Modelling the Dynamics of Credit Spreads of European Corporate Bond Indices

by

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A Thesis Submitted for the Degree of PhD in Finance

City University Cass Business School
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July 2010

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ACKNOWLEDGEMENTS

It is not possible to name all the people who have helped me at the various stages of this thesis.

I would like to express my greatest gratitude to my supervisor, Dr. Amir Alizadeh, for his patience and guidance throughout these years. My special thanks also go to Professor Costas Grammenos, Head of Department of Shipping, Trade and Finance for his support and encouragement. I am also grateful to Gladys Parish and John M. Hadjipateras for their valuable comments on different parts of the thesis.

I am in great debt to my parents, Maria and Nikos, for their compassion and support not only through the course of my studies but also during my life and to Christina Christopoulou for her support and limitless understanding during my work.

DECLARATION

ABSTRACT

Credit spreads are important financial tools, since they are used as indicators of economic progression, investment decisions, trading and hedging, as well as pricing credit derivatives. Their role has become more significant for the European fixed income markets since the introduction of the Euro, which reshaped the mechanics of the financial environment. The introduction of single currency provided the means for a pan-European economic growth and cross-border development, liberalized a vast inflow of capital which was once fragmented into different currencies, and provided the dynamics of cross-border investments around a unified legislative framework. Thus, the main subject of the thesis is to provide further insight into and investigate the nature and the dynamics of credit spreads of European corporate bond indices during the credit crisis period.

Traditional quantitative credit risk models assume that changes in spreads are normally distributed but empirical evidence shows that they are likely to be skewed and fat-tailed, and if they are ignored then the calculation of loss probabilities will be seriously compromised. Therefore, the first area of investigation aims to provide further insight into the dynamics of higher moments and regime shifts in credit spread changes by applying a GARCH-type model that allows for time-varying volatility, skewness and kurtosis, as well as a Markov regime-switching GARCH specification to capture the structural changes in the volatility of credit spreads. Furthermore, a comparison of the different specifications is undertaken in order to assess which model better fits the empirical distribution of the data and produces best Value-at-Risk estimates. The results presented have significant implications for risk management, as well as in the pricing of credit derivatives.

The second area of investigation is to assess and evaluate time-varying correlation of credit spreads. Different multivariate GARCH models, such as Orthogonal-GARCH, the Constant and Dynamic Correlation GARCH models, Risk Metrics and Diagonal-BEKK, are applied to examine the behaviour and dynamics of time-varying correlation. Additionally, the performance of the proposed models is examined by determining whether they produce accurate VaR estimates. The study finds evidence in support of time-varying correlation coefficients between credit spreads which appears to be market dependent and has implications for pricing of derivatives, portfolio selection, trading and hedging activities, as well as risk management.

Finally, the impact of economic determinants of credit spreads such as the risk-free rate, inflation, as well as equity and commodity indices and volatilities, are investigated over different market conditions using regime switching models. The results highlight how the effect of the determinants on credit spreads varies across different market conditions and point to the existence of non-linear relationship between the determinants and credit spread changes. The study reveals that the regime dependent determinants have significant explanatory power only in the high volatility regime. Finally, it is shown that the feed-forward neural network model out-performs the other specifications applied in this study in terms of estimating out-of-sample mean forecasts.

LIST OF ABBREVIATIONS

ADF	Augmented Dickey-Fuller unit root test
AIC	Akaike Information Criterion
ARCH	Autoregressive Condtional Heteroscedasticity
ARCH Test	Engle's Test for Heteroscedasticity
ARMA	Autoregressive Moving Average
BEKK	Baba, Engle, Kraft and Kroner
BIC	Bayesian information criterion
BSM	Black-Scholes and Merton specifications
CC-GARCH	Conditional Correlation GARCH
D-BEKK	Diagonal-BEKK
DCC-GARCH	Dynamic Conditional Correlation GARCH
ΔL	Change in the level of the risk-free rate
ΔS	Change in the slope of yield curve
ΔV	Change in the EURO STOXX 50 Volatility Index
ΔI	Change in the EuroMTS Inflation Index
ΔC	Change in the Goldman Sachs S&P Commodity Index
EVT	Extreme Value Theory
FHS	Filtered Historical Simulation
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GJR-GARCH	Glosten-Jagannathan-Runkle GARCH
GARCH-SK	Generalised Autoregressive Conditional Heteroscedastocoty, Skewness and Kurtosis
HS	Historical Simulation
LB-Q-Test	Ljung-Box Autocorrelation *Q*-test
LR	Likelihood Ratio Test
LRUC	Likelihood Ratio Test of the Unconditional Coverage
LRIND	Likelihood Ratio Test of the Independence
LRCC	Likelihood Ratio Test of the Conditional Coverage
MAE	Mean Absolute Error
MGARCH	Multivariate GARCH
MRS	Markov Regime Switching Regression
MRS-GARCH	Markov Regime Switching GARCH
MSE	Mean Squared Error
NN	Neural Networks
O-GARCH	Orthogonal GARCH
OLS	Ordinary Least Squares
PCA	Principal Component Analysis
PCSP	Percentage Correct Sign Predictions
*R*E	Return on the MSCI Berra Pan-Euro Index
RMSE	Root Mean Square Error
TUFF	Time Until First Failure
VaR	Value-at-Risk
VAR	Vector Autoregressive Model

Chapter 1

Introduction

1.1 Introduction

The aim of this chapter is to introduce the European bond markets and to establish the significance of the research in this thesis. It starts by briefly discussing the history of the European Union that ultimately led to the creation of the single market and the single European currency - the Euro. The chapter then describes how the single currency acts as a catalyst in reshaping the mechanics of the European financial markets, the means of promoting economic integration between the member states and encouraging the development of a deep and liquid European bond market, which is significantly larger than the equivalent U.S. and Japanese bond markets. In addition, the chapter introduces the notation of the debt instrument and the different types of risk which affect this instrument. However, it argues that the most important type of risk to which market participants are exposed is credit risk and, also discusses the importance of modelling this type of risk. Finally, the objectives, significance and contribution of the thesis are presented.

1.2 Introduction to the European Markets

The European Union, as we know it today, is a unique economic and political partnership between twenty-seven European countries, working towards the achievement of common goals, including peace and political stability, economic prosperity, financial stability and to overcome the economic and political divisions between member-states by promoting the development of a single market. The restrictions on trade, competition, and taxation between the member states have been gradually eliminated through a number of important treaties. The Treaty of Rome, in 1957, established the European Economic Community (EEC) with the aim of removing customs barriers within the Community and establishing a common customs tariff on goods from non-EEC countries. The key instrument in establishing the single market was the Single European Act, signed in Luxembourg and Hague and came into force in July 1987; its purpose being to stimulate the industrial and commercial expansion under a unified taxation and economic legislation framework. The Treaty on European Union, signed in Maastricht on 7 February 1992, introduced new forms of co-operation between the member states and led to the creation of the single European currency - the Euro.

The objective of the Euro is to promote the economic activity of the single market by eliminating exchange rates, improving transparency of the price mechanism, promoting price stability and finally protecting economies from large negative shocks which would otherwise affect the individual currencies negatively. Improving transparency of the price mechanism enables consumers and investors to recognise changes in relative prices and to make informed consumption and investment decisions in order to allocate resources more efficiently. Promoting price stability has a threefold effect: first, it reduces the inflation risk premia requested by investors in interest rates, and therefore, reduces the real interest rates and offers incentives for further investments; second, it reduces prolonged inflation and deflation periods which can distort the economic activity and the behaviour of the single market; and third it prevents the arbitrary redistribution of wealth and income as a result of unexpected inflation or deflation.

Prior to the introduction of the European Monetary Union (EMU), the European fixed income markets were a fragmented network due to the non-uniform taxation regime, the different transaction costs and high level of complexity in creating different kinds of positions, such as cross-market hedging. The studies of Ozcan *et al.* (2009), Lane, (2008), Fabozzi and Choudhry, (2004), among others, have shown that the integration of the European markets has been achieved at a substantial level. Although it varies across different product segments, integration is complete for unsecured interbank deposits such as EONIA (Euro Overnight Index Average) and EURIBOR (Euro Inter-Bank Offer Rate), as well as interest rate swaps whose bid/ask spreads have narrowed. Furthermore, functional money markets are the first step to the development of bond markets, as they not only price liquidity, which is used as a benchmark for pricing fixed income instruments, but also assist the development of forward and spot rates. Forward rates, for example, are essential ingredients for pricing a variety of instruments such as OTC (Over-The-Counter) instruments, interest rate swaps, forward rate agreements (FRA) and option contracts.

1.3 Introduction to the European Bond Markets

The introduction of the single European currency in January 1999 was the catalyst for reshaping the mechanics of the European financial markets. According to Holder *et al.* (2001), the launch of the Euro as a single currency was heralded as a major opportunity for the development of European bond markets. This introduction has resulted in the development of a deep and liquid bond market, which is promoted by: the once fragmented capital under the different currencies, the economic growth and development on a pan-European level, and the issuance of cross-border securities and instruments under a unified legislative framework. Specifically, it has allowed financial institutions to access a larger pool of investors which enabled them to diversify their liabilities away from the traditional country-specific loan structures and to distribute credit risk on a wider base and in a more efficient manner.

The European government bond market has become the largest government bond market both in size and issues. In 2003, the European government bond market had € 2.5 trillion outstanding bonds which was 40% and 30% larger than the respective US Treasury and Japanese markets (see Fabozzi and Choundhry, 2004). In addition, in

February 2003, there were 250 European government issues of more than € 1 billion outstanding compared to 108 issues in the US treasury market. The European corporate bond market has significantly increased and in 2004 amounted to 45% of the international bond markets (Bank of International Settlements, Quarterly Review, 2004). Finally, the introduction of the Euro as a single currency has enabled the development of a deep and liquid European government and corporate bond markets, and has greatly benefited other European Fixed Income markets such as the quasi-sovereigns, high grade and high yield bonds, asset-backed securities (ABS), mortgage-backed securities (MBS), collateralised debt obligations (CDO), and repackaged securities.

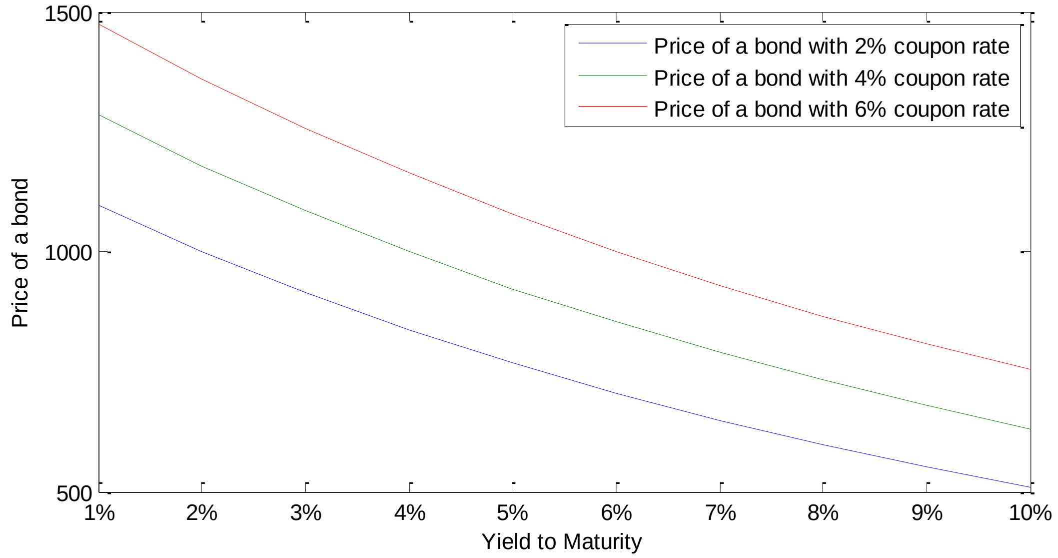
1.4 Introduction to Debt Securities

A bond, also known as a fixed income instrument, is a debt capital market instrument by which the issuer or borrower is committed to repaying the lender or bondholder the amount borrowed, called principal, plus interest over a specified period of time. The most common type of bond is a bond without any embedded option, which makes periodic payments at a fixed rate, known as coupon, and the principal on the maturity date. The price of such a bond is the present value of the cash flows (coupons and principal) and can be calculated in three steps. First, the cash flows to which the bond holder is entitled are estimated; second, the discount rates for the maturities corresponding to the cash flow payment dates are calculated; and third, the bond is obtained as the discounted value of these cash flows. The pricing formula is given as:

$$P=\sum_{t=1}^{T}\frac{CF}{\left(1+y_t\right)^t}=\frac{C}{\left(1+y_1\right)}+\frac{C}{\left(1+y_2\right)^2}+\ldots+\frac{C+FV}{\left(1+y_T\right)^T} \qquad (1.1)$$

where CF denotes the cash flows of the bond, C denotes the coupon, FV the face value or principal and $1/\left(1+y_t\right)^t$ the discount rate corresponding to the appropriate maturity. Figure 1.1 presents the relationship between the bond prices of a five-year maturity coupon bearing bond with a face value of £1000 for different coupon rates and yield-to-maturities. It shows that the price of the bond increases as the coupon rate increases, assuming that interest rates are kept constant; and that the price of the bond decreases as interest rates increase, assuming constant coupon rates.

Figure 1.1 Relationship between bond prices, coupon rates and interest rates



The risk of investing in fixed income instruments is characterised by the impact of different market risks on the return of the security. A typical bond has two sources of return: first, through changes in the market value of the bond; and second, through the cash flows and their reinvestments over the holding period. Both of these sources are affected by a number of risks[1], such as: interest-rate risk; reinvestment risk; call risk; yield curve risk; credit risk; liquidity risk; exchange-rate risk; inflation or purchasing power risk; event risk and sovereign risk.

Interest-rate risk is the risk associated with the movement of interest rates that results in capital loss when buying or selling a fixed income security (for example, an investor who may wish to sell a bond prior to the maturity date, may experience a capital loss when interest rates increase). *Reinvestment risk* is the risk that the proceeds received from interest and principal are reinvested at a lower interest rate than the fixed income instrument that generated these proceeds. *Call risk* is the risk associated with bonds that have a provision that allows the issuer to retire (call) in full or in part the issue before the maturity date if interest rates decline below the coupon rate (which means that in case the issuer calls the bond the investor will have to reinvest the proceeds at lower interest rates and the price of a callable will be reduced relative to the price of a comparable option-free bond). *Yield curve risk* is the risk that affects the price of the

[1] For additional information on the individual risks refer to Fabozzi, F, J., The Handbook of Fixed Income Securities, 2005, McGraw-Hill Professional, 7-th Edition.

fixed income instrument when the yield curve shifts (for instance if an upward parallel-shift in the yield curve is observed then the bond price according to equation (1.1) will decrease and will result in a capital loss). *Liquidity risk* is the risk that an investor wishing to sell a bond prior to the maturity date is concerned with whether the bid price from a broker will be close to the indicated value of the issue. Liquidity is measured as the size between the bid and ask prices quoted by a dealer, and the wider the bid-ask spread, the greater the risk. *Exchange-rate risk* is the risk an investor will receive less of the domestic currency, when investing in a fixed income instrument that makes payments in a currency other than the investor's domestic currency. *Inflation or purchasing power risk* arises from the decline of a bond's cash flows due to inflation as the interest rate the issuer promises to make is fixed for the life of the issue. *Event risk* is the risk associated with unforeseen events that impair the ability of an issuer to meet his obligations. Such events may be: natural disasters or industrial accidents; takeover or corporate restructuring; and regulatory changes. *Sovereign risk* is the risk an investor faces when purchasing a fixed income security from a foreign entity (for example a UK investor purchasing a U.S. Treasury bill). *Credit risk* is one of the most important types of risk to which the investors or traders in the bond markets are exposed. It is the risk that a borrower may fail to satisfy the terms of the contractual obligations with respect to the timely payment of the interest (coupon) and the amount borrowed (principal). Therefore, before entering a contract, the counter-party is evaluated in terms of its capacity to fulfil its contractual obligation based on its credit risk and creditworthiness.

1.5 Introduction to Credit Risk

Credit risk is characterized by three types of risk, namely, *default risk; downgrade risk;* and *credit spread risk*. *Default risk* is the risk that the issuer will be unable to honour his contractual obligations in full and on time. *Downgrade risk* is the risk that a recognized rating agency will reduce the credit rating of an issuer. This deterioration of credit-worthiness reflects the issuer's capacity to honour his debt obligation and affects the price of the security issued by the issuer. *Credit spread risk* is the risk that the yield premium or the spread over a reference rate will increase for a debt obligation due to adverse changes in market conditions. These risks do not appear, at first, to be interrelated; however, default is the product of a series of downgrades and credit spread widening which reflects the gradual inability of the issuer to honour his debt obligations (see Anson *et al*. 2004).

Credit spreads are defined as the difference between the yield to maturities of a corporate and a comparable government bond. They are important variables in the financial markets as they reflect the likelihood of failure of an entity to honour its obligation. The behaviour of credit spreads is counter-cyclical to that of the economy, which means that they narrow during business cycle expansions and widen during contractions (see Guha and Hiris, 2002, Altman, and Bana, 2004, among others). The economic interpretation for this kind of behaviour is that during a contracting economy, corporations experience a decline in cash flows, increasing the likelihood that the bond or debt issuers might be unable to service their debt obligations. At the same time, governments adopt monetary policies, such as reducing interest rates in order to boost the economy. Therefore, the combination of these interest rate cuts and the higher yield demanded by investors, in order to hold fixed income instruments as credit quality deteriorates, widens credit spreads. On the other hand, on the beginning of an expanding economy, governments increase interest rates, corporations expand and their cash flows increase which in turn decreases the likelihood that the bond issuers might be unable to honour their contractual obligations which results in reducing credit spreads.

Even before the appearance of credit derivatives there were mechanisms for protecting against credit risk, such as collateral guarantees, private mortgage insurance, insurance wraps and letters of credit. These mechanisms are embedded within bond structures and other loan agreements by design and, therefore, are not tradable in the secondary market. Thus, the development and rapid expansion of the credit derivatives markets can be attributed, first, to their effectiveness and efficiency to disperse credit risk to other investors, who are willing to accept this type of risk for the potential of an enhanced yield; and, second, to the information they have contributed for the enhancement of risk management.

1.6 Introduction to the European Benchmark Yield Curves

Government bonds have long been used as benchmark instruments in the global fixed income markets. This is because they present unique features not monitored in any other security in the market. The most important of these are that they are assumed to be default-risk free; they span a wider range of maturities thus making it easy to compute long-term instruments; and they present higher liquidity compared to non-government papers. What is more, with the development of the repo and derivatives markets for government securities, participants are allowed to undertake a variety of positions that reflect future expectations and device trading and hedging strategies to optimize their risk-return portfolios.

Before the introduction of the common monetary policy, the Euro market was without a uniform benchmark yield curve. The study by the International Monetary Fund (IMF) (2001) argues that none of the German, French, or to a lesser extent, Italian securities that have emerged can fulfil all purposes of pricing, hedging and investment as they are too modest in size, compared to the overall pan-European market. Dunne *et al.* (2002) argue that the benchmark yield curve should consist of a basket of bonds rather than a single instrument. In addition, the European fixed income markets considered interest rate swaps as benchmark rates, because they offered a simpler way to compare returns or borrowing costs in different markets and could provide comparability across the fragmented European fixed income markets. Blanco (2001) argues that interest rate swaps were the preferred means of hedging cash positions in non-government bonds.

After the introduction of the Euro, the German government securities market has benefited the most from improvements in liquidity making it an attractive benchmark yield curve. In addition, the launch of the Eurex Bond and Eurex Repo[2] trading platforms has greatly assisted in the expansion of the German bond market by lowering the transaction costs and increasing significantly its liquidity compared to the other reference securities in the Euro market. The studies of Ejsing and Sihvonen (2009) and Schknecht, von Hagen, and Wolswijk (2010) find that German benchmark

[2] More information on Eurex Bonds and Eurex Repos can be found in:
http://www.eurex-bonds.com/
http://www.eurexrepo.com/

status securities have superior liquidity compared with those in the U.S. treasury market, and that during the credit crisis period the German Government market gained a safe-haven status in the international financial markets similar to the U.S. Treasury market.

There are also a few other non-government securities, which may have the potential to be used as benchmarks yields, although none of them has been widely applied in the Euro market. Possible candidates could be the yield index of similarly rated corporate bonds, average yield on collateralized obligations, Pfandbriefe or covered bonds and, finally, debt instruments issued by government-sponsored enterprises (GSEs).

1.7 Aims, Objectives and Contribution of the Thesis

Accurate assessment of credit risk depends on methods to accurately measure and control potential or expected losses resulting from default. This includes the estimation of credit exposure, the probability of default, and the fraction of market value recoverable at default. Credit spreads, which are defined as the difference between the yield to maturities of a corporate and a comparable government bond, is believed to reflect the credit risk of the issuer. Credit spreads change over time for reasons such as varying market conditions, changes in the credit ratings of issuers, or changes in expectations regarding the recovery rate (see Campbell and Huisman, 2003, Longstaff and Schwartz, 1995, among others). The role of credit spreads has become even more important in the European fixed income markets since the launch of the single currency, because credit spreads are used as indicators of economic progression, investment decisions, trading and hedging activities, as well as pricing credit derivatives.

Traditional quantitative credit risk models assume that changes in spreads are normally distributed but empirical evidence shows that they are likely to be skewed and fat-tailed[3] (see Campbell and Huisman, 2003, Phoa, 1999, among others). Negative skeweness, therefore, indicates a bias towards downside exposure, which means that there are more negative changes or large negative returns than positive ones (see

[3] Skewness is a measure of the asymmetry and kurtosis is a measure of the peakedness of a probability distribution.

Anson *et al.* 2004). Furthermore, the probability distribution of credit spreads at any given time is of paramount importance for risk management, trading and hedging activities, as Pedrosa and Roll (1998) argue that the calculations of loss probabilities, such as VaR, will be seriously compromised by the assumptions of normality, constant higher moments, and by ignoring the possibility of shifting regimes. Therefore, the aim of this thesis is to investigate the nature and dynamics of credit spreads and to evaluate the forecasting performance of the different econometric approaches during the credit crisis period; which according to Neftci (1996) all higher moments need to be taken into consideration when pricing derivatives during rare market events.

