Playing Outside: Excursions from the Tonality in Jazz Improvisation

Volume II of III:
Chapters 2 & 3

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Chapter 2

John Coltrane: Bars 137-172 of "Acknowledgement" from "A Love Supreme" (1964)

A journey of transpositions

This chapter concerns bars 137-172 of "Acknowledgement" of "A Love Supreme", the first part of the four-section religious suite by the John Coltrane quartet (Impulse 77, 1964). This is a relatively well-known example of a musician "playing outside" the underlying tonality and, as we shall see, has been examined by other writers. In these bars, Coltrane takes the motif central to the entire album (F, Ab, F, Bb = a {0, 3, 5} set) and, having stated it twice in the tonic key of F, proceeds to work his way through various diverse transpositions of the motif, methodically playing one transposition per bar. Finally, he returns once again to the motif in F, repeating it 8 times (see Figure 2-1). Pianist McCoy Tyner, bassist Jimmy Garrison and drummer Elvin Jones accompany. Tyner initially performs chords (mostly quartals) that resolve with Coltrane's transpositions of the motif. That this example is (a) played slowly and (b) is highly systematic makes it relatively easy to study.

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1 This short section of music has been the inspiration for much of the work that I have undertaken regarding "playing outside".
Part I: "Acknowledgement"

from "A Love Supreme"
(1964, Impulse 77)

Fig. 2-1 "Acknowledgement", bars 137-172
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Coltrane’s transposition of the motif in this way is more complex and prolonged than the mostly polarised in/out activity explored in Chapter 1, and might usefully be described as a “journey”.¹ That is to say, in bars 137-172 there are various transpositions of this motif which have a range of relationships with the underlying tonality: this is not just a “there and back again” move to the bII and return to the I, for example. I suggest that this section might be seen as a microcosm of the entire album in that it represents a kind of musical pilgrimage or is symbolic of a spiritual journey towards God and righteousness.²

Before examining this “outside” section of bars 137-172 of “Acknowledgement”, let us first consider (a) its context within the album as a whole and (b) some other pieces recorded by Coltrane in the preceding period which share some if its characteristics.

The Album

Released in 1964, “A Love Supreme” was extremely well received and, like Coltrane’s previous hit LP, “My Favorite Things” (1960), “drew buyers from beyond the jazz market”. Before the end of the 1960s it had achieved “gold” status, an incredible achievement for a jazz record in such a length of time.³ Bob Thiele saw the success of the album thus:

¹ However, there is one example from Chapter 1 which shows some similarities with Coltrane’s approach: “Latin Shuffle”, where John Medeski gradually moves away from the (black note) key of Eb minor, introducing one or two more white notes in each successive chord and then suddenly returns to the tonic scale. However, as can be seen, bars 137-172 in fact represent a sophisticated example of motivic, not spatial “out” playing.
² Coltrane explores these themes in his notes on the album cover.
“Coltrane’s album sales were around 25,000 to 50,000 over a year’s time, which was tremendous for a jazz album. *A Love Supreme* was his best seller, going into six figures. I was amazed, because I’d never heard much of his music on the radio and I wondered, “Who’s buying all these records?” Then, I visited several colleges as part of an educational package, and I saw that almost all the student musicians had Coltrane records. I think young musicians were probably the first to buy his records.”

“A Love Supreme” made full use of the LP format, introduced in 1948 by Columbia Records, being a four-part integrated suite that filled the record. In fact, its musical integration and subject mean that we might see it as a jazz “concept album”. Coltrane said:

“For the first time in my life, I have the whole album from beginning to end.”

In 1965, it was selected as “Record of the Year” (*Down Beat* Readers Poll), “Record of the Year” (*Down Beat* International Jazz Critics Poll), “Jazz Composition of the Year” and “Jazz Album of the Year” (*Jazz* Readers Poll). As a performer on the tenor and soprano saxophones Coltrane won many similar awards that year.

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4 Bob Thiele, quoted in Thomas (1975) p. 179
5 John Coltrane, quoted in Thomas (1975) p. 184
6 Coltrane’s soprano saxophone playing gained him second place in the “miscellaneous instrument” *Down Beat* Poll, a list usually exclusively populated by violin, vibraphone and harmonica players.
The popularity and influence of this album continues up to the present day: for a number of weeks in 1998 this album was Jazz FM’s (London) top selling record, and in a recent TV documentary about Coltrane many jazz artists spoke about “A Love Supreme”.7 Lewis Porter spends an entire chapter describing this work in his recent (1998), broad-ranging biography of Coltrane. Several internet sites relating to Coltrane are subtitled “A Love Supreme”, equating Coltrane’s musical career in general with the album’s title. A turn-of-the-century novel has recently (1997) been retitled “A Love Supreme”, thus encoding it with a reference to the album.8 The record has even spawned forms of religious worship in a couple of instances, mostly based upon reverence of Coltrane as an intercessor.9 We will now briefly examine analysis of bars 137-172 of “Acknowledgement” made by other authors.

“...Through all keys...”

The album’s success makes me unsurprised that, whenever I have spoken to musicians about the particular bars which we shall examine (137-172 of “Acknowledgement”) they nearly always know which section I am referring to, and many have sung the music to me. Indeed, with regard to “playing outside”, many see this section as seminal in terms of such a tradition, or crucially influential upon their own technique. We shall see, perhaps unsurprisingly, that whilst there are many examples of (motivic) “outside” playing which precede “A Love Supreme”, rarely has there been such a clearly audible or openly stated example. It seems that it was Ekkehard Jost in “Free Jazz” (1974) was the first to record that Coltrane

7 “Jazz Heroes: John Coltrane” Channel 4, broadcast June 6th 1998
8 Pauline Hopkins (1900) “Contending Forces: A Romance Illustrative of Negro Life North and South” (Schomberg Library of Nineteenth-Century Black Women Writers)
9 Porter (1998) pp. 296-297; see also below
"sequences [the central motive] through all keys."\textsuperscript{10}

For Jost, this section represents

"...an emphatic cumulation created by relatively simple means, and one of the essential expressive features of the recordings [Coltrane] made in the ensuing years."\textsuperscript{11}

Indeed, Jost goes on to compare the musical material and structure found here with later Coltrane works. However, Jost is mostly concerned with the transposition of the motif per se and not the precise list of transpositions. He gives an incomplete notation of this section, using dotted quavers and semiquavers and does not pursue an analysis of the details of the transpositions involved (p. 33). Jost describes "A Love Supreme" as

"...the consummate product of an assimilation process in which Coltrane sums up five years of musical experiences and perceptions."\textsuperscript{12}

Without providing specific evidence, here Jost makes an important point about this piece. In fact, Coltrane chose very specific, and, I will argue, quintessential, musical material as the basis for both the melodic and harmonic patterns of this piece. Jost's choice of a timeframe of five years is also important, and generally accurate, since this would take

\textsuperscript{10} Jost (1974) p. 33  
\textsuperscript{11} Jost (1974) p. 33
us back to the recording of "Kind of Blue" under Miles Davis in 1959. As we will see, there are close comparisons that may be made between these two pieces with regard to the contrast between consonant and dissonant material.

Barry Kernfeld (1988) chose Andrew White's more complete notation of this section (made in 1973), which uses more conventional jazz rhythm notation (straight quavers as swung quavers) and shows all the transpositions accurately. There are some small errors, either in Andrew White's original notation, or the copying of his work. Firstly, all anacrusis in the recording is actually in quavers: however White/Kernfeld have the first two phrases starting with semiquavers (these are, in point of fact, very quiet quavers: perhaps the relative volume of these events caused the confusion). Secondly, the final note of the last bar is an F, not a G as given.13

Kernfeld notes that:

"Motivic improvisation in jazz rarely involves the kind of systematic repetition and transposition heard in classical music...As [this notation] demonstrates. Coltrane repeats the motif more than 30 times, eventually transposing it into all 12 keys. Far more characteristically in jazz, the rhythmic and intervallic shape of a motif is not repeated literally but is subjected to the processes of variation described [above in the article]. Fine examples occur in Coltrane’s solo on So What from Miles Davis’ album Kind of Blue (1959, Columbia CL1355)."14

12 Jost (1974) p. 32
In fact, in the notated example in Kernfeld’s article the motif occurs exactly 30 times (i.e. not “…more than…”). Kernfeld is correct in pointing out the rarity of this approach. However, he distances it from the jazz tradition (and thus emphasises its shock value) by (a) comparing it directly with a Coltrane solo improvised some five years earlier (although this too centres around the minor pentatonic) and (b) suggesting that “motivic improvisation rarely involves the kind of systematic repetition and transposition heard in classical music” (p. 559). However, he seems to recognise that the exclusivity and simplicity of this section are bound up with its fame and importance to students of Coltrane: we should note that this analysis is in the section of the Jazz Grove dedicated to “Improvisation”, and is not part of the Coltrane biography in that same book.

Perhaps the most thorough analysis of this section, and indeed, of the album as a whole, occurs in Porter’s biography of Coltrane. Porter, as mentioned above, dedicates an entire chapter to “A Love Supreme”, and there gives excellent technical musical analysis of the album as well as many details regarding its recording. In this thesis I extend some of his remarks concerning interval frequency (see below) as well as refine and give detail to Coltrane’s relationship to the motif and its transposition in bars 137-172. Like Kernfeld, Porter chooses to show White’s notation, and although the F note error in Kernfeld is not there, the notation of the anacrusis is identical. Note that White’s/Porter’s bar numbers assume that bar 1 is the start of the head (I have numbered the bars from the first statement of the theme in the bass, since this is where the pulse is initiated; thus my bar 137 = Porter’s bar 121).

14 Kernfeld (1988) pp. 559-560
16 Porter (1998)
17 Chapter 17. pp. 231-249
The Chant

The insistent, steady repetition of the motif invokes the mood of a mantra to bars 137-172, and, indeed. this section is immediately followed by members of the group chanting the words “A Love Supreme”, using the notes of the motif. Porter (1998) points out that it is only when we hear this chanting that we can associate the title of the album. “A Love Supreme” with this four-note melodic pattern (F, Ab, F, Bb), although we have heard it repeated extensively in the bass.19 Ian Carr once sang the words “So What?” as he played the two chords central to that Miles Davis piece to me, but, aside from this example, I cannot think of just a title being used in this way.20 Thomas claims that this chant occurs 19 times and assigns numerological significance to this, relating it to the Kabbala:

“By the numbers, then: if you listen carefully to the continuous chanting of “a love supreme” that separates the first and second sections of the composition, you can count the repetition of this phrase nineteen times. Separate that number: one means alone: nine stands for universal. One creative man alone, either with or against the universe, but definitely of the universal consciousness. One plus nine equals ten. And, according to the Kabbala [books of Jewish mysticism], there are ten manifestations of God.”21

Thomas is almost correct to say that the motif is chanted 19 times. Careful listening reveals the chant starting quietly at bar 173, and occurring 20 times (16 times in F. 4

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18 Porter (1998) p. 242
19 Porter (1998) p. 242
20 Personal conversation at a workshop in 1983 at the University of Brighton
21 Thomas (1975) p. 185
times in Eb). The following plan shows clearly how the chant is divided, like much of the previous music, into groups of four bars:

<table>
<thead>
<tr>
<th>bar:</th>
<th>173</th>
<th>177</th>
<th>181</th>
<th>185</th>
<th>189</th>
</tr>
</thead>
<tbody>
<tr>
<td>key:</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td>Eb</td>
</tr>
<tr>
<td>count:</td>
<td>/</td>
<td>///</td>
<td>/</td>
<td>///</td>
<td>/// (= 20)</td>
</tr>
</tbody>
</table>

Table 2-1 Four-bar Structure of bars 137-172

This is clearly a natural extension of the (immediately preceding) 8-bar vamp on the motif in F (bars 165-172), and, indeed, the whole of bars 137-172, which divides neatly into 4-bar sections. This has a clear precedent in the (conventional 4-bar structure of the) Head of the piece. Although I disagree with Thomas’ analysis of this section, I would agree with his implied opinion that there may be a self-conscious mathematical dimension to Coltrane’s compositional choices in constructing this spiritual piece, and, indeed, that it may also contain encoded information directly describing Coltrane’s views regarding the individual and the universal. Cole (1976) also spends much time exploring Coltrane’s interest in numerology.

The “out” section at bars 137-172 of “Acknowledgement” is built upon the \{0, 3, 5\} set of the motif, and, as we will now see, this set, along with \{0, 5, 7\} are central to both the album in general, and other themes of pieces composed and recorded by Coltrane just before “A Love Supreme”.
Part I: “Acknowledgement”

Introduction

As a rubato introduction to the piece, Coltrane uses the notes B, E, F#, and an upper B to form a rapid, shifting Bsus4 arpeggio over an E bass; the same notes are used by McCoy Tyner to generate chords beneath this arpeggio.

\[
\text{Bsus}^4
\]

Fig. 2-2 Bsus4 arpeggio

Note that this set \{0, 5, 7\} also opens the head that follows immediately afterwards, but a semitone higher: \{C, F, G, C\}. This introductory section has a dual function. Let us note that the key of E is a relatively rare/difficult key on the tenor saxophone, and that the introduction is not in the same key as the rest of the piece. In this light, I suggest that the \{E, F#, B\} set is an autobiographical reference to what was at this time the greatest (financial) success in Coltrane’s career, “My Favorite Things” (1960), in that this set in this key forms the theme of this piece:

\[
\text{Em}^5
\]

\[
\text{C}^5
\]

Fig. 2-3 Theme of “My Favorite Things”
Indeed, at the time of the writing of "A Love Supreme", Coltrane had spoken of trying to create "...another 'My Favorite Things'". That Coltrane chooses this part of "My Favorite Things" is partly because it is the opening theme of that piece, but also because of its high frequency of use in music in general: this is a theme I will develop below. Indeed, this set is not only a reference to a piece that Coltrane played: it is also a more detailed autobiographical reference to a way of working that Coltrane had undertaken for many years.

"[Coltrane] is mostly self-taught as a writer, and generally starts his work at the piano. "I sit there and run over chord progressions and sequences, and eventually I get a song - or songs - out of each little problem."" 22

Further, Demsey (1991) has shown that Coltrane was interested in the mathematical integrity of musical patterns to a great extent, and that these formed the basis of much of his work, including the Major 3rd interval structure at the centre of "Giant Steps" (and other pieces); the cover of this album is where the above quotation is taken.

At the end of this rubato introduction, the bass begins to play the four-note motif. the drums enter and the head begins.
Part I: "Acknowledgement"

from "A Love Supreme" (1964, Impulse 77) J. Coltrane

Tenor Saxophone

Head 17

T. Sax.

T. Sax.

T. Sax.

T. Sax.

T. Sax.

T. Sax.

T. Sax.

T. Sax.

Fig. 2-4 Head of "Acknowledgement"
Head

Examining Fig. 2-4, we can see that the melody of the 16-bar head is essentially constructed from the \( \{0, 5, 7\} \) set used in the introduction (e.g. bars 16 and 17), and a new one, sourced in the bass line: \( \{0, 3, 5\} \) (first heard at the end of bar 22). This \( \{0, 3, 5\} \) set is to be the central motif of the album, until Part IV, where these two sets are altered/combined to become the minor triad \( \{0, 3, 7\} \) (this replacement of the conventionally dissonant IV by the V may be read as symbolic of the "completion" of the spiritual quest that is "A Love Supreme"). All other material (i.e. interpolations and decorations) in the head which is not derived from these sets may be seen to belong to F minor pentatonic (i.e. the entire head is in F minor pentatonic, apart from the G notes found in the opening C, G, F motif). Before examining how Coltrane uses this material beyond the head, let us examine these two sets in more detail.

Characteristics of the \( \{0, 3, 5\} \) and \( \{0, 5, 7\} \) sets

Even a casual examination of these sets shows them to be at the heart of melodic and harmonic structure in much blues and jazz music. \( \{0, 3, 5\} \) is a member of a limited set of three-note sets that occur in (all modes of) the Major. Jazz melodic. Harmonic minor. Pentatonic, Blues and diminished scales:
set: notes:

\((0, 2, 5) = \text{inversion of } (0, 3, 5)\)

\((0, 3, 5)\)

\((0, 3, 6) = \text{diminished triad}\)

\((0, 3, 7) = \text{minor triad}\)

\((0, 4, 7) = \text{Major triad}\)

Table 2-2 Three-note Subsets common to Commonly-used Scales

Similarly, \((0, 5, 7)\) can be added to this list if we ignore the diminished scale. In a sense, then, the \((0, 3, 5)\) and \((0, 5, 7)\) sets might be regarded as common triads.\(^1\) Now let us note a simple theory that shows a connection between the \((0, 3, 5)\) and \((0, 5, 7)\) sets.

There are only a few 3-note subsets of the minor pentatonic scale that can sum to that scale after a single transposition, and, of these, only \((0, 3, 5)\), its inversion \((0, 2, 5)\), and \((0, 5, 7)\) do not add new notes:

<table>
<thead>
<tr>
<th>3-note subset</th>
<th>transposition(s) to sum to C minor pentatonic</th>
<th>sets:</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1)) C Eb Bb</td>
<td>G Bb F</td>
<td>((0, 2, 5))</td>
</tr>
<tr>
<td>((2)) C Eb F</td>
<td>G Bb C</td>
<td>((0, 3, 5))</td>
</tr>
<tr>
<td>((3)) C Eb G</td>
<td>Bb Db F</td>
<td>((0, 3, 7)); adds Db</td>
</tr>
<tr>
<td>((4)) C F G</td>
<td>(a) Eb Ab Bb</td>
<td>((0, 5, 7)); adds Ab</td>
</tr>
<tr>
<td></td>
<td>(b) Bb Eb F</td>
<td>((0, 5, 7))</td>
</tr>
</tbody>
</table>

Table 2-3 Three-note Subsets summing to the minor pentatonic

This theory separates these two sets from all other 3-note subsets of the minor pentatonic scale, and has a direct correlation with the summing of two (minor) pentatonic chords to form the Dorian mode that I described in my analysis of "So What?" in Chapter 1.
above. Porter (1998) notes that the minor pentatonic scale can be seen as being constructed of two \{0, 3, 5\} sets, and is thus easy to learn/transpose.\(^2\)

Here are a few more points which may have added to the appeal of these sets to Coltrane. Firstly, both of these sets can be seen as possible phrases starting on the I of the (commonly-used) minor pentatonic. Blues scale and Dorian mode (but not the (classical) Major scale). Secondly, the \{0, 5, 7\} set is at the heart of all quartal harmony used extensively by the pianist, McCoy Tyner (occurring previously to "A Love Supreme" in, for example, Bill Evans' piano work behind Miles Davis and Coltrane on the album "Kind of Blue" (1959): see the analysis of "So What?" in Chapter 1. above). Thirdly, \{0, 3, 5\} is the set that "connects" the upper notes of a 7 and m7 chord voicing (e.g. G, Bb, C as V bVII and I of C7 and Cm7). Lastly, the \{0, 3, 5\} and \{0, 5, 7\} sets share elements 0 and 5, i.e. the I and IV so frequently used in blues, and, transposed, the II V progression central to bebop. In this regard, let us recall that Coltrane wrote many pure, almost minimalist blues and was a bebop musician before he joined Miles Davis in the late 1950s.

As we have just seen, the minor pentatonic scale, used so much by Coltrane and other jazz musicians, can be regarded as being constructed of two \{0, 3, 5\} sets. This structural relationship is expressed by Coltrane throughout Part I and, being an equivalent of the I > IV relationship at the centre of the blues, forms the emphatic theme for Part III, "Pursuance", essentially a blues in Bb. Note that, in Table 2-3 above, (2) results in a duplicate I, whereas (4b) results in a duplicate IV (these marked in italics).

\(^1\) Haerle (1980) includes the sus4 chord (= \{0, 5, 7\}) as a common triad since "...it occurs so frequently [in jazz] that it might almost be considered a [basic] type" (p. 7)
\(^2\) Porter (1998) pp. 233-234. The minor pentatonic scale had become a core element in Coltrane's improvisational approach well before "A Love Supreme", as we have seen in the previous chapter.
This is analogous to the key duality inherent to the \{0, 3, 5\} set (i.e. any \{0, 3, 5\} set belongs to two keys of the minor pentatonic scale).

Thus, we can say that these sets are appropriate choices for a piece that is (a) a summary of jazz music and (b) as autobiographical as "A Love Supreme". Further, what Coltrane is achieving is the dislocation of these blues sets from a lyric/melodic association characterised by the viewpoint that the blues is all about, for example, "...women and work..." and the construction of music that is more mathematical, more deconstructive, less autonomous, less self-referential. The mathematically “perfect” musical structures thus act as a representation of the (Godly) “perfection” to which Coltrane aspires in the album notes.

Examples of the \{0, 3, 5\} and \{0, 5, 7\} sets occurring in Coltrane’s work previous to “A Love Supreme”

“Miles’ Mode”

The use of these two sets to construct the head has a strong resonance with Coltrane’s 1962 composition, “Miles’ Mode” (later retitled “Red Planet”) (see Fig. 2-5). The head of this piece is constructed around a 12-tone row, which is then retrograded and appended with a minor pentatonic tag. Thus, Miles (Davis’) “Mode”, from the title, and thus, his territory, is, in fact, the complete chromatic scale, which has 12 modes, all of which are identical.

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3 Thomas (1975) p. 23
4 From “Coltrane” (1962) (Impulse 21)
5 This is in contrast with the more restricted use of modes (of the Major scale) made famous by Davis. Coltrane. Evans. etc. on “Kind of Blue”.
Miles' Mode

from "Coltrane"
(1962, Impulse 21)

Fig. 2-5 Theme of "Miles' Mode"
If we examine the tone row carefully, we can see that it is made up from a pair of sets identical to those used in the head of "Acknowledgement": \{0, 3, 5\} and \{0, 5, 7\}. carefully transposed to complete the chromatic set. Demsey (1991) correctly notes that this 12-tone row does not occur verbatim in Slonimsky's "Thesaurus of Scales and Musical Patterns" (1947), a major resource for Coltrane's compositions. However, he suggests that Slonimsky's division of the chromatic scale into the chords of Cm, Dm, E and F# may have been influential, even source material. It is true that the tone-row for "Miles' Mode" is divided into 4 sets of 3 notes: however Demsey has to adjust its order and transposition to find the chords of Cm, Dm, E and F#. Further, let us note that his paper does not make any connection between this row and "A Love Supreme".

There are a couple of other points that we should note about "Miles' Mode", that seem to prefigure some of the structures that we will examine in bars 137-172 of "Acknowledgement". Firstly, let us note that the opening seven notes of the melody comprise B Dorian, whilst the latter five comprise Eb Pentatonic (interestingly: the names of these scales occur at opposite ends of the row: B is at the start, and Eb at the end). This suggests that the piece is constructed from a tonal "duality" of "in" and "out" scales: B Dorian (not Aeolian, as suggested by Demsey (1991)) and Eb Pentatonic (= C minor pentatonic). Secondly, Coltrane has separated out the tone row into "in" and "out" material. The retrograde statement reorders this to become "out" followed by "in" material. Thus, the use of a tone row and its retrograde form allows a move away from and back to a (modal) tonic, whilst covering the chromatic set (in the manner of a classical serialist).
"Alabama"

The stylistic connections between parts of "A Love Supreme" and Coltrane's 1963 piece, "Alabama," have been noted, for example by Kernfeld:

“At this time Coltrane also developed a kind of meditative, slow, rubato melody based on black gospel preaching. In Alabama (1963), he interpreted a speech by Martin Luther King, Jr., later, in Psalm from A Love Supreme (1964), he instrumentally "narrated" his own prayer.”

Cole suggests that the relationship is even closer, writing that ...

... the melodic line of the piece was developed from the rhythmic inflections of speech given by Dr. Martin Luther King.”

1 From “Live At Birdland” (1963) (Impulse 50). There has been some confusion regarding the structure/recording of “Alabama”. Michael Cuscana has attempted to resolve this in his sleeve notes for the re-release of “John Coltrane: Live at Birdland”: “When this album originally came out, “Alabama” inadvertently included take 4 which states the theme and has Coltrane briefly launching into a solo before it breaks down, followed by take 5 which is just a straight statement of the theme. Subsequent pressings included just take 5. Since then, both versions have appeared at various times. For this re-issue, we have included the full performance of both takes.” (Cuscana, Michael. additional sleeve note to “John Coltrane: Live at Birdland” (1963. Impulse AS-50, re-released as Impulse 11982)). However, I believe that take 4 and 5 are in fact one continuous performance. Cuscana’s description ignores the fact that the rubato theme of take 4 is performed by Coltrane and Tyner only, whereas take 5 includes bass and drums. Also, Coltrane’s solo does not exactly “break down”, he merely stops after 16 bars. this being followed by a further 4 bars on bass and short (reflective?) pause. Relatedly, there is the video evidence of a TV appearance by the Coltrane quartet (1963. San Francisco). where the group play the 'complete' form. Of course, it is possible that Coltrane simply copied the (eventual) pattern of takes from the LP, but this seems unlikely.

3 Cole (1976) p. 150
Regrettably, I have as yet been unable to find a specific speech by King that might have been the stimulus for the rhythms of the melodic lines in "Alabama".\footnote{I have only examined the text of several of King’s more famous speeches. Of course, this rhythm might have come from any speech that appealed to Coltrane.} Note that Coltrane also recorded a piece called "Reverend King" in San Francisco on February 2nd, 1966.

The (much-recorded) inspiration for "Alabama" is as follows:

"On the Sunday morning of September 15, 1963, a dozen sticks of dynamite were planted in the basement of the 16th Street Baptist Church in Birmingham. Alabama. At 10:25 A.M. an explosion took place, blowing several man-sized holes in the church walls, injuring fourteen parishioners, and killing four black girls, aged eleven through fourteen, who had just finished their Sunday school lesson. The Love that Forgives."\footnote{Thomas (1975) p. 167. The television programme "Jazz Heroes: John Coltrane", broadcast on Channel 4 on 21 June 1998 (19:30pm) suggested that the Klu Klux Klan was responsible for the bombing.}

Coltrane’s "Alabama", then, seeks the language of (one of) his spiritual leader(s) to express his feelings about this terrible event.\footnote{In fact, the bombing was just one of many events that may mean that Americans now see Alabama as "...an especially shameful spot on the racial map...After all, Birmingham is where racist bombers killed four little black girls at the 16th Street Baptist church, and where Martin Luther King was imprisoned and the infamous white police chief, Bull Connor, unleashed water cannon and police dogs on peaceful protestors. Selma is where state troopers beat up unarmed marchers, putting 16 in hospital. The "freedom riders", practising desegregation on Greyhound buses, had their worst moments in Alabama. When George Wallace lost a governor’s primary he famously declared that he would never be "out-niggered" again. All that is engraved on the collective memory of America, indeed the world." ("Alabama" The Economist. October 18th 1997, p. 67).} He at once supports and employs King’s views, without implicitly stating them. In the music and album notes of "A Love Supreme", Coltrane presents a personal philosophical view of his struggle. However, just as the album notes derive in part from the King James Version of the Bible, we shall see...
that bars 137-172 of “Acknowledgement” may also rely upon an external, published source.

Whilst there are clear precedents re the subject matter of “A Love Supreme” to be found in “Alabama”, let us also note scalar and motif constructions that link the two pieces. Firstly, the melody of “Alabama” is in the minor pentatonic, and, secondly, it uses cadential melodic forms similar to the \{0, 3, 5\} set in “Acknowledgement”.

There are two melodic phrases that might be seen to fit the word “Al-a-ba-ma”:

- Eb C Eb C
- Eb Eb Bb C - this is \{3, 10, 0\}, i.e. the inversion of the \{0, 3, 5\} set used in “A Love Supreme”; note that these are both 4-note phrases constructed from a 3-note set. This phrase seems to be the one picked out by Budds (1990) when he writes: “The rhythm of the word “Alabama” (long-long-short-long) can be heard in a phrase that recurs several times as the piece unfolds.” (p. 124).

Indian Scales

I have found another possible source/influence upon Coltrane in his search for quintessential sets thanks to the fact that Carl Grubbs (cousin of Coltrane’s first wife Naima) once copied some Indian scales from one of Coltrane’s notebooks, and that these are reproduced by Porter (1998). Porter shows them in order to evidence Coltrane’s interest in scales and modes from around the world. However, after each of three of the scales there are small (upward) three-note subsets that come from various parts of these
scales. As it happens, two of these three are the \{0, 3, 5\} set used by Coltrane in "A Love Supreme". and the other ("Night, Power and Majesty") is similar in that it has a Major 3rd followed by a minor 2nd (i.e. \{0, 4, 5\}). My supervisor, Dr. Gerry Farrell, has suggested to me that these subsets do not refer to constructions used in the performance of Indian music per se. However, this search for prototypical musical material is Coltrane's attempt to find common ground between various world musics (including jazz). Let us recall that in 1961 alone Coltrane recorded a range of albums with "international" titles: "Africa/Brass", "India" and "Olé". All of these albums see Coltrane using the minor pentatonic scale in various contexts. In "A Love Supreme", we can see Coltrane further refining his relationship to such scale material.
"Integration" of the \{0, 3, 5\} and \{0, 5, 7\} sets as a measure of their "Independence"

There is an important difference between these two sets that makes \{0, 3, 5\} a more appropriate set than \{0, 5, 7\} for transposing in the way that Coltrane does at bars 137-172. If we transpose the \{0, 3, 5\} set to all keys, we can measure the number of common notes (or the "integration") of these transpositions with the set at I. In fact, for the \{0, 3, 5\} set, there is never an instance where we will get more than one note in common with the original key, and, indeed, often there are no notes in common:

<table>
<thead>
<tr>
<th>key</th>
<th>notes</th>
<th>#/ notes found in I (integration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>C Eb F</td>
<td>3</td>
</tr>
<tr>
<td>bII</td>
<td>Db E F#</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>D F G</td>
<td>1</td>
</tr>
<tr>
<td>bIII</td>
<td>Eb F# Ab</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>E G A</td>
<td>0</td>
</tr>
<tr>
<td>IV</td>
<td>F Ab Bb</td>
<td>1</td>
</tr>
<tr>
<td>#IV</td>
<td>F# A B</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>G Bb C</td>
<td>1</td>
</tr>
<tr>
<td>bVI</td>
<td>Ab B Db</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>A C D</td>
<td>1</td>
</tr>
<tr>
<td>bVII</td>
<td>Bb Db Eb</td>
<td>1</td>
</tr>
<tr>
<td>VII</td>
<td>B D E</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2-4 Self-integration of \{0, 3, 5\} set under transposition

Let us note that, under transposition, the maximum result here is 1, which in itself is the absolute minimum possible result for any scale (of the 2048)\(^1\) under transposition to any

---

\(^1\) I first outlined these 2048 scales in my paper "How Weird Can Things Get?" (1997). Essentially, scale #1 = C; scale #2 = C Db; scale #3 = C D; and so on until scale #2048 = C Db D Eb E F F# G Ab A Bb B (the chromatic scale). See Appendix 2 for the complete list.
of 11 other keys. This contrasts with the \{0, 5, 7\} set, which, under transposition, has
more various numbers of notes in common with I, thus:

<table>
<thead>
<tr>
<th>key</th>
<th>notes</th>
<th>#/ notes found in I (integration):</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>C F G</td>
<td>3</td>
</tr>
<tr>
<td>bII</td>
<td>Db F# Ab</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>D G A</td>
<td>1</td>
</tr>
<tr>
<td>bIII</td>
<td>Eb Ab Bb</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>E A B</td>
<td>0</td>
</tr>
<tr>
<td>IV</td>
<td>F Bb C</td>
<td>2</td>
</tr>
<tr>
<td>#IV</td>
<td>F# B Db</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>G C D</td>
<td>2</td>
</tr>
<tr>
<td>bVI</td>
<td>Ab Db Eb</td>
<td>0</td>
</tr>
<tr>
<td>VI</td>
<td>A D E</td>
<td>0</td>
</tr>
<tr>
<td>bVII</td>
<td>Bb Eb F</td>
<td>1</td>
</tr>
<tr>
<td>VII</td>
<td>B E F#</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2-5 Integration of \{0, 5, 7\} set under transposition

Thus, we can see that by choosing a set such as \{0, 3, 5\} as the melodic material for
much of the piece, Coltrane is able to maintain a higher sense of separation and
difference between transpositions as he moves through all 12 keys in bars 137-172. This
has the advantage that we are less likely to hear the transpositions as being linked to
form other, larger structures (e.g. a commonly-used scale). Let us name this
characteristic “independence”, and name sets that exhibit this characteristic “independent
sets”. Relatedly, let us say that \{0, 5, 7\} shows a wider “range” of possible results.
Indeed, the \{0, 5, 7\} set shows not only minimum (0) and maximum (3) results re
integration but also all other levels between: (1) and (2).
Discovery of the “independence” characteristic of the \{0, 3, 5\} set made we wonder how many such independent sets exist, and how they fit within the jazz scales commonly used by Coltrane at this time. I have undertaken this search with my computer. Here are the results:

<table>
<thead>
<tr>
<th>#:</th>
<th>scale:</th>
<th>freq. of occurrence in scales:</th>
<th>interval vector:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dorian</td>
<td>Blues</td>
</tr>
<tr>
<td></td>
<td>count:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>one-note:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>two-notes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>C Db</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>C D</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>C Eb</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>C E</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>C F</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>three-notes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>C Eb G</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>42</td>
<td>C E G</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>C D F</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>C Eb F</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>C Db F</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>C E F</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>C Db E</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>C Eb E</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>C Db Eb</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>C D Eb</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2-6 Occurrence of ‘independent’ sets in Dorian, Blues and Pentatonic Scales (modes stripped out of sets)
The "#" column refers to the scale number (from 1 to 2048; see Appendix 2). Scale #33 (marked in bold) represents the "A Love Supreme" set of \( \{0, 3, 5\} \), and scale #25 is its inversion \( \{0, 2.5\} \). Important to an understanding of these data is that the results for the Dorian mode are good for all modes of that scale (i.e. good for the Major scale (Ionian mode) as well as the Phrygian, Lydian, Mixolydian, Aeolian and Locrian modes.

