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Automatic Tracing of Blood Flow Velocity in Pulsed Doppler Images

Zhe Wang, Student Member IEEE, Greg Slabaugh, Member IEEE Mengchu Zhou, Fellow, IEEE and Tong Fang, Member IEEE

Abstract—Assessment of blood flow velocity in Doppler images is of great importance in clinical studies and research. From the Doppler waveform envelope, numerous indices can be obtained, such as the pulsatility index, resistance index, and systolic/diastolic ratio, as well as acceleration of the blood through valves. The evaluation for the Doppler images is usually conducted off-line and manually by the physicians. Fully-automatic detection of the envelope has the advantages of being convenient, time and labor saving. The main objective of this paper is to propose an automated technique based on image processing and computer vision algorithms for real-time tracing of the waveform envelope in a sequence of pulsed Doppler images. To this end, first we establish an information-theoretic image model and a statistical shape-driven dynamical model, which are used to address the large degree of noise and poor contrast common in this application. Relying upon these two models, we construct a discrete Kalman filter for the recursive estimation of the blood velocity envelope, while taking into account the measurement noise from these two sources. The models and Kalman filter form an adaptive weighting, closed-loop envelope tracing framework. We present the theory and implementation of our methodology, and demonstrate its ability to accurately trace the blood flow velocity in pulse wave Doppler images as well as its robustness to noise and computational efficiency.

I. INTRODUCTION

In recent decades, the feasibility of measuring blood flow in the heart and vessels using the Doppler effect in ultrasonic waves has become well known [3]. Since pulse wave Doppler, also known as pulsed Doppler, allows users to obtain the flow information at any depth on the sound beam axis simultaneously with B-mode and M-mode images, it is widely used at present.

The velocity of blood in intact blood vessels can be tracked non-invasively by identifying the Doppler shifts from a backscattered ultrasound signal. And the envelope of pulsed Doppler signal has been the important feature that characterizes blood flow. From the envelope, various clinical indices can be computed, such as the pulsatility index, resistance index, and systolic/diastolic ratio, as well as acceleration of the blood through valves. Currently, only manual methods are used clinically in order to extract these indices. Well-trained physicians trace the flow images by hand. Therefore, the manual extraction method is both time and labor consuming. Moreover, it has limited reproducibility and is subject to observer variability. An automated method to analyze the Doppler signal can improve its accuracy and can result in a powerful tool for clinical noninvasive evaluation of the blood flow. The real-time tracing of blood flow velocity for pulsed Doppler image sequences, especially the slope of envelope, considered as the acceleration of the blood through valves, is of great importance to physicians for diagnostic use. There have been many tracing algorithms proposed recently. However, few of them are designed specifically for ultrasound pulsed Doppler image sequences.

Moreover, the envelope of the signal may be smeared due to low SNR of the acquisition apparatus or a small percentage of fast moving blood. Under such conditions, without a prior model to assist the tracing, most classical image processing algorithms may fail to produce a valid result.

This work is inspired by a number of model-based image segmentation algorithms in the literature. Having some expectation of the shape can greatly assist in the tracing from image sequences. A statistical model provides a means of automatically deriving more complex information directly from a training set, once the important relationships have been identified. Leventon et al. [2] proposed a shape-based segmenter. They incorporated shape information as a prior model to restrict the flow of an active contour. Staib and Duncan [4] used an elliptic Fourier decomposition of the boundary and placed a Gaussian prior to incorporate shape information into the segmentation. Tsai et al. [5] calculated the parameters of an implicit representation of the curve to minimize the energy functions for image segmentation.

This paper proposes a model-based approach for ultrasonic pulsed wave Doppler tracing. Using a noise model, our method applies an information-theoretic edge detector to robustly identify changes in intensity distributions that appear at the envelope. The edge detector results form the image model. Our method also applies a statistical shape model to help guide the tracing when the edge detector results are ambiguous. We incorporate these components into an overall system using a Kalman filter for Bayesian tracking. The use of a Kalman filter algorithm is advantageous for the dynamic measurement of the envelope of blood velocity in Doppler image sequences for the following reasons. First, Kalman filter is a recursive procedure, it requires minimum amount of storage for the past samples. Second, it provides an efficient mechanism for modelling slowly time-varying noisy systems. Third, the accuracy of estimation can be assessed by monitoring the error covariance. The Kalman filter is well
known for its capability of supporting estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown [1]. Our resulting approach is robust to noise and computationally efficient, allowing tracing to occur in real-time as data is input to the system.

