Risk Management for Property/Casualty Insurance Companies

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BY

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DECLARATION

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This thesis addresses the need to reduce inefficiencies in management of insurance company risk capital. The laxity in managing the cost of capital is a result of dysfunctional property/casualty risk classification and capital accumulation practices in the insurance industry. We reclassify risk based on both peril and financial functional features, in order to capture all the facets of risk affecting a firm and ultimately to achieve optimal capital allocation.

With the purpose of reducing inefficiencies in mind, we explore and isolate the impact of regulation on insurance company profitability. We use barrier option pricing models to mimic the impact of solvency requirements on firm-wide risk. This methodology of measuring risk is better than plain vanilla option pricing models, in that, through the option to an early default, we are able to capture the economic significance of financial distress, and allocate firm-wide risk capital. The firm-wide risk is incidentally used to empirically test the impact of risk on the cost of carry, the quality of operational profitability and forward asset commitment per unit of liabilities.

Our empirical test confirms a strong relationship between firm-level risk, and the cost of carry, return on policyholders' surplus and the cost of capital per contract underwritten. The results are better than previous results obtained using plain vanilla option-pricing models and reveal the importance of incorporating solvency requirements in defining the economic significance of insolvency. The results also points to the importance of advised risk classification procedures to the whole process of integrated risk measurement and financing, which we explore in this study.
1.1 Overview of the Study

According to Santomera and Babbel (1997) the insurance industry is evolving to a higher level of risk management techniques and approaches of which much improvement is needed. The objective of this study is to show how insurance companies seek to reduce the costs of transacting risk in their portfolios using integrated risk management systems to engineer optimal cash flows. We examine the problems caused by various risks on insurance company profitability and solvency thresholds. We also isolate the impact of integrated risk management systems on cost structures of insurance payoffs.

The main reason for us carrying out this study is underpinned by a barrage of criticism labelled against the insurance industry for their lax management of risk capital. Inefficiencies in capital management are a result of excessive accumulation of relative capital against all the risks defining the loss distribution. Insurance companies accumulate capital relative to risks they face in the portfolios, in order to satisfy
regulatory and rating agents’ solvency requirements. Risk classification by the insurance industry and regulators fail to take into consideration financial attributes of each liability account. The classification adopted in this study recognises duration and convexity risks defining each liability account and matching it to assets with similar duration. It helps establish effective insurance transfer pricing systems that link insurance risk to capital required. Empirical evidence from our study supports the idea of using transfer pricing systems to improve spreads on both asset and liability accounts.

Relative capital does not specify what capital is really required to sponsor each section of the loss distribution, which is why it is inefficient in financing all cash flows of the loss distribution. Thus, the lack of specifications in capital requirements based on specific risk category in the insurance portfolio is the main reason behind the under-performance of the property/casualty insurance sector compared other financial sectors. Absolute capital composed of bespoken equity-to-risk components over the loss distribution is the only efficient way of reducing the cost of risk capital. The definition of risk capital based on absolute capital gives insight into the problem of inadequate capital for sponsoring certain sections of the loss distribution. In order to improve the efficiency of risk financing programmes, we define instruments used in balancing the relation between risk-capital-usage and loss distribution characteristics.

Another important aspect of our study is the derivation of firm-wide risk by using barrier option-pricing theory. Firm-wide risk is computed from the insolvency put option incorporating an exogenously determined solvency margin or knock-out barrier. The knock-out barrier is used to define value enshrined in the option to an early default.
brought about by an intervention of regulatory authorities when a company's assets drop below this barrier. Risk defined by this extended version to the standard Mertonian option pricing methodology brings us closer to insurance cash flow behaviour in practice as confirmed by our empirical analysis. It helps explain why insurance companies hold at most thrice the statutory solvency margins. The study also confirm that most of the contingent capital purchased by insurance companies is used for surplus relief, due to the consequences brought about by trading at asset levels close to the solvency threshold. We are also able to explain why many insurance companies fail to fully comprehend the whole distribution they are faced with, hence the tendency to resort to relative capital accumulation rather than capital allocation.

The study also attributes the firm-wide risk as the single most influential factor on risk-taking behaviour and financial structures of insurance companies. Insolvency risk is used to explain the strength of the relationship between firm-wide risk, carry traits, return on policyholder surplus and the cost of capital per contract underwritten. Our findings confirm a strong correlation between default risk and these variables, which is quite consistent with our theorems. These results reveal the real drivers of value creation within an insurance company, and how they are managed to enhance profitability and improve efficiency in risk capital utilisation. They also point to the importance of advised risk classification procedures on the whole process of integrated risk measurement and financing, which we also explore in this study.

1.2 Motivation for this Study

Insurance literature has established that intrinsic cost of trading risk is the ruin probability an insurance company is bound to face the moment it enters into the
business of trading risk. The intrinsic cost is looked at as the starting point of analysing insurance company profitability, since the various cost components are then used to inflate this distribution into a more encompassing distribution. The theory that underlies our analogy is based on the fact that the price of risk is equivalent to the intrinsic value of the firm. The theory further asserts that given risk is carried from origination to settlement date, the price at time should incorporate the cost of capital required to service this risk to settlement date. It is from this insight that we can segregate the intrinsic and time value components, in any insurance pricing contract. This distinction is important since insurance pricing currently concentrate more on factors affecting the intrinsic value rather than the cost of capital element, hence the under-pricing and failure of insurance companies to manage risks for the policyholders.

Value creation over a holding period depends on the ability of management to capture costs associated with these components into the pricing of individual risks. These two components determine the level of capital required supporting a risk portfolio and its cost during the holding period. It is also apparent that these two key components should be targeted and controlled within an insurance company in order to stabilise pay-off profiles. The reason for this being that both the intrinsic and time value components vary from one risk holding period to another. That's why it is important to establish what the key drivers are as far as their impact on economic value volatility is concerned. This is done in order to effectively control risk in a portfolio by targeting volatility at its very source.
It is within company cash flow patterns that risks ingrained can be pinpointed, controlled and new pay-off profiles engineered to alternatively and effectively finance risk. Portfolios using integrated risk management systems have stable earnings and capital structure, with the efficacy of reducing the cost of debt, and consequently the overall cost of capital (Doherty, 1997). Integrated risk management impacts a company’s financial structure through earnings and costs configuration. We measure the impact of the key drivers on value creation, by striking a balance between the cost of capital and profitability. We address the implications of under-pricing and how the model proposed in this study could be used to stabilise earnings and profitability within an insurance company.

The main objective of this study is to establish the relationship between risk and the essential value drivers within an insurance company brought about by the nature of the financial structure. Barclay and Smith (1999) pointed out that financial structure determines risk-taking behaviour within an insurance company, as the level of capital available depends on what each stakeholder perceive to be adequate protection/return. As pointed above previous literature by Doherty and Garven (1986), Daykin and Hey (1989) and Babbel and Santomera (1997) on the subject of insurance risk management ignored this intricate relationship to the detriment of results obtained. They failed to explain insurance cash flows behaviour in practice, because risk taking behaviour defines the endeavour of companies in trying to resolve conflicting stakeholder interests.

We establish what different stakeholders of an insurance company consider as the cost of risk capital required for them to release their funds, or regard adequate for
healthy transaction of underwriting risk. Capital needed to carry a claim forward to settlement date is referred to as risk capital. Merton and Perold (1993) defined risk capital, as the smallest amount that can be invested to insure the value of an insurance company's net assets against a loss in value, relative to the risk-free investment of those net assets. Given fixed liabilities, riskiness in the net assets is similar to the riskiness of gross assets, which mean that risk capital requirements are the same. Capital required to support risk assumed has been defined in insurance literature by the probability of ruin, risk-based capital (RBC), expected policyholder deficit (EPD) and Value at Risk (VaR); (Cummins (1988), Jorion (2000), Bustic (1994), Sommer and Cummins (1996)). Our model derived from the intrinsic value component is extended to incorporate the cost of capital, a more comprehensive economic value distribution approach.

The EPD is an option pricing methodology first used by Bustic (1994) in measuring the cost of default in insurance companies. This method has been herald as a closed form solution to measuring the level of capital at risk, since it does not only account for the probability of default but also the severity of default. In fact it's a better measure of the level of capital required by insurance companies than the Value at Risk (VaR) and ruin probability methodologies. However, the EPD methodology assumes that default occurs at the end of the period, which is not true for insurance companies. Default in insurance companies occurs at any time during the life of the company, especially when large losses with very low probabilities of occurrence, exceed the going concern value of the firm.
The EPD methodology is opaque in its computation of default in that, it fails to clearly link the pricing formula to the state of the firm. Under this methodology the probability of default remains generally positive, during the life of the company. In practice the probability of default is low for firms that have survived through the underwriting cycles. Mutenga, Dinenis and Hatgionnides (2001) developed a model that takes into account the price of early default and the influence of regulators in both the cost and probability of default. Their methodology brings us closer to the actual behaviour of insurance cash flows as driven by the cost of compliance.

Option pricing theory in insurance literature is used to value assets and liabilities of insurance firms, pricing of individual insurance contracts and to show the portfolio effect on risk and capital requirements (Doherty and Gavern (1986), Cummins (1988) & (1991), Cummins and Danzon (1997) and Cummins, Allen and Phillips (1998)). Merton's default model envisages that the value of equity increase by virtue of a put option that allows shareholders to transfer wealth from policyholders when things go pear shaped. The asymmetric nature of shareholders' claim on the company's assets is due to limited liability option, which allows them to walk away if the value of assets falls below the value of liabilities. This leaves human equity without jobs, and policyholders with a depleted portfolio and limited guarantee funds available to personal lines policyholders.

Default rate brought about by increased risk in a portfolio reduces the value of debt, but it also increases the value of equity and the cost of carry with it. This prompts policyholders to require insurance companies to pay them a premium commensurate with the risk of default, a value well captured in the cost of carry methodology. Thus
the lower the asset values the more expensive it will be to borrow money from the policyholders, since high cost of carry ratios are reminiscent of poor underwriting cash flows. Also the higher the probability of default, the higher are the bankruptcy costs and the lower will be the net asset values (NAV). This means policyholders will demand a premium from the company for committing their funds upfront to buy securities promising to deliver in the future.

Capital required supporting risks assumed by a company is derived from premiums, equity and leveraged through engineered risk financing payoffs. It is a condition for entering the market to comply with minimum solvency requirements and this cost is embedded in the intrinsic/fair price of every insurance contract. The cost of minimal security capital should first be factored in the fair price of a contract as propounded by Mutenga, Dinenis and Hatgioannides (2001), Cummins (1988), Cummins, Allen and Phillips (1997), and Barth (2000). This unit of capital factored in the price of risk is equal to the cost of financial distress to all stakeholders, should one occur either during or at the end of the holding period.

1.3 Theoretical Background to this Study

Our work parallels that of Black and Cox (1976), Merton (1977), Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995), and Briys and De Verene (1997). We introduce the impact of solvency margins on risk premiums, return on policyholder funds, portfolio risk and the total cost of risk management. By taking into consideration the impact of solvency margins and stochastic liabilities, our approach is not only consistent with financial literature in Myers and Read (1999), and Merton and Perold (1993), but also actuarial in Daykin, Pentikainen and Pesonen.
(1994), Bustic (1994) and Barth (2000). This means that risk levels computed by our model are close to those observed in practice. We are also able to explain the reason why it is expensive and difficult to finance extreme asset values close to the barrier for insurance companies using this methodology.

We also extend the option pricing methodology in Cummins, Allen and Phillips (1997) by introducing variable interest rates, in measuring the firm-wide risk embedded in insurance cash flows. Our methodology does not depend on a single boundary, which assume that default occur at the end of the period as envisaged by Black and Scholes (1973), Merton (1974), Cummins (1988), Bustic (1994) and Cummins, Allen and Phillips (1997). In this thesis we allow for the effects of solvency requirements on the default profile of the firm, given the possibility of regulators coming in to take control of the company if the required solvency margin is reached. The solvency requirements define the boundary that should not be breached; i.e. asset values fall below the solvency margin, if the company is to continue operating. This barrier is determined by the regulators and is a structural barrier that defines the economic meaning of the insolvency-causing event. The barrier defines the policyholder’s payoff upon bankruptcy.

The role that regulators play is similar to the objective achieved by covenants in corporate bonds, which gives the bondholders the right to bankrupt the company if its asset values fall below a pre-specified solvency threshold. This feature in insurance companies resemble the characteristics of barrier options, which knocks out Equityholders’ option on the company’s assets, if asset values go down and reach the pre-specified insolvency threshold. The pre-condition to risk trading which triggers
insolvency upon being breached is enshrined in the minimum solvency margin requirements.

Minimum solvency margins are a safety mechanism that gives regulators the right to intervene in the company’s operations, force reorganisation or liquidate the company if its performance fails to match the threshold specified. The threshold at which insolvency will occur follow that of Black and Cox (1976), Merton (1977), Cummins (1991), and Longstaff and Schwartz (1995). This constitutes the barrier that need not be breached, if the company is to continue with its operations. It means that the value of insurance assets are path-dependent, in that the payoff is dependent on the realised asset path, which trigger certain parts of the contract if the asset price becomes too low. This barrier when breached invokes action from the authorities to suspend operations to limit the dissipation of assets. In other words, default occurs the first time when the value of assets is lower than the stochastic barrier. Upon achievement of this out-strike price of assets, it is assumed that all other liability classes are simultaneously defaulted.

The out-strike price of assets is set as a pre-condition to underwriting a specified amount of liabilities, upon which asset values should not go below during the life of the option. If they go below the specified asset values, the regulators will take over the company for the policyholders. In order to avoid the eventuality of a knock-out, the insurance company should continue meeting their contractual obligations to all policyholders irrespective of the class of business. If regulators intervene at such a point, the option of Equityholders on the firm’s assets is extinguished and they will receive nothing from their investments.
The fact that regulators intervene before liabilities are greater than assets mean that shareholders should give up the company before they have recouped the residual assets earmarked for this cushion for policyholders. Early intervention takes us a step closer to how insurance company cash flows behave in practice, given a higher exit price than envisaged under the perfect market scenario of the standard firm Black Scholes model. This makes insurance cash flows unique from cash flows of ordinary firms, because they have to give up the company even before the face value of assets is not yet equal to the value of liabilities.

This mean that there is no possibility of equityholders recovering value in the company and all liabilities are considered to have equal bargaining power and there is no priority over settling another. In other words, policyholders receive an exogenously specified fraction of the remaining assets; asset values will usually be lower after take over by regulators than it would be the case if the company had remained in the hands of equityholders. Regulatory company take over reduces liquidity, which tend to dissipate asset values (dead-weight cost of bankruptcy), a phenomenon well known practice because most of the companies placed in administration rarely survive and become operational again. What regulation does is that it tries to reduce bankruptcy costs, if it is efficient according to consumer protection theories (Skipper, 2001). Usually, regulation does not totally reduce bankruptcy costs, because it does not only reduce liquidity, but also the firm is not run under a capital market regime, where positive NPV projects are considered. The aim of regulators of preserving assets when they intervene defeats the whole purpose of asset building, because the best strategy is
not to preserve but to build. In preserving no value is added in the form of net asset value (NAV), but value is destroyed.

The contribution we are making in this paper is underlined in the ability of our risk measurement model to better explain the behaviour of insurance cash flows than methods used before. The fact that more variables are captured by our option-pricing methodology means risk levels computed are closer to what is actually observed in practice. The risk classification methodology used also help capture peculiar financial aspects of insurance cash flows, rather than the mere underlying risks embedded in perils insured. What we are able to show for the first time is that insurance cash flows are financial and they should be viewed as such, as we are able to show that under-performance of the insurance industry has been due to cash flow misspecification. This thesis enable insurance companies to better measure their firm-wide risk with precision and help to improve risk control, financing and capital allocation for the firm. The paper also help broaden our understanding of firm-wide risk as measured under regulatory constraint as the major driver in insurance company operational profitability.

**Theorems and conditions:**

I. The risk premiums measured by default risk are inversely related to the cost of carry of the insurance company and the exogenously determined insolvency threshold.

II. The price of liabilities is an increasing function of the default-risk, and the barrier-to-asset value ratio, as policyholders’ surplus is inversely related to risk and cost of compliance.
III. The cost of risk capital factored in liabilities contracts is a function of both default risk and hedge instruments used.

- The price of risk is equal to the intrinsic cost plus the cost of servicing capital required carrying risk forward to claim settlement date;
- The distant price of liabilities is equal to the nearby cost of liabilities plus the cost of capital required to support the liabilities from a nearby to a distant settlement date;
- Insurance companies' value creation bounds are a function of the cost structures and the cost of borrowing external capital, and;
- Default risk is reduced through hedging arrangements that reflect the characteristics of underlying risky cash flows

Our theorems are consistent with antecedent literature by Cummins (1991), Kim, Ramaswamy and Sundaresan (1993), Briys and De Varene (1997) and Klein and Inglis (1999), who emphasised the importance of incorporating the cost of default when pricing contingent liabilities. Cummins, Allen and Phillips (1998) pointed out that liability prices computed under the option pricing methodology are close to prices observed in practice. Irrespective of the fact that they were using vanilla option pricing methods (Merton (1973), Cummins (1988) and Sommer (19996)) in deriving their prices, this methodology capture risks that have not been captured by the Myers and Cohen (1987) and other financial economics pricing methodology.

1.4 Methodology

The model used in this study to measure portfolio risk is designed to capture risk levels defined by the financial structure. Our extended model for measuring insurance
risk using the barrier option pricing captures the real drivers behind insurance company operational profitability. Solvency margins reduce the value of claim shareholders have on the company’s assets following default, whilst increasing the value of liabilities. Risk increases for shareholders and human capital, since the level of solvency margins determines the probability of default. The higher the level of solvency margins the higher will be the probability of default, but the lower will be the level of loss to policyholders.

The methodology used is robust and has produced consistent results when used to price insurance liabilities by Doherty and Garven (1986), Cummins (1988) and (1991), Sommer (1996) and Cummins, Allen and Phillips (1997). Our model is an extension to previous work on option pricing of insurance company by these authorities, whose empirical work is based on the Mertonian option pricing methodology. Their models although insightful, are limited in their ability to capture the actual dynamics of an insurance company, when subjected to the cost of compliance. They are based on the assumption that liabilities are not guaranteed, which is difficult to justify in practice given the role regulators play in protecting policyholders.

1.5 Implications of this Study

This thesis develops a simple new framework for measuring insurance companies’ firm-wide risk that incorporates an early default option. The option pricing methodology is applied to derive the closed form valuation for overall firm risk as measured by the insolvency put option. The main advantage of using the option pricing is that it can easily be used to value insurance companies even for companies with complex financial structures and loss settlement pattern.
Through this study, we are able to establish the relationship between risk and cost of carry traits that are observed in practice, and the ultimate financial structure adopted by a firm. The risk measurement methodologies used in this study capture the following functional features missed by preceding risk measurement models:

- Risks classified according to financial attributes of liabilities rather than perils insured;
- Risks are decomposed into components that are economically modelled into loss distributions on a standalone basis;
- The effects of variable interest rates on an insurer's risk profile;
- The effects of management quality;
- Market price movements;
- Hazard seasoning;
- Liquidity embedded in solvency ratios and the price of illiquidity is defined and built into illiquid positions, in order to capture this risk;
- The dynamic features allowing us to measure an insurance company's risk profile over time; and
- The effects of firm-wide risk on operational profitability and cost of capital per unit of contract insured.

The establishment of a functional relationship between these components and risk gives greater insight into the deficiencies of current practices in the market. It helps us establish the missing link between absolute risk, required capital and cost of carry traits. This is crucial to establishing the importance of insurance transfer pricing systems in insurance companies, which could be used to control spreads. Transfer
pricing systems are required to improve the cost of originating funds from policyholders, as liability accounts are obliged to generate commensurate return based on functional financial aspects of risk rather than insured perils. It also brings the classification of insurance risk in line with the treatment of risk by other financial institutions, by using duration and convexity attributes. Our empirical tests confirm that most of the risk in insurance portfolios originates from liability classes, reinforcing the need for discipline in liability classes.

One important aspect yielded from this study is that firm risk measured by the insolvency put is equivalent to expected policyholder's surplus deficit and an even more efficient tool for measuring risk than VaR. We show that the correlation between assets and liability have significant effect on the overall risk profile of the firm, as well as the overall spread paid for originating liabilities. We also show that the model provides three primary empirical evidence: firstly that spreads paid for originating insurance business are negatively related to the level of cost of carry and default risk; secondly that insolvency risk with an early default option is functionally related both to operational profitability and the cost of capital required to carry liabilities to settlement date; and finally, our model has many implications for hedging default risk.

1.6 Limitations of the Study

The major limitation of this study is our inability to use market values of equity and liabilities to measure portfolio risk. We use balance sheet figures because there are too few insurance companies trading on the London Stock market. Differences in accounting practices before the introduction of the EU Insurance Directive made it statistically suicidal to use European insurance companies’ data. A similar study using
publicly traded insurance companies data for European companies will be carried out in the future, that is for the period after the adoption of the directive by each member state.

Another limitation is enshrined in our model adopted, which uses lognormal distribution to characterise insurance liability losses. This tends to understate the real risk embedded in the tails of liability accounts prone to internal contamination or of longer duration. The Weibull or Pareto distribution would be more appropriate to use, but studies using standard option pricing methodology by Cummins, Allen and Phillips (1997) show that the predictive power of lognormal distribution methodologies is still good. Our method incorporating the effect of solvency margins provides stronger predictive power of insurance cash flow behaviour and gives spreads closer to those observed in reality than these standard models.

We are also unable to classify assets on balance sheets based on their duration, as statutory accounts used for empirical analysis do not specify asset duration. However, we believe that our classification can be used in practise easily as duration of assets bought by the company is known in advance. The default asset classification used in this study do not compromise the quality of our results as every attempt was made to put assets in their right class.

We also did not manage to capture current risk levels in the industry since most of the companies that merged after 1989 were excluded from our study. Such a study would be able to capture the changing trends in risk capital costs in the industry. The inclusion of data for merged companies would have distorted our results, since our aim
was to capture going concern companies' data without any alterations in profiles due to start-ups.

1.7 Structure of the Thesis

The second Chapter of this study deals with the issue of insurance risk classification, risk measurement and the market for insurance risk. We begin in the first section of this Chapter by defining risk and risk capital, with respect to the way various practitioners perceive risk in the insurance industry. A review of literature on risk classification is carried out culminating in the development of a generalised risk classification structure used in this study. The main purpose of our model is to identify various cost components within the loss distribution domain, their impact on payoffs and how integrated risk management can be used as a vehicle to deliver optimal and stable payoffs.

In the second section of Chapter (2) we explore the market for insurance risk and its implications on insurance company profitability and cash flow volatility. The first section reviews literature on risk financing instruments used to manage insurance company cost structures. A broad classification based on the section of the loss distribution they target is discussed to give insight on the efficiency of these instruments at matching risk attributes to equity by blending risk financing instruments. Theorem (3) proposed in Chapter (3) is based on the discussion in this section. It is empirically tested in Chapter (4) to establish the relationship between, the cost of risk capital per contract written to the cost of risk financing and firm-wide risk.
The third chapter proposes a risk measurement model for firm-level risk exposure. The model is developed from a standard Black-Scholes model, which is then extended to incorporate the effects of solvency thresholds on the value of the default put. The default put option is used as a measure for financial distress or policyholder surplus deficit, which is the value likely to be lost at anytime for a given probability. We consider asset and liability risk distributions on a stand-alone basis and the aggregate risk after taking into consideration the covariances. We also make propositions for theorems 1 and 2 tested in Chapter 4, based on the impact of default risk and liability spreads on the quality of operational profitability and the cost of borrowing.
CHAPTER 2:

INTEGRATED RISK MANAGEMENT SYSTEMS

2.1 Introduction

In this chapter we are going to look at risk classification as the gateway to integrated risk management. Without a comprehensive risk classification some risks will be overlooked in the risk measurement process and will be left unfunded because they are not all quantified. Risk classification is an essential part in the risk management process, because each risk component can easily be measured and its importance in the portfolio known. It also brings enlightenment to management on the characteristics of risks faced by the company, their correlation features, and natural hedges.

There has been failure among insurance practitioners to dimension risk according to its financial functional features, and to agree on a method that links risk and capital. These disagreements are a result of the diversity in the methods that are used in classifying insurance risk. The harmonisation in the classification of risk from either a
regulatory or financial economics point of view means the same standards will be used to quantify risk across the industry. In the USA, harmonisation of risk categorisation in the industry from regulatory framework come from the quantities defined in the NAIC's Risk-Based-Capital formula (RBC). In fact this formula has been adapted by a number of firms in measuring insurance company risk, by making it the basis of dynamic financial analysis (DFA) methodology (AM Best, 1999). This makes the RBC risk classification the basis for an all-encompassing risk classification and quantification methodology.

This all-encompassing methodology does not only classify risk according to the hazards underlying the contract sold, as is with under an underwriting and regulatory viewpoint, but also looks at the financial functional features, such as the duration of cash flows being underwritten. Our all-encompassing methodology seeks to capture both functional features not only in the classification but also the measurement processes, so as to capture all the risk inherent in cash flow exposures.

This chapter is divided into two sections. We classify risk in section one by making reference to three main sources of insurance risk, actuarial, financial and operational. We also propose a risk classification model with the aim of enhancing risk quantification and risk financing. The model isolates risks that are used in Chapter 3 for risk measurement using option-pricing theory. We wrap up the chapter by looking at the market for insurance risk.
2.2 Risk and Risk Capital

Risk is generally defined as the volatility ($\sigma$) of net cash flows of a business unit arising from uncertainties in outcomes. Insurable risk volatility has only one dimension, whilst financial risk has dual dimensions. Insurance companies generate cash flows with both functional features, for example underwriting cash flows are exposed not only to underlying perils but also to duration risk.

Risk also relates to fluctuations in the value of shareholder investments in portfolios, in both absolute and relative terms to benchmarks. Traditionally risk has been measured in absolute terms by the standard deviation of portfolio returns, which computes historical dispersion of returns around a portfolio’s average return after subtracting a portfolio’s risk free return.

A variant definition of risk is found in traditional risk management, where risk is defined as the possibility of that positive expectation of a goal-oriented system that will not be fulfilled. This definition is similar to the one given in FRS5 of 1994 Reporting the Substance of Transactions, whereby risk is defined as uncertainty to the benefits that a business will derive from pursuing its objectives and strategies. Individual insurance business risks are constituents of uncertainty because business objectives and strategies generally relate to creation of future values. These definitions capture the two-way dimensions to risk; the probability of a financially favourable deviation or the possibility of loss. A mathematical definition of risk for a portfolio with two exposures is given by

$$\text{Var}[L_1 + L_2] = \text{Var}[L_1] + \text{Var}[L_2] + 2 \text{Cov}[L_1, L_2]$$  \hspace{1cm} (2.1)
Where: \( \text{Var}[L_1+L_2] \) is the variance of two liability exposures,
\( L_1, L_2 \) are liability cash flows for exposures 1 and 2,
\( \text{Cov}[L_1,L_2] \) is the covariance of liabilities 1 and 2.

The risk of the portfolio is obtained by taking the square root of the equation (2.1) above, which is the standard deviation of the portfolio. This equation (2.1) reveals that when two exposures are combined there are benefits to be reaped, resulting from the covariance component, which defines the diversification benefits. This is called the covariance risk, which defines the resilience of a portfolio's net worth to extreme movements in the market. A portfolio with greater diversification is should experience less vulnerability to covariance risk than a less diversified portfolio. Thus, even when the portfolio size grows a covariance matrix is used to measure the uncertainty between any two risk exposed cash flows.

It is apparent from the above discussion that in order for a portfolio to be well diversified, it either has to write many policies or hold many assets. In order for this to hold the law of large numbers should apply, but because of the central limit theorem, there is a limit to the amount of risk diversified away. The diversification benefits depend on how the risk classes co-move, and on the weights of assets or liabilities in the same risk category. Risk for a portfolio with \( N \) risk exposures taking into account adjustments for exposure weights is given by

\[
\text{Var}[p] = \sum_{i=1}^{N} \sum_{j=i}^{N} V_i V_j \text{Cov}(i,j)
\]

(2.2)

Where \( V \) is the value of the cash flow exposure
The risk of the portfolio is measured by the size of its net worth relative to its standard deviation. This is the relativity capital to volatility of the portfolio, measuring the resilience of the portfolio to any adverse market movements. The riskiness of a portfolio defined by the number of standard deviations of capital in the portfolio is captioned by

\[ C_{SD} = \frac{(NA - D)}{SD_p} \]  \hspace{1cm} (2.3)

Where:  
\( C_{SD} = \text{standard deviation of capital}, \)
\( SD_p = \text{portfolio standard deviation (\( \sqrt{\text{Var(p)} } \))}, \)
\( A = \text{assets}, \)
\( D = \text{debt}. \)

Thus, the probability of default depends on the number of standard deviations of capital, supporting the level of risk in the portfolio\(^1\). The number of standard deviations depends not only on the level of capital held but the level of risk in the portfolio. The level of debt, which should be deducted when calculating the risk score, also affects the level of capital used in the calculation. The higher the level of debt the lower will be the risk score, which defines the financial strength of the company. This score should be used to balance the level of capital that is held and the return that is required, since excessive capital dampens rate of return on capital. In other words capital held should correspond to the level of risk in the portfolio. The more risky a portfolio is the more capital is required to support the business activity.

\(^1\)\( VaR(T; c) = -\alpha \sigma V \)
Wilson (1997) defined risk capital as the amount of economic equity which must be held to support that particular level of risky business activity. This capital provides a given level of safety (the "solvency standard") to policyholders for a given maximum possible loss within a known confidence interval over a given holding period. This level of capital held relative to risk in the portfolio has been defined as the value at risk (VaR). This capital in an insurance company provides policyholders with a level of protection against default.

Economic capital for a given portfolio is tied to a hurdle rate of return that is acceptable to shareholders on the commitment of their capital to this investment. The hurdle rate is derived from the capital asset pricing model, which measures market return on a traded insurance stock with an equivalent risk level. An activity that will generate return above the hurdle rate will widen the risk trading bounds and generate shareholder value, whilst those below the hurdle reduce the risk trading bounds, destroying shareholder wealth in the process. Return generated on economic capital is termed risk-adjusted return on capital (RAROC). It is calculated as the present value of expected net income as a percentage of economic capital attributed to the activity.

2.2.1 Risk and Risk Capital — An Actuarial Science’s viewpoint

An insurance definition of risk is based on the expected value of losses. It has also been characterised by Cummins (1993) in terms of relativity, by extending the expected value definition to capture the contamination characteristics of portfolios. The definition of risk in terms of relativity is based on the implications of the application of the law of large numbers and the central limit theorem on the overall insurance risk. His definition characterises an insurer’s risk exposure as relative and
absolute, depending on the operation of the law of large numbers and the implication of the central limit theorem on the overall risk portfolio. Relative risk operates under the assumptions that exposure units are independent and identically distributed (IID), that the law of large numbers successfully operates, and that average loss exposure unit becomes arbitrarily close to the true mean of the loss distribution with probability approaching 1 as N approaches infinity. According to Cummins it means that we can define relative risk either by the standard error of the portfolio or the ratio of the standard error of the mean to the distributional mean loss per exposure unit².

The fact that both parameters tend to zero as N goes to infinity, implies that this risk will be insignificant in large portfolios, due to diversification benefits. On the other hand Cummins defined an insurer's absolute³ risk in terms of correlation characteristics of the exposures in the portfolio, in other words the covariance risk as it is known in finance. In this case risk is looked at from the point of view of imperfect portfolio correlation, due to contamination. Contamination results in poor resilience of net worth from severe movements in the markets arise from individual exposure units coming into the portfolio, being correlated to risks already in the portfolio. This enigma risk is embedded in the tails of the portfolio loss distribution, and is difficult to finance using all-purpose financing due to the drawback of its accumulation characteristics, which makes it expensive.

\[ IRR_1 = \frac{\sigma}{\sqrt{N}} \]
\[ IRR_2 = \frac{\sigma}{(\mu \sqrt{N})} \]

Where: IRR is the Insurer's risk;
N is the number of exposure units comprising the portfolio;
\( \sigma \) is the standard deviation of loss of each exposure unit;
\( \mu \) is the mean loss per exposure unit.

³ IAR = \( \sigma \sqrt{N} \)
Where: IAR is the insurer's absolute risk.
Due to the covariance risk, absolute risk tends to increase with the size of the portfolio (N) approaches infinity, implying it cannot be eliminated in large portfolios. The distinction between these two risks is that whilst relative risk can be reduced by portfolio diversification, this cannot be said for absolute risk as the portfolio grows. These concepts are linked to capital required to fund the risk carried in a portfolio, in so far as maintaining an acceptable level of financial distress is concerned. Total capital required for a portfolio is a function of absolute risk, whereas relative risk is important when considering the per policy capital requirements. Therefore capital requirement for individual policies declines as the portfolio becomes large, but the total capital for the portfolio tends to infinity with the portfolio size.

Adding risk does not reduce absolute risk, but through subdividing it, making insurance risk management similar to that achieved through portfolio management of financial risks. When exposure units within a portfolio are not IID, they lend themselves to contamination. This internal contamination is a result of covariance risk, which is the non-diversifiable component similar to systematic risk in capital asset pricing. Cummins used Samuelson's "1/√N Law" to show that risk cannot be eliminated if the element of independence does not exist and that variance does not vanish with infinite subdivision, but approaches the common variance between the units.

Meyers and Heckman (1983), Wang (1999) and Meyers (1999a, b, c) used parameter uncertainty to illustrate the impact of portfolio contamination in property insurance, emanating from correlation because of geographic proximity and exposure
to catastrophes. This entails that a book concentrated in an area exposed to catastrophe
generating perils is bound to consume more capital, than a portfolio that is
geographically diversified. In fact the geographic diversification of a portfolio is
important when determining which type of risk financing instrument to buy and the
rate on line that should be paid to finance a portfolio. Marginal risk increments to a
portfolio mean that it might not be economic to accept an exposure unit that is
correlated to exposure units in the portfolio, as this has the effect of increasing the
absolute risk. Therefore, allocating capital to incoming risk exposures might not be
economic, due to higher marginal risk compared to return generated by the business
already underwritten. Meyers (1999), Myers and Read (1998) used the marginal risk
factor as a determinant of the level of capital consumed by an exposure.

This is done on the understanding that the insurer’s potential liability on an
exposure unit is limited by its entire capital, not the capital allocated to individual
exposure units. The allocated capital based on the marginal capital argument is a result
of the behaviour of risk as the portfolios grow. Its shortcomings are endowed in its
inability to accurately measure the equity that is necessary to fund an activity while
maintaining a target default probability, as it ignores the interest paid by debtholders.

This approach to risk management in insurance companies denotes the need for
the distinction of these risks in order to determine the level of capital required and to
whom the burden of paying for this cost should fall. Cummins’s study asserts that the
decline in the insurer’s relative risk linked to per policy capital does not mean the
insureds should not pay for the cost of capital. On the other hand, Meyers (1999) and
Myers and Read (1998) used marginal risk factors to measure capital requirements in
an insurance portfolio, their methods places the burden for paying the cost of carry on the policyholders. The observation is that capital requirements tend to a mean value for exposure units with different risk characteristics and that if they are correlated the level of capital required will be unusually high.

2.2.2 Risk and Risk Capital – A Financial Viewpoint

Financial risk anchors on two pillars the mean and the variance of a portfolio, and the contribution of each risk exposure is measured by its contribution it makes to the portfolio, according to these two parameters. According to Markowitz (1952) risk of an investment is measured in terms of covariability of its rate of return, to the rate of return of the portfolio. As pointed in equation (2.1), whilst variance measures the potential dispersion of future rate of return of a security, it is the portfolio risk that is important in measuring risk, as specific risk can easily be offset against the returns of other securities. In finance according to Sharpe (1964) and Lintner (1965) a risky asset is priced according to its relative contribution to the total risk of the market portfolio. The total risk of the market portfolio is measured by the variance of its rate of return distribution, is given by

\[ \beta = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} = \frac{\sigma_i}{\sigma_m} \rho_{i,m} \]  

(2.4)

Where:  
\( R_m \) & \( R_i \) = return rates on market portfolio and asset i,
\( \sigma_m \) and \( \sigma_i \) = standard deviations of returns on the market portfolio and asset i,
\( \rho_{i,m} \) = correlation coefficient between i and m.
This capital asset pricing risk denoted the ratio \( \beta \) (beta) measures how a security co-moves with the market i.e. the systematic risk of the asset, and is priced based on the non-diversifiable component. Total market portfolio risk is obtained by the weighted sum of all covariances:

\[
\sum_{i=1}^{N} x_i \text{Cov}(R_i, R_m) = \sigma_p^2
\]  
(2.5)

Where:  

\( x_i \) = weight of security I in the market portfolio,

\( N \) = number of assets in the portfolio.

\[ \sum_{i=1}^{N} x = 1 \]

Similarities with the insurance risk arise from the fact that they both assimilate absolute risk as the portfolio becomes large. The most important point is beta being used as a relative measure of co-movements of the securities for which the shareholders are compensated. The same concept has been used in insurance literature to value how much policyholders are paid, for buying an insurance policy with a risky company. Cummins (1990) used the \( \beta \) coefficient in financial pricing models for insurance companies. The betas of individual business units were linearly decomposed as a measure of risk and were also used in the beta-based capital allocation by Albrecht (1997). The problem with these methods is that they tend to deconstruct company risk from a single source, like the underwriting betas (Cummins and Harrington (1985)) or market betas. By taking the portfolio risk and coming up with portfolio betas a company’s overall risk can be measured.