The following sections discuss in detail the significance of each empirical chapter, the econometric and statistical techniques applied, and highlight the contribution of this thesis in the existing literature. The first empirical chapter provides further insight into the dynamics of higher moments and regime shifts in credit spread changes. The second empirical chapter investigates the dynamics of time-varying correlation between the credit spread indices. Finally, the third empirical chapter examines the impact of different macroeconomic drivers on the credit spread changes.

1.7.1 Modelling the Dynamics of Credit Spread Moments

Modelling the dynamics of higher moments and regime shifts of financial time series is important not only for risk management, as this study shows, but for derivative pricing, investment decisions, as well as trading and hedging strategies. Campbell and Siddique (1999), León *et al.* (2004) and Brooks *et al.* (2005) argue that the dynamics of higher moments of financial variables can be affected by frequency, seasonality and return aggregation which consequently can affect the result of trading and hedging activities. Moreover, they argue that models which account for higher moments can better describe and capture the distributional properties of underlying asset returns.

Kostika and Markellos (2007) show that models, which allow for time variation in volatility, skewness and kurtosis, outperform conventional hedge ratio estimation methodologies. The studies of Prakash, *et al.* (2003), Sun and Yan (2003), Harvey, *et al.* (2004) and Jondeau and Rockinger (2006) argue the importance of incorporating higher moments in portfolio allocation as they provide superior approximation of the expected utility. Additionally, as Brooks *et al.* (2005) show, models accounting for time-varying higher moments can be used to compute separate estimates of the market-required risk premium associated with variance, skewness and kurtosis risk. In risk management, the studies of Burns (2002), Angelidis *et al.* (2003), Brooks and Persand (2003), Brooks *et al.* (2005), Perignon and Smith (2006) and Wilhelmsson (2007) illustrate that after examining a variety of GARCH models only those that account for higher moments in their distributions provide significantly better volatility and Value-at-Risk estimates. In derivative pricing, Heston and Nandi (2000), Christoffersen, *et al.*, (2006), Jacobs and Li (2004), and Tahani (2006) illustrate the importance of incorporating the dynamics of higher moments in option pricing as they find substantial pricing improvements compared with the conventional pricing models. Finally, León *et al.* (2004) argue that the dynamics of higher moments can be applied in analyzing the information content of option-implied coefficients of skewness and kurtosis.

Another approach that allows the dynamics of volatility to change is Markov regime switching models. For example, the studies of Hamilton and Susmel (1994), Hamilton (1994), Klaassen (2002), and Marcucci (2009) have shown that regimes capture the dynamics of the underlying returns better and produce superior forecasts, as well as more accurate estimates of VaR. Alizadeh and Nomikos (2004) illustrate that Markov Regime Switching hedge ratios outperform alternative models in terms of reducing portfolio risk.

However, the existing literature on credit risk models and credit risk management is limited and existing studies do not consider the dynamics of credit spread moments. In this respect, Chapter 5 aims to provide further insight into the dynamics of higher moments and regime shifts in credit spread changes by applying a GARCH-type model that allows for time-varying volatility, skewness and kurtosis. In addition, a regime-switching specification is used to capture the structural changes in volatility of credit spread changes, while allowing the degrees-of-freedom to be state dependent to account for time-varying kurtosis. Finally, Chapter 5 compares the different specifications to assess which model better fits the empirical distribution of the data and produces best Value-at-Risk estimates.

1.7.2 Modelling the Dynamics of Correlation between Credit Spreads

Correlation between asset prices is the most important parameter in portfolio selection, asset allocation, pricing of derivatives, risk management, as well as trading and hedging activities. Numerous studies have shown that correlation between different asset classes varies over time (e.g. Engel and Rosenberg, 1995, Bollerslev, 1990, Kroner and Claessens, 1991, Karolyi, 1995, Lien and Luo, 1994, amongst others) and although correlation between yields and credit spreads of different maturities can be relatively high, it has been shown that correlation between them can vary over time (see Dai and Singleton, 2003, Singleton, 2006, Berndt *et al.*, 2008). The time-varying correlation in yields and credit spreads is of paramount importance in risk management, credit portfolio modelling, the evaluation of credit derivatives (i.e. collateralized debt obligations (CDOs)), trading and hedging default risk.

The significance of modelling the time-varying correlation of credit spreads has been illustrated by a number of studies. Berndt *et al.* (2008) highlight the importance of credit spread correlation in the pricing of credit derivatives by incorporating interest rate and credit spread correlation in their model. Roscovan (2008) constructs a hedging strategy by relating bond portfolio returns to changes in credit spreads. Friewarld and Pichler (2008) propose a spread based model to price credit derivatives that incorporates the correlation of credit spreads. They compare their model with other conventional approaches and find that their model is superior during turbulent market conditions. Bobey (2009) investigates the relationship between systematic default correlation and corporate bond credit spreads and finds that credit spreads are positively related to the CDO market implied default correlation.

However, the literature on understanding the dynamics of correlation in credit spreads is limited and has only recently become an area of interest. The aim of Chapter 6 is to investigate the behaviour of time-varying correlation in credit spreads and to compare the properties and performances of the different multivariate GARCH models. The performance of the proposed models is examined by determining whether they produce accurate VaR estimates.

1.7.3 Determinants of European Corporate Credit Spread Indices

Over the last few years a large body of literature has been devoted to investigating the behaviour of the drivers behind credit spreads. The literature on the determinants of credit spreads can be classified into two categories, the theoretical; and empirical. The theoretical literature can be further separated into two categories, structural and reduced form models. The former spring from the work of Black and Scholes (1973) and Merton (1974), who assume that the value of the firm follows a stochastic process and default occurs when the value of the firm falls below a predetermined boundary. The latter spring from the studies of Jarrow and Turnbull (1995) and Duffie and Singleton (1999) who treat default as a pure jump process. The theoretical literature also argues in favour of the existence of an inverse relationship between interest rates and credit spreads. This means that an increase in the risk-free rate would narrow credit spreads, as it would increase the risk neutral drift and reduce the default probability.

The studies of Delianides and Gerske (2001), Collin-Dufresne and Martin (2001) and Brown (2001), among others, approach the determinants of credit spreads in a more empirical framework, by applying a variety of econometric models and using a number of different factors as determinants. They find that an increase in the risk-free rate would induce a widening in the credit spreads as opposed to the theoretical literature. In addition, empirical models have illustrated the existence of a common factor that explains the remaining large proportion of variation in credit spreads. Finally, the theoretical and the empirical approaches have generated an interesting dichotomy regarding the perception of the risk-free rate and its possible relationship to credit spreads.

In Chapter 7 the effect of the risk-free rate and other important determinants, such as the inflation and commodity prices (measured as the EuroMTS inflation and Goldman and Sachs Commodity Indices) which have not been previously considered as macroeconomic drivers of credit spread changes are investigated. The EuroMTS Inflation Index measures the performance of the Eurozone inflation-linked sovereign debt, allowing the examination of the daily impact on credit spread changes. The S&P Goldman Sachs Commodity Index is a composite index of the commodity sector returns and is broadly diversified across the spectrum of commodities. In addition, this thesis models the nonlinear effects of the determinants on credit spread changes and examines whether the influence of the determinants on credit spread changes varies during different market conditions by applying two statistical approaches: the Markov regime switching regression model and the feed-forward neural networks. The former approach is able to capture structural breaks in the time series, which may be due to changes in government policy, market microstructure, seasonality, and business cycles, to name but a few, and springs from the work of Goldfeld and Quandt (1973) and Hamilton (1989).

The latter statistical approach, which has not been previously applied in the modelling of the non-linear relationship between determinants and credit spreads, is Neural Networks. Neural Networks emerged in the late-1800s as an attempt to describe the processing behaviour of the human mind; since then they have been expanded and applied in many scientific areas with varying degrees of success. This type of neural networks models nonlinear relationships between input and output layers while the information moves in only one direction, from the input layer (i.e. the determinants) through the hidden layer and, finally, to the output layer (i.e. credit spread changes).

1.8 Structure of the Thesis

In this section the outline and structure of the thesis will be presented. Chapter 2 reviews the literature as follows: a general overview of the approaches that are applied in evaluating credit risk is presented and past studies that investigate the determinants of credit spreads are discussed; the distributional properties of credit spreads are presented and the different econometric and statistical approaches proposed by a number of studies are discussed in detail. In addition, the shortcomings of these studies are discussed and those areas that need further investigation are highlighted.

Chapter 3 discusses the different econometric and statistical methodologies that are applied throughout the thesis. The building blocks of time series modelling, the univariate time series models, including stationarity and unit root tests are presented.

Topics on multivariate and non-linear analysis of time series such as VAR and Markov regime switching models are also presented. The ARCH and GARCH models as well as their variations, which are used to estimate time-varying variance, skewness and kurtosis along with the Markov regime switching GARCH models are discussed in detail. Finally, the multivariate volatility models such as BEKK, O-GARCH, CC-GARCH and DCC-GARCH as well as the principal component analysis are presented.

Chapter 4 introduces the data set used for empirical analysis, reviews the statistical properties of the different variables and examines the univariate properties of the series such as stationarity and unit roots. The credit spreads examined are computed as the difference between the yield on iBoxx Euro Corporate Indices and the yield on equivalent German government bonds. The German government bonds are selected because they are the most liquid instruments compared to the other European and U.S. benchmark rates, in addition to having gained a safe-haven status in the international financial markets.

Chapter 5 investigates the dynamics of credit spread moments and compares such behaviour across different credit ratings and maturities. To achieve this, a series of models including simple asymmetric GARCH models, time-varying volatility, skewness and kurtosis models known as GARCH-SK, as well as variants of Markov Regime Switching GARCH models are utilised. The analysis captures the dynamics of the shape of the distribution of credit spreads over time. Furthermore, the forecasting performance of these models, in estimation of Value-at-Risk, is examined.

Chapter 6 is devoted to examining and comparing the properties and performance of the different multivariate GARCH models in capturing the correlation between credit spreads. The models examined within this study are the Orthogonal-GARCH, the Constant and Dynamic Correlation GARCH, Risk Metrics and Diagonal-BEKK formulation. The performance of these models is examined by determining whether they produce accurate VaR estimates..

Chapter 7 investigates the impact of the risk-free rate and other important determinants on credit spreads. In order to examine whether the influence of the determinants on credit spread changes varies under different market conditions a Markov regime switching model is proposed and estimated. In addition, a feed-forward neural network, which models the nonlinear relationship between input and output layers while the information moves in only one direction, from the input layer (i.e. the determinants) through the hidden layer and finally to the output layer (i.e. credit spread changes), is utilized.

The final section of this chapter presents the main conclusions of the study as well as the importance and implications of each empirical study. Finally, the last section of the chapter is devoted to suggesting limitations in the empirical investigation as well as suggestions for future research.

1.9 Conclusions

This chapter introduced the unique characteristics of the European fixed income markets and familiarised the reader with the topics discussed and analysed in this thesis. In particular, a brief historical description of the events that led to the creation of the European Union and the ways in which the single currency has affected the developments of the European fixed income markets are outlined. What is more, the different types of risk that affect a debt instrument are introduced, presenting the argument that credit risk is one of the most important types of risk to which market participants are exposed and discussing the importance of modelling this type of risk.

The main areas of research analysed and discussed in this thesis are identified and the contribution of this thesis to the existing literature is highlighted. The aim of the first empirical chapter is to provide further insight into, and enhance our understanding of, the dynamics of higher moments and regime shifts in credit spread changes. Several econometric techniques and models are applied to investigate these dynamics and regime shifts. Different specifications and models are also compared in order to establish which model better fits the empirical distribution of credit spread series and produces the best volatility and VaR estimates based on risk management loss functions. The second empirical chapter examines and compares the properties and performance of the different multivariate GARCH models in capturing the correlation between the credit spreads and which specification produces best Value-at-Risk estimates based on risk managements' loss functions. The third empirical chapter examines the impact of different market and macroeconomic drivers on the Euro credit spreads. It highlights how the effect of the determinants on credit spreads varies across different market conditions and applies a statistical model that has not been previously employed in capturing the non-linear relationship between the determinants and credit spread changes.

Chapter 2

Literature Review

2.1 Introduction

One of the most important types of risk to which market practitioners are exposed is credit risk. Credit risk is defined as the risk that a borrower will fail to satisfy the terms of the contractual obligations with respect to the timely payment of the amount borrowed. More importantly, these obligations not only refer to the incomplete fulfilment of the borrower's obligation, but also to the delay or postponement in fulfilling them, even though they are later satisfied. Therefore, before entering into a contract the counter-party is evaluated in terms of its capacity to fulfil its contractual obligation based on its credit risk and creditworthiness.

Credit risk incorporates three types of risk, namely: *default risk; downgrade risk;* and *credit spread risk*. *Default risk* is the risk that the issuer of an obligation will be unable to pay the outstanding debt. This means that the issuer will be unable to repay either the full amount or a portion of the debt. *Downgrade risk* is the risk associated with the deterioration of an issuer's credit status, reported by recognized statistical rating agencies, reflecting its capacity to honour its debt obligation. *Credit spread risk* is the risk that the yield premium or the spread over a reference rate will increase for a debt obligation, due to changes in market conditions; and also is the market's reaction to the perceived credit deterioration, which does not necessarily mean that the issue will default.

This chapter is structured as follows. Section 2.2 introduces and reviews the two most common approaches that are proposed and developed to model credit risk, the structural and reduced form models, Section 2.3 describes the determinants of credit spreads, Section 2.4 discusses the importance in modelling the dynamics of credit spreads, Section 2.5 reviews the ARCH / GARCH and the variants as well as the Markov regime switching GARCH models, Section 2.6 reviews the multivariate GARCH models and Section 2.7 presents the summary of the review and concludes.

2.2 Credit Models

Both the structural and reduced-form approaches allow for the evaluation of risky claims and explaining credit risk. Structural models assume a stochastic process for the evolution of the firm's value, and default occurs when the value of the firm reaches a predetermined boundary. On the other hand, according to Duffie and Singleton (1999), the reduced form models treat default as a pure jump process following an intensity-based or a hazard-rate process. In the first instance default is treated endogenously, while, in the second, exogenously.

2.2.1 Structural Models

According to Black and Scholes (1973) a corporate debt is a contingent claim on the firm's value, and corporate liabilities can be viewed as a covered call[4]. In the Black, Scholes (1973) and Merton (1974) setting (see Chapter 8 in Anson, *et al*, 2004), a firm

[4]A covered call owns the asset and sells a call option

has issued one type of debt, a zero coupon bond with face value K and maturity T. The market value of its equity is E(t) at time t, the market value of its debt is D(t,T) at time t, and the value of its assets at time t is given by A(t). Default is assumed to occur at maturity if and when the value of the assets falls below the value of debt, K. If the value of assets at maturity is greater than the face value of debt, then the debt holders are paid in full and equity owners receive the remaining value; otherwise a default event is triggered and bond holders take control of the firm and equity owners receive nothing. Thus at maturity (T), the value of the firm's equity is the amount remaining after debts have been paid out of the assets value:

$$E(T) = \max\{A(T)-K,0\} \tag{2.1}$$

This payoff resembles a long call option on the firm's assets with a strike price equal to the face value of the debt. Thus, the value of debt at maturity is:

$$D(T,T) = \min\{A(T),K\} = A(T)-\max\{A(T)-K,0\} \tag{2.2}$$

In the Black and Scholes (1973) framework the assets follow a lognormal stochastic process:

$$\frac{dA(t)}{A(t)} = rdt + \sigma dZ(t) \tag{2.3}$$

where

r : is the risk-free rate which is assumed constant,

σ : is the volatility, and

Z(t) : is a Brownian motion.

Moreover, the value of the equity, given by BSM, which is equal to the value of a call option as seen above is:

$$E(t) = A(t)N(d1)-e^{-r(T-t)}KN(d2) \tag{2.4}$$

and its debt value equals:

$$D(t,T) = A(t) - E(t) = A(t)[1-N(d1)]+e^{-r(T-t)}KN(d2) \tag{2.5}$$

where

N(.) : is the cumulative normal probability, with

N(d2) : the survival probability,

1- N(d2): the default probability

The first term in the above equation represents the recovery value in case of default while the second term represents the present value of the debt K should no default occur.

The yield is given by solving $D(t,T) = Ke^{-y/(T-t)}$ for y, thus:

$$y = \frac{\ln K - \ln D(t,T)}{(T-t)} \tag{2.6}$$

Although the BSM model captures realistically the shapes of the credit spread term structures resembling those in the market and provides information regarding the debt structure, it also has a few disadvantages regarding the simplistic assumptions that formulate its basis. Firstly, the BSM model assumes that a firm has only one type of debt (a zero-coupon bond) with a certain maturity. Merton (1974) presents a model that is able to price a callable coupon bond and, by extension, to reflect coupon payments. Moreover, Geske (1977) extends the BSM to model default at different times, using the compound option model he proposes. He illustrates that corporate liabilities with different maturities can be viewed as a sequence of compound options. Next, BSM does not allow for default event prior to maturity. Black and Cox (1976) introduced an absorbing barrier structural model. This model considers default to be triggered when the firm's value reaches a certain default threshold and it is modelled as a knock-out barrier option.

Furthermore, under the BSM model, default cannot occur by surprise. Zhou (1997) and Huan and Huang (2003) inserted a jump – diffusion model in the BSM framework. The advantage of this type of model is that it makes the default a surprise. As a result of this approach, low leverage firms appear to have significant spreads at the short end of the credit term structure.

Finally, BSM assumes that interest rates are constant. Shimko *et al.* (1993) illustrate that a stochastic interest rate process can be used in the BSM framework as long as the volatility of the return of the zero coupon bond depends only on time to maturity. Other models that introduced stochastic interest rates are that of Longstaff and Schwartz (1995) which assumes a stochastic mean reverting interest rate model (see Vasicek, 1977) and has exogenously specified a default boundary while Dufresne *et al.*

(2001) proposed a model with stochastic interest rates, with the ability to capture both the firm's current debt structure and its possibility to alter in the future.

More complicated characteristics of the BSM framework are presented by Ericsoon and Reneby (1997) and Rainer (1999) who consider both stochastic interest rates and stochastic default boundaries, where default can occur prior to maturity. Unfortunately, these kinds of models are very difficult to be implemented empirically and are also computationally intensive.

Another focus concerns the specification of the asset value that triggers bankruptcy. Leland (1994) endogenizes the value of assets that trigger bankruptcy by introducing taxes and bankruptcy costs, as factors, in determining the optimal asset value at which a firm should declare bankruptcy. Leland and Toft (1996) extend this model to derive a term structure of credit spreads. However, there are problems when using these types of models, cince the information required to price debt claims may be unavailable to the market, especially data on taxation and bankruptcy costs.

All these models demonstrate the potential of the BSM framework to become more realistic but at the same time more complex, which results in an increase in the difficulty of its tractability and, in extreme cases, there may be no closed-form solution.

2.2.2 Reduced Form Models

Reduced form models are arbitrage free, employ the risk-neutral measure and consider default as an unpredictable event governed by an intensity-based or hazard-rate process. As a result, they simplify the constraints of defining the causes of default. Jarrow and Turnbull (1995) present one of the first reduced-form models where default and recovery are based on the Poisson default process.

The Poisson process is described in order to proceed to the Jarrow-Turnbull model and it is derived from Anson, *et al*. (2004). The process at time t takes value N_t which is an increasing set of integers 0,1,2,... and the probability of default over a small time interval dt is given by:

$$\Pr\left[N_{t+dt} - N_t = 1\right] = \lambda dt \tag{2.7}$$

where λ is the intensity parameter of the Poisson process and represents the annualized instantaneous forward default probability at time t and, in the Jarrow-Turnbull model, is assumed to be constant. The default time distribution is the time until the first default event occurs and it is given as:

$$Q(t,T) = \Pr\left[T > t\right] = e^{-\lambda(T-t)} \tag{2.8}$$

The Jarrow-Turnbull model assumes that recovery payment is paid at maturity time T regardless of when default occurs. This removes any dependency between the price of the bond and the conditional default probability. The value of the coupon bond is then given as:

$$B(t) = P(t,T)R(T)\int_t^T -dQ(t,u)du + \sum_{j=1}^{n} P(t,T)c_j e^{-\lambda(T_j-t)} \tag{2.9}$$

where $P(t,T)$ is the risk-free discount factor, c_j the j-coupon, $Q(t,T)$ the survival probability given by equation 2.8, R the recovery rate. However, in practice the Jarrow-Turnbull model is adjusted to allow the intensity parameter to be a function of time and the recovery to be paid upon default or coupon dates. The advantage of Jarrow-Turnbull model is its calibration ability. As the default probabilities and recovery rates are exogenously specified, the default probability curve and by extension the spread curve can be derived from risky zero-coupon bonds.

Jarrow, Lando and Turnbull (1997) extend the Jarrow and Turnbull (1995) model to incorporate migration risk. Migration risk is the risk of a downgrade of the firm's credit rating, resulting in widening of the credit spreads, but not causing a default. Default is modelled under the Jarrow, Lando and Turnbull (1997) as a first time continuous-time Markov chain with K states, where states 1 to K-1 are credit ratings and the K state is an absorbing state (default). To make the model tractable, as the Markov chain greatly increases the number of parameters to be estimated, Jarrow, Lando and Turnbull (1997) suggest using the historical transition matrix which is published by rating agencies such as Standard & Poor and Moody's. While the historical transition matrix consists of real probabilities which are different from the risk-neutral probabilities, Jarrow, Lando and Turnbull (1997) assumed that the risk-neutral probabilities are proportional (linear) to the real probabilities. However, the empirical validity of the aforementioned methodology is yet to be demonstrated (see Anson, et al, 2004).

Duffie and Singleton (1999) take a different approach compared with Jarrow and Turnbull (1995). Duffie and Singleton (1999) assume that the payment of recovery may occur at any time, but the recovery rate is restricted as a proportion of the bond price at the time of default. Hence Duffie and Singleton (1999) model is known as a fractional recovery model. The recovery rate is given as:

$$R(t) = \delta D(t,T) \tag{2.10}$$

where δ is a fixed recovery rate ratio and $D(t,T)$ the price of the bond as if default did not occur. The price of the bond at time t is given as:

$$D(t,T) = E_t\left[e^{\left(-\int_t^T (r(u)+s(u))du\right)} \right] X(T) \tag{2.11}$$

where $s(u) = p_u(1-\delta)$, p is the default probability which follows a Poisson distribution, r is the risk-free rate process and X is the terminal payoff of the bond. Moreover, the product $p(1-\delta)$ is considered as a spread over the risk-free rate. The advantage of the Duffie-Singleton model is the derivation of the spread curve which is based upon the probability curve and the recovery rate. The disadvantage of the Duffie-Singleton approach is that, if a claim does not have a payout at maturity, such as credit default swaps, the value of the claim today is zero, making such claims impossible to value.