Similarly, the data for the Pentatonic scale is accurate for its modes, which, of course, includes the minor pentatonic).

Thus we can see from these data that the \{F, G, Bb\} and \{F, Ab, Bb\} sets are equally "independent" and, further, occur as frequently as each other in the commonly-used scales examined in Table 2-6 above. Perhaps Coltrane's choice of the \{F, Ab, Bb\} set as the motif for "A Love Supreme" may have been influenced by the following characteristics that distinguish it from the \{F, G, Bb\} set:

1. \{F, Ab\} defines the motif as melodic and chordal
2. \{F, G, Bb\} does not exist with F as I in the Blues or minor pentatonic scale, and this confirms the essential source of this motif as being from these (relatively African?) scales, although occurring in the (relatively European?) Dorian.

In this table, the scales are ordered out of numerical order in order to compare inversionally equivalent scales (e.g. the first three-note scales: #35 and #42). Indeed, let us note that #33 and its inversion #25 have the same results, and that only #15 and its inversion #32 share this characteristic. By examining the "count" column, and recognising that each scale has as many modes as notes, we can see that there are actually \((1 + (5 \times 2) + (10 \times 3) = 41\) independent scales from a total of 67 (= 1 + 11 + 55) one-, two- and three-note scales. Thus the "independent" quality of the \{0, 3, 5\} set
(= scale #33) is far from rare. The interval vector data is from Forte (1973), and shows that these independent sets are actually all those sets where the greatest value for any interval vector is 1. Compare this with interval vector of \{0, 5, 7\} which is 010020. \{0, 5, 7\}, as we have seen, is not an independent set. Atonal theory shows that the value of 2 means that under transposition to the Perfect 4th or Perfect 5th that there will be two common notes.\(^2\)

However, despite the fact that the independent nature of #33 is not rare, there is another factor that we can see here that confirms its prototypicality. If we examine the frequency of occurrence data for the these sets within the Dorian-Blues and Pentatonic scales we can see that, of the 3-note independent sets, the \{0, 3, 5\} and its inversion, \{0, 2, 5\} show the highest results. This is especially notable regarding Dorian, where it is in fact unequalled in frequency. Also, if we sum these frequency data we see that #33 outstrips even the ubiquitous major and minor triads #33: 8: #35 = 6: #42 = 4.

So, these two strands of research show that the \{0, 3, 5\} set shows a simultaneous independence from transpositions of itself and a high level of integration with the underlying tonality of (a) this piece and (b) jazz music in general. This has the effect of aiding each transposition in bars 137-172 to sound separate and distinct: it is this that helps these bars represent the personal journey of struggle described by Coltrane in his album notes. Importantly, Coltrane has chosen a set of three notes in length for a four syllable phrase ("A Love Supreme"), and this search has also revealed that there are in fact no 4-note (or larger) scales that are "independent". Thus here we may have an muso-mathematical explanation for (at least part of) what Cole sees as an essentially spiritual obsession with the number 3 on the part of Coltrane.\(^3\)

\(^3\) Cole (1976)
Coltrane’s Solo

Throughout his extended solo in “Acknowledgement”, Coltrane mostly uses F minor pentatonic, with interpolated transpositions of the \{0, 3, 5\} set. These transpositions mostly use intervals of a Perfect 4th or a semitone. As the piece develops beyond the head, the \{0, 3, 5\} set increasingly becomes the domain of Coltrane, whereas the \{0, 5, 7\} is used more by Tyner (he almost exclusively plays quartal harmony: formed by inverting \{0, 5, 7\} \text{(sus4)}, e.g. F, Bb, C to \{7, 0, 5\} \text{(quartal)}, e.g. C, F, Bb). Note however the Db, Ab, Eb, Db pattern at bar 82 \text{ (= \{0, 5, 7\})}. I will now show a table of occurrences of this \{0, 3, 5\} set in Coltrane’s solo (each letter name in this table represents the lower note of each transposition of the \{0, 3, 5\} set). In this opening part of solo (bars 33-121) the \{0, 3, 5\} sets are performed as simple, ascending three-note pattern, e.g. F, Ab, Bb:

bars 33-48: F minor pentatonic

bars 49-60: \{0, 3, 5\} sets in keys of:

C F
C F
C F
C F Bb
C F
Eb Eb
Bb Eb
Bb
C F
D G G C F Bb
D G C F Bb Eb
D G C F
bars 61-68: F minor pentatonic
(F sets at bars 65 & 66)

bars 69-81: \{0.3.5\} sets in keys of:
F Bb Eb Ab Ab Db Ab Db Ab Db F# B E A A A A D D G C F Bb

bars 82-95: F minor pentatonic

bars 96-114: F minor pentatonic
(including \{0.3.5\} sets in keys of):
F F Bb Bb...Bb...Bb...Bb
C
D D G...G...Cm...Cm7...Cm...Cm...C(m) - less clear
Bb Bb...Eb...

bars 115-121: \{0,3,5\} sets in keys of:
C
D
Eb
E
F
F#
G
Ab
A
Bb
B
C...

Table 2-7 Transposition of \{0,3,5\} set in Coltrane's solo, bars 33-121

We can see, then, that the \{0,3,5\} set is the central motif of this solo. Note that at bars 69-81 Coltrane covers all twelve keys, and that these occur sequentially through the
circle of fifths, including some repetitions for emphasis, and add to the tension of this journey. I have marked the twelve keys in bold. Note that this is a section where the set is transposed through all twelve keys which precedes that mentioned by Jost (1974) and Kernfeld (1988) (although here it is the set, not the motif being transposed). Similarly, coverage of all twelve keys is very nearly achieved in the sideslipping section of bars 115-121 (there is no Db).

Then, from bar 137 we hear the motif expanded to become F, Ab, F, Bb (the bass line that we have heard at the beginning, although this has been variously altered up to this point), this moving through the following list of transpositions, methodically stated, lasting a bar each, thus:
bar:  

<table>
<thead>
<tr>
<th>Bar</th>
<th>Up/Down</th>
<th>Motif Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>138</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>139</td>
<td>up G</td>
<td>G</td>
</tr>
<tr>
<td>140</td>
<td>down D</td>
<td>D</td>
</tr>
<tr>
<td>141</td>
<td>up Ab</td>
<td>Ab</td>
</tr>
<tr>
<td>142</td>
<td>down Db</td>
<td>Db</td>
</tr>
<tr>
<td>143</td>
<td>down C</td>
<td>C</td>
</tr>
<tr>
<td>144</td>
<td>oct+up D</td>
<td>D</td>
</tr>
<tr>
<td>145</td>
<td>up Eb</td>
<td>Eb</td>
</tr>
<tr>
<td>146</td>
<td>down Ab</td>
<td>Ab</td>
</tr>
<tr>
<td>147</td>
<td>up Db</td>
<td>Db</td>
</tr>
<tr>
<td>148</td>
<td>down A</td>
<td>A</td>
</tr>
<tr>
<td>149</td>
<td>down F</td>
<td>F</td>
</tr>
<tr>
<td>150</td>
<td>up D</td>
<td>D</td>
</tr>
<tr>
<td>151</td>
<td>down Ab</td>
<td>Ab</td>
</tr>
<tr>
<td>152</td>
<td>up B</td>
<td>B</td>
</tr>
<tr>
<td>153</td>
<td>down E</td>
<td>E</td>
</tr>
<tr>
<td>154</td>
<td>up B</td>
<td>B</td>
</tr>
<tr>
<td>155</td>
<td>down Db</td>
<td>Db</td>
</tr>
<tr>
<td>156</td>
<td>oct+up Eb</td>
<td>Eb</td>
</tr>
<tr>
<td>157</td>
<td>down Bb</td>
<td>Bb</td>
</tr>
<tr>
<td>158</td>
<td>up C</td>
<td>C</td>
</tr>
<tr>
<td>159</td>
<td>down G</td>
<td>G</td>
</tr>
<tr>
<td>160</td>
<td>down D</td>
<td>D</td>
</tr>
<tr>
<td>161</td>
<td>up E</td>
<td>E</td>
</tr>
<tr>
<td>162</td>
<td>up F#</td>
<td>F#</td>
</tr>
<tr>
<td>163</td>
<td>up G</td>
<td>G</td>
</tr>
<tr>
<td>164</td>
<td>up Ab</td>
<td>Ab</td>
</tr>
<tr>
<td>165</td>
<td>down F</td>
<td>F</td>
</tr>
<tr>
<td>166</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>167</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>168</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>169</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>170</td>
<td>same F</td>
<td>F</td>
</tr>
<tr>
<td>171</td>
<td>same F</td>
<td>F</td>
</tr>
</tbody>
</table>

Table 2-8: Transposition of “A Love Supreme” motif in Coltrane’s solo, bars 137-172

The up/down column in the above table details that although Coltrane visits some keys fairly frequently, he never does so in quite the same way. For example, he plays D followed by Ab twice (bars 140-141 and 150-151). However, the second time, he moves downwards to the Ab where previously he had moved upwards. Let us examine
this journey as a graph showing transpositions of the motif above and below the tonic note F (i.e. F= 0):

![Graph showing start notes of transpositions of motif in bars 137-172 (F=0)](image)

Fig. 2-6 Graph Showing Start Notes of Transpositions of Motif in bars 137-172 (F=0)

Note bars 160-164, which show only gradual upward movement, before the rapid return to the tonic F at bar 165. Apart from this final cadential preparation, this graph reveals that Coltrane’s strategy is to keep changing direction and interval, maintaining surprise.

Let us now examine the frequency of use of each key (not permitting octave equivalence):
(above tonic)
Eb 2
D 2
Db 1
C 1
B 2
Bb 1
A 1
Ab 4
G 3
F# 1
F 11 (tonic)
E 2
Eb 0
D 2
Db 2
C 1
(below tonic)

sum=36 bars

Table 2-9 Frequency of use of each transposition of “A Love Supreme” motif by pitch, bars 137-172

Here is a graph of these data:
Fig. 2-7 Graph Showing Frequencies of Transpositions of Motif in bars 137-172

Clearly, there is a general consistency here, but not a strict equality of use that might suggest that this section was either notated or not (substantially) improvised. Note that the extraordinary frequency of I1 at F would be only 2 if we ignore all repeats, i.e. only include the number of times it is actually transposed to (i.e. at bars 149 and 165). Thus, Coltrane gives the tonic key an equality of importance within bars 139-164, despite its use at bars 137-138 and 165-172 as a tonic anchor. In fact, it was only after notating this section that I knew for certain that he played the motif in F within the “journey” (bars 139-164).
Avoidance of redundancy

As we have seen, it seems that Coltrane is not strictly concerned with maintaining an equal frequency of use for all twelve keys. Of course, this is not surprising in that such order might suggest that the (symbolic) "struggle" is in some way predictable, and thus has the potential for mitigation. Rather, I believe that the motivating factor that supports this roughly equal occurrence of keys is an interest in an avoidance of redundancy. This is achieved by the maintenance (and balance) of the following concerns:

1. frequent change of interval between transpositions
2. frequent change of direction of transposition
3. continual shift between:
   (a) keys that are subsets of F Dorian
   (b) keys that are partially related to F Dorian
   (c) keys that are unrelated to F Dorian

All of these factors interact to maintain a level of surprise and interest commensurate with the preceding solo material, thus offsetting the self-inflicted limited range and rhythmic simplicity found in bars 137-172. Let us examine these concerns in turn:

1. frequent change of interval between transpositions

The transpositions in bars 137-172 have the following interval distribution:
interval: frequency:
min2 4
Maj2 5
min3 2
Maj3 2
Perf4 5
Aug4 2
Perf5 4
min6 0
Maj6 1
min7 0
Maj7 0
octave 0
min9 0
Maj9 2

(average = 2.45)

Table 2-10 Frequency of use of interval type in transposition of "A Love Supreme" motif, bars 137-172

Porter (1988) suggests that Coltrane is extensively using "fifth relations" to construct this section:

"Although Coltrane does not follow a regular key scheme, there are many fifth relations - the first statement in G is followed by one in D, A-flat is followed by D-flat, E-flat by A-flat by Db-flat, B-natural by E-natural and back to B-natural. These fifth relations derive from the relations of the two fourths within the basic scale, and each pair of fifth-related statements represents a variation of that scale." ¹

¹ Porter (1998) p. 242
That Porter describes only “fifth relations” is interesting in that, as raw data in the table above, the min2 and Maj2 are just as numerous as the Perfect 4\textsuperscript{th} and Perfect 5\textsuperscript{th} (these four intervals all clearly exceeding the average of 2.45). However, by the phrase “fifth relations” Porter is clearly referring to both Perfect 5ths and Perfect 4ths in the table above. Let us, then, sum the above data by interval and inversion. Note that all oct\textsuperscript{ups} movements are Major 9ths. On the saxophone, of course, these can be read as Major 2nds with the addition of the octave key, and not the large spatial leaps as they appear as on a piano or in notation. Given this, I think that Coltrane is deliberately using the Major 9th as an alternative to the Major 2nd; however saving this smaller interval for the (fundamental) opening (F, G), the (Bb, C) at bars 157-158, and the cadential closing (D, E, F\# etc.). Note that all such Major 2nd/Major 9th movements are upwards. Thus, here I equate Major 9ths with Major 2nds (/minor 7ths):

<table>
<thead>
<tr>
<th>interval</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>min2/Maj7</td>
<td>4</td>
</tr>
<tr>
<td>Maj2/min7/Maj9</td>
<td>7</td>
</tr>
<tr>
<td>min3/Maj6</td>
<td>3</td>
</tr>
<tr>
<td>Maj3/min6</td>
<td>2</td>
</tr>
<tr>
<td>Perf4/Perf5</td>
<td>9</td>
</tr>
<tr>
<td>Aug4</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2-11 Frequency of use of inversionally-equivalent interval types in transposition of “A Love Supreme” motif, bars 137-172

This new table shows that Porter is correct to say that these “fifth relation” intervals (with the Perfect 4\textsuperscript{th} and Perfect 5\textsuperscript{th} seen as an equivalent pair) are the most common. Indeed, we should be surprised if we were to hear this many in a random selection of keys over this many bars. To check this expectation, first let us note that in his description Porter has omitted the Eb to Bb movement in bars 156-157 and the C to G
to D movements in bars 158-160. Assuming there are no adjacent keys, we would expect to hear a Perfect 4th once in every 11 transpositions, and the same for the Perfect 5th. Thus, we would expect to hear either Perfect 4ths or Perfect 5ths twice in every 11 bars, or, 4.91 times in 27 transpositions. In fact, examining the above table, we can see that Coltrane plays a fifth relation 9 times out of 29, which is nearly twice as often as we would expect if these keys were randomly generated. However, I have gone to some lengths to show the details of this frequency data because I believe that the fact that Porter's (and my own) ear/mind is drawn to the fifth relations is as much due to their frequency as a number of other reasons:

- They are comparatively easy to hear because they are central to traditions of jazz harmony
- Relatedly, Coltrane has saturated our ears already with the set being transposed through intervals of a Perfect 4th in the solo section prior to bars 137-172
- They are used in the first sequence made by Coltrane in this section (bars 145-147)
- Upon reflection, as Porter points out, they form minor pentatonic scales (speaking personally, however, I do not hear these scales as discrete sets; I just know that they are there from a theoretical standpoint: apart from bars 146-147 – see below).

Also, let us recognise that the interval of a Perfect 4th up (and Perfect 5th down) between transpositions emphasises a common note, and that this might be seen as unusual in that one of the characteristics (as I have described above) of the \( \{0, 3, 5\} \) set is that it is normally "independent". Let us now examine the cases where the last note of a given transposition is the same pitch class as the first note of the transposition that follows immediately afterwards:

<table>
<thead>
<tr>
<th>bar</th>
<th>up/down</th>
<th>key</th>
<th>interval</th>
<th>common note</th>
</tr>
</thead>
</table>
Note that, by including up/down data in our analysis, we can see that (a) each of these events concerns a unique set of transpositions, and (b) only once does Coltrane use an octave-specific common pitch (i.e. the repeated Db at bar 146). This suggests that although Coltrane often uses “fifth relations” between transpositions of the motif (as described by Porter), he is actually more concerned with avoiding redundant sets of transpositions. The repeated Db at bar 146 is so noticeable and, further, unifies bars 146 and 147 into a Db minor pentatonic list, that Coltrane only uses this “up a Perfect 4th” strategy once in the entire section (bars 137-172). Further, let us note that both the motif of Db, E, Db, F# and the Db minor pentatonic scale are relatively distant to F Dorian, and thus, we might argue, this distance compensates Coltrane’s use of the Db as a pivot note between the motif transposed to Ab and Db. That is, the “easily heard” common note is compensated by the relative “outness” of the motif at Ab and Db. Let us note at this juncture that 4 out of the 9 fifth relations concern the keys of G, D, Ab and Db; thus these fifth relations are not evenly distributed throughout all keys. This hints at possible structures beyond those that we have so far examined; we will return to this below.
Thus, to extend Porter’s analysis, I suggest that Coltrane is not primarily emphasising any particular movement (i.e. fifth relations) in choosing transpositions of the motif in bars 137-172; rather, he is concentrating upon the creation of surprise by constantly shifting our expectation of what key will come next. In fact, it seems to me that Coltrane’s use of some (and short) sequences of intervals in sets of transpositions of the motif is actually yet another part of his strategy for surprise and avoidance of redundancy. (Indeed, it is in these sequences that most of these fifth relations occur.)

This is because the pitch and contour context, as well as the tonal resolution, of such sequences is constantly changed by Coltrane. Let us examine these sequences in detail:

145 up Eb
146 down Ab
147 up Db

Table 2-13 Sequence (a), bars 145-147

This represents a conventional “rocking” movement through Perfect 4ths (in the manner of a popular song/bebop).

147 up Db
148 down A
149 down F

Table 2-14 Sequence (b), bars 147-149

Here is a downward movement in Major 3rds, redolent of Coltrane’s “Giant Steps” chord progression. That the motif transposed back to the tonic key of F immediately follows the motif at A is perhaps explained by the fact that the motif at A creates a theoretical (major) F6 chord, whilst the motif at F neatly reasserts the underlying
tonality of F minor. As I mentioned above, I did not hear this F as such until I had notated it, despite it being the tonic key (nor did I hear the descending sequence of Major 3rds).

152 up B
153 down E
154 up B

Table 2-15 Sequence (c), bars 152-154

This movement shows the only immediate return to a given key, and is not strictly sequential.

154 up B
155 down Db
156 oct+up Eb

Table 2-16 Sequence (d), bars 154-156

This section could be seen a variation upon a sequence, due to the Major 9th interval between the Db and Eb transpositions.

158 up C
159 down G
160 down D

Table 2-17 Sequence (e), bars 158-160

This is a sequence of downward Perfect 4ths (in contrast with the “rocking” movement of Perfect 4ths at bars 145-147 above).
Here we find an upward sequence of Major 2nds. The transpositions then continue in an upward direction, the intervals compressed to minor 2nds, thus:

<table>
<thead>
<tr>
<th>Bar</th>
<th>Direction</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>down</td>
<td>D</td>
</tr>
<tr>
<td>161</td>
<td>up</td>
<td>E</td>
</tr>
<tr>
<td>162</td>
<td>up</td>
<td>F#</td>
</tr>
</tbody>
</table>

Table 2-18 Sequence (f), bars 160-162

The transpositions at bars 160-165 function in quite a separate way to those heard previously. That is, the gradual upward rise of keys functions as a cadence towards the tonic of the motif starting on F (at bar 165). This can be seen clearly in the graph of transpositions, where the general zigzag pattern gives way to a curve at bars 160-164:
The very fact that these bars operate in a different way to their immediate predecessors is a (retrospective) signal that Coltrane is thinking of this section as a cadence. The fact that the F is approached downwards after five upward movements is in itself a sign that the tonic has been reached, and the cadence has closed. The compression of the initial Major 2nd intervals into minor 2nd intervals asserts a feeling of increasing tension through these few bars, and allows Coltrane to continue moving upwards without creating a sequence longer than three keys in length (e.g., not D E F# Ab). Note also that the lowest and highest transpositions of this section (to D (bar 160) and Ab (bar 164)) are a conjugate pair to F. Each of these keys being a minor 3rd distant. Thus, the "up/up/up/up/down" movements of these transpositions (at bars 161-165) reminds me of a technique employed by pianists Bill Evans and Dave Brubeck to end upward arpeggios of chord/scale tones at the piano at the end of a piece (especially Pentatonic at V or II (= Lydian/#i1 resolution)). This technique involves playing notes in a long upward arpeggio only, and then playing one note lower, thus announcing the conclusion. In contrast with bar 165 of "Acknowledgement", this lower note may be dissonant, but the change of direction (combined with rhythmic placement) asserts a similar sense of completion.

Further, let us note that the keys of D and Ab are symmetrical by direction in a way that the Db and A at bars 147 and 148 are not.
bars 160-164 surround, but do not state the transposition at F: this tension is released at bar 165.

The fact that all of these sequences are 3 keys in length, the fact that they sometimes overlap, the fact that they fall in various position re the four-bar blocks of 137-172 (see below for more on this), and the fact that they are all different in some respect all show that Coltrane is attempting to avoid us anticipating his next move. Of course, that he permits sequences at all may be indicative of either (a) their importance to him in general; they may just be “under his fingers”, or (b) these sequences (especially the highly common “fifth relations”) may be the by-product of some other scheme. We will examine this second possibility later.

Transposition of the motif in a manner similar to that found in bars 137-172 of “Acknowledgement” occurs at three other points in the album. Coltrane performs the first of these in Part II, “Resolution”. At the very climax of his solo (6’12”), Coltrane he uses the {0, 3, 5} motif at Eb, F and G, followed by the first two notes of a further transposition to Bb; this last transposition is completed by a high Ab, not an Eb note. It is McCoy Tyner who performs the latter two, in Part III, “Pursuance”. Early in his solo, McCoy Tyner plays two short sections based upon transposition of the motif: at 2’26” and 2’55”. At 2’26”. Tyner moves through transpositions of the motif at F, C, F, B variation (B, Eb and F notes), B, A, Ab, F#, Bb, Eb, Db, D, C, E, F#, Ab subset (Ab and B notes), Bb (+ the note F). At 2’55, Tyner moves through transpositions of the motif at C, F, C, D, E, F#. Ab (+ a repeated B note). None of these three later lists of transpositions of the motif show anything like the organisation found in Coltrane’s solo in Part I, “Acknowledgement”. However, the reason I am describing these events here is because they all use of an ascending list of transpositions using the interval of a whole tone (these are marked in bold above), similar to the cadential structure we have
seen in bars 160-164 of Part I, "Acknowledgement". Seen together, the upward transposition of the motif through a sequence of whole tone intervals (i.e. the keys of D, E, F# and Ab) in order to form cadences appear related and premeditated.

2. Frequent change of direction of transposition

From the list of transpositions shown in Table 2-8 it is also clear that Coltrane is trying to surprise the listener by altering the direction of key movements through this section. In fact, where he repeats any of the 12 keys of the transpositions, each required movement tends to be from the opposite direction (or uses the octave), as can be seen from the following table:
Table 2-20 Directions by which transpositions are approached, bars 137-172

We can see from this table that only the three keys of F, Ab and B have adjacent
directional repeats (these pairs of movements are underlined in the table above). I
would expect more keys to show this characteristic if the transpositions were randomly
generated. The two movements to F may simply exhibit a predilection for a downward
cadence (like the bII7 to I7 cadence, commonly used in jazz). Only the two
transpositions to B show any kind of "simplicity" in that have only a single key
between (bars 152-154). However, this is the only such occurrence of such a "simple"
pattern, and thus this, it might be argued, further emphasises the unpredictable nature
of this section. Let us examine the up/down data more carefully as a graph (note that
the cumulative average data on the following graph for the first two points is 0; the
curve at this early stage is duplicated by the transposition data):
Fig. 2-9 Graph Showing Start Notes of Transpositions of Motif in bars 137-172 and Cumulative Average Data

The zigzag line on this graph represent the transpositions up/down from the key of F (i.e. F = 0) that we have seen before, whilst the smoother curve shows the cumulative average of these up and down movements. We can see from this latter curve that although Coltrane is striking a general balance between up and down movements, he clearly prefers keys above the tonic F (i.e. above 0). Of course, we could argue that Coltrane is limited in a sense by the relationship between the key of the piece and the range of the saxophone. However, it is striking that he does not use the two possible lower transpositions of B (-7) and Bb (-6), nor does he move above the Eb transposition an octave and a minor third above middle C. This use of a limited upper range makes his highest note an Ab (an octave and a minor sixth above middle C). On the tenor saxophone, this is a tone higher, of course (= Bb), but we have heard much higher notes in the preceding solo. Coltrane was already associated at this time as one of several musicians whose concerns included extending the conventionally-used range.
of the instrument (something he was to continue to do). Even a basic saxophone fingering chart will tend to show notes at least a Perfect 5th above this Bb. Upon reflection, it seems that Coltrane is sticking to the middle register in order to impart a feeling of calm, measured consideration of the material at hand. Coltrane is performing a kind of mantra, and is plainly more concerned with playing every key of the motif than (a) every note that he is able to on his instrument and (b) maintaining a strict balance between keys above and below the tonic F.

Noting that he does not play the low Bb and B transpositions, and also does not play, for example, the motif at E above Eb, let us note that these three keys (Bb, B and E) are all “out” keys with regard to F minor and that the following key upwards from E is the resolved F. It is conceivable that Coltrane would have only wanted to have a single F transposition of the motif within bars 137-172 in that it then represents a unique (and thus “supreme”) focus of resolution by specific octave location. Thus an extension of the range to include E might tend to destroy this by encouraging an upper F. That Coltrane chose the low F is representative of a desire to present an atmosphere of calm prayer/worship; also we first hear this motif played by the bass and Coltrane is thus engaging his resolution octave within roughly the same pitch space. Further, this means there is a range of keys above and below a central F (although, as we have seen, Coltrane tends towards using keys above this F).

---

4 For example, Mel Bay (1983)

5 Let us note that the range of keys starts on low C (moving through the tonic F), and up to high Eb. These three keys form the \{0, 3, 5\} set at C. I suggest this is a coincidence, but I am not certain of this.
3. continual shift between:

(a) keys that are subsets of F Dorian
(b) keys that are partially related to F Dorian
(c) keys that are unrelated to F Dorian

We shall see that this is a most important rationale behind bars 137-172, operating in parallel with the strategies described above. Coltrane creates gradual “outward” contours of transpositions re their integration with the underlying mode of F Dorian. I suggest that F Dorian is the underlying tonality (the “in” scale), because although Coltrane’s solo mostly uses F minor pentatonic, where he does extend this to include the II and the VI these tend to be G natural and D natural. Further, this is quite usual; as can be seen in “Miles’ Mode” (1961) above. Coltrane (as well as Tyner and Garrison) considers (if not always plays) Dorian to be the resolved tonality in this context.¹ Let us assess the relative dissonance of each transposition of the set against this scale:

¹ Indeed, this is true of “modal” jazz in general. Not only is Dorian mode the most commonly-used mode, but a musician using the minor pentatonic as a subset of either the Phrygian or
motif starting on:  | number of notes in F Dorian (integration):
---|---
F | 3
F# | 0
G | 3
Ab | 1
A | 2
Bb | 2
B | 1
C | 3
Db | 0
D | 3
Eb | 2
E | 1

(sum = 21; average = 21/12 = 1.75)

Table 2-21 Integration of \{(0, 3.5)\} set in F Dorian under transposition

This simple analysis shows that, for this motif, the keys of F#, Ab, B, Db and E (= E Pentatonic list) have below average integration with F Dorian, and thus could be considered as “out” keys, whilst F, G, A, Bb, C, D and Eb (= F Mixolydian list) have above average integration with F Dorian, and thus could be considered as “in” keys. This kind of analysis makes A and Ab appear strange in that they each seem to be in the wrong scale group! What I mean is that the list of “in” keys would in fact be F Dorian if it were not for the A (natural); also, the Ab occurs in the list of “out” keys. Thus, I suggest that simple experience of assessing integration by counting common notes shows A and Ab to be strange cases; we have already noted that Coltrane uses these two keys to return to F. Indeed we shall see below that Coltrane seems to have a special place for these two keys in the hierarchy that he employs to create this section.

Aeolian modes (as an underlying tonality) will usually emphasise the characteristic notes of these modes with enthusiasm.
Before examining these issues, I will group the above data by result and examine Coltrane's frequency of use for each group:

<table>
<thead>
<tr>
<th>#/notes in F Dorian: keys</th>
<th>frequency</th>
<th>frequency sum</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Db, F#</td>
<td>3, 1</td>
<td>= 4</td>
</tr>
<tr>
<td>1</td>
<td>E, Ab, B</td>
<td>2, 4, 2</td>
<td>= 8</td>
</tr>
<tr>
<td>2</td>
<td>Eb, A, Bb</td>
<td>2, 1, 1</td>
<td>= 4</td>
</tr>
<tr>
<td>3</td>
<td>C, D, F, G</td>
<td>2, 4, 2, 3</td>
<td>= 11</td>
</tr>
</tbody>
</table>

Table 2-22 Frequency, frequency sum and average data for {0, 3, 5} set in bars 137-172, ordered by integration in F Dorian

We can see from this table that there is a fairly even spread of key choice by level of integration with F Dorian. By comparison with the previous model, we can say that the average number of occurrences of “in” keys (i.e. 2 and 3 notes in F Dorian) is 4.08, and that of “out” keys (i.e. 0 and 1 notes in F Dorian) is 4.66. This shows a trend towards “out” keys, despite their relative rarity (5 “out” to 7 “in”). Note that, as above, this assesses Coltrane as having transposed to F twice (at bars 149 and 165), although this key actually occurs more commonly in bars 137-172. This approach makes sense if we accept that Coltrane had always intended to start and finish on F. Let us now adapt the data in the table above, such that we can see the “out” value of the transposition keys against F Dorian, thus:
motif starting on: number of notes *not* in F Dorian ("out" value):

<table>
<thead>
<tr>
<th>Note</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>F#</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
</tr>
<tr>
<td>Ab</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>Bb</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>Db</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
</tr>
<tr>
<td>Eb</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
</tr>
</tbody>
</table>

(sum = 15; average = 15/12 = 1.25)

Table 2-23 Number of notes not in F Dorian for transpositions of the \{0, 3, 5\} set

Let us now use this new data to examine bars 137-172 expressed as a graph (this new approach to the data means that "in" events appear low in the graph, and "out" events appear high in the graph):
As we can see, there is a high level of organisation to this graph. The peaks tend to contain more than one piece of data, and the troughs in between are not solitary keys (except at bar 163, which is within the "cadence" section). Further, the first three peaks have increasingly larger bases. This evidence suggests that Coltrane was perhaps grouping the "in" and "out" material in more complex ways than this graph can show. This has encouraged me to construct a more detailed graph showing the "contours of difference" between the transpositions of the motif. To pursue this idea, I have adapted the work of Watkins and Dyson (1985, p. 77) who (in turn) adapted the work of Longuet-Hughes (1976, 1978) in order to analyse melodies based on the major scale. Watkins and Dyson assign each note of the chromatic scale (arranged in Perfect 5ths) with a "sharpness value", or $q$, ranging from $-5$ to $+6$, thus:
Watkins and Dyson use this table on the grounds that a large number of melodies contain successive interval spans of less than 6 fifths and, thus, that the strength of a scale schema in a listener’s mind is influenced by the comparative bunching of notes from within this circle of fifths. Thus, they argue, the 3-note set \{0, 5, 7\} (= sus4 chord (C F G)) is better at defining a tonality in the listeners mind than the major or minor triad (p. 78). Thus, this technique allows us to analyse the relative dissonance and consonance of note sets. Since the underlying tonality in “Acknowledgement” is F Dorian, we need to further adapt this approach in order to examine the contours of difference employed by Coltrane. Thus, noticing that in Watkins’ and Dyson’s table the tonic takes a value of zero, and that notes are organised in perfect 5ths, we can construct a similar table based on the F Dorian mode, thus:

<table>
<thead>
<tr>
<th>note:</th>
<th>Ab</th>
<th>Eb</th>
<th>Bb</th>
<th>F</th>
<th>C</th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
<th>B</th>
<th>F#</th>
<th>Db</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(Dor)</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2-25 Adapted Watkins and Dyson/Longuet-Hughes table: Chromatic scale arranged by fifths with associated “sharpness value” for q(Dor).

