, in time, the selection effect will wear off and that the lives will move to an unstable form of the healthy state (State 2) from which they may become sick enough to make a claim under the PHI policy.

As with "healthy", we have a sub-divided the "sick" state between "short term sick" (State 3) from which it is possible to recover (and therefore return to State 2) and "long term sick" (State 4) from which it is assumed that recovery is impossible.

We have also introduced a lapsed state (State 5) into the model. We assume that policyholders will only enter this state from State 1, on the grounds that policyholders in any other state would find it worthwhile to continue their PHI policy.

The approach adopted here is to assume, initially, that the forces of transition between the states are all constant (i.e. not dependent on age). We then relax this assumption by using force of transition functions which are piecewise constant (i.e. constant only within age bands).

3. Constant Forces of Transition

As implied above, as a first step, it is possible to use a time-homogeneous continuous time Markov process to represent the transitions between states. Hence, we assume that a person in any state is subject to a constant force of progression into the other states which it is possible to enter according to Figure 1.

Let $p_{ij}(t)$ be the probability that a life currently in state i will be in state j in t years time. The constant force of transition between states i and j, μ_{ij}, is defined as

$$\mu_{ij} = \lim_{t \to 0+} \frac{p_{ij}(t)}{t} \quad i \neq j$$

It is convenient also to define, for each i,

$$\mu_{ii} = -\sum_{j:j \neq i} \mu_{ij}$$

where the summation is over the states j which are linked directly to state i.

The constant transition intensities are depicted in Figure 1. The fact that these intensities are constant means that the length of time already spent in the current state has no effect on the future length of time which the policyholder will remain in the state. Hence, a "memoryless" property exists (Haberman, 1992).

A common approach used to set up equations for the required transition probabilities (for example, Jones (1994) and Haberman (1995)) is to use the Chapman-Kolmogorov backward system of difference-differential equations (Cox and Miller, 1965). The fact that the transition intensities are assumed to be constant means that explicit solutions to the equations can be found.

The backward system of equations is derived by considering the interval $(0, t+h]$ as being comprised of subintervals $(0, h]$ and $(h, t+h]$ and then letting $h \to 0$.

These equations lead to a set of difference-differential equations. For illustration purposes, some of the differential equations are set out below:

$$\left.\begin{aligned}
\frac{d}{dt}p_{11}(t) &= -\left(\mu_{12}+\mu_{15}+\mu_{16}\right)p_{11}(t)\\
\frac{d}{dt}p_{12}(t) &= -(\mu_{12}+\mu_{15}+\mu_{16})p_{12}(t)+\mu_{12}p_{22}(t)\\
\frac{d}{dt}p_{22}(t) &= -(\mu_{23}+\mu_{26})p_{22}(t)+\mu_{23}p_{32}(t)\\
\frac{d}{dt}p_{23}(t) &= -(\mu_{23}+\mu_{26})p_{23}(t)+\mu_{23}p_{33}(t)\\
\frac{d}{dt}p_{33}(t) &= \mu_{32}p_{23}(t)-(\mu_{32}+\mu_{34}+\mu_{36})p_{33}(t)\\
\frac{d}{dt}p_{32}(t) &= \mu_{32}p_{22}(t)-(\mu_{32}+\mu_{34}+\mu_{36})p_{32}(t)\\
\frac{d}{dt}p_{44}(t) &= -\mu_{46}p_{44}(t)\\
\frac{d}{dt}p_{46}(t) &= -\mu_{46}p_{46}(t)+\mu_{46}p_{66}(t)\\
\frac{d}{dt}p_{55}(t) &= -\mu_{56}p_{55}(t)\\
\frac{d}{dt}p_{56}(t) &= -\mu_{56}p_{56}(t)+\mu_{56}p_{66}(t)\\
\frac{d}{dt}p_{66}(t) &= 0
\end{aligned}\right\} \quad (1)$$

The most straightforward way to solve these differential equations is to follow the method outlined by Jones (1994).

