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Creditor Coordination, Liquidation Timing, and Debt Valuation

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Abstract

This paper derives closed-form solutions for values of debt and equity in a continuous-time structural model in which the demands of creditors to be repaid cause a firm to be put into bankruptcy. This allows discussion of the effect of creditor coordination in recovering money on the values of debt, equity, and the firm, as well as on optimal capital structure. The effects of features of bankruptcy codes that prevent coordination failures between creditors, such as automatic stays and preference law, are also considered. The model suggests that such features, while preventing coordination failures, can decrease welfare.

I. Introduction

Leland (1994) argues that the decision to put a firm into bankruptcy depends on the value of its debt and the value of its equity, and that in turn, the values of debt and equity of a firm depend on when that firm will be put into bankruptcy. Therefore, models need to solve for the optimal decision to put a firm into bankruptcy and the values of debt and equity jointly. In Leland's approach, the firm promises a perpetual coupon payment to debt holders. The assumption is that if a coupon payment is missed, the firm is put into bankruptcy immediately. In this kind of framework, bankruptcy happens when the *equity holder* optimally decides to stop injecting funds to ensure that coupons are paid in full.

However, some firms do in practice miss coupon payments without being put into bankruptcy immediately. For example, Varma and Cantor (2005) report that of slightly more than 1,000 "initial default events" recorded by Moody's for the

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period 1983–2003, more than 55% consist of missed interest payments and grace period defaults, which occur when firms are not in formal bankruptcy (the remaining cases are direct Chapter 11 filings, distressed exchanges, missed principals, direct Chapter 7 filings, and Chapter 11 prepacks).

Once payments are missed, the most important provision of a debt contract, the provision to make timely payments, has been breached. In this kind of situation, bankruptcy can happen when *creditors* decide to take legal steps to demand repayment.[1] Furthermore, whether or not creditors will be able to coordinate will matter, and hence features of bankruptcy codes such as automatic stays and preference law that affect the coordination of creditors will matter. This paper asks the following questions: How are the values of debt, equity, and the firm affected if it is the decision of *creditors* to demand repayment that causes bankruptcy? How do features of bankruptcy codes such as automatic stays, preference law, and a policy of equality of distributions to creditors affect creditor coordination, the decision to demand repayment, and hence the values of debt, equity, and the firm? How do they affect optimal capital structure and welfare? The paper presents answers to these questions in the context of a continuous-time structural model of debt and equity that produces closed-form values of debt, equity, and the firm.

In the model, the equity holder cannot inject funds, and default happens when cash flows are insufficient to make the coupon payments. The equity holder does not receive a bankruptcy payoff, and therefore has incentives to gamble for resurrection (i.e., to delay putting the firm into bankruptcy as long as possible). Once the firm is in default, creditors have a right to demand full payment, either collectively or individually. Successful legal action of creditors leads to bankruptcy, which is taken to be synonymous with liquidation.[2] The model considers different outcomes that might arise depending on whether or not creditors can coordinate.

First consider the case in which creditors cannot coordinate. Then the features of bankruptcy codes that affect creditor coordination such as automatic stays, preference law, and equality of distributions are important. To develop an intuition about the effect of such features, I first discuss how uncoordinated creditors would interact in the absence of these features, and then I discuss how uncoordinated creditors interact in the presence of such features.

When creditors cannot coordinate, they will compare the costs and benefits of individually grabbing assets to decide when to act, taking into account the possible actions of other creditors. Consider initially the case in which automatic stays and preference law do not apply, and there is no policy of equality of distributions to creditors. Suppose that whenever the firm is defaulting on at least some of the promised interest payments on its debt, creditors have to decide whether to individually hire a costly lawyer who will attempt to obtain a judgment lien (i.e., attempt to grab assets). In doing so, they consider the probability of being

[1]For example, LoPucki (1983) provides empirical evidence that this in fact happens in practice.

[2]The majority of bankruptcy filings in the United States are Chapter 7 filings (bankruptcy liquidation) as opposed to Chapter 11 (bankruptcy reorganization). For example, the News Release of the Administrative Office of the U.S. Courts of Nov. 14, 2003, reports that 21,008 businesses filed for Chapter 7 in the fiscal year 2003, whereas only 9,185 filed for Chapter 11. Chapter 11 (bankruptcy organization) is discussed informally in Section II.B. A formal discussion in the given framework is likely to be a fruitful area for future research.

successful in recovering money, which is assumed to depend negatively on the cash available to the firm, and the number of other creditors filing claims; if many creditors file claims and the firm does not have sufficient cash to fight in court, it will be liquidated. Creditors who did file claims will receive a higher liquidation payoff. If few creditors file claims and the firm has sufficient cash to fight in court, the creditors who did file claims have to pay their lawyers but are unsuccessful in grabbing assets.

The central features of this game are strategic complementarities and imperfect information about the actions of other creditors.[3] They produce a critical point that describes how much default creditors are willing to tolerate before a sufficient number of them will attempt to grab assets such that the firm is liquidated. The location of this critical point is determined by the payoffs in the game. For example, for very low legal costs of grabbing assets, individual creditors rush to grab assets very early.

Now consider the case in which automatic stays and preference law do apply, and there is a policy of equality of distributions to creditors. This changes the payoffs of individual creditors. Once a sufficient number of creditors file claims, the firm is put into bankruptcy, an automatic stay applies to most creditors, and preference law ensures that most of the money previously obtained by creditors has to be returned to the trustee, to be shared equally among creditors. This reduces incentives to grab assets individually, leading to later liquidation.

Is this later liquidation always a good thing? In the model, there is an optimal point to liquidate the firm, optimal in the sense that liquidating at this point maximizes firm value. The point at which uncoordinated creditors liquidate a firm in an asset grab can lie either above or below that point (i.e., uncoordinated creditors can produce liquidation that is "too early," or "too late") depending on their incentives to grab assets.

Now consider the benchmark case when creditors can coordinate. In contrast to the case when they cannot coordinate, the features of bankruptcy codes that affect creditor coordination are not important. Coordinated creditors will try to select the time to grab assets to maximize the value of debt. If the share of the liquidation value that goes to creditors is large, creditors will always want the firm to be liquidated "too early."[4] Compared to this benchmark case, a lack of coordination between creditors can increase firm value if it leads to later liquidation, but it will always decrease firm value if it leads to even earlier liquidation.

The model also suggests that incentives to grab assets affect optimal capital structure. Consider a standard debt-equity trade-off between a tax advantage and expected financial distress costs. In addition to this, when deciding on the optimal level of debt, agents now also need to consider the effect of the level of debt on firm value via liquidation timing: Since the critical point at which the firm is

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[3]A technical contribution of the paper is proposing a way of integrating a private information game in a standard continuous-time public-information pricing model with a single filtration via some specific assumptions on the timing of the game.

[4]Here, the model is a good representation of environments in which creditors have large influence over the timing of liquidation, such as, for example, in the United Kingdom, where floating charge holders can (and often do) cause liquidation of a firm.

liquidated depends on how much default creditors tolerate, and how much default creditors tolerate for a given cash flow is a function of how much debt has been issued, the cash flow at which the firm is liquidated is also a function of the level of debt.

In the model, optimal leverage can be a nonmonotonic function of incentives to grab assets. Low incentives to grab assets mean late liquidation, which makes debt somewhat unattractive and medium leverage optimal. Medium incentives to grab assets mean slightly earlier liquidation, which makes debt more attractive and high leverage optimal. High incentives to grab assets mean early liquidation, which makes debt very unattractive and low leverage optimal. Since the incentives to grab assets that a given bankruptcy code produces are related to "creditor rights scores" such as that of La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1998), which contain information on, for example, whether or not automatic stays apply in a given country, this is a testable implication of the model.

In terms of welfare, the model suggests that bankruptcy codes that strongly disincentivize asset grabs can decrease welfare (for any welfare function that attaches some positive weights to firm values and expected tax revenues). The argument is as follows: Strong disincentives against asset grabs lead to very late liquidation, which reduces firm value. Weaker disincentives mean earlier liquidation, which can improve firm value. Earlier liquidation also has 2 effects on expected tax revenues: Earlier liquidation means the firm is likely to pay taxes for a shorter time. At the same time, however, earlier liquidation can imply lower optimal leverage, which means a lower tax shield, and hence higher tax revenues while it is paying. Overall, the 2nd effect can outweigh the 1st effect, and there are situations in which earlier liquidation increases expected tax revenues. So weaker disincentives to grab assets can increase both firm value and expected tax revenues.

Related Literature

In terms of the dynamic pricing literature, the model presented here is related to those of Leland (1994) and other papers that propose extensions of Leland's model. For example, Broadie, Chernov, and Sundaresan (2007) or François and Morellec (2004) augment the Leland model by allowing for a period prior to liquidation in which the firm defaults on payments, which they label as (Chapter 11-type) bankruptcy; this happens when the asset value falls below a bankruptcy barrier that is endogenously determined and chosen by equity holders. François and Morellec assume that the firm is liquidated once it has spent enough time below the bankruptcy boundary.[5] In the model of Broadie et al., liquidation happens at an ex post optimal liquidation boundary below the bankruptcy boundary, or if the firm spends a sufficient amount of time under the bankruptcy boundary. In contrast, the model in this paper follows the approach of Naqvi (2008), in which negative dividends or asset sales are not allowed, which implies default occurs when cash flows are insufficient to cover coupon payments. In this approach,

[5]Generalizations of this approach are considered by Moraux (2002) and Galai, Raviv, and Wiener (2007).

default is not synonymous with bankruptcy, and efforts to collect on debt are closely related to the actual incidence of liquidation.

The model here is also related to dynamic pricing models that incorporate strategic debt service games or renegotiation games as in the papers of Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).[6] In practice, the possibility of renegotiation between equity holders and creditors can be an important additional factor determining actual liquidation outcomes, over and above the decision of creditors to collect on their debt. In order to focus on the decision of creditors to collect on the debt, how the ability to coordinate influences this decision, and how features of bankruptcy codes influence incentives to collect individually, however, the model presented in this paper considers a situation in which renegotiation or strategic debt service is not possible. This is useful not only for understanding what might drive liquidation outcomes in situations in which renegotiating costs would be prohibitively high, but also possibly for understanding the bargaining positions of the parties in renegotiation.

