Export Diversification through Resource-Based Industrialization: The Case of Natural Gas☆

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Abstract

For small resource-rich developing economies, specialization in raw exports is usually considered to be detrimental to growth and Resource-Based Industrialization (RBI) is often advocated to promote export diversification. This paper develops a new methodology to assess the performance of these RBI policies. We first formulate an adapted mean-variance portfolio model that explicitly takes into consideration: (i) a technology-based representation of the set of feasible export combinations, and (ii) the cost structure of the resource processing industries. Second, we provide a computationally tractable reformulation of the resulting mixed-integer nonlinear optimization problem. Finally, we present an application to the case of natural gas, comparing current and efficient export-oriented industrialization strategies of nine gas-rich developing countries.

Keywords: OR in natural resources; OR in developing countries; mean-variance portfolio model; efficiency measure; development planning; resource-based industrialization; export earnings volatility; natural gas.

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1. Introduction

Export diversification has long been a stated policy goal for many commodity-dependent developing economies. During the last 40 years, many analysts and policy makers have also advocated export-oriented industrialization centered on primary products obtained from resource processing (e.g., ESMAP, 1997; MHEB, 2008). Natural resources generally offer multiple export-oriented monetization opportunities in addition to raw exports. For example, natural gas can be exported in a raw form using transnational pipelines or Liquefied Natural Gas (LNG) vessels but it can also be: used as a source of power in electricity-intensive activities (e.g., aluminum smelting); converted into liquid automotive fuels; or processed as a raw material for fertilizers, petrochemicals or steel.

This paper develops a methodology to assess the performance of resource-based export diversification strategies. In the case of natural gas, we observe a wide variety of possible patterns of monetization. At one extreme, Yemen has recently adopted a whole specialization strategy in raw gas exports. At the other extreme, Trinidad & Tobago experienced a total diversification through gas-processing industries during the 1980s (Gelb, 1988, p. 105). Our point of departure is the seminal contribution of Brainard and Cooper (1968), who adapt Markowitz's (1952, 1959) Mean–Variance Portfolio (hereafter MVP) theory to analyze the trade-offs of export diversification policies. On the one hand, a wisely selected export diversification may look desirable as a means of moderating the variability of export earnings. But, on the other hand, such a policy can also have a negative and substantial impact on the perceived rents if it involves shifting resources from a highly profitable industry into substantially less profitable uses.

The first contribution of this paper is to formulate an adapted MVP model that explicitly takes into consideration: (i) a technology-based representation of the set of feasible export combinations for a resource-rich developing economy, and (ii) the cost structure of the resource processing industries. Paradoxically, previous development studies based on the MVP concepts have disregarded processing costs.[1] Such an omission seems reasonable in the case of export goods with comparable production costs but can hardly be advocated when processing costs differ significantly, as is likely to be the case with resource-based industries.[2] Indeed, any optimal portfolio obtained while focusing solely on export earnings could be largely suboptimal from the perspective of a governmental planner concerned with both the variability of export earnings and the expected amount of resource rents to be perceived.

[1] To justify this omission, Bertinelli et al. (2009) underline the unavailability of complete information on the costs of producing one unit of each of the products that could be exported to the world market.

[2] In the case of natural gas, the processing costs differ significantly from one type of gas-based industry to another (Auty, 1988; ESMAP, 1997).

The second contribution focuses on the computational issues faced when applying this adapted MVP model to realistic cases. This model is a mixed-integer nonlinear optimization problem (MINLP) which is very challenging to solve. So, we detail a reformulated version of that problem that is computationally tractable in realistic cases. This reformulation makes the MVP model at hand a valuable tool for both development analysts and scholars interested in the design of an export-oriented industrialization policy in a small, open, developing economy. It is also of paramount importance for public decision makers in resource-rich countries who have to deal with politically sensitive issues concerning the monetization of national resources.

The third contribution of this paper is to detail an application to the case of natural gas. Using our MVP approach, we examine the efficient export-oriented industrialization strategies that can be implemented in a sample of nine gas-rich countries. In order to assess the performance of the industrialization strategies adopted in these countries, we also develop an adapted gauging methodology. In this case study, we make use of cost information derived from engineering studies because, despite their inherent limitations, these data reflect the information available to governmental planners (e.g., ESMAP, 1997). These engineering studies convey some interesting features of the industries under scrutiny, such as an order of magnitude for the economies of scale that can be obtained at the plant level, and the ranges of possible capacities for the processing plants. Our findings: (i) suggest that a diversification away from raw exports and toward other resource processing industries is not necessarily a *panacea*; (ii) indicate that some countries should investigate the possibility of modifying their current resource monetization strategy; and (iii) question the relevance of certain gas-based industries that have recently received an upsurge in interest. More precisely, we show that the raw exports of natural gas can provide a country with the highest level of expected returns, suggesting that any attempts to diversify the economy away from raw export using Resource-Based Industrialization (hereafter RBI) indubitably result in a lowered level of expected returns.

In finance, a voluminous Operations Research (OR) literature has examined the numerous practical problems encountered when applying portfolio theory to financial management activities.[3] In this sense, OR has played a pivotal role in the widespread use of MVP theory in the finance industry.[4] In contrast, portfolio concepts are generally overlooked by both development experts and

[3] A non-exhaustive clustering of that literature includes the contributions aimed at: (i) enriching the original MVP problem by including a dynamic framework and/or the constraints faced by real-world investors such as the need to diversify the investments in a number of sectors, the nonprofitability of holding small positions, and the constraint of buying stocks by lots (e.g.: Perold, 1984; Bonami and Leujeune, 2009); (ii) developing the quantitative methods required to solve large-scale portfolio problems (e.g.: Crama and Schyns, 2003; Bonami and Leujeune, 2009); and (iii) considering alternative risk measures such as the mean absolute deviation, a piecewise linear risk function used in Konno and Yamazaki (1991), or downside risk measures like the semi-variance (see e.g., Grootveld and Hallebach, 1999) or the Value-at-Risk that measures the worst losses which can be expected with certain probability (e.g.: Castellacci and Siclari, 2003). We also refer to the articles published in 2013 special issue of this journal "*60 Years Following Harry Markowitz's Contribution to Portfolio Theory and Operations Research*" (Zopounidis et al., 2013) for an overview of the recent research efforts in that field.

[4] Now, MVP concepts are also frequently used in the electricity sector. For example, Roques et al. (2008) apply a MVP model to determine efficient power generation portfolios in a liberalized electricity industry.

governmental planners. To our knowledge, only a handful of development economic studies have applied the MVP approach to analyze the export diversification problems found in a commodity-dependent developing economy (Love, 1979; Caceres, 1979; Labys and Lord, 1990; Alwang and Siegel, 1994; Bertinelli et al., 2009). The discussion in Alwang and Siegel (1994, p. 410) offers a credible explanation for this lack of consideration: these early portfolio studies are based on simple adaptations of the original MVP model that lack a sensible representation of the country's export possibility frontier (i.e., how the outputs of the various exporting sectors are related to each other). This is a strong limitation that: (i) questions the validity of the policy recommendations that can be prescribed from these simple MVP models and, (ii) largely explains the current lack of interest by development practitioners. To our knowledge, this paper is the first to detail an adapted MVP model that includes an enhanced representation of the export possibility frontier of a resource-rich country.

This paper is part of a limited, but rapidly growing, OR literature aimed at examining the public policy problems of developing economies (White et al., 2011). These contributions typically highlight the positive role operational researchers can play in using quantitative techniques to address crucial development issues such as: public finance and debt management (Balibek and Köksalan, 2010), health care system design (Rahman and Smith, 2000), water resource development plans (Abu-Taleb and Mareschal, 1995), infrastructure planning (Brimberg et al., 2003), natural resource policy (Kalu, 1998), or rural electrification problems (Henao et al., 2012; Ferrer-Martí et al., 2013). So far, the export planning issues faced in developing economies have received very little attention. One notable exception is Levary and Choi (1983) who designed a linear goal programming model to identify an optimal industrialization strategy for South Korea, a then-emerging nation with high population density and poor resource endowment. In the present paper, we examine the export policies that can be implemented in a resource-rich economy using an MVP approach. Our approach thus clearly differs from this older contribution both in terms of context and methodology.

The paper is organized as follows: Section 2 clarifies the background of our analysis. Section 3 presents a modified MVP model that incorporates an engineering-inspired representation of resource processing technologies. That section also examines the associated computational issues and details a reformulation of this MVP model that is computationally tractable in realistic cases. Section 4 details an application of this methodology to the case of natural gas and clarifies the implementation of the modified model. Section 5 discusses the gas-based industrialization strategies implemented in nine countries with the help of an adapted non-parametric measure of their inefficiencies. Finally, the last section offers a summary and some concluding remarks. For the sake of clarity, all the mathematical proofs are in Appendix A, and Appendix B presents a technical discussion that documents the computational gains offered by our reformulated MVP model.

2. Economic background

In this section, an overview of the existing development economics literature is provided so as to clarify the motivation of our analysis. We first highlight the negative effects of the volatility of export revenues on the development of a resource-dominated economy. Then, we justify the use of an export diversification centered on the installation of resource-based industries as a possible remedy. Lastly, we introduce the methodology that has been used to analyze export diversification policies. We also outline the practical limitations of the existing models when making policy recommendations.

2.1 The resource curse and its explanations

Experience provides numerous cases of commodity-dependent economies, particularly countries with a sizeable endowment of hydrocarbons whose economic performances are nonetheless outperformed by other resource-poor economies (Gelb, 1988; Sachs and Warner, 1995), a phenomenon coined the "resource curse". What mechanisms might explain this negative relationship between resource abundance and economic performance? Unsurprisingly, this question has motivated a rich literature (Stevens, 2003; Frankel, 2010). The proposed explanations can be roughly regrouped in two main categories. A first line of research focuses on governance issues and typically emphasizes the effects of rapacious rent-seeking, of corruption, or those of weakened institutional capacity (Ross, 1999). A second type of transmission mechanism emphasizes the importance of economic effects such as the "Dutch Disease" effect detailed in Corden and Neary (1982).

This latter category also includes recent explanations based on the volatility of primary commodity prices. The empirical analyses reported by Mendoza (1997), Blattman et al. (2007), and van der Ploeg and Poelhekke (2009) indicate that price fluctuations have a significant negative effect on growth. Several economic arguments may justify these empirical findings. For example, the literature on irreversible investment suggests that the uncertainty associated with this volatility can delay aggregate investment and thus depress growth (Bernanke, 1983; Pindyck, 1991). Alternative explanations emphasize either the influence of terms-of-trade variability on precautionary saving and consumption growth (Mendoza, 1997), or the interactions between trade specialization and financial market imperfections (Hausmann and Rigobon, 2003). Independent of the mechanism at work, these contributions indicate that the variability of natural resource revenues induced by volatile primary commodity prices could be harmful for those economies with the highest concentrations of commodity exports.

2.2 Export diversification to address export revenue volatility

Three different types of strategies can be proposed to handle these volatile export revenues. Firstly, the use of market-based financial instruments (e.g., commodity price futures, or options contracts) can make it possible to hedge commodity price risk for a given period of time. This solution has been widely advocated during the last 25 years (Borensztein et al., 2009) but the use of these instruments for macro-hedging purposes has so far remained limited.[5] A second possible strategy consists of the creation of a dedicated stabilization fund, i.e., an accumulation of precautionary savings aimed at buffering these economic variations. Yet, experience suggests that the effectiveness of stabilization funds in mitigating economic volatility is variable depending on the type of stochastic process followed by the commodity prices (Deaton, 1999).[6] Lastly, a third strategy involves the promotion of a "physical" diversification in export trade aimed at moderating the instability of the export earnings. It is supported by the empirical observations of Love (1983) which indicate a positive linkage between commodity concentration and the volatility of export earnings. According to that perspective, countries should consider the implementation of an export diversification policy.

In this paper, we focus on the export diversification policies centered on RBI. Thus, our analysis concentrates on processed primary goods and disregards the diversification policies centered on the expansion of manufactured exports. At least three lines of arguments motivate that perspective. Firstly, the "Dutch Disease" effect (Corden and Neary, 1982) may compromise the chances of a successful wave of export-oriented industrialization based on manufactured goods. Secondly, the empirical findings of Love (1983) suggest that a broad diversification into manufacturing does not necessarily lead to greater earnings stability for a commodity-dominated economy. Lastly, Owens and Wood (1997) build on the Heckscher-Ohlin (H-O) trade theory and indicate that resource-rich countries can have a comparative advantage in processed primary goods.