The rest of this paper is organized as follows. Section II describes our shape models, first the information-theoretic image model, and then the PCA (Principal Component Analysis)-based shape-driven dynamic model. Section III introduces the system framework in detail and Section IV provides experimental results that demonstrate the ability of our proposed algorithm to trace the envelope in pulsed Doppler image sequences, even for low signal to noise ratio images.

II. Multi-modal Shape Models

This work proposes a model-based, closed-loop algorithmic framework for Pulsed Doppler tracing. The system diagram is depicted in Fig. 1. In our algorithm, both a information-theoretic image model and a statistical shape-driven dynamical model are developed for assisting the tracing process. In order to constrain or direct the model advancing and evolving, the Kalman filter is employed.

A. Information-theoretic Image Model

Due to speckle and electronic noise, Pulsed Doppler images typically have a noisy appearance that makes edge detection rather difficult. This issue is compounded by the fact that the blood moving through the region of interest has varying speed, and much of the blood can be moving with the velocity slower than the peak velocity that we are interested in tracing. For these reasons, standard edge localization operations (gradient, Sobel, Canny edge detectors) applied to the Pulsed Doppler data failed to produce useful outputs for our image model, particularly when the signal to noise ratio was low.

Therefore, we developed a new edge detection technique that compares, using information theory, the intensities of a known noise region to that of the image. The objective is to find regions in the image that differ, statistically, from the background noise. Let a region of the the known background be denoted as $B(x,y)$. Using non-parametric density estimation, we form a probability distribution $p_b(I)$ from the histogram of $B(x,y)$. This probability distribution describes the probability of a pixel of intensity $I$ occurring in $B(x,y)$ and serves as an intensity model of the image background. Similarly, we select a sub-window of the image $I(x,y)$ around pixel $(x,y)$ and form a probability distribution $p(I)$, to form a model of the intensities around the pixel. Using information theoretic concepts, we then compute the symmetric Kullback-Liebler divergence (also known as the $J$-divergence) as

$$J = \frac{1}{2} \left[ \int p_b(I) \log \frac{p_b(I)}{p(I)} dI + \int p(I) \log \frac{p(I)}{p_b(I)} dI \right] \quad (1)$$

Intuitively, $J$ provides a measure that describes how “different” the intensity distributions are. Regions of the image that match the background model will have a small $J$, while regions that differ significantly from the background model will have a large $J$. Since the computation of $J$ includes the background noise model, this method works effectively even when the noise in the image is strong. Eq. (1) is evaluated at each pixel in the image, forming a statistical comparison map $M(x,y)$. Next, we apply a standard edge localization algorithm to $M(x,y)$, such as the difference of Gaussian (DOG) filter, to produce a feature map $F(x,y)$, which is then used in our image model.

B. Statistical Shape Model

The Pulsed Doppler data may contain an incomplete and ambiguous envelope of blood velocity. In particular, the problem of determining what is and what is not an edge is confounded by the fact that edges are often partially hidden or distorted by various effects such as electronic noise, ultrasound speckle noise and so on. Due to the above reasons, an envelope tracing approach that uses solely edge detector outputs will not be robust. Therefore, we introduce PCA-based statistical shape model [6] into our algorithmic framework. The justification of this choice is that the shapes of pulse waves in the specific dataset share some common shape pattern as observed in Fig. 2. The statistical shape model is learned in an offline process using manually traced envelopes for different applications.

Since the size or structure of the shape model in the training dataset may be quite different, we separate them into several categories based on the valve being studied. Within each application, $n$ separate shapes in the database are aligned to one coherent coordinate frame.

In particular, the position in $y$ axis of each of the $n$ aligned shapes is formed as $n$ separate $d$ dimension vectors $\{s_1, s_2, ..., s_n\}$. We compute $\mu$, the mean shape of the shape database as follows,

$$\mu = \frac{1}{n} \sum_{i=1}^{n} s_i \quad (2)$$

The mean value is then subtracted from each vector to create $n$ centroid-aligned shape models, $\{\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n\}$. These mean-offset shapes are then formed into the training matrix to capture the variabilities of the training shapes. Specifically, we define the training matrix $M$ as
\[
M = [\tilde{s}_1 \tilde{s}_2 \cdots \tilde{s}_n]
\]

Singular Value Decomposition is employed to the matrix \(\Omega\),
\[
\Omega = \frac{1}{n} MM^T
\]
\[
\frac{1}{n} MM^T = U \Lambda U^T
\]
where \(\Lambda\) is a diagonal matrix of eigenvalues and columns of \(U\) are the corresponding eigenvectors, representing the \(n\) orthogonal modes of variation in the shape.