The model here is also related to some papers outside the dynamic pricing literature. The asset grab game is a variant of a model that has been used in the context of (static) debt pricing by Morris and Shin (2004), which in turn is closely related to models used, for example, in the context of currency crises (Morris and Shin (1998)) or bank runs (e.g., Goldstein and Pauzner (2005)).[7] Papers that examine creditor coordination include the paper by Gertner and Scharfstein (1991), which looks at the aggregate effects of individually optimal decisions to accept an exchange offer. In contrast, here the focus is on the decision to individually collect on debt on which a firm is defaulting. Furthermore, Bolton and Scharfstein (1996) examine a model in which inefficiencies in liquidation or renegotiation that are the result of having a large number of creditors can reduce moral hazard; and Bris and Welch (2005) look at how free riding in debt collection efforts between uncoordinated creditors can be beneficial in reducing socially wasteful expenditures that result from equity holders and debt holders fighting over the liquidation value of the firm. Papers that examine the relationship between capital structure and liquidation decisions include that by Titman (1984). However, he only considers the case that would correspond to the benchmark case of coordinated creditors here.

In the next sections, a discrete-time model of an asset grab game between uncoordinated creditors is presented (Section II), and its solution is sketched (Section III). With this discrete-time solution, values of debt and equity cannot be calculated in closed form. (This means, for instance, that in the discrete-time model, it is not possible to consider the benchmark case of coordinated creditors who time their asset grab to maximize the value of debt.) In order to allow valuation, the continuous-time limit is taken. Section IV then reports and discusses the liquidation outcomes in the case of uncoordinated creditors, and compares this with the liquidation outcomes in the (benchmark) case of coordinated

[6] Other versions of renegotiation games are considered (e.g., by Mella-Barral (1999), Fan and Sundaresan (2000), Hege and Mella-Barral (2005), or Hackbarth, Hennessy, and Leland (2007)).

[7] See also Morris and Shin (2003) for a survey of these models.

creditors. Section V discusses the implications of the coordination of creditors for the optimal capital structure and welfare. Section VI concludes.

II. The Model

Time is discrete and increases in steps of size Δ. A firm engages in a productive activity that uses a single productive asset and generates cash flow (net of costs) until the firm is liquidated (i.e., the productive asset is sold). The cash flow at time t is $x_t\Delta$, where x_t evolves according to the following stochastic process:

$$(1) \quad x_{t+\Delta} = x_t + (\mu + \sigma\nu)x_t\Delta + x_t\eta_{t+\Delta}, \qquad \eta_{t+\Delta} \sim \text{NID}\left(0, \sigma^2\Delta\right).$$

Here, the rate of growth of x_t (i.e., $(x_{t+\Delta} - x_t)/x_t$) has a deterministic component that is a sum of a risk premium $\sigma\nu$ and a parameter μ, and normally distributed disturbance term $\eta_{t+\Delta}$, with mean 0 and variance $\sigma^2\Delta$. Here, μ and σ are constants, and ν is (potentially) a function of the state variable x_t and time. There is a risk-free asset that pays a constant rate of return $r\Delta$, where $0 < r < \mu$.

The firm is initially set up with equity and debt; debt is issued with a promised perpetual coupon of $c\Delta$ per time period. At periods in which the cash flow $x_t\Delta$ is sufficient to pay the coupon, the coupon is paid in full, and any remainder is paid out to the equity holder as a dividend. The equity holder pays taxes at rate τ on this dividend; for simplicity, debt is assumed not to be taxed.[8] When the cash flow $x_t\Delta$ is insufficient to pay the coupon, creditors receive the cash flow (net of a financial distress cost) in partial payment of the coupon, and the equity holder receives nothing (i.e., the firm partially defaults on coupon payments). I assume that the financial distress cost is a fraction $\lambda \in (0, 1)$ of the cash flow $x_t\Delta$, such that the cash flow that creditors receive when the firm defaults is $(1 - \lambda)x_t\Delta$. (The presence of a tax on dividends and financial distress costs will produce a standard debt-equity trade-off in the model.)

This setup ignores that defaults have to be cured (i.e., when the debtor is in arrears, interest payments that have been missed have to be paid at a later date if a sufficient amount of money becomes available) and is therefore an approximation.[9]

The setup also assumes that the equity holders do not inject cash to prevent default. This assumption could be relaxed without overly affecting results, at the cost of a substantial increase in model complexity.[10]

[8]Here, τ should therefore be interpreted as a net tax advantage to debt in the sense of Miller (1977).

[9]Although allowing for cures would make the model more realistic, there is a trade-off in terms of complexity versus obtaining closed-form solutions, since introducing cures would introduce path dependency. The model of Broadie et al. (2007) is an example of a similar model that allows for cures but can only be solved numerically.

[10]Bruche and Naqvi (2010) develop a similar model in which the equity holder (acting as a Stackelberg leader) decides how long to prevent default by injecting cash, and the (coordinated) creditors subsequently decide when to liquidate the firm once it is defaulting. In this framework, the equity holder weighs two things when deciding how long to inject cash to prevent default: On the one hand, defaulting earlier means saving more cash; on the other hand, defaulting earlier means giving more incentives to creditors to liquidate. In the model, the equity holder will in general inject some cash to prevent default, but coordinated creditors will still want to liquidate prematurely. Moreover, for plausible parameter values, the "saving cash" motive is dominant, meaning that default happens almost immediately after cash flows are insufficient to pay coupons.

If the firm is liquidated, the productive asset is sold, that is, the cash flows are swapped irreversibly for a constant liquidation value $K > 0$, of which a fraction $s > 0$ covers legal costs, such that the net liquidation value is $(1 - s)K$. Note that choosing a constant liquidation value rather than arguing that the liquidation value is a fraction of the preliquidation going-concern value of the firm, as is often done in the literature, is significant in that it means that liquidation (even in the unlevered firm) will be optimal at some positive level of the going-concern value.

I assume that the net liquidation value $(1 - s)K$ is less than or equal to the face value of debt c/r;[11] that in the event of liquidation, absolute priority is respected;[12] and that the net liquidation value therefore goes to creditors.[13] Formally, the payoffs to debt and equity are as follows:

$$\text{(2)}\quad \text{payoff to equity at } t = \begin{cases} (1-\tau)(x_t - c)\Delta, & \text{before liquidation if } x_t\Delta \geq c\Delta, \\ 0, & \text{before liquidation if } x_t\Delta < c\Delta, \\ 0, & \text{at liquidation (final payoff),} \end{cases}$$

and

$$\text{(3)}\quad \text{payoff to debt at } t = \begin{cases} c\Delta, & \text{before liquidation if } x_t\Delta \geq c\Delta, \\ (1-\lambda)x_t\Delta, & \text{before liquidation if } x_t\Delta < c\Delta, \\ (1-s)K, & \text{at liquidation (final payoff).} \end{cases}$$

Note that the liquidation payoff of the equity holder is 0, but that her continuation payoff is always positive. This implies that she will always avoid liquidation when possible.

A. The Asset Grab Game with No Bankruptcy

In the absence of a bankruptcy procedure that acts as a formal coordination device, creditors can still recover money by uncoordinated individual legal action. Suppose that creditors can choose to hire or not to hire a lawyer who tries to recover money by obtaining a judgment lien against the firm (grabbing or not grabbing assets), and that the firm is liquidated if a "sufficient" fraction of the creditors hire lawyers.

Formally, suppose there exists a continuum of creditors with mass 1. Creditors play a stage of the game just before the beginning of each time period $t + \Delta$.

[11] Since debt is perpetual, there is no payment of principal. Interpreting c/r as the face value of debt (or principal) in this case is common in the literature (see, e.g., Mella-Barral and Perraudin (1997)).

[12] In practice, absolute priority is typically respected in liquidation, although it is often not respected in reorganization.

[13] One can also consider the less interesting case where $(1 - s)K > c/r$, where the liquidation payoff to creditors is the full face value c/r, and the liquidation payoff to the equity holder is the remaining liquidation value after creditors have been paid off, $(1 - s)K - c/r$. Details are available from the author.

Suppose a court agrees to liquidate when the fraction l of creditors who decide to hire a lawyer and attempt to grab assets is larger than or equal to $x_{t+\Delta}/c$. This formulation ensures that it will be impossible for the firm to be forcibly liquidated when $x_{t+\Delta} > c$.

All creditors know the cash flow in the previous period t, such that one can interpret it as public information about the cash flow $x_{t+\Delta}$, where the precision of this information is $\alpha = (\sigma^2 \Delta)^{-1}$. When they are about to play a stage, creditors also receive a private signal ξ_i (subscript i indexes the different creditors) about the value that the cash flow x will take at $t + \Delta$, given by

$$(4) \qquad \xi_i = x_{t+\Delta} + x_t \epsilon_i, \qquad \epsilon_i \sim \text{NID}\left(0, \frac{1}{\beta}\right),$$

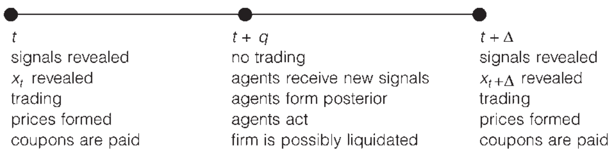
where $\text{Cov}(\eta_{t+\Delta}, \epsilon_i) = 0$ (i.e., the noise is orthogonal to the innovations in the cash flow). From the signal ξ_i and the public information x_t, creditors form a posterior about the cash flow of the firm in period $t + \Delta$. The differences in posteriors resulting from the differences in the private signal create uncertainty about the actions of other creditors and hence coordination failure. Once creditors have formed their posterior, they act.

The following assumptions about timing of the game will allow for derivation of closed-form valuation formulas via standard techniques, once the continuous-time limit is taken: Markets open at times $t, t + \Delta, t + 2\Delta, \ldots,$ and the asset grab games are played at some intermediate time periods $t + q, (t + \Delta) + q, \ldots,$ etc., in which markets are closed. Assume that whenever trading occurs, the cash flow at that time is public information. As a consequence of these timing assumptions, only public information will be incorporated into prices (see Figure 1). To simplify the pricing argument later, one can also assume that coupons or partial coupons, etc. are paid at the times at which markets are open (i.e., $t, t + \Delta, \ldots,$ etc.).[14]

FIGURE 1

Timing Assumptions

Markets open at times $t, t + \Delta, \ldots,$ and the asset grab games are played at some intermediate time periods $t + q, \ldots,$ etc., in which markets are closed.



