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The Probationary Period as a Screening Device: Competitive Markets

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Abstract

Seminal papers about asymmetry of information in a competitive insurance market, and the monetary deductible as a screening device show that any existing equilibrium is of a separating type. High risks buy complete insurance whilst low risks buy partial insurance. Rothschild and Stiglitz (1976) deal with insurance companies showing Nash behaviour, while Miyazaki (1977) and Spence (1978) consider firms with Wilson foresight. In this paper, we analyze the strength of the probationary period as a screening device. We show that in such a case a) under Nash behavior, low risks may prefer not to purchase any insurance at all in equilibrium and b) under Wilson foresight, a pooling equilibrium may exist.

1 Introduction

In insurance markets with adverse selection and two different risk types, insurers can implement a screening tool to separate low risk and high risk individuals. The most common device is the monetary deductible. Rothschild and Stiglitz (1976) have shown that, if this instrument is applied in a competitive market with Nash behavior, an equilibrium exists if the proportion of high risks in a population exceeds a certain threshold. This equilibrium, in the sequel called a *Cournot-Nash equilibrium*, is always of a separating type. High risks buy full coverage and low risks buy partial coverage, both at actuarially fair terms. Miyazaki (1977) and Spence (1978) demonstrate that, in a competitive market with *Wilson foresight*[1], an equilibrium always exists. Like the Cournot-Nash equilibrium derived in Rothschild and Stiglitz (1976), such an equilibrium is always of a separating type, except that high risks may be subsidized by low risks. Crocker and Snow (1985)

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[1]Wilson foresight (see Wilson, 1977) implies that the insurance companies take into account that existing contracts may be withdrawn as a result of a new contract under consideration.

show that, according to the definition of efficiency developed in Harris and Townsend (1981), such a *Wilson-Miyazaki equilibrium* is second best efficient.

An alternative screening device, which will be the theme of this paper, is the probationary period. Such a device excludes coverage for events that occur during a predefined period after the inception of the policy. The method, aiming to rule out preexisting conditions, has found applications in some dental and medical policies. Besides, over recent years it has gained popularity among Dutch group life companies, as a consequence of new legislation concerning the medical examination of employees. By the new law which came into force at the beginning of 1998, insurance companies are strongly restricted in their possibilities to perform medical tests on individual members of a group life scheme. A probationary period may then be an appropriate instrument to identify individuals who are likely to make a claim soon after inception.

Several characteristics of the probationary period, like for example its implication for the expected utility of consumers, have been investigated in Eeckhoudt et al. (1988). The authors' main conclusion is that most of the basic properties of the above mentioned monetary deductibles do not carry over to probationary periods. For example, the equilibrium on a competitive market (with symmetric information regarding risk class) is completely ambiguous when there is a positive loading factor - as opposed to the monetary deductible case where the optimal solution involves incomplete yet positive coverage. The reason behind this result is that with a probationary period, it is not possible to rank different degrees of coverage according to their riskiness (in the sense of Rothschild & Stiglitz, 1970). Hence, preferences will typically exhibit non-convexities with regard to premium rates and the period of probation.

Fluet (1992) applies the concept of a probationary period in a competitive insurance market with asymmetric information and firms exhibiting Nash behavior. He adopts the screening device of a time-dependent monetary deductible. Assuming that the proportion of high risk agents is large enough, the high risks buy full coverage, while the low risks buy partial coverage in monetary terms. The monetary deductible may vary over time but is always positive. Hence, the contract with a pure probationary period can never be an equilibrium, but a combination of an initial probationary period and subsequent deductibles may well arise.

Fluet's finding suggests a certain kind of inferiority of the probationary period as a screening device, when compared with the monetary deductible. This has been confirmed in Spreeuw (2005), who shows that using this instrument in a monopolistic insurance market may lead to a pooling equilibrium, where both classes of risk buy full coverage. This would never be possible with a monetary deductible, as shown in Stiglitz (1977).

This paper deals with a competitive insurance market and focuses on the probationary period, rather than the monetary deductible, as an instrument to let individuals self-select. In this respect, the approach is less general than Fluet (1992) as contracts incorporating both time and monetary deductibles are not considered. It should be stated, however, that a monetary deductible always implies some non-linear pricing which may be difficult to implement in practice. Moreover, Fluet (1992) has shown that the combination of both devices makes it difficult to draw conclusions, unless restricting assumptions are being made. By concentrating on the probationary period, one can get an idea of possible equilibria resulting if, just as in Fluet (1992), allowance is made for partial coverage in monetary terms. We will, however, deal with both the cases of Nash behavior and Wilson foresight. We will show that, in the former case, a separating equilibrium may be degenerate even if the proportion of high risks is high, and in the latter case a pooling equilibrium may exist.

The model is described in Section 2, where also the basic assumptions are listed and the details about the main example in this paper are given. In Section 3, it is assumed that firms are myopic in the sense that they do not take subsequent withdrawals of contracts into account when designing policies. We will repeat a result obtained in Spreeuw (2005), namely that, for full coverage the absolute value of the slope of any individual's indifference curve exceeds the firm's corresponding marginal profit. This is an important property which will be used throughout the remainder of the section. We show that if the low risks' probability of incurring a loss is high and the distinction between the high risks and low risks is strong, the separating equilibrium will be degenerate in the sense that the low risk type gets no insurance coverage at all. Such an equilibrium exists provided that the share of high risks in the population is sufficiently high.

Section 4 deals with Wilson foresight. There we show that, if the insurer's strategy is restricted to pooling contracts, full coverage may be optimal. This is a result which contradicts Miyazaki's and Spence's findings. Finally, the restriction of offering pooling contracts only is dropped and hence the insurer can offer any pair of policies. It is shown that even then a pooling contract may be optimal. Under the assumptions stated in this paper, such a pooling contract would involve complete coverage.

Conclusions are given in Section 5.

2 The basic assumptions and the nature of a probationary period

In this section we will start with an overview of the general assumptions and definitions in Subsection 2.1. Thereafter, in Subsection 2.2, we will indicate the relationship between the time-of-accident of the high risks and low risks and motivate our choice.

2.1 General assumptions

The basic assumptions are mainly derived from Fluet (1992). For the ease of exposition they are listed below:

- A population consists of two risk classes, namely the high risks and the low risks. In the remainder of this paper all variables pertaining to high risk and low risk individuals will be accompanied by the subscripts H and L, respectively. All individuals have an initial wealth equal to W.

- All individuals within the population are identical, except with respect to the probability of having an accident in the period $[0, n]$, where 0 is the current time. In case an individual is faced with an accident, there is a monetary loss D. The probability of having an accident for an individual of risk class i is denoted by η_i, $i \in \{H, L\}$, with $\eta_L < \eta_H < 1$. It is assumed that an accident can occur to each individual at most once.

- All risks are insurable.

- The population consists of N individuals, of which N_H and N_L belong to the category of high risks and low risks respectively. Hence $N = N_H + N_L$. The proportion of high risks among the entire population is denoted by ρ, so $N_H = \rho N$.

- The time at which any accident occurs is perfectly observable by both the individual concerned and the company.

- The probability for an individual of risk class i, $i \in \{H, L\}$, of facing an accident before time t $(0 \leq t \leq n)$ is denoted by $F_i(t)$ (hence $F_i(n) = \eta_i$), and it is assumed to be differentiable in $[0, n]$, with derivative $f_i(t) > 0$, $\forall t \in [0, n]$. All individuals fully know these probabilities. These probabilities are exogenous, so that the risk of moral hazard is non-existent.

- To each individual, the same utility function $U(\cdot)$ applies, which is assumed to be increasing, strictly concave, twice continuously differentiable and independent of time.

- Insurance companies are risk neutral profit maximizers and can offer any set of contracts which result in a nonnegative expected profit.

- There are no transaction costs involved in the supply of insurance and no administrative costs for the insurance business. Nor are there costs of obtaining classification information on a potential insured when it is possible to do so.

- Contracts are specified by (t, P), with t and P denoting the probationary period and the premium respectively. For the given contract, no indemnity is paid if an accident occurs in the period $[0, t]$, nor will the premium P, to be paid at time 0, be refunded to the insured. On the other hand, if an accident occurs during the period $(t, n]$, the insured will get a benefit equal to D (= loss).

- We denote the expected utility resulting from taking out the policy (t, P) by $E_i\,(t, P)$. This implies:

  $$E_i\,(t, P) = F_i\,(t)\, U\,(W - P - D) + (1 - F_i\,(t))\, U\,(W - P)\,; \qquad i \in \{H, L\}\,. \quad (1)$$

  The special case case of no insurance is denoted by E_i, so:

  $$E_i = E_i\,(n, 0) = \eta_i U\,(W - D) + (1 - \eta_i)\, U\,(W)\,, \qquad i \in \{H, L\}\,. \quad (2)$$

2.2 Relationship between distribution functions of time-of-loss

We have already defined $F_H(t)$ and $F_L(t)$, the c.d.f.'s of time of loss, in the previous subsection. We will now establish a relationship between those functions by assuming that the ratio between $F_H(t)$ and $F_L(t)$ is defined by the function $b(t)$, i.e.:

$$F_H(t) = b(t) F_L(t), \text{ with } b(t) > 1; 0 \leq t \leq n \tag{3}$$

where $b(t)$ is a real valued and differentiable function.