Since the dimension of \(\Omega\) can be extremely large in most cases, to obtain the eigenvalues and eigenvectors of \(\Omega\) is computationally expensive. Instead, the eigenvalues and eigenvectors of \(\Omega\) can be easily computed from those of a much smaller matrix, \(\Psi\), as defined by
\[
\Psi = \frac{1}{n} M^T M
\]

Let \(\eta\) be an eigenvector of \(\Psi\) with non-zero corresponding eigenvalue \(\lambda\). It is straightforward to show \(M \eta\) is an eigenvector of \(\Omega\) with eigenvalue \(\lambda\).

The eigenvectors represent the principal axes of variations in the training set, with the corresponding eigenvalues indicating the amount of influence its eigenvector has in determining the shape.

The matrix \(U\), consisting of eigenvectors, can be used to project an input signal into the eigenspace. Let \(k \leq n\) be the number of modes to consider, or the dimension of the reduced-rank eigenspace. Since there is no universal \(k\) that can be set, we chose \(k\) empirically for our experiments. Using these \(k\) principal modes, we calculate the weight for the input signal \(z\) on each eigenshape.
\[
\omega_i = v_i^T \cdot z
\]
where \(i = 1, 2, \cdots, k\), and \(v_i = M \eta_i\), representing the eigenshape.
\[
\bar{z} = \mu + \sum_{j=1}^{k} \omega_i (z - \mu)
\]

We now obtain a PCA-based shape model as well as image model for the tracing process.

\[\text{Fig. 2. Examples of the envelope shape.}\]

III. SHAPE MODELS INTO ENVELOPE TRACING

Given a frame of pulsed Doppler video, the prior shape information can be embedded into the tracing process. In this section, we describe the framework for envelope tracing.

A. Kalman Filtering

The problem at hand is to track the envelope of blood flow velocity from a sequence of Doppler images. We choose the vertical position of the envelope as an appropriate tracing parameter. A variety of tracking methods can be used to solve it. Among these, the Kalman filtering method has many advantages. First, it is a computationally efficient recursive procedure requiring minimum amount of storage for the past samples. The results of the previous step is used to predict the current states. Furthermore, the accuracy of the estimation can be assessed by monitoring the error covariance. Finally, a priori knowledge about the system can be readily incorporated into the Kalman filter.

The Doppler tracing problem using the Kalman filter can be formulated as follows. Let the state vector be \(x_k = [y_k \ \dot{y}_k]^T\), where \(k\) is the current step, \(y_k\) denotes the vertical position of blood flow velocity envelope, and \(\dot{y}_k\) is the derivative of \(y_k\).

The system equation can be modeled as
\[
x_k = Ax_{k-1} + w_{k-1}
\]
where \(w_k\) is assumed to be Gaussian white noise with zero mean, i.e., \(w_k \sim N(0, Q)\) and \(A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\) denotes the state transition matrix.

With the observation \(z_k = \begin{bmatrix} z_{i,k} \\ z_{p,k} \end{bmatrix}\), the observation equation is
\[
z_k = Hx_k + v_k
\]
where \(z_{i,k}\) and \(z_{p,k}\) are from the image model and PCA-based model, respectively, \(H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\) is the observation matrix and \(v_k\) is assumed to be Gaussian noise, i.e., \(v_k \sim N(0, R)\).

The Kalman filter enables one to realize a motion model taking into account system noise and observation noise. The information from the image model and PCA-based model are incorporated into the system via observation vector and observation noise covariance. The usual task of the Kalman filter is the estimation of the state vector \(x_k\) given only the observation \(z_k\). The filter operation consists of two parts, time update (prediction) and measurement update (correction).