[14]One way to interpret these assumptions is the following: During the day (t), trading occurs and prices reveal all information. In the evening ($t + q$), markets close. The creditors now receive private information about the financial situation of the firm (e.g., via a tip-off). They have to decide immediately whether or not to call their lawyer. If called, a lawyer starts billing his client immediately and works throughout the night to prepare the papers to be filed. In the morning, before markets open, all lawyers who have been hired congregate in front of the courthouse; if the number of lawyers is large, it is clear that the firm will be liquidated. If the firm is not liquidated, when markets open (at $t + \Delta$), trading occurs, and prices reveal all private information of creditors.

Attempting to grab assets produces an immediate cost sK. If the firm is pushed into liquidation, creditors who have grabbed assets receive the liquidation value $(1 - s)K$, whereas creditors who have not grabbed assets receive 0. If the firm is not pushed into liquidation, creditors who have attempted to grab assets still incur the cost. Table 1 illustrates the instantaneous payoffs that creditors consider (together with any possible payoffs in the future) when making the decision whether to attempt to grab assets at the intermediate time periods $t + q$.

TABLE 1

Payoffs to Creditors in the Discrete-Time Game

Payoffs to creditors at the interim stage $t + q$.

	Liquidation	No Liquidation
Grab assets	$(1 - s)K$	$-sK$
Do not grab assets	0	0

While payoffs assumed here produce the fundamental feature of the game, which is strategic complementarities, potentially more complicated, and possibly more realistic payoffs, especially those that depend on the actual fraction of creditors attempting to grab assets, are conceivable. It would, for example, be natural to argue that in the event of liquidation, assets will first be shared among those creditors who attempted to grab assets, and all remaining assets will be shared between those who did not attempt to grab assets. In this kind of setup with *one-sided strategic complementarities* (Goldstein and Pauzner (2005)), it is possible to prove the existence of equilibria in monotone strategies, and uniqueness for particular structures of noise (Morris and Shin (2003), Goldstein and Pauzner), but this greatly complicates solutions and does not change the qualitative flavor of the game. In the continuous-time limit derived later, the payoff does not depend on the fraction of creditors that attempt to grab assets in any case.

Since creditors maximize expected utility, payoffs will have to be converted into utility before solving the game. I assume that utility is additively separable across time, such that expected utility can be decomposed into an instantaneous (Bernoulli) utility associated with instantaneous payoffs, and a (discounted) continuation utility associated with future payoffs.[15]

B. The Asset Grab Game with Bankruptcy

If the costs of attempting to grab assets is low enough, this can lead to uncoordinated asset grabs for firms that still have very high cash flows: Any creditor will worry about all other creditors grabbing and will therefore grab.

Bankruptcy codes attempt to prevent such asset grabs via automatic stays, preference law, and a policy of equality of distributions to creditors, thus protecting debtors. For example, part of the stated purpose of preference law in the United States is the following:

[15] A specific choice of utility function will imply a specific form for the risk premium $\nu\sigma$.

> [B]y permitting the trustee to avoid prebankruptcy transfers that occur within a short period before bankruptcy, creditors are discouraged from racing to the courthouse to dismember the debtor during his slide into bankruptcy.[16]

In practice, firms do apply for protection from their creditors exactly when these attempt to individually grab assets (LoPucki (1983)). This suggests modifying the game by allowing the equity holder to explicitly put the firm into formal (Chapter 7-type) bankruptcy, in which case an automatic stay, preference law, and a policy of equality of distributions to creditors apply.

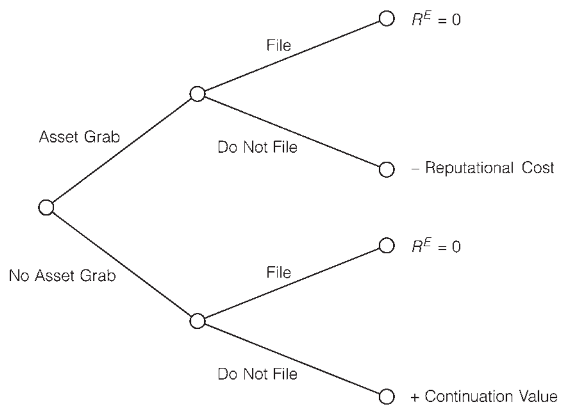
A modeling issue then is whether the equity holder and creditors should move simultaneously, or whether either creditors or the equity holder should move first. In the United States, preference law (United States Code, Title 11, Ch. 5.III § 547) specifies that any preferential transfer that a creditor manages to obtain in the period up to 90 days before formal bankruptcy can be avoided; that is, if a creditor manages to grab assets within 90 days before formal bankruptcy, the grabbed amount has to be returned to the trustee, to be shared equally across all creditors. This suggests that the equity holder can put the firm into formal bankruptcy *after* observing the actions of creditors, but that this move would still affect the payoffs to creditors.

Suppose, therefore, that just before each time period $t + \Delta$ one now has the following 2-substage sequential move game: First, creditors decide whether or not to grab assets. Then, the equity holder decides whether or not to file for bankruptcy, having observed the actions of all creditors (see Figure 2).

FIGURE 2

Payoffs to Equity in Bankruptcy Game at $t + q$

First, creditors decide whether or not to grab assets. The equity holder observes whether a sufficient number of creditors have grabbed assets such that the firm will be liquidated piecemeal ("asset grab") or not ("no asset grab"), and will decide to put the firm into bankruptcy or not depending on the given payoffs. As long as the reputational cost is not 0, and the continuation value is above 0, the equity holder puts the firm into bankruptcy only when an asset grab has happened.



[16]H.R. Rep. No. 595, 95th Cong., 1st sess., 1977, 177 describing the Bankruptcy Reform Act of 1978. Note that the verb "to avoid" is used here in its legal sense of "to repudiate, nullify, or render void."

A 2nd modeling issue is how to describe the payoffs to the equity holder and creditors in bankruptcy. If absolute priority is respected in Chapter 7, the payoff to the equity holder is 0. In practice, it is likely that an equity holder would have at least a weak preference for orderly Chapter 7 liquidation over a disorderly asset grab liquidation (e.g., because of reputational costs associated with the latter). Suppose that such reputational costs exist, but that they are arbitrarily small. When a large number of creditors grab assets such that an asset grab liquidation would result, the equity holder would then file for bankruptcy, since this is weakly preferred. When a small number of creditors grab assets such that no asset grab liquidation would result, the equity holder does not file for bankruptcy, since in this situation the continuation value to equity (which is always positive) is always higher than the liquidation value (which is 0).

Bankruptcy codes strive to achieve equality of distributions to creditors but do not necessarily achieve full equality of distributions. For example, preference law does not prevent all types of prebankruptcy transfers, and requests of the trustee to return grabbed funds are open to a legal challenge. To model this in a reduced-form manner, suppose one modifies the payoffs that result from a successful asset grab (that now provokes bankruptcy): Assume that creditors who grabbed assets obtain $(1-s)K$ as before, but assume that creditors who did not grab assets now obtain $(1-\varepsilon)(1-s)K$, where $0 \leq \varepsilon \leq 1$ (i.e., that payoffs "differ by an epsilon").[17] The new payoff matrix for creditors, already anticipating the actions of the equity holder in the 2nd substage, is then given in Table 2.

TABLE 2

Payoffs to Creditors in Bankruptcy Game at $t+q$

The optimal choice of equity holders in the 2nd stage (put the firm into bankruptcy when there is a successful asset grab) produces the given payoff matrix for creditors in the 1st stage.

	Bankruptcy	No Bankruptcy
Grab assets	$(1-s)K$	$-sK$
Do not grab assets	$(1-\varepsilon)(1-s)K$	0

If $\varepsilon = 1$, the payoff to creditors who did not grab assets is 0. In this case, the payoffs are the same as in the asset grab game without bankruptcy codes. (Since the asset grab game without bankruptcy codes turns out to be a special case, it will not be necessary to describe its solution separately later.)

If $\varepsilon = 0$, the payoffs to all creditors are the same if the grab is successful (i.e., bankruptcy codes achieve full equality of distributions). Since creditors who do not grab assets do not incur legal costs if the asset grab is unsuccessful, not grabbing assets now is a (weakly) dominant strategy; the incentives to grab assets therefore are much weaker then before.

For intermediate values of ε, the higher is ε, the larger is the difference in payoffs for creditors who grab and those who do not grab when the grab is successful, and the stronger are the incentives to grab assets. In this sense, one

[17]For example, ε could be related to the probability that the bankruptcy trustee successfully challenges a prebankruptcy transfer.

can think of ε as measuring the incentives to grab assets produced by a bankruptcy code.

Chapter 11

A key assumption in the model presented here is that the payoff to the equity holder in bankruptcy is 0. This a reasonable description of Chapter 7, but not necessarily of Chapter 11: In Chapter 7, absolute priority is likely to be respected, whereas in Chapter 11, there are likely to be deviations from absolute priority in favor of equity holders (Bris, Welch, and Zhu (2006), Franks and Torous (1989)).

How would relaxing this assumption affect the model? For situations in which the bankruptcy payoff to equity holders is positive, the equity holders could have a motive to voluntarily file for bankruptcy. If this payoff is relatively large, the equity holder could have a motive to file before an asset grab occurred. In that case, creditor coordination and features of bankruptcy that affect creditor coordination would not affect when the firm is put into bankruptcy. Conversely, if this payoff is relatively small, the equity holders would not have a motive to file before an asset grab occurred, and the game would work essentially as described previously.

In this sense, the flavor of the game would not change substantially when considering Chapter 11 explicitly, as long as Chapter 11 bankruptcy payoffs to equity holders are small. As noted previously, there is some empirical evidence that firms are forced to file for bankruptcy precisely when creditors attempt to individually grab assets (LoPucki (1983)), which suggests that payoffs to equity holders are small. A more formal examination of Chapter 11 in the context of the type of model proposed here is likely to be a fruitful area for future research.