Remark 1 *Assumption (3) is less restrictive than (and hence implied by) the assumption made by Fluet (1992) that the high risk type has a higher hazard rate for each $t \in [0, n]$*

This strict inequality between $F_H(t)$ and $F_L(t)$ ensures that all contracts which will be purchased by the low risks are also acceptable for the high risks. Consider the contract (t, P). Then, for $b(t)$ monotone non-decreasing, we have:

$$\begin{aligned}
& E_H(t, P) - E_H \\
= \; & U(W-P) - U(W) \\
& + b(t) F_L(t) (U(W-P-D) - U(W-P)) \\
& - b(n) \eta_L (U(W-D) - U(W)) \\
> \; & U(W-P) - U(W) \\
& + F_L(t) (U(W-P-D) - U(W-P)) \\
& - \eta_L (U(W-D) - U(W)) \\
= \; & E_L(t, P) - E_L \\
\geq \; & 0.
\end{aligned} \tag{4}$$

Remark 2 *If we allow for the function $b(t)$ to be decreasing for some $t \in [0, n]$, there may be cases where inequality* (4) *does not hold.*

2.3 Details about the examples

Throughout the paper, we will use examples to illustrate the implications of our findings. These examples are all based on the same specifications concerning the utility functions of the individuals. We assume it to be within the CARA class; more specifically the exponential function

$$U(x) = -\alpha e^{-\alpha x}. \tag{5}$$

We use the following numbers for the numerical illustrations: $\alpha = 0.00001$, $D = 200,000$. Findings depend on α and D only through αD.

In the examples, we also assume that the difference between the two groups of consumers is constant in t, hence $b(t) = b$. Concerning the time of loss function $F_L(t)$ is is actually not necessary to specify any functional form apart from the obvious fact that $f_L(t) \geq 0 \; \forall t \in [0, n]$.

3 Nash behavior

In this section, we work with the assumption that insurance companies are myopic in the sense that they do not take potential withdrawals of competitors' contracts into account when offering policies. We will show that a separating equilibrium, if it exists, may have quite different properties from the equilibrium in the monetary deductible case. Finally, we analyze the existence of different types of equilibria with an exponential utility function.

3.1 Indifference curves and iso-profit curves

In this subsection we will show that, at any point of full coverage, an individual's marginal rate of substitution of premium for time deductible exceeds the firm's marginal profit. Recall that, in case of a monetary deductible, these two quantities are always equal to each other.

We define $\Gamma(t, P)$ as the expected profit resulting from offering a contract (t, P) to an individual. This gives

$$\Gamma(t, P) = P - (\eta - F(t)) D. \tag{6}$$

The marginal profit, in terms of the probationary period, is equal to

$$\frac{\partial \Gamma(t, P)}{dt} = f(t) D. \tag{7}$$

The individual's marginal rate of substitution of premium for probationary period (the slope of the indifference curve) is equal to

$$-\frac{dP}{dt} = f(t) \frac{U(W-P) - U(W-P-D)}{F(t) U'(W-P-D) + (1-F(t)) U'(W-P)}. \tag{8}$$

For $t = 0$ (full coverage), appealing to the strict concavity of $U(\cdot)$, this leads to

$$-\left(\frac{dP}{dt}\right)_{(t=0)} = f(0) \frac{U(W-P) - U(W-P-D)}{U'(W-P)} > f(0) D. \tag{9}$$

This inequality follows because the introduction of a positive probationary period introduces a wedge between the utility experienced during the probationary period (in the event of an accident) and the utility experienced afterwards. The actuarially fair premium moves smoothly around $t = 0$, however. This result suggests that the effectiveness of the probationary period as a screening device can be quite poor. We will use this result in the next sections.

3.2 The Nash Equilibrium

Rothschild and Stiglitz (1976) have shown that, if all firms in the insurance market are myopic and the proportion of high risks exceeds a certain level, a Cournot-Nash separating equilibrium exists. High risks buy full insurance while the policy for the low risks is subject to a monetary deductible.

In this section we will show that, with the probationary period as a screening device, a separating Cournot-Nash equilibrium may involve having low risks purchasing no insurance at all, no matter what the proportion of high risks is. Such an equilibrium would involve the contract $(0, \eta_H D)$ for the high risks (i.e. full coverage, at an actuarially fair premium), in combination with a certain contract $(t_L, (\eta_L - F_L(t_L)) D)$ (partial coverage at an actuarially fair premium) for the low risks.

The latter contract satisfies both the self-selection constraint for the high risks

$$U(W - \eta_H D) \geq E_H(t_L, (\eta_L - F_L(t_L)) D), \tag{10}$$

and the reservation constraint for the low risks

$$E_L(t_L, (\eta_L - F_L(t_L)) D) \geq E_L. \tag{11}$$

Taking the low type's marginal utility with respect to the probationary period, we get

$$\begin{aligned}
& d\frac{E_L(t_L, P_L(t_L))}{dt_L} \\
= \; & f_L(t_L) D \\
& \cdot \{F_L(t_L) U'(W - P_L(t_L) - D) + (1 - F_L(t_L)) U'(W - P_L(t_L)) \\
& - \left. \frac{U(W - P_L(t_L)) - U(W - P_L(t_L) - D)}{D} \right\}.
\end{aligned} \tag{12}$$

So that the individual trades off the increase in utility due to a reduction in the premium (the first term in equation 12) against the negative utility loss due to reduced coverage (the second term). Let $\beta(\cdot)$ be a real valued function, such that $0 \leq \beta(t_L) \leq D$ for any $t_L \in [0, n]$. Consequently:

$$\begin{aligned}
U'(W - (\eta_L - F_L(t_L)) D - D) & \leq U'(W - (\eta_L - F_L(t_L)) D - \beta(t_L)) \\
& \leq U'(W - (\eta_L - F_L(t_L)) D).
\end{aligned} \tag{13}$$

where we make use of the definition of the actuarially fair contract: $P_L(t_L) = (\eta_L - F_L(t_L)) D$. Then, according to the mean value theorem, equation (12) can be rewritten as:

$$\begin{aligned}
& d\frac{E_L(t_L, (\eta_L - F_L(t_L)) D)}{dt_L} \\
= \; & f_L(t_L) D \\
& \cdot \{F_L(t_L) U'(W - (\eta_L - F_L(t_L)) D - D) + (1 - F_L(t_L) U'(W - (\eta_L - F_L(t_L)) D)) \\
& - U'(W - (\eta_L - F_L(t_L)) D - \beta(t_L))\},
\end{aligned} \tag{14}$$

For $t_L = n$, equation (14) reduces to:

$$\begin{aligned}&\left(d\frac{E_L\left(t_L,\left(\eta_L-F_L\left(t_L\right)\right)D\right)}{dt_L}\right)_{(t_L=n)} \\ &= f_L\left(n\right)D\left(\eta_L U'\left(W-D\right)+\left(1-\eta_L\right)U'\left(W\right)-U'\left(W-\beta\left(n\right)\right)\right). \qquad (15)\end{aligned}$$

For large η_L, this derivative is positive, indicating that in such cases the individual prefers no insurance to coverage with a long probationary period. Hence, for t_L in a neighborhood of n, and a relatively smooth behavior of $F_L(t_L)$, the derivative in (14) will be positive as well. So there are at least some actuarially fair contracts which a low risk will not purchase. Now consider equation (10), recalling that $\eta_H = b(n)\eta_L$. The greater $b(n)$, the higher the premium for the high risks and the longer the probationary period for the low risks. Note that for $b(n) = \frac{1}{\eta_L}$ (the maximum value $b(n)$ can take), the high risk individual is indifferent between full insurance and no insurance, as shown in Eeckhoudt et al. (1988) and therefore there will be no coverage for the low risks. Hence, the following lemma.

Lemma 3 *For sufficiently high η_L and $b(n)$, the separating equilibrium will involve low risks purchasing no insurance at all, whereas high risks purchase complete coverage.*

This result is different from the monetary deductible case, where the low risks always get some, albeit incomplete, coverage. It should be noted that this result does not depend qualitatively on the sign of the derivative $b'(t)$. In general, for $b'(t) < 0$, the probationary period becomes more efficient as a screening device as the self-selection constraint (10) is less costly to satisfy when high risks have proportionately more mass concentrated in the beginning. When the shadow cost of the self-selection constraint decreases, the low risk type will be more likely to prefer the separating menu over no coverage at all. The conclusion in Lemma 3 still holds, however, since a high value of η_L implies that $b'(t)$ must be relatively small in absolute value. Furthermore, our conclusion will not be altered even if inequality (4) is reversed for some $t \in [0, n]$ when $b'(t) < 0$. The reason is that, unless a degenerate separating equilibrium occurs, the self-selection constraint of the high risk type will be binding in equilibrium. If this would not be the case, the low risk type would opt for full coverage ($t_L = 0$); i.e. a contract that will always be preferred by the high risk type, irrespective of the shape of the $b(t)$ function.

It remains to be shown that the separating equilibrium actually exists, however. Since the equilibrium we have sketched above is the most preferred separating equilibrium, we only need to ensure that there is no pooling contract that can attract both types of clients and make non-negative profits. Such a contract would need to be preferred over no insurance by the low risk type, and preferred to the actuarially fair full coverage contract by the high risk type. Just as in Rothschild & Stiglitz (1976), the profitability of such a pooling contract is decreasing in the share of high risk types (ρ) and hence the degenerate separating equilibrium we have sketched above only exists provided that the share of high risk types is sufficiently high.

3.2.1 Example: The exponential utility function

For illustrative purposes, we now use the specification of the utility function provided in subsection 2.3. It transpires, however, that even with this simple specification of the utility function, it is not possible to find analytical solutions to the problem that determine parameter values for which the different types of equilibria arise. What we can do, however, is, firstly to derive parameter values for which the equilibrium is unambiguously of one type or the other (with an ambiguous region in between) and, secondly, to use simulation techniques to arrive at a complete partitioning of the parameter space. All proofs concerning the analytical results will be provided in the appendix.

First, we define the regions for which analytical solutions are attainable. A useful finding in this endeavour is that no interior extreme point will be a maximum; hence, there are only two points to compare in order to determine which equilibrium arises: $t_L = n$ and $t_L = \widetilde{t}_L$, where $\widetilde{t}_L$ is defined according to the self selection constraint of the high risk type:

$$e^{\alpha\eta_H D} = e^{\alpha\left(\eta_L - F_L\left(\widetilde{t}_L\right)\right)D}\left[bF_L\left(\widetilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right] \tag{16}$$

It can be shown that $\widetilde{t}_L$ is unique. This property will also be important in the analysis of the Wilson case in section 4 below.

