“Waiting for Carnot”: Information and complexity

David Bawden and Lyn Robinson
Centre for Information Science
City University London

Consequently: he who wants to have right without wrong,
Order without disorder,
Does not understand the principles
Of heaven and earth.
He does not know how
Things hang together
Chuang Tzu, *Great and small*.

Thomas Merton (ed). *The way of Chuang Tzu*. New York NY: New Directions, 1965, p. 133.

Water is bland; ice crystals are beautiful
Frank Close, The Void, Oxford: Oxford University Press, 2007, p. 128.

Abstract
The relationship between information and complexity is analysed, by way of a detailed literature analysis. Complexity is a multi-faceted concept, with no single agreed definition. There are numerous approaches to defining and measuring complexity and organisation, all involving the idea of information. Conceptions of complexity, order, organization and ‘interesting order’ are inextricably intertwined with those of information. Shannon’s formalism captures information’s unpredictable creative contributions to organized complexity; a full understanding of information’s relation to structure and order is still lacking. Conceptual investigations of this topic should enrich the theoretical basis of the information science discipline, and create fruitful links with other disciplines which study the concepts of information and complexity.

Introduction

Complexity is a seemingly intuitively obvious concept, which is very difficult to define and understand. Batty, Morphet, Masucci, and Stanilov (2014, p. 364) roundly declare that “complexity, by its very nature is an impossible term to define … complex systems defy definition”. Yet, although the concept of complexity nay be difficult to define with precision, it is nonetheless clear that it is closely related to – indeed, intertwined with – the concept of information. The purpose of this paper is to examine this relationship by means of a detailed analysis of the pertinent literature. In doing so, we shall make extensive use of quotations from the original texts in order to ensure that the - often somewhat subtle - points made by their authors are not misinterpreted through the diluting medium of paraphrase; for a justification of this approach, see Bawden and Robinson (2015).

Analysis of the information-complexity relationship has often invoked the concept of entropy. In an earlier paper (Bawden and Robinson, 2015), we examined the information-entropy relation, noting in particular the different ways in which information had been related to structure and disorder, since the early days of information theory. For Claude Shannon, information was equivalent to entropy, and so was associated with randomness, uncertainty, and disorder (Shannon and Weaver, 1949). For Norbert Wiener (1948) it was the opposite, or negative, of entropy, and so was associated with order and structure. This polarity in views on the nature of information, first clearly identified and discussed in an information science context by Qvortrup (1993), is relevant to the complexity issue, but insufficient to resolve it. Information is associated with both pattern and randomness, Shannon's perspective capturing unpredictable and creative aspects, Wiener's concept capturing aspects of structured organization (Bawden and Robinson, 2015). Both perspectives are needed as a basis for analysis of the links between information and complexity, although all the information-based approaches to complexity discussed below have taken the Shannon perspective by default; this may be seen as one reason for their limitations.

The idea that systems rich in entropy, and in information, may experience self-organisation, and the growth of interesting complexity, is generally considered to have been first clearly stated by Prigogine and Stengers (1984), and later developed by writers such as Hayles (1990) and Goonatilake (1991). Most studies of the detailed nature of complexity has been carried out within the physical sciences, where links between complexity, entropy and information have been recognized for over two decades (see, for example, Zurek, 1990). But the relevance of the link between these concepts is by no means restricted to the physics domain. For example, Luciano Floridi makes use of them in his formulation of information ethics (Floridi, 2013). He argues, as a fundamental ethical basis, that entropy should be minimized in the infosphere, using entropy here to mean the destruction or corruption of entities understood as complex informational objects. When we reflect how different this usage of the concepts is from that of the study of objective, physical, information, and that scholars such as Brier (2010, 2013) have seen them relevant to the study of information in all its manifestations, we may feel justified in proposing that these issues are relevant to information science across the whole scope of the discipline.

Although we believe that analyses of this kind will ultimately have a practical value in contributing to the design of better information systems, this paper is an unapologetically theoretical and conceptual offering. We believe that the information sciences, no less than any other academic discipline, should seek to understand their foundational concepts as fully as possible, and how these interact with similar concepts in other disciplines.

Order, complexity, information

Shannon's equation yields a measure of information associated with uncertainty, unpredictability, randomness and disorder; Wiener's conception of information, calculated in essentially the same way, is associated with the opposite; with predictability and order. Neither of these can adequately capture the ideas of complexity and structure (Bawden and Robinson, 2015). This is a reflection of a fundamental difference between the ideas of simple 'order', on the one hand, and 'organisation' or 'complexity' on the other (Davies, 1987; Wicken, 1987A; Schneider and Sagan, 2005; Bawden. 2007). "Order and complexity", wrote Wicken (1987A, p. 43), "are cognates – carved from the same conceptual space, yet opposite in meaning". Order implies a simple, predictable structure; complexity an intricate arrangement of interacting entities. Entropy, or Shannon information, is a measure of the information needed to give a complete, ordered, description of any system; complexity is a measure of the information needed to specify the relationships between the elements of organized systems.

Both order and organization are associated with information, but in a rather different way: order, and entropy, may be viewed as a measure of the quantity of information, and organisation as a measure of its quality. The latter becomes important when we deal with complex systems, involving many interactions among their constituent elements, where, informally stated, the whole becomes more than the sum of the parts, and emergent properties become significant. However, as West (2013, p. 14) puts it "The trouble is, we don't have a unified, conceptual framework for addressing questions of complexity. We don't know what kind of data we need, nor how much, or what critical questions we should be asking."; for technical examination of this point, see Badii and Politi (1997), Feldman and Crutchfield (1998), Ellis (2004), Mitchell (2009), Zuchowski (2012), Gershenson and Fernández (2012), Gao, Liu, Zhang, Hu and Cao (2013), and Theurer (2014), and for a gentler introduction see Holland (2014).