III. Solving the Discrete-Time Asset Grab Game

In the repeated game, the continuation utility does not depend on the current action of an individual creditor, because creditors are atomistic. This implies that the repeated game can be solved as a series of one-shot games, which greatly simplifies the analysis.[18] I give a summary of the procedure here; for details see Appendix A.

I solve a single-stage asset grab game using the same procedure as in Morris and Shin (2004). Suppose that creditors follow a switching strategy (i.e., attempt to grab assets if their posterior mean over the cash flow is below a critical level). Given the critical level of the posterior mean, one can work out a critical level of the cash flow below which a sufficient number of creditors will attempt to grab, and hence the firm will be liquidated. Given the critical level of the cash flow, one can work out around which critical level of the posterior mean creditors will switch. This gives 2 equations in 2 unknowns, which are solved. It can be shown that an equilibrium in switching strategies in this type of game is the only

[18]For any creditor for whom the utility of grabbing assets is compared to the utility of not grabbing assets, the *same* expression for the continuation utility will appear on both sides of an inequality. The continuation utility therefore cancels out, and only Bernoulli utilities of instantaneous payoffs matter.

equilibrium that survives iterated deletion of dominated strategies (see, e.g., Morris and Shin (2003)).

Solving the game here is therefore equivalent to finding the critical level of the cash flow $\overline{x}_{\mathrm{AG}}$ (AG for "asset grabs"), such that when the cash flow falls below this critical level, a sufficient number of creditors decide to grab assets, and the firm is liquidated. I show in Appendix A.5 that this critical value of the cash flow $\overline{x}_{t+\Delta}^{\mathrm{AG}}$ is given by a complicated implicit function, which is not repeated here since a more easily interpretable expression will be derived later in the continuous-time limit.

As is standard in this type of game, the equilibrium will be unique, as long as the private information is "precise enough" in relation to the public information (see Appendix A.6). Basically, for coordination failure to arise, creditors need to be sufficiently uncertain about the actions of other creditors. They will only be uncertain about the actions of other creditors if these actions reflect mostly private information. The actions will reflect mostly private information if that information is relatively useful (precise) in relation to public information.

If the private information is "precise enough" and the equilibrium is unique, it could, for example, be shown that as payoffs to creditors who grab assets and payoffs to creditors who do not grab assets become more similar (ε decreases), the critical boundary $\overline{x}_{\mathrm{AG}}$ decreases. The interpretation here is that as the opportunity cost of not grabbing assets decreases, creditors will be more reluctant to grab assets, and will do so only for lower cash flows. However, since the solution will turn out to be much simpler once the continuous-time limit is taken, I delay a more detailed discussion until Section IV.

IV. Liquidation Decisions in the Continuous-Time Limit

In order to simplify the solution of the game, and also to allow the derivation of the values of debt, equity, and the firm, I take continuous-time limits. This will produce a simplified formula for the boundary at which asset grabs happen, which I discuss in Section IV.A. The closed forms for the values of debt, equity, and the firm also allow looking at the benchmark case in which creditors choose a liquidation boundary to maximize the value of debt, which I discuss in Section IV.B.[19] Section IV.C compares the 2 types of liquidation outcomes.

Loosely speaking, in order to take the continuous-time limit, the size of the time-step Δ needs to tend to dt. As described in detail in Appendix B, limits can be taken here in such a way that i) the equilibrium in the game is guaranteed to be unique, ii) all creditors grab assets immediately when the cash flow hits the critical level $\overline{x}_{\mathrm{AG}}$, iii) the strategic uncertainty of the marginal creditor over the actions of other creditors (and hence the flavor of the equilibrium in the game) is preserved, and iv) valuation via standard contingent claims techniques is possible. Closed forms for the value of debt, equity, and the firm are derived in Appendix C.

[19]They will also later allow looking at capital structure that is optimal in the sense of maximizing firm value.

A. The Case of Uncoordinated Creditors

In the continuous-time limit, the expression for the critical boundary at which uncoordinated asset grabs cause a liquidation of the firm is simplified substantially. The following proposition states this critical boundary:

Proposition 1. In the continuous-time limit, the critical level of the cash flow at which an asset grab occurs is given by

$$\overline{x}_{\text{AG}} = \theta c, \tag{5}$$

where $0 < \theta < 1$ is given by

$$\theta := \frac{u((1-s)K) - u((1-\varepsilon)(1-s)K)}{u((1-s)K) - u((1-\varepsilon)(1-s)K) + u(0) - u(-sK)}. \tag{6}$$

Proof. See Appendix B. □

This means that in continuous time, the critical level of the cash flow is a fraction θ of the coupon, where this fraction reflects the utility of grabbing assets versus the utility of not grabbing assets, both in situations in which the firm is liquidated and in situations in which it is not liquidated.

Here, $\overline{x}_{\text{AG}}$ is below c. Liquidation of the firm will then happen in the following way: As the cash flow falls below c, the firm will begin defaulting on part of the coupon payments. As the cash flow falls further to $\overline{x}_{\text{AG}}$, creditors will grab assets, and the firm will be liquidated.

Now, θ, and hence $\overline{x}_{\text{AG}}$, is decreasing in s; if it is expensive to hire a lawyer to file a claim, creditors will be reluctant to attempt to grab assets, and the cash flow has to be lower before a sufficient number of them act to force liquidation.

Here, θ, and hence $\overline{x}_{\text{AG}}$ is increasing in ε. Lowering ε makes payoffs in bankruptcy for creditors who grabbed and creditors who did not grab more similar, and therefore reduces incentives to grab, and consequently reduces the critical level of the cash flow at which asset grabs occur. As ε tends to 0, not grabbing assets becomes a (weakly) dominant strategy, illustrating how bankruptcy codes can prevent asset grabs. If $\varepsilon > 0$, then creditors grab assets at some positive level of the cash flow, after which the equity holder puts the firm into bankruptcy, as has been observed to happen in practice (LoPucki (1983)).

B. The Benchmark Case of Coordinated Creditors

With closed-form solutions for the values of debt, equity, and the firm, it is also possible to consider the benchmark case in which coordinated creditors choose a liquidation boundary that maximizes the value of debt.[20] More generally, one can consider the liquidation boundaries that maximize either the value of debt, the value of equity, or the value of the levered firm. I label these as $\overline{x}_D$, $\overline{x}_E$, and $\overline{x}_V$, respectively.

[20]See Appendix C for the formulas for the value of debt, equity, and the firm.

To maximize the value of debt, coordinated creditors would want to force liquidation once the cash flow hits $\overline{x}_D$. They will be able to do so if the firm is defaulting on coupon payments at this point and the firm has not already been liquidated by the equity holder. This means that whether or not the firm is actually liquidated at $\overline{x}_D$ depends on the ordering of $\overline{x}_D, \overline{x}_E$, and c. The relationship between these boundaries is summarized in the following proposition:

Proposition 2. Let $\overline{x}_D$, $\overline{x}_E$, and $\overline{x}_V$ denote the liquidation boundaries that maximize the values of debt, equity, and the firm, respectively. Then

$$0 = \overline{x}_E < \overline{x}_V < \overline{x}_D \leq c. \tag{7}$$

Proof. An intuitive argument for why the proposition holds is as follows:[21] Since the net liquidation proceeds $(1-s)K$ are assumed to be less than the face value of debt, and absolute priority is assumed to be respected, the liquidation payoff to the equity holder is 0, and the liquidation payoff to creditors is equal to the net liquidation value $(1-s)K$.

This implies that in liquidation, the equity holder never gains anything, but always loses a positive continuation value—associated with the possibility that the cash flow can always return to the region where it exceeds the promised coupon payment, and that the firm will therefore pay dividends at some point in the future. Hence the equity holder never wants to liquidate. Mathematically, the value of equity is maximized for a choice of liquidation boundary $\overline{x}_E = 0$, which will be hit with probability 0.

Conversely, in liquidation, creditors do gain a positive liquidation payoff but lose a positive continuation value. The point at which this gain and loss are traded off optimally is given by $\overline{x}_D$. Mathematically, the value of debt is maximized for a choice of liquidation boundary $0 < \overline{x}_D \leq c$. This is above 0 because the liquidation payoff is positive, and hence creditors will want to liquidate for very low cash flows/continuation values. It is below c because the liquidation payoff is less than the value of receiving the full coupon c forever (c/r), and hence creditors will not want to liquidate unless the firm is defaulting.[22]

Since the market value of the firm is defined as the sum of the value of debt and equity, the liquidation boundary that maximizes this value ($\overline{x}_V$) represents a compromise between what debt holders and equity holders want, and lies in between the liquidation boundary that maximizes the value of debt ($\overline{x}_D$) and the liquidation boundary that maximizes the value of equity ($\overline{x}_E$). □

Now, $\overline{x}_D$ is below c and above $\overline{x}_E$. Liquidation of the firm will then happen in the following way: As the cash flow falls below c, the firm will begin defaulting on part of the coupon payments. As the cash flow falls further to $\overline{x}_D$, creditors will grab assets in a coordinated fashion and cause liquidation. (At this point, equity holders would prefer continuation, but since the firm is defaulting on part of the coupon payments, creditors have the legal right to grab assets.)

[21] A full formal proof is available from the author.

[22] $\overline{x}_D = c$ exactly when $(1-s)K = c/r$.

Compared to the liquidation decision that would maximize the market value of the firm ($\bar{x}_V$), creditors want excessive liquidation. This is because the continuation value of creditors is lower than the combined continuation value of all parties.[23]

C. A Comparison of Coordinated and Uncoordinated Liquidation

In the benchmark case of coordinated creditors, the firm will be liquidated at too high a cash flow, in the sense that later liquidation would increase the total value of all claims on the firm.

In the case of uncoordinated creditors, the firm could be liquidated either at too high a cash flow or at too low a cash flow, depending on the parameter s, the legal cost of grabbing assets, and the parameter ε, the difference in payoffs in liquidation between creditors who did grab assets and creditors who did not grab assets.