The forces of transition and transition probability functions are expressed in matrix form such that:

Q is the 6×6 matrix with (i, j) entry μ_{ij}

$P(t)$ is the 6×6 matrix with (i, j) entry $p_{ij}(t)$

The Chapman-Kolmogorov forward and backward systems of equations may then be written respectively

$$P'(t) = P(t).Q \tag{2}$$

and $$P'(t) = Q.P(t) \tag{3}$$

where $P'(t)$ is the matrix with (i, j) entry $\frac{d}{dt}p_{ij}(t)$ and with boundary condition $P(0) = I$, the identity matrix.

Equations (2) and (3) have the solution

$$P(t) = e^{Qt} \tag{4}$$

Now, as noted by Cox and Miller(1965), if Q has distinct eigenvalues $d_1, d_2, \ldots, d_6$ then

$$Q = A.D.A^{-1} \tag{5}$$

where $D = diag\ (d_1, \ldots, d_6)$ and the ith column of A is the right-eigenvector associated with d_i.

Furthermore,

$$P(t) = A.diag(e^{d_1 t}, \ldots e^{d_6 t}).A^{-1} \tag{6}$$

The transition probability matrix, $P(t)$, can then be obtained from equation (6) using standard computer software, for example S-PLUS.

We should then note that $p_{ij}(t)$ is the (i, j) element of the matrix $P(t)$.

4. Piecewise Constant Forces Of Transition

In Section 3. above, we assumed that the forces of transition were constant with respect to time which enabled us to obtain the transition probabilities in a relatively straightforward manner. However, when considering Permanent Health Insurance it is unrealistic to assume that the forces of transition do not depend on age. To deal with this problem, we again adopt the general methodology described in Jones(1994).

By assuming that the forces of transition are piecewise constant, we are able to use the theory described in Section 3, whilst permitting the forces of transition to vary by age. For illustrative purposes in this paper, we have assumed that the forces of transition are constant over successive 5-year age bands (i.e. age 30-34, 35-39, ...60-64).

We now define $P_x(1)$ to be the transition probability matrix $P(1)$ applicable to the year of age x to $x + 1$. In other words, the (i, j) element of $P_x(1)$ is the probability that the policyholder currently aged x and in State i will be in State j in one year's time (aged $x + 1$).

If we let $p_{ij}^x(t)$ be the probability that a life currently aged x and in State i will be in State j in t years time then

$p_{ij}^x(t) = (i, j)$ element of the matrix resulting from the matrix product:

$$\prod_{k=0}^{t-1} P_{x+k}(1) \qquad \text{provided } t \text{ is an integer.}$$

5. PHI Data

Initially, we wish to test the effect which the force of transition from state 1 to state 5 (described here as the lapse rate) has on the net premium emerging from the model. In order to do this, we need to assign parameter values to all the forces of transition in the model and allow the lapse rate, μ_{15}, to vary.

The data used in the investigation has been taken from Report No 12 of the Continuous Mortality Investigation Reports published by the Institute and Faculty of Actuaries in 1991 ("CMIR 12").

As mentioned in Section 4 above, we have assumed that the parameter values applicable to the initial age of each 5-year age band (ie ages 30, 35, ..., 60) are constant over the whole of the 5-year age band.

6. Parameter Values

We set out below the way in which each parameter value has been chosen.

μ_{23}, **Unstable Healthy** $\longrightarrow$ **Short Term Sick**

We consider the sickness inception rate, σ_x, described in Part C of CMIR 12. We could have used values assuming a deferred period of 1 week; however, in view of the fact that this would include trivial sickness claims, we have instead used values for a deferred period of 4 weeks which are found in Table C16 of CMIR 12 (page 74).

