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# Higgs Doublets, Split Multiplets and Heterotic $SU(3)_C \times SU(2)_L \times U(1)_Y$ Spectra

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## Abstract

A methodology for computing the massless spectrum of heterotic vacua with Wilson lines is presented. This is applied to a specific class of vacua with holomorphic  $SU(5)$ -bundles over torus-fibered Calabi-Yau threefolds with fundamental group  $\mathbb{Z}_2$ . These vacua lead to low energy theories with the standard model gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and three families of quark/leptons. The massless spectrum is computed, including the multiplicity of Higgs doublets.

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An important goal of heterotic string theory is to demonstrate the existence of vacua consistent with low energy particle physics phenomenology. This has been discussed within the context of  $G(\subset E_8)$ -bundles on simply connected elliptic Calabi-Yau threefolds in [1, 2, 3, 4]. These vacua correspond to GUT theories with gauge groups such as  $SU(5)$  and  $Spin(10)$ . However, to introduce Wilson lines one must extend these results to  $G$ -bundles on torus-fibered Calabi-Yau threefolds  $X$  with non-trivial fundamental group. This was done in [5], where  $G = SU(5)$  bundles on Calabi-Yau spaces with  $\pi_1(X) = \mathbb{Z}_2$  were constructed. These vacua have low energy theories with standard model gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and three families of quarks and leptons.

However, to complete the analysis of these vacua, it is essential to compute the entire massless spectra. This was done within the GUT context in [6], where methods for determining the spectrum were introduced and used in an  $SU(5)$  example. An interesting result is that the non-chiral part of the spectrum was shown to jump on isolated subspaces of the  $G$ -bundle moduli space. In this paper, these methods are extended to vacua admitting Wilson lines. We show how one computes the massless spectrum, and give an explicit example using the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  standard model vacuum introduced in [5]. Here, we simply outline our results, presenting the specific details in [7].

A vacuum of heterotic string theory is determined by specifying a Calabi-Yau threefold  $X$  with fundamental group

$$\pi_1(X) = F \tag{1}$$

and a stable, holomorphic vector bundle  $V$  with structure group

$$G \subset E_8. \tag{2}$$

Denote by

$$H = Z_{E_8}(G) \tag{3}$$

the commutant of  $G$  in  $E_8$ .  $V$  is constrained to satisfy the anomaly cancellation condition that

$$c_2(TX) - c_2(V) \text{ is effective} \tag{4}$$

and

$$c_3(V) = 6, \tag{5}$$

leading to three families of quark/leptons. When  $F \neq \mathbb{1}$ , a Wilson line  $W$  can be introduced. A Wilson line is a flat  $H$ -bundle. The bundle

$$V' = V \oplus W \tag{6}$$

has structure group  $G \times F$ , which spontaneously breaks  $E_8$  to the gauge group

$$S = Z_H(F) = Z_{E_8}(G \times F). \quad (7)$$

$X$  can be constructed as the quotient

$$X = \tilde{X}/F, \quad (8)$$

where  $\tilde{X}$  is a simply connected Calabi-Yau threefold and  $F$  acts freely on  $\tilde{X}$ .  $V$  and  $V'$  coincide when pulled back to  $\tilde{X}$  and are denoted by  $\tilde{V}$ . The structure group of  $\tilde{V}$  is  $G$ , while (4) and (5) become

$$c_2(\tilde{X}) - c_2(\tilde{V}) \text{ effective}, \quad c_3(\tilde{V}) = 6|F| \quad (9)$$

respectively. With respect to the subgroup  $G \times H \subset E_8$ ,  $\text{ad}\tilde{V}$  decomposes as

$$\text{ad}\tilde{V} = \bigoplus_i U_i(\tilde{V}) \otimes R_i, \quad (10)$$

where  $U_i(\tilde{V})$  are the vector bundles associated with the irreducible representation  $U_i$  of  $G$  and  $R_i$  are the corresponding representations of  $H$ .