If ε and hence the differences in payoffs between creditors who grab and creditors who do not grab are very large, and/or the legal costs of grabbing assets are very low, there are very strong incentives to grab assets. In such a situation, the firm will be liquidated at too high a cash flow. If ε and hence the differences in payoffs between creditors who grab and creditors who do not grab are very small, and/or the legal costs of grabbing assets are very high, there are very weak incentives to grab assets. In such a situation, the firm will be liquidated at too low a cash flow.

It is possible that a lack of coordination between creditors can actually lead to better liquidation timing. This is because coordinated creditors always liquidate too early, whereas uncoordinated creditors might or might not liquidate too early, depending on the incentives to individually grab assets.

Of course, all of this discussion assumes that Coasian bargaining is not feasible. If it is, then it is clear that liquidation at the firm-value maximizing liquidation point could always be achieved (Haugen and Senbet (1978), Mella-Barral (1999)). While factors that prevent creditor coordination are likely to also prevent Coasian bargaining, one might think that at least in the case of coordinated creditors, Coasian bargaining could be feasible. For example, if a firm has borrowed from one bank only, then this bank might accept some delay in liquidation in exchange for an equity stake.

However, there is at least anecdotal evidence that the kind of premature liquidation described in the coordinated benchmark can occur, suggesting that in practice things such as, for example, holdup problems, asymmetric information, or high bargaining costs could be preventing Coasian bargaining from taking place. For instance, in the United Kingdom, floating charge holders (typically banks) are a good example of coordinated creditors with strong rights. It has been argued that there, "the bank may decide against keeping a good company going because it does not see the upside potential" (Hart (1995), p. 168), or that floating charge holders "apply themselves ruthlessly to the realization of assets to satisfy

[23]This is essentially the point made by Hart ((1995), p. 166); the version presented here is a variation of the version of the argument presented by Naqvi (2008).

the charge [...] in some cases with scant regard for the future of the company" (Woolridge (1987)).

V. Liquidation, Capital Structure, and Welfare

This section discusses optimal capital structure choices and a social planner's problem: Bankruptcy codes affect how a firm is liquidated and hence affect the debt-equity trade-off. Also, a social planner could affect bankruptcy codes and hence indirectly affect the choice of capital structure and, hence, welfare.

In the context of a concrete numerical example, I first discuss how the different liquidation boundaries are affected by the level of debt in the firm (as measured by the coupon c), then discuss how these liquidation mechanisms affect the debt-equity trade-off and hence capital structure choices, and then consider the social planner's problem.

For this exercise, input parameters are chosen as follows: The initial cash flow is $x(0)=5$, the risk-free rate is $r=5\%$, the rate of growth under the pricing measure is $\mu=3\%$, the volatility is $\sigma=20\%$, the tax parameter is $\tau=40\%$,[24] and the financial distress cost parameter is $\lambda=50\%$.[25] The liquidation value is $K=60$, and the legal cost of grabbing assets is $s=10\%$. (Together, these parameters imply that it would be optimal to liquidate an *unlevered* firm once the cash flow falls to around 1.17.) I assume that investors are approximately risk neutral. This simplifies the calculation of the liquidation boundary $\bar{x}_{\text{AG}}$ slightly (it will then be given by $\bar{x}_{\text{AG}}=\varepsilon(1-s)c/(\varepsilon(1-s)+s)$).

Later, I will plot liquidation boundaries and firm values against the coupon c. The coupon will vary between $r(1-s)K=2.7$, and $x(0)=5$. For the lowest coupon, $c/r=(1-s)K$, and debt is fully collateralized and hence risk free. For the highest coupon, the company is on the verge of default and hence very risky.

I also allow ε to vary. As argued previously, this parameter measures differences in payoffs between creditors who grab assets and creditors who do not grab assets, and hence the strength of incentives to grab assets. (I will consider low, medium, and high incentives to grab assets: $\varepsilon_L=0$, $\varepsilon_M=0.025$, $\varepsilon_H=1$.)

A. Liquidation Boundaries

Figure 3 plots liquidation boundaries as a function of the coupon. The solid line corresponds to the liquidation boundary that maximizes the firm value ($\bar{x}_V$). In general, this liquidation boundary can either be increasing or decreasing in c, depending on the relative size of the tax and distress cost parameters (τ and λ). There are 2 opposing effects: On the one hand, higher levels of debt increase the size of the tax shield and hence increase the continuation value of the firm, which makes later liquidation optimal. On the other hand, higher levels of debt

[24] Kemsley and Nissim (2002) estimate the debt tax shield to be approximately 40% of debt balances.

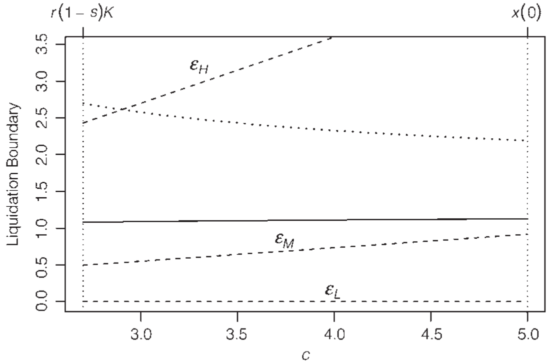
[25] This implies a reduction in percentage of the gross cash flow that is paid out in distress by ⅙. To see this, note that the percentage of gross cash flow x that ends up in the hands of equity holders outside of distress is $(1-\tau)=60\%$, and the percentage of gross cash flow x that ends up in the hands of creditors in distress is $(1-\lambda)=50\%$.

increase expected financial distress costs and hence decrease continuation value of the firm, which makes earlier liquidation optimal. For the given parameters, these 2 effects almost completely neutralize each other, and the boundary is essentially flat.

FIGURE 3

Liquidation Boundaries as a Function of Coupon

Liquidation boundaries that maximize firm value ($\overline{x}_V$, solid line) and debt value ($\overline{x}_D$, dotted line), and liquidation boundaries that result from asset grabs in the presence of bankruptcy codes ($\overline{x}_{AG}$, dashed lines), with varying ε ($\varepsilon_L = 0$, $\varepsilon_M = 0.025$, $\varepsilon_H = 1$), all as functions of coupon c. Other parameters are as in the main text.



The dotted line corresponds to the liquidation boundary that maximizes the value of debt ($\overline{x}_D$). This is always decreasing in c. For low c, the continuation value of creditors is low in relation to their liquidation payoff $(1-s)K$. This means that they prefer to liquidate early, and hence $\overline{x}_D$ is large. As c increases, the continuation value of creditors increases, and they therefore prefer to liquidate later, and hence $\overline{x}_D$ decreases.

Although this is not plotted here, the liquidation boundary that maximizes the value of equity ($\overline{x}_E$) is flat at 0, because the continuation value of the equity holder is always positive, but the liquidation payoff of the equity holder is 0. One can see that the boundaries $\overline{x}_D, \overline{x}_V$, and $\overline{x}_E$ satisfy $\overline{x}_E < \overline{x}_V < \overline{x}_D \leq c$ as suggested by Proposition 2.

The dashed lines correspond to the liquidation boundaries that result from asset grabs when bankruptcy codes affect the interaction of creditors ($\overline{x}_{\text{AG}}$) for low, medium, and high incentives to grab assets ($\varepsilon_L = 0, \varepsilon_M = 0.025, \varepsilon_H = 1$). It can be seen that for the larger incentives to grab assets (larger ε), the corresponding boundary lies higher. Also, the boundaries with positive ε (i.e., with ε_M and ε_H) are increasing functions of c. This is because the game essentially determines the fraction of the coupon on which the firm can default before creditors grab (i.e., the liquidation boundary is a fraction of the coupon).

B. Optimal Capital Structure

In the context of the numerical example, one can now discuss optimal capital structure in the ex ante sense of Leland (1994); that is, one can consider the owner of an all-equity firm that wants to exit by selling debt and equity in proportions that maximize the overall proceeds of the sale, taking a liquidation mechanism as given. The choice of capital structure in this context is synonymous with a choice of the level of coupon c.

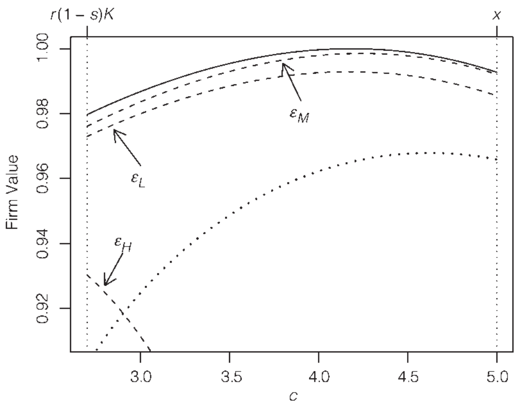
In the model, there is a trade-off as in standard trade-off theory: Increasing the level of debt c increases the size of the tax shield, but it also increases expected financial distress costs. Here, however, there is an additional effect of c on firm value, which is via liquidation boundaries.

Figure 4 shows the firm values for different liquidation boundaries. The solid line corresponds to a firm that is liquidated at the firm-value maximizing liquidation boundary (at $\bar{x}_V$). The firm value is maximized in this case for a coupon of $c \approx 4.2$.[26]

FIGURE 4

Firm Values as a Function of Coupon

Firm values (measured as fraction of maximum attainable firm value) as a function of coupon c, for different liquidation boundaries: firm value for liquidation at $\bar{x}_V$ (solid line), firm value for liquidation at $\bar{x}_D$ (dotted line), and several firm values for liquidation at $\bar{x}_{AG}$ corresponding to different ε, as in Figure 3.



The dotted line corresponds to a firm that is liquidated at the debt-value maximizing liquidation boundary (at $\bar{x}_D$). One can see that first, the firm value in this case is lower for all c. This is because in this case, the firm is liquidated very early, which decreases firm value. Second, one can see that the firm value in this case is maximized for a higher coupon of around $c \approx 4.6$. This is because increasing c here has the additional effect of decreasing $\bar{x}_D$, bringing it closer to firm-value maximizing liquidation boundary $\bar{x}_V$ (see Figure 3).