μ_{16}, **Superhealthy** $\longrightarrow$ **Dead**
μ_{26}, **Unstable Healthy** $\longrightarrow$ **Dead**

CMIR 12 assumes the morality rate amongst healthy lives to be that of Male Permanent Assurances 1979 - 82, duration 0. The rates are shown in Table E17 (page 132) under the column headed $m(x)$.

In our model, we have sub-divided healthy lives into "Superhealthy" and "Unstable healthy" states. Since the latter will experience higher mortality rates than the former, we have decided to assume:

μ_{16} = 80% of the mortality rates for Male Permanent Assurances of 1979-82, duration 0.
μ_{26} = 120% of the mortality rates for Male Permanent Assurances of 1979-82, duration 0.

The overall effect can therefore be considered to be broadly consistent with CMIR 12. As suggested by Cordeiro (1995), net premium values are likely to be insensitive to the parameter values chosen for the forces of mortality.

μ_{32}, **Short Term Sick** $\longrightarrow$ **Unstable Healthy**

We consider the recovery rates described in Section 3, Part B of CMIR 12. Page 34 of CMIR 12 sets out values of $\rho_{y+z,z}$ at various ages where:

$\rho_{y+z,z}$ = The transition intensity from sick to healthy at current age $y + z$ and current duration of sickness z.

It is clear that these recovery rates vary considerably by duration of sickness (measured in weeks). In view of the relatively simplistic approach deliberately adopted in our model, we will use parameter values which we consider to be reasonable. Initially we will assume a constant value of 2.0 at all ages. We will then analyse the effect of assuming $\mu_{32} = 3.0$ at all ages instead in Section 8.4.

μ_{36}, **Short Term Sick** $\longrightarrow$ **Dead**

We consider the mortality intensities described in Section 6, Part B of CMIR 12. Page 39 of CMIR 12 sets out values of $\nu_{y+z,z}$ at various ages where:

$\nu_{y+z,z}$ = The transition intensity from sick to dead at current age $y + z$ and current duration of sickness z.

For our calculations, we will use the values at 15 weeks duration of sickness which is when the values reach their peak. Interpolated values have been used where necessary.

μ_{34}, **Short Term Sick** $\longrightarrow$ **Long Term Sick**

As a proxy for this force of transition, we use the sickness inception rate, σ_x, for a 26 week deferred period (i.e. a suitably long period of deferment). As with μ_{23}, the rates can be found in Table C16 of CMIR 12 (page 74).

μ_{46}, **Long Term Sick** $\longrightarrow$ **Dead**

We can again consider the mortality intensities $\nu_{y+z,z}$ which were described under μ_{36} above. It seems appropriate to use these intensities at a suitably long sickness duration. We will use the values at duration 5 years which are shown on page 39 of CMIR 12.

μ_{56}, **Lapse** $\longrightarrow$ **Dead**

In view of the fact that we are assuming that only "Superhealthy" policyholders lapse their policies, we assume that :

$$\mu_{56} = \mu_{16}$$

μ_{12}, **Superhealthy** $\longrightarrow$ **Healthy**

It is clear that CMIR 12 will not be able to provide explicit parameter values for μ_{12}. However, it seems reasonable to ensure that our estimates of μ_{12} should be such that the aggregate mortality rates implied within the model approximately reflect the mortality table Male Permanent Assurances 1979-82, duration 0. We have, therefore, chosen values for μ_{12} which meet this constraint. This has been done by inspection, since more sophisticated techniques would lead to spurious accuracy. For the purpose of carrying out this exercise, we assume that μ_{15}, the lapse rate, takes the value 0.01 at all ages.

μ_{15}, **Superhealthy** $\longrightarrow$ **Lapse**

Finally, having fixed the other parameters, we will let the lapse rate, μ_{15}, vary in order to investigate the effect on the net premium.

We set out a summary of the parameter values, other than μ_{15} in Appendix 1. Appendix 2 shows the number of lives in each state at sample ages given 100 healthy lives entering the superhealthy state (State 1) at age 30. It can therefore be seen for example that, by age 65, 17.4% of the lives have died, 19.9% have lapsed and 13% are still in the "superhealthy" state.

