The IASB Insurance Project for life insurance contracts: impact on reserving methods and solvency requirements

Laura Ballotta[*], Giorgia Esposito[†‡], Steven Haberman[§]

April 4, 2006

Abstract

In this communication, we review the fair value - based accounting framework promoted by the IASB Insurance Project for the case of a life insurance company. In particular, for the case of a simple participating contract with minimum guarantee, we show that the fair valuation process allows for the identification of a suitable safety loading to hedge against default risk; furthermore, we show that, when compared with the "traditional" accounting system based on the construction of mathematical reserves, the fair value approach offers a more sound reporting framework in terms of covering of the liability, implementation costs, volatility of assets and liabilities and solvency capital requirements.

Keywords: Black-Scholes option pricing formula, fair value, Lévy processes, mathematical reserves, participating contracts, shortfall probability, solvency requirements.

JEL Classification: G13, G23

Insurance branch codes: IB10

Subject codes: IM10

1 Introduction

In response to an increasingly difficult economic climate, in which the financial stability of the insurance industry has been affected by events such as the crash in the equity markets in 2001 and 2002, a steady fall in bonds yields, as well as increased longevity,

[*]Faculty of Actuarial Science and Statistics, Cass Business School, City University London, e-mail: L.Ballotta@city.ac.uk.

[†]Department of Actuarial and Financial Sciences, University of Rome La Sapienza, e-mail: giorgia.esposito@uniroma1.it

[‡]Corresponding Author

[§]Faculty of Actuarial Science and Statistics, Cass Business School, City University London, e-mail: S.Haberman@city.ac.uk.

the focus of regulators on accounting rules, capital adequacy and solvency requirements for insurance companies has increased.

In particular, the three common themes behind the activity of many regulatory bodies around the world are a comprehensive financial reporting framework for the appropriate assessment of the specific risks that insurance companies are running; the standardization of approaches between countries and industries, where sensible; and an improved transparency and comparability of accounting information. To this purpose, the International Accounting Standards Board (IASB) in Europe and the Financial Accounting Standards Board (FASB) in the US have been working over the last few years towards the proposal of a model for the valuation of assets and liabilities which produces comparable, reliable and market consistent measurements. As such, the focus of this model has to be on the "economic" value of the insurance companies business. This theme has been followed by IASB and FASB with the proposal of a "fair value" accounting system for all assets and liabilities, where "fair value" means "the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction" (IASB, 2004).

In Europe, phase 1 of the IASB Insurance Project has been completed with the issuance of the new International Financial Reporting Standard (IFRS) 4 in March 2004, which establishes the changes in accounting rules as of January 2005. It is not our aim to describe here the technicalities of the new IFRS 4 (for a comprehensive exposition of its main features, we refer for example to FitchRatings, (2004)). However, we note that phase 1 requires significantly increased disclosure of accounting information, but only relatively limited changes to the accounting methodology, as the majority of the liabilities that have to be recorded at fair value are those originated by derivatives embedded in insurance contracts, such as life products offering a guarantee of minimum equity returns on surrender or maturity. The changes to the treatment of the assets side of the balance sheet is, instead, the direct result of the implementation of IAS 39, under which investments have to be classified as "available for sale" or "held for trading", and hence marked to market, unless the insurer is able to demonstrate the intent to forego future profit opportunities generated by these financial instruments (in which case investments can be classified as "held to maturity" and consequently reported at amortized historic cost). Hence, these changes are to be considered as the basis for the transition period leading up to the proper fair value accounting framework, which will be implemented in phase 2 (expected at the time of writing) to be completed by 2009-2010. We note, however, that the regulatory bodies in the UK, the Netherlands and Switzerland have introduced, or are in the process of introducing from January 2006, accounting rules based on the full mark-to-market of assets and liabilities, to be accompanied by the assessment of risk capital on a number of relevant adverse scenarios. In particular, the Swiss Solvency Tests (FOPI, (2004)) developed by the Swiss Federal Office of Private Insurance, and the Twin Peaks/Individual Capital Adequacy Standard implemented by the Financial Service Authority in the UK (FSA, (2004)) are designed to offer compatibility with regulatory demands on other market players like banks.

In the financial literature, the topic of market consistent valuation of life insurance products is well known and goes back to the work of Brennan and Schwartz (1976) on unit-linked policies. Since then, a wide range of contributions have followed, specifically on the issue of the fair valuation of the different contract typologies available in the insurance markets around the world (for a comprehensive review of these studies, we refer for example to Jørgensen (2004) and the references therein). However, it is recognized that the adoption of the fair value approach in the financial reporting system may have a significant impact on the design of some life insurance products; the premia charged to policyholders; the methodologies for the construction of reserves; and, more generally, the solvency profiles of companies.

In light of these considerations, the purpose of this paper is to analyze in detail some of these aspects by means of a simple participating contract with a minimum guarantee. In particular, we focus on three important issues. Firstly, we discuss how the fair valuation principle can be used to identify the extra premium that needs to be charged as a solvency loading to cover the insurance company's default option. Secondly, we note that no specific recommendation has yet been made by IASB as to which stochastic model is the most appropriate as the target accounting model. Hence, we adopt the standard Black-Scholes framework as a benchmark and, following the guidelines of IASB, we also propose a methodology for the analysis of the model risk and the parameter risk arising from this approach. Finally, we explore some possible alternative schemes for the construction of the mathematical reserves, and consider the advantages of adopting the fair value approach for solvency assessment purposes. Our focus on this last aspect is because of the ongoing EU Solvency II review of insurance firm's capital requirements, which is expected to come into effect at the same time as phase 2 of the IASB Insurance Project.

The paper is organized as follows. In section 2, we describe the design of the simple life insurance policy, the approach to fair valuation that we are considering in this paper and how it leads to a fair premium for the contract. In section 3, we introduce a possible methodology for the analysis of the model error and the parameter error components of the Market Value Margin, as requested by the IASB. In sections 4 and 5, we provide a comparative, quantitative study of the performance of the fair valuation method. In particular, we propose a range of deterministic reserving methods (static and dynamic) for comparison with the fair value of the liability in section 4. In section 5, we then introduce the concept of Risk Bearing Capital and use this as a means of assessing the solvency of the insurance company. Section 6 provides some concluding comments.

2 The participating contract and market consistent valuation

In this section, we consider a participating contract with a minimum guarantee and a simple mechanism for the calculation of the reversionary bonus. Following the general recommendation from the IASB accounting project, we then calculate the fair value of this contract using a stochastic model to represent the market dynamics of the fund backing the policy. In particular, we choose as valuation framework the traditional Black and Scholes (1973) model; we also focus on the implication of the fair valuation process on the insurer's solvency and on the definition of the contract premium. In the remaining parts of this paper, we ignore lapses, mortality and any further benefit that the policyholder may receive from the participating contract, such as death benefits or a terminal bonus.

2.1 The general framework

Consider as given a filtered probability space $\left(\Omega, \mathcal{F}, \{\mathbb{F}_t\}_{t\geq 0}, \mathbb{P}\right)$, and let us assume that the financial market is frictionless with continuous trading (i.e. there are no taxes, no transaction costs, no restrictions on borrowing or short sales and all securities are perfectly divisible). Furthermore, let $r \in \mathbb{R}^{++}$ be the continuously compounded risk-free rate of interest.

The policyholder enters the contract at time 0 paying an initial single premium[1] P_0, which is invested, together with paid-in-capital, E_0, by the insurance company in an equity fund, A. This is assumed to follow the traditional geometric Brownian motion

$$dA(t) = \mu A(t)\, dt + \sigma A(t)\, dW(t), \tag{1}$$

under the real objective probability $\mathbb{P}$. The parameters $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^{++}$ represent respectively the expected rate of growth (i.e. the expected rate of return) and the volatility of the fund.

In return, the policyholder has an account, P, which accumulates annually at rate $r_P(t)$, so that

$$P(t) = P(t-1)(1 + r_P(t)) \quad t = 1, 2, ..., T, \tag{2}$$

with

$$\begin{aligned} P(0) &= P_0 = \theta A(0); \\ r_P(t) &= \max\{r_G, \beta r_A(t)\}, \end{aligned}$$

where r_A denotes some smoothed return on the reference portfolio. The parameter β represents the participation rate of the policyholder in the returns generated by the reference portfolio, whilst r_G denotes the fixed guaranteed rate. The parameter θ represents instead the proportion of the initial reference portfolio financed by the policyholder. Ballotta et al. (2006) refer to θ as the cost allocation parameter or leverage coefficient.

We assume that the policyholder receives the benefit only at maturity, T; however, if at maturity, the insurance company is not capable of paying the full amount $P(T)$, then the policyholder seizes the assets available. At the expiration date, T, the policyholder will also receive a terminal bonus based on a percentage, γ, of the final surplus earned by the insurance company:

$$\gamma R(T) = \gamma\left(\theta\left(A(T) - P(T)\right)^+\right);$$

[1] We show in the next sections that in order to prevent arbitrage opportunities from arising, the premium actually charged has to be readjusted to include default risk.

the parameter γ represents the terminal bonus rate.

Hence, the liability, L, of the insurance company at maturity is

$$L(T) = \begin{cases} A(T) & \text{if } A(T) < P(T) \\ P(T) & \text{if } P(T) < A(T) < \frac{P(T)}{\theta} \\ P(T) + \gamma R(T) & \text{if } A(T) > \frac{P(T)}{\theta}. \end{cases},$$

or, equivalently

$$L(T) = P(T) + \gamma R(T) - D(T), \tag{3}$$

where

$$D(T) = (P(T) - A(T))^{+}$$

is the payoff of the so-called default option.