It should be borne in mind that each of the methods of measuring risk discussed above in an insurance company only measure risk in part, with the financial economics methods concentrating on the market risk and actuarial methods on underwriting risk.
Cummins's relativity method is good for internal control purposes because it stipulates risk factors that are taken into account when allocating capital. Capital requirements per individual policy decline, as the portfolio becomes large because of the decline in relative risk. Conversely, growth in a portfolio entails contamination arising from the correlation between exposure units, meaning more capital will be required because of the increase in absolute risk as diversification benefits will cease. Therefore, what drives capital consumption in insurance companies is this absolute risk component; it occupies the upper tail of the loss distribution.

The cost trading insurance risk is a function of individual exposure units or the overall risk characteristics of the exposure units of a portfolio. In fact there is need for the insured to pay for the cost of carrying risk forward to a settlement date, though they pay close to the risk premium, insurance companies arbitrage risk trading by exploiting the margins earned from holding insureds' funds in the form of technical reserves. What is really paid for by the insureds should be the cost of utilising these reserves because in a competitive environment it is difficult to charge a rate above the risk premium. Otherwise what this study will show is that contribution to equity by the insured is not through direct utilisation of rating methods but through reserve utilisation. The shortcoming of this way of defining risk is that it doesn't take into consideration other operational risks affecting insurance cash flows, as it only considers capital requirement costs based on liability risk.

As we saw from the discussion above risk level is also a function of the covariance risk characteristics of the business being underwritten into the portfolio [Meyers (1999)]. This means that capital requirement for renewing an exposure unit is
related to the marginal risk added to the portfolio by writing an extra exposure unit [Myers and Read (1999)]. Marginal risk depends upon the properties of risks already in the portfolio, because exposure units already in the portfolio have a bearing on the magnitude of risk added to the portfolio by an incoming risk. Thus whatever impact an exposure unit might have on a portfolio's absolute risk, the determining factor is a portfolio's risk characteristics, irrespective of underwriting standards and financial goals. The nature of capital required to sponsor risks in the portfolio is determined by the characteristics of the absolute risk. In a way not only capital requirements are defined by absolute risk, but also by the type of capital used to effectively quell volatility in the return profile.

Option pricing methodology has also been used to measure firm-wide risk, by taking into account the effects of both assets and liabilities on the return profile of the portfolio (Cummins 1988). The portfolio risk parameter $\sigma_p$, by using portfolio theory to compute risk arising from both assets and liabilities. $\sigma_p$ is then used in the option-pricing model to compute default risk which represents the overall risk of the firm. The advantage of this risk measurement parameter is the incorporation of asset-liability risks and leverage risks and liquidity risks, which are not captured by other risk measurement methods above. Therefore, our more encompassing methodology discussed in chapter (3) has a stronger predictive power of fair insurance contract prices and provides an efficient way of allocating risk capital.

2.3 Risk Classification

Risk classification has evolved from a disintegrated form where only two broad categories were considered (pure and speculative or financial risks), into a holistic
approach encompassing total risk faced by a company. According to Brenner (1996) theories about risk aversion originated from the days of disintegration of risk. Under this theory those who like risk, gamble, while those who are risk averse, insure, and the theory stressed out that one couldn’t both insure and gamble. Speculation is seen as an action motivated by the desire to increase wealth, whilst insurance is an act of protecting the wealth which has already been achieved.

These acts both involve risk and its management, as well the cost of carry to be paid by those off-loading risk to those willing to assume these risks in the form of premiums, profits, interest, which is deemed to be sufficient for the level of risk. The only difference is the nature of risk being sold at each point of trading, the profiles underlying these risks and risk dimensioning. Otherwise, managing positions exposed to these risks is beneficial to a company because it preserves shareholder value. In each case risk is costly because it destroys value and should be managed in order to preserve value.

2.3.1 Traditional and Regulatory Risk Classification:

Risk is looked at in a contextual and integrative manner in the works of Levin and Schneider (1997). Their classification is based on the processes of handling risk - exposure management and risk management. Exposures were considered to involve the same events as risk, but at a different level depending on materiality of its effect on an organisation’s finances. Risk analysis done using a variety of actuarial, statistical or financial techniques and their management is not a strategic function. The management of risk entails planning, organising, directing and controlling risk to respond to chance; since it undermines a business’s management process. This
classification of events, exposures and risks is more amenable to integration of risks but it does not tell us what the nature of the risks are, the measurement methods whether based on embedded hazards or their impact on cash flows.

Kloman (1992) classified risk by dividing risk faced by an organisation into two major groups, global and organisational risks. The global risks segment is composed of risks related to the factors that affect the organisational risk environment. Organisational risks which under his model drive the global risks are further classified into four divisions financial or market operational, political and legal risks. Kloman also distinguished the extent to which these classifications have to go to be strictly delimited to business risks or hazard risk. However, his method of classification still separates speculative risks from pure risks by only looking at the causative factors and not their effects on cash flow structures within an organisation. In this study we believe cash flow behaviour should be the overriding factor in the determination of a risk segment. His classification based on global risks was also echoed by Troy (1995) who pointed out that increasingly globalised business operations has led to the development of a new economy, which is transitional, fluid and characterised by extreme complexity. Insurance business is global in nature but our perception is that risk is indivisible by nature and this dimension of risk is just one component of a host of risks faced by an organisation. For example when underwriting a line of business, the company will not only be exposed to liability risk, but also duration risk in relation to the tail of business. Thus, classifying exposure to risk, according to all exposures a stream of cash flow is exposed to, makes the classification more robust and complete. This methodology bridges the gap between actuarial and financial risk measurement methodology.
Insurance-based classifications are diverse in nature, the commonly used being the quantities in the standard NAIC’s Risk-based-capital formula. The risk categories used to calculate minimum capital requirements termed R₀ to R₅ are given as follows:

R₀: Investments in insurance affiliates
- Non-controlled assets, Guarantees for affiliates and Contingent liabilities

R₁: Fixed income securities
- Cash, Bonds, Bond size adjustment factor, Mortgage loans, Short term investments, Collateral loans and Asset concentration adjustment for fixed income securities.

R₂: Equity investments
- Common stocks, Preferred stocks, Real estate, Other invested assets, Aggregate write-ins for invested assets and Asset concentration adjustment for equity investments

R₃: Credit risk
- Reinsurance recoverables, Other receivables

R₄: Reserving risk
- Basic reserving risk charge, Offset for loss-sensitive business, Adjustment for claims-made business, Loss concentration factor and Growth charge for reserving risk.

R₅: Written premium risk
- Basic premium risk charge, Offset for loss-sensitive business, Adjustment for claims-made business, Premium concentration factor and Growth charge for premium risk.

Total capital requirements = R₀ + (R₁² + R₂² + R₃² + R₄² + R₅²)⁰.₅⁵  (2.5)
Feldblum (1996) raised three issues pertaining to this formula are particularly important:

- the lack of covariance terms in the square root rule;
- the exclusion of the Ro charge from the square root rule; and
- the marginal capital effects of each risk element.

According to Bustic (1994) the square root rule, used under the RBC methodology overestimates the amount of capital needed to achieve a given "expected policyholder deficit" ratio if the risk elements have normal or lognormal probability distributions. Furthermore, it has been observed that the correlation among the risk factors is weak, to such an extent that they underestimate the need for capital, which is small. The movement of one-half of the credit risk charge into the reserving risk category accounts for correlation between the risk of adverse reserve development and reinsurance credit risk.

The RBC requirements are largely dominated by the underwriting risk charges; particularly by the reserving risk charge. According to Feldblum (1996) reserving risk charges are just ad hoc extrapolations from historical happenstance, they do not adequately distinguish financially-troubled companies from sound companies, and they provide perverse incentives that may raise insolvency risks.

This classification is based on the regulatory framework; it goes to some extent in harmonising the way risk is classified within an insurance company. Regulatory classification has formed a basis for dynamic financial analysis techniques used in
quantifying risk as well as RAROC. Modifications of risk classification as envisaged under RBC, is due to shortcomings in the regulatory framework arising from rigidity and failure to capture actual characteristics of absolute risk. This method of classification also fails to take into account the functional cash flow exposures, because underwriting cash flows are exposed to both liability and duration risks. Our classification methodology will try to bridge this gap.

2.3.2 Actuarial Risk Classification

Insurance risk is also anatomised from an actuarial point of view, by adopting a framework used by the Society of Actuaries' Committee on Valuation and Related Matters, which put risk into categories C-1, C-2, C-3, and C-4:

- C1 risk is defined as volatility in cash flows arising from invested assets other than volatility brought about by interest rate risk.
- C2 risk is defined as volatility in cash flows arising from obligations or underwriting aspects of insurance risk trading portfolios.
- C3 risk is defined as volatility in cash flows brought about by interest rate movements in the presence of a mismatch of assets and liabilities and risk of disintermediation caused by embedded options that are sensitive to changes in interest.
- C4 risk is defined as volatility in cash flows emanating from management decisions, fraud, and errors or omissions.

Risks pertaining to the C-1 category are asset risks which include interest rate risks, credit risk, market risk, and currency risk. The second set of risks C-2 is pricing and reserving risk based on the premise of inadequacy to meet policyholder obligation.
C-3 risk is asset/liability-matching risks, with its underlying causative factors based on movements in interest and inflation rates affecting both values of assets and liabilities. The last sub-set of risk C-4 is a miscellaneous component and is similar to what Kloman classified as global risks, and what we classify as operational risks.

The Casualty Actuarial Society also uses the same classification of grouping risk from c1 to c4 in their dynamic financial analysis (DFA) models. These models seek to integrate separate functional areas of insurance risk trading, to reflect the interplay between assets and liabilities and the resultant effects on income, cash flows, overall return structure and the cost of capital. DFA is a process of analysing the financial condition of an insurance company. Financial condition refers to the ability of the company's capital and surplus to adequately support the company's future operations. This model is good at linking between strategies and results. It uses scenarios to illustrate the impact of the risk environment on strategies and decisions in the context of information about risk exposure in insurance portfolios.

DFA models are complex but good for internal controls when it comes to the value of insurance portfolios and assessing performance given a range of economic environments. Thus it provides a platform for strategic planning, tax planning, risk financing, pricing or market strategy and isolates those risks exposing equity to financial distress. Since it is not standardised it is affected by the model input, and it also determines whether deterministic or stochastic models are the most appropriate given the resources and data at hand. This technique has been criticised for being descriptive rather than prescriptive, since it starts with a methodology and not the framework for linking the actual levels of capital required to risk as the decision is left
to the user (Nakada et. al. 1999). Its complexity and flexibility means that it gives tailor made solutions to individual risk portfolios, but detailed DFA models require significant expenditure on software, time in assumption determination, maintaining the model and interpreting the results.

The drawback for the actuarial view which focuses on risks in isolation and their impact on statutory accounting statements is that it tends to foster risk measures that do not aggregate well at the firm level and leads to a piecemeal approach toward risk management (Santomera and Babbel - 1997). Actuarial classification and models fail to measure the risk management engagement. What insurance companies need is a framework that links the cost of carrying risk forward to settlement date to the level of capital required to optimise risk in an economic way. Our risk classification model will address these shortcomings.

2.3.3 Financial Risk Classification

Risk has also been classified on a financial basis. The generic groups financial risk is dimensioned into are actuarial, systematic, liquidity, operational and legal risks. Financial risk classification looks at risk only insofar as it impacts firm economic value; aggregation and covariance are the focus. This view is the one, actuarial practitioners are now considering in their risk quantification methods, especially under the DFA techniques where financial techniques are used. Therefore, financial risk classification can be seen as the window to greater risk integration in insurance. However, these models fail to capture risk characteristics of individual risks as blending of the portfolio increases.
A risk classification we are looking at is the one that should not only enable us to integrate the firm-level risk but also enable us to model the risks for frequent reporting intervals. Santomera and Babbel (1997) raised a view we also share, when they pointed out that these analyses are complex, difficult, and not easily communicated to non-specialist in the risks considered. Another problem is that risks are not dimensioned in similar ways, and management’s technical expertise to appreciate the true nature of both the risks themselves and the analyses conducted to illustrate the insurer’s exposure to them is limited. They also pointed out that aggregate risk exposure is receiving greater attention, with risk being measured in terms of variability of outcome and cash flow or earnings effect of risky positions. These methods are both amenable to the examination of correlation of different risks and the extent to which they can or should be viewed as offsetting. At the moment most insurers due to their categorisation of risk evaluate these risks separately and aggregate total exposure by simple addition [Santomera and Babbel (1997)]. Achieving integration of risks within an insurance set up however, requires significant work to be done on these risks on an individual basis.

Another classification adopted by Nakada et al (1999) divides risk into three broad categories namely asset, liability and operating. These risks are further put into subgroups based on characteristic inherent in each risk, of which we have credit and market, catastrophe and non-catastrophe, and business and event risks respectively. The advantage of this classification is the ease with which value distributions can be integrated, as the underlying distributions are developed based on differences in risk drivers and distribution type. Furthermore this classification enables the generation of standalone distribution for each risk component, and the measurement of economic
capital on each standalone risk, which can be used in determining the contribution to
economic capital of each component to an aggregated risk component. The risk
measurement techniques used incorporating both actuarial and financial risks, help to
capture the characteristics of each risk component, as depicted in the specific loss
distributions, matching the frequency to the severity. This classification has been used
in determining economic capital requirements; the conceptual framework followed in
generating and aggregating individual distributions by Nakada et al (1999) is robust,
which makes this classification an anchoring point in our study.

2.3.4 Classifying Insurance Risk a Pragmatic Approach

The classification used in this study derives from various models elaborated
above, especially the VaR methodology which is consistent with the current practice.
We view this as the starting point in our classification process because this
methodology helps us aggregate the risk easily, as well as enabling us to capture all the
functional features of cash flows. Our classification goes a step further by categorising
risk, not only with respect to cash flow behaviour, but also on pliability to
measurement and coalescence into the firm-risk-level framework. The model used is a
decomposition of the collective risk profile, in that; it looks at risk from the source, its
consumption and its layoff at acceptable levels. This methodology directly measures
the total variability of potential outcomes through a priori distribution specification
and does not depend on subjectively pre-specified range of risky environments to
derive the worst-case scenario. We propose a broad-based risk classification in this
thesis, so as to address the shortcomings of both traditional and regulatory risk
treatment in the insurance industry. The proposed risk segments, which we can use to
aggregate insurance risk, are captioned as follows:
These two major groups are further subdivided into segments and sub-segments according to the nature of underlying causative factors, and their impact on cash flow structure (as shown in Table 2.1). The model also takes into consideration the correlation between the sub-sets, requiring interlocking risk measurement, control and financing tools. A step-model is considered the most appropriate model to employ, since it eliminates rigidities in risk perception, dimensioning and management. We decompose insurance risk, based on underlying cash flow characteristics, and present it in an integrated way using the step model depicted in Table (2.1).

The impact of risk in insurance risk trading ranges from the expected losses to unexpected losses leading to financial distress or insolvency. Insolvency occurs when a company’s capital has eroded to the point where its ability to generate cash to pay outstanding claims is jeopardised. The materialisation of insolvency results from the lack of market liquidity or the demise of liabilities exceeding the assets backing the liabilities, thereby preventing quick or effective liquidation of positions or portfolios and limited access to funds. Liquidity risk has two components, the timing and pricing adequacy element. Adverse movements in these components can lead to a funding crisis, resulting from any of the following:

- occurrence of unexpected events such as large claims,
- assets write down,
- loss of confidence or legal crisis,
- early settlement of claims than expected,
• failure to make quick recoveries from reinsurers, and
• accumulation of claims due to natural catastrophes.

The framework that is used in this integrated risk classification framework is underlined by the basic principles of Modigliani and Miller that the value of the firm does not depend on the reengineering of the financial structure, given the prevalence of perfect markets, but on choosing projects with positive Net Present Values. Therefore, the identification of risk factors that are used in this model depends on their potential to destroy value in the firm. It is also those risks that are actively managed by insurance companies at the moment, though in a haphazard manner. The risk that are identified and categorised into the groups below are easy to measure and manage because historical data are available for use in quantification and management.

Table 2.1: An Integrated Risk Classification Framework

<table>
<thead>
<tr>
<th>Cash Flow Engineering Risk</th>
<th>Credit Risk</th>
<th>Interest Rate Risk</th>
<th>Liability Risk</th>
<th>Equity Risk</th>
<th>Currency Risk</th>
<th>Operations Risk</th>
<th>Event Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Asset Ratings</td>
<td>Asset and liability Duration</td>
<td>Line of business and size</td>
<td>Project Financing</td>
<td>Insurance Operations held in foreign currency</td>
<td>Expenses Risk</td>
<td>Event: fraud, error</td>
</tr>
<tr>
<td></td>
<td>Reinsurer Ratings</td>
<td>Reserve Duration</td>
<td>Reserve type and size</td>
<td>Public and private equity</td>
<td>Currency matching</td>
<td>Incompetence</td>
<td>Catastrophic Risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Duration Matching</td>
<td></td>
<td></td>
<td>Assets and liabilities held in foreign currencies</td>
<td>Compliance</td>
<td>Financial Crisis &amp; Liquidity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Business risk</td>
<td>Terrorism</td>
</tr>
</tbody>
</table>
We called the first set of risks cash flow engineering risks because; these risks have to be traded in order to create value. They arise during the process of generating value for shareholders and their operation has the effect of reducing shareholder value or making decisions uncertain. For a risk to be classified under this heading there must be a stream of cash flow exposed, because the very existence of such a stream is to generate value. Operational risks on the other hand are inherent in the processes of engineering cash flows; for example, fraud or errors arise not from the cash flows traded but from process risks. Therefore, we envisage two measurement processes in this thesis. The first entails the measurement of engineering risks using portfolio theory methodology. Operational risks are measured as residual risk using the top-down approach, like the option pricing methodology and stress testing. In fact, the model developed in Chapter 3 goes a long way to incorporate even the cost of compliance to an insurance company.

The models that have been used to measure risk have concentrated on certain aspects of the insurance company, like the underlying perils in liability classes and market risk in assets. One such methodology that looks only at asset and liability classes without looking at all the risks affecting a firm’s cash flows is the award winning option pricing paper by Cummins, Phillips and Allan (1997). Their methodology failed to take into account all the functional features of insurance cash flows, in that only perils insured and market risk were used to measure firm-wide risk, whilst other important risks are ignored. A more encompassing methodology like ours accounts not only for perils affecting an account but also the duration of reserves held for each account.
The same concept of making operational risk part of the risk measurement methodology is also shared by Shepheard-Walwyn and Litterman (1998), who integrated this risk component in expected rate of return and volatility determination. Another model showing the importance of operational risks in value creation is the one that further developed Myers's model by Tigeorgis (1988) and Pindyck (1988). Equation (2.6) is more reminiscent of dynamic cash flows, with momentum stored within an insurance portfolio over a period of time is dependent upon how managers exploit the growth opportunities. According to Shepheard-Walwyn and Litterman (1998) the Sharpe ratio can be maximised in year (t), by determining the expected return \( E(R_{pt}) \) and standard deviation \( \sigma_{pt} \). Shepheard-Walwyn and Litterman's equation is used to show not only the interaction between assets and liability accounts but also the role that operational risk play in optimising risk-return as given in equation (2.6) below:

\[
E(R_{pt}) = E(\Delta A_t + Y_t - L_t) \tag{2.6}
\]

Where: \( E(R_{pt}) \) represents forecast value for earnings at time (t); 
\( \Delta A_t \) represents the change in the value of the insurance company's portfolio of assets in time (t).

---

\[^4\text{NPV} = \text{"Static" NPV} + \text{Increment in Future Growth Opportunities.}\]

\[^5\text{Sharpe Ratio}_t = \frac{R_{pt} - R_f}{\sigma_{pt}}\]

Where: \( R_{pt} \) is the company-wide return on invested finds, in time t. \( R_f \) is the return in the risk-free rate at time t, and \( \sigma_{pt} \) is the standard deviation of \( R_{pt} \) measured at time t.

This ratio is an expression of return in relation to risk, measuring return relative to the total risk of an insurance company's portfolio, where total risk is the standard deviation of portfolio returns. What we will be evaluating is the ability of management to alter the portfolio, given the risk level at which they it is constrained to operate. By comparing Sharpe ratios for the portfolio and the benchmark, we can establish whether an insurance company has performed positively, that is when the ratio is greater than that of the benchmark.
$\textbf{Y}_t$ is the value of the insurance company’s new business revenue (growth) in time (t).

$L_t$ is the cost that the firm incurs in time (t).

Equation (2.6) gives an insight into the value stored in the goodwill (future growth prospect) of an insurance company, which is mainly determined by the prudence in management decisions. If decisions by management are poor, this will soon be reflected in the firm’s ability to generate and maintain existing business. An example of a management decision that will affect both profitability and risk levels might be that of under-reserving which will not only affect capital structure but also the credit ratings and share prices. Portfolio risk outlined in equation (2.7) considers factors under the cost of management actions in as far as they affect the realisation of growth, because value enhancement is a function of $\textbf{Y}_t$, which should efficiently be exploited. The risk parameter for a portfolio that takes into account the effect of management decisions is given in equation (2.7) as follows:

$$\sigma_t^2 = \sigma^2 \Delta A_t + \sigma^2 Y_t + \sigma^2 L_t + 2[\text{Cov} (\Delta A_t, Y_t) - \text{Cov} (\Delta A_t, L_t) - \text{Cov} (Y_t, L_t)] \quad (2.7)$$

The portfolio risk parameter computed using risk classification based on the major groups of assets, liability and operational risks shows the feasibility of this type of classification for practical application. We use this methodology for measuring portfolio risk in this study. We are able to show that this classification is easy to apply and to be understood by those using the model to implement policy. The advantage of this classification over the other methods is its simplicity when it comes to application and the ease with which problem areas can be diagnosed and corrected.
2.3.5 Cash flow Engineering Risks

Risk is identified and measured according to the process of generating cash flows, which has two dimensions, the underwriting and market risks. Insurance companies borrow funds from policyholders at a rate directly connected to the underwriting risk, and this rate determines the cost of carry. The products underwritten by an insurance company generate premiums received at time zero, and losses are paid at time one [the assumption that all premiums (claims) are received (paid) at time zero (one) is not true in practice]. The time lag between receipt and payment of losses means that the premiums will be available for investment, with the interval for holding the funds depending on the loss payment pattern.

The payoff profiles of insurance companies are downward sloping, and derive from the underlying risks and how they are managed. The building blocks of any financial engineering is its strength in describing any risk trading position by making reference to the underlying cash flows. The definitions of an exposure can be a claim settlement date, amount of investment, or a risk financing return profile or credit profile of counter-party. The funds generated might not be enough to cover the losses that will be paid, the deficit which is usually known as the underwriting loss is what we call the liability risk. It is the variation in cost of risk trade arising from various risk factors that constitute underwriting risk. The utilisation of the trade (funds raised but not yet paid out as losses) is usually affected by market risk factors, because it is invested in uncertain markets, which mean that these underwriting cash flows are also exposed to duration and convexity risk.
We have pointed out above that cash flow engineering risk is the uncertainty in the surplus of a company associated with changes in variables used to engineer cash flows in order to create value. These variables have been identified in Table (2.2) above, as the drivers of value in an insurance company. The risks in column two define the main exposure that will be used in our portfolio risk measurement formula. The formula that is going to be used in defining risk of the cash flows of a portfolio is given by

\[
\sigma_p^2 = V^2 \sigma_A^2 + V_B^2 \sigma_B^2 + 2 V_A V_B \text{Cov}(A, B) \\
\sigma_p^2 = V^2_A \sigma_A^2 + V^2_B \sigma_B^2 + 2 V_A V_B \rho \sigma_A \sigma_B \\
= \begin{bmatrix} V^2_A & V_A V_B \\ V_B & V_B \end{bmatrix} \begin{bmatrix} \sigma_A^2 & \sigma_A \sigma_B \rho \\ \sigma_A \sigma_B \rho & \sigma_B^2 \end{bmatrix} \begin{bmatrix} V^2_A & V_A V_B \\ V_B & V_B \end{bmatrix} = V \Omega V^T
\]

\(2.8\)

Where:  
\(V = 1 \times 2\) value vector,  
\(\Omega = \text{Variance-covariance matrix},\)  
\(V^T = \text{the transpose of } V,\)  
\(C = 2 \times 2\) correlation matrix,
\[ \sigma = 2 \times 2 \text{ diagonal standard deviation matrix.} \]

This formula as pointed out in equation (2.8) can also be applied to a portfolio with N risks, the dimensions of the vectors and matrices are adjusted accordingly. This portfolio extension is done to incorporate risks identified in Table (2.2). In order to capture all the functional features of the risks above, the risk categories in column 2 above are further divided into further sub-categories. These subcategories enable us to capture all functional cash flows as captioned in column 3 of Table (2.2), i.e. the causative factors behind these risks. For example, in the credit risk category we will be able to know whether the risk is coming from assets held or contingent capital. This however, does not tell us the market they are coming from, the currency of that credit exposure, the industry the obligor is in or the duration of the bond. By doing this, a whole range of risks are identified and included in the measurement process. The same process is used to identify risks in each category, until all the possible exposures are identified. We discuss the processes of mapping these risks and measurement in the following sections. We first of all adopt a descriptive approach of the risks before looking at the computation of variance and the covariance in each risk category.

2.3.5.1 Liability Risk

| Liability Risk | Line of business and size | Reserve type and size |

This risk pertains to uncertainty in policyholder surplus associated with future insurance liabilities. This risk is largely dependent on the lines of business underwritten, the pricing policy and the size of reserves set aside for each risk. This
was quite clear in the Independent Insurance case, when the size of reserves and the type of business it was underwriting lead to its demise. Its demise was not a result of an event risk, but the normal on the grain underwriting risk assumed by the company.

The fact that this risk is contained in products sold by an insurance company means that, it derives from contingent events underwritten, exposure of insurance portfolios to hazard losses, reserve risk, and catastrophic risk. Underwriting risks are unique to specific contracts sold and are primarily related to the terms in the contract document. The timing and amount of liabilities associated with engineering insurance cash flows vary from expectations or assumptions underlying models employed. These variations bring with them obligations that must be met by capital commitments from shareholders and the cost of trade paid by policyholders.

Liability risk is not limited to actuarial risk only, but also extends to include financial risks on the liability side of the balance sheet. Insurance markets are evolving into complete risk trading machines, underwriting both actuarial and financial markets business. Therefore, liability risk can no longer be adequately assessed based on actuarial liability risks only, but also on their primary risk participation in the financial markets. What we have in mind is an insurance company that has a dual role of being a conventional insurance underwriter and that of being an investment banker. The primary cash flows engineered bring with them obligations, which will have the same effect on the underwriting loss profile as any conventional insurance portfolio.

In order to compute the risk for liabilities for all the liabilities underwritten by an insurance company, incurred losses for each line of business have been used in
determining liability growth rates. Cummins (1988) and Sommers (1996) computed liability growth rates by using the variance-covariance approach assuming that changes in incurred losses are normal. This characterisation of losses by their mean and variance mean that these parameters can be derived from the multivariate distribution of the risk factors and the composition of the portfolio. The risk factors are the losses incurred by the insurance company, which we assume to be log-normally distributed, so the log growth rates during a holding period is given by

\[ R_{lt} = \ln \left( \frac{L_t}{L_{t-1}} \right) \]  

(2.9)

Where:  
\( L_t \) denotes the value of the incurred losses at time \( t \),  
\( L_{t-1} \) is the value of incurred losses at time \( t-1 \),  
\( R_{Lt} \) denotes growth rates of liabilities at time \( t \).

Liability growth rates for the portfolio follow a multivariate normal distribution with mean \( \mu_L \), volatility and correlation between two growth rates of two lines of liabilities \( \sigma_L \), and \( \rho \), respectively. We know that since the marginal distribution for each liability growth rate is univariate normal, the portfolio liability growth rates on the portfolio, being a linear combination of normal distributions, is also normally distributed:

\[ R_L \sim N(\mu_L, \sigma_L) \]  

(2.10)

The mean growth rates are the weighted average all the lines of business in the portfolio. It can also be said that the standard deviation is computed using equation (2.8) above, with the dimension of the vectors and matrices adjusted accordingly. We
can calculate the value at risk for liabilities using the expression in equation (2.8) -
\[ \text{VaR}_L = \alpha [\text{Var} \sigma \text{Var}^T]^{1/2} \cdot L. \] This is just part of the model; we need to compute the value at risk of all the cash flow exposures in the portfolio.

When we incorporate financial risk the cash flows are not usually correlated, inasmuch as they have different cash flow attributes, which bring with them a window of diversification benefits as envisaged by our model. Traditional insurance portfolios are difficult to diversify as the assets backing liabilities are generated mostly from cash flows attributed to the liabilities. In any case, either of the engineered cash flows has to perform in order to maintain profitability, which shows the limitations of diversification in conventional portfolios. The expansion of the scope of liability risk in an insurance portfolio brings with it the diversification of the liabilities faced by an insurance company that are not correlated in any way. It's this principle which we apply to insurance companies in order to improve portfolio performance and diversify liabilities portfolio, rather than just the asset/liability match applied at the moment. So as long as we view the trading of risk from the liabilities side and the creation of a portfolio from both actuarial and financial liabilities, better returns can be expected in insurance portfolios.

2.3.5.1.1 Non-Catastrophe Risk

This risk component relates to volatility embedded in absolute loss costs. The determinants of this risk are diverse in nature resulting mainly from changes in loss costs and loss adjustment expenses and claim randomness. Risk resulting from expected losses is categorised into two major distinct classes, catastrophe and non-
catastrophe, according to the characteristics of loss distributions and the level of surplus required underwriting the policy.

The level of claims in the market are assumed to be represented by the burning cost component of premiums written and claims cost in respect of supplementary cover provided. Premiums are determined in advance and based on expectations of carry commitments. They are a function of the attributes of liabilities written and each assuming distinct parameters depending on the growth rate, claim settlement pattern, volatility of claims, and market share initially held by the company. This means that in determining them we face the challenge of choosing a good model that will be able to explain the process risk affecting underlying fundamentals of carry assumptions.

Apart from the model risk, the determination of premiums is also at the mercy of competitive pressures, regulatory interventions, and market cyclicality. Some variables like cyclicality are incorporated into models, with positive results of reducing the overall risk exposure to underwriting cycles. Therefore what we are concerned with when dealing with risk embedded in premiums, is whether the underlying premium assumptions are adequate to cover the underlying carry conditions. Otherwise, deviations from the underlying risk carry conditions makes fair premiums hard to obtain and widen spreads characterising the cost of capital required to buy insurance risk.

Claims level should vary with underlying stochastic nature of the risk profile, the length and shape of the cycles linked to the type of liability. Claims outgo for non-cat risks is not highly skewed as it follows a lognormal distribution, whilst that of cat risks
follow the Pareto or Weibull distributions. The model we use in measuring insurance firm-wide risk in chapter (3) does not take into account the effect of super-cat risks. The idea is to keep the model simple, without losing the intended goal of providing a framework that is not too complex and easy to communicate to those implementing it on a day to day basis.

It is important to note that since actuarial literature has concentrated on dealing with the problems associated with forecasting losses, we will not attempt to explore this area as it is well documented. What we shall make mention of in this case is how this risk is segregated into cat and non-cat components as this has a bearing on capital requirements. Whilst not-cat risk is closely related to factors affecting reserve risk, mix of policies in the book, moral hazard and adverse selection, cat risk is attributed to nature coverage, international tort, geographical concentration of book and legal jurisdiction. Modelling the size of cat losses and their probabilities has lead to improved certainty in prediction levels, though their frequency, severity and nature remains uncertain. This is the basis of insurance liability, because both non-cat and cat risks affect the overall cost of borrowing funds from policyholders and hence the pricing of individual policies.

Another important risk is reserve risk, which is a result of disparities in the actual cost of losses for liabilities incurred under the valuation date from expected values. A number of factors can be attributed to this risk ranging from the credibility of the database, the volatility of the claims process, currency fluctuations, inflation, legal environment, corporate culture and pricing patterns. Risks embedded in the model used to calculate reserves, for premium reserves otherwise defined as a portion of
premiums that is equivalent to unexpired risks, depends on the model used and its appropriateness to the cash flows patterns. In other words, the earned premium is the average of premiums written in year (t) and (t-1), and unearned premium reserve is given at end of year (t). Reserve risk is a function of the size of the company, as large companies hold colossal reserves than in small companies where volatility in loss reserves is more conspicuous (A.M. Best, 1998). In fact big reserves are more diversified and give greater flexibility due to greater resources available to the company to better control reserve and price risks.

Required capital depends on the variance in incurred loss development between accident years, and the variance in accident year of the incurred loss development between companies. These two measures of variance provide an indication of volatility in loss development for each line of business. Growth in premium income is an additional risk incorporated in measuring variance in reserves, to reflect the change in exposure profile. However, the provisions for outstanding claims and future claims are made on a nominal basis and then discounted at a risk-free rate of return (treated in a similar way by Myers and Cohn (1987)).

Case Study 1:
First Central Insurance Company started operations writing commercial lines in 1979 as a stock insurance company in New York. It had an A.M. Best rating of A (excellent) in 1986 because of good underwriting results in the preceding four years, a trend, which continued in the 1990s. However, its initial reserves or rates to cater for the rapid growth it was experiencing were inadequate. Premium underwriting doubled during the four years to $65.6 million during the four years but capital to support the growth only rose $2.6 million to $20.3 million. A.M. Best lowered their rating to A- due to the company's weakening capital position continued high growth and the quality of investment. The company held significant equity position in each portfolio; it had straining dividend requirements to the holding company and market losses in the carrier's portfolio, but still showing positive underwriting and operating performance. The company's loss ratio increased by 20% despite an unusually low-accident year, which was lower than each of the preceding growth years. The 20% loss ratio was masked by an excellent portfolio increase that provided a 30% growth in surplus, the slither in underwriting results was a result of reserve deficiencies in the previous years. First Central continued to pay dividends to its parents, limiting any further capital growth. A.M. Best downgraded the rating to B++ due to the continued deterioration in capital levels, reserve deficiencies and substantial dividend requirements to service the parent's debt.
First Central tried in 1996 to restore profitability and stability by reducing exposure through reinsurance protection, cancelling policies and restructuring its investment portfolio to reduce volatility. After announcing a reserve strengthening in early 1996, halving the surplus, the company was place under surveillance by the New York Insurance Department. A.M. Best further reduced their rating to D and in 1997, the company was placed into rehabilitation. (A.M. Best 1999).

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Orig. Loss Reserve</th>
<th>Develop Reserve Thru'97</th>
<th>Develop to Orig. (%)</th>
<th>Develop to PHS (%)</th>
<th>Develop to NPE (%)</th>
<th>Unpaid Reserves @ 12/97</th>
<th>Unpaid Res. to Dev. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>27,269</td>
<td>44,170</td>
<td>62</td>
<td>93.6</td>
<td>150</td>
<td>12,677</td>
<td>28.7</td>
</tr>
<tr>
<td>1993</td>
<td>35,127</td>
<td>59,732</td>
<td>70</td>
<td>111.9</td>
<td>139.5</td>
<td>23,955</td>
<td>40.1</td>
</tr>
<tr>
<td>1994</td>
<td>45,968</td>
<td>75,643</td>
<td>64.6</td>
<td>145.9</td>
<td>151.1</td>
<td>42,446</td>
<td>56.1</td>
</tr>
<tr>
<td>1995</td>
<td>65,607</td>
<td>84,879</td>
<td>29.4</td>
<td>73.1</td>
<td>160.0</td>
<td>62,441</td>
<td>73.6</td>
</tr>
<tr>
<td>1996</td>
<td>86,229</td>
<td>86,229</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>86,229</td>
<td>100</td>
</tr>
<tr>
<td>1997</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Table 2.4: First Central: Loss and LAE Reserve Development ($000)**

Source: Best’s Averages & Aggregates 1998.

The collapse of First Central is a classic case of how inadequate reserving can affect a company’s long term financial viability especially when it is writing unseasoned business. As shown in table (2.4) above there was a severe adverse reserve development in 1996, which exceeded the original reserves by about $20million or...
The first nine months of 1997 also underlined the problem of inadequate reserving which saw adverse reserve development crowning the year with a total of $24 million. It can be inferred from the same table that if history is to go by reserve development for this company has been unfavourable, with deficiencies recorded in each of the past seven calendar and accident years.

