SYSTEMS OF SYSTEMS: A CONTROL THEORETIC VIEW

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Abstract—This paper considers the notion of SoS as an evolution of the standard notion of systems, provides a clear distinction to the standard notion of composite systems and aims to provide an abstract and generic definition that is detached from the particular domain as well as a classification of the families of SoS. We present a new abstract definition of the notion of System of Systems as an evolution of the notion of Composite Systems, empowered by the concept of autonomy and participation in tasks usually linked to games. Control theoretic concepts and methodologies are used to provide the characterization of the notion of “systems play” that is used as the evolution of the notion of the interconnection topology. In this set up the subsystems in SoS act as autonomous intelligent agents in a multi-agent system that is defined by a central task and possibly a game.

Keywords- Systems, Complexity, System of Systems, Control Theory

I. INTRODUCTION

The concept of “System of Systems” has emerged in many and diverse fields of applications and describes the integration of many independent, autonomous systems, frequently of large dimensions, which are brought together in order to satisfy a global goal and under certain rules of engagement. These complex multi-systems exhibit features well beyond the standard notion of system composition represent a synthesis of systems which themselves have a degree of autonomy, but this composition is subject to a central task and related rules. The term has been linked to problems of complex nature, but so far it has been used in a very loose way, by different communities with no special effort to give it a precise definition and link it to the rigorous methodologies concepts and tools of the Mathematical System Theory. Establishing the links with the traditional approaches is essential, if we are to transfer and appropriately develop powerful and established analytical tools to a field that is still unstructured and where little progress has been made in developing a generic and unifying methodology.

The main objective is to place the concept of “Systems of Systems” within the standard framework of Systems Theory that is suitable for some subsequent further formal development (mathematical formulation). Such systems emerge in different and diverse domains and their classification, is also crucial, since different domains may require alternative modeling tools. A central part of our effort is to explain the difference of SoS from that of Composite Systems which leads to the generalization of the standard notion of interconnection topology (linked to composite systems) to the new notion of “systems play”. We introduce the notion of the integrated system, as a system with intelligence and explain the context of the new notion of “systems play” which provides the global compositions leading to what we refer as SoS. The description of the systems play then emerges as a central task or game and ways this may be characterized is defined.

II. THE NOTION OF A SYSTEM

The development of a systems framework for general systems is not a new activity [1, 2, 30]. However, such developments have been influenced predominantly by the standard engineering paradigm and as a result they failed to cope with new paradigms such as those of the business processes, data systems, biological systems, and emerging complex systems paradigms. Our task here is to reconsider existing concepts and notions from the general Systems area [1], detach them from the influences of specific paradigms, generalise them appropriately to make them relevant for the new challenges and then use them to define the notion of “System of Systems”. We follow a conceptual systems approach that may lead to formal notions as described in [2]. Our work relies on existing methodologies, but aims at redefining notions, concepts and introduce new ones reflecting the needs of the new paradigms.

Definition (2.1): A system is an interconnection, organisation of objects which are embedded in a given environment.

This definition is general and uses as fundamental elements the primitive notions of: objects, connectivities – relations (topology), and environment and it is suitable for the study of “soft”, and “hard” systems. The concept of a system refers to the level of reality (physical or manmade construction) and this is an essential observation, to distinguish it from the notion of system model, which refers to the sphere of abstraction. An object is a general unit (abstract, or physical) defined in terms of its attributes and the possible relations between them. For a given object, we define...
its environment as the set of objects, signals, events, structures, which are considered topologically external to the object, and are linked to the object in terms of a structure, relations between their attributes. The existence of the objects environment implies crossings of the imaginary boundary and such crossings indicate the connectivities of the object to objects in its environment. The set of objects in a system are related between themselves and to the system environment through relationships referred to as interconnection topologies. The internal linking between the objects of the system defines the internal interconnection structure, whereas that part expressing the links of the objects to the system’s environment will be called external interconnection topology. The internal and external interconnection topology structure may be fixed or evolving and their nature gives to the system a specific character and identity. The nature of the external interconnection topology is crucial in defining the embedding of the system in its environment and it is the central notion in characterising the difference between composite systems and system of systems. If $\mathcal{F}$ denotes the interconnection topology, $S_a$ the system aggregate (collection of objects) and by * the action of $\mathcal{F}$ on $S_a$ we may represent the system as

\[ S = S_a \ast \mathcal{F} \]