From standard contingent claim theory, it follows that the market value at time $t \in [0, T]$ of the participating contract, C, is

$$\begin{aligned} V_C(t) &= \hat{\mathbb{E}}\left[e^{-r(T-t)}\left(P(T) + \gamma R(T) - D(T)\right)\middle|\,\mathbb{F}_t\right] \qquad (4) \\ &= \hat{\mathbb{E}}\left[e^{-r(T-t)} P(T)\middle|\,\mathbb{F}_t\right] + \gamma \hat{\mathbb{E}}\left[e^{-r(T-t)} R(T)\middle|\,\mathbb{F}_t\right] - \hat{\mathbb{E}}\left[e^{-r(T-t)} D(T)\middle|\,\mathbb{F}_t\right] \\ &= V_P(0) + \gamma V_R(0) - V_D(0), \end{aligned}$$

where $\hat{\mathbb{E}}$ denotes the expectation under the risk-neutral probability measure $\hat{\mathbb{P}}$, and $\mathbb{F}_t$ is the information flow up to (and including) time t.[2] Under these circumstances, the premium P_0 charged is fair to the policyholder if the following equation is satisfied

$$P_0 = V_C(0) = V_P(0) + \gamma V_R(0) - V_D(0). \tag{5}$$

As far as the equityholders are concerned, their claim at maturity (under the assumption of limited liability) is:

$$E(T) = \begin{cases} 0 & \text{if } A(T) < P(T) \\ A(T) - P(T) & \text{if } P(T) < A(T) < \frac{P(T)}{\theta} \\ A(T) - P(T) - \gamma R(T) & \text{if } A(T) > \frac{P(T)}{\theta}, \end{cases}$$

or

$$\begin{aligned} E(T) &= (A(T) - P(T))^{+} - \gamma R(T) \\ &= A(T) - P(T) + D(T) - \gamma R(T); \end{aligned}$$

therefore, the fair contribution to the company's capital should satisfy the following

$$V_E(0) = E_0 = (1 - \theta) A(0), \tag{6}$$

which implies that the fair value condition is the same for both classes of stakeholders, since equation (5) and (6) are equivalent.

[2]Note that we implicitly assume the existence of the risk-neutral probability measure $\hat{\mathbb{P}}$ (which, given the specification of the market is unique). In other words, we assume that the participating policy is attainable and therefore the replicating portfolio exists. This is consistent with the spirit of FASB recognition of the Level 3 fair value estimates for insurance liabilities with complex contingencies and embedded options elements (FASB, 2004).

2.2 The specifics of the contract and fair valuation

In order to implement a market consistent valuation framework, we need to fully specify the accumulation rate in equation (2). In particular, we assume that

$$r_A(t) = \left(\frac{A(t) - A(t-1)}{A(t-1)}\right).$$

This accumulation mechanism is a special case (for $n = 1$) of the more general smoothing scheme analyzed by Ballotta et al. (2006); in their contribution, Ballotta et al. (2006) consider the problem of finding a fair set of design parameters for the participating contract, and then they analyze the interactions between each of these parameters, as well as the behaviour of the full set when the market conditions change. In this paper, however, our focus is on the impact of fair valuation on the company's solvency profile, and therefore, for ease of exposition of the main results, we have chosen this simplified mechanism. Based on the same reasoning, we assume in the remainder of this paper that $\theta = 1$, i.e. the reference portfolio is fully funded by policyholders; further, we focus only on the guaranteed part of the benefit at maturity, $P(T)$, and assume that the terminal bonus feature is not included in the contract's design; in other words from now on, we fix $\gamma = 0$.

We note that, under this set of assumptions, equation (5) reduces to

$$P_0 = V_P(0) - V_D(0), \tag{7}$$

whilst the value of the equityholders claim is zero, since $E_0 = 0$.[3]

Given our set of assumptions, and the features of the policy design, a closed-form analytical formula for the value of the policy reserve $V_P(t)$ can be obtained. The same, however, does not apply for the value of the default option, $V_D(t)$, due to the recursive nature of P (see equation (2)) and the fact that P is highly dependent on the path followed by the reference fund A. Therefore, the market price of the default option will be approximated by numerical procedures.

In more detail, from equation (2), it follows that the benefit at maturity can be rewritten as

$$P(T) = P_0 \prod_{k=1}^{t} (1 + r_P(k)) \prod_{k=t+1}^{T} (1 + r_P(k)) = P(t) \prod_{k=t+1}^{T} (1 + r_P(k)).$$

Moreover,

$$r_P(t) = \max\left\{r_G, \beta\left(\frac{A(t) - A(t-1)}{A(t-1)}\right)\right\} = r_G + \left(\beta\left(\frac{A(t) - A(t-1)}{A(t-1)}\right) - r_G\right)^+,$$

which implies that, in distribution,

$$r_P(t) \stackrel{D}{=} r_G + \left(\beta\left(e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma W_1'} - 1\right) - r_G\right)^+ \qquad \text{under } \mathbb{P},$$

[3]We refer to Ballotta et al. (2006) for a fuller treatment of the more general case when $\theta < 1$ and $\gamma > 0$.

and

$$r_P(t) \overset{D}{=} r_G + \left(\beta\left(e^{\left(r-\frac{\sigma^2}{2}\right)+\sigma\hat{W}_1} - 1\right) - r_G\right)^+ \qquad \text{under } \hat{\mathbb{P}},$$

where W_1' is an independent copy of the $\mathbb{P}$-Brownian motion, and $\hat{W}$ is a standard one-dimensional Brownian motion under the risk neutral probability measure $\hat{\mathbb{P}}$.
This implies that the annual rate of return $r_P(t)$ generates a sequence of independent random variables $\forall t \in [0,T]$. Therefore, the market value of the policy reserve is

$$\begin{aligned}
V_P(t) &= P(t)\,\hat{\mathbb{E}}\left[e^{-r(T-t)} \prod_{k=t+1}^{T} (1 + r_P(k)) \middle| \mathbb{F}_t\right] \\
&= P(t) \prod_{k=t+1}^{T} \hat{\mathbb{E}}\left\{e^{-r}\left[1 + r_G + \left(\beta\left(e^{\left(r-\frac{\sigma^2}{2}\right)+\sigma\hat{W}_1} - 1\right) - r_G\right)^+\right]\right\} \\
&= P(t)\left[e^{-r}(1 + r_G) + \beta N(d_1) - e^{-r}(\beta + r_G) N(d_2)\right]^{T-t} \qquad (8)
\end{aligned}$$

where

$$d_1 = \frac{\ln\frac{\beta}{\beta+r_G} + \left(r + \frac{\sigma^2}{2}\right)}{\sigma}; \quad d_2 = d_1 - \sigma.$$

The pricing equation (8) follows as an application of the Black-Scholes option formula (see also Bacinello, 2001, Ballotta, 2005, and Miltersen and Persson, 2003, for similar results).

2.3 Default probability and the premium for the participating contract

Based on the valuation formula (8), in this section, we implement a scenario generation procedure in order to analyze the evolution of the market value of the participating contract and the reference fund over the lifetime of the policy. Since these quantities represent the liabilities and the assets in the balance sheet of the insurer, this study will allow us to understand the consequences of the market based accounting standards on the solvency profile of the life insurance.

The numerical procedure is organized in the following steps:

1. using the $\mathbb{P}$-dynamic of the underlying asset A, we generate possible trajectories of the reference fund from the starting date of the policy till time $t \in (0,T]$. Each trajectory consists of 1 observation per month.

2. Then, we calculate the annual returns on the reference fund A to obtain the amount accumulated by the policy till time t, $P(t)$.

3. The output from step 2 is used to calculate $V_P(t)$ according to equation (8).

Fair value of the participating contract			
P_0	$V_C(0)$	$V_P(0)$	$V_D(0)$
100	100	222.73	122.73
Relative solvency loading coefficient	$\delta = 1.2273$		

Table 1: The fair value of the participating contract with minimum guarantee for the benchmark set of parameters. The Monte Carlo simulation for the price of the default option is based on 10,000 paths; the antithetic variate technique and the control variate method are used for variance reduction purposes.

4. Finally, we use the output from step 1 and 2 to compute the market value of the default option, $V_D(t)$, using Monte Carlo techniques. The Monte Carlo experiment is based on 10,000 paths; antithetic variate methods and the control variate procedure are implemented in order to reduce the error of the estimates.

Unless otherwise stated, the initial single premium is $P_0 = 100$ and the base set of parameters used for our numerical example is as follows:

$$\mu = 10\% \text{ p.a.}; \quad \sigma = 15\% \text{ p.a.}; \quad \beta = 80\%; \quad r_G = 4\% \text{ p.a.}; \quad r = 4.5\% \text{ p.a.}; \quad T = 20 \text{ years}.$$

The no arbitrage values of the contract and its components corresponding to the above set of parameters are shown in Table 1; we note that, for the given set of parameters, the premium charged to the policyholders satisfies the fair condition given in equation (7), which is consistent with the no arbitrage principle.

A possible scenario resulting from this numerical experiment is illustrated in Figure 1.(a). In this plot, we show the evolution of the reference fund A, the fair value of the benefit V_P, the value of the default option V_D, and the total value of the contract V_C. We note that the assets of the life insurance company are not enough to guarantee the payment of the full benefit promised at maturity; consequently, the curves representing the assets' price and the total value of the contract coincide (see equation (4)). The default probability arising from this model, calculated on the basis of 100,000 scenarios, is in fact 74.42% (see Table 2).