For the degenerate case, we look at regions of the parameters where the low risk's utility is increasing over all the values of t_L that are admissible according to the self selection constraint. In such a case, the equilibrium will clearly have to be degenerate (if it exists). For the standard Rothschild-Stiglitz type of equilibrium, we look for parameter values for which the marginal utility at $t_L = n$ is non-positive. Whenever this condition is fulfilled, it follows from the shape of the second derivative that the utility of the low risk type is decreasing for the entire interval $t_L \in \left(\widetilde{t}_L, n\right)$. Accordingly, we can establish the following lemma:

Lemma 4 *For $\eta_L \geq \eta_L^*$, the equilibrium strategy, if it exists, is to offer only the contract $(0, \eta_H D)$ (degenerate equilibrium) whereas for $\eta_L \leq \tilde{\eta}_L$ it is optimal to offer the contracts $(0, \eta_H D)$ and $\left(\widetilde{t}_L, \left(\eta_L - F_L\left(\widetilde{t}_L\right)\right)D\right)$ (standard equilibrium), where $\widetilde{t}_L$ is defined by equation (16) and the cutoff points of η_L are defined as follows:*

$$\eta_L^* = \frac{\left(\ln\left(\frac{\alpha D}{b\left(e^{\alpha D}-1\right)(b-1)\alpha D}\right)-1\right)\left(e^{\alpha D}-1\right)+\alpha D}{\left(e^{\alpha D}-1\right)(b-1)\alpha D} \tag{17}$$

$$\tilde{\eta}_L = \frac{1}{\alpha D} - \frac{1}{\left(e^{\alpha D}-1\right)} \tag{18}$$

Proof. See Appendix A. ■

Hence, this lemma leaves a certain region of parameter values ambiguous in terms of the equilibrium that prevails. For a complete partitioning of the parameter space by equilibrium type, we would need to have an explicit solution for $\tilde{t}_L$. This is not possible, since it contains Lambert's W function that does not have analytical solutions:

$$F_L\left(\tilde{t}_L\right) = -\frac{b\left(e^{\alpha D}-1\right)\text{LambertW}\left(-\frac{\alpha D}{b\left(e^{\alpha D}-1\right)}\exp\left(\alpha D\frac{b\eta_L\left(e^{\alpha D}-1\right)(b-1)}{b\left(e^{\alpha D}-1\right)}\right)\right)+\alpha D}{b\left(e^{\alpha D}-1\right)\alpha D} \tag{19}$$

where *LambertW* signifies Lambert's W function, i.e. the solution to $z = W(z)\, e^{W(z)}$, defined for $z \in \left(-\frac{1}{e}, \infty\right)$. By means of the algorithm provided by Corless et al (1996), we can simulate the value of $F_L\left(\tilde{t}_L\right)$ for a range of parameter values. Some examples are provided in *Figure 1*.

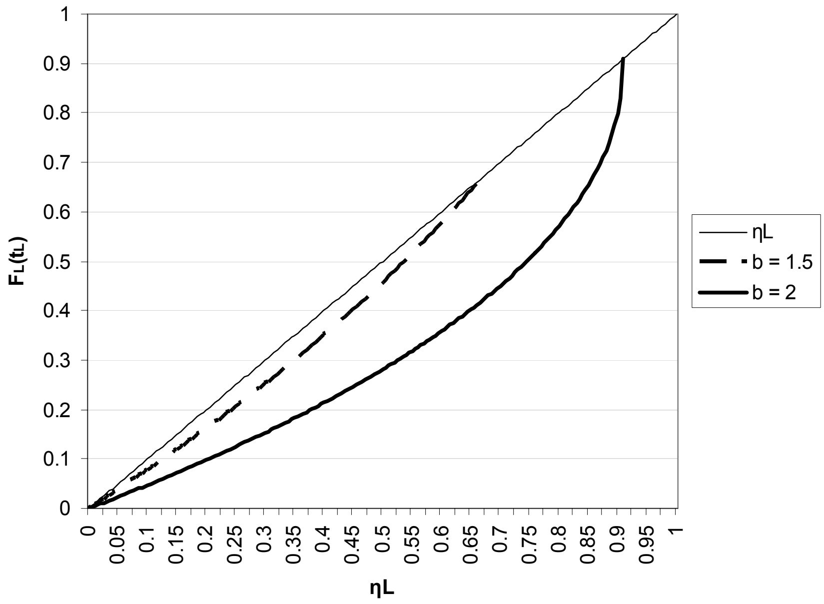

Figure 1: Critical values for the probationary period.

The full curve shows values of $F_L\left(\tilde{t}_L\right)$ when b, the risk markup of the high risk type equals 2. Quite intuitively, a higher value of b allows for lower values of $\tilde{t}_L$ (mimicking the low risk type is less appealing), which is also reflected in the figure.

Finally, we use the simulated values from *Figure 1* in order to partition the parameter space according to the different equilibria. First, however, we establish the conditions for existence:

Lemma 5 *A separating Nash equilibrium only exists if*

$$\rho > \frac{\ln\left(\eta_L\left(e^{\alpha D}-1\right)+1\right)-\eta_L\alpha D}{\eta_L\alpha D\,(b-1)} \tag{20}$$

and a degenerate equilibrium only exists is

$$\rho > \frac{\ln\left(F_L\left(\widetilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)-F_L\left(\widetilde{t}_L\right)\alpha D}{\eta_L \alpha D\left(b-1\right)} \tag{21}$$

Proof. See appendix B. ∎

The simulation results are presented in *Figure 2*. Just as we established in *Lemma 3*, the Nash equilibrium involves low risk types not purchasing any insurance at all for high values of η_L and b.

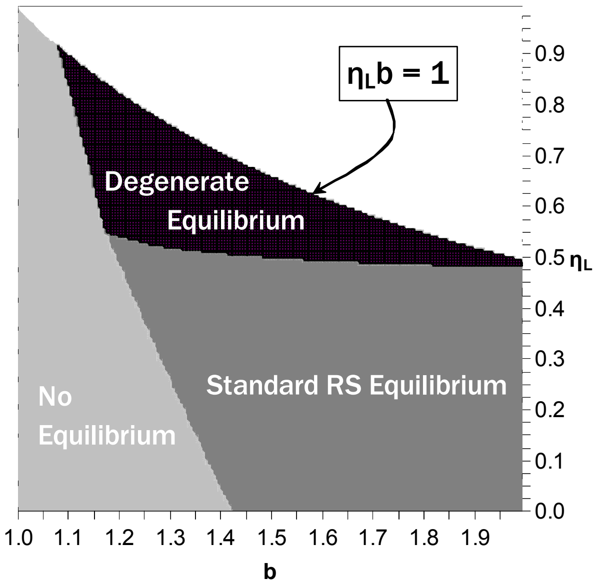

Figure 2: Equilibrium Types, Nash Case. Rho = 0.25

4 Wilson foresight

If firms behave with Wilson foresight, an equilibrium, based on maximal welfare for the low risks always exists. In this section, we show that such an equilibrium may be of a pooling type. We also provide some examples for the exponential utility function.

4.1 Optimal pooling contract

Miyazaki (1977) and Spence (1978) have shown that, for the monetary deductible as a screening device the equilibrium is always of a separating type. Firstly, from the insurer's point of view, a pooling strategy without full coverage will always be inferior to offering a pair of different contracts, with the original pooling contract designed for the low risks and full coverage for the high risks.

Secondly, a pooling strategy can never involve complete coverage. In case of pooling, the low risks would pay a loaded premium for constant b. As discussed for example in Arrow (1963) and Pashigian et al. (1966), full insurance cannot be optimal if a premium is loaded. This point is also stressed in Eeckhoudt et al. (1988).

In this subsection, we will concentrate on the latter conclusion. We show that, with the probationary period as screening device, offering full coverage could provide the low risks with optimal welfare if the firms' choice were restricted to selling pooling contracts.

Assume that the insurer can only offer a pooling contract (t, P). Let's denote the objective function by $\widehat{V}_L(t, P)$, which is defined as the expected utility for a low risk type agent resulting from offering such a contract. Then we have

$$\widehat{V}_L(t, P) = F_L(t)\, U(W - P - D) + (1 - F_L(t))\, U(W - P). \tag{22}$$

Such contracts satisfy the binding non-profit constraint:

$$P = (\rho\, (b(0)\, \eta_L - b(t)\, F_L(t)) + (1-\rho)(\eta_L - F_L(t)))\, D. \tag{23}$$

So we can express P as a function of t. We will use the notation $P(t)$ and, consequently, $\widehat{V}_L(t)$ to denote $\widehat{V}_L(t, P)$ in (22) expressed as a function of t only:

$$\widehat{V}_L(t) = F_L(t)\, U(W - P(t) - D) + (1 - F_L(t))\, U(W - P(t)). \tag{24}$$

We analyze the function (24) by taking its derivative with respect to t. This returns:

$$\begin{aligned}
& d\frac{\widehat{V}_L(t)}{dt} = -f_L(t)\,(U(W - P(t)) - U(W - P(t) - D)) \\
& -\frac{dP(t)}{dt}\,(F_L(t)\, U'(W - P(t) - D) + (1 - F_L(t))\, U'(W - P(t))) \\
= \;& f_L(t) \left\{ \left(\rho \left(b(t) - 1 + \frac{b'(t)\, F_L(t)}{f_L(t)} \right) + 1 \right) D \begin{pmatrix} F_L(t)\, U'(W - P(t) - D) \\ + (1 - F_L(t))\, U'(W - P(t)) \end{pmatrix} \right. \\
& \left. - (U(W - P(t)) - U(W - P(t) - D)) \right\}.
\end{aligned} \tag{25}$$

