A Fiscal Stimulus with Deep Habits and Optimal Monetary Policy

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Abstract
A New-Keynesian model with deep habits and optimal monetary policy delivers a larger-than-one multiplier and consumption crowding-in. Optimized Taylor-type rules dominate a conventional Taylor rule. Consumption is crowded out if the Taylor rule is sub-optimal or if commitment is absent.

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Keywords: Deep habits, Optimal monetary policy, Price-level rule.
1 Introduction

The efficacy of a fiscal stimulus remains a controversial issue in applied macroeconomics. In particular the range of empirical government spending multipliers is wide – Ramey (2011) surveys the literature and argues that this is between 0.8 and 1.5 – and the sign of the effect on private consumption is controversial. In fact, part of the empirical literature finds evidence for a crowding-out of consumption, while many Structural Vector-Autoregressions (SVARs) provide evidence for a crowding-in effect. Canonical Dynamic Stochastic General Equilibrium (DSGE) models typically predict fiscal multipliers well below the empirical range and the crowding-out of private consumption.

A modelling device that has been used to obtain the consumption crowding-in and higher fiscal multipliers in Real Business Cycle (RBC) models is the assumption that external ‘deep habits’ à la Ravn et al. (2006) are formed in private and public consumption, i.e. habits on the average consumption level of each variety of goods. Jacob (2012) shows that in a New-Keynesian (NK) model with deep habits, increasing degrees of price stickiness soften the expansionary effects of a fiscal stimulus and may overturn the results obtainable in a RBC model.

This paper also investigates these issues in a NK model with deep habits but pays particular attention to the subtle interactions between fiscal and monetary policy that determine the outcome of a fiscal stimulus. In particular, we study a boost to government spending alongside a number of possible interest rate policies: first, the welfare-optimal (Ramsey) policy; second, a time-consistent policy; third, a conventional Taylor interest rate rule which prescribes an immediate and strong response to the output gap; fourth, an empirically based rule with a much weaker response to output; and finally an optimized simple Taylor type rule (of which a price-level rule is a special case) that turns out to closely mimic the optimal policy.1

2 Model

The model is a standard NK model with Rotemberg price stickiness and convex investment adjustment costs augmented with deep habit formation. It can be considered as an extension of the model employed by Jacob (2012) given that it also includes investment.2

1In the companion working paper version of this article we also examine the outcome of these simple rules with a zero lower bound constraint for an initial period.

2To retain a sharp focus on the issue of deep habit we abstract from unemployment. A number of recent papers examine fiscal multipliers having introduced Mortensen-Pissarides search-matching frictions into otherwise standard NK models (but without deep habit) – see Campolmi et al. (2011); Monacelli et al. (2010).
2.1 Households

A continuum of identical households \( j \in [0, 1] \) has preferences over differentiated consumption varieties \( i \in [0, 1] \). Following Ravn et al. (2006), households exhibit external deep habit formation in consumption, i.e. on the average consumption level of each variety of good. Their optimisation problem is

\[
\max \left\{ (X_t^j)^j, K_{t+1}^j, B_{t+1}^j, I_t^j, h_t^j \right\} E_t \sum_{s=0}^{\infty} \beta^{t+s} U((X_{t+s})^j, 1 - H_{t+s}^j),
\]

subject to constraints

\[
(X_t^c)^j + \Omega_t + I_t^j + \frac{B_{t+1}^j}{P_t} = W_t P_t H_t^j + R_t K_t^j + \frac{R_{t-1} B_t^j}{P_t} + \int_0^1 J_{it} di - T_t,
\]

\[
K_{t+1}^j = (1 - \delta) K_t^j + I_t^j \left[ 1 - S \left( \frac{I_t^j}{I_{t-1}^j} \right) \right],
\]

where \( \beta \in (0, 1) \) is the discount factor, \( (X_t)^j = X \left((X_t^c)^j, X_t^g \right) \) is a composite of habit-adjusted differentiated private and public consumption goods similar to that in Pappa (2009), and \( H_t^j \) are hours of work. The private component of \((X_t)^j\) is