2.3 Designing optimal export diversification policies: an MVP approach

Following Brainard and Cooper (1968), a handful of development economic studies have applied the MVP concepts originally developed in Markowitz (1952, 1959) to identify the set of optimal export diversification policies that can be implemented in a given economy (Love, 1979; Caceres, 1979; Labys and Lord, 1990; Alwang and Siegel, 1994; Bertinelli et al., 2009). In these analyses, commodity prices are assumed to be the unique source of uncertainty and these random variables are supposed to be jointly normally distributed with known parameters (i.e., the vector of expected values and the variance-covariance matrix). The decision variables are the non-negative shares of the various products in the country's total export earnings and together constitute the country's export portfolio. The country's utility to be maximized is modeled using a mean-variance utility function that captures the trade-offs between the risks measured by the portfolio's variance and returns measured by the

[5] Several factors may explain this lack of consideration for market-based insurance techniques, including: (i) the illiquidity of futures markets at long-term maturities (Borensztein et al., 2009); (ii) the restricted range of products traded on futures markets (compared to those of exported commodities) that generates imperfect hedging strategies; or (iii) the reluctance of policy makers to adopt price insurance schemes that may put them under strong political pressure in the event of a short-term loss whereas stable earnings generally provide them with little political support (Hogan and Sturzenegger, 2010).

[6] If volume variability can be neglected and if the price of a non-renewable resource follows a random walk, the adoption of an inflexible reference price for the stabilization mechanism will eventually cause the fund to either accumulate indefinitely or become exhausted in finite time (Deaton, 1999).

expected amount of export earnings. The country's optimization program is subject to the equivalent of a budget constraint as the sum of the shares has to be equal to one. This analytical framework is thus equivalent to the standard MVP model with no riskless assets and no short sales permitted. By continuously varying the coefficient of absolute risk aversion, it is possible to determine a set of optimal portfolios and draw an efficient frontier in the plane (variance of the country's export earnings, expected value of these export earnings).

These previous studies are based on a simple adaptation of the original MVP model that lacks a detailed representation of the technology used in the export industries. As result, two strong limitations hamper the policy prescriptions that may be derived from these models. First, these studies do not take into account the production possibilities of the economy (Alwang and Siegel, 1994, p. 410). Second, the industries' cost structures are ignored in these applications of the MVP approach. This omission is seldom justified but Bertinelli et al. (2009) explicitly proposed an explanation: the unavailability of cost data for all industries. Fortunately, technology and cost information rooted in process engineering studies can be obtained for most resource-processing technologies. According to these data, resource-based industries exhibit huge differences in their processing costs (e.g., Auty, 1988, for the case of natural gas). As these differences matter for a governmental decision maker concerned with both the variability of export earnings and the expected amount of resource rents to be perceived, this paper aims to take advantage of this available information.

3. Model

This section adapts the standard MVP model to analyze the export diversification strategies focusing on RBI. Our approach can be decomposed into three successive steps. First, we outline the general setup considered in this paper. In a second step, we formulate a modified MVP model that embeds an engineering-inspired representation of the resource processing technologies. Third, we examine the computational issues associated with this modified MVP model and provide a reformulated version of that problem that is computationally tractable in realistic cases.

3.1 General setup

We consider the risk-averse government of a small, open economy endowed with a unique resource[7] and examine the government's export-oriented options to monetize that resource. In most countries, the government claims an ultimate legal title to the nation's resources, even those located in a private domain.[8] It can grant users rights as concessions if it so chooses. Nonetheless, it remains the exclusive

[7] The extension to the more general case of more than two unrelated resources (i.e., resources that can be processed using industries that have no more than one resource in their list of inputs) does not cause any conceptual difficulty.

[8] This institutional framework is very common for underground resources (both mineral and petroleum). It can also be occasionally observed with above-ground resources (a famous example is provided by the case of hydropower resources in Norway which are tightly controlled by the state).

or almost exclusive recipient of the resource rents and thus has considerable influence on the monetization of the country's domestic resources. Potentially, there are m exported goods produced domestically and derived from the processing of the country's resource. Hence, we assume that the influences of the other non-resource-based exports can be neglected so that attention can be entirely focused on the export earnings generated by these m resource-based industries. There are no joint products in these resource processing industries. We denote by $M = \{1,...,m\}$ the set of these exported goods.

The government's decision amounts to choosing a resource diversification policy for a given planning horizon, i.e., the flow of products exported during the planning horizon. We assume that the strategy selected at the beginning of this planning horizon remains unchanged to the terminal date. This assumption is coherent with the irreversible nature of the capital investments required for the implementation of a resource processing industry. During the planning horizon, the instantaneous flow of domestic resource aimed at being either exported or processed is constant and known. This simplifying assumption could easily be relaxed to deal with a known, but unsteady, pattern of resource flow during the planning horizon. This flow of resource is denoted by $PROD$.

The country in question is small and is a price taker in the sense that it is unable to influence international prices. This assumption seems appropriate for numerous resources and their associated processed primary products. The government makes its economic decisions before international prices are known. We assume away other types of uncertainty. Hence, our analysis concentrates on price risk and does not consider other technical or operational risks (e.g., through domestic input price, plant outages, construction cost overruns). Given that domestic conditions are usually better known, it seems reasonable to assume that foreign prices are less likely to be known with certainty. The international prices of the exported goods are assumed to be jointly normally distributed.

3.2 Taking processing technologies into account

For each exported good, governmental planners have to decide on an industrial configuration i.e., the number of plants to be installed and the positive resource flows to be processed in these various plants.

We now detail the technology of each resource processing industry. For an individual plant j aimed at producing the exported good i, we denote: y_{ij} the output, q_{ij} the amount of resource used as an input and x_{ij} a vector that gathers all the other inputs (capital, labor, other intermediate materials). The resource input q_{ij} and all the combinations of the other inputs x_{ij} are assumed to be perfect complements.[9] Thus, the productivity of the resource input y_{ij}/q_{ij} is equal to a constant positive

[9] Hence, we are implicitly assuming (i) that the supply of any of the other inputs is perfectly elastic and (ii) that all the other inputs are as a group separable from the resource input so that the plant's production function has the following nested form:

coefficient a_i that is invariant with the activity level y_{ij}. Using this linear relation, the plant's cost function can be reformulated as a single-variable function of the resource input q_{ij}. The present value of the total cost of installing and operating a plant capable of processing any given flow of resource q_{ij} during the planning horizon is $c_i\left(q_{ij}\right)$ where $c_i(.)$ is a positive, monotonically increasing, twice continuously differentiable, concave cost function of the variable q_{ij}.

Because of technological constraints on the feasible combinations of the other inputs x_{ij}, some lumpiness is at work at the plant level and the plant's cost function is defined on the exogenously restricted domain $\left[\underline{Q}_i,\overline{Q}_i\right]$, where $\underline{Q}_i$ (respectively $\overline{Q}_i$) is positive and represents the plant's minimum (respectively maximum) implementable size. If the output were to be null, there is no need to build a plant and we impose that $c_i(0)=0$.

We let $M_{2\underline{Q}\leq\overline{Q}}=\left\{i\in M:2\underline{Q}_i\leq\overline{Q}_i\right\}$ denote the subset of goods that can be produced using plants with a range of implementable sizes that is large enough to verify $2\underline{Q}_i\leq\overline{Q}_i$.[10] For notational simplicity, we also let $M_{2\underline{Q}>\overline{Q}}=M\setminus M_{2\underline{Q}\leq\overline{Q}}$ denote its complement. We assume that the exported goods are ordered so that any index in $\left\{1,...,\left|M_{2\underline{Q}\leq\overline{Q}}\right|\right\}$ refers to a good in $M_{2\underline{Q}\leq\overline{Q}}$ and thus $M_{2\underline{Q}>\overline{Q}}=\left\{\left|M_{2\underline{Q}\leq\overline{Q}}\right|+1,...,m\right\}$.

For a government that wishes to process a given flow of domestic resource $PROD$, these size restrictions suggest a maximum bound on the number of processing plants worth being considered. For each exported good i, there can be at most $\left\lfloor PROD/\underline{Q}_i\right\rfloor$ (where $\lfloor . \rfloor$ is the floor function) processing plants with a positive output. So, we denote $N_i=\left\{1,...,\left\lfloor PROD/\underline{Q}_i\right\rfloor\right\}$ the set of the processing plants that can be installed for each good i.

The government has a constant absolute risk aversion utility which, coupled with the normal assumption above, leads to a mean-variance utility function. Thus, we are assuming that plant-level resource processing decisions, and thus export decisions, can be derived from the following aggregate utility-maximization problem:

$y_{ij}=a_i\min\left(q_{ij},k_i\left(x_{ij}\right)\right)$ where the first stage corresponds to a Leontief fixed-proportion technology, and the second stage is described by an intermediate production function k_i that is assumed to be well-behaved (i.e., positive, monotonic, twice continuously differentiable and quasi-concave). The resource input and the bundle $k_i\left(x_{ij}\right)$ are used in fixed constant proportions and are thus perfect complements.

[10] There are numerous examples of resource processing technologies with a range of implementable sizes that verify this assumption. For example, all six of the gas-based industries considered in Section 4. A list of other examples includes: the blast furnace in the metallurgical industry, the distillation processes in oil refineries, and the steam cracking technologies used in the petrochemical industry...

Problem (P0):

$$\max \quad \overline{R}^T \left(\sum_{j \in N_i} q_{ij} \right)_{i \in M} - \sum_{i \in M} \sum_{j \in N_i} c_i \left(q_{ij} \right) - \frac{\lambda}{2} \left(\sum_{j \in N_i} q_{ij} \right)_{i \in M}^T \Phi \left(\sum_{j \in N_i} q_{ij} \right)_{i \in M} , \tag{1.a}$$

$$\text{s.t.} \quad \sum_{i \in M} \sum_{j \in N_i} q_{ij} = PROD , \tag{1.b}$$

$$q_{ij} \in \{0\} \cup \left[\underline{Q_i}, \overline{Q_i} \right], \qquad \forall i \in M \ , \ \forall j \in N_i , \tag{1.c}$$

where the vector $\left(\sum_{j \in N_i} q_{ij} \right)_{i \in M} = \left(\sum_{j \in N_1} q_{1j}, ..., \sum_{j \in N_m} q_{mj} \right)$ aggregates the plant-level resource processing decisions to provide the total resource flows transformed into each good, $\overline{R} = \left(\overline{R_1}, ..., \overline{R_m} \right)$ is the vector of expected values for the discounted sums of future unit revenues, Φ is the associated variance-covariance matrix, and λ is the coefficient of absolute risk aversion.

The objective function (1.a) captures the trade-offs between the gains in terms of reduction in export earning instability and the gains in terms of increase in the expected value of the perceived resource rents.[11] Equation (1.b) is the resource constraint. The disjunctive constraints (1.c) represent the lumpiness of the resource processing technologies.[12] Thus, the problem (P0) is a single-period mean-variance portfolio problem under separable concave transaction costs with minimal share constraints on the continuous variables.[13]

The problem (P0) is a mixed-integer nonlinear optimization problem (MINLP) that has a non-convex nature (because of the concavity of the processing cost functions). From a computational perspective, the solution of this class of problems is reputed to be very challenging (Floudas, 1995; Horst and Tuy, 1996; Floudas and Gonaris, 2009). Moreover, the size of the MINLP at hand is large as there are: $\sum_{i \in M} \left\lfloor PROD / \underline{Q_i} \right\rfloor$ non-negative variables, $\sum_{i \in M} \left\lfloor PROD / \underline{Q_i} \right\rfloor$ binary variables and $1 + 2 \sum_{i \in M} \left\lfloor PROD / \underline{Q_i} \right\rfloor$ linear constraints. To gain some insights into these computational issues, a series of experiments were conducted using small numerical instances of that problem. The results reported in Appendix B show that, *ceteris paribus*, a modest increase in the country's resource

[11] We can remark that cost and revenue are kept separated in this specification. This model thus differs from conventional portfolio selection models that commonly consider the unit expected return (i.e. the expected value of the difference between unit revenues and unit costs). However, the formulation used in these conventional MVP problems implicitly posits a cost function that exhibits constant returns to scale, an assumption that can hardly be invoked to model resource processing industries.

[12] Each of these $\sum_{i \in M} \left\lfloor PROD / \underline{Q_i} \right\rfloor$ disjunctive constraints can be implemented using a dedicated binary variable and two linear constraints.

[13] This situation offers some resemblances to Perold (1984) who discusses the case of a financial portfolio manager seeking to prevent the holding of very small active positions (because small holdings usually involve substantial holding costs while offering a limited impact on the overall performance of the portfolio).

endowment $PROD$ (from $\sum_{i\in M}\left\lfloor PROD/\underline{Q}_i \right\rfloor$=21 to $\sum_{i\in M}\left\lfloor PROD/\underline{Q}_i \right\rfloor$=28) can be sufficient to cause an explosion of the CPU time needed to solve that problem. From a practitioner's perspective, the applicability of the problem (P0) is thus extremely limited.