As discussed in [7], the massless spectrum is identified as

$$\ker(\mathcal{D}) = \bigoplus_{q=0,1} \bigoplus_i \left( H^q(\tilde{X}, U_i(\tilde{V})) \otimes R_i \right)^{\rho'(F)}, \quad (11)$$

where  $\mathcal{D}$  is the Dirac operator on  $X$ ,  $\rho'(F)$  specifies the  $F$  action on both  $H^q(\tilde{X}, U_i(\tilde{V}))$  and  $R_i$  and the superscript indicates the invariant part of the expression. Decomposing  $R_i$  in terms of its irreducible  $F$ -representations  $A_j$ ,

$$R_i = \bigoplus_j (A_j \otimes B_{ij}), \quad (12)$$

expression (11) becomes

$$\ker(\mathcal{D}) = \bigoplus_{q=0,1} \bigoplus_{i,j} (H^q(\tilde{X}, U_i(\tilde{V})) \otimes A_j)^{\rho'(F)} \otimes B_{ij}. \quad (13)$$

Here,  $B_{ij}$  carries a representation of the gauge group  $S$ . Therefore, to compute the massless spectrum it suffices to determine the dimension of the space of  $F$ -invariants in  $H^q(\tilde{X}, U_i(\tilde{V})) \otimes A_j$ .

In this paper, we choose

$$F = \mathbb{Z}_2, \quad G = SU(5). \quad (14)$$

Then

$$H = SU(5) \tag{15}$$

and (11) becomes

$$\begin{aligned} \ker(\mathcal{D}) = & \left( H^1(\tilde{X}, U_i(\tilde{V})) \otimes \mathbb{1} \right)^{\rho'(\mathbb{Z}_2)} \oplus \left( H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 24 \right)^{\rho'(\mathbb{Z}_2)} \oplus \left( H^1(\tilde{X}, \tilde{V}) \otimes \overline{10} \right)^{\rho'(\mathbb{Z}_2)} \\ & \oplus \left( H^1(\tilde{X}, \tilde{V}^*) \otimes 10 \right)^{\rho'(\mathbb{Z}_2)} \oplus \left( H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 5 \right)^{\rho'(\mathbb{Z}_2)} \oplus \left( H^1(\tilde{X}, \wedge^2 \tilde{V}^*) \otimes \bar{5} \right)^{\rho'(\mathbb{Z}_2)} \end{aligned} \tag{16}$$

To determine the massless spectrum, one must compute the cohomology groups in (16), the action of  $\mathbb{Z}_2$  on these groups and the action of  $\mathbb{Z}_2$  on each representation  $R_i$ . Since the last of these is straightforward, we discuss it first.

For  $F = \mathbb{Z}_2$ ,  $W$  spontaneously breaks  $H$  to the standard model gauge group

$$S = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{17}$$

The action of  $\mathbb{Z}_2$  on each representation  $R_i$  of  $H$  is easily computed. For example, for  $R_i = 5$  expression (12) is

$$5 = 1 \otimes (3, 1)_{-2} \oplus (-1) \otimes (1, 2)_3, \tag{18}$$

where  $\pm 1$  are the representations  $A_j$  of  $\mathbb{Z}_2$  while  $(a, b)_w$  are representations of  $S$ . For notational simplicity, we display  $w = 3Y$ . The action of  $\mathbb{Z}_2$  on each representation  $R_i$  in (16), as well as the corresponding representations  $B_{ij}$  of  $S$ , are listed in Table 1.

To compute the cohomology groups in (16), we must construct  $\tilde{X}$  and  $\tilde{V}$ . Choose  $\tilde{X}$  to be the fiber product

$$\tilde{X} = B \times_{\mathbb{P}^1} B' \tag{19}$$

of two  $d\mathbb{P}_9$  surfaces  $B$  and  $B'$ .  $\tilde{X}$  is elliptically fibered over both surfaces with the projections

$$\pi' : \tilde{X} \rightarrow B, \quad \pi : \tilde{X} \rightarrow B'. \tag{20}$$

$B$  and  $B'$  are themselves elliptically fibered over  $\mathbb{P}^1$  with the maps

$$\beta : B \rightarrow \mathbb{P}^1, \quad \beta' : B' \rightarrow \mathbb{P}^1. \tag{21}$$