7. Valuation Functions

For illustrative purposes, we calculate the annual net premium required in respect of the following PHI policy:

We consider a 35 year term PHI policy issued to a life aged 30. A payment of £1,000 is paid at the end of each year if the policyholder is sick at that time. Premiums are waived during periods of sickness and there is no deferred period. The benefits and premiums cease at the age of 65.

We use a valuation rate of interest of 6% pa.

In order to calculate the annual net premium, we need to set up the standard equation of value, based on the equivalence principle:

Expected present value of net premiums = Expected present value of benefits

We note that a premium is only payable if the policyholder is either in State 1 ("Superhealthy") or State 2 ("Unstable healthy") at the start of the policy year in question. Hence the expected present value of the premium is

$$P\left(\ddot{a}^{11}_{30:\overline{35}|} + \ddot{a}^{12}_{30:\overline{35}|}\right)$$

where P = annual net premium

and $\ddot{a}^{1j}_{30:\overline{35}|} = \sum_{k=0}^{34} p^{30}_{1j}(k)v^k \qquad j = 1, 2$

We also note that the PHI benefit is only payable if the policyholder is in either State 3 ("Short Term Sick") or State 4 ("Long Term Sick") at the end of the policy year in question. Hence, the expected present value of the PHI benefits is

$$1000\left(a^{13}_{30:\overline{35}|} + a^{14}_{30:\overline{35}|}\right)$$

where $a^{1j}_{30:\overline{35}|} = \sum_{k=1}^{35} p^{30}_{1j}(k)v^k \qquad j = 3, 4$

Therefore, we can find P from

$$P = \frac{1000\left(a^{13}_{30:\overline{35}|} + a^{14}_{30:\overline{35}|}\right)}{\left(\ddot{a}^{11}_{30:\overline{35}|} + \ddot{a}^{12}_{30:\overline{35}|}\right)}$$

8. Results

8.1 Sensitivity to μ_{15} the lapse rate

We set out in Figure 2 below a graph showing how the theoretical net premium varies as the lapse rate, μ_{15}, takes values between 0 and 1.

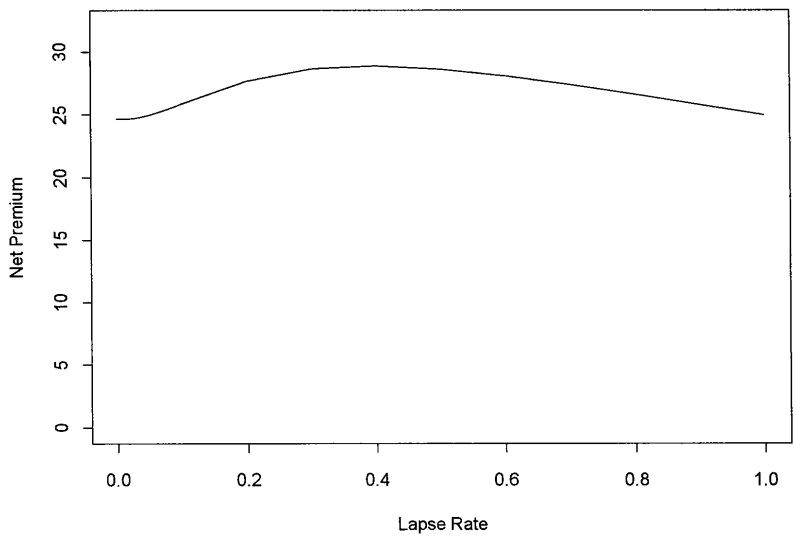

Figure 2: Sensitivity of net premium to the lapse rate, μ_{15}

It can be seen that, perhaps surprisingly, the net premium is relatively insensitive to the lapse rate. The net premium remains fairly constant at around £25 per annum despite large changes in the lapse rate.