The dashed lines correspond to a firm that is liquidated as a result of asset grabs, in the presence of bankruptcy codes ($\bar{x}_{AG}$), for various values of ε. One can see that for a high ε_H (high incentives to grab assets), the resulting very early asset grabs produce a low firm value. Also, firm value in this case is maximized for a

[26] Since $\bar{x}_V$ is defined as the $\bar{x}$ that satisfies $\partial V/\partial\bar{x} = 0$, one knows that the total derivative $dV/d\bar{x} = \partial V/\partial c + (\partial V/\partial\bar{x})\partial\bar{x}/\partial c = \partial V/\partial c$ at this point. The 1st-order condition associated with maximizing V therefore only considers the partial derivative $\partial V/\partial c$, so in this sense, if the firm is liquidated at $\bar{x}_V$, only a standard trade-off (effect of c on V via tax advantage and expected financial distress cost) is considered, and the effect of c on the liquidation boundary has no effect on the optimal choice of c.

very low c. This is because decreasing c has the additional effect of decreasing $\bar{x}_{\mathrm{AG}}$, bringing it closer to the firm-value maximizing liquidation boundary $\bar{x}_V$ (see Figure 3).

For a low ε_L ($=0$, which implies no incentives to grab assets), one can see that the resulting very late asset grabs produce a higher firm value. Also, firm value in this case is maximized for a c very close to the c that maximizes firm value in the case in which the firm is liquidated at $\bar{x}_V$. This is because adjusting c has no effect on $\bar{x}_{\mathrm{AG}}$, just as adjusting c has (almost) no effect on $\bar{x}_V$ (see Figure 3).

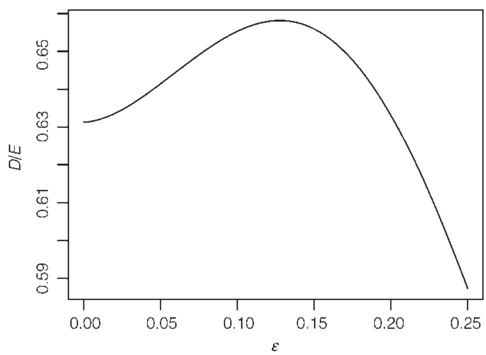
Lastly, for an intermediate ε_M, one can see that even higher firm values can be achieved. For the particular value of ε here ($\varepsilon_M = 0.025$), firm value in this case is maximized for a slightly higher coupon than in the case where the firm is liquidated at $\bar{x}_V$. This is because increasing c again has the additional effect of increasing $\bar{x}_{\mathrm{AG}}$, bringing it closer to the firm-value maximizing liquidation boundary $\bar{x}_V$ (see Figure 3).

The previous argument suggests that when the firm is liquidated at $\bar{x}_{\mathrm{AG}}$, the optimal level of debt (c) is nonmonotonic in ε, the parameter that describes the differences in payoffs of creditors who grab assets and creditors who do not grab assets and hence describes the incentives to grab assets. As it turns out, this nonmonotonicity of the optimal c in ε also implies that optimal leverage (D/E) of a firm is nonmonotonic in ε, as illustrated in Figure 5.

FIGURE 5

Optimal Leverage as a Function of ε

Optimal leverage (optimal D/E) as a function of the parameter ε that describes the differences in payoffs between creditors who grab assets and creditors who do not grab assets, and hence the incentives to grab assets, as achieved by bankruptcy codes.



This is a potentially testable implication of the model. Consider, for example, the "creditor rights score" proposed by La Porta et al. (1998) and used in much subsequent work. This score is based in part on whether creditors are subject to an automatic stay (higher score if they are not). Being subject to an automatic stay or not affects incentives to grab assets, which in the context of the model are described by the parameter ε. The analysis here suggests that the relationship between a "creditor rights" score and leverage might very well be nonmonotonic: Very weak "creditor rights" make debt unattractive and make low leverage optimal. Medium "creditor rights" make debt more attractive and make higher leverage optimal. Very strong "creditor rights" can lead to very early asset grabs, which make debt less attractive and make low leverage optimal.

C. A Social Planner

One can now ask the question, "How would a social planner optimally adjust bankruptcy law to maximize welfare?" A social planner could, for example, adjust features of bankruptcy codes such as preference law to adjust the incentives to grab assets, and hence liquidation timing (the parameter ε), and hence the choice of capital structure, and hence welfare.

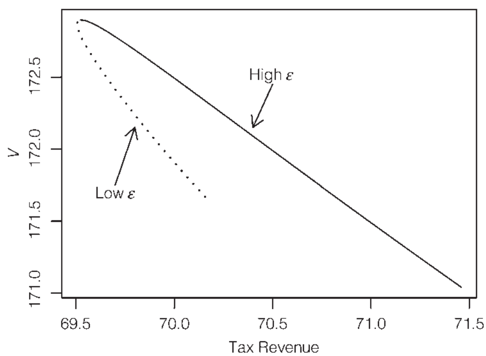
A key question in this context is how welfare should be measured. A social planner is likely to care about firm values. However, a social planner is also likely to care about the (expected) tax revenues raised from firms. In a fully fledged discussion, one could set up a formal problem where the objective of the social planner is, for example, to maximize firm value subject to the constraint that a certain amount of tax revenues has to be raised. Reasonable choice variables in such a problem could be tax rates, and the parameter ε that describes incentives to grab assets. In general, this would lead to a welfare function in which the social planner implicitly attaches different (positive) *weights* to firm value and tax revenues.

Even without such a fully fledged discussion, it is possible to make some statement about which values of ε a social planner would not want: One can consider how different ε lead to different combinations of both firm value and tax revenues (taking the tax parameter τ as given), and find those that are Pareto dominated. For such combinations, welfare can always be improved, regardless of the precise weights on firm value and tax revenues, as long as these are positive. Essentially, a social planner would choose combinations that lie on an "efficient frontier," as in Figure 6.

FIGURE 6

Firm Values and Tax Revenues

Combinations of firm value and tax revenues traced out by varying $\varepsilon \in [0, 0.1]$. The solid line denotes Pareto-dominating combinations ($\varepsilon \gtrapprox 0.04$), and the dotted line denotes Pareto-dominated combinations ($\varepsilon \lessapprox 0.04$).



As it turns out, for the given parameters, low values of ε produce Pareto-dominated combinations of firm value and tax revenues (dotted part of the line in Figure 6). Why is this? For very low ε, liquidation happens very late, and firm values are low. For slightly higher ε, liquidation happens earlier, and firm values can be higher. Earlier liquidation reduces expected tax revenues, ceteris paribus, since no taxes are paid after the firm has been liquidated. However, earlier liquidation can make lower debt levels optimal, which decreases the tax shield, increasing expected tax revenues.

From a practical perspective, this indicates that bankruptcy codes that achieve (close to) full equality of distributions to creditors, and hence produce very small incentives to grab assets, can lead to low welfare.

VI. Conclusion

This paper presents a continuous-time structural model in which defaulting firms are liquidated when creditors attempt to enforce claims against these firms. It considers how the values of debt, equity, and the firm are affected by the actions of coordinated creditors and uncoordinated creditors. In the case of uncoordinated creditors, the effect of features of (Chapter 7-type) bankruptcy codes such as automatic stays, preference law, and policies of equality of distributions that affect creditor coordination are also discussed. Closed-form solutions are derived for the values of debt, equity, and the firm.

In the model, coordinated creditors can have incentives to liquidate prematurely, in the sense that firm value would be higher if the firm was liquidated later. Uncoordinated creditors care about payoffs in an asset grab game. If legal costs of grabbing assets are low, they can have incentives to grab assets too early. Features of Chapter 7-type bankruptcy codes that affect creditor coordination (automatic stays, preference law, policies of equality of distribution) change the payoffs in the asset grab game such that grabbing assets becomes less attractive, protecting debtors. This leads to later liquidation.

The level of debt has an effect on when the firm is liquidated, both in the case in which creditors are coordinated as well as in the case where creditors are uncoordinated. This means that debt levels should ideally be adjusted to improve liquidation timing. The paper shows how this motive for adjusting debt levels can interact with a standard debt-equity trade-off of a tax advantage versus expected financial distress costs.

Since the strength of incentives for asset grabs produced by bankruptcy codes affects the sensitivity of liquidation timing to debt levels, it affects the optimal capital structure decision. In the model, optimal leverage can be first increasing and then decreasing in the strength of incentives to grab assets, suggesting that in empirical comparisons, the relationship between "creditor rights" (La Porta et al. (1998)) and capital structure can be nonmonotonic.

Finally, one can consider the problem of a social planner that decides to adjust the strength of incentives to grab assets to maximize welfare. As long as the welfare function attaches positive weight to both firm values and tax revenues, very weak incentives to grab assets (produced, e.g., by automatic stays combined with a strong policy of equality of distributions to creditors) can be suboptimal: Low incentives to grab mean very late liquidation. Increasing incentives to grab assets means earlier liquidation. This can increase firm values. In addition, although it reduces expected tax revenues ceteris paribus (because no taxes are paid once the firm is liquidated), it can decrease optimal leverage, which decreases the tax shield and hence increases expected taxes ceteris paribus. So earlier liquidation can increase both firm values and expected tax revenues.

Appendix A. Solution of the Asset Grab Game

1. Basic Procedure

The solution procedure for a single stage of the discrete-time repeated game is the same as that of Morris and Shin (2004). Suppose that creditors follow a switching strategy around a certain posterior belief. Given the posterior belief around which creditors switch, it is possible to derive the critical next-period cash flow for which the firm will be liquidated. Given a critical cash flow for which the firm will be liquidated, it is possible to derive the belief around which creditors switch. This produces 2 equations in 2 unknowns, which can then be solved for the critical cash flow for which the firm is liquidated.