In accordance with the mean value theorem:

$$\frac{U(W - P(t)) - U(W - P(t) - D)}{D} = U'(W - P(t) - \beta(t)), \tag{26}$$

for some $\beta(t) \in [0, D]$, $\quad t \in [0, n]$. Note that, compared to the previous subsection, P is a function of t, so β is a function of t as well. Consequently:

$$U'(W - P(t) - D) \le U'(W - P(t) - \beta(t)) \le U'(W - P(t)). \tag{27}$$

So we can rewrite (25) as

$$
\begin{aligned}
& d\frac{\widehat{V}_L(t)}{dt} \\
= \; & f_L(t)\, D \\
& \left\{ \left(\rho \left(b(t) - 1 + \frac{b'(t)\, F_L(t)}{f_L(t)} \right) + 1 \right) \left(\begin{array}{c} F_L(t)\, U'(W - P(t) - D) \\ + (1 - F_L(t))\, U'(W - P(t)) \end{array} \right) \right. \\
& \left. - U'(W - P(t) - \beta(t)) \right\}. \qquad (28)
\end{aligned}
$$

For $t = 0$, expression (28) reduces to

$$
\left(d\frac{\widehat{V}(t)}{dt} \right)_{(t=0)} = f_L(0)\, D \left\{ (\rho(b(0) - 1) + 1)\, U'(W - P(0)) - U'(W - P(0) - \beta(0)) \right\}, \qquad (29)
$$

which is negative for $\rho = 0$ or $b(0) = 1$. By continuity, it is therefore negative at $t = 0$ for ρ and $b(t)$ in a neighborhood of 0 and 1, respectively. This implies that, for such values of ρ or $b(0)$, the pooling contract with complete coverage is at least not the worst among all the pooling contracts. Recall that, for the monetary deductible as a screening device, the above expression is always positive, so complete coverage can never be optimal.
Furthermore, note that if η_L is small, then $F_L(t)$ is small for each $t \in [0, n]$ and hence, again by continuity, we have for each $t \in [0, n]$:

$$
U'(W - P(t) - \beta(t)) \geq F_L(t)\, U'(W - P(t) - D) + (1 - F_L(t))\, U'(W - P(t)). \qquad (30)
$$

For $b'(t) < 0$, this inequality implies that $d\frac{\widehat{V}(t)}{dt}$ is negative everywhere for ρ and $b(t)$ in a neighborhood of 0 and 1, respectively. Even if $b'(t) > 0$, the term $\frac{b'(t)F_L(t)}{f_L(t)}$ in equation (29) is small whenever η_L is small. It follows that the best pooling strategy may be indeed to provide full coverage.

Of course, the insurer's choice is not restricted to pooling contracts; the general case where the insurer can offer any pair of policies, is dealt with in the next section.

4.2 The equilibrium

Obviously, the low risks' opportunities for a higher welfare are enhanced if, unlike the previous subsection, its choice is not restricted to pooling contracts.

Note that, just as in Miyazaki (1977) and Spence (1978),

- any contract acceptable for the low risk is also acceptable for the high risks (as follows from equation (3)), and
- complete coverage is optimal in case of symmetric information (as shown in Eeckhoudt et al. (1988)).

This implies that, again just as in Stiglitz, (1977), any strategy of offering a pooling contract with incomplete coverage to both risk types is inferior to offering the original pooling contract to the low risks and full coverage to the high risks.

Definition 6 *The Wilson Equilibrium is a set of contracts* $(0, P_H)$ *and* (t_L, P_L) *that maximizes expected welfare of the low risk type* $E_L(t_L, P_L)$, *subject to the following constraints:*

$$E_H(0, P_H) \geq E_H(t_L, P_L) \tag{31}$$

$$\rho(P_H - \eta_H D) + (1-\rho)(P_L - (\eta_L - F_L(t_L)) D) = 0 \tag{32}$$

$$P_H - \eta_H D \leq 0 \tag{33}$$

$$t_L \in [0, n] \tag{34}$$

It should be noted that the above constraints allow for a pooling equilibrium, since if we impose $t_L = 0$ and $P_H = P_L$, constraint (31) above becomes an identity. Considering the three first constraints, two of them are relatively straightforward since they require self-selection by the high risk type (31) and non-negative profits (32), respectively. The third constraint (33) states that the contract offered to the high risk type in equilibrium may not earn positive profits. This constraint is required since otherwise a competitor could offer a contract that generates slightly lower profits, that attracts the high risk types, and that remains profitable once the original set of contracts has been withdrawn. It follows from a similar argument that the overall profit constraint has to be binding with equality.

In what follows, we will denote a set of contracts *feasible* if it satisfies the three constraints in *Definition 6*.

The Lagrangian to be optimized has the following shape:

$$\begin{aligned}\mathcal{L} &= E_L(t_L, P_L) + \lambda_1 \{E_H(0, P_H) - E_H(t_L, P_L)\} \\ &\quad + \lambda_2 \{\rho(P_H - b(n)\eta_L D) + (1-\rho)(P_L - (\eta_L - F_L(t_L)) D)\} + \lambda_3 \{b(n)\eta_L D - P_H\} \\ &\quad - \lambda_4 (t_L - n) - \lambda_5 t_L\end{aligned} \tag{35}$$

In (35), the Lagrange multiplier λ_1 applies to the self-selection constraint for the high risks, the multiplier λ_2 corresponds to the nonnegative profit constraint for the firm and the multiplier λ_3 corresponds to the cross-subsidization constraint. Finally, the multipliers λ_4 and λ_5 refer to the condition that $t_L \in [0, n]$.

Next, we analyze the properties of potential Wilson equilibria in two different steps. First, we carry out local analysis in the neighborhood of $t_L = 0$ in order to find out under what circumstances the pooling contract is locally preferred to the separating contracts. Secondly, we look at more general properties of the potential Wilson equilibria.

4.2.1 Local Analysis

Notice that the pooling contract outlined above satisfies all of the constraints, and has constraint (33) satisfied with inequality as long as $\rho < 1$. Hence we have $\lambda_3 = 0$ in a neighborhood of $t_L = 0$. Logically, we have $\lambda_4 = 0$ and $\lambda_5 = 0$ as well. Then the Kuhn-Tucker conditions may be simplified as

$$\frac{\partial \mathcal{L}}{\partial P_H} = \lambda_2 \rho - \lambda_1 U'(W - P_H) \tag{36}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial P_L} &= -F_L(t_L) U'(W - P_L - D) - (1 - F_L(t_L)) U'(W - P_L) \\ &\quad +\lambda_1 (b(t_L) F_L(t_L) U'(W - P_L - D) + (1 - b(t_L) F_L(t_L)) U'(W - P_L)) \\ &\quad +\lambda_2 (1 - \rho) \end{aligned} \tag{37}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t_L} &= f_L(t_L) (U(W - P_L - D) - U(W - P_L)) \\ &\quad -\lambda_1 (b(t_L) f_L(t_L) + b'(t_L) F_L(t_L)) (U(W - P_L - D) - U(W - P_L)) \\ &\quad +\lambda_2 (1 - \rho) f_L(t_L) D \end{aligned} \tag{38}$$

Using the conditions (36) and (37) we can solve for the Lagrange multipliers:

$$\lambda_1 = \frac{\rho (F_L(t_L) U'(W - P_L - D) + (1 - F_L(t_L)) U'(W - P_L))}{g(t_L)} \tag{39}$$

$$\lambda_2 = \frac{(F_L(t_L) U'(W - P_L - D) + (1 - F_L(t_L)) U'(W - P_L)) U'(W - P_H)}{g(t_L)} \tag{40}$$

where

$$\begin{aligned} g(t_L) &= U'(W - P_L)(1 - \rho) + \rho F_L(t_L) b(t_L) U'(W - P_L - D) \\ &\quad +\rho (1 - b(t_L) F_L(t_L)) U'(W - P_L). \end{aligned} \tag{41}$$

Using the mean value theorem, condition (38) simplifies to:

$$\frac{\partial \mathcal{L}}{\partial t_L} = f_L(t_L) D \left[\left(\lambda_1 \left(b(t_L) + \frac{b'(t_L) F_L(t_L)}{f_L(t_L)} \right) - 1 \right) U'(W - P_L - \beta(t_L)) + \lambda_2 (1 - \rho) \right] \tag{42}$$

And if we insert the multipliers, we get

$$\left.\frac{\partial \mathcal{L}}{\partial t_L}\right|_{t_L=0} = \frac{f_L(0)\, DU'(W-P_L)\begin{bmatrix} (1-\rho)\left(U'(W-P_L)-U'(W-P_L-\beta)\right) \\ +\rho\,(b(0)-1)\,U'(W-P_L-\beta) \end{bmatrix}}{g(0)} \tag{43}$$

The first term in the bracket is clearly negative, whereas the other one is positive but has an upper bound. The entire expression is increasing in ρ and in $b(0)$. Hence, for low values of ρ and $b(0)$, the expression will be negative, implying that the pooling contract is preferred to some feasible separating contracts in the neighborhood of $t_L = 0$.

Lemma 7 *For low values of ρ and $b(0)$, the pooling contract provides the low risk type with higher expected welfare than at least some of the feasible separating menus.*

4.2.2 Global Analysis

In order to draw more general conclusions concerning the properties of potential equilibria, it is useful to assume constraints binding and then inserting them into the Lagrangian. In this subsection, we start out assuming that constraint (31) is binding, as it is in a neighborhood of $t_L = 0$ onwards. Then we analyze the alternative setting where constraint (33) is binding, as it might eventually. Our aim is to analyze for what parameter values the potential equilibria in the two settings have similar properties, so that general conclusions may be drawn.