(Ruelle 1991, pp. 136-137) sums it up like this: "An entity is complex if it embodies information that is hard to get. We have not said what 'hard to get' means, and therefore our definition of complexity has no sharp meaning. … . As a consequence, there will be not one but several definitions of complexity, depending on the background in which we place ourselves." Theurer (2014, p. 283) comments that "complexity can be measured in many ways … . A system that is less complex on one measure may turn out to be more so on another." Theurer suggests that the complexity of any system may be assessed in a number of general ways: by number of parts of the system; by degree of interactivity between parts; by the difficulty of predicting the system's behaviour from a knowledge of the properties of its parts; by computational complexity; by the extent to which to which it demonstrates emergent properties; and by the degree of non-linear behaviour exhibited.

In order to examine the relation between information and the slippery and multi-faceted concept of complexity, we can begin by noting that there is one aspect in which complexity, in whatever way it may be defined, is similar to entropy, and one in which it is very different.

The similarity is that there is a subjective element to complexity, as there is to entropy; its value changes as our knowledge of a system changes. As Simon (1962, p. 481) writes, "how complex or simple a structure is depends critically upon the way we describe it. Most of the complex structures found in the world are enormously redundant, and we can use this redundancy to simplify their description. But to use it, to achieve the simplification, we must find the right representation." And, as Simon points out, if structures in the world are not complex in a way that we find natural, e.g. in the form of a hierarchy, then we may not be able to understand and describe them, nor even necessarily recognize them as complex. Nor can there be a simple relation between the information available about a system and its perceived complexity; as we gain more information about a system, we may perceive it to be more organized and complex, or less so.

The difference is essentially the same as the distinction between entropy and organization (Bawden and Robinson, 2015). Complexity, like organization but unlike entropy, is not an extensive additive property. If we measure the physical entropy of a system, say a container of mixed gases, then if we add another identical container then the entropy doubles. If we have a complex object – a watch, a Siamese cat – then it will have a certain complexity; if we have five watches or cats, it certainly does not seem sensible to say we have five times the complexity. With some measures of complexity, such as logical depth, which we will introduce later, the amount of complexity in five items is not five times that of the original, but the original plus an amount to allow for the complexity of the production of the copies (Lloyd and Pagels 1988, p. 208; Crutchfield and Shalizi, 1999). In general, however, there is no clear relation between complexity and number of complex items.

Shannon's formalism is generally accepted as the appropriate metric for the 'order' sense of information, or entropy, a measure of the amount of information in a message or physical system, abstracted from any context and meaning-free, and reaching its maximum in a random set of symbols (Bawden and Robinson, 2015). A variety of approaches to defining organization and complexity, taking these concepts as essentially similar, if not synonymous, in a quantitative and objective way has been proposed; see Gell-Mann (1995A) for a brief introduction, Sporns (2007) and Mitchell (2009) for overviews, and McShea and Brandon (2010) for a distinction, in the biological realm, between what they term "colloquial complexity" and "pure complexity", the latter based on counts of the number and diversity of the elements of a system. Some complexity measures are formally defined, others are not, and it remains a challenge to find the most useful. Indeed there is still debate as to whether complexity is a concept more closely associated, in logical terms, with objects or processes; some analogy can be seen with the debates as to whether information is best regarded as a thing or as a process (Buckland, 1991; Case, 2012).

And indeed, conceptions of complexity are very often associated with information, particularly *mutual information*, the amount of information shared between random variables, correlating their variability and measuring the amount which knowing one tells us about the other (Furner, 2014, pp. 154-155). Those who study complex systems often use the concept of information to characterize and measure order and disorder, complexity and simplicity (Mitchell 2009). Cohen (2006, p. 1218) expresses it straightforwardly: “complex systems sense, store, and deploy more information than do simple systems”. The economist Eric Beinhocker (2006, p. 12) writes that “evolution can perform its tricks not just in the ‘substrate’ of DNA but in any system that has the right information processing and information storage characteristics.” The physicist Murray Gell-Mann (1995B, p. 21) said of complex adaptive systems that “although they differ widely in their physical attributes, they resemble one another in the way they handle information. That common feature is perhaps the best starting point for exploring how they operate”. Gell-Mann and Lloyd (1996) similarly regard entropy, Shannon information and various measures of complexity as all being ‘information measures’. Tegmark (2014, p. 292) gives a nuanced account of their inter-relations:

> “Whereas the complexity of an object measures how complicated it is to describe, its information content [mutual information between the object and the rest of the world] measures the extent to which it describes the rest of the world. In other words, information is a measure of how much *meaning* complexity has. If you fill your hard drive with random numbers, then it contains no information about the outside world, but if you fill it with history books or with movie clips of your family, then it does.”

It is worth remembering that those who have studied this topic are generally not interested in all complexity, but in organized complexity which is interesting and meaningful, in the usual sense of being comprehended by, and conveying meaning to, a human recipient. Here, there is a necessary extension of the scope of the concept of quantitative objective information, regarded as free of meaning in Shannon’s original conception. As long ago as 1948, Warren Weaver distinguished between unorganized and organized complexity. The former is seen in situations with many variables, and where each of the variables has an individual erratic and unpredicable behaviour, but where the system as a whole has orderly statistical properties; these are precisely the situations analysed by statistical mechanics, using concepts of entropy and Shannon information. The latter is seen in situations intermediate between disorganized complexity and the simplicity of a small number of deterministic variables, and where there is regularity, meaningful interaction between components and “a sizable number of factors which are interrelated into an organic whole” (Weaver, 1948, p. 539).

In 2001, Seth Lloyd produced what he described as a “non-exhaustive” list of complexity measures then extant, arranged by three dimensions along which complexity can be measured: how hard is a system to describe?; how hard is it to create?; and what is its degree of organization? The list includes over 40 measures; many are based on information theoretic considerations, although there are others, for example simple – the size of the system, i.e. the number of components that it encompasses – or complicated – e.g. its fractal nature, based on concepts from dynamic systems theory.

So, how can we understand and measure complexity, focusing on measures involving information? We consider below some examples of these measures, though by no means all of those which have been suggested.

Algorithmic information content

We could simply use Shannon entropy, or its negative, as a measure of complexity; Batty, Morphet, Masucci, and Stanilov (2014), for example, use this measure for the complexity of spatial geographic systems, such as cities. Or we could, as Gatlin (1972) and Layzer (1977) did, use the difference between maximum possible entropy and observed entropy, denoting this difference as 'information' or 'organisation'.