On one hand, such a high default probability is a consequence of the mechanism adopted for the accumulation of the benefit in the sense that the reversionary bonus scheme does not offer an adequate smoothing of the fund's returns; on the other hand, however, equation (7) shows that the policy reserve is not the only component affecting the value of the participating contract, as we need also to take account of the fact that the insurance company's liability is limited by the market value of the reference fund. This feature is captured by the payoff of the so-called default option D. Consequently, V_D is, as Ballotta et al. (2006) observe, an estimate of the market loss that the policyholder incurs if a shortfall occurs. However, equation (7) also implies that

$$P_0 + V_D(0) = V_P(0).$$

Hence, as already observed by Ballotta (2005), the value V_D of the default option can be considered as the extra premium that the insurer has to charge the policyholder

	Probability of default at maturity	
GBM model	$\mathbb{P}(P(T) > A(T)) = 74.42\%$	$\mathbb{P}(P(T) > A_{tot}(T)) = 6.97\%$
GLP model	$\mathbb{P}(P(T) > S(T)) = 81.71\%$	$\mathbb{P}(P(T) > S_{tot}(T)) = 12.74\%$

Table 2: Default probability at maturity under the geometric Brownian motion framework presented in section 2.1 and the Lévy process framework introduced in section 3.1. Calculations have been performed for the benchmark set of parameters under the real probability measure, on the basis of 100,000 simulations.

for no arbitrage opportunities to arise. Without receiving this extra amount, in fact, the insurer would be offering the guaranteed benefits too cheaply, as the solvency risk attached to the contract would be ignored.

Based on these considerations, we redefine the overall premium paid by the policyholder at inception of the contract to be $P_0' = P_0 + V_D(0)$, where P_0 is the part of the premium on which the benefit will be based, and V_D is instead the part of the premium that the policyholder has to pay in order to be "insured" against a possible default of the insurance company. In this sense, the fair value of the default option associated to the participating contract, $V_D(0)$, could be regarded as a solvency, or safety, loading to the premium. If we write $V_D(0) = \delta P_0$, then $P_0' = (1+\delta) P_0$, with δ representing a relative solvency loading coefficient (Daykin et al. (1994)). If this additional premium is invested for example in the market to purchase another share of the reference portfolio, at time $t = 0$, the value of the fund backing the policy is $A(0) = P_0$, whilst the total value of the assets available to the insurer is $A_{tot}(0) = P_0' = P_0 + V_D(0)$. Both A and A_{tot} evolve as described in equation (1).

The dynamics of each component of the contract resulting from this readjustment and corresponding to the scenario presented in Figure 1.(a), are illustrated in Figure 1.(b). If the insurance company invests the additional premium in the same fund backing the policy, the default option moves out of the money, whilst the value of the contract converges to the value of the promised benefits. The probability of a shortfall occurring at maturity is 6.97% as reported in Table 2 (on the basis of 100,000 scenarios). Panels (a) and (c) of Figure 2 show the corresponding shortfall distribution for both situations, i.e. when the premium for the default option is ignored (panel (a)), and when instead it is charged and invested in the fund (panel (c)). The reduction in the right-tail of the distribution in the latter case is evident.

We observe that in the case study analyzed in this paper, the insurance company is "passive" in terms of risk management, i.e. it does not implement any hedging strategy. The results discussed, however, show that if a suitable solvency loading is charged, the reduction in the default probability is such that a carefully designed hedging strategy can successfully reduce the default risk further, so that the contract is fully honoured, even if the smoothing mechanism is as weak as the one used in this note. By charging V_D, in fact, the insurer would have enough funds to acquire a hedging portfolio which fully covers all of the risks incorporated in the contract.

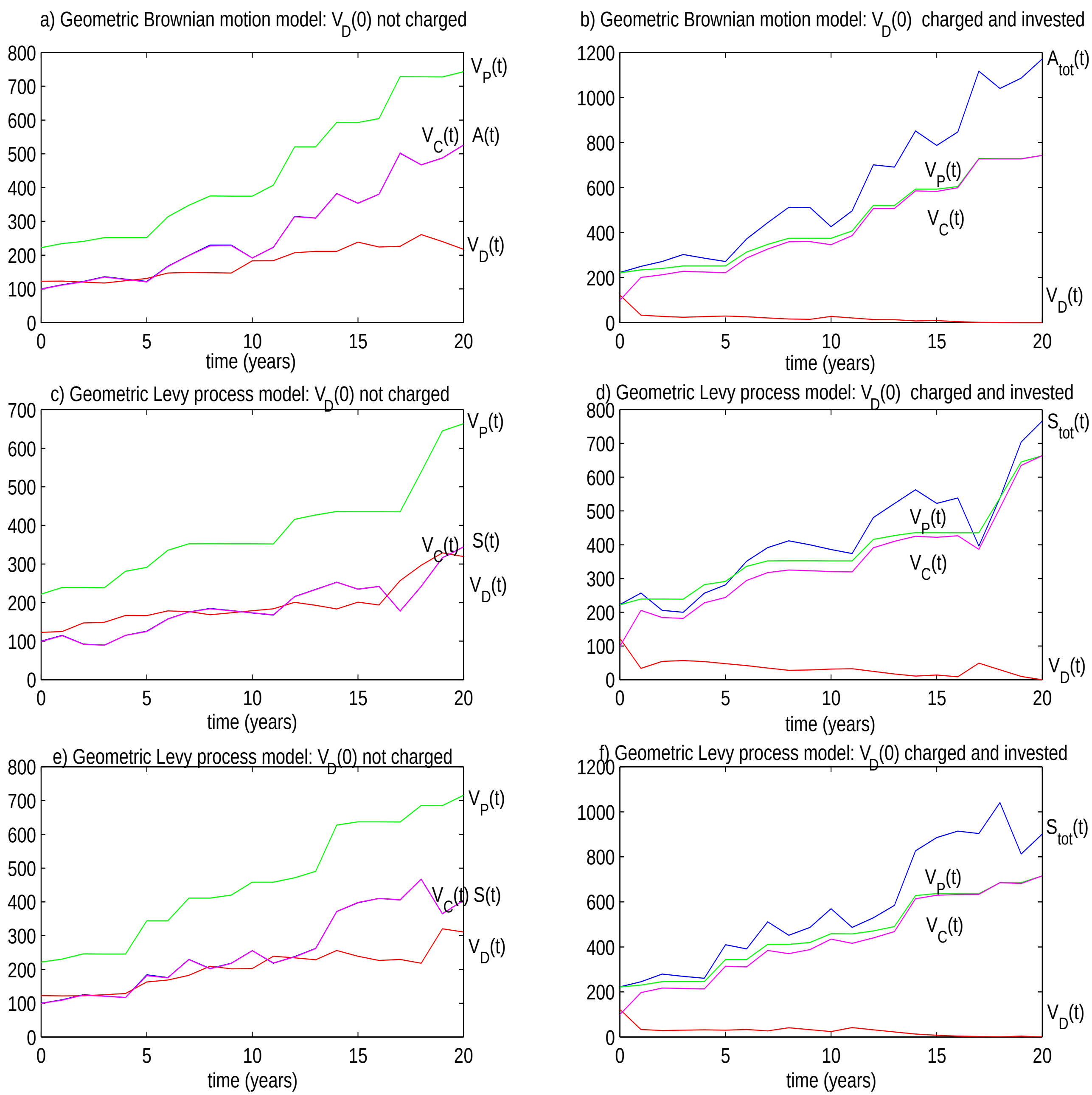

Figure 1: Scenario generation: a possible evolution of the reference fund and the fair value of the participating contract, the benefits and the default option. Panels (a)-(b) show one possible scenario under the geometric Brownian motion paradigm presented in section 2.1. Panels (c)-(d) and (e)-(f) illustrate two possible scenarios under the Lévy process paradigm discussed in section 3.1. The scenario generation has been obtained for the benchmark set of parameters under the real probability measure $\mathbb{P}$.

3 Market value margin

The valuation model presented in the previous section, and therefore also the results discussed in section 2.3, relies on a number of assumptions (most notably, the assumption that the equity assets follow a log-normal distribution[4]), and the specification of a number of external variables, like the expected return of the assets or their volatility, that can move significantly over the lifetime of the contract. It is widely acknowledged that it is difficult to classify accurately the distribution of market prices and to assess the probability of extreme events, especially falls, concerning stock prices. This implies that, as discussed in Ballotta (2005), there are biases in the fair value measurements originating from the model developed in the previous section.

The aim of this section of the paper is to examine the impact of this type of uncertainty, affecting the fair valuation of insurance liabilities, on the expected cash flows and the solvency profile of the insurer. We note that the IASB intends to take into account these elements of risk, the so-called model risk and parameter risk, during phase 2 of the implementation of the accounting standards project, by requiring insurance companies to calculate a Market Value Margin (see, for example, FitchRatings, 2004) as a buffer to reflect the risks and uncertainties that are inherent in insurance contracts.

3.1 The model error

As previously mentioned, the calculations of fair values presented in section 2 are dependent on the assumption of normal distributed log-returns. However, it is well known that, in the real financial market, this is not the case, as the dynamic of the assets is not continuous but appears to consist of jumps only. A recent analysis offered by Carr et al. (2002), in fact, shows that, in general, market prices lack a diffusion component, as if it were diversified away, implying that the Brownian motion-based representation of the equity fund proposed in section 2.1 might not be realistic. Hence, in this section, we analyze the impact of the biases in the fair value-based estimates of the liability implied by the participating contract described in section 2.1-2.2, due to a misspecification of the distribution of the underlying fund. In particular, we assume that the "true" equity asset, which we denote by S, as opposed to the equivalent quantity A that the insurance company assumes to be log-normal, is in reality driven by a geometric Lévy process. The amount $P(t)$ accumulated by the policyholder's account till time t will be determined by the evolution of S over the same period of time. Hence, the error in the model will affect the value of the policy reserve, $V_P(t)$, and the value of the default option, $V_D(t)$, to the extent to which $P(t)$ is affected. The cost of the time value of the guarantees and the options to be exercised over $(t, T]$ is instead calculated according to the original model.