If initially we assume that constraint (31) is binding in equilibrium, we may express the objective function in terms of only one of the variables t_L, P_L or P_H just by substituting (31) and (32) into (35). In the same way as in the previous section, we will express it as a function of t_L only. Likewise, P_L is a function of t_L, so we will use the notation $P_L(t_L)$. Denote the low risks' welfare function by $\widehat{V}_L(t_L)$. Then, substitution leads to:

$$\widehat{V}_L(t_L) = E_L\left(P_L(t_L), t_L\right), \tag{44}$$

with the relationship between P_L and t_L shown as:

$$\begin{aligned} & U\left(W - b(t_L)\,\eta_L D + \frac{(1-\rho)}{\rho}\left(P_L - (\eta_L - F_L(t_L))\, D\right)\right) \\ = \;& b(t_L)\, F_L(t_L)\, U(W-P_L-D) + (1 - b(t_L)\, F_L(t_L))\, U(W-P_L). \end{aligned} \tag{45}$$

This leads to:

$$\begin{aligned} & \frac{d\widehat{V}_L(t_L)}{dt_L} \\ = \;& \left(-\frac{dP_L(t_L)}{dt_L}\right)\left(F_L(t_L)\, U'(W-P_L(t_L)-D) + (1-F_L(t_L))\, U'(W-P_L(t_L))\right) \\ & - f_L(t_L)\left(U(W-P_L(t_L)) - U(W-P_L(t_L)-D)\right), \end{aligned} \tag{46}$$

with

$$\left(-\frac{dP_L\left(t_L\right)}{dt_L}\right)=\frac{h\left(t_L\right)}{k\left(t_L\right)}. \tag{47}$$

In (47),

$$\begin{aligned} h\left(t_L\right) &= \left(\left(1-\rho\right)f_L\left(t_L\right)D-\rho b'\left(t_L\right)\right)\cdot U'\left(W-b\left(t_L\right)\eta_L D+\frac{\left(1-\rho\right)}{\rho}\left(P_L-\left(\eta_L-F_L\left(t_L\right)\right)D\right)\right) \\ &\quad +\rho\left(b\left(t_L\right)f_L\left(t_L\right)+F_L\left(t_L\right)b'\left(t_L\right)\right)\left(U\left(W-P_L\left(t_L\right)\right)-U\left(W-P_L\left(t_L\right)-D\right)\right), \end{aligned} \tag{48}$$

and

$$\begin{aligned} k\left(t_L\right) &= \left(1-\rho\right)U'\left(W-b\left(t_L\right)\eta_L D+\frac{\left(1-\rho\right)}{\rho}\left(P_L-\left(\eta_L-F_L\left(t_L\right)\right)D\right)\right) \\ &\quad +\rho\left(\begin{array}{c} b\left(t_L\right)F_L\left(t_L\right)U'\left(W-P_L\left(t_L\right)-D\right) \\ +\left(1-b\left(t_L\right)F_L\left(t_L\right)\right)U'\left(W-P_L\left(t_L\right)\right) \end{array}\right). \end{aligned} \tag{49}$$

By defining

$$g\left(t_L\right)=F_L\left(t_L\right)U'\left(W-P_L\left(t_L\right)-D\right)+\left(1-F_L\left(t_L\right)\right)U'\left(W-P_L\left(t_L\right)\right) \tag{50}$$

equation (46) reduces, after some rewriting to:

$$\begin{aligned} \frac{d\widehat{V}_L\left(t_L\right)}{dt_L} &= \frac{f_L\left(t_L\right)D}{k\left(t_L\right)} \\ &\cdot\left\{U'\left(W-b\left(t_L\right)\eta_L D+\frac{\left(1-\rho\right)}{\rho}\left(P_L-\left(\eta_L-F_L\left(t_L\right)\right)D\right)\right)\right. \\ &\cdot\left(\left(1-\rho\left(1+\frac{b'\left(t_L\right)}{f_L\left(t_L\right)}\right)\right)g\left(t_L\right)-\left(1-\rho\right)U'\left(W-P_L\left(t_L\right)-\beta\left(t_L\right)\right)\right) \\ &\left.+\rho\left(\left(b\left(t_L\right)-1\right)U'\left(W-P_L\right)U'\left(W-P_L\left(t_L\right)-\beta\left(t_L\right)\right)+b'\left(t_L\right)F_L\left(t_L\right)g\left(t_L\right)\right)\right\}, \end{aligned} \tag{51}$$

for some $\beta\left(t_L\right)\in\left[0,D\right]$. Now consider the expression between the curly brackets of (51), which apparently determines the sign of the derivative. The first term in the curly brackets reflects the trade-off between the utility that the low risk type looses due to an extension of the probationary period, and the potential gain from a reduction in premium rates that such an extension may cause. For a small value of η_L, the value of $F_L\left(t_L\right)$ will be small as well. It follows that the 'premium reduction' effect will be relatively small compared with the 'reduced coverage' effect (i.e. the indifference curves of the low risk type are relatively steep). Hence, if $b'\left(t_L\right)\geqslant 0$ and η_L small, the first term in the curly brackets will be negative for all $t_L\in\left[0,n\right]$.

The second and the third terms, on the other hand, represent the shadow cost of the high risk type's self-selection constraint. If η_L is low, the third term becomes insignificant (since both $F_L(t_L)$ and $g(t_L)$ are relatively small in this case). The second term, on the other hand, is positive but has an upper bound. For low values of ρ, however, this term becomes relatively insignificant as well. It follows that as long as $b'(t_L) \geqslant 0$, sufficiently small values of ρ and η_L will imply that derivative (51) is negative for all $t_L \in [0, n]$ and hence the pooling contract may be a Wilson equilibrium.

To establish this result, however, we need to consider what happens if constraint (33) is binding instead. Such an equilibrium may come about and, as noticed by Harris and Townsend (1981), the set of equilibria that arise coincide with the corresponding Nash equilibria. Hence, from Section 3 we know that the derivative of the low risk type's utility with respect to the probationary period is:

$$\begin{aligned}
& d\frac{E_L(t_L, (\eta_L - F_L(t_L)) D)}{dt_L} \\
= \; & f_L(t_L) D \\
& \cdot \{F_L(t_L) U'(W - (\eta_L - F_L(t_L)) D - D) + (1 - F_L(t_L) U'(W - (\eta_L - F_L(t_L)) D)) \\
& - U'(W - (\eta_L - F_L(t_L)) D - \beta(t_L))\}, \qquad (52)
\end{aligned}$$

Comparing equation (51) with equation (52), we notice that both are negative for sufficiently small values of ρ and η_L. Hence, we may state the following lemma.

Lemma 8 *For $b'(t) \geq 0$, ρ and η_L sufficiently small, the pooling contract may provide the low risks with maximal welfare.*

Now consider the case where $b'(t) < 0$. In this case, the sign of equation (51) is more ambiguous, and it is not possible to extend the conclusions of Lemma 8 to this case. For certain parameter values, a pooling equilibrium may still be possible. We can make the same observation as in Section 3, however, i.e. that when $b'(t) < 0$, the probationary period is a more efficient screening device, and hence a separating equilibrium is more likely.

4.2.3 An example: the exponential utility function

Again, we use the exponential utility function outlined in subsection 2.3. For the Wilson case, there are actually four different types of equilibria that can arise, depending on the parameter values. Hence, the equilibrium may involve any of the following:

- The *pooling* contract $(0, \rho b \eta_L D + (1 - \rho) \eta_L D)$
- *Separating* contracts *without* cross-subsidization: $(0, b\eta_L D)$, $\left(\tilde{t}_L, \left(\eta_L - F_L\left(\tilde{t}_L\right)\right) D\right)$ (with $\tilde{t}_L$ defined by equation (16) above).
- *Separating* contracts *with* cross-subsidization: $(0, P_H^*)$, (t_L^*, P_L^*)

- *Degenerate* equilibrium: contract $(0, b\eta_L D)$ only

Among the possible equilibria listed above, three correspond to corner solutions: consistently with the argument in subsection 3.2.1 above, a separating menu without cross-subsidization cannot be an interior solution since the second derivative of the low risk type's utility function is positive at any extreme point. Hence, only the separating contract with cross-subsidization is based on an interior solution for t_L^*. The contracts offered in that case are defined by

$$t_L^* = F_L^{-1}\left(\frac{\begin{array}{c}(1-\rho)\left(b\left(e^{\alpha D}-1\right)-(b+1)\left(e^{\alpha D}-1\right)\right)- \\ \sqrt{(1-\rho)\left[(b-1)\alpha D-b\left(e^{\alpha D}-1\right)\right]^2-\rho\left[(b-1)\alpha D+b\left(e^{\alpha D}-1\right)\right]^2}\end{array}}{2\left(e^{\alpha D}-1\right)(1-\rho)\,b\alpha D}\right) \tag{53}$$

$$P_L^* = \rho\left(b\eta_L D + \frac{1-\rho}{\rho}\left(\eta_L - F_L\left(t_L^*\right)\right)D - \frac{1}{\alpha}\ln\left(bF_L\left(t_L^*\right)\left(e^{\alpha D}-1\right)+1\right)\right) \tag{54}$$

$$P_H^* = b\eta_L D - \frac{1-\rho}{\rho}\left(P_L^* - \left(\eta_L - F_L\left(t_L^*\right)\right)D\right) \tag{55}$$

The equilibrium *with* cross-subsidization is only possible in case b, the difference between the two risk groups, is large enough - hence implying that it is beneficial for the low risk group to subsidize the high risk group in order to reduce the probationary period t_L. This threshold, b^*, equals:

$$b^* = \frac{\alpha D}{e^{\alpha D}-1-\alpha D} \tag{56}$$

which depends on α, the absolute risk aversion parameter, and D, the size of the loss, only.