But to apply Shannon's formalism in this way, we first have to express whatever system we are examining in the form of a message, which may be a forced analogy. Even then, we will find that these measure equate greatest complexity with greatest randomness. This does not seem to make sense, since one of the things that makes at least some interesting things complex is precisely that they are not random, but have evolved or been created to be useful or beautiful (Bawden and Robinson, 2015). This view has been expressed clearly by several authors, for example:

> "The most complex entities are not the most ordered or random ones but somewhere in between. Simple Shannon entropy doesn't capture our intuitive concept of complexity." (Mitchell, 2009, p. 98)
>
> "Dynamical systems range in a continuum from completely ordered, regular systems like the arrangement of carbon atoms in a diamond crystal to completely disordered, chaotic systems like molecules in a gas. The intuitive notion of complexity … is that complex systems lies somewhere in the continuum between order and chaos. Polymers, cells, brains and chickens are all structurally complex – they are neither wholly ordered or wholly disordered. Any reasonable measure of complexity should therefore vanish for the extremes of complete order or disorder and not vanish for the structurally intricate systems between these extremes" (Lloyd and Pagels, 1988, p. 187).

One interesting and relevant extension of Shannon's formalism is *algorithmic information content* (AIC), devised independently by Gregory Chaitin and by Andrei Kolmogorov (Gleick, 2011). This assesses a message, or other set of information, according to the length of the shortest algorithm which can reproduce it (formally, to ensure consistency, this is taken to mean a program for a universal Turing machine in binary code). When the algorithm is short, i.e. AIC is low, the message is simple and ordered and contains little information (in the Shannon sense); when it is long, the message is complex and random, and information-rich. This measure formalises the ideas that a simple situation is easy to describe, needing only a little information for a complete description, while a complex situation is hard to describe, needing a lot of information to achieve a complete account, and that the amount of information in a system can be quantified by the length of its most concise description (Gell-Mann and Lloyd, 1996). This idea had been clearly stated by

Simon (1962, p. 478) decades earlier: “If a complex structure is completely unredundant – if no aspect of its structure can be inferred from any other – then it is its own simplest description. We can exhibit it, but we cannot describe it by a simpler structure." A wide survey of the applications of algorithmic complexity is given by Burgin (2010, pp. 364-372).

Unlike Shannon’s information, AIC does not deal with probabilities over an ensemble of messages, but is a property of individual messages. If the message is part of an ensemble, however, the average AIC of the messages is closely related to the Shannon measure (Gell-Mann and Lloyd, 1996).

Wicken (1987B) commends AIC on the grounds that what Shannon’s formula measures is the complexity of structural relations, that ‘complexity’ is a better term than ‘entropy’, and that Chaitin’s formalism makes clear that Shannon information is related to this kind of complexity. However, “like entropy, algorithmic information content assigns higher information content to random objects than ones we would intuitively consider to be complex” (Mitchell, 2009, p. 98). Any random sequence of letters of the Latin alphabet will have a higher algorithmic content than will any of Shakespeare’s plays. The algorithmic definition of complexity is at root a definition of randomness (Lloyd and Pagels, 1988). As Deacon and Koutroufinis (2014, pp. 407-408) put it, “This understanding of complexity therefore produces a paradoxical problem in that it effectively treats a maximally disordered system and the random string of characters that describes it as more complex than ones that exhibit interesting and/or unprecedented properties, such as being alive or being conscious. Intuition suggests instead that a thoroughly random and maximally unpredictable, i.e. maximally incomprehensible, sequence is simple in its organization”.

This may be illustrated by an architectural example, given by Smolin (2013) and extended by Bawden and Robinson (2015). We may compare Frank Gehry’s Guggenheim Museum in Bilbao, a building whose outer surface is composed of individual elements, each unique, so that each element must go into a specific place, with Battersea Power Station in London, the largest brick building in Europe, with over 60 million bricks, whose structure is unaffected if any sets of particular bricks are inter-changed. According to Shannon's information theory, the brick building would possess a higher information content, whereas in Wiener's interpretation it would have a lower information content (Bawden and Robinson, 2015). Extending this further, we can say that the Gehry building has greater AIC than Battersea Power Station, because the instructions given to its builders must necessarily be more detailed and hence extensive. Greater AIC is generally understood as more randomness, but the Gehry building is anything but random, in the usual sense of the word. Rather it looks complicated, irregular, and unstructured, while the power station looks simple, uniform, and structured; this is the aspect measured by their respective AIC.

AIC gives a measure of complexity associated with the amount of Shannon information. It is however a rather limited one, which Gell-Mann characterizes as “crude complexity … [which] does not correspond to what is meant by complexity in most situations … to define effective complexity, one needs something quite different from a quantity that achieves its maximum in random strings" (Gell-Mann, 1995B, p. 50). This, and similar measures, differs from entropy, in that any conception of entropy, as we have seen deals with sets of states, while AIC can be measured for a single state.

Before considering what these might be, we should note one particularly relevant application of AIC. Zurek (1989A, 1989B) represented the physical entropy of a system as the sum of the missing information, calculated by Shannon's formula, and the AIC of a bit string representing the available data about the system. By virtue of this insight, which was further developed by others (Gell-Mann and Lloyd, 1996), Zurek is believed by many commentators to have finally solved the problem of Maxwell's Demon, and hence the problem of subjectivity of entropy (Leff and Rex, 1990; Leff and Rex, 2002; Bawden and Robinson, 2015). However, there is continuing dissent as to whether information theoretic arguments have indeed solved the problem of the Demon; see, for example, Norton (2013). The contribution of AIC to entropy is tiny in comparison to that of the Shannon information (of course, if there were total knowledge of every microscopic aspect of the system, the entropy due to missing information would be nil), but this would seem to finally confirm the status of information as a constituent of the physical universe (Bawden and Robinson, 2013). In general, the contribution of what we might call 'intellectual information' is very small, compared with physical entropy: compressing a mole [a standard measure of chemical quantity] of gas to half its original volume decreases our ignorance by 10^{23} bits, a far larger change in entropy than that produced by memorizing all the books ever written (Sethna, 2006, p. 85). This illustrates clearly the meaning-free (again in the usual sense of failing to convey meaning to a human recipient) nature of information entropy, and its inability to characterize interesting information.