Here the minimum q value is -3 because Dorian is two flats distant from Ionian mode (-5 + 2 = -3). Notice how this packs to the left the seven notes that make up F Dorian.

---

2 Actually, I worked out this q(Dor) table by trial and error, and this theory is retrospective.
We can now apply these \( q(\text{Dor}) \) values to all of the notes from each motif, and calculate sums for each transposition, thus:

<table>
<thead>
<tr>
<th>bar:</th>
<th>motif:</th>
<th>( q(\text{Dor}) ):</th>
<th>sum:</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td>F Ab Bb</td>
<td>0 -3 -1</td>
<td>-4</td>
</tr>
<tr>
<td>138</td>
<td>F Ab Bb</td>
<td>0 -3 -1</td>
<td>-4</td>
</tr>
<tr>
<td>139</td>
<td>G Bb C</td>
<td>2 -1 1</td>
<td>2</td>
</tr>
<tr>
<td>140</td>
<td>D F G</td>
<td>3 0 2</td>
<td>5</td>
</tr>
</tbody>
</table>

etc.

...  

172  F Ab Bb 0 -3 -1 -4

Table 2-26 Sums of \( q(\text{Dor}) \) data for motif transpositions, bars 137-172

These data allow us to construct the following graph:

![Graph](image)

Fig. 2-11 Graph Showing Sums of Sharpness Value \( (q(\text{Dor})) \) for Transpositions of Motif in bars 137-172
This graph shows much more detail than Fig. 2-10 with regard to the relationships that each transposition has with the Dorian scale and shows (generally speaking) that Coltrane is gradually moving upwards away from F and then relatively quickly back down to/towards it. However, although we can say that this approach is theoretically "pure", there are some surprising "notches" in what otherwise could be four smooth curves moving gradually away from and then swiftly back towards the tonality. These notches are at bars 144, 152 and 157 (keys of D, E and Bb). Assuming that Coltrane is gradually moving "out", away from tonic F, we might expect that the curves would appear smoother. Further, if we find that these notches are consistent deviations by key and/or context, then we should be able to construct a model that shows Coltrane's personal key hierarchy. After consideration of various alternatives, I propose the following "out" factor table, which shows the hierarchy that I believe controls this section:

<table>
<thead>
<tr>
<th>key</th>
<th>&quot;out&quot; factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
</tr>
<tr>
<td>Eb, Bb</td>
<td>4</td>
</tr>
<tr>
<td>Ab, A</td>
<td>5</td>
</tr>
<tr>
<td>E, B</td>
<td>6</td>
</tr>
<tr>
<td>Db, F#</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 2-27 Proposed "out factor" table for bars 137-172

Having shown this table, I now wish to explain my approach in "teasing out" such a hierarchy. First of all, we must adapt the "integration" table above to create a "non-integration" data table, thus:
#/notes not in F Dorian: keys:

<table>
<thead>
<tr>
<th>#</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Db, F#</td>
</tr>
<tr>
<td>2</td>
<td>E, Ab, B</td>
</tr>
<tr>
<td>1</td>
<td>Eb, A, Bb</td>
</tr>
<tr>
<td>0</td>
<td>C, D, F, G</td>
</tr>
</tbody>
</table>

Table 2-28 Showing “non-integration” of \{0, 3, 5\} sets in F Dorian

Then, two subtleties are incorporated (Subtleties 1. and 2.):

**Sublety 1.** C, D, F and G are separated out and given discrete values, thus (these derive from the order provided by the q(Dor) “sum” table above; changes marked in bold):

<table>
<thead>
<tr>
<th>key</th>
<th>“out” factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
</tr>
<tr>
<td>Eb, A, Bb</td>
<td>4</td>
</tr>
<tr>
<td>E, Ab, B</td>
<td>5</td>
</tr>
<tr>
<td>Db, F#</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2-29 Separation of C, D, F and G in construction of proposed “out factor” table

These F, G, C and D transpositions are the resolved \{0, 3, 5\} sets in F Dorian (at I, II, V and VI respectively). The relative positions of these four keys in this new hierarchy reflect the following rationales:
- F is perceived as the tonic transposition, most resolved and the "goal" of the whole section
- C is the next most resolved transposition, ending on the note F, and forming the F minor pentatonic scale when combined with the motif at F
- G and D are more problematic; however, we may note that G contains Bb and C (both from F minor pentatonic), and D contains F and G (only the former of which is from F minor pentatonic). Note that by ordering them G, then D, we follow the precedent set by the first two keys; i.e. moving in Perfect 5ths (F, C, G, D)

**Sublety 2.**  Ab is "promoted" and A is "demoted" to create a new factor, thus (changes marked in **bold**):

<table>
<thead>
<tr>
<th>key</th>
<th>&quot;out&quot; factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
</tr>
<tr>
<td>Eb, Bb</td>
<td>4</td>
</tr>
<tr>
<td>Ab, A</td>
<td>5</td>
</tr>
<tr>
<td>E, B</td>
<td>6</td>
</tr>
<tr>
<td>Db, F#</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 2-30 Promotion of Ab and demotion of A in construction of proposed "out factor" table

This reflects the following rationales:
- A is subjectively (visually) "out" when compared to Eb and Bb, starting, as it does, on the Major 3\textsuperscript{rd} of F, and thus appears (as well as sounds) antagonistic to the minor tonality. Note that Coltrane only chooses this key once at bar 148.

- Ab (by being the minor 3\textsuperscript{rd} of F) comparatively confirms F minor pentatonic (when compared with E and B).

- The keys immediately before the (only) two movements down to F at bars 149 and 165 are in fact A and Ab respectively and thus, it might be argued, they have a special cadential role.

Let us now express the "out" factor table of transpositions in bars 137-172 as a graph:

![Graph Showing "Out" Factor Data for Transpositions of Motif in bars 137-172](image)

We can see that whilst this maintains the overall shape of the previous graph, here the curves are much smoother, whilst the returns downwards are similarly steep (in fact, the average move upwards on this graph is 1.7, whereas the average movement
downwards is 4.1). We can see that the promotion of Ab has done most to smooth out the bumps previously found in the curves. Specifically, the Ab at bar 151 now appears below the B E B section that immediately follows it (bars 152-154), leading further "out" to the Db above at bar 155.

Generally speaking, Coltrane seems to be applying a similar approach to the "out" contour of this section as he might the pitch contour of a conventional improvised melody, or a harmonic journey. There is a strong contrast, however, between Coltrane gradually moving away from the home key of F minor in "Acknowledgement" and the tone-row of "Miles' Mode" (1961). In that piece, Coltrane's melody starts "in" the B minor tonality (the first 7 notes of the row are B Dorian), then suddenly moves "out" (the last 5 notes from C minor pentatonic). Here is a graph showing "outness" of the tone-row of "Miles' Mode", measured using q(B Dorian):

![Graph Showing "Out" Factor Data for "Miles' Mode"](image)

Note the highly contrasting leap from A (-2) to G (+8), as well as the grouping of the B Dorian set with q < 4 and the Eb Pentatonic as q > 3. Thus we can say that whilst these two pieces are constructed from the same two sets (\{0, 3, 5\} and \{0, 5, 7\})
incorporated into a fully chromatic approach, they display distinctively different “outness” contours.

Let us describe the outness contour of bars 137-172 shown in Fig. 2-12 as being made up of four main peaks, and note that they cover a range of at least 6 bars, and that the highest factor of the last key in all cases is usually 1 or 0:

<table>
<thead>
<tr>
<th>bars:</th>
<th>range (bars):</th>
<th>factor of last key:</th>
</tr>
</thead>
<tbody>
<tr>
<td>138-143</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>143-149</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>149-158</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>158-165</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2-31 Peaks of outness contour, bars 138-165

In fact, these last key factors indicate the only appearances of the keys of F and C in the section, further justifying our earlier separation of the keys of F and C from G and D by “out” factors. From this perspective the motif at F and C may be seen as the resolved transpositions (let us recall that these sets sum to F minor pentatonic). There is what at first glance appears to be a rogue peak-valley-peak pattern at bars 162-164 (this has not been removed from the previous Watkins and Dyson/Longuet-Hughes graph by the “out factor” table, Table 2-27). However, we should recall that this is the ascending cadential section, and note the fact that Coltrane still maintains a generally peak-like contour of outness, despite a shift of focus towards ascending through the keys (re pitch) from bar 160.

Thus, I suggest that the “out factor” graph above represents Coltrane’s hierarchy of keys (informal or formal) for this section. Of course, over the course of a lifetime a musician’s attitude to the gravity of a given transposition is likely to change, given that
those gravities are likely to be affected by given scales/motifs per se, and that opinions
cchange in the light of developing musical experience and access to theory. Indeed, it is
possible to see a situation where a musician could change their mind about the ordering
of such a hierarchy even during a *performance* of a given piece. However, the
ubiquitous nature of the \{0, 3. 5\} set, and the familiarity that Coltrane had with the
Dorian mode lead me to suggest that the key hierarchy shown above may be a
relatively stable model, and I wonder whether other pieces by Coltrane (outside the
scope of this work) reveal adherence to such a hierarchy.

So far, then, I have shown that Coltrane displays a relatively specific tonal hierarchy for the transpositions of the motif, and that he engages in strategies that avoid redundancy. However, in the process of my research I have postulated other questions beyond these analyses regarding the structure of bars 137-172 of “Acknowledgement”. These further questions, and their answers, map out my journey towards suggesting two possible sources for the series of transpositions in this section. As we shall see, this further analysis does not discount the theories described above. Rather, it suggests that Coltrane was also referring to a still deeper structure in his performance of bars 137-172. The questions are as follows:

- Why do the keys occur in this specific order?
- Are some keys deliberately used early or later on?
- Did Coltrane cover all 12 keys by chance?
- Did Coltrane attempt to cover the chromatic set by using specific lists of transpositions of the “A Love Supreme” set?
- Is there a correlation between chromatic saturation and key hierarchy?
- Did Coltrane attempt to cover the chromatic scale as fast as possible by transposing “A Love Supreme” motif as he did?
- Is the “A Love Supreme” set the fastest 3-note set at covering the chromatic set?
- Can we find evidence of a repeated section and/or a mnemonic?
- What are possible sources of this repeated section?
  - Possible Source 1: A Construction of Transpositions Designed to Cover the Chromatic Set
  - Possible Source 2: Slonimsky’s “Mother” and “Grandmother” Chords
- Is there a correlation between the “out factor” graph and the occurrences of G, D, Ab and Db?
• What is the probability that Coltrane's “core” list was randomly generated?
• How many strings of length 2, 3 or 4 from a specific tone-row should we expect to find on average in 27 transpositions?

I have also examined the following areas of study:

• A Proposed Chronology of Composition
• Eric Dolphy's Addresses
• “245” and “A Love Supreme”

I will answer these questions one at a time, beginning with:

Why do the keys occur in this specific order?

In thinking about whether (and/or how) Coltrane designed the framework of this section let us recall John Cage’s reflection:

“When I thought thoroughly about it, I realised that there is usually an essential difference between composing a piece and listening to it. The composer knows his piece as a forest wanderer knows his path, but the listener is confronted with the same creation the same way as a stranger in a forest.”

---

1 John Cage, quoted in Thomas (1975) p. 171
Generally speaking, we shall see that Coltrane continually shifts between these two roles in bars 137-172; much as we would expect an improviser to operate. We shall see that some of the material is methodically planned, other material less so. For example, I do not believe that Coltrane had prepared the entire list of transpositions, and/or performed it from a written score. As it happens, such a compositional approach would be outside of the codes of improvisation that Coltrane had learnt and developed up to this point in his career. Indeed, a prepared list of keys would remove the struggle of negotiating the journey, of which this section is symbolic. We shall examine a range of formulae and strategies that may produce the list of keys actually heard, which Coltrane may have used. As noted above, the avoidance of redundancy seems to be Coltrane's primary ambition in his choice of keys. However, of course, this theory does not explain the choice of the first few transpositions (i.e. before redundancy could take effect).

Thus, before examining the section in more general terms, let us examine the opening bars (137-143; i.e. the first peak on the “out factor” graph) in a little more detail.

To begin with, let us note that Coltrane performs the first two F motifs (bars 137-138) in one breath. This contrasts with the initial transposition to G (and all others), characterised by its the staccato fourth note. This suggests that Coltrane always intended to play two Fs at the start. Relatedly, I think it unlikely that Coltrane had not prepared to use G as the initial transposition (at least).

This initial transposition to G (bar 139) is remarkable in that it sounds like a distinct shift of key, although all the notes from this motif {G, Bb, C} exist in F Dorian. In retrospect, it seems clear that Coltrane has specifically decided to start his journey through all twelve keys with a relatively consonant movement, given the “out factor” data that we have previously examined. In this sense, he might equally well have chosen any of the three other transpositions of the motif that exist in F Dorian (C, G or D). The
motif at C is clearly the most resolved, forming the upper part of the minor pentatonic scale. However it shows the disadvantage of ending on the note F, perhaps suggesting an immediate completion or resolution of the symbolic “struggle”. Thus, perhaps Coltrane’s motivation in choosing G over D is because, of the four transpositions of the motif that exist in F Dorian. G is the only one that does not contain the tonic note F:

\{F, Ab, Bb\}; \{G, Bb, C\}; \{C, Eb, F\}; \{D, F, G\}.

Table 2-32 Only one \{0, 3, 5\} set in F Dorian does not contain the note F.

The choice of G thus immediately creates a real sense of movement away from the tonic, despite both F and G belonging to F Dorian. Thus, G anticipates the “independent” (to repeat a term) nature of this journey. The choice of G also requires the smallest possible interval (amongst these alternatives) of a Major 2nd upwards.

There is a further reason why Coltrane may have started with G, based in the tradition of bebop, a jazz tradition with which he was very familiar. As the first of these transpositions, the motif at G is can only be perceived in direct relationship with the F that precedes it, and, as such a pair, these two keys might be seen to refer to bebop musicians’ (Parker et al) use of the II chord (e.g. D over a C) as a resolved tonality that includes a raised, and thus resolved, fourth (i.e. using Lydian mode, with its characteristic \#11). Relatedly, the minor 13th chord can be seen as a minor triad atop a m7 chord. Such chords are sometimes defined as “polychords”, and split between the two hands of a pianist in theory texts, thus:
That is, Cmaj13#11 = D triad above a Cmaj7 chord, and Cm13 = Dm triad above a Cm7 chord. In both cases, the upper triad (a) is a tone higher, (b) shares no common notes with its source, and (c) does not include the tonic. Characteristics (a) and (c) can be found in an examination of the "A Love Supreme" motif at I and II, thus:

I: F, Ab, Bb II: G, Bb, C

Table 2-33 Common note Bb in I and II transpositions of \{0, 3, 5\} set

As we can see, the repeated note Bb (in bold) means that characteristic (b) is not to be found here. However, let us note that the upper triads in the Cmaj13#11 and Cm13 chords are often used by bop musicians to perform short melodic phrases (this often extends to the Major pentatonic and minor pentatonic scales, respectively). It is this practice (represented by the splitting of the chord into the pianist's two hands in the theory books) that I am reminded of by Coltrane's use of the II as the first transposition of the motif. In a sense, then, in choosing G, Coltrane is referring to a harmonic tradition of jazz, which thus mitigates the perceived dissonance of this initial transposition.

The downward move to D that follows (at bar 140) balances the initial move upwards to G, and is a further key resolved within F Dorian. However, this D transposition has none
The downward move to D that follows (at bar 140) balances the initial move upwards to G, and is a further key resolved within F Dorian. However, this D transposition has none of the special bebop characteristics described above regarding G. The transposition to Ab (bar 141) is special in that it starts on the resolved bIII (Ab; also a note from the motif in F), and then gradually leaves the tonality (B, Db). This B note may be seen as a #IV blues note, matching the Ab’s bIII role over the F tonic. By contrast, the following Db note is outside jazz scalar conventions, and thus is the first truly “out” note. However, this relative dissonance is offset by the motivic nature of this section. Further, as we have seen, Ab is the first transposition (bar 141) to include a note outside the mode of F Dorian, and, let us note in advance, is also the final transposition before the eventual return to F at bar 165; this final movement, then, might be seen to have a (bIII) “blues” feel about it, characteristic of the motif itself (= I bIII I IV). These placements and structure suggest a special importance may have been assigned to this transposition by Coltrane. The Db transposition that follows, as seen on the “out factor” graph above, takes Coltrane even further away from the home key of F minor. The following C transposition acts as an extreme contrast, rapidly resolving what had been the steadily increasing dissonance of bars 139-142.

McCoy Tyner accompanies Coltrane with chords that resolve with the transpositions of the motif through this “first peak” until bar 141. This harmonic unity suggests that Tyner knew, roughly speaking, what Coltrane was likely to do in these first few bars. However, at bar 142 (the Db transposition), Tyner seems to go adrift. On the other hand, he hears Coltrane return to the resolved C transposition in bar 143, and accompanies appropriately. The following bars see Tyner attempting to follow Coltrane’s movements through the keys: sometimes they are in the same key, sometimes not. However, Tyner dissipates any sense of harmonic disunity by giving the chord progressions their own
structural integrity. In a sense, then, Tyner’s accompaniment models the “struggle” to return to the tonic undertaken by Coltrane in bars 137-172.

Thus, I suggest that Coltrane was very careful in his choice of opening transpositions in this section, even to the extent that his pianist was able to develop a harmonic accompaniment that matched these opening movements. The transpositions that follow, as noted above, pursue a goal of avoidance of redundancy. But, I suggest, there are deeper structures which can explain why the keys occur in this specific order. And it is these structures that will be revealed in answering the remaining questions posed above.

Are some keys deliberately used early or later on?

If we examine bars 137-172 and list the keys by order of appearance, we find:

F, G, D, Ab, Db, C, Eb, A, B, E, Bb, F#

Here is this order of appearance assessed against the “out factor” values, with a (polynomial) trendline added to show the gradual increase in use of dissonant transpositions of the motif over bars 137-172:
Fig. 2-15 Transpositions of the Motif in Order of Appearance, with “Out Factor” Data, assessed by Polynomial Trendline

Beyond this structure, which parallels Coltrane’s use of the dissonance hierarchy to create the curves in the “out factor” graph examined earlier, I have found no real evidence that some keys occur early in the list of 26 transpositions, and then are not used again. On the contrary, the earlier in the list that a key occurs, the more likely it is to be repeated. Thus, Coltrane seems to be gradually adding new keys to the list. However, there is one special characteristic of the motif and its relationships to other keys that Coltrane seems to use early in the list in order to accentuate dissonant movements, which I will describe below. Further, there is a single case where, I suggest, Coltrane may be deliberately restricting the use of a transposition of the motif until late in the list. Here are descriptions of these deliberate early and late uses of certain keys:

**Early Use**

The motivic set \{F, Ab, Bb\} can be followed by any note except E (VII) or B (#IV) and be found in at least one Dorian key. This means that, if we accept the Dorian mode as
the core scale for the piece, transposition to the VII or #IV of a given motif may add an extra level of dissonance to this action. Examples of transpositions of the "A Love Supreme" motif where the following note is the VII or #IV of the preceding motif are:

140 D
141 up Ab
142 Db
143 down C
150 D
151 down Ab

Table 2-34 Transpositions of \{0, 3, 5\} set where following note is VII or #IV

This would seem to be a fairly average frequency of occurrence for this kind of event (we have 3, and randomly speaking, we would expect 4.73 (2 d.p.) of these kinds of movements in 26 bars of transposition), and, thus, there seems to be no evidence that Coltrane is deliberately focusing upon these movements in general, despite their value in inferring a change of key for the motif. If anything, their shortage supports the premise that Coltrane is avoiding redundancy of interval. However, the first two examples do occur very near the start of this section, and neatly act as an extra dissonance "pointer" to an initially surprised listener. It seems possible, then, that Coltrane was aware of the special role of these events, and used them appropriately. Clearly, however, this theory relates to transposition interval and not to a specific key.
Late Use

Given the importance of "a semitone up" in "outside playing" (see previous chapter), it is interesting that the motif in F# (the last to complete the 12) occurs at the end of this list, is performed only once, and appears as part of the ascending cadence at bars 160-164. F# is, of course, the bII of the tonic key of F, and thus can be seen as being sourced in the tradition of using this transposition as cadential material. Thus, I suggest that Coltrane deliberately restricts the use of this key until last.

Did Coltrane cover all 12 keys by chance?

In undertaking this research I recognised the possibility that Coltrane only covers all twelve keys by chance. This is a possibility if we accept that his target is to continually change key 26 times before returning to the tonic key. It is 26 times because 26 = the number of possible transpositions in bars 137-172, but ignoring the final move to F, which we will take as being non-random. The following experiments were undertaken using Microsoft Excel on my PC.

Let us imagine a fictitious musician, name of Charlie Chance, who will function as a random key generator. For this stage of the experiment, we assume that an accident has left him with no memory. This is not to say that Coltrane has a similar memory to Charlie Chance, of course. Rather, let us suppose that Coltrane may be spending so much mental energy (a) creating smooth dissonance curves and (b) avoiding redundancy that recall of the list of played and unplayed keys may be relatively difficult. To begin

---

2 I had the idea for such a character after reading Barrow's (1995) chapter on randomly generated music.
with, let us recognise that the probability that Charlie Chance will cover all 12 keys from 12 completely random selections is extremely low:

\[ P = 0.000000002... \]
\[ = \text{roughly } 1 \text{ in } 500 \text{ million}^3 \]

However, if we allow Charlie to make 26 random selections (26 = the number in transpositions in bars 137-172 of “Acknowledgement”), we find that the likelihood that within those 26 selections that all 12 keys are covered is quite high:

\[ P = 0.215471706 \]
\[ = \text{roughly } 1 \text{ in } 5 \]

Of course, to refine this model with regard to “Acknowledgement” we must examine Coltrane’s strategy more carefully. For example, let us note that there are no adjacent repeats of transpositions in bars 137-172; that is, Coltrane is always moving from key to key. If we factor this in to our calculations we find that the probability of Charlie Chance covering all 12 keys and not performing any adjacent repeats in 26 selections is:

\[ P = 0.033573114 \]
\[ = \text{roughly } 1 \text{ in } 30 \]

---

3 Please see Appendix 3 for the formulae for this and the following probability calculations.
This result is so much lower than the 1 in 5 above simply because adjacent repeated notes are such (relatively) likely events. But, of course, in this model, Charlie is randomly generating selections, and is not discounting the selection of repeated keys. And so we must ask Charlie, as he moves from selection to selection, to choose any key except the key that he has just chosen (i.e. he is recovering from the aforementioned accident, and now has a very short term memory). Note that Coltrane does not separate keys any more than this. For example, bars 152-154 show Coltrane moving from B to E and back to B; thus the smallest time that it takes for Coltrane to repeat a key is two bars. In this scenario:

\[
P = 0.295600825
\]

\[= \text{roughly } 1 \text{ in } 3\]

So, we can say that for Coltrane to have covered all 12 keys in 26 transpositions without an adjacent repeat simply by chance would appear to be a clear statistical possibility (\(P = 0.3\) (1 d.p.)). Thus, in the light of this analysis, we have the intriguing situation of not being able to state with any real statistical confidence whether Coltrane covers all 12 keys deliberately or by chance.

However, if we accept that this section represents a journey “away from the esteemed path” during “a period of irresolution” (from Coltrane’s album notes), then we can see the deliberate use of all 12 keys as a natural symbol of a diverse, difficult journey. On the other hand, a random journey, that might or might not cover all 12 keys, would accurately symbolise the chaos of such a difficult period in his life. I believe that Coltrane enjoys both of these symbols by maintaining a pseudo-random structure. That is, he transposes the motif 26 times in such a way that he deliberately attempts to cover
all 12 keys, yet makes the listener/analyst (a group that includes Coltrane) feel that the selections are essentially random in nature.

From the experience of imagining Charlie Chance, I suggest that the fact that Coltrane never immediately repeats a key can, in itself, be read as a further sign that he is consciously acting non-randomly with regard to key selection, and that he is not, for example, performing a previously-generated list of keys in the performance. As we have seen, we would fully expect Charlie Chance to perform at least one adjacent key most of the time, if he is allowed to operate in a totally random manner. Coltrane, by contrast, is undertaking various strategies to surprise us with the next key: the deliberate, unique, repetition found at bars 152-154 (B E B) is just such a case in point. Thus, upon reflection, we can say that Coltrane has deliberately avoided the strategy of immediately repeating a key as a further generator of surprise. Such an immediate repetition would also disrupt the sense of a journey without rest. Of course, it seems likely that Coltrane had always decided to repeat the motif at F over and over at the end of this section, and thus, we can argue, just such a section of repeated keys does indeed symbolise to him the resolution of the journey.

Did Coltrane attempt to cover the chromatic set by using specific lists of transpositions of the “A Love Supreme” set?

We have seen that Coltrane covers all 12 keys in bars 137-172. Let us again recall the structure of Coltrane’s earlier composition, “Miles’ Mode”(1961). In this piece, the chromatic set is covered not by a list of transpositions of the sets concerned, but rather by the sum of all notes used in the opening bars; that is, as a conventional tone row. Equally, we might suppose that in bars 137-172 of “Acknowledgement” Coltrane’s goal
the sum of all notes used in the opening bars; that is, as a conventional tone row. Equally, we might suppose that in bars 137-172 of "Acknowledgement" Coltrane's goal was to cover the chromatic set by the sum of all notes used, using specific lists of transpositions of the motif to this end. Let us examine this possibility. That is, we will note that the key of F provides the notes F, Ab and Bb (3 of the 12 needed), the key of G provides G, Bb and C (a further 2, the Bb having been already sounded in the motif at F), and so on, until we find that the chromatic set is covered. Let us examine how fast Coltrane covers the chromatic set, firstly by notation:

Fig. 2-16 Chromatic Saturation achieved in bars 137-148

We can see that the 12 semitones arrive in the order F Ab Bb G C D B Db E F# Eb A, and that it is the transposition to A at bar 148 that completes this "string" of transpositions; thus this string requires 11 keys (bars 138-148) to complete the chromatic set. Clearly, we can examine the entire section of bars 137-172 for such strings. In the following table, vertical columns of "x"s (strings) show the shortest collections of transpositions necessary to form the chromatic set, starting from each bar (thus the first vertical column of "x"s refers to the notation above):
Table 2-35 Speed of Chromatic Saturation in bars 137-172

We can see, therefore, that every bar from 137-158 has the potential to fulfil the chromatic set in less than 12 transpositions of the motif. Let us note that the average length of these strings is 8.33 (2 d.p.) keys. Further, let us note that Coltrane covers the chromatic set in six keys on 3 separate occasions. Of course, this chromatic saturation
can be seen as occurring simply as a by-product of transposition based upon the concepts (examined above) of:

1. frequent change of interval between transpositions
2. frequent change of direction of transposition
3. continual shift between:
   (a) keys that are subsets of F Dorian
   (b) keys that are partially related to F Dorian
   (c) keys that are unrelated to F Dorian.
Is there a correlation between chromatic saturation and key hierarchy?

Recalling the “peaks” of outness found in the “out factor” graphs above, it is interesting to note a correlation between this and the chromatic saturation data above:

<table>
<thead>
<tr>
<th>“peak” bars</th>
<th>range</th>
<th>factor of last key</th>
<th>chromatic saturation?</th>
</tr>
</thead>
<tbody>
<tr>
<td>138-143</td>
<td>6</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>143-149</td>
<td>7</td>
<td>0</td>
<td>yes (“perfect fit”)</td>
</tr>
<tr>
<td>149-158</td>
<td>10</td>
<td>1</td>
<td>yes (“perfect fit”)</td>
</tr>
<tr>
<td>158-165</td>
<td>8</td>
<td>0</td>
<td>yes (c.s. achieved at bar 164; bar 165 = F final 8 bars)</td>
</tr>
</tbody>
</table>

Table 2-36 Range, factor of last key and Chromatic Saturation data for peaks, bars 138-165

By “perfect fit” I mean that chromatic saturation could be seen to actually signal the end of that hill. Note that peaks are defined to have resolved by the occurrence of a transposition with an “out factor” of just 0 or 1.

Bars 138-143 only fail to meet the “perfect fit” criteria in that they lack the note A. Further, bars 158-165 only fail to meet this “perfect fit” criteria in that chromatic saturation is achieved a bar early at bar 164. If we recall that bar 165 is the start of the final 8 bars of the motif at F, then we can imagine a state of mind that focused more upon this cadential event than the chromatic “count”. Let us now further refine our search by posing a more detailed question. That is:
Did Coltrane attempt to cover the chromatic scale as fast as possible by transposing “A Love Supreme” motif as he did?

In order to pursue this idea, let us first divide the strings into two types: (a) those where there are repeated keys within a given string and (b) those where there are no repeated keys with a given string. Here is Table 2-37, a new version of Table 2-35 with the letter “R” showing repeats of a transposition within each string:
Table 2-37 Speed of Chromatic Saturation in bars 137-172, showing Repeated Keys

Clearly the frequent occurrences of repeated keys within strings shown here suggests that Coltrane was not concerned with covering the chromatic scale as fast as possible.

However, there are three strings in the table above where Coltrane only takes 6 keys to cover the chromatic set:
bars: keys: intervals:
144-149 D Eb Ab Db A F min2 P5 P4 Maj3 Maj3
145-150 Eb Ab Db A F D P5 P4 Maj3 Maj3 Maj6
153-158 E B Db Eb Bb C P5 min7 Maj9 P4 Maj2

Table 2-38 Chromatic Saturation achieved in three six-transposition strings

We can see that there is a wide range of keys and intervals here; from this it also seems that Coltrane has no preset plan re chromatic scale coverage. Further, this is not the quickest way to saturate the chromatic scale by transposing the motif. In fact, just five transpositions are all that is required. The following table shows this:

for “A Love Supreme” set (= scale #33):

<table>
<thead>
<tr>
<th># of transpositions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td># of LOTs that sum to c. scale</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>87</td>
<td>231</td>
<td>258</td>
<td>156</td>
<td>55</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>P (2 d.p.)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.19</td>
<td>0.5</td>
<td>0.78</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2-39 Chromatic Saturation data for {0, 3, 5} set (= scale #33)

Here “LOTs” means List Of Transpositions. This refers to the 2048 scales, which may also be seen as lists of transpositions of a given motif. (That is, there are 2048 scales, any of which may be transposed through 2048 Lists Of Transpositions. For example, if we transpose scale #1361 (the Major scale) through the list of transpositions provided by LOT #2 (C, Db), we cover the chromatic set.) The term “c. scale” means the chromatic scale. “P” means probability. Thus, it can be seen that the minimum number of transpositions of the motif that can cover the chromatic scale is 5. Further, these are relatively precise LOTs (P = 0.02 (2 d.p.)), and are highly unlikely to be randomly generated. By contrast note that all LOTs of 10 and more keys always saturate the “A
Love Supreme" motif. The five LOTs that could result in the "A Love Supreme" set (scale #33) summing to the chromatic set in just five transpositions are as follows:

LOT#   keys:
248    C Db D F F#
302    C Db E F B
343    C Db G Ab A
457    C Eb E Bb B
550    C F# G Ab B

Table 2-40 LOTs with five members which achieve Chromatic Saturation data for {0, 3, 5} set

Note that these are, in fact, modally equivalent (i.e. the five modes of a single scale). Let us choose LOT #248 (C, Db, D, F, F#) as our "prime form" since it is the most "packed to the left". in atonal parlance. Further, LOT #248 also has the practical attraction that it is fairly simple to recall. When in use, this LOT can be used in any order, of course, with the set itself occurring in various inversions, giving many possibilities.