[4]We observe that, although this assumption is not supported by empirical evidence, it forms the basis of the RiskMetrics (Mina and Xiao, (2001)) model, which is recommended by the Swiss Solvency Test (FOPI, 2004) as a standard asset model.

Quantity	Process	Parameters
A	Equation (1)	$\mu = 10\%$ $\quad \sigma = 15\%$
S	Equation (9)	$\mu_X = -0.0537;$ $\quad \sigma_X = 0.07;$ $\quad \lambda = 0.68;$ $a = 0.1254;$ $\quad \gamma = 0.1312.$

Table 3: Base set of parameters for the full model (Bakshi et al. (1997)).

For simplicity, we use a Lévy process with jumps of finite activity, i.e.

$$
\begin{aligned}
dS(t) &= \left(a + \frac{\gamma^2}{2} + \int_{\mathbb{R}} z\nu(dz)\right) S(t_-)\, dt + \gamma S(t_-)\, dW(t) \\
&\quad + S(t_-) \int_{\mathbb{R}} z\left(N(dz, dt) - \nu(dz)\, dt\right), \qquad (9)
\end{aligned}
$$

where N is an homogeneous Poisson process, ν is the $\mathbb{P}$-Lévy measure specified as $\nu(dx) = \lambda f_X(dx)$, $f_X(dx)$ is the density function of the random variables X modelling the size of the jumps in the Lévy process, and $Z = e^X - 1$ is the proportion of the stock jump. Furthermore, we assume that the jump size is normally distributed, that is $X \sim N(\mu_X, \sigma_X^2)$. Finally, for the numerical experiment, we use the parameters as in Bakshi et al. (1997), i.e.

$$\mu_X = -0.0537; \quad \sigma_X = 0.07; \quad \lambda = 0.68.$$

In order to produce a sensible comparison, we need to ensure that the expected rate of return on the equity and the instantaneous volatility of its driving process are kept equal for both A and S. Specifically, the instantaneous variance of the log-returns resulting from the dynamics of A and S respectively is σ^2 and $\gamma^2 + \lambda(\mu_X^2 + \sigma_X^2)$. Hence, given the set of parameters used, $\gamma = 0.1312$. The expected return on the assets is instead described by the drift of the corresponding stochastic differential equations, i.e. μ and $a + \frac{\gamma^2}{2} + \lambda\left(e^{\mu_X + \frac{\sigma_X^2}{2}} - 1\right)$ for A and S respectively. Therefore, $a = 0.1254$. The full base set of parameters is given in Table 3.

We make again the distinction between the fund backing the policy for which $S(0) = P_0$, and the total of the assets available to the insurer, for which $S_{tot}(0) = P_0' = P_0 + V_D(0)$. Both S and S_{tot} evolve as described in equation (9). The results from this model are summarized by the two possible scenarios illustrated in panels (c) - (f) of Figure 1. Both plots show that the introduction in the dynamic of the assets of a jump component increases the default option premium, and therefore the severity of the shortfall risk as measured by the shortfall probability (which is 81.71%, as shown in Table 2). However, if the additional premium $V_D(0)$ is paid by the policyholder and invested by the insurer in the fund S, the default option moves out of the money and the shortfall probability reduces to 12.74%. The corresponding shortfall distributions are shown in Figure 2.(b) and Figure 2.(d). Also in these cases, we can observe the reduction in the right-tail of the distribution.

Finally, we observe that the default probabilities generated by the Lévy process-based model are higher than the ones generated under the standard Black-Scholes

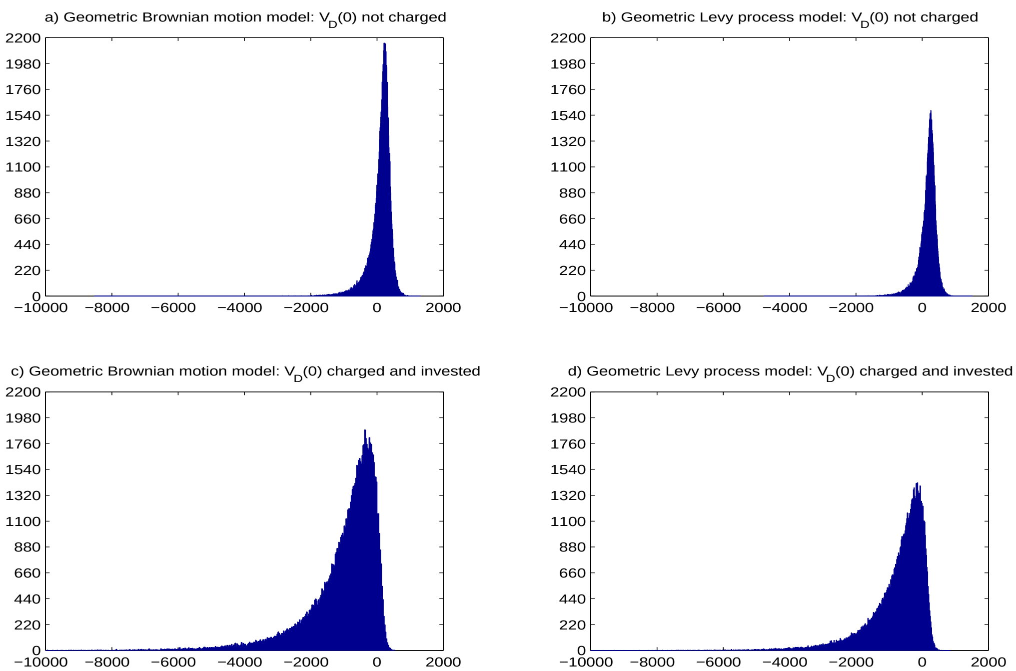

Figure 2: Shortfall distribution, $\mathbb{P}(D(T) > 0)$, for the benchmark set of parameters under the real probability measure, based on 100,000 scenarios.

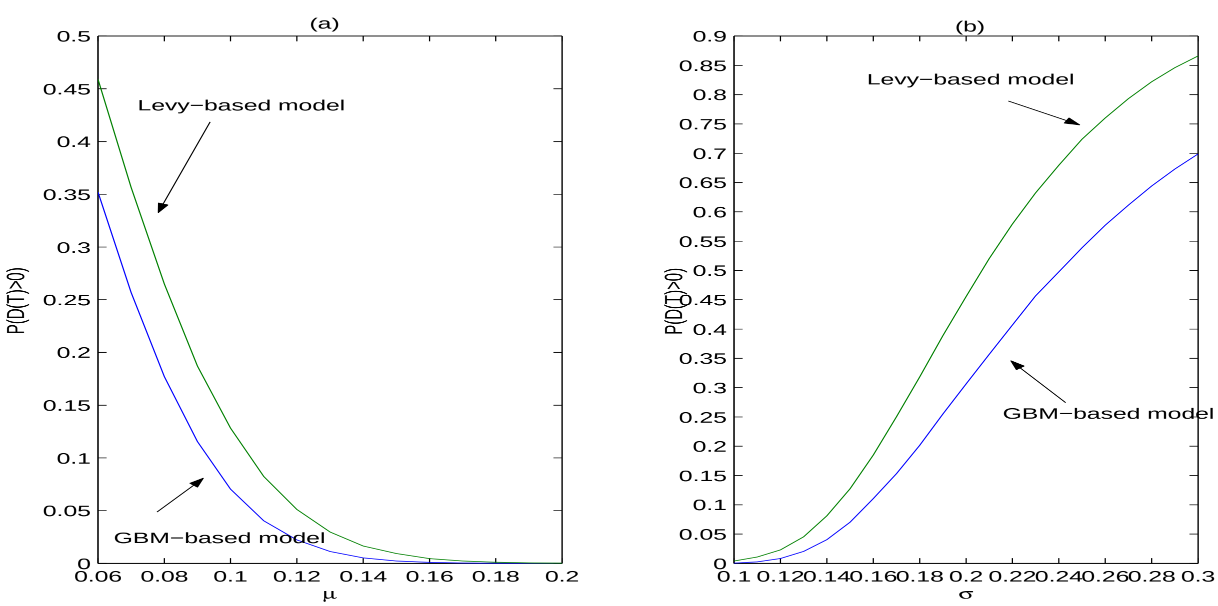

Figure 3: Parameter error: impact on the shortfall probability of changes in the expected rate of return and volatility of the equity portfolio backing the policy.

framework. This is due to the fact that the latter ignores the possibility that market prices can jump downwards, which would lead to a deterioration in the financial stability of the insurers (see Ballotta, 2005, for further details).

3.2 The parameter error

In this section, we show how the probability of default changes under both market paradigms considered in the paper when the expected return on assets, μ, and their volatility, σ, are allowed to change. Results are shown in Figure 3. In panel (a), we observe that the higher the expected return on assets, the lower the probability of default under both market paradigms. This is expected, since policyholders benefit from a rise in the expected returns on the reference fund which is moderated by the participation rate β. Hence, the assets grow more than the corresponding liability, ensuring a reduced default risk.