So what measure might characterize 'interesting' information? Shannon's information entropy and AIC both distinguish clearly the opposite poles of order, repetition, and certainty, and of disorder, randomness, and uncertainty, whether in the context of a message, a physical system, or the whole universe. But both, these opposite poles, as we have seen, embody simplicity; complex interesting things happen in the intermediate regions, in what Baranger (2001) characterizes as an interplay between chaos and non-chaos. The idea that interesting things, organized complexity, happen at the interface between order and chaos is seen in many contexts, from art to science. This was Erwin Schrodinger's insight when he suggested an aperiodic crystal as the carrier of information in living systems (Schrödinger, 1944).

Ruelle (2007, p. 127) expresses this dual nature of interesting information particularly clearly, in the context of mathematics:

> "I think that the beauty of mathematics lies in uncovering the hidden simplicity and complexity that coexist in the rigid logical framework that the subject imposes. Of course, the interplay and tension between simplicity and

> complexity are an element of art and beauty also outside of mathematics. Indeed, the beauty that we find in mathematics must be related to the beauty that our human nature sees elsewhere. And the fact that we are attracted by both simplicity and complexity, two contradictory concepts, befits our illogical human nature. But the remarkable thing here is that the shock of simplicity and complexity is intrinsic to mathematics; it is not a human construction. One may say that this is why mathematics is beautiful: it naturally embodies the simple and the complex that we are yearning for."

Mathematicians, says Ruelle, look not merely for correct statements, logically derived from axioms, but for interesting results: "These interesting results, or theorems, organize themselves into meaningful and natural structures, and one may say that the object of mathematics is to find and study these structures." (Ruelle 2007, p. 8). As examples of such structures, he names groups, algebraic varieties, categories and functors. Similarly, Byers (2007, p. 316) argues that interesting results in mathematics, and indeed much more generally, are to be found in regions that are neither completely ordered nor completely random, but where there is complexity that is to say, ordered randomness. Without the existence of the phenomenon we are calling randomness, life would be boring: there would be no evolution, no innovation, no creativity. On the other hand, a world that consisted only of the random would be terrifying, unpredictable, chaotic with no regularities and therefore no life.

One promising idea is to use AIC to capture the random element of complexity, complementing it by some measure of the difficulty of creating the system at hand; such a measure will be probabilistic, and will therefore naturally use Shannon's conception of information (Lloyd and Pagels, 1988). We consider this idea in the next section.

Effective and statistical complexity

How are we to define and quantify information structures which are interesting, and which reflect internal regularities and orders which are not merely repetitive? There are a variety of suggestions, but as yet no agreement, on finding measures which reach a maximum when the AIC is neither very large nor very small (Gell-Mann, 1995B; Devine, 2009; Gleick, 2011).

One such is Gell-Mann's 'effective complexity', a measure related to AIC, but which accords better than AIC with our intuitive ideas about complexity (Gell-Mann, 1995A; Gell-Mann, 1995B; Gell-Mann and Lloyd, 1996). This assumes that any given entity is composed of a combination of regularity and randomness. The randomness is uninteresting, so we apply the AIC formalism to the non-random parts of the system. "The amount of information needed to describe the set of identified regularities of an entity is that entity's *effective complexity*. An information or entropy term describing the random component can be added to the effective complexity to yield what we call the *total information*" (Gell-Mann and Lloyd, 1996, p. 45). 'Total information' is also referred to as 'augmented entropy', emphasizing the link between the two. "Effective complexity measures knowledge, in the sense that it quantifies the extent to which an entity is taken to be regular, non-random and hence predicatable. The remaining features of the entity are taken to be irregular and probabilistic." (Gell-Mann and Lloyd, 1996, p. 49).

"To calculate the effective complexity, first one figures out the best description of the regularities of the entity: the effective complexity is defined as the amount of information contained in that description, or equivalently, the algorithmic information content of the set of regularities … The best set of regularities is the smallest one that describes the entity in question, and that at the same time minimizes the remaining random component of the entity" (Mitchell, 2009, p. 99). This is an appealing idea, and Fuentes (2014), pointing out that it is a way of measuring one of the relations between information and complexity, has applied it to define emergent properties of complex systems.

"Effective complexity is a compelling idea, though like most of the proposed measures of complexity, it is hard to actually measure. Critics have also pointed out that the subjectivity of its definition remains a problem" (Mitchell, 2009, p. 99). How do we decide what the regularities are, and what if we disagree? They will depend on which regularities we can detect, in which of them we are interested, and how we describe them; McAllister (2003) gives examples. Gell-Mann and Lloyd (1996) argue that a formally 'best' set of regularities may be identified as that which mimimises the effective complexity for the least value of total information. This is certainly an objective measure for choosing between alternative regularities, but offers no way of identifying them, or deciding which are interesting.

A different approach to assessing complexity, though still based on Shannon, is 'statistical complexity' (Crutchfield and Young, 1989) and the very similar 'effective measure complexity' (Grassberger, 1986), both of which assess the minimum amount of information about the past behaviour of a system which is needed to predict its behaviour in the future, in statistical terms. These are closely related to Shannon entropy, in that the system is regarded as a source of messages, and its behaviour as a series of such messages. A model of the system is created so that the set of 'messages' it produces is the same over time, as a statistical distribution, as that of the actual system. The statistical complexity is then the Shannon information content of the simplest such model which predicts the system's behaviour. Like effective complexity, statistical complexity is low for both highly ordered and random systems and is high for systems in between; those that would normally be considered as complex.

The problem with this general approach is that it is not always obvious how a system may be represented as a message source. An alternative, or, better, complementary, approach to defining complexity examines how difficult an object is to make; essentially identifying its complexity with the amount of information (measured in Shannon terms) processed in its creation. This relies on the seemingly reasonable idea that complex things been created with some difficulty or have evolved over a long period of trial and error. This is considered in the next section.