In general, we have that for high values of ρ, the separating equilibrium without cross-subsidization will prevail. This is down to the fact that when the proportion of high risks in the population is particularly high, efficiency would require redistribution from the high risk to the low risk. This possibility is ruled out by constraint (33), however. The pooling equilibrium, on the other hand, arises for low values of ρ. This is the opposite situation where the high risks represent such a low proportion of the population that the welfare loss due to subsidization is very small compared to the gains from having no probationary period at all. In between these two scenarios, there may be separating contracts with cross-subsidization.

The exact conditions for the various equilibria are provided in the Appendix. Here, we will simply make use of the algorithm introduced in subsection 3.2.1 in order to derive some pictorial representations of the different equilibria. In *Figure 3*[2] we partition the parameter space by equilibrium type when $\rho = 0.25$. Just as we established in Lemma 8, the pooling equilibrium is more likely the lower is the value of η_L.

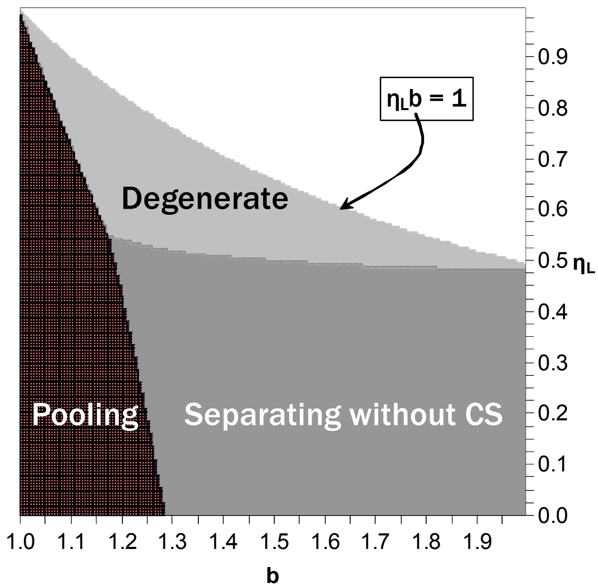

Figure 3: Equilibrium Types, Wilson Case. Rho equals 0.25

Comparing *Figure 3* with *Figure 4*, that is based on $\rho = 0.75$, we can also confirm that the area where pooling equilibria arise is shrinking in ρ.

5 Conclusions and final remarks

In this paper, we have investigated the effectiveness of a probationary period as a screening device in an insurance market with adverse selection problems. In general, we find evidence that the probationary period is a relatively poor instrument, and this finding seems to be robust to varying assumptions on the extent of forward-looking behaviour on the part of the insurance companies.

Accordingly, we find that the Cournot-Nash separating equilibrium, if it exists, may entail no insurance coverage at all for the low risks. This outcome is quite different from the equilibrium derived in Rothschild and Stiglitz (1976), where low risks always get some degree of coverage. This degenerate equilibrium comes about if the low risks' probability of incurring an accident is large, and the difference between the low risks' and the high

[2]The region denoted 'Separating without CS' shows the parameter values for which a separating equilibrium without cross-subsidisation occurs.

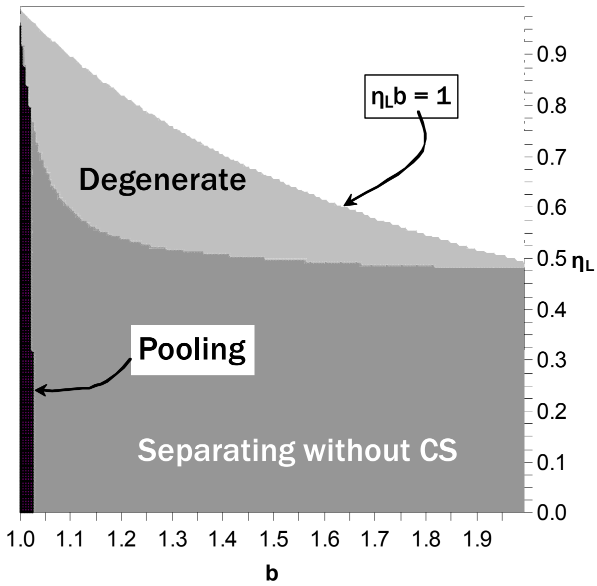

Figure 4: Equilibrium Types, Wilson Case. Rho equals 0.75

risks' probability is large as well. The general conditions for existence of an equilibrium are quite similar to those found by Rothschild and Stiglitz (1976), i.e. that the proportion of high risks in the population is sufficiently high.

Moreover we find that, in contradiction to the findings in Miyazaki (1977) and Spence (1978), a pooling contract with complete coverage may provide the low risks more welfare than any other pooling contract. Offering such a contract may even be more beneficial to this class of risk than offering any set of separating policies. A strategy involving a pooling full coverage contract can be Pareto-efficient if the low risk's probability of incurring an accident, the differences between the low risks' and the high risks' probabilities, as well as the proportion of high risks within the entire population are all relatively low. This is a remarkable finding as the assumptions of the model imply that the single-crossing property in the (t, P) space is always satisfied.

In fact, our conclusions show consistency with the findings in Fluet (1992). Consider a contract with a probationary period only. Then the monetary deductible is equal to the loss during that period and zero thereafter. Apparently, in many cases this maximal difference between the deductibles does not work out very well for the individual. Fluet has shown that any time-dependent monetary deductible for the low risks is always strictly positive. It is interesting to note that, in case $F_H(t) = bF_L(t)$ (as considered in the examples this paper), Fluet establishes that the monetary deductible for the low risks is constant over time. This implies that the separating menu does not involve a probationary period. Fluet only considers the case of Nash behavior in detail. However, he rightly points out in his concluding remarks that the low risks will purchase the same type of contract if firms behave with Wilson foresight.

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A Proof of Lemma 4

First we establish that no interior contracts, with $t_L \in \left(\tilde{t}_L, n\right)$ can be offered in equilibrium. First consider the utility of the low risk type from a certain actuarially fair contract:

$$E_L\left(t_L, \left(\eta_L - F_L\left(t_L\right)\right) D\right) = -\alpha e^{-\alpha(W-(\eta_L - F_L(t_L))D)} \left[\left(e^{\alpha D}-1\right) F_L\left(t_L\right)+1\right] \tag{57}$$

Taking the derivative with respect to t_L, we get:

$$\frac{\partial E_L\left(t_L, \left(\eta_L - F_L\left(t_L\right)\right) D\right)}{\partial t_L} = \alpha f_L\left(t_L\right) e^{-\alpha(W-(\eta_L - F_L(t_L))D)} \left(\left(\alpha D F_L\left(t_L\right)-1\right)\left(e^{\alpha D}-1\right)+\alpha D\right) \tag{58}$$

And the second derivative equals:

$$\begin{aligned}\frac{\partial^2 E_L\left(t_L, \left(\eta_L - F_L\left(t_L\right)\right) D\right)}{\partial\left(t_L\right)^2} &= \frac{f_L'\left(t_L\right)}{f_L\left(t_L\right)}\frac{\partial E_L}{\partial t_L} + \left(\alpha f_L\left(t_L\right)\right)^2 D e^{-\alpha(W-(\eta_L - F_L(t_L))D)} \cdot \\ &\quad \left(\left(2-\alpha D F_L\left(t_L\right)\right)\left(e^{\alpha D}-1\right)-\alpha D\right)\end{aligned} \tag{59}$$

from which follows that $\frac{\partial E_L(t_L,(\eta_L - F_L(t_L))D)}{\partial t_L} \Rightarrow \frac{\partial^2 E_L(t_L,(\eta_L - F_L(t_L))D)}{\partial (t_L)^2} > 0$; hence, no interior extreme point can be a maximum. Accordingly, we need to compare the low risk type's utility from contract $\left(\tilde{t}_L, \left(\eta_L - F_L\left(\tilde{t}_L\right)\right) D\right)$ with the utility of no insurance at all, E_L.

A condition for the former to be preferred is

$$e^{\alpha\left(\eta_L - F_L\left(\tilde{t}_L\right)\right)D}\left[\left(e^{\alpha D}-1\right) F_L\left(\tilde{t}_L\right)+1\right] < \left(e^{\alpha D}-1\right)\eta_L + 1 \tag{60}$$

For

$$\eta_L \leq \frac{1}{\alpha D} - \frac{1}{e^{\alpha D}-1} \tag{61}$$

the LHS of (60) is increasing in $\tilde{t}_L$ and reaches a maximum at $\tilde{t}_L = n$. Hence, for $\eta_L < \tilde{\eta}_L$, the inequality (60) holds for sure and the resulting equilibrium will be the standard separating type.