The net premium increases from £24.67 per annum to £28.86 per annum as μ_{15} increases from 0 to 0.4. The reason for this is that when the lapse rate is small, there are a large number of lives in the system who are in the "superhealthy" state and therefore continue to pay premiums without receiving any PHI benefit payments. This tends to suppress the net premium averaged over all the policyholders in the system. As the lapse rate increases, more of the "superhealthy" lives leave the system by lapsing which will tend to increase the average premium payable in respect of the remaining, relatively unhealthy, insured population.

It is then necessary to explain why the net premium decreases as the lapse rate increases beyond 0.4. The reason for this counter-intuitive result is that, at very high lapse rates, most of the population will lapse in the first few years of the policy which makes the total premium paid in advance at the outset by <u>all</u> the insured lives a significant amount. The fact that a large amount of premium income is received at the start of the policy, but that the insured population then reduces dramatically over the first few years, will tend to reduce the average premium required.

It should be noted that lapse rates of more than 0.4 would appear to be unrealistic. For example, it can be shown that if $\mu_{15} = 0.4$, over 83% of the insured population aged 30 at the outset would have lapsed their policy during the first 5 years of the policy.

Finally, before discussing other sensitivity issues, it is worth comparing the net premiums calculated using the model described in this paper with those derived from the data in CMIR 12.

The data contained in Table F1 on page 228 of CMIR 12 suggests that the net premium for a policy similar to that described in Section 7, but with premium and benefit payments made continuously and with a deferred period of 1 week, should be £24.24 per annum. The net premium figures shown in Figure 2 are clearly of the same order of magnitude and hence provide some comfort that our model (including the parameter values chosen) is consistent with the model described in CMIR 12.

8.2 Sensitivity to μ_{12}

We set out in Figure 3 below a graph showing how the net premium changes when the parameter values for μ_{12} given in Appendix 1 are increased or decreased by 10%.

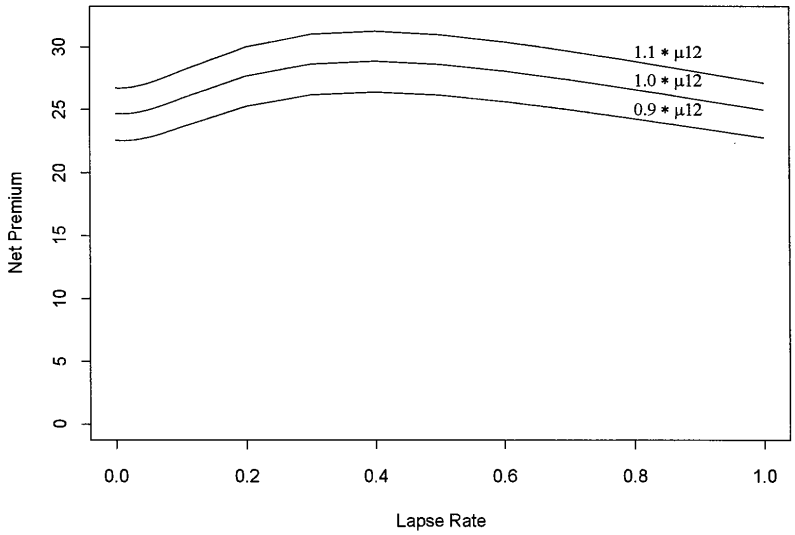

Figure 3: Sensitivity of net premium to μ_{12}

The net premium increases by 8.5% when the μ_{12} values are increased by 10% and decreases by 8% when the μ_{12} values decrease by 10%. In both cases, it can be seen that the effect is insensitive to the lapse rate, μ_{15}.

It is to be expected that the net premium moves in the same direction as μ_{12}. This is because an increase in μ_{12} causes more lives to move from the "superhealthy" to the "healthy" state

where they are exposed to the risk of sickness, which in turn will lead to an increase in the premium required.

