Multinational Learning under Asymmetric Information

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The aim of this paper is to analyze the competition between a multinational and the incumbent firm in a foreign market under asymmetric information about demand and unobservable outputs. It is shown that the incumbent firm increases its production in the first period to signal to the multinational that the demand is low. The multinational reduces its output in the foreign market in order to signal-jam. In addition, the multinational increases its production in the other market. However, total production of the multinational is lower. Implications for research and development expenditure by the multinational are examined.

1. Introduction

Multinationals seem to be ubiquitous, increasing their presence in many markets. It has long been believed that multinational firms exist in order to exploit economies of scale in production, that is, declining marginal costs. However, if that were the case, these “efficient” firms would take over the entire market. Yet such behavior does not seem to occur. Instead, multinationals seem to be encountering increasing competitive pressures as foreign markets mature. It is the premise of this paper that multinationals, like all other firms, suffer from decreasing returns to scale, that is, increasing marginal costs.

In this paper, we take a different view of multinational firms. Generally, there are political and regulatory reasons for market structures to be different in each country, and politics is often used by larger firms to enter new markets. However, once in a market, multinational firms can exploit their flexibility derived from supplying several markets. At the same time, multinational firms suffer from lack of knowledge about the fundamentals of local markets. Indeed, the local firms usually have better information about their own markets than an “outside” multinational.1 We study the behavior of multinational firms focusing on the market structure in various markets and informational asymmetries that characterize the foreign markets. In particular, we examine how the multinational firm can use its flexibility over several markets in order to learn about foreign markets and manipulate the beliefs of the local firms.

Very little theoretical work has been done to analyze the effect of this informational advantage of the local firms in a dynamic model, especially in the context of international economics (Brander 1995 surveys static models based on asymmetric information in the context of trade policy). The exceptions are Eaton and Mirman (1991) and Horstmann and Markusen

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1 In fact, this has been given as one of the reasons why multinationals may choose to enter new markets through joint ventures with the local firms (see Helpman and Krugman 1986; Desai and Hines 1996; Horstmann and Markusen 1996).
(1996). The former analyzes a two-period model of learning. However, it does not deal with multinational firms, and more important, the informational assumptions made there are not suited to the multinational analysis. Horstmann and Markusen allow learning by multinationals to be instantaneous and do not allow any competition between the multinational and the local, informed firm.

In order to study the effects of informational differences on firm behavior and the implications of the market structure in new markets, we examine duopolistic competition under asymmetric information where the uninformed firm is a multinational that operates in more than one market.

Specifically, we consider a multinational that produces a good in its home market as a monopolist and in a foreign market as a duopolist. The home demand conditions are common knowledge except for the random shocks, while a foreign demand parameter is known only to the foreign firm. The two markets are segmented in that the prices are determined independently. However, the multinational’s costs of production in the two countries are interdependent, reflecting the main feature of multinational firms, namely, the existence of a firm-specific input that can be utilized in production across locations (Caves 1982). One source of such economies and externality is the research and development (R&D) expenditure that contributes to the production both at home and in the foreign market. This element has been captured through the cost function of the multinational for convenience. 2 We also present a model explicitly showing the relationship between the cost function and the multinational’s expenditure on R&D as a guideline for other applications of the model.

We analyze a two-period model in order to study learning by the multinational. The multinational is assumed to have a prior on the unknown parameter of the demand function. It updates this prior by using the observed price and its own output choice. We assume that the firms observe only their own output. Thus, the updating by the multinational is based on conjectures about the foreign firm’s first-period output. In addition, the inability to observe the other firm’s output also implies that the foreign firm cannot infer the multinational’s actual posterior.

The dynamic aspect of the model, combined with the assumption that the first-period outputs are not observable, adds interesting and complex elements of information manipulation by the two firms to the static or myopic analysis. The foreign firm has the incentive to distort its first-period production to prevent the multinational from learning about the true state of demand and thus enjoy greater profits in the second period. Thus, the informed firm “signals.” However, since the quantities produced are not observable, the foreign firm cannot signal as effectively as it would like. Instead, unobservable quantity has the effect of providing the multinational with an incentive to signal-jam by producing less. The equilibrium is a result of these opposing tendencies.

Specifically, the main findings of this paper are that the foreign firm increases its first-period output to make the multinational believe that the demand is low, while the multinational reduces its output to make the incumbent believe that it believes that the demand is high. The multinational increases its output in other markets. However, total output of the multinational falls. We illustrate applications of the model by showing that the level of R&D expenditure undertaken by the multinational also changes as a result of learning. In particular, we show that the first-period R&D expenditure falls because of signal jamming. The increased output of the

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2 This paper abstracts from the R&D advantage of the multinational to focus on the learning process.
foreign firm in the first period can be viewed as predatory dumping (in the sense of Eaton and Mirman 1991) since it is intended to discourage the multinational from producing more in the second period.

Previous work on signal jamming has considered firms that compete in the same market as well as in segmented markets (Mirman and Urbano 1988; Eaton and Mirman 1991). This work differs from those in introducing uninformed multinationals and thus allowing us to discuss the implications for the form of multinational entry into foreign markets. This paper is most closely related to Mirman and Urbano (1988), Eaton and Mirman (1991), and Horstmann and Markusen (1996). Although seemingly very similar, the question asked in Eaton and Mirman is very different. While they show how a firm with private information in one, monopolistic market can enjoy a strategic advantage in other, duopolistic markets, this paper shows how a firm faced with an information disadvantage in a duopolistic market manipulates its output in all markets it operates in to enhance its learning. In terms of analysis, asymmetric information in the duopolistic market, coupled with the unobservability of outputs, leads to the possibility of information manipulation not only by the informed firm (as in Eaton and Mirman) but also by the multinational. Now the informed firm must conjecture the multinational’s beliefs about the unknown parameter. Moreover, Eaton and Mirman’s dumping result (overproduction by the informed firm in the first period) is only one of the possibilities due to nonuniqueness of equilibrium. Thus, it is possible that a counterintuitive equilibrium (the opposite of what they claim) can also be supported in their model. In contrast, in this paper we show that when markets are integrated, the nonuniqueness problem does not arise and that dumping occurs in equilibrium. Thus, the properties of equilibrium are fundamentally different, depending on which market is characterized by the informational asymmetry. Mirman and Urbano (1988) study integrated markets as we do. We use that model as a basis to study multinationals.

This work contributes to the theory of multinational enterprises in the spirit of Horstmann and Markusen (1996). These authors examine the mode of entry of the multinational in the face of an information disadvantage. They compare foreign direct investment with hiring a local firm to serve as its agent. However, they do not allow any competition between the uninformed multinational and the agent who is more informed. Our paper fills that gap and provides a different approach to studying the mode of entry.