3.3 A computationally tractable formulation

In this subsection, we propose a reformulation of the problem (P0), with the aim at reducing the size of the problem and making it tractable in realistic numerical instances. Our approach is based on the following remark. Consider a feasible vector of plant-level resource processing decisions that satisfies the constraints (1.b) and (1.c). Conceivably, there exist several plant-level resource processing decisions $\left(q_{ij}\right)_{i\in M,\, j\in N_i}$ that: (i) verify these constraints, and (ii) provide, for each good i, the same level of the aggregate flow of resource transformed into that good $q_i = \sum_{j\in N_i} q_{ij}$. By construction, all these vectors of plant-level resource processing decisions offer the same level of expected total revenues and the same total risk. However, they can differ in terms of processing costs. According to the objective function (1.a), some of these plant-level resource processing decisions should be preferred to others: those that minimize the processing costs. As the total processing cost function is separable, a pre-identification of these cost-minimizing plant-level decisions $\left(q_{ij}\right)_{j\in N_i}$ for some goods could potentially pave the way for a more efficient specification of the MVP problem at hand. This subsection is aimed at providing such a pre-identification for all the exported goods $i \in M_{2\underline{Q}\le\overline{Q}}$.

We let q_i denote the aggregate resource flow aimed at being transformed into each exported good $i \in M_{2\underline{Q}\le\overline{Q}}$. To begin with, we are going to establish that, for any good $i \in M_{2\underline{Q}\le\overline{Q}}$, and any value $q_i \in \left[\underline{Q}_i, +\infty\right)$, it is possible to decompose that aggregate flow into at least one feasible vector of plant-level resource processing decisions. Then, we are going to provide, for any good $i \in M_{2\underline{Q}\le\overline{Q}}$, a characterization of the cost-minimizing plant-level decisions that are able to process that aggregate flow q_i. Lastly, we will show how this characterization can be used to construct a reformulated version of the MVP problem at hand.

We let $n_i : q_i \mapsto \left\lceil q_i / \overline{Q}_i \right\rceil$ where $\lceil . \rceil$ is the ceiling function. By construction, $n_i\left(q_i\right)$ provides a lower bound on the number of plants that are needed to transform the aggregate resource flow q_i.

Proposition 1: *For any good $i \in M_{2\underline{Q}\le\overline{Q}}$ and any aggregate resource flow $q_i \in \left[\underline{Q}_i, +\infty\right)$ to be processed in the country, there exists at least one vector of plant-level resource processing* *decisions* $\left(q_{ij}\right)_{j\in\{1,\dots,n_i(q_i)\}}$ *that satisfies both: (i)* $\sum_{j=1}^{n_i(q_i)} q_{ij} = q_i$, *and (ii)* $q_{ij} \in \left[\underline{Q_i}, \overline{Q_i}\right]$ *for any plant* $j \in \{1,\dots,n_i(q_i)\}$.

This proposition indicates that $n_i(q_i)$ is the smallest number of processing plants that can be installed to transform any aggregate resource flow $q_i \in \left[\underline{Q_i}, \infty\right)$ into a given good $i \in M_{2\underline{Q}\le\overline{Q}}$.

Proposition 1 also provides some insights regarding the feasibility of the problem (P0). Given the restrictions imposed by: (i) the lumpy nature of the processing technologies, and (ii) the resource equation (1.b), one could wonder whether there exists a feasible industrial configuration $\left(q_{ij}\right)_{i\in M, j\in N_i}$. The following corollary addresses this concern.

Corollary 1: *If* $M_{2\underline{Q}\le\overline{Q}} \neq \emptyset$, *for any level of the country's resource flow* $PROD$ *with* $PROD \geq \min_{i\in M_{2\underline{Q}\le\overline{Q}}} \left\{\underline{Q_i}\right\}$, *there exists at least one vector of plant-level resource processing decisions* $\left(q_{ij}\right)_{i\in M, j\in N_i}$ *that satisfies the conditions stated in equations* (1.b) *and* (1.c).

With this remark in mind, we now focus on a given good $i \in M_{2\underline{Q}\le\overline{Q}}$ and identify a cost-minimizing vector of plant-level resource processing decisions that is capable of processing any given aggregate resource flow $q_i \in \left[\underline{Q_i}, +\infty\right)$. Hereafter, we denote $r_i(q_i) = q_i - \left(n_i(q_i) - 1\right)\overline{Q_i}$ as the size of the residual plant if $n_i(q_i) - 1$ plants were to be installed with the largest implementable size.

Proposition 2: *We consider an exported good* $i \in M_{2\underline{Q}\le\overline{Q}}$ *that is processed in plants with a plant-level cost function* $c_i(x_i)$ *that satisfies the assumptions above (concavity, twice differentiability). For any flow of resource* $q_i \in \left[\underline{Q_i}, +\infty\right)$ *aimed at being transformed into good* i, *a cost-minimizing industrial configuration for that particular good has:*

- *if* $r_i(q_i) \geq \underline{Q_i}$ *:* $\left(n_i(q_i) - 1\right)$ *plants of size* $\overline{Q_i}$ *and a residual plant of size* $r_i(q_i)$ *;*
- *otherwise if* $r_i(q_i) < \underline{Q_i}$ *:* $\left(n_i(q_i) - 2\right)$ *plants of size* $\overline{Q_i}$, *a plant of size* $\underline{Q_i}$ *and a residual plant of size* $\left[r_i(q_i) + \overline{Q_i} - \underline{Q_i}\right]$.

If we denote $\delta_i(q_i)$ as an indicator function that takes the value 1 if $r_i(q_i) \geq \underline{Q_i}$ and 0 elsewhere, this proposition can be used to define $C_i\left(q_i, n_i(q_i), \delta_i(q_i)\right)$ the function that gives the minimum total cost to transform any flow of resource q_i, with $q_i \geq \underline{Q_i}$, using the industry $i \in M_{2\underline{Q}\le\overline{Q}}$:

$$C_i\left(q_i,n_i,\delta_i\right)=\begin{cases}\left(n_i-2\right).c_i\left(\overline{Q_i}\right)+\delta_i.\left[c_i\left(\overline{Q_i}\right)+c_i\left(q_i-\left(n_i-1\right).\overline{Q_i}\right)\right]\\ \quad+\left(1-\delta_i\right).\left[c_i\left(\underline{Q_i}\right)+c_i\left(q_i-\underline{Q_i}-\left(n_i-2\right).\overline{Q_i}\right)\right].\end{cases} \tag{2}$$

More importantly, this proposition suggests a simplification of the original problem (P0). Rather than using the individual plants' inputs $\left(q_{ij}\right)_{j\in N_i}$ as decision variables to model the processing of each good $i\in M_{2\underline{Q}\leq\overline{Q}}$, we can use the total flow of resources q_i aimed at being transformed into these goods together with the structure of the cost-minimizing industrial configurations provided in Proposition 1. As a result, we now propose a revised specification for the problem (P0):

Problem (P1):

$$\max \quad \overline{R}^T\begin{pmatrix} q_i \\ \sum_{j\in N_i} q_{ij}\end{pmatrix} - \left[\sum_{i\in M_{2\underline{Q}\leq\overline{Q}}}\varsigma_i\, C_i\left(q_i,n_i,\delta_i\right)+\sum_{i\in M_{2\underline{Q}>\overline{Q}}}\sum_{j\in N_i} c_i\left(q_{ij}\right)\right] - \frac{\lambda}{2}\begin{pmatrix} q_i \\ \sum_{j\in N_i} q_{ij}\end{pmatrix}^T \Phi \begin{pmatrix} q_i \\ \sum_{j\in N_i} q_{ij}\end{pmatrix} \tag{3.a}$$

$$\text{s.t.} \quad \sum_{i\in M_{2\underline{Q}\leq\overline{Q}}} q_i+\sum_{i\in M_{2\underline{Q}>\overline{Q}}}\sum_{j\in N_i} q_{ij} = PROD, \tag{3.b}$$

$$\underline{Q_i}\;\varsigma_{ij} \leq q_{ij} \leq \varsigma_{ij}\;\overline{Q_i}, \qquad \forall i\in M_{2\underline{Q}>\overline{Q}},\ \forall j\in N_i \tag{3.c}$$

$$\left(n_i-1\right)\overline{Q_i} \leq q_i \leq n_i\overline{Q_i}, \qquad \forall i\in M_{2\underline{Q}\leq\overline{Q}} \tag{3.d}$$

$$\left(n_i-1\right)\overline{Q_i}+\underline{Q_i}\;\delta_i \leq q_i \leq \left(n_i-1\right)\overline{Q_i}+\underline{Q_i}+\delta_i\overline{Q_i}, \qquad \forall i\in M_{2\underline{Q}\leq\overline{Q}} \tag{3.e}$$

$$\underline{Q_i}\;\varsigma_i \leq q_i \leq \varsigma_i\; PROD, \qquad \forall i\in M_{2\underline{Q}\leq\overline{Q}} \tag{3.f}$$

$$q_{ij}\geq 0,\ \varsigma_{ij}\in\{0,1\}, \qquad \forall i\in M_{2\underline{Q}>\overline{Q}},\ \forall j\in N_i \tag{3.g}$$

$$q_i\geq 0,\ n_i\in\mathbb{N}^*,\ \delta_i\in\{0,1\},\ \varsigma_i\in\{0,1\}, \qquad \forall i\in M_{2\underline{Q}\leq\overline{Q}} \tag{3.h}$$

where $\begin{pmatrix} q_i \\ \sum_{j\in N_i} q_{ij}\end{pmatrix}$ is used as a short notation for the stacked vector $\left(\left(q_i\right)_{i\in M_{2\underline{Q}\leq\overline{Q}}},\left(\sum_{j\in N_i} q_{ij}\right)_{i\in M_{2\underline{Q}>\overline{Q}}}\right)$. In Problem (P1), the decision variables are as follows: for each good $i\in M_{2\underline{Q}\leq\overline{Q}}$, the non-negative aggregate flow of resource q_i, a binary variable ς_i associated with the disjunctive choice "*export at least a certain amount, or not at all*"; the number of plants n_i to be implemented; and δ_i a binary variable that indicates whether $r_i\left(q_i\right)$ is larger than $\underline{Q_i}$ or not; and for each good $i\in M_{2\underline{Q}>\overline{Q}}$ and each plant $j\in N_i$, the non-negative flow of resource q_{ij} processed in that plant, the binary variable ς_{ij} indicating whether or not the flow of resource to be processed in plant j attains that plant's minimum implementable size. In this program, the objective is to maximize the value of the mean-variance utility (3.a) and this optimization program is subject to a series of linear constraints aimed at describing the set of possible export combinations. Here, (3.b) is the resource constraint. The constraints (3.c) and (3.g) model the disjunctive choice (1.c) using the binary variable ς_{ij} that is forced to be equal to 1 if and only if that plant j has a size in the feasible range $\left[\underline{Q_i},\overline{Q_i}\right]$. Thanks to constraints of type (3.d), n_i has to be related to q_i so that $n_i = \left\lceil q_i / \overline{Q_i} \right\rceil$ for any good i. The constraints (3.e) insure that the binary variable δ_i takes the value 1 if and only if $r_i(q_i) \geq \underline{Q_i}$. The constraints (3.f) force the binary variable ς_i to be equal to 1 if the country wishes to process a strictly positive flow of resources (i.e., $q_i > 0$) and guarantee that the export of product i will be impeded if the desired flow q_i is strictly less than the prescribed minimum size $\underline{Q_i}$.

Proposition 3: *Suppose that: (i) the plant-level cost functions* $c_i(x_i)$, $\forall i = 1,...,m$ *satisfy the assumptions above (concavity, twice differentiability), (ii) that* $M_{2\underline{Q}\leq\overline{Q}} \neq \varnothing$, *and (iii) that the overall flow of resource is large enough to verify* $PROD \geq \min_{i\in M_{2\underline{Q}\leq\overline{Q}}} \left\{\underline{Q_i}\right\}$. *In that case, the problem (P1) has a global solution.*

3.4 Size and computational differences

The problem (P1) provides a reformulated version of the original problem (P0). Compared to the initial problem (P0), the problem (P1) includes a modified representation of the decisions related to the exported goods i that can be produced in plants with a large range of implementable sizes that verifies $2\underline{Q_i} \leq \overline{Q_i}$. Table 1 summarizes the sizes of the two problems (P0) and (P1) and shows that the difference in size is only related to the goods in the subset $M_{2\underline{Q}\leq\overline{Q}}$.