\[
(X_t^c)^j = \left[ \int_0^1 (C_{it}^j - \theta^c S_{it-1}^c)^{1-\eta} di \right]^{\frac{1}{1-\eta}},
\]

where \( \theta^c \in (0, 1) \) is the degree of deep habit formation on each variety, and \( S_{it-1}^c \) denotes the stock of habit in the consumption of good \( i \), which evolves over time according to

\[
S_{it}^c = \varphi^c S_{it-1}^c + \left( 1 - \varphi^c \right) C_{it},
\]

where \( \varphi^c \in (0, 1) \) implies persistence. The optimal level of demand for each variety, \( C_{it}^j \), for a given composite is obtained by minimizing total expenditure \( \int_0^1 P_t C_{it}^j di \) over \( C_{it}^j \), subject to (3). This leads to

\[
C_{it}^j = \left( \frac{P_{it}}{P_t} \right)^{-\eta} (X_t^c)^j + \theta^c S_{it-1}^c,
\]

where \( P_{it} \) is the price of variety \( i \), \( P_t \equiv \left[ \int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}} \) is the nominal price index and \( \eta \) is the intratemporal elasticity of substitution. Multiplying (5) by \( P_{it} \) and integrating, real consumption expenditure \( C_t^j \) can be written as a function of the consumption composite and the stock of habit: \( C_t^j = (X_t^c)^j + \Omega_t \), where
\( \Omega_t = \theta^c \int_0^1 \frac{P_t}{P_t^c} S_{it-1}^c di \). Households hold capital holdings, evolving according to (2) where \( \delta \) is the capital depreciation rate, \( I^j_t \) is investment, and \( S(\cdot) \) represents an investment adjustment cost satisfying \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \). Investment is also a composite of goods, i.e. \( I^j_t = \left[ \int_0^1 \left( I^i_{it} \right)^{1-\frac{\eta}{\sigma}} di \right]^{\frac{1}{1-\eta}} \), but does not feature habit formation. Expenditure minimisation leads to the optimal level of demand of private investment goods for each variety \( i \):

\[
I^j_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} I^j_t. \tag{6}
\]

Households buy consumption goods, \( C^j_t \), invest in investment goods, \( I^j_t \), and nominal bond holdings, \( B^j_t \), receive the hourly wage, \( W_t \), the rental rate of capital, \( R^K_t \), the return on nominal bond holdings, \( R_t \), and firms’ profits, \( \int_0^1 J_{it} di \); and pay lump-sum taxes \( T_t \).

The first-order condition (FOC) with respect to (w.r.t.) the private consumption composite \( (X^j_{it})^j \) implies that the Lagrange multiplier on the household’s budget constraint (1) is equal to \( \Lambda^j_t = U^j_{X^c, t} \), where \( U^j_{X^c, t} \) is the marginal utility of the private consumption composite. Let \( \Lambda^j_t Q^j_t \) be the multiplier on the capital accumulation equation (2), and \( Q^j_t \) represent Tobin’s Q. Then, the FOC w.r.t. capital, \( K^j_{t+1} \), implies \( Q^j_t = E_t \left\{ D^j_{t,t+1} \left[ R^K_{t+1} + (1 - \delta)Q^j_{t+1} \right] \right\} \), where \( D^j_{t,t+1} \equiv \beta^j_{X^c, t} \) is the stochastic discount factor; the FOC w.r.t. investment \( I^j_t \) yields

\[
Q^j_t \left( 1 - S \left( \frac{I^j_t}{I^j_{t-1}} \right) \right) - S' \left( \frac{I^j_t}{I^j_{t-1}} \right) \left( \frac{I^j_{t-1}}{I^j_{t-1}} \right) + E_t \left( D^j_{t,t+1} Q^j_{t+1} S' \left( \frac{I^j_{t+1}}{I^j_t} \right) \left( \frac{I^j_{t+1}}{I^j_t} \right) \right) = 1;
\]

the FOC w.r.t. the bond holdings delivers \( 1 = E_t \left\{ D^j_{t,t+1} \frac{R_{t+1}}{\Pi_{t+1}} \right\} \), where \( \Pi_t \equiv \frac{P_t}{P_{t+1}} \) is the gross inflation rate. Finally the FOC w.r.t hours implies: \( -U^j_{H, t} = \frac{U^j_{X^c, t} W_t}{P_t} \).