Another extension of the reduced-form models is that of Duffie and Lando (1997), who formulate a structural model that can be estimated as a reduced-form model using the framework of Duffie and Singleton (1999). A diffusion process is used for the firm’s asset value, the default barrier is given by the imperfect accounting system and the hazard rate is given in terms of the asset’s volatility. Taking these types of models further, Cathcar and El-Jahel (1998) propose a model where default occurs when a signalling process hits a lower barrier. Their default-risk free rate then follows a Cox, Ingersoll and Ross (1985) process where they assume that the default-risk free rate and process are uncorrelated.

Empirical results by Duffee (1998) and Tufano and Das (1996) illustrate that, by modelling the intensity function as a Cox process, the credit spread depends on both the default free term structure and an equity index. In addition, Duffie and Singleton (1999), Jarrow, Turnbull and Lando (1997) and Jarrow and Turnbull (1995) imply that for many credit derivatives only the expected loss needs to be modelled.

Reduced-form models lay a solid foundation in an attempt to model the risk-neutral probability of default. However, Anson, *et al*. (2004) show that to a large extent default can be anticipated, as it is the product of a series of downgrades and spread widenings, making the spread-based diffusion models popular.

2.2.3 Spread-Based Models

Spread-based models are very similar to barrier structural models, their difference lying in the use of a different state variable to detect default. A spread model uses bond spreads, while default barriers are exogenously specified and need to be calibrated with real data. Models that apply spread dynamics generally take par asset swap spreads as the fundamental spread variable. Asset swap spreads are a spread over LIBOR which represents the credit quality of a specific security.

Asset swap spreads follow a lognormal process:

$$\frac{ds(t)}{s(t)} = \sigma dZ(t) \tag{2.12}$$

where

σ : is the percentage volatility of spread changes

dZ(t) : is a Brownian motion.

Hull (2000) shows that the spread is usually centred around its spot value:

$$s(t) = s(0)\exp\left(-\frac{\sigma^2 t}{2} + \sigma Z(t)\right) \tag{2.13}$$

A default boundary is specified in terms of a spread and is quite arbitrary as the boundary can be time dependent. The price of a digital default swap paying out 1$ at maturity in the event of default when interest rates and spreads are independent is:

$$D(t,T) = E_t\left[\exp\left(\int_t^T r(u)du\right)\right]F(t,T) \tag{2.14}$$

where F(t,T) is the price of a digital barrier option whose payoff equals the probability that the spread crosses the barrier at some time before maturity. The problem with this type of approach is that it lacks theoretical under pinning.

2.2.4 Hazard Models

Shumway (2001) argues that a good default model should be an econometric model, illustrating that many theoretical models do not take into consideration factors such as accounting rations, market size and historical returns, which in turn have great value in predicting defaults. Thus, the proposed model contains both theoretical and empirical factors and its survival and hazard functions are given as:

$$S(t,x;\theta) = 1 - \sum_{j<t} f(j,x;\theta) \tag{2.15}$$

$$\varphi(t,x;\theta) = \frac{f(t,x;\theta)}{S(t,x;\theta)} \tag{2.16}$$

where $f(t,x;\theta)$ is the probability density function of x which is a set of explanatory variables. The maximum likelihood function is then given as:

$$L = \prod_{i=1}^{n} \varphi(t_i,x_i;\theta)^{y_i} S(t_i,x_i;\theta) \tag{2.17}$$

$$y_i = \begin{cases} 1, & if \quad default \quad occurs \\ 0, & otherwise \end{cases} \tag{2.18}$$

2.2.5 Credit Rating Transition Matrices

Ratings represent an issuer’s credit quality by reflecting his capacity to honour his debt obligations, and are assigned by rating agencies such as Standard and Poor’s, Moody’s and Fitch. A credit rating system applies a number of rating grades to rank issuers according to their default probability, which are presented in the transition matrix. Transition matrices describe the changes in credit quality and serve as input to many credit risk applications, including the measurement of credit portfolio risk, modelling the term structure of credit risk premia, and the pricing of credit derivatives (see Jarrow, Lando and Turnbull, 1997). There are two procedures in estimating the transition matrices from observed historical transitions - the cohort and hazard approaches.

The transition matrix in the cohort approach is estimated by dividing the sum of obligators that have migrated from grade *i* to *j* by the overall number of obligators that was in grade *i* at the start of the considered periods and is given as:

$$p_{ij} = \frac{\sum_t N_{ij,t}}{\sum_t N_{i,t}} = \frac{N_{ij}}{N_i} \tag{2.19}$$

where $N_{i,t}$ denotes the number of obligators in grade *i* at the beginning of period t, $N_{ij,t}$ denotes the number of obligators that have migrated from grade *i* to *j* at the end of the period t.

Although the cohort approach is able to estimate transition probabilities from transition frequencies, it does not consider the timing and sequence of transitions within the examined period. The hazard rate approach is an alternative to the cohort approach, as it is able to capture the timing of transitions. According to Lando and Skodeberg (2002), in the case of time-homogeneity the first step in estimating the Markov transition matrix in the hazard rate approach is to formulate the generator matrix **Λ.** The off-diagonal entries of the generator matrix over a time period $\left[t_0, t_1\right]$ are given as:

$$\lambda_{ij} = \frac{N_{ij}}{\int_{t_0}^{t_1} Y_i\left(s\right) ds} \tag{2.20}$$

where N_{ij} denotes the number of transitions from *i* grade to *j*, $Y_i\left(s\right)$ denotes the number of obligators rated *i* at time s, and the integral in the denominator represents the time of obligators belonging to rating grade *i* before migrating. The on-diagonal entries of the generator matrix are given as:

$$\lambda_{ii} = -\sum_{i \neq j} \lambda_{ij} \tag{2.21}$$

Finally, the T-year transition matrix **P**(T) is estimated from the generator matrix **Λ** in the following manner:

$$\mathbf{P}(T) = e^{(\Lambda T)} = \sum_{k=0}^{\infty} \frac{\mathbf{\Lambda}^k T^k}{k!} \tag{2.22}$$

A number of studies argue that the assumption of time homogeneity may not hold over the long run (see Carty and Fons, 1993, Altman and Kao, 1992 among others). Lando and Skodeberg (2002) propose a non-homogeneous continuous time Markov process whose transition probability matrix for the period from t_0 to t_1 is estimated by $\mathbf{P}(t_0, t_1)$ given there are m transitions:

$$\mathbf{P}(t_0, t_1) = \prod_{i=1}^{m} \left(1 + \Delta \widehat{\mathbf{A}}(T_i)\right) \tag{2.23}$$

where T_i is a jump time in the interval $[t_0, t_1]$ and

$$\Delta \widehat{\mathbf{A}}(T_i) = \begin{bmatrix} -\frac{\Delta N_1(T_i)}{Y_1(T_i)} & \frac{\Delta N_{1,2}(T_i)}{Y_1(T_i)} & \frac{\Delta N_{1,3}(T_i)}{Y_1(T_i)} & \cdots & \frac{\Delta N_{1,\rho}(T_i)}{Y_1(T_i)} \\ \frac{\Delta N_{2,1}(T_i)}{Y_2(T_i)} & -\frac{\Delta N_2(T_i)}{Y_2(T_i)} & \frac{\Delta N_{2,3}(T_i)}{Y_2(T_i)} & \cdots & \frac{\Delta N_{2,\rho}(T_i)}{Y_2(T_i)} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \frac{\Delta N_{\rho-1,1}(T_i)}{Y_{\rho-1}(T_i)} & \frac{\Delta N_{\rho-1,2}(T_i)}{Y_{\rho-1}(T_i)} & \cdots & -\frac{\Delta N_{\rho-1}(T_i)}{Y_{\rho-1}(T_i)} & \frac{\Delta N_{\rho-1,\rho}(T_i)}{Y_{\rho-1}(T_i)} \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \tag{2.24}$$

where $\Delta N_{h,j}(T_i)$ denotes the number of transitions observed from state *h* to *j* at time T_i, $\Delta N_k(T_i)$ counts the total number of transitions away from state k at time T_i and $Y_k(T_i)$ is the number of firms in state k right before time T_i. The on-diagonal elements measure the fraction of the exposed firms $Y_k(T_i)$ which leaves the state at time T_i. The off-diagonal elements count the specific types of transitions away from the state divided by the number of exposed firms. The last row of the $\Delta \widehat{\mathbf{A}}(T_i)$ is zero because it indicates an absorbing barrier which means that, when a firm defaults, it remains defaulted.

2.3 Determinants of Credit Spreads

This section describes the empirical investigation of the behaviour of corporate bond prices as well as the determination of the drivers behind credit spreads. Credit spreads are defined as the difference between the yield to maturities of a corporate and a comparable government bond. Credit spreads are important variables in the financial markets, as they reflect the likelihood of failure of an entity to honour its obligation.

Delianedis and Geske (2001) investigate and identify the components of US corporate spreads from November 1991 to December 1998, using taxation, liquidity, recovery risk, and account for jumps, volatility and market factors. They conclude that default and recovery risks are only partial components of corporate spreads. Dufresne *et al.* (2001) study US credit spreads for the period from July 1988 through December 1997 and argue the existence of a systematic factor after examining a variety of determinants such as changes in spot rates, changes in slope and level of the yield curve, firm's leverage, volatility (changes in the VIX index), business climate, and movement and magnitude of a downward jump in the firm's value.

Christiansen (2000) and Huan and Huang (2003) study the dynamics of correlation between credit spreads and interest rates on days of macroeconomic announcements, such as employment situation report, GDP and PPI reports among others. They find that credit spreads and interest rates are negatively correlated, except on the announcement days were they become uncorrelated.

Nerin *et al.* (2002) study the components of credit risk, which is derived by credit default swaps and US corporate spreads, during the period of January 1998 to February 2000. The components considered are credit ratings, level and slope of the yield curve, stock prices, volatility of the firm's value, index returns, and other market factors. They find that these variables are able to explain a large percentage of variation both in credit default swaps and corporate spreads (approximately 60%), with equity market factors explaining up to 50% of the variation.

Brown (2001) examines the explanatory power of the Treasury yield, consumer confidence, the VIX index and a liquidity measure on US corporate bond spreads for different credit quality and maturity portfolios over the 1984 to 1999 period. His findings reveal that these variables can explain up to 30% of yields changes, yield spread volatility is higher for lower credit quality portfolios, yield spreads are more volatile for shorter-maturity portfolios keeping credit quality constant; and finally, a considerable portion of yield spread volatility is due to changes in the non-default margin components of the corporate bond yield spread.

Longstaff and Schwartz (1995) assume that the value of the firm and risk-free interest rate follow a correlated diffusion process. This means that credit spreads depend on the firm's asset value, interest rate and the correlation between these factors. They find that a negative relationship exists between credit spreads and interest rates. Duffee (1998) finds that there is a negative relationship between credit spreads and the level and slope of the term structure of interest rates.

Changes in government policy, market microstructure, seasonality, business cycles, to name but a few, can affect the properties of financial and economic time series both in terms of their mean value and volatility and are known as structural breaks. Structural breaks characterize the non-linear dynamics of time series and are well documented in the literature (see Nefci, 1982 and 1984, Sichel, 1987, Hamilton, 1985, among others). One popular approach in modelling nonlinear dynamics is the Markov regime switching models. These types of models spring from the work of Goldfeld and Quandt (1973) and Hamilton (1989) who model the structural breaks of the ARCH specification and are later extended into a regime-switching specification both for the mean and volatility process.

Bansal *et al*. (2004) incorporate a regime-shift term structure model based on the Hamilton (1989) specification and apply it to monthly U.S. Treasury yield data from 1964 to 2001 with maturities 1, 3, 6, monthly and 1, 2, 3, 4, 5 annually. They find that their model can justify the transition dynamics of the Treasury yields and find a link between business cycles and regimes. Davies (2004) analyses the determinants of Moody's AAA and BAA credit spread indices using regime switching techniques. He finds that the risk-free rate, equity return, slope of yield curve and industrial production growth have explanatory power over the short-term credit spreads. Specifically, risk-free rate has a negative effect on credit spreads during periods of low volatility and has no effect during periods of high volatility. Cheung and Erlandsson (2004) apply a Markov regime switching model to monthly exchange rates (DEM/USD, GBP/USD, FFR/USD) for the period from 1973 to 1988. They find strong evidence of regime switching in exchange rates. Clarida *et al.* (2006) test a multivariate asymmetric Markov switching model on the weekly euro-rates for German, Japanese and U.S. bonds with different maturities for the period between 1982 to 2000. They find strong evidence of non-linearity in the term structures, and of regimes being related to business cycles and to inflation.

In theory all these variables would predict credit spread changes; however, in practice they have little explanatory power, leading to the belief that a common factor exists which explains the remaining variation. This study examines the impact of the risk-free rate and other important determinants on the credit spreads of Euro bond indices over different market conditions. It introduces two macroeconomic indicators that have not been considered previously as drivers of credit spreads and they are found to suggest a consistent effect in the determination of credit spreads.

2.4 Importance in Modelling the Dynamics of Credit Spread Moments

Financial series including the European credit spreads and their returns, used in this study[5], exhibit non-Gaussian return distributions (e.g. leptokurtosis), volatility clustering, regime switches and time-varying higher moments (see Campbell and Siddique (1999) and León *et al.* (2004) among others). A number of studies delineate the importance of modelling the dynamics of higher moments in risk management, hedging, portfolio allocation and option pricing.

Kostika and Markellos (2007) show that models which allow for time variation in variance, skewness and kurtosis outperform conventional hedge ratio estimation methodologies such as OLS, error-correction, exponentially weighted moving averages, and GARCH models. Their analysis was based on spot and futures data on FTSE, Dow Jones, and DAX equity indices for the period from January 1999 to September 2004. Alizadeh and Nomikos (2004) illustrate that Markov Regime Switching hedge ratios outperform alternative models such as GARCH, error-correction and OLS using the spot and future weekly data on FTSE 100 and S&P 500 from May 1984 to March 2001.

[5] For more information refer to Table 4.1 of the Data Chapter.

The studies of Prakash *et al.* (2003), Sun and Yan (2003), Harvey *et al.* (2004) and Jondeau and Rockinger (2006) argue the importance of incorporating higher moments in portfolio allocation as they provide superior approximation of the expected utility function. Specifically, Prakash *et al.* (2003) and Sun and Yan (2003) apply a polynomial goal programming which incorporates investor preferences for skewness in determining the optimal portfolio. The former test the model on 17 international stock market indices over the period from July 1993 to December 2000 and the latter in stocks from the US and Japanese market over the period from May 1975 to December 1997; and both find that skewness should not be neglected in portfolio allocation. Harvey *et al.* (2004) propose a Bayesian model which employs a skew normal distribution[6] in optimal portfolio selection. They test the model on the daily returns of four equity stocks (General Electric, Lucent Technologies, Cisco Systems and Sun Microsystems) for the period from April 1996 to March 2002, on weekly returns over the period from January 1989 to June 2000 on four equity portfolios (Russell 1000, Russell 2000, Morgan Stanley Capital International non-U.S. developed countries and Morgan Stanley Capital International Emerging Markets) and three fixed income portfolios (government bonds, corporate bonds and mortgage backed bonds). They find that it is important to incorporate higher-order moments in portfolio selection. Finally, similar results are presented in the study of Jondeau and Rockinger (2006), who propose a Taylor series expansion of the expected utility which is able to capture higher-moments. They apply it on weekly returns for dollar-denominated stocks indices for the main geographical areas (North America, Europe and Asia) from January 1976 to December 2001, and weekly returns for stocks included in the S&P 100 index from January 1974 to January 2002.

In risk management, the studies of Burns (2002), Angelidis *et al.* (2003), Brooks and Persand (2003), Brooks *et al.* (2005), Perignon and Smith (2006) and Wilhelmsson (2007) illustrate that, after examining a variety of GARCH models, only those that account for higher moments in their distributions provide significantly better volatility and Value-at-Risk estimates. In addition, the studies of Hamilton and Susmel (1994), Hamilton (1994), Klaassen (2002), and Marcucci (2009) have shown that regimes

[6] Sahu, S. K., Branco, M. D., and Dey, D. K. (2003), "A New Class of Multivariate Skew Distributions with Applications to Bayesian Regression Models," *Canadian Journal of Statistics*.

capture the dynamics of the underlying returns better and produce superior forecasts as well as more accurate estimates of VaR.

The importance of modelling the dynamics of higher moments in option pricing is first presented by Heston and Nandi (2000) who propose a GARCH specification that incoporates conditional skewness. They test their model on the S&P 500 option intraday data over the period from 1992 to 1994 and find substantial pricing improvement compared with the Black and Scholes (1973) model. Christoffersen *et al.* (2006) propose a model that allows for conditional skewness, as well as conditional heteroskedasticity and a leverage effect which controls the asymmetry of the distribution. They test the performance of the proposed model in pricing S&P 500 index options over the period from February 1989 to December 2001 and conclude that their model’s performance is superior to the nested models for out-of-the money puts. Tahani (2006) examines the log-spreads between Moody’s AAA and BAA with maturities 10 and 20 years bond indices, and US treasury bond yields of 10 and 30 years to maturity, over the 1986 to 1992 period. The estimation of GARCH effects with a skewness parameter reveals better data fit than the simple mean-reverting models and concludes that the difference between the proposed GARCH specification and Longstaff and Schwartz (1995) is more important for at-the-money credit spread call options.

2.5 ARCH/GARCH and Markov Regime Switching GARCH models

Engle (1982) introduces the Autoregressive Conditional Heteroscedasticity (ARCH) model a formal approach in modelling the variance of a time series. One of the first studies in modelling ARCH effects in financial series is Weiss (1983), who finds significant evidence of ARCH effects in US AAA rating bond yields. In another study, Bierens, Huang and Kong (2003) propose an ARCH model that incorporates portfolio rebalancing, jumps[7] and market factors. They apply their model to nine Merrill Lynch daily series of option-adjusted spreads corporate bond indices with ratings from AAA to C for the period of January 1997 to August 2002 and find that their proposed model

[7] According to the authors: The jump probability is allowed to depend on the lagged market conditions and is assumed to be a logistic function.

outperforms the ARCH specification in producing out-of-sample forecasts, as it is able to capture the extreme movements in credit spreads.

In a later study Bollerslev (1986) extends the ARCH model to the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) class of models. Manzoni (2002) finds significant evidence of ARCH effects and nonlinearities in credit spreads on the Sterling Eurobond index for the period 1991 to 1999. He applies both the ARCH and GARCH specifications to model time-varying volatility and captures the persistence in the conditional variance. Brooks *et al.* (2005) use a GARCH specification with a time varying-kurtosis, (GARCHK). They apply the model to daily returns of the S&P 500 and UK FTSE 100 indices, as well as the total return of the US and UK ten-year maturity benchmark bond indices, over the period from January 1990 until June 2000 and find significant evidence indicating persistence in time-varying kurtosis of the series.

Changes in government policy, seasonality, and business cycles among others can affect the properties of financial and economic time series both in terms of their mean value and variance. Lamourex and Lastrapes (1990) attribute the volatility persistence to the presence of structural breaks in the variance. Hamilton and Susmel (1994), Gray (1996), Dueker (1997), Klaassen (2002) and Alizadeh and Nomikos (2004), to name but a few, extend the Goldfeld and Quandt (1973) and Hamilton (1989) models which allow both the mean and the variance to switch between different states of the market.

Gray (1996) proposes a Generalized Regime-Switching Model, (GRS), of the short-term interest rate and applies it to weekly U.S. Treasury bill rates from January 1970 to April 1990. He finds that his model out-performs other models of the short-rate, both in fit of the empirical distributions and in forecasting volatility. Perignon and Smith (2006) propose a yield factor volatility model that includes level[8], GARCH effects and regime swifts. They test their specification on monthly U.S bond yields for the 1970 to 2002 period and find that when volatility is allowed to switch from low to high-volatility regimes, the model’s fit improves dramatically and strengthens the level

[8] According to Perignon and Smith (2006) the variance of the yields factors is estimated by using a level effect, in which the conditional volatility is a nonlinear function of the level of short-term interest rates.

effect. In addition, their model provides the best out-of-sample forecasting performance of yield volatility.

The literature in modelling the time-varying higher moments and non-linearities in credit spreads is limited. This study models the time-varying skewness and kurtosis of credit spreads as well as estimating a Markov regime switching GARCH model which allows its Student-t degrees-of-freedom parameter to switch in such a manner that the conditional variance and kurtosis are subject to discrete shifts.

2.6 Modelling the Dynamics of Correlation of Credit Spreads

Correlation is defined as a statistic applied extensively in portfolio analysis and asset allocation, to indicate whether the returns of two or more assets are likely to present a systematic, linear relationship over time. The studies of Bollerslev *et al.* (1988), Engel and Rosenberg (1995), amongst others, have shown that covariances and correlation of financial series change over time and have an important role in the pricing of derivatives, portfolio selection, trading and hedging and risk management.

The first study to model time-varying covariances is Bollerslev *et al.* (1988). They test their model on quarterly data of 3- and 6-month Treasury bills, 20-year Treasury bond and the New York Stock Exchange Index from 1959 to 1984. They find that the conditional covariances change over time and are able to explain the time-varying risk premia. Alexander (2001) applies an Orthogonal-GARCH (OGARCH) model on the daily prices of the WTI crude oil with different maturities from February 1993 to March 1999, daily UK zero coupon yield data with different maturities from January 1992 to March 1995, and daily European equity index returns (France, German, Holland, Spain, Sweden and UK indices). She finds that a small number of factors can explain a significant amount of variability and fares good compared with the BEKK and VEKK specifications. Chiang and Jiandong (2008) examine the dynamics of correlation between stock and bond markets. They test a variety of models such as a rolling regression model and a BEKK on the Vanguard Total Bond and Stock Market Index Funds, from June 1996 to June 2008, and find that correlation is time-varying and negative.

Recent studies reveal the importance of modelling the time-varying correlation of credit spreads in the pricing of credit derivatives and credit portfolio hedging and trading. Berndt, Ritchken and Sun (2009) develop a Markovian model based on the Heath, Jarrow and Merton (1992)[9] framework for pricing credit derivatives. They highlight the importance of incorporating the correlation between interest rates and credit spreads, the correlation between credit spreads of different firms, and the dynamics of credit spreads in the structure of the model. Roscovan (2008) constructs a hedging strategy by relating bond portfolio returns to changes in credit spreads. Friewarld and Pichler (2008) propose a spread-based model to price credit derivatives that incorporates the correlation of credit spreads. They compare their model with other conventional approaches and find that their model is superior during market turbulences. Bobey (2009) investigates the relationship between systematic default correlation and corporate bond credit spreads and finds that credit spreads are positively related to the CDO market implied default correlation.