In attempting to create the chromatic set by purely random transposition of the "A Love Supreme" set, I have found the following (this is experimental data; circa +/-5% accurate):
#/goes: successes/attempts = probability (to 3 significant figures):

<table>
<thead>
<tr>
<th>#</th>
<th>Successes/Attempts</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>428/73259</td>
<td>0.00584...</td>
</tr>
<tr>
<td>6</td>
<td>439/9033</td>
<td>0.0486...</td>
</tr>
<tr>
<td>7</td>
<td>863/6296</td>
<td>0.137...</td>
</tr>
<tr>
<td>8</td>
<td>722/2678</td>
<td>0.270...</td>
</tr>
<tr>
<td>9</td>
<td>1823/4854</td>
<td>0.376...</td>
</tr>
<tr>
<td>10</td>
<td>4663/9615</td>
<td>0.485...</td>
</tr>
<tr>
<td>11</td>
<td>7732/12956</td>
<td>0.597...</td>
</tr>
<tr>
<td>12</td>
<td>42255/62729</td>
<td>0.673...</td>
</tr>
</tbody>
</table>

Table 2-41 Probability of achievement Chromatic Saturation data for \{0, 3.5\} set under random transposition

Or, put more simply:

#/transpositions: roughly 1 in:

<table>
<thead>
<tr>
<th>#</th>
<th>Roughly 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>171</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>1.7</td>
</tr>
<tr>
<td>12</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 2-42 Probability of achievement Chromatic Saturation data for \{0, 3.5\} set under random transposition (as 1 in x)

---

1 For example, Forte (1973) defines this set as prime form "5-6": \{0,1,2,5,6\} with an interval vector of [311221] (p. 179).
This table emphasises the point made above that random generation of LOT #248 (or its modes) is highly unlikely. Further, it suggests that achievement of the average of 8.33 (2 d.p.) transpositions in the strings (noted above), has a probability of roughly 1 in 2.8 (this result interpolated between those for 8 and 9 “goes”), even when ignoring issues such as memory, etc.

The chances of choosing keys (without repeats; i.e. not completely random) that cover the chromatic set are (as we saw above):

<table>
<thead>
<tr>
<th>keys</th>
<th>probability: roughly 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.02 66</td>
</tr>
<tr>
<td>6</td>
<td>0.19 5.3</td>
</tr>
<tr>
<td>7</td>
<td>0.5 2</td>
</tr>
<tr>
<td>8</td>
<td>0.78 1.3</td>
</tr>
<tr>
<td>9</td>
<td>0.95 1.06</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2-43 Probability of achievement Chromatic Saturation data for {0, 3, 5} set under random transposition – without repeats

If we examine the opportunities for, say, 6 keys, we find...

There are 28 keys in bars 137-172 (ignoring repeats for this exercise), thus 23 opportunities. Thus, with probability for 6 keys being 0.19, we would expect (23 x 0.19 =) 4.37 lots of 6 successful LOTs. Actually there are 3! If we extend this approach to all
sizes of LOTs we may compare these “expected” results with those actually found in bars 137-172, and compare the two sets, thus:

<table>
<thead>
<tr>
<th>LOT size:</th>
<th>P x opportunities:</th>
<th>we expect:</th>
<th>we get:</th>
<th>“Error”:</th>
<th>“% Error”:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 x 28</td>
<td>= 0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>0 x 27</td>
<td>= 0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>3</td>
<td>0 x 26</td>
<td>= 0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>0 x 25</td>
<td>= 0</td>
<td>0</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>5</td>
<td>0.02 x 24</td>
<td>= 0.48</td>
<td>0</td>
<td>0.48</td>
<td>100%</td>
</tr>
<tr>
<td>6</td>
<td>0.19 x 23</td>
<td>= 4.37</td>
<td>3</td>
<td>1.37</td>
<td>31.35%</td>
</tr>
<tr>
<td>7</td>
<td>0.5 x 22</td>
<td>= 11</td>
<td>7</td>
<td>4</td>
<td>36.36%</td>
</tr>
<tr>
<td>8</td>
<td>0.78 x 21</td>
<td>= 16.38</td>
<td>12</td>
<td>4.38</td>
<td>26.74%</td>
</tr>
<tr>
<td>9</td>
<td>0.95 x 20</td>
<td>= 19</td>
<td>14</td>
<td>5</td>
<td>26.32%</td>
</tr>
<tr>
<td>10</td>
<td>1 x 19</td>
<td>= 19</td>
<td>17</td>
<td>2</td>
<td>10.53%</td>
</tr>
<tr>
<td>11</td>
<td>1 x 18</td>
<td>= 18</td>
<td>18</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>12</td>
<td>1 x 17</td>
<td>= 17</td>
<td>17</td>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

(all data to 2 d.p.)

Table 2-44 Showing “Error” and “Percentage Error” data between expected and actual occurences of Chromatic Saturation in bars 137-172

Here, then, “Error” assesses the difference between the expected and found results, and “Percentage Error” (= “% Error”) shows what percentage the obtained result is of the expected result. These data suggest that these chromatic scale collections are merely formed by chance, and are not the result of part of Coltrane's strategy.
Both of these graphs show smooth changes in error and percentage error data (but for the 100% error of no 5 key LOTs), adding weight to the idea that these chromatic sets are formed as a necessary result of the transpositions of the motif, and, thus, that rapid chromatic saturation is not high on Coltrane's mind, but, rather, a result of other strategies.
Further, let us note that the average number of keys of the motif required to achieve chromatic saturation (under random transposition) is 8.24 (2 d.p.). Let us assess how close Coltrane is to this average. The following graph shows the sum of the cumulative number of transpositions required to achieve chromatic saturation divided by the number of bars concerned (a sort of rolling average):

![Graph Showing Trend Towards Average Required for Chromatic Saturation](image)

From this graph we can see that Coltrane's choices show a trend which moves towards the expected average: note that the final entry is 8.33 (recurring). This is remarkably close to the expected average of 8.24 (2 d.p.) over 26 transpositions. Thus Coltrane, on average, is no faster or slower at achieving chromatic saturation than we might expect. Thus Coltrane's transpositions choices accurately impose quasi-random characteristics upon this section.

However, the following graph shows that Coltrane gradually tends towards reducing the number of transpositions required to cover the chromatic set over bars 137-172 (the
linear trendline suggests he is below the average of 8.24 (2 d.p.) for the second half of this section:

![Graph Showing Reduced Number of Transpositions of the Motif to achieve Chromatic Saturation in bars 137-172 (Linear Trendline)](image)

Therefore, we can say that although Coltrane does not seem to have calculated (and used) the (supremely fast) 5-key LOTs, there is some evidence that he exhibits an overall desire to control movements between keys that will create shorter lists that sum to the chromatic set as he moves from bar to bar. This gradual evolution would seem to point to a desire to heighten the chromatic nature of the section through time by shortening the time taken to sum to the chromatic set. However, we should note that this trend is subtle, and otherwise this section covers the chromatic set remarkably in line with expected average probability.
Is the "A Love Supreme" set the fastest 3-note set at covering the chromatic set?

We have seen that the set used by Coltrane to construct the "A Love Supreme" motif will achieve chromatic saturation within as few as five transpositions. However, let us now note that there are several 3-note sets that can saturate even faster, if the correct List Of Transpositions is chosen. For example, Scale #13 (C, Db, D) will achieve chromatic saturation within as few as 4 transpositions (LOT#164 = C, Eb, F#. A). Note that both this scale and LOT are highly symmetrical, and thus their use would have a relatively high aural predictability. This speed of saturation is also true of several other 3-note scales and associated LOTs, as can be seen in the following table:

<table>
<thead>
<tr>
<th>scale #</th>
<th>LOT #</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>164</td>
</tr>
<tr>
<td>16</td>
<td>164</td>
</tr>
<tr>
<td>19</td>
<td>164</td>
</tr>
<tr>
<td>22</td>
<td>164</td>
</tr>
<tr>
<td>24</td>
<td>98, 164, 202</td>
</tr>
<tr>
<td>27</td>
<td>164</td>
</tr>
<tr>
<td>30</td>
<td>98, 164, 202</td>
</tr>
<tr>
<td>40</td>
<td>164</td>
</tr>
<tr>
<td>46</td>
<td>164</td>
</tr>
<tr>
<td>48</td>
<td>164</td>
</tr>
<tr>
<td>51</td>
<td>164</td>
</tr>
<tr>
<td>58</td>
<td>164</td>
</tr>
<tr>
<td>61</td>
<td>164</td>
</tr>
<tr>
<td>63</td>
<td>98, 164, 202</td>
</tr>
<tr>
<td>67</td>
<td>164</td>
</tr>
</tbody>
</table>

Table 2-45 The 3-note Scales and LOTs that achieve Chromatic Saturation
Note, again, the underlying symmetry of many of the commonly-occurring LOTs:

- LOT #98 is C, Db, F, F# (min2, Maj3rd, min2)
- LOT #164 is C, Eb, F#, A (min3, min3, min3)
- LOT #202 is C, F, F#, B (Perf4, min2, Perf4)

Table 2-46 LOTs with underlying symmetry

This requirement for a highly symmetrical structure is perhaps most highlighted by the fact in Table 2-45 we can see that scale #43 (C, E, Ab) has the highest list of possible LOTs. This symmetry contrasts with the (relatively slower) set of the “A Love Supreme” motif (scale #33) and its associated fastest LOTs (e.g. LOT #248). We might expect the “independent” nature of the “A Love Supreme” set to be responsible for its specific character re LOTs, due to its relatively asymmetrical structure. However, there is not a direct correlation between independence and speed of chromatic saturation, generally speaking. For example, scales #16 and #40 (C, Db, F and C, E, F) are both “independent” under transposition, and can sum to the chromatic set in just four transpositions - under LOT#164 (C, Eb, F# A). However, this is a rare case: the remainder of the 3-note “independent” scales (#14, 15, 16, 23, 25, 32, 33, 35, 40, 42) have 5 as the minimum number of transpositions required to achieve the chromatic set. What this shows is that there are many three-note sets that Coltrane could have chosen for the “A Love Supreme” motif that saturate faster than scale #33, and that two of these sets (scales #16 and #40) are, just like scale #33, “independent”.

The above analysis is based upon the assumption that the correct LOT may be chosen in order to achieve chromatic saturation most speedily. However, we might wonder which sets of notes achieve chromatic saturation quickest if their transpositions are generated
utterly randomly. I have undertaken a series of experiments on all 2048 scales using my computer as a random transposition generator. Those sets which are (a) the slowest and (b) the fastest at achieving chromatic saturation under random transposition have been thus discovered, and I present the results of this work in Appendix 4. These data show that the "A Love Supreme" set (represented as scale 3(d) in Table 1 of Appendix 4) is in fact amongst the very slowest of 3-note sets at achieving chromatic saturation under random transposition.

What these analyses show conclusively is that a high speed of chromatic saturation was not a priority for Coltrane when composing the "A Love Supreme" motif. Further, we can say that one of the advantages of the "A Love Supreme" motif is that when it is transposed in a quasi-random manner (as in bars 137-172: see below) it is less likely to achieve chromatic saturation than most other 3-note sets.
Can we find evidence of a repeated section and/or a mnemonic?

An examination of bars 137-172 reveals the following frequency of occurrence for each of the 12 keys (i.e. by pitch class):

<table>
<thead>
<tr>
<th>key</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>Db</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
</tr>
<tr>
<td>Eb</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>11</td>
</tr>
<tr>
<td>F#</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>3</td>
</tr>
<tr>
<td>Ab</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>Bb</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2-47 Frequency of use of each transposition of "A Love Supreme" motif by pitch class, bars 137-172

Let us note the relatively high occurrence of the keys of Db, D, G and Ab (ignoring the tonic F), which are marked above in bold. In fact, if sum the results for Db, D, G and Ab, we find that these 4 keys occur more often than the other 7 keys put together (again, ignoring the tonic F), thus:

\[ Db + D + G + Ab = 3 + 4 + 3 + 4 = 14 \text{ times} \]
\[ C + Eb + E + F# + A + Bb + B = 2 + 2 + 2 + 1 + 1 + 1 + 2 = 11 \text{ times} \]

Table 2-48 Frequency of use of \{Db, D, G, Ab\} and \{other\} transpositions of motif
This discovery suggested that this set of \{Db, D, G, Ab\} might represent a core list of keys from a mnemonic used by Coltrane to cover all 12 keys. Let us now examine the specific occurrences of these 4 keys within bars 137-172:

<table>
<thead>
<tr>
<th>bar</th>
<th>up/down</th>
<th>motif key</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>138</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>139</td>
<td>up</td>
<td>G</td>
</tr>
<tr>
<td>140</td>
<td>down</td>
<td>D</td>
</tr>
<tr>
<td>141</td>
<td>up</td>
<td>Ab</td>
</tr>
<tr>
<td>142</td>
<td>down</td>
<td>Db</td>
</tr>
<tr>
<td>143</td>
<td>down</td>
<td>C</td>
</tr>
<tr>
<td>144</td>
<td>oct+up</td>
<td>D</td>
</tr>
<tr>
<td>145</td>
<td>up</td>
<td>Eb</td>
</tr>
<tr>
<td>146</td>
<td>down</td>
<td>Ab</td>
</tr>
<tr>
<td>147</td>
<td>up</td>
<td>Db</td>
</tr>
<tr>
<td>148</td>
<td>down</td>
<td>A</td>
</tr>
<tr>
<td>149</td>
<td>down</td>
<td>F</td>
</tr>
<tr>
<td>150</td>
<td>up</td>
<td>D</td>
</tr>
<tr>
<td>151</td>
<td>down</td>
<td>Ab</td>
</tr>
<tr>
<td>152</td>
<td>up</td>
<td>B</td>
</tr>
<tr>
<td>153</td>
<td>down</td>
<td>E</td>
</tr>
<tr>
<td>154</td>
<td>up</td>
<td>B</td>
</tr>
<tr>
<td>155</td>
<td>down</td>
<td>Db</td>
</tr>
<tr>
<td>156</td>
<td>oct+up</td>
<td>Eb</td>
</tr>
<tr>
<td>157</td>
<td>down</td>
<td>Bb</td>
</tr>
<tr>
<td>158</td>
<td>up</td>
<td>C</td>
</tr>
<tr>
<td>159</td>
<td>down</td>
<td>G</td>
</tr>
<tr>
<td>160</td>
<td>down</td>
<td>D</td>
</tr>
<tr>
<td>161</td>
<td>up</td>
<td>E</td>
</tr>
<tr>
<td>162</td>
<td>up</td>
<td>F#</td>
</tr>
<tr>
<td>163</td>
<td>up</td>
<td>G</td>
</tr>
<tr>
<td>164</td>
<td>up</td>
<td>Ab</td>
</tr>
<tr>
<td>165</td>
<td>down</td>
<td>F</td>
</tr>
<tr>
<td>166</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>167</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>168</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>169</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>170</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>171</td>
<td>same</td>
<td>F</td>
</tr>
<tr>
<td>172</td>
<td>same</td>
<td>F</td>
</tr>
</tbody>
</table>

Table 2-49 Locations of \{Db, D, G, Ab\} transpositions in bars 137-172
From these data we can see that:

- the keys of G, D, Ab and Db occur right at the start of this section in sequence (bars 139-142)

- occurrences of G, D, Ab or Db are mostly grouped together in small ordered subsets (bars 146-147; 150-151; 159-160; 163-164)

- these subsets generally follow the strict order of G, then D, then Ab, then Db, and use adjacent members of this strict order (the exception to this being found in bars 163-164, which is within the cadential section, but even there Ab follows G, thus maintaining the list's direction)

- there seems to be controlled variation of the use of this set of keys: for example, bar 145 (Eb) might be read as a simple interpolation in what is otherwise a D, Ab, Db pattern (bars 144-147)

Thus, the set {Db, D, G, Ab} seems to have a specific order of use: G, D, Ab, Db. There is no other set of keys to be found in bars 137-172 that has these characteristics. Note, however, the use of controlled variation described above; Coltrane does not use any pair of keys from this set of four in precisely the same way through the section. Further, we cannot, for example, state that every D follows a G, or that every Ab follows a D, or that every Db follows an Ab. Thus the implementation of this set is highly integrated within Coltrane's attempts to avoid redundancy and thus surprise the listener. Thus, this theory seems compatible with my previous analyses.
What are the possible sources of this repeated section?

We have seen that Coltrane had a specific hierarchy for the transpositions of the motif with regard to the tonic F. However, the above analysis suggests that Coltrane may have simultaneously seen the list of G, D, Ab and Db as a “core” list of transpositions from which to build this section. That this list is four keys long, and that it is constructed from two chromatic pairs (Db and D, G and Ab) means that is relatively simple to recall. I now propose two possible sources for this “core” list of keys:

1. a construction of transpositions designed to cover the chromatic set (probably calculated by Coltrane himself)
2. material found within Slonimsky’s “Thesaurus”: the “Mother” and “Grandmother” chords

Let us examine these two possible sources one at a time:

**Possible Source 1: A Construction of Transpositions Designed to Cover the Chromatic Set**

This is a calculated (/composed) attempt to organise transpositions of the “A Love Supreme” set in such a way as they form the chromatic set. If we start by transposing the motif chromatically upwards, we find it takes 7 keys to cover the chromatic set, thus:
This gives us a chromatic list of keys, thus:

C, Db, D, Eb, E, F, F# (7 transpositions)

With this approach there is much redundancy, shown in the final row of the table above (for example, the notes F and F# occur 3 times), and the list of keys only stops with the key of F#, which provides us with the B note. However this illusive B note could also be found in the keys of Ab and B, thus suggesting the following alternative sets of seven keys:

C, Db, D, Eb, E, F, Ab
C, Db, D, Eb, E, F, B

Having taken this step, it is easy to see that there are many possible patterns of 7 keys that will achieve the chromatic set. I suggest that the following search is a logical strategy (extending from this initial approach) to reduce the number of necessary keys.
This new search is undertaken by recording the notes covered by the key of C, then choosing Db as the second key, since it contains no common notes with the motif at C:

notes: C  Db  D  Eb  E  F  F#  G  Ab  A  Bb  B  
C  x  x  x  
Db x  x  x  

Table 2-51 Search for Chromatic Saturation strategy (a)

(The starting note of each motif is marked in bold.) So far, then, we have covered 6 of the 12 notes of the chromatic scale. Realising that the (next, logical) motif at D will contain the note F, which we have already had from the motif at C, we might look for another key. We might then ignore the keys of Eb, E, F and F# because, similarly, those notes have already been "used" by the motifs at C and Db\(^1\). From this perspective, G appears attractive because the notes G and Bb have not been used (although it actually no better than D or any other key; it also contains the note C, of course, which is already covered), thus:

notes: C  Db  D  Eb  E  F  F#  G  Ab  A  Bb  B  
C  x  x  x  
Db  x  x  x  
G  (x)  x  x  
((x) = a duplicate)

Table 2-52 Search for Chromatic Saturation strategy (b)

Matching the previously successful \{C, Db\} chromatic relationship, we might add the key of Ab next, thus:

\(^1\) In fact, no third key is possible without covering one of the notes already used.
### Table 2-53 Search for Chromatic Saturation strategy (c)

The remaining notes D and A can only be covered by a single transposition at A, thus:

<table>
<thead>
<tr>
<th>notes: C</th>
<th>Db</th>
<th>D</th>
<th>Eb</th>
<th>E</th>
<th>F</th>
<th>F#</th>
<th>G</th>
<th>Ab</th>
<th>A</th>
<th>Bb</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Db</td>
<td></td>
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<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>G</td>
<td>(x)</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Ab</td>
<td>(x)</td>
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<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
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<td></td>
<td></td>
<td>x</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### Table 2-54 Search for Chromatic Saturation strategy (d)

Here is a summary (duplicate notes removed, transpositions in bold):

<table>
<thead>
<tr>
<th>notes: C</th>
<th>Db</th>
<th>D</th>
<th>Eb</th>
<th>E</th>
<th>F</th>
<th>F#</th>
<th>G</th>
<th>Ab</th>
<th>A</th>
<th>Bb</th>
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<tbody>
<tr>
<td>C</td>
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<td>x</td>
<td>x</td>
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<td>x</td>
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<td></td>
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<tr>
<td>Db</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
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<tr>
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<td></td>
<td>x</td>
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<td>x</td>
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<td></td>
<td>x</td>
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</tr>
</tbody>
</table>

### Table 2-55 Summary of search for Chromatic Saturation strategy
This strategy, then, provides us with a list of 5 keys, thus:

C, Db, G, Ab, A (=5 transpositions).

This is a clear improvement on our initial, purely chromatic, attempt (see Table 2-50). As we have seen above, five is in fact the fewest number of keys of the motif that will sum to the chromatic set. However, we have already noted that, aside from the tonic F, it is the keys of Db, D, G and Ab that Coltrane plays more frequently than any others. How then do we explain the high frequency of the key of D, which is not found in the list of five keys above (see Table 2-55)? It is possible that Coltrane may have chosen to use an approach that uses both keys of D and A in order to cover those final two notes (D and A), thus:

notes: C Db D Eb E F F# G Ab A Bb B
key:
C x x x
Db x x x
G x x
Ab x x
D+A x x

Table 2-56 Version of search for Chromatic Saturation strategy which includes both D and A

Coltrane may have done this to compensate for the fact that the key of A has both its upper notes “wrapped around” C; thus he may have simply not have spotted that the key of A could cover both notes. Although this seems unlikely at first glance, this kind of strategy would be analogous to the one that we noted at the use of the transposition of the motif to G as the third key in our construction of the list, above (i.e., as noted above, that the key of G is, in fact, no better than the key of D - or any other key). All of the
other keys chosen have at the most one of their upper note “wrapped around” the C: the key of A, however, has both of its upper notes occurring to the left of the tonic (in **bold**) in the diagram above. In summary, we find that this approach produces a list of six keys that includes D:

C. Db, D, G, Ab. A

Further, there are structures within this 6-key list that may have appealed to Coltrane. Firstly, note that this list can be neatly divided into two halves of 3-note chromatic sets:

C   Db   D   G   Ab   A

Also, let us note that the symmetrically centred set of four keys are the most commonly occurring (C and A occur twice and once only, respectively, and never occur adjacent to Db or Ab respectively), i.e. (marked in **bold**):

C   Db   D   G   Ab   A

Note also its structural relationship with the positions of the “A Love Supreme” set at I and V of the minor pentatonic, thus (both of these theoretical models at C):

keys:   C   Db   D   G   Ab   A

minor pentatonic:   C   Eb   F   G   Bb   C

Table 2-57 Comparison of six-key set and minor pentatonic structure

In fact, Coltrane moves through this “core” list in an oscillating motion, thus:
We might postulate, then, that Coltrane plays the transpositions in this order as a variation of a more symmetrical G, D, Ab, Db, A, C order; that is, an order that shows a "rocking" motion between the keys. Thus, adapting the above diagram, we can postulate the following:

<table>
<thead>
<tr>
<th>C</th>
<th>Db</th>
<th>D</th>
<th>G</th>
<th>Ab</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>4</td>
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<tr>
<td>5</td>
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</tr>
</tbody>
</table>

Table 2-58 Oscillating structure of six-key set

This oscillating motion, expanding outwards, produces an "out factor" curve very similar to that found in the first peak (described above). This theoretical model thus matches the idea that this section could be seen as being symbolic of a gradual moving "away from the esteemed path", characteristic of a "period of irresolution" (these quotations are from Coltrane's album notes to "A Love Supreme"), drawn from the transpositions actually chosen. Indeed, if we accept that Coltrane had calculated this list
of C, Db, D, G, Ab, A, had learnt it as G, D, Ab, Db, A, C (i.e. rocking between transpositions, gradually moving away from F Dorian) and had altered the ordering of last two keys of A and C, then this may explain why there is a relatively high frequency of fifth relations in bars 137-172 (as noted by Porter). This is because, if we examine the intervals that lead between this list of keys, we find that Perfect 4ths/Perfect 5ths are the most common interval, occurring twice between adjacent members of this list (Perfect 4ths and 5ths marked in bold):


However, recalling Coltrane’s apparent desire to avoid redundancy, we might note that this “rocking” pattern begins to sound very predictable if we include the motif at A. Thus, perhaps Coltrane edited out the A transposition in order to emphasise the characteristic of a chaotic journey by moving straight on to the C. In fact, if Coltrane had played the motif in A at bar 143, then a graph of the “out factors” for these first few bars show an even smoother curve than that found in the first peak (see above). That is, from Db to A to C is a gentler journey than straight from Db to C. Let us note that the only time in bars 137-172 that the key of A actually occurs (bar 148) is immediately following the second occurrence of Db, perhaps suggesting that Coltrane felt the need to account for this transposition fairly soon after its absence (?) at bar 143:
Table 2-60 Context of transposition to the key of A, bar 148

It is also striking that it is immediately following this A that Coltrane plays the motif at F (bar 149), which perhaps he is using as a kind of tonic punctuation to a now complete (i.e. fully represented) "core" list of G, D, Ab, Db, A, C. The only other occurrence of Db in this section is at bar 155. Here it is followed by further different key: an Eb, a Major 9th above. Could it be that Coltrane uses this relatively large upward interval in order to show contrast with the previous movements that follow Db?

Table 2-61 Transposition of "A Love Supreme" motif in Coltrane's solo. bars 155-156

This suggestion calls for a reassessment of the only other previous Major 9th interval, at bar 144. That this occurs immediately after the initial G, D, Ab, Db, C section implies
that Coltrane, similarly, may be trying to create contrast with the end of this list by making the following transposition such a large interval:

142 down Db
143 down C
144 oct+up D

Table 2-62 Transposition of “A Love Supreme” motif in Coltrane’s solo, bars 142-144

So, this use of the tonic F and the Major 9th interval add further weight to the theory that Coltrane is using the oscillating pattern of G, D, Ab, Db, A, C as a “core” list. It is also possible, of course, that Coltrane rejected the motif at A at bar 143 for the simple reason that it forms an F6 chord when combined with an F bass. This conventional resolution is further emphasised in that it occurs as the fifth transposition of the “rocking” list defined above, if we see G as the first transposition (4 bar blocks being a common ground between blues, popular song and many jazz originals):

bar #: // 1 2 3 4 // 5
key: // G D Ab Db // A etc.

Table 2-63 Position of (unperformed) transposition to A as first bar in a four-bar bloc

An argument against Possible Source 1

Although this theory shows a possible source for the list of G, D, Ab, Db, there is a further issue I should like to address which challenges this theory.
Let us note that the search described here initially provides the list in a different order than that performed by Coltrane, and it is only the invention of a "rocking" movement through the list that creates a better match (this reordering of the core list seems to me to be the biggest "leap of faith" in the above theory).

Perhaps, then, although this theory has some initial simplicity (and, thus, charm). I believe that it is more likely that the keys existed in the strict order G, D, Ab, Db in the source itself. It is this possibility that I propose as the second possible source for the commonly occurring list of G, D, Ab, Db.
Possible Source 2: Slonimsky’s “Mother” and “Grandmother” Chords

“In a lecture at the University of Iowa on April 6, 1976, Nicolas Slonimsky reported that his publisher had informed him that Coltrane had recommended Slonimsky’s “Thesaurus of Scales and Melodic Patterns” (New York: Coleman-Ross, 1947) to all his colleagues and pupils. This work, which contains nearly 1,000 traditional and contrived scales, is described by its author as ‘a reference book for composers in search of new materials’.1

Nicolas Slonimsky’s “Thesaurus of Scales and Melodic Patterns”, published in 1947, was one text among many that Coltrane is reported to have used during his practise routines; Thomas (1975), for example, describes how Coltrane used saxophone, piano (specifically Slonimsky’s book), violin and harp books.2 Indeed, this book is often associated uniquely with Coltrane, and seemed to have an especial importance to him. Thomas describes Coltrane’s use of Slonimsky’s “Thesaurus” in a practise session thus:

“He begins with tritone progressions, intervals of augmented fourths that contain three whole tones. He continues with a series of scales that call for an interpolation of four or more notes, constructed from a complex series of sixteenth notes and an amazing number of accidentals, for almost every other note seems to have a sharp or flat preceding it, indicating an immediate change of tone, either up or down a half step.”3

1 Budds (1990) p. 50
2 Thomas (1975) pp. 102-103
Exactly when and how Coltrane came across Slonimsky's book is unclear. Porter (1998), citing McCoy Tyner, suggests that Coltrane was likely to have first seen it sometime during the mid-1950s. Thomas (1975) says Barry Harris (piano) showed Slonimsky's book to Coltrane; Porter (1998) reports that Harris denies this. Jimmy Heath (tenor saxophone) has said that Coltrane was not using Slonimsky's book during their joint practise sessions, for which they borrowed books from the Philadelphia Library, which started in 1948.

It is clear that Dennis Sandole was at least partly responsible for encouraging Coltrane's interest in scale constructions per se:

"I taught John Coltrane advanced musical techniques. I started him in theory, then moved him along. He asked me about bitonalities and polytonalities, combining more than one key signature. I suggested tetrachord techniques and pentatonic as well as diatonic scales, and he was playing arpeggios on all of them. I taught him from my thesaurus of scales that I've compiled through the years, using notes from foreign scales and mixing them with Western scales until everything was just right. He studied semitonal scales, modal scales, pedal point clusters, and harmony derived from melodic lines, with no chord structure involved."

Sandole had written his own text book of scales ("Scale Lore"), which, interestingly, he saw as being based on "an aural approach" in contrast to Slonimsky's "intellectual

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3 Thomas (1975) p. 102; Thomas seems to be referring to pp. 1-11 of Slonimsky (1947).
5 Porter (1998) p. 149 and p. 63
6 Dennis Sandole, quoted in Thomas (1975) p. 51
approach". Thus, Sandole may have advised Coltrane of the cultural neutrality of Slonimsky’s book, although it seems that they did not use it in their lessons together.

Supportive of, and inspiration to, the theory outlined here is the revelation by David Demsey (1991) that Coltrane’s use of Slonimsky’s book was not limited to practising melodic patterns and developing technical skills on the saxophone. Demsey shows that the second half (8 bars) of Coltrane’s composition “Giant Steps” is in fact taken from p. vi of the introduction of the “Thesaurus of Scales and Melodic Patterns”. Although “Giant Steps” was recorded in 1959, McCoy Tyner has suggested that this composition may have been written as early as May 1957.8 Porter succinctly describes Slonimsky’s purpose in showing this example as being “to show that one can make...abstract patterns more tonal by underpinning them with tonic-dominant chords”.9 Coltrane had already revealed an interest in fully-chromatic melody in “Miles’ Mode”. Let us recall that it was the album “Giant Steps” that launched Coltrane’s career as leader10; thus, Slonimsky’s book had become valuable to him, both musically and from the perspective of his career.

Aside from this example, Slonimsky’s book must have appealed to Coltrane in many ways. In the introduction, Slonimsky gives several examples of composers’ use of “unconventional” scales and patterns, which, like the harmony of the “Giant Steps” model on p. vi, may have not seen so strange to Coltrane: for example, Slonimsky notes Busoni’s enthusiasm for the scale C Db Eb Fb Gb Ab Bb which provides a “novel harmonic sensation” (jazz musicians know this as the Super Locrian.

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7 Porter (1998) p. 51
8 Porter (1998) p. 150, quoting Priestley (p. 53)
9 Porter (1998) p. 150
10 See Thomas (1975) p. 115
conventionally used over a V7Alt.(b9,b13) chord). Relatively, Slonimsky's theories relating to harmonisation of the melodic patterns centres around the use of 3-note voicings of Major triads, dominant 7th, Major 9th and Whole Tone chords (pp. 240-241; the pages immediately before the occurrence of the “Mother” and “Grandmother” chords - see below). In general, the book provides page after page of non-hierarchical (from a stylistic perspective), systematically generated scalar and melodic material. In this sense, this book shows neither a strong classical nor jazz inclination (cf. Russell (1953)). Thus the material is non-stylistic, culturally pluralistic, anti-prejudicial, universal. Just as the King James' Version of the Bible is the source for much of the text on the album cover, so Slonimsky's text might be seen as the musical “bible” from which Coltrane draws inspiration.