A  $\mathbb{Z}_2$  action  $\tau$  on  $\tilde{X}$  can be obtained as the lift

$$\tau = \tau_B \times_{\mathbb{P}^1} \tau_{B'} \tag{22}$$

of two involutions  $\tau_B$  and  $\tau_{B'}$  on  $B$  and  $B'$  respectively. It is sufficient to know that  $\tau_B$  acts on  $\mathbb{P}^1$  as

$$t_0 \rightarrow t_0, \quad t_1 \rightarrow -t_1, \tag{23}$$

$U_i$	$H^q(\tilde{X}, U_i(\tilde{V}))$	$R_i$	$\chi_{A_j}$	$B_{ij}$
24	$H^1(\tilde{X}, \text{ad}\tilde{V})$	1	0	$(1, 1)_0$
1	$H^0(\tilde{X}, \mathcal{O}_{\tilde{X}})$	24	0	$(8, 1)_0 \oplus (1, 3)_0 \oplus (1, 1)_0$
			1	$(3, 2)_{-5} \oplus (\bar{3}, 2)_5$
10	$H^1(\tilde{X}, \wedge^2 \tilde{V})$	5	0	$(3, 1)_{-2}$
			1	$(1, 2)_3$
$\bar{10}$	$H^1(\tilde{X}, \wedge^2 \tilde{V}^*)$	$\bar{5}$	0	$(\bar{3}, 1)_2$
			1	$(1, 2)_{-3}$
5	$H^1(\tilde{X}, \tilde{V})$	$\bar{10}$	0	$(3, 1)_4 \oplus (1, 1)_{-6}$
			1	$(\bar{3}, 2)_{-1}$
$\bar{5}$	$H^1(\tilde{X}, \tilde{V}^*)$	10	0	$(\bar{3}, 1)_{-4} \oplus (1, 1)_6$
			1	$(3, 2)_1$

Table 1: The decomposition of  $H^q(X, \text{ad}V')$  where  $G = SU(5)$  and  $F = \mathbb{Z}_2$ . The  $\chi_{A_j}$  are the characters of the  $\mathbb{Z}_2$  action on  $R_i$ . The  $a, b$  in  $(a, b)_w$  are the representations of  $SU(3)_C$  and  $SU(2)_L$  respectively, whereas  $w = 3Y$ .

where  $t_0, t_1$  are projective coordinates. This action has two fixed points,  $p_0$  and  $p_\infty$ . The fiber  $f_0 = \beta^{-1}(p_0)$  is acted on freely by  $\tau_B$ , whereas  $f_\infty = \beta^{-1}(p_\infty)$  has four fixed points. The  $\tau_{B'}$  action on  $B'$  has similar properties. In order for  $\tau$  in (22) to act freely on  $\tilde{X}$ , one must “twist” the two  $\mathbb{P}^1$  lines in (21) when identifying them in (19). This twist sets  $p'_0 = p_\infty$  and  $p'_\infty = p_0$ .

Stable, holomorphic vector bundles  $\tilde{V}$  on  $\tilde{X}$  with structure group  $G = SU(5)$  can be constructed as the extension

$$0 \rightarrow V_2 \rightarrow \tilde{V} \rightarrow V_3 \rightarrow 0 \quad (24)$$

of two vector bundles

$$V_i = \pi'^* W_i \otimes \pi^* L_i \quad (25)$$

with  $\text{rk}V_i = i$  for  $i = 2, 3$ .  $W_2$  and  $W_3$  are vector bundles on  $B$  with rank 2 and 3 respectively, while  $L_2$  and  $L_3$  are line bundles on  $B'$ . We need to lift the  $\mathbb{Z}_2$  action on  $\tilde{X}$  to an action on  $V_2, V_3$  and  $\tilde{V}$ . For  $\tilde{V}$  to be  $\mathbb{Z}_2$  invariant, it is necessary to restrict both  $V_2$  and  $V_3$  to be invariant. This is done by choosing  $W_i$  and  $L_i$  to be  $\tau_B$  and  $\tau_{B'}$  invariant respectively. There are many line bundles  $L_i$  that are invariant under  $\tau_{B'}$ . However, we now impose the