### 2.2 Government

As in Ravn et al. (2006) deep habits are present also in government consumption.\(^3\) In each period \( t \), the government allocates spending \( P_t G_t \) over differentiated goods sold by retailers in a monopolistic market to maximize the quantity of a habit-adjusted composite good:

\[
X^g_{it} = \left[ \int_0^1 (G_{it} - \theta^g S^g_{it-1})^{1-\frac{\eta}{\sigma}} di \right]^{\frac{1}{1-\eta}},
\]

\(^3\)This can be justified by assuming that households derive habits also on consumption of government provided goods. One can also argue that public goods are local in nature and households care about the provision of individual public goods in their own constituency relative to others; or that procurement relationships are formed between government and firms.
subject to the budget constraint $\int_0^1 P_{it} G_{it} \, di \leq P_t G_t$, where $\theta^g$ is the degree of deep habit formation in government spending and $S_{it-1}^g$ denotes the stock of habits for this expenditure, which evolves as:

$$S_{it}^g = \theta^g S_{it-1}^g + (1 - \theta^g) G_{it}, \quad (7)$$

and exhibits persistence $\rho^g$. At the optimum

$$G_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} X_t^g + \theta^g S_{it-1}^g. \quad (8)$$

Aggregate real government consumption, $G_t$, is an autoregressive process

$$\log \left( \frac{G_t}{G} \right) = \rho_G \log \left( \frac{G_{t-1}}{G} \right) + \epsilon^G_t, \quad (9)$$

where $\rho_G$ is an autoregressive parameter and $\epsilon^G_t$ is a mean zero, i.i.d. random shock with standard deviation $\sigma^G$. The government budget constraint is simply $G_t = T_t$.

### 2.3 Firms

A continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ rents capital, $K_{it}$, and hires labour, $H_{it}$ to produce differentiated goods $Y_{it}$ with convex technology $F(H_{it}, K_{it})$, which are sold at price $P_{it}$. Firms face quadratic price adjustment costs $\frac{1}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2 Y_t$, as in Rotemberg (1982) – where parameter $\xi$ measures the degree of price stickiness – and maximize the following flow of discounted profits:

$$J_{it} = E_t \left\{ \sum_{s=0}^{\infty} D_{t,t+s} \left[ \frac{P_{it+s}}{P_{it+s}} (C_{it+s} + G_{it+s} + I_{it+s}) \right. \right.$$

$$\left. - \frac{W_{it+s}}{P_{it+s}} H_{it+s} - R_{it+s} K_{it+s} - \frac{\xi}{2} \left( \frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 Y_t \right] \right\},$$

with respect to $K_{it+s}$, $H_{it+s}$, $C_{it+s}$, $S_{it+s}^C$, $G_{it+s}$, $S_{it+s}^g$ and $P_{it+s}$ subject to (4), (5), (6), (7), (8), and the firm’s resource constraint

$$C_{it+s} + G_{it+s} + I_{it+s} = F(H_{it}, K_{it}) - FC = Y_{it}, \quad (10)$$
where $FC$ are fixed production costs, set to ensure that the free entry condition of long-run zero profits is satisfied. The corresponding first-order conditions for this problem are:

\[
\begin{align*}
R_t^K &= MC_t F_{K,it}, \\
W_t &= MC_t F_{H,it}, \\
\nu^c_t &= \frac{P_{it}}{P_t} - MC_t + (1 - \varphi^c)\lambda^c_t, \\
\lambda^c_t &= E_t D_{t,t+1}(\theta^c \nu^c_{t+1} + \varphi^c \lambda^c_{t+1}), \\
\nu^g_t &= \frac{P_{it}}{P_t} - MC_t + (1 - \varphi^g)\lambda^g_t, \\
\lambda^g_t &= E_t D_{t,t+1}(\theta^g \nu^g_{t+1} + \varphi^g \lambda^g_{t+1}),
\end{align*}
\]