It is from the demand curve risk that by undercutting rates in a bid to increase market share; First Central failed to provide for the long-term cost of risk. The company's investment policy and continued demand from its holding company for it to pay dividends even in the lean years further exacerbated their weak financial position. The reserving risk should have been managed by taking cognisance of the company's long term cost structures vis-à-vis the prevailing risk factors so as to adequately rate business written into the book. In the case of First Central it was the impact of demand curve risk and the failure of sponsoring capital to meet the actual capital needs of its risk portfolio.

Risk consumption is supposed to be matched with the capacity. Available capacity was not adequate because it did not equate to the actual risk factors faced by the company. The combination of under-pricing and excessive growth rates was a cocktail for disaster, and spell out the underlying factors behind the silent assassin, demand curve risk. It is apparent in Table (2-5) that the fundamentals went wrong in 1995, becoming worse in 1996 when the combined ratio was 110.6% and 133.3% respectively. The operating ratios of 104.7% and 128.2% in these years, lead to a decline in the overall liquidity during 1996, from 101.2% to 95.65, a further decline ensued in the first nine months of 1997 to about 87%.
If this demand curve risk was well managed, growth rates should have been the first indicators to management that their fundamentals were wrong and that there was excessive utilisation of all-purpose equity. This case shows that irrespective of good underwriting results, if the fundamentals are missed, a company is bound to face the consequences of failing to define its risk parameters and the type of equity it should be utilising for each sector of its loss distribution. A proper risk classification should have warned them of the danger of relying on accumulated equity to sponsor the whole loss distribution. In Table (2.5) it is clear that good performance in the earlier years with rates of return on equity (ROE) of 12% and 13.5%, did not reflect the realised five-year return of 0.4%. Therefore, the false impression painted by the high rate of return on equity did reflect the quality of risk management, but the problem of relying on ROE as a measure of performance. We believe management compensation systems in insurance should be based on the cost of risk or spreads they deliver on equity supplied to them.

In this study we distinguish between performance and risk management, because performance management without risk management always fail to deliver long-term quality returns. This issue is discussed in detail in Chapter (3) when we deal with default/liability spreads. It measures the cost at which management borrows funds from the policyholders and how these funds are utilised to earn a commensurate return. The management of spreads on both liability and asset accounts is a virtue, since through this art quality operational profits can be delivered to shareholders.
2.3.5.1.2 Event Risk

This risk is prevalent in portfolios underwriting property classes covering natural perils such as hurricane, flooding, earthquakes, drought, hail, windstorm, tornadoes or cyclones, international tort, and liability classes. It's also prevalent in asset portfolios as a financial crisis, resulting from unusual increases in the covariance risk that reduces the resilience of the surplus. Thus a single financial crisis (a recession in the US due to a terrorist act at the World Trade Centre, dampening other major financial markets) changes the correlation between various assets in the portfolio to unity. This raises the level of risk in the portfolio as well as the capital requirements.

Losses resulting from these perils are uncertain as to their occurrence and are usually full of unpleasant surprises as to their magnitude. This entails that the distribution profile of this risk is highly skewed with fat tails. Risk measured in our model, can further be developed to capture catastrophe risk using scenario analysis and stress testing, by looking at the magnitude of extreme historical events, both financial and catastrophic. This is necessary because the overriding assumption in this study is that risk factors are log-normally distributed, so a smooth behaviour is implied, which excludes the possibility of jumps. Stress testing and scenario testing will help identify the vulnerabilities of the surplus to a variety of catastrophic events. The fact that when these events occur, historical correlations change with increases in volatilities, mean that resilience of the surplus to market movements is reduced.

The vulnerabilities depend on the particular characteristics of the portfolio, with insurance companies more susceptible to liability catastrophic risks if they are underwriting most of their business in geographically prone areas. A word of caution
is that there are a lot of permutations on the basic stress shocks which might be ignored, for example whoever thought that the twin towers of the World Trade Centre (WTC) will be obliterated by the same event and accelerate a world-wide economic recession. This methodology is just useful for highlighting those hot spots in the portfolio, because it does not tell us the likelihood of the events.

The existence of this risk within a portfolio exerts more strain on the cost of carry, meaning extra capital requirements. This risk has high capital consumption attributes, especially when the degree of contamination is high within a portfolio that is the units insured within a portfolio have a high correlation. The contamination usually arises from geographical concentrations of the exposure units insured by a portfolio.

It has been observed from the US market that the cost of carry has always been high in catastrophic years, from 1938 to 1998 on the overall industrial statistics. The number of insolvencies occurring in respective catastrophic years also supports this observation (A. M. Best, 1998). This show how the cost of carry is affected by risk throughout the entire loss distribution profile, adding the catastrophic risk component to the non-catastrophe, increase the cost of carry as the distribution profile becomes more risky. The distribution shown below becomes more risky because of the correlation inherent in the units modelled in each separate distribution. We believe that in the case of insurance risk, combining separate loss distributions on the cost of carry profile increase, rather than diversify the risk away.
An insurance company underwriting a portfolio characterised by loss distribution (A) will experience less volatility in its cost of carry, than a company underwriting the loss distribution (B) in figure (2.1). In fact these distributions show portfolios with different loss distributions and how these when combined affect the overall cost structure of an insurance company. The three loss distributions also determine the level and type of capital required sponsoring inherent risk. It is evident that more capital is required for the catastrophe distribution than for the non-catastrophe distribution and even more capital is required for the combined loss distribution (C). This also entails that the required return on capital supplied is directly linked to the characteristics of the distribution as its representative of the level of volatility...
embedded within a portfolio. Combining the cat and the non-cat distributions does not improve the predictability of cost of carry trends, because the risky element is not diversified away as it is still present within a portfolio and will to a large extent influence volatility.

**Case Study 2:**

MCA started trading risk in Tulsa, Oklahoma as a conservative insurance company in 1929. In 1988, MCA decided to expand its business into writing homeowner and other property coverage in Florida, this was to prove a costly decision. In August 1992, Hurricane Andrew devastated south Florida coast resulting in insurance losses more than $17 billion. The initial estimates revealed that MCA suffered losses exceeding $50 million, with reinsurance arrangements expected to release capital up to the tune of $20 million the net loss falling back on MCA was $30 million. MCA after the catastrophe agreed with the Oklahoma Insurance Department to suspend writing new business. In September 1992, it was placed under state supervision to guarantee that claims from catastrophe event victims would not exhaust the company’s resources. In October 1992, MCA was placed into receivership, with claims estimated to be 3000, with a pecuniary value of more than $90 million. This far outstripped its financial reserves of $21 million in reinsurance and $15 million in capital. Insurance claims covered by Florida Insurance Guaranty Association were only up to the limit of $300,000.

(Source: A.M. Best, 1999)

Case study (2) shows how risk-equity mismatch wrecked a sound company on paper. The company changed its business mixture bringing with it new risks and a different loss distribution to the one faced by MCA previously. This required the company to redefine its risk profile and equity requirements to reflect changes in its business mixture. The company’s perception of risks inherent in its new profile was impaired by the omission of a thorough risk and equity strategy redefinition. This meant that equity funding the new portfolio was inadequate, hence its demise. We believe that, had the company reclassified its risk profile and redefined its risk parameters properly, it would have survived this catastrophe.
Risk is classified according to the tier it occupies on the loss distribution, so it should be financed from an economic point of view by equity with matching risk characteristics. This was ignored in the MCA case. This characterisation of risk according to the level of severity and probability of occurrence makes the task of matching risk to equity easier and the utilisation of equity more efficient. The paucity of equity for risks occupying the super-cat tier of the loss distribution, has led to the development of new risk financing instruments aimed at diversifying sponsoring equity. The alternative risk financing methods for risks in the super-cat region targets the financial markets as the source of diversified equity suitable for off-loading insurance risks.

Alternative capitalisation methods have come in the form of insurance derivatives traded at the Chicago Board of Trade (CBOT), swaps traded at Catastrophic Risk Exchange (CATEX) and the securitisation of non-performing assets and/or losses. No single instrument is efficient in financing insurance risks faced over the entire loss distribution, but different instruments deliver different coverages at costs reflective of the underlying risk financed. The decision on the type of sponsoring equity used in financing risk throughout the loss distribution should be based on the underlying risk characteristics and the economics of using that particular instrument.

Reinsurance capital is inadequate to finance super-cat risks because its equity accumulation characteristics are not amenable to financing all risk in the upper tail. Risk financing methods discussed below are intended to improve cost effectiveness in equity utilisation whilst delivering desired portfolio stability. This entails that the cost
of risk financing should be factored in the fair price charged the policyholders and in capital allocation to activities.

2.3.5.2 Interest Rate Risk

Interest rate risk is the uncertainty in surplus associated with the change of interest rates in the future. It emanates from four distinct sources which expose insurance cash flows; these are repricing risk, yield curve risk, basis risk and optionality. These mismatches expose insurance company income and economic value to unexpected fluctuations, as underlying economic factors change. Movements in interest rates resulting in repricing rematches expose an insurance company to changes in the shape and slope of the yield curve (steeping or fattening). Yield curve risk results from changes in the slope and shape of the yield curve when unexpected shifts have adverse effects on the income and economic value of an insurance company. In this case, an insurance company that is funding a long-tail liability with a short term interest bearing asset could face a decline in income arising from the position and its underlying value if interest rates increase.

Yields vary differently depending on the quality of instruments employed, liquidity and maturity, which may not be in line with the liabilities, as they fluctuate independently. The reinvestment rate has a significant impact on the result of the model, which means that there is bound to be under- or over-statement of risk because of the inexact choice of reinvestment rate. Another source of risk crop up from imperfect correlation in adjustment of rates earned and paid on different instruments.
with otherwise similar repricing characteristics. Basis risk gives rise to unexpected changes in the cash flows and earnings spread between assets, liabilities and off-balance sheet instruments of similar maturities or repricing frequencies.

Insurance companies use options in trading and non-trading accounts, with instruments having embedded options bearing greater significance in non-trading activities. Instruments like bonds and notes have option provisions, as well as policies issued to policyholders, that give them the right to claim recovery or bonuses on the occurrence or non-occurrence of pre-specified events; the recourse to reinsurance and other capital market payoffs also have embedded options. These cash flows are characterised by asymmetrical payoffs as they are generally exercised to the advantage of the buyer and to the disadvantage of the seller. There is also a high level of leverage in these instruments, which tend to magnify exposure/risk level to portfolio, hence, the need for prudence.

The timing of exercising these securities is uncertain, which exposes the portfolio to the risk of repricing, as these options are exercised in a pattern that does not usually follow the repricing process of backing assets. Thus at any stage when the repricing of assets happens, there will be a change in their profile, the liability portfolios at each stage should be readjusted to match backing assets, as if the pricing was done at the initial period. This interaction between liabilities and assets points to an internal insurance risk pricing process that takes into account the rollover risk encountered at each repricing stage. Dinenis and Mutenga (1999) used a model that disintegrates the insurance risk pricing process into various risk cost components, by deciphering spreads inherent in both asset and liability accounts. This helps in accounting for the
effect of risk dimensionally, according to time and against a benchmark for re-trading a risk portfolio.

A study by Santomera and Babbel (1997) on risk management in general insurance companies pointed out that the management of risk appears to be concentrated on the asset side, ignoring the effects of duration variations of liabilities. Another important observation by these two authors is the failure by current duration and convexity measurements to incorporate the equity risk in asset/liability models. Assets available should not only be looked at as a means of sustaining value within a company through meeting all future claims outgo but also act as a stabilising factor over the long run. This means that assets should be able to create a surplus in good underwriting years, whilst asset realisation should be effected in lean years when a shortfall occur.

If the asset/liability duration model was complete in characterising asset portfolio risk then a factor could be assigned to asset/liability risk determined by the probability that desired level of mismatch is achieved ceteris paribus. The problem with such modelling is that it does not fully characterise the loss distribution of asset risk, so the default-put option method will be more advantageous to use as it go some way to fully characterising the loss distribution of assets and liabilities. The reason for this is that asset portfolios contain more options, that simple linear measures such as a basis point value or duration are inappropriate; even if convexity is included, the measures are not accurate enough to estimate risk associated with large fluctuations in underlying prices (Smithson and Minton, 1996). It is practise to use the market value for assets, though it is difficult to do this for liabilities, past loss data or simulated data is all that is close to
the market value of a liability. A continuous time valuation of insurance liabilities as 
the one utilised in some dynamic financial analysis would have been best to use, but 
due to a lack of standardisation and the economic rationale behind such a model, we 
will use models that model historical losses to measure insurance liabilities.

Adjustments should also be done to reflect altering liquidity horizons in the 
default-option model, so that the anticipated time to close or hedge positions is taken 
into account. Factors inherent in these risks are dependent on the purpose of the cash 
flow altering instrument and the potential to deliver stored value required backing 
liabilities. Therefore, the following should be considered when making adjustments in 
the default-option model:

- The liquidity structure of the instrument;
- Tradability of these instruments;
- Elasticity of demand and supply of a product, e.g. some products used to back 
  liabilities in highly liquid markets may become illiquid when they are 
  substantially in the money or out of the money.
- Credit rating of the product issuer or the company selling the contingent 
  capital,
- Scale of the company's presence within a geographical insurance market or 
  the financial markets (the MCA demise).
- Size of individual positions relative to the issue size; insurance companies 
  have limitations as to the proportions they can invest in a security, offered by 
  the same issuer, such limitations also apply to reinsurance capital sourcing.
- Bid-offer spreads within a market are also an indication of typical liquidity of 
  individual securities;
• Daily turnover of the securities market is also important.

In the VaR at risk model this risk is measured by changes\(^6\) in duration\(^7\) or convexity\(^8\), which is also a measure for asset/liability risk\(^9\). This method will work perfectly well for those companies investing in the bond market like Munich Re, but is not reflective of the risk of assets backing liabilities in the then Commercial Union's (now CGNU), which was heavily invested in the equity markets. This means Munich Re will be more interested in how adverse movements in interest rates markets affect its assets and liabilities.

The investment policy of a company is a major determinant factor on how these cash flows interact, for Munich Re Company performance is highly correlated to bond market performance. Conversely CGNU’s investment policy made it more prone to equity market performance. Thus, the current measure of immunisation of using interest rate risk cannot be a good measure for those companies with liabilities backed by assets invested in the equity market.

\[
\frac{dP}{dy} = -P \frac{D}{1 + y} = -PD^* \\
\text{Where: } P = \text{price of the security,} \\
\text{Y = yield to maturity,} \\
D = \text{Macaulay Duration of the security,} \\
D^* = \text{modified duration,} \\
dP/dy = \text{change in the security price for a change in the yield to maturity.}
\]

\[
D^* = \frac{D}{1 + y} \\
\]

\[
CX = \frac{1}{P} \cdot \frac{\Delta P}{\Delta y} \left( \frac{\Delta P}{\Delta y} \right) \\
\text{Where: } CX = \text{convexity of the bond,} \\
\Delta P = \text{Change in price corresponding to a change in yield,} \\
\Delta y = \text{Change in yield}
\]

\[
\Delta P = -PD^* \Delta y + \frac{1}{2} P \cdot CX \cdot \Delta y^2
\]
Margins or mismatch reserves in an asset portfolio can effectively be established if the cash flows are unpacked and variability attributed to the underlying risk factors of the market they are placed in. Assets are considered individually under this valuation method and their behaviour evaluated to establish their effectiveness in immunising the portfolio. Beard (1974) considered margins, which are required for the various components of liabilities, and indicates that assets and liabilities must be considered when assessing capital strength. The inclusion of various investments in a portfolio to measure and manage duration improves its precision as a measure and on limiting exposure. This study shall also endeavour to see how various investment instruments can be incorporated in the measure of asset/liability risk. The issue of substantiality should be emphasised since the out-laying of resources to track and manage these risks over and above the existing measures should be economic.

The asset/liability duration or convexity model does not completely characterise the asset portfolio risk due to inherent options, so the value at risk (VaR), as defined by the default-put option is used as it goes some way in characterising the loss distribution of assets (Smithson and Minton, 1996). Default-put risk is defined as the largest likely loss from asset and liability risk that a portfolio will suffer over a time interval, and with a degree of certainty. It is used to test the cost of risk capital in this study.

2.3.5.3 Equity and Currency Risk

\[ \sigma \Delta P = PD^* \sigma \Delta y \]
These risks arise from uncertainty in net-worth associated with the risk of equity market valuation and future exchange rates. Insurance companies invest their assets in publicly traded equity holdings, in affiliated companies that might not be publicly traded and venture capital projects. Movements in the stock markets will expose an insurance company's cash flows to risk specific to the industrial sector and the market the stock traded. Equity risk will be computed using equation (2.1), based on the market and industry, and we also take into account the weightings in each market category so as to capture the contribution of each market and industry.

In order to compute the risk for shares in the portfolio of an insurance account, we have to use the prices of the stocks. The price of shares are assumed to be log-normally distributed, which we use to compute log-returns, for a chosen holding period. The log returns computed as follows are log-normally distributed,

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$ (2.11)

Where:  
$S_t$ denotes the price of the share at time $t$,  
$S_{t-1}$ is the price of the share at time $t-1$,  
$R_t$ denotes that rate of return of the share at time $t$.

Since the distribution of these returns is characterised the mean and the variance, this is true with the portfolio since it follows the multivariate normal distribution of the risk factors. This describes return as a linear combination of normal distributions with
a mean $\mu_E$, a standard deviation $\sigma_E$, and $\rho$ is the correlation coefficient between returns of two shares. The portfolio return ($R_E$) normally distributed and is given by

$$R_E \sim N(\mu, \sigma) \quad (2.12)$$

The computation of the mean and standard deviation are important at the computation of equity portfolio risk, we extend Markowitz's (1952) model and use equation (2.1) to compute portfolio risk. The portfolio mean is just a weighted average return ($\mu_E$) of all the shares. We can use this to compute the value at risk for equity the equity portfolio $[\text{VaR}(T; c) = \alpha \sigma_E \cdot E_V]$, but this is just part of a jigsaw in our risk computation puzzle. In other words we are more interested in all the risk factors affecting an insurance company's operations.

Currency risk results from assets and liabilities, and insurance operations held in foreign currencies, as well as from currency matching. This risk significantly affects big insurance companies trading mostly in developed markets and the so-called emerging markets. Currency exposure in the EU has been greatly reduced because at most 12 currencies were reduced to a single currency on the 1st of January 2002. However, insurance companies are still exposed to currency risk from emerging markets, which do not have floated currencies and even if they are floated it is very volatile and hard to repatriate.

The computation process of these risks is quite simple, we just choose a time period in which we want to measure the risk, two currencies at a time is all you want to use in the computation of the covariance matrix and repeat the same process to
compute the portfolio risk. The data is readily available even on an hourly basis, but in our case we use the daily data.

2.3.5.4 Credit Risk

<table>
<thead>
<tr>
<th>Credit Risk</th>
<th>Asset Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reinsurer Ratings</td>
</tr>
</tbody>
</table>

This is the inability (default) or unwillingness on the part of the counterparty to perform on their pre-committed terms, or uncertainty regarding the collectibility of recoveries arising from the financial condition of the reinsurer or the ambiguity about cover provided. This is how we view this exposure in our valuation model. Direct credit risk results from counterparty default on traditional on-balance sheet products, such as reinsurance; the exposure is the financial value. Counterparties may also default on unmatured off-balance sheet products such as risk exchange contracts, finite risk contracts, swaps, cat-options; with the credit equivalent exposure being a function of the prevailing prices. Another dimension to the credit risk is the one related to settlement where a counterpart defaults on transactions in the process of being settled and value has been delivered to the counterpart but not yet received in return.

Credit risk is low frequency and high severity and is correlated to the magnitude of losses incurred by a reinsurer, because there is a tendency to default reinsurance contracts when catastrophic losses affect a counterparty. On financial instruments like bonds, credit returns are highly skewed and fat tailed, because improvements in credit quality brings limited upside to an investor, while downgrades bring with them substantial downsides. This makes credit risk in insurance quite unique in that the very purpose for obtaining reinsurance, which is to cover losses on the tails are the very
losses that are most likely to lead to a default. Thus, there is a high correlation between the size of cover obtained under a reinsurance arrangement, and the probability of default as it is usually arranged to portfolios with high sums insured.

In order to compute credit risk for an insurance company's portfolio the exposures should be classified according to their rating. The information of ratings on bonds, loans and reinsurance bought, is obtained from credit rating companies like Standard and Poors', A.M. Best or Moody's. These rating agencies rate obligors based on more than 20 years of data, on which default probabilities are published. We use this information to identify non-rated bonds, project loans and mortgages, and map them to a rating. We specify the transition matrix from the chosen rating system, which mean that if we choose the S&P rating, we will have seven categories from a AAA to a CCC and then default. The transition matrix is a summary of the historical pattern of migration for bond ratings over a holding period, which is usually specified as one year. An adjustment is made on the average historical values so as to make the data consistent with the assessment of the current economic environment.

We first have to compute the volatility of each instrument in a rating category, before moving on to measure portfolio credit risk exposure. The overriding assumption is that all issuers with the same rating category class are homogeneous; they have the same transition probabilities and the same default probability [Crouhy, Galai and Mark (2001)]. Using equity return correlation to generate the joint migration probabilities generates the portfolio diversification effect. There are a number of methods used to measure credit risk, like the modified Merton option pricing method, the actuarial approach, the KMV™ and the reduced form approach. These are alternative methods
to the CreditMetrics™ used to address the shortcoming of using equity as a proxy for company risk determination. The mathematical methodology for measuring credit risk exposure is beyond the scope of this study and the RiskMetrics™ group gives details of the methodology in the CreditMetrics™ technical document.

On a prudence note, the financial position of the counterparty must be assessed thoroughly when deciding on buying capital or exchange risk in the secondary risk market. This is because the risk of default declines with the strength of the balance sheet of a company, and the consistency in their underwriting and accounting policies. However, it should also be borne in mind that default risk might also result from systematic risk factors. Correlation between default risks in an insurance portfolio is low on the asset risk side but high on the reinsurance securities side. This correlation mentioned above is also affected by the asymmetric nature of loss distributions, which means that when losses occur they will be in many multiples of the expected loss, hence the need for more economic capital to absorb them. At times reinsurers default where the balance sheet indicators do not predict the possibility of default; but default might arise from variance of portfolio performance from the mean value. Even if we use the mark to market methodology, it does not protect the balance sheet, because a position with a nil exposure can turn into a multi-million exposure within seconds of catastrophe event occurring. The levels of asymmetry, arising from the way exposure values grow within a short period of time at devastating rates makes it very difficult to hedge this risk. Therefore, this risk cannot be completely diversified away and some other instruments as credit derivatives can also be used in managing it, where the underlying risk can be marked to market.
Operational risks are inherent in management decision and agency problem, in the form of spread costs incurred by virtue of being in the business of trading risk. Compliance, quality of human capital, ingenuity of staff and reputation are at the centre of this model. The ability to continue creating wealth is enshrined in the quality of management decisions and rationality in management of risk taking behaviour. A model similar to the one above (equation 2.6) is the one given by Myers (1977), which divides the value of a company into two components, value of assets in place \((V_a)\) and value of future growth opportunities \((V_{go})\).

\[
V = V_a + V_{go} \quad (2.13)
\]

\(V_a\) is defined as the value at the current balance sheet date which is not dependent on further discretionary management efforts in the form of investments, contrary \(V_{go}\) does depend on profitability of these investments. In equation (2.13) cash flows in \(V_a\) represent the Net Present Values (NPVs) of asset and liability, which are considered static. On the other hand \(V_{go}\) represents the company’s ability to gain access to markets, the abilities of its qualified personnel, goodwill, security and ability to provide capacity on demand. Risk ingrained in operations are diverse, including but not limited to strategic changes in competitive advantage, due diligence, inflation,
unemployment, technological and legal changes, economic depression, terrorism and political risk due to globalisation. We will measure this risk in Chapter 3, where we compute residual risk using the option pricing methodology.

2.3.7 Functional Risk Classification

The classification proposed above focuses on all risks faced by an insurance company. However, it fails to recognise financial aspects of cash flows generated on liability and asset accounts. So, within this traditional structure, we further propose a classification that endeavours to group liability and asset risks according to their duration and convexity rather than just the underlying perils insured. This classification divides risks according to the time they take to settle and risks affecting them during the carry period, and not just risks arising from underlying perils insured.

It sees insurance cash flows as a fund affected by risks of financial nature, brought about by the time they take to extinguish. This classification categorises asset and liability risks into three major accounts based on the duration of cash flows, i.e. short-term, medium-term and long-term risks. Each liability risk category is matched to assets with similar functional features. This makes it easier to manage duration and convexity risk in insurance portfolios as risks are matched according to their functional attributes not diverse and irreconcilable risks embedded in perils insured.

It also helps insurance companies establish better transfer pricing systems, where liability accounts are paid interest by asset accounts for lending underwriting originated funds for investment purposes. The rate paid to each liability account is
based on the duration of cash flows, irrespective of the nature of liabilities in that account. The rate paid to liability accounts for these funds is the rate prevailing in the financial markets of cash flows with similar functional characteristics. This means that both liability and asset accounts should be able to generate commensurate return to shareholders after taking into account spreads paid to policyholders for originating funds and interest rates to liability accounts respectively.

Transfer pricing systems instil discipline in liability accounts, as their interest rate is not based on underlying perils but competitive rates in the market. If the underwriting policy is poor, the cost of borrowing funds from policyholders will be high and the account will fail to generate return commensurate to the risk inherent in the portfolio. The same applies to asset accounts if investment return is lower than the interest rate they pay for borrowed funds from liability accounts (they will be destroying value). The main purpose of this classification is to make accounts perform based on financial aspects as each is forced to generate a return commensurate to the portfolio risk. We are able to bring insurance risk classification in line with banks that already classify their risks this way and have been performing much better than insurance companies. We use this classification in chapter 4 and 5 when developing our risk measurement model and empirical analysis respectively.

2.3.8 The Impact of Risk on Insurance Companies

The cost of financial distress becomes apparent when a company is placed under some form of regulatory supervision, such as receivership, liquidation or conservatorship (A.M. Best 1999). A study by A.M. Best in the US in the period 1969 to 1998 the property/casualty companies failure rate peaked in 1975 at 1.4%, then in
1985 and 1992 at 2.3%, and 1989 at 2.0%. In the table (2.6) it is apparent that 640 insurance company insolvencies were identified in the US since 1969, with the annual failure frequency of 0.89%.

<table>
<thead>
<tr>
<th>Primary Causes</th>
<th>Number of Companies</th>
<th>% of Total Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Catastrophe Risk (Deficient Loss Reserves)</td>
<td>145</td>
<td>34</td>
</tr>
<tr>
<td>Non-catastrophe risk (Rapid Growth)</td>
<td>86</td>
<td>20</td>
</tr>
<tr>
<td>Operational Risk: Alleged Fraud</td>
<td>44</td>
<td>10</td>
</tr>
<tr>
<td>Asset Risk: Overstated Assets</td>
<td>39</td>
<td>9</td>
</tr>
<tr>
<td>Operational Risk: Significant Change in Business</td>
<td>28</td>
<td>7</td>
</tr>
<tr>
<td>Credit risk (Reinsurance Failure)</td>
<td>22</td>
<td>5</td>
</tr>
<tr>
<td>Liability Risk (Catastrophe)</td>
<td>36</td>
<td>8</td>
</tr>
<tr>
<td>Asset Risk: Impaired Affiliate</td>
<td>26</td>
<td>6</td>
</tr>
<tr>
<td>Total Identified</td>
<td>426</td>
<td>100</td>
</tr>
<tr>
<td>Miscellaneous/Unidentified</td>
<td>214</td>
<td></td>
</tr>
<tr>
<td>Total Insolvencies</td>
<td>640</td>
<td></td>
</tr>
</tbody>
</table>

Source: A.M. Best - 1999

The major cause accounting for 34% of insolvencies is attributed to deficiency in loss reserves, which in this case is directly linked to inadequate premium rates. If this finding is to go by demand curve risk then it is a major cause of insurance company failures. Demand curve risk results from deterioration in the underwriting book when an insurance company tries to increase its market share by undercutting premium rates, thereby leading at times to inadequate loss reserves. Demand curve risk is usually accompanied by rapid growth, the second greatest cause of insolvencies in this study, which accounts for 20% and usually reflects the quality of management decisions.

A.M. Best found that 38 out of the 44 insolvencies caused by alleged fraud occurred in the past 15 years. However, with better regulatory methods demanding ever increasing tighter audits and market conduct reviews, this cause can now be detected earlier. Middle of the road causes like the overstatement of assets and a
significant change in business has also contributed to the demise of 39 and 28 companies respectively. Asset risk as defined in Table (2.6) occurred when assets are reduced greater than the value of a company’s surplus or when an audit reveals a misstatement of asset values. On the other hand, operational risks are a result of a shift in the carrier’s territorial or product focus (A.M. Best 1999).

Insolvencies resulting from affiliate impairment all arose in the past 10 years, confirming that insurance companies are more likely to use the corporate veil when faced with bankruptcy in its associated companies (Cummins, Allen and Phillips 1997). Failure in reinsurance occurred mainly in the 1980s when insurers were riding the demand curve risk. It has also been observed that there is a strong correlation between the industry’s combined ratio, and reinsurance failure frequency rate. Since the combined ratio is a benchmark to the industry’s profitability, it provides the impact of various risks on the industry. Therefore, this ratio reflects the financial health of a company and the probability of insolvency.

2.4 Market for Insurance Risk

Instruments targeting underwriting risk reallocation have been developing over the years. The evolution of these instruments gives a vivid picture of how insurance practitioners have been developing their knowledge of trying to understand cash flow behaviour. The understanding of certain cash flow bundles have led to the development of each instrument, starting with reinsurance in the 19th century to the late 20th century instruments of securitisation. It gives insight as to how the developers of these instruments have sought to mitigate negative cash flow movements, given their perception and understanding of insurance cash flows at each stage. The essence
of developing instruments at each stage is to engineer cash flows that optimise return structures best understood by developers of these instruments at each stage. We can construe that each instrument developed at each stage shows us the level of understanding developers had on the underlying insurance cash flows. The level of understanding of underlying cash flows is essential to the acknowledgement of the limitations each instrument has in solving the super-cat cash flow puzzle in insurance.

**Figure 2.2: Cash Flow Engineering Instruments**

Insurance cash flows display characteristics that either provide opportunity or threaten return structures if they are not managed. If they provide opportunity then companies opt to either retain more cash flows if there are arbitrage opportunities or transfer if they sub-optimise risk-return profiles. Figure (2.2) shows stages in the risk financing process, which we believe to be essential to an efficient way of utilising equity.

Given the nature of risk ingrained in each section of the loss distribution, each method is more efficient when applied to its matching layer of the risk profile. It also
reflects the attributes of engineered cash flows and their efficiency in optimising underlying cash flows. In the diagram it is apparent that retention based techniques are more likely to be used than contingency financing and securitisation; because of the layer they occupy in the financing structure. Whilst it is important according to Lamm (1998) that the frequency of use declines as one moves from one risk tier to the next, this is to do more with efficiency and costs rather than the tier occupied.

Higher layers have low and uncertain loss probabilities of occurrence, so the use of these instruments is not economically viable if the company is small (Froot, 1998). The main reason for high cost structures in these instruments is due to the paucity of credible data, which limit the level of knowledge we have on these cash flows. Structuring risk financing entails understanding specific characteristics at each layering, because efficient risk financing structures recognise underlying cash flow features. What makes an instrument efficient in financing is its equity accumulation features matching to the level of risk at each layering. Therefore, the new paradigm in insurance risk financing is that of matching cash flow characteristics to instruments (Figure 2.3) attributes, to utilise equity available in financing insurance risks.

Given the existing knowledge on insurance cash flows behaviour our risk finance programming is designed to reallocate risk to those who have the competitive advantage to bear it, enabling those seeking to strengthen their financial position to source leveraged capital at reduced costs. Within this framework also present is the issue of replicating cash flows to create hedge positions. This measure is intended to stabilise risk portfolios through the enhancement of the cost of risk trading, lower default probability, and making contingent capital available, whilst maintaining target
financial structures. Insurance companies should satisfy the appetites of their cash flows for funds required to trade risk at lower rates than those prevailing in the alternative money market, by doing so they will be able to trade narrow their non-value adding bounds and stabilise return. We believe that risk financing should be able to release assets committed to liabilities and should reduce the cost of risk capital on sponsoring all-purpose equity.

The use of different risk financing instruments in sourcing contingent capital is mainly aimed at smoothing the cost of carry component, thereby stabilising return. For effective and optimal risk, trading an instrument should target a specific bundle of cash flows. That is why it is difficult to mitigate negative cash flows arising from catastrophic risks, through traditional insurance and reinsurance systems alone. There is need to use instruments that explain the underlying cash flows because reinsurance and retention instruments have failed to fully mitigate risk inherent in these cash flows. Retention and reinsurance based instruments do not go far enough in satisfying the risk transfer requirements of cash flows occupying the upper tails of the loss distribution.

Instruments used in hedging firm-wide risk are also categorised into three main groups according the nature of cash flows they target. The first group is composed of those instruments that target expected cash flow deviations; they are termed retention-based because the original carrier retains risk in total or in part. Retention-based techniques are efficient at financing risk in this region using the all-purpose equity supplied by shareholders or accumulated from retained earnings. The second in figure (2.3) is composed of reinsurance-based instruments like traditional reinsurance,
financial reinsurance and swaps. These are used to provide protection for cash flows occupying the unexpected loss region up to the tails of the loss distribution.

Losses in the tails of the loss distribution are best financed by equity based instruments which are the final group in figure (2.3). Equity-based instruments are bought from the financial market with the sole purpose of using diversified equity to fluff up equity levels when depleted after a cat event. They are efficient at financing the upper part of the loss distribution because capital is superficially accumulated, making it cheaper than accumulated reinsurance and retention-based equity.

Various methods of financing risk are listed in fig. (2.3) above will be analysed as to their effectiveness in stabilising cash flows. Another test they have to pass is their

\[ \text{Expected loss} = \sum_{\text{events}} \text{Loss}_i \times \text{Likelihood of loss}_i \]
ability to enhance capitalisation in the event of a catastrophe occurring. We believe risk-financing instruments should be able to bridge the gap between sound asset-to-liability ratios and the solvency threshold. The figure (2.3) represents the major instruments available to an insurance company to manage risks affecting cash flow behaviour. The use of any instrument is determined by the requirements of the underlying risky insurance cash flows, meaning all instruments ought to at least to explain some elements of these underlying cash flows. A brief description of each instrument shows how each instrument is designed to interact with the underlying cash flows, in the process delivering optimal return structures.

2.4.1 Retention-Based Techniques

This method of finance focuses on the profitability of retaining pound swapping risks, usually the high frequency and low severity occupying the expected loss layers of the loss distribution. The perception of insurance cash flows under this method is different from a reinsurance perspective in that retained cash flows are viewed as being efficiently financed by all-purpose equity. The levels of retention in all the major markets have been increasing at a faster rate than that of the overall capacity limit as shown in figure (2.4). Retaining risk within a portfolio is a healthy thing as it is essential for profit generation to assume risk commensurate to the capital supplied; it is from risk assumed that a competitive advantage is generated. In fact, insurance companies add value by underwriting risk, because the essence of being in the business of trading insurance risk is to earn a commensurate return from liability risks. Funds generated from underwriting insurance risks are a source of investment income,

\[ \text{Unexpected loss} = \sum_{\text{event} i} \text{Likelihood of loss}_i \times (\text{Loss}_i - \text{Expected loss})^2 \]
which means that the more a company originates underwriting risk the bigger would be its investment fund, but also the greater the risk.

**Figure 2.4: Average Retention and Limit Per Programme and Rate on Line**

![Graph showing average retention and limit per programme and rate on line from 1989 to 1999.](image)

The level of retention is based among other factors on the financial capacity to bear risk, that is, on the ability of sponsoring equity to explain the nature of underlying cash flows. It should be noted that the retention of risk is done when cash flows spell out that the equity available is efficient in sponsoring the risk at hand. If retention levels are not in line with cash flow characteristics; i.e. target financial structures should be satisfied and return structures be stabilised, then will the cost of excessive risk levels become apparent in cash flow behaviour. The cost of excessive or inadequate risk retention can be measured in terms of missed target cash flows, over a period of time say five years, because that’s where most risk factors are captured.
Various risk retention techniques have been developed in order to make cash flow structures more profitable and less risky when retained against a level of equity. Figure (2.4) above shows that average retention levels of insurance companies have grown at a faster rate than the overall capacity offered, irrespective of the fall in the rate on line since 1994. It shows why retention based techniques are important in engineering cash flows. They are at the core of any efficient risk financing programme as the evidence in figure (2.4) suggest, because more companies are increasing their retention levels, irrespective of the reductions in the rate on line. The decline in the rate on line propounds a cheaper cover, this has not been accompanied with increases in cessions but more is being retained. It is not only the price of cover that matters, but also the efficiency of equity in financing risks within a portfolio.