This paper contributes to the existing literature on signaling, signal jamming, and experimentation (Grossman, Kihlstrom, and Mirman 1977; Milgrom and Roberts 1982; Matthews and Mirman 1983). In our model, the decisions are made simultaneously, and the output choices of the other firm are not observable. Together, these features lead to interesting differences in the results from these other models. First, the assumption of unobservable outputs leads the multinational to reduce its output in response to signaling by the incumbent. Thus, both firms distort their outputs in the first period to affect the flow of the information in their favor. Second, since the output choices are simultaneous, the value of the second-period game depends on the information available to the multinational, unlike the Matthews and Mirman model, where the entry decision decides the value of the second-period game. Finally, in our paper, although there is an opportunity for the multinational to experiment, that is, manipulate its output to acquire information for use in future periods, the experimentation does not occur. This is because all

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3 For another interpretation, see Fischer and Mirman (1994).
4 There seems to be an error in the proof of the lemma in their paper (pp. 70–3).
levels of output of the incumbent firm give the same information to the multinational because of the specific type of uncertainty assumed.

This paper is organized as follows. Section 2 presents the model. Section 3 provides the analysis of the second-period competition. Section 4 presents the analysis of the two-period competition. Section 5 contains discussion and implications of the results, especially with respect to R&D. We conclude in section 6.

2. Model

There are two firms: a multinational firm, denoted by H (for home), and a foreign incumbent firm, denoted by F. There are two segmented markets in the same good: the home market, where only the multinational supplies output, and the foreign market, where both firms supply output. That is, the multinational is a monopolist in the home market and a Cournot duopolist in the foreign market.\(^5\) There are two time periods, 1 and 2. The two periods are connected only by the process of learning and information.

*The Home Market*

The inverse demand function in the home market is

\[ P_t = a - bQ_t^H + \epsilon_t^H \]

where \( P_t \) is the price of the good in the home market at time \( t \) and \( Q_t^H \) is the quantity produced by the multinational in the home market at time \( t \). The term \( \epsilon_t \) is a random variable with support over the entire real line, with probability density function \( g(\epsilon) \), which has mean 0. The intercept \( a \) and slope \( b \) are known to both firms and are not time dependent.

*The Foreign Market*

The multinational competes with the foreign firm in the foreign market. Further, the competition is assumed to be Cournot. Thus, the inverse demand function in the foreign market is

\[ P_t^F = a - bQ_t^H - bQ_t^F + \epsilon_t^F, \]

where \( P_t^F \) denotes the price in the foreign market, \( Q_t^H \) denotes the quantity produced by the multinational in the foreign market, and \( Q_t^F \) denotes the quantity produced by the foreign firm in the foreign market. The term \( \epsilon_t^F \) is a random variable with probability density function \( f(\epsilon) \), which has support on the entire real line and mean 0. In addition, \( f \) is assumed to satisfy the monotone likelihood ratio property. Moreover, the noise term \( \epsilon_t^F \) is independent over time and of \( \epsilon_t \). Thus, the two time periods are independent except through the process of learning.

Finally, \( \bar{a} \in \{a, \bar{a}\}, \bar{a} > a \), is the intercept of the foreign demand function. (For convenience, the slope coefficient is assumed to be the same in the two markets.) We refer to \( \bar{a} \) as the high-demand condition and to \( a \) as the low-demand condition in the foreign market.

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\(^5\) We do not distinguish between trade and foreign direct investment. The only issue discussed in this paper is the effect of asymmetric information, in a given market structure, on firm behavior.
Cost Functions

Firm H produces $Q$ and $Q'^H$ at cost $c(Q + Q'^H)^2$, where $c$ is a positive constant. For simplicity, firm F is assumed to have zero costs.

Information Structure

The parameters of the demand function in the home market of the multinational are common knowledge, as are the distributions of the random variables $\epsilon'^H$ and $\epsilon^F$. The intercept of the foreign demand function is assumed to be known only to the foreign firm. The multinational has a prior belief, at the beginning of period 1, that $\bar{a}$ (high demand) occurs with probability $\rho$ and $a$ (low demand) occurs with probability $1 - \rho$. At the end of period 1, both firms are assumed to observe the price of the good in each market. In addition, each firm is assumed to observe only its own output and not the output of the other firm. The multinational uses these observations to learn about the state of the foreign demand. Learning occurs through updating of beliefs. Using Bayes’s rule, the multinational updates its beliefs about the state of the foreign demand using the observation of the market prices in both markets and its own output in the first period. We let $\rho_2$ denote the posterior probability of the multinational that the foreign demand intercept is $\bar{a}$. Specifically,

$$
\rho_2 = \frac{\rho f(P - \bar{a} + bQ'^F + bQ'^{H})}{\rho f(P - a + bQ'^F + bQ'^H) + (1 - \rho)f(P - \bar{a} + bQ'^F + bQ'^H)},
$$

where $Q'^H$ is the actual output produced by the multinational in the first period and $\hat{Q}'^F$ and $Q'^F$ are the conjectures of the multinational about the foreign firm’s first-period output.

The fact that the multinational learns from observations of the market price and its own output gives the foreign firm an incentive to manipulate its output in the first period to signal to the multinational that the demand is low. However, due to the unobservability of each other’s output, the multinational has the incentive to signal-jam by manipulating its output in the first period. Thus, the updating is also important for the foreign firm. Although the foreign firm knows the true value of the parameter $\bar{a}$, the updated belief of the multinational plays an important role in deciding its (the foreign firm’s) output strategy in the Cournot equilibrium in the foreign market. However, since the output of the multinational in the first period is not known to the foreign firm, the latter must conjecture it in order to compute the updated belief of the multinational. On the basis of this conjectured output, the foreign firm believes that the multinational’s posterior probability that $\bar{a}$ is equal to $\bar{a}$ is $\rho_2$. That is,

$$
\rho_2 = \frac{\rho f(P - \bar{a} + b\hat{Q}'^F + bQ'^{H})}{\rho f(P - a + b\hat{Q}'^F + bQ'^H) + (1 - \rho)f(P - \bar{a} + b\hat{Q}'^F + bQ'^H)},
$$

where $Q'^H$ is the foreign firm’s conjecture of the multinational’s first-period output and $\hat{Q}'^F$ and $Q'^F$ are the conjectures of the multinational about the foreign firm’s output.

Note that both $\rho_2$ and $\rho_2^F$ are calculated using Bayes’s rule, the publicly observed price of the good, and the prior belief of the multinational, which is common knowledge. Thus, these

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4 This is an out-of-equilibrium point of view necessary to determine the equilibrium outputs. This notion of “beliefs about beliefs” is necessary when the outputs are not observed in order to study how the multinational manipulates its output. In equilibrium, the conjectured output must be the same as the actual output produced. Note that the unobservability of outputs results in the difference between Equations 1 and 2 and thus is the source of signal jamming.
beliefs differ when the actual output choice of the multinational is different from the expected output conjectured by the foreign firm. However, in equilibrium, the beliefs are the same.