In panel (b), we show the sensitivity of the default probability to the reference portfolio's volatility; we observe that by increasing the volatility, the shortfall probability becomes higher under both market models considered in this paper. In fact, more volatile prices mean a higher probability of substantial variations in the value of the assets; however, the policyholder's benefits are only affected by upwards movements of the reference fund because of the presence of the minimum guarantee, r_G (in the definition of $r_P(t)$). We also note that the gap between the default probabilities obtained under the two market paradigms tends to increase as σ increases, which is due to the additional instability in asset prices generated by the presence of jumps in the Lévy-based model.

4 Analysis of reserving methods

The introduction of a full fair value reporting system is considered a controversial issue by many insurance company representatives. From a survey carried out by Dickinson and Liedtke (2004) involving 40 leading international insurance and reinsurance companies, it emerges that one of the most critical issues is the so called "artificial volatility problem" (see Jørgensen (2004) as well for a discussion of this specific issue). According to the survey's results, the respondents agree that the adoption of the fair value approach in preparing balance sheets will result in more volatile reported assets and liability values, and consequently more volatile reported earnings and cost of capital. The implication of this fact is that it would then be more difficult to provide earnings forecasts or forward-looking information to the investment community.

Given these criticisms, in this section, we consider a number of alternative methods for the calculation of the "deterministic" reserves which are then compared against the fair value of the liability imposed by the participating contract, i.e. $V_P(t)$. This comparison is aimed at analyzing the adequacy of the reserves in terms of covering the liabilities, their cost of implementation and their impact on the volatility of the balance sheet. Any resulting solvency issue is, instead, discussed in section 5.

We follow established actuarial theory and practice for the determination of "deterministic" reserves (Fisher and Young (1971), Booth et al. (2005)), based on the equivalence principle. Thus, we define the reserves as the present value of the future benefits (net of the present value of the future premia paid by the policyholder, which we do not consider as our contract has a single premium paid at inception). Hence the value of the deterministic reserve at time $t \in [0, T]$ is

$$V_R(t) = P_R(T)\, e^{-r(T-t)},$$

where $P_R(T)$ is a "fictitious" policyholder account which provides the insurer with a best estimate of the benefit due at maturity. For the discount rate, we use the risk free rate of interest, r, since this can be considered as the lower bound for any prudential discount rate chosen by the life insurance company. The alternative methods for the construction of the reserves, discussed below, differ depending on the rule which we use to calculate $P_R(T)$.

Static method Here we assume that the life insurance company adopts a passive reserving strategy, so that

$$P_R(T) = P_0 (1 + r_R)^T,$$

where $r_R = 8.5\%$. The choice of this value is justified on the basis of a prudential estimate of the asset returns that will be credited to the policyholder account. In fact, we are assuming that the asset has an expected rate of return $\mu = 10\%$ and that the participation rate of the policyholder in the asset performance is $\beta = 80\%$. By assuming a constant accumulation rate, we are considering the simplest case of smoothing over the whole lifetime of the contract. The results are presented in Figures 4 and 5, in which we show the probability that, at the end of each year, the value of the reserves is below the fair value of the liability, i.e. $V_P(t)$, for the cases of the geometric Brownian motion and Lévy process paradigms respectively. The plots show that $\mathbb{P}(V_R(t) < V_P(t))$ is consistently high over the term of the contract. However, since $V_P(T) = P(T)$, the probability that the static reserve is less than the amount of the benefit due at expiration is above 90% under both market paradigms, which indicates that the rate used to calculate the best estimate of the liabilities is not sensitive enough to the dynamics of the effective liabilities.

Dynamic method In this case, we assume that the life insurance company adopts an active risk management strategy, so that the rate at which the "fictitious" account $P_R(T)$ accumulates is readjusted every n years, in order to take into account the performance of the reference fund and the accumulated liability P. In the following analysis, we consider the cases $n = 1, 3, 4, 5$. We assume that the readjustment occurs at the reset dates $t_k, k = n, 2n, ..., \left[\frac{T}{n}\right]$, where $[x]$ denotes the smallest integer not less than x. Then, for $t \in [0, t_n)$

$$\begin{aligned} P_R(T) &= P_0 (1 + r_R)^T \\ r_R &= 8.5\%. \end{aligned}$$

Hence, for the first n years of the contract, the reserves are identical to the static case. For $t \geq t_n$ instead, we have

$$P_R(T) = P(t_k)(1 + r_R(t_k))^{T-t_k},$$

where $P(t_k)$ is the value of the benefit cumulated till the reset date t_k, and $r_R(t_k)$ is related to the dynamics of the portfolio backing the policy and the value of the benefits. We consider below four possible alternative definitions of $r_R(t_k)$. This list is not meant to be exhaustive but to serve as an illustration only. Firstly, let

$$\begin{aligned}
\bar{\mu}_n(t_k) &= \frac{1}{n}\sum_{k=1}^{n} \frac{A_{tot}(t_k) - A_{tot}(t_{k-1})}{A_{tot}(t_{k-1})}; \\
\bar{\mu}_n^{(P)}(t_k) &= \frac{1}{n}\sum_{k=1}^{n} \frac{P(t_k) - P(t_{k-1})}{P(t_{k-1})},
\end{aligned}$$

i.e. the average at time t_k respectively of the annual returns achieved by the reference fund over the last n years, and of the annual rates at which the policyholder benefit accumulated over the last n years. Then, the alternative definitions under consideration are as follows.

1. As observed above, a rate of growth of the reserves fixed at 8.5% is not sufficient to cover the liabilities generated by a participating contract written on an asset with an expected annual rate of return of 10% and a 15% annual volatility. Hence, we construct a new accumulation rate which uses the prudential estimate of 8.5% as a floor, to which an addition is made in the case in which asset returns perform better than expected. Therefore, we consider:

$$r_R(t_k) = \max\{r_R, \beta\bar{\mu}_n(t_k)\},$$

where $r_R = 8.5\%$, as given by the static method, and β is the participation rate.

2. An alternative to the previous methods could be the replication of the benefit's accumulation rate, so that:

$$r_R(t_k) = \max\{r_G, \beta\bar{\mu}_n(t_k)\},$$

where $r_G = 4\%$ is the minimum guarantee and β is the participation rate.

3. Since the principal goal for the establishment of reserves is to enable the company to build over time sufficient resources to pay the benefits to the policyholder when due, an alternative approach would be to make the accumulation rate $r_R(t_k)$ dependent on the evolution of the benefit itself, rather than on the reference fund. In this case, we consider:

$$r_R(t_k) = \max\left\{r_G, \bar{\mu}_n^{(P)}(t_k)\right\},$$

4. Finally, we propose an accumulation rate based on the extent to which the assets perform above or below expectations. Let

$$s_n(t_k) = \bar{\mu}_n(t_k) - \mu$$

be the spread at time t_k between the average of the last n years' returns on the asset and its expected rate of return. Then we consider:

$$\begin{aligned} r_R(t_k) &= r_R + \alpha s_n(t_k), \\ \alpha &= \begin{cases} \beta & \text{if } s_n(t_k) > 0 \\ b < \beta & \text{if } s_n(t_k) < 0, \end{cases} \end{aligned}$$

where β is the participation rate. Hence, we allow for a smoothing method that weights differently upwards or downwards movements of the reference fund. This asymmetric way of considering positive and negative movements in the asset returns enables us to have at maturity the exact amount of resources to pay the full benefit to the policyholder, without overcharging the company during unfavourable periods. In the simulations, we assume $b = 40\%$ against a participation rate $\beta = 80\%$.

Similarly to the analysis carried out for the static reserves, we plot in Figures 4-5 the probabilities that the reserves calculated on the basis of the proposed methods are below $V_P(t)$. We observe that $\mathbb{P}(V_R(t) < V_P(t))$ drops very quickly, especially if the readjustment of the accumulation rate occurs every year. The "step-shape" of the curves, though, suggests that the trajectory followed by the reserves might not be smooth. An example is shown in Figures 6-7, where we illustrate the evolution over the lifetime of the policy of the proposed dynamic reserves corresponding to the specific scenarios presented in Figure 1.(b), (d) and (f) respectively. As expected, for both market paradigms considered in this paper, the evolution of the reserves is very unstable and volatile, due to a number of high peaks occurring over the term of the contract. The reason for this behaviour comes from the definition of the fictitious account $P_R(T)$, which represents the projection to maturity of the expected liability. This projection, in fact, is calculated using the information available at the reset dates, t_k, about the past performance of the asset returns. Hence, the value of the options still to be exercised is calculated on the basis of the past n years' returns and the realized volatility over the same period, but, at the same time, ignoring the possible impact of the future market dynamic. This is particularly evident when the readjustment of the accumulation rate, $r_R(t)$, occurs every year, i.e. for $n = 1$. As n increases, the smoothing effect becomes stronger and the peaks reduce in terms of both frequency and magnitude. The fact that the annual amount of the dynamic reserves is often greater than the fair value of the liability with a significant probability, means that, when compared to $V_P(t)$, the proposed reserving schemes are, in a sense, more expensive for the company to implement.