2. Posteriors

Given the assumptions on the information structure in the main text, posteriors can be worked out as follows: From the signal ξ_i and the public information x_t, creditors form a posterior about the cash flow in period $t+\Delta$, $x_{t+\Delta}$ that is normal, with mean and variance given by

$$\text{(A-1)} \qquad \rho_i = \mathrm{E}[x_{t+\Delta}|\xi_i] = \frac{\alpha(1+(\mu+\sigma\nu_t)\Delta)x_t+\beta\xi_i}{\alpha+\beta}, \quad \text{and}$$

$$\mathrm{Var}(x_{t+\Delta}|\xi_i) = \frac{(x_t)^2}{\alpha+\beta}.$$

3. Critical Value of $x_{t+\Delta}$ for Which the Firm Is Liquidated

Suppose creditors follow a switching strategy around ρ^* (i.e., creditors grab assets when their posterior mean is below ρ^*). Then a creditor will not grab assets if and only if the private signal is bigger than

$$\text{(A-2)} \qquad \xi^* = \frac{\alpha+\beta}{\beta}\rho^* - \frac{\alpha}{\beta}(1+(\mu+\sigma\nu_t)\Delta)x_t.$$

Conditional on state $x_{t+\Delta}$, the distribution of ξ_i is normal with mean $x_{t+\Delta}$ and precision β/x_t^2. So the ex ante probability for any creditor of grabbing assets is equal to $\Phi\left\{(1/x_t)\sqrt{\beta}(\xi^*-x_{t+\Delta})\right\}$, where $\Phi(\cdot)$ is the cumulative standard normal density function. The fraction of creditors that grab assets will be equal to this ex ante probability for any individual creditor by some version of the law of large numbers.

Since the firm fails if the fraction that grabs assets is $l \geq x_{t+\Delta}/c$, the critical value of $x_{t+\Delta}$ (denoted by $\bar{x}^{\mathrm{AG}}_{t+\Delta}$) for which the firm fails at t is given by $\bar{x}^{\mathrm{AG}}_{t+\Delta} = c\Phi\left\{(1/x_t)\sqrt{\beta}\left(\xi^*-\bar{x}^{\mathrm{AG}}_{t+\Delta}\right)\right\}$ or

$$\text{(A-3)} \qquad \bar{x}^{\mathrm{AG}}_{t+\Delta} = c\Phi\left\{\frac{1}{x_t}\left(\frac{\alpha}{\sqrt{\beta}}(\rho^*-(1+(\mu+\sigma\nu_t)\Delta)x_t)+\sqrt{\beta}\left(\rho^*-\bar{x}^{\mathrm{AG}}_{t+\Delta}\right)\right)\right\}.$$

4. Critical Value of ρ around Which Creditors Switch

Payoffs in a stage of the game are as described in the payoff matrix in the main text.

Creditors will switch if they believe that they will obtain a higher utility from doing so. One finds the critical ρ^* around which creditors switch by considering the marginal creditor, for whom the expected utility of not grabbing assets should just equal the expected utility of grabbing assets.

Let F denote the posterior cumulative distribution (given the belief) over the cash flow $x_{t+\Delta}$, let $u(\cdot)$ denote the (Bernoulli) utility function that maps instantaneous payoffs into instantaneous utility, let δ be the subjective discount factor, and let U_{L} and U_{NL} denote the future utility associated with the firm being liquidated and not liquidated in the current period, respectively. Then for the marginal creditor,

$$\begin{aligned}\int_{-\infty}^{\overline{x}_{t+\Delta}^{\mathrm{AG}}} u((1-s)K)dF + \delta \int_{-\infty}^{\overline{x}_{t+\Delta}^{\mathrm{AG}}} U_{\mathrm{L}}dF + \int_{\overline{x}_{t+\Delta}^{\mathrm{AG}}}^{\infty} u(-sK)dF + \delta \int_{\overline{x}_{t+\Delta}^{\mathrm{AG}}}^{\infty} U_{\mathrm{NL}}dF \quad &= \\ \int_{-\infty}^{\overline{x}_{t+\Delta}^{\mathrm{AG}}} u(0)dF + \delta\int_{-\infty}^{\overline{x}_{t+\Delta}^{\mathrm{AG}}} U_{\mathrm{L}}dF + \int_{\overline{x}_{t+\Delta}^{\mathrm{AG}}}^{\infty} u((1-\varepsilon)(1-s)K)dF + \delta\int_{\overline{x}_{t+\Delta}^{\mathrm{AG}}}^{\infty} U_{\mathrm{NL}}dF.&\end{aligned}$$

The terms in U_{L} and U_{NL} cancel, since future payoffs do not depend on the actions of an individual creditor. Also, since the Bernoulli utilities do not depend on x directly, one can move them out of the integrals to obtain

$$\begin{aligned}u((1-s)K)\Pr\left(x_{t+\Delta} \leq \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) + u(-sK)\Pr\left(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) \quad &= \\ u((1-\varepsilon)(1-s)K)\Pr\left(x_{t+\Delta} \leq \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) + u(0)\Pr\left(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}}\right).&\end{aligned}$$

Noting that $\Pr\left(x_{t+\Delta} \leq \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) = 1 - \Pr\left(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}}\right)$, one can rewrite this as

$$\text{(A-4)} \quad \begin{aligned}\Pr\left(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) \quad &= \quad \frac{u((1-s)K) - u((1-\varepsilon)(1-s)K)}{u((1-s)K) - u((1-\varepsilon)(1-s)K) + u(0) - u(-sK)} \\ &:= \quad \theta,\end{aligned}$$

where θ is defined by this expression. Note that $\theta = \varepsilon(1-s)/(\varepsilon(1-s)+s)$ when creditors are risk neutral.

For the marginal creditor, the probability $\Pr(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}})$ is given by

$$\text{(A-5)} \qquad \Pr\left(x_{t+\Delta} > \overline{x}_{t+\Delta}^{\mathrm{AG}}\right) \quad = \quad \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\rho^* - \overline{x}_{t+\Delta}^{\mathrm{AG}}\right)\right\},$$

where $\Phi(\cdot)$ denotes the cumulative normal density function.

One can equate expressions (A-5) and (A-4) to obtain

$$\text{(A-6)} \qquad \rho^* - \overline{x}_{t+\Delta}^{\mathrm{AG}} \quad = \quad \frac{x_t}{\sqrt{\alpha+\beta}}\Phi^{-1}(\theta).$$

5. Solving for $\overline{x}_{t+\Delta}^{\mathrm{AG}}$

Combining expressions (A-6) and (A-3), one can solve for the critical level of the cash flow in the next period that causes failure in the preceding intermediate period:

$$\text{(A-7)} \qquad \overline{x}_{t+\Delta}^{\mathrm{AG}} \quad = \quad c\Phi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{\overline{x}_{t+\Delta}^{\mathrm{AG}}}{x_t} - 1 - (\mu + \sigma\nu_t)\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta)\right\}.$$

Reorganization at time $t+q$ in the stage just before $t+\Delta$ will occur when x hits $\overline{x}_{t+\Delta}^{\mathrm{AG}}$ at $t+\Delta$.

6. Uniqueness

Lemma 1. The critical value of the cash flow $\overline{x}_{t+\Delta}^{AG}$ is unique if

$$c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1. \tag{A-8}$$

Proof. This is a version of the proof in Morris and Shin (2004). A sufficient condition for a unique solution is that the slope of the right-hand side (RHS) of equation (A-7), seen as a function of $\overline{x}_{t+\Delta}^{AG}$, is less than 1 everywhere. This slope is equal to

$$c\varphi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{\overline{x}_{t+\Delta}^{AG}}{x_t}-1-(\mu+\sigma\nu_t)\Delta\right)+\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta)\right\}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t}. \tag{A-9}$$

The standard normal density reaches a maximum of $1/\sqrt{2\pi}$ at 0, hence a sufficient condition for a unique solution is that $c(1/\sqrt{2\pi})(\alpha/\sqrt{\beta})(1/x_t) < 1$, as stated in expression (A-8). □

This type of condition is standard for this type of game. Basically, for coordination failure to arise, creditors need to be sufficiently uncertain about the actions of other creditors. They will only be uncertain about the actions of other creditors if these actions reflect mostly private information. The actions will reflect mostly private information if that information is relatively useful (precise) in relation to public information. Mathematically, this means that the ratio $\alpha/\sqrt{\beta}$ has to be "relatively low."

7. Uncertainty in the Limit

Conditional on the cash flow in the next period, the probability that a creditor receives a signal that prompts her to grab assets is $\Phi\left((1/x_t)\sqrt{\beta}\,(\xi_t^*-x_{t+\Delta})\right)$. It is easily seen that this tends either to 0 or 1 as $\beta\to\infty$, depending on whether the critical level of the signal ξ_t^* is below or above the cash flow $x_{t+\Delta}$. Hence creditors will either all grab assets or all refrain from doing so.

However, in the continuous-time limit, the marginal creditor (the creditor who receives a signal $\xi_t=\xi_t^*$) views the fraction of creditors that attempt to grab assets as a random variable that is uniformly distributed (i.e., the marginal creditor is completely uncertain about this fraction). It is in this sense that strategic uncertainty remains present in the limit. This is a special case of a general result in coordination failure games in which the precision of private information goes to infinity. The general result is discussed in more detail, for example, in Morris and Shin (2003).

Lemma 2. The belief of the marginal creditor over the fraction that attempt to grab assets l is uniform in the limit.

Proof. The proportion of creditors that receive a signal lower than ξ^* is

$$l = \Phi\left\{\frac{\sqrt{\beta}}{x_t}(\xi^*-x_{t+\Delta})\right\}. \tag{A-10}$$

What is the probability that a fraction less than y of the other bondholders receive a signal higher than that of the marginal creditor, conditional on the marginal creditor's signal, or what is $\Pr\left((1-l)<y\mid\xi^*\right)$?