Pure retention is also a function of a firm’s capital structure. The capital structure spells the level of capital available to sponsor liabilities carried forward to settlement date and its suitability for use on risks underwritten on a day to day basis and the realisation that value can be added by retaining cash flows. Certainly, the use of all-purpose equity becomes inefficient when risks coming into a portfolio are correlated to risks already in the portfolio. We have already shown how contaminating classes could be identified using integrated risk classification and portfolio theory based risk analysis. In the same way, insurance companies can compute these statistics to identify classes correlated to the whole portfolio and decide the best risk-financing tool to use.

Retentions also serve as a disciplinary measure required by those providing risk capital on higher layers, to encourage a company to be more prudent in managing its loss portfolio. Retention is essential in any risk-trading situation, as it is a means of
reducing moral hazard and adverse selection arising from information asymmetry inherent in insurance cash flows. Traditionally reinsurance requires cedants to retain a portion of the risk portfolio before participation; it is also a requirement for securitisation to pitch a high retention in order to make catastrophe bonds tradable. Retention is also important in cost determination, as a layered portfolio is easier and cheaper to finance. Voluntary or pure retention can not easily be extended to risks in the upper tail, as equity supplied as an all-purpose instrument is not efficient to fund catastrophic risks characterising this region in insurance cash flows.

Retention-based methods can be implemented in two different ways; those that are executed within a portfolio and those that are engineered off-balance sheet under a separate financial set up. The decision to engineer retention within or outside a portfolio depends on the attributes of the risk presented the objectives of managers and the profitability of resultant cash flows. The discussion above is based on retention done within a portfolio; cash flows retained are financed by the all-purpose equity available and are viewed as a source of float. These cash flows are important to the intrinsic value of firm, retaining them enhances the value of the firm if the cost of float is lower than the cost of raising funds prevailing in alternative finance markets. The underwriting profit or loss determines the cost of float on the cash flows.

Secondly, retention-based instruments point to efficiency in financing risk through the separation of portfolios, by financing risk through a separate equity arrangement. The intention behind using this later method is to arbitrage certain cash flow components that cannot be extracted when risk is actively retained in the portfolio. Cash flow attributes which could be arbitraged are those pertaining to tax-efficiency,
financing flexibility, development of new financing products, pricing efficiency and integrating the risk financing structure. Such arrangements like captives are used to reinsure risks of the parent company in an offshore location (not necessarily always), in a way achieving an efficient equity-to-risk financing. They enable an insurer to have access to the risk reallocation market at terms that are more favourable, the key is to reduce the cost of risk financing over a period of years and ensure the availability of cover after major catastrophes. With an integrated structure in mind, it enhances the vision that an insurer has towards risk management and makes it more focused on risk management issues.

Berkshire Hathaway is retaining super-cat risks on its books, on the basis that major catastrophes are rare occurrences; they expect this business to be profitable in most of the years and occasionally to show huge losses. There is paucity of information on cat losses, which makes estimates fuzzier, the loss for a worst-case scenario for Berkshire on its California Earthquake Authority (CEA) is about $600 million, which is only 1% of the company market value. Thus, retention-based techniques can be used relative to the capital in the portfolio, as long as they are efficient in financing the risk characteristics displayed.

Portfolio diversification has its limits as it tends to a systematic risk profile that is denominated by risks present in the upper tail. Therefore, it would not be efficient for an insurer to carry a portfolio with an undiversifiable cat risk and financing it with an all-purpose equity accumulation, suitable for risks in the lower layers. In both cases retention based methods are inefficient in transferring risk embedded in these cash flows, as they do not redistribute upper tail or cat risk. Therefore, shareholders are
implicitly made to insure as they are the first in the line to suffer when a cat event occurs, implying that they will require a higher return if all-purpose equity rather than other methods of funding finance this risk. Obtaining high returns in the insurance industry has proved elusive, Figure (2.5) below shows that the property/casualty and reinsurance markets have over the past 15 years up to 1997 under-performed the Fortune 500 Index and other stock markets.

When deciding to use retention methods, target cost of capital must be considered with the benefit of arbitraging these cash flows accruing to shareholders and human equity. Excessive risk taking activity would be viewed as threatening by managers, putting pressure on the cost of capital due to missed targets emanating from management decisions. The right retention is just right for the target financial structure and should only be utilised up to the level where equity attributes indicates when inefficiency in financing is setting in.

**Figure 2.5: The Profitability of Property/Casualty and Reinsurance Markets**
2.4.2 Reinsurance Based Techniques

Reinsurance based techniques can be classified into two broad groups those with a traditional flair and those that are financial in nature. The difference is a transitional one, in that financial instruments evolved from encumbrances inherent in traditional techniques. In other words, they are a result of a better and informed understanding of insurance cash flows, not well catered for under traditional reinsurance. Traditional reinsurance will be referred to as reinsurance as in its own right its evolving from an underwriting risk focused instrument to a more integrated financial technique of managing all the risks.

Source: Guy Carpenter
2.4.2.1 Reinsurance

Reinsurance has been the main vehicle used by insurance companies to finance underwriting risk. The main forms of reinsurance are proportional reinsurance, composed of facultative, quota share and surplus share, and non-proportional reinsurance composed of facultative, per risk, catastrophe and aggregate excess of loss. Under Quota share the ceding company transfers a specified percentage of all the premiums originating from a defined portfolio of business to a reinsurer, meaning the loss is also in the same proportion to the premium received. Surplus share are characterised by cession of risks individually in the reinsurance treaty. Thus, premiums and losses are distributed between the ceding company and the reinsurer according to the ratio of the retention to the sum insured and therefore, variegate from one risk to another.

The trading of non-proportional reinsurance implies that there is no proportionality; in as far as the distribution between premiums and the portion of loss is concerned. The determining factor of this form of reinsurance is the level of loss and not the premium, since the loss should exceed retention before the reinsurer could be called upon to pay up to a certain limit. The retention may be for an individual risk, that is a loss arising from an individual risk known as working excess of loss (XL) cover, an accumulation of individual losses per event (catastrophic XL covers), an accumulation of losses or loss ratio within a portfolio known as a aggregate XL or stop loss cover. A detailed description of how these treaties operate is outside the scope of this thesis. The aim of this section is the determination of how reinsurance explains the
behaviour of insurance cash flows, as it is apparent that each form of reinsurance is tailored to alter specific cash flow attributes in the payoff structure.

The cash flow structure targeted by reinsurance is that embedded with underwriting risk, which is the risk that the actual losses paid differ from those expected due to changes in the nature of risk, stochastic nature of random events or the risk of error in the calculations. Froot (1993) pointed out that hedging is driven by the interaction between investment and financing consideration. Establishing the relation between the existing assets and cash flow behaviour is essential as pre-existing assets determines the firm’s capacity to contain risk. What then can be said of reinsurance is that it targets a section of the loss distribution below the tails that affect economic capital, profitability and buffer capital.

**Figure 2.6: Expense Ratios For US Reinsurance Companies**

![Expense Ratios For US Reinsurance Companies](image)

*Source: Guy Carpenter*
Losses determine the level of spreads paid by insurers on liabilities so they should be managed. Reinsurance as a means of managing spreads targets that section of the loss distribution that could be altered to deliver an acceptable level of risk. Reinsurance can only effectively alter the loss distribution up to the level where the cost of contingent funds is optimal as compared to other sources of financing. The problem with reinsurance is that equity in sponsoring risks is also accumulated, putting it in the same class with all-purpose equity used in funding retentions. Therefore, it is still dogged with inefficiencies encountered under retention-based techniques, because accumulating equity for an event occurring once in a hundred or thousand years is uneconomic and prohibitively expensive. Furthermore, reinsurance is traded on a piecemeal basis making it more costly than other risk financing methods.

It is evident from the graph in Figure (2.6) above that expenses in the reinsurance sector are high and have been rising dramatically in the recent years despite a galore of cost reduction mergers in the industry (the number of professional reinsurers in Guy Carpenter’s composite declined from 64 in 1992 to 38 in 1997). Such types of expenses and levels of compensation for business defy economics, those who feel comfortable with such inefficiency in pricing reinsurance contracts, should be reminded that the cost of capital is the determinant of profitability.

The combination of all risks within a portfolio brings in diversification across the exposure units; a lot of literature has been devoted to the effects of diversification within a portfolio. So if ever reinsurance is used emphasis should not only be on what it does to the loss distribution, but what the overall payoff looks like when we apply any reinsurance instrument. It is possible to simulate the overall payoff structures for
any an instrument applied. The effectiveness of any instrument should not only be limited to the local distribution but on the overall and distant distribution. The whole distribution process shows the appetite for capital, how much the instrument cost and the levels of return brought by the instrument.

2.4.2.2 Financial Reinsurance

Finite risk reinsurance represents a combination of risk transfer and risk financing techniques by emphasising on the time value of money. These instruments are used to cover underwriting and timing risks. The first part has been discussed under reinsurance above; the second part pertains to those risks resulting from erroneous expectations regarding the rapidity of loss settlement. Since loss payments may occur earlier than expected insurers are exposed to liquidity risk and also suffers loss of interest bearing capital, in the form of loss reserves. They can also be treated as a retention-based method, which deals with risky cash flows off the balance sheet, and suitable to finance losses in the middle layers of the loss distribution. The following discussion on these covers supports this fact as the underlying cash flows points to a specific equity accumulation attributes compatible with these instruments. It means that they are efficient in financing risk, if they are used in funding lower and middle layers, because these instruments can not meet the equity attributes amenable to catastrophe risk.

The multi-year nature of most finite reinsurance contracts means that the prospective and retrospective covers can be provided. Prospective contracts cover current and future business underwritten, catastrophic losses and act as a device for cost-effective covers smoothing future fluctuations of asset-to-liability ratios. On the
other hand retrospective covers in addition to providing cost-effective coverage, they enable the insurer to exploit the possibility of high investment income from long tail business.

This multi-year nature of contracts also means that greater credit risk, should be taken into account when pricing these instruments, as it is difficult to assess with precision the financial status of the counterparty. Risk and the cost effectiveness of these covers should be balanced, when taking the decision to arbitrage cash flows retained under such an arrangement. Such decisions depend on the nature of cash flows as a source of float and how these cash flows affect the financial structure, had they been managed on the balance sheet. Retaining them helps preserve float as a company is able to benefit from investment income generated and improvement in the efficiency of all-purpose equity financing. The goal at the end of the day is to reduce the cost of capital by matching cash flows to the risk-funding instrument.

Loss Portfolio Transfers (LTPs) relieve the insurer of its existing obligations to pay losses already incurred on the book, by transferring these obligations to a third party in exchange of a premium. They are retrospective in nature, in that, the transfer of liabilities is only for outstanding losses. They remove that portion of the book which is seen as unprofitable, or a threat to the stability of the portfolio. The premium ceded is approximately equivalent to the net present value of the ceded loss reserves. The reinsurer charges a profit and cost margin, for underwriting risks reflecting the timing and subsequent reserves assumed. They provide a means of managing timing risk, relating to claims settlement over time, and provide a good vehicle when the uncertainty of claim settlement pattern proves costly to the insurer. The insurer will be
able to reduce the overall volatility of the portfolio indirectly reducing return and security levels required by both shareholders and suppliers of human equity.

Since they provide a means of converting future income from investments to current underwriting income, the pressure on solvency margins is reduced. Implied discounting of the loss reserves strengthens equity and effectively increases solvency since liabilities ceded will be greater than premiums ceded, if the loss development pattern follows the estimated trend. Insurers also use portfolio transfers when withdrawing from certain lines of business or closing peripheral business to concentrate on the core business. This cash flow engineering technique have also been useful in bringing precision to planning and in facilitating mergers and acquisitions as well as controlling latent liabilities spiralling out of control (Sigma 1996/99). Equity release under this technique improves financing efficiency, reduces the cost of capital by alleviating pressure on components of the financial structure, and it makes cash flows left in the portfolio easier to manage.

Adverse Development Covers (ADCs) provide cover for losses resulting from contracts concluded in the past, they do not cede loss portfolios in this case. They provide protection against losses that have been incurred but not reported (IBNR) and for protection against inadequate loss reserves (IBNER). The premium paid reflects the scope of the underwriting risks assumed and takes into account the net present value of the loss payments expected during the term of the contract. In this case, the time value of money can be used to come up with a more cost-affective way of funding this risk. They also facilitate acquisitions or mergers of insurance companies since long latent claims can be partially protected using these covers. Considering a
company with liabilities that are difficult to assess, ADCs offer an information arbitrage to shareholders and rating agencies in that the company will trade the unknown for the known. Finite risk reinsurers at times do assume credit risk for the primary insurer in circumstances where one or more traditional reinsurers are insolvent. These covers through assumption of subsequent reserve risk also further enhance the insurer’s access to traditional excess of loss covers (Sigma, 1997/99).

Finite Quota Shares (FQSs) provide cover for business of the current and future underwriting years, by ceding a part of unearned premiums in return for commission. They are a result of a US statutory instrument (Statutory Accounting Principles) which did allow for accruals of acquisition costs as in the accounting principles, they were accounted for immediately. FAQs can correct the inter-temporal reduction in equity, by removing distortions on the balance sheet, and profit and loss account arising from volatile acquisition costs. Volatility in cost components has the impact of undermining target capital structures, reducing underwriting capacity and profitability, as well as increasing the overall capital costs. The provision in advance of the profit expected by the insurer on that business future underwriting years, help smooth results the same function performed by traditional build-up quota shares that finance acquisition costs. However, they are a source of moral hazard and adverse selection as this risk is subjective, even though there might be safeguards of liability linked sliding scale commission and specific limits on liability (Sigma, 1997/99). FAQs are essential engineering cash flows affected by disparities in acquisition costs and can only be used on cash flows displaying yearly disparities significantly affecting return on equity.
Our final risk engineering technique under this section targets uncertainty of loss distribution from year to year, by dealing with underwriting risk and balancing risk over time effectively smoothing fluctuations in the financial structure. This technique known as Spread Loss Treaty (SLT) derives from accumulation of bespoken annual premiums into an experience (loss experience) account over the whole term of the treaty (funded cover). Apart from being a reinsurance-based technique, it also has retention-based attributes in that interest income is credited to the ceding company, with the payment of losses and reinsurers’ margin being paid from the experience account. The losses incurred are distributed over a number of years, with the result in the account each year determining the level of premiums to be paid and which of the parties is liable to pay if the contract is in its last year.

The fact that the ceding company is not compelled to totally balance the experience account means that underwriting risk can be financed efficiently. The stabilising effect of the timing risk management component is similar to that unlimited equalisation reserves do to a risk portfolio (Sigma, 1997). They enable companies to adopt an underwriting policy oriented toward continuity, refocusing and concentrating on core business, efficient utilisation of equity, profitability improvement, improving certainty and utility in pursuing strategic goals, and stabilisation of the costs of financing risk by insulating the insurer from market cycles. Improvement in equity financing efficiency is the core to the success of any instrument, these instruments work well when used to reengineer specific cash flows which sub-optimise equity usage if they are allowed to remain in their original state. By targeting specific cash flows, these instruments are easily bespoken to suite the characteristics displayed by risks underlying the portfolio. Remember that the management of insurance risks is
done on the assumption of understanding the cash flow behaviour, which makes financial reinsurance special to those explaining any specific cash flow in a portfolio.

2.4.2.3 Swaps

This technique involves reciprocation of risk though an exchange, with the intention of either eliminating risky business from the portfolio or arbitraging from ceded business. They involve two or more risk-bearers assuming partial and/or reciprocal liability for a defined component of each other's risk. The early risk exchange mechanism was done in London at a Lloyd's coffee shop where traders came and posted or advertised their risks on a notice board; assessments were done on site and risk acceptance communicated by indication of proportion assumed and a signature at the bottom of the sheet. If such a mechanism were maintained we would not be talking of the revolutionary Catastrophic Risk Exchange (CATEX). Insurance market inefficiency structure was incorporated into the distribution system, making Lloyd's more of a reinsurance market than a swap market.

CATEX and CATEX Bermuda began operating in early 1996 and 1997 as a reinsurance intermediary facilitating reinsurance transactions and licensed by the New York Insurance Department and Bermuda Parliament respectively. Trading is based on an electronic trading system with subscribers gaining access to the trading system on a global scale. The exchanges allow members to buy, sell or exchange insurance risk and trade index-based insurance derivative products.
The CATEX is a world-wide system for the exchange of risk by insurers and other financial service firms on an exchange. Under the CATEX mechanism subscribers place and advertise risks they seek to exchange with other companies around the world on the exchange. Risk bearers and their brokers negotiate and complete trades through the communications systems set, and these trades are registered, published to the system and archived with CATEX. Risks are exchanged on a risk-for-premium or risk-for-risk basis with rights and obligations set forth in CATEX-traded contracts flowing only between the contracting parties. Thus, it is not a risk-bearing facility as the exchange of risk between subscribers is treated as a reinsurance transaction; existing insurance statutory accounting practices are utilised to record CATEX trades. Therefore, this market provides a platform for electronic reinsurance transactions, where subscribers swap risks by allowing exposed insurers to obtain additional financial protection antecedent to a catastrophic event. In this case, the risk bearer’s ability to propagate its risk across geographic regions, perils and business lines is enhanced.

This risk-spreading feature protects the insurer's capital and surplus in case of a major catastrophe, and reduces a company's value at risk for particular catastrophic events. CATEX facilitates flexibility in risk financing by providing an opportunity for insurers to adjust and rebalance their risk portfolios on a real-time basis in response to market forces (Sweeney 1997). This new distribution channel reduces risk distribution costs than under the piecemeal reinsurance approach, and acts as a way of obtaining coverage for risks that are difficult to place. It allows prompt and easier analysis of transactional information by company underwriters.
Capital accumulation characteristics now are similar to those under reinsurance; hence, the classification under reinsurance based techniques. Most of the participating companies are from the insurance market, meaning an undiversified insurance surplus portfolio is being used to fund these risks, making capital generated from this market suitable only for middle layers. The nature of subscribers in this market makes the funding of risk in the upper layers inefficient, because surplus generation is done through the accumulation of equity.

2.4.3 Equity Based Instruments

The main purpose of equity-based instruments is to match equity type to the nature of cash flow characteristics displayed by an insurance risk portfolio. Risk portfolios currently traded in the insurance industry behave in a way which can not be explained by a single or all the instruments in use at the moment. The enigma risk within an insurance portfolio is found in the upper tail of the loss distribution, which makes capital expensive if it is financed by the all-purpose equity supplied by shareholders. The nature of this risk is that it makes it expensive to finance by traditional techniques, when long-term risk financing and risk transfer consideration is considered.

Capital required to service risks presented to the insurance market by insureds and society is not adequate. The insurance market surplus before the WTC attacks was estimated at round US$465 billion, which is only a fraction of capital market capitalisation, with volatility levels in a single day quoted around US$75 billion. Fluctuations of this magnitude do not affect the viability of the market. A similar loss in the insurance industry will reduce the industry surplus by a significant amount,
given that competitive pricing is done in line with a normal loss year and usually conforms to cost structures of companies with the lowest possible cost structures.

The reason for contemplating losses of such a magnitude emanates from the fact that catastrophic activity have been on the increase and losses arising from a single event are ever on the increase. So in order to stabilise rates and capital availability when such events occur (see Figure 2.3); a more robust way of accumulating equity to finance the risk is needed and this is found in the capital markets. The magnitude and attributes underlying risk capital provided by capital markets in relation to spread of risk over time and incidence of risk occurring means that they have the stamina to deal with these risks effectively.

The cost of capital required to finance all catastrophic risks in exposed areas is phenomenal and providing such capital require instruments that package these risks in such a way that the return they give is equivalent to the risk they present. Figure (2.4) illustrates that both the reinsurance and the property/casualty markets have been underperforming other financial institutions, given the higher level of risk insurance companies, as compared to other financial institutions.

A cost-effective funding method is that which spreads risk between exposure and time, by appropriately matching portfolio risk characteristics with equity accumulation attributes. These equity based instruments provide features, where capital can be supplied at an affordable rate, considering that it will be required to support an event which would occur at anytime in the future even before the build up of reserves. Therefore, insurers should augment their traditional risk taking approach and expertise
with the skill set and expertise more traditionally associated with the capital markets.

2.4.3.1 Securitisation

Securitisation techniques can be classified into three broad groups, synthetic reinsurance, credit risk based and event puts. These techniques are based on risk transfer and financing principles, aimed at preserving the value of the firm post loss. Principally it is an assets hedge geared to replenish depleted assets after a catastrophic event; thus it preserves the positive net present value (NPV) cash flows embedded within an insurance risk portfolio. This method focuses on the loss distribution and its impact on the collective cash flows of an insurance company, by separating exposures that make the distribution more expensive to fund using retention and reinsurance based instruments from the overall portfolio and finances it on its own. It is essential to understand that the driving force behind the use of this instrument is the inefficiency and overall capital costs in retention or reinsurance funded risk portfolios.

The way the first technique operates is that an insurer having identified exposed cash flows repackages them, to make them tradable to potential investors in the capital markets. In the case of credit risk securitisation investors receive contingent surplus notes with the insurer having collateral accession rights to the Special Purpose Vehicle (SPV) capital and replacing it with its own surplus notes, from which investors will receive their return if the right is exercised. The funding of catastrophic risk is done by simultaneously selling cat bonds to investors, through the SPV and buying reinsurance coverage from the SPV. The SPV invests the principal received from investors in high-
grade investments, and issue reinsurance to the insurer effectively ceding its cat event risk to the investors.

Access to capital is afforded when a catastrophic loss event or an exotic pre-specified event is triggered. The SPV synthetically replaces the traditional reinsurer, with premiums and revenue receivables on the catastrophe cash flows sold to investors and proceeds from the fund only made available to the insurer on the occurrence the event. The funding arrangement runs for a specified period of time if no event occurs, paying interest, premium and the principal to the investors, a partial amount if a triggering event occurs or be wound up if the event exhausts the fund. The reinsurance is purchased in the capital markets so it is a memoryless transaction (the Markovian principles), with no judgements about past events (Froot, 1998).

Event-puts are securities issued by institutions and bought by insurance companies; they give the insurers the right to put the securities in exchange for cash if the specified events occur. These were issued by European institutions in the early 1990s to Japanese insurers; however since then they have lost their appeal.

The main advantage of securitisation is the elimination credit risk (only investors carry credit risk of the insurer if there is a collateral accession rights). It transfers event risk totally and separates the funds from any other claims, since they are only triggered by the occurrence of a specific event. On the other hand event puts transfer event risk with the bond issuer but retain credit risk with insurers as they have a right to put the bonds back at par to the issuing counter party in exchange for cash, if the event specified occur. The issuer must be liquid at the time of occurrence, due to the
uncertainty of earthquakes. Since they have a liquidity-financing component, they have been preferred to paying a line of credit, as they are also an investment (Lamm, 1998).

The effectiveness of this method anchors on the probability of loss, which is directly linked to the layer covered, higher layers have low probability of loss importing the notion of low premiums, which might not be enough to cover the costs involved in sponsoring the SPV. Therefore, there is a trade-off between effectiveness and the costs involved as institutional investors favour high grade bonds which coincidentally fall on the higher layers which have low probability of loss, as those with a higher probability of loss/low recovery levels are bound to be below investment grade. USAA in its issue managed to secure protection for lower layers by incorporating the principal protection clause, which delayed the realisation of the principal as opposed to higher layers which put the principal at risk.

The problem with principal protection is that more funds should be raised than required to transfer the risk, since the other portion is needed to finance the principal protected as it will be invested in risk free assets. This makes the cost of financing small issues prohibitive; the cost may be in excess of 150 basis points above the actuarial cost of risk depending on the size of the issue (Froot, 1998). The balancing of effectiveness and cost is central as higher layers are seen to be beyond the influence of the insurer, also its these layers that require capital amounts made available at competitive rates as bond returns reflect those prevailing in the reinsurance markets. Enigma risk in the upper-tail is ruinous, which makes this source of funding more
effective as large capital amounts can easily be made available for capital appetite satiation of these monstrous risky cash flows.

Securitisation removes catastrophic cash flows from the portfolio, by transferring a portfolio of catastrophic liabilities from its balance sheet. The impact of the transaction on the overall operating result is important, it must reduce the cost of capital associated with the cash flows in the portfolio, since the risky cash flows would be separated and transferred to a suitable and specific capital vehicle. Remaining cash flows can be funded by the all-purpose equity, because they are less risky meaning a lower rate of return is required, and more business can be written given increased capacity. The use of this method of course depends on the willingness by investors to buy these securities, the precondition of investment grade must be satisfied, which makes the role of rating agents important as investors rely on them to understand the nature of securities issued.

It also has to be pointed out that the yields of the bonds have to be high, the regulation governing such trades should be in place and the cost of producing information must be low. By 1998 the Bank of England had accepted repackaging of receivables, but it was not yet clear whether the Department of Trade and Industry (DTI) accepted this. In the other markets these instruments have been traded successfully and much can not be said about their effectiveness as the reinsurance market is still soft (as shown in figure (2.5) rate on line fell for the fifth year in succession and to below pre-hurricane Andrew levels).\textsuperscript{13}

\textsuperscript{13} The bonds cover the highest-end risk typically damage from 1-in-100-year or 1-in-200-year events, but they pay several percentage points above competing investments.
Raising external capital does not always resolve a low internal capital generation problem; usually corrective measures are required within the loss portfolio. Therefore, an understanding of insurance cash flows should be the first thing before any instrument is chosen for transfer and financing of risk; otherwise the funding process will be inefficient no matter how complex and appealing it is.

2.4.3.2 Contingent Equity

This instrument identifies the post loss leverage as the problem that has to be addressed; in as far as the impact of catastrophic losses on a portfolio is concerned. It is true that losses of unexpected proportions deplete assets, with the result of disproportionate increase in liabilities. Such increases alter the overall financial structure, impairing the ability to raise capital at competitive rates, credit rating and the capacity to write risks of certain magnitude. After the 1992 UK indemnity catastrophic losses, it was notable that Independent insurance company was one of the few that had the capital to write more business, for other big companies it was hard to trade competitively after suffering such huge losses.

It is characteristic of insurance cash flows that debt levels rise, when there are disturbances in upper tail of the distribution, exposing the company to a higher probability of ruin. What can be inferred from the use of equity put is that the decision should initially be based on the nature of the loss distribution. Thereafter, can the focus be on other factors like, the current capital structure, its relation with the risk portfolio, the target capital structure and the cost of deviation from the target financial structure. Given that post loss investments have a positive net present value, it's necessary to use
techniques that redress the balance between equity and debt, so that the potential embedded in the value of the firm may be realised.

The fact is, it is difficult and expensive to raise capital when wounded, so the best way to protect value embedded in the firm is to prearrange equity to be put at favourable rates, when a financially impairing event occur. The insurer is assured that, after a severe catastrophic event which reduces the surplus and possibly their credit standing, the company will be able to procure capital up to the agreed limits to help refinance business operations. Capital raised through equity puts is treated as equity which means it adds directly to surplus, providing a stronger balance sheet and protection at a cost that is lower than a traditional secondary equity offering and competitive when compared to a top layer catastrophe cover.

Contingent equity\textsuperscript{14} therefore, seeks to finance and transfer insurer’s losses with a pre-negotiated sale of securities linked to exchange-traded shares, at a fee for maintaining liquidity when the equity is on demand. The cost of contingency capital varies from one issuer to another, for example a reinsurer in Bermuda had a variable spread over Libor deriving from its credit rating at the time of issue, high when credit rating is low and lower when it is high. This instrument is designed to complement existing risk market trade payoffs, so it offers the insurer exceptional high layer coverage to augment their existing reinsurance/risk trade payoff protection.

\textsuperscript{14} Aon Re Inc. and Centre Reinsurance first introduced the contingent equity product to the market under the trade name CatEPuts (Catastrophic equity puts). Deals in RLI & Horace Mann and the first syndicated deal followed this with the option writers in the La Salle Re being European Re (lead investor), Allianz, Aon & CAN, the equity was in the form of convertible preferred shares.
If a catastrophic event exhausts the insurer’s traditional reinsurance coverage, the insurer will exercise the option to put contingency equity to investors, with the added advantage of retaining the ability to raise capital in alternative markets. In the case of La Salle Re a Bermuda reinsurer writing global based risks, it provided a $100m contingent equity facility following either a single catastrophic event exceeding $200m or an aggregation of $250m from smaller catastrophes, at a cost of $2.35m p.a. for three years. This deal ensures that following a major catastrophe likely to hit the whole market, La Salle will be able to spend their time most profitably underwriting risk in a hard market without having to raise capital first (Sayers et al 1998). This instrument has not been used much, as some big insurance companies believe that they can easily raise capital on-the-spot, without the expense of paying for a contingency fee.

What can be said of both arguments for and against the use of an equity put is that it all depends on what the underlying cash flows are dictating, certainly a company would not use it if it is an expensive way of financing, given alternatives available. A thorough understanding of the underlying cash flows is therefore essential, because what equity puts does is just pointing at leverage deficiency given the state of cash flows under strenuous conditions, and how to address this deficiency with a replenishing strategy.

This might not be the best strategy for those desiring to transferring risk totally, as this strategy only transfers it to shareholders, the use of equity to finance a catastrophic risk in the long run will be more expensive. Maybe that is why those who designed this instrument applied the pecking order theory where it is only used when other cheaper instruments are exhausted. This point to the fact that it works well in a blended state
than as a stand-alone instrument, it explains some parts of the cash flow structure but not the whole cash flow process. It works well in part; therefore, it should be applied on those unique cash flow attributes, which derive from the interaction between loss distributions and the overall financial structure.

2.4.3.3 Derivatives

Options traded in insurance transactions are usually call options, as they display similar characteristic to excess of loss reinsurance risk transfer techniques. Call options in insurance risk trading locution are securities purchased by the insurer (investor/seller) giving them the right (obligation), but not the obligation to receive (pay) funds from a seller (to an insurer) if the index value exceeds a specified level during the exercise period. The specified level is known as the attachment point, where cover starts to operate by virtue of exercising the right, when the index is in the money.

Insurance option trading is currently done at Chicago Board of Trade (CBOT)\(^\text{15}\), with the underlying trigger being the Property Claims Services (PCS) index. In the case of PCS options, an investor will sell this right to an insurer at a premium, with the insurer receiving cash payment equal to the settlement value of the PCS index, which is above the strike price. Call option spreads entail the simultaneous purchase and sale of two or more option contracts, buying one at a lower strike price and selling the other at a higher price.

\(^{15}\) The Bermuda Commodities Exchange which was trading cat-options using the Guy Carpenter Catastrophe Index (GCCI), ceased trading in 1999.
The trading of contracts on an index brings a number of risks to the insurer, namely basis risk, credit risk, model and timing risk. These risks vary from one insurer to another and should be addressed if the intended purpose of the hedge is to be satisfied. Credit risk under reinsurance is reduced by evaluating creditworthiness and regulatory requirements for collateral balances from unauthorised counter-parties, it is minimal when trading options on the exchange, as exchanges have a system of securing recoveries from counter-parties trading on the exchange. The other risk is that the models used in evaluating the effectiveness of these contracts might not accurately predict the results of the company or the index. This risk underlies a company’s capacity to determine the expected recovery and distribution of recovery from an option trade, as models do not always fit complex real life scenarios.

Timing risk comes into play when there are errors in the reporting periods and the timing of the cash settlement of the derivative between the event and the settlement of the option. It also has to do with delays in receipt of funds to pay claims between the date of the catastrophe and the settlement date of the derivative. The option maybe liquidated if the market is liquid, with the effect of further exposure to other risks or funds may be borrowed until the value of the option is realised.

A crucial risk is that of basis, which arises when a company’s recoveries from the catastrophic index contract may be different from its own account’s experience from catastrophe losses. Such failure by a hedge replicate exactly the losses accruing to the insured constitute what is called "basis-risk". Every company is not similar as corporate policy affects how much risk is assumed, underwriting ethics, portfolio composition and claims management, resulting in company’s loss experience differing
from an associate index. Indices should be able to incorporate company experiences in order for hedging to be economic, so their data composition and breadth are key factors to be looked at when determining hedge effectiveness. The GCCI index’s granularity was different from PCS index in that it was broad-based, and closely resembled placing companies’ risk portfolios. When the nature and intensity of the cat event being covered are broader, event risk is more devastating than localised, hence a lower basis risk.

A company should also be able to precisely determine changes that will happen in the composition of the book, as changes that happen during the underwriting period imply a change in exposure base underlying the hedge. This reduces the effectiveness of a hedge, as there will be a mismatch between the exposure and the hedge. Insurers always want their loss experience to correlate with the index, as is with the case with some other tailor-made contracts like reinsurance. Another way of improving on hedge effectiveness will be to look at the detail in the index and try to match it with the company’s experience or to use various strategies to improve on recovery without actually reducing the basis risk. In this case, recovery on the option is improved by shifting the distribution of recovery such that the probability of loss is reduced and the probability of gain is improved.

The correlation coefficient has been traditionally used to measure the effectiveness of a hedge. It is considered in practice that the level of correlation decrease as you move for reinsurance based instruments that have a correlation coefficient of 1 for quota shares, to options that have a high degree of basis risk. A study by the American Academy of Actuaries (2000), pointed out that since correlation
is a statistic between two variables, it does not consider relative magnitudes of movements between the two variables, i.e. it is not always a one-to-one basis. In order to address this phenomenon, they used a framework that defines the risks to be hedged, identifies the suitable index-based index structure and the causal relationship between exposure to be hedged and the underlying index, in mathematically determining the effectiveness of a hedge. In defining risks it is essential to consider only those transferred by the hedge, outlining the business covered, territory, retentions and limits, perils, underwriting classifications and exposure period of losses. This will eliminate overlaps in coverage and establish a strong base for monitoring and measurement of effectiveness.

The effectiveness in the hedge instrument will mean delivery of the desired equity accumulation components that brings in a well-diversified capital base. Upper layers need capital that can spread super-cat risk without any strain on the underlying surplus. The separation of risk depicted in figure (2.3), points to the suitability of options to the financing of risk in the upper tail, as the cost structure of equity supplied to fund these risks is different to that used for all-purpose equity financing. Only an effective hedge can deliver unique elements required by upper layer cash flows, a hedge with gaps puts pressure on the all-purpose equity, making the overall cost of capital higher. If an effective hedge could be obtained for a portfolio, insurance companies can deliver a required return that is commensurate with the risk that is embedded in their portfolios.

The failure of the reinsurance market to deliver a commensurate return on equity is due to the nature of risks assumed in the upper layers and more costly in terms of capital requirements. Risk in the reinsurance sector is high, but the price needed for
trading that risk in the insurance market declines as we move from lower and less risky layers to higher risky layers. Thus, with the margins squeezed on insurance risk traded in this market, the new markets taking up this risk have exposed the inefficiencies in reinsurance pricing.

Efficiency in equity funding risks is needed in the upper tail cash flows, because volatility brought about by activity in the upper tail is viewed as undesirable by management, by adopting strategies aimed at protecting their human equity they sub-optimise cash flows in the process. It has also been noted that these pressures on human equity and all-purpose equity will make the stakeholders demand a high return on their equity, making the cost of capital high, lowering the value creation bounds (Mutenga and Dinenis - 1999). That is why it is important to obtain an effective hedge with equity accumulation characteristics that explain well the behaviour of cash flows in the super-cat tails.

Risks in this area are violent, so handling them requires a thorough understanding of what their next move will be and the Academy of Actuaries’ findings are a step forward in finding the best instruments that managing these cash flows by explaining underlying behaviour. We believe that this is the only way forward for improving the return on equity supplied to arbitrage insurance risk trading, because the best way to manage risk embedded in cash flows is by explaining them. This is what is needed in using these latest techniques; we should be so bound to reinsurance methodology in coming up with these instruments, as this has hampered development in the trading of somewhat efficient risk financing techniques.
2.4.4 Blended Risk Financing Techniques

The fusion of risk financing instruments offers a great deal of advantages to insurers. It helps insurers take their risk profiles more realistically, flexibly and strictly account for capital costs in an insurance company’s portfolio. It is their impact on the portfolio’s cost of capital that we are interested in, as they should be able to explain underlying cash flows. It has been established above that no single instrument explains the behaviour of an insurance risk portfolio, so blending techniques discussed above will not only improve profitability but also removes unnecessary pressure on equity.

It can be conferred that the use of certain instruments enables insurance companies to manage their target returns with greater intuition. The fact here is that at each stage instruments used to correct post-loss capital structures have failed to deliver the desired structures that optimise shareholder value. These deviations occur irrespective of the use of reinsurance and other financial reinsurance techniques. What is the cost of such deviations from targets given that implemented techniques fail to deliver and the continual threat of a specific risk? This risk has been identified as costly to finance using the all-purpose equity, firstly, large amounts must be amassed in order to reduce the risk level, secondly, amassed funds are costly to maintain, may be misused and inefficient given the already low returns in the industry. It should be noted that such a method would make sense in as far as explaining insurance cash flow is concerned, but what we are interested in, is what constitutes the delivery of optimal returns, making the selection of the right instruments essential.

The problem with insurance traders is that they have been bewitched by their over-reliance on reinsurance such that they are so much focused on it as the centre
point for development of new instruments. They have to distance themselves from reinsurance first before deciding on any new solution on the problem areas in insurance cash flows. Even though the level of capitalisation in the reinsurance industry in the US in 1998 stood at a premium to surplus ratio of 0.67, meaning more capital is supporting premium written; catastrophic risk still poses problems. The trends from 1989 have been against the grain as the once upheld standard for regulatory attention was 3 units of premium to a unit of surplus, but since then it has been below the 0.89.