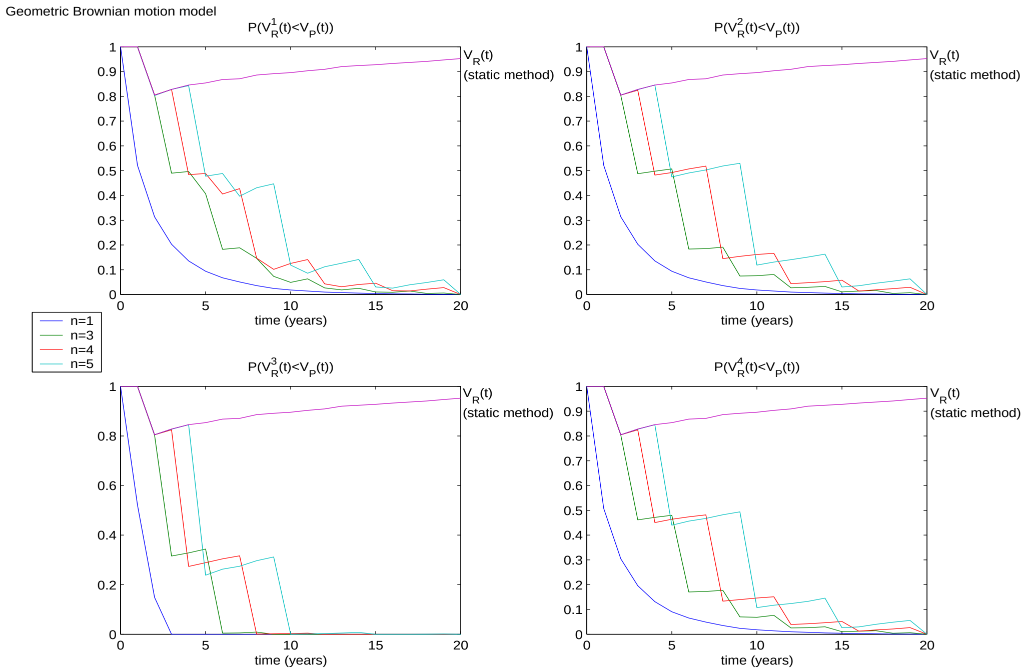

Figure 4: $\mathbb{P}\left(V_R\left(t\right) < V_P\left(t\right)\right)$ corresponding to the benchmark set of parameters. The case of the deterministic reserves under the geometric Brownian motion paradigm. The probabilities are calculated under the real probability measure using 100,000 scenarios.

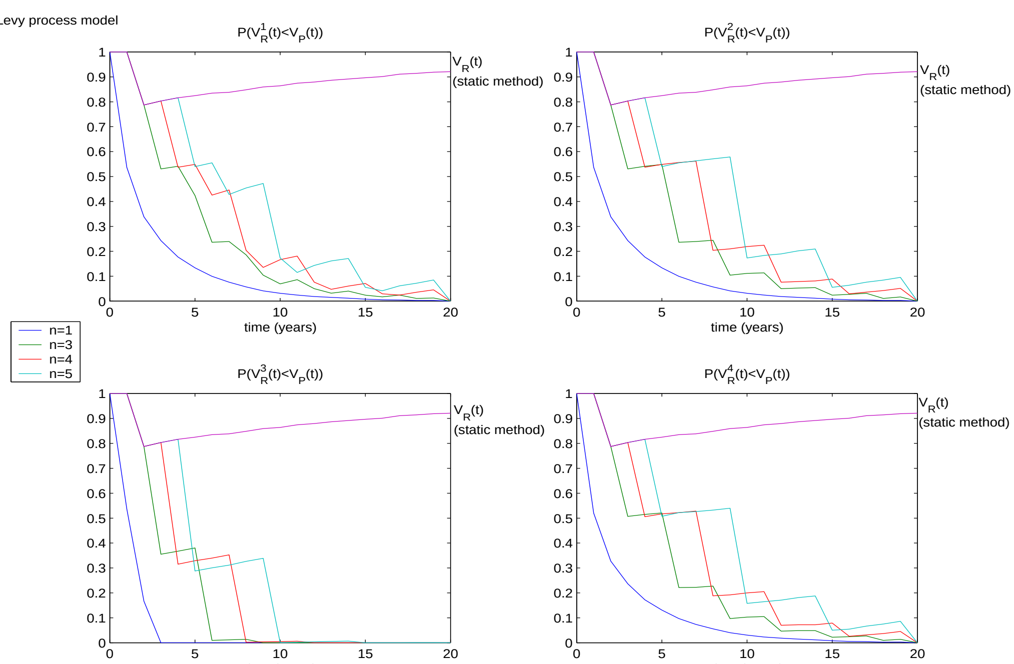

Figure 5: $\mathbb{P}\left(V_R\left(t\right) < V_P\left(t\right)\right)$ corresponding to the benchmark set of parameters. The case of the deterministic reserves under the Lévy process paradigm. The probabilities are calculated under the real probability measure using 100,000 scenarios.

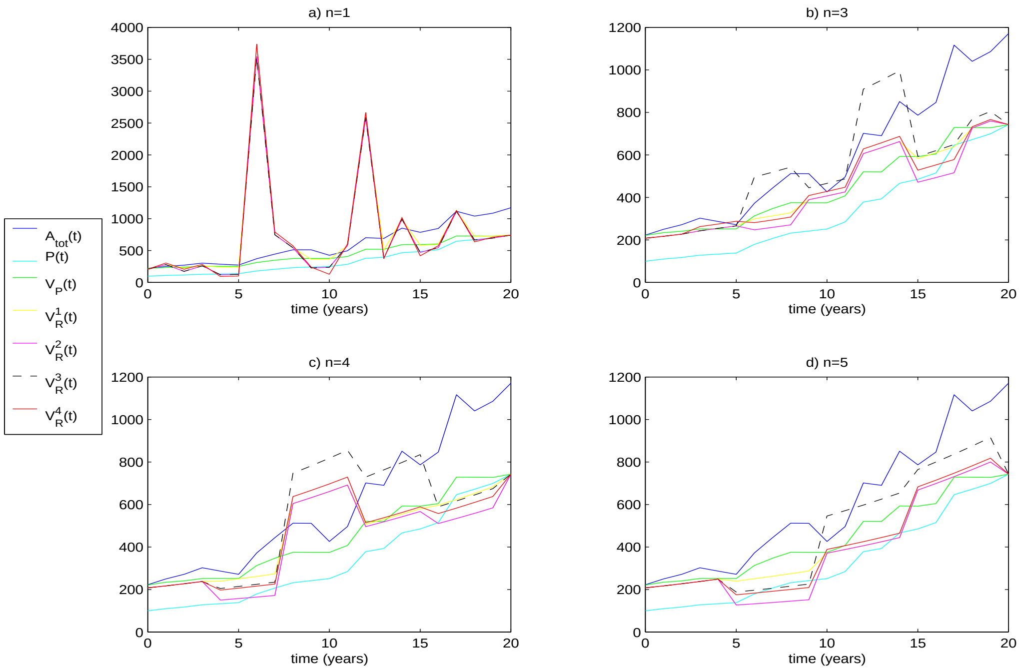

Figure 6: Scenario generation: the case of the dynamic reserves under the geometric Brownian motion paradigm. The trajectories of the market values are the same as in Figure 1.(b).

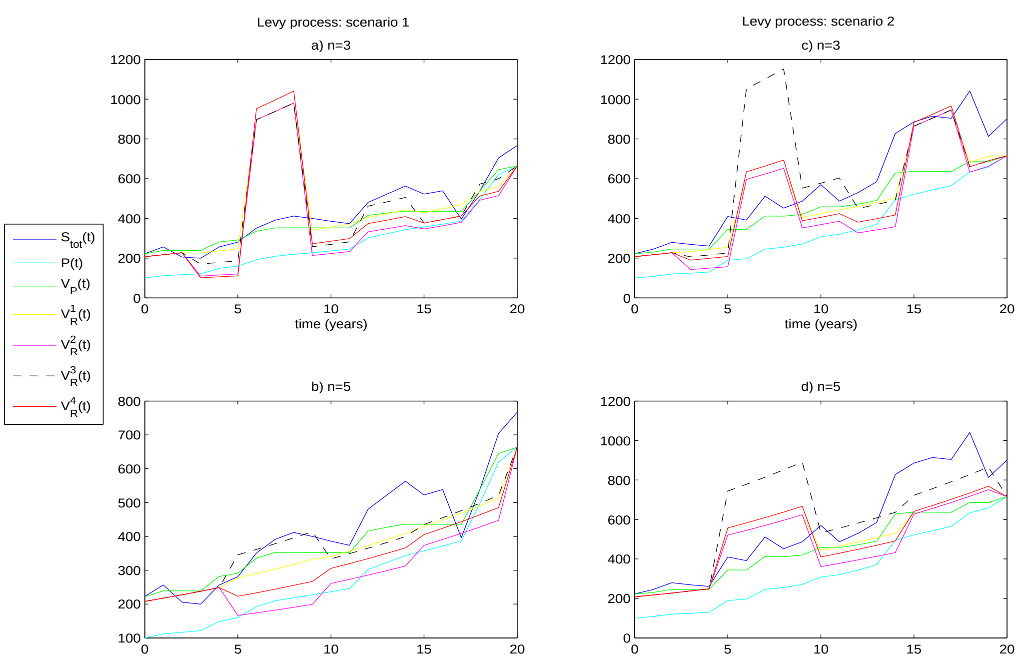

Figure 7: Scenario generation: the case of the dynamic reserves under the Lévy process paradigm. The trajectories of the market values in panels (a) and (b) are the same as in Figure 1.(d); the trajectories in panels (c) and (d) are the same as in Figure 1.(f) .

Retrospective method Finally, we consider the retrospective reserve, which is calculated as the current amount of the policy reserve, i.e. the current amount of the accumulated benefits, $P(t)$, at time t.

The evolution of the retrospective reserve corresponding to the scenarios considered, is shown as well in Figures 6-7. We observe that $P(t)$ is always below the estimated value of the liabilities calculated using both the fair value approach and the dynamic reserving schemes discussed above, and the market value of the assets. This is as expected since, by construction, the retrospective reserve does not account for the full cost of the time value of the options and guarantees included in the contract design (see equation 8).