Variables $MC_t$, $\nu^c_t$, $\lambda^c_t$, $\nu^g_t$, $\lambda^g_t$ are the Lagrange multipliers associated with constraints (10), (5), (4), (8) and (7) respectively. In particular, $MC_t$ is the shadow value of output and represents the firm’s real marginal cost.

### 2.4 Monetary policy

Monetary policy is set either (i) optimally as the solution to a Ramsey problem, in which the monetary authority maximizes households’ welfare or (ii) to be welfare-optimal subject to a time-consistency constraint or (iii) according to a Taylor-type interest-rate rule:

\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_r) \left[ \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{\bar{Y}} \right) \right],
\]

where $\rho_r$ is the interest rate smoothing parameter and $\rho_\pi$ and $\rho_y$ are the monetary responses to inflation and the ‘output gap’, $\frac{Y_t}{\bar{Y}}$, where $\bar{Y}$ is the deterministic steady state. 4 or (iv) as a price-level rule:

\[
\log \left( \frac{R_t}{R} \right) = \log \left( \frac{R_{t-1}}{R} \right) + \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{\bar{Y}} \right);
\]

\footnote{However, strictly speaking the output gap in the original Taylor rule is $\frac{Y_t}{\bar{Y}}$ where $Y^*$ is the flexi-price output. The former type of rule has the advantage that it is readily observed, so for the most part the rules examined take this form. However we compare the two forms at the end of the paper.}
Both sub-optimal and welfare-optimal forms of these simple rules are examined.

2.5 Equilibrium

In equilibrium all markets clear. The resource constraint completes the model:

\[ Y_t = C_t + I_t + G_t + \frac{\xi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t. \]

3 Functional forms

The utility function specializes as

\[ U(X_t, 1 - H_t) = h X_t^{(1-\rho)(1-H_t)^\rho} \frac{1-\sigma_c-1}{1-\sigma_c}, \]

where \( \sigma_c > 0 \) is the coefficient of relative risk aversion, and \( \rho \) determines the weight of leisure and the consumption composite in agents’ utility. The consumption composite in turn is a Cobb-Douglas aggregate of private and public consumption with \( \nu_x \) representing the share of private consumption in the aggregate. Investment adjustment costs are quadratic:

\[ S \left( \frac{I_t}{I_{t-1}} \right) = \gamma \left( \frac{I_t}{I_{t-1}} - 1 \right)^2, \quad \gamma > 0, \]

while the production function is Cobb-Douglas:

\[ F(H_t, K_t) = (A_t H_t)^\alpha K_t^{1-\alpha}, \]

where \( A_t \) is a labour-augmenting technology shock and \( \alpha \) represents the labour share of income.

4 Parameter choice

Most parameter values are taken directly from the calibration exercise of Ravn et al. (2006): \( \beta = 0.9902, \eta = 5.3, \delta = 0.0253, \sigma_c = 2, \theta^c = \theta^g = 0.86, \rho^c = \rho^g = 0.85, \rho_G = 0.9 \). Parameters \( \rho \) is set at the steady state to target \( h = 0.33 \) and \( \nu_x \) to make \( G/Y = 0.20 \) welfare-optimal, the labor share of income \( \alpha = 0.67 \), and the investment adjustment cost parameter \( \gamma = 5 \) as estimated by Christiano et al. (2005).