Table 1. A comparison of the sizes of the two problems

	Problem (P0)	Problem (P1)
number of non-negative real variables	$\sum_{i\in M}\left\lfloor PROD/\underline{Q_i}\right\rfloor$	$\left\lvert M_{2\underline{Q}\leq\overline{Q}}\right\rvert + \sum_{i\in M_{2\underline{Q}>\overline{Q}}}\left\lfloor PROD/\underline{Q_i}\right\rfloor$
number of binary variables	$\sum_{i\in M}\left\lfloor PROD/\underline{Q_i}\right\rfloor$	$2\left\lvert M_{2\underline{Q}\leq\overline{Q}}\right\rvert + \sum_{i\in M_{2\underline{Q}>\overline{Q}}}\left\lfloor PROD/\underline{Q_i}\right\rfloor$
number of integer variables (binary excepted)	0	$\left\lvert M_{2\underline{Q}\leq\overline{Q}}\right\rvert$ (*)
number of equations	$1+2\sum_{i\in M}\left\lfloor PROD/\underline{Q_i}\right\rfloor$	$6\left\lvert M_{2\underline{Q}\leq\overline{Q}}\right\rvert + 1 + 2\sum_{i\in M_{2\underline{Q}>\overline{Q}}}\left\lfloor PROD/\underline{Q_i}\right\rfloor$

Note: [(*)] In the reformulated problem (P1), the constraints (3.b) and (3.d) together impose some restrictions on the values of these integer variables: $n_i \in \left\{1,\ldots,\left\lceil PROD/\overline{Q_i}\right\rceil\right\}$ for all $i \in M_{2\underline{Q}\leq\overline{Q}}$.

According to Table 1, the problem (P1) requires a smaller number of real-valued variables in the realistic cases where the plant's minimum implementable sizes of the goods in $M_{2\underline{Q}\leq\overline{Q}}$ are smaller than the country's flow to be processed $PROD$ (because $\left|M_{2\underline{Q}\leq\overline{Q}}\right| \leq \sum_{i\in M_{2\underline{Q}\leq\overline{Q}}} \left\lfloor PROD/\underline{Q_i}\right\rfloor$). Regarding the number of discrete variables, these figures suggest a tree structure with a total of $2^{\sum_{i=1}^{\left|M_{2\underline{Q}\leq\overline{Q}}\right|}\left\lfloor PROD/\underline{Q_i}\right\rfloor}$ end nodes for the problem (P0) compared to $2^{2\left|M_{2\underline{Q}\leq\overline{Q}}\right|}\prod_{i=1}^{\left|M_{2\underline{Q}\leq\overline{Q}}\right|}\left\lceil PROD/\overline{Q_i}\right\rceil$ end nodes for (P1). For very small problems (typically the ones based on a limited production level compared to the size of the plants' minimum implementable sizes), these figures are likely to remain comparable. For example, if the plants' implementable sizes verify $\left\lfloor PROD/\underline{Q_i}\right\rfloor = 2$ and $\left\lceil PROD/\overline{Q_i}\right\rceil = 1$ for any good $i \in M_{2\underline{Q}\leq\overline{Q}}$, the number of end nodes are equivalent. In contrast, the reduction in size provided by the reformulated problem (P1) becomes significant when larger production levels are considered. For example, if $\left\lfloor PROD/\underline{Q_i}\right\rfloor = 5$ and $\left\lceil PROD/\overline{Q_i}\right\rceil = 3$ for any good $i \in M_{2\underline{Q}\leq\overline{Q}}$, the problem (P1) systematically involves a smaller tree for any number of goods $\left|M_{2\underline{Q}\leq\overline{Q}}\right|$. Regarding the number of equations, a similar discussion also indicates that problem (P1) requires a smaller number of equations when considering large problems. To gain further insight into the computational performances of the two problems, a series of numerical experiments have been conducted (cf. Appendix B). Our findings confirm that the reformulated problem (P1) provides considerable reductions in solution times.

4. Application to the case of natural gas

In this section, we detail an application of the proposed methodology to assess the performances of the export-oriented industrialization possibilities offered by natural gas.

4.1 Background and data

We aim at analyzing the gas monetization strategies implemented in a sample of nine economies endowed with significant reserves of natural gas (Angola, Bahrain, Brunei, Equatorial Guinea, Nigeria, Oman, Qatar, Trinidad & Tobago, and the UAE). The gas-processing industries implemented in these countries are overwhelmingly export-oriented.

In this study, we focus on six resource-based industries that represent the major monetization options offered by natural gas and neglect the influences of other exports. The list includes: (i) the liquefaction train (a dedicated cryogenic infrastructure used to export natural gas in an LNG form); metal processing industries like (ii) aluminum smelting or (iii) iron and steel plants producing Direct

Reduced Iron (DRI); petrochemical plants that convert natural gas into (iv) diesel oil (using the so-called Gas-To-Liquid (GTL) techniques) or (v) methanol (a basic non-oil petrochemical); and (vi) fertilizer industries producing urea.

Table 2 summarizes the gas monetization strategies implemented in these countries, namely (i) the overall flow of natural gas aimed at being processed in these six export industries, and (ii) the composition of the country's portfolio. In addition, a quantitative measure of diversity may be useful for providing an overall picture of the implemented portfolio and may thus ease cross-country comparisons. Because of its simplicity, the Herfindahl-Hirschman Index (HHI), defined as the sum of the squared shares, constitutes an attractive choice. Indeed, the HHI reflects both variety (i.e., the number of industries in operation) and balance (the spread among these industries).

According to Table 2, the overall gas flows to be processed differ a great deal from one country to another but these figures remain modest as they represent less than 5% of the annual world gas production. We can notice that export diversification is at work in these countries as all of them have implemented at least two industries. Looking at the HHI scores, one may notice that the two most diversified portfolios are those implemented in the UAE and Bahrain. Interestingly, Bahrain is the only country that does not export LNG (i.e., natural gas in a liquefied form) and has thus implemented a complete diversification away from raw exports. In contrast, a significant share is allocated to LNG export facilities in all the other eight countries. In seven countries, the LNG share is around or above 75% and this preponderance largely explains their high HHI scores.

Table 2. The size and composition of the planned portfolios

	Gas flow *PROD* (MMCFD)	Allocated Shares (%)						HHI
		Aluminum smelters	GTL plants	DRI plants	LNG trains	Methanol plants	Urea plants	
Angola	938.4	16.0%	-	-	84.0%	-	-	73.2%
Bahrain	342.5	63.5%	-	14.6%	-	11.1%	10.7%	44.9%
Brunei	1 165.8	-	-	-	93.7%	6.3%	-	88.1%
Equatorial Guinea	656.8	-	-	-	85.4%	14.6%	-	75.1%
Nigeria	3 582.6	1.3%	8.9%	-	88.9%	-	0.8%	79.8%
Oman	2 016.2	4.5%	-	2.5%	83.5%	2.4%	7.1%	70.5%
Qatar	12 722.6	1.1%	13.3%	0.6%	82.8%	0.6%	1.5%	70.4%
Trinidad & Tobago	3 069.6	-	0.7%	4.7%	74.6%	18.7%	1.4%	59.4%
U.A.E.	1 454.0	29.0%	-	7.9%	53.7%	-	9.4%	38.8%

Note: For each country, this table details: (i) the overall flow of natural gas used as an input in these six processing industries measured in millions of cubic feet per day (MMCFD), (ii) the shares of this flow allocated to these industries, and (iii) the associated Herfindahl-Hirschman Index. The overall flow is that which is required for the operation of all the country's gas-processing plants at their designed capacities. It has been obtained using the gas input values that will be given in Table 3 together with a detailed inventory of the projected output capacities (in tons of output) for the processing plants already installed, those under-construction and the projects for which a "Final Investment Decision" was formally announced as of 1 January 2011. These inventories have been obtained from IHS Global Insight and project promoters.

4.2 Numerical hypotheses

We now detail and discuss the numerical assumptions used in our analysis.

a - Planning horizon and discount rates

To begin with, we clarify the chronology. Gas-based industrialization typically entails the installation of capital-intensive industries. As the corresponding investment expenditures are largely irreversible, planners have to consider an appropriately long planning horizon. We thus follow ESMAP (1997) and consider a construction time lag measured from the moment of the actual start of construction of three years followed by 25 years of operations (this latter figure is supposed to be equal to a plant's entire lifetime).

Because of this long planning horizon, the use of discounted values is required for future cash flows. The real economic discounting rate for development projects in emerging economies is country-specific and is usually evaluated by the local planning agency. However, these data are not publicly available. That's why we follow the World Bank's standard methodology for project evaluations and consider a 10% figure for the real economic discounting rate (World Bank, 2004, p.35). Sensitivity analyses of the results to both a lower (8%) and a higher (12%) cost of capital have also been carried out but did not greatly modify the conclusions. For the sake of brevity, these sensitivity results are not reported hereafter.

b - Resource extraction

In this study, the stream of future gas extraction is assumed to be imposed by exogenous geological considerations. For a given country, the flow of natural gas that will be extracted during the whole planning horizon is assumed to be known and to remain equal to $PROD$ during that horizon.[14] For each of the countries under scrutiny, we have used the flow figures listed in Table 2.

Here, the country's total extraction cost is a given that does not vary with the composition of the portfolio. Given that publicly available data on E&P costs are rather scarce (these costs vary greatly by region, by field, and scale) compared to those available in gas-processing technologies, E&P costs have been excluded from the analysis. That's why we have adopted the "netback value" approach that is commonly used in the gas industry.[15] The netback value overestimates the amount of resource rent because the E&P costs have not been deducted. However, adopting either a resource rent perspective, or a netback one for the objective function used in our MVP model has no impact on the composition of the optimal portfolios.

c - Processing technologies

In our model, there is no variability in the resource flows allocated to the various industries during the planning horizon. This assumption is coherent with the contractual features observed in the natural gas

[14] Of course, a more complex extraction profile could be considered if the appropriate data were available. Nevertheless, this so-called "plateau" profile is very common in the natural gas industry.

[15] The netback value per unit volume of gas is defined as the difference between discounted export revenues and discounted processing and shipping costs (Auty, 1988) and is often interpreted as a residual payment to gas at wellhead.

industry. Both gas fields and gas-based industries are vertically-related, specialized assets in the sense of Klein et al. (1978). Accordingly, investment in these assets generates appropriable quasi-rents and creates the possibility of opportunistic behavior in the case of separate ownership. Against this backdrop, transactions involving gas producers and gas processing industries are usually governed by long-term contracts with a very long duration that include binding "take-or-pay" clauses aimed at tightly limiting the variability of the purchased gas flow.

The sizes of the individual plants can be continuously drawn within the ranges listed in Table 3. We can remark that, for each technology i, the condition $0 < 2\underline{Q}_i \leq \overline{Q_i}$ holds. Thus, $M = M_{2\underline{Q} \leq \bar{Q}}$ in this application.

Table 3. Cost parameters for the individual gas-processing plants

Gas use *(gauging equipment)*	Gas input (Mcf/ton)	Range of implementable processing capacities (10^3 tpa) Minimum	Maximum	Investment cost function $C_i\left(q_{ij}\right) = \alpha_i . q_{ij}^{\beta_i}$ (US$ with q_{ij} in tpa) α_i	β_i	O&M cost (US$/ton)	Freight (US$/ton)	Cost of raw minerals (if any) (% of output price)
Aluminum *(line pot)*	91.13	50.00	386.00	12 424.33	0.941	745.25	28.57	26.74
Gas-to-Liquid *(Fischer-Tropsch reactor)*	71.82	110.99	838.61	3 517.74	1.000	44.40	22.20	-
Direct Reduced Iron *(shaft furnace)*	12.17	310.00	1 950.00	2 276.56	0.840	16.16	17.14	63.00
Liquefied Natural Gas *(liquefaction train)*	55.35	2 500.00	7 100.00	3 843.43	0.853	9.71	29.00	-
Methanol *(methanol reactor)*	31.76	204.00	3 400.00	3 023.30	0.875	41.00	24.00	-
Urea *(urea reactor)*	21.61	170.00	1 500.00	4 161.45	0.832	62.00	22.86	-

Note #1: tpa = tons per annum. All cost figures are in 2010 US dollars. All plants are assumed to be at a port location with adequate infrastructure. For aluminum, these figures correspond to an integrated project (smelter + gas power plant). It is also assumed that the price of alumina in US$/t is equal to 14% of those of aluminum (Rio Tinto) and, that 1.91 t of alumina is required for each t of aluminum (US DoE). For DRI, we assume that 1.5 tons of fine iron ore are required for each ton of DRI (ESMAP, 1997), and that the price of iron ore in US$/t is equal to 42% of those of scrap steel (the mean value observed during the last five years). For GTL, a conversion factor of 1 barrel of diesel oil per day = 49.33 metric tons per year has been used. For LNG, a conversion factor of 1 ton of LNG = 48.572 MMBTU has been used.