It would be correct to say that much of the book's philosophy and content is distinctly associable with modern classical music (indeed modern American classical music: the “Pandiatonic” examples have a distinctive Copland-esque flavour). However, this is not incompatible with Coltrane's position at the point at which he seems to have first used the book. Let us recall the interest that the bebop musicians showed in self-consciously expanding their musical vocabulary, partly by developing relationships (of various intensities) with modern classical music, and the influence that this had upon later musicians. Indeed, Coltrane had himself listened to “The Firebird Suite” and “The Rite of Spring” because he had heard that Parker carried around miniature scores of Stravinsky:

\[\text{\footnotesize 11 Slonimsky (1947) p. iii} \]
\[\text{\footnotesize 12 See, for example. Budds (1990)} \]
\[\text{\footnotesize 13 Jimmy Heath, quoted in Porter (1998) p. 63} \]
“John never spoke more highly of any classical composer than Stravinsky. Once, when we were visiting his mother, he brought out a record and said, “I think I’ve found my universal musician. Then he played Stravinsky’s Firebird Suite.”14

Coltrane must have seen Slonimsky’s “Thesaurus” as a natural extension of this approach, with the distinct advantage that its systematic nature makes it apparently descriptive of - and applicable to - all musics. Every possible scale (all 2048) is accounted for in some way, and Slonimsky devised a specific, non-hierarchical language to describe melodic structures, based in the divisions of the octave. Each scale and melodic pattern is given a number, and there is some useful cross-referencing.

This completeness has a parallel in Jimmy Heath description of Coltrane’s desire to play fluently in all keys.15 Such an approach seem to stem from an initial goal of technical fluency, but has the advantage of allowing a musician to develop a vocabulary of transferable formulae, which enable a given scale/motif to be used in a multitude of contexts; this idea is pursued in detail by Berliner (1994). This is an extension of the use of “riffs”, common to much jazz, which, however, tend to be fixed in their relationship with a given key. What Coltrane contributed is a concentration upon, firstly, those motifs that occur in a range of functional positions within a scale, and, secondly, a conscious search for such motifs. We have seen such a structure in the “A Love Supreme” sets of {0, 3, 5} and {0, 5, 7}. “Alabama”, so closely related to “A Love Supreme”, shows thematic use of {0, 3, 5}. Similarly, the solo of “Giant Steps” and, later, “Meditations” show heavy use of {0, 2, 4, 7}.16

14 Alice Coltrane, quoted in Thomas (1975) p. 192
15 Porter (1998) p. 64
Incorporated within, and after, the list of nearly 1000 numbered scales and melodic patterns (pp. 1-191), Slonimsky gives several sections of musical constructions that may have appealed to Coltrane, simply because they describe approaches similar to his own compositional and improvisational practices:

- Division of Twelve Tones into Four Mutually Exclusive Triads (pp. 177-178)
- Pandiatonic Progressions (pp. 192-195)
- Pandiatonic Cadences (p. 194) - cf. Miles Davis and Coltrane’s use of what we might term the “Panchromatic Cadences” of “So What?” and “Impressions”
- Twelve-Tone Patterns (p.173)
- Polytonal Scales (p. 220); all of these consist of a pair of major scales a minor 3rd or a Major 3rd apart
- Polyrhythmic Scales (p. 224) - Coltrane’s music, and his drummer, Elvin Jones, were frequently described as “polyrhythmic”
- Polytonal Polyrhythmic Scales (p. 226)
- Palindromic Canons (p. 234) - cf. palindromic quality of “Miles’ Mode”

At the rear of the book (pp. 242-243), beyond the main text, Slonimsky shows (in large type) 12 heroic chords which completely fill two pages, the final two of these being the “Mother Chord” and the “Grandmother Chord” – see Fig. 2-21. It is within these chords that we will find the “core” list of G, D, Ab, Db used by Coltrane in bars 137-172 of “Acknowledgement”. Slonimsky describes the Mother Chord in his introduction thus:

\[\text{17 I have discovered a small omission in this section of Slonimsky's book – see Appendix 5.}\]
“The idea was first introduced by the Austrian musician Fritz Klein in 1921 in a curious composition entitled *Die Machine*, with the subtitle *Ex-Tonal Self-Satire*. The name of the composer was concealed behind a characteristic nom de plume *Heautontimorumenus* which means Self-Torturer. In this piece Klein introduced a Mother Chord which contains not only all 11 intervals, but 12 different notes as well.”

18 Slonimsky (1947) p. iii
Mother and Grandmother Chords

from Slonimsky (1947)
"Thesaurus of Scales and Melodic Patterns" p. 243

Fig. 2-21 Slonimsky's Mother and Grandmother Chords
Slonimsky then reveals his “further elaboration” of Klein’s Mother Chord: an invertible 11-interval, 12-tone chord. This he named the “Grandmother Chord”.¹ It has all of the properties of the Mother Chord, but also has the intervals ordered in such a way that they

“are alternately odd-numbered and even-numbered when counting in semitones, with the row of odd-numbered intervals forming a decreasing arithmetical progression and the row of even-numbered intervals forming an increasing arithmetical progression”.²

Slonimsky also points out in his introduction that “the order of notes in the Grandmother Chord is identical with the 12-tone Spiral Pattern No. 1232a [p. 175].”³ This cross-reference could have provided Coltrane with a melodic form of the chord (which would be easier to read) prior to transposition to the key of F.

I will now show how Coltrane’s “core” list of transpositions can be found in a set of notes common to both Klein’s “Mother” and Slonimsky’s “Grandmother” chords. Here is table showing my numbering of the Mother and Grandmother chords (note No.1 is lowest in the chord, note No.12 is highest; i.e. these lists are upside down with regard to the notated chords), with their original transpositions (as published), and transposed with F as the root (i.e. transposed to the key of “Acknowledgement”):

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¹ Is it possible that the names of these chords held an interest for Coltrane, beyond their chromatic nature? Coltrane’s mother (who was a widow) had recently become a grandmother, with the birth of John W. Coltrane, Jr. on August 26, 1964. Let us note that this birthday is just a few months before the recording of “A Love Supreme” (December 9, 1964).

² Slonimsky (1947) p. iii
<table>
<thead>
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<th>$x$</th>
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<th>$B$</th>
<th>$E$</th>
<th>$D$</th>
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<tr>
<td>8</td>
<td>F</td>
<td>Ab</td>
<td>Db</td>
<td>Db</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>G</td>
<td>E</td>
<td>Eb</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Bb</td>
<td>G</td>
<td>F#</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>D</td>
<td>F</td>
<td>Bb</td>
<td>Bb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Eb</td>
<td>F#</td>
<td>B</td>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2-64 Published and transposed (to F) Mother and Grandmother Chords

(Note that in this, and following tables, $x$ refers to the position of a particular note within a given transposition of the Mother or Grandmother chord.) We have already seen that Coltrane uses the keys of G, D, Ab and Db most frequently in bars 137-172 (aside from the tonic F). Let us note that, amongst other similarities, this specific list (and order) of keys occurs as the elements 5, 6, 7 and 8 in both the Mother and Grandmother chords:

---

3 Slonimsky (1947) p. iii
Mother (at F)  Grandmother (at F)
\[ = M(\text{at F}) = G(\text{at F}) \]

\begin{tabular}{ll}
1 & F  \\
2 & E  \\
3 & C  \\
4 & A  \\
5 & G  \\
6 & D  \\
7 & Ab \\
8 & Db \\
9 & Eb \\
10 & F# \\
11 & Bb \\
12 & B \\
\end{tabular}

Table 2-65 Showing common \{G, D, Ab, Db\} set in both Mother and Grandmother Chords

We might expect to find somewhere in Slonimsky’s book any set of notes used by Coltrane (or any other musician); the “Thesaurus” is a complete list of melodic sets, after all. However, for this string of 4 keys to occur in the key of F for both of these chords and in the correct order seemed a large enough a coincidence to warrant further investigation. Let us at this point recall that these chords appear in the book as obviously as that section used by Coltrane for “Giant Steps”: i.e. within 5 pages of the front and rear covers and away from the body of the text.

Here is a table which shows subsets (i.e. two elements or larger) of these lists found in bars 137-172 as they occur in the Mother and Grandmother chords, which shows that transposition of the chords to the key of F matches Coltrane’s performance much more highly than if they are left in their original form:
<table>
<thead>
<tr>
<th>bar</th>
<th>up/down:</th>
<th>motif</th>
<th>key:</th>
<th>Mother</th>
<th>Grandmother</th>
<th>Mother</th>
<th>Grandmother</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td>same</td>
<td>F</td>
<td></td>
<td>F</td>
<td></td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>138</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>139</td>
<td>up</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>down</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>up</td>
<td>Ab</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>down</td>
<td>Db</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>143</td>
<td>down</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>oct+up</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>145</td>
<td>up</td>
<td>Eb</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146</td>
<td>down</td>
<td>Ab</td>
<td></td>
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<tr>
<td>147</td>
<td>up</td>
<td>Db</td>
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<tr>
<td>148</td>
<td>down</td>
<td>A</td>
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<td></td>
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<tr>
<td>149</td>
<td>down</td>
<td>F</td>
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<td></td>
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<td></td>
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<tr>
<td>150</td>
<td>up</td>
<td>D</td>
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<td></td>
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<tr>
<td>151</td>
<td>down</td>
<td>Ab</td>
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<td></td>
</tr>
<tr>
<td>152</td>
<td>up</td>
<td>B</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>153</td>
<td>down</td>
<td>E</td>
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<td></td>
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<tr>
<td>154</td>
<td>up</td>
<td>B</td>
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<tr>
<td>155</td>
<td>down</td>
<td>Db</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>156</td>
<td>oct+up</td>
<td>Eb</td>
<td></td>
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<td></td>
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<tr>
<td>157</td>
<td>down</td>
<td>Bb</td>
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<tr>
<td>158</td>
<td>up</td>
<td>C</td>
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<tr>
<td>159</td>
<td>down</td>
<td>G</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>160</td>
<td>down</td>
<td>D</td>
<td></td>
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<td></td>
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<tr>
<td>161</td>
<td>up</td>
<td>E</td>
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<tr>
<td>162</td>
<td>up</td>
<td>F#</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>163</td>
<td>up</td>
<td>G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>164</td>
<td>up</td>
<td>Ab</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>165</td>
<td>down</td>
<td>F</td>
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<tr>
<td>168</td>
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<tr>
<td>169</td>
<td>down</td>
<td>F</td>
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<tr>
<td>170</td>
<td>down</td>
<td>F</td>
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<tr>
<td>171</td>
<td>down</td>
<td>F</td>
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<td>172</td>
<td>down</td>
<td>F</td>
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</tr>
</tbody>
</table>

Table 2-66 Matches between list of transpositions of bars 137-172 and transposed Mother and Grandmother Chords

In noting this transposition away from their original form, let us note that, firstly, that this allows a comparison of the two chords as melodic lists (in their original forms, the two chords start on different notes: A and C), and, secondly, that Coltrane had transposed Slonimsky’s example on p. vi up a minor 3rd when using it for “Giant
Steps”. Perhaps this transposition is an attempt by Coltrane to contribute to the form. Demsey (1991) notes that the V > I relationships in the Slonimsky example have been extended to the (jazz) II > V > I pattern of “Giant Steps”.

There is also some evidence that Coltrane also recalls the core list’s context within the Mother and Grandmother chords. We can see this if we note that Coltrane plays the motif at Db on three occasions. These transpositions to Db are followed by the following choices of keys: C (bar 143), A (bar 148) and Eb (bar 156). These three keys are the notes that follow the core set of G, D, Ab, Db in both the Mother and Grandmother chords, thus:

<table>
<thead>
<tr>
<th></th>
<th>Mother (at F)</th>
<th>Grandmother (at F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>E</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>F#</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
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<tr>
<td>5</td>
<td>G</td>
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</tr>
<tr>
<td>6</td>
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<td>7</td>
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<tr>
<td>8</td>
<td>Db</td>
<td>Db</td>
</tr>
<tr>
<td>9</td>
<td>Eb</td>
<td>A</td>
</tr>
<tr>
<td>10</td>
<td>F#</td>
<td>C</td>
</tr>
<tr>
<td>11</td>
<td>Bb</td>
<td>Bb</td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

Table 2-67 Showing that C, A and Eb are immediately adjacent to core \{G, D, Ab, Db\} sets
We might usefully play the “devil’s advocate” at this point and suppose that Coltrane may have either transposed one or other of Slonimsky’s chords to match their tonics, or transposed them both up a tone for his (Bb) tenor saxophone. However, we would discover no real parallel with either the Mother or Grandmother chord with this approach, the data are clearly sparse, and the match is nowhere near as good as that to be found in the “(at F)” columns in Table 2-66 above:
<table>
<thead>
<tr>
<th>bar</th>
<th>up/down</th>
<th>motif</th>
<th>key</th>
<th>Mother</th>
<th>Grandmother</th>
<th>Mother</th>
<th>Grandmother</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Original)</td>
<td>(Original)</td>
<td>(at F)</td>
<td>(at F)</td>
</tr>
<tr>
<td>137</td>
<td></td>
<td></td>
<td></td>
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<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>138</td>
<td>same</td>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>139</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>Bb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td>Eb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>143</td>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>oct+up</td>
<td></td>
<td></td>
<td></td>
<td>E</td>
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<td></td>
</tr>
<tr>
<td>145</td>
<td>up</td>
<td></td>
<td></td>
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<td>F</td>
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<tr>
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<td>Bb</td>
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<td></td>
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<td>Eb</td>
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<tr>
<td>148</td>
<td>down</td>
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<td>149</td>
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<td>up</td>
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<td>A</td>
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<td>151</td>
<td>down</td>
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<td>Bb</td>
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<tr>
<td>152</td>
<td>up</td>
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<td></td>
<td>Db</td>
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<tr>
<td>153</td>
<td>down</td>
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<td></td>
<td></td>
<td>F#</td>
<td></td>
<td></td>
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<tr>
<td>154</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>Db</td>
<td></td>
<td>M(at F)8</td>
</tr>
<tr>
<td>155</td>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td>Eb</td>
<td></td>
<td>M(at F)9</td>
</tr>
<tr>
<td>156</td>
<td>oct+up</td>
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<td></td>
<td>F</td>
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<tr>
<td>157</td>
<td>down</td>
<td></td>
<td></td>
<td></td>
<td>C</td>
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<td></td>
</tr>
<tr>
<td>158</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>D</td>
<td></td>
<td>G(O)5</td>
</tr>
<tr>
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<td></td>
<td>A</td>
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<td>G(O)6</td>
</tr>
<tr>
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<td>down</td>
<td></td>
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<td>E</td>
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<tr>
<td>161</td>
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<td>F#</td>
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<tr>
<td>162</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>Ab</td>
<td></td>
<td>G(at F)2</td>
</tr>
<tr>
<td>163</td>
<td>up</td>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td></td>
<td>G(at F)3</td>
</tr>
<tr>
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<td>up</td>
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<td>Bb</td>
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<td>165</td>
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<td>172</td>
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<td>G</td>
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</tr>
</tbody>
</table>

Table 2-68 Published and transposed (for Bb sax) Mother and Grandmother Chords

This suggests that Coltrane made no attempt to transpose the material in the Mother and Grandmother chords; such an approach would suggest that Coltrane saw these chords as neutral, atonal structures, i.e. tone rows, in the manner of "Miles' Mode".
Further, if we examine the occurrence of ‘strings’ from bars 137-172 for all transpositions of the Mother and Grandmother chords, we still find that the transposition of these chords to F shows the highest correspondence with bars 137-172 for both Mother and Grandmother (in the following tables, an entry number refers to $x$, where $x =$ the number of the note from the given chord; see above):
Appearance of string from bars 137-172 in Mother chord rooted at:

bar: motif key: C Db D Eb E F F# G Ab A Bb B

<p>| | | | | | | | | | | | |</p>
<table>
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</tr>
</thead>
<tbody>
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(Mother)  2  8  8  2  2  12  2  0  4  4  7  0

Table 2-69 Location of "strings" from Mother Chord in bars 137-172
Appearance of string from bars 137-172 in G’mother chord rooted

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**Table 2-70 Location of “strings” from Grandmother Chord in bars 137-172**
If we sum the frequency of occurrence of each of the strings from Tables 2-69 and 2-70 we find the following result:

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<td>2</td>
</tr>
</tbody>
</table>

Table 2-71 Sums of “string” data for Mother and Grandmother Chords

Although F is clearly the largest entry here (marked in bold), supporting our initial assumption, other keys (i.e.: Db and D) are not especially remote, statistically speaking. However, I believe that we can discount the relative closeness of these results because only when transposed to the key of F do the Mother and Grandmother chords display the following characteristics in the above tables:

- a “string” of four transpositions in length
- this string being common to both Mother and Grandmother chords
- this string being played at the very start of the transpositions of this section (in this sense we can discount the preceding movements from F to G; both of these occur in chords and keys which do not even remotely challenge the supremacy of F in the above table)
- limitation of entries to a core list: 5, 6, 7 and 8 (2, 3 and 9 also occur, but rarely).

By contrast, Db and D show much more diverse results. These characteristics are made clear by the following “tally” chart (compiled from Tables 2-69 and 2-70):
<table>
<thead>
<tr>
<th>x:</th>
<th>Db</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>//</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>///</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>///</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>///</td>
<td></td>
<td>///</td>
</tr>
<tr>
<td>6</td>
<td>///</td>
<td></td>
<td>///</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>///</td>
</tr>
<tr>
<td>8</td>
<td>/</td>
<td>/</td>
<td>///</td>
</tr>
<tr>
<td>9</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-72 Tally Chart showing frequency of occurrence of x for Mother and Grandmother Chords rooted at Db, D and F

In fact, the \( \{5, 6, 7, 8\} \) data in Table 2-72 are fairly specific to the key of F. Let us note that there are, in fact, only 2 occurrences of any subsets of the \( \{5, 6, 7, 8\} \) data in both the Mother and Grandmother chords (from the above tables) at keys other than F, and that these are only 2 keys in length (Table 2-73):
Appearance of string from bars 137-172

in both Mother and Grandmother chords rooted at:

bar: motif key: C Db D Eb E F F# G Ab A Bb B

145 Eb  7
146 Ab  8

152 B  7
153 E  8

Table 2-73 Occurrences of strings for both Mother and Grandmother Chords rooted at notes other than F

That there are only these two occurrences of subsets of the \{5, 6, 7, 8\} data not only supports the theory that the "core" list of G, D, Ab, Db is possibly derived from the Mother and Grandmother chords, but also indicates that, once it was calculated, this "core" list was not then transposed as a set by Coltrane in order to further develop chromatic saturation. In both cases in Table 2-73, the occurrence of the \{7, 8\} pattern refers to a downward interval of a Perfect 5th. This suggests that many of the "fifth relations" described by Porter (1988) are directly caused by Coltrane's use of the G, D, Ab, Db list (G to D is a Perfect 4th downwards, Ab to Db is a Perfect 5th downwards). In this sense, then, we can say that the high occurrence of fifth relations is a direct by-product of Coltrane's reliance upon this "core" list.

Let us now examine the frequency of use of each of the transpositions, with these data ordered by the notes of the Mother and Grandmother chords; i.e. M(at F) and G(at F):
frequency of use:
M(at F)  G(at F)
x:
1  0  0
2  0  1
3  0  1
4  0  0
5  2  2
6  3  3
7  3  3
8  3  2
9  1  1
10  0  0
11  0  0
12  0  0

Table 2-74 Frequency of use of elements of strings for Mother and Grandmother Chords

Table 2-74 clearly shows the high frequency of the data for x = 5, 6, 7 and 8 (marked in bold), and the low frequency of other possible strings from the Mother and Grandmother chords. Further, there are one or two points that can be made concerning this table.

Firstly, given the zero results in the line x = 4 in the frequency of use (M (at F) and G (at F)) table above, we could equally assume (a) that G(at F)x = 2 and G(at F)x = 3 are random creations (they exist in a solitary way compared to all other results), or (b) that they are the proto-material for the cadential rise at bars 160-164.

Secondly, the results in line x = 9 are interesting (1, 1), in that they suggest that Coltrane may have remembered (/deliberately prepared) to play the next transpositions after 5, 6, 7 and 8 from each of the Mother and Grandmother chords respectively (i.e.
the A at bar 148 and the Eb at bar 156, as mentioned above). Thus, this analysis suggests that Coltrane practised both the Mother and Grandmother chords with equal emphasis, and saw A and Eb as equally appropriate keys with which to follow Db. Similarly, however we must accept the possibility that these results are simply due to chance: in a random situation, each individual entry has a 1 in 6 chance of being either an A or Eb, of course. Although this is a statistically weak suggestion, it may therefore be representative of a “weak” memory on Coltrane’s behalf of the position of the keys of A and Eb in the Mother and Grandmother chords which contrasts with a “stronger” memory of the G, D, Ab, Db “core” list.

From this we might speculate that Coltrane rehearsed the motif through the lists of keys given by a transposition of the Mother and/or Grandmother chords to the key of F. Let us note that such practice only produces one case where the final note of one key is identical to the first note of the following key; thus, for the Mother Chord (common note data marked in bold):
<table>
<thead>
<tr>
<th>Mother Chord</th>
<th>at F</th>
<th>motif as #s:</th>
<th>is this note in next key?</th>
<th>is 3rd note the tonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>key:</td>
<td>1st note</td>
<td>2nd note</td>
<td>3rd note</td>
<td>of next key?</td>
</tr>
<tr>
<td>F</td>
<td>6 9 11</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>5 8 10</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1 4 6</td>
<td>1 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>10 1 3</td>
<td>0 1 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>8 11 1</td>
<td>1 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>3 6 8</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ab</td>
<td>9 12 2</td>
<td>0 0 1 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Db</td>
<td>2 5 7</td>
<td>0 0 1 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Eb</td>
<td>4 7 9</td>
<td>0 1 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F#</td>
<td>7 10 12</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bb</td>
<td>11 2 4</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>12 3 5</td>
<td>0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

sумы: 2 2 2 1

Table 2.75: Showing single occurrence of common note using Mother Chord
Similarly, for the Grandmother Chord (common note data marked in bold):

<table>
<thead>
<tr>
<th>Grandmother Chord</th>
<th>is this note in next key?</th>
<th>is 3rd note the tonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>at F key: motif as #s:</td>
<td>1st note</td>
<td>2nd note</td>
</tr>
<tr>
<td>F</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>F#</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Eb</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>G</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Ab</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Db</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>A</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Bb</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

sums: 2 2 2 1

Table 2-76 Showing single occurrence of common note using Grandmother Chord

These occurrences represent the only position of a Perfect 4th interval. Thus, transposition of the motif through these two lists confirms the "independent" nature of the set.

Let us recall that the Perfect 4th is:

- the most commonly occurring interval in the Dorian mode
- the interval that "sum" two motifs to become a minor pentatonic scale: i.e. {C, Eb, F} and {F, Ab, Bb} become F Dorian
from the jazz tradition of VI > II > V > I etc.: this familiarity might be said to be offset by the fact that the keys of the motif concerned (Ab and Db) are relatively distant from the underlying tonality of F Dorian.

In each of the Mother and Grandmother chords, each interval occurs once only (this characteristic is noted by Slonimsky). As we have seen already, in bars 137-172, Coltrane uses the last note of a transposition as the first of the next (i.e. interval of a Perfect 4th) exactly four times:

<table>
<thead>
<tr>
<th>bar:</th>
<th>up/down:</th>
<th>key:</th>
<th>interval:</th>
<th>common note:</th>
</tr>
</thead>
<tbody>
<tr>
<td>141</td>
<td></td>
<td>Ab</td>
<td></td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>down</td>
<td>Db</td>
<td>Perf 5</td>
<td>octave Db</td>
</tr>
<tr>
<td>145</td>
<td></td>
<td>Eb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>146</td>
<td>down</td>
<td>Ab</td>
<td>Perf 5</td>
<td>octave Ab</td>
</tr>
<tr>
<td>147</td>
<td>up</td>
<td>Db</td>
<td>Perf 4</td>
<td>Db</td>
</tr>
<tr>
<td>152</td>
<td></td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>153</td>
<td>down</td>
<td>E</td>
<td>Perf 5</td>
<td>octave E</td>
</tr>
</tbody>
</table>

Table 2-77 Occurrences of adjacently repeated pitch classes, bars 137-172 (copy of Table 2-12)

Although this seems like a relatively high frequency for these intervals (as noted above), let us note that all of these movements are comprised of transpositions which are similarly distant (by “out factor”) from the underlying tonality of F Dorian. Here is a graph which shows that all of these transpositions have an “out factor” of at least 4 (in fact the average is roughly 5.6):
Thus, both (a) the familiarity of the interval from jazz and the piece in general and (b) the lack of independence created by a transposition of an (upward) Perfect 4\textsuperscript{th} or (downward) Perfect 5\textsuperscript{th} are offset (compensated) by the use of relatively distant (i.e. "out") transposition sets. This section contrasts sharply with the earlier section of his solo (bars 49-114), where Coltrane uses almost exclusively transpositions of a Perfect 4\textsuperscript{th} (common to jazz and the interval that sums the motif into a minor pentatonic scale, as noted above). By using the (upward) Perfect 4\textsuperscript{th} or (downward) Perfect 5\textsuperscript{th} intervals in the relatively distant manner described here, Coltrane has imparted a hierarchy of interval use into bars 137-172.
Is there a correlation between the "out factor" graph and the occurrences of G, D, Ab and Db?

Let us postulate that Coltrane played the motif through the lists of transpositions provided by the Mother and Grandmother chords at F to test their suitability as chromatic lists before the recording of "A Love Supreme". Let us now compile "outness" data for these two journeys using the adapted Longuet-Hughes \( q(Dor) \) method (described above), thus:

<table>
<thead>
<tr>
<th>note</th>
<th>Ab</th>
<th>Eb</th>
<th>Bb</th>
<th>F</th>
<th>C</th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
<th>B</th>
<th>F#</th>
<th>Db</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(dor)</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2-78 Adapted Watkins and Dvson/Longuet-Hughes table: Chromatic scale arranged by fifths with associated "sharpness value" for \( q(Dor) \) (copy of Table 2-25)

Let us now construct the following graphs which describe the "outness" of the various keys derived from the Mother and Grandmother chords at F:
Crucially, the G, D, Ab, Db string, common to both chords, is the longest string of keys (i.e. four keys) that shows a gradually increasing dissonance. The curve produced by this common, central G, D, Ab, Db string is clearly similar to the four peaks that we noticed in the “out factor” graph above. Relatedly, the key of Db (the last entry of this list) shows the highest dissonance of all keys. The Mother chord shows no other coherent section of increasing dissonance; all movements in the curve are alternately upward and downward. By contrast, the Grandmother curve shows two (smaller, 3-key long) sections of gradually increasing dissonance. These are caused by the strings F, E, F# and C, Bb, B. Further, let us note that the long central strings of keys that show increasing dissonance are followed by relatively steep returns to resolved transpositions, and that the keys that mark these downward slopes are Eb, A and C: we saw above that the keys of C, A and Eb are the only three chosen by Coltrane after a statement of the motif at Db.

We might argue, then, that Coltrane’s approach in maintaining curves of gradually increasing dissonance with the underlying tonality of F Dorian may be either (a)
sourced in, or, (b) supported by this characteristic of gradually increasing dissonance shown by the common, central G, D, Ab, Db section of both the Mother and Grandmother chords. Further, this characteristic is emphasised by the F, E, F# and C, Bb, B sections found in Slonimsky's Grandmother chord.

There is a further level of connection between the core list of G, D, Ab, Db and Coltrane's control of gradually increasing dissonance. To show this, let us mark the peaks in a list of transpositions in bars 137-172, and note all occurrences of G, D, Ab, Db within those boundaries (thus, the peak start/end points are marked here in bold, whilst occurrences of G, D, Ab and Db are in *italics*):
bar: up/down: motif key:

137  F
138  same  F  (start of peak 1)
139  up    G
140  down  D
141  up    Ab
142  down  Db
143  down  C  (start of peak 2)
144  oct+up  D
145  up    Eb
146  down  Ab
147  up    Db
148  down  A
149  down  F  (start of peak 3)
150  up    D
151  down  Ab
152  up    B
153  down  E
154  up    B
155  down  Db
156  oct+up  Eb
157  down  Bb
158  up    C  (start of peak 4)
159  down  G
160  down  D
161  up    E
162  up    F#
163  up    G
164  up    Ab
165  down  F  (end of peak 4)
166  same  F
167  same  F
168  same  F
169  same  F
170  same  F
171  same  F
172  same  F

Table 2-79 Showing peak start/end points and occurrences of G, D, Ab and Db transpositions in bars 137-172

What is immediately obvious is that (a) all five of the transpositions marked in bold are either F or C, and (b) each peak start point is immediately followed by a key from the G, D, Ab, Db list. A subtler characteristic is that the start of a new hill “resets” this list of keys back (i.e. G or D) to a point previous in the list. That is, the set order G. then D. then Ab. then Db is strictly maintained until the occurrence of a new start/end point.
This latter theory breaks down at bar 163, where we hear G before (the expected) Ab. However, let us recall that bar 163 is in the final cadential section, and further that the G itself causes a deep trough in the “out factor” graph. This suggests that we should see the start/end points of the peaks as merely temporary resting stations of resolution, before Coltrane reaches for either G or D from the formulaic pattern of G, D, Ab, Db.

What is the probability that Coltrane’s “core” list was randomly generated?

Whilst noting the caveat that Slonimsky’s book is “complete”, further support for the argument that Coltrane obtained his “core” list from the Mother and/or Grandmother chords may be derived from an assessment of the probability that such a string of keys should occur as frequently as it does in bars 137-172. This assessment will allow us to see how many times we would expect to see a specific transposition “string” such as G, D (in that order) in 27 transpositions (similarly, G, D, Ab; G, D, Ab, Db; and so on). The odds of such a string of 4 transpositions matching an identical string of 2, 3, or 4 notes from a randomly generated 12-tone row similar to the Mother and Grandmother chords may be calculated using the following equation:

\[
P = \frac{\binom{11}{S} \cdot \frac{T-(S-1)}{12}}{\binom{12}{S}}
\]

Here, \( P \) = Probability, \( S \) = String length and \( T \) = Number of Transpositions in the section of music. We will say that \( T = 27 \) in order to reveal the relevant probabilities for bars 137-172. Thus, to calculate the probability that, say, the 2-note ordered string G, D might appear simply by chance in 27 bars, we need to use the above formula, thus (\( S = 2 \)):
\[
\frac{P}{1} = \frac{11!}{10!} + 2 \cdot 1.666666 = 5.0769232
\]

\[
P = 0.1969696
\]

In this way, we can construct a table for all values of \( S \) (from 1 through 12), and can assess the probability that strings of these lengths from the Mother or Grandmother chords will be found in a randomly generated list of transpositions, and then compare these results with the actual pattern of transpositions employed by Coltrane in bars 137-172. The results are as follows (I have converted the data into "= 1 in x" format in order to make direct comparisons with the actual frequencies in bars 137-172):

<table>
<thead>
<tr>
<th>( S )</th>
<th>( P = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>= 1 in 1</td>
</tr>
<tr>
<td>2</td>
<td>(0.1969696) = 1 in 5</td>
</tr>
<tr>
<td>3</td>
<td>= 1 in 53</td>
</tr>
<tr>
<td>4</td>
<td>= 1 in 495</td>
</tr>
<tr>
<td>5</td>
<td>= 1 in 4,132</td>
</tr>
<tr>
<td>6</td>
<td>= 1 in 30,240</td>
</tr>
<tr>
<td>7</td>
<td>= 1 in 190,080</td>
</tr>
<tr>
<td>8</td>
<td>= 1 in 997,9208</td>
</tr>
<tr>
<td>9</td>
<td>= 1 in 4,201,769</td>
</tr>
<tr>
<td>10</td>
<td>= 1 in 13,305,600</td>
</tr>
<tr>
<td>11</td>
<td>= 1 in 28,176,566</td>
</tr>
<tr>
<td>12</td>
<td>= 1 in 29,937,600</td>
</tr>
</tbody>
</table>

Table 2-80 Probability that a string from the Mother or Grandmother Chord of length \( S \) will occur in randomly generated transpositions

What these data show is that we would only expect to find one occurrence of, say, the G. D string in every 5 separate randomly-generated lists of 27 transpositions (i.e. 5
separate versions of bars 137-172). Similarly, it would take, on average, 52.8 such lists (on average) to generate the specific 3-note string G, D, Ab, and 495 such lists (on average) to generate the specific 4-note string G, D, Ab. Db. What this shows is that for the string G, D, Ab, Db (and its subsets) to occur as frequently as they do is unlikely to be the result of random generation and is thus likely to have been caused by some compositional influence. Having noted this, let us now compare these probability results with what we find at bars 137-172, focusing especially upon the frequency of strings of length 2, 3 and 4.