The Rotemberg parameter \( \xi \) is set equal to 25.304, which corresponds to Calvo contracts of an average duration of 3 quarters. For the conventional Taylor rule (Taylor, 1993) there is no persistence \( (\rho_r = 0) \) and \( \rho_y = 0.5 \). Estimated Taylor rules typically reveal considerable persistence and a less aggressive response to output: we choose an empirical rule from Smets and Wouters (2007) (SW) where \( \rho_r = 0.81, \rho_x = 2.04, \rho_y = 0.08 \). The ‘quasi-empirical’ rule is a compromise, i.e., the same \( \rho_r \) and \( \rho_x \), but \( \rho_y = 0.5 \) as in the conventional Taylor rule.
5 Results

We report the impulse responses to a government expenditure shock of size 1 percent of steady-state output to be able to interpret the output response as a fiscal multiplier.

Before proceeding to the main results in a NK model, Figure 1 examines this fiscal stimulus in its flexi-price core. The main feature of Ravn et al. (2006) is borne out by these impulse responses: in the absence of deep habits the multiplier is well below unity, but when deep habits are added, the multiplier is over 2, brought about by a reduction in the mark-up. Investment and consumption are now crowded in rather than crowded out. Demand for labour rises bringing about a substantial increase in the real wage and hours worked.

Turning to the NK model with monetary policy, Table 1 reports the rules set out in Section 2.4 and welfare outcomes compared with the optimal policy.\(^5\) To compute welfare-optimal policy subject to constraints in the case of time-consistent and optimized simple rules, we employ a linear quadratic approach as for example in Woodford (2003, Section 6.5) and Levine et al. (2008b). In the chosen environment the steady state is distorted by the price mark-up under monopolistic competition and external deep habits and we employ a numerical procedure to carry out an accurate LQ approximation for this case.\(^6\) We do not assume taxation policy is in place to remove steady-state inefficiency, but even if we did the fiscal shock itself is inefficient since the model is calibrated so that the observed share of government spending is optimal. With either inefficient shocks or a distorted steady state it is now well-established in the literature that using monetary policy alone, perfect stabilization (keeping prices stable and real outcomes at their flexi-price level) is not optimal.\(^7\)

There are four sources of forward-looking behaviour in our model: the Euler consumption equation, investment, and habit in both consumption and government services. This feature introduces a considerable degree of time inconsistency into the optimal Ramsey policy as can be seen by the substantial welfare loss in percentage terms if the monetary authority cannot commit to some form of interest rate rule. Our optimized simple rule turns out to be a quasi-price-level rule is only slightly welfare-superior to an optimized price-level rule; both come very close (well below 1%) to mimicking the welfare outcome of optimal policy.

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\(^5\)See Levine et al. (2008a) for details of these three monetary policy regimes. Note that these optimized simple rules are shock-dependent and here only apply to a fiscal shock with the assumed persistence. In a stochastic environment facing many shocks they need to be redesigned and will be dependent on the relative persistence and variances of all shocks. It then becomes important to estimate the model, including the properties of the shocks, before proceeding to the design of such rules.

\(^6\)Leith et al. (2012) show that the distortion generated by external deep habits create particularly strong trade-offs for optimal monetary policy.

\(^7\)Woodford (2003, Section 6.5) employs a simple NK model to demonstrate this.
<table>
<thead>
<tr>
<th>Rule</th>
<th>$[\rho_r, \rho_y]$</th>
<th>Welfare Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal (Ramsey)</td>
<td>not applicable</td>
<td>0</td>
</tr>
<tr>
<td>Time Consistent (TCT)</td>
<td>not applicable</td>
<td>197</td>
</tr>
<tr>
<td>Conventional Taylor</td>
<td>[0, 1.5, 0.50]</td>
<td>76.6</td>
</tr>
<tr>
<td>Empirical Taylor (SW)</td>
<td>[0.81, 2.04, 0.08]</td>
<td>8.0</td>
</tr>
<tr>
<td>Quasi-Empirical Taylor</td>
<td>[0.81, 2.04, 0.50]</td>
<td>17.6</td>
</tr>
<tr>
<td>Optimized Simple</td>
<td>[0.95, 0.099, 0.000]</td>
<td>0.22</td>
</tr>
<tr>
<td>Optimized Price Level</td>
<td>[1.00, 0.0064, 0]</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Notes: The welfare loss is reported as a % increase of that under optimal policy. For integral simple rules with $\rho_r = 1$, the rule is expressed as $\log \left( \frac{R_t}{R_t} \right) = \rho_r \log \left( \frac{R_{t-1}}{R_t} \right) + \rho_y \log \left( \frac{R_t}{P_t} \right) + \rho_y \log \left( \frac{Y_t}{Y_t} \right)$.