The event $1-l<y$ is equivalent to the event $1-\Phi\left\{(\sqrt{\beta}/x_t)(\xi^*-x_{t+\Delta})\right\}$ $< y$ or (rearranging) $x_{t+\Delta} < \xi^*+(x_t/\sqrt{\beta})\Phi^{-1}\{y\}$. So the probability in question can be expressed as

$$\Pr\left(x_{t+\Delta}<\xi^*+\frac{x_t}{\sqrt{\beta}}\Phi^{-1}\{1-y\}\,\middle|\,\xi^*\right). \tag{A-11}$$

The posterior of the marginal creditor over $x_{t+\Delta}$ has mean ρ^* and variance $x_t^2/(\alpha+\beta)$, hence this probability is

$$\text{(A-12)} \qquad \Pr\left((1-l) < y \mid \rho^*\right) = \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\{y\} - \rho^*\right)\right\}.$$

Now as one takes limits, $\rho^* \to \xi^*$, since private information becomes infinitely more precise than public information (the creditor attaches all weight to the signal and none to the mean of the prior), and $\sqrt{\alpha+\beta}/\sqrt{\beta} \to 1$. It follows that

$$\text{(A-13)} \qquad \Pr\left((1-l) < y \mid \rho^*\right) = y,$$

so the cumulative distribution of $1-l$ is the identity function, which implies that the density of $1-l$, and hence also l, will be uniform. □

Appendix B. The Continuous-Time Limit

The process given in expression (1) is a Euler discretization of the stochastic differential equation

$$\text{(B-1)} \qquad dx(t) = (\mu + \sigma\nu(t))\,x(t)dt + \sigma x(t)dW(t).$$

Consequently, if one takes limits as $\Delta \to 0$, the solution of the discretization (1) will converge both weakly (with order 1) and strongly (with order 0.5) to the solution of equation (B-1) (see, e.g., Kloeden and Platen (2000)).

The variance $\sigma^2\Delta$ needs to be $O(\Delta)$ as it tends to 0, and the precision $\alpha = (\sigma^2\Delta)^{-1}$ needs to be $O(1/\Delta)$ as it tends to infinity. It is noted in Appendix A.6 that the critical boundary $\overline{x}_{t+\Delta}^{\text{AG}}$ (and hence the equilibrium of the game) is unique if

$$\text{(B-2)} \qquad c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1.$$

Obviously, if the precision of public information α goes to infinity, a necessary condition for the equilibrium to be unique as described previously is that the precision of private information β also goes to infinity. A sufficient condition for the equilibrium to be unique as described previously is that the precision of private information β goes to infinity, but at a rate faster than $1/\Delta^2$ (i.e., that $1/\beta = o\left(\Delta^2\right)$), such that $\alpha/\sqrt{\beta}$ tends to 0.

Proof of Proposition 1. Take limits as described (i.e., let $1/\alpha = \sigma^2\Delta$ be $O(\Delta)$, let $1/\beta = o\left(\Delta^2\right)$, and take limits as $\Delta \to 0$). One can see that under these assumptions $\alpha/\sqrt{\beta} \to 0$, and that $\sqrt{\alpha+\beta}/\sqrt{\beta} \to 1$. This, in turn, implies that the limit of expression (A-7) becomes $\overline{x}_{\text{AG}} = \theta c$, as stated in Proposition 1. □

It remains to show that this limiting solution can be appropriately used for pricing. First, it will be necessary to describe the payoffs to creditors, and second, it will be necessary to describe the information set (filtration) on the basis of which prices will be formed.

One notes that in the limit, as $\beta \to \infty$, the probability that a creditor receives a signal that prompts her to grab assets will tend to either 0 or 1 (Appendix A.7). What this means is that because creditors essentially all receive the same information (as the signal becomes infinitely precise), creditors will either all grab assets or will all refrain from doing so. For any creditor, the ex ante probability of grabbing assets when the other creditors do not tends to 0. Also, the probability of not grabbing assets if all other creditors are grabbing assets tends to 0. In the limit, creditors receive the same signals, and there is no uncertainty about the cash flow. However, as shown in Appendix A.7, strategic uncertainty remains in the sense that the *marginal* creditor, that is, the creditor who receives a signal such that the posterior mean is equal to the critical posterior mean around which creditors switch, has maximum uncertainty about the actions of other creditors. Hence the quality of the equilibrium is preserved in the limit. At the same time, since in equilibrium the creditors act in unison, the payoffs are the same for all creditors and are essentially as described in Section II.

Also, due to the timing assumptions, no private information is available at the time at which prices are formed. The filtration relevant for pricing here will be that generated by the cash flow process $x(t)$.

Appendix C. Valuation

The valuation problem here is one of working out the values of the payoffs as described in Section II, contingent on the cash flow $x(t)$, for a given liquidation boundary $\bar{x}$. One can first change to the pricing measure $\mathbb{Q}$ defined by the money market account. The differential of the cash flow $x(t)$ can be expressed in terms of a Brownian motion under this measure as

$$dx(t) = \mu x(t)dt + \sigma x(t)d\tilde{W}(t), \tag{C-1}$$

where $d\tilde{W}(t) := dW(t) + \nu dt$.

In general, for any value (value of debt, equity, or the firm), there will be formulas for 2 regions, 1 for the region where the firm is not defaulting on promised coupon payments ($x(t) \geq c$), and 1 for the region where the firm is defaulting on part of the promised coupon payments ($x(t) < c$).

Within a region, the method for solving the valuation problem is standard (see, e.g., Dixit (1993)). Let $F(x)$ denote the value of a perpetual claim to flow payoffs of the type $a + bx$, where x is an Itō process, and a, b are constants. Then requiring that the discounted gains process associated with the claim is a martingale under $\mathbb{Q}$ produces a pricing ordinary differential equation (ODE). For a differential of the cash flow $x(t)$ as in equation (C-1), the ODE will have the form

$$\frac{1}{2}\sigma^2 x^2 F''(x) + \mu x F'(x) + a + bx = rF(x). \tag{C-2}$$

Solutions to this ODE have the form

$$F = Ax^{-\gamma} + Bx^{\delta} + \frac{a}{r} + \frac{bx}{r-\mu}, \tag{C-3}$$

where A and B are constants of integration to be determined via boundary conditions, and $\delta > 1$ and $-\gamma < 0$ are the positive and negative roots, respectively, of the characteristic equation of the ODE, and are therefore functions of μ, σ, and r. Since there are 2 regions per claim, there are 4 constants of integration per claim.

Let $D_1(x)$ denote the value of debt in the no default region, and let $D_2(x)$ denote the value of debt in the default region. To obtain the 4 constants of integration, one can apply the following boundary conditions:

$$\lim_{x\to\infty} D_1(x) = \frac{c}{r}, \tag{C-4}$$

$$D_1(c) = D_2(c), \tag{C-5}$$

$$D_1'(c) = D_2'(c), \tag{C-6}$$

$$D_2(\bar{x}) = (1-s)K. \tag{C-7}$$

Expression (C-4) states that as the cash flow becomes very large, debt essentially becomes riskless. Expression (C-5) is a value-matching condition that states that at the point where the dynamics of the discounted gains process change (when the cash flow is just equal to the coupon), the value of the solution to both differential equations has to be the same.

Expression (C-6) is required to rule out arbitrage as the cash flow falls below the coupon (see, e.g., Dixit (1993)), and expression (C-7) is another value-matching condition that states that when the firm is liquidated, the price of debt must be equal to the liquidation payoff.

Let D_1 denote the value of debt in the no default region, and let D_2 denote the value of debt in the default region. Applying the 4 boundary conditions, the 4 constants of integration can be found to obtain the following formulas:

$$D_1(x;\bar{x}) = \frac{c}{r} + \left[D_2(c;\bar{x}) - \frac{c}{r}\right]\left(\frac{x}{c}\right)^{-\gamma}, \tag{C-8}$$

for the value of debt in the no default region, and

$$\begin{aligned} D_2(x;\bar{x}) &= \left\{\frac{(1-\lambda)x}{r-\mu} - Z((1-\lambda)c)\left(\frac{x}{c}\right)^{\delta}\right\} \\ &\quad + \left[(1-s)K - \left\{\frac{(1-\lambda)\bar{x}}{r-\mu} - Z((1-\lambda)c)\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}, \end{aligned} \tag{C-9}$$

for the value of debt in the default region, where the function $Z(y)$ is given by

$$Z(y) = \frac{\gamma}{\delta+\gamma}\left(\frac{1+\gamma}{\gamma}\frac{y}{r-\mu} - \frac{c}{r}\right). \tag{C-10}$$

Let E_1 denote the value of equity in the no default region, and let E_2 denote the value of equity in the default region. To obtain the 4 constants of integration, one can impose the following boundary conditions:

$$\lim_{x\to\infty} E_1(x) - (1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) = 0, \tag{C-11}$$

$$E_1(c) = E_2(c), \tag{C-12}$$

$$E_1'(c) = E_2'(c), \tag{C-13}$$

$$E_2(\bar{x}) = 0. \tag{C-14}$$

The interpretation of these boundary conditions is obvious, and parallels that of the boundary conditions for the value of debt. Applying these 4 boundary conditions, one obtains the following formulas:

$$\begin{aligned} E_1(x;\bar{x}) &= \left\{(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right)\right\} \\ &\quad + \left[E_2(c;\bar{x}) - \left\{(1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right)\right\}\right]\left(\frac{x}{c}\right)^{-\gamma}, \end{aligned} \tag{C-15}$$

for the value of equity in the no default region, and

$$E_2(x;\bar{x}) = \left\{(1-\tau)Z\left(\frac{x}{c}\right)^{\delta}\right\} + \left[0 - \left\{(1-\tau)Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}, \tag{C-16}$$

for the value of debt in the default region.

Finally, the value of the firm can be calculated as the sum of the value of debt and the value of equity, in each region:

$$\text{(C-17)} \qquad \begin{aligned} V_1(x;\bar{x}) &= E_1(x;\bar{x}) + D_1(x;\bar{x}) \\ &= \left\{(1-\tau)\frac{x}{r-\mu} + \tau\frac{c}{r}\right\} \\ &\quad + \left[V_2(c;\bar{x}) - \left\{(1-\tau)\frac{c}{r-\mu} + \tau\frac{c}{r}\right\}\right]\left(\frac{x}{c}\right)^{-\gamma}, \end{aligned}$$

and

$$\text{(C-18)} \qquad \begin{aligned} V_2(x;\bar{x}) &= E_2(x;\bar{x}) + D_2(x;\bar{x}) \\ &= \left\{\frac{(1-\lambda)x}{r-\mu} + A\left(\frac{x}{c}\right)^{\delta}\right\} \\ &\quad + \left[(1-s)K - \left\{\frac{(1-\lambda)\bar{x}}{r-\mu} + A\left(\frac{\bar{x}}{c}\right)^{\delta}\right\}\right]\left(\frac{x}{\bar{x}}\right)^{-\gamma}, \end{aligned}$$

where

$$\text{(C-19)} \qquad A = (1-\tau)Z(c) - Z((1-\lambda)c).$$

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