The foreign firm uses its computation of $\rho \varepsilon$ to derive the reaction function of the multinational and then to determine its own optimal output.\footnote{Note that the price in the home market does not provide any information about the state of the foreign market since the multinational is the only firm in this market and the two markets are segmented. The home output depends on beliefs of the multinational about the foreign market, but the home output decision contains no information about the foreign market.} Both firms maximize the sum of profits over the two periods. The discount factor is assumed to be zero for convenience. To determine the equilibrium of the model, we first solve for the equilibrium outputs produced by the two firms in the second period and then determine their value functions. We then solve the two-period maximization problems of the two firms and determine their outputs in the first period.

3. Second-Period Competition

In the second period, firm F chooses its output $Q^f_2$ to maximize its second-period profits given the multinational’s output $Q^H_2$. However, since firm F does not observe the multinational’s output in period 1 and thus cannot determine $\rho_2$, it derives the reaction function of the multinational using $\rho \varepsilon$. On the other hand, the multinational chooses its second-period outputs $Q_2$ and $Q^H_2$, using its actual posterior $\rho_2$, to maximize its second-period profits. Thus, to derive the equilibrium outputs of the two firms in the second period, we must first determine the foreign firm’s optimal output, which requires solving for the multinational’s reaction function using $\rho \varepsilon$ and then solving the actual maximization problem of the multinational using the beliefs $\rho_2$. In solving for its optimal output, the multinational takes into account the fact that the foreign firm uses $\rho \varepsilon$ rather than $\rho_2$. We solve for firm F’s optimal output first.

Firm F chooses $Q^f_2$ to maximize

$$(\bar{a} - bQ^H_2 - bQ^f_2)Q^f_2,$$

where $\bar{a} \in \{\bar{a}, a\}$. The first order conditions are

$$\bar{a} - bQ^H_2 - 2bQ^f_2 = 0 \rightarrow Q^f_2(\bar{a}) = \frac{\bar{a} - bQ^H_2}{2b}. \quad (3)$$

Now firm F believes that firm H’s posterior is $\rho \varepsilon$ rather than the actual posterior $\rho_2$. Thus, firm F believes that firm H chooses $Q_2$ and $Q^H_2$ to maximize

$$(a - bQ_2)Q_2 + [\hat{a}(\rho \varepsilon) - bQ^H_2 - b\hat{Q}^f(\rho \varepsilon)]Q^H_2 - c(Q_2 + Q^H_2)^2.$$

Here

$$\hat{a}(\rho \varepsilon) = \rho \varepsilon \bar{a} + (1 - \rho \varepsilon) a$$

is the expected demand intercept of the multinational as perceived by firm F and

$$\hat{Q}^f(\rho \varepsilon) = \rho \varepsilon \hat{Q}^f + (1 - \rho \varepsilon) Q^f$$

is the expected output of firm F that the multinational is believed to be using.

The perceived first-order conditions for the multinational are

$$Q^f_2(\rho \varepsilon) = \frac{\rho \varepsilon \bar{a} + (1 - \rho \varepsilon) a}{2b}.$$
\[ a - 2bQ_2 - 2c(Q_2 + Q_{1F}^e) = 0 \quad \text{and} \quad \hat{\alpha}(\rho_5^f) - 2bQ_{1F}^e - b\hat{Q}_x^e(\rho_5^f) - 2c(Q_2 + Q_{1F}^e) = 0. \]

Solving these two equations gives us the reaction function that firm F believes it is facing:

\[ Q_{1F}^e = \frac{b + c}{2(b^2 + 2bc)} \left[ \hat{\alpha}(\rho_5^f) - b\hat{Q}_x^e(\rho_5^f) - \frac{ac}{b + c} \right]. \quad (5) \]

Substituting for \( Q_x^e \) from Equation 3 into Equation 4, we obtain

\[ \hat{Q}_x^e(\rho_5^f) = \rho_5^f \left[ \frac{\hat{a} - bQ_{1F}^e}{2b} \right] + (1 - \rho_5^f) \left[ \frac{a - bQ_{1F}^e}{2b} \right] = \frac{\hat{a}(\rho_5^f) - bQ_{1F}^e}{2b}. \]

Substituting this in Equation 5 yields

\[ Q_{1F}^e(\rho_5^f) + \frac{2(b + c)}{b(3b + 7c)} \left[ \frac{\hat{a}(\rho_5^f)}{2} - \frac{ac}{b + c} \right]. \quad (6) \]

This is the output that firm F expects the multinational to produce in the second period in the foreign market. Substituting \( Q_{1F}^e \) from Equation 6 for \( Q_x^e \) in Equation 3, we get the profit-maximizing output of firm F in the second period:

\[ Q_x^f(\rho_5^f, \hat{a}) = \frac{1}{2b} \left[ \hat{a} - \frac{b + c}{3b + 7c} \hat{a}(\rho_5^f) + \frac{2ac}{3b + 7c} \right]. \quad (7) \]

Note that the choice of output of firm F, given by Equation 7, depends negatively on \( \rho_5^f \), that is, on its belief of the multinational’s belief about the state of the demand in the second period.

Substituting for \( Q_x^f \) and \( Q_{1F}^e \) from Equations 7 and 6, respectively, into firm F’s second-period profit function, we get the value function of firm F that is derived from the assumption that the multinational’s posterior is \( \rho_5^f \), that is,

\[ \hat{V}(\rho_5^f) = b(Q_x^f)^2 \left[ \frac{1}{2} \left( \hat{a} - \frac{b + c}{3b + 7c} \hat{a}(\rho_5^f) + \frac{2ac}{3b + 7c} \right)^2, \right. \]

where \( \hat{a} \in \{\hat{a}, a\} \).

**Properties of Firm F’s Value Function**

In the following lemma, we present some properties of the value function of the foreign firm F that are used later.

**Lemma 1.**

(a) Firm F’s second period profits are higher in the high-demand state than in the low-demand state, for all possible beliefs of the multinational. That is,

\[ \hat{V}(\rho_5^f) > V(\rho_5^f) \quad \text{and} \]

(b) Firm F’s loss of second-period profits due to a higher \( \rho_5^f \), that is, a more optimistic multinational, is higher if the true state is high demand than if it is low demand. That is,

\[ \hat{V}'(\rho_5^f) < V'(\rho_5^f) < 0. \]

**Proof.**
(a) is obvious from the expression for firm F’s value function, and (b) can be shown as follows:

$$\tilde{V}'(\rho_f) = 2b\tilde{Q}_f^\xi \frac{d\tilde{Q}_f^\xi}{d\rho_f}.$$ 

Now

$$\frac{dQ_f^\xi}{d\rho_f} = \frac{dQ_f^\xi}{d\rho_f} < 0$$

and \(\tilde{Q}_f^\xi > Q_f^\xi\). Thus the result. QED.