Next, define $\widehat{t}_L$ by $\left.\frac{\partial E_L(t_L,(\eta_L - F_L(t_L))D)}{\partial t_L}\right|_{t_L=\widehat{t}_L} = 0$. It follows that if $\widehat{t}_L < \tilde{t}_L$, $\left.\frac{\partial E_L(t_L,(\eta_L - F_L(t_L))D)}{\partial t_L}\right|_{t_L=\tilde{t}_L} > 0$, and hence the only admissible equilibrium (since interior maxima do not exist) is the degenerate one where $t_L = n$. A sufficient condition for $\widehat{t}_L < \tilde{t}_L$ is that

$$\eta_L \geq \frac{\left(\ln\left(\frac{\alpha D}{b(e^{\alpha D}-1-\alpha D)+\alpha D}\right) - 1\right)\left(e^{\alpha D} - 1\right) + \alpha D}{\left(e^{\alpha D} - 1\right)(b-1)\,\alpha D} \tag{62}$$

which completes the proof.■

B Proof of Lemma 5

The actuarially fair pooling contract will have premium rate

$$P = (1 - \rho(1-b))(\eta_L - F_L(t))\,D \tag{63}$$

Existence of the degenerate equilibrium then requires

$$E_H(0, b\eta_L D) \geq E_H(t, P) \tag{64}$$

$$E_L(n, 0) \geq E_L(t, P) \tag{65}$$

Notably, we have established already that $E_L(n,0) \geq E_L(t, (\eta_L - F_L(t_L))\,D)$ for all $t_L \in \left[\widetilde{t}_L, n\right]$. Since $P > (\eta_L - F_L(t_L))\,D$ it follows that inequality (65) holds with inequality for $t \in \left[\widetilde{t}_L, n\right]$. Hence, any pooling contract able to destabilize the equilibrium will have $t \in \left[0, \widetilde{t}_L\right)$. Now consider the derivative of the two risk types' expected utility from the pooling contract with respect to t.

$$E_L(t, P) = -\left(F_L(t)\left(e^{\alpha D} - 1\right) + 1\right)\alpha e^{-\alpha(W-(1-\rho(1-b))(\eta_L - F_L(t))D)} \tag{66}$$

$$\frac{\partial E_L(t,P)}{\partial t} = f_L(t)\,\alpha e^{-\alpha(W-(1-\rho(1-b))(\eta_L - F_L(t))D)}\begin{bmatrix} (1-\rho(1-b))\,\alpha D\left(F_L(t)\left(e^{\alpha D}-1\right)+1\right) \\ -\left(e^{\alpha D}-1\right)\end{bmatrix} \tag{67}$$

The sign of this expression depends on the sign of the expression in square brackets. Denote this expression $s(t,\rho)$. Hence,

$$s(t,\rho) = (1-\rho(1-b))\,\alpha D\left(F_L(t)\left(e^{\alpha D}-1\right)+1\right) - \left(e^{\alpha D}-1\right) \tag{68}$$

Taking the derivative of this expression with respect to t, we get:

$$\frac{\partial s(t,\rho)}{\partial t} = (1-\rho(1-b))\,\alpha D f_L(t)\left(e^{\alpha D}-1\right) > 0 \tag{69}$$

Hence, the expression determining the sign of the marginal utility is increasing everywhere. Accordingly, there are three possibilities: either $\frac{\partial E_L(t,P)}{\partial t} > 0$ for all t, or it is negative for all t, or it attains an extreme point in that interval. For the third case, consider the second derivative of the expected utility with respect to t at that extreme point:

$$\left.\frac{\partial^2 E_L(t,P)}{\partial t^2}\right|_{t=t^*} = (f_L(t))^2\,\alpha^2(1-\rho(1-b))\,De^{-\alpha(W-(1-\rho(1-b))(\eta_L-F_L(t))D)}\cdot \tag{70}$$

$$\left[2\left(e^{\alpha D}-1\right)-(1-\rho(1-b))\,\alpha D\left(F_L(t)\left(e^{\alpha D}-1\right)+1\right)\right] \tag{71}$$

This expression is positive at the extreme point; hence, the interior point will be a local minimum. The only candidate that remains, then, is the pooling equilibrium with $t=0$. Inserting the premium rate

$$P = (1-\rho(1-b))\,\eta_L D \tag{72}$$

in the expected utility function of the low risk type, we get

$$E_L(0,(1-\rho(1-b))\,\eta_L D) = -\alpha e^{-\alpha(W-(1-\rho(1-b))\eta_L D)} \tag{73}$$

which is to be compared with

$$E_L(n,0) = -\alpha e^{-\alpha W}\left(\eta_L\left(e^{\alpha D}-1\right)+1\right) \tag{74}$$

And the relevant cutoff point for the parameter rho is:

$$\rho < \frac{\ln\left(\eta_L\left(e^{\alpha D}-1\right)+1\right)-\eta_L\alpha D}{\eta_L\alpha D(b-1)} \tag{75}$$

Since the high risk type will purchase the pooling contract with $t=0$, this is the only condition for non-existence of the degenerate equilibrium.

Secondly, consider the separating equilibrium. In this case we have

$$E_L\left(\widetilde{t}_L,\left(\eta_L-F_L\left(\widetilde{t}_L\right)\right)D\right) \geq E_L\left(t_L,\left(\eta_L-F_L(t_L)\right)D\right) \tag{76}$$

for all $t_L \in \left[\widetilde{t}_L, n\right]$. It follows that the contract $\left(\widetilde{t}_L,\left(\eta_L-F_L\left(\widetilde{t}_L\right)\right)D\right)$ is better than any pooling contract in that region as well. Hence, again we can confine our interest to the interval $t \in \left[0,\widetilde{t}_L\right)$. By the same reasoning as above, the only contract that can challenge the separating contract is one with $t=0$. Hence, we have to compare

$$E_L(t,P) = -\left(F_L\left(\widetilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)\alpha e^{-\alpha\left(W-\left(\eta_L-F_L\left(\widetilde{t}_L\right)\right)D\right)} \tag{77}$$

with $E_L(0,(1-\rho(1-b))\eta_L D)$. We get the condition

$$\rho_2 < \frac{\ln\left(F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)-F_L\left(\tilde{t}_L\right)\alpha D}{\eta_L D\alpha(b-1)} \tag{78}$$

C Derivation of Wilson Equilibria

In this appendix, we characterize the conditions for different equilibria in the Wilson case, when individuals have an exponential utility function. Our analysis will make use of the properties of the utility function of the low risk type, which is the maximand in the derivation of the Wilson equilibrium.

First, if we consider the case where the self-selection constraint is binding, and impose the zero profit constraint, we can solve for the premium rate of the low risk type. In this case, the premium rate is equal to

$$P_L = \rho\left(\eta_H D + \frac{1-\rho}{\rho}\left(\eta_L - F_L(t_L)\right)D - \frac{1}{\alpha}\ln\left(bF_L(t_L)\left(e^{\alpha D}-1\right)+1\right)\right). \tag{79}$$

It is important to notice that as $t_L \to 0$, this expression approaches the pooling premium. Accordingly, the expected utility function of the low risk type behaves smoothly in a neighborhood of $t_L = 0$. Next, we insert (79) into the expected utility function. After some rearrangement, what we get is:

$$V(t_L,\rho) = e^{-\alpha W}\left(bF_L(t_L)\left(e^{\alpha D}-1\right)+1\right)^{-\rho}\left(F_L(t_L)\left(e^{\alpha D}-1\right)+1\right)\cdot e^{\alpha D(\rho\eta_H+(1-\rho)(\eta_L-F_L(t_L)))} \tag{80}$$

which is the expected utility the low risk enjoys from admissible contracts in the range $t_L \in \left[0,\tilde{t}_L\right]$.

We have established in appendix A that for separating contracts without cross-subsidization (which in the Wilson case require $t_L \in \left[\tilde{t}_L, n\right]$) the equilibrium cannot be an interior point. Hence, we have the following alternatives to consider:

1. The pooling contract
2. An interior point in the interval $t_L \in \left[0,\tilde{t}_L\right]$
3. A separating contract without cross-subsidization at $t_L = \tilde{t}_L$
4. The degenerate equilibrium where the low risk type purchases no insurance at all.

It transpires that the relevant equilibria depend in a quite intricate manner on the value of the parameters ρ and b. Hence, for future reference, we will now define some useful cutoff values for ρ. Firstly, we take the derivative of equation (80) with respect to t_L. This equals:

$$\frac{\partial V(t_L,\rho)}{\partial t_L} = e^{-\alpha W}\left(bF_L(t_L)\left(e^{\alpha D}-1\right)+1\right)^{-\rho} e^{\alpha D(\rho\eta_H+(1-\rho)(\eta_L-F_L(t_L)))} F_L'(t_L)$$
$$\left\{ \begin{array}{c} \left(F_L(t_L)\left(e^{\alpha D}-1\right)+1\right)\left(\rho\frac{b\left(e^{\alpha D}-1\right)}{bF_L(t_L)(e^{\alpha D}-1)+1}+(1-\rho)\alpha D\right) \\ -\left(e^{\alpha D}-1\right) \end{array} \right\} \tag{81}$$

The sign of this derivative is determined by the sign of the expression in the large curly brackets. Therefore, we define this expression as $s(t_L,\rho)$. Hence,

$$s(t_L,\rho) = \left(F_L(t_L)\left(e^{\alpha D}-1\right)+1\right)\left(\rho\frac{b\left(e^{\alpha D}-1\right)}{bF_L(t_L)(e^{\alpha D}-1)+1}+(1-\rho)\alpha D-\left(e^{\alpha D}-1\right)\right). \tag{82}$$

Taking the derivative of this expression with respect to t_L gives:

$$\frac{\partial s(t_L,\rho)}{\partial t_L} = F_L'(t_L)\left(e^{\alpha D}-1\right)\left((1-\rho)\alpha D-\rho\frac{b\left(e^{\alpha D}-1\right)}{\left(bF_L(t_L)(e^{\alpha D}-1)+1\right)^2}\right). \tag{83}$$