8.3 Sensitivity to μ_{23}, the sickness inception rate

We set out in Figure 4 below a graph showing how net premium changes when the parameter values for μ_{23}, the sickness inception rate, are altered by 10%.

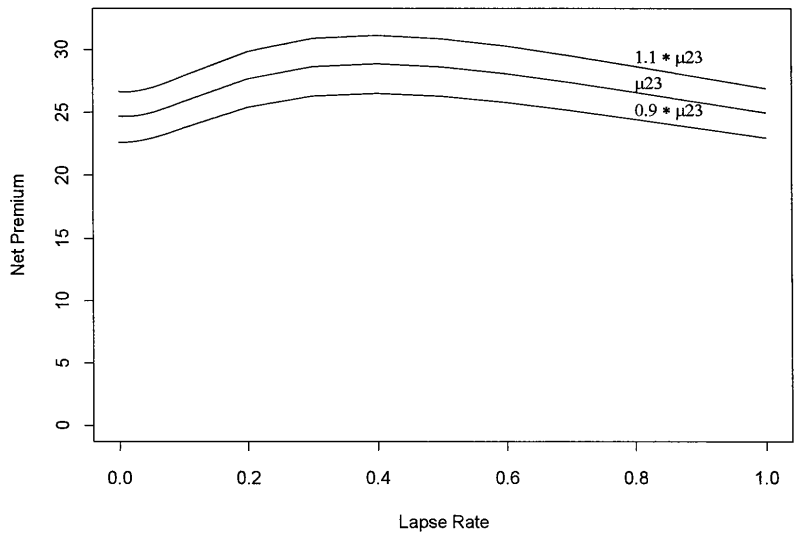

Figure 4: Sensitivity of net premium to sickness inception rate, μ_{23}

The net premium increases by 8% when the μ_{23} values are increased by 10%, and decreases by 8% when the μ_{23} values are decreased by 10%. Again, the results are largely unaffected by the level of lapse rate assumed.

As expected, an increase in the sickness inception rate causes an increase in the net premium required.

8.4 Sensitivity to μ_{32}, the recovery rate

We set out in Figure 5 below a graph showing how the net premium changes when the parameter value for μ_{32}, the recovery rate, is increased or decreased by 10% (i.e. changed from 2.0 at all ages to 2.2 or 1.8, respectively).

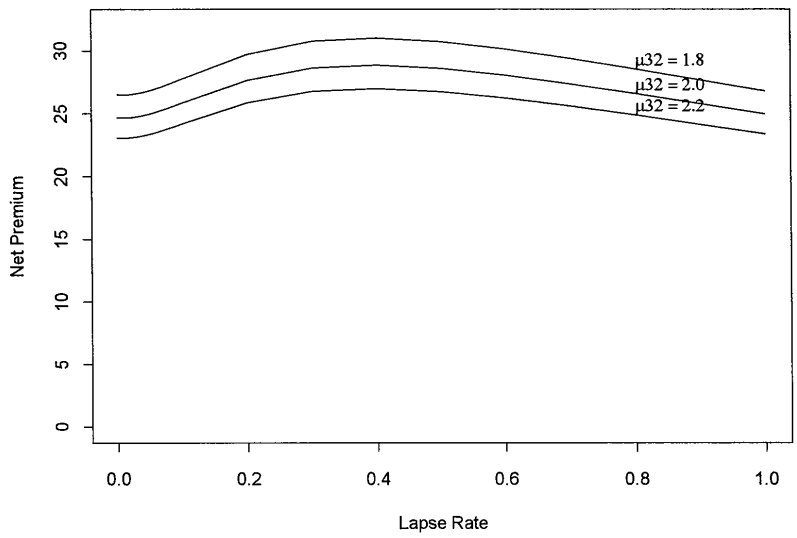

Figure 5: Sensitivity of net premium to the recovery rate, μ_{32}

The net premium increases by 7.5% when the recovery rate is reduced by 10%, and decreases by 6.5% when it is increased by 10%. Again, the level of lapse rate has very little effect on these results.