If the industry is overcapitalised in as far as premiums underwritten to surplus is concerned, why then is the industry undercapitalised when it comes to risks faced on the upper tail? Froot (1998) pointed out the distinctness of catastrophic risk, that it is expensive to fund when it is included in the portfolio, since those who will be supplying funds will require compensation for the extra risk. Therefore, the way forward for this problem is to extricate upper-tail risk from the whole portfolio and market it in a different market where adverse movements in these cash flows can effectively be hedged.

The problem with reinsurance, captive reinsurance and financial reinsurance is that it does not totally transfer risk, credit risk means the company will have to pick up losses when a counter-party defaults and capitalisation in the market is not suitable for risks embedded in the upper-tail of the distribution. As shown below, the capacity utilised in the six markets vary, the US uses a large chunk of reinsurance capacity available in the private sector which is not even enough to cover a single catastrophic event hitting Miami. This point to the fact that reinsurance is a misfit financing tool.
given the capital requirements inherent in the upper-tail of insurance loss distributions, reinsurance is suitable and efficient to a certain level of exposure.

Table 2.7: Property Cat Reinsurance Capacity Limit For the Private Sector From Ground Up (FGU) - 1999

<table>
<thead>
<tr>
<th>Country</th>
<th>Limit (in Billion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>$27.0</td>
</tr>
<tr>
<td>Japan</td>
<td>$12.5</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>$10.5</td>
</tr>
<tr>
<td>Canada</td>
<td>$6.0</td>
</tr>
<tr>
<td>France</td>
<td>$4.5</td>
</tr>
<tr>
<td>Australia</td>
<td>$4.5</td>
</tr>
</tbody>
</table>

Source: Guy Carpenter – Data for Private Sector only.

There are risks within insurance cash flows that are suitable for certain instruments and these should be matched in order to reduce the overall cost of capital. The overall capital cost in the industry is of concern, there is need to reduce it in order to make the sector more competitive and in line with other sectors on the Fortune 500. The use of the all-purpose equity and reinsurance-based instruments has bedevilled the industry with the problem of under-funding due to the use of misfit instruments in financing insurance upper-tail risks.

Certainly there must be a way of holistically funding risk, but our problem is why the industry places so much faith on instruments that have proved misfits in the past in order to engineer new instruments. There is need to look at the cash flows define them over the overall risk trading process, in order to identify the peculiarities within each cash flow component. It is true that investors should only be compensated for systematic risk. Certainly, the systematic risk should not be catastrophic risk in the portfolio, since reinsurance based instruments are inefficient there are instruments that
can remove this risk from the portfolio, and make insurance return structures more efficient.

2.5 Conclusion

The goal of any risk consumption exercise in insurance is to maximise the long-term risk-adjusted return to shareholders, subject to a risk profile. Thus, some balance has to be stricken between optimal return and the level of risk accepted. Excessive assumption might over-stretch the financial capability of an insurance company, whilst an over-cautious stance on risk might result in loss of revenue and assumption of a sub-optimal position. What is that optimal position, which enhances shareholder value without exposing the company to the risk of insolvency? What is the cost of financial distress to an insurance company? Such a position is achieved if the price of risk factors faced by a firm is set at an arbitrage free rate. This is the subject of the remaining chapters of this thesis.

This study gives an insight into the way that risk is arbitrated through the engineering of insurance cash flows. The engineering of insurance cash flows is a process, which starts from the identification of risk levels and distinct distributions at these levels within a loss distribution. Risk segmentation is a virtue as it allows an insurance company to match specific risk attributes with equity that is efficient in financing risk at that specific layer. Segmented risk is amenable to instruments that explain the behaviour of cash flows, without these explanatory elements an instrument is bound to leave gaps in the cover exposing equity to unnecessary risk levels.
When equity is exposed to high level of risk those supplying financial and human equity, also require colossal risk premiums. Managers supplying human equity increase the cost of capital by sub-optimising cash lows, rating agencies’ ratings will also reflect the level of risk premium commensurate with risk levels in the portfolio, and those supplying equity will require a fair return. The inefficient use of equity has been identified as the underlying cause of under-performance of the insurance industry against peer industries. The realisation that risk capital consumption in insurance cash flows is different led to the development of different risk hedge instruments through out the centuries spanning from the 19th century. These developments emanated from a greater understanding of insurance risk cash flow behaviour, with the resulting effect of the creation of hedge instruments with explanatory qualities matching the underlying risk embedded in the cash flows. Therefore, inefficiencies embedded in cash flows meant that the cost of equity is directly linked to their attributes, engineering specific hedge instruments is the answer to this inefficiency.

This study applies these hedge instruments to portfolios using retention-based techniques in varying proportions to the underlying all-purpose equity, efficiency of instruments is seen to be largely dependent on the efficiency of equity in financing retained risks. It has also been established that in the cash flow engineering process, risk is arbitraged by exploiting equity efficiencies at each layer in the loss distribution. The effectiveness of a programme is measured by the extent to which a payoff stays over and above the knockout barrier and the gradients of the upside and downside payoffs. The target of every insurance company is to attain stability; cash flow engineering provides the solution as long as risk segments are matched to cash flows with amenable equity accumulation structures.
CHAPTER 3:

MEASUREMENT OF RISK AND COST STRUCTURE MANAGEMENT

3.1 Introduction

Insurance companies like banks borrow funds from policyholders which they use to support their investment activities. Policies sold to policyholders are promises upon occurrence of a contingent event, insured in the contract. Policyholders pay premiums in return for the securities issued to them by insurance companies. These features of an insurance contract are similar those of zero coupon bonds, where the promised payment is made at the end of the period. The value of the promise to pay at the end of the period depends on the financial strength of the company and volatility of insured liabilities. Volatility in insurance asset values is a function of time, liability and asset growth rates, as the time of payment and payout ratios are not known in advance, making an insurance contract a risky debt.

Investment funds raised by selling securities to policyholders require insurance companies to commit shareholder funds as risk capital. This is done to assure policyholders that claims will be met and as a cushion to the funds raised. Risk is
contained in contingent event products sold by insurance companies and exposure of insurance portfolios to hazard losses, investments and risk financing tools. The uniqueness of risk characteristics are specific to contract type, the terms in the contract document and has payoff profiles similar to those embedded in options. These variations bring with them obligations that must be met by capital commitments from shareholders and the cost of float paid by policyholders.

3.2 Motivation and Theoretical Background

The economic methodology on pricing insurance liabilities before Cummins (1988) is based either on the probability of ruin, the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT) or discounted cash flow techniques, Ferrari (1967), Bigger and Kahane (1978), Fairley (1979), D'arcy (1983), Witt and Urrutia (1983), Derrig (1985) and Doherty and Garven (1986). Those that use the weighted average cost of capital techniques did not have a precise method of calculating the cost of debt at the beginning of the period, Venezian (1988), Urrita (1987), Myers and Cohn (1981 & 87), and Garven and D’Arcy (1991). In order to come up with a fair price for liabilities, the cost of equity needed to fund the origination of liabilities and credit risk inherent in the originating firm need to be taken into account.

The building blocks of any risk measurement model is its strength in describing any risk trading position by making reference to the underlying cash flows. It should be able to explain the relationship between liabilities and equity required backing the origination of the liabilities. The EPD is an option pricing methodology derived by Bustic (1994) for measuring the cost of insolvency in insurance companies. This
method has been herald as a closed form solution to measuring the level of capital at risk, since it does not only account for the probability of default but also the severity of default. In fact, it is a better measure of the level of capital required by insurance companies than the Value at Risk (VaR) and ruin probability methodology. However, the EPD methodology assumes that default occurs at the end of the period, which is not true for insurance companies. In practice, default in insurance companies occurs at any time during the life of the company, especially when large losses with very low probabilities of occurrence, exceed the going concern value of the firm.

We, however, start by defining the ownership structure of an insurance company using as basic model given by the following equation (3.1):

\[ V_t = E_t + L_t \]  

Where:  
- \( V_t \) = Value of the firm's assets,  
- \( E_t \) = Equity,  
- \( L_t \) = Liabilities/debt at par value.

The ownership structure in equation (3.1) above tells us that the value of the firm depends on the relationship between equity and liabilities. The assumption underlying the financial structure is that, at time \( t=0 \), an insurance company issues two types of securities; namely a homogeneous liability with cash flow characteristics similar to those of a zero coupon bond and a residual claim in the form of equity. Equity is provided for by shareholders at a cost commensurate to the level of risk in the business being traded, with a value \( E_t \) at time \((t = T)\). Liabilities, the other form of financing in our equation represents money due to policyholders in the form of technical reserves.
They have a face value of \( L \), with a holding period of \( (t = T) \), and they have a value of \( L_t \) derived at time \( t \). Therefore, total asset values \( (V_t) \) at time \( t \), are financed at the beginning of the holding period by equity \( (E) \) and liabilities \( (L) \). Time is defined in the interval; \( 0 \leq t \leq T \), with the boundary conditions true at time \( (t = T) \), since both stakeholders have an option on the value of the firm that is expected to be exercised at expiry date. Therefore, the total value of the firm \( (V_t) \) is independent of the capital structure, financing and investment decisions, which means that the Modigliani-Miller (1958) theorem holds, as cash outflows are financed by new securities. It can also be construed that risk management in either form is irrelevant, and that management risk manipulation will not add economic value, if the M&M assumptions holds.

Since the face value of liabilities \( (L) \) is the strike price on the option to pay on the occurrence of an insured contingent event, value recovered depends on whether the insurance company is solvent or insolvent. The valuation of corporate assets is done to establish the financial state of the firm, as this is necessary for the determination of the payoffs for each stakeholder in the firm. In order to value the assets, we need to establish the asset processes they follow. We base our essential assumptions on Merton (1974) and the subsequent application of his methodology by Cummins (1988) on insurance cash flows. The firm's assets are assumed to be lognormally distributed, and are governed by a risk neutral probability \( Q \)-martingale, given by the following stochastic differential equations

\[
dV_t = r_t V_t dt + \sigma_V V_t dW_t
\]

(3.2)

Where: \( r_t = \) instantaneous interest rate (risk free rate);
\( \sigma_V \) & \( \sigma_L \) = Diffusion parameters for assets and liabilities;
$W_v \text{ and } W_L = \text{Brownian increments under the risk-adjusted probability measure } Q, \text{ normally distributed with variance } dt, \text{ and correlated, denoted } \rho.\

In the above dynamics, both processes of the drift and randomness are scaled with $V_t$ and $L_t$, by multiplying these through out the equations. Markets are assumed complete and frictionless, no taxation, no transaction costs, and trading is continuous as envisaged under Black-Scholes (1973) and Merton (1973).

The advantage of using Merton’s methodology in pricing insurance liabilities is that it takes into account the effect of limited liability vested with shareholders. The fact is that shareholders have limited liability, as they have an option to default if things go pear shaped. This option depends on the assessed value of assets at the end of the holding period ($T$), which determines whether the firm is solvent or insolvent. If the company is solvent, then the firm will honour its promises to policyholders in full, that is, the guaranteed face value of liabilities ($L$). In this case, the assets would have generated enough value to match guaranteed liabilities ($V_t \geq L$). If the company is insolvent, that is the assets have failed to generate enough to match the face value of liabilities $L$, this implies that $V_t \leq L$. In this case, policyholders will not be paid their promised amount, but the residual value of the firm. The total value policyholders expect to receive at time ($t = T$) is defined by the following identity:

$$L_t = \begin{cases} V_t & \text{if } V_t < L \\ L & \text{else.} \end{cases}$$

3.3
The level of payment expected by shareholders also depends on the financial state of the company. Whilst, shareholders enjoy the upside risk, their limited liability means they take the residual liability and have the following payoffs:

\[ E_t = \begin{cases} V_t - L & \text{if} \quad V_t > L \\ 0 & \text{else.} \end{cases} \]

The realization of these payoffs is dependent on the probability of reaching the critical asset values that could lead to default. We know that the value risky equity and liabilities, is closely linked to the critical asset levels \(V_t\), which should not be exceeded by liabilities if default is to be averted. This makes default a function of the financial structure of the firm, i.e. the leverage ratio given below as follows:

\[ l_t = \frac{L e^{-r t}}{V_0} \]

Where: \( l_t \) = leverage ratio,

\( L e^{-r t} \) = Present value of liabilities at matuity

(liabilities are not risk-adjusted),

\( V_0 \) = Present value of the firm's assets,

\( r \) = risk-free interest rate.

In the above equation the ratio of liabilities to assets, directly affect the probability of default, which mean that the financial structure is an important factor in the measuring firm-level risk. The assumption is that equity and liabilities are the only securities financing the assets, and that no further cash is raised if the \( L > V_T \). Therefore, the relativity between asset and liability is an important indicator of the level of credit risk inherent in the firm. This credit risk affects the amount due to policyholders, as defined by the probability of exceedence \( \text{Prob}(V_T < L) > 0 \).
The cost of originating liabilities like any other debt is linked to the credit risk of the company. Credit risk depends on the level of risk capital invested by shareholders. Thus, whilst the cost of liabilities depends on the level of capital, the cost of equity is determined by the riskiness of liabilities. Therefore, the overall cost of capital to the firm is dependent on the financial structure of the firm. This is because risky liabilities determine the level of capital required, in order to maintain the value of assets above the critical level. The overall cost of capital to an insurance company, is therefore the weighted average cost of equity and liabilities.

This assertion points to the fact that return and financing of an insurance company’s business are dependent on its financial structure. The cost of shareholder funds is measured by the idiosyncratic risk embedded in the business the company is underwriting. In other words, it is the risk premium paid over and above the risk free rate as a reward to shareholders for supplying funds required to originate liabilities. The basic assumption in the standard Merton (1974) option pricing methodology is that equity is the only source of funding for liabilities. As we have already seen in Chapter 2, equity is not efficient in financing the whole risk profile of an insurance company, other forms of funding come in to improve efficiency and enhance the return profile. Therefore, the usage of the term equity in this thesis implies that other forms of risk financing are already incorporated, unless we specifically state the distinction. This means that the cost of equity is equal to the weighted average cost of all risk financing used in funding the origination of investment funds. A weighted average cost of equity is a better measure of the overall cost of hedging default than just considering shareholders’ funds.
It is apparent from the payoffs above that both equityholders and policyholders have contingent claims on the value of the firm. If we retrace our steps, we can rewrite equation (3.1) using the put-call-parity formula, as a simple option-pricing model.

\[ V_t = C(V_t, L) + [L e^{-rT} - D(V_t, L)] \]  

(3.6)

Where: 

- \( C(V_t, L) \) = call option on assets (V), with exercise price of L, and time to maturity T.
- \( D(V_t, L) \) = insolvency put option on assets (V), with exercise price of L, and time to maturity T.

These are European options with the following payoff with an exercise price of L at maturity (t = T):

\[ C(V, L) = \text{Max}[0, V_T - L] \]  

(3.7)

\[ D(V, L) = \text{Max}[0, L - V_T] \]  

(3.8)

The value of the firm is divided between the shareholders (call option) and the policyholders (the second bracket - representing the risk-free debt minus the value of the insolvency put). The option pricing model for insurance companies tells us that shareholders have the right to receive the residual value of the company at the expiration date by virtue of the call option they have on the company's assets. This option is exercised if \( V_t \geq L \), in which the residual value (V-L) is received, the owners default if \( V_t < L \), giving up the firm's assets to policyholders.

The second bracket represents discounted liabilities, with the first part defining a long position on risk-less liabilities and a short position on a put to default. The default option discounts liabilities, because policyholders might not be able to receive the full
amount of liabilities if the company is insolvent. We call these bankruptcy costs, which are costs incurred after the firm is delivered to policyholders. These costs are borne by the policyholders, since the claim of shareholders on the firm expire worthless. The payoff for liabilities which capture the effect of limited liability is priced as follows:

\[
L_t = L e^{-r^*} - D(V_t, L, \tau) \quad (3.9)
\]

This relationship points to the fact that the value of debt in the firm devalues when the probability of default increases. The cost of insolvency to policyholders is defined by the value of the put option in equation (3.9), which is rewritten in equation (3.10) using the Black-Scholes option-pricing model.

\[
D(V_t, L) = -V_N(-d_1) + L e^{-r^*} V_N(-d_2) \quad (3.10)
\]

Where:

\[
d_1 = \frac{\ln \left( \frac{V_t}{L} \right) + \left( \frac{r_p + \sigma^2}{2} \right) \tau}{\sigma \sqrt{\tau}} \quad (3.11)
\]

\[
d_2 = d_1 - \sigma \sqrt{\tau} \quad (3.12)
\]

\[N(.) = \text{Standard normal distribution}\]

This defines the value of policyholder claims in the company as being the asset values multiplied by the probability of a shortfall if \(V_t \leq L\), and the present value of liabilities multiplied by the probability of full recovery. The value of the put option represents the premium required for insurance against insolvency risk. The model depicts factors that determine the firm's insolvency risk and its costs. Therefore, the
overall cost of insolvency risk is a function of the holding period, the leverage ratio, risk free interest rate and asset volatility (\(\sigma\)). An insurance company can reduce its insolvency risk by reducing asset volatility and the leverage ratio. Variations in the risk free rate of return also affect the cost of default, with a decrease in interest rate having an opposite effect on insolvency risk.

If it is true that the put to default is enticing, then shareholders would prefer portfolios that are risky to that are more stable. This is due to the fact that, they benefit from keeping the upside risk and transferring the downside risk to policyholders if the firm is bankrupt \((V_t \leq L)\). However, in practice, shareholders rarely benefit from this asymmetric scenario of enjoying the upside, whilst walking away from the company when things go pear-shaped. This is because such payoffs come at a cost, which is reflected in the cost of borrowing funds from policyholders and raising external capital. Remember that the premium required to originate liabilities is directly linked to the perceived insolvency risk in the insurance company and certainly will be reflected in the cost of capital. This has impact an on the portfolio’s Sharpe ratios, due to the effect of non-systematic risk, which will be higher given a level of excess return per unit risk. Therefore, management will be obliged to maximise shareholder value, by earning a higher margin on investments over and above its marginal cost of capital, by reducing risk inherent in a portfolio. Competition in the market, the need to continue creating value for shareholders and the costs embedded in regulation deters value arrogation activities.

Whilst Merton’s model captures the impact of leverage, it does not explicitly account for agency costs. It implicitly accounts for the limited liability option vested in
the equityholders, which is equal to the value of the put option equity holders have on
the value of the firm’s assets (V), at a strike price L, at maturity. The value of the
default option vested with the equityholders is equivalent to the cost of hedging risky
liabilities. There are other factors that affect the cost of insolvency besides the leverage
ratio, interest rate, and the holding period. Volatility in both liability and asset
accounts and the correlation between these accounts also affect the cost of default and
ultimately the weighted average cost of capital. Therefore, in the next sections we
extend this model, in order to capture the unique features of insurance cash flows like,
the interactions between assets and liabilities in a multi-line, multi-asset company, the
liability growth rates and solvency margins. We address these issues in turn under each
of the following sections.

3.2.1 Adjusting for Inherent Insurance Cash Flow Risk

In this section we will price contingent claims of an insurance company by
extending standard Mertonian option pricing model to take into account the effect of
liability payout ratios. This mirrors the relationship established above where liabilities
and assets interact in creating value, but we relax the condition that liabilities are
constant. Insurance companies in practise are faced with variable payout ratios, hence
the need to readjust the risk-free rate used above. From the previous section, we define
the dynamics governing returns on both asset and liability accounts by the following
differential equations:

\[
\begin{align*}
dV_t &= (r_f + \pi_v) V_t dt + \sigma_v V_t dW_v \\
dL_t &= (r_L + \pi_l) L_t dt + \sigma_L L_t dW_L
\end{align*}
\]

Where: \( r_L = \) payout ratio of liabilities (inflation rate).
\[ \pi_V = \text{the market risk premium for assets.} \]

\[ \pi_L = \text{the market risk premium for liabilities.} \]

An Intertemporal Capital Asset Pricing Model (ICAPM)\(^{16}\) prices assets and liability portfolios above applied in Cummins (1988 & 1991), Cummins and Danzon (1997), and Cummins, Allen & Phillips (1998) is used in defining the expected return for the drift parameters. The rate at which liabilities are discounted is based on actuarial principles, due to uncertainty of insurance cash flows. The first consideration would be the payout ratio for liabilities, which account for the overall cost of liabilities. We compute the payout ratio as follows:

\[ r_L = -\ln \frac{L_0}{L_t} \quad (3.15) \]

Where:  
- \( L_0 = \text{The present value of liabilities,} \)
- \( L_t = \text{The face value of liabilities at time T.} \)

This is used as a positive expected risk load that should be added to take into consideration uncertainty associated with liabilities. If the value of liabilities (\( L_0 \)) at time zero is less than the present value of risk-adjusted liabilities (\( L e^{-r_T} \)), then the cost of liabilities is less than the risk free rate. According to Cummins (1988), the dynamics are generalised in such a way that the economy’s inflation rates are accounted for in the risk-adjusted rate \( r_p \), since insurance inflation rate tends to grow faster than that of the economy. This tenet seems to deviate from financial principles.

---

\(^{16}\) The market risk premium \( \pi \) is defined by the following equation:

\[ \pi = \rho_{jm} \left( \frac{\sigma_j}{\sigma_m} \right) (r_m - r_r) \]

Where: \( r_m \) and \( \sigma_m \) = the drift and diffusion parameters of the Brownian motion processes for the market respectively.
which state that the discount rate should only be fixed above the risk free rate when systematic risk is expected. The reason for using the actuarial principle is for us to take into account the stochastic nature of insurance liabilities, which makes the cash flows costly as far as risk taking, is concerned.

There are also financial reasons for us using the risk-adjusted rate; though systematic risk might not be revealed in the traditional underwriting beta which rewards a firm for bearing systematic risk. Campbell and Mei (1993) gave the main reason why we have to discount at a positive risk load. The reason pertains to the nature of long tail liabilities which might not display any correlation features with economic variables, whilst concealed in the underwriting cash flows is systematic risk. They identified the sources of systematic risk by decomposing the CAPM beta into three broad categories, the insurance underwriting beta, the economy-wide beta and the company specific beta. Their result shows that the correlation between a company’s cash flows and market returns are not the primary determinant of the firm’s equity beta. Cornell (1999) reported similar results to those recorded by Campbell and Mei, with average betas for classes of assets tested significantly different from zero over the entire period of the study.

\[ \rho_m = \text{the instantaneous correlation coefficient between the Brownian motion process, the market portfolio and the assets.} \]

\[ 17 \beta_{L,m} = \beta_{cfL,m} - \beta_{r,m} - \beta_{eL,m} \]

Where: \( \beta_{cfL,m} \) = CAPM beta arising from the correlation between the company’s cash flows and market returns (insurance underwriting beta).

\( \beta_{r,m} \) = Company specific beta resulting from the correlation between the expected excess return for the company’s stock and future expected excess market returns.

\( \beta_{eL,m} \) = Economy wide beta, resulting from correlated innovations in future realisations of the short-term real-rate of interest and future expected excess market returns.
Therefore, the existence of systematic risk means that underwriting cash flows have to be discounted at a rate below the risk free rate of interest in order to produce the positive risk load, \( r_p \). By discounting at a risk adjusted rate of return we account for any systematic risk contained in the cash flows, and the market imperfections which makes risk costly to bear. Therefore, the risk adjustment to the discount rate has the effect of mathematically eliminating risk so that the risk adjusted portfolio rate of return that results is the riskless rate. Failure to achieve this desired riskless state means that further risk remains in the portfolio, implying an incomplete risk adjustment process.

In this rate is also reflected the level of risk transferred to the reinsurers and how much it will cost to finance it. Thus, depending on the level of volatility in the portfolio reinsurance companies will also demand compensation from the insurance company in the form of a risk adjusted discount rate that is below the risk free rate. This entails that the magnitude of the risk-adjusted discount rate below the risk-free rate depends on the surplus to liability ratio, which spells the level of equity beta.

On the same vein, Bingham (2000) argued that fair premiums in a state where liability betas are always negative entail the readjustment of equity betas and the cost of capital in line with changes in leverage. This is done to reduce the unrealistic burden imposed on insurance pricing. Therefore, what our model gives is a contract price computation, which incorporates not only the loss payout ratio but also the investment yield, market risk premium and the leverage. The achievement of target returns given low levels of leverage, should either be met by increases in premiums, or reduced market risk premiums or cost of capital. What all this means is that total return
with risk-adjustments must equal the risk-free rate; otherwise without such adjustments it must equal the target cost of capital-based return.

3.2.2 Modelling Risk Factors for Multi-line Insurance Companies

Insurance companies are not mono-line liability companies but they write a number of lines generating cash flows with diverse liability attributes. We consider a company that does not only sell insurance contracts in a number of lines, but also invests generated income in a number of assets.

What we have not done though is the specification of the dynamics of each class of business and asset account, as these certainly follow a different dynamic process. This enables us to develop a model that capture the portfolio effects of diversification on capital allocation and insurance contract pricing, which is constructed on the principle of transfer pricing between liability and asset accounts. In order to simplify our model the offsetting risk-taking incentive of guarantee funds is ignored. This makes our model applicable to all insurance companies with different capitalisation levels. In developing this model, we maintain the assumption assets and liabilities accounts follow the Wiener processes (Cummins, 1988), defined by

\[
dV = (r_f + \pi_{AI}) A_i dt + \sigma_{A_i} A_i dW \\
dL_i = (r_L + \pi_{LI}) L_i dt + \sigma_{L_i} L_i dW
\]

(3.16) (3.17)

\(A_i\) and \(L_i\) are the values of the \(i^{th}\) asset and the \(i^{th}\) liability class, \(i = 1, ..., d\) and \(r + \pi_A\), \(r_L + \pi_L\), \(\sigma_{A_i}\), \(\sigma_{L_i}\) are the drift and volatility of the asset and liabilities respectively, and \(dW_{AI}\) and \(dW_{LI}\) are the increments of the Weiner process. As before
we considered $dW$ as a random number drawn from the normal distribution with mean zero and standard deviation $\sqrt{dt}$ so that:

$$E(dW_i) = 0 \quad \text{and} \quad E(dW_i^2) = dt \quad (3.18a,b)$$

These Brownian increments for assets and liabilities, $dW_i$, $dW_j$, $dW_l$, and $dW_l$ are instantaneously correlated with the following dynamics:

$$dW_{A_i}dW_{A_j} = \rho_{A_iA_j}, \quad dW_{l_i}dW_{l_j} = \rho_{l_il_l}, \quad \text{and} \quad dW_{A_i}dW_{l_i} = \rho_{A_iL_i} \quad (3.19a-c)$$

In this case $i \neq j$.

We also believe that investment funds should be split between surplus and premium accounts, as done by previous researchers on this same subject, Doherty and Gavern (1986), Cummins (1988 & 1991) and Cummins, Allen and Phillips (1998). Investment funds are defined as the initial capital required originating the business plus the premiums generated on the capital committed up-front. This approach ties capital to liabilities; since originated liabilities are seen as loans with varying durations loaned to the asset accounts at a fee. It makes sense to view the interaction between assets and liabilities and the risk capital attaching thereto, as an incremental charge incorporated in each contract. The reason for this lies in the fact that a marginal approach to capital allocation will allocate all the capital available, 100% to each line of business (Myers and Read, 1999). Certainly, this supports our main theorem that asset accounts are equal to the liabilities originated plus the capital allocated to each line of business.

Our method of viewing insurance business as that of borrowing from the policyholders and then lending to asset accounts is similar to the way banks classify
their business, depending on the type of liabilities. In this thesis we are not going to follow the traditional approach to classifying insurance business based the perils insured under each contract like property, casualty or marine. We base our classification as pointed out in Chapter (2), on the duration and convexity of liabilities originated. This classification brings in line insurance classification with that of other financial institutions, in that origination is linked to investment decisions. The present classification of business does not offer us this important element in capital allocation and the pricing of insurance contracts. Therefore, by making assets liability specific the issue of duration and convexity risk is resolved, this makes its specification and management easier. Thus, the proposed classification of a line of business based on the time it takes to settle a liability brings into play the impact liabilities have on the investment policy.

We divide liabilities into three broad groups in this thesis, short-term liabilities, medium-term liabilities and long-term liabilities, depending on their duration and convexity. Originated funds in liability accounts are assumed borrowed to assets account with similar duration and convexity structures, through a transfer pricing system as proposed by Cummins (2000). This transfer pricing system depends on how much an insurance company pays to originate the business, since it has a bearing on how much the assets accounts should earn in order to sustain this system.

The rate at which insurance companies pay to policyholders in order to originate business is defined as the cost of float. It is the interest rate paid by liability accounts to policyholders for the business underwritten in respect to the underlying perils in each contract. These generated funds when lend to asset accounts are expected to earn
a rate of interest reflecting the duration embedded in the funds, irrespective of the underlying perils insured. Liability accounts are expected to underwrite risk at interest rates below those envisaged in the financial markets for cash flows with similar duration and convexity characteristics. If the cost of origination is higher than return generated from the asset accounts, shareholder value will be destroyed and the reverse is true, with a qualification on the condition that the benchmark cost of equity is met.

In order to manipulate multi-liability and multi-asset random variables, a multidimensional version of Ito's Lemma is used to value the option of the firm, \( V_i(A_i, L_i, t) \):

\[
\begin{align*}
\frac{dV}{dt} &= \left( \frac{1}{2} \sum_{i,j} \sigma_i \sigma_j \rho_{ij} A_i A_j \frac{d^2 V}{dA_i dA_j} + \frac{1}{2} \sum_{i,j} \sigma_i \sigma_j \rho_{ij} L_i L_j \frac{d^2 V}{dL_i dL_j} + \frac{d^2 V}{dA_i dL_i} \right) dt \\
&+ \sum_{i=1}^{n} \frac{dV}{dA_i} \frac{dA_i}{dt} + \sum_{i=1}^{n} \frac{dV}{dL_i} \frac{dL_i}{dt}
\end{align*}
\]

\[ (3.2) \]

We use the hedging argument, by assuming that assets are available that could provide a perfect dynamic hedge, within the same time interval, \( T-t \). We set a portfolio of assets and liabilities options on an insurance trading portfolio and short a number of assets and liabilities.

\[
\begin{align*}
\Pi &= V(A_i, L_i, t) - \sum_{i=1}^{n} A_i - \sum_{i=1}^{n} L_i \\
\frac{d\Pi}{dA_i} &= \left( \frac{dV}{dA_i} - \Delta_{A_i} \right) + \sum_{i=1}^{n} \left( \frac{dV}{dL_i} - \Delta_{L_i} \right) dL_i
\end{align*}
\]

\[ (3.21) \]

\[ (3.22) \]
Let:
\[ \Delta A_i = \frac{dV}{dA_i} \quad \text{and} \quad \Delta L_i = \frac{dV}{dL_i} \]

Now for each \( i \), it is implies that the portfolio is hedged and is risk free. Setting return equal to risk free date, we arrive at:

\[
\frac{dV}{dt} + \frac{1}{2} \sum_{i=1}^{d} \sum_{j=1}^{d} \sigma_{ij} \rho_{ij} A_i A_j \frac{d^2V}{dA_i dA_j} + \frac{1}{2} \sum_{i=1}^{d} \sum_{j=1}^{d} \sigma_{ij} \rho_{ij} L_i L_j \frac{d^2V}{dL_i dL_j} + \frac{d}{dL_i} \sum_{i=1}^{d} \sigma A_i \rho A_i L_i \frac{d^2V}{dA_i dL_i} + \frac{d}{dL_i} \sum_{i=1}^{d} \sigma A_i \rho A_i L_i \frac{d^2V}{dA_i dL_i} + \frac{d}{dL_i} \sum_{i=1}^{d} \sigma A_i \rho A_i L_i \frac{d^2V}{dA_i dL_i}
\]

\[
= \frac{d}{dA_i} \sum_{i=1}^{d} \sigma A_i \rho A_i L_i dt - r_f \sum_{i=1}^{d} \sigma A_i \rho A_i L_i dt + r_f V = 0
\]  

(3.23)

In our equation, \( \rho_{ij} \) and \( \rho_{AL} \), are correlation coefficients between the \( i^{th} \) and \( j^{th} \) assets and liability returns. We can also calculate the covariance, which is denoted \( \sigma^{ij} \) and \( \sigma_{AL} \) for returns on \( i \) assets and lines of liabilities. Suppose we have five liability lines and five assets in our portfolio, the following covariance matrix \( \Sigma \) will represent \( \sigma^{2} \) below.

\[
\Sigma = \begin{pmatrix}
\sigma_{A_1}^2 & \sigma_{A_1 A_2} & \sigma_{A_1 A_3} & \sigma_{L_1 A_1} & \sigma_{L_2 A_1} & \sigma_{L_3 A_1} \\
\sigma_{A_1 A_2} & \sigma_{A_2}^2 & \sigma_{A_2 A_3} & \sigma_{L_1 A_2} & \sigma_{L_2 A_2} & \sigma_{L_3 A_2} \\
\sigma_{A_1 A_3} & \sigma_{A_2 A_3} & \sigma_{A_3}^2 & \sigma_{L_1 A_3} & \sigma_{L_2 A_3} & \sigma_{L_3 A_3} \\
\sigma_{L_1 A_1} & \sigma_{L_1 A_2} & \sigma_{L_1 A_3} & \sigma_{L_1}^2 & \sigma_{L_2 L_1} & \sigma_{L_3 L_1} \\
\sigma_{L_2 A_1} & \sigma_{L_2 A_2} & \sigma_{L_2 A_3} & \sigma_{L_2 L_1} & \sigma_{L_2}^2 & \sigma_{L_3 L_2} \\
\sigma_{L_3 A_1} & \sigma_{L_3 A_2} & \sigma_{L_3 A_3} & \sigma_{L_3 L_1} & \sigma_{L_3 L_2} & \sigma_{L_3}^2
\end{pmatrix}
\]  

(3.24)

Note: \( \sigma_{A_i} = \sigma_{A_j} \); \( \sigma_{L_i} = \sigma_{L_j} \), \( i \neq j \) and \( \sigma_{AL_1} = \sigma_{L_1 A_i} \)

This covariance matrix is important in as far as assessing the level of risk and risk capital required for a portfolio.Whilst correlation between lines of business and assets
gives a company the benefits of internal portfolio diversification, there is need to extend our assessment to how individual lines of business correlate with the whole portfolio ($\rho_{LI}$). Capital allocation and overall risk taking behaviour within a company depend on how each line of business correlates with the overall portfolio. This means an extension to the definition of risk per line of business, which is based not only on the volatility parameter of the line and the correlation between the lines' liabilities but also the entire insurer's portfolio of liabilities and assets, Merton and Perold (1993) and Myers & Read (1999).

The EPD and Cummins (1988) methodologies are opaque in their computation of default in that, they fail to link clearly the pricing formula to the state of the firm. Under these methodologies, the probability of default remains generally positive during the life of the company. In practice, the probability of default is low for firms that have survived through the underwriting cycles. In this thesis, we develop a model that takes into account the price of early default and the influence of regulators in both the cost and probability of default. This methodology brings us closer to the actual behaviour of insurance cash flows as driven by the cost of compliance.

3.3 Modelling for the Economic Significance of Insolvency

Current literature on the on the pricing of liabilities though insightful, is however, limited in capturing the actual dynamics of an insurance company, when subjected to the cost of compliance. Most of the literature assumes that liabilities are not guaranteed, which is difficult to justify in practice given the role regulators play in protecting policyholders. The role which regulators play is similar to the objective achieved by covenants in corporate bonds, which gives the bondholders the right to
bankrupt the company if its asset values fall below a pre-specified solvency threshold. This feature in insurance companies resembles the characteristics of barrier options, which knocks out equityholders’ option on the company’s assets, if asset values go down and reach the pre-specified insolvency threshold. The pre-condition to risk trading which triggers insolvency upon being breached is enshrined in the minimum solvency margin requirements.

Minimum solvency margins are a safety mechanism that gives regulators the right to intervene in the company’s operations, force reorganisation or liquidate the company if its performance fails to match the threshold specified. The threshold at which insolvency will occur or the outstrike price of assets follow that of Black and Cox (1976), Cummins (1991), and Longstaff and Schwartz (1995), with the value of $H(t)$. This constitutes the barrier which need not be breached, if the company is to continue with its operations. This means that the value of insurance assets are path-dependent, in that the payoff is dependent on the realised asset path, which triggers certain parts of the contract if the asset price becomes too low.

This structural barrier determined by the regulators defines the economic meaning of the insolvency-causing event. It determines the policyholder’s payoff upon bankruptcy. This barrier when breached invokes action from the authorities to suspend operations to limit the dissipation of assets. In other words, default occurs the first time when the value of assets is lower than the stochastic barrier. Liabilities upon achievement of this outstrike price of assets; it is assumed that all other liability classes are simultaneously defaulted.
The outstrike price of assets is set as a pre-condition to underwriting a specified amount of liabilities, upon which asset values should not go below during the life of the option. If they go below the specified asset values, the regulators will take over the company for the policyholders. In order to avoid the eventuality of a knockout, the insurance company should continue meeting their contractual obligations to all policyholders irrespective of the class of business. If regulators intervene at such a point, the option of equityholders on the firm’s assets is extinguished and they will receive nothing from their investments.