Factor decomposition through principal component analysis (PCA) has been applied in many empirical investigations in finance and, specifically, in interest rates. Pearson (1901) developed Principal Component Analysis (PCA), a statistical technique for transforming correlated variables into a smaller number of uncorrelated variables called the “principal components”.

One of the first studies are those of Litterman and Scheinkman (1991) who employ PCA to identify the common factors that affect the returns of the Treasury and bond securities for the period from February 1986 to June 1988. They find that three principal components explain the variation in bond returns. Soto (2004) applies PCA on daily Spanish government bond prices and yields with different maturities for the period from January 1990 to December 1999. He finds that three principal components are able to explain the movements of the Spanish term structure. Malava (2006) applies PCA on daily quotes of market data and interest rate swaps for EUR, USD, JPY and GBP for the period from April 2006 to July 2006. He finds that three

[9] For additional information regarding the HJM framework see: Heath, D., Jarrow, R., A., and Morton, A., (1992). “Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation.” *Econometrica*, 60(1), 77-105

principal components explain up to 86% of the interest rate variation, and suggests the existence of a common global interest rate factor.

Research in the time-varying correlation of credit spreads is limited and only recently has the importance of modelling the correlation dynamics been revealed. This study aims to model the time-varying correlation of the European credit spreads and to evaluate the performance of different multivariate GARCH models in risk management. It reveals how correlation changes over time, and especially during the credit crisis period, when its behaviour changes.

2.7 Conclusions

This chapter highlighted credit risk as the most important risk market to which participants are exposed. This is due to the fact that two parties enter into a contractual agreement after evaluating each other's capacity to honour that agreement. The chapter explained in detail, and highlighted, the different methods to measure credit risk such as the structural and reduced form models as well as other approaches, such as spread-based and hazard models. The chapter briefly described the two approaches - the cohort and hazard - of estimating transition matrices from observed historical transitions. These transition matrices are later used as inputs to many credit risk applications, including the measurement of credit portfolio risk, and the pricing of credit derivatives. The chapter then discussed the importance of modelling the dynamics of higher moments and correlation in risk management, hedging, portfolio allocation and option pricing and presented the empirical findings of the behaviour of corporate bond prices, as well as the determination of the drivers behind credit spreads.

Chapter 3

Methodology

3.1 Introduction

The aim of this chapter is to introduce the wealth of econometric and statistical formulations that model the dynamic behaviour and the impact of a set of determinants on credit spread changes. Section 2 introduces the building blocks of time series modelling and the univariate time series models. This class of specifications attempts to capture the linear relationship between a financial time series and the information available prior to time t. As such, the correlations between the time series and its past values become the building blocks for studying financial and economic time series and are referred to as serial correlations or autocorrelations.

Section 2 also introduces the multivariate time series models, which attempt to explain changes in a financial variable, by reference to the movements in the current or past values of other explanatory variables. Consequently, the dynamic relationships between different asset returns play an important role in portfolio management and allocation as well as trading and hedging. Finally, as the behaviour of a financial time series can change over time either permanently or temporarily, Markov regime switching models are applied. These type of models capture such changes in the behaviour of a time series, and are known as a structural breaks or regime shifts.

Section 3 introduces the econometric models for modelling the time-varying volatility, skewness and kurtosis of an asset return. Volatility is measured by the variance or standard deviation and it is a crude estimate of an asset's risk, since volatility may change over time. Therefore, it is important to model the dynamics of volatility, such as volatility clustering or volatility pooling. Mandlebrot (1963) was the first to notice the phenomenon of volatility clustering in the stock market. He noticed the tendency of large changes to follow large changes and small changes to follow small changes. In other words, volatility tends to be positively correlated with its level from previous periods. Inspired by Mandlebrot's observation, a series of studies began to model the behaviour of volatility in financial time series.

Engle (1982) introduces the Autoregressive Conditional Heteroscedasticity (ARCH) to model the time-varying volatility of a time series, which is later generalized by Bollerslev (1986), who proposes the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. Since then numerous studies have been introduced in modelling the time-varying volatility of equity returns, commodity prices, interest rates, exchange rates and other financial assets. However, financial time series including the European credit spreads exhibit leptokurtosis and leverage effects, and the GARCH models are extended to capture the dynamics of higher moments.

In addition, similar to the univariate time series models, structural breaks can affect the properties of the mean and variance processes. As described in the work of Lamourex and Lastrapes (1990), the persistence of conditional volatility is attributed to structural breaks. As such, Section 3 presents the Markov regime switching GARCH models that are able to capture changes in the dynamics of the volatility of financial time series.

Section 4 presents the multivariate volatility models and Section 5 the principal component analysis which are able to capture the dynamics of covariances and correlation of the European credit spread indices. Correlation is a statistic that indicates whether the returns of two or more assets are likely to present a systematic, linear relationship. However, a number of studies have shown that covariance and correlation of financial series vary over time and have an important role in the pricing of derivatives, portfolio selection, trading and hedging, and risk management. The first study to model time-varying covariances is Bollerslev *et al.* (1988), which is a springboard for a number of other models, such as Bollerslev (1990), who proposes the constant Conditional Correlation Multivariate GARCH model, Alexander (2001), who presents an Orthogonal-GARCH, Engle and Kroner (1995), who propose the Baba, Engle, Kraft and Kroner representation (BEKK), and Engle and Shephard (2001) who developed a Dynamic Conditional Correlation Multivariate GARCH model.

Section 6 presents the univariate and multivariate Value-at-Risk estimation procedures as well as RiskMetrics and Historical Simulation that are applied in order to examine the performance of the different models in risk management. Finally it presents Christoffersen's (1998) back testing procedure to examine the adequacy of the VaR estimates.

3.2 Mean Process Models

3.2.1 Introduction to Linear Time Series Analysis and Basic Notation

Univariate time series analysis provides a framework to model and forecast the dynamic structure of a financial time series. The econometric models introduced in this section are the autoregressive (AR), moving-average (MA) and mixed autoregressive moving-average (ARMA) models. These models capture the linear relationship between its current value, y_t, and the information available in its past; this linear relationship is known as serial correlation or autocorrelation. Understanding these simple models provides the building blocks of the more sophisticated financial econometric models of the later chapters.

One of the important concepts of time series analysis is stationarity. The importance of stationarity is described in Brooks (2002), who argues that stationarity of a series can influence its behaviour and properties, the same way as non-stationary data can lead to spurious regressions and the standard assumptions for asymptotic analysis will be invalid under this case. A time series is said to be strictly stationary when its joint distribution does not change under a time shift. This is a strong condition and according to Tsay (2001) it is hard to empirically verify it; instead weakly stationarity is assumed. A time series is weakly stationary when the mean, variance and covariance between y_t and y_{t-l} are constant and finite. The covariance $Cov(y_t, y_{t-l}) = \gamma_l$ is called the lag-l autocovariance or autocovariance function. However, another measure to describe the linear relationship between observations is applied - the autocorrelation function:

$$\rho_l = \frac{Cov(y_t, y_{t-l})}{\sqrt{Var(y_t)Var(y_{t-l})}} = \frac{Cov(y_t, y_{t-l})}{Var(y_t)} = \frac{\gamma_l}{\gamma_0} \tag{3.1}$$

where the weakly stationarity property is used for $Var(y_t) = Var(y_{t-l})$ and autocorrelation takes values from $-1 \leq \rho_l \leq 1$. A weakly stationary time series is not serial correlated when $\rho_l = 0$ for all l>0.

One approach to test for a unit root [10]would be to evaluate the autocorrelation function of the financial time series. In the case of a unit root process, where shocks remain indefinitely in the system, the autocorrelation function decays slowly to zero. Therefore, the process can be mistaken for a highly persistent but stationary process. However, there are standard techniques for unit root test and the most widely applied are Dickey and Fuller (1979)[11] and Phillips-Perron (1988).

[10] A process is non-stationary when the roots of the characteristic equation lie inside the unit circle and is stationary otherwise. To better understand what a unit root is, consider the equation: $y_t = 1.2y_{t-1} + \varepsilon_t$. The first step is to express the equation using lag operation notation: $(1-1.2L)\,y_t = \varepsilon_t$ and the characteristic equation becomes: $1-1.2L = 0$ solving towards L, the root is: $L = 0.83 \prec 1$. As L is smaller than 1 and the process therefore is not stationary.

[11] For instance, the simple test of Dickey and Fuller (1979) examines the null hypothesis that a series contains a unit root, φ=1, against the alternative that the series is stationary, φ<1, in the following equation: $y_t = \varphi y_{t-1} + \varepsilon_t$

3.2.2 Moving Average, Autoregressive and ARMA Processes

A moving average process (MA) is the simplest class of models as it is described by a linear combination of white noise processes[12]. The q order of a moving average model denoted as MA(q) is given by:

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}, \quad \varepsilon_t \sim IID\left(0, \sigma^2\right) \tag{3.2}$$

The properties of a moving average process of order q are:

$$E\left(y_t\right) = \mu \tag{3.3}$$

$$Var\left(y_t\right) = \gamma_0 = \left(1 + \theta_1^2 + \ldots \theta_q^2\right)\sigma^2 \tag{3.4}$$

$$\gamma_s = \begin{cases} \left(\theta_s + \theta_{s+1}\theta_1 + \ldots + \theta_q \theta_{q-s}\right)\sigma^2, & for \quad s = 1, \ldots, q \\ 0, \quad for \quad s > q \end{cases} \tag{3.5}$$

Thus a MA process has a constant and finite mean and variance, and autocovariances which may be non-zero up to q lag and will be zero thereafter.

An autoregressive (AR) process models and predicts variables by conditioning the current value of a variable on its past values and an error term. The autoregressive model of order p, AR(p), is given as:

$$y_t = \mu + \varphi_1 y_{t-\iota} + \ldots + \varphi_p y_{t-p} + \varepsilon_t = \mu + \sum_{i=1}^{p} \varphi_\iota y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim IID\left(0, \sigma^2\right) \tag{3.6}$$

The properties of the autoregressive process of order p are:

$$E\left(y_t\right) = \frac{\mu}{1 - \varphi_1 - \ldots - \varphi_p} \tag{3.7}$$

$$Var\left(y_t\right) = \frac{\sigma^2}{\left(1 + \varphi_1^2 + \ldots + \varphi_p^2\right)} \tag{3.8}$$

The autocovariances and autocorrelation functions are estimated throught the Yule-Walker equations. They express the correlogram $\left(\tau_s\right)$ as a function of the autoregressive coefficients $\left(\varphi_1\right)$ such that:

[12] A series y_t is called white noise if y_t is a sequence of independent and identically distributed random variables with mean zero, variance σ^2 and autocorrelation function zero.

$$\begin{aligned}
&\tau_1 = \varphi_1 + \tau_1\varphi_1 + \ldots + \tau_{p-1}\varphi_p \\
&\vdots \\
&\tau_p = \tau_{p-1}\varphi_1 + \tau_{p-2}\varphi_2 + \ldots + \varphi_p
\end{aligned} \tag{3.9}$$

For the simple AR(1) model the autocovariance and autocorrelation, for any s lag, are:

$$\begin{aligned}
&\gamma_s = \frac{\varphi_1^s \sigma^2}{\left(1-\varphi_1^2\right)} \\
&\tau_s = \varphi_1^s
\end{aligned} \tag{3.10}$$

For a stationary AR model, the autocorrelation function will decay geometrically to zero.

An ARMA process is a combination of an autoregressive model with order p, AR(p), and a moving average model with order q, MA(q). The properties of an ARMA process will be a combination of the AR and MA processes. Therefore, an ARMA(p,q) process will have geometrically declining autocorrelation and partial autocorrelation functions. The autocorrelation function of the ARMA process will exhibit characteristics from both the AR and MA models, but beyond the q-th lag, autocorrelation function becomes identical to the AR model.

The identification of the order of p and q is undertaken with the help of a set of criteria, known as information criteria. The two most popular criteria are Akaike's (1974) information criterion (AIC) and Schwarz's (1978) Bayesian information criterion (BIC) and are given as:

$$\begin{aligned}
&AIC = \ln\left(\hat{\sigma}^2\right) + \frac{2k}{T} \\
&BIC = \ln\left(\hat{\sigma}^2\right) + \frac{2k}{T}\ln\left(T\right)
\end{aligned} \tag{3.11}$$

where $\hat{\sigma}^2$ denotes the residual variance which is given by the residual sum of squares, RSS, divided by the number of degrees of freedom, $\left(T-k\right)$, where k is given as: $k = p + q + 1$and T is the sample size. The choice of the order of p and q is made on the number of parameters that minimize the information criteria. Therefore, adding an extra term will reduce the information criteria only when the RSS is reduced more than the penalty term.

3.2.3 Vector Autoregressive Models

The univariate models examined previously try to model the linear relationship between a financial time series and the information available prior to time t. However, movements in one market may affect movements in another market, or changes in a financial variable may be attributable to movements in the current or past values of other explanatory variables. Multivariate analysis jointly models financial time series in order to understand their dynamic structure and has an important role in portfolio management and allocation, as well as, trading and hedging.

A Vector autoregressive (VAR) model belongs to the multivariate time series analysis class of models and describes the dynamic structure of a financial time series. The VAR(1)[13] model for a multivariate time series $\mathbf{y_t}$ with k-dimensions is given as:

$$\mathbf{y_t} = \boldsymbol{\alpha_0} + \mathbf{A}\mathbf{y_{t-1}} + \boldsymbol{\varepsilon_t} \tag{3.12}$$

where $\mathbf{A}$ is a N x N matrix, $\boldsymbol{\varepsilon_t}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix $\boldsymbol{\Sigma}$. In a bivariate case (i.e. k=2, $\mathbf{y_t} = \left(y_{1t}, y_{2t}\right)'$ and $\boldsymbol{\varepsilon_t} = \left(\varepsilon_{1t}, \varepsilon_{2t}\right)'$) the VAR(1) consists of the following equations:

$$\begin{aligned} y_{1t} &= a_{1,0} + a_{1,1}y_{1,t-1} + a_{1,2}y_{2,t-1} + \varepsilon_{1t,} \\ y_{2t} &= a_{1,0} + a_{2,1}y_{1,t-1} + a_{2,2}y_{2,t-1} + \varepsilon_{2t,} \end{aligned} \tag{3.13}$$

For example, the coefficient $a_{1,2}$ expresses the conditional effect of $y_{2,t-1}$ on y_{1t} given $y_{1,t-1}$. In case $a_{1,2} = 0$, in the first equation, the y_{1t} does not depend on $y_{2,t-1}$. However, if $a_{1,2} = 0$, in the first equation, and $a_{2,1} \neq 0$, in the second equation, then there is an unidirectional relationship from y_{1t} to y_{2t}. If $a_{1,2} = a_{2,1} = 0$ then there is no relationship between y_{1t} to y_{2t} and if $a_{1,2} \neq a_{2,1} \neq 0$ then there exists a feedback relationship between y_{1t} and y_{2t}.

[13] An increase in the number of parameters may result in over-parameterisation and difficulty in estimation and forecasting. For example if there are N variables with K lags then $N + KN^2$ parameters need to be estimated.

3.2.4 Markov Regime Switching Models

Even though Vector autoregressive models capture the relationship between financial series, Markov regime switching models are able to identify the non-linear dynamics of a time series. These types of models spring from the work of Goldfeld and Quandt (1973) and Hamilton (1989) who model the structural breaks of the ARCH specification. They are extended by Hamilton (1993 and 1994) and Hamilton and Susmel (1994), who propose that the influence of explanatory variables can be allowed to be state-dependent, with the universe of possible occurrences being categorised by s_t different states.

Hamilton (1994) presents the general Markov regime switching regression model, which is given as: let $\mathbf{y_t}$ be an (N x 1) vector of observed endogenous variables and $\mathbf{x_t}$ a K x 1) vector of observed exogenous variables. If the process is governed by regime $s_t = j$ at time t, then the conditional density of $\mathbf{y_t}$, is given as:

$$f\left(\mathbf{y_t} \mid s_t = j, \mathbf{x_t}, \Omega_{t-1}; \mathbf{a}\right) \tag{3.14}$$

where **a** is a vector of parameters characterizing the conditional density. If there are *N* different regimes, then there are *N* different conditional densities and are collected in an (N x 1) vector denoted as $\boldsymbol{\eta}_\mathbf{t}$. For example if the model for the mean equation is given by:

$$y_t = a_{0,s_t} + a_{1,s_t} x_{1t} + ... + a_{k,s_t} x_{kt} + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma^2\right) \tag{3.15}$$

where x_{1t} to x_{kt} are the explanatory variables. Assuming normality of the error terms and the existence of two states j=1, 2 the conditional density vector $\boldsymbol{\eta}$ whose elements are given as:

$$\boldsymbol{\eta}_\mathbf{t} = \begin{bmatrix} f\left(y_t \mid s_t = 1, \Omega_{t-1}; \mathbf{a}\right) \\ f\left(y_t \mid s_t = 2, \Omega_{t-1}; \mathbf{a}\right) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sigma_1\sqrt{2\pi}} \exp^{\left[-\frac{\left(y_t - \bar{y}_{t,1}\right)^2}{2\sigma_1^2}\right]} \\ \dfrac{1}{\sigma_2\sqrt{2\pi}} \exp^{\left[-\frac{\left(y_t - \bar{y}_{t,2}\right)^2}{2\sigma_2^2}\right]} \end{bmatrix} \tag{3.16}$$

where $\bar{y}_{t,j}$ is the fitted mean equation at time *t* and state *j*.

Movements of the state variable between regimes are governed by a Markov process, which states that the probability of transition from state *i* at time *t-1* to state *j* at time *t* depends only on the state at time *t-1* and not on any other previous states. Thus, the time independent transition probabilities are given by:

$$\Pr\left\{s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots, \mathbf{x_t}, \Omega_{t-1}\right\} = \Pr\left\{s_t = j \mid s_{t-1} = i\right\} = p_{ij} \tag{3.17}$$

and these probabilities can be summarized in a matrix: $\mathbf{P} = \left(p_{ij}\right)$. The population parameters $\boldsymbol{\theta}$ that describe a time series governed by (3.14) consist of the vector **a** and the transition probabilities p_{ij}**.** The Markov chain is described by the random vector $\widehat{\boldsymbol{\xi}}_{\mathbf{t}}$, whose i^{th} element equals one if $s_t = i$. The Markov chain is assumed to be unobservable and, hence, the regime at time *t* is unknown. Therefore only probabilities are assigned to regimes. The optimal inference and forecast can be found by iterating the following pair of equations:

$$\widehat{\boldsymbol{\xi}}_{\mathbf{t/t}} = \frac{\widehat{\boldsymbol{\xi}}_{\mathbf{t/t\text{-}1}} \circ \boldsymbol{\eta}_{\mathbf{t}}}{\mathbf{1}'\left(\widehat{\boldsymbol{\xi}}_{\mathbf{t/t\text{-}1}} \circ \boldsymbol{\eta}_{\mathbf{t}}\right)} \tag{3.18}$$

$$E\left(\widehat{\boldsymbol{\xi}}_{\mathbf{t+1}} / \widehat{\boldsymbol{\xi}}_{\mathbf{t}}\right) = \mathbf{P}\widehat{\boldsymbol{\xi}}_{\mathbf{t}} \tag{3.19}$$

where $\circ$ denotes the element-by-element multiplication. The set of optimal parameters $\boldsymbol{\theta}$ can be obtained by maximizing the conditional log likelihood function:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log\left(f\left(\mathbf{y_t} \mid \mathbf{x_t}, \boldsymbol{\Omega}_{\mathbf{t\text{-}1}}, \boldsymbol{\theta}\right)\right) \tag{3.20}$$

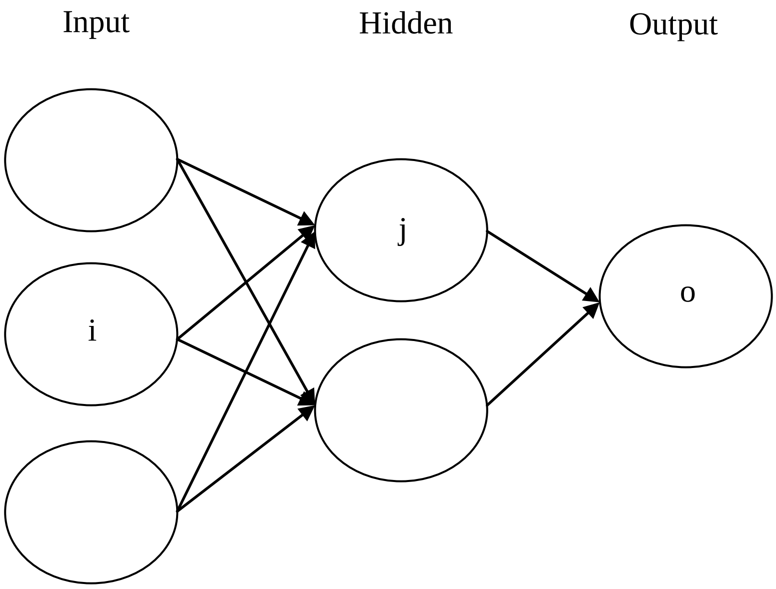
where

$$f\left(\mathbf{y_t} \mid \mathbf{x_t}, \boldsymbol{\Omega}_{\mathbf{t\text{-}1}}, \boldsymbol{\theta}\right) = \mathbf{1}'\left(\widehat{\boldsymbol{\xi}}_{\mathbf{t/t\text{-}1}} \circ \boldsymbol{\eta}_{\mathbf{t}}\right) \tag{3.21}$$

3.2.5 Feed Forward Neural Network

Another branch of the literature that allows for non-linear dynamics in a financial time series is the feed-forward neural network. A feed-forward neural network models complex relationships between input and output layers while the information moves in only one direction, from the input layer through the hidden layer or layers and, finally, to the output layer. Figure 3.1 presents an example of a 3-2-1 feed-forward neural network. This network has three nodes in the input layer, one hidden layer with two nodes and one node in the output layer.