**How many strings of length 2, 3 or 4 from a specific tone-row should we expect to find on average in 27 transpositions?**

As we have just seen, the probability for a specific string of 2 keys in length (e.g. G, D) is 0.1969696 (= roughly 1 in 5). There are 11 such strings of 2 keys in length derivable from a specific tone-row, so we would expect to see (in 27 transpositions) one or two non-specific strings of 2 keys simply created by chance. In fact, from the tables above, we can see that this happens with the Mother Chord when transposed to C, Eb, E, F#, Ab and A; and, similarly, to the Grandmother Chord when transposed to C, Eb, F#, G, Ab, A, Bb and B. However, for the key of F, 2-note strings occur at a greater rate.

Similarly, we would expect to see a three-note string (e.g. G, D, Ab) once in every 52.8 transpositions; thus, in a diagram made up of 12 keys, each moving through 27 transpositions, we would expect to see an average of about 6 three-note strings ((12 x 27)/52.8 = 6.1363636). Again, for the key of F, 3-note strings occur at a greater rate. Regarding the four-note string (e.g. G, D, Ab, Db), these are expected to occur every 1 in 495 transpositions ((12 x 27)/495 = 0.6545454). So, although we should not be
surprised to see such a string in so many bars, (on average) roughly 13 would appear in 20 pieces with as many transpositions, i.e. just more than half of the time. By comparison, we can say that in bars 137-172, 4-note strings occur at a greater rate. For the G, D, Ab, Db string to be specifically the central 4 keys of both the Mother and Grandmother chords (if they are randomly generated) is 9 times less likely (= 0.0727272 = roughly 1 in 14) Also, for this G, D, Ab, Db string to occur as the first four transpositions of a piece (as it does at bars 139-142) is 24 times less likely (= 0.0030303 = 1 in 330). Similarly, for its subsets to occur so prominently is still less likely (i.e. P < 1 in 330). This additional evidence further supports the premise that the existence of this 4-note string and the relatively high occurrence of its subsets are the results of conscious decisions, and not the result of chance operations.

Thus, 2-note, 3-note and 4-note strings from the Mother and Grandmother chords (at F) all occur at a greater rate than we would expect in a randomly generated version of bars 137-172. This suggests that the order of notes found in both the Mother and Grandmother chords have a strong correlation with the order of transpositions used by Coltrane in bars 137-172.

Summary of Possible Source 2

We saw above that only when we transpose the Mother and Grandmother chords to F do we find a string of four transpositions in length in bars 137-172. As we have seen, the chances of this happening randomly in a piece of music with as many transpositions as bars 137-172 are poor. It would seem to be a good guess, then, that Coltrane is deliberately using G, D, Ab, Db as a core set, and, further, may have derived this set from Slonimsky’s Mother and/or Grandmother chords. We have also seen that Coltrane
uses C, Eb, and A as transpositions after each Db, and that these may be seen to derive from the Mother and Grandmother chords at F. However, these data alone are not proof of Coltrane's exact methodology. It is conceivable that Coltrane memorised the G, D, Ab, Db section, recalling this at will. It is equally conceivable that Coltrane, having studied and developed his piece with Slonimsky's chords, then undertook to create a similar pattern to the sequential pattern of the Grandmother chord (i.e., the alternately descending and ascending chromatic movement away from the root), and came across this particular group of four transpositions independently. In accepting this latter possibility, we should recall the frequency of occurrence of subsets of this string; if he has conceived of this row of transpositions "on the fly", then Coltrane has also made it a central element of the entire journey from bar 137-172 at very short notice. This would seem to be unlikely, given that we have seen that Coltrane is concentrating on (a) producing coherent "out" contours whilst moving through the various keys and (b) attempting to cover all twelve keys (i.e. this would seem to be quite enough to be thinking of during a performance without constructing subsets of a brand new, improvised core string of transpositions).

However, on balance, it seems reasonable to suggest that Coltrane may have used the Mother and Grandmother chords to generate the "core" list of transpositions found in bars 137-172. In the light of this conclusion I propose the following chronology of composition.
A Proposed Chronology of Composition

The following outlines the process that Coltrane may have underwent in the construction of “A Love Supreme”, specifically with regard to those parts of “Acknowledgement” central to this study. Thus, Coltrane:

1. composed/chose motif as “quintessential”, “independent” and non-autonomous (in concert key of F)
2. improvised using superset of motif: F minor pentatonic/F Dorian
3. decided to transpose motif through many (all) keys
4. decided to refer to Slonimsky’s “Thesaurus” for an equally non-autonomous 12-note “journey”; all interval (etc.) Mother and Grandmother chords found at the rear of book
5. transposed Mother and Grandmother chords to F, since motif starts on F
6. played motif through Mother and Grandmother chord lists (possibly using pattern No. 1232a, p. 175)
7. in doing so and finding (a) sequential nature of Grandmother chord unsatisfactory (...G, D, Ab, Db, C, A, Bb, B) and (b) start of both Mother and Grandmother chord lists too steep a contour of “outness”, decided to use the central, common string of four transpositions: G, D, Ab, Db
8. recognising the contour of these four keys as more satisfactory, Coltrane designed/allowed himself to improvise a rising cadential section for the final few bars, resolving finally to the motif in F. This cadence creates a specific disruption to what is (avoiding a pre-prepared serial list, anathema to improvisational code) a potentially infinite list of transpositions. Previous to this, Coltrane improvised transpositions through various keys, resorting to subsets of the G, D, Ab, Db list in the same way as musicians resort to licks and riffs in conventional improvisation
(these subsets are deliberate and carefully produced in order to avoid redundancy; i.e. in order to avoid the repetition of a given string).
Eric Dolphy's Addresses

My research has also revealed that Eric Dolphy, a colleague of Coltrane, encoded house numbers into his compositions: "245" (recorded April 1, 1960) and "17 West" (recorded August 16, 1960); he also played on "111-44" by Oliver Nelson. In fact, I was first attracted to examine the piece "245" because this number might be seen to represent, say D, F, G (if C is 1), i.e. the \{0, 3, 5\} set from "A Love Supreme", and I wondered whether there was a relationship between these two pieces. Here we will examine "245", "17 West" and "111-44", before moving on to a consideration of possible connections between "245" and "A Love Supreme".

"245" (Fig. 2-25)

"245" was a popular piece, recorded by Dolphy on no less than five separate dates (for example, on the album "Outward Bound" (1960, New Jazz 8236)). Simosko and Tepperman (1996) describe these various recordings (p. 136). It is Ron Eyre who points out that "245" is Dolphy's address on the album notes. Let us examine how the number "245" is encoded in the piece by Dolphy. If we see the first six notes (ignoring the anacrusis C, presuming that this was added later) as three pairs, and examine the intervals of these (descending) pairs we find:
from "Outward Bound"
(1960) New Jazz 8236

Eric Dolphy

Fig. 2-25 "245"

subito p mf
Ab  G  = mi2
F   C  = P4
Eb  Ab = P5

I.e.: (mi)2 (P) 4 (P) 5 = "245"

Table 2-81 Interval structure of opening melody of "245"

Amusingly, at the start of his solo, Freddie Hubbard plays an altered version of this six-note phrase, swapping the positions of the F and Eb. This brings both importance to the device (he quotes or paraphrases no other part of the head that I can find in his solo) and creates a personal interpretation of the melody - perhaps Dolphy had explained its meaning to Hubbard.

Let us admit, then, that the \{0, 3, 5\} set found in "A Love Supreme" is clearly not the material that Dolphy chooses with which to construct "245". We will now examine "17 West" and "111-44" in an attempt to add weight to the supposition that the connection between the melodic structure of "245" and its title is not merely coincidental. ¹

¹ Of course, "245" is an appealing number for this kind of approach: when the British saxophonist Julian Argüelles composed "500 Broadway", he "simply did not think of" encoding this number within the piece (personal conversation).
17 West

from "Out There"
(1960) New Jazz 8252

Eric Dolphy

Fig. 2-26 "17 West"
"17 West" (Fig. 2-26)

Similarly, "17 West" (from "Out There" (1960) New Jazz 8252), according to the liner notes by Joe Goldberg, was "a former Manhattan address" of Dolphy. By examining the number of notes in each phrase (and the rests that separate them) we find the following pattern:

rest 7 rest 14 rest 7 rest 19

Table 2-82 Preliminary note and rest structure of melody of "17 West"

The number of 7's found here (and that 14!) made me suspect that there might be a correlation between numbers of notes and the title. For this to become clearer, we must accept the following:

- the repeat of motif $a$ at the end of bar 3 signals the end of the previous melody; also, as notated in "The Source", the C in bar 3 is played staccato (possibly actually a quaver followed by a quaver rest?)
- the trill in bar 9 is actually a decoration (added later?) of the (two) notes F and Eb (nowhere else in the piece is there a rhythmic element smaller than a quaver), thus suggesting that this section was originally 17, and not 19, notes long.

This revised interpretation gives the following (rests marked as "1"s):
from "Straight Ahead" (1961) New Jazz 8255

last 4 bars of Out Head:

Alto Sax

Bass Clarinet

Coda

Fig. 2-27 "111-44"
Fig. 2-27 "111-44"

Coda Resolved in 12 bars (theoretical):
Note the mirror symmetry (1 7 1 7 answered by 7 1 7 1) before the final 17 notes. The “7 1 7 1” pattern can be seen as a pair of “westerly” (i.e. right-to-left) readings of “17”. Thus, 17 West is conceptually more comprehensive than “245”, affecting the whole head. In “245”, the encoding is limited to just the first phrase of the piece. Perhaps this is not surprising, given that it was (according to the recording dates) composed later.

“111-44” (Fig. 2-27)

Further, I suggest that Dolphy may have had a hand in the titling of “111-44” by Oliver Nelson, as well as the composition of the coda. This piece was recorded on March 1, 1961 on what was Oliver Nelson’s album “Straight Ahead” (1961, New Jazz 8255). However, the album notes make it clear that Dolphy is the “star” of this particular release; there seems little doubt in the mind of Joe Goldberg, who wrote the album notes, that Nelson, despite being the leader of the session, is working under the shadow of Dolphy. Apparently, “111-44” also took its title from a house number, where Nelson and Dolphy had both lived. The piece is a 32-bar composition based on conventional bop melodic and harmonic formulae. Careful study of the chart has revealed no similar construction to Dolphy’s “245” or “17 West” (i.e. a reference to the title within the interval/metrical structure of the head). However, the coda has an interesting form, more radical, than the rest of the piece. This coda is in three sections:
bars: description:

1-2 two bars Db9 (bI1) (there is a tag that leads to this at bar 32 of the out head)

3-4 motif transposed at minor 3rd intervals contrapuntal section

5-11 repeated Bb and Db motifs - stops after coda has lasted 11 bars

Table 2-84 Coda structure of “111-44”

This final pattern could easily have been extended twice more to fit exactly 12 bars, with the last note (a Db) falling on beat 1 of the 12th bar (see last line of notation: “Coda Resolved in 12 Bars”). Thus, I suggest that “111-44” may have been subject to a last-minute (in the studio?), partial extension by this coda, possibly composed by Dolphy, of 11 bars of 4/4 (i.e. four crotchets to a bar). Let us note that, although there are similar patterns to bars 1-4 of the coda to be found elsewhere on the album (e.g. “Images” and “Ralph’s New Blues” have similar contrapuntal structures, with instruments overlapping each other), the insistent repetition of the Bb and Db motif is unique. However, Goldberg also says that the entire recording session for the album took only two and a half hours - this reduces the chance that revision/addition was undertaken to any of the pieces in the studio, but does not remove the possibility, and, of course, there was likely to have been rehearsal elsewhere.

Bar 11 of the coda sees a Db and C performed simultaneously (i.e. I and bII as a simultaneous event); Nelson plays the Db, Dolphy the C. Is Dolphy showing that he knows which note is the last? Is he counting, and not Nelson? This would not be surprising given the fact that Dolphy’s part comes in chunks - it is only from bars 9-11 that he plays the pair of motifs three times (a relatively simple operation compared to Nelson’s nine pairs starting at bar 3). Similarly, if Dolphy had conceived of the coda himself, he is perhaps more likely to count correctly. The pause that I have notated
himself; he is perhaps more likely to count correctly. The pause that I have notated above
the final chord in bar 11 allows this to be notated in 11 bars on the page. It is noteworthy
that the drummer (Roy Haynes) continues for a further 2 bars. This encourages me in my
belief that this coda was not well known to the rhythm section at the time of the recording.
Finally, let us note that this final chord is held for exactly 11 beats until someone
(Dolphy?) cues the ending.

In conclusion, then, having examined these three pieces, I feel confident that my
analysis which shows a connection between the melodic structure of “245” and its title
is correct.

“245” and “A Love Supreme”

Although Dolphy does not use the \{0, 3, 5\} set in composing “245”, there is a link
between this piece and “A Love Supreme” that can be found in Slonimsky’s book. If
we examine Pattern No. 245 from Slonimsky’s “Thesaurus of Scales and Melodic
Patterns” (1947), p. 35, we find that it is in fact the \{0, 3, 5\} set transposing through
Major 3rds:

![Pattern No. 245 from Slonimsky (1947). p. 35](image)

Initially, when I noticed this, I assumed that the similarity between the title of “245”
and the motif that makes up the Slonimsky Pattern No. 245 was simply a coincidence.
to do with each other (let us recall that the number “245” is encoded in a quite different way, i.e. by the intervals that make up the first phrase). However, upon reflection, I now think that there may be a connection between these pieces founded in a possible sharing (and even collaborative use) of Slonimsky’s book by Dolphy and Coltrane. This idea was promoted by Porter’s (1988) description of the musical tradition and artistic environment which permeated house number 245, Carlton Avenue, Brooklyn:

“In 1959 John’s [Coltrane] cousin Mary moved to New York City. Mary stayed with the trombonist Charles Greenlee – also known by his Muslim name Harnifan Majid – who had been with Gillespie when Coltrane was there. They lived for a while in the huge brownstone house that trombonist Slide Hampton owned at 245 Carlton Avenue in Brooklyn. Hampton’s other tenants included at times trumpeter Freddie Hubbard, guitarist Wes Montgomery, bassist Larry Riley – all friends from Indianapolis – saxophonist Eric Dolphy, and the painter known as Prophet, whose surrealist work graces the covers of Dolphy’s first two LPs as a leader in 1960. “It was wonderful,” Mary recalls. Dolphy dedicated a piece to the house – “245”. Coltrane would drop by at times, and he strengthened significant associations there. He had used Hubbard on his last Prestige recording session, December 26, 1958; later he would perform with Montgomery. And Dolphy, already a close friend and musical soulmate – they’d discuss music over the phone – became a regular member of his group.”

1 Porter (1998) p. 140
Given this environment, let us speculate that Coltrane and Dolphy may have composed/used Slonimsky’s book together in this actual house, and looked to see what patterns have given numbers in Slonimsky’s book. This would be in line with Coltrane’s well-recorded interests in numerology. I suggest that pattern No. 245 may have appealed to both Coltrane and Dolphy for two reasons. Firstly, because it was the number of this house, and secondly, because an examination of this pattern reveals a couple of amusing coincidences:

- The title of the pattern ("245") is encoded within the motif that forms the pattern itself (as II IV V of a scale) – thus, the pattern is self-referential.
- The motif is transposed through Major 3rds to form the pattern – this key relationship is central to Coltrane’s “Giant Steps” progression method – Coltrane often played motifs transposed in this way during this period, and, as shown by Demsey (1991), this structure itself comes from elsewhere in Slonimsky’s book.

Thus, I suggest that Coltrane, like Dolphy, sought to express the number of this shared house in some way in his music, and, further, that it is possible that a joint study of Pattern No. 245 may have resulted in both Coltrane and Dolphy enjoying a neutral artistic “ownership” of the set \{0, 3, 5\} (they had similar rights over the space in the house) due to its coincidental title and motif structure. Did they then develop this pattern in some way together? This postulation might go some way to resolve the confusion regarding the authorship of “Miles’ Mode”/”Red Planet”. As we have seen, this piece is constructed from a 12-tone row based upon the \{0, 3, 5\} and \{0, 5, 7\} sets (these might be seen thus: \{0, 3, 5\} as II IV V and \{0, 5, 7\} as Maj2, Perf4, Perf5). Although it was thought for a long time that Coltrane was the
composer of this piece, simply re-titling it at some point. David Wild’s album notes for “Live at the Village Vanguard” suggest that the piece “was almost certainly written” by Dolphy.² Similarly, Alan Saul has collected some valuable data that suggests that Dolphy is the composer of this piece.³ I suggest that the fact that both points of view have some historical/musical evidence, and that the material could be seen as being neutral to both parties, possibly based in a study of Slonimsky’s book, means that this piece may have been developed by both musicians, and, further, that neither musician (who were close friends, after all) would then do anything to stop the other claiming the material as their own in a claim for royalties. The fact that there are two titles for the piece, erratically located, is a natural outcome of this scenario. Further, if we examine Pattern No. 245 in Slonimsky’s book in detail, we find that the first 5 notes form a C9 chord and that the next 4 notes form a B6/Abm7 chord; this is analogous to the B Dorian/C minor pentatonic structure of the tone row of “Miles’ Mode”, both in organisation and key.

But what relevance does this have upon our analysis of “Acknowledgement”? I now suggest that the significance of this pattern No. 245 from Slonimsky’s book was maintained up until the time that Coltrane was composing his autobiographical suite “A Love Supreme”. We have seen already how important this book was to Coltrane, for both practise and composition. Perhaps Coltrane was frustrated by Dolphy’s choice to use 2.4 and 5 as (relatively imprecise) interval data compared with what I am suggesting is Coltrane’s use of the numbers as (relatively precise) scale step data, and saw “A Love Supreme” as an opportunity to state his own preferred use of the number “245”. Finally, there are several other postulations that I wish to present that

² Wild (1997) p. 33
³ See <http://farcry.neurobio.pitt.edu/Discographies/RedPlanet.html>
support the idea that Coltrane may have used pattern No. 245 as material for “A Love Supreme”:

1. Noting that the motif that forms the pattern No. 245 is II IV V of some key, let us note that (for the first and final transpositions in Pattern No. 245) this key is F, and that Part I of “A Love Supreme” (i.e. “Acknowledgement”) is in the key of F.

2. Further, let us note that Coltrane’s normally highly selective use of the minor pentatonic scale is sometimes interpolated with the note G in the solo of Part I, perhaps derived from the sum of the motif transposed to the II with tonic note, thus:

\{F\} + \{G, Bb, C\}

3. Further, let us recall that the first transposition of the “A Love Supreme” motif in bars 137-172 (i.e. bar 139) is at G, thus:

Notes: G Bb C
Step in F Dorian: 2 4 5

Table 2-85 “A Love Supreme” motif in G as steps 2, 4 and 5 in F Dorian

Thus, perhaps Coltrane was celebrating the set’s source by choosing G as the first transposition of this section.

4. Relatedly, the final bar of Part IV (that is, the final bar of the record) sees Coltrane playing the set at II (thus forming II IV V), choosing this over, say, (a) V bVII I (which would end on the tonic note), or (b) the C minor triad that is the core of the rest of Part IV.
5. The characteristic transpositional movement in Major 3rds is *downwards* in all of the following contexts:

<table>
<thead>
<tr>
<th>Keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Giant Steps”, bars 1-3</td>
</tr>
<tr>
<td>“Acknowledgement”, bars 147-149</td>
</tr>
<tr>
<td>Slonimsky pattern No. 245</td>
</tr>
</tbody>
</table>

Table 2-86 Downward Major 3rd movements

6. Porter (1998) has shown that the key structure of the four parts of “A Love Supreme” may be seen as symmetrical about a central axis:

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (Maj2)</td>
<td>Eb (Perf5)</td>
<td>Bb (Maj2)</td>
<td>C</td>
</tr>
</tbody>
</table>

Table 2-87 Showing Porter’s description of key structure of “A Love Supreme”

Finally, I suggest that there may be some possible relationships between “245” and the key structure of “A Love Supreme”, thus:

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Direction</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Eb</td>
<td>down</td>
<td>Maj 2</td>
</tr>
<tr>
<td>F</td>
<td>Bb</td>
<td>up</td>
<td>Perf 4</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>up</td>
<td>Perf 5</td>
</tr>
</tbody>
</table>

Table 2-88 Possible relationship of “245” and key structure of “A Love Supreme”

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4 See Porter (1998) p. 236
Note how this relates to Dolphy’s use of the number 245 to generate intervals for his piece “245”. However, this table is untidy in that it contrasts the movement down to Eb with the other movements up to Bb and C. If we allow all of these movements to be up, we get:

**Part I:** F
**Part II:** G (incorrect)
**Part III:** Bb
**Part IV:** C

Table 2-89 Showing application of upward intervals from “245” does not result in the key structure of “A Love Supreme”

However, there is another construction that may have influenced the choice of Eb as the second key, namely the number of flats in the relevant Dorian modes (and shows the number 245 twice):

<table>
<thead>
<tr>
<th>From:</th>
<th>to:</th>
<th>direction</th>
<th>interval</th>
<th>number of flats in Dorian mode:</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Eb</td>
<td>down</td>
<td>Maj 2</td>
<td>Eb Dorian: 5</td>
</tr>
<tr>
<td>F</td>
<td>Bb</td>
<td>up</td>
<td>Perf 4</td>
<td>Bb Dorian: 4</td>
</tr>
<tr>
<td>F</td>
<td>C</td>
<td>up</td>
<td>Perf 5</td>
<td>C Dorian: 2</td>
</tr>
</tbody>
</table>

Table 2-90 Possible use of “245” to define the number of flats for the keys of “A Love Supreme”

Keyboardist Robert Irving first recorded with Miles Davis on the trumpeter’s “comeback” album, “The Man With The Horn”, released in 1980. At the time Robert Irving III was 26 years old, and had recently graduated from the University of North Carolina. \(^1\) Robert Irving III composed two pieces for this album: the title track, “The Man With The Horn” (with Randy Hall) and “Shout” (with Randy Hall and Glen Burris). Davis chose Irving again as keyboardist on “Decoy” (1984) and “You’re Under Arrest” (1985). \(^2\) It is these two later albums that are characterised by the use of a special “outside” strategy employed by Irving. This strategy involves the use of sets of chords, often chromatically linked into structures that I have called “chains”. Before examining these “chain” strategies in detail, let us first consider the two earlier pieces, mentioned above, “The Man With The Horn” and “Shout”. These have several characteristics which suggest the derivation of the “chain” strategies. These two pieces are jointly composed and it is not easy to ascribe specific parts of the composition to specific writers. \(^3\) However, whether Robert Irving III was or was not responsible for the elements which I will focus upon below, there are clear compositional links between

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\(^1\) Chambers (1989) p. 299

\(^2\) Further, for the album “Decoy”, Irving is named as co-producer as well as arranger of the tracks “Decoy” and “Code M.D.”. For the album “You’re Under Arrest”, he is named as producer as well as arranger of all of the tracks apart from “You’re Under Arrest”, which is arranged by John Scofield and Miles Davis.

\(^3\) In fact, Davis himself singled out Randy Hall as the key composer of “Shout” when he was interviewed by Cheryl McCall ((1982) “Miles Davis” Musician, Player and Listener 41. March): quoted in Chambers (1989) p. 301.
these elements and the later works that are more certainly composed (and/or performed) by Irving.

"The Man with the Horn" (Randy Hall/Robert Irving III) (Fig. 3-1)

(from "The Man with the Horn" (1980) Columbia FC 36790)

This composition, essentially constructed with conventional jazz/rock chords, anticipates the "chain" strategies in various ways. Here let us note:

- the use of a limited set of right hand chords played above various bass notes (e.g. Db/Eb (with additional Gb note) as a tonic Ebm11 chord of the piece at bar 1, also the common right hand chord with two different bass notes at bars 3 and 4 of the Introduction)
- the use of complex chromatic passing chords as (a) core (e.g. Cb/Fb chord in bar 4 of the A section) and (b) variation material (the equivalent Bbm9 chord in bar 2 of the A2 section used for the trumpet solos)
- an underlying tonality (Ebm), with use of the bII as functional cadential harmony (i.e. Fbmaj7#11, Cb/Fb and Fb69(#11) chords)
- some use of harmonies which incorporate quartals (e.g. AbQ at bars 3 and 4 of introduction, and BbQ at bar 3 of the B section)
The Man With The Horn

from "The Man With The Horn"
(1980, Columbia FC 36790) Randy Hall, Robert Irving III

Fig. 3-1 "The Man With The Horn"
Fig. 3-1 "The Man With The Horn"
Shout

from "The Man With The Horn"
(1980, Columbia FC 36790)

Piano

Pno

Fig. 3-2 "Shout"
Similarly, there are several precedents for the "chain" strategies to be found here. Let us note:

- the use of three-note chords moving over a given bass note (bars 1-4 of A section)
- the use of chord progressions constructed as much by premeditated symmetry as functional harmony. For example, in the second line of the A section, the right hand moves up a tone from a Db to an Eb triad, whilst the left hand moves down a tone from Bb to Ab. Next, the right hand moves up a tone from an E to a F# triad, whilst the left hand moves down a tone from F# to E. Note that this progression moves gradually away from the home key, before returning Eb/F in the following bar.
- the use of chromatic passing chords (e.g. the D/E in A2). Crucially, there is more sophisticated use of such chords in this piece. Note that the D/E chord (at bar 2 of A2) and the E/F# chord (at bar 2 of A3) are chromatic deviations down and up (respectively) from the tonic chord of Eb/F found at bar 1 of A2 and A3. This has the effect of highlighting the role of these two chords as passing chords, and creates a distance between them and the piece.
- the use of Eb and Db triads, these performed with identical hand shapes

Further, there is heavy use of Db, Eb and Gb triads in both of these pieces. These are important chords (and hand shapes) in the "chain" strategy, examined below. Most chords in "The Man With The Horn" consist of four notes performed in the right hand with
another in the bass, and in “Shout” there are mostly three-note right hand chords with another in the bass. We will find a similar consistency in the “chain” system. We should note that both of these pieces are essentially tonal: “The Man with the Horn” is in Eb minor, and “Shout” is in F minor. By contrast, what we will see in the harmony for “Decoy” and “You’re Under Arrest” is use of identical right hand chordal material, but used over white note bass lines; i.e. in a more dissonant manner.
Decoy

(from "Decoy"
(1984, Columbia FC 38991))

R. Irving III

Intro

Trumpet

Synth

Bass

(percussion: vibra-slap)

Tpt

MD Syn.

Syn.

Bass

Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
Fig. 3-3 "Decoy"
"Decoy" (Robert Irving III) (Fig. 3-3)

(from "Decoy" (1984) Columbia FC 38991)

This is the title, and opening, track of Miles Davis' album.

Introduction (bars 1-4) and A Section (bars 5-36)

Four bars of drums and percussion act as the Introduction to this piece, the start of which is overlapped by a chromatic synth motif. Although this motif is seemingly abstract, we shall see it relates to other material in the A section which follows it. Darryl Jones plays a bass line centered on A (ending with a striking pair of Ebs at bar 12). The organ (a Yamaha DX-7 synthesiser preset) chord at bar 7 confirms this bass line as symbolic of the structure at the heart of the piece; i.e. the tritone relationship of A and Eb. The additional note G in the organ pad, as well as being the functional m7 of A seems to be present so that each hand is playing a three-note chord, and to maintain a visual symmetry at the keyboard. This chord alternates with an A quartal chord (AQ) throughout this A section (first heard at bar 9).

Returning to the opening chromatic synth motif, we can now see that it too has a relationship with both A and Eb. Indeed, it is constructed from two distinct sets: (a) an Eb quartal and (b) the I and bIII notes of A:
Note that these two sets are ordered to create chromatic movement towards the A and C (up from the Ab and down from the Db), and that the whole motif is placed metrically so that the A falls on the first beat of the bar, the C on the second. This matches the general on-beat metric emphasis of the piece. Further, note that the quartal is in the dissonant key of Eb whereas the (sublimated) triad is in the resolved key of A minor. This is interesting in that later we shall see the quartal used as resolved harmony in this piece and (Major and) minor triad used as dissonant material. I suggest that there is a direct correlation between the title of the piece and the ambiguous structure inherent in these various uses of the tritone relationship of A and Eb.

B Section (starts at bar 37)

Here the tritone relationship found in the Introduction and the A Section is further explored. Note the Eb diminished melody played by trumpet and synthesiser at bar 37, which starts on Eb and ends on A (this is a repeated theme). Further, it is here in the B Section that we will find the first examples of the “chain” strategies mentioned above, and these, at least initially, also relate to the tritone. The bass part is essentially in the key of A minor (with Ebs taking the place of the conventional V), and an Eb Major triad is the opening harmony event in the keyboard part. At one level this chord of Eb can be seen to
come from the preferred chordal repertoire of the composer (see the pieces above): in this sense, it is the tonality of the bass line that is actually new ("The Man with the Horn" has an F bass, "Shout" is in Eb). However, this chord is in fact also the first element of a chromatic "chain" of three chords (marked a). Further, let us note that whereas in "The Man With The Horn" and "Shout", Eb is a relatively resolved chord, this piece is in A minor, and thus the Eb Major triad is much more dissonant. Whilst maintaining the top note G of this Eb chord, Irving sideslips downwards the other two notes to form an A quartal (AQ) which is relatively resolved against the A bass. Thus, the Eb chord is seen, retrospectively, as a dissonant chord awaiting release. The common note G at the top of these chords is used as a pedal note to connect these otherwise contrasting "in" and "out" chords. Irving then returns to the Eb triad, recreating the sense of tension. This initial chain reveals important precedents:

- freedom to move up and/or down the chain
- importance of Eb triad as tension and AQ as resolved triad - this parallels the bass line built on A and Eb and the previously heard organ chord (see above)
- the lower two notes are either both black or both white
- the chain elements will be gradually revealed through the piece

For pattern b at bar 40 Irving plays a new, yet related, chord. This is derived by flattening the lower two notes of the A quartal (heard in bar 39). We shall call this chord an "Altered Quartal" (abbreviated to "xAQ"), thus, in this case, we have "AbAQ". Thus, we have now heard three types of chords which may be seen to be related to a single.

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1 This chord is, intriguingly, the inverse of the right hand voicing of the 7#9 chord, which the author associates most directly with another Miles Davis' piece: "All Blues" from "Kind Of Blue" (1959. Columbia 1355). The D7#9 chord found at bar 9 and 10 of this piece is
chromatic "chain". These chords are all connected (by the upper G pedal note), yet they relate to the underlying tonality in highly contrasting ways. This AbAQ chord is followed by the familiar AQ and Eb triad; thus pattern b, just like a, ends on an Eb triad.

It is pattern c that confirms this Pedal Chain as a motif in that here we hear a transposition (up a Perfect 4th) of pattern b. Whilst this transposition aurally relates to commonly-used blues structures, we should note that the hand at the keyboard has an identical look and feel for both b and c (B = black note, W = white note):

AbAQ, DbAQ: BBW
AQ, DQ: WWW
Eb, Ab: BBW

Table 3-I Showing identical Black/White structure of chords at b and c

Pattern d further extends the repertoire of material by including a CQ. Note that this continues the pattern of the descending pair of notes heard in the DQ and DbAQ that start pattern d. In a sense, then, it is the upper note that has changed (from a C to a Bb). Thus, it might be argued, by using a CQ, Irving avoids a C unison at the top and bottom of the chord. (Although, as we shall see later, he also has another solution for this.) Further, this CQ does not move to a Gb Major triad, or a BAQ chord. Rather, it simply descends chromatically to the BQ in bar 45. This has the effect of resolving the chain back to the home key of A. Note that the lower two notes of this chain always descend chromatically, and, thus, despite this difference, the majority of the chain strategy is maintained.

characteristically voiced F#. C, E# (Augmented 4th, Perfect 4th). By contrast an F#AQ chord would be voiced F#. B, E# (Perfect 4th, Augmented 4th).