Note #2: These data have been gathered from institutions (The International Energy Agency, The Energy Sector Management Assistance Program, The U.S. Department of Energy), associations (Cedigaz, International Aluminum Institute, GIIGNL, Society of Petroleum Engineers) and companies (Qatar Fertilizer Co., HYL/Energiron, Marathon, Midrex, Rio Tinto, Sasol, Shell, Stamicarbon). The inter-industry coherence has been checked using detailed cost-engineering studies available at IFP Energies Nouvelles, a French R&D center focused entirely on the energy industries.

Concerning processing costs, project engineers typically evaluate a plant's total investment expenditure using a smoothly increasing function. The specification $c_i\left(q_{ij}\right) = \alpha_i . q_{ij}^{\beta_i}$, where q_{ij} is the processing capacity of plant j and β_i represents the (non-negative) constant elasticity of the total investment cost with respect to production, is a popular choice. With the gas-processing technologies at hand, plant-specific economies of scale are at work. Hence, $\beta_i \leq 1$ for all i. In addition, maintenance and operating (O&M) costs are assumed to vary linearly with output. This specification of the plant level cost functions is thus compatible with our modeling framework. In this study, these investment expenditures are assumed to be equally distributed during the construction period (ESMAP, 1997).

From a numerical perspective, all the results presented hereafter are derived from the figures listed in Table 3. In this study, common technologies and cost parameters have been assumed for all countries, which is consistent with the method usually applied in preliminary cost estimations of resource processing projects (e.g., ESMAP, 1997).

d - Revenues

Any application of our MVP approach requires some information on the joint distribution of the random revenues. To our knowledge, previous studies use the descriptive statistics computed from the world market price series as inputs (Brainard and Cooper, 1968; Labys and Lord, 1990; Alwang and Siegel, 1994; Bertinelli et al., 2009). Accordingly, international prices are supposed to follow a strictly stationary process and the average prices and the estimated variance-covariance matrix are directly used as proxies for the true, but unobserved, values of the expected value and the variance-covariance matrix.

However, two caveats must be mentioned. Firstly, serial correlation is frequently observed in individual commodity price series. Secondly, the commodities at hand are clearly related[16] and their price trajectories are likely to exhibit some significant co-movements. As a result, we have to look for an empirical model capable of: (i) generating individual price trajectories that are consistent with the observed dynamics, and (ii) capturing the intricate dynamic interdependences among these prices.

Table 4. The parameters of the distribution of future unit revenues

	Expected value $\bar{R}$ ($/CFD)	Standard deviation ($/CFD)
Aluminum	37.780	16.652
Diesel oil	20.982	13.698
DRI	15.317	9.678
LNG	20.229	11.892
Methanol	22.728	13.259
Urea	26.412	18.579

	Correlation Matrix					
	Aluminum	Diesel oil	DRI	LNG	Methanol	Urea
Aluminum	1.000					
Diesel oil	-0.014***	1.000				
DRI	-0.240***	0.627***	1.000			
LNG	-0.059***	0.983***	0.677***	1.000		
Methanol	0.830***	0.500***	0.263***	0.479***	1.000	
Urea	0.655***	0.524***	0.538***	0.552***	0.927***	1.000

Note: This table details the parameters of the distribution of the discounted sum of future revenues per unit of gas input. For both aluminum and DRI, these figures are net of the purchase costs of the raw mineral materials. All figures are in 2010 US dollars. *** indicates significance at the 0.01 level.

[16] Numerous linkages exist among these commodities. For example: natural gas is a major input into the production of urea or methanol. Natural gas and oil are co-products in numerous cases and gas prices are also notoriously influenced by the oil products' indexed pricing formulas used in numerous long-term importing contracts. Both aluminum smelting and steel production are well-known energy intensive activities. Besides, these two mineral commodities can be considered as imperfect substitutes in numerous end uses.

A parsimonious multivariate time-series model of the monthly commodity prices has thus been specified and estimated.[17] Monte Carlo simulations of this empirical model allow us to generate a large number (100,000) of possible future monthly price trajectories (evaluated in constant US dollars per ton of exported product). These trajectories are used in combination with a Discounted Cash Flow (DCF) model based on the assumptions detailed in Table 3 (gas input values, conversion factors, cost of raw minerals for aluminum smelting and iron ore reduction) to derive a sample of present values of the revenues obtained when processing one unit of resource with the six industries at hand. This sample is in turn used to estimate the parameters of the multivariate distribution of these present values: the expected value $\overline{R}$ and the variance-covariance matrix Φ. These values are detailed in Table 4.

4.3 The efficient frontier

All these data on both revenues (the estimated parameters $\overline{R}$ and Φ) and costs are used as inputs in our modified MVP model. Hence, we can identify the optimal portfolios of gas-processing technologies for a country that considers a given value for the coefficient of absolute risk aversion.

From a computational perspective, at least three lines of arguments indicate that we are dealing with a favorable numerical instance of the MVP problem (P1). Firstly, we can notice that $M_{2\underline{Q} \leq \overline{Q}} = M$ and that all the industries under scrutiny verify the assumptions used in Proposition 1. In such a numerical instance, the computational results in Appendix B indicate that the reformulated problem (P1) is far less demanding to solve than the original problem (P0). Secondly, the number of gas-based industries under consideration remains limited ($m = 6$) which indicates that there are only six non-negative variables in the reformulated problem (P1). Thirdly, the maximum implementable sizes of the gas-processing plants listed in Table 3 are relatively large compared to the countries' gas flows listed in Table 2. As a result, the sizes of the corresponding instances of the problem (P1) are small enough to be successfully attacked by modern global solvers such as BARON (Sahinidis, 1996; Tawarmalani and Sahinidis, 2004). Thanks to recent developments in deterministic global optimization algorithms (branch and bound algorithms based on outer-approximation schemes of the original non-convex MINLPs, range reduction techniques, and appropriate branching strategies), an accurate global solution for this problem can be obtained in modest computational time.

By varying the coefficient of absolute risk aversion, it is possible to determine the efficient frontier, i.e., the set of feasible optimal portfolios whose expected returns (i.e., the expected present values of future export earnings net of processing costs) may not increase unless their risks (i.e., their variances) increase. Hence, this approach does not prescribe a single optimal portfolio combination, but rather a set of efficient choices, represented by the efficient frontier in the graph of the portfolio

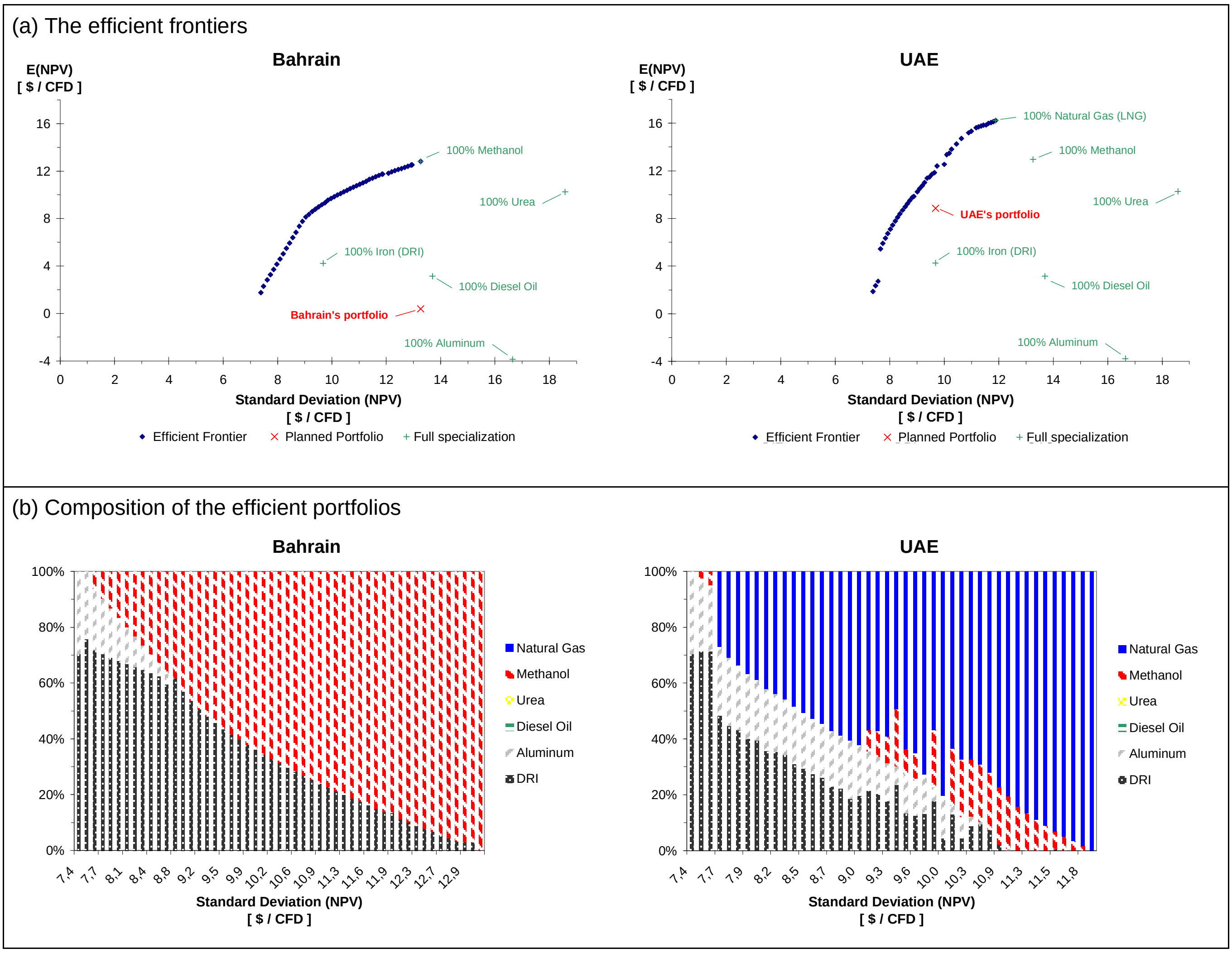
[17] The construction of this empirical model is detailed in a supplementary appendix.

expected return against the portfolio standard deviation. Depending on the country's own preferences and risk aversion, planners can choose an optimal portfolio (and thus a risk-return combination).

Figure 1.(a) shows the obtained efficient frontier for Bahrain and the UAE and Figure 1.(b) details the composition of these efficient portfolios. From Figure 1, several facts stand out. First, the efficient frontier illustrates the presence of trade-offs between risk and reward: the higher returns are obtained at a price of a larger variance. This figure also confirms that RBI-based export diversification policies cannot totally annihilate the commodity price risks as the total risk associated with a minimal risk portfolio remains strictly positive.

Second, we can notice that, contrary to the frontier obtained using the standard MVP formulation, the efficient frontiers at hand exhibit some discontinuities. Given that the modified MVP model includes some binary/integer variables, continuously varying the coefficient of absolute risk aversion from a given value to a neighboring one may cause the model to switch from an initial optimal industrial configuration (described by a combination of binary and integers) to another one that can be quite different in terms of processing costs.

Figure 1. The efficient frontier, an illustration of Bahrain and the UAE



Lastly, we can compare these two frontiers. For low levels of risks (a standard deviation lower than 7.7 $/cfd), the expected returns are similar. For these points, the composition of the efficient portfolios is similar (a combination of mineral processing activities: aluminum smelting and iron ore processing). In contrast, for large enough levels of risk (a standard deviation larger than 9 $/cfd), UAE's efficient portfolios obtain a larger expected return than those of Bahrain's. According to Figure 1.(b), these greater returns have to deal with the presence of natural gas exports in these portfolios. Interestingly, if for each technology we evaluate the range of the expected net present value of the export earnings net of processing costs in $/cfd as a function of the plant's size, we find that raw gas exports based on the LNG technology systematically provide the largest returns. Because of this absolute domination of LNG exports, the greater the appetite for returns of planners, the more LNG plants there would be in the optimal portfolio.[18] However, a full specialization in the export of LNG is not necessarily feasible because of lumpiness issues. Indeed, a comparison of the minimum implementable sizes (measured in terms of resource flow requirements) indicates that LNG export facilities have a very large-scale nature compared to alternative monetization options. So, the LNG option is only implementable in countries with sufficiently large resource endowments, which is not the case for Bahrain. As a corollary, we note that for a country with a specialized export structure fully concentrated on LNG (i.e., on raw exports of natural gas), any attempt to diversify will involve some trade-offs: a lower risk will be obtained at a price of a smaller return.