remaining constraint that  $\tilde{V}$  satisfy (9) with  $|F| = 2$ . This restricts the allowed line bundles to be

$$L_2 = \mathcal{O}_{B'}(3r'), \quad L_3 = \mathcal{O}_{B'}(-2r'), \quad (26)$$

where  $r'$  is a specific divisor of  $B'$  with  $\deg(r') = 2$  when restricted to a fiber. By construction,  $\tilde{V}$  corresponds to an extension class

$$[\tilde{V}] \in \text{Ext}_{\tilde{X}}^1(V_3, V_2). \quad (27)$$

$\text{Ext}_{\tilde{X}}^1(V_3, V_2)$  is the direct sum of two subspaces which are invariant and anti-invariant under the action of  $\tau$ .  $\tilde{V}$  will be invariant if  $[\tilde{V}]$  lies in the invariant subspace.

We can now construct the cohomology groups in (16). However, one group,  $H^1(\tilde{X}, \text{ad}\tilde{V})$ , corresponding to vector bundle moduli, requires techniques beyond those developed in this paper and will not be discussed. Let us consider  $H^0(\tilde{X}, \mathcal{O}_{\tilde{X}})$ . Since  $\mathcal{O}_{\tilde{X}}$  is the trivial bundle, it follows that

$$H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \simeq \mathbb{C}. \quad (28)$$

Note that since  $\mathcal{O}_{\tilde{X}}$  is independent of  $\tilde{V}$ ,  $\mathbb{Z}_2$  acts trivially on  $H^0(\tilde{X}, \mathcal{O}_{\tilde{X}})$ .

Next, we determine  $H^1(\tilde{X}, \tilde{V})$ . From the long exact sequence associated with (24), we find that

$$H^1(\tilde{X}, \tilde{V}) \simeq H^1(\tilde{X}, V_2). \quad (29)$$

Using (25) and pushing this down from  $\tilde{X}$  to  $\mathbb{P}^1$  gives

$$H^1(\tilde{X}, V_2) \simeq H^0(\mathbb{P}^1, R^1\beta_*W_2 \otimes \beta'_*L_2). \quad (30)$$

We find that  $R^1\beta_*W_2 \simeq \mathcal{O}_{p_\infty}$ . From (26) it follows that  $\beta'_*L_2$  has degree 6 along  $f'_0$ . We conclude that

$$H^1(\tilde{X}, \tilde{V}) \simeq \mathbb{C} \otimes \mathbb{C}^6 = \mathbb{C}^6. \quad (31)$$

Now consider  $H^1(\tilde{X}, \tilde{V}^*)$ . This can be determined from (31) using the Atiyah-Singer index theorem which, together with Serre duality, gives

$$h^1(\tilde{X}, \tilde{V}^*) = 6 + h^1(\tilde{X}, \tilde{V}), \quad (32)$$

where we have used (9) with  $|F| = 2$ . This and (31) then imply

$$H^1(\tilde{X}, \tilde{V}^*) \simeq \mathbb{C}^{12}. \quad (33)$$

We now turn to the computation of  $H^1(\tilde{X}, \wedge^2 \tilde{V})$ . One can show that  $H^1(\tilde{X}, \wedge^2 \tilde{V})$  lies in the exact sequence

$$0 \rightarrow H^1(\tilde{X}, \wedge^2 V_2) \rightarrow \boxed{H^1(\tilde{X}, \wedge^2 \tilde{V})} \rightarrow H^1(\tilde{X}, V_2 \otimes V_3) \xrightarrow{M^T} H^2(\tilde{X}, \wedge^2 V_2) \rightarrow \dots \quad (34)$$