2 I am thinking here of the sus4(8) chord (e.g. at bar 1 of "Code M.D.").
At this point, we need to separate and define two types of chromatic chord "chains" found here in Irving's playing: "Pedal Chains" and "Quartal Chains". Pedal Chains are those which contain a pedal note such as the AbAQ / AQ / Eb set found at bars 38-41 (pedal note G). "Quartal Chains" are simply sets of chromatically adjacent quartals, like the CQ and BQ found in pattern d (in bars 44-45).

Note Miles Davis' solo in bars 47-48. Here we see rising semitone pairs, repeated at various positions (G Ab, C Db, etc.), followed by a descending run from A (marked *). Thus, although fully chromatic, Miles' melody is carefully linked to the key of the piece.

Pattern e introduces a fourth chord type to the Pedal Chain system: the Em heard at the end of bar 48-49. Note its long duration when compared to earlier members of pattern e. It seems clear that Irving is concerned with a gradual revelation of this Pedal Chain strategy though the piece (let us recall that this is the first, and title track on the album).

Pattern f is, similarly, a combination of the Pedal Chain of AbAQ / AQ / Eb and an additional quartal (GQ). However, this is clearly a tone below the AQ that precedes it, and thus does not form a Quartal Chain, as defined above. There also seems to be a (very faint) occurrence of an AbAQ within bar 51. At the end of bar 55 Miles Davis starts a short solo, starting on A and ending on Eb (this is the reverse of the structure of the theme at bars 37-38 and 45-46; see above).

Pattern g is noteworthy for several reasons. The sustained 13th voicing starting at bar 57 may be the result of an error; it is, after all, an identical (all-white) chord to many of the

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3 Chains in this chapter are notated in the text with spaces between the chords and the forward slashes, thus: Eb / AQ. This is to avoid any possible confusion with chords that have unusual bass notes, such as F/Eb.
quartals seen here. Irving takes a relatively complex route through these bars (note the separate parts at bars 59 and 60, as well as the two distinct chains), seemingly in order to resolve this alien event, and the rhythmic phrasing is not as confident as my notation suggests. Note the unprecedented falling E to D note pattern at bars 58-59. By the end of bar 59, however, things seem to be back under control, and Irving performs a (resolved) BQ followed by its partner, Db Major. Bar 60 is noteworthy for containing a subtle variation on the core Pedal Chain, i.e. sustaining the G note as the chords change beneath (rather than replaying the G for each chord). Beyond this small additional complexity, Irving also moves on to include a further logical extension of the Pedal Chain strategy, namely the sus4 chord at bar 61. We should notice at this point that whenever new chords (AQ, m, sus4) have been introduced over and above those found in the original pattern a in bars 38-39, they all share the pedal note G with this original pattern.

Pattern h seems to act as compensation for the complexity of pattern g. Note how the AQ appears after the AbAQ in bar 64: this shows a new flexibility re ordering of the Pedal Chain sets.

Pattern i (bars 67-72) is interesting in that it contains a (newly heard) Gb chord as the start of a chain; here it appears above an Eb bass, these two parts sharing the same rhythm. Here, then, at the start of this pattern, we find a simple, resolved Ebm7 chord. However, we should really see the chord as independent of the bass: from this point onwards in my analysis, we will see that Irving uses this Gb triad as the most dissonant chord (symbolically, if not actually) in his chains. However, Irving does not continue the chain (i.e. thus: Gb / CQ / DbAQ), preferring instead Gb / CQ / BQ.
There then follows the longest Quartal Chain so far: AbQ, GQ, AbQ, AQ, BbQ, BQ.
Note that (a) all of these quartals consist entirely of either black or white notes, and (b) the chain set from GQ to BQ represents the longest possible such chain of quartals that has this characteristic:

![Figure 3-5 The Twelve Quartals, showing those which consist of all Black or all White Notes](image)

Pattern j at bar 73 shows further use of a subset of the core Pedal Chain: AQ / Eb / Em, with the Eb triad acting as dissonant material, resolving to the AQ and Em alternately. Pattern k, by contrast, introduces a new key. Notice how this is offset by the familiarity of the structure of the chain: F / BQ / BbAQ. In a sense, the use of this chain justifies the previous appearances of the BQ chord. Notice how the structure of previous keys is maintained with this chain; i.e. the lower two notes of each chord are either both black or both white. Pattern l, just as h above, acts as compensation by using a subset of the core chain: AbAQ / AQ.

Pattern m sees Irving permit a variation on the Quartal Chain, namely a Major triad Chain. Let us note that these chords have been carefully chosen: they relate to pattern i (bars 67-72) in that in both cases (a) Gb is the starting chord, and (b) a resolved quartal is the final destination of the phrase. Note the A to EQ movement at bar 93. Here Irving is showing more flexibility in his application of the chain strategy in moving between chords.
by this point in the piece; it is the top note which ascends chromatically, whilst the lower
two notes remain fixed (see also pattern n that follows). Before continuing, let us consider
this A major chord a little further. Note its position directly after the Eb triad, its staccato
ccharacter acting as a kind of punctuation. The choice of these two chords would seem to
be a further reference to the Eb / A tritone used so much in this piece. However, Irving
quickly resolves this section with the EQ; a stark A Major triad is not a consistent
harmonic resolution in this piece.⁴

As suggested above, in pattern n there is a further example of lower notes being
maintained between a pair of chords, with the upper note ascending chromatically, turning
an Eb chord into BbAQ. Note the BQ that follows. As we have seen, this is often used as
a chord to resolve phrases.

Between bars 97 and 136 (not notated), Irving uses material from the Pedal and Quartal
Chains described above more freely. That is, he employs shorter subsets of these chains,
sustains them for shorter durations and leaps from key to key more rapidly. Further, there
is a wider range of rhythmic placement re the consonant and dissonant chords from these
chains. However, we should note that, with regard to the Pedal Chains, Irving continues to
concentrate upon those keys which have most, if not all, of their lower two notes either all
black or all white.

Pattern o at bar 137 consists entirely of the GQ to BQ set examined above (see i in bars
67-72), and fulfils a similar cadential function, tension built by upwards chromatic
movement. Let us recall that this set is the largest complete string of all white/all black
quartals. By contrast with i, however, the consistency of performance re the rhythmic

⁴ Under these events the chord from the A section can be heard faintly. This may have been
multitracked, since this chord seems to be complete, requiring both hands to be performed.
structure seems a little more premeditated here. At bar 142 we hear the chromatic motif which opened the piece at various octaves, confirming that this is not just an abstract pattern of notes.

Up until the section which I have dubbed the “Interlude” (here the drums are fade out), there is again further free use of material from the chains already examined. At bar 223 of the Interlude, Irving performs a dramatic chain (pattern p), which gets louder, along with the rest of the mix. This consists of a real sequence of a pair of AQ > Q pairs (DbAQ / DQ and EbAQ / EQ), followed by an Esus4 chord. This has the effect of providing the ascending melody notes of C, D and E. Note that the Esus4 chord contains the same notes as the BQ, which, as we have noted before, is often used as a resolved chord in this piece: this is the point at which the bass and drums reenter.

Summary of the Patterns

Here is a summary of the patterns (all of which are annotated in the notation), with Pedal Chains underlined and Quartal Chains in bold:

\[
a (\text{bars 38-39}) \\
\text{Eb} \quad \text{AQ} \quad \text{Eb}
\]

\[
b (\text{bars 40-41}) \\
\text{AbAQ} \quad \text{AQ} \quad \text{Eb}
\]

\[
c (\text{bars 42-43}) \\
\text{DbAQ} \quad \text{DQ} \quad \text{Ab}
\]
$d$ (bars 44-45)

\[ DQ \quad DbAQ \quad CQ \quad BQ \]

$e$ (bars 47-50)

\[ AbAQ \quad AO \quad Eb \quad Em \quad Eb \quad AQ \]

$f$ (bars 52-54)

\[ AbAQ \quad AO \quad Eb \quad AQ \quad GQ \]

$g$ (bars 57-62)

\[ (G)13 \quad Bdim \quad (G)13 \quad GQ \quad Db \quad AbAQ \quad AO \quad Eb \quad Em \quad Csus4 \]

$h$ (bars 62-66)

\[ Em \quad Eb \quad AbAQ \quad AO \quad Eb \]

$i$ (bars 67-72)

\[ Gb \quad CQ \quad BQ \quad AbQ \quad GQ \quad AbQ \quad AO \quad BbQ \quad BQ \]

$j$ (bars 75-76)

\[ Eb \quad AO \quad Eb \quad Em \]

$k$ (bars 77-78)

\[ F \quad BQ \quad BbAQ \]

$l$ (bars 79-80)

\[ AbAQ \quad AO \]

$m$ (bars 91-93)

\[ Gb \quad F \quad E \quad Eb \quad A \quad EQ \]

$n$ (bars 95-96)

\[ Eb \quad BbAQ \quad BQ \]
Whilst noting that this is only a selection (representative, I believe) of material performed in “Decoy”, I present the following summaries of the use of Pedal and Quartal Chains in this piece.

Pedal Chain Summary

The Pedal Chains have a core pair of two chords: Eb Major triad and AQ. These two chords are sourced in the same Eb/A tritone relationship used in the Introduction, the A Section and Miles Davis' parts and solos. As we have seen above, as Irving introduces the other Pedal Chain material (AQ; m: sus4), he does so from this particular base: i.e. the AQ / Eb pair (thus: AbAQ, Em and Csus4 are the first occurrences of these chord types).

It can be seen from the summary of the patterns (above) that most of them commence with either a Major triad or an altered quartal chord (this is true of 12 of the 16 patterns that are o to p). Further, these Major triads and altered quartals usually resolve to the relevant adjacent quartal, e.g.: 
a Eb > AQ
b AbAQ > AQ

Table 3-3 Resolutions to adjacent quartals

Noting that Decoy is in the key of A minor, we can see that these structures follow the convention of dissonant material resolving to consonant material. That is, the quartals are generally used as resolved chords, with the Major triad and altered quartal chords seen as chromatic, dissonant variations. The resolved quartals used here are AQ, DQ, EQ and BQ. Note that the only other quartal is CO, and that this is followed by BQ on both occasions that it occurs above (patterns d and i).

Let us now examine the frequency of the Pedal Chain chords (AQ / Q / M / m / sus4) as they appear in chains in these notated sections of “Decoy” by means of a graph:

![Graph Showing the Frequency of Pedal Chain Chords in “Decoy”](image)
This graph shows high use of AbAQ/AQ/Eb/Em/Csus4 as core chain, with AQ and Eb as the clear favourites. Note that the lower two notes of all of the chords assessed in this graph are either BB or WW. Further, we should note that the limited extent of some of the chains seen here may be due to this BB/WW structure not being maintained across the full range of the Pedal Chain in all of these keys.

**Quartal Chain Summary**

Let us now examine the following graph, which shows the frequency of occurrence of quartals in the notated sections of “Decoy” (whether or not they fall within annotated chains):

![Fig. 3-7 Graph Showing the Frequency of Quartals in “Decoy”](image)

Note the relative high occurrence of quartals from GQ to BQ (i.e. the last five columns in graph). This is partly due to the two occurrences of these five quartals as a Quartal
Chain in bars 67-72 and 137-140. As we noted above, all of these five chords use only either black or white notes in their construction. Further, most of the quartals found in this piece do actually fall within chains; from GQ to CQ is the complete set of quartals used in chains. Also, let us note (a) that the two most frequently occurring quartals are AQ and BQ, and, (b) respectively, that these chords have A as their lowest and highest members.

**Pedal and Quartal Chains in “Decoy”**

Both of these chain systems have been designed to use simple B/W structures at the keyboard. Although the harmonic value of each chord is generally understood, Irving constantly shifts between “in” and “out” of the home key of A minor in an independent manner. Further, even in some other pieces on these albums (to be examined below) which are in different keys, we will see these chords occurring frequently. Thus, it might be argued, spatial issues are here prioritised over harmonic issues. However, the structure of these chains and the fact that the keys of these pieces are mostly white note keys: C Blues, E minor, G, A minor (compare this with the use of flatter keys for “The Man With The Horn” and “Shout”: Fm and Ebm) permits a roughly consistent use of them. We can see from the summary of the patterns (above) that Pedal and Quartal Chains are usually performed separately. The only section of the notation that can be seen to act contrarily to this rule is the relatively lengthy pattern i at bars 67-72.

“Decoy” is not only the first track on the first of the two albums which we are examining, it is also the piece in which these two chain strategies appear in their most prototypical state. We should note that there are places within this track where Irving uses material from the core chains in a relative free manner, avoiding verbatim
restatement of chain “lists”. However, we shall see as we examine other tracks that these chain strategies (and the material generated by them) are used increasingly freely. From the album “Decoy”, we will examine two other tracks: “Code M.D.” and “That’s Right”. From the album “You’re Under Arrest” we will examine “Katia” and “You’re Under Arrest”.
Code M.D.

from "Decoy"
(1984, Columbia FC 38991)

synth
bass synth
bass
fig. 3-8 "code m.d."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
Fig. 3-8 "Code M.D."
"Code M.D." (Robert Irving III) (Fig. 3-8)

(from "Decoy" (1984) Columbia FC 38991)

This piece, the second track on the album “Decoy”, immediately introduces a new element to the Pedal Chain at bar 1 (as the first element of pattern a). This new element, as we might expect, is revealed as a chromatic extension of the core AQ / Eb chain, being a descendant of the Altered Quartal. The additional D note (GCDG) seems to be included in order to maintain a sense of tonal unity between this new chord and the others in the chain: without it, the simpler (GCG) chord sounds too resolved, and relatively “thin”. Note also the direct relationship between this (GCDG) chord and the “highest” chord chain found so far, i.e. the sus4 chord (see bar 9 (etc.) of “Decoy”). Relatedly, this new chord contains both a lower and an upper Perfect 4th interval, a characteristic that it shares with the quartal chords. We will call this new chord a “sus4(8)” chord, the “(8)” referring to the octave note.

Pattern b at bars 8-9 does not contain a chain, but note that the DbAQ and BbAQ share a BBW visual structure, simplifying (perhaps even suggesting) this movement. Further, these chords are adjacent on the keyboard by visual structure. This pattern b is compositionally linked to pattern c that follows, these two progressions acting as “question and answer” in an eight-bar ostinato. This ostinato is repeated many times as a key motif of the composition and is often accompanied by guitar and/or saxophone. After much careful listening, I have decided that the first four chords in pattern c are simple dyads (Perfect 4th intervals), moving in a real sequence (c’ and c’), and not, as I first expected, quartals. However, these Perfect 4th dyads maintain a preference for quartal harmony, even when three-note chords are seen as inappropriate.
The section annotated $d$ at bars 36-39 shows pattern $c$ extended by an Eb triad. Over the A bass this chord adds an air of tension (as in “Decoy”), and this new complexity is matched by the cadential pattern $e$, which leads into the B Section. This pattern $e$ is noteworthy for a new type of chromatic chain; an Altered Quartal Chain of CAQ, BAQ and BbAQ. To obtain this progression (as notated here) requires a combined analysis of the guitar and synth parts. The synth uses dyads of a Perfect 4th interval, descending chromatically, whilst the guitar plays a melodic form partially disassociated with Irving’s general chain systems. Note that, by comparison with other chains examined above, this is a relatively complex mix of black and white notes, its performance made easier by its division between the synth and guitar. Indeed, the synth uses, as always, only either black or white notes. Note that the bass part adds a further level of complexity to this progression with the use of a symmetrical structure, moving down a semitone from Eb to D and up a semitone from A to Bb. This use of the bass part as a tool that alters (and complicates) otherwise straightforward chords recalls “The Man With The Horn” and “Shout”.

Pattern $f$ covers the first 8 bars of the B Section (annotated by five brackets), and consists of a set of now familiar chords (Gsus4(8), Eb, Gb, AbAQ and F). However, the only chromatic chain-like structure is the final cadence at bar 48 where the chords of Gb and F are accompanied in the bass by Fb and Eb. This pair of inverted dominant 7th chords have an abstract aural quality, but may be seen to closely relate to the home key of the B Section, which is D minor. The final F voicing in the keyboard acts a Dm7 subset, and the bass falls chromatically from Fb to Eb in preparation for the D at the start of bar 52. This 8-bar structure repeats three more times at bars 49-56, 57-64 and 65-72. However, there are a few small differences to be found at each repeat. GQ is the first chord in these three later repeats (e.g. beat 4 of bar 49), not the (new) Gsus4(8) chord. The repeat at bars 57-64 includes a short two-part melody (at bar 54-55). This is
noteworthy for the fact that it relates to previous material by mostly using intervals of a Perfect 4th. However, it also uses the complete set of Eb Dorian, the bII of the home key of D minor (this scale may also be seen to include the Gb triad in bar 53). Thus, here we see Irving composing with a relatively conventional scalar "out" strategy (as seen in Chapter 1). At bar 62 this two-part melody also appears, but here Irving changes two notes (an F and an Eb) to Gbs (marked *s). The complete set of Eb Dorian is, however, maintained between the synth and guitar parts. Further, note the complete list of D minor pentatonic that precedes this repeat in bar 61, which acts as a contrast to the "out" Eb Dorian set.

At the end of bar 72, the drum (machine) and (synth) bass parts return to the A Section, and the whole piece repeats, with an extended B Section for the sax and trumpet solos which follow.1

Summary of the Patterns

Here is a summary of the patterns described above, with the Pedal Chain underlined and Altered Quartal Chains in bold:

1 Note that the bass part is performed by a computer triggering a synthesiser/sampler using MIDI, despite Darryl Jones being credited as the bassist on this track on the album sleeve. There is a precedent for this use of a verbatim repeat of music in Miles' "SHHH/Peaceful" on "In a Silent Way" (1969), where a single take is used (at least) twice on the same record (see Chambers (1989) p. 157). Similarly, "Yesternow" on "A Tribute to Jack Johnson" (1970) incorporates one and a half minutes of "SHHH/Peaceful" (1969) (see Chambers (1989) pp. 202-203).
A Section

$ a $ bars 1-6

Gsus4(8) AbAQ AO Eb

$ b $ bars 8-9

Gb DbAQ BbAQ GQ Gsus4(8)

c bars 12-13

Bb4 G4 Ab4 F4 Gsus4(8)

d bars 36-39

Gsus4(8) Eb

e bar 40

Gb CAQ BAQ BbAQ (note A)

B Section

$ f $ bars 41-48 (also with variation as bars 49-56, 57-64 65-72)

Gsus4(8) Eb Gb EbAQ Gb F

Table 3-4 Summary of the patterns in “Code M.D.”

Summary

Although “Code M.D.” contains far fewer chains than “Decoy”, Irving uses a remarkably similar chord vocabulary, with regard to quality, key and voicing. Further, we have seen extension of the chain strategies to include visually identical (and adjacent) Altered Quartals (pattern $ b $) and an Altered Quartal Chain (pattern $ e $).
That's Right

from "Decoy"
(1984, Columbia FC 38991)

M. Davis/J. Scofield
(arr. M. Davis/Gil Evans)

Trumpet

Synth

Bass

Tpt

Syn.

Bass

Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
Fig. 3-9 "That's Right"
"That's Right" (Miles Davis/John Scofield - arranged by Miles Davis and Gil Evans)
(Fig. 3-9)

(from "Decoy" (1984) Columbia FC 38991)

This piece, like "Decoy" and "Code M.D.", employs Eb as a core chord. However, unlike the previous pieces, here this Eb chord is performed by Irving with a C bass note, forming a tonic Cm7 chord (bar 1). Thus, suddenly, the Eb triad takes on a conventional role, and (for the first time) is a resolved chord. Examination of the first two chords of the piece which form the vamp reveals a variation upon a Pedal Chain: the right hand (Dm) element of the F6 chord at bar 2 is a variation on the AQ that we have seen associated with Eb in the earlier pieces. After various lengths of time there is a (III / bIII / II / V) turnaround (the first being at bar 21 - see below).

Pattern a in bars 7-8 reveals the first of only a few (verbatim) chains to be found in this piece. Here Irving is using the Gb triad as to construct a (conventional) C7b9#11 chord, creating great dissonance over the C bass. That he is familiar with this jazz convention, and is not using the Gb chord as an iconic “out” chord with regard to an underlying tonality (as seen in “Decoy” and “Code M.D.”) is suggested by its strong placement at beat 1 of bar 7, and the immediate repeats of this progression at bars 9-10 and 11-12. The CQ/F is (effectively) a resolved chord; the dissonant note Bb softened by its being a suspension from the previous chord (in the manner of all previously heard Pedal Chains). (Note that the chords marked in the score from bar 9 represent the core bass note.) Pattern b in bars 13-16 has no Pedal or Quartal Chain: instead we find chromatic sideslapping of Major triads (E and Eb). Note the placement of an Eb triad over an F bass at bar 14. This right hand voicing, of course, generally belongs over a C bass in this
piece. Further, let us recall that “Shout” is characterised by such an Eb/F chord. The Bbm chord in bar 15 (marked *) is highly unusual; it has a BWB visual structure and would not be the conventional voicing of this chord (the conventional voicing would be BBW in an “A” Pedal Chain, thus: DAQ / EbQ / A / Bbm / Gbsus4). Similarly, the movement from BbAQ to Bbsus4 is also achieved with a previously unheard technique: sustaining a lower dyad and moving the upper note down a Major 3rd interval. Note, however, that these two upper notes (A and F) relate directly to the bass line.

Pattern c reveals the second Pedal Chain to be found in this piece: the core progression of AQ to Eb. Note also the rhythmic repeats of the Gb and Dm chords. This is the first situation where Irving has decided to repeat a chord rather than move to another. This may be due to the slow tempo rather than representing a shift in chain strategy. This section has a density similar to the previous four bars (13-16): the final Eb/C and Dm/F being a thematic reference before the forthcoming turnaround annotated as pattern d. Note how Irving uses Major and quartal chords moving as a real sequence (marked d' and d'') to create a turnaround that has the hallmark tonal qualities of his chain strategies. This turnaround includes the Pedal Chain of Gb / CQ (previously heard at bars 7-8). Although we have noted that this cadence is a real sequence, let us also note that this structure sees Irving apparently avoid a chain commonly found in “Decoy”, namely Gb / CQ / BQ. Note, however, that the E chord found here at bar 24 (a) shares its lower two notes with BQ, and (b) has a WWB visual construction in common with the CQ that precedes it. Bars 25-36 see Irving perform a sustained Eb triad over the alternating C and F bass line. He adds the notes D and Eb with the left hand, and a second Synth (an overdub? or Davis?) performs a C minor pentatonic melody. This use of the Eb triad as functional over both C and F (e.g. bars 1 and 14) seems to be the catalyst for the next section. Here, however, Irving takes a further liberty which has the effect that another member of the band seems to start to doubt that Irving knows
“where” he is in regard to the two-bar progression which alternates between C and F. In bars 37-39, Irving plays a lower bass melody, mostly using the notes C and F. This use of a C bass note for the F chord (bar 38) and an F bass note for the C chord seems to suggest to Darryl Jones (the bassist) that Irving has got lost, so he takes action and plays a clear F at bar 42. The Dm (/F) chord (as at bar 2) from Irving at bar 44 seems to be there to let everyone know that they are together, and indeed, that the duality of function that he is assigning to nearly all the chords in his repertoire is deliberate. The recording, of course, does not reveal any visual signals that may have occurred at this point (this band was well known for its use of subtle visual cues, encouraged by Davis).¹

Pattern e at bars 40 to 46 shows a Gb to CQ Pedal Chain, and, again, as in pattern d, this CQ (WWB) is followed by an E Major triad (also WWB). Note here how Irving uses this E triad as sideslip to the Eb chord which follows. Irving then alternates between two elements of a Pedal Chain: the Db Major and Dm minor triads at bars 43-46. Further, let us note that the first five chords in this pattern (Gb, CQ, E, Eb, and Dm) cover the chromatic scale (the three repeated notes being an F and two Bbs).

For the “Theme” section that follows (bars 47-58), Irving returns to the more conventional Eb and Dm chords that form Cm7 and F6 chords with bass part. Pattern f at bars 69 to 76 repeats the Gb to CQ Pedal Chain, but this progression is completed by an unusual event: a BbAQ chord (bar 76) (I might have expected BAQ or BQ here). However, we have already heard this BbAQ chord over an F bass at bar 16 (pattern b). Bars 81 to 84 represent the next turnaround, and it is striking that there is no chordal accompaniment from Irving at this point. However, at the following turnaround (bars 101-104), he performs a simple, but carefully designed part. The D Major triad which

¹ I saw this group perform in London in 1986 at the Wembley Conference Centre. This experience and viewing a video of this groups’ performance at the Montreal Jazz Festival revealed many such examples of on-stage cueing by Davis.
completely satisfies the D bass note at bar 103 is also performed with the same rhythmic placement a bar earlier (bar 102), over an Eb bass. Thus, here Irving is using the chromatically sideslipping bass line to create a sense of tension and resolution. This variation is in direct opposition to his usual chain strategy of moving chords chromatically up and down against a fixed bass note/toneality. Similarly, the next turnaround at bars 143 to 147 (pattern h) reveals a further surprise. Here Irving sideslips large, conventional chords. These chords have conventional 13#11 voicings (as described by Mehegan\textsuperscript{2}), and they are performed with a conventional bop blues “comping” rhythm pattern and melodic decoration. There is sense in which the use of these chords might be described as a development of the chain strategy in that the four 13#11 chords are enlargements of AbQ, GQ, F#Q and FQ chords respectively. (On the other hand, the use of these classic voicings seems retrogressive in relation to Irving’s usual chordal palette.) Thus, here (pattern h, bars 143-147), and in the previous turnaround (pattern g, bars 101-104), we have seen deliberate attempts to surprise with developments of the chain strategies. In retrospect, then, perhaps the blank bars 81 to 84 may be read as indicative of Irving’s desire to add interest to these turnarounds. His silence there, as noted above, comes as just such a “surprise”.

Pattern i consists of a (rare) melodic fragment, mostly built with Perfect 4ths, which may be seen to be derived from Eb Dorian (no Ab). This is immediately followed by a sideslipping major chord pattern. We might note the symmetry of this progression, and, further, that these three (right-hand) chords are used similarly in “Shout” (bars 1-2 of A2: bars 1-2 of A3). I suggest that here, however, this chord progression is the result of a single chord of Eb being played, with a pitch bend of a semitone applied first upwards, then downwards.

\textsuperscript{2} Mehegan (1959b)
Summary of the Patterns

Here is a summary of the patterns described above, with the Pedal Chain **underlined** and (new) Chromatic Chains in **bold**:

<table>
<thead>
<tr>
<th>a bars 7-8</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: Gb [= C7b9##11] CO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: C</td>
<td></td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b bars 13-16</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: / E / Eb / E Bbm E / BbAQ Bbsus4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: / C / F / C</td>
<td></td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c bars 17-20</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: / Gb / Gb / BbQ / AO Eb / Dm Dm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: / C / F / C</td>
<td></td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d bars 21-24</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: / DQ / Gb [= Ebm7] / CO / E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: / E / Eb / D / G</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>e bars 40-46</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gb CQ E Eb Dm Db Dm Db Db</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>f bars 69-76</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: / Gb / CQ / / / / / / / BbAQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: / C / F / C / F / C / F / C / F</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>g bars 101-104</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>synth: / / D / D /</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bass: / E / Eb / D / G</td>
<td></td>
<td></td>
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</tbody>
</table>
Thus, in “That’s Right”, Irving extends the chain strategies seen in “Decoy” in several ways. Most characteristic are the use of a fixed chord with a moving bass line (pattern g), the use of Mehegan voicings (pattern h), and the use of the pitch bend wheel on the keyboard (pattern i).

The frequent use of the E Major triad in this piece deserves a little more attention, I believe. We see this E Major chord used (a) adjacently to Eb Major triads (patterns b and e) and (b) as the third chord in a Gb, CQ, E chain (patterns d and e). In “Decoy”, the Eb Major triad was often used with an E minor (i.e. not Major) triad as part of the core Pedal Chain. I suggest that the reason that Irving prefers the E Major triad here is that an E minor triad performed over a C bass would form a Cmaj7 chord (and an Fmaj9#11 over an F bass), and that these would be unacceptable in a C (minor) Blues composition. Similarly, as noted above, we normally saw a Gb, CQ chain completed with a BQ chord in “Decoy”. Such a BQ chord would have a similar effect to the Em triad over a C bass; it would form a Cmaj13 chord, equally unacceptable, and for the same reason. Thus, Irving is using the (bII) E Major triad as dissonant material in a piece where the (previously dissonant) Eb Major triad is the tonic chord. Further, as we noted above.
this E Major triad shares a WWB visual characteristic with the CQ that precedes it in this progression.
Katia

from "You're Under Arrest"
(1985. Columbia CB 26447)

M. Davis/R. Irving III

Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
Fig. 3-10 "Katia"
"Katia" (Miles Davis/Robert Irving III) (Fig. 3-10)

(from "You're Under Arrest" (1985) Columbia CB 26447)

This piece is the opening track on Side B of the album "You're Under Arrest", and is a continuation of the last track on Side A (called "Katia Prelude"). "Katia" fades up, beginning *in medias res*. Bar 1 in my notation refers to the first heard notes in this track. For this piece, I have examined both (a) Robert Irving's chain strategies at the keyboard, and (b) Miles Davis' contribution at his own keyboard, showing how the trumpeter's choice of chords and voicings relate to Irving's work. My annotation of their performances is organised as follows: patterns a to k are performed by Irving; patterns l to s are performed by Davis. We will examine Irving's contributions first.

Robert Irving III

Pattern a is an Asus(8) chord, which acts as a drone. Note that this, like all other sus4(8) chords, consists entirely of white notes. I have found pattern b difficult to notate (both Irving and Davis are using an identical sound on their Oberheim OBXA synths). I suggest that Irving performs the Ab to DQ chain, and the chord heard before (the Esus4 in bar 58) is played by Davis (see below for further analysis). The two musicians play a similar progression as pattern c (bars 68-69). Pattern d shows a long chromatic Quartal Chain, from AQ down to DQ. Irving creates a cadence by sustaining the last two chords (EbQ to DQ), although (a) the length of the chain and (b) the dissonance of the final DQ

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1 This analysis ignores the possibility that Irving may have overdubbed a keyboard part, matching Davis' trumpet. Indeed, there are several events which suggest that parts may have been overdubbed in other ways. For example, at bars 126-136, where the DX-7 (electric piano) is in the foreground, we can hear an OBXA Synth in the background playing chain parts (these are relatively dissonant to Davis' "stabs", and may thus have been supplanted).
over an E(m7) bass line means that this chain assumes a very neutral mood; these sustained chords act as a foil to Davis' "stabs" of the previous few bars. Whereas at d Irving follows an EbQ with the dissonant DQ, at e he moves a comparatively large distance down to the more resolved AQ. The CQ to DbQ annotated as pattern f is achieved by means of the pitch bend wheel on the synth. The AbQ at the start of pattern g is clearly an extension of f, yet is adopted rhythmically by the following Gb/CQ/BQ chain, this already having been heard many times in "Decoy" (bars 67-72, etc.). This chain is then repeated twice as pattern h. This is the first example of such a verbatim adjacent repeat of a chain that I have found in these pieces. Pattern i, an antiphonal response to Davis (bars 173-174), has a similar neutral quality to d (indeed i is a subset of d). Note, however, that here Irving ends on a more resolved EQ. Pattern j is a continuation of this antiphony. Here we can see unprecedented activity in several ways.

Never before has Irving used an AbQ to GAQ chain. Similarly, the following Gb4(b9) and E4(9) chords maintain the syntax of the Pedal Chain strategy, yet extend this basic vocabulary. Note that, in the case of the latter two chords, this extension is beyond the octave. As we might expect, these new chords follow the convention of having "easy" hand shapes; the common upper black note of the relatively large 4(b9) and 4(9) chords aids their performance. Pattern k sees a Gb chord replace a DbQ, forming a Pedal Chain with the CQ that follows. This would seem to suggest that Irving is discontented with the DbQ for some reason. Perhaps this is due to the fact that DbQ is a resolved Major69 chord over the E (minor) bass, forming most of the Mehegan voicing for an Emaj7 chord (this recalls to mind Irving's avoidance of the Em triad over the C bass notes in the C Blues piece "That's Right", examined above). Further, the Gb to CQ movement would seem to be a little easier to perform than the sideslipping (BBW) DbQ to (WWB) CQ progression. Also, Irving surprises the listener by failing to fall to the AQ directly from the BbQ; instead he inserts an AbQ chord, the whole functioning as an
interrupted cadence. The AbQ and BbQ act as an “out” symmetrical pair, each chromatically distant from the resolved AQ. This structure has the advantage of affirming the AQ’s function as a resolved chord, something less likely to be understood by a simpler descending chromatic progression, and functions like the \{x, x-1, x+1\} sets” examined in Chapter 1. The WWW and BBB visual structure of these four chords makes this symmetrical pattern easy to perform. This AQ is accompanied at bar 221 with a collection of (WWW) Dsus4 and (BBB) Dbsus4 triads performed on the DX-7 (electric piano preset). Finally, let us note that this resolved AQ chord (a) ends the piece (bar 246 onwards) and (b) that it is sustained beyond the final bass note, thus asserting its independence.