**The Multinational’s Value Function**

The multinational’s true posterior belief about the state of foreign demand is \(\rho_2\) rather than \(\rho_f\). Thus, the multinational solves for its optimal output in the second period in the same way as firm F believes, except that it uses its true posterior \(\rho_2\) in the maximization problem. That is, it chooses \(Q_2\) and \(Q_2^\eta\) to maximize

$$(a - bQ_2)Q_2 + [\tilde{a}(\rho_2) - bQ_2^\eta - b\tilde{Q}_f^\xi(\rho_2, \rho_f)]Q_2^\eta - c(Q_2 + Q_2^\eta)^2.$$

Here

$$\tilde{a}(\rho_2) = \rho_2\tilde{a} + (1 - \rho_2)a$$

is the expected demand intercept of the multinational and

$$\tilde{Q}_f^\xi(\rho_2, \rho_f) = \rho_2\tilde{Q}_f^\xi(\rho_f) + (1 - \rho_2)Q_f^\xi(\rho_f)$$

is the expected output of the foreign firm as actually calculated by the multinational.

The analysis of the multinational’s actual maximization problem is similar to the one carried out in the previous section. We present the key equations here. The analogue of Equation 5 of the previous section is

$$Q_2^\eta = \frac{b + c}{2(b^2 + 2bc)}\left[\tilde{a}(\rho_2) - b\tilde{Q}_f^\xi(\rho_2, \rho_f) - \frac{ac}{b + c}\right].$$

(9)

Recall from the discussion at the beginning of this section that the multinational knows that the foreign firm uses \(\rho_f\) to derive its (the foreign firm’s) optimal output. Thus, we substitute for \(Q_2^\xi(\rho_f, \tilde{a})\) from Equation 7 into Equation 8 and obtain

$$\tilde{Q}_f^\xi(\rho_2, \rho_f) = \frac{1}{2b}\left[\tilde{a}(\rho_2) - \frac{b + c}{3b + 7c}\tilde{a}(\rho_f) + \frac{2ac}{3b + 7c}\right].$$

(10)

This equation shows that the multinational believes that the average output of the foreign firm depends not only on its true posterior (i.e., \(\rho_2\)) but also on the foreign firm’s conjectured posterior (i.e., \(\rho_f\)). This is important since it induces the multinational to manipulate the foreign firm’s beliefs about its (the multinational’s) beliefs through its choice of the first-period output. In particular, it induces the multinational to signal-jam.

Substituting for \(\tilde{Q}_f^\xi\) from Equation 10 in Equation 9 yields the actual output of the multinational firm,
\[ Q^H(y, \rho^F) = \frac{b + c}{2b(b + 2c)} \left[ \frac{1}{2} a(p_2) - \frac{b + c}{2(3b + 7c)} a(p^F) - \frac{ac}{3b + 7c} - \frac{ac}{b + c} \right]. \]

This equation shows that the multinational produces more in the second period if \( p_2 \) is higher as well as if \( p^F \) is higher. On the other hand, the multinational believes the average output of the foreign firm to be higher if \( p_2 \) is higher but lower if \( p^F \) is higher.

Finally, the multinational’s second-period output in the home market, \( Q_2 \), can be easily derived from the first-order condition, that is,

\[ Q_2(p_2, p^F) = \frac{a - 2cQ^H(p_2, p^F)}{2(b + c)}. \]

Since the multinational’s outputs in the home market and the foreign market, as well as its expectation of the foreign firm’s second period output, are all functions of both \( p_2 \) and \( p^F \), the multinational’s second-period profits are a function of both \( p_2 \) and \( p^F \). Simplifying the multinational’s second-period profits expression (and suppressing this dependence on beliefs), its value function, denoted by \( W \), is given by

\[ W(p_2, p^F) = b[(Q_2)^2 + (Q^H)^2] + c(Q_2 + Q^H)^2. \]

Lemma 2 shows an important property of the multinational’s value function that gives the multinational an incentive to manipulate the beliefs of the foreign firm, that is, \( p^F \), through its (unobserved) output in the first period.

**Lemma 2.** The more optimistic the multinational firm is believed (by the foreign firm) to be about the state of the demand, regardless of its true beliefs, the higher are its profits:

\[ \frac{dW(p_2, p^F)}{dp^F} > 0. \]

**Proof.** (For convenience, the subscript for time period 2 is omitted for outputs throughout the proof.) Differentiating the multinational’s value function with respect to \( p^F \), we obtain

\[ \frac{dW(p_2, p^F)}{dp^F} = 2b \left( Q \frac{dQ}{dp^F} + Q^H \frac{dQ^H}{dp^F} \right) + 2c(Q + Q^H) \left( \frac{dQ}{dp^F} + \frac{dQ^H}{dp^F} \right). \]

Substituting for \( Q \) and \( dQ/dp^F \) yields

\[ \frac{dW(p_2, p^F)}{dp^F} = \left[ \frac{2b(b^2 + 2c^2 + 3bc)}{(b + c)^2} \right] Q^H \frac{dQ^H}{dp^F}. \]

Since

\[ \frac{dQ^H}{dp^F} > 0, \]

the result follows. **QED.**

Lemmas 1 and 2 show that the second-period profits of both firms depend on the beliefs of the multinational. Indeed, Lemma 1 shows that firm F’s second-period profits depend negatively on its perceived posterior probability of the multinational. This is because the more optimistic the multinational is believed to be about demand conditions, the more it is expected to produce. In Cournot equilibrium, more production by the multinational has the effect of lowering the foreign firm’s profits. This negative effect provides firm F with an incentive to
take actions in the first period that lead to a lower $\rho_f$ for the multinational. Quite interestingly, the multinational’s value function reflects the fact that it is aware that firm F does not know its beliefs, and thus its profits depend on $\rho_f$ as well as $\rho_2$. In addition, its profits increase as $\rho_f$ increases since a higher $\rho_f$ implies that the foreign firm produces less output in the second period. This provides the multinational with an incentive to take actions in the first period to make firm F believe that $\rho_f$ is higher than it actually is. This is signal jamming. Thus, while the foreign firm signals low demand, there is also scope for signal jamming by the multinational, which contributes to the multinational’s learning. In the next section, we show that the foreign incumbent firm indeed signals by producing more than in a myopic equilibrium, while the multinational signal-jams by producing less than in a myopic equilibrium. That is, the multinational produces less due to a lower output of the foreign firm but also due to the incentive to manipulate its own output, that is, due to signal jamming.

4. First-Period Competition

Firm F chooses how much to sell in the foreign market in the first period, corresponding to the high- and low-demand conditions, to maximize its two period profits. The multinational decides how much to sell at home and in the foreign market, in the first period, to maximize its two-period profits. Both firms take into account the effect of their choices in the first period on their second-period profits through the updated beliefs of the multinational. Each possible output in the first period leads to a different price distribution of the foreign good and thus different information and different levels of expected future profits.