5 Solvency

The scenarios presented in section 4, and illustrated in Figures 6-7, show an additional feature deserving a more detailed analysis. The specific trajectories considered, in fact, suggest the possibility that the proposed dynamic reserves could be at a higher level than the total of the assets available to the company. Based on these scenarios, it is reasonable to expect periods in which the level of the reserves is above the total value of the available assets, followed by periods in which the reserves drop below it. In terms of portfolio management, this implies that the insurer needs to adopt a very aggressive investment strategy in order to balance the high level of the reserves, followed by large disinvestments when these move below the assets level. Hence, compared to the fair value approach, estimating the value of the liabilities using a deterministic reserving scheme, such as the ones proposed above, could prove more expensive in terms of the cost of capital, with consequences for the solvency profile of the insurance company.

As mentioned in section 1, a new solvency regime, named "Solvency II", is currently discussed by the European Union with the aim of creating incentives to develop internal models for measuring the risk situation within insurance companies. The aim of this section is to analyze, in the light of the guidelines arising from the Solvency II project, the capital requirements for an insurance company issuing a participating contract with minimum guarantee such as the one described in the previous sections. We consider here only the valuation paradigm based on the Lévy process model, with the premium for the default option invested in the same equity portfolio backing the policy (similar conclusions can be obtained for the geometric Brownian motion model as well; results are available from the authors).

Solvency II follows the three-pillar approach used in Basel II, so that Pillar 1 comprises quantitative minimum requirements for equity capital provision; Pillar 2 deals with supervisory review methods and the development of standards for sound internal risk management; finally, Pillar 3 defines the requirements for disclosure obligations and market transparency in order to ensure a high degree of market discipline. More specifically, one of the focus areas of Solvency II is to develop a formula for the calculation of the target capital, i.e. the amount needed to ensure that the probability of failure of the insurance company within a given period is very low. This formula should be based on the assessment of the future cashflows related to the policies in force and assets held. The assessment should take into account all relevant risks, and should be carried out by building on the IASB proposals and the latest IFRS. The time horizon proposed is one year, whilst longer term elements should be taken into account in Pillar 2 (European Commission, 2004). Although a specific assumption for the most suitable confidence level is still under discussion, the current practice in some countries such as the Netherlands and the UK, suggests a 99.5% probability (i.e. the ruin probability of 1/200), which, broadly speaking, implies that the insurance company should be at least of investment grade quality ("BBB") in terms of rating.

Hence, as the target capital depends on the annual ruin probability, i.e. the probability that the insurer will have not enough assets to cover the liabilities at the end of the observation period, we consider as a relevant index for our analysis, the difference between the market consistent value of the assets and the best estimate of the liabilities generated by the contract. As the best estimate of the liability we use its fair value as calculated in section 2.2, and the estimates produced by the deterministic reserving methods introduced in section 4. In the language of the Swiss Solvency Test (FOPI, 2004), this index is called Risk Bearing Capital (RBC). For easy of exposition, we express the RBC as a proportion of the estimated liability, i.e.

$$RBC = \frac{S_{tot}(t) - V(t)}{V(t)},$$

where V is the best estimate of the liability.

We note that a reserving method which leads to an artificially low value of the liabilities will have a $\mathbb{P}\left(RBC > 0\right)$ with desirable characteristics. We comment on this possibility in a later paragraph.

In Figure 8, we represent the probability that at the end of each year the RBC is positive. These plots are accompanied by the analysis of the moments of the distribution of the RBC in each year, shown in Figures 9-11. Figures 9 and 10 relate to the four dynamic reserving methods described in section 5, and Figure 11 relates to the static reserving method. We observe that the fair value approach not only generates the highest probability of being solvent (i.e. of having a positive RBC), but also guarantees a stable RBC over the lifetime of the contract and, consequently, reduced additional injections of capital from the shareholders. This also implies that the fair value reporting system could prove helpful for the definition of an effective ALM policy.

In contrast, the probabilities of being solvent originated by any of the four dynamic reserving methods discussed in section 4, deteriorate very quickly with time (especially if the accumulation rate, $r_R(t)$, is readjusted every year). Moreover, the step-shape of the curves suggests that the capital requirements for the insurance company would be very different one year from the other in terms of magnitude. This emerges also from the analysis of the moments of the distribution of the RBC. The irregular behaviour of the RBC's variance, in particular, represents for the insurance company a significant problem for the development and implementation of an effective strategy of internal risk management. Finally, in Figure 8 we note that the static reserving method leads to the highest probability of solvency; however, this plot should be read together with

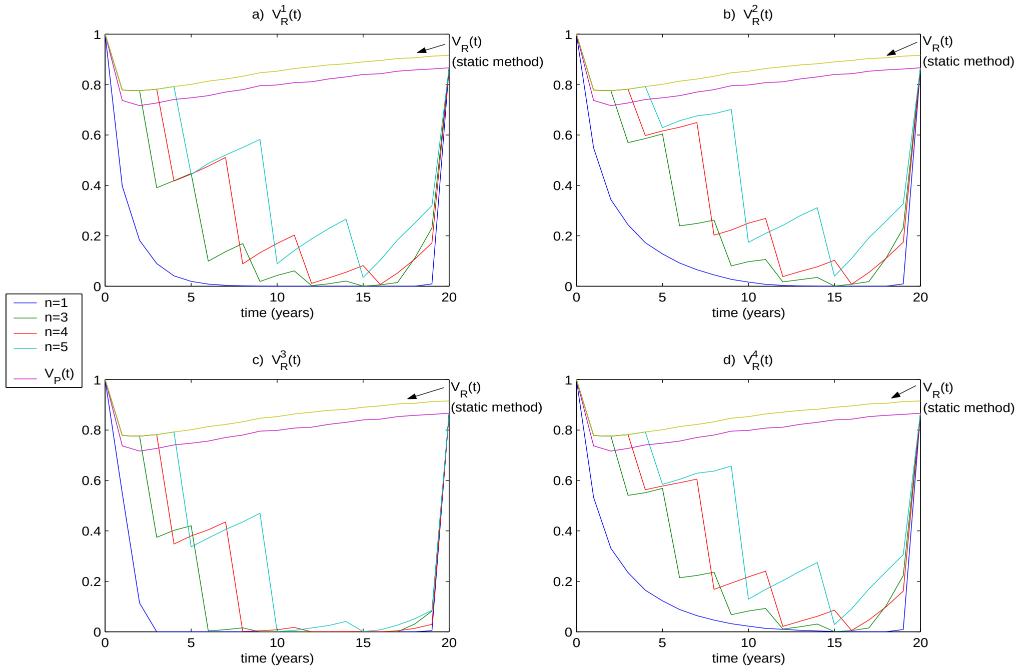

Figure 8: $\mathbb{P}(RBC > 0)$ for the benchmark set of parameters under the real probability measure. The distribution has been obtained using 100,000 scenarios. Each panel corresponds to one of the dynamic reserving method described in section 4.

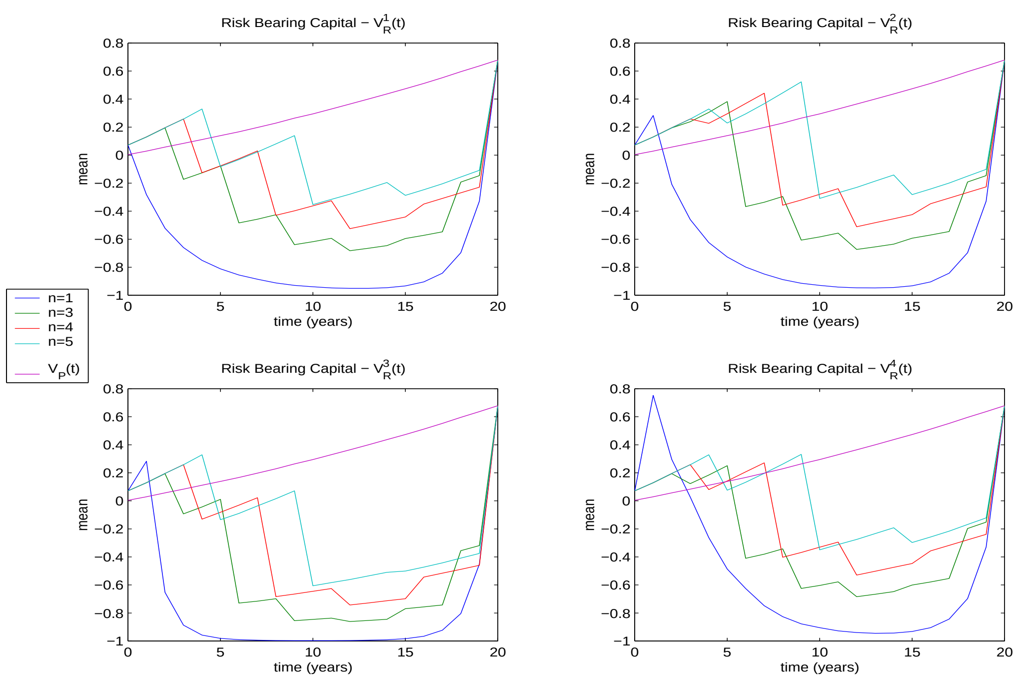

Figure 9: The mean of the risk bearing capital distribution resulting from the numerical experiment described in Figure 8. The risk bearing capital is calculated as $(S_{tot} - V)/V$, where V is the best estimate of the liability provided by the four dynamic reserving schemes described in section 4 and by the fair value $V_P(t)$.