It is to be expected that an increase in the recovery rate should lead to a reduction in the amount of PHI premium required.

8.5 Sensitivity to μ_{34}

We set out in Figure 6 below a graph showing how the net premium changes when the parameter values for μ_{34} are increased or decreased by 10%.

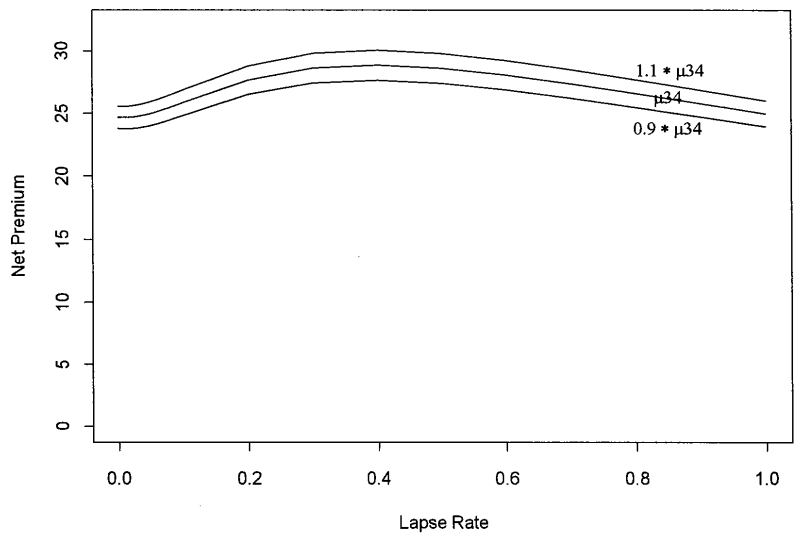

Figure 6: Sensitivity of net premium to μ_{34}

It can be seen that the net premium is relatively insensitive to changes in μ_{34} since a 10% increase/decrease in the latter causes only a 4% increase/decrease in the net premium.

As expected, an increase in the long term sickness inception rate leads to an increase in the net premium required.

8.6 Analysing the relationship between μ_{12} and μ_{32}

In Section 6, we explained how the parameter values for μ_{12} were chosen so that the aggregate mortality rates within the model broadly reflected the Male Permanent Assurances 1979-82, duration 0.

We now analyse how sensitive the values of μ_{12} are to a change in the other parameters, in particular to a 50% increase in the recovery rate, μ_{32}. In other words, we retain all the parameter values summarised in Appendix 1 except for μ_{32}, which we increase from 2.0 at all ages to 3.0, and μ_{12} which we need to recalibrate in order to ensure that the aggregate mortality rates still reflect the mortality table mentioned above.

As before, we derive the values for μ_{12} by inspection and the results can be summarised as follows:

Table 1: Comparison of μ_{12} values

	$\mu_{32} = 2.0$	$\mu_{32} = 3.0$
Age	μ_{12}	μ_{12}
30-34	0.022	0.026
35-39	0.005	0.009
40-44	0.015	0.021
45-49	0.015	0.035
50-54	0.055	0.070
55-59	0.085	0.110
60-64	0.129	0.430

It can be seen that a 50% increase in μ_{32} requires an increase in μ_{12} of approximately the same order of magnitude in order to leave the aggregate mortality rates within the model unaltered.

It can be seen from Figure 7 below that the changes in the two sets of parameter values would leave the net premium at approximately the same level as before (i.e. approximately £25 per annum).

Figure 7 below shows the effect on the net premium of changing the values of μ_{32} and μ_{12} from the first column of Table 1 to the second column.