Figure 3.1 Presents a 3-2-1 feed-forward neural network



Consider a feed-forward neural network with one hidden layer, the j^{th} node in this hidden layer is specified as (see Tsay, 2001):

$$h_j = f_j\left(a_{0,j} + \sum_{i\to j} w_{i,j}x_i\right) \tag{3.22}$$

where x_i is the value of the i^{th} input node, $w_{i,j}$ is the weight of the i^{th} input node feeding to the j^{th} hidden node, $a_{0,j}$ is called the bias and f_j is called an activation function as it processes information from one layer to the next and is assumed to be a logistic function such that:

$$f_j(z) = \frac{e^z}{1+e^z} \tag{3.23}$$

The output node is defined as:

$$o = f_o\left(a_{0,o} + \sum_{j\to o} w_{j,o}h_j\right) \tag{3.24}$$

where h_j is the value of the j^{th} node in the hidden layer, $w_{j,o}$ is the weight of the j^{th} node of the hidden layer feeding to the o^{th} output, f_o is assumed to be a linear activation function such that the output node becomes:

$$o = a_{0,o} + \sum_{j=1}^{n} w_{j,o}h_j \tag{3.25}$$

where n is the number of nodes in the hidden layer. Therefore, from equations (3.22) and (7.10) the output node becomes:

$$o = f_o\left[a_{0,o} + \sum_{j \to o} w_{j,o} f_j\left(a_{0,j} + \sum_{i \to j} w_{i,j} x_i\right)\right] \tag{3.26}$$

In order to apply the neural network the first step is to train it by determining the number of nodes, and estimating their biases and weights; and the second step is forecasting. Training a neural network entails the following two steps: the first step is to partition the input and output vector into three disjoint sets: *training, validation* and *test*. The neural network is trained on the *training set* and the *validation set* is used to examine the generalization ability of the network and to stop training before over-fitting. The generalization ability of a network refers to its ability to capture unforeseen inputs, i.e those on which the network has not been trained. Finally, the *test set* is used as a completely independent test of the network's generalization ability.

The second step entails the estimation of the biases and weights by minimizing the following nonlinear function:

$$S^2 = \sum_{t=1}^{T} \left(r_t - o_t\right)^2 \tag{3.27}$$

where r_t denotes the financial time series. The estimation of Equation (3.27) can be solved by a number of iterative methods such as: Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton backpropagation and Levenberg-Marquardt backpropagation among others[14]. Backpropagation[15] is a popular learning algorithm for network training which was introduced by Bryson and Ho (1969). The backpropagation algorithm starts by selecting a pair of input and output vectors with an equal size to the number of network inputs and outputs. It then selects an input and estimates the activation functions starting from the input layer up to the output layer. It then computes the difference between the estimated outputs and the real outputs of the time series, called the *output error*, and propagates the output error backward from the output layer to the input layer, while changing the weights at each node. Each iteration is called an epoch and lasts until a predetermined maximum number of epochs are achieved or the output error falls below a pre-specified threshold or both.

[14] For more information refer to Gill, Murray, and Wright, *Practical Optimization*, 1981.
[15] For a detailed derivation of backpropagation see Ripley, B., D., (1993). "Statistical Aspects of Neural Networks" In O. E. Barndorff-Nielsen, J., L., Jensen and W., S., Kendall, Chapman and Hall, London.

3.3 Univariate Volatility Models

3.3.1 ARCH/GARCH Models

Financial series exhibit a number of important properties such as non-Gaussian return distributions (e.g. leptokurtosis), volatility clustering and leverage effects. In his pioneering study, Engle (1982) introduces a formal approach in modelling the variance of a time series by conditioning its variance on the square lagged disturbances in an autoregressive form known as Autoregressive Conditional Heteroscedasticity (ARCH) model. The ARCH(q) model is given by:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}^2, \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \tag{3.28}$$

In order to ensure that the conditional variance is positive and stationary, the coefficients must satisfy the conditions: $\beta_0 > 0$ and $0 < \sum \beta_i < 1$. The ARCH model is extended into the Generalized Autoregressive Condition Heteroskedasticity (GARCH) framework by Bollerslev (1986), where variance is conditioned on lagged residuals as well as lagged variance itself. The GARCH(p,q) formulation is given by:

$$\sigma_t^2 = \beta_o + \sum_{i=1}^{q} \beta_{1,i} \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_{2.j} \sigma_{t-j}^2, \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \tag{3.29}$$

Similar to ARCH the parameters must satisfy the non-negativity constraints $\beta_0 > 0$, $\beta_{1,i} > 0$ and $\beta_{2,j} > 0$ and the sum $\sum \beta_{1,i} + \sum \beta_{2,j} < 1$ must be less than one in order for the unconditional variance $\left(\sigma_0^2 = \beta_0 \big/ \left(1 - \sum \beta_{1,i} - \sum \beta_{2,j}\right)\right)$ to be stationary and non-explosive. Although a variety of GARCH models has been proposed in the literature, the most commonly applied model in a financial time series is GARCH (1,1), as it is able to adequately capture the dynamics of variance (see Hansen and Lunde, 2005).

One of the restrictions of GARCH models is that they enforce a symmetric response of volatility to positive and negative shocks. In other words it is assumed that negative and positive shocks impact the volatility by the same magnitude. However, the literature in finance has shown that positive shocks impact volatility differently from negative shocks of the same magnitude. This phenomenon is called the leverage effect. GARCH models, in the presence of leverage effects, may lead to biased estimates of variance and inaccurate forecasts. Glosten *et al.* (1993) propose an extension of the GARCH model, the GJR-GARCH, to account for possible asymmetries and is specified as:

$$\sigma_t^2 = \beta_o + \sum_{i=1}^{p} \beta_{1,i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_{2,j} \sigma_{t-j}^2 + \beta_3 \varepsilon_{t-1}^2 I_{t-1}, \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \tag{3.30}$$

where I_{t-1} is an indicator function taking the value of one when the innovation term is negative, and zero otherwise. Therefore, the significance and sign of the β_3 coefficient in the estimated model should suggest whether the time varying volatility of the financial series responds differently to positive or negative shocks.

Another approach in this direction is Nelson (1991), who proposes the exponential GARCH or E-GARCH model, of which one possible specification can be given as:

$$\sigma_t^2 = \exp\left(\beta_0 + \sum_{i=1}^{p} \beta_{1,i} \left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^{q} \beta_{2,j} \log\left(\sigma_{t-j}\right) + \sum_{i=1}^{p} \beta_{3,i} \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) \right| \right), \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \tag{3.31}$$

The EGARCH model does not need to impose non-negativity constraints and, therefore, the parameters β_0, $\beta_{1,i}$, $\beta_{2,j}$ and $\beta_{3,i}$ can take any real value.

Although the asymmetric GARCH models can be specified in such a way as to capture the excess kurtosis by using Student-t or GED distributions, they fail to capture the dynamics of kurtosis and, more importantly, the variation of skewness over time. The studies of Bond (2001), Ricardo (2003), Bera and Kim (2002), Angelidis *et al.* (2003) among others suggest the application of non-Gaussian distributions that capture the higher moment dynamics.

3.3.2 The impact of non-Gaussian distributions in GARCH estimation

One of the distributions that has met with wide empirical success is Hansen's Skewed t-distribution. Hansen (1994) proposes an autoregressive conditional density model which allows its shape parametetrs to vary over time, within a set of permitted boundaries. Bond (2001) finds that GARCH models using asymmetric non-normal distributions - especially Hansen's Skewed t-distribution - provide better estimation and VaR results when compared with the Gaussian or the mixed jump-diffusion distributions. Daal and Yi (2005) examine the performance of a variety of alternative distributions such as Hansen's skewed t-distribution in GARCH specifications in comparison to a mixture of GARCH-Jump models using daily returns of the US, Korean, Indonesian, Mexican and Brazilian Indices during the May 1995 to July 2002 period. They report that models with leptokurtic distributions perform better in the high volatility series of the emerging markets, and that Hansen's (1994) skewed t-distribution provides the best overall VaR forecast estimates.

Jondeau and Rockinger (2003) extend Hansen's (1994) model by relaxing the parametric restrictions and, thus imposing, a time-varying structure for the two parameters that control the probabilistic mass of the density function. They test the model using daily forex data (DM/US, UK/US, YEN/US and FF/US) over the period from July 1991 to September 1999, as well as daily equity index returns (S&P 500, NIKKEI, DAX30, CAC40 and FTSE 100) over the period from August 1971 to September 1999; and on the 3-month and 10-year rates of the US, UK, Germany and France from January 1975 to September 1999. They find evidence of higher-moment persistence in all series and their cross-sectional analysis reveals that the higher-moments between the index returns and foreign exchange are strongly related.

Ricardo (2003) tests a variety of GARCH models that account both for symmetry and asymmetry as well as incorporating heavy tails distributions that estimate the probability of a maximum loss occurring. His results over daily returns of the Buenos Aires Stock Exchange (MERVAL) and Dow Jones Index reveal that asymmetric GARCH models provide better VaR estimates compared with other models.

Bera and Kim (2002) propose the Pearson Type IV distribution which accounts for asymmetry and excess kurtosis along with heteroskedasticity. Pearson Type IV distribution has three parameters that can be interpreted as variance, skewness and kurtosis and according to Bera and Kim (2002) they can be considered as different components of risk premiums. They apply their model in the daily returns of NYSE during the August 1991 to April 1996 period and find that the distribution is able to fit the data.

Another approach is Theodossiou (1998), who develops a skewed extension of the Generalized t-distribution[16]. He tests the model in S&P 500, TSE300 and TOPIC equity indices as well as in exchange rates (Canadian dollar / U.S. dollar, Japanese yen / U.S. dollar) and finds that his proposed distribution provides excellent fit of the empirical distributions.

Angelidis *et al.* (2004) examine the predictive accuracy of a variety of GARCH models with various distributions in producing accurate VaR estimates. After examining the daily returns of CAC40, DAX30, FTSE100, NIKKEI225 and S&P 500 from July 1987 to October 2002, they find: firstly, that the mean process has no important role in the forecast of VaR estimators; secondly, the more flexible the GARCH model the better its volatility forecasts; and thirdly, leptokurtic distributions provide better VaR estimators as they are able to capture the series characteristics.

Wilhelmsson (2007) proposes an Inverse Normal Gaussian (NIG) distribution which allows its parameters that resemble skewness and kurtosis to vary over time. His proposed model outperforms the Gaussian GARCH model in terms of VaR estimators of the daily returns of S&P 500 over the July 1962 to September 2005 period.

However, these parametric distributional models do not allow for any dynamic in higher-moments of estimated residuals and, consequently, dependent variables.

3.3.3 The importance of time-varying higher moments

One of the first studies in modelling the dynamics of higher-moments is that of Campbell and Siddique (1999). They present an autoregressive conditional variance and skewness specification, assuming a non-central t-distribution for the error term in the mean equation. They examine the daily, weekly, and monthly returns of different global equity index returns (S&P 500 from January 1969 to December 1997, Dax 30 from January 1975 to December 1997, Nikkei 225 from January 1980 to December

[16] The Generalized t-distribution is proposed by Mcdonald, J., B., and Newey, W., K.., 1988. Partially adaptive estimation of regression models via the generalized T distribution. *Economic Theory,* 4**,** 428-457.

1997 and Mexico, Chile, Thailand and Taiwan over the period from January 1989 to January 1998) and find a number of stylized facts. Firstly, the asymmetric variance is consistent with conditional skewness. Secondly, conditional skewness is not only time-varying, but also has an impact on the persistence of conditional variance. Finally, the dynamics of higher-moments are affected by frequency and seasonality.

Brooks *et al.* (2005) use a GARCH specification with a time varying-kurtosis, (GARCHK) which allows the fourth moment of the Student's t-distribution to vary over time. The application of the model to daily returns of the S&P 500 and UK FTSE 100, as well as the total return of the US and UK ten-year maturity benchmark bond indices over the period from January 1990 until June 2000, revealed that kurtosis is time-varying.

Other approaches on modelling the fourth moment specification include He and Terasvirta (2002b), who examine the fourth moment structure of the GARCH(1,1) model with conditionally non-normal innovations, and Rubio *et al.* (2005), who derive expressions for the kurtosis of GARCH and stochastic volatility in the presence of conditionally non-normal leptokurtic innovations.

León *et al.* (2004) propose a GARCH model which allows for time-varying volatility, skewness and kurtosis. The model is estimated assuming a variation of the Gram-Charlier series expansion of the normal density function for the standardized residuals η_t:

$$
\begin{aligned}
& y_t = a_0 + \sum_{m=1}^{n} a_m y_{t-m} + \varepsilon_t, \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \\
& \varepsilon_t = \eta_t \sqrt{\sigma_t}, \quad \eta_t \sim (0,1) \\
& \sigma_t = \beta_0 + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, \\
& s_t = \gamma_0 + \sum_{i=1}^{q} \gamma_i \eta_{t-i}^3 + \sum_{j=1}^{q} \gamma_q s_{t-q}, \\
& k_t = \delta_0 + \sum_{i=1}^{q} \delta_i \eta_{t-i}^4 + \sum_{j=1}^{p} \delta_j k_{t-j},
\end{aligned}
\tag{3.32}
$$

where s_t is skewness and k_t kurtosis. The density function for the standardized residuals η_t condition on the information available in *t-1* is given as:

$$f(\eta_t) = \varphi(\eta_t)\psi^2(\eta_t) / \Gamma_t \tag{3.33}$$

where,

$$\begin{aligned} \varphi(\eta_t) &= \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{\eta_t^2}{2}}, \\ \psi(\eta_t) &= \left[1 + \frac{s_t}{3!}\left(\eta_t^3 - 3\eta_t\right) + \frac{k_t - 3}{4!}\left(\eta_t^4 - 6\eta_t^2 + 3\right)\right], \\ \Gamma_t &= 1 + \frac{s_t^2}{3!} + \frac{(k_t - 3)^2}{4!} \end{aligned} \tag{3.34}$$

where $\varphi(\cdot)$ denotes the probability density function of the standard normal distribution and $\psi(\cdot)$ the polynomial part of fourth order. Their proposed probability density function ensures that the density is positive for all parameter values in (3.32) and integrates to one (see Appendix 3.A for proof).

León *et al.* (2004) test the model in exchange rates (British Pound / USD, Japanese Yen / USD, German Mark / USD and Swiss Franc / USD) over the period from January 1990 to May 2002, and on daily index returns (S&P 500, NASDAQ, DAX30, IBEX35 and MEXBOL) over the period from January 1990 to May 2003. Not only do they find strong evidence of time-varying skewness and kurtosis but their model outperforms other specification with constant skewness and kurtosis in terms of in-sample predictive power.

3.3.4 Markov Regime Switching GARCH Model

Structural breaks characterized by changes in government policy, seasonality, business cycles, among others, can affect the properties of financial and economic time series both in terms of their mean value and volatility. Models that allow changes in the state of the market to be incorporated in the model spring from the work of Goldfeld and Quandt (1973) and Hamilton (1989), who model the structural breaks of the mean process. They are later extended by Hamilton and Susmel (1994), Gray (1996), Dueker

(1997), Klaassen (2002) and Alizadeh and Nomikos (2004), to name but a few, to allow both the mean and the variance to switch between different states.

The simple GARCH(1,1) specification is extended into a two-state Markov regime switching GARCH[17] such that:

$$\begin{aligned} y_t &= a_{0,s_t} + \sum_{m=1}^{n} a_{m,s_t}\varepsilon_{t-m,s_t} + \varepsilon_{t,s_t}, \quad \varepsilon_{t,s_t} \sim IID\left(0,\sigma_{t,s_t}^2\right) \\ \sigma_{i,t}^2 &= \beta_{0,s_t} + \sum_{j=1}^{p} \beta_{j,s_t}\sigma_{t-j,s_t}^2 + \sum_{i=1}^{q} \beta_{i,s_t}\varepsilon_{t-i,s_t}^2 \end{aligned} \tag{3.35}$$

where s_t denotes the state of the market is and can take two values: one denotes the high-volatility regime and the other the low volatility regime. Movements of the state variable between regimes are governed by a first-order Markov process. This means that the probability distribution of the state at any time t depends only on the state at time t-1 and not on any previous time intervals and a transition probability of $\Pr(s_t = j \mid s_{t-1} = i) = p_{ij}$ relates the two states. Hence the transition matrix $\boldsymbol{\pi}$ for a two state Markov chain is given by:

$$\boldsymbol{\pi} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{3.36}$$

p_{12} represents the probability that state 1 will be followed by state 2, p_{21} is the probability that state 2 will be followed by state 1. Transition probabilities p_{11} and p_{22} are the probabilities that there will be no change in the state of the market in the following period. Hamilton (1989 and 1994) shows that the unconditional probabilities of being in regime one, P_1, or regime two P_2, are given by:

$$\mathrm{P}(\mathrm{s_t}=1) = \mathrm{P}_1 = \frac{1-p_{22}}{2\text{-}p_{11}\text{-}p_{22}} \quad \text{and} \quad \mathrm{P}(\mathrm{s_t}=2) = \mathrm{P}_2 = \frac{1-p_{11}}{2\text{-}p_{11}\text{-}p_{22}} \tag{3.37}$$

In the Markov regime switching GARCH models an issue of path-dependence arises, which renders the estimation procedure intractable. This happens because the conditional variance depends not only on past information but also on the current regime and its previous states. This results in the integration of a number of regime paths that grow exponentially with the sample size, thus, rendering the estimation tractability infeasible.

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[17] An increase in the number of regime sometimes results in over-parameterisation and difficulty in estimation due to regime-switching models being non-linear.

Gray (1996) suggests integrating the unobserved regime path in the GARCH specification by using the conditional expectation of the past variance. The drawback of this approach lies in the complexity it generates when forecasting multi-step-ahead volatility. Klaassen (2002) suggests the usage of the conditional expectation of the lagged conditional variance with a broader information set than Gray (1996) and presents the following expression for the conditional variance, for simplicity the GARCH(1,1) is presented:

$$\sigma_{i,t}^2 = \beta_{0,i} + \beta_{1,i}\varepsilon_{t-1}^2 + \beta_{2,i}E_{t-1}\left\{\sigma_{i,t-1}^2 \mid \tilde{s}_{t-1}\right\} \tag{3.38}$$

where $\tilde{s}_{t-1}$ is the history of the regime path ($\tilde{s}_{t-1} = s_{t-1},\ s_{t-2},\ s_{t-3},\ \cdots$). The log-likelihood function for the above regime-switching GARCH model is defined as:

$$LL = \sum_{t=-R+\omega+1}^{T+\omega} \ln\left[P_{1,t}\, f(r_t \mid s_t = 1) + (1 - P_{1,t})\, f(r_t \mid s_t = 2)\right] \tag{3.39}$$

where $\omega = 0,1,...,n$ and $f(r_t \mid s_t = i)$ is the conditional distribution given that regime i occurs at time t. The advantages of Klaassen's (2002) specification when compared with the other models are the high degree of flexibility for capturing the persistence of shocks to volatility, and the fact that it allows the computation of multi-step ahead volatility forecasts calculated recursively as in standard GARCH models. The multi-step-ahead volatility forecasts are computed as a weighted average of the multi-step ahead volatility forecasts in each regime, where the weights are the prediction probabilities.

$$\hat{\sigma}_{T,T+\tau}^2 = \beta_{i,0} + \left(\beta_{i,1} + \beta_{i,2}\right)E_T\left\{\hat{\sigma}_{T,T+\tau-1}^2 \mid s_{T+\tau}\right\} \tag{3.40}$$

where $\hat{\sigma}_{T,T+\tau}^{(i)}$ indicates the τ-step-ahead volatility forecast at time T.

3.4 Multivariate Volatility Models

This section generalizes the univariate models and presents multivariate models which capture the dynamic relationships between multiple volatility processes. Multivariate volatility models have a wide application in the financial world, ranging from asset and option pricing, to hedging and risk management, as well as portfolio construction and asset allocation.

This section adopts a similar notation to the one already presented for the univariate models. Consider a multivariate process $\mathbf{y_t} = \{y_{1t}, \ldots y_{nt}\}$ such that:

$$\mathbf{y_t} = \boldsymbol{\mu}_\mathbf{t} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{3.41}$$

where $\boldsymbol{\mu}_\mathbf{t}$ is the conditional mean vector and

$$\boldsymbol{\varepsilon}_\mathbf{t} = \mathbf{H_t^{1/2} z_t} \tag{3.42}$$

where $\mathbf{H_t^{1/2}}$ is a N x N positive definite matrix containing all variance and covariances of $\mathbf{y_t}$ and $\mathbf{z_t}$ is an random vector with:

$$\begin{aligned} \mathbf{E(z_t) = 0} \\ \mathbf{Var(z_t) = I_N} \end{aligned} \tag{3.43}$$

where $\mathbf{I_N}$ is an order N identity matrix. The conditional variance matrix of the multivariate process is then given by:

$$\mathbf{Var(y_t) = Var(\boldsymbol{\varepsilon}_t) = H_t^{1/2} Var(z_t) H_t'^{1/2} = H_t} \tag{3.44}$$

where $\mathbf{H_t}$ is the conditional variance matrix with $\mathbf{H_t^{1/2}}$ being a NxN positive definite matrix and estimated by the Choleksy factorization of $\mathbf{H_t}$. The Cholesky decomposition is the decomposition of a symmetric, positive definite matrix, $\mathbf{H_t}$, into the product of a lower triangular matrix , $\mathbf{L_t}$, with unit diagonal elements and a diagonal matrix, $\mathbf{G_t}$, with positive diagonal elements such that: $\mathbf{H_t = L_t G_t L_t'}$ (see Tsay, 2001).

The following paragraphs denote the different parameterizations of the time evolution of the covariance matrix $\mathbf{H_t}$. There are three categorizations for multivariate GARCH models. The first is the generalizations of the univariate GARCH model; the second and third are linear and nonlinear combinations of the univariate GARCH models. The first includes the VEC, BEKK, RiskMetrics and factor models; the second includes the orthogonal models and; the third category the constant and dynamic correlation models.