Firm F’s first period expected profits are

$$\tilde{\Pi}_f(\bar{Q}_f^f, Q_1^{1e}, \bar{a}) = (\bar{a} - bQ_1^{1e} - bQ_f^f)Q_1^f,$$

where $\bar{a} \in \{\bar{a}, a\}$ and $\bar{Q}_f^f = Q_f^f(\bar{a})$.

The multinational’s first period expected profits are

$$\Pi_2(Q_1^f, Q_1, \bar{Q}_f^f, Q_f^f) = (a - bQ_1)Q_1 + [\bar{a}(\rho) - bQ_1^{1e} - b\bar{Q}_f^{1e}(\rho)]Q_f^f - c(Q_1 + Q_f^f)^2.$$ 

Here, $\rho$ is the prior belief of the multinational about the state of the foreign demand,

$$\bar{a}(\rho) = \rho\bar{a} + (1 - \rho)a, \quad \text{and} \quad \bar{Q}_f^{1e}(\rho) = \rho\bar{Q}_f^f + (1 - \rho)Q_f^f.$$

Firm F chooses $\bar{Q}_f^f$ to maximize

$$\tilde{\Pi} = \Pi_f(\bar{Q}_f^f, Q_1^{1e}, \bar{a}) + EV(\rho_f),$$

where

$$EV(\rho_f) = \int \tilde{V}[\rho_f(P)]f(P - \bar{a} + bQ_f^f + bQ_1^{1e})dP$$

is the expected value of future profits of firm F. The first-order conditions for the foreign firm’s two-period maximization problem are

$$\frac{d\tilde{\Pi}_f(\bar{Q}_f^f, Q_1^{1e}, \bar{a})}{d\bar{Q}_f^f} + \frac{d}{d\bar{Q}_f^f} \int \tilde{V}[\rho_f(P)]f(P - \bar{a} + bQ_f^f + bQ_1^{1e})dP = 0,$$

where $\bar{a} \in \{\bar{a}, a\}$. That is,
\[
\frac{d\Pi(\bar{Q}_f, Q''_f, \bar{a})}{dQ''_f} = -b \int \tilde{V}[p_\Sigma(P)]f'(P - \bar{a} + b\bar{Q}_f + bQ''_f)\,dP. \tag{11}
\]

Note that we do not differentiate \(\tilde{V}\) with respect to \(Q''_f\). This is because \(p_\Sigma(P)\) is not a function of what the foreign firm actually produces but of the multinational’s conjecture of it. The foreign firm thus cannot change \(p_\Sigma(P)\) as it changes its first-period output.

The multinational chooses \(Q_1\) and \(Q''_f\) to maximize

\[
\Pi'' = \Pi''(Q''_1, Q''_1, Q''_f, Q''_f) + EW(p_2, p_\Sigma),
\]

where

\[
EW(p_2, p_\Sigma) = \int W[p_2(P), p_\Sigma(P)][pf(P - \bar{a} + b\bar{Q}_f + bQ''_f) + (1 - p)f(P - \bar{a} + b\bar{Q}_f + bQ''_f)]\,dP
\]

is the expected value of the future profits of the multinational. The first-order conditions of the multinational’s two-period problem are as follows.

First, with respect to \(Q_1\),

\[
\frac{d\Pi''(Q''_1, Q''_1, Q''_f, Q''_f)}{dQ_1} = 0.
\]

Note that there is no direct effect of the second-period on the first-period output in the home market since the price in the home market conveys no information about the foreign demand conditions.

Second, with respect to \(Q''_f\),

\[
\frac{d\Pi''(Q''_1, Q''_1, Q''_f, Q''_f)}{dQ''_f} + \frac{d}{dQ''_f} \int W[p_2(P), p_\Sigma(P)]h(P)\,dP = 0.
\]

That is,

\[
\frac{d\Pi''(Q''_1, Q''_1, Q''_f, Q''_f)}{dQ''_f} = -\frac{d}{dQ''_f} \int W[p_2(P), p_\Sigma(P)]h(P)\,dP,
\]

where

\[
h(P) = pf(P - \bar{a} + b\bar{Q}_f + bQ''_f) + (1 - p)f(P - \bar{a} + b\bar{Q}_f + bQ''_f).
\]

Now, letting \(W_1\) and \(W_2\) denote the partial derivatives of the function \(W\) with respect to its two arguments, we obtain

\[
\frac{d}{dQ''_f} \int W[p_2(P), p_\Sigma(P)]h(P)\,dP = b \int W[p_2(P), p_\Sigma(P)]h'(P)\,dP + \int W_1[p_2(P), p_\Sigma(P)]\frac{dp_2(P)}{dQ''_f}h(P)\,dP. \tag{12}
\]

Note that the multinational cannot influence \(p_\Sigma(P)\) through its choice of first-period output, and thus the derivative of \(p_\Sigma(P)\) with respect to \(Q''_f\) is zero. Thus, there is no \(W_2\) term in the derivative.

The following theorem is one of the central results of this paper. The first part of the
theorem shows that the foreign firm signals that the demand is low by increasing its output in the first period for each quantity level of the multinational, regardless of the true state of the demand. It does so to depress the market price to make the multinational believe that the demand is low. The second part of the theorem shows that for each output level of the foreign firm, the multinational signal-jams by producing less in the first period than the optimal level in a myopic setting. It does so to make the foreign firm believe that it (the multinational firm) believes the foreign demand to be good.

In proving this theorem, the monotone likelihood ratio property of the error term in the foreign demand function is needed in order to ensure that higher prices lead to more optimistic posterior beliefs about demand (i.e., \(dp_1/dP > 0\)). Recall that we have assumed that the distribution of the error term \(\epsilon_i^f\) has the monotone likelihood ratio property. In order to use this assumption and show that \(dp_1/dP > 0\), it is necessary to ensure that the sufficient condition in Theorem 1, that is, \(\bar{a} - b\bar{Q}_i^f > a - bQ_i^f\), holds. We will prove in Theorem 2 that this condition holds.

Let \(\bar{Q}_i^f, Q_i^f, \bar{Q}_i^n, Q_i^n\) be the equilibrium quantities produced by the two firms in the first period.

**Theorem 1.** If \(\bar{a} - b\bar{Q}_i^f > a - bQ_i^f\), then
(a) For \(\bar{a} \in (\bar{a}, a)\) and for a given output of the multinational firm, the foreign firm produces more in the first period than in the static (no learning) case. That is,

\[
\frac{d\Pi_1^f(\bar{Q}_i^f, Q_i^n, \bar{a})}{d\bar{Q}_i^f} < 0.
\]

(b) For a given output of the foreign firm, the multinational produces less in the first period than in the static case.

That is,

\[
\frac{d\Pi_1^n(Q_i^n, \bar{Q}_i^f, \bar{Q}_i^f, Q_i^f)}{dQ_i^n} > 0.
\]

In order to prove this theorem, we need Lemmas 3 and 4.