A new value of the state vector \(\hat{x}_k\) is estimated during the prediction step based on a posteriori state estimate at step \(k-1\) given measurement \(z_{k-1}\). Further on, the estimate error covariance matrix \(P\) is arranged according to Eq. (12).
\[
\hat{x}_k^- = A \hat{x}_{k-1}
\]
\[
P_k^- = AP_{k-1}A^T + Q
\]
where \(\hat{x}_k^-\) (note the superscript minus) is our a priori state estimate at step \(k\) given knowledge of the process prior to step \(k\) and \(P_k^-\) is a posteriori state estimation at step \(k\) given measurement \(z_k\). \(P_k^-\) is the a priori estimate error covariance.
matrix at step $k$ and $P_{k-1}$ is the a posteriori estimate error covariance matrix at step $k-1$.

Every time a new observation $z_k$ is available, this value is used to update the estimation $\hat{x}_k$. In addition, the covariance matrix $P$ and the Kalman gain matrix $K$ are recalculated as follows.

$$K_k = P_k^{-1}H^T(HP_k^{-1}H^T + R)^{-1}$$  \hspace{1cm} (13)

$$\hat{x}_k = \hat{x}_k^- + K_k(Z_k - H\hat{x}_k^-)$$  \hspace{1cm} (14)

$$P_k = (I - K_kH)P_k^-$$  \hspace{1cm} (15)

**B. Feedback Information and Adaptive Weighting**

Observation $z_i$ comes from the image model, referring to the top point of the detected edge for each time point. The other observation $z_p$ denotes the reconstructed signal from the decomposed input signal in the eigenspace. The input signal is actually the output of the Kalman filter so that the prediction information from the Kalman filter can be fed back to help direct the PCA-based shape model. However, since the dimension of the input signal into eigenspace can not be too small, the Kalman filter needs to accumulate enough output for PCA model. During this period, the image model takes over the responsibility of providing inputs for the PCA model.

Since the impact of these two models on the tracing process can be varying with time, we use the observation noise matrix $R_s = \begin{bmatrix} r_i & 0 \\ 0 & r_p \end{bmatrix}$ to indicate our “trust index” for each step such that the system can weight the two models adaptively. The “trust index” is determined by the performance of edge detection. Specifically, the value of J-divergence, as shown in Eq. (1), at the detected edge are used to define the “trust index” for both image and PCA models. In (16) and (17), $\alpha$ and $\beta$ are constant and chosen at the beginning of the process empirically. If $J_k$ is large enough, it will be considered as a sign that the image model is more reliable at the current step and thus should be trusted more than PCA model, and vice versa.

$$r_p = \alpha \times \lfloor J_k / \max(J) \times 10 \rfloor / 10$$  \hspace{1cm} (16)

$$r_i = J_k - r_p$$  \hspace{1cm} (17)

**IV. EXPERIMENTAL RESULTS**

The experiments for the proposed algorithm are performed on a real, Doppler images of heart valves, depicted in Fig. 3 and 4. In particular, we tested our envelope detection algorithm on images from an aortic insufficiency (AI) dataset. The training data for building a pulse shape space consists of 10 pulses that have the envelopes drawn manually.

The tracing occurs causally as the data becomes available; therefore the figures show the tracing output as a function of time as data comes into the system. The result in Fig. 3 shows a successful tracing using the automatic tracing approach. Even in locations where the maximal blood flow velocity is not well defined, our method produces a good result due to the use of our statistical shape model. Fig. 4 shows a more challenging example where the background noise is strong, and the resulting signal to noise ratio is low. Despite these challenges, our algorithm produces a good result that is suitable for automatic computation of indices and diagnostic measurements.

**V. CONCLUSION**

The problem of envelope tracing in Pulsed Doppler images has been addressed in this paper. This is of great importance to physicians for diagnostic use. We propose a novel model-based, feedback and adaptive weighting tracing algorithm using the Kalman filter. It incorporates a non-parametric statistical comparison of image intensities in order to estimate edges in noisy Pulsed Doppler data, as well as a statistical shape model learned from manual tracings for different applications. The experimental results demonstrate that the proposed approach can be successfully applied to blood flow velocity envelope tracing for Doppler images as it is robust to noise and computationally efficient, suitable for real-time applications as the algorithm can be executed fast enough to meet real-time requirements.

**REFERENCES**

Fig. 3. Illustration of tracing blood flow velocity in Doppler images. (a) Original complete display (b)-(e) 30%, 60%, 75%, 100% tracing completed

Fig. 4. Illustration of tracing blood flow velocity in Doppler images. (a) Original complete display (b)-(e) 30%, 60%, 75%, 100% tracing completed