To continue, we must compute the terms  $H^i(\tilde{X}, \wedge^2 V_2)$ ,  $i = 2, 3$  and  $H^1(\tilde{X}, V_2 \otimes V_3)$ , as well as the linear map  $M^T$ . Pushing  $H^i(\tilde{X}, \wedge^2 V_2)$ ,  $i = 2, 3$  down to  $\mathbb{P}^1$ , we find

$$H^1(\tilde{X}, \wedge^2 V_2) \simeq \bigoplus^5 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1})^* \quad (35)$$

and

$$H^2(\tilde{X}, \wedge^2 V_2) \simeq \bigoplus^7 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1})^* \oplus \bigoplus^5 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))^*. \quad (36)$$

It follows that

$$H^1(\tilde{X}, \wedge^2 V_2) \simeq \mathbb{C}^5, \quad H^2(\tilde{X}, \wedge^2 V_2) \simeq \mathbb{C}^{17}. \quad (37)$$

Calculating  $H^1(\tilde{X}, V_2 \otimes V_3)$  is more difficult. Using (25) and pushing down to  $\mathbb{P}^1$ , we find

$$H^1(\tilde{X}, V_2 \otimes V_3) \simeq H^0(\mathbb{P}^1, R^1 \beta_*(W_2 \otimes W_3) \otimes \beta'_*(L_2 \otimes L_3)). \quad (38)$$

Here, we simply state that  $R^1 \beta_*(W_2 \otimes W_3)$  is a sheaf supported at each of 12 points  $p_r \in \mathbb{P}^1$ ,  $r = 1, \dots, 12$  and at  $p_\infty$ . Specifically,

$$R^1 \beta_*(W_2 \otimes W_3) \simeq \bigoplus_{r=1}^{12} \mathcal{O}_{p_r} \oplus \bigoplus^3 \mathcal{O}_{p_\infty}. \quad (39)$$

Furthermore, it follows from (26) that  $\beta'_*(L_2 \otimes L_3)$  is a rank two vector bundle on  $\mathbb{P}^1$ . Combining these results, (38) becomes

$$H^1(\tilde{X}, V_2 \otimes V_3) \simeq \mathbb{C}^{15} \otimes \mathbb{C}^2 = \mathbb{C}^{30}. \quad (40)$$

Finally, we must know the rank of  $M^T$ . It follows from (37) and (40) that  $M^T$  is a  $30 \times 17$  matrix. In addition, one can show it depends on 150 vector bundle moduli. At a generic point in moduli space, we find that

$$\text{rk}(M^T) = 17. \quad (41)$$

Putting (37), (40) and (41) into (34), we conclude that

$$H^1(\tilde{X}, \wedge^2 \tilde{V}) \simeq \mathbb{C}^{18}. \quad (42)$$



Finally, we need to compute  $H^1(\tilde{X}, \wedge^2 \tilde{V}^*)$ . Again, this is easily determined using the Atiyah-Singer index theorem. In this context, we find

$$h^1(\tilde{X}, \wedge^2 \tilde{V}^*) = 6 + h^1(\tilde{X}, \wedge^2 \tilde{V}). \quad (43)$$

Combining this with (42) yields

$$H^1(\tilde{X}, \wedge^2 \tilde{V}^*) \simeq \mathbb{C}^{24}. \quad (44)$$

Having computed all the cohomology groups in (16), we now determine the explicit action of  $\mathbb{Z}_2$  on each of them. Let us begin with  $H^1(\tilde{X}, \tilde{V})$ , which was given in (31). To begin with, consider the second factor,  $\mathbb{C}^6$ . This can be shown to be parametrized by the polynomials

$$\{x_0^{3-i}x_1^i, \quad yx_0^{1-j}x_1^j\}, \quad (45)$$

where  $i = 0, \dots, 3$  and  $j = 0, 1$ . Here,  $x_0, x_1$  and  $y$  are sections of specific bundles on the base  $\mathbb{P}^1$ , which transform as

$$x_0 \rightarrow x_0, \quad x_1 \rightarrow -x_1, \quad y \rightarrow y \quad (46)$$

under  $\tau_{B'}$ . Applying these transformations to (45), we see that  $\mathbb{C}^6$  decomposes as  $\mathbb{C}_{(+)}^3 \oplus \mathbb{C}_{(-)}^3$  under the action of  $\mathbb{Z}_2$ . Since this is evenly split between  $+$  and  $-$ , the  $\mathbb{Z}_2$  action on the first factor  $\mathbb{C}$  in (31) is irrelevant. We conclude that

$$H^1(\tilde{X}, \tilde{V}) \simeq \mathbb{C}_{(+)}^3 \oplus \mathbb{C}_{(-)}^3. \quad (47)$$