**Lemma 3.** If \(\bar{a} - b\bar{Q}_i^f > a - bQ_i^f\), then

\[
\int \tilde{V}[\rho_1(P)]f_1(P - \bar{a} + bQ_i^f + bQ_i^n)\ dP > 0.
\]

**Proof.** Since \(f\) satisfies the monotone likelihood ratio property, the condition \(\bar{a} - b\bar{Q}_i^f > a - bQ_i^f\) implies that \([dp_1(P)]/dP > 0\). Now using integration by parts, we can write,

\[
\int \tilde{V}[\rho_1(P)]f_1(P - \bar{a} + bQ_i^f + bQ_i^n)\ dP = -\int \tilde{V}'[\rho_1(P)]\frac{dp_1(P)}{dP} f(P - \bar{a} + bQ_i^f + bQ_i^n)\ dP.
\]

The right-hand side is positive since \([dp_1(P)]/dP > 0\) and, by Lemma 1, \(\tilde{V}'[\rho_1(P)] < 0\). Thus the result. \(QED\).

**Lemma 4.** For firm H, if \(\bar{a} - b\bar{Q}_i^f > a - bQ_i^f\), then (a)

\[
\int W_1[\rho_1(P), \rho_1(P)]\left[\frac{dp_1(P)}{dQ_i^n} - b\frac{dp_1(P)}{dP}\right]h(P)\ dP = 0
\]

and (b)
\[ \int W_2[\rho_2(P), \rho_5(P)] \frac{d\rho_5(P)}{dP} \cdot h(P) \, dP > 0, \]

**Proof.** From the expression for the posterior \( \rho_2 \), it is clear that

\[ \frac{d\rho(P)}{d\theta^*} = b \frac{d\rho_2(P)}{dP}. \]

Thus, (a) follows.

For (b), recall from Lemma 2 that \( W_2[\rho_2(P), \rho_5(P)] > 0 \). Further, the condition \( \bar{a} - b\hat{Q}_f > a - b\hat{Q}_f \) yields \( [d\rho_5(P)]/dP > 0 \) because of the monotone likelihood ratio property of \( f \). Thus, (b) follows. \textit{QED.}

Note that the first part of Lemma 4 implies that the multinational does not experiment. That is, it does not manipulate its own output to learn. This is due to the fact that the intercept of the demand curve is assumed to be unknown to the multinational firm. This assumption implies no experimentation because changes in the multinational’s first-period output translate completely into changes in the price of the good.

We now prove Theorem 1.

**Proof of Theorem 1.** Recall the first-order condition of the foreign firm’s two-period maximization problem, given by Equation 11. By Lemma 3, the right-hand side of Equation 11 is negative. Thus (a) follows. For (b), recall the first-order conditions of the multinational’s two-period problem, given by Equation 12. Integrating the first term (on the right-hand side of Equation 12) by parts gives

\[ b \int W[\rho_2(P), \rho_5(P)] h'(P) \, dP \]

\[ = -b \left[ \int \left[ W_1[\rho_2(P), \rho_5(P)] \frac{d\rho_2(P)}{dP} + W_2[\rho_2(P), \rho_5(P)] \frac{d\rho_5(P)}{dP} \right] \cdot h(P) \, dP \right]. \]

By Lemma 4, the result follows. \textit{QED.}

Theorem 2 shows that the sufficient condition for Theorem 1 holds. The proof is given in the Appendix.

**Theorem 2.** The condition \( \bar{a} - b\hat{Q}_f > a - b\hat{Q}_f \) holds.

Theorems 1 and 2 together show that for every quantity produced by the foreign firm, the multinational firm produces less in the foreign market than in the static model, and for every quantity of the multinational firm in the foreign market, the foreign firm produces more than in the static model. This result lays the foundation for making a comparison between the equilibrium quantities produced in the Bayesian Nash equilibrium as described previously and the quantities produced in a myopic setting as described in the following.

Consider the situation when the two firms choose their first-period production without taking into account the effect of their decisions on the second-period profits, that is, when all the informational effects are ignored. We call the resulting production levels “myopic” in contrast to the Bayesian Nash equilibrium quantities that are the solution to the two-period maximization problems. Theorem 3 shows that the myopic equilibrium quantity produced by the multinational in the home market is less than the corresponding Bayesian Nash equilibrium quantity, the myopic quantity produced in the foreign market is more than in the Nash equilibrium, and the quantity produced by the foreign firm in the myopic equilibrium is less than the
quantity produced in the Nash equilibrium. We also show that the total output produced by the multinational in the two markets is lower in the Nash equilibrium than in the myopic setting.

Using the letter "M" to denote "myopic," let \( \hat{Q}^{FM} \) and \( Q^M \) denote the equilibrium quantities produced by the incumbent firm in the first period corresponding to the high- and low-demand intercept, respectively. Let \( Q^{HM} \) and \( Q^H \) denote the quantities produced by the multinational in the foreign market and in the home market, respectively.

**Theorem 3.** The following relationship holds between the myopic output levels and the quantities produced in the Bayesian Nash equilibrium:

(a) \( \hat{Q}^{FM} < \hat{Q}^F \),

(b) \( Q^{FM} < Q^F \),

(c) \( Q^{HM} > Q^H \),

(d) \( Q^M < Q_1 \),

(e) \( Q_1 + Q^H < Q^M + Q^{HM} \).

The proof is in the Appendix.

This theorem shows that the quantities produced by the two firms are different when learning is allowed by the uninformed firm than when it is not. More important, we are able to establish the direction in which these quantities are distorted by the two firms. The results are the outcome of an information manipulation process.

5. Implications for Research and Development

The model of learning by the multinational presented in this paper can be used to study various other interesting issues. In this section, we illustrate this by outlining a model that shows the connection between the outcome of learning by the multinational, established in the paper, and the R&D expenditure of the multinational.

Thus far we have assumed that the multinational’s cost of producing output in the home market and the foreign market is quadratic and given by

\[
c(Q + Q^H)^2.
\]

This specification enables us to study the learning behavior of the multinational in a simple setting while capturing the essential feature of a multinational enterprise, namely, a relationship between its home and its foreign production. We now apply the results of the model to address the question of the expenditure on R&D. In this model of R&D, the expenditure on R&D lowers the cost of production of both goods (produced for the home market and for the foreign market). Since these costs apply to total outputs in both markets, the R&D expenditure serves to connect the activities of the multinational across markets.