Logical and thermodynamic depth
One way of considering complexity in terms of creation or evolution is Bennett's 'logical depth', which formally defines the complexity of an entity in terms of the simplest Turing machine (that with the least number of states and rules) that could produce it, when the entity is encoded in binary form (Bennett, 1990; Lloyd, 1990). By this definition, both random numbers and ordered regular numbers are logically shallow, the desired property. However, this approach has severe practical problems with regard to application: "Logical depth has very nice theoretical properties that match our intuitions, but it does not give any practical way of measuring the complexity of any natural object of interest, since there is typically no practical way of finding the smallest Turing machine that could have generated a given object, not to mention determining how long that machine would take to generate it. And this doesn't even take into account the difficulty, in general, of describing a given object [in binary form]" (Mitchell, 2009 p. 101).

An extension of this idea, which does not require the encoding of physical systems as numbers, is Lloyd and Pagels' (1988) 'thermodynamic depth'. Based on a similar conception to logical depth, namely that complex objects are harder to construct, it measures the total amount of thermodynamic and informational resources required by the physical construction process, information again understood in Shannon's terms (Lloyd and Pagels 1988; Lloyd 1990). This is appealing, but requires us to know, in some detail, the process of creation. It also shares a property of entropy, namely that its value clearly varies according to our knowledge of the situation; something which appears simple at first glance may be revealed as complex on closer inspection (Bawden and Robinson, 2015). In fact, this measure of complexity is equal to the difference between coarse-grained, macroscopic, entropy, and fine-grained entropy (Lloyd and Pagels, 1988, p. 191). It is proportional to the amount of information, by Shannon's measure, needed to characterise the way in which a system came to its current state – "the amount of information shunted between the various parts of the system in the process of constructing a particular state" (Lloyd and Pagels 1988, p. 208) - and is therefore closely related to logical depth, and indeed to AIC (Crutchfield and Shalizi, 1999). Other somewhat similar measures which focus on the diversity of the physical components of a system, though still with information as a fundamental component, include *physical complexity*, *predictive information,* and *dynamical depth* (Deacon and Koutroufinis, 2014).

There are, however, other measures of complexity, starting from quite different premises, though still related in various ways to information, which address aspects not included by the Shannon or AIC formalisms. These are considered in the next section.

Structural and hierarchical complexity
One such alternative measure of complexity is termed 'incomparability', a measure of the degree of similarity in a set of individuals, which may be used to measure the way in which the similarity of parts of a system change over time, and which was initially suggested by Seitz and Kirwan (2014). They give a very simple example of this as follows: consider a group of people throughout their lives, from birth to a

great age. As babies, and as very elderly people, they will have many aspects of life in common; in middle age, following different careers and roles, they will be at their most different, and hence their group will be in its most complex state.

Yet other measures of complexity consider a different expression of information, the degree of hierarchy in the organization of a system. The idea that complexity is associated with repeated division into sub-systems was introduced by Herbert Simon's famous paper of 1962, 'The architecture of complexity'. Ellis (2004) and Mitchell (2009) note, with examples, that many others have explored the idea of hierarchy as a way of quantifying complexity. Hierarchy is, of course, a central concept in several areas of LIS, such a classification, resource description and information architecture; as examples, see Jinfang (2013), Hall, Fernando, Clough, Soroa, Agirre and Stevenson (2014), Neelameghan (2002), and Wright, Nardini, Aronson and Rindflesch (1999). That such a vital tool, and well-nigh universally used, principle for organizing and representing meaningful recorded information is also an objective measure of complexity is a further indication that complexity and information are closely linked.

Simon introduced the idea that "that complexity frequently takes the form of hierarchy, and that hierarchic systems have some common properties that are independent of their specific content" (Simon 1962, p. 468). Essentially, this amounts to the idea that complex systems have many sub-units, hierarchically arranged, and that interactions and information exchange happen mainly within, rather than between, sub-units. However Simon presented no quantitative formula for measuring complexity in this way, suggesting rather that the study of complex systems needs a 'theory of hierarchy'.

"There are many measures of complexity … Each of these measures captures something about our notion of complexity but all have both theoretical and practical limitations, and have so far rarely been useful for characterizing any real-world systems. The diversity of measures that have been proposed indicates that the notions of complexity that we're trying to get at have many different interacting dimensions and probably can't be captured by a single measurement scale."
(Mitchell 2009, pp. 110-111)

Universal laws of complexity may be too ambitious or too vague. Defining complexity may be the wrong approach. Maybe complexity is the wrong word. There is concern that "the field of complex systems will share the fate of cybernetics … - that is, it will pinpoint intriguing analogies among different systems without producing a coherent and rigorous mathematical theory that explains and predicts their behaviour" (Mitchell, 2009, p. 299). "An in-joke in [complexity science] is that we're 'waiting for Carnot'. Sadi Carnot was a physicist of the early nineteenth century who originated some of the key concepts of thermodynamics. Similarly, we are waiting for the right concepts and mathematics to be formulated to describe the many forms of complexity we see in nature" (Mitchell, 2009, p. 302). We now go on to consider what forms these might take, with the proviso that such ideas are still speculative.

Beyond entropy and order

Power laws are ubiquitous in the information sciences, and in many other contexts in the physical, biological and social sciences (Egghe, 2005; Mitzenmacher, 2004). The physical and informational mechanisms by which they arise are still open problems, and may have significant relations to the understanding of complexity (Mitchell, 2009, p. 272). They are found to occur in many self-organised complex systems.

Stuart Kauffman put forward the idea that the evolution of complex organisms is due in part to self-organization, which may predominate over natural selection (Kauffmann, 1993, 2000). He has argued for a "fourth law of thermodynamics": life has an innate tendency to become more complex, which is independent of any tendency of natural selection. Others argue that there is no need for any 'fourth law'; only the extension of the second law to open systems, away from equilibrium, in which complexity may spontaneously arise (see, for example, Schneider and Sagan, 2005).