**Figure 1: Description of an embedded system**

An aggregate of systems leads to the creation of new forms of systems which may be either described within the framework of composite systems, or demonstrate additional features which add complexity to the description and may be referred to as system of systems. The term system of systems (SoS) has been used in the literature in different ways [7], [8]. Most definitions ([7], [9], [10]) describe features or properties of complex systems linked to specific examples. The class of systems exhibiting behaviour of Systems of Systems typically exhibit aspects of the behaviour met in complex systems; however, not all complex problems fall in the realm of systems of systems. Problem areas characterized as System of systems exhibit features such as [8]: Operational Independence of Elements; Managerial Independence of Elements; Evolutionary Development; Emergent Behaviour; Geographical Distribution of Elements; Inter-disciplinary Study; Heterogeneity of Systems; etc. The definitions that have been given so far [10], contain elements of what the abstract notion should have, but they are more linked to specific features and are linked to areas of applications. A literature survey and discussions on these definitions are given in [8], [9]. A more generic definition that captures the key features and which is a good basis for further development is given below [8]:

**Definition (2.2):** (i) Systems of systems are large-scale integrated systems which are heterogeneous and independently operable on their own, but are networked together for a common goal. The goal, as mentioned before, may be cost, performance, robustness, etc.

(ii) A System of Systems is a “super system” comprised of other elements which themselves are independent complex operational systems and interact among themselves to achieve a common goal. Each element of a SoS achieves well-substantiated goals even if they are detached from the SoS.

The above definitions are descriptive and they capture crucial features of what the notion should involve; however, they do not answer the question, why the new notion different than that of composite systems. The distinctive feature of our approach is that we treat the notion of System of Systems (SoS) as an evolution of the standard notion in engineering of Composite Systems (CoS) [13]. Making the transition from CoS to SoS requires to identify the commonalities and differences between the two notions. We note:

- Both CoS and SoS are compositions of simpler objects, or systems.
- Both CoS and SoS are embedded in the environment of a larger system.
- The objects, or sub-systems in CoS do not have their independent goal, they are not autonomous and their behaviour is subject to the rules of the interconnection topology.
- The interconnection rule in CoS is expressed as a graph topology.
- The subsystems in SoS may have their own goals and some of them may be autonomous, semi-autonomous, or organised as autonomous groupings of composite systems.
- There may be a connection rule expressed as a graph topology for the information structures of the subsystems.
- The SoS has associated with it a global operational task where every subsystem enters as an agent with their individual Operational Set, Goals.

### III. A NEW CHARACTERISATION FOR THE SYSTEM OF SYSTEMS

Developing a generic definition for SoS that transcends specific domains of applications is essential for the development of systems engineering framework [14]. In the system representation of Figure (1) [2], the system appears as an autonomous agent (internal system structure together with its inputs and outputs), having its operational instructions and goals and a pair of information vectors expressed by the input and output influences vectors. Additional properties may be
introduced by assuming that the system under consideration has the control, modelling and supervisory capabilities integrated within it which enable the system to act as an agent with independence capabilities and act as a player in games. We may represent such systems as illustrated in Figure (2). Such a form of the system will referred to as an integrated system. The latter term is used to distinguish it from systems which have no integrated control and information processing capabilities and which may be referred to as simple systems. If such a system is embedded in a larger system (Composite, System of Systems) relations with other systems may be defined in two different ways:

(i) An interconnection topology of the graph type defined on the set of input-, output- influences subsystem information structures.

(ii) A global game where every subsystem enters as an agent with their individual Operational Set, Goals.

The distinguishing feature of the SoS is that the subsystems participate in the composition as intelligent agents with a relative autonomy and may act as players in a game. The latter property requires that the systems entering the composition are of the integrated type, since this requires capabilities for control, estimation, modelling and supervisory capabilities. Features, such as large dimensionality, heterogeneity, network structure, Operational, Adaptability, Emergent Behavior etc may be also present in the case of CoS as well. We define:

**Definition (3.1):** Consider a set of systems \( \Sigma = \{ S_i, i=1,2,...,\mu \} \) and let \( \mathcal{F} \) be an interconnection rule defined on the information structures of \( S_i \) systems. The action of \( \mathcal{F} \) on \( \Sigma \), called a Composite System, or the composition of \( \Sigma \) under \( \mathcal{F} \).