We now compute the  $\mathbb{Z}_2$  action on  $H^1(\tilde{X}, \tilde{V}^*)$  in (33) using the Atiyah-Singer index theorem. First, consider the index theorem for  $V$  on  $X = \tilde{X}/\mathbb{Z}_2$ . Using (9) with  $|F| = 2$ , the fact that  $H^q(\tilde{X}, \tilde{V})_{(+)} = H^q(X, V)$  for any  $q$  and Serre duality, we find

$$h^1(\tilde{X}, \tilde{V}^*)_{(+)} = 3 + h^1(\tilde{X}, \tilde{V})_{(+)}. \quad (48)$$

Using (9), Serre duality and (48), the index theorem for  $\tilde{V}$  on  $\tilde{X}$  becomes

$$h^1(\tilde{X}, \tilde{V}^*)_{(-)} = 3 + h^1(\tilde{X}, \tilde{V})_{(-)}. \quad (49)$$

It then follows from (47), (48) and (49) that

$$H^1(\tilde{X}, \tilde{V}^*) \simeq \mathbb{C}_{(+)}^6 \oplus \mathbb{C}_{(-)}^6 \quad (50)$$

under the action of  $\mathbb{Z}_2$ .

Now consider  $H^1(\tilde{X}, \wedge^2 \tilde{V})$  in (42). It follows from (34) that to find the  $\mathbb{Z}_2$  action on  $H^1(\tilde{X}, \wedge^2 \tilde{V})$ , one must determine its action on  $H^i(\tilde{X}, \wedge^2 V_2)$ ,  $i = 1, 2$  in (37),  $H^1(\tilde{X}, V_2 \otimes V_3)$  in (40) and on the map  $M^T$  satisfying (41). Since the decomposition of each of these cohomology groups under  $\mathbb{Z}_2$  is computed using methods similar to those leading to (47), we simply state the results. We find

$$H^1(\tilde{X}, \wedge^2 V_2) \simeq \mathbb{C}_{(+)}^3 \oplus \mathbb{C}_{(-)}^2, \quad H^2(\tilde{X}, \wedge^2 V_2) \simeq \mathbb{C}_{(+)}^9 \oplus \mathbb{C}_{(-)}^8 \quad (51)$$

and

$$H^1(\tilde{X}, V_2 \otimes V_3) \simeq \mathbb{C}_{(+)}^{15} \oplus \mathbb{C}_{(-)}^{15}. \quad (52)$$

Furthermore, one can show that  $M^T$  can be taken to be invariant under  $\mathbb{Z}_2$ , corresponding to choosing  $[\tilde{V}]$  to be in  $\text{Ext}_{\tilde{X}}^1(V_3, V_2)_{(+)}$ . Then, it follows from (51) and (52) that

$$(\ker M^T)_{(+)} = \mathbb{C}_{(+)}^6, \quad (\ker M^T)_{(-)} = \mathbb{C}_{(-)}^7. \quad (53)$$

Putting (51) and (53) into the exact sequence (34), we conclude that

$$H^1(\tilde{X}, \wedge^2 \tilde{V}) \simeq \mathbb{C}_{(+)}^9 \oplus \mathbb{C}_{(-)}^9. \quad (54)$$

Finally, we can compute the  $\mathbb{Z}_2$  action on  $H^1(\tilde{X}, \wedge^2 \tilde{V}^*)$  using the Atiyah-Singer index theorem. This computation is very similar to that leading to (50), so we will simply state the result. We find that

$$H^1(\tilde{X}, \wedge^2 \tilde{V}^*) \simeq \mathbb{C}_{(+)}^{12} \oplus \mathbb{C}_{(-)}^{12}. \quad (55)$$

We now possess all of the ingredients necessary to compute the massless spectrum. Combining (47), (50) and (54)-(55) with the results in Table 1, one can determine the  $\rho'(\mathbb{Z}_2)$  invariant subspace for each cohomology group in (16). The associated multiplets descend to  $X = \tilde{X}/\mathbb{Z}_2$  to form the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  particle physics spectrum. The results are tabulated in Table 2.