Such measures go beyond the entropy/information/order/disorder issue, and reflect the realization that although such considerations are the basis for complexity and creativity, they are not sufficient to capture it. As Kaufmann (2000, pp. 4-5) puts it, the conception of organization which they embrace is "not covered by matter alone, energy alone, entropy alone, or information alone"; in particular "our concepts of entropy and its negative, Shannon's information … entirely miss the central issues". This may be seen in physics, as summarized by Ford (2103, p. 3):

> "The attributes of the universe are being mixed up in a manner that is hard to follow and our failure to grasp and retain the detail of all this is what is meant by the growth of uncertainty…. The second law is a reflection of an underlying imperative to mix, share and explore [and to evolve complexity, feeding off energy flows] and the growth of entropy is our rationalization of this complexity."

and by Smolin (2013):

> "The universe is a process for breeding novel phenomena and states of organization, which will forever renew itself as it evolves to states of ever higher complexity and organization" (Smolin, 2013, p. 194) … "A universe in equilibrium cannot be complex, because the random processes that bring it to equilibrium destroy organization. But this does not mean that complexity itself can be measured by the absence of entropy. To fully characterize complexity we need notions beyond the thermodynamics of systems in equilibrium" (Smolin, 2013, p. 202) …. "Highly complex systems cannot be in equilibrium, because order is not random, so high entropy and high complexity cannot coexist. Describing a system as complex does not just mean that it has low entropy. A row of atoms sitting in a line has low entropy, but is hardly complex. A better characterization of complexity, invented by Julian Barbour and myself, is what we call variety: a system has

> high variety if every pair of its subsystems can be distinguished from each other by giving a minimal amount of information about how they are connected or related to the whole. A city has high variety because you can easily tell from looking around which corner you are on. Such conditions arise in nature in systems far from equilibrium as a result of processes of self-organisation" (Smolin, 2013, p.219).

The emphasis on the conflict between complexity and equilibrium is worth noting. Complexity cannot arise, or continue, in a static state, where all elements are balanced; it needs continual change and activity to exist. For more detail on variety as a measure of complexity, see Barbour and Smolin (1992).

Something analogous in the limitations of the measure of order and disorder as a useful dimension for describing a system is also observed in the realm of subjective meaningful information, where ordered and disordered surroundings – the classic, and often quoted examples being tidy and messy desks - both promote different, and equally positive, behaviours (Vohs, Redden and Rahinel, 2013; Bawden and Robinson, 2015). Tidy and messy environments exhibit complexity in different ways.

That this limitation of the order-disorder dimension is observed whether information is regarded as subjective of objective is an important issue for those interested in looking at links between information in the physical and social worlds. Here is an area where the information sciences may contribute to physical sciences and vice versa, as advocated by Furner (2014) and Robinson and Bawden (2014). What makes 'interesting complexity' is the relevant question, not how information relates to entropy, order and disorder, and this may be linked with the concept of emergent properties.

An emergent property is a property which becomes manifest only at a certain scale or level of complexity. A well-known example in the physical world is temperature. This can be understood informally as the measure of the amount of heat in some physical material, or defined formally as a function of the way the atoms of the material are distributed among available energy levels. It is meaningless to speak of the temperature of a single atom, or a few isolated atoms. This is not to say that the temperature of an isolated atom is impossible to measure, or has an indeterminate value; rather that the idea is has no meaning. The property of temperature only emerges when a sufficient number of atoms are gathered together.

Information similarly has emergent properties, when we consider the elements of information formally, as bits as defined by Shannon, or informally, as isolated facts or figures. A simple example is the *consistency* or *reliability* of a collection of information, a property which cannot be possessed by an isolated item of information, however defined. It may be objected that we might form a view of a single fact or figure, because we are aware of the source from which it comes, and the meaning which it is intended to impart; but then it is no longer isolated, but part of a larger system of information and knowledge.

It may be that all interestingly complex information structures may amount to emergent information properties and may go some way to answering one of the 'Big Questions' posed by physicist John Wheeler: "what makes meaning?" (Bawden, 2008; Clayton, 2004). However, we must here add the caveat, expressed clearly by Theurer (2014, p. 277) that "there is a broad consensus that emergence is somehow linked with complexity. But the precise nature of that relationship is less than clear. Some argue that emergence is a fundamental feature of complex systems. Others argue that emergence is itself complex". Perhaps future analyses of emergent properties in information-rich systems may shed light on this fundamental issue.

Conclusions

Information is inextricably associated with pattern, but also with randomness (Bawden and Robinson, 2015). We can now see clearly that there are two aspects to the role of information in creating complexity. For the unpredictable, and arguably creative, aspects, Shannon information, particularly as expressed in AIC, is appropriate. For the aspects of structure and organisation, Wiener's negative entropy concept is too simplistic to be appropriate. We need a better way of understanding and formalising this face of information to capture the structural aspects of the relation between information and complexity.

Kauffman (1993, p. 173) suggests that eighteenth century science dealt with organized simplicity through the Newtonian paradigm, nineteenth century science dealt with disorganized complexity through statistical mechanics, while twentieth century science began to deal with organized complexity. We might say that the first ignored complexity through reductionism and a focus on the most simple cases, the second introduced the idea of entropy, while the third is still looking for some equivalent concept for organized complexity. It seems inevitable that this will be associated with information.

We still do not understand complexity, nor how best to measure and compare it in any particular system or context. In that sense, we are indeed still waiting for Carnot. But we do know that conceptions of complexity, order, organization and 'interesting order' are inextricably intertwined with those of information. Ellis (2004, p. 607) puts it strongly: "True complexity involves vast quantities of stored information and hierarchically organized structures that process information in a purposeful manner". It is not possible to discuss complexity without information concepts, nor is it possible to discuss information, except as the simplest meaning-free Shannon measure of communication capacity, without invoking ideas of complexity. Meaning is inherently complex, and arguably best understood as an emergent property within complex information-rich systems.

This relation is not a simple one. Information concepts have been powerful aids in understanding and quantifying entropy, but that does not mean that information and entropy are the same; nor are they opposites (Bawden and Robinson, 2015). The same is true for the informational bases of complexity. There are profound and subtle relations between entropy and information, objective and subjective, order and chaos, simplicity and complexity. These may point to the possibility that there may be underlying, and wide-reaching, information laws, applicable across many domains of enquiry, and levels of reality, as suggested by authors such as Bawden and Robinson (2013), Robinson and Bawden (2014), Brier (2010, 2013), and Duncan and Semura (2007). This is surely a worthwhile focus for study over the coming decades, in information science as much as in any other area. Such studies will achieve the aims expressed at the outset, of strengthening the theoretical base of the information science discipline, with the consequent prospect of advances in practice.