3.4.1 Generalizations of the univariate GARCH model: DVEC, BEKK and Factor Models

Bollerslev *et al*, (1988) propose the VEC Multivariate Generalized Autoregressive Conditional Heteroscedastic model (VEC-MGARCH), in which the elements of the covariance matrix are linear function of the lagged squared errors and cross products of errors and the lagged values in the covariance matrix. The model is defined as:

$$\mathbf{H_t} = \mathbf{C} + \sum_{\mathbf{i=1}}^{\mathbf{q}} \mathbf{A_i} \circ \boldsymbol{\varepsilon}_{\mathbf{t\text{-}i}} \boldsymbol{\varepsilon}'_{\mathbf{t\text{-}i}} + \sum_{\mathbf{j=1}}^{\mathbf{p}} \mathbf{B_j} \circ \mathbf{H_t} \tag{3.45}$$

where p and q are positive integers, $\mathbf{A_i}$ and $\mathbf{B_j}$ are symmetric matrices with dimension, and $\circ$ denotes the Hadamard product; that is, element-by-element multiplication. However, the large number of estimated parameters, $(p+q)\left(N(N+1)/2\right)^2 + N(N+1)/2$, makes the estimation procedure computationally intense and demanding. Therefore, Bollerslev *et al.* (1988) propose a more simplidfied version of the VEC-MGARCH model by assuming that $\mathbf{A_i}$ and $\mathbf{B_j}$ to be diagonal matrices and each element of $\mathbf{H_t}$ depends only on its own lag and the previous value of $\boldsymbol{\varepsilon}_{\mathbf{t\text{-}i}} \boldsymbol{\varepsilon}'_{\mathbf{t\text{-}i}}$. Under this formulation the number of estimated parameters drops to $(p+q+1)N(N+1)/2$. The drawback of the VEC models is that they do not guarantee a positive definite covariance matrix without the application of strong parametric restrictions.

Engle and Kroner (1995) propose the Baba, Engle, Kraft and Kroner (BEKK) specification with the attractive property that the conditional covariance matrix is positive definite by construction. The BEKK specification is given as:

$$\mathbf{H_t} = \mathbf{CC'} + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{A_i} \left(\boldsymbol{\varepsilon}_{\mathbf{t\text{-}i}} \boldsymbol{\varepsilon}'_{\mathbf{t\text{-}i}}\right) \mathbf{A'_i} + \sum_{\mathbf{j=1}}^{\mathbf{q}} \mathbf{B_j} \mathbf{H_{t\text{-}j}} \mathbf{B'_j} \tag{3.46}$$

where $\mathbf{A_{kj}}$, $\mathbf{B_{kj}}$ and $\mathbf{C}$ are N x N parameter matrices and $\mathbf{C}$ is a lower triangular matrix. The decomposition of the constant term into a product of two triangular matrices is to ensure positive definiteness of $\mathbf{H_t}$.

However, this formulation has a number of disadvantages. The first drawback is the difficulty of interpreting the parameters $\mathbf{A_i}$ and $\mathbf{B_j}$ and secondly, since the estimated number of parameters of the model increases as the number of series and dimensions of p and q increase $\left[N^2\left(p+q\right)+N\left(N+1\right)/2\right]$. A simplified version of the BEKK specification is the diagonal BEKK model such that $\mathbf{A_i}$ and $\mathbf{B_j}$ are diagonal matrices. The scalar BEKK is the most restricted version of the diagonal BEKK with $\mathbf{A}=a\mathbf{I}$ and $\mathbf{B}=b\mathbf{I}$, where a and b are scalars.

The large number of parameters renders the numerical estimation of the VEC and BEKK formulation difficult. Instead factor models impose the dynamics of common factors on the elements of the covariance matrix $\mathbf{H_t}$. Engle *et al.* (1990) were the first to propose a $\mathbf{H_t}$ parameterization based on the co-movements of a small number of common factors. Bollerslev and Engle (1993) extend the above parameterization and model the common factor conditional variances. Lin (1992) proposes a factor model that is a based on the BEKK formulation, the F-GARCH(1,1,K) by assuming k=1,....,K, $\mathbf{A_k^*}$ and $\mathbf{B_k^*}$ have rank of one and have the same left and right eigenvectors.

$$\mathbf{A_k^*}=a_k\mathbf{w_k}\boldsymbol{\lambda}'_\mathbf{k} \quad \textit{and} \quad \mathbf{B_k^*}=\beta_k\mathbf{w_k}\boldsymbol{\lambda}'_\mathbf{k} \tag{3.47}$$

where a_k and β_k are scalars, $\mathbf{w_k}$ and $\boldsymbol{\lambda}'_\mathbf{k}$ are Nx1 vectors satisfying:

$$\mathbf{w}'_\mathbf{k}\boldsymbol{\lambda}_\mathbf{i}=\begin{cases}0 & k\neq i\\ 1 & k=i\end{cases} \tag{3.48}$$

$$\sum_{n=1}^{N}w_{kn}=1 \tag{3.49}$$

By substituting the above equations into the BEKK formulation (3.46) the following expression is derived with the identification restriction of equation (3.49):

$$\mathbf{H_t}=\mathbf{C}+\sum_{\mathbf{k=1}}^{\mathbf{K}}\boldsymbol{\lambda}_\mathbf{k}\boldsymbol{\lambda}'_\mathbf{k}\left(a_k^2\mathbf{w}'_\mathbf{k}\boldsymbol{\varepsilon}_\mathbf{t-1}\boldsymbol{\varepsilon}'_\mathbf{t-1}\mathbf{w_k}+\beta_k^2\mathbf{w}'_\mathbf{k}\mathbf{H_{t-1}}\mathbf{w_k}\right) \tag{3.50}$$

The $\mathbf{H_t}$ is a reduced rank K matrix but remains full ranked as $\mathbf{C}$ is positive definite. The $\boldsymbol{\lambda}_\mathbf{k}$ vector and $w'_k\varepsilon_t$ scalar are called the k-th factor loading.

Another variant of the factor models is Vrontos *et al.* (2003) who propose the full-factor GARCH model, defined as:

$$\mathbf{H_t} = \mathbf{W}\mathbf{\Sigma_t}\mathbf{W}' \quad (3.51)$$

where $\mathbf{W}$ is an N by N triangular matrix, with ones on its diagonals and the diagonals of $\mathbf{\Sigma_t} = diag\left(\sigma_{1,t}^2, \ldots, \sigma_{N,t}^2\right)$ contain the conditional variances of the j-th factor. By construction $\mathbf{H_t}$ is positive definite

3.4.2 Orthogonal Models: O-GARCH and GO-GARCH

Orthogonal models try to simplify the dynamic structure of a multivariate volatility process. Orthogonal models assume that a large number of interrelated variables can be linearly transformed into a set of uncorrelated components by the means of an orthogonal matrix. The first model to be examined is the O-GARCH model which is proposed by Alexander (2001). It allows N x N GARCH covariance matrices to be estimated from just *m* univariate GARCH models; where *N* are the number of variables and *m* the number of principal components (see Chapter 3 section 4.6 for more details regarding principal component analysis). The time-varying covariance matrix $\mathbf{H_t}$ is given as:

$$\mathbf{H_t} = \mathbf{A}\mathbf{D_t}\mathbf{A}' \quad (3.52)$$

where $\mathbf{A} = \left(w_{ij}^*\right)$ is the normalised factor weight vector, **W**, from principal component analysis, $\mathbf{D_t}$ is the diagonal matrix of variances of the principal components estimated by a GARCH model and requires the estimation of $N(N+5)/2$ parameters. Alexander (2001) illustrates that Equation (3.46) yields a positive semi-definite matrix at every point in time even when the number of principal components is less than the variables of the original system.

van der Weide (2002) presents the GO-GARCH model which is a generalization of the O-GARCH model. The GO-GARCH specification does not impose the transformation matrix **W** to be orthogonal as in the case of the O-GARCH model, which in GO-GARCH is given as:

$$\mathbf{W} = \mathbf{P}\mathbf{\Lambda}^{1/2}\mathbf{U} \quad (3.53)$$

where $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_N), \quad \lambda_1 > \ldots > \lambda_N$ and λ are the eigenvalues, **P** is the matrix of corresponding eigenvectors to eigenvalues, and **U** is given as:

$$\mathbf{U} = \prod_{\mathbf{i<j}} \mathbf{R_{ij}}(\theta_{ij}), \quad -\pi \le \theta_{ij} \le \pi, \quad i, j = 1, \ldots, n \tag{3.54}$$

where $R_{\mathbf{ij}}(\theta_{ij})$ performs a rotation in the plane spanned by e_i and e_j over an θ_{ij} angle. The conditional correlation of the error term is defined as:

$$\mathbf{R_t} = \mathbf{D_t^{-1} V_t D_t^{-1}}, \quad where \quad \mathbf{D_t} = (\mathbf{V_t} \circ \mathbf{I})^{\mathbf{1/2}} \; and \quad \mathbf{V_t} = \mathbf{W \Sigma_t W'} \tag{3.55}$$

where $\circ$ denotes the Hadamard product and $\mathbf{V_t}$ denotes the conditional covariances of the variables. Finally, the O-GARCH and GO-GARCH are covariance stationary if the *m* univariate GARCH process is stationary.

3.4.3 Conditional Correlation Models: CC-GARCH, DCC-GARCH and GDC-GARCH

Under this category, models are able to estimate separately the individual conditional variances and the covariances. These types of models require the estimation of far less parameters than those compared in the previous sections. Among these models are CC-GARCH, DCC-GARCH and GDC-GARCH formulations.

Bollerslev (1990) proposed the Constant Correlation MGARCH model in which the conditional correlations are constant. Let a multivariate GARCH model with returns from N classes exhibit a conditional multivariate normal distribution of zero mean and covariance matrix, $\mathbf{H_t}$ and $\mathbf{r_t} \sim N(0, \mathbf{H_t})$ where $\mathbf{r_t}$ are the returns, with either zero mean or the residuals from a filtered time series. The covariance matrix is defined as:

$$\mathbf{H_t} = \mathbf{D_t R D_t} = \left(\rho_{ij}\sqrt{\sigma_{iit}\sigma_{jjt}}\right) \tag{3.56}$$

where $\mathbf{D_t}$ is a N x N diagonal matrix of time varying standard deviations from a univariate GARCH model with $\sqrt{\sigma_{it}}$ for the i^{th} diagonal element and $\mathbf{R}$ is the constant correlation matrix which is a symmetric positive definite matrix with $\rho_{ii} = 1 \quad \forall \quad i$.

While the assumption of the constant correlation ensures the positive definiteness and provides computational simplicity and estimation, it may not be appropriate and too restrictive in real life applications. In this respect Tsui and Yu (1999), Tse (2000), Bera and Kim (2002) and Engle (2002) highlight the dynamic correlation of bonds and other financial instruments. To address this problem, Tse and Tsui (2002) and Engle and Shephard (2001) propose the Dynamic Conditional Correlation Multivariate GARCH model, by retaining the CC's decomposition but making the conditional correlation matrix in Equation (3.56) time-varying. Engle and Shephard (2001) test their model on 100 assets of the S&P 500 sector indices including the composite and the 30 Dow Jones Industrial Average Stock including the average. They find strong evidence of time-varying correlation and illustrate the flexibility of the DCC-GARCH model to capture asymmetric effects in volatility or to incorporate long memory volatility models.

Tse and Tsui (2002) propose the following dynamic correlation structure:

$$\mathbf{R_t} = \left(\mathbf{1} - \boldsymbol{\theta}_1 - \boldsymbol{\theta}_2\right)\mathbf{R} + \boldsymbol{\theta}_1 \boldsymbol{\Psi}_{\mathbf{t-1}} + \boldsymbol{\theta}_2 \mathbf{R_{t-1}} \tag{3.57}$$

where θ_1 and θ_2 satisfy the $0 \le \theta_1 + \theta_2 < 1$, $\mathbf{R}$ is a time-invariant N x N positive definite parameter matrix with $\rho_{ii} = 1$, and $\boldsymbol{\Psi}_{\mathbf{t-1}}$ is an N x N well-defined correlation matrix of the standardised residuals at time *t-1*, with the following specification:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^{M} \varepsilon_{i,t-m}\varepsilon_{j,t-m}}{\sqrt{\sum_{m=1}^{M} \varepsilon^2{}_{i,t-m} \sum_{h=1}^{M} \varepsilon^2{}_{j,t-m}}} \tag{3.58}$$

Engle and Shephard (2001) propose the following dynamic correlation structure:

$$\begin{aligned} \mathbf{H_t} &= \left(\mathbf{1} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{a_m} - \sum_{\mathbf{j=1}}^{\mathbf{q}} \boldsymbol{\beta}_\mathbf{n}\right)\bar{\mathbf{H}} + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{a_i}\left(\boldsymbol{\varepsilon}_{\mathbf{t-i}}\boldsymbol{\varepsilon}'_{\mathbf{t-i}}\right) + \sum_{\mathbf{j=1}}^{\mathbf{q}} \boldsymbol{\beta}_\mathbf{j}\mathbf{H_{t-j}}, \\ \mathbf{R_t} &= \mathbf{H_t^{-1}H_tH_t'^{-1}} \end{aligned} \tag{3.59}$$

where $\bar{\mathbf{H}}$ is the unconditional covariance of the standardized residual resulting from the first stage estimation. An element of the DCC or $\mathbf{R_t}$, consists of time-varying correlation as $\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}}$ and $\mathbf{H}_{\mathbf{t}}'^{\mathbf{-1}}$ is a N x N diagonal matrix consisting of the square root of the diagonal element of $\mathbf{H}_{\mathbf{t}}$, where:

$$\mathbf{H}_{\mathbf{t}}' = \begin{bmatrix} \sqrt{h_{11}} & 0 & . & 0 \\ 0 & \sqrt{h_{22}} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \sqrt{h_{kk}} \end{bmatrix} \tag{3.60}$$

The estimation of the DDC-GARCH is performed over two steps. The first entails the estimation of univariate GARCH models for series and the second stage, the standardized residuals from the first stage, are used to estimate the parameters of the dynamic correlation. The likelihood function of the second stage is given as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(2\log\left(\left|\mathbf{R}_{\mathbf{t}}\right|\right) + \boldsymbol{\varepsilon}_{\mathbf{t}}'\mathbf{R}_{\mathbf{t}}^{\mathbf{-1}}\boldsymbol{\varepsilon}_{\mathbf{t}}\right) \tag{3.61}$$

Kroner and Ng (1998) propose a general dynamic covariance model that nests several of the already presented models. The GDC model is defined:

$$\mathbf{H}_{\mathbf{t}} = \mathbf{D}_{\mathbf{t}}\mathbf{R}_{\mathbf{t}}\mathbf{D}_{\mathbf{t}} + \boldsymbol{\Phi} \circ \boldsymbol{\Theta}_{\mathbf{t}} \tag{3.62}$$

where $D_t = \left(d_{ijt}\right), \quad d_{ijt} = \sqrt{\theta_{iit}} \quad \forall i, \quad d_{ijt} = 0 \quad \forall i \neq j$, R_t can be specified either by Tse and Tsui (2002) or Engle and Shephard (2001) formulations, $\boldsymbol{\theta}_{\mathbf{ijt}} = \boldsymbol{\omega}_{\mathbf{ij}} + \mathbf{a}_{\mathbf{i}}'\boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}_{\mathbf{t-1}}'\mathbf{a}_{\mathbf{j}} + \mathbf{g}_{\mathbf{i}}'\mathbf{H}_{\mathbf{t-1}}\mathbf{g}_{\mathbf{j}}, \quad \mathbf{i}, \mathbf{j}$, $\mathbf{a}_{\mathbf{i}}$ and $\mathbf{g}_{\mathbf{i}}$ are N x 1 vector of parameters.

3.4.4 Testing for Constant Correlation

Engle and Shephard (2001) claim that testing the hypothesis of constant correlation can be difficult to evaluate, due to the time-varying volatilities of correlations and the misspecifications of the estimated models. Tse (2000) proposes a test for the null of constant conditional correlation against the alternative of a dynamic correlation structure, and Bera (1996) tests the null of constant conditional correlation against a diffuse alternative. But neither of these tests cannot generalize to higher dimensions. In this regard, Engle and Shephard (2001) proposed a test of whether the estimated correlation matrix $\mathbf{R}_{\mathbf{t}}$ is constant over time. Let the null $H_0 : \mathbf{R}_{\mathbf{t}} = \bar{\mathbf{R}}$, $\forall t \in T$ and the alternative hypotheses be:

$$H_1 : vech^u\left(\mathbf{R_t}\right) = vech^u\left(\bar{\mathbf{R}}_\mathbf{t}\right) + \beta_1 vech^u\left(\mathbf{R_{t-1}}\right) + \beta_2 vech^u\left(\mathbf{R_{t-2}}\right) + ... + \beta_p vech^u\left(\mathbf{R_{t-p}}\right) \quad (3.63)$$

where $vech^u$ is a modified $vech$ which only selects elements above the diagonal. The next step is the estimation of univariate GARCH processes and the standardization of the residuals. A variable of N x 1 vector such as:

$$\mathbf{Y_t} = vech^u\left[\left(\frac{\mathbf{r_t}}{\mathbf{D}\sqrt{\bar{\mathbf{R}}}}\right)\left(\frac{\mathbf{r_t}}{\mathbf{D}\sqrt{\bar{\mathbf{R}}}}\right)^{\mathbf{T}} - \mathbf{I_k}\right] \quad (3.64)$$

where $\mathbf{D}$ is the N x N diagonal matrix of time varying standard deviations from univariate GARCH models, and $\mathbf{r_t} / \mathbf{D}\sqrt{\bar{\mathbf{R}}}$ is a N x 1 vector of residuals standardized under the null. Finally the following Vector autoregression is computed:

$$\mathbf{y_t} = a + \beta_1 \mathbf{Y_{t-1}} + ... + \beta_n \mathbf{Y_{t-n}} + \boldsymbol{\eta}_\mathbf{t} \quad (3.65)$$

Under the null hypothesis, the constant and all of the lagged parameters in the VAR model will be zero, where the test statistics are asymptotically distributed as χ^2_{s+1}.

3.5 Principal Component Analysis

Another statistical technique that has been applied extensively in interest rate modelling is the Principal Component Analysis (PCA), and it is the basis of the Orthogonal-GARCH specification described in the previous sections. PCA is developed by Pearson (1901) and is a statistical technique for transforming correlated variables into a smaller number of uncorrelated variables called the principal components. Given a *n*-dimensional multivariate process $\mathbf{Y} = \{Y_1, \ldots Y_n\}$ the principal components which are a linear combination of the multivariate process are given as:

$$\mathbf{P = YW} \quad (3.66)$$

where $\mathbf{W}$ is called factor weight vector and the *i-th* principal component of the multivariate process is given as:

$$\mathbf{P_i} = w_{1i}Y_1 + w_{2i}Y_2 + \ldots w_{ni}Y_n = \mathbf{YW_i} \quad (3.67)$$

In order to estimate the factor weights the variance of the vector $\mathbf{P}$ given by the following equation is maximized given the constraint that $\mathbf{W'W = 1}$:

$$Var\left(\mathbf{P}\right) = \mathbf{W'\Lambda W} \quad (3.68)$$

where $\mathbf{\Lambda}$ is a symmetric matrix of order n of the sample correlations between the variables and it is given as:

$$\mathbf{\Lambda} = \mathbf{Y}'\mathbf{Y} \tag{3.69}$$

The maximization method applied is known as the method of Lagrange multiplier and it is written as:

$$\mathbf{L} = \mathbf{W}'\mathbf{\Lambda}\mathbf{W} - \boldsymbol{\lambda}\left(\mathbf{W}'\mathbf{W} - \mathbf{I}\right) \tag{3.70}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier. The eigenvalues of the symmetric matrix $\mathbf{\Lambda}$ which are ranked according with their magnitude $\lambda_1 > \lambda_2 > \ldots \lambda_n$; and the corresponding eigenvectors $W_1, W_2 ..., W_n$ are estimated by solving the characteristic equation:

$$\left(\mathbf{\Lambda} - \boldsymbol{\lambda}\mathbf{I}\right)\mathbf{W} = 0 \tag{3.71}$$

where $\mathbf{I}$ is the identity matrix. Finally, the percentage of the variance that is explained by the first k-components is given by:

$$\frac{\sum_{j=1}^{k} \lambda_j}{\sum_{i=1}^{n} \lambda_i} \tag{3.72}$$

In conclusion, principal component analysis is able to reduce the dimensionality of a data set which consists of a large number of inter related variables by transforming the data set into a new set of variables, the principal components, which are uncorrelated and are in order so that the first few account for most of the variation in the original data set.

3.6 Value at Risk

Value-at-Risk is the maximum loss expected to occur over a given time period with a given probability. With the adoption of BASEL (2005) the number of exceptions is used to determine the levels of capital requirements. These exceptions are the number of occasions when the actual loss is larger than the predicted VaR model.

3.6.1 Univariate Value at Risk Estimation

The VaR at time t at a% significance level is calculated as follows:

$$VaR_t = \mu_{t+n} + F^{-1}\left(a\right) \cdot \sqrt{\hat{\sigma}_{t+n}} \tag{3.73}$$

where $\mu_{t+1|t}$ is the conditional mean, $F^{-1}(a)$ is the corresponding empirical quantile of the assumed distribution, *n* is the investment horizon and $\hat{\sigma}_{t+n}$ is the volatility forecast at time *t+n*. Although, many implementations of VaR assume that asset returns are normally distributed, the Gram-Charlier expansions are a convenient tool to account for departures of normality and capture the distributional characteristics of financial series. Gram-Charlier expansions produce a density function that can be viewed as an expansion of the standard normal density function augmented with terms that capture the effects of skewness and excess kurtosis. The inverse cumulative density function[18] is given as:

$$F_{GC}^{-1}(a) = \varphi_{1-a}^{-1}\left(1 + \frac{s}{6}\left[\left(\varphi_{1-a}^{-1}\right)^2 - 1\right] + \frac{k}{24}\left[\left(\varphi_{1-a}^{-1}\right)^3 - 3\varphi_{1-a}^{-1}\right]\right) \tag{3.74}$$

where $\varphi(\cdot)^{-1}$ denotes the inverse cumulative density function of the standard normal distribution and *s* and *k* the sample's skewness and kurtosis.

3.6.2 Multivariate Value-at-Risk Analysis

Value-at-Risk can be considered as a measurement of a portfolio's market risk and measures the maximum portfolio's loss expected to occur over a given time period with a given degree-of-confidence. The VaR at time *t* at alpha% significance level is estimated as follows:

$$P\left(\mathbf{w}_\mathbf{t}'\mathbf{P}_\mathbf{t} < VaR\right) = a \tag{3.75}$$

where $\mathbf{w}_\mathbf{t}$ are the weights at time *t* and $\mathbf{P}_\mathbf{t}$ the vector of the portfolio returns at time t.