Summary of the Patterns (Robert Irving III)

Here is a summary of the patterns described above, with the Pedal Chain underlined and Quartal Chains in bold:

a bar 49...
Asus4(8)

b bars 59-60
Ab DQ

c bars 68-69
Ab DQ

d bars 134-138
AQ AbQ GQ F#Q FQ EQ EbQ DQ
$e$ bars 160-162
F#Q  FQ  EQ  EbQ  AQ

$f$ bars 163-165
Esus4(8)BbQ  BQ  CQ  DbQ  BQ  BbQ  AQ

g bars 165-167
AbQ  Gb  CO  BQ

$h$ bars 168-172
Gb  CO  BQ  Gb  CO  BQ

$i$ bars 175-178
AbQ  GQ  F#Q  FQ  EQ

$j$ bars 212-214
AbO  GAQ  Gb4(8)  F4(b9)  E4(9)

$k$ bars 215-223
Ebo  DQ  Gb  CO  BQ  BbQ  AbQ  AQ

Table 3-6 Summary of the patterns in “Katia” (Robert Irving III)

Miles Davis

We can see from the notation that Davis plays trumpet and keyboard “stabs” simultaneously, interspersed with chromatic solo material. Davis first used an Oberheim synth this way in 1982 (around the time of “Star People”). On stage, this activity is facilitated by a (radio) microphone fixed to Davis’ trumpet, permitting greater mobility. I have found this music difficult to notate, partly due to the crash cymbal usually added

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$^2$ See Chambers p. 342-343 and 349
by Vince Wilburn to these stabs, but mostly due to the presence of the (real) trumpet over the synth. Davis has said "...if I don't play over it, it will sound mechanical". Indeed, the complex timbres of the trumpet sometimes made the specific chord and/or voicing very tricky to pin down. I present the following analysis with this caveat.

From bars 49 to 70 (pattern I) Davis uses an Esus4 chord to accompany this chord's highest member, the note B, on the trumpet. Davis stops playing these chords at the start of his chromatic trumpet solo sections (both starting with a high C, at bars 59 and 68), and Irving takes over as keyboardist with chain material. Pattern m (bars 83-87) sees Davis use a set of quartals. Again, the highest note represents the trumpet pitch. Note that the choice of the final EQ is driven by melodic concerns, and thus requires an interval between the final two chords of a Perfect 4th. We will see several examples of this below. This is not a characteristic which Irving's strategies generally share. Bars 106 to 115 (pattern n) see Davis use mostly Dbsus4(8) and Esus4(8) chords. This creates a pair of contrasting Major and minor modes for the E bass. At bar 111 Davis performs his first chord which is not BBBB or WWWW: an Fsus4(8) (= WBWW). Note the slight sustain on the Fsus4(8), drawing attention to this chord. Pattern o continues in the same vein. In the eight bars of pattern p (bars 148-155) we hear Davis play a mix of sus4, quartal and (newly heard) sus2 voicings. However, these new complexities are offset by repeats following a four-line structure (each line lasting two bars) that follows an A, B, A, B form. Let us note that the chords of Dsus2 and Dbsus2 are simply inversions of the AQ and BbQ that follow them. Further, we might note the number of sideslips found in this section:

4 We might note that Davis has suggested that his combined trumpet and synth technique is a substitute for five trumpets (Williams, Richard (1983) "On top of all the beat" The Times (London) 28 April; quoted in Chambers p. 343). In this regard let us note that a sus4(8) voicing, combined with Davis' trumpet, sums to five notes. However, five is the maximum, and not the usual, number of voices.
Esus4 > Ebsus4
Dsus2 > Dbsus2
AQ > BbQ

Table 3-7 Sideslips by Miles Davis in “Katia”

Pattern q is interesting in that here, for the first time, Davis plays the lowest note of the chord on his trumpet. He continues this strategy in bars 173-174 (pattern r). Note that both of these patterns move chromatically, offsetting this complexity. However, at bar 174, Davis uses an F#sus4 chord (marked *), which has a rare BWB visual structure.

Pattern s (bars 204-211) sees the return to Davis using the trumpet pitch as the upper note in the accompanying chord. Pattern s, like p, has an (A, B, A, C) form. However, it is performed more freely against the bars, slowing down, and with a looser feel: the trumpet slightly anticipates the synth for the last two chords (Esus4 and Fsus4). This delay is very expressive as the trumpet rises in pitch, and extends the emotional range of these otherwise metrically strict “stabs”.

Let us note that nearly all of the chords played by Davis consist entirely of either black or white notes, and, further, that all of the voicings used (sus2, sus4 and quartal) are inversions of each other. Thus, Davis entirely limits himself to chords built on Perfect 4th intervals. Further, those two chords which are not completely black or white are always approached chromatically.

Finally, let us note that the chords chosen by Davis facilitate the combined use of trumpet and synth. For example, when playing a quartal on the keyboard, then the lower note of that chord supplies Davis with the note that he should think of when playing trumpet in order to sound the top note of the chord, the trumpet being a Bb transposing instrument. For example, a Bb quartal is made up of the notes Bb, Eb and Ab.
trumpeter is invited to play a Bb (the bottom note in the quartal), it will sound like a concert Ab (the top note in the quartal). The other chords, as we have seen, are inversions of such a quartal (sus2 and sus4). In these cases, it is the upper of the two adjacent notes that provides Davis with the note to be played on trumpet. Thus, for the chords of BbQ, Absus2 and Ebsus4, Davis can “think” a Bb and “sound” an Ab.

Summary of the Patterns (Miles Davis)

Here is a summary of the patterns described above, with chords that do not have a BBB or WWW visual characteristic marked in bold:

l bar 53...
Esus4

m bars 83-87
BbQ  BQ  BQ  BbQ  AQ  EQ

n bars 106-115

o bars 122-135
Esus4  Ebsus4  Esus4  Dbsus4  Esus4  Ebsus4  Esus4  Esus4  Esus4

p bars 148-155
A:  Esus4  Dbsus4  AQ  BbQ
B:  Dsus2  Dbsus2  AQ  Esus4(8)
A':  Dsus2  Dbsus2  AQ  BbQ
B:  Dsus2  Dbsus2  AQ  Esus4(8)

q bars 169-170
Asus4  Absus4  Gsus4  Absus4  Asus4
Table 3-8 Summary of the patterns in “Katia” (Miles Davis)

<table>
<thead>
<tr>
<th>Bars</th>
<th>Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>173-174</td>
<td>Gsus4, Absus4, Gsus4, F#sus4</td>
</tr>
<tr>
<td>204-211</td>
<td>A: Dsus4, Dbsus4, Csus4, Dbsus4</td>
</tr>
<tr>
<td></td>
<td>B: Dsus4, Dbsus4, Csus4</td>
</tr>
<tr>
<td></td>
<td>A: Dsus4, Dbsus4, Csus4, Dbsus4</td>
</tr>
<tr>
<td></td>
<td>C: Dsus4, Ebsus4, Esus4, Fsus4</td>
</tr>
</tbody>
</table>

Summary

Pattern j in Irving’s work represents a further development of the Pedal Chain, employing Gb4(8) and F4(b9) chords. However, despite this, there is not the same range and fluency of performance of Pedal Chains in this piece as we have seen earlier. Indeed, here Irving just uses (a) the Major triad and quartal elements from two Pedal Chains, and (b) relatively lengthy Quartal Chains. Note that in this piece Irving uses all of the four quartals that do not show all black or all white notes; i.e. CQ, DbQ, FQ and F#Q. In fact, he does this in most of the Quartal Chains listed above (d, e, f, and i). By contrast, in the three previous pieces Irving mostly uses WWW quartals, resolved to the bass note. Further, all 12 possible quartals may be found here, and with fairly similar frequency. For example, members of the (black and white) CQ, DbQ, FQ and F#Q set occur surprisingly commonly: 9 times between themselves. In contrast with Irving, Davis uses chords that consist entirely of black or white notes more usually, and relatively exclusively. In fact, if we invert all of the chords he uses to be quartals, we find that he uses CQ, DbQ, DQ, EbQ, EQ, GQ, AbQ, AQ, BbQ and BQ. The CQ and DbQ found here represent the (relatively rare) Fsus4 and F#sus4 chords marked in bold.
in the table above. Further, their partners, FQ and F#Q (WBB and BWB) do not appear. Thus, whilst Davis concentrates on visual characteristics, Irving does not seem to be using the same consonance/dissonance rules that we found in the Pedal and Quartal Chains in, say, “Decoy”, although he is using a related chord vocabulary. I will examine this issue further, below.
You're Under Arrest

from "You're Under Arrest"
(1985, Columbia CB 26447)

J. Scofield

Fig. 3-11 "You're Under Arrest"
Fig. 3-11 "You're Under Arrest"
John Scofield's piece "You're Under Arrest", is, like "Katia", also in the key of E minor. Pattern a shows Irving use an AQ, BbQ, BQ set that will be a feature of this piece. This set, we should note, is also common to, and used cadentially in "Decoy" (in A minor). Note that the final AQ (in bar 51) in pattern a is further resolved with a single E note; AQ is not as resolved here as it was in "Decoy". EQ would be the equivalent chord in this piece, and I suggest that this additional E note is compensation for this difference. Pattern b (bars 53-55) also ends with an AQ and an E note, but is also characterised by the Gb / CQ / BQ chain used in "Decoy" and "Katia". Pattern c see Irving complete a progression initialised by Davis (similarly to bars 175-178 of "That's Right"); here Davis continues his strategy of using his trumpet note to define the upper element of BBB and WWW quartals, examined in "Katia", above. Note the variation of the Gb / CQ / BQ progression found at b by the interpolation of a BbQ. A further variation of this progression happens soon after at pattern d, where Irving replaces the BbQ with an AbQ in a "call and response" structure (again, we have seen this progression before: at bars 215-223 of "That's Right"). Patterns e, g and i show further use of the quartal progressions found at a, b and d, and the Gb / CQ / BQ chain is repeated as patterns f and h.

Summary of the Patterns

Here is a summary of the patterns described above, with the Pedal Chains underlined and quartal chains in bold:
Table 3-9 Summary of the patterns in “You’re Under Arrest”
Summary

Here Irving restricts himself still further than in “Katia”: there is a single Pedal Chain subset of Gb to CQ, and the range of the quartals is reduced from all twelve to just five chords (AbQ, AQ, BbQ, BQ and CQ). Further, in contrast with “Katia”, and in common with other previous pieces, most of the quartals here are BBB or WWW. Indeed, there is a clear tonal difference between “Katia” and “You’re Under Arrest”, despite (a) the use of many similar chords, and (b) the fact that both pieces are in E minor. Whereas “Katia” at times sounds neutral and less harmonically focused than the previous pieces, “You’re Under Arrest” seems to have a greater confidence with regard to consonant and dissonant chords and their use in the construction of cadences. We will examine the possible reasons for these characteristics later in this chapter.
Summary of Pedal Chain Theory

We might usefully construct a complete (theoretical) Pedal Chain, showing the gradual enlargement of the set, thus:

Type:

1

AO Eb

2

AbAO AO Eb

3

AbAO AO Eb Em

4

AbAO AO Eb Em Csus4

5

Gsus4(8) AbAO AO Eb Em Csus4

6

F4(9) F#4(b9) G4(8) AbAO AO Eb Em Csus4

Table 3-10 Summary of Pedal Chain Theory

I will refer to this Pedal Chain from now on as the “Eb” Pedal Chain, since it, and no other, contains the Eb Major triad. Pedal Chain Types 1 to 4 above are revealed in this order in “Decoy”. The sus4(8) chord required for Type 5 first occurs in “Code M.D.”, and the 4(9), 4(b9) and 4(8) chords of Type 6 occur in “Katia”. Thus, the first 5 Types occur on the album “Decoy” (1984), and the 6th Type is only revealed on the album “You’re Under Arrest” (1985). This gradual revelation, then, matches the track order and release dates of these two albums. Note that in Chain Type 6 the 4(8) chord replaces

---

1 As mentioned above, all of the following analysis is based upon the (incomplete) notation that I have undertaken of these pieces. Where I have omitted sections of notation this is where Irving is not playing chain material or where he is playing music already notated and examined. Thus, events described here as “rare” may be viewed as rarer than the data show.
the sus4(8), thus maintaining the three-note structure of these chords, and promoting movement up and down the chain. Also, let us note that it is only Pedal Chain Type 6 that covers the chromatic scale. Note that nearly all of the first occurrences of a new Pedal Chain element in this chapter occur in the key specifically defined here, suggesting that the Eb Pedal Chain is the original source. The exception to this is, again, Type 6, which contains the trio of chords described as 4(9), 4(b9) and 4(8). These occur a semitone lower on their one and only appearance (i.e. at bars 213-214 of “Katia”).

This difference seems to be for a couple of reasons. The first reason is that the hand shape of E4(9), F4(b9) and F#4(8) is easier than that needed to play these chords a semitone higher (i.e. as F4(9), F#4(b9) and G4(8)). The second reason relates to the key of the piece (E minor), and will be examined later in the chapter. Before moving on, let us note how the visual structure of this “Eb” Pedal Chain extends to other Pedal Chains used by Irving in “Decoy” (extended theoretically):

```
"Db"       GQ   Db   Dm
           WWW BBW WWW

"Eb"       Gsus4(8) AbAQ  AQ   Eb   Em   Csus4
           WWWWW BBW WWW BBW WWW WWW

"Ab"       Csus4(8) DbAQ  DQ   Ab   Am
           WWWWW BBW WWW BBW WWW
```

Table 3-11 Showing related visual structures of Pedal Chains at Db, Eb and Ab

Further, the “F” and “Gb” Pedal Chains have related visual structures:
Let us now construct a simple graph which shows the frequency of occurrences of the chord types (of any key) to be found within the notated chains:

![Graph Showing Frequencies of Chord Types](image)

Table 3-12: Showing related visual structures of Pedal Chains at F and Gb

This graph clearly shows the chords of the Major triad and Quartal as the most common chord type, followed by the Altered Quartal and minor triad. The sus4(8) chord, although fairly common and harmonically important within several pieces, does not commonly occur within the performed Pedal Chains. Further, as noted above, the 4(9), 4(b9) and 4(8) chords occur once only in those sections examined above. Thus, Figure
3-12 shows that most of the Pedal Chains are based around the core pair of a Quartal and Major triad, whilst most of the other chords are introduced locally to this core pair.

A Dissonance Hierarchy for the Pedal Chain Chords

As I examined these pieces, I found some (often visually related) chains appearing over and over again, whatever the key of the piece. However, alongside these visual and hand shape strategies, I believe that Irving is also making certain tonal decisions with regard to one particular tonality in two of these pieces. The tonality is E minor and the pieces are "Katia" and "You're Under Arrest".

In order to explore this idea, I propose the following "dissonance hierarchy" for the chains. This will allow us to assess the "in-ness" and "out-ness" of the chords against the three keys of A minor, C minor (Blues) and E minor. Such a hierarchy may be revealed in a similar fashion to that examined in bars 137-172 of John Coltrane's "Acknowledgement" (see Chapter 2). That is, we can use an adaptation of Watkins' and Dyson's version of Longuet-Hughes' theory to produce a dissonance value for each of the chords used by Irving, paying regard to the various keys in which these pieces are written. Thus, we will again use the \( q(\text{Dorian}) \)\(^2 \) tonal model, thus:

---

\(^2\) I have chosen Dorian as the core scale, since all of the pieces here are either in a minor or Blues mode, and experimentation has shown this mode to provide a consistent model across Irving's use of his chain strategies in these pieces.
Whereas for “Acknowledgement” F = 0, here the key of the piece = 0. Thus, if the key of the piece is A (such as “Decoy”), then we can use the following table:

<table>
<thead>
<tr>
<th>note:</th>
<th>C</th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
<th>B</th>
<th>F#</th>
<th>Db</th>
<th>Ab</th>
</tr>
</thead>
<tbody>
<tr>
<td>q(Dor):</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3-14 Adapted Watkins and Dyson/Longuet-Hughes table: Chromatic scale arranged by fifths with associated “sharpness value” for q(Dor): A=0

From this we can now calculate a dissonance value for any given chord by summing the absolute values\(^3\) of (a) each note in the chord (ignoring octave duplicates, i.e the “(8)” of the sus4(8) and 4(8) chords) and (b) the bass note. For example, Eb Major triad and A Quartal (both over an A bass) have dissonance values of 15 and 3 respectively, calculated thus:

---

\(^3\) By “absolute”, I mean that we ignore any negative symbols in the data. Thus, for example: 2 + -1 = 3 (absolute) and -3 + 1 = 4 (absolute).
Table 3-15 Showing calculation of “dissonance values” for Eb/A and AQ/A

These results reflect the relative dissonance of Eb/A (value = 15) versus the relative consonance of AQ/A (value = 3). Having calculated dissonance values for all of the chord types in all keys, I have been able to examine the dissonance contour of a given chain against any bass note. For example, the following graphs show the values for the opening chains in “Decoy” (patterns a to f):

Fig 3-13 Graphs Showing the Dissonance Values of patterns a, b and c (bars 38-43)
Fig 3-14 Graphs Showing the Dissonance Values of patterns d, e and f (bars 44-54)

It can be seen from these graphs that Irving tends to use highly resolved and highly dissonant chords, alternating between the two types (this is reflected in the steep zigzag structure of the graphs above). Further analysis shows three trends that reveal the importance of this contrasting dissonance/consonance characteristic to Irving. Firstly, Irving repeats most frequently those chord sets from chains that have such a polarised dissonance structure. Secondly, chain phrases tend to end on a highly consonant or dissonant chord. The exception to this is the final GQ/A (at bar 49). However, this is the first appearance of this chord, and, although it may be seen to belong to the visually similar "Db" Pedal Chain, it is only used as such in the unusual pattern g (at bar 59). This suggests that although Irving fluently employs the (similar) hand shapes of these chains, he may not be completely familiar with the consonant/dissonant nature of these chords against a given bass. Indeed, I suggest that he develops a hierarchy in his mind as the piece progresses. Thirdly, it may be seen that the Eb and AQ (and AbAQ) core set is more diversely consonant and dissonant than chord sets that have a similar visual structure (e.g. Ab and DQ (and DbAQ)). Here are further graphs that show the
dissonance data of the “Db”, “Eb”, “F”, “Gb” and “Ab” Pedal Chains, all over an A bass, as found in “Decoy”:

Fig 3-15 Graph Showing the Dissonance Values for the “Db” Pedal Chain

Fig 3-16 Graph Showing the Dissonance Values for the “Eb” Pedal Chain
Fig 3-17 Graph Showing the Dissonance Values for the "F" Pedal Chain

Fig 3-18 Graph Showing the Dissonance Values for the "Gb" Pedal Chain

Fig 3-19 Graph Showing the Dissonance Values for the "Ab" Pedal Chain
Note that the earliest, and most commonly occurring, of these Pedal Chains are the “Eb” and “Ab” chains (in that order). The graphs shown here exhibit a strong (and similar) zigzag pattern for these two chains, most especially around the core pair of Quartal and Major triad (and their adjacent chords: the Altered Quartal and minor triad), suggesting premeditation of these consonance/dissonance relationships. However, (here in the key of A), the (often visually similar) “Db”, “F” and “Gb” chains show much shallower curves around the Quartal and Major chord points. This is due to less difference (i.e. lower contrast) in the dissonance values for these chords in this key. However, note that the BbAQ and BQ from the “F” chain are also commonly occurring chords in “Decoy” and that these have dissonance values (and a difference) identical to the chords of Eb and AQ.

There is one occurrence of the Gb and CQ chain in “Decoy” (as pattern i at bar 67). We can see from the “Gb” Pedal Chain graph that these chords are not highly contrasted with each other. However, the chords of Gb and CQ from this Pedal Chain is used extensively in “That’s Right”. There, of course, these chords are transpositional equivalent of the Eb and AQ chain in “Decoy” (“That’s Right” being in C and “Decoy” being in A), and thus they show a greater degree of contrast.

Table 3-16 (below) shows the dissonance values for all of the Pedal Chain chords. Shown against a common C bass note. I have marked in bold those sections where there is highest contrast between adjacent chords. I have defined the range of highest contrast as being those points where the absolute difference between three adjacent chords is greater than or equal to 18. For example, in the first line of the table: (15-7)+(15-4)=18 and (15-5)+(14-5)=19).
Table 3-16 Dissonance values for all Pedal Chain chords (C bass)

It is these most dissonant Pedal Chain subsets (marked in bold) that form the basis of Irving's accompaniment in "Decoy" and "Code M.D.". By contrast, Table 3-16 also reveals that the chords which form the vamped head progression of "That's Right" (i.e. Cm7 and Dm/F) have little contrast with each other, thus:4

---

4 The value for Dm/F is found at Am/C in the table above.
Further, we noted above that the Dm/F is an alteration of the (expected) AQ/F chord, and now we can see that this would show a more dissonant value, and thus create a steeper curve between these two chords, thus:

What matters here is not the specific value for each chord, but the relative contrast between them. Thus, in “That's Right”, Irving uses closely related (i.e. similarly dissonant) chords for the head (replacing the expected AQ with the less dissonant Dm), and then goes on to use more highly contrasting chords in the solo sections of the piece.

---

5 Similarly, the value for AQ/F is found at EQ/C in the table above.
We might note that the last six chords in row four of the table above show low contrast, and that it is this key which is chosen for “That’s Right” (although it is row ten which generally shows the least contrast; none of its chords are marked in bold).

Irving’s tendency to use Pedal Chains that have these highly contrasting dissonance values extends to his choice and use of Quartal Chains. For example, this Quartal Chain...

\[
\begin{array}{ccc}
AQ & BbQ & BQ \\
3 & 18 & 3
\end{array}
\]

Table 3-17 Highly contrasted Quartal Chain

...has a highly contrasted structure (over an A bass). Indeed, it has a three-chord difference value of 30 \(((18-3)+(18-3))\); the maximum possible.\(^6\) This set is often used in “Decoy”, frequently as a cadential resolution to longer Quartal Chains, for example in patterns \(i\) and \(o\) (set shown in bold):

\(i\) (bars 67-72)

\[
\begin{array}{ccccccc}
Gb & CQ & BQ & AbQ' & GQ & AbQ & AQ & BbQ & BQ \\
3 & 18 & 3 & & & & & & \\
\end{array}
\]

\(o\) (bars 137-140)

\[
\begin{array}{cccc}
GQ & AbQ & AQ & BbQ & BQ \\
3 & 18 & 3 & & & \\
\end{array}
\]

Table 3-18 Patterns \(i\) and \(o\) from “Decoy”
Chains in “Katia” and “You’re Under Arrest”

With the dissonance hierarchy defined above in mind, let us return to an examination of “Katia” and “You’re Under Arrest”, which, as noted above, are both in E minor.

“Katia”

Irving uses the Ab to DQ Pedal Chain subset (from “Decoy”, etc.) at patterns b and c of “Katia”. However, here this progression has a much lower contrast due to the E bass, as shown by the following graph (the steeper the angle, the greater the contrast):

![Graph showing difference in contrast between DQ and Ab chords over A and E bass notes](image)

Fig 3-22 Graph Showing the difference in contrast between Dissonance Values for DQ and Ab Chords over (a) A and (b) E bass notes

However, the Gb to CQ progressions which follow (patterns g to k) have a higher contrast than this, and, in fact, even show an increase in contrast (i.e. the progression shows a steeper angle on the graph below) over “Decoy” and “Code M.D.”, with their A bass notes:

6 This theory has a loose relation with the bII quartal characteristics which I made in the
Fig 3-23 Graph Showing the difference in contrast between Dissonance Values for CQ and Gb Chords over (a) A and (b) E bass notes

I suggest that these graphs may represent why Irving aborts the use of the Ab to DQ progression after just two hearings (ending at bar 69), and why the Gb to CQ progression occurs four times over a much longer period (patterns g to k). However, we should note that both of these progressions show a lower contrast than the core “Eb” Pedal Chain used in “Decoy”, with its characteristic zigzag graph (see Fig. 3-16 above). This lower contrast between the elements of commonly used Pedal Chains in “Katia” has the effect of creating the more neutral mood in this piece, noted above. It would be possible, of course, for Irving to use the equivalent “Eb” Pedal Chain from “Decoy”. Transposed to the key of E, this would be:

EbAQ   EQ   Bb   Bm
11     3     15   5
BBW    WWW  WBW  BWW

Table 3-19 Bb Pedal Chain (E bass)
However, this would require using the WBW voicing of Bb Major. However, Irving takes two steps to improve the contrast in this piece, and both of these involve temporary abandonment of the strict B/W visual structures which are otherwise so common. Firstly, he performs long Quartal Chains, often using the most dissonant set of \{EQ, FQ, F#Q\}, which appears at \(d\), \(e\) and \(i\), thus:

\[
\begin{array}{cccccccc}
d \text{ bars 134-138} \\
AQ & AbQ & GQ & F#Q & FQ & EQ & EbQ & DQ \\
& & & & & 3 & 18 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
e \text{ bars 160-162} \\
F#Q & FQ & EQ & EbQ & AQ \\
& & & & & 3 & 18 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
i \text{ bars 175-178} \\
AbQ & GQ & F#Q & FQ & EQ \\
& & & & & 3 & 18 & 3 \\
\end{array}
\]

Table 3-20 Patterns \(d\), \(e\) and \(i\) from “Katia”

This \{EQ, FQ, F#Q\} set is the equivalent of the \{AQ, BbQ, BQ\} set over an A bass. However, to include this set Irving is not maintaining the B/W visual structure central to his strategies:

\[
\begin{array}{cccc}
EQ & FQ & F#Q \\
WWW & WBB & BWB \\
\end{array}
\]

Table 3-21 Black/White structure of EQ, FQ and F#Q
Further, these chords occur no more frequently than other quartals; as noted above, in "Katia" all 12 quartals appear. Here is a simple table showing their frequency of occurrence:

<table>
<thead>
<tr>
<th>CQ</th>
<th>DbQ</th>
<th>DQ</th>
<th>EbQ</th>
<th>EQ</th>
<th>FQ</th>
<th>F#Q</th>
<th>GQ</th>
<th>AbQ</th>
<th>AQ</th>
<th>BbQ</th>
<th>BQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 3-22 Frequency of occurrence of quartals in "Katia"

Thus, I suggest that Irving is moving through these long quartal chains, even at the expense of using three-note chords where the lower two notes are WB or BW in order to fulfil his desire to perform the (more limited) quartal set with the high consonance/dissonance contrast. Clearly, he is listening to each chord and assessing its value by ear rather than transposing a known highly contrasted structure (this is also true of the "Eb" Pedal Chain from "Decoy") to the key of "Katia". Note that whilst "Decoy" opens with a fully formed (and generally highly contrasted) set of Pedal Chains, these most dissonant events occur relatively late in "Katia". Further, Irving does actually use the \{AQ, BbQ, BQ\} set in "Katia", as the cadential element of pattern $f$:

$f$ bars 163-165

<table>
<thead>
<tr>
<th>BQ</th>
<th>BbQ</th>
<th>AQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3-23 Pattern $f$ from "Katia"

Note that this progression has relatively high contrast against an E bass. This reliance upon these chords, and their B/W visual structures, is extended at pattern $k$ (bars 215-
223), where Irving uses the \{AQ, BbQ, BQ\} set and interpolates a (similarly BBB) AbQ chord, thus:

\[
k \text{ bars } 215-223
\]
\[
\begin{array}{cccc}
BQ & BbQ & AbQ & AQ \\
6 & 15 & 9 & 2 \\
\end{array}
\]

Table 3-24 Pattern \(k\) from "Katia"

Let us now examine the second step that Irving takes to improve the contrast of this piece. We find this at pattern \(j\) (bars 212-214). By introducing the new chords of 4(8), 4(b9) and 4(9), Irving gains access to a highly contrasting chord chain, thus:

\[
j \text{ bars } 212-214
\]
\[
\begin{array}{ccccccc}
AbQ & GAQ & Gb4(8) & F4(b9) & E4(9) \\
9 & 8 & 3 & 15 & 3 \\
\end{array}
\]

Table 3-25 Pattern \(j\) from "Katia"

Note that these last three chords are at their most highly contrasted only in this key \((15-3)+(15-3)=24\); see row 11 of Table 3-16 above. Shown as a graph, we can see the characteristic zigzag shape that these latter three chords form over an E bass:
Thus, I suggest that Irving performs these new chords as compensation for the relative low contrast of the preceding AbQ/E and GAQ/E chords. Let us note how the latter three chords in this chain are formed upon the II, bII and I of the key. Perhaps this cadential list was the catalyst for the new material, the bII and I in particular suggesting to Irving that a contrasting structure might exist there. Thus, similarly to the \{E Q, F Q, F\#Q\} set described above, here Irving forsakes his usual B/W visual structures in order to effect a high level of contrast between chords.

In pointing out the differences between the Pedal and Quartal Chains used here and those used in “Decoy” I recognise that “Decoy” and “Katia” are, of course, different pieces with different compositional requirements. However, I feel there is sufficient evidence to suggest that in “Katia” Irving is relying upon a palette of chords and a (not always compatible) consonance/dissonance strategy designed for a different key, rather than a distinct set of chords specifically composed for this piece with its E minor bass part.
"You’re Under Arrest"

This piece, like “Katia”, is in E minor. However, if we construct a table showing the frequency of quartals in “You’re Under Arrest” (similar to that constructed for “Katia”, above), we find a very different structure:

<table>
<thead>
<tr>
<th>CQ</th>
<th>DbQ</th>
<th>DQ</th>
<th>EbQ</th>
<th>EQ</th>
<th>FQ</th>
<th>F#Q</th>
<th>GQ</th>
<th>AbQ</th>
<th>AQ</th>
<th>BbQ</th>
<th>BQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>11</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3-26 Frequency of occurrence of quartals in “You’re Under Arrest”

Whereas in “Katia” all 12 quartals were present, there is a much more local, limited set in use here. Clearly based around AQ and BQ, with local chromatic “out” quartals of AbQ, BbQ and CQ, there is no use whatsoever of the most dissonant set {EQ, FQ, F#Q}. This further encourages me to believe that the appearance of this most dissonant set in “Katia” is due to quasi-random sideslipping through all 12 quartals, rather than premeditation. However, we do find several appearances of the {AQ, BbQ, BQ} set here (which was also used in “Decoy”):

a bars 49-51

<table>
<thead>
<tr>
<th>AQ</th>
<th>BbQ</th>
<th>BQ</th>
<th>BbQ</th>
<th>AQ (E note)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c bars 62-64

<table>
<thead>
<tr>
<th>Gb</th>
<th>CQ</th>
<th>BQ</th>
<th>BbQ</th>
<th>AQ (E note)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
At pattern d (bars 75-80) and g (bars 187-191) Irving replaces the BbQ with an AbQ (similar to the interpolation of the AbQ at pattern k (bars 215-223) in “Katia”):

\[
\begin{array}{cccccc}
\text{d bars 75-80} \\
\text{AQ} & \text{BbQ} & \text{BQ} & \text{AQ} \\
\text{Gb} & \text{CQ} & \text{BQ} & \text{AbQ} & \text{AQ} \\
& & 6 & 9 & 2 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\text{g bars 187-191} \\
\text{Gb} & \text{CQ} & \text{BQ} & \text{AbQ} & \text{AQ} & \text{BbQ} & \text{BQ} & \text{AbQ} & \text{AQ (E note)} \\
& & & & & 6 & 9 & 2 \\
\end{array}
\]

Thus, in these two pieces in E minor, “Katia” and “You’re Under Arrest”, we have found two forces at work which are not always mutually supportive. One is the desire to perform from the restricted palette of chords derived from the chains of “Decoy” and thus, by necessity, maintain the B/W visual characteristics of the chords employed. The second is to maintain the level of consonance/dissonance contrast heard in “Decoy”. In these two pieces, however, Irving has clearly supplanted the B/W visual strategy in order to employ highly aurally contrasted chord sets.