Acknowledgements
We thank two anonymous referees for detailed and insightful comments which greatly helped to improve this paper. We are grateful to the staff of the British Library, and of the libraries of City University London and University College London for assisting in access to the wide range of material examined.

References

Badii, P. and Politi, A. (1997). Complexity: hierarchical structures and scaling in physics. Cambridge: Cambridge University Press.

Baranger, M. (2001). Chaos, complexity and entropy. Unpublished presentation, available at http://necsi.edu/projects/baranger/cce.pdf.

Barbour, J. and Smolin, L. (1992). Extremal variety as the foundation of a cosmological quantum theory. arXiv:hep-th/92-3041. Retrieved 7 July 2014 from http://arxiv.org/abs/hep-th/9203041.

Batty, M., Morphet, R. Masucci, P. and Stanilov, K. (2014). Entropy, complexity and spatial information. Journal of Geographical Systems, 16(4), 363-385.

Bawden, D. (2007). Organised complexity, meaning and understanding: an approach to a unified view of information for information science. Aslib Proceedings, 59(4/5), 307-327.

Bawden, D. (2008). Really big questions and the meaning of documentation: in memoriam John Archibald Wheeler (1911-2008). Journal of Documentation, 64(5), 641-642.

Bawden, D. and Robinson, L. (2015). “A few exciting words”: information and entropy revisited. Journal of the Association for Information Science and Technology, in press.

Bawden, D. and Robinson, L. (2013). “Deep down things”: in what ways is information physical, and why does it matter for LIS?. Information Research, 18(3), paper C03 [online]. Retrieved from http://InformationR.net/ir/18-3/colis/paperC03.html 18 October 2014.

Beinhocker, E.D. (2006). The origin of wealth: evolution, complexity, and the radical remaking of economics. Cambridge MA: Harvard Business School Press.

Bennett, C.H. (1990). How to define complexity in physics and why. In Zurek, W. H. (ed.). Complexity, entropy and the physics of information, Boulder CO: Westview Press, pp 137-148.

Brier, S. (2013). The transdisciplinary view of information theory from a cybersemiotic perspective, in Theories of information, communication and knowledge: a multidisciplinary approach, Ibekwe-SanJuan, F. and Dousa, T. (eds.), Dordrecht: Springer, pp. 23-49.

Brier, S. (2010). Cybersemiotics: an evolutionary world view going beyond entropy and information into the question of meaning. Entropy, 12(8), 1902-1920.

Buckland, M.K. (1991). Information as thing. Journal of the American Society for Information Science. 42(5), 351-360.

Burgin, M. (2010). Theory of information: fundamentality, diversity and unification. World Scientific: Singapore.

Byers, W. (2007). How mathematicians think: using ambiguity, contradiction and paradox to create mathematics. Princeton and Oxford: Princeton University Press.

Case, D.O. (2012). The concept of information. Chapter 3 of Looking for information: a survey of research on information seeking, needs, and behaviour (3rd edn.). Bingley, Emerald, pp. 45-75.

Clayton, P.D. (2004). Emergence: us from it. In Barrow, J.D., Davies, P.C.W. and Harper, C.L. (eds.) Science and ultimate reality: quantum theory, cosmology and complexity. Cambridge: Cambridge University Press, pp. 577-606.

Cohen, I. (2006). Informational landscapes in art, science, and evolution. Bulletin of Mathematical Biology, 68, 1213-1229.

Crutchfield, J.P. and Shalizi, C.R. (1999). Thermodynamic depth of causal states: when paddling around in Occam's pool shallowness is a virtue. Physical Review E, 59(1), 275-283.

Crutchfield, J.P. and Young, K. (1989). Inferring statistical complexity. Physical Review Letters, 63(2), 105-108.

Davies, P. (1987). The Cosmic Blueprint: Order and Complexity at the Edge of Chaos. Penguin, London.

Deacon, T. and Koutroufinis, S. (2014). Complexity and dynamical depth. Information, 5(3), 404-423.

Devine, S. (2009). The insights of algorithmic entropy. Entropy, 11(1), 85-110.

Duncan, T.L. and Semura, J.S. (2007). Information loss as a foundational principle for the second law of thermodynamics. Foundations of Physics, 37(12), 1767-1773.

Egghe, L. (2005). Power laws in the information production process: Lotkaian informetrics. Amsterdam: Elsevier.

Ellis, G.F.R. (2004). True complexity and its associated ontology. In Barrow, J.D., Davies, P.C.W. and Harper, C.L. (eds.) Science and ultimate reality: quantum theory, cosmology and complexity. Cambridge: Cambridge University Press, pp. 607-636.

Feldman, D.P. and Crutchfield, J.P. (1998). Measures of statistical complexity: why? Physics Letters A, 238(4/5), 244-252.

Floridi, L. (2013). The ethics of information. Oxford: Oxford University Press.

Ford, I. (2013). Statistical physics: an entropic approach. Chichester: Wiley.

Fuentes, M.A. (2014). Complexity and the emergence of physical properties. Entropy, 16(8), 4489-4496.

Furner, J. (2014). Information without information studies. In Ibekwe-SanJuan, F. and Dousa, T. (eds.). Theories of information, communication and knowledge: a multidisciplinary approach. Dordrecht: Springer, pp. 143-179.

Gao, J., Liu, F., Zhang, J., Hu, J. and Cao, Y. (2013). Information entropy as a building block of complexity theory. Entropy, 15(9), 3396-3418.

Gatlin, L. (1972). Information theory and the living system. New York NY: Columbia University Press.

Gell-Mann, M. (1995A). What is complexity? Complexity, 1(1), 16-19.

Gell-Mann, M. (1995B). The quark and the jaguar: adventures in the simple and the complex. London: Abacus.

Gell-Mann, M. and Lloyd, S. (1996). Information measures, effective complexity and total information. Complexity, 2 (1), 44-52.

Gershenson and Fernández (2012). Complexity and information: measuring emergence, self-organization, and homeostatis at multiple scales. Complexity, 18(2), 29-44.