According to Rombouts and Verbeek (2009) VaR is a function of a confidence level a, a density function, the portfolio weights $\mathbf{w}_\mathbf{t}$, a functional form of the mean vector $\boldsymbol{\mu}_\mathbf{t}$ and of a covariance matrix $\mathbf{H}_\mathbf{t}$. In the special case where the density function is the multivariate normal density function, the VaR estimators are computed as:

$$\mathbf{w}_\mathbf{t}'\boldsymbol{\mu}_\mathbf{t} + \left(\sqrt{\mathbf{w}_\mathbf{t}'\mathbf{H}_\mathbf{t}\mathbf{w}_\mathbf{t}}\right)Z_a \tag{3.76}$$

where Z_a is the ath quantile of the univariate standard normal distribution.

[18] For further details see Christoffersen, P., F., and Goncalves, S., (2005). "Estimation Risk in Financial Risk Management." Journal of Risk, 7(3), 1-28.

3.6.3 RiskMetrics

RiskMetrics was developed by J.P. Morgan in 1989 for the computation of VaR. The RiskMetrics (1996) uses the exponential weighted average model (EWMA) for the estimation of variance. The decay factor takes values between 0.9 and 1. J.P. Morgan suggests that for daily data a decay factor of value 0.94 should be used and for monthly 0.97. The EWMA variance can be specified as:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \tag{3.77}$$

and in a portfolio as:

$$\sigma_{ij,t}^2 = \lambda\sigma_{ij,t-1}^2 + (1-\lambda) r_{t-1}^i r_{t-1}^j \tag{3.78}$$

Engle and Bollerslev (1986) suggested the variances and covariances can be specified within RiskMetrics as an IGARCH specification, defined as:

$$\sigma_{ij,t} = (1-\lambda)\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \lambda\sigma_{ij,t-1} \tag{3.79}$$

The T-period VaR is then computed by the square root of time rule:

$$VaR(T) = \sqrt{T} * VaR \tag{3.80}$$

3.6.4 Historical Simulation

The Historical Simulation uses past returns to infer the cumulative distribution function. Such an approach allows the incorporation of asymmetries, fat tails and other unique distributional characterizations for the specific asset class. The VaR of the historical simulation is defined as:

$$VaR_t = (F_a)\left(\{r_{t-i}\}_{i=t-i-N}^{t-1}\right) \tag{3.81}$$

where the right hand of the equation is the a percentile of N past returns. The historical simulation faces two shortcomings. Firstly, it assumes that returns are independent and identically distributed; and secondly, it attributes equal weights to all past returns. An extension of the historical simulation is the Filtered Historical Simulation, which a semi-parametric technique. FHS is defined as:

$$VaR_t = F_a\left(\{z_{t-i}\}_{i=t-i-N}^{t-1} \mid \theta\right)\sigma_t \tag{3.82}$$

where z_{t-i} are the standardized residuals and σ_t is the standard deviation of the returns.

3.6.5 Extreme Value Theory

Extreme value theory is a branch of statistics devoted in modelling extreme deviations of the return distribution. The central result in extreme value theory (EVT) states that the extreme tail of a wide range of distributions can approximately be described by the generalized Pareto distribution. Christoffersen (2003) presents extreme value theory in the following manner. Consider the standardized returns such as:

$$z_{t+1} = r_{t+1}/\sigma_{t+1}, \quad z_{t+1} \sim i.i.d.(0,1) \tag{3.83}$$

The probability of the standardized returns *z* less a threshold *u* being below a value *x* given that the standardized returns are beyond the threshold *u*, for *x*>*u*, is given as:

$$\begin{aligned} F_u(x) &= \Pr\{z-u \le x / z > u\} = \\ \frac{\Pr\{z-u \le x\}}{\Pr\{z>u\}} &= \frac{F(x+u)-F(u)}{1-F(u)} \end{aligned} \tag{3.84}$$

The distribution $F_u(x)$ depends on the choice of threshold. As the threshold *u* becomes larger the $F_u(x)$ converges to the generalized Pareto distribution, for $\beta > 0$ is given:

$$G(x;\xi,\beta) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi}, & \xi \ne 0 \\ 1-(1+x/\beta), & \xi = 0 \end{cases} \tag{3.85}$$

and

$$\begin{cases} x \ge u & \xi \ge 0 \\ u \le x \le u-\beta/\xi & \xi < 0 \end{cases} \tag{3.86}$$

In order to estimate the parameters using maximum likelihood estimation method, a rearrangement of equation (3.84) is undertaken. Let $y = x+u$ then:

$$F(y) = 1-[1-F(u)][1-F_u(y-u)] = 1-\frac{\mathrm{T}_u}{\mathrm{T}}\left(1+\xi(y-u)/\beta\right)^{-1/\xi} \tag{3.87}$$

where *T* denotes the total sample size, T_u the number of standardized observations beyond the threshold *u*; while the term $[1-F(u)]$ is estimated by the proportion of standardized observations which are beyond the threshold, T_u/T and, finally, $\xi \ne 0$.

Christoffersen (2003) argues that for financial series ξ is positive and thus the Hill estimator exists:

$$\Pr(z > y) = 1 - F(y) \approx cy^{-1/\xi}, \quad y > u \tag{3.88}$$

Thus, the likelihood function for all observation y_i larger than the threshold *u* is:

$$\ln L = \prod_{i=1}^{T_u} f(y_i) \big/ \left[1 - F(u)\right] = \prod_{i=1}^{T_u} \frac{-\frac{1}{\xi} y_i^{-1/\xi - 1}}{cu^{-1/\xi}} \tag{3.89}$$

and therefore the log-likelihood function is:

$$\ln L = -\sum_{i=1}^{T_u} - \ln(\xi) - \left(\frac{1}{\xi} + 1\right) \ln(y_i) + \frac{1}{\xi} \ln(u) \tag{3.90}$$

Estimating the derivative with respect to ξ and *c* and setting it to zero, yields:

$$\begin{aligned} \xi &= \frac{1}{T_u} \ln(y_i / u), \\ c &= \frac{T}{T_u} u^{1/\xi} \end{aligned} \tag{3.91}$$

The cumulative distribution function then becomes:

$$F(y) = 1 - cy^{-1/\xi} = 1 - \frac{T_u}{T} (y/u)^{-1/\xi} \tag{3.92}$$

Finally, the loss quantile, $F^{-1}(1-p)$, which is applied in Value-at-Risk is defined as:

$$F^{-1}(1-p) = u\left[p \big/ (T_u / T)\right]^{-\xi} \tag{3.93}$$

The crucial component in EVT is the choice of the threshold *u*. If the threshold *u* is set too high then only a few observations are used to estimate the tail and thus the tail parameter, ξ will remain uncertain. On the other hand, if the threshold *u* is set too narrow then biased estimates of the tail parameter, ξ are computed. Christoffersen (2003) proposes that the threshold *u* should be the 95$^{\text{th}}$ percentile of the data set.

3.6.6 Other Value-at-Risk Approaches

The Monte Carlo simulation generates numerous paths for the evolution of the returns of an asset and assumes that they follow a stochastic process. The VaR is then estimated by the difference between the expected value of the distribution of the simulated returns and the a% lower percentile of the distribution. The advantage of this approach is the incorporation of the underlying dynamics of the return series, such as mean reversion and seasonality amongst others.

The peaks over threshold technique measures the excessiveness of returns over a high threshold and the occurrence of an event following a Poisson process. The peaks over threshold overcomes the disadvantages met by the extreme value theory, which include having difficulty selecting the appropriate length period and not considering the return dynamics.

The non-parametric family of VaR estimation techniques does not make any specific assumptions of the asset's return distribution, rather it utilizes the historical return's distribution in the estimation of the tail's distribution. The advantage of the non-parametric techniques is their ability to capture asymmetries and fat tails. The quantile estimation, the historical simulation and the filtered historical simulation belong to this approach.

The quantile estimation can be distinguished by two separate categories: the empirical quantile and the quantile regression. The first approach assumes that the return distribution in the prediction period is the same as in the sample period. The disadvantages of this approach are: firstly, the assumption that the return distribution remains constant from the sample period over to the prediction period; and secondly, the empirical quantiles are not efficient estimates of the theoretical ones.

Quantile regression takes into consideration the return's mean or variance dynamics. Koenker and Bassett (1978) suggest that the estimation of quantiles of the distribution function of returns should incorporate explanatory variables. Engle and Mangavelli (2004) suggest that Value at Risk should incorporate the same properties as volatility, for example clustering. The advantage of this approach is that it does not make any assumption of the return's distribution.

3.6.7 Risk Management Loss Functions

The adequacy of the VaR estimates is examined by the Christoffersen (1998) back-testing procedure. This test can be divided into three sub-tests: correct unconditional coverage; independence; and correct conditional coverage. In this respect, the rejection of the model can be categorized as the unconditional coverage failure or the exception clustering, or both.

Christoffersen (1998) tests are computed under the likelihood ratio specification and are the following:

The LR statistic for the correct unconditional coverage:

$$LR_{UC} = LR_{PF} = 2\left[\log\left(\pi_1^{n_1}\left(1-\pi_1\right)^{n_0}\right) - \log\left(\left(1-a\right)^{n_1} a^{n_0}\right)\right] \sim \chi^2\left(1\right) \qquad (3.94)$$

where n_1 is the number of 1's in the indicator series, n_0 is the number of 0's in the indicator series, α is the tolerance level of the VaR estimates and $\pi_1 = \dfrac{n_1}{n_1 + n_0}$.

The LR statistic for test of independence:

$$LR_{IND} = 2\left[\log\left(\left(1-\pi_{01}\right)^{\left(n_0-n_{01}\right)} \pi_{01}^{\;n_{01}} \left(1-\pi_{11}\right)^{\left(n_1-n_{11}\right)} \pi_{11}^{\;n_{11}}\right) - \log\left(\pi_1^{n_1}\left(1-\pi_1\right)^{n_0}\right)\right] \sim \chi^2\left(1\right) \qquad (3.95)$$

where n_{ij} is the number of i values followed by a j value in the indicator series,

$\pi_{ij} = \Pr\left\{I_t = i/I_{t-1} = j\right\}\left(i, j = 0,1\right)$ and $\pi_{01} = {n_{01}}/{n_0}, \pi_{11} = {n_{11}}/{n_1}$.

The LR statistic for the correct conditional coverage is given as the sum of the correct unconditional coverage and the independence test:

$$LR_{CC} = LR_{UC} + LR_{IND} \sim \chi^2\left(2\right) \qquad (3.96)$$

The best models are those that generate a coverage rate less than the nominal; and a model is considered to be adequate for risk management when it is able to pass both the conditional and unconditional coverage tests.

3.7 Conclusions

This chapter introduces the econometric and statistical formulations necessary to model the dynamic behaviour of credit spread changes, their conditional higher moments and their correlation. It introduced the building blocks of time series analysis, the uniavariate models which forecast and model the dynamic structure of a financial time series. It extends into multivariate time series analysis and Markov regime switching models. The former tries to model the dynamic structure of a financial time series by reference to movements in other explanatory variables, while the latter models changes in the mean process of time series.

However, the financial time series exhibit properties such as non-Gaussian return distributions, volatility clustering and time-varying higher moments. As such GARCH models introduced by Bollerslev (1986) are extended to incorporate higher moments, GARCH-SK, and changes in the dynamics of volatility, MRS-GARCH. In order, to examine the dynamic relationship of covariances and correlations the multivariate volatility models, such as: MGARCH, BEKK, O-GARCH among others, are introduced. Finally, it presents the risk management loss functions and back testing techniques depicted by Christoffersen (1998) to measure the efficiency of the different VaR estimates.

APPENDIX 3.A

Proof that León et al. (2004) proposed pdf integrates to one

This appendix illustrates that the function $f\left(\eta_t\right)$ in (3.33) integrates to one. $\psi\left(\eta_t\right)$ from (3.34) can be rewritten as:

$$\begin{aligned}\psi\left(\eta_t\right) &= 1+\frac{s_t}{3!}\left(\eta_t^{\,3}-3\eta_t\right)+\frac{k_t-3}{4!}\left(\eta_t^{\,4}-6\eta_t^{\,2}+3\right)= \\ &1+\frac{s_t}{\sqrt{3!}}H_3\left(\eta_t\right)+\frac{k_t-3}{\sqrt{4!}}H_4\left(\eta_t\right)\end{aligned} \tag{3A.1}$$

where $\left\{H_i\left(x\right)\right\}_{i\in N}$ represents the Hermite polynomials such that for $i \geq 2$ they hold the following recurrence relation:

$$H_i\left(x\right)=\left(xH_{i-1}\left(x\right)-\sqrt{i-1}H_{i-2}\left(x\right)\right)\Big/\sqrt{i} \tag{3A.2}$$

and they satisfy the following conditions:

$$\begin{aligned}&\int_{-\infty}^{\infty} H_i^2\left(x\right)\varphi\left(x\right)dx=1, \quad \forall i \\ &\int_{-\infty}^{\infty} H_i\left(x\right)H_j\left(x\right)\varphi\left(x\right)dx=0, \quad \forall i\neq j\end{aligned} \tag{3A.3}$$

where $\varphi\left(\cdot\right)$ denotes the standard normal density function. The integration of the conditional density function in (3.33) given the condition in (3A.3) becomes:

$$\begin{aligned}&\frac{1}{\Gamma_t}\int_{-\infty}^{\infty}\varphi\left(\eta_t\right)\left[1+\frac{s_t}{\sqrt{3!}}H_3\left(\eta_t\right)+\frac{k_t-3}{\sqrt{4!}}H_4\left(\eta_t\right)\right]^2 d\eta_t = \\ &=\frac{1}{\Gamma_t}\left[\int_{-\infty}^{\infty}\varphi\left(\eta_t\right)d\eta_t+\frac{s_t^2}{3!}\int_{-\infty}^{\infty}\varphi\left(\eta_t\right)H_3^2\left(\eta_t\right)d\eta_t+\frac{\left(k_t-3\right)^2}{4!}\int_{-\infty}^{\infty}\varphi\left(\eta_t\right)H_4^2\left(\eta_t\right)d\eta_t\right]= \\ &=\frac{1}{\Gamma_t}\left[1+\frac{s_t^2}{3!}+\frac{\left(k_t-3\right)^2}{4!}\right]=1\end{aligned} \tag{3A-4}$$

Chapter 4

Data Collection & Processing

4.1 Introduction

The aim of this chapter is to introduce the data set used for empirical analysis in the thesis and to review the statistical properties of the different variables selected. The data comprising this study is daily and includes the yields on Markit iBoxx Euro Corporate Indices[19] for the AAA, AA, A and BBB ratings and 1-3, 3-5, 5-7 and 7-10 maturities, the yields on German Government Bonds with maturities with 3, 5, 7 and 10 years to maturity, the MSCI[20] Berra Pan-Euro Index, the EURO STOXX 50 Volatility Index[21] (Vstoxx), EuroMTS Inflation Index[22] and Goldman Sachs S&P

[19] iBoxx Euro Corporate indices are part of the Markit iBoxx portfolio of indices which covers the cash bond markets. Markit was founded in 2001 as the first independent source of credit derivative pricing and some of their indices are: Markit iTraxx, Markit CDX and Markit iBoxxFX, among others. For more information visit: http://indices.markit.com/

[20] MSCI Barra is a provider of investment decision support tools to investment institutions. More information can be found in: http://www.mscibarra.com

[21] For more information on EURO STOXX 50 Volatility Index visit: http://www.stoxx.com/indices/index_information.html?symbol=V2TX

[22] For more information regarding EuroMTS Inflation Index visit: http://www.euromtsindices.com/index_new/content/inflation_linked/overview.php

GSCI Commodity Index[23] from 03/01/2000 to 30/04/2009, in total 2433 daily quotes which are available from DataStream.

The data set used covers the European bond market, which is a market with unique characteristics and dynamics. This is mainly due to the introduction of the Euro as a single currency and the introduction of Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia as new member states in 2004. The introduction of the single currency provided the means to reshape the mechanics of the European financial markets, by liberating vast inflows of fragmented capitals under the different currencies, and providing the means for cross-border investments around a unified legislative framework which promoted the economic expansion of the single market, making it the largest fixed income economy in the world[24].

The chapter is organised as follows: Section 4.2 presents the credit spreads and introduces their distributional properties. Section 4.3 presents the different variables applied in the determination of the drivers of credit spreads as well as the descriptive statistics and unit root tests.

4.2 The Credit Spreads

The credit spreads are computed as the difference between the yield on iBoxx Euro Corporate Indices and the yield on equivalent German government bonds. The selection of the German government bonds was based primarily on their liquidity and large size, and also on the fact that during the credit crisis period the German government market gained a safe-haven status in international financial markets, a status similar to that of the U.S. Treasury market (see Ejsing and Sihvonen, 2009, and Schknecht, von Hagen, and Wolswijk, 2010). In order to obtain the credit spreads of the rating indices, the difference between the yield on each iBoxx rating index and the

[23] For information regarding S&P GSCI refer to:
http://www2.goldmansachs.com/services/securities/products/sp-gsci-commodity-index/index.html

[24] Source International Monetary Fund World, Economic Outlook Database, April 2010:
http://www.imf.org/external/pubs/ft/weo/2010/01/weodata/weorept.aspx?sy=2004&ey=2009&scsm=1&ssd=1&sort=country&ds=.&br=1&c=998&s=NGDP_RPCH%2CNGDPD%2CPPPGDP%2CPPPPC&grp=1&a=1&pr.x=58&pr.y=1

3-year to maturity German government bond yield is selected, as it is the most liquid government bond. Finally, the credit spreads of the maturity indices are estimated as the difference between the yield on each iBoxx maturity index and the respective maturity of the yield on German government bonds.

The inclusion of the iBoxx indices in this study was decided on the premise that they would provide accurate and high quality bond prices. These indices are used as a proxy for the underlying market and serve as a basis for derivative products and portfolio valuation. Their construction abides by the highest market standards as indicated by the EFFAS European Bond Commission Standardised Rules on Constructing and Calculating Bond Indices. The inclusion criteria incorporate fixed and zero-coupon bonds including step-up[25] and event-driven bonds with a minimum time to maturity of one year. Each bond is assigned to an index based on the average ratings generated by Fitch, Moody and S&P. Lastly, the outstanding amount for corporate bonds to be included is € 500 million.

A number of financial institutions provide the bond prices for the calculation of the Markit iBoxx Indices, including: ABN Amro, Barclays Capital, BNP Paribas, Deutsche Bank, Dresdner Kleinwort, Goldman Sachs, HSBC, JP Morgan, Morgan Stanley, Royal Bank of Scotland, and UBS. Index rebalancing occurs on two specified dates. The first takes place on a quarterly basis and is concerned with coupon changes and coupon payments; the second rebalancing takes place on a monthly[26] basis and takes into account the changes in credit-worthiness and rating of the bond, which considers whether the issue or issuer has defaulted, downgraded or reduced the out-standing amount below threshold level.

Figure 4.1, 4.2 and 4.3 present the time series of the yield on German Government Bonds, the yield on the iBoxx indices, and credit spread indices respectively. For the German Government bonds, yields decrease from 2000 to the middle of 2005 and then increase until 2009, when they decrease to levels not observed previously in the sample period. Furthermore, the yields on the iBoxx Euro Corporate indices, decrease

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[25] A step-up bond is a bond that its coupon increases at regular intervals.

[26] The impact of the monthly rebalancing on the yield indices was examined by estimating a regression model with dummy variables reflecting these dates. The results showed that the rebalancing dates have no impact on the iBoxx indices.

from 2000 to the middle of 2005, from which point they increase up to 2009 at levels not previously seen. The credit spread indices widen during 2002 to 2004, and narrow during 2004 to 2007. In 2004 Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia join the European Union, which results in a large economic expansion of the single market and as a result narrows credit spreads up until 2007. Finally, the credit spreads increase during 2007 to 2009 to levels again not reached before in the sample period, signalling the credit crisis period.

By visually examining the credit spread time series of Figures 4.4 and 4.5 and their first differences in Figures 4.6 and 4.7, it can be seen that the credit spread indices show structural shifts both in the mean and volatility processes and appear to be highly correlated. Figures 4.4 and 4.5 reveal a number of structural shifts in the properties of credit spreads; such breaks may have been in the middle of 2001 and in the middle of 2003, while another may appear in the middle of 2007. Figures 4.6 and 4.7 present the changes in credit spreads and reveal that the period of 2000 to middle of 2005 is one of high volatility during which credit spreads changes exhibit volatility persistence. Volatility seems to decline from 2006 up until 2007 from which point it increases and exhibits spikes for the changes in credit spreads of the rating indices.