The fact that regulators intervene before liabilities are greater than assets ($L>V$) means that equityholders should give up the company before they have recouped the residual assets earmarked for this cushion for policyholders. Early intervention takes us a step closer to how insurance companies behave in practice, given a higher exit price than envisaged under the perfect market scenario of the standard firm Black Scholes model elaborated above. This makes insurance cash flows unique from cash flows of ordinary firms, because they have to give up the company even before the face value of the assets is not yet equal to the value of liabilities.

### 3.3.1 Firm-level Risk Measurement Model for Insurance Companies

We will derive the valuation expression for risky liabilities in this section and examine their implications on risk premiums paid by insurance companies in order to underwrite business. In this section we will price contingent claims on an insurance company by using the extended option pricing model, also used by authors specified above on insurance portfolios and corporate bonds. The model used defines value in a
multi-line insurance risk portfolio as $V(A_i, L_i, r, t)$. This mirrors the relationship established above where liabilities and assets interact in creating value.

In order to capture solvency threshold $H(t)$, over which default will be triggered we relax the assumptions made by Doherty and Gervem (1986), Cummins (1988, 1991), Cummins and Danzon (1996), Cummins and Sommer (1996) and Cummins, Allen and Phillips (1997) that liabilities are not guaranteed. Let $V_u$ denote the value of assets, if the value remain above the solvency margin $H(u)$, and the time period $t \leq u \leq T$. The value of the option $V_d(A_i, L_i, t)$ is equal to the standard option for insurance companies denoted by $V(A_i, L_i, t)$, because it retains the same characteristics if it is not knocked out during the holding period. We assume that cash outflows are financed first by premiums and thereafter by equity, which has a residual claim, and affords limited liability to its owners. Default is triggered simultaneously for all policies issued the first time the value of assets reach the critical level, $H(t)$. The risk-adjusted dynamics of $H(t)$, assuming that the barrier corresponds to the value of liabilities of the firm, is modelled using the following diffusion:

$$dH(t) = (r_f + \pi_h)H(t)dt + \sigma_{hv}H(t)dW_v(t) \quad (3.25)$$

Where $\sigma_{hv}$ is a positive constant and $\pi_h$ adjusting risk for barrier is equivalent to $r_L$ the payout ratio to policyholders, used by Cummins (1988). The relationship between the knockout barrier and liabilities as envisaged in equation (3.25) means that uncertainty in this variable is also directly related to asset values of the firm. Therefore, this solvency threshold is stochastic as is evident for differing solvency
requirements between big and small companies under the European Union (EU) solvency requirements.

We have made mention of the barrier denoted by $H(t)$ above, it defines the policyholder’s payoff upon bankruptcy. This is an exogenously determined constant, specified as follows:

$$H = \alpha L e^{-r t}$$  
(3.26)

Where: $0 \leq \alpha \leq 1$

The value for $\alpha$ is specified as zero in Cummins’s model, which defines a situation, where policyholders have no guarantees at all. On the other polar liabilities are totally guaranteed and riskless. This is attainable under a scenario when $\alpha$ is equal to 1, but in this thesis we adopt the intermediate case where $\alpha$ varies between 16% and 23% for European companies and variable depending on the required risk-based capital for USA insurance companies. It is apparent that the insolvency barrier is stochastic because it is discounted at a risk free rate net of the growth in liabilities up to maturity rate.

The cost associated with the level at which the barrier is fixed is reflected in the credit spreads of the company, as it applies to each case. A breach of the barrier ($V_t \leq H$) forcing reorganisation or bankruptcy is as a means of allocating a fraction of the exogenously determined assets of the insurance company to various classes of liabilities, assuming application of the strict priority rule. When asset values go down, reaching the barrier denoted by $H$, regulators take over the company and the call
option of equityholders on the asset values of the firm will become worthless. The boundary conditions for the value of equity are given as follows:

\[ E = \begin{cases} 0 & \text{if } V_t \leq H \\ V_t & \text{else.} \end{cases} \]  \hspace{1cm} (3.27)

Where: \( E \) = the down-and-out call option.
\( V_t \) = Asset terminal price.
\( H \) = Barrier

It is apparent that the payoff of a down-and-out call option on insurance asset values is zero when, \( V_t = aL e^{-rT} \). It can also be pointed out that irrespective of the method of restructuring an insurance company, policyholders will swap their original claim in a distressed company for a set of new assets. Furthermore, if default occurs we assume that the strict priority rule is observed, because regulators rather than the policyholders will take over the firm and in the process determine what is due to each debtholder. This mean that there is no possibility of equityholders recovering value in the company and all liabilities are considered to have equal bargaining power and there is no priority over settling another. This is a sensible assumption because most insurance companies do not use senior debt to finance their liabilities. In other words, policyholders receive an exogenously specified fraction of the remaining assets; asset values will usually be lower after take over by regulators than it would be the case if the company had remained in the hands of equityholders. Regulatory company take-over reduces liquidity, which tends to dissipate asset values, a phenomenon well known in practice because most of the companies placed in administration rarely survive and become operational again.
Let us assume that in case of default before maturity, and a reorganization of the company, policyholders will be paid a fixed value of \( 1 - w \) multiplied by the face value of the liability at maturity. The factor \( w \) defines the proportion received by policyholders if there is a reorganization of the insurance company during the holding period of the liability. Longstaff and Schwartz (1995) pointed out that value of \( w \) is constrained by the adding-up constraint which stipulates that the total settlement of liability claims cannot exceed \( H(t) \), and its an exogenous constant. This will mean a certainty equivalent payoff at maturity of

\[
1 - w I_{t < T}
\]

(3.28)

Where: \( I \) = indicator function taking the value of one when the barrier is breached and zero otherwise.

\( t \) = First passage time of \( V(u) \) through \( H(u) \)

The first time passage of \( V(u) \) through the barrier \( H(u) \), is defined as follows:

\[
\gamma = \inf \left\{ u \geq t, V(u) = H(u) = aLe^{-rT} \right\} = \inf \left\{ u \geq t, I(u) = \log V(u) - \log H(u) = 0 \right\}
\]

(3.29)

The value of equity and liabilities depends not only on the value of \( V(t) \) and \( Le^{-rt} \), through the solvency ratio \( Le^{-rt} / V(t) \), denoted \( l_e \) but also by the relationship between \( H(t) \) and \( V(t) \), through the early insolvency ratio \( V(t)/H(t) \), denoted \( q_t \). Therefore, the payoff of each stakeholder on the value of an insurance company can be derived directly from both the solvency and the early insolvency ratios, without having to specify \( V(t) \) and \( H(t) \). In this context, \( l_e \) and \( q_t \) can be seen as a proxy to the measure of default risk of an insurance company.
The value of liabilities and equity at maturity is given by

\[ L_T = \alpha L \cdot I_{\gamma < T} + L \cdot I_{\gamma \geq T, V_T \geq L} + V_T \cdot I_{\gamma > T, V_T < L} \]  
(3.30)

and,

\[ E_T = (V_T - L) \cdot I_{\gamma \geq T, V_T \geq L} \]  
(3.31)

respectively.

The first part to equation (3.30) defines a scenario where default occurs prior to maturity, that is, policyholders receive an amount equal to the externally determined solvency threshold and nothing more. The second and third parts to this equation represent the payoff if there is no default during the holding period, but at time \( T \). In other words the value of the firm is always above the solvency threshold \( H(u) \) and insolvency occurs at maturity. The whole equation is a summary of the cash flow at maturity given all these possibilities, of first passage time during the holding period.

Using the risk-neutral pricing technique, the price of liabilities as at time \( t \), is given by the discounted value of future expected cash flows under the risk-neutral probability \( Q \):

\[ L_t = E_Q \left[ e^{-r_T \cdot (\alpha L \cdot I_{\gamma < T} + L \cdot I_{\gamma \geq T, V_T \geq L} + V_T \cdot I_{\gamma > T, V_T < L})} \right] \]  
(3.32)

This equation on the value of liabilities collapses to the following closed form solution, by using the methodology of the change of numeraires and time change:

\[ L_t = e^{-r_T} \left[ 1 - D_E (l_t) + \frac{q_t}{l_t} D_E (q_t) \right] \]  
(3.33)

Where:
The second and third parts of equation (3.33) are European put options priced at time \( t \), with maturity at time \( T \), and standard normal distributions given as follows:

\[
D_E(l_t) = -\frac{1}{l_t} N(-d_1) + N(-d_2) \tag{3.34}
\]
\[
D_E(q_t) = -\frac{q_t^2}{l_t} N(-d_3) + N(-d_4) \tag{3.35}
\]

\[
d_1 = \frac{-\ln l_t + \left( r_p + \sigma^2 / 2 \right) \tau}{\sigma \sqrt{\tau}} = d_2 + \sigma \sqrt{\tau} \tag{3.36a; b}
\]
\[
d_3 = \frac{-\ln q_t + \left( r_p + \sigma^2 / 2 \right) \tau}{\sigma \sqrt{\tau}} = d_4 + \sigma \sqrt{\tau} \tag{3.36a; b}
\]

Equation (3.33) is the closed form solution for the value of liabilities within an insurance company, constituted of three possible positions that could be taken by policyholders. The first part to the equation is equivalent to risk free liabilities, which is attained under the circumstances of \( q_t = l_t \) and \( \alpha = 1 \). Under such a scenario, default is forced when the value of assets is equal to the present value of liabilities discounted at the risk free rate (the term multiplied to the bracket). The term \( l_t \) is an unbiased estimate of the real asset-liability-ratio under the Q economy. The term \( q_t \) is the ratio
of the current insolvency barrier to the current value of the company, and this ratio is known as the early insolvency ratio.

The option pricing model for insurance companies tells us that shareholders have the right to receive the residual value of the company at the expiration date by virtue of the call option they have on the company's value. This option is exercised if $V_t > L_t$, in which they receive $V_t - L_t$; they default either when $V_t < L_t$ at expiry or when the threshold is breached ($V_t < H_t$). The default option discounts debt, because the value of equity increases with the value of default as this reduces the value of liabilities, but early default option increases the value of debt.

The default put is enticing if there are no guarantees to policyholders, making shareholders prefer portfolios that are risky due to the benefits reaped from keeping the upside risk and transfer of the downside risk to policyholders. However, such a payoff comes at a cost, which will be reflected in the cost of borrowing funds from policyholders and raising external capital. This has an impact on the portfolio's Sharpe ratios, due to the effect of non-systematic risk which will be higher given a level of excess return per unit risk. Therefore, management will be obliged to maximise shareholder value, by earning a higher margin on investments over and above its marginal cost of capital, by reducing risk inherent in a portfolio.

This equation (3.33) confirms the point we put forth in equation (3.1) that the value of the firm is divided between the shareholders (call option) and the policyholders (the second bracket - representing the risk-free debt minus the value of the insolvency put - incorporating the value of an option to an early default). The
value of policyholders' or debtholders' claim on the firm is represented by the relationship in the big bracket. This relationship tells that the value of debt in the firm devalues when the probability of default increases, but the existence of a guarantee increases the value of debt. This implies that an increase in the level of risk within a company is bound to reduce the value of equity. Shareholders rarely benefit from any asymmetric scenario existing when guarantees are absent, where they gain from the upside and walk away from the company when things go pear-shaped. Competition in the market, the need to continue creating value for shareholders and the costs embedded in regulation deters value arrogation activities.

When an insurance company fails to meet the criterion stipulated by the regulators on the liabilities underwritten, or if a company fails to meet its obligations, default ensues. At this point the barrier $H(t)$ is equal to the value of the firm $V_t$, which mean that default occurs when $q_t$ is equal to unit. The second part to equation (3.33) represents a standard put-to-default at maturity as given by Cummins (1988 & 1991). The final cash flow in the equation is as result conditional upon the possibility of early default being triggered by premature forced insolvency, and represents a long position on a European put. It palliates the effects of the traditional Mertonian and Cummins put to default, due to the possibility of earning an early default.

3.3.2 Risk Premium Computation on Insurance Cash Flows

In this section we are going to develop theorems based on the cost structure of the firm, which can be tested from empirical data, using the option pricing theory in insurance firms based on the extended version of our option pricing model. Option pricing theory in insurance literature have been used to value assets and liabilities of

The contribution which we are making by using option-pricing methodology is spelt out in the theorems outlined below, and the subsequent use of empirical data in testing these theorems. Our model is an extension to previous work on option pricing of insurance company by Cummins, whose empirical work is based on the Mertonian option pricing methodology.

Our work is parallel to that of Black and Cox (1976), Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995), and Briys and De Verene (1997) in that we introduce the impact of solvency margins on risk premiums, pricing, overall risk of a portfolio and the total cost of risk management. By taking into consideration the impact of solvency margins and the stochastic nature of insurance liabilities, our approach is not only consistent with financial literature in Myers and Read (1999), Merton and Perold (1993), Campbell and Mei (1993), Cornell (1999), Fama and French (1993, 1996, 1997), MacKinlay and Pastor (1999), and Stambaugh (1999) but also actuarial in Daykin, Pentikainen and Pesonen (1994). This means our prices will be close to those observed in practice. This leads us to making our first proposition on performance measurement within a portfolio trading insurance risk.

**Proposition 1:**
The risk premium as measured by default risk is inversely related to the cost of carry of an insurance company and the exogenously determined insolvency threshold.
This is consistent with previous literature in that the fortunes of stakeholders on assets of an insurance company are primarily determined by the cost of solvency requirements, Black and Cox (1976), Cummins (1991), and Longstaff and Schwartz (1995). This cost arises from the cost of up-front capital required by regulators in order for the company to write business, with the guarantee that it will honour its promises to debtholders and remain solvent.

Solvency margins are set over and above the breakeven point of assets and liabilities as a percentage of liabilities, meaning liabilities need not exceed assets before regulators come in and intervene in the running of the company. The margin which is set at 18% means that equityholders will have to surrender the company to the debtholders before the market value of assets is equal to the market value of liabilities. This has the effect of reducing the value of equity and increasing the value of debt within an insurance risk-trading portfolio.

The methodology we developed above tells us a different story, of the desire by equityholders to increase risk within a portfolio in order for them to increase the value of equity. What we know is that equityholders are interested in value creation than value arrogation, because the only way they can recoup their investment is through value that is created from underwriting risks. Thus, risks carried by investors of capital are rewarded by the creation of value through underwriting insurance risks. The methodology showing the dynamics of an insurance company subject to constrains of statutory margins is developed to show how it affects the behaviour of price of risk in insurance companies.
We adopted the methodology used in pricing risky corporate bonds by Briys and De Verene (1997), Black and Cox (1976), and Longstaff and Schwartz (1995), to compute risk premiums paid by insurance companies to policyholders when originating business. This methodology is used because it computes credit risk embedded in insurance cash flows more efficiently, by taking into consideration the impact of default on the price of liabilities. The advantage which this method has over those used in the past to price insurance risk is that risk premiums are computed as a function of credit risk, portfolio risk ($\sigma_t$), liability risk ($\sigma_L$) and asset risk ($\sigma_A$). Let risk premium be denoted by $\pi_T$ or YSPREAD (in Chapter 4). The cost of liabilities is derived, which is computed by the difference between the price of liabilities maturing in time $T$, and the price of an equivalent risk-free liabilities which have a face value of 0. Remember that we derived the cost of liabilities is given by

$$\pi_T = r_L - r_f = -\frac{1}{T} \ln \left[ 1 - D_e(l_1, 1) + D_e(q_t, 1) \right]$$

(3.37)

Where: $\pi_T$ = default or policyholders' deficit spread for a standard default (SDSPRE) or for an early default spread (EDSPRE).

$$r_L = -\ln \frac{L_e^{-r_T} - D_e}{L_t} \frac{1}{T}$$

(3.38)

This equation tells us that the insolvency-triggering mechanism is directly related to the payoff received by policyholders when early bankruptcy is forced upon the insurance company. It has been elaborated above that the three parts of this equation represent all solvency scenarios of an insurance company; a standard put-to default at maturity, and the conditional possibility of early default being triggered by premature
forced insolvency. This makes our equation more incorporative of all the possible scenarios an insurance company operates under, bringing us a step closer to reality without unnecessary complexities. Therefore, the level at which the barrier is set is paramount to the risk taking activity within an insurance company, because under this asset pricing model volatility is bad for equityholders.

The level at which the barrier is set is inversely related to the value of equity. In other words as the barrier level is increased, the value of debt in the company increases as this reduces the value of the insolvency put. By increasing the solvency margin, the barrier is brought closer to the face value of assets, this increases the probability of breaching the barrier knocking-out the claim of shareholders on the company’s assets. Whilst this increases the probability of full recovery by debtholders/policyholders; it increases the probability of equityholders forfeiting their investment to debtholders.

3.3.3 Regulatory Regime and the Price of Insurance contracts

In this section, we propose the theorem on the effects of regulatory regimes on the cost of liabilities. We believe that regulatory regimes impose external costs on the price of liabilities, since they act as a form of structural guarantee against default. The theorem given below is a building to the fair price of insurance liabilities; we extend it in the next section to incorporate the cost of equity and other contingent risk financing methods.
Proposition II:

The price of liabilities is an increasing function of the default-risk variable \( l_i \) and the barrier-to-asset value ratio \( q_i \), as policyholders' surplus is inversely related to risk and cost of compliance.

This proposition is consistent with financial literature as given by Black and Cox (1976), Longstaff and Schwartz (1995), Briys and De Varenne (1997) and Klein and Inglis (1999). This body of literature points out that the levels of \( l_i \) dictates the leverage in a company, which is negatively related to risk premiums required to underwrite a unit of business. Thus, lower values of the asset-to-liability ratio imply that the value of the firm is closer to default threshold, meaning higher discounts for default risk, the reverse is true with high values of \( l_i \). On the other hand, if the volatility of assets is higher irrespective of a high value of \( l_i \), default risk increases making liabilities more risky.

Insurance companies hold excess capital over and above that required by regulators in order to reduce the impact of volatility on the value of equity. An increase in asset value volatility means that there is a greater possibility of breaching the knock-out barrier set by the regulators, which spell a reduction in the value of equity and lower premiums policyholders are prepared to pay. This is consistent with the theory envisaged under barrier options in that volatility in assets values tends to increase when the value of assets moves close to the statutory margin barrier, so that equityholders will desire to keep as far away from the knock-out barrier as possible.
It evident from equation (3.33) that the value of liabilities depends on the asset-liability-ratio, which can be interpreted as an instrumental variable for the credit rating of the insurance company (Briys, Bellalah, Mai and De Varenne – 1998). The default measure $I_n$ according to Ogden (1987) explains about 78% of the variations in agency ratings of corporate bonds, which makes this theorem central to how risk is managed within an insurance risk trading portfolio. The default-risk measure certainly affects the level of risk premium discounting required by policyholders in order for them to place their business with the company, as well as the structure of risk financing programmes. We are also able to explain the survival instincts by insurance companies, which are encapsulated by their desire to keep as far away from the knockout barrier as possible. The reason why insurance companies tend to hold excess capital than necessary is governed by their perception of the risk and the difficulties associated with hedging extreme values around the barrier.

3.3.4 Modelling the Cost of Financing Contingent Liabilities Origination

The intrinsic cost of trading risk is the Default put an insurance company is bound to face the moment it enters the business of trading risk. The intrinsic cost is looked at as the starting point of analysing insurance company profitability, since the various cost components are then used to inflate this distribution into a more encompassing distribution. The theory that underlies our analogy is based on the fact that the price of risk at $(t = 0)$ is equivalent to the intrinsic value. We further assert that, given that risk is to be carried to settlement date $t$, the price at time zero should incorporate the cost of capital required to service this risk to settlement date. It is from this insight that we can segregate the intrinsic and time value components, in any insurance pricing contract. This distinction is important since insurance pricing currently concentrate more on the
intrinsic value than the time value element, culminating to the under-pricing and failure of insurance companies to manage risks for the policyholders.

Value creation over a holding period depends on the ability of management to capture costs associated with these components into the pricing of individual risks. These two components determine the level of capital required supporting a risk portfolio and its cost during the holding period. It is also apparent that these two key components should be targeted and controlled within an insurance company in order to stabilise pay-off profiles. The intrinsic and time value components vary from one risk holding period to another. That’s why it is important to establish what the key drivers are as far as their impact on economic value volatility is concerned. This is done in order to effectively control risk in a portfolio by targeting volatility at its very source.

It is within company cash flow patterns that risk can be pinpointed, controlled and new pay-off profiles engineered to alternatively and effectively finance risk. It has been observed that portfolios using integrated risk management systems are more stable in their earnings and capital structure, with the efficacy of reducing the cost of debt, and consequently the overall cost of capital (Doherty, 1997). To secure such a position risks should be measured and financed in an integrated manner, by striking a balance between the cost of capital and profitability.

Capital needed to carry a claim forward is referred to as risk capital. Merton and Perold (1993) defined risk capital, as the smallest amount that can be invested to insure the value of an insurance company’s net assets against a loss in value, relative to the risk-free investment of those net assets. They argued that given fixed liabilities,
riskiness in net assets is similar to riskiness of gross assets and they both require the same risk capital. Capital required for internal control to support risk assumed could be defined by any of these methods; probability of ruin, policyholder deficit, Value at Risk, cost of float, cost of carry, or as the standard deviation (See Meyers, 1999). These methods derive from the intrinsic value component; a more comprehensive model is the one that take into consideration of the economic significance of default risk, which lead us to the following proposition.

**Proposition III:**
The cost of risk capital factored in liability contracts is a function of both default risk and hedge instruments used.

Capital is required to support risks assumed by a company and is derived from premiums, equity and leveraged through engineered risk financing payoffs. It is a condition for entering the market to comply with minimum solvency and this cost is embedded in the intrinsic/fair price of every contract. The cost of minimal security capital should first be factored in the fair price of a contract as propounded by our model above. This is concordant with antecedent literature by Cummins (1991), Kim, Ramaswamy and Sundaresan (1993), Briys and De Varenne (1997) and Klein and Inglis (1999), who emphasised the importance of incorporating the cost of compliance when pricing contingent liabilities.

It has been observed under this methodology that the prices computed are close to prices observed in practice than those computed using vanilla option pricing methods by Merton (1973), Cummins (1988) and Cummins, Allen and Phillips (1998). There is
no empirical literature on the pricing of insurance contracts that specifically isolates the cost of compliance as a primary constituent of a fair price, except for the proposal by Cummins (1991) on which he stressed the need for further research on the subject. Other literature quoted above is mainly based on the pricing of corporate bonds, which have striking similarities with insurance companies’ contingent liabilities.

**Condition I:**
The price of risk is equal to the intrinsic cost (credit spread) plus the cost of capital required carrying risk forward to claim settlement date.

This is consistent with antecedent literature by Cummins (1991), Kim, Ramaswamy and Sundaresan (1993), Briys and De Varenne (1997) and Klein and Inglis (1999), who emphasised the importance of incorporating the cost of default when pricing contingent liabilities. Cummins, Allen and Phillips (1998) pointed out that liability prices computed under the option pricing methodology are close to prices observed in practice. Irrespective of the fact that they were using vanilla option pricing methods (Merton (1973), Cummins (1988) and Sommer (19996)) in deriving their prices, this methodology capture risks that have not been captured by the Myers and Cohen (1987) and other financial economics pricing methodology.

Under the risk measurement model developed above insurance companies are obliged to keep their asset values above the knock-out barrier. This capital is a cost to the company because regulators will come in and take control of the firm even when assets are still greater than liabilities; that is why, the cost of risk capital should be accounted for in the fair price of insurance contracts. The cost exists ex ante, and every
company bears it in the business of trading insurance risk, which makes it an intrinsic part of any portfolio where insurance risk is assumed.

The definition of a fair price is all encompassing in that not only did we incorporate underwriting risk but also other risks, and of importance, the incorporation of idiosyncratic risk in our discounting factors. This means the intrinsic value definition is not only based on losses, but also categorically accounts for the cost of entering the business of trading insurance risk. However, the definition of intrinsic value is relative in that it depends on a number of factors, the risk environment, risk appetite and the regulatory regime. The stricter the regulatory regime is, the higher will be the solvency threshold and the more costly will be the capital supplied due to a higher risk of default. This renders the cost of risk portfolios different, depending of course on the regulatory regime of the country and the expectations of rating agents. A general equation for the economic cost of carrying risk for an uncertain future settlement date is represented by equation (3.39) given below:

\[
\Omega_{0,t} = L_0 (1 + \kappa)
\]

and

\[
\kappa = (\pi_T + r_E)
\]

Where: \( \Omega_{0,t} \) = price of liabilities at \( (t=0) \) for delivery within time interval \( T \);

\( L_0 \) = The intrinsic value of trading insurance risks or the expected value of losses;

\( \pi_T \) = The credit spread derived using the option pricing model above;

\( r_E \) = Rate of return required by shareholders (CAPM based);

\( \kappa \) = The ratio of cost of capital required supporting risk until delivery date, to the expected value of losses.
As elaborated above, the value of risk-free liabilities at time zero, is the present value of liabilities minus the default put option, \( D_E(l, q, r, \tau, \sigma) \). The advantages of using our extended option pricing methodology is its ability to capture the overall volatility in assets and liabilities through the portfolio risk parameter SIGMA (\( \sigma \)) and the impact of early forced bankruptcy. This is an important element because we are able to determine the level of risk in the portfolio, by taking into account the diversification effect across the lines, as well as the impact of safeguards brought about by the early bankruptcy option.

3.3.4.1 **Duration and the Price of Liabilities**

In this case, we are able to price the overall risk embedded in a portfolio through the quantity \( \kappa \), and the cost of capital attached to it. This variable is similar to that used by Cummins (2000) in allocating economic capital to insurance operations. The economic capital used in deriving “\( \kappa \)” is obtained by dividing the default put to liabilities and equalising this result at a designated target rate. The role played by this quantity is that of measuring the capital required in order to maximise shareholder value in the firm, which makes this methodology consistent with financial pricing theory. It is apparent from the equation above that as the value of \( \kappa \), increases the value of the default put declines and the more equity dominates the pricing of the contract. If our proposition should hold, the following condition is also true.
Condition II:

The distant price of liabilities is equal to the nearby cost of liabilities plus the cost of capital required to support the liabilities from a nearby to a distant settlement date.

We express this condition by the following equation:

\[ \Omega_{0,d} = \Omega_{0,n} (1 + \kappa_d), \quad d > n \]  

(3.40)

Where:  
\( \Omega_{0,n} = \) the cost of risk at time zero for the nearby contingent liability recoverable in the time interval \( n \).  
\( \Omega_{0,d} = \) the price of liabilities at time zero for distant contingent liability recoverable in time interval \( d \); and  
\( \kappa_d = \) the percentage cost of servicing liabilities from time \( n \) to time \( d \).

This condition arises from the fact that insurers, when trading risk, seek not only to diversify away underwriting risk, but also to spread risk across time as in finite insurance. Therefore, the longer it takes to settle a liability the more its pricing fundamentals derive from the cost of capital required to service it. Short tailed risk’s pricing is fundamentally underpinned on the behaviour of underwriting risk or the intrinsic cost component, with the level of capital needed determined by volatility and the magnitude of aggregate losses.

This brings out the fact that the price of any liability insured has two distinct components, namely its intrinsic value and the value-adding component. This analogy distinguishes two elements of volatility, the first one deriving from the underlying portfolio as it relates time and the second one the cost of capital also as it relates to the lock-up cost of time. Underlying volatility is brought about by the nature of assets and
liabilities in a risk portfolio; we call it the drift element of the assets and liabilities. The intrinsic value in any risk trading contract is used to cover the basic cost of liabilities, whilst the value adding component is related to the role time plays in holding that contact to settlement date.

Time volatility on the other hand is the effect which time has on the volatility of the net assets and liabilities of a risk portfolio, vis-à-vis capital needed to support the liabilities. The effect of time on volatility depends on the time lag between the assumption of risk and the settlement date. Therefore, provided the intrinsic cost and the cost of capital are well provided for, a risk trading contract is deemed to supply the required return on capital. This means that the elements of volatility which concerns an insurer are those pertaining to risk associated with the intrinsic cost and the cost of capital as they move in relation to the cost of borrowing funds from policyholders.

If we are to account for the cost of capital required to support liabilities, then the discounting in the price should be done on the intrinsic cost component rather than the capital cost component. In order for our price to reflect what shareholders really require, when their funds are tied to liabilities with a long duration, the capital cost component should be higher than it would be for liabilities with shorter durations. Therefore, for liabilities with the same underlying intrinsic cost, their prices should differ in two ways. Firstly, the intrinsic cost component should be discounted to take into account the benefits of investment income. Secondly, the capital cost component should reflect the costs embedded in holding capital over a longer and risky holding period.
It is evident that more capital is needed to service a portfolio with a longer settlement time than the one with a shorter one, due to the level of uncertainty brought in by greater exposure to risks as to time. In order to allow for uncertainty embedded in the time element, companies have to operate at high solvency thresholds so as to be assured that volatility in losses would not eclipse the underlying pricing structure. Therefore, the efficient pricing of a risk portfolio entails the proper accounting of the cost of capital required to carry the risk from the nearby date to an uncertain date in the future. Long tailed risks require a longer-term commitment of capital; pre-empting greater exposure to stochastic risk elements brought in by time. The same can be said for portfolios affected by catastrophe risk, in that the intrinsic and capital cost components will be high because of the need to accumulate capital and the nature of risk factors.

Our method of allocating capital according to the entire risk of a line, helps us capture the risk characteristics of each line and prompts different equity requirements across lines. This is based on the advantages of Myers-Reed's method and that of Bustic (1994) who allocated equity as a linear function of the line's beta\(^\text{18}\), is also echoed by Cummins et al (2000) that they resolve the problem of optimal capital allocation by allocating all equity to line of business, since the weighted sum of lines of liability betas is unit. It is true that capital allocation per line of business is directly proportional to the correlation with the loss portfolio (\(\rho_{\text{LIA}}\)) and inversely proportional to its correlation with the asset portfolio (\(\rho_{\text{LIA}}\)), Cummins (2000).

\[^{18}\beta_{\text{Li}} = \rho_{\text{LiL}} \frac{\sigma_{\text{Li}}}{\sigma_{\text{L}}},\text{Where: } \sigma_{\text{Li}} = \text{Line's volatility parameter.}\]
An insurance company’s risk levels increase as the correlation between a line of business and total liability portfolio increases. This implies that more equity is needed to carry the risk forward to loss settlement date (this subject is discussed in detail at the end of this chapter). This means that lines of business that have high-risk parameters will receive the same treatment with those that are highly correlated with the portfolio. Conversely, a high correlation between a line of business and that of assets require less capital, by virtue of the fact that the correlation between assets and liabilities is inversely related to the risk of the whole portfolio. The reason for this lies in the natural hedge that is created due to a positive correlation between assets and liabilities, which reduces the overall risk of the firm (Cummins, 2000).

This point is certainly important when underwriting homeowners’ policies in catastrophe prone regions, because risk underwritten from the same geographical location tends to correlate with the portfolio, increasing the absolute risk in the process. That is why it is important not to base our assessment entirely on individual line risk and inter-line diversification but the consideration of systematic risks resulting from the introduction of a line of business to the overall portfolio. Merton and Perold (1993) used a macro-firm capital allocation approach to argue that the stand-alone line allocation of equity is inefficient, in that it fails to take into account the effects of diversification. This approach is similar to our proposal, which asserts that capital allocation should be done on a marginal basis, by adding on a line of business to the portfolio, and measures the marginal capital required.

\[ \sigma_L = \text{Entire loss portfolio's volatility parameter.} \]
A macro-firm approach whilst theoretically appealing in as far as taking into account the effect of diversification is concerned, this method does not reflect the underwriting decisions taken in the insurance industry on a day to day basis. Lines of business are not simply added to the portfolio, but business comes in trickles, resulting in instantaneous changes in the liability as well as the risk profiles. This behaviour is more reflective of the micro changes in risk with respect to the trickles of business coming into the portfolio, a sensible idea because each set of assets derive from a set of liabilities, by tying their duration and convexity together.

A micro approach to the way capital is allocated as it comes into the portfolio, originating from each line of business will mean the cost of capital will be actually factored in each contract written. It will also enable an insurance company to decide the optimal point at which it should stop writing more business in a line, given the rates prevailing in the market and the threshold which should be met. This approach to allocating capital which leads to optimal capital structures, high RAROC and EVAOC, is consistent with recent literature on the subject by Myers and Read (1999). Their methodology leads to 100 percent capital allocation to each line of business. Therefore, our methodology is an efficient and cost effective way of allocating capital, because the accumulation of excessive capital has taxation and free agency cost disadvantages. Since our methodology avoids these pitfalls through optimal capital allocation structures, this cost of risk management is limited.
3.3.4.2 Contingent Risk Financing and the Price of Liabilities

What equation (3.39) portrays is that in every premium paid there is a cost of carry that is taken into account before value could be created. If the price charged for the holding period is not matched to the outflow at settlement date, value would be created or destroyed depending on whether liabilities are fairly priced, $L_t \leq V_t$. Equation (3.39) accounts for the internal cost that arises due to the need to comply with and guarantee that policyholders will recover their claims on promises sold to them by insurance companies. This cost is synonymous with the cost of borrowing funds from policyholders. A quality book lowers not only the cost of borrowing but also improves the value adding bounds of the insurer.

As we will see from the empirical analysis, the risk cost of carry factored portfolio is only an absolute cost, which is controlled through risk financing and expense management. This leads us to our next condition on the impact of risk financing on the overall cost of risk management and the importance of attacking risk from the source.

**Condition III:**

Insurance companies' value creation bounds are a function of the cost structures and the cost of borrowing external capital.

The cost of capital required to support assumed risk consumed should be in line with that of other players in the market. Otherwise, the cost of risk will not remain within the bounds of the insurer with the lowest cost structure. This means that as long as the price of risk drifts (Fig. 3.1) from the lowest bounds, insurers with the lowest...
cost structures will start exploiting the value adding opportunity. This is illustrated in Figure (3.1), where the 45 degrees line gives the optimum price of risk and cost structures. Companies with low cost structures will exploit the quasi-value addition, the moment cost of risk curves begin to drift in either direction.

The objectives for mergers and acquisitions are designed to make operating bounds narrower, by reducing the cost of capital associated with transacting insurance risks. The thinner the non-cost saving bounds are the greater the probability of adding value given the price movements in the risk market. This is implied by the potential to quasi-add value from a place and retain strategy or retain and carry strategy. Inter market correlation indicate that those trading in these markets ought to manage the cost of transacting risk in line with players in other markets in order to continue trading beyond the non-cost saving bounds. This is why French insurance companies are now using catastrophe bonds to finance risk, because it is cheaper than reinsurance and does not have any credit risk. The initial reinsurance programmes were exhausted by the 2000 French storms, a cost, which was embedded in the risk factors bur not considered in the pricing. By using catastrophe bonds to supplement capacity, the risk of default is reduced, so also is the cost of risk. Therefore, cost structures derive not only the underlying perils insured, but also the instruments used in financing. Those companies that are efficient in both managing the physical risk and risk financing structures, will occupy the lowest cost structures.

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Figure (3.1) gives us an illustration of how price drifts lead to quasi-value addition given a constant supply of risk capital and risk factors (default risk). The lower part of the diagram shows that insurers with low cost structures (CS2) will be adding value to shareholders wealth if price of risk (RPe) is equal to the intrinsic value (Risk Factors). However, those with high cost structures will be shading value, and have to compensate this through other areas where they have competitive advantage e.g. investment management or risk financing. In competitive insurance markets, the intrinsic risk price curve is not a replica of the risk factors curve. It is represented by the region between curve RPe and curve RP2, which coincides with cost curves of insurance companies with low cost structures. Companies trading risk at cost curves CSe and CS2 will have to either consolidate or compete based on areas of their competitive advantage.