Specifically, let \( c_i = 1/k_i \) and \( c_2 = 1/k_2 \), where \( c_i \) and \( k_i \) denote the marginal cost and the R&D expenditure, respectively, in period \( i, i = 1, 2 \). That is, the marginal cost of production of the multinational falls as its R&D expenditure in the current and/or the previous period increases. The multinational’s profits in the second period are (omitting the second-period subscripts on outputs for convenience)
\[(a - bQ)Q + (\hat{a} - b\hat{Q}^F - bQ^H)Q^H - \frac{1}{k_1 k_2} (Q + Q^H)^2 - k_2,\]

where \(\hat{a}\) and \(\hat{Q}^F\) are the expected demand intercept and the output produced by the foreign incumbent, respectively, based on the multinational's posterior, \(p_2\). The multinational chooses \(Q, Q^H\) and \(k_2\) to maximize profits. The first-order conditions of the multinational's maximization problem in the second period are

\[a - 2bQ - \frac{2}{k_1 k_2} (Q + Q^H) = 0,\]

\[\hat{a} - 2bQ^H - b\hat{Q}^F - \frac{2}{k_1 k_2} (Q + Q^H) = 0, \quad \text{and} \quad \frac{1}{k_1 k_2} (Q + Q^H)^2 - 1 = 0.\]

Thus,

\[k_2 = \frac{Q + Q^H}{\sqrt{k_1}}.\]

The process of calculating the value functions of the two firms in the second period turns out to be identical to the one used in the model of this paper thus far. In fact, the value functions are easily found to be

\[\bar{V}(\rho_f) = b(Q^F)^2 = \frac{3\hat{a} - \hat{a}(\rho_f) + \frac{4}{\sqrt{k_1}}}{36b},\]

for the foreign firm and

\[W(\rho_2, \rho_f) = b(Q^2 + (Q^H)^2),\]

where

\[Q = \frac{a - 2}{2b} \quad \text{and} \quad Q^H = \frac{3\hat{a}(\rho) + \hat{a}(\rho^c) - \frac{16}{\sqrt{k_1}}}{12b}.\]

Incorporating the investment in R&D in this way has the effect of simplifying the value functions without changing their properties with respect to beliefs. That is, Lemmas 1 and 2 continue to hold. Furthermore, the updating of beliefs is unaffected. Thus, the two-period problem of each firm remains the same, except that the multinational also chooses the investment in R&D, \(k_1\), in the first period. It is straightforward to show that Theorems 1 to 3 continue to hold.

Now, to analyze the effect of learning on \(k_1\), we consider a benchmark case where \(k_1\) has no effect on the future production costs of the multinational. That is, we consider the model in which \(c_i = 1/k_i, i = 1, 2\). In this model, the value functions are independent of \(k_1\). Denoting the myopic level of R&D by \(k^m\), we have the following result.

**Theorem 4.** The multinational undertakes less R&D in the first period of a two-period model than in the static model. That is, \(k_1 < k^m\).
Proof. Since Theorems 1 to 3 continue to hold, total output produced by the multinational in the first period is less than in the myopic equilibrium (see Theorem 3 [e]). Now the first-order condition with respect to \( k_1 \) of the multinational's two-period maximization problem is

\[
    k_1 = Q + Q^H.
\]

Thus, the result follows. QED.

This result shows that learning by the multinational reduces its expenditure on R&D or "knowledge capital." This occurs because the expenditure on R&D is exactly equal to the total output of the multinational (Equation 14). Since the effect of learning is to reduce total output, R&D expenditure falls as well.

Next, we consider the case where \( k_1 \) lowers \( c_1 \) as well as \( c_2 \). Since the value functions are now functions of beliefs, as well as \( k_1 \), the first-order condition of the two-period maximization problem of the multinational with respect to \( k_1 \) changes to

\[
    \frac{1}{k_1^2} (Q + Q^H)^2 - 1 + \frac{k_1^{-3/2}}{18b} \int \left[ 9a + 4a(p) - \frac{34}{\sqrt{k_1}} \right] h(P) \, dP = 0,
\]

where \( h(P) \) is as defined in the preceding section of the paper. A full analysis of this problem is beyond the scope of this paper. However, we report the following results.

Theorem 5. When the first-period R&D lowers the future costs of production,

(a) the multinational firm undertakes more R&D in the first period of a two-period model than in the static model. That is, \( k_1 > k^M \). Further,

(b) Signal jamming lowers R&D in the first period: The first-period R&D is lower in a two-period model with signal jamming than without. That is, \( dk_1/d(Q + Q^H) > 0 \).

Thus, while in the absence of future effects of current R&D signal jamming reduces the current R&D, Theorem 5 shows that when the current R&D affects future costs, the effect of learning on the expenditure on R&D depends on the benchmark. Theorem 5(a) shows that the future reduction in costs due to current R&D leads to a higher R&D in the first period compared to the static (or myopic) case. On the other hand, Theorem 5(b) shows that the multinational undertakes lower R&D in the first period because of signal jamming since signal jamming reduces \( Q + Q^H \) compared to a dynamic model with no signal jamming.

Thus, if a multinational enters a new market about which it has less information than its local competitors, the model predicts that it would produce more output in the home market but less total output in the first period. It does so to make the foreign firm believe that it is optimistic about the demand conditions. This has the effect of offsetting the overproduction by the foreign firm. Further, the multinational would undertake less R&D in the first period because of signal jamming even when the first-period R&D lowers the future costs.

This paper has other implications for the theory of multinational enterprises. For example, it is commonly believed that asymmetric information puts the multinational at a disadvantage with respect to the local competitors, lowering its profits compared to markets in which it is fully informed. However, an implication of the results of this paper is that the possibility of learning and the existence of other markets that the multinational serves mean a lower cost of learning for the multinational than another informationally disadvantaged firm that operates in only one market and is unable to learn. Thus, the ability of the multinational to move resources across markets helps it lower the costs of learning.

Finally, the model implies that, in the absence of any R&D advantage, entering a new
market about which it has less knowledge than its local competitors is costly for the multinational in terms of lower profits due to the information disadvantage. This work provides an alternative framework to that of Horstmann and Markusen (1996) to quantify the costs of gathering information by the multinational on its own.

6. Conclusion

In this paper, we have analyzed the dynamics of learning when a multinational firm and a local incumbent compete in a market about which the incumbent knows more than the multinational firm does. The main result of the paper is that the learning process entails lower production by the multinational in the foreign market, higher production in the home market, but lower overall production and higher production by the incumbent firm compared to the situation when no learning is allowed. The purpose of these distortions is to influence the market price that is the basis of information gathering for the multinational. The paper provides insight into the costs of entry in new markets by the multinational when it is informationally disadvantaged.