The information structure of each system is defined by the pair of the input and output influence vectors and the interconnection rule may be represented by a graph topology [2], [11]. The resulted system is embedded in a larger system and it is treated as new system with its own system boundary.

In the above definition the systems considered are simple and not necessarily integrated. This definition may now be extended as follows:

**Definition (3.2):** Consider a set of integrated systems \( \Sigma = \{ S_i, i=1,2,...,\mu \} \), \( \mathcal{F} \) be an interconnection rule defined on the information structures of \( S_i \) systems and let \( S_c = \Sigma^{*} \mathcal{F} \) be the resulting composite system. If \( \mathcal{G} \) is a general rule of operations, referred to as “systems play” that is defined on the systems \( S_i \) then the action of \( \mathcal{G} \) on \( S_c \) is a new system \( S_c^{*} \mathcal{F} \mathcal{G} \) which will be called a System of Systems, or the \( \mathcal{F}, \mathcal{G} \) composition of \( \Sigma \).

In the above definition the notion of SoS emerges as an evolution of CoS since the systems are assumed to be integrated, ie having capabilities for information processing and thus they are capable to act as agents and participate in games of some type. We assume an interconnection topology defined on the information structures of the components, but this may not necessarily be strong and some sub-systems may be entirely autonomous. Note that the transition from the CoS to SoS involves moving from simple to integrated systems as far as the subsystems, and the introduction of the new notion of “systems play” which emerges as a generalization of the notion of topological composition. The nature of the applications defines the systems play, which frequently may be expressed as a game defined on intelligent agents. If not all subsystems are integrated we may define:

**Definition (3.3):** Consider a set of systems \( \Sigma = \{ S_i, i=1,2,...,\rho \}, S'_i, i=1,2,...,\sigma \} \) where the \( S_i, i=1,2,...,\rho \) subset is integrated and the \( \{S'_i, i=1,2,...,\sigma \} \) subset is simple. We consider \( \mathcal{F} \) to be an interconnection rule defined on the information structures of sub-systems of \( \Sigma \) and let \( S_c = \Sigma^{*} \mathcal{F} \) be the resulting composite system. If \( \mathcal{G} \) is a systems play that is defined on the integrated systems \( S_i \) then the action of \( \mathcal{G} \) on \( S_c \) is a new system \( S^{*} = \Sigma^{*} \mathcal{F} \mathcal{G} \) which will be called a Weak System of Systems, or the weak \( \mathcal{F}, \mathcal{G} \) composition of \( \Sigma \).

The essence of the new definition is that SoS emerges as a two dimensional notion. At the lower level it appears as a composite system with some interconnection topology defined on the subsystems, which are now assumed to possess information processing capabilities. It is the latter property that allows these subsystems to act as agents and SoS to emerge as a multi-agent system (MAS) composed of multiple interacting intelligent agents (the subsystems). This multi-agent systems view allows SoS to act as vehicle to solve problems which are difficult or impossible for an individual
agent. The multi-agent dimension of SoS has important characteristics such as [16]:

- Autonomy: the agents are at least partially autonomous
- Local views: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge
- Decentralization: there is no designated single controlling agent, but decision and information gathering is distributed.

It is these properties that allow SoS to develop “self-organization” capabilities and find the best solution to the problems defined on them.

IV. CLASSIFICATION OF SOS

The major challenge in the development of a unifying approach to the study of SoS is the quantitative characterisation of the new notion of the systems play. Taking into account that SoS problems emerge in many and diverse domains, it is clear that some classification of the general SoS family into sub-families with common characteristics is essential before we embark to the characterisation of notions such as systems play and subsequently address issues of design, re-design and then study of emergence for such systems. The classification of SoS may be achieved according to different criteria such as the origin:

(i) Physical, or natural SoS (N-SoS)
(ii) Engineered or Constructed SoS (E-SoS)

The first category involves problems of the natural world, and social-economic problems and are the results of evolution of physical, or socio-economic processes. Problems such as the “ecosystem” of a geographical region, and issues such as “social phenomena” are typical examples. The common characteristic of these classes is that they are the results of a “natural evolution” and they are not the by-products of some notion of design. Of course, there are grey areas between the two classes such is the case “global economy” where evolution is accompanied by some effort to intervene and affect the economic processes (government policies etc). It is important to note that in N-SoS some “goals”, “principles” drive the development of the systems play, whereas in E-SoS the “goal” is driven some coordination effort. This leads to another way of classifying SoS based on structural and operational characteristics. This classification refers to the mechanisms defining the relations between the subsystems. We may distinguish the following distinct classes:

(a) Goal Driven and Unstructured (GU-SoS)
(b) Goal Driven with Central Coordination (GC-SoS)

In GU-SoS class the central goal for the system operation is set, as well as the environment within which the system operations will take place. In this case the nature of the system play is entirely defined by the set goal. In such cases the goal may define a form of a game where the intelligent agents may participate. Typical examples are problems related to “ecosystems”, where there is no coordinated human interference. A further classification for this class is into:

- Pure Goal Driven (P-GU-SoS)
- Goal and Scenario Driven (S-GU-SoS)

In the P-GU-SoS class the subsystems, as intelligent agents, interpret the central goal, may assign to themselves sug-goals and they then develop actions and self-organisation to achieve the central goal, which may be expressed as optimization of a performance index, subject to satisfaction of their individual goals as well. In S-GU-SoS a scenario linked to the goal is given, the subsystems as intelligent agents undertake roles which aim to optimize a central performance index and satisfy their own particular goals. Clearly, in all such cases appropriate games have to be defined.

The GC-SoS class on the other hand has the same features as the P-GU-SoS and similar subclasses with the additional feature the existence of coordination. The presence of coordination imposes a structure to the interpretation of the goal by the subsystem and the development of appropriate scenarios to achieve the central goal and partial goal. Coordination is common to E-SoS and may be viewed as an interpreter for the development of operational activities. The nature of coordination also introduces special features to SoS characterization since it introduces a structure to the resulted systems play. Coordination is a form of organization and there may be different types such as “Hierarchical”, “Heterarchical” and “Holonic” [19]. Such forms of organization structure the systems play and the development of scenarios. Note that in N-SoS self-organization has evolved and there is no coordination; the evolved structure may look like an optimal scenario and acts as a natural substitute for the coordination. Man-made systems usually involve coordination which drives the development of the system play. These classes define sub-families of SoS; further classification may be introduced by the nature of the origins of the overall SoS. Types of SoS where the subsystems are of the engineering type without human action involvement are referred to as “hard”. Systems involving human presence and behaviour will be referred to as “soft” and those involving a mixture of the two types will be called “hybrid”.

V. METHODOLOGIES FOR SYSTEMS PLAY

The system-wide coordination of real-world systems of systems is a challenging and open problem. The development of a description for the systems play depends on the nature of the particular SoS. In the following we outline different methodologies may provide the required framework for such task. In the following, we will investigate several methods that have emerged in different domains to manage systems of systems which involve: Co-Operative Control, market based coordination techniques, population control methodologies,
and coalition games. Each of these methodologies provide formal descriptions of the notion of systems play.

A. Co-Operative Control

The notion of Co-Operative Control has been used in a number of ways in the literature. A typical case describing a class of SoS very close to technological problems is the Vehicle Formation Problem [17],[18] defined as the control of the formation of k vehicles that are performing a shared task; the task depends on the relationship between the locations of the individual vehicles and the task defines the scenario that has to be realized. It is assumed that the vehicles are able to communicate with the other vehicles in carrying out the task and they have capabilities to control their position in the effort to perform the task. Each vehicle is described as a rigid body moving in space and a state vector x may be associated with each one; by x = (x1,..., xN) we may represent the complete state for the set of N vehicles. The collection of all individual states defines the state of the system and the execution of the assigned task requires the assignment of additional states that can make the system an SoS. The development of the scenario, task is handled by introducing for each vehicle an additional discrete state, αi, which defines the role of the vehicle in the task and this is represented as an element of a discrete set A. The definition of A depends on the specific cooperative control problem.

It is assumed that the vehicles are able to communicate with some set of other vehicles and the set of possible communication channels is represented by a graph G. The nodes of the graph represent the individual vehicles and a directed edge between two nodes represents the ability of a vehicle to receive information from another vehicle. Given a collection of vehicles with state x and roles α, we may define a task or scenario in terms of a performance function J the optimization of which is equivalent to the completion of the task. Clearly, such problems may also have constraints which make the problem a constrained optimization problem. The execution of the scenario requires a strategy and for this case this expressed as an assignment of the inputs ui for each vehicle and a selection of the roles of the vehicles. For SoS the problems of interest are those involving cooperative tasks that can be solved using a decentralized strategy.