Solvency thresholds affect the value adding bounds, in which insurance companies operate within. The cost of trading risk within the market is dependent on how best capital available is utilised to borrow funds from insureds at favourable rates and aid surplus at lower as well as stable costs. In order for an insurance company to acquire risk for its retention theorem (III) defined by the following equation must be satisfied:

\[ L_E (1 + \kappa) \geq \Omega_{E_{\text{ai}}} \]  
\( \text{Where: } L_E = \) the intrinsic cost of risk grossed up for the cost structure of the insurance company.  
\( \Omega_{0,t} = \) Price of liabilities taking into account grossed up intrinsic cost at time zero for delivery within time interval T
In equation (3.41) we factor the cost of transacting risk, which includes among other things, cost structure, taxation, limitations on investment instruments, negative impact of trading in the alternative market (e.g. derivatives), credit risk, liquidity risk, etc. The decision to originate liabilities depends not only on the cost of capital supplied by equityholders, but also risk capital bought from the risk financing markets. The overall cost of capital should enable an insurance company to trade within its value adding bounds, which are defined by the following equation (3.42):

\[ L_{E_l} \left(1 + \kappa \right) \leq \Omega_{0,t} \leq L_{E_u} \left(1 + \kappa \right) \]  

(3.42)

The subscripts \( E_l \) and \( E_u \) in the equation are notations for lower and high cost structures respectively. The cost of transacting risk has a loosening effect on the price relationship elaborated in equation (3.42). The level risk a company assumes should be equivalent to the level of capital in the portfolio. Optimal capital allocated to risks assumed defines the lower bounds for adding value. The upper bounds are defined by a sub-optimal risk capital position that is able to maintain minimal return on equity to a company. An optimal capital-to-risk position generates the highest possible returns earned on any portfolio with similar risk characteristics. What makes an equity-risk position unique is the way risk characteristics are matched with risk financing techniques. The equity to risk position determines whether the company is optimising risk in the portfolio or not. It also helps define the return volatility structure of the firm, with those companies optimising their equity-risk positions having more stable portfolios.
3.3.4.3 The Value of Cash Flow Engineering to the Firm

In their analogy, Mutenga and Dinenis (1999) pointed out that the cost of capital is company specific and follows the market trends tending toward that of the company with the lowest cost of capital structure, irrespective of the risk category. It is company specific because insurance companies have different cost structures in as far as transacting insurance risk is concerned. They observed that bounds also echo the relationship between company risk and cost of capital. Their analysis reveals that the cost of capital is a major factor in determining value creation, there was a significant relationship between the cost of capital and the cost structures under which companies operate. That is why we believe it is essential to manage the cost of transacting risk in order for a company to improve its value adding bounds. This cost of transacting risk advocated explains the whole process of insurance pricing, so it should be incorporated in pricing insurance contracts.

Default risk is reduced through hedging arrangements that reflect the characteristics of underlying risky cash flows. In fact, value is added when capacity required to write business can be generated at a lower cost than the return of funds raised. Therefore, given that the cost of generating capacity should be lower than that of lending capacity, this relationship is given by the following condition (IV) defined by equation (3.43):

\[ L \left( 1 + \kappa_B \right) \leq \Omega_F \]

(3.43)

Where: \( \kappa_B = \) the cost of capital less the net cost risk financing;

\( \Omega_F = \) Price of liabilities after considering the net cost of risk financing.
As pointed above when pricing a portfolio of insurance risks, consideration should be also be taken on the other charge on capital supplied, the cost of renting capital to liabilities. This cost is considered in relation to the effect of hedging on the level of capital released on the payoff profile and the cost of carry. It follows that when pricing primary contracts, the cost of engineering an optimal payoff profile should be incorporated, since this is part of the cost of servicing the risk presented. Capital supplied through cash flow engineering in order to leverage, is dependent on the characteristics of the risk portfolio, but from an economic point of view, it is determined by the cost of renting capital to liability accounts.

Hedging insurance risks is a function of both the existence of default and the process followed by the default boundary. Hedging is the art of explaining and sizing the behaviour and attributes of underlying cash flows and minimising corporate risk premium spreads. Cost structures are the major determinants of profitability and internal capital generation, the more efficient they are managed the greater the possibility of generating internal capital at lower spreads.

The current hedging techniques for insurance portfolios fail to explain the characteristics of insurance cash flows as divulged in volatility paths under the barrier option pricing methodology. Hedging the extreme values that are characteristic of barrier option cash flows is difficulty, due to the sign taken by gamma under different volatility paths. This makes it difficulty to hedge, due to the discontinuity of delta at the barrier. The discontinuity of delta means that the gamma is instantaneously infinite.

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20 A general derivation of the cost of risk financing is by subtracting expected ceded premium from ceding commissions and expected ceded losses. The denominator is obtained from the reduction in gross capital requirements resulting from the risk-financing payoff; otherwise, reduction in capital requirement is equal to risk financing.
at the barrier. This explains why it is costly and difficult to delta hedge insurance portfolios at the extreme value, and the best hedging techniques.

Insurance companies always attempt to generate capacity at a lower rate, than what they tend to earn from lending generated funds to asset accounts. Insurers need capital as security in order for them to raise funds through premiums and make sure that the policy provisions are met. Thus when generating capacity, the price an insurance company is prepared to pay depends on how much in terms of leverage it needs in order to optimise value addition. Therefore, the cost of creating capacity, the cost at which generated investment funds are borrowed and how much return is generated from these funds is important when deciding to use a risk-financing instrument.

A general derivation of the cost of risk financing is by subtracting expected ceded premium from ceding commissions and expected ceded losses. The denominator is obtained from the reduction in gross capital requirements resulting from the risk-financing payoff; otherwise, reduction in capital requirement is equal to risk financing. All that matters is the effectiveness of each instrument in as far as correlation between their payoff and portfolio losses in question is concerned. The cost of capital rented to liabilities should be stable in the long-run. This will help yield the required return within an insurance company's portfolio and establish an effective transfer pricing system. However, some risks by their very nature require more capital than others, this of course will be based on the loss distribution and capacity consumption rate. If buying capital to support a portion of risk assumed is expensive, from, say, the
reinsurance market, then those placing risk will retain more or seek placement in alternative markets, such action should generate cost savings.

The level of savings generated is dependent on whether funding obtained can effectively better reinsurance, which currently scores high due to its standings at law. However, reinsurers tend to be reactive rather than proactive when it comes to the supply of such capital. On the other hand shareholders require those who manage their capital to be proactive, meaning they should not move with the tides. It is important for management to manage the cost of trading risk economically, since the cost of capital utilised to create various cash flows has a bearing on the level of profitability. The use of different instruments in sourcing contingent capital is mainly aimed at smoothing the cost component, thereby stabilising return.

Traditional markets like reinsurance have a positive aspect, in that they receive favourable treatment at law than other alternative cash flow engineering methods. This makes them an expensive means of funding contingent claims and less appealing, hence making the capital provided by alternatives less competitive or equivalent. Market imperfections also make alternative risk financing instruments different from reinsurance as far as capital released is concerned. That is why it is difficult to adjudge arbitrage opportunities because the alternatives are not a replica. Partialities in treatment at law now per the regulatory framework mean that the level of capital released is not the same.

Value is added if cost reduction is achieved by obtaining capital in alternative markets at lower rates than those prevailing in traditional markets. In perfect markets
the price of risk should be the same throughout the markets supplying payoff-engineering funds; otherwise those purchasing such funds will arbitrage. Payoff engineering should also enable a risky portfolio to deliver stable rates required by those who sell risk, in exchange for securities to be delivered at settlement date. However, it is still possible to create value where the cost of risk in one market is lower than it is sold in another market, provided minimum solvability levels are met. Thus, with different price levels on the same tranche of risk in two markets releasing the same level of capital, an insurer can arbitrage by buying risk low and selling high in another market. In this case, an insurer can sell the burning cost of risk in the traditional market, thereby obtaining capital to service the risk at a lower price than would have been supplied to it by alternative markets.

Payoff engineering helps insurers manage risk more effectively, in an integrated manner and helps them earn an acceptable holding period return on risk securities issued to insureds. Due to a high intrinsic value, the cost of capital required to finance catastrophic risks is high, making it difficult to source all the capital from the traditional reinsurance market. Seeking alternative sources of financing are all attempts to control the cost of capital supplied, so that the return could be earned on underwriting risks rather than from shareholder funds. This is what makes an insurer's payoff profile different from that of conventional investment trusts. This distinction of capital generation is important in as far as, understanding when an insurance company is justified to reduce prices to gain market share, because the cost of generating capital as pointed above depends on the instrument used.
The cost of capital to service risk is embedded in the risk characteristics of a portfolio and the cost of leverage funds to engineer the desired payoff profiles. A portfolio susceptible to catastrophic losses requires more capital and depending on the level of capital available in the market, there is a higher tag in terms of risk premium. The level of capital required for a single catastrophic event in the US is now estimated at $100billion, whilst the insurance industry’s equity and surplus is only $240billion. A loss of this magnitude will wipe away the insurance industry capitalisation. Capital markets are capitalised at $33trillion, with an average daily standard deviation of around the same figure as the capitalisation of the insurance industry. Making underwriting risks tradable in financial markets enhances the capacity of the insurance markets and brings transparency in the pricing of insurance risk. It will also bring the required equilibrium in the pricing of risk in insurance. However, the current insurance-linked securities traded have a higher mark-up than the traditional reinsurance instruments. This is due to conflicting methods of pricing these risks; as well, there are few speculators prepared to take positions on these risks.

**Figure 3.2: Insurance Company Payoffs with the Introduction of Hedging**
In Figure (3.2), the optimal a-a is the one that is more risky than the others, a reflection of the level of retention vis-à-vis capital levels in a portfolio. This portfolio is however desirable on the upside, unlike a conservative portfolio that under utilise equity compatible with risks in the underlying portfolio, as reflected in the gradient of the upside which is flatter and a less risky downside curve b-b1. An important factor here to note is the impact of hedging on the portfolios; firstly it turns a risky portfolio into an efficient one c-c1. It does improve the other portfolios but not to the level that a portfolio efficiently utilising retention-based instruments is pegged, because the essence here is not just to reduce risk but to arbitrage risk-trading activity.

The new payoff profiles are flat, given the assumption that the risk-financing programme works for each portfolio. The hedge is effective if it eliminates all the underwriting risk, unfortunately this is not the case since insurance risk hedge instruments only alter the gradient of the slope of the payoffs. The curve retain the downward slope due to the existence of some systemic and basis risk as there are clauses of co-participation on losses exceeding a certain ratio, reinstatement limitations, reinstatement premiums, event limitations, inflationary control clauses and credit risk arising from counter parties. If this cover is the best deal for an insurance company then the curve can not be flexed upwards further beyond the cost of risk factors. Thus, the gradient of the lower part of the curve determines how effective the risk financing arrangement has been in providing the needed security, which might be at the expense of profitability on the upside. Thus the new upside and downside payoff can either be represented by curve “a-a1”, if the company is optimising its cash flows and curve “b-b1” if there is over protection.
An attempt to retain more risk should be justified by the underlying strength provided by instruments used in altering the risk profiles. This is because, any increase in return means that a company would be assuming more risk. The risk assumption should be done when the company has a competitive advantage in carrying more risk than the party it wants trade risk with. The trend in the insurance industry is that companies are retaining more risk than ever before (Figure 2.4), hence, the marked reduction in business being reinsured. Instruments used in financing retained risks are the one that enables the insurer to have more access to the investment income where good risks are rewarded.

If a company has a competitive advantage in financing retained risk, then curve “c-c1” will be a more representative pay-off curve, where the curve tilts upwards (gradient increased). A point to be noted is that the break-even point assumed is now at a higher operating ratio, implying a better stability in returns from trading risk. Therefore, any risk-financing strategy endeavours to push and flex the pay-off profiles as far to the right as possible. The longer the lower curve (security) stays above the knock-out barrier; the more stable will be the results of the insurance company. Beyond this threshold is the non-value adding bounds, a point reached by many companies in the UK in 1992, when losses arising from mortgage indemnity claims eroded reserves.

The figures (3.3 & 3.4) below elaborate cash flow alteration through the implementation of different techniques on an insurance risk profile, using curves more representative of insurance payoffs. The best results are obtained by utilising any of the instruments discussed above individually or in combination, depending on the
nature of underlying cash flows. It is interesting to note that from such cash flow profiling techniques, insurance companies can optimise their position by exploiting their competitive advantage in trading both the upside and downside risks.

**Figure 3.3: Insurance Company Payoff Profiles under Traditional Hedging**

![Diagram showing traditional hedging strategies with payoffs, reinsurance, knock-out barriers, and solvency levels.]

**Figure 3.4: Insurance Company Pay-off Profiles Using Blended Hedging**

![Diagram showing blended hedging strategies with risk-financing programmes, payoffs, and solvency levels.]

The figures above show the impact of two risk financing programmes when applied to portfolios with distinct risk retention policies. The cost of each programme
is indicated by the level it occupies, with the cost effective one occupying the upper option profile and a more expensive the lower of the curves. The pay-off profiles above show that traditional risk financing techniques in their own right are inefficient in delivering the lowest possible costs of financing risk. A reinsurance-based instrument whilst delivering the desired correlation characteristics, it does not however totally alter the distribution of losses in the portfolio because of credit and systematic risk. The instrument only alters that part of the loss distribution efficiently funded by the arrangement; otherwise it does not effectively alter losses in the super-cat region.

Figure (3.3) show this characteristic of reinsurance based instruments in the new pay-off profile which whilst deflecting the lower part of the curve upwards, the cost of over conservatism is paid by throwing away a lot of float. The risk financing programmes applied to the underlying portfolios are a reflection of cost structures and hedge effectiveness. Traditional methods are more expensive than integrated programmes, because they do not utilise equity efficiently, making the ultimate cost of capital high. Therefore, an effective financing integrated programme is bound to have a lower cost of capital structure, than the one that leaves gaps and exposes other equity components of the financial structure.

In Figure (3.3) and (3.4) the two programmes are applied to portfolios using different retention based techniques, with the result of payoff profiles with unique characteristics. A portfolio adopting a conservative policy is faced with an asymmetric payoff, of a flatter upside payoff and a steeper downside profile. This is due to the fact that an impaired internal capital generating potential, does not only affect profitability but also stability in the portfolio. An unstable portfolio will mean high marginal tax
rates, increased agency problem and a higher risk premium on equity funding risks in the portfolio. In our study, we discovered that companies with poor internal capital generation, usually experience capitalisation problems after a prolonged spell of experiencing such a problem. This is also observed in company economic value added (EVA) and market value added (MVA), as pointed out by (Mutenga and Dinenis, 1999).

Curves to the right are more efficient payoffs than the ones to the left, though they do not reflect the level of retention but the efficiency of equity embedded in the portfolios in question. Insurance companies use risk-financing instruments in order to stabilise their portfolios, this expectation is satisfied in Figure (3.4) by the curve that is further to the right. The unique characteristic of payoff profiles generated by cash flow engineering is the steeping of the upside, a stretched effect on the middle section and a flatter profile when the curve is tapering. The stretched middle section represents stability endowed in a portfolio, the longer this section stays over the knock-out barrier the more stable the portfolio will be and the shorter this profile will be the more volatile will be the portfolio. This is the most critical factor to look at when you are measuring the impact of cash flow engineering, it must stretch this region as far as possible. The cost of failing to achieve optimum payoffs could be measured by comparing the attained profile and what is deemed an optimal profile.

3.4 Conclusion

In Chapter, (2) we classified risks faced by an insurance company, in this chapter we isolate and use the grouped risks to derive the firm-wide risk. The model developed in this chapter presents a new risk measurement framework that incorporates the
effects of solvency margins on the overall risk of firm. The model is developed from robust techniques, which have been used to measure default risk in corporate bonds and insurance companies alike. Our model is an extension to an already existing body of literature, which use option pricing to value insurance liabilities. This model provides a better measure of firmwide risk than models based on standard option pricing models, because it captures all the variables that determine cash flow variability in insurance companies. Furthermore, risk derived from the default put does not capture cash flows affecting shareholder interests only, but also risks that affect policyholders and spreads that insurance companies are prepared to pay for investment funds.

This model is empirically tested in Chapter (4), to establish the relationship of default risk and spreads derived from our model, to risk capital cost variability and the quality of operational profitability. We are able to show that this model is efficient in explaining the theorems proposed in this chapter. The only drawback with this model is its use of the normal distribution in modelling the loss distribution. It is not efficient at capturing risk embedded in the tails; enigma risk in insurance cash flows is better explained by Weibull or Pareto distribution. Irrespective if this, our model's efficiency in measuring firmwide risk is not diminished, because the liability classes manifesting cat risk characteristics only form part of the total loss distribution which is derived from different asset and liability accounts.

This methodology is also a more practical approach to measure risk, as it captures the additional risk associated with illiquidity when the company trades close to its solvency threshold. This is a well-established phenomenon in barrier options that
volatility increases drastically as asset-to-liability ratios approaches the knockout barrier. We realise this phenomenon and that financial structure decisions are primarily based the level of solvency thresholds, and are made to believe that risk-taking behaviour is underpinned by solvency margin levels. That is why we incorporated solvency thresholds in our option-pricing methodology to capture this risk factor that has been missed by many insurance risk measurement models. The strength of our findings in Chapter (4) is a result of the incorporation of this factor, as you will discover.
CHAPTER 4:

THE COST OF INSURANCE COMPANY RISK

4.1 Introduction

The purpose of this thesis is to explore and isolate the impact of default risk on the cost of carry and spreads paid by an insurer to its policyholders. The cost of carry is the amount that an insurance company pays for a unit of investment funds borrowed from policyholders. In this chapter we look at the empirical evidence using the model developed above to be established whether default risk and the safety margins introduced by the regulators affect the cost of carry and incidentally the behaviour of constituents of the profitability profile of an insurance company, and if it does to what extent.

We also develop a transfer pricing system, which we consider necessary for improving performance in property/casualty insurance companies, based on the observations, and conclusions raised in this thesis. This thesis also establishes the link between the insolvency put, the cost of carry and the investment return. This chain is
the driving force for sustainable performance in an insurance company, because
instability in any of these variables either increases the cost of equity, or destabilises
the whole portfolio and ultimately drags down return on equity with it. As you read
the chapter you will realise that the quality of investment return is a function of the
cost of carry as this is considered a good measure of quality in the underwriting book.
Therefore, volatility in the cost of carry is always reflected in the overall return on
average invested assets, because funds invested in assets are tied to liabilities.

We have seen that the success of Berkshire Hathaway is underpinned by its cost
of float/cost of carry management, as this ratio determines what it takes to acquire a
unit of investible funds. This ratio is a measure of the quality of underwriting as it
contributes to investment funds, which makes it a critical ratio, because profitability is
determined by the quality of underwriting and investment cash flows. Through this
ratio we are be able to pinpoint that the under-performance of property and casualty
insurance companies against the S&P 500 and other indices is more to do with the
quality of underwriting cash flows than merely limiting the argument to excessive
capital. The reason behind performance is not merely underpinned in the disparities to
the costs of borrowing as compared to other financial institutions, but the quality of
cash flows relative to the risks these sub-sectors face. Therefore, the cost of carry is at
the heart of insurance companies’ operations, because a company that does not have
good underwriting cash flows is more likely to fail in the long run, even though its
investment earnings might be stable. The length of time before a company with poor
underwriting cash flows fails depends on the quality of investment income and the
magnitudes of cancerous cost of carry structures.
The cost of carry has been ignored as a performance measure, in preference to absolute factors like the combined and operating ratios. Whilst we are not disputing the potency of these factors in performance measurement, this paper puts the cost of carry at the forefront of financial indicators attributing profitability to the way cost structures are efficiently managed in insurance companies. Ratios capturing the absolute performance of underwriting structures like the combined ratio do not actually tell us what it costs us to underwrite the next risk, but relative terms like the cost of carry enable us to see the potential profitability of the next generated unit of investible funds. The level of borrowed funds affects the ratio propounded in this paper more than the magnitude of combined ratios, because the cost of borrowing increase with each unit of investment funds generated beyond the equilibrium point. This point does not diminish the importance of combined ratios, our aim is to extend the platform on which underwriting profitability is measured and develop tools that enable companies to easily define and isolate costs inherent in their portfolios.

The cost of carry is an important quantity in as far as the quality of cash flows is concerned, because insurance companies unlike investment trusts; generate wealth through funds originated from underwriting activities. This quantity enables insurance companies to identify lines of business they can arbitrage liability risk, those they have to exit or keep in order to diversify away certain risks and balance their portfolios. The fact that we are computing the cost of carrying a risk forward to settlement date makes it easy to link and compare this to quantities like the cost equity, risk free rate of borrowing, the cost of funds in other markets and whether investment income is enough to cover the cost of borrowing. It can be said that the cost of carry is the amount paid to policyholders for the cost of default, the costs are in the form of
spreads reflecting premium to be paid for the value of the put option shareholders hold against the value of the firm.

Merton's default model envisages that the value of equity increases by virtue of a put option that allows shareholders to transfer wealth from policyholders when things go pear shaped, by virtue of the limited liability option. This asymmetric nature means that shareholders can walk away if the value of assets falls below the value of liabilities, leaving human equity without jobs, and policyholders with a depleted portfolio and individuals having recourse to guarantee funds. Therefore, whilst the default rate brought about by increased risk in a portfolio reduces the value of debt on one hand, and increases the value of equity and the cost of carry with it on the other. This prompts policyholders to require insurance companies to pay them a premium commensurate with the risk of default, a value well captured in the cost of carry methodology. Thus the lower the asset levels the more expensive it will be to borrow money from the policyholders, since the high cost of carry ratios are reminiscent of poor underwriting cash flows.

The under-performance of insurance companies against other financial institutions has been linked to excessive capital, principally correct, but we believe this is more to do with cash flow quality rather than by just reducing the size of portfolios. We have more to learn from the artefacts of the cost of carry, than we have attributed to the celebrated measures like the combined and operational ratios. The slumps in return and underperformance of insurance companies linked to tight margins are conspicuous in this ratio, as well as the impact of catastrophic losses and underwriting cycles. This factor can easily and reliably be used as a measure of liability risk because movements
in borrowing costs are always captured in the overall profitability profile and the quality of underlying cash flows.

4.2 Data and Methodology

In this section, we specify the variables and the data that will enable us to do an empirical analysis on the influence of the cost of carry. We first outline our sample and data sources before we move on to the specification of variables necessary for the computation of our statistic and analysis. Our analysis is based on company data rather than market figures because the purpose of this analysis is to improve internal controls in an insurance company and the establishment of efficient internal financial structures. Another reason for not considering market data is that very few property/casualty insurance companies publicly trade on the London Stock Exchange, that they will not constitute a good sample. The methodology used help establish the relationship between the quality of underwriting cash flows and borrowing costs, and underlying risks as captured by spreads in the cost of default.

4.2.1 Endogenous Variables: - Risk and

4.2.2 Return Estimations

The data sets used in this study are the UK General Insurance Companies Analysis data is obtained from A.M. Best International data CD-ROM version 7.7 published in November 2000, and the DataStream financial data is obtained online from DataStream financial data updated everyday. UK Insurance companies data obtained is for the period 1981 to 1999, though our analysis is based on the period 1986 to 1998. This sample period chosen eliminates companies which were in
operation in 1981 but have cease operations as the sample period progresses. It also helps us include new concept and specialist companies like Directline, DAS and Independent Insurance Company which have outperformed the traditional insurance companies since they commenced their operations. These companies started operating around the period 1984 to 1986; our sample period choice enables us to capture risk characteristics associated with all the players in the London market today. Data obtained from this source is drawn out from statutory annual accounts filed by insurers with the Department of Trade and Industry (DTI).

The sample period chosen also enables us to avoid distortions brought about by the mergers and start-ups occurring mainly after 1995. This entails that companies recently formed due to mergers in the UK in the past few years are not included, as these do not satisfy the five-year or ten-year selection criteria. In 1998 and 1999, a total of 344 and 323 companies respectively traded on the London companies market, of which only 13 companies are composites forming our initial population. Companies that meet our ten-year period selection criteria were 316 (See Appendix: A for the list of companies considered in this study). These samples are larger than those used by Cummins, Allen and Phillip (1997) in their analysis with only 90 companies and 315 observations of which only 270 were for the whole five-year period. Our data gives us at most 3,160 observations for each variable for the ten-year sample period. The number of observations in our sample is sufficient to give us robust results, since the number of observations is larger than those used in recent insurance literature, Cummins, Allen and Phillips (1997) and in Ronn and Verma (1987) where only 43 banks were used in assessing volatility in banks using option pricing methodology. The data extracted from underwriting results, technical reserves, surplus, assets,
liabilities and pre-tax return are explained in detail under the specific sections defining the variables.

For composite companies, only the general business statistics is considered, and the data is easy to extract since regulatory authorities require separation in the accounting of these businesses. The criteria set for the selection of a company in our data set is based on the companies having been trading as an authorised insurer in the London market for the entire period and their financial data being recorded on the A.M. Best Companies analysis CD-ROM published in November 2000. This is necessary for the computation of liability growth rates.

4.2.2.1 Liability Growth Rates

Liability growth rates are calculated using market loss data for each line of business as reported in the statutory accounts. UK statutory accounts record eleven lines of business namely Health and Accident, Property, Motor, Marine, Aviation, Miscellaneous and Pecuniary, Transportation, Third Party Liability, Proportional Treaty, Non-Proportional Treaty, and Marine, Aviation and Transportation (MAT) Treaty. These classes are grouped into two major accounts depending on the duration of liabilities irrespective of the line of business. In the statutory accounts as recorded in A.M. Best data tapes, each line of business is accounted in either one year or three year account. These accounts are used to distinguish both premiums and liabilities falling in short tail classes from those falling in long-tail classes. This classification means that liabilities, losses incurred and premiums are properly classified according to the average time period it takes to settle losses. This approach is therefore, based on the
predominant feature of liability characteristics rather than an approximation on the volume of losses settled within a specified period.

The classification based on one-year and three-year account basis is also supported by the argument of the volume of losses settled before or after three years, which is more than 80% for these accounts, respectively. The information pertaining to the duration of liabilities in each class was obtained from the total claims paid percent to total claims paid and outstanding for one-year and three year account triangles. This type of classification is common in insurance and has been used in insurance literature to capture risk characteristics associated with each group. The method of classification used in the UK reduces errors of misclassification than those based on estimations. The only shortcoming with this grouping methodology is our inability to account for fat tails, common in classes prone to catastrophic risk contamination.

The computation of liability growth rates is based on the losses incurred market data for the one-year and three-year accounts. This is consistent with a body of actuarial and financial economics literature, since the aim is to eliminate non-synchronous characteristics of individual companies' data. The main assumption here is that insurance companies are price takers and do not have any influence on the overall rates charged in a competitive market. Thus, in order to compute growth rates that are likely to be repeated in the future, insurance market data is used in the computation of yearly growth rates for each account. Liability growth rates are calculated by taking the natural logarithms market total incurred losses for year T, and T-1, given by
Where: \( r_{Li} = \text{growth rates on liability class } i, \) and 
\( L_{it} = \text{loss index value of liability class } i, \text{ at year } T. \)

Annual data representing the index for incurred losses is used because there is no comprehensive quarterly recorded data for each company. The use of quarterly recorded data would substantially improve the quality of our results, and move us closer to the assumptions in our model; however, the use of annual does not significantly affect the purpose of our empirical analysis. The proportions for each company's liability account is based on total for losses outstanding and loss adjustment expenses (LAE) reserves, unearned premium reserves (UPR), unexpired risk provisions (URP), IBNR provisions and acquisition costs provisions for each year constituted. The provisions for each year for account of business are divided by the total for one-year and three-year accounts to obtain the liability allocations for each account for each year. The proportions of liabilities in each of the portfolios is then multiplied by market growth rates obtained in equation (4.1), to obtain annual liability growth rates for each company, for each year and for each account of business. The liability growth rates for each of these two accounts are abbreviated LIBGRO1 and LIBGRO3 for short tail (one-year account) and long tail (three-year account) lines of business, respectively. Line growth rates obtained for each company and account of business are summed up to obtain annual liability portfolio growth rates. This is the \( r_L \) derived in Chapter (3), it is used in adjusting the risk-free rate when computing the insolvency put and default spreads.
4.2.2.2 Asset Returns

The data central to the computation of asset returns and the risk-free rates is obtained from two sources the DataStream Online files and A.M. Best UK General Insurance Companies Analysis CD-ROMs. The main factor that influences our choice of data on assets from Datastream is based on the way assets are classified on the statutory DTI accounts. Unlike the liabilities assets on UK insurance companies' balance sheets are not classified according to their duration, but on the type of security. The dilemma we face is that we do not know the duration of variable and fixed interest securities which are just classified into these major groups. Therefore, our classification is based on the approach used in statutory accounts, by classifying assets into five major groups, namely Fixed Income Securities, Variable Income Securities, Equities, Real Estate and Insurance debtors.

We assumed that insurance debt, mainly represented by balances due from brokers and other insurance companies receives a rate of return equal to zero since no interest is paid. This argument is consistent with mainstream insurance literature, meaning there is no total return index representing return on insurance debt. If we use this assumption it eliminates insurance debt from our analysis of return on assets and asset risk. This makes our risk measurement method incomplete, because balances from agencies still carry credit risk. In order to cater for this phenomenon, the total return index is based on the 90 days NCDs, because duration of this type of insurance debt is usually equal to three months. Even though this will be a negative return account it gives us the return and risk structure of these insurance debts.
The grouped assets for each company for each year are added together to obtain annual totals, from 1986 to 1999. The annual asset portfolio totals are used to obtain the asset allocation in each year by dividing it by each asset grouping constituting the portfolio for each year. Market asset data used in computing returns was obtained from the FTSE Real Estate Index, the Financial Times Fixed Interest – price index (FTFixed), the FTSE-All Share Index, and the FTA British Government Index Link five year price index from 1985 to 1999. The reason for using asset indices is based on the assumption that insurance companies have perfectly diversified portfolios that closely resemble the index of each of the asset they are investing in. Furthermore, in competitive markets insurers are price takers meaning they do not have any influence on the prices offered in these markets. Quarterly market data is used to compute returns on each asset class by taking their natural logarithms for quarter t, and t-1, given by

$$r_{Ai} = \ln\left(\frac{A_{it}}{A_{i,t-1}}\right)$$  \hspace{1cm} (4.2)

Where: \(r_{Ai}\) = return on asset i, at quarter t; and

\(A_{it}\) = index value of asset i, at quarter t.

Adding returns for each quarter and taking their average figure for the year annualises these returns. The annual figures are then multiplied by asset proportions to obtain asset returns for each company based on its portfolio allocations. Thus, asset returns for each year for an insurance company are determined by its investment
policy, as reflected in the asset allocations. The same procedures are followed when computing asset risk, which is determined by asset allocations.

4.2.3 Portfolio Risk

The model developed in Chapter (3) points to carry spreads paid by insurance companies to originate investment funds as a function of the risk inherent in the company. In this section we take the necessary steps required to measure portfolio risk, so as to test empirically the implications of the developed theoretical methodology. Since our concern is on internal controls, we are looking at balance sheet risk measures, as they influence the carry or default spreads. This is a sensible measure due to limitations in market data on stock trading insurance companies in the UK; very few companies trading insurance risk are listed on the London Stock Exchange. It is therefore necessary to assess risk embedded in balance sheet cash flows, since all the stakeholders assessing the strength of each company use the balance sheet and statutory accounts in their analysis. One such group that uses statutory data to assess risk embedded in insurance cash flows are the rating agencies.

Our computation of internal cash flow risk measure is consistent with the model presented in Chapter (3), and option pricing methodologies in Briys et al (1998), Cummins and Danzon (1997) and Sommer (1996). This study extends the option pricing methodology to the examination of the effect of solvency margins in computing the insolvency put and ultimately default spreads. It captures not only the stochastic nature of liabilities ingrained in liability growth rates, but also a stochastic knockout barrier, and more importantly absolute risk embedded in contaminated portfolios. It enables us to investigate the relationship between the cost of carry and
risk-taking activities within a company given the existence of implicit and explicit bankruptcy costs. We use the risk parameter obtained from our multi-line/asset option-pricing model in Chapter (2), given as follows:

\[
\text{SIGMA} = \sqrt{\sum_{i=1}^{d} \sum_{j=1}^{d} (\sigma_{Ai}\sigma_{Aj} + \sigma_{Li}\sigma_{Lj}) - 2\sigma_{Ai}\sigma_{Li}\rho_{AiLi}} 
\]

(4.3)

Where: SIGMA = Portfolio Risk Parameter.

The problem with the data we obtained is that assets are not matched specifically to liabilities due to general balance sheet data on assets, which makes the computation of a portfolio risk parameter difficult. It is also difficult to ascertain the investment policy of a company, but in order to be consistent with previous literature we assume that every insurance company matches assets to liabilities based on their duration. We go on to do the estimates of volatilities in assets, and liability accounts using market returns and growth rates computed using equation (4.1) and (4.2) above. Proportions of asset and liability accounts are used to calculate the share of risk based on the portfolio allocations. This means that risk in each company depends on the variations in the portfolio asset and liability mixtures annually for each company. The measurement method is consistent with financial literature as propounded by Cummins and Sommer (1996) and helps us in testing our hypothesis on the efficiency of insurance companies transfer pricing systems.

The covariance matrix elaborated in Chapter (3) was produced to account for the co-movement among asset indices and liability indices, and between assets and liabilities market indices. This results in us obtaining asset (liability) risk parameter
after only taking into account individual asset (liability) risks and their covariance, and
the portfolio risk by accounting for these risk parameters and the covariance between
the asset and liability accounts. The risk parameter obtained from our computation is
used in computing both the insolvency put and the default spreads using following
equation.

\[
Y_{SPRE} = \frac{1}{T} \ln \left[ 1 - P_E(l_t, 1) + P_E(q_t, \frac{1}{q_t}) \right]
\]  \hspace{1cm} (4.4)

Where: \( Y_{SPRE} \) = Default or Policyholder Deficit Spread for a Standard Default
Spread (SDSPRE) or for an Early Default Spread (EDSPRE).

This risk parameter could also be used as a regressor in our regression model
presented in equation (4.4), to test our hypotheses specified in chapter three. The use
of the risk parameter \( \Sigma \) as a regressor helps us account for risk information
pertaining both to liabilities and assets and provides a better measure of portfolio risk
than the one used by regulatory authorities in the EU or USA. However, the overall
firm risk as represented by the insolvency put (surplus deficit) provides a better
measure, since it does not only take into account portfolio risk for assets and liabilities
but also the risk embedded in the leverage and solvency margins.

Another piece of information required in the calculation of both the insolvency
put and the default spreads are the asset-to-liability ratio \( (l_t) \) and the early default ratio
\( (q_t) \). The data on both assets and liabilities are obtained from assets and total liabilities
as reported for the same period on the financial accounts as recorded in the A.M. Best
international General Insurance Companies Analysis CD-ROM. The first ratio is
obtained by dividing the book value of assets with the book value of liabilities for each
The purpose of this ratio has been pointed out as an indicator of gearing in a portfolio. In the absence of early default solvency margin this ratio represents Cummins's default put option. In order to take into account the impact of solvency requirements by regulatory authorities the required solvency ratio on the statutory accounts is used to represent $q_t$. These two variables are important in the demand for insurance especially from the corporate quarters, which is more concerned with the level of capitalisation of the company. These variables are important because insurance markets are so responsive to changes in safety levels, so their stability is important since they influence spreads paid when borrowing funds from policyholders.

Our model requires us to risk-adjust the risk-free rate as denoted by Treasury Bills (TBs) for asset accounts and by the liability growth rates ($r_L$) for liability accounts. The information on the risk free rate is obtained from DataStream British Government 3 months TBs returns. The total value of liability growth rates for each year is the annual weighted average of liability growth rates for each company. The risk-adjusted figure is then used in the computation of the insolvency put and the default spreads as propounded in chapter three.

The procedure in Chapter (3) is followed in the computation of both insolvency put and default spreads, by feeding the empirical information into the option pricing models. The values for $d_1$ to $d_4$ are first obtained, before $D_E(l_t)$ and $D_E(q_t)$ are computed using the normal distribution function in excel for the 'N's. The figures for $D_E(l_t)$ and $D_E(q_t)$ are then used to compute the standard insolvency puts and default

\[ r_p = r_f - \left( z_{L_1} r_{L_1} + \ldots + z_{L_d} r_{L_d} \right) \]

\[ 21 \text{ A detailed account on the computation of liability growth rate is given in Cummins (1988), and the following formula shows how risk-adjusted discount rates are obtained:} \]
spreads, and the early default option puts and spreads, for each company, and for each year. It is apparent from the model that both the puts and spreads obtained are more representative of the risk profile of the firm, since they incorporate all the components that determine the riskiness of a firm. While the insolvency put reveals the risk perception from the policyholder's viewpoint, spreads represent the risk premium required for investing funds in that portfolio. The reason for this argument about spreads is elaborated in the next section, which specifies the dependent variable used in this analysis.

Through this model we are able to show that the solvency level influences risk-taking activity in an insurance company. It has been pointed that the higher the solvency margins, the lower the value of equity due to higher insolvency put values as a result of the early default option value. It therefore, means that if a company is to retain cheaper internal generated funds it has to control its portfolio risk, in the process reducing spreads paid on borrowed funds. In order to substantiate our proposition on the impact of solvency margins on the overall risk taking activity within a company, we use standard option pricing in our computation of the insolvency put and carry spreads and the extended version proposed in this thesis. This enables us to effectively isolate the impact solvency margin has on the cost of carry, by comparing the results of the two methods.

4.2.4 The Explanatory Variable: Cost of Carry Estimations.

In this section we specify our dependant variable, which is composed of two components, the cost of float and return on invested assets. The reason for requiring

\[ \text{Where } r_p = \text{portfolio risk adjusted rate; } r_f = \text{risk free rate; } Z_{ij} = \text{proportion of liability } i, \text{ to the whole} \]
these components to be present at the same time is that not only are liability growth rates considered in the model but also asset returns in the computation of risk. This means that liability growth rates will not be considered as regressors in our analysis, because they have already been accounted for in the model. Our approach differs from that of Cummins et. al. (1997) who incorporated liability growth rates in the regression, even though it was used in the risk adjustment process in the model. We will use fewer variables in our regression model than those used in previous studies since we believe our model is all encompassing for regressors; insolvency puts and default spreads which explains most of the variability in insurance cash flows.