5. Policy performance appraisal

An efficient frontier graph can also be used to visually appraise the efficiency of a country's export diversification policy. For example, in Figure 1.(a), the countries' efficient frontiers are graphed together with a point representing the performances of the countries' planned portfolios in terms of risks and returns. So, a simple visual evaluation of distance from the efficient frontier provides an indication of the inefficiencies resulting from the chosen policy. To complete these visual indications, we now provide a quantitative evaluation of the efficiency of the planned portfolio.

5.1 Methodology

We use an adapted version of the non-parametric portfolio rating approach proposed in Morey and Morey (1999) that has been extended and further generalized in Briec et al. (2004), Briec et al. (2007), Briec and Kerstens (2009), and Briec and Kerstens (2010). According to this approach, the

[18] Incidentally, the fact that exports of natural gas through LNG technologies provide the largest returns explains why risk-neutral project promoters generally perceive this option as being the most attractive. As an illustration, we can quote the case of Yemen where LNG exports started in 2009 and those of Cyprus, Cameroon, Mozambique, Namibia, and Papua New Guinea where major LNG projects are actively promoted by international petroleum companies.

inefficiency of a given portfolio is evaluated by looking at the distance between that particular element in the production possibility set and the efficient frontier.

Formally, we analyze the case of a country that considers a set of exported goods $M = M_{2\underline{Q}\leq\overline{Q}}$ that can be processed using plants with a large range of possible implementable sizes. We assume that the country considers a feasible[19] gas monetization policy $q_0 = (q_{01}, ..., q_{0m})$ that has a given level of expected return E_0 and a given risk V_0. Starting from this portfolio with unknown efficiency, we apply a directional distance function that seeks to increase the portfolio's expected net present value while simultaneously reducing its risk. If we consider the direction given by the particular vector $g = (-g_V, g_E) \in (-\mathbb{R}_+) \times \mathbb{R}_+$, this distance is given by the solution of the following MINLP:

Problem (P2):

$$\max \quad \theta \tag{4.a}$$

$$\text{s.t.} \quad \sum_{i \in M_{2\underline{Q}\leq\overline{Q}}} \left[\overline{R_i} q_i - \varsigma_i \; C_i(q_i, n_i, \delta_i) \right] - g_E \; \theta \; \geq \; E_0 \tag{4.b}$$

$$\sum_{i \in M_{2\underline{Q}\leq\overline{Q}}} \sum_{i' \in M_{2\underline{Q}\leq\overline{Q}}} q_i \Phi_{ii'} q_{i'} + g_v \; \theta \; \leq \; V_0 \tag{4.c}$$

$$\sum_{i \in M_{2\underline{Q}\leq\overline{Q}}} q_i \; = \; PROD \tag{4.d}$$

$$(n_i - 1)\overline{Q_i} \; \leq \; q_i \; \leq \; n_i \overline{Q_i} \qquad \forall i \in M_{2\underline{Q}\leq\overline{Q}} \tag{4.e}$$

$$(n_i - 1)\overline{Q_i} + \underline{Q_i} \; \delta_i \; \leq \; q_i \; \leq \; (n_i - 1)\overline{Q_i} + \underline{Q_i} + \delta_i \overline{Q_i} \qquad \forall i \in M_{2\underline{Q}\leq\overline{Q}} \tag{4.f}$$

$$\underline{Q_i} \; \varsigma_i \; \leq \; q_i \; \leq \; \varsigma_i \; PROD \qquad \forall i \in M_{2\underline{Q}\leq\overline{Q}} \tag{4.g}$$

$$\theta \geq 0, \; q_i \geq 0, \; n_i \in \mathbb{N}^*, \; \delta_i \in \{0,1\}, \; \varsigma_i \in \{0,1\} \qquad \forall i \in M_{2\underline{Q}\leq\overline{Q}} \tag{4.h}$$

In this problem, the goal is to find an optimally rebalanced portfolio so as to maximize the value of the non-negative variable θ. Because of the inequalities (4.b) and (4.c), θ measures the optimal improvements that can be obtained in terms of increasing returns and decreasing risks in the direction g. Of course, such a rebalanced portfolio must be a feasible one, which means that the combination of resource flows $q = (q_1, ..., q_i, ..., q_m)$ and the associated binary and integer variables must satisfy the resource constraint (4.d) and the technological constraints (4.e)-(4.g), i.e., those already used in Problem (P1).

[19] That is, it verifies both $\sum_{i=1}^{m} q_{0i} = PROD$ and $\{i \in \{1, ..., m\}, 0 < q_{0i} < \underline{Q_i}\} = \varnothing$.

For a country that has to compare several gas monetization policies, this approach provides a simple gauging procedure: applying the same distance function to evaluate the efficiency of the proposed portfolios allows it to rate and compare the performances of the various options. Arguably, the portfolio with the smallest distance possible is deemed the best. If no improvements can be found (i.e., at the optimum, we have $\theta = 0$), then the initial portfolio q_0 belongs to the efficient frontier and is thus reputed to be efficient. Incidentally, we can remark that this program has a nonempty feasible set.[20]

5.2 Results

In applications, an arbitrary choice must be made for the direction vector g (Briec et al., 2004). In this study, we have chosen the direction $g = (0, E_0)$ which is the "return expansion" approach introduced in Morey and Morey (1999). Accordingly, the thrust is on augmenting the expected amount of perceived resource rents with no increases in the total risk. This methodology has been applied to gauge the efficiencies of the portfolios implemented in these nine countries. In Table 5, we report the obtained results: the optimal improvements and the composition of the optimally rebalanced portfolio.

Table 5. Efficiency evaluation of the export policy in terms of return expansion

	θ	Allocated shares for the rebalanced portfolio (%)						HHI
		Aluminum smelters	GTL plants	DRI plants	LNG trains	Methanol plants	Urea plants	
Angola	5.1%	-	-	13.1%	58.6%	28.3%	-	44.1%
Bahrain	3275.2%	-	-	-	-	100.0%	-	100.0%
Brunei	1.0%	-	-	-	92.3%	7.7%	-	85.9%
Equatorial Guinea	0.0%	-	-	-	85.4%	14.6%	-	75.1%
Nigeria	9.5%	0.3%	-	-	99.7%	-	-	99.3%
Oman	6.2%	-	-	3.7%	84.5%	11.7%	-	73.0%
Qatar	13.5%	-	-	-	92.3%	7.7%	0.1%	85.7%
Trinidad & Tobago	1.6%	-	-	-	64.6%	35.4%	-	54.2%
U.A.E.	37.9%	4.3%	-	20.9%	56.0%	18.8%	-	39.5%

Note: θ is the achievable improvement. These figures have been obtained using a "return expansion" direction. The allocated shares detail the composition of the portfolio that provides the optimal "return expansion" while preserving the same level of total risk. HHI is the associated Herfindahl-Hirschman Index.

Several policy recommendations can be derived from these results. First, the results obtained for Bahrain and the UAE confirm the impression derived from the visual observation of Figure 1: the chosen diversification policies exhibit significant inefficiencies. As we are dealing with industrial assets, any modification of a previously decided portfolio is likely to generate some rebalancing costs that have not been taken into consideration in this approach. Nevertheless, the magnitude of the gains

[20] The initial portfolio q_0 belongs to the feasible set. So, $\theta = 0$, and for any i: $q_i = q_{0i}$, $n_i = \lceil q_{0i} / \overline{Q_i} \rceil$, $\delta_i = 1$ if $\underline{Q_i} + (n_i - 1)\overline{Q_i} \leq q_{0i}$, and $\varsigma_i = 1$ if $q_{0i} > 0$ satisfy the conditions (4.d)-(4.h). Moreover, the expected net present value of that portfolio is E_0 and its variance is V_0 which is coherent with the satisfaction of the conditions (4.b) and (4.c).

obtained with the rebalanced portfolios is large enough to suggest that, in both countries, it might be useful to further investigate the possibility of revising the current RBI policies.

Second, a comparison of the HHI figures listed in Table 2 and Table 5 indicates that diversification is not necessarily a *panacea.* Out of these nine countries, only Angola could derive some benefit from a more diversified use of its gas as its rebalanced portfolio has both a lower HHI score and substantial gains in expected returns. By contrast, countries like Bahrain, Oman, Nigeria or Qatar could obtain substantial risk-preserving gains in expected returns by using significantly less diversified portfolios than those actually implemented. In the case of Bahrain, a complete specialization in methanol processing would even be preferred to the planned portfolio. For Nigeria, an almost complete specialization in LNG could provide a substantial gain without any impact on risk.

Third, we note that some countries like Brunei, Equatorial Guinea and Trinidad & Tobago cannot expect large gains from a return improving rebalancing of their actual portfolios. In the particular case of Equatorial Guinea, no improvement can be obtained, meaning that this country's portfolio belongs to the efficient frontier. This latter finding may be explained by the fact that, in this country, the decision to construct both an LNG train and a methanol plant resulted from an integrated planning approach. For Brunei, the improved portfolio solely involves a minor rebalancing between the shares of the chosen technologies: methanol and LNG. Concerning Trinidad & Tobago, these findings can be used to inform a local debate of the opportunity to install an aluminum smelter. During the last decade, this large project generated a controversial debate in the Caribbean nation before being officially canceled by a governmental decision in 2010. Interestingly, the government's motivation for halting this project explicitly mentioned concerns about the optimal use of the nation's gas resources. Our results indicate that aluminum smelting is not part of the country's optimal portfolio and thus provide some support for that decision.

Lastly, the relative attractiveness of the various technologies deserves a comment. We focus on the GTL technology because this extremely capital-intensive technology is experiencing an upsurge in interest. In addition to the large GTL plants recently installed in Nigeria and Qatar, several GTL projects are currently under review in Algeria, Bolivia, Egypt, Turkmenistan and Uzbekistan (MHEB, 2008; IEA, 2010, p. 145). Interestingly, our findings indicate that the GTL option is never selected in any of the optimally rebalanced portfolios listed in Table 5. Moreover, a meticulous examination of the composition of the portfolios located on the nine efficient frontiers has been carried out and has confirmed that the GTL technology has never been chosen in these efficient portfolios. The fact that the export revenues derived from this technology are highly correlated with those of LNG, though being far less lucrative, can explain these poor results. These findings may have a country-specific nature. Nevertheless, they suggest that it might be preferable to initiate some further studies aimed at meticulously assessing the economics of these GTL projects before authorizing their construction.

6. Concluding remarks

For small open economies which are unusually well-endowed with natural resources, the positive role played by export diversification in improving economic outcomes is part of the conventional wisdom among analysts, policy makers and the population at large. The MVP model presented in this paper would be useful in planning an export diversification centered on the deployment of resource-based industries. In this regard, attention is focused on the extent to which a wisely selected RBI strategy may reduce the variability of the country's export earnings and/or enhance the expected level of perceived resource rents.

From a methodological perspective, the challenge of this paper is twofold. First, it details a modified MVP model that: (i) explicitly takes into consideration the main features of resource-based industries (differences in the processing costs, existence of economies of scale at the plant level, and lumpiness), and (ii) includes an adapted representation of the country's export possibility frontier. This model is thus immune to the limitation pointed out in Alwang and Siegel (1994, p. 410). Second, this paper addresses the computational challenges associated with this MVP model and provides a reformulated version of this MVP model that is more parsimonious and thus easier to solve. Hence, we believe that this model is able to provide valuable guidance for the decision makers involved in the design of an export-oriented RBI strategy.

As an application of the methodology, the paper analyzes the optimal export portfolios that can be considered by a country endowed with significant deposits of natural gas. This study allows us to present some clarifications on the practical implementation of the proposed approach (e.g., on the modeling of the random export revenues). At an empirical level, we have evaluated the efficient export frontier of nine gas-rich economies. We observe that the countries' efficient frontier varies with the countries' endowment and that a larger endowment offers many more options for policy planners. Besides which, our findings confirm that, if technically possible, a complete specialization in liquefied natural gas systematically provides not only the highest returns, but also the highest risk among the set of efficient portfolios. In addition, we conduct a quantitative assessment of the efficiency of the export portfolio implemented in these nine countries. The results indicate that, in all countries but one (Equatorial Guinea), the RBI strategy that has been implemented is outperformed by an optimally rebalanced portfolio.

Yet, the message in this paper is broader than the findings obtained for these nine gas-rich countries. Recently, a series of significant natural gas deposits were discovered in small developing economies (e.g., Cameroon, Cyprus, Mauritania, Mozambique, Namibia, Papua New Guinea, Tanzania) generating an upsurge of interest for gas-based industrialization policies. More generally, although our application focuses on the case of natural gas, it should be clear that a similar approach could apply to other resources as well (for example: oil and petrochemicals, cotton and textile, agricultural commodities and the agro-industries).