Appendix

**Theorem 2.** The condition \( a - b\hat{Q}_f > a - bQ_f \) holds.

**Proof.** Suppose not. Then

\[
\hat{a} - b\hat{Q}_f < a - bQ_f.
\]

Or

\[
b(\hat{Q}_f - Q_f) > \hat{a} - a.
\]

Since \( \hat{Q}_f \) and \( Q_f \) are equilibrium quantities, for all \( \hat{Q} \neq \hat{Q}_f \) and \( Q \neq Q_f \), the following must be true:

\[
(\hat{a} - b\hat{Q}_f^n - b\hat{Q}_f)\hat{Q}_f + \int \hat{V}[p(P)]f(P - \hat{a} + b\hat{Q}_f^\tau + bQ_f^n)\,dP
\]

\[
> (a - bQ_f^\tau - bQ_f)Q_f + \int V[p(P)]f(P - a + bQ_f + bQ_f^\tau)\,dP,
\]

and

\[
(a - bQ_f^\tau - bQ_f)Q_f + \int V[p(P)]f(P - a + bQ_f + bQ_f^\tau)\,dP
\]

\[
> (a - bQ_f^\tau - bQ_f)Q_f + \int V[p(P)]f(P - a + bQ_f + bQ_f^\tau)\,dP.
\]

Rearranging the two inequalities, we get

\[
(\hat{a} - b\hat{Q}_f^\tau - b\hat{Q}_f)\hat{Q}_f - (a - bQ_f^\tau - b\hat{Q}_f)Q_f
\]

\[
> \int \hat{V}[p(P)]f(P - \hat{a} + b\hat{Q}_f + bQ_f^n)\,dP - \int V[p(P)]f(P - a + b\hat{Q}_f + bQ_f^n)\,dP
\]

\[
= \int \hat{V}[p(P)]\hat{Z}(P)\,dP.
\]

and
\[ \int V[p(P)]Z(P) \, dP = \int V[p(P)]f(P - a + bQ^f + bQ^{Qf}) \, dP - \int V[p(P)]f(P - a + bQ + bQ^{Qf}) \, dP > (a - bQ^f - bQ)^2 - (a - bQ^f - bQ^f)Q^f. \]

Now let \( \hat{Q} \) and \( \check{Q} \) be chosen as follows:

\[ \hat{Q} - Q^f = \frac{\hat{a} - a}{b} = \hat{Q}^f - Q^f. \]

This implies that

\[ \hat{Z}(P) = Z(P) = Z(P). \]

Next, we show that

\[ \int V[p(P)]Z(P) \, dP \geq \int V[p(P)]Z(P) \, dP. \]

Rewrite this as

\[ \int (V[p(P)] - V[p(P)])Z(P) \, dP \geq 0. \]

Now, note that \( V[p(P)] - V[p(P)] \) is a positive and increasing function of \( P \) by Lemma 1 and by the monotone likelihood ratio property of \( f \). Furthermore, by the maintained assumption \( \hat{a} - \hat{bQ^f} < a - bQ^f, Z(P) \) goes from negative to positive and integrates to 0. This establishes the previous inequality. Using this inequality, we get

\[ (a - bQ^f - b\hat{Q}^f)(\hat{Q}^f - (a - bQ^f - bQ)(\hat{Q}^f - (a - bQ^f - bQ^f))Q^f). \]

Substituting for \( \hat{Q} \) and \( \check{Q} \), we get

\[ [\hat{a} - bQ^f - b\hat{Q}^f - (a - bQ^f - bQ^f)]\frac{\hat{a} - a}{b} > 0. \]

This can be rewritten as

\[ \hat{a} - a > b(\hat{Q}^f - Q^f), \]

a contradiction. \( QED. \)

Proof of Theorem 3. Under myopic equilibrium, the first-order condition of the incumbent firm is

\[ \hat{a} - bQ^f - 2bQ^{Qf} = 0. \]

Under Bayesian Nash equilibrium, the first order condition is

\[ \hat{a} - bQ^f - 2bQ^f = -\frac{dE\hat{V}}{dQ^f} < 0. \]

Here the inequality is due to Theorem 1a. Subtracting one from the other gives us

\[ 2b(Q^f - Q^{Qf}) + 2b(Q^f - Q^{Qf}) = \frac{dE\hat{V}(p)}{dQ^f}. \]

Taking expectation with respect to \( \rho \), we get

\[ 2b(\hat{Q}^f - Q^{Qf}) + b(Q^f - Q^{Qf}) = \frac{dE\hat{V}(p)}{dQ^f}. \]  \hspace{1cm} (15)

Similarly, the multinational firm's first-order conditions for a myopic equilibrium are

\[ a - 2bQ^f - 2c(Q^f + Q^{Qf}) = 0 \text{ and} \]

\[ \hat{a}(p) - bQ^{Qf}(p) - 2bQ^{Qf} - 2c(Q^f + Q^{Qf}) = 0. \]  \hspace{1cm} (16)

Under the Bayesian Nash equilibrium, the first-order conditions are
a - 2bQ_i - 2c(Q_i + Q''_i) = 0 \text{ and }
\frac{d}{dQ''_i} - b\frac{\dot{Q}''_i}{\dot{Q}_i} - 2bQ''_i - 2c(Q_i + Q''_i) = -\frac{dE\nu(Q', p_1)}{dQ''_i} > 0. \tag{17}

The last inequality follows by Theorem 1(b). Subtracting Equation 17 from Equation 16, we get
\begin{equation}
\begin{aligned}
b\left(\dot{Q}'_i - \dot{Q}''_i\right) + 2b(Q''_i - Q''_i) + 2c(Q_i - Q''_i + Q''_i - Q''_i) = \frac{dE\nu(Q', p_1)}{dQ''_i} < 0. \\
\end{aligned}
\end{equation}

Now
\begin{equation}
Q_i - Q''_i = \frac{c(Q''_i - Q''_i)}{b + c}. \tag{19}
\end{equation}

Substituting for $\dot{Q}'_i - \dot{Q}''_i$ from Equation 15 and for $Q_i - Q''_i$ from Equation 19, we get
\begin{equation}
\begin{aligned}
\frac{3}{2} b(Q''_i - Q''_i) + \frac{2bc}{b + c} (Q''_i - Q''_i) = \frac{dE\nu(Q', p_1)}{dQ''_i} - \frac{1}{2} \frac{dE\nu(Q', p_1)}{dQ'_i} < 0.
\end{aligned}
\end{equation}

Thus,
\begin{equation}
Q''_i - Q''_i < 0 \Rightarrow Q''_i < Q''_i.
\end{equation}

This proves (c).

Then, by the inequalities for $Q'_i$ and $Q''_i$, and theorem 1(a),
\begin{equation}
\dot{Q}'_i > \dot{Q}''_i.
\end{equation}

This proves (a) and (b).

From Equation 19, we obtain
\begin{equation}
Q_i > Q''_i.
\end{equation}

This proves (d).

Finally,
\begin{equation}
Q_i + Q''_i = \frac{a - 2cQ''_i}{2(b + c)} + Q''_i = \frac{a + 2bQ''_i}{2(b + c)}.
\end{equation}

A similar expression holds for the myopic case, hence, by (c) proved previously, it follows that $Q_i + Q''_i < Q''_i + Q''_i$. Thus, (e) is proved. \textit{QED.}

References