To begin with, the spectrum contains one copy of vector supermultiplets transforming under  $SU(3)_C \times SU(2)_L \times U(1)_Y$  as

$$(8, 1)_0 \oplus (1, 3)_0 \oplus (1, 1)_0. \quad (56)$$

Furthermore, it contains three families of quarks and lepton superfields, each family transforming as

$$(3, 2)_1, \quad (\bar{3}, 1)_{-4}, \quad (\bar{3}, 1)_2 \quad (57)$$

$R_i$	$(\chi_{H^q}, A_j)$	$(H^q(\tilde{X}, U_i(\tilde{V})) \otimes A_j)^{\rho'(F)}$	$B_{ij}$
1			
24	(0, 0)	$\mathbb{C}_{(+)}^1$	$(8, 1)_0 \oplus (1, 3)_0 \oplus (1, 1)_0$
5	(0, 0)	$\mathbb{C}_{(+)}^9$	$(\mathbf{3}, 1)_{-2}$
	(1, 1)	$\mathbb{C}_{(-)}^9$	$(1, 2)_3$
$\bar{5}$	(0, 0)	$\mathbb{C}_{(+)}^{12}$	$(\bar{\mathbf{3}}, 1)_2$
	(1, 1)	$\mathbb{C}_{(-)}^{12}$	$(1, 2)_{-3}$
$\bar{10}$	(0, 0)	$\mathbb{C}_{(+)}^3$	$(3, 1)_4 \oplus (1, 1)_{-6}$
	(1, 1)	$\mathbb{C}_{(-)}^3$	$(\bar{\mathbf{3}}, 2)_{-1}$
10	(0, 0)	$\mathbb{C}_{(+)}^6$	$(\bar{\mathbf{3}}, 1)_{-4} \oplus (1, 1)_6$
	(1, 1)	$\mathbb{C}_{(-)}^6$	$(\mathbf{3}, 2)_1$

Table 2: The particle spectrum of the low-energy  $SU(3)_C \times SU(2)_L \times U(1)_Y$  theory. The  $\chi_{H^q}$  are the characters of the  $\mathbb{Z}_2$  representations on  $H^q(\tilde{X}, U_i(\tilde{V}))$ . The  $U(1)$  charges listed are  $w = 3Y$ .

and

$$(1, 2)_{-3}, \quad (1, 1)_6 \quad (58)$$

respectively. However, there are additional chiral superfields in the spectrum. It follows from Table 2 that these occur as conjugate pairs of the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  representations

$$(3, 1)_{-2}, \quad (1, 2)_3 \quad (59)$$

and

$$(3, 1)_4 \oplus (1, 1)_{-6}, \quad (\bar{\mathbf{3}}, 2)_{-1}. \quad (60)$$

These multiplets arise as  $\mathbb{Z}_2$  invariants in the 5 and  $\bar{10}$  representations of  $H = SU(5)$ . The spectrum has

$$n_{(3,1)_{-2}} = 9, \quad n_{(1,2)_3} = 9 \quad (61)$$

and

$$n_{(3,1)_4 \oplus (1,1)_{-6}} = 3, \quad n_{(\bar{\mathbf{3}}, 2)_{-1}} = 3 \quad (62)$$

copies of (59) and (60) respectively. The multiplicity  $n_{(1,2)_3}$  corresponds to the number of Higgs doublet conjugate pairs in the low energy spectrum. The remaining multiplets in (59) and (60) are exotic. Clearly, the number of Higgs doublets and the exotic multiplets is not

consistent with phenomenology. However, we emphasize that these results were computed within a specific context, which is but a small subset of the possible standard model heterotic vacua. These generalized vacua and their spectra will be presented in future publications.

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