B. Market-Economics Based Coordination Techniques

The distinguishing feature of SoS is that there are autonomous units with their own management and control functions that are coupled by resource flows which need to be balanced, over appropriate periods of time depending local or global storage capacities. The performance of the subsystem consumption and production is influenced by availability of these resources [27]. To perform an arbitration of these flows requires economic balancing mechanisms [20], [21]. The management of the resource flows may be expressed as a network management problem, given that the resource flows define some generic network structure within which we define the flows. Clearly, the overall system performance and behaviour is influenced by discrete decisions taken. Two different approaches that can be used for the management of such flow-coupled SoS are: economics-driven coordination and market-based mechanisms. In both cases, the coordinator has only limited information about the behaviour and the constraints of the local units which perform a local optimization of their operational policies.

In the economics-driven coordination, it is assumed that the control of SoS involves the setting of production / consumption constraints or references between the global SoS coordinator and the controllers of individual systems. The SoS coordinator utilizes simplified models of the sub-systems, and a model of the connecting networks to compute references or constraints on the exchanged flows. The resulting optimization is based on the dynamic price profiles for the resources that are consumed or produced by the subsystems over the planning horizon. An alternative approach is to use mechanisms employing the concepts of economic markets to distribute limited resources between subsystems. The market is defined as a population of agents consisting of producers selling goods and consumers buying these goods [20], where the consumers' demand depends on the usefulness or utility of a good for the completion of its task. The prices of the resources which are set by the market affect the utility and, thus, the demand side. The goal of a market-based coordination mechanism is to generate equilibrium between the producers and the consumers such that the overall supply equals the overall demand. A popular mechanism to compute such equilibria is an auction and many different kinds of auction mechanisms have been developed [20]. Market-based mechanisms are inherently decentralized and can thus be mapped directly to systems with autonomous subsystems.

C. Population control methods

Population control refers to systems that comprise a large number of semi-independent subsystems, which macroscopically are viewed in terms of their emergent behaviour. Such systems are used in ecology to capture the fluctuations in the populations of interacting species and the relevant models use continuous variables to capture populations and differential equations to capture their evolution. There are extensions to hybrid models [23] and to delay and/or stochastic differential equation models. Of special interest is the class of mixed-effect models [22], which address the evolution of a heterogeneous population of individuals, which deploy ordinary differential equations, but with parameters linked to appropriate probability distributions. Population systems dynamics are gaining in importance, as man-made systems become increasingly complex and large-scale and control of the emergent behaviour of large collections of semi-autonomous subsystems becomes an issue. Such methods are primarily motivated by biological applications, but have potential for the engineering field of SoS. These methods need to be adapted and extended, if they are to be made applicable to engineered SoS.
**D. Coalition Games**

The basic idea of SoS is to consider the overall system as a set of subsystems that are controlled by local controllers or agents which may exchange information and cooperate. This feature demonstrates the link of SoS to distributed and decentralized control schemes with the additional property that the interaction between the subsystems may indicate a time-varying coupling. It is this special feature that indicates the links to a rather new category of management and control schemes referred to as *coalitional management schemes.* In this paradigm different agents cooperate when there is enough interaction between the controlled systems and they work in a decentralized fashion when there is little interaction. A coalition is a temporary alliance or partnering of groups in order to achieve a common purpose or to engage in a joint activity [26]. A coalition of systems is a temporary system of systems built to achieve a common objective. Coalition building is the process by which parties come together to form a coalition. Forming coalitions requires that the groups have similar values, interests, and goals which may allow members to combine their resources and become more powerful than when they each acted alone.

**VI. CONCLUSION**

The new definition for the SoS is the starting point for the development of methodology that may lead to systematic design. Examining the rules of composition of the subsystems and their coordination as agents in a larger system defines a challenging new area for research and requires links across many disciplines. Examining in detail the special features of the different classes of SoS is crucial in the effort to provide a quantitative formulation of the notion of “systems play” which may take different forms in the different classes. This is also crucial in quantifying the notion of *emergence* in the SoS context. The potential for applications is well beyond the traditional engineering field, when powerful modeling tools are defined that may allow the study of design and decision problems of the respective classes of SoS. It is worth mentioning at this point that the majority of SoS are products of “physical”, or “technological “ evolution, rather than products of systematic design and understanding evolutionary processes leading to the formation of SoS is crucial.

**VII. REFERENCES**