As in any modeling effort, we made some simplifying assumptions. The three main ones are: (i) that volume variability is negligible, compared to the variability of international prices; (ii) that the flow of resource is determined exogenously without taking into consideration that sector's economics (e.g., depletion in the case of a non-renewable resource); and (iii) that diversification policies do not generate externalities, such as an increase in human capital or a promotion of learning-by-doing. Conceivably, other assumptions typically used in a MVP model can become more controversial when applying it to determine export mixes than when applying it to financial assets choice. In finance, the assumption of price-taking financial investors, for example, is usually sensible, even for deep-pocketed investors. This is less clear when analyzing the export mix of a country that controls a large share of the world's endowment of a given resource (especially, if that resource can hardly be substituted to process certain goods, and if there are little substitution possibilities for these goods on the demand side). Such a country could exert market power on the market of these processed goods, a strategic behavior which is completely omitted in the usual MVP framework. It is clearly of interest to relax these assumptions in future research.

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Appendix A – Mathematical Proofs

Proof of Proposition 1

We consider a given good $i \in M_{2\underline{Q} \le \overline{Q}}$*. Two cases have to be considered depending on whether* $q_i \in \left[\underline{Q_i}, \overline{Q_i}\right]$ *or* $q_i \in \left(\overline{Q_i}, +\infty\right)$*. A straightforward proof can be given for the case* $q_i \in \left[\underline{Q_i}, \overline{Q_i}\right]$ *as a single plant processing exactly the entire flow* q_i *verifies the two conditions. So, we now examine the case* $q_i \in \left(\overline{Q_i}, +\infty\right)$*. Given that the range of implementable sizes is large enough to verify* $2\underline{Q_i} \le \overline{Q_i}$*, it also verifies* $t\underline{Q_i} \le (t-1)\overline{Q_i}$ *for any integer* t *with* $t \ge 2$*. So, we have* $\underline{Q_i} \le \left(\left(n_i(q_i)-1\right)/n_i(q_i)\right)\overline{Q_i}$*. By definition, the integer* $n_i(q_i)$ *verifies* $\left(n_i(q_i)-1\right)\overline{Q_i} < q_i \le n_i(q_i)\overline{Q_i}$ *and thus* $\underline{Q_i} < q_i / n_i(q_i) \le \overline{Q_i}$*. So, a collection of* $n_i(q_i)$ *plants of type* i *with an equal size* $q_{ij} = q_i / n_i(q_i)$ *for any* $j \in \{1, ..., n_i(q_i)\}$ *satisfies all the conditions.* *Q.E.D.*

Proof of Corollary 1

We consider a full specialization based on the technology $i \in M_{2\underline{Q} \le \overline{Q}}$ *with the smallest implementable size, i.e.* $q_k = PROD$ *for the technology* $k = \arg\min_{i \in M_{2\underline{Q} \le \overline{Q}}} \left\{\underline{Q_i}\right\}$*. Using Proposition 1, we know that* *there exists at least one vector of plant-level resource processing decisions denoted* $\left(\tilde{q}_{kj}\right)_{j\in\{1,\dots,n_k(PROD)\}}$ *that satisfies both: (i)* $\sum_{j=1}^{n_k(PROD)} \tilde{q}_{kj} = PROD$ *, and (ii)* $\tilde{q}_{kj} \in \left[\underline{Q}_k, \overline{Q}_k\right]$ *for any plant* $j \in \{1,\dots,n_k(PROD)\}$*.*

Remarking that $\lfloor x \rfloor \geq \lceil x/2 \rceil$ *for any* $x \in [1,+\infty)$*, we have* $\left\lfloor PROD/\underline{Q}_k \right\rfloor \geq \left\lceil PROD/\left(2\underline{Q}_k\right) \right\rceil$ *(because of the condition* $PROD \geq \min_{i\in M_{2\underline{Q}\leq\overline{Q}}} \left\{\underline{Q}_i\right\}$*). As the range of implementable sizes verifies* $2\underline{Q}_k \leq \overline{Q}_k$ *, we have* $\left\lfloor PROD/\underline{Q}_k \right\rfloor \geq \left\lceil PROD/\overline{Q}_k \right\rceil = n_k(PROD)$ *which proves that* $\{1,\dots,n_k(PROD)\} \subseteq N_k$*.*

So, a multi-plant industrial configuration $\left(q_{ij}\right)_{i\in M, j\in N_i}$ *with: (i)* $n_k(PROD)$ *plants of type* k *with a size* $q_{kj} = \tilde{q}_{kj}$ *for any* $j \in \{1,\dots,n_k(PROD)\}$*, (ii)* $q_{kj} = 0$ *for any* $j \in N_k \setminus \{1,\dots,n_k(PROD)\}$*, and (ii) no processing at all for the other goods* $i \in M \setminus \{k\}$ *(i.e.,* $q_{ij} = 0$ *for any* $j \in N_i$*), satisfies all the conditions stated in equations (1.b) and (1.c).* *Q.E.D.*

Proof of Proposition 2

We consider a given good $i \in M_{2\underline{Q}\leq\overline{Q}}$*. The proof requires four independent steps.*

<u>STEP #1:</u> To begin with, we provide a cost-minimizing allocation of an exogenously determined flow of resource S_i*, with* $2\underline{Q}_i \leq S_i \leq 2\overline{Q}_i$ *, that involves exactly two plants. We consider a given pair of plants* $j \in \{1,2\}$ *, each processing a flow* $q_{ij} \in \left[\underline{Q}_i, \overline{Q}_i\right]$ *at a cost* $c_i\left(q_{ij}\right)$*. To avoid index permutations, we assume that plants are ordered in decreasing sizes. So, we are facing the following non-convex nonlinear optimization problem (NLP):*

$$\min \quad c_i\left(q_{i1}\right) + c_i\left(q_{i2}\right) \tag{5.a}$$

$$\text{s.t.} \quad q_{i1} + q_{i2} = S_i \tag{5.b}$$

$$q_{i1} \geq q_{i2} \tag{5.c}$$

$$q_{ij} \in \left[\underline{Q}_i, \overline{Q}_i\right] \qquad \forall j \in \{1,2\} \tag{5.d}$$

Using (5.b), we can reformulate this NLP as a single-variable optimization problem and let $\alpha = q_{i1}/S_i$ *be that variable. Equation (5.b) imposes that* $q_{i2} = (1-\alpha)S_i$*. Because of (5.c),* α *must verify* $\alpha \geq 1/2$*. Because of (5.d), we have* $\alpha \in \left[\underline{Q}_i/S_i, \overline{Q}_i/S_i\right]$ *and* $\alpha \in \left[1-\overline{Q}_i/S_i, 1-\underline{Q}_i/S_i\right]$*. Given that* $S_i \leq 2\overline{Q}_i$ *, we have* $1-\overline{Q}_i/S_i \leq 1/2$*. Moreover, we have* $\underline{Q}_i/S_i \leq 1/2$ *because* $2\underline{Q}_i \leq S_i$*. Accordingly, the NLP can be simplified as follows: find* $\alpha \in \left[1/2, \min\left\{\overline{Q}_i/S_i, 1-\underline{Q}_i/S_i\right\}\right]$ *that minimizes the overall cost* $c_i\left(\alpha S_i\right) + c_i\left((1-\alpha)S_i\right)$*. Given that* $S_i \leq 2\overline{Q}_i$ *and that* $2\underline{Q}_i \leq S_i$*, this latter interval is nonempty. Given* *that c_i is a twice continuously differentiable concave function, we have $c_i^{'}(\alpha S_i) \le c_i^{'}((1-\alpha)S_i)$ for any $\alpha \ge 1/2$. The derivative of $c_i(\alpha S_i)+c_i((1-\alpha)S_i)$ with respect to α (i.e., $c_i^{'}(\alpha S_i)-c_i^{'}((1-\alpha)S_i)$), is thus negative which indicates that the total cost $c_i(\alpha S_i)+c_i((1-\alpha)S_i)$ is a decreasing function of α for any $\alpha \ge 1/2$. Hence, an optimal solution α^* is given by the upper bound i.e., $\alpha^* = \min\left\{\overline{Q_i}/S_i, 1-\underline{Q_i}/S_i\right\}$. Using words, this solution is such that: (i) if the quantity to be processed is large enough (i.e., $\overline{Q_i}+\underline{Q_i} \le S_i$), we have $\alpha^* = \overline{Q_i}/S_i$ indicating that the plant $j=1$ has the maximum implementable size; (ii) otherwise (i.e., $\overline{Q_i}+\underline{Q_i} > S_i$), we have $\alpha^* = 1-\underline{Q_i}/S_i$ indicating that the plant $j=2$ has the minimum implementable size.*

As a corollary, this result indicates that, for any S_i with $2\underline{Q_i} \le S_i \le 2\overline{Q_i}$, there exists a cost-minimizing allocation of S_i between two plants that has at least one plant with a size equal to the bounds (either $\underline{Q_i}$ or $\overline{Q_i}$).

<u>STEP #2 (existence of a solution):</u> Now, we consider the number of processing plants n_i as a given parameter. We consider a flow of resource q_i with $q_i \ge \underline{Q_i}$ aimed at being processed using these n_i plants and denote q_{ij} the flow processed in plant $j \in \{1,...,n_i\}$. Furthermore, we assume that n_i is such that there exists at least one industrial configuration $(q_{ij})_{j\in\{1,...,n_i\}}$ that verifies: $q_{ij} \in \left[\underline{Q_i}, \overline{Q_i}\right]$ for any $j \in \{1,...,n_i\}$, and $\sum_{j=1}^{n_i} q_{ij} = q_i$. So, we have $n_i \in T_i$ where $T_i := \left\{n_i \in \mathbb{N} : n_i\underline{Q_i} \le q_i \le n_i\overline{Q_i}\right\}$.

The feasible set $F_{n_i} := \left\{(q_{ij})_{j\in\{1,...,n_i\}} \in \prod_{j=1}^{n_i}\left[\underline{Q_i}, \overline{Q_i}\right], \sum_{j=1}^{n_i} q_{ij} = q_i\right\}$ is closed and bounded. As $n_i \in T_i$, this set is also nonempty. Given that the total cost function $\sum_{j=1}^{n_i} c_i(q_{ij})$ to be minimized on F_{n_i} is continuous and real-valued, there exists at least one industrial configuration $(q_{ij}^{n_i})_{j\in\{1,...,n_i\}} \in F_{n_i}$ of the overall flow q_i among the n_i plants that minimizes the total cost (Weierstrass Theorem).

<u>STEP #3:</u> Now, that the existence of a cost-minimizing industrial configuration $(q_{ij}^{n_i})_{j\in\{1,...,n_i\}}$ has been established. We assume that such a configuration has at least two plants indexed k_1 and k_2 with $\underline{Q_i} < q_{ik_1}^{n_i} < \overline{Q_i}$ and $\underline{Q_i} < q_{ik_2}^{n_i} < \overline{Q_i}$. Applying the result obtained in Step #1 to the plants k_1 and k_2, it is possible to reallocate the total flow $q_{ik_1}^{n_i} + q_{ik_2}^{n_i}$ between two plants, with no increase in the total cost, so that one of the plants has a size equal to the bounds (either $\underline{Q_i}$ or $\overline{Q_i}$). Thus, for any cost-minimizing industrial configuration $(q_{ij}^{n_i})_{j\in\{1,...,n_i\}}$, we can propose an industrial configuration $(\hat{q}_{ij}^{n_i})_{j\in\{1,...,n_i\}} \in F_{n_i}$ that has at most one unique plant $k \in \{1,...,n_i\}$ processing $\hat{q}_{ik}^{n_i}$ with $\underline{Q_i} < \hat{q}_{ik}^{n_i} < \overline{Q_i}$ and that verifies

$\sum_{j=1}^{n_i} c_i\left(\hat{q}_{ik}^{n_i}\right) \leq \sum_{j=1}^{n_i} c_i\left(q_{ij}^{n_i}\right)$. *As* $\left(q_{ij}^{n_i}\right)_{j\in\{1,\dots,n_i\}}$ *is a cost-minimizing allocation, these two total costs have to be equal.*

In short, for a given $n_i \in T_i$, *there exists a feasible cost-minimizing industrial configuration that has at most one unique plant with a size that is not equal to the bounds (either* $\underline{Q_i}$ *or* $\overline{Q_i}$ *).*

<u>*STEP #4:*</u> *Here,* c_i *is a continuous single-variable concave cost function with* $c_i(0)=0$. *Thus,* c_i *is subadditive and verifies* $2c_i\left(\underline{Q_i}\right) \geq c_i\left(2\underline{Q_i}\right)$. *So, replacing two plants of minimum size* $\underline{Q_i}$ *by a single plant of size* $2\underline{Q_i}$ *is: (i) technically feasible because* $2\underline{Q_i} \leq \overline{Q_i}$, *and (ii) at least as cost-efficient.*