Gleick, J. (2011). The information: a history, a theory, a flood. London: Fourth Estate.

Goonatilake, S. (2001). The evolution of information: lineages in gene, culture and artefact. London: Pinter Publishers.

Grassberger, P. (1986). Toward a quantitative theory of self-generated complexity. International Journal of Theoretical Physics, 25(9), 907-938.

Hall, M.M., Fernando, S., Clough, P.D., Soroa, A., Agirre, E. and Stevenson, M. (2014). Evaluating hierarchical organisation structures for exploring digital libraries. Information Retrieval, 17(4), 351-379.

Hayles, N.K. (1990). Chaos bound: orderly disorder in contemporary literature and science. Ithaca NY: Cornell University Press.

Holland, J.H. (2014). Complexity: a very short introduction. Oxford: Oxford University Press.

Jinfang, N. (2013). Hierarchical relationships in the bibliographic universe. Cataloging and Classification Quarterly, 51(5), 473-490.

Kauffman, S.A. (1993). The origins of order. New York NY: Oxford University Press.

Kauffman, S.A. (2000). Investigations. New York NY: Oxford University Press.

Layzer, D. (1977). Information in cosmology, physics and biology. International Journal of Quantum Chemistry, 12(supplement 1), 185-195.

Leff, H.S. and Rex, A.F. (1990). Maxwell's Demon: entropy, information, computing. Bristol: IOP Publishing.

Leff, H. S., and Rex, A.F. (2002). Maxwell's Demon 2: Entropy, classical and quantum information, computing. Bristol: IOP Publishing.

Lloyd, S. (1990). The calculus of intricacy. The Sciences, 30(5), 38-44.

Lloyd, S. (2001). Measures of complexity: a non-exhaustive list. IEEE Control Systems Magazine, August 2001.

Lloyd, S. and Pagels, H. (1988). Complexity as thermodynamic depth. Annals of physics, 188(1), 186-213.

McAllister, W. (2003). Effective complexity as a measure of information content. Philosophy of Science, 70(2), 302-307.

McShea, D.W. and Brandon, R.N. (2010). Biology's first law: the tendency for diversity and complexity to increase in evolutionary systems. Chicago IL: Chicago University Press.

Mitchell, M. (2009). Complexity: a guided tour. New York NY: Oxford University Press.

Mitzenmacher, M. (2004). A brief history of generative models for power law and lognormal distribution. Internet Mathematics, 1(2), 226-251.

Neelameghan, A. (2002). Hierachy, hierarchical relation and hierarchical arrangement. Information Studies, 8(1), 9-23.

Norton, J.D. (2013). All shook up: Maxwell's Demon and the thermodynamics of computation. Entropy, 15(10), 4432-4483.

Prigogine, I. and Stengers, I. (1984). Order out of chaos. London: Heinemann.

Qvortrup, L. (1993). The controversy over the concept of information. An overview and a selected and annotated bibliography. Cybernetics and human knowing, 1(4), 3-24.

Robinson, L. and Bawden, D. (2014). Mind the gap: transitions between concepts of information in varied domains. In Ibekwe-SanJuan, F. and Dousa, T. (eds.). Theories of information, communication and knowledge: a multidisciplinary approach,. Dordrecht: Springer, pp. 121-141.

Ruelle, D. (2007). The mathematician's brain. Princeton: Princeton University Press.

Ruelle, D. (1991). Chance and chaos. Princeton NJ: Princeton University Press.

Schneider, E.D. and Sagan, D. (2005). Into the cool: energy, flow, thermodynamics and life. Chicago IL: University of Chicago Press.

Schrödinger, E. (1944). What is life? The physical aspect of the living cell. Cambridge: Cambridge University Press.

Seitz, W. and Kirwan, A.D. (2014). Entropy vs. majorization: what determines complexity? Entropy, 16(7), 3793-3807.

Sethna, J.P. (2006). Entropy, order parameters and complexity. Oxford: Oxford University Press.

Shannon, C.E. and Weaver, W. (1949). The mathematical theory of communication. Urbana IL: University of Illinois Press.

Simon, H.A. (1962). The architecture of complexity. Proceedings of the American Philosophical Society, 106(6), 467-482.

Smolin, L. (2013). Time reborn. London: Allen Lane.

Sporns, O. (2007). Complexity. Scholarpedia, 2(10): 1623.

Tegmark, M. (2014). Our mathematical universe. London: Allen Lane.

Theurer, K.L. (2014). Complexity-based theories of emergence: criticisms and constraints. International Studies in the Philosophy of Science, 28(3), 277-301.

Vohs, K.D., Redden, J.P. and Rahinel, R. (2013). Physical order produces healthy choices, generosity and conventionality, whereas disorder produces creativity. Psychological Sciences, 24(9), 1860-1867.

Weaver, W. (1948). Science and complexity. American Scientist, 36(4), 536-544.

West, G. (2013). Wisdom in numbers: new forms of complexity, integrating information with energy. Scientific American, 308(5), 14.

Wicken, J.S. (1987A). Evolution, thermodynamics and information: extending the Darwinian program. New York NY: Oxford University Press.

Wicken, J.S. (1987B). Entropy and information: suggestions for common language. Philosophy of Science, 54(2), 176-193.

Wiener, N. (1948). Cybernetics, or control and communication in the animal and the machine. New York, NY: John Wiley and Sons.

Wright, L.W., Nardini, H.K.G., Aronson, A.R. and Rindflesch, T.C. (1999). Hierarchical concept indexing of full-text documents in the Unified Medical Language System Information Sources Map. Journal of the American Society for Information Science, 50(6), 514-523.

Zuchowski, L.C. (2012). Disentangling complexity from randomness and chaos. Entropy, 14(2), 177-212.

Zurek, W. H. (ed.) (1990). Complexity, entropy and the physics of information. Boulder CO: Westview Press.

Zurek, W. H. (1989A). Algorithmic randomness and physical entropy. Physical Review A, 40(8), 4731-4751.

Zurek, W.H. (1989B). Thermodynamic cost of computation, algorithmic complexity and the information metric. Nature, 341(no. 6328), 119-124.