Table 1: Optimal and ad hoc Monetary Rules Compared

As discussed before, a conventional Taylor rule involves an instantaneous and over-aggressive response to output compared with optimized rules, resulting in a significant welfare loss. The estimated empirical rule, by contrast, is much closer to being welfare-optimal whilst the quasi-empirical rule is somewhere in between.

Figure 2 shows the impulse response functions (equivalent to fiscal multipliers) to a fiscal shock when monetary policy is either ex ante optimal, time-consistent or conducted using either the optimized or the conventional Taylor simple commitment rule reported in Table 1. We see that the model delivers a fiscal multiplier above one for a prolonged period and the crowding-in effect on private consumption if the monetary authority can commit to some ex ante optimal rule. Comparing Figures 1 and 2 it is interesting to note that the ex ante optimal commitment and flexi-price outcomes are different, the reason being the non-optimality of perfect stabilization, as discussed. In particular, the fiscal multiplier is slightly smaller than in the flexi-price case. If it cannot commit, then the model provides some support for fiscal stimulus pessimism with a crowding-out effect on private consumption. The same applies to a fiscal stimulus alongside the conventional Taylor rule.

Finally we re-examine the optimized simple rule using Taylor’s original definition of the output gap $Y_t^*$ where $Y_t^*$ is the flexi-price output. Figure 3 and 4 compare non-optimal and optimized forms. We have seen from Figures 1 and 2 that the fiscal multiplier is higher for a flexi-price compared with the NK model, which is in line with the point made by Jacob (2012). It follows that the output gap falls calling for a ceteris paribus decrease in the nominal interest rate with non-optimal rules. As a result, consumption is crowded in also under the non-optimal Taylor rule (Figure 3). This rule tends to stabilize both the output gap and inflation. However, this is not optimal for the reasons discussed earlier. In fact, when the
two types of rules are designed to be optimal (Figure 4), they result in almost identical real and inflation outcomes, though by means of quite different interest rate paths. There is then clearly an advantage in using a rule that respond to output relative to its steady state rather than its flexi-price level: the former is observable and, when welfare-optimal, it turns out mimic the latter.

6 Conclusions

This paper shows that (i) for an empirically relevant degree of price stickiness, when a RBC à la Ravn et al. (2006) is turned into a NK model and monetary policy is set optimally, the model delivers a fiscal multiplier above one and the crowding-in effect on private consumption obtainable in a RBC; (ii) an optimized simple Taylor-type interest-rate rule yield results close to optimal policy and dominates a conventional Taylor rule; (iii) private consumption is crowded out and the fiscal multiplier experiences a sizeable contraction if the Taylor rule negates the fiscal stimulus with an immediate and high response to the output gap that, we show, is implausible from both a normative and positive perspective, or if the government cannot commit; (iv) the original optimized Taylor rule responding to output relative to its flexi-price level can be mimicked by an observable optimized rule responding to output relative to its steady state.

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References


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8 The optimized rule responding to $\frac{Y_t}{Y}$ is similar to the one reported in the table responding to $\frac{Y_t}{P}$ with $\rho_r = 0.96$, $\rho_e = 0.047$ and $\rho_y = 0.000$. 


Figure 1: A government spending expansion under flexible prices

Figure 2: A government spending expansion under alternative monetary regimes
Taylor Rule on Output relative to Steady State

Taylor Rule on Output Gap

Figure 3: A government spending expansion under Non-Optimal Taylor-Type Rules

Taylor Rule on Output relative to Steady State

Taylor Rule on Output Gap

Figure 4: A government spending expansion under Optimized Taylor-Type Rules