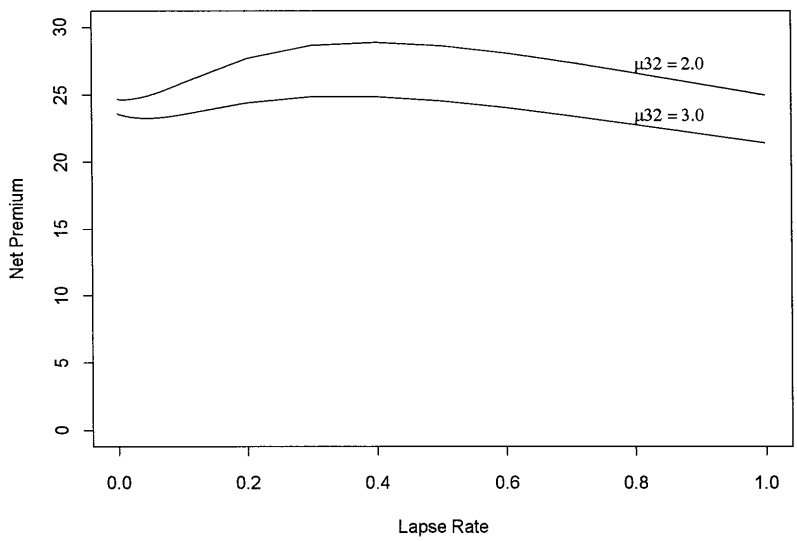

Figure 7: Effect of increasing μ_{32}

It can be seen that the net premium values are very similar when the lapse rate, μ_{15}, is very small. This can be explained by the fact that both sets of values of μ_{12} are calibrated when μ_{15} is set equal to 0.01. The net premiums diverge slightly at higher lapse rates, although they are clearly still of the same order of magnitude (i.e. £25 per annum). As noted in Section 8.1, this level of premium is consistent with the net premium calculated using the model described in CMIR 12.

9. Conclusions

In modelling Permanent Health Insurance, one of the main difficulties that needs to be overcome is that the forces of transition between different states may depend not only on the age of the policyholder but also on the time spent in the current state (e.g. sickness state). This usually leads to a semi-Markov model being used in which case convenient expressions for the transition probabilities are hard to obtain.

In this paper, we have adopted a model where the problems of duration-dependence described above are dealt with by increasing the number of states to differentiate between short-term and long-term stays in a particular status. This enables the model to be Markov, rather than semi-Markov, and therefore leads to tractable solutions. We have also taken the opportunity to consider lapses within the model.

We have tested the sensitivity of the net premium for a particular policy to changes in the model parameter values. The most significant result is that the net premium is insensitive to changes in the lapse rate.

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Appendix 1: Summary of Parameters

Age	$\mu_{56}=\mu_{16}$	μ_{26}	μ_{46}	μ_{36}	μ_{32}	μ_{34}	μ_{23}	μ_{12}
30-34	0.0003	0.0005	0.0172	0.1108	2.0000	0.1183	0.2346	0.0220
35-39	0.0004	0.0006	0.0190	0.1180	2.0000	0.1065	0.2240	0.0045
40-44	0.0006	0.0010	0.0215	0.1251	2.0000	0.1007	0.2059	0.0150
45-49	0.0011	0.0017	0.0239	0.1379	2.0000	0.1000	0.1901	0.0150
50-54	0.0019	0.0028	0.0271	0.1507	2.0000	0.1042	0.1834	0.0550
55-59	0.0031	0.0046	0.0303	0.1694	2.0000	0.1140	0.1929	0.0850
60-65	0.0049	0.0073	0.0343	0.1880	2.0000	0.1310	0.2302	0.1290

Appendix 2: Percentage of Lives in Each State at Sample Ages

Age	State					
	1	**2**	**3**	**4**	**5**	**6**
30	100	0	0	0	0	0
31	96.9	2.0	0.1	0	1.0	0
32	93.7	3.9	0.3	0	2.0	0.1
.						
.						
.						
.						
50	61.0	16.6	1.4	1.6	15.6	3.8
.						
.						
.						
65	13.0	40.6	4.0	5.1	19.9	17.4

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