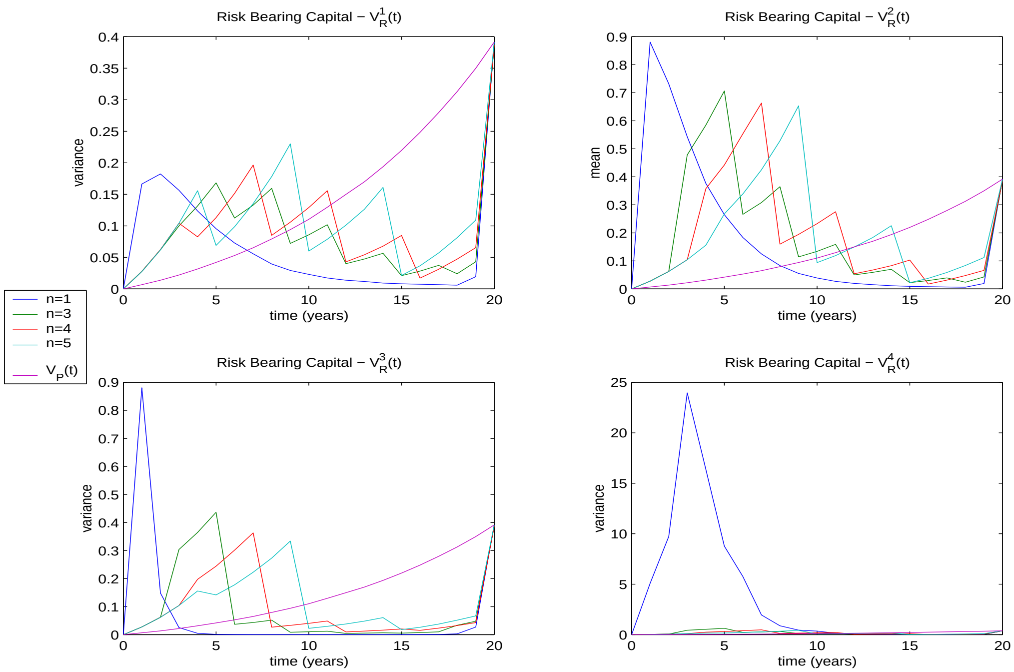

Figure 10: The variance of the risk bearing capital distribution resulting from the numerical experiment described in Figure 8. We note the magnitude of this moment of the distribution for the reserving scheme 4.

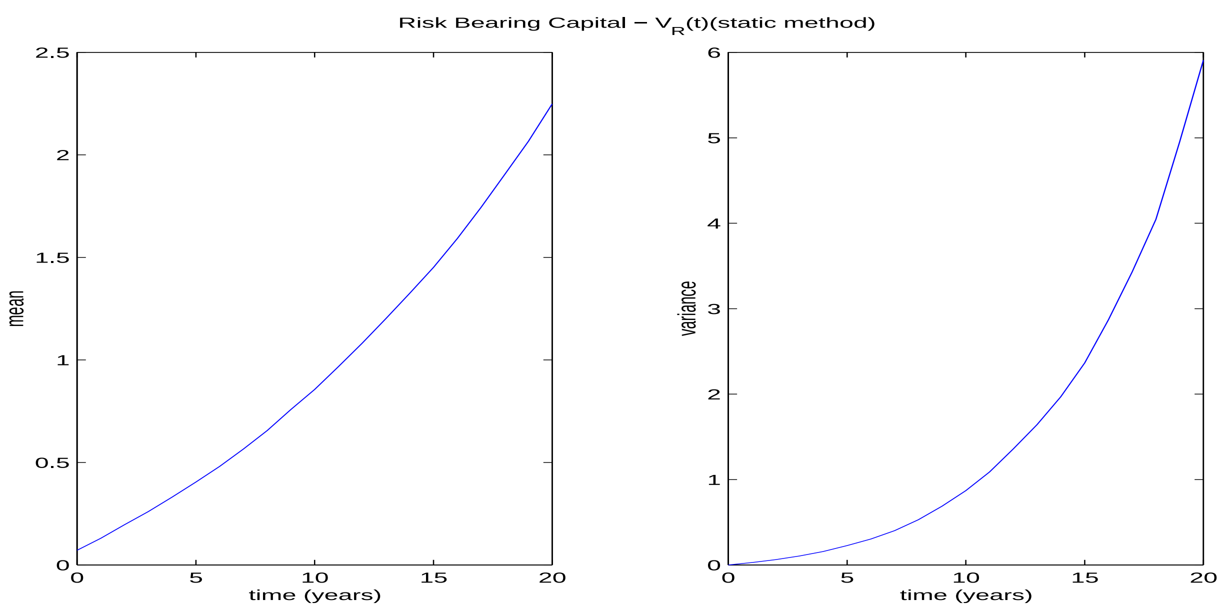

Figure 11: Mean and variance of the risk bearing capital distribution when the best estimate of the liability is given by the static reserve.

Figure 5 discussed in section 4. This latter Figure, in fact, shows that the reason for such a consistently high RBC is due to the fact that the static reserve consistently underestimates the fair value of the liability, to the extent that the reserves accumulated till maturity according to this method, fail to cover the actual amount of the benefits in 92% of the cases. The very high variance of the RBC (in Figure 11) is instead due to the fact that the static reserves are completely uncorrelated with the movements of the assets (and hence the market volatility).

6 Concluding remarks

In this paper, we describe a fair valuation approach to setting the premium and reserves for a simple, participating insurance policy, which includes in its design a minimum guaranteed rate and a participation rate (analogous to a reversionary bonus).

The decomposition of the contract into its basic components, i.e. the policy reserve and the default option, allowed by the fair value procedure, enables us to identify an allowance for the default option in the premium (via a solvency loading). We also demonstrate how such an allowance leads to a dramatic reduction in the probability of default for the contract at maturity, for reasonable parameter values. This is also relevant in terms of the risk management of the contract under consideration: the probability of default for the case without a solvency loading is, in fact, too high for any realistic hedging strategy to be effective.

An analysis of the model error and parameter error for the contract indicates the sensitivity of the results to the underlying assumptions and point to the need for a Market Value Margin. In order to represent model error, we use the idea that the insurance company assumes that its invested assets are accumulating according to a geometric Brownian motion process whereas, in fact, they are driven by a geometric Lévy process.

An important conclusion of the analysis is that attempts to use deterministic reserving methods for such a typical contract (defined along traditional lines) lead to undesirable results when compared with the fair valuation approach. These conclusions are demonstrated in section 4 by our considering the probability that, at the end of each year, the value of the reserves is below the fair value of the liability – a probability that we find to be unacceptably high. These conclusions are also supported by the analysis in section 5 which considers the moments of the Risk Bearing Capital distribution and the probability that the Risk Bearing Capital is positive at the end of each year, thereby demonstrating solvency.

References

[1] Bacinello, A. R. (2001). Fair pricing of life insurance participating contracts with a minimum interest rate guaranteed. *Astin Bulletin*, 31, 275-97.

[2] Bakshi, G. S., Cao, C. and Z. Chen (1997). Empirical performance of alternative option pricing models, *Journal of Finance*, 52, 2003-49.

[3] Ballotta, L. (2005). A Lévy process-based framework for the fair valuation of participating life insurance contracts, *Insurance: Mathematics and Economics*, 32, 2, 173-96.

[4] Ballotta, L., Haberman, S. and N. Wang (2006). Guarantees in with-profit and unitised with profit life insurance contracts: fair valuation problem in presence of the default option, *Journal of Risk and Insurance*, 73, 1, 97-121.

[5] Black, F. and M. Scholes (1973). The pricing of options on corporate liabilities, *Journal of Political Economy*, 81, 637-59

[6] Booth, P., Chadburn, R., Haberman, S., James, D., Khorasanee, Z., Plumb, R. H., and B. Rikayzen (2005). *Modern Actuarial Theory and Practice*, 2^{nd} edition, Chapman & Hall, London.

[7] Brennan, M. J. and E. S. Schwartz (1976). The pricing of equity-linked life insurance policies with an asset value guarantee, *Journal of Financial Economics*, 3, 195-213.

[8] Carr, P., Geman. H., Madan D. B. and M. Yor (2002). The fine structure of asset returns: an empirical investigation, *Journal of Business*, 75, 305-32.

[9] Daykin, C. D., Pentikainen, T. and E. Pesonen (1994). *Practical risk theory for actuaries*. 1^{st} edition, Chapman & Hall, London.

[10] Dickinson, G. and P. M. Liedtke (2004). Impact of a fair value financial reporting system on insurance companies: a survey. *The Geneva Papers on Risk and Insurance*, 29, 3, 540-81.

[11] European Commission (2004). Solvency II-Organization of work, discussion on Pillar I work areas and suggestions of further work on Pillar II for CEIOPS. Note to the IC Solvency Committee (February 2004).

[12] FASB (2004). Project update: fair value measurements.

[13] Fisher, H. and J. Young (1971). *Actuarial Practice of Life Assurance*, Cambridge University Press, Cambridge.

[14] FitchRatings (2004). Mind the GAAP: Fitch's view on Insurance IFRS.

[15] FOPI (2004). White Paper of the Swiss Solvency Test.

[16] FSA (2004). PS04/16. Integrated Prudential Sourcebook for Insurers.

[17] IASB (2004). International Financial Reporting Standard 4 - Insurance Contracts.

[18] Jørgensen, P. L. (2004). On accounting standards and fair valuation of life insurance and pension liabilities, *Scandinavian Actuarial Journal*, 5, 372-94.

[19] Miltersen, K., R. and S. A. Persson (2003). Guaranteed investment contracts: distributed and undistributed excess return, *Scandinavian Actuarial Journal*, 4, 257-79.

[20] Mina, J. and J. Y. Xiao (2001). Return to RiskMetrics: the evolution of a standard, *RiskMetrics*, New York.