The first component of our dependent variable, the cost of float (COF), is used to measure the quality of the cash flows from the underwriting side. It represents the cost of borrowing funds from policyholders. These funds are treated as liabilities in financial statements, because they are a legitimate property of policyholders, obliging insurance companies to pay a certain percentage denoted by COF to the owners of these funds. This liability on insurance books termed, float or carry, provides insurance companies with funds for investments, in return for the risks assumed under the contracts. In fact these funds recorded as technical funds on insurance companies’ books have been a major contributor (80%) of investment funds for insurance companies, according to the AM Best Averages and Aggregates USA Property/Casualty statistics for the period 1938 to 2000. This is the essence of being in insurance business — arbitraging underwriting risks. A ratio that incorporates these funds captures the cost embedded in utilising the funds; a better measure of prudence
than combined ratios and enables a company to set-up effective risk transfer pricing systems.

The percentage rate that insurance companies pay in order to secure investment funds is a product of the underwriting result divided by the technical reserves (See equation 4.6). This is the cost of carry or precisely the cost of borrowing funds from policyholders. The numerator is a net figure of what is actually paid to the policyholders for putting their faith in the company and denominator the level of funds that belongs to them.

$$\text{COF}_{t+1} = \frac{L_{t+1} - P_{t+1}}{U_{t+1}} \times 100\% \quad (4.6)$$

Where: \( \text{COF} \) = the cost of float;

\( P \) = premiums earned net of risk financing expenditure;

\( L \) = liabilities (incurred losses net of risk financing payoffs and expenses);

\( U \) = technical reserves.

Another way of defining the numerator is to visualise it as being net cash flows paid by policyholders in exchange for contingent risk capital. If this net figure is negative it entails that an insurance company is actually paying policyholders for holding their funds and if positive, these funds will be generated at a negative implied interest rates. The higher insurance companies pay policyholders the less will be their ability to generate large Sharpe ratios, leading to less flexible financial structures and constrained internal capital generation. This method of computing the cost of liabilities
has been used successfully by Berkshire Hathaway in defining the quality of underwriting cash flows.

The data that is used to compute this ratio is extracted from A.M. Best UK General Business Data CD-ROMs published in November 2000. Data is extracted from the Revenue Account and Balance Sheet of both one-year and three-year accounts, from the declared underwriting results and General Business Reserves for the years 1985 to 1999. Ratios for each company and for each year are then obtained by dividing the underwriting results by the figure for borrowed fund to compute the cost of borrowing expressed as a percentage.

The second component is the return that is generated on borrowed funds, termed return on invested assets (ROIA). This measures the average return on the company’s invested assets by dividing the company’s annual net investment income by the mean of the net invested assets. In the same vein, data used in the computation is obtained from the Revenue Account and Balance Sheets. The ratio obtained is then added to the cost of borrowing to obtain the explanatory variable used in the regression model specified below. Another way of looking at this variable from the point of view our option-pricing model is to consider it as a return on policyholders’ surplus otherwise termed cost of carry (CCRRY) in this analysis. This measures the overall company’s profitability from underwriting and investment activity after tax divided by the mean of prior and current year-end surplus. This should represent a good proxy for the spread generated from our model, viewed from the policyholder’s point of view. Our empirical analysis is done using a regression model and is specified in the next section.
4.3 Analysis and Results

In this section we test the ramifications of the model we developed in Chapter (3), by carrying out a number of empirical tests. We establish the relationship between the cost of carry and risk, and default-spread and the level of protection provided by solvency margins. In order to achieve this we run regressions for each financial year for both accounts with solvency margin restrictions and those without such protections. The variables used in our regression have been expounded above, the summary statistics of which are presented below and the result of our analysis in the ensuing sections. This analysis will also enable us to isolate the importance of efficient transfer pricing systems, capital allocation, risk financing systems and the development of recommendations with regards to findings.

4.3.1 Descriptive Statistics

We report descriptive statistics for all the variables used in this study in Table (4.1). The average portfolio risk for insurance companies during the period is 0.27. This compare with volatility figures for publicly traded insurance companies based on USA data of 0.379 and implied volatility of 0.115 on studies by Cummins, Allen and Phillips (1997). The main account that contributes to portfolio volatility is the liabilities account, rather than assets account. This is due to internal contamination of insurance liability portfolios arising in catastrophic years, which tend to magnify the level of volatility. The assets account on the other hand, is composed of diversified portfolios mainly invested in fixed income securities that are more stable. This confirms the need to control liability accounts by using transfer pricing systems. There is greater variability in the level of risk in liability accounts mainly based on the lines
of business, as short-tailed lines are more volatile than long-tailed lines. The same story is inferred from the liability growth rates, which show that short-tailed accounts, have higher liability growth rates than long-tailed lines. This is quite consistent with duration features and pricing practices in the insurance industry, which tend to affect short-tailed lines more than the long-tailed lines.

The values of standard and early default put options are 8.98 million and 51 million respectively. These are 1.07% and 6.109% of average policyholder surplus respectively. The average figures, of course, vary from company to company and are quite high for EDPUT in years with catastrophic losses. As expected, the default put that incorporates the effects of solvency thresholds is higher than the standard one, which confirms the notion that solvency margins increase the cost of default on Equityholders. Therefore, solvency thresholds increase the overall risk of a firm, by virtue of the option to an early default. It also confirms the notion that solvency margin levels determine not only the risk taking behaviour of insurance companies but also the ultimate capital levels. This also includes economic capital, which imposes greater capital requirements with its primary reference being the equity requirements imposed by the solvency threshold.

Default spreads paid on standard default risk are on average −0.035, whilst those paid on the early default spread (EDSPRE) are −0.229. This shows that policyholders are prepared to pay less when there are no safety margins in place and they are quite high when there is protection. We believe that by using ‘A’ equivalent rating as the solvency margin the spreads paid by policyholders for insuring with a company with greater protection will be higher than those with protection based on regulatory
solvency margins. Spreads computed using EDSPRE are also closer to the cost of float are observed in practice, averaging 0.203 during catastrophic years when most insolvencies occur and 0.05 during good underwriting years. Our study therefore, confirms the importance of solvency margins in both determining firm risk and spreads paid by policyholder for contingent securities sold to them.

<table>
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<th>Table 4.1: Results - Descriptive Statistics</th>
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<td><strong>Notation</strong></td>
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<td>Net Premium to Asset Ratio</td>
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<td>Standard Insolvency Put</td>
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<td>Insolvent Put - Early Default</td>
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<td>Default Spread Standard</td>
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<tr>
<td>Spread With Early Default Option</td>
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<tr>
<td>Asset Risk</td>
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<td>Liability Risk</td>
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<td>Portfolio Risk</td>
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<td>Liab/Asset Ratio</td>
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<td>Short Tail Risk Adjusted Rate</td>
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<td>Long Tail Risk Adjusted Rate</td>
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<td>Rate of Return on Inv. Assets</td>
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<td>Policyholders’ Surplus</td>
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<td>Total Assets</td>
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<td>Net Asset Values</td>
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<td>Total Liabilities</td>
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<td>L/Tailed Liability Growth Rate</td>
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<td>S/Tailed Liability Growth Rates</td>
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<td>Solvency Ratio</td>
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<td>Reserves</td>
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<td>Risk Financing (% of Net Assets</td>
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<td>Std. Deviation</td>
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<td>CCRRY&lt;sup&gt;d&lt;/sup&gt;</td>
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<tr>
<td>PRASS&lt;sup&gt;f&lt;/sup&gt;</td>
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</table>

*Predictors:* (Constant), Spread with Early Default Option, Default Spread Standard

<sup>a</sup>Predictors: (Constant), Rate of Return on Inv. Assets, Spread With Early Default Option, Insolvency Put With Early Default

<sup>b</sup>Predictors: (Constant), Spread With Early Default Option, Rate of Return on Inv. Assets, Insolvency Put With Early Default, Risk Financing

<sup>c</sup>Predictors: (Constant), Spread With Early Default Option, Rate of Return on Inv. Assets, Insolvency Put With Early Default, Risk Financing

<sup>d</sup>Dependent Variable: Cost of Carry (CCRY)

<sup>e</sup>Dependent Variable: Risk-Adjusted Return on Capital (RAROC)

<sup>f</sup>Dependent Variable: Premium to Asset Ratio (PRASS)
### Table 4.3: Results - Coefficients and Collinearity Statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardised Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>95% Confidence Interval for B</th>
<th>Correlations</th>
<th>Collinearity Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
<td>Zero-order</td>
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<td>CCRY (Constant)</td>
<td>-300.839</td>
<td>31.816</td>
<td>-8.641</td>
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<tr>
<td>CCRY Spread With Early Default</td>
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<td>-.601</td>
<td>-27.05</td>
<td>.000</td>
<td>-.517</td>
<td>-.447</td>
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<td>CCRY Standard Default Spread</td>
<td>-317.895</td>
<td>36.054</td>
<td>-.196</td>
<td>-8.82</td>
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<td>-388.633</td>
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<td>PSY (Constant)</td>
<td>-25.348</td>
<td>5.120</td>
<td>-4.951</td>
<td>.000</td>
<td>-35.388</td>
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<tr>
<td>PSY Insolvency Put With Early Default</td>
<td>-4.909E-07</td>
<td>.000</td>
<td>-.839</td>
<td>-75.367</td>
<td>.000</td>
<td>.000</td>
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<td>PSY Spread With Early Default Option</td>
<td>9.532E-05</td>
<td>.000</td>
<td>.076</td>
<td>6.792</td>
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<tr>
<td>PSY Rate of Return on Inv. Assets</td>
<td>2.578</td>
<td>.588</td>
<td>.049</td>
<td>4.382</td>
<td>.000</td>
<td>.1424</td>
<td>3.731</td>
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<tr>
<td>PRS (Constant)</td>
<td>143.125</td>
<td>5.529</td>
<td>25.887</td>
<td>.000</td>
<td>132.283</td>
<td>153.968</td>
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<tr>
<td>PRS Insolvency Put With Early Default</td>
<td>3.189E-07</td>
<td>.000</td>
<td>.706</td>
<td>46.804</td>
<td>.000</td>
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<td>.000</td>
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<tr>
<td>PRS Spread With Early Default Option</td>
<td>-4.101E-05</td>
<td>.000</td>
<td>-.042</td>
<td>-2.788</td>
<td>.005</td>
<td>.000</td>
<td>.000</td>
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<tr>
<td>PRS Risk Financing</td>
<td>-2.021E-02</td>
<td>.004</td>
<td>-.072</td>
<td>-4.779</td>
<td>.000</td>
<td>.029</td>
<td>.012</td>
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<tr>
<td>PRS Rate of Return on Inv. Assets</td>
<td>-1.242</td>
<td>.645</td>
<td>-.029</td>
<td>-1.926</td>
<td>.054</td>
<td>-2.507</td>
<td>.023</td>
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</table>
4.3.2 Cost of Carry Aberration

The empirical analysis on the model proposed in Chapter (3) seeks to establish the linear relationship between risk premiums on policyholders’ deficit and the model. The explanatory variable used is termed the cost of carry for both portfolios with and without solvency restrictions. Models without solvency restrictions have been tested on the pricing of insurance contracts; our regression analysis incorporates the early default option arising from the provisions of solvency restrictions as an endogenous variable. The regression used to estimate the relationship between the risk-premium, cost-of carry and in general the portfolio risk is given as follows:

\[
CCRY_{kt} = \alpha + \beta_1 EDSPRE_{kt} + \beta_2 SDSPRE_{kt} + \varepsilon_{kt} \quad (4.7)
\]

Where: \( CCRY_{kt} \) = Cost of Carry for company k, in year t.

\( EDSPRE_{kt} \) = The insolvency spread with solvency restrictions to net assets for company k, in year t,

\( SDSPRE_{kt} \) = The insolvency spread without solvency restrictions to net assets for company k, in year t,

\( \varepsilon_{kt} \) = Error term for company k, in year t.

The above regression equation only shows a model for the estimation of the cost of carry or precisely the cost of risk. The variables to this equation are standard default and early default spreads, the results of which are presented in table (4.2) and (4.3). This is done in order to observe the model that provides the best explanation for insurance cash flows given the incorporation of safety margins. In the process, we are
able to test proposition (I) which states that value creation is a function of the rate at which a company borrows from its policyholder and the utilisation of generated underwriting funds. We are also able to test this proposition which attributes policyholder deficit risk premium to movements in the liability-to-asset ratio and the exogenously determined insolvency threshold, which is already, incorporated in both our standard and early defaults spreads.

The data used to run our regressions is the time-series pooled panel data for the period 1992 to 1999. The pooled data is assessed for the entire period and for all the companies with available data over the period. The software used in our analysis allows us to exclude cases on a pair-wise basis if data for either of the variables is missing. This method calculates the correlation coefficient between a pair of variables based on all cases with complete information for the two variables, regardless of whether the cases are missing for any other variables. This helps us capture data in both long-tail and short-tail business accounts since some companies only write one of these business accounts. This scenario could lead to a lot of cases being eliminated if we use listwise missing-value treatment which eliminates cases with missing values completely. The pair-wise treatment ensures that we do not eliminate too many cases unnecessarily, thus maintaining a large sample. The sample is robust and has more observations than the one used by Cummins et. al. on similar studies.

The proposition we made is that the standard default and early default spreads are negatively related to the cost of carry. This cost determines what actually should be paid to ultimate stakeholders, given the costs associated with raising investment funds and the return earned on invested assets. The estimated spreads from put option pricing
model are regressed against the cost of carry. The computation of these variables clearly shows that not only is leverage risk and asset-liability risk incorporated but also the risk associated with the solvency margins.

SIGMA used in computing volatility in both assets and liability returns includes their covariances as well. We also use risk-adjusted interest rates in our model, which incorporates the impact of growth rates in both shorttail and longtail business. What the model pointed out is that volatility increases as the company’s assets drop toward the required solvency threshold, a phenomenon common to barrier options. This has the effect of increasing the cost of borrowing funds from policyholders, and that of shareholders in the form of higher profit margins to shield against borrowing costs. The results we report in table (4.2) and (4.3) on proposition (I) are based on these two factors.

The results provide support for propositions I. As pointed out in proposition 1, the signs of the coefficients support the proposition that the cost of carry is negatively related to both the standard default and early default spreads that incorporate the cost of compliance and leverage ratios. The selection of these variables into our model was based on the criteria that enter a variable with an ‘F’ probability of $0.050 \leq F \geq 0.1$. The results reject the null hypothesis that there is no linear relationship between default spreads and the cost of carry, with an adjusted R square of 0.419 and high ‘F’ values, significant at one percent or better. It also supports the assertions that solvency thresholds are important in determining the rate of return required by equityholders. This evidence can be gleaned from the R square change, which had a 0.382 and 0.038
contribution rate to the model for early default and standard default spreads respectively.

The student 't' statistic test for the coefficients support our conclusion that default spreads contribute significantly to the cost of carry. This observation of 't' statistics for coefficients significant at 1 percent or better is consistent with our observation on contributions to risk parameter (SIGMA), from asset and liabilities accounts. We observed that on average, liability accounts were contributing 75% to SIGMA, with the remainder coming from asset accounts and the asset-liability covariance. With this we can say there is a strong link between the cost of carry and the default spreads paid to policyholders when originating insurance risks. Thus, we also confirm what has been established by the broad body of literature that the fortunes of an insurance company depend more on its underwriting philosophy than on its investment side. The main reason for this lies in their contribution to SIGMA; investments accounts contributes less because they are more diversified than liabilities accounts which tend to suffer from portfolio contamination.

These results confirm a strong link between default spreads and spreads earned by equityholders as they relate to the interaction between the cost of borrowing and investment return on these invested funds. They also support our observations that a transfer pricing system could improve profitability within an insurance company, by charging each centre that uses funds originated from underwriting insurance risk. The strong relationship between risk spreads and the cost of carry composed of the cost of float and return on invested assets, supports the proposition for transfer pricing.
systems in insurance companies. This will enable insurance companies to effectively control the cost embedded in borrowing and match these to asset returns.

Therefore, if spreads paid on liabilities on origination of business are important in determining the cost of carry, our argument for rewarding these accounts when lending originated funds to asset accounts is a noble one. Our findings justify the proposition for the reclassification of insurance liabilities into short-term, medium-term and long-term classes rather than on the perils insured basis. This we said would enhance the management of duration risk between assets and liabilities, as argued by Cummins, Allen and Phillips (1997), that, prices do not vary by line after adjusting for line-specific liability growth rates, so our classification will not alter class specific risk profiles.

With this strong link between the cost of carry and spreads paid on liability accounts, our proposed classification will enhance class performance as liabilities could easily be matched to assets based on duration and convexity profiles rather than insured risk perils. The underperformance of the insurance market against other financial institutions is anchored in the misclassification of liabilities which makes it difficult to finance them as this is based on underlying perils rather than the financial aspects of the risk. This should provide food for thought for those who are trying to securitize insurance risk, the question is should we stick to the traditional way of financing peril-based risk or the proposed duration-based classification. Our results support our proposed duration-and-convexity-based classification, because it addresses the real factors that determine the cost of carry for an insurance company.
4.3.3 Variability in Return on Policyholder Surplus

In this section we specify and test for the model which proposes that return on policyholder surplus is a function of default risk, liability spreads and return on invested assets. The reason we are using the return on policyholder surplus is enshrined in the idea that an insolvency put option is equivalent to expected policyholder deficit. What we intend to test in this section is whether the perceived policyholder deficit is a determinant of the year to year realised return on policyholder's surplus (ROPHS).

This variable is used to measure an insurance company's operational profitability, before capital gains and losses and before income tax. It is computed as given in equation (4.8) below:

\[
\text{ROPHS}_t = \frac{\text{PHOI}_t}{[\text{PHS}_{t-1} + \text{PHS}_t]/2)}
\]

Where: \(\text{ROPHS}_t = \text{Return on Policyholders' Surplus in year } t.\)

\(\text{PHS}_t = \text{Policyholders' Surplus in year } t.\)

\(\text{PHOI}_t = \text{Policyholders' Operating Income in year } t.\)

The advantage of using this variable is that it is based on the recurring internal earnings and captures the impact of items like the level and mix of business writings, its geographical orientation and regulatory environment, investment philosophy and financial market environment. It also captures other factors such as growth, taxes, expenses, persistence of reinsurance coverage, and premium and loss reserve adequacy. Operational profitability is the single most important source of surplus.
growth, an important element in providing protection against shocks from unexpected loss events. So our test on proposition number (II) seek to establish the relationship between persistent quality return on policyholders’ surplus and firm default risk, as well as spreads paid to policyholders for the perceived default risk and return on invested assets. The model is specified in equation (4.9) below:

\[ \text{ROPHS}_{kt} = \alpha + \beta_1 \text{EDPUT}_{kt} + \beta_2 \text{EDSPRE}_{kt} + \beta_3 \text{ROIA}_{kt} + \epsilon_{kt} \]  

(4.9)

Where:
- \( \text{ROPHS}_{kt} \) = Return on Policyholders’ Surplus for company \( k \), in year \( t \).
- \( \text{EDPUT}_{kt} \) = The ratio of insolvency put with solvency restrictions to net assets for company \( k \), in year \( t \).
- \( \text{ROIA}_{kt} \) = Ratio of Investment Income to Invested Assets for company \( k \), in year \( t \).
- \( \epsilon_{kt} \) = error term for company \( k \), in year \( t \).

Regression results that are reported in Table (4.2) and (4.3) are for the test of the relationship between return on policyholders’ surplus and default risk and spreads as envisaged in equation (4.9). The default put option with provisions for an early default arising from the existence of required solvency margins is used to represent the overall firm risk. This variable is a better measure of risk, because its computation is based not only on portfolio theory derived asset-liability risk, but also risk present in asset-liability gearing as it interacts with required solvency margins.

The first part of our proposition in Chapter (3) points to the importance of the asset-liability ratio (ASSLIBR) and solvency margins (SOLVR) in determining the quality of returns. Tests using ASSLIBR and SOLVR, and default spreads as
regressors were carried out to confirm whether the quality of return linearly related to these factors. The results which have not been reported in Table (4.2) and (4.3), support the hypothesis that the return on policyholder surplus is negatively related to a stochastic knock-out barrier. This observation of a negative coefficient with ‘t’ statistics significant at 1 percent or better is quite consistent with the observation from financial mathematics literature by Wilmot (1999) that, volatility increases as the asset-to-liability ratio approaches the knock out barrier.

The rationale here is that higher solvency margins lead to companies accumulating more equity to provide for the cushion, so as to maintain a safe margin away from the knockout barrier. On average, UK insurance companies maintain margins three times larger than the required solvency margins. Albeit each time the solvency margin is raised, more capital is accumulated to provide a buffer zone. That is why we are incorporating the option to an early default in our insolvency put, because risk-taking activity within an insurance company revolves around the solvency threshold. In fact companies have to tread carefully because trading close to the solvency threshold, has long-term survival and profitability ramifications.

In equation (4.9) we use the insolvency put with an option to early default, as a proxy for firm risk incorporating the leverage and solvency threshold risk. We address the second part of proposition (II), which states that the quality of operational profitability is a function of default risk as defined by the insolvency put with an option to early default. As the computation of the insolvency put shows, there is no need for us to include SOLVR and ASSLIBR as variables in the equation because they have already been catered for. We also believe that due to the number of variables
entered when computing our insolvency put, it is able to explain most of the variability in return on policyholders’ surplus. Therefore, we have only limited our independent variables to three.

We selected the independent variables in equation (4.9) using as similar technique of setting the interval for the probability of ‘F’ to fall between 0.05 and 0.1 for a point in and out respectively. The results confirm our proposition which asserts that the return on policyholders’ surplus is negatively related to overall firm default risk, incorporating both leverage and knock-out barrier risk factors. The positive coefficients for the other independent variables in Table (4.3) also confirm that there is a positive relationship between the quality of operational profitability and liability and asset spreads. The ‘t’ statistics for the coefficient are also significant at 1 percent or better, the standard errors are also very low. It is also not surprising that the insolvency put is a major contributor to the R square. The adjusted R square of 0.89 and an ‘F’ value 4847.461 supports our hypothesis, and reject the null hypothesis that there is no linear relationship. These values were either significant at 1 percent or better, confirming the linear relationship between default risk and the rate of return required by equityholders.

4.3.4 Variability in Liabilities Price per Pound of Assets Committed

In this section we proceed to test proposition (III) by using the Premium-to-Asset ratio as the independent variable. The first part of proposition (III) states that the price of liabilities is an increasing function of both the default-risk variable (ASSLIBR) and the barrier-to-asset ratio (SOLVR). Proposition (III) goes further to assert that embedded in the price of liabilities is the cost of capital required to carry risk forward
to settlement date. The cost of this capital committed to liabilities underwritten is measured by the per unit assets committed to premiums net of risk financing. This ratio is obtained by dividing net premiums underwritten to average net assets for the year, for both long- and short-tail accounts. We do not segregate between long-tail and short-tail lines because insurance prices across lines of business for a given insurer are equal when default risk and line-specific liability growth rates are controlled (Cummins et. al. 1997). The regression model linking the price of an insurance contract to capital is given by equation (4.10) below:

\[
\text{NPRASS}_{kt} = \alpha + \beta_1 \text{EDPUT}_{kt} + \beta_2 \text{ROIA}_{kt} + \beta_3 \text{EDSPRE}_{kt} + \beta_4 \text{RFIN}_{kt} + \varepsilon_{kt} \quad (4.10)
\]

Where:

- \(\text{NPRASS}_{kt}\) = Net Premium to Asset Ratio for company \(k\), in year \(t\).
- \(\text{EDPUT}_{kt}\) = The ratio of insolvency put with solvency restrictions to net assets for company \(k\), in year \(t\),
- \(\text{ROIA}_{kt}\) = Return on Invested Assets for company \(k\), in year \(t\),
- \(\text{EDSPRE}_{kt}\) = The insolvency Spread with solvency restrictions to net assets for company \(k\), in year \(t\),
- \(\text{RFIN}_{kt}\) = Risk Financing for company \(k\), in year \(t\),
- \(\varepsilon_{kt}\) = Error term for company \(k\), in year \(t\).

Similar selection procedures for the variables used in the regression models above were also adopted in this model. All the other three independent variables used in the analysis are derived as defined above. Risk financing (RFIN) is defined as the percentage of ceded reinsurance premiums to gross premiums. Data for this variable is also obtained from statutory revenue accounts recorded on the A.M. Best International
UK General Insurance Business data tapes of the year 2000. Risk financing is used in our model because it reduces the pound assets committed to liabilities, and when well-sourced it improves the strength of the balance sheet. It is also one variable looked at closely by rating agencies when determining the strength of a company's book, whether risk financing is adequate or there is over reliance on it. So we expect risk financing to play a part on determining the cost of an insurance contract per unit of assets committed.

In the same vein we expect firm default risk to continue playing a major role in determining this cost of carrying risk forward to settlement date. Our measurement method is quite crude though, because as the life of an insurance contract is reduced during the course of the year assets committed to these liabilities will also be reduced. Reduced commitment of assets at any point in time during the life of the contact means reduced cost of carrying the risk forward to settlement date. This issue was raised in our proposition, and we believe that an arbitrage situation is avoided if this condition is met. However, based on the assumption that insurance companies business does not flow in at one go, but it trickles in throughout the underwriting period, capital commitment should not change significantly from the average at any point in time. Therefore, with capital released, being committed to new risks trickling in throughout the underwriting period, our dependent variable is a good proxy for the cost of carrying the risk forward to settlement date.

The results for this regression equation are also reported in Tables (4.2) and (4.3) in the lower half. The evidence from these tests confirms our proposition that the ratio of assets committed to premiums underwritten is positively related to firm default risk.
The betas also confirm risk financing, liabilities spread and ROIA as increasing functions of premium-to-asset ratio. The student ‘t’ statistic for the coefficients for a two-tailed test is significant at 1% or better, except for ROIA which is significant at 10% level. The model is a good fit, with an adjusted R square of 0.507 significant at 1% or better for a two-tailed test.

The results establish a piece of evidence that capital commitment to liabilities underwritten is a linear function of default risk and spreads in the form of risk financing, liabilities growth rates and asset returns. The dominance of default risk is also not surprising, as this result is consistent with observations by Briys et al. (1998) that firm risk explains 78% of the ratings given to companies by rating agencies. What makes these results special though is that ours is the first empirical study on insurance data using option-pricing theory with an early default option. The results are better than those in previous studies using the standard option pricing methodology because of the increased explanatory power of our models. Our model though not directly comparable to those of Cummins, Allen and Phillips (1997), Cummins and Sommer (1996) and Sommer (1996), because we used different independent variables, different market data and sample sizes, our result looks more superior than the ones they reported, due to lower beta standard errors.

Another important piece of evidence is the positioning of risk financing in the model, it is neither dominant nor insignificant, it just occupies the middle ground it does in the day to day trading of insurance business. Insurance companies retain most of the business up to 85% for big companies and ranging from 70-80% for medium sized companies. This high retention quotient is well reflected in the contribution risk-
financing make to insurance companies' portfolio. High retentions also mean high liability risk, an issue well-captured by our model, since default risk tends to dominate the contributions to the model as evidenced in the coefficients.

4.3.5 The Implications of Autocorrelation and Multi-Collinearity

The data used in our regression analysis for testing proposition (I), (II) and (III) is time series data, hence the need to test for autocorrelation. The null hypothesis we test against is that there is autocorrelation in the error terms of our regression models. We use the Durbin-Watson statistic to test for any sequential correlation of error terms in our models. The results for the Durbin-Watson statistic test are presented in Table (4.2). We reject the null hypothesis in all the three models tested, as the values for the statistic are insignificantly different from the value of 2. A Durbin-Watson statistic with the value of 2 shows that there is no autocorrelation in the error terms, so our values of 2.001, 1.995 and 1.972 are not significantly different to 2, confirming the robustness of our results.

We also carried out statistical tests for collinearity for all the tested models. The tests for co-linearity are done in order to check for correlation in the independent variables. The null hypothesis against which this is tested against is that the significant adjusted $R^2$ are a result of correlation in the independent variables. The method we use to test for multi-collinearity is the proportion of variability not explained by the variables when the $i^{th}$ independent variable is considered the dependent variable and
the regression equation between it and the other independent variables is calculated. The proportion of variability not explained by the other variables is $1 - R_t^2$. This quantity, known as the tolerance of the variable, is the one we use to test for multicollinearity. The co-linearity statistics measuring the level of tolerance for each variable that is reported in Table (4.3) reject the null hypothesis that significant adjusted R square is a result of independent variables correlation. This also confirms the robustness of our result in that all the tolerance of the variables for all the regression models tested are all close to the value of 1.

$$
D = \frac{\sum_{i=2}^{N}(e_i - e_{i-1})^2}{\sum_{i=1}^{N}e_i^2}
$$
CHAPTER 5:

CONCLUSION AND FUTURE RESEARCH

This thesis develops a simple novel framework for measuring insurance companies' firm-wide risk that incorporates an early default option. The option pricing methodology is applied to derive the closed form valuation for overall firm risk as measured by the insolvency put option. The main advantage of using the option pricing is that it can easily be used to value insurance companies even for companies with complex financial structures and loss settlement pattern.

One important aspect yielded by this study is that firm risk measured by the insolvency put is equivalent to expected policyholder's surplus deficit an even more efficient tool for measuring risk than VaR. We show that the correlation between assets and liabilities has a significant effect on the overall risk profile of the firm, as well as the overall spread paid for originating liabilities. We also show that the model provides three primary empirical results that:

a) spreads paid for originating insurance business are negatively related to the level of cost of carry and default risk;
b) insolvency risk with an early default option is functionally related both to operational profitability and the cost of capital required to carry liabilities to settlement date; and

c) our model has many implications for hedging default risk.

The evidence from our empirical results strongly supports that the implications of this model are consistent with implicit risk levels considered by rating agencies when awarding ratings to insurance companies. In particular, the default risk explains more than 50% of the variability in insurance cash flows. There is compelling evidence that solvency consciousness in insurance companies is a major factor in determining the risk taking behaviour and the incorporation of the impact of the solvency threshold is an important step to a more comprehensive, and realistic firm-wide risk measurement technique.

Our models manage to unravel the reasons behind the tendency by insurers to maintain high asset-to-liability ratios even if their default risk is quite low. This behaviour is consistent with barrier option pricing theory which asserts that trading at assets-to-liability ratios close to the solvency threshold leads to a steep increase of volatility in the cash flows. So the desire by human capital to trade as far away from the solvency threshold usually spells this risk-taking behaviour, which provides further evidence why insurance companies accumulate rather than allocate capital to liabilities.

The implications implicit in our results are that insurance companies can measure risk inherent in their portfolios with precision provided they incorporate the stochastic
knockout barrier in their risk measurement models. We honestly believe that capital should be allocated rather than accumulated against liabilities, because such practices lead to underperformance of insurance companies, as is the case now. It is rather bizarre that the return on equity in our findings is negatively related to risk, it should be the other way round, and this evidence strongly supports the culture of accumulating capital against liabilities. Excessive capital in the insurance industry has been the major factor in poor performances, against other financial institutions where risk capital is allocated to liabilities and not accumulated. The results of the models suggest that there are great rewards to be reaped by adopting a disciplined risk management structure, ranging from measurement to hedging default risk. This will extricate insurance companies from the trap of capital accumulation, as they could easily assess their risk positions and allocate capital based on their default risk. We have shown that firm default risk is the major driver in insurance companies' operational profitability, so focusing on the elements contributing most to cash flow variability should control it.

Other important insights about our valuation of liability spreads emerge from this analysis. We show that the cost of carry is inversely related to spreads paid for originating liabilities. The evidence from this analysis provides strong support for the need for insurance companies to reclassify liability risks to bring them in line with those of other financial institutions. Our proposition is also supported by evidence in mainstream insurance literature that places liabilities as the main determinant of performance within an insurance company, with operational profitability depending on the quality of underwriting results 75-85% of the times. This piece of evidence really spells the business insurance companies are really in, the business of trading
underwriting risk. The reclassification of liabilities according to their financial nature, rather than perils insured enables insurance companies to match liabilities to asset accounts and makes it easier for a transfer pricing system to be established.

The way liability and asset spreads interact to create wealth, provides further evidence for the need to reclassify insurance liabilities. The implications of a classification structure proposed in this thesis are consistent with the current structures in the banking sector, where the classification of liabilities is based on the duration and convexity risk, and thereafter matched with appropriate asset accounts. This structure will enable insurance companies to better control the cost of borrowing investment funds from policyholders, because insurers can see what they are really paying to acquire the funds. A transfer pricing system links assets and liabilities based on their financial risk characteristics rather than insured perils. Such a structure has wider implications on the way asset and liability spreads could be controlled to improve performance and the nature of risk financing tools used to manage risk in these accounts. Since spreads determine the superiority of insurance company performance, it is not good enough to lose money in the liability accounts, to recover it from investment accounts.

Companies that have been posting good results like Swiss Re, Aegon and Directline, to mention just a few, have learnt to control their spreads through prudent underwriting, hence their good performance. Under a transfer pricing system, how much an insurance company pays to borrow investment funds, determines how much it must earn from its assets accounts in order to satisfy its shareholders. A more onerous position will be a failure to control the liabilities accounts, with the hope of
recovering from investments, because how much you borrow at determines the rate of interest they should be lend at to asset accounts, and at how much asset accounts should earn to satisfy shareholders. A transfer system injects prudence in the way liability accounts are managed, because they are the main drivers of quality operational profitability. The results provide insurance managers with great benefits to be reaped when they devise effective ways of controlling liability accounts, and a transfer pricing system is one of them. After all, there is strong evidence that most of the volatility in the firm originates from this quarter; good underwriting practice is the only way forward to reduce unnecessary excessive accumulation capital backing the liabilities. A transfer pricing system will also greatly enhance the performance of risk management techniques and the management of duration and convexity risk more effectively.

We were also able to establish a strong link between the cost of risk capital allocated to liabilities carried forward by insurance companies to settlement date and default risk. As expected from our propositions, default risk affects immensely the cost of capital used to back liabilities, with risk financing also playing a role as a means of releasing assets committed to liabilities. The contribution risk financing plays in our model is quite consistent with the risk financing policy of high retention quotients in the insurance industry. The diversity and effectiveness of risk financing techniques is not well captured in our model. However, there is strong evidence to suggest that risk financing is used for two main reasons to avoid financial distress by strengthening the balance sheet position or for pure financial reasons by reducing the cost of capital per liability commitments. By using the diverse risk financing techniques available, insurance companies are able to shore up their knockout barriers, relieving surplus in
the process and enabling the company to superficially trade as further away from the
from the solvency threshold as possible.

Risk financing tools like contingent equity puts, are geared to address the problem
of asset values encroaching into the solvency threshold after a catastrophic event. This
equity put fluffs up the company’s financial position away from the solvency threshold
as soon as the equity put is triggered. The only shortcoming of measuring the cost of
capital per liability contract is the failure by our option-pricing model to capture fat-
tails unique to insurance business. However, this does not denigrate our findings as the
model is quite robust in explaining the sole aim of our study, which is to establish the
relationship between default risk with an early default option, and components key to
operational profitability and that includes risk financing. Further, evidence is also
available from insurance risk pricing literature which shows that the option pricing
methodology outperforms any other insurance risk pricing models in its predictive
power of rates actually charged in practice.

Finally, we observed that while traditional models of valuing insurance company
risk are conceptually true, they do not go far enough to capture all the risks embedded
in insurance portfolios. The main advantage of our valuation procedure is that it
provides a solid practical basis for valuing risk in multi-line insurance companies and
can be used to provide specific solutions for asset and liability spread problems, and
better risk financing results. Through the use of this risk measurement methodology
and transfer pricing systems, companies will be able to choose the best risk financing
solutions best suited to the financial aspects of the risk rather than the effect of insured
perils. In particular, the model provides means of isolating problem areas that could
easily be tackled, by tailoring risk-financing solutions that best explain the observed properties of the underlying risk. This should be able to provide new avenues for new risk financing techniques that do not solely focus on perils but the financial aspects of insurance liabilities. We believe that future research should focus on models incorporating the jump processes, variable interest rates and the implications of a three year period model for companies operating at Lloyds. A model with jump processes will be able to explain the effect of catastrophic risks, something not captured in our model.
APPENDIX A:

INSURANCE COMPANIES USED FOR THIS STUDY

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</tr>
<tr>
<td>Aegon Insurance Co (UK) Ltd</td>
<td>Churchill Insurance Co Ltd</td>
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Sovereign Insurance (UK) Ltd

Yasuda Kasai Ins Co of Europe Ltd
Zeneca Insurance Co Ltd
Zurich GSG Ltd
Zurich Insurance Co (UK Br)
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