The factor in the large brackets increases monotonously in t_L. Besides, the expression is decreasing in ρ. Now we can define the following cutoff values:

$$\rho^* = \frac{\alpha D}{(b-1)b(e^{\alpha D}-1)+\alpha D} \tag{84}$$

which is the solution to $\frac{\partial s(t_L,\rho)}{\partial t_L}_{(t_L=0)} = 0$. Hence, for $\rho > \rho^*$, we have $\frac{\partial s(t_L,\rho)}{\partial t_L}_{(t_L=0)} < 0$. Next, we define

$$\rho^{**} = \frac{\alpha D\left(bF_L\left(\tilde{t}_L\right)(e^{\alpha D}-1)+1\right)^2}{(b-1)b(e^{\alpha D}-1)+\alpha D\left(bF_L\left(\tilde{t}_L\right)(e^{\alpha D}-1)+1\right)^2} \tag{85}$$

which is the solution to $\frac{\partial s(t_L,\rho)}{\partial t_L}_{\left(t_L=\tilde{t}_L\right)} = 0$. Again, the implication is that for $\rho > \rho^{**}$, we have $\frac{\partial s(t_L,\rho)}{\partial t_L}_{\left(t_L=\tilde{t}_L\right)} < 0$. Furthermore, we define

$$\rho_{POOL} = \frac{\left(e^{\alpha D}-1\right)-\alpha D}{b(e^{\alpha D}-1)-\alpha D} \tag{86}$$

as the solution to $\frac{\partial V(t_L,\rho)}{\partial t_L}_{(t_L=0)} = 0$. Evaluating (81) at $t_L = 0$, we find that the expression in the brackets is increasing in ρ. Hence, if $\rho > \rho_{POOL}$, we have $\frac{\partial V(t_L,\rho)}{\partial t_L}_{(t_L=0)} > 0$, and the pooling contract is not even locally optimal. Next, we define

$$\rho_{RS}=\frac{\left(bF_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)\left(\left(e^{\alpha D}-1\right)-\alpha D\left(F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)\right)}{\left(F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)\left(b\left(e^{\alpha D}-1\right)-\alpha D\left(bF_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)\right)} \tag{87}$$

which is the solution to $\frac{\partial V(t_L,\rho)}{\partial t_L}_{\left(t_L=\tilde{t}_L\right)}=0$. Finally, define

$$\rho_2=\frac{\ln\left(F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1\right)-F_L\left(\tilde{t}_L\right)\alpha D}{(b-1)\,\eta_L\alpha D} \tag{88}$$

which is the point at which the low risk type is indifferent between the separating contract without cross-subsidization $\left(\tilde{t}_L,\left(\eta_L-F_L\left(\tilde{t}_L\right)\right)D\right)$ and the pooling contract. For higher values of ρ_2, the separating contract is preferred.

Having defined these cutoff values in ρ, we characterize the equilibria that prevail under different circumstances in the following two lemmas:

Lemma 9 *if $b\eta_L<1$ and $b<b^*$*

the following conclusions hold concerning ρ:

1. If $\rho<\rho_2$, the only contract that is offered in equilibrium is the pooling contract $(0,\rho b\eta_L D+(1-\rho)\,\eta_L D)$, provided that

$$\rho<\frac{\ln\left(\eta_L\left(e^{\alpha D}-1\right)+1\right)-\eta_L\alpha D}{(b-1)\,\eta_L\alpha D} \tag{89}$$

If the above condition is not satisfied, only the actuarial contract of the high risk type $(0,b\eta_L D)$ is offered in equilibrium.

2. If $\rho>\rho_2$, the equilibrium menu includes $(0,b\eta_L D)$. Besides, the separating contract $\left(\tilde{t}_L,\left(\eta_L-F_L\left(\tilde{t}_L\right)\right)D\right)$ will be offered, provided that

$$e^{\alpha D\left(\eta_L-F_L\left(\tilde{t}_L\right)\right)}<\frac{\eta_L\left(e^{\alpha D}-1\right)+1}{F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1} \tag{90}$$

Proof. From the condition that $b<b^*$ follows that

$$\rho_{POOL}<\rho^* \tag{91}$$

which implies that for all $\rho<\rho^*$, *if* the marginal utility in equation (81) changes sign in the interval $t_L\in\left[0,\tilde{t}_L\right]$, it changes from negative to positive ($s\left(t_L,\rho\right)$ is monotonously increasing in the interval). In other words, this condition precludes an interior equilibrium in the interval $t_L\in\left(0,\tilde{t}_L\right)$. Furthermore, as we established in Appendix A above, there can be no interior equilibrium when there is no cross-subsidization either (i.e. in the interval $t_L\in\left(\tilde{t}_L,n\right)$).

Hence, for $\rho<\rho^*$, there are two cases to consider:

1. For $\rho < \rho_2$, the pooling contract (with no cross-subsidization) is preferred over the separating contract. Hence it will be offered provided that the low risk type will actually purchase it, which will be the case if:

$$E_L\left(0, \rho b\eta_L D + (1-\rho)\,\eta_L D\right) > E_L \tag{92}$$

implying condition (89).

2. For $\rho > \rho_2$, the separating contract (with no cross-subsidization) is preferred, provided that it will actually be bought by the low risks, which requires

$$E_L\left(\tilde{t}_L, \left(\eta_L - F_L\left(\tilde{t}_L\right)\right) D\right) > E_L \tag{93}$$

implying condition (90).

For the case where $\rho > \rho^*$, finally, the derivative of the low risk type's utility with respect to t_L is strictly positive for all $t_L \in \left[0, \tilde{t}_L\right]$, implying that the separating contract will be offered provided that condition (90) holds. ■

Lemma 10 *if $b\eta_L < 1$, $b > b^*$ and*

$$F_L\left(\tilde{t}_L\right) < \frac{b\left(e^{\alpha D}-1\right) - \alpha D}{2b\alpha D\left(e^{\alpha D}-1\right)} \tag{94}$$

the following conclusions hold concerning ρ:

1. If $\rho < \rho_{POOL}$, only the pooling contract will be offered in equilibrium, provided that

$$\rho < \frac{\ln\left(\eta_L\left(e^{\alpha D}-1\right)+1\right) - \eta_L \alpha D}{(b-1)\,\eta_L \alpha D} \tag{95}$$

2. If $\rho > \rho_{RS}$, it is optimal to offer the separating (not cross-subsidizing) menu $(0, b\eta_L D)$ and $\left(\tilde{t}_L, \left(\eta_L - F_L\left(\tilde{t}_L\right)\right) D\right)$, provided that

$$e^{\alpha D\left(\eta_L - F_L\left(\tilde{t}_L\right)\right)} < \frac{\eta_L\left(e^{\alpha D}-1\right)+1}{F_L\left(\tilde{t}_L\right)\left(e^{\alpha D}-1\right)+1} \tag{96}$$

3. If $\rho_{POOL} < \rho < \rho_{RS}$, it is optimal to offer (t_L^*, P_L^*) together with $(0, P_H^*)$, provided that

$$e^{\alpha P_L^*}\left(F_L\left(t_L^*\right)\left(e^{\alpha D}-1\right)+1\right) < \eta_L\left(e^{\alpha D}-1\right)+1 \tag{97}$$

If the conditions of none of these three cases are satisfied, it is optimal to offer only $(0, b\eta_L D)$.

Proof. The common conditions for b and $F_L\left(\tilde{t}_L\right)$ above imply that

$$\rho^{**} < \rho_{RS} \tag{98}$$

$$\rho_{RS} > \rho_{POOL} \tag{99}$$

and

$$\rho_{POOL} > \rho^{*} \tag{100}$$

There are two different cases to consider, depending on whether $\rho_{POOL} \lesseqgtr \rho^{**}$:

1. $\rho_{POOL} < \rho^{**}$.

a) $\rho \leqslant \rho_{POOL}$. In this case, it follows from $\rho < \rho_{POOL}$ that the pooling contract is locally optimal, since $\frac{\partial V(t_L,\rho)}{\partial t_L}_{(t_L=0)} < 0$. Furthermore, since $\rho < \rho_{RS}$, the marginal utility is negative at $t_L = \tilde{t}_L$ as well. Consequently, the pooling contract will be offered, provided that the low risk prefers it over no insurance, which is assured by condition (95).

b) $\rho_{POOL} < \rho \leqslant \rho_{RS}$. The condition $\rho > \rho_{POOL}$ implies that the pooling contract is not locally optimal (i.e. $\frac{\partial V(t_L,\rho)}{\partial t_L}_{(t_L=0)} > 0$), hence it can be ruled out as a candidate. Furthermore, since $\rho < \rho_{RS}$, marginal utility is negative at $t_L = \tilde{t}_L$. Hence, the equilibrium is either a separating menu with cross-subsidization or a degenerate equilibrium. The separating menu is preferred by the low risk provided that condition (97) is fulfilled.

c) $\rho > \rho_{RS}$. This condition implies that $\frac{\partial V(t_L,\rho)}{\partial t_L}$ is positive throughout $t_L \in \left[0, \tilde{t}_L\right]$. Hence, the equilibrium menu is the separating one without cross-subsidization, provided that it is purchased by the low risk (which is assured by condition (96) above.

2. $\rho_{POOL} \geqslant \rho^{**}$.

a) $\rho \leqslant \rho^{**}$. Since $\rho < \rho_{POOL}$ the pooling contract is locally optimal and since $\rho < \rho_{RS}$, $\frac{\partial V(t_L,\rho)}{\partial t_L}$ is decreasing in t_L at $t_L = \tilde{t}_L$. The condition $\rho \leqslant \rho^{**}$ assures that no interior extreme point will be a maximum. Hence, the pooling contract is offered in equilibrium, provided it is purchased by the low risk type (which follows from condition (95)).

b) $\rho^{**} < \rho < \rho_{POOL}$. Again, $\rho < \rho_{POOL}$ assures that the pooling contract is locally optimal, and $\rho > \rho^{**}$ assures that the marginal utility with respect to t_L is decreasing throughout. The equilibrium involves offering the pooling contract.

c) $\rho_{POOL} < \rho < \rho_{RS}$. $\rho > \rho_{POOL}$ precludes the pooling contract, and $\rho < \rho_{RS}$ assures that the marginal utility is decreasing at $t_L = \tilde{t}_L$. Hence, the low risk utility is concave in t_L over $t_L \in \left[0, \tilde{t}_L\right]$, and the only equilibrium candidates are a separating menu with cross-subsidization, and a degenerate equilibrium.

d) $\rho > \rho_{RS}$. The marginal utility of the low risk type is positive in t_L for all $t_L \in \left[0, \tilde{t}_L\right]$. Hence, the equilibrium menu is the separating one without cross-subsidization, provided that it is purchased by the low risk (which is assured by condition (96) above. ■

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