<u>*Conclusion:*</u> *In the preceding steps, we have shown that, for any* $n_i \in T_i$, *there is a cost-minimizing industrial configuration denoted* $\left(\hat{q}_{ij}^{n_i}\right)_{j\in\{1,\dots,n_i\}}$ *capable of processing* q_i *at a total cost* $\underline{Cost}_{n_i} = \sum_{j=1}^{n_i} c_i\left(\hat{q}_{ij}^{n_i}\right)$. *As* n_i *is in the finite set* T_i, *we can enumerate and compare these costs.*

So, we assume that a given $\hat{n}_i \in T_i$ *provides the least costly industrial configuration* $\left(\hat{q}_{ij}^{\hat{n}_i}\right)_{j\in\{1,\dots,\hat{n}_i\}}$ *: i.e., its total cost verifies* $\sum_{j=1}^{\hat{n}_i} c_i\left(\hat{q}_{ij}^{\hat{n}_i}\right) = \min_{n_i\in N_i}\left\{\underline{Cost}_{n_i}\right\}$. *If that configuration* $\left(\hat{q}_{ij}^{\hat{n}_i}\right)_{j\in\{1,\dots,\hat{n}_i\}}$ *has more than two plants with a size in the open interval* $\left(\underline{Q_i},\overline{Q_i}\right)$, *and/or more than two plants with a minimum size* $\underline{Q_i}$, *the results obtained in Steps #3 and #4 can be iteratively invoked to claim that there exists a more parsimonious feasible industrial configuration that is at least as cost-efficient (i.e., an integer* $n_i^* \in T_i$ *with* $n_i^* \leq \hat{n}_i$ *and a configuration* $\left(\hat{q}_{ij}^{n_i^*}\right)_{j\in\{1,\dots,n_i^*\}} \in F_{n_i^*}$ *that minimizes the total cost) and has at most one unique plant with a size in the open interval* $\left(\underline{Q_i},\overline{Q_i}\right)$, *and at most one unique plant with a size equal to* $\underline{Q_i}$. *Accordingly, this cost-efficient parsimonious configuration must satisfy one of these four conditions for any level* q_i *with* $q_i \geq \underline{Q_i}$ *:*

case 1: $n_i^*\overline{Q_i} = q_i$,

case 2: $\left(n_i^*-1\right)\overline{Q_i} + \underline{Q_i} = q_i$,

case 3: $\left(n_i^*-1\right)\overline{Q_i} + r_i = q_i$ *with* $\underline{Q_i} < r_i < \overline{Q_i}$,

case 4: $\left(n_i^*-2\right)\overline{Q_i} + r_i + \underline{Q_i} = q_i$ *with* $\underline{Q_i} < r_i < \overline{Q_i}$.

The value $n_i^* = \left\lceil q_i / \overline{Q_i} \right\rceil$ *together with the configurations listed in Proposition 1 systematically satisfy one of these four conditions.* *Q.E.D.*

Proof of Proposition 3

When considering the integer and binary variables (ς_{ij}), (n_i), (δ_i) *and* (ς_i) *as parameters and rearranging, this problem can be rewritten as a nonlinear optimization problem (NLP) that has an interesting form:*

$$\text{Problem } (NLP_{n,\delta,\varsigma}) \qquad \max \quad \overline{R'}^T z - Cost_{n,\delta,\varsigma}(z) - \frac{\lambda}{2} z^T \Phi' z \tag{6.a}$$

$$\text{s.t.} \qquad x \in D_{n,\delta,\varsigma} \cap S_n \tag{6.b}$$

where $z = \left((q_i)_{i \in M_{2\underline{Q} \leq \overline{Q}}}, (q_{ij})_{i \in M_{2\underline{Q} > \overline{Q}}, j \in N_i} \right)$ *is the stacked vector of all the non-negative resource processing decisions,* $\overline{R'} = \left((\overline{R}_i)_{i \in M_{2\underline{Q} \leq \overline{Q}}}, (\overline{R}_i)_{i \in M_{2\underline{Q} > \overline{Q}}, j \in N_i} \right)$ *is the stacked vector of expected revenues,* Φ' *is the associated variance-covariance matrix, and* $Cost_{n,\delta,\varsigma}(z) = \sum_{i \in M_{2\underline{Q} \leq \overline{Q}}} \varsigma_i C_i(q_i, n_i, \delta_i) + \sum_{i \in M_{2\underline{Q} > \overline{Q}}} \sum_{j \in N_i} c_i(q_{ij})$ *is the sum of twice continuously differentiable, concave, univariate functions. The set* $D_{n,\delta,\varsigma}$ *is a polytope defined by a series of linear inequalities associated with the collection of linear constraints of type (3.b), (3.e), (3.f), (3.g) and (3.h). The set* S_n *is a rectangle of upper and lower bounds on the vector* x *that corresponds to the constraints of type (3.c) and (3.d).*

If the feasible set $D_{n,\delta,\varsigma} \cap S_n$ *is nonempty, the objective function is continuous and real-valued on a closed and bounded set and thus the problem* $NLP_{n,\delta,\varsigma}$ *has a solution (Weierstrass Theorem).*

In addition, the number of combinations of integer and binary variables that have to be considered is bounded because, for any good $i \in M_{2\underline{Q} \leq \overline{Q}}$, *any integer value* n_i *larger than* $\left(\lceil PROD / \overline{Q_i} \rceil + 1 \right)$ *cannot jointly satisfy equations (3.b) and (3.d).*

Moreover, we can prove that there exists at least one combination of discrete parameters that verifies the conditions for a nonempty feasible set. As $M_{2\underline{Q} \leq \overline{Q}} \neq \emptyset$ *and* $PROD \geq \min_{i \in M_{2\underline{Q} \leq \overline{Q}}} \{\underline{Q_i}\}$, *we can consider the case of a full specialization in the good* $k = \arg\min_{i \in M_{2\underline{Q} \leq \overline{Q}}} \{\underline{Q_i}\}$. *If we consider the following list of discrete parameters: (i)* $\varsigma_{ij} = 0$ *for any good* $i \in M_{2\underline{Q} > \overline{Q}}$ *and any plant* $j \in N_j$; *(ii)* $n_i = 1$, $\delta_i = 0$, *and* $\varsigma_i = 0$ *for any good* $i \in M_{2\underline{Q} \leq \overline{Q}} \setminus \{k\}$ *together with* $n_k = \lceil PROD / \overline{Q_k} \rceil$ *plants of type* k, $\varsigma_k = 1$ *and* $\delta_k = \min\left(\left\lfloor \frac{PROD - (n_i - 1)\overline{Q_k}}{\underline{Q_k}} \right\rfloor, 1 \right)$, *then the vector* z *with* $q_{ij} = 0$ *for any good* $i \in M_{2\underline{Q} > \overline{Q}}$ *and any plant* $j \in N_j$, $q_i = 0$ *for any* $i \in M_{2\underline{Q} \leq \overline{Q}} \setminus \{k\}$ *and* $q_k = PROD$ *verifies all the conditions (3.b), (3.c),* *(3.d), (3.e), (3.f), (3.g) and (3.h). So, for these discrete parameters, the feasible set* $D_{n,\delta,\varsigma} \cap S_n$ *is nonempty.*

So, given that (i) the number of combinations that are worth being considered is bounded, and (ii) there exists at least one combination of discrete parameters that provides a real-valued solution, an enumeration of the solutions of $\left(\mathrm{NLP}_{n,\delta,\varsigma}\right)$ *for the various combinations of discrete parameters provides the global solution to the problem (P1).*

Remark that, from a computational perspective, the non-convex problem $\left(\mathrm{NLP}_{n,\delta,\varsigma}\right)$ *is a Difference of Convex (DC) programming problem (Horst and Tuy, 1996; Horst and Thoai, 1999) that has a favorable structure because: (i) the objective function to be maximized is the difference between a concave quadratic function* $\overline{R'}^T z - (\lambda/2) z^T \Phi' z$ *and a separable concave function* $Cost_{n,\delta,\varsigma}(z)$, *(ii) this cost function is a finite sum of twice continuously differentiable, monotonically increasing, concave univariate cost functions that are defined over a closed interval (cf. the rectangle above), and (iii) the constraints are the intersection of a polytope and a rectangle.*[21] *Q.E.D.*

[21] Following Falk and Soland (1969) and Konno and Wijayanayake (2001), an iterative procedure can be used to compute an ε-optimal solution to that particular DC problem. The procedure is based on the construction of a convex relaxation of the original problem (by replacing each univariate concave cost function by an underestimating envelope function which is linear and univariate). The relaxed problem is a linearly constrained, convex, quadratic programming problem and its solution provides both a lower bound (the value of the original objective function at that point) and an upper bound (the value of the relaxed objective function at that point) for the optimal objective value of the original problem (Xue and Xu, 2005). Using this framework, a branch and bound scheme can be applied to minimize the difference between these two bounds (see Xue et al., 2006). This scheme is aimed at refining the quality of the outer approximation by: (i) generating successive partitions of the initial rectangle into rectangular subsets, (ii) redefining the linear envelopes of the univariate cost functions over each of these subsets, (iii) solving the relaxed models, and (iv) recording the respective maxima. The procedure continues by partitioning the subset that corresponds to the largest maxima and again maximizing the convex relaxation to the original problem over each of the resulting subsets. As proved in Xue et al. (2006), this procedure provides an efficient computational method to obtain an ε-optimal solution to that specific DC problem.

Appendix B – Computational experiments

In this Appendix, we report a series of computational experiments conducted with the two problems (P0) and (P1). As the reformulated problem (P1) modifies the modeling of the resource processing decisions of all the goods in the subset $M_{2\underline{Q}\leq\bar{Q}}$ *but does not change those of the other goods, our discussion is centered on the polar case* $M = M_{2\underline{Q}\leq\bar{Q}}$. *The comparisons are based on a series of small diversification problems involving* $m = 3$ *goods. The two problems have been implemented in GAMS (Brook et al., 1988) and solved using the BARON solver. The data instances used for these numerical experiments are summarized in Table B.1.*

Table B.1. Data instances for the computational experiments

	Range of implementable processing capacities		Expected value	Cost function $C_i(q_{ij}) = \alpha_i . q_{ij}^{\beta_i}$		Standard deviation	Correlation matrix		
Exported Good	Minimum	Maximum	$\bar{R}$	α_i	β_i		A	B	C
A	20.0	45.0	2.00	1.50	0.90	0.10	1.00	0.20	-0.10
B	10.0	22.0	2.10	1.00	0.99	0.10	0.20	1.00	-0.05
C	5.0	12.0	2.00	1.00	0.95	0.12	-0.10	-0.05	1.00

Table B.2 presents convergence results for four instances corresponding to various production levels and assuming a value $\lambda = 1$ *for the coefficient of absolute risk aversion. A comparison of execution times required to solve the small problem* $PROD = 60$ *indicates that the two problems (P0) and (P1) are comparable (though (P1) is slightly faster). However, the CPU time needed to solve the problem (P0) increases very rapidly with the production level. Problem (P0) is solved in 58.4 minutes when considering a production level* $PROD = 100$, *and cannot be solved in 8.5 hours when an enlarged value* $PROD = 115$ *is considered. In contrast, less than two seconds are needed to obtain a converged solution when using the reformulated problem (P1). These results clearly illustrate the superiority of the reformulated specification (P1). Given that users typically have to solve a series of numerical instances of the MVP problem at hand (for example, to generate sensibility analyses or to determine the efficient frontier by varying the coefficient of absolute risk aversion to gain insights on the composition of the efficient portfolios), the reformulated problem (P1) offers a significant computational advantage.*

Table B.2. Convergence results for the numerical experiments

		Production level $PROD$			
		60.0	85.0	100.0	115.0
Problem (P0)	Number of real variables	14	21	25	28
	Number of discrete variables	14	21	25	28
	Number of end nodes for the associated tree	*16,384*	*2,097,152*	*33,554,432*	*268,435,456*
	Number of equations	29	43	51	57
	Total CPU time (s)	4.39	196.17	3,506.49	>30,600.00
	Objective value at the optimum	57.527	76.562	87.387	NA
Problem (P1)	Number of real variables	3	3	3	3
	Number of discrete variables	9	9	9	9
	Number of end nodes for the associated tree	*1,024*	*2,304*	*5,376*	*6,144*
	Number of equations	19	19	19	19
	Total CPU time (s)	0.66	0.90	1.03	1.19
	Objective value at the optimum	57.527	76.562	87.387	97.546

Note: NA indicates that the solver failed to provide a converged solution within the allotted maximum CPU time (30,600.00 s). The relevant computer specifications are: AMD Turion 64X2 TL-60, 2.00 GHz, 2 GB RAM, 32-bit OS.