Figure 4.1 Presents the yield-to-maturity of German government benchmark bonds

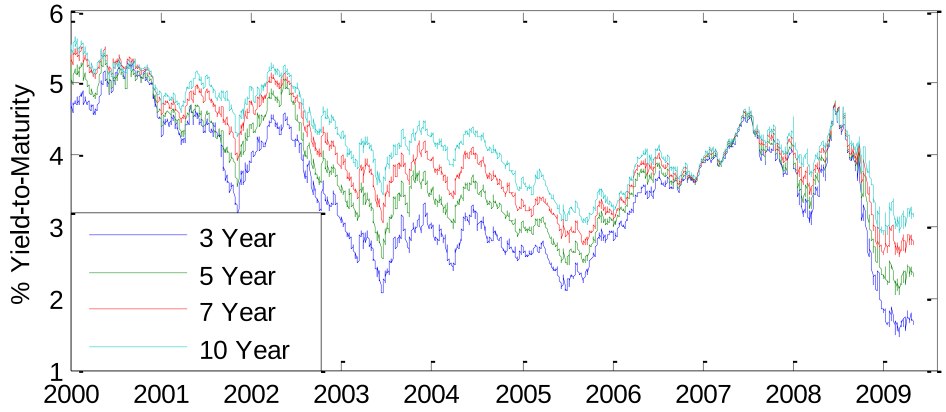


Figure 4.2 Presents the yield-to-maturity of the rating indices

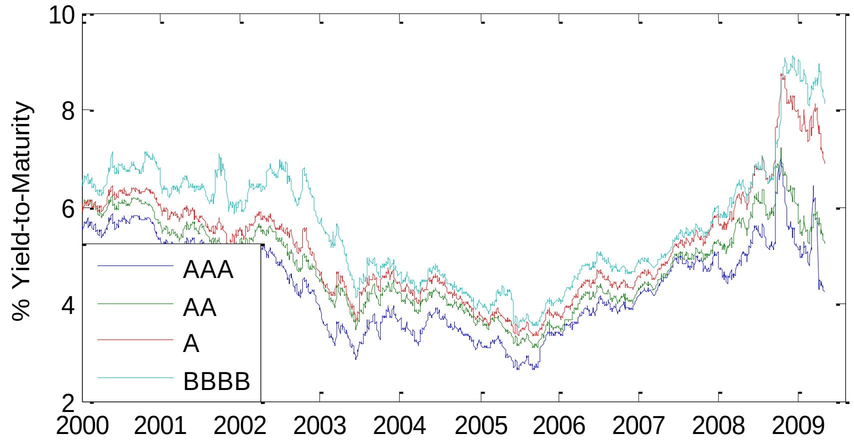


Figure 4.3 Presents the yield-to-maturity of the maturity indices

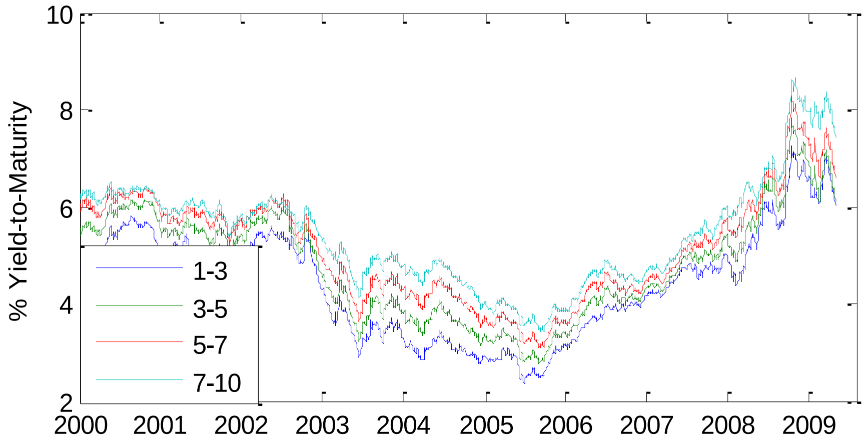


Figure 4.4 Presents the credit spreads of the rating indices

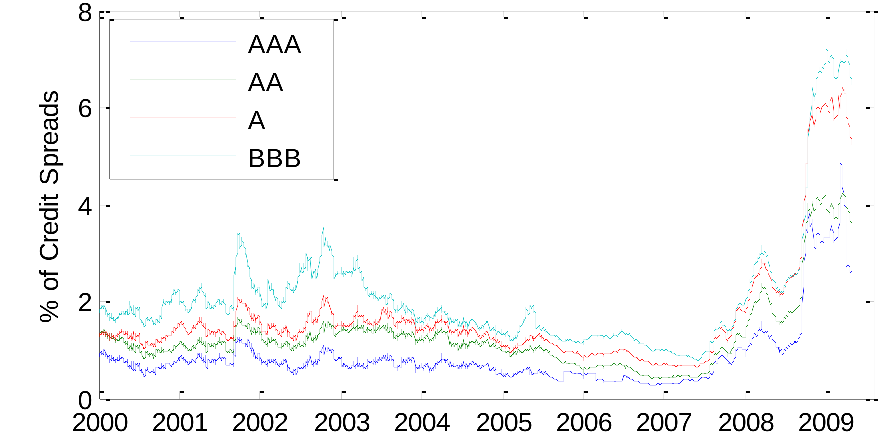


Figure 4.5 Presents the credit spreads of the maturity indices

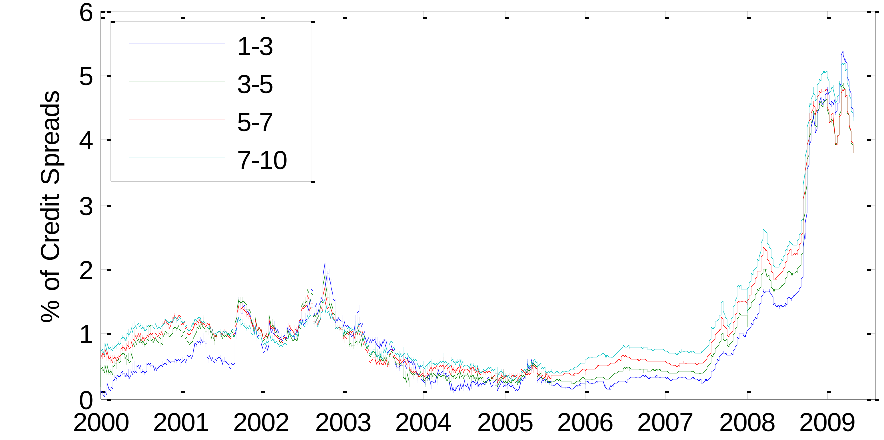


Figure 4.6 Presents the changes in credit spreads of the rating indices

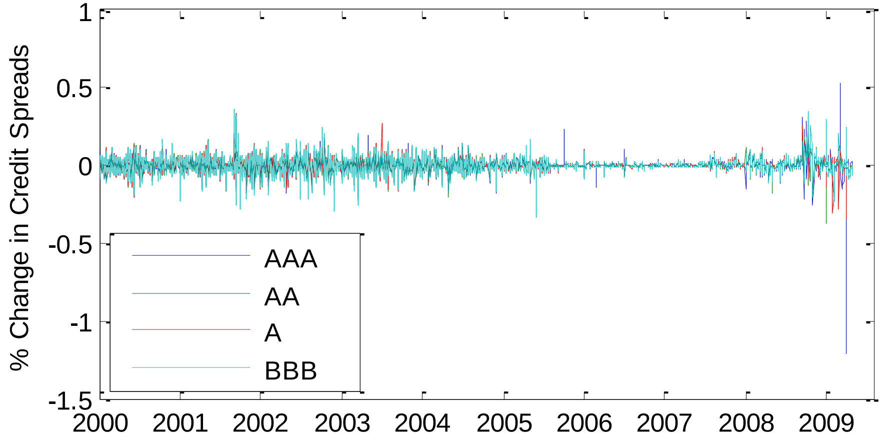


Figure 4.7 Presents the changes in credit spreads of the maturity indices

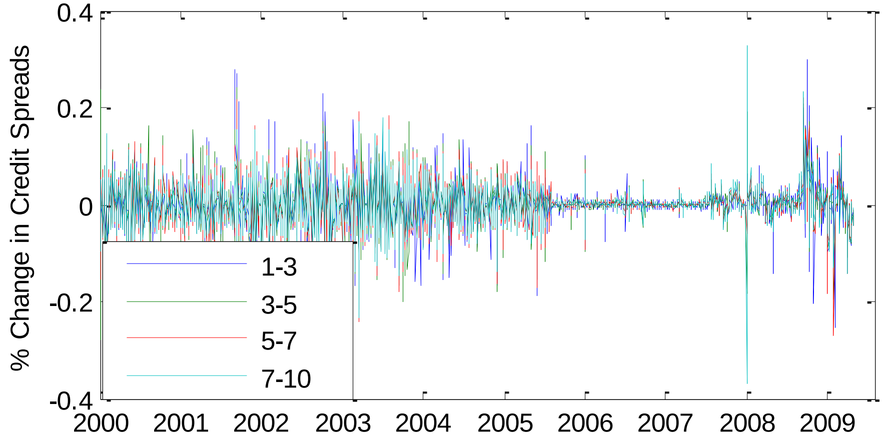


Table 4.1 Reports the descriptive statistics of yield indices, credit spreads and changes in credit spreads

Panel A: Yield Indices

Statistics	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
Mean	4.182	4.583	4.827	5.313	4.104	4.497	4.824	5.087
Std. Dev.	0.894	0.861	0.874	1.055	0.981	0.987	0.946	0.852
Skewness	0.300	0.332	0.242	0.155	0.056	0.125	0.110	-0.013
Kurtosis	1.830	1.833	1.781	1.557	1.667	1.642	1.661	1.786
Jarque-Bera[27]	436	429	447	578	535	532	494	412
Probability	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Panel B: Credit Spread Indices

Statistics	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
Mean	0.635	1.036	1.280	1.766	0.557	0.656	0.730	0.793
Std. Dev.	0.197	0.306	0.319	0.552	0.403	0.373	0.328	0.276
Skewness	0.165	-0.465	-0.094	0.619	1.156	0.737	0.537	0.135
Kurtosis	2.918	2.360	2.572	2.998	3.641	2.492	2.038	1.912
Jarque-Bera	31	339	58	408	1532	647	553	334
Probability	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Panel C: Changes in Credit Spreads

Statistics	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
Mean	-0.077	-0.119	-0.086	-0.139	0.021	-0.013	-0.030	-0.011
Std. Dev.	0.679	0.671	0.700	0.824	0.729	0.746	0.673	0.592
Skewness	0.197	0.096	0.077	-0.128	0.143	-0.258	-0.118	0.003
Kurtosis	7.749	7.656	7.872	9.305	7.852	7.237	5.860	5.910
Jarque-Bera	6047	5782	6327	10601	6290	4850	2193	2255
Prob	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Anson *et al*, (2004) argue that credit downgrades, defaults and bankruptcies of credit-risky assets can be described in terms of their skewness and kurtosis. Skewness is a measure of the asymmetry of a probability distribution. However, negative skeweness indicates a bias towards downside exposure, which means that there are more negative changes or large negative returns than positive ones. Kurtosis is a measure of the peakedness of a probability distribution. A distribution with positive kurtosis is called

[27] The Jarque-Bera test, tests for departure from normality and it's statistic is defined as:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4} \right)$$

where n is the number of observations, S is the sample skewness and K the sample kurtosis. The JB tests the null hypothesis of data normality and is chi-square asymptotic with two degrees-of-freedom.

leptokurtic, which means that the distribution has a more acute peakedness around the mean and has fatter tails.

The descriptive statistics for yields, credit spreads and the annualized first difference of credit spreads are presented in Panels A, B and C of Table 4.1, respectively. Panel A presents the different statistics of the yields on iBoxx Euro Corporate indices. Both the average yield and volatility, as measured by the standard deviation, increase as ratings decline and maturities increase. These results are in accordance with the literature, as an investor requires a higher return for assuming lower-rated and long-term bonds. Coefficients of skewness for all indices is positive but decreases as ratings decline, increases for the 1-3 and 3-5 indices and decreases for the 5-7 and 7-10 indices, indicating that lower ratings and higher maturities are prone to more negative movements due to credit downgrades, defaults and bankruptcies. The coefficients of the kurtosis reveal that distribution of the yield indices is platykurtic and appear to increase as maturities increase. Finally, the Jarque-Bera normality of the data test is rejected at 1% level for all indices.

The descriptive statistics of credit spreads presented in Panel B reveal that the average spread and volatility increase as ratings decline and maturities increase. Coefficients of skewness decrease for the higher rating (AAA and AA) indices and then increase for the lower rating (A and BBB) indices; finally they decrease as maturities increase. Negative coefficients of kurtosis indicate that the distribution of credit spreads is platykurtic. Finally, annualized changes in credit spreads presented in Panel C reveal that the volatility of credit spread changes increases as the ratings decline and decreases as the maturities increase. The coefficients of skewness decrease as the ratings decline and decrease for the lower-term maturity indices (1-3 and 3-5), but increase for the longer-term maturity indices (5-7 and 7-10). The coefficients of kurtosis suggest that the distribution of credit spread changes is leptokurtic and decrease as ratings improve and as maturities increase. Finally, the Jarque-Bera test for data normality is rejected for all indices at a 1% level.

Table 4.2 presents the unit root tests, which were performed by the application of the Augmented Dickey-Fuller test and the Phillips-Perron test allowing for a time trend both for the levels and first differences of the series. The levels of the credit spread do not reject the null hypothesis of a unit root at the 1% level while the first differences have an entirely opposite pattern. All series reject the unit root hypothesis at 1%, and most by a significant margin.

Table 4.2 Reports the estimation results of the unit root tests

	Augmented Dickey-Fuller Test		**Phillips-Perron Unit Root Test**	
Indices	Level	First Difference	Level	First Difference
AAA	-1.797	-42.636	-2.697	-68.314
AA	-0.043	-41.731	-0.782	-68.299
A	0.274	-39.432	-0.400	-65.482
BBB	-0.723	-33.606	-0.980	-54.955
1-3Y	-1.547	-37.602	-1.800	-59.888
3-5Y	-0.757	-41.938	-1.222	-66.949
5-7Y	0.300	-42.450	-0.310	-69.745
7-10Y	0.888	-43.838	0.225	-70.385
Critical values: (1%) = -3.962, (5%)=-3.411, (10%)=-3.127				

4.3 Determinants of Credit Spreads

One of the objectives of this study is to examine the impact of the risk-free rate and other important determinants on the credit spreads over different market conditions. Following the empirical and theoretical work presented in chapter two, the set of determinants selected to capture the default risk are the: risk-free rate; slope of the yield curve; MSCI Berra Pan-Euro Index; EURO STOXX 50 Volatility Index (Vstoxx); EuroMTS Inflation Index; and Goldman Sachs S&P Commodity Index (GSCI).

The risk-free rate is included because the theoretical models suggest that increases in the risk-free rate can lead to a reduction of credit spreads, by increasing the risk-neutral drift rate which decreases the probability of default. On the other hand, the slope of the yield curve is included, since it is considered a proxy for the future interest rate movements. A steep yield curve may imply a future increase in the interest rates which, in turn, may lead to a tightening of credit spreads. The risk-free rate and the yield curve slope considered in this study follow the literature presented by previous studies (see Alexander and Kaesk (2007), Collin-Dufresne, Goldstein and Martin

(2001) among others). They consider the former as the level of the 5-year benchmark rate and the latter as the difference between the 10-year and 2-year of the benchmark rate. In this study the benchmark rate is the yield on German Government bonds.

Davies (2004) considers as a proxy of the firm's equity value, the returns of the S&P 500 index and finds it to be inversely related to credit spreads. He argues that an expected positive stock market return may reduce the firm's leverage and thus lower the probability of default and, as a result, reduce credit spreads. Dufresne *et al*. (2001) argue that even if the probability of default remains constant for a firm, changes in credit spread can occur due to changes in the expected recovery rate, which is a function of the overall business climate. They consider the S&P 500 index as a proxy for the overall state of the economy. This study considers the MSCI Berra Pan-Euro Index as a proxy of the firm's equity value. The MSCI Berra Pan-Euro index was created to serve as the basis for derivative contracts, exchange traded funds and other passive investment products. The index comprises large and liquid securities with the goal of capturing 90% of the capitalization of the broader benchmark. It was developed with a base value of 1000 as of December 31, 1998, while, in June 2008, the index contained 227 securities from across the European market.

The VSTOXX index is considered as a variable that can be used as a proxy for market volatility and it is a measure of expected future volatility. VSTOXX provides a key measure of market expectations of near-term volatility based on the DJ Euro STOXX 50[28] options prices. Davies (2004) suggests that the VIX index, a similar index as to the VSTOXX but based on the implied volatilities of S&P 500 index options, represents a more forward-looking volatility variable than the simple derived volatility measures.

The core macroeconomic indicators used in this study are the inflation and commodity prices indices (measured as the EuroMTS and Goldman Sachs Commodity Indices). These indicators measure the direction of future market conditions and they have not been considered previously as drivers of credit spreads. Increasing values of inflation

[28]DJ Euro STOXX 50 Index is a Blue-chip representation of Sector leaders in the Euro Market. The index covers Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

and commodity prices could suggest a deterioration in the wider macroeconomic outlook which may affect the default probability and recovery rates, resulting in the widening of credit spreads. Another possible explanation may be that investors demand a higher yield, as option-free bonds do not protect investors from unexpected changes in inflation (see Risa, 2001). It may be possible to show that a future increase in inflation may widen credit spreads (see proof in Appendix 4.B). The EuroMTS Inflation Index measures the performance of the Euro zone's inflation linked sovereign debt. The EuroMTS Inflation Index includes only Euro zone's inflation linked government bonds issued with the minimum outstanding amount of €2 billion and have at least one year until final maturity. Finally, the S&P GSCI Index is a composite index of commodity sector returns representing an unleveraged, long-only investment in commodity futures that is broadly diversified across the spectrum of commodities. In April 2010 there were 24 commodities included in the index, which are grouped into the following categories: energy (71.53%), industrial metals (8.76%), precious metals (3.13%), agriculture (11.85%) and livestock (4.73%)[29].

Table 4.3 presents the descriptive statistics of the variables which are considered in the study as determinants of credit spreads: MSCI Pan-European Equity Index; Vstoxx Volatility Index; GSCI Goldman Sachs Commodity Index; EuroMTS Pan-European Inflation Index; level; and slope.

Multicollinearity occurs when explanatory variables in a regression model are highly correlated. Although, multicollinearity does not reduce the predictive power of the model, in its presence the coefficient estimates of the regression model become very sensitive to small changes in the specification. This means that adding or removing an explanatory variable leads to large changes in the coefficient values or the significances of other variables and, therefore, inferences regarding the estimated coefficients and significance of the explanatory variables may not hold. In order to investigate the presence of multicollinearity, the correlation between the variables is examined (see Brooks, 2002). Table 4.4 presents the correlation between the determinants. The returns of the equity index and volatility index are negatively

[29] Source: http://www2.goldmansachs.com/services/securities/products/sp-gsci-commodity-index/tables.html

correlated with a value of -0.771, while the remaining determinants are not highly correlated.

Table 4.3 Reports the descriptive statistics of the determinants

	Equity	Volatility	Commodity	Inflation	Level	Slope
Panel A: Reports the price levels of the determinants						
Mean	103.357	24.198	298.295	132.275	3.874	0.966
Std. Dev.	21.842	10.341	99.447	21.419	0.737	0.531
Skewness	0.073	1.335	0.598	-0.137	0.209	-0.325
Kurtosis	1.831	4.401	1.874	1.508	1.989	1.946
JBstat	369.314	2420.678	718.054	612.253	318.602	408.300
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Reports the first differences of the determinants						
Mean	$2.47*10^{-05}$	$-3.65*10^{-04}$	$4.63*10^{-04}$	$2.27*10^{-04}$	0.001	-0.001
Std. Dev.	0.012	0.050	0.014	0.003	0.050	0.029
Skewness	-0.164	0.889	-0.180	-0.340	0.448	-0.432
Kurtosis	6.201	7.744	4.353	4.596	7.294	10.278
JBstat	2756.978	6833.427	522.108	801.112	5123	14303
Probability	0.000	0.000	0.000	0.000		

Notes:
* Equity refers to MSCI Pan-Euro Index,
* Volatility to VSTOXX index,
* Commodity to S&P GSCI Index,
* Inflation to EuroMTS Inflation index,
* Level to the 5-year yield on German government bond,
* Slope to the difference between the 10-year and 2-year yields of the German government bonds.

Table 4.4 Reports the correlation coefficients between the determinants

	Level	Slope	Equity	Volatility	Commodity	Inflation
Level	1.000	-0.166	0.054	-0.040	-0.089	0.001
Slope	-0.166	1.000	0.003	-0.004	-0.046	0.034
Equity	0.054	0.003	1.000	-0.771	-0.114	0.209
Volatility	-0.040	-0.004	-0.771	1.000	0.049	-0.197
Commodity	-0.089	-0.046	-0.114	0.049	1.000	0.075
Inflation	0.001	0.034	0.209	-0.197	0.075	1.000

4.4 Conclusions

This chapter introduced the data set utilized in this study. It showed that the credit spreads involved in the study exhibit non-Gaussian return distributions as the Jarque-Bera test for normality is rejected for all rating and maturity indices, and revealed that the properties of credit spreads exhibit structural shifts. For instance, credit spreads widen during 2002 to 2004, narrow with the introduction of the new member states from 2004 to 2007 and, finally, during 2007 to 2009 increase to levels which have not been previously seen in the sample period. Therefore, the calculation of loss probabilities, such as VaR, would be seriously compromised by the assumptions of normality and by ignoring the possibility of shifting regimes.

Additionally, the chapter introduced the set of determinants selected to capture the default risk and presented key macroeconomic indicators, which have not been previously considered as drivers of credit spreads, namely, the inflation and commodity price indices. These indicators measure the direction of future market conditions; therefore, increasing values of inflation and commodity prices may suggest that the economic conditions deteriorate which may affect the default probabilities and recovery rates, which may result in credit spreads to widen.

APPENDIX 4.A

The Impact of Inflation on Credit Spreads

In Chapter 1 Section 4 a coupon bearing bond without any embedded options does not protect investors from unexpected changes in the Consumer Price Index (CPI). Investors therefore demand compensation for inflation risk, in the form of a higher yield. Let us assume we have a coupon bearing bond and at time *t+1* a future increase in inflation is expected. Investors, therefore, demand a higher yield, which can be described as follows:

$$y_{t+1} > y_t \qquad (4A.1)$$

However, the yield to maturity consists of the risk free rate and a compensation for the default risk, the credit spread. Equation 4A.1 then becomes:

$$\begin{aligned} &y_{t+1} > y_t \Leftrightarrow r_{t+1} + cs_{t+1} > r_t + cs_t \Leftrightarrow \\ &cs_{t+1} - cs_t > -\left(r_{t+1} - r_t\right) \end{aligned} \qquad (4A.2)$$

In addition, according to Fisher equation, the interest rate is given by:

$$r = r_n - i \qquad (4A.3)$$

where r_n is the nominal rate and i is inflation. If we further assume that the nominal rate between the period *t* and *t+1* it either remains constant or increases[30] then:

$$i_{t+1} > i_t \Leftrightarrow -i_{t+1} < -i_t \Leftrightarrow r_n - i_{t+1} < r_n - i_t \Leftrightarrow r_{t+1} < r_t \Leftrightarrow r_{t+1} - r_t < 0 \qquad (4A.4)$$

Therefore, from equations (4A.2) and (4A.4):

$$cs_{t+1} - cs_t > -\left(r_{t+1} - r_t\right) \Leftrightarrow cs_{t+1} - cs_t > 0 \Leftrightarrow cs_{t+1} > cs_t \qquad (4A.5)$$

Equation (4A.5) concludes that when inflation increases so do credit spreads.

[30] Governments usually raise interest rates in order to dampen inflationary pressure.

Chapter 5

Modelling Dynamics of Credit Spread Moments of European Corporate Bond Indices

5.1 Introduction

In the fixed income market, *Yield Spread* is defined as the difference between the yield to maturity of a corporate and the yield to maturity of a comparable government bond. Yield spreads are important variables in the financial markets as they are used for assessing the relative credit or default risk of issuers. As a result, yield spreads are also referred to as credit spread since they reflect the likelihood of failure of an entity to honour its obligation.[31] Yield spreads are used as indicators of economic progression, investment decisions, trading and hedging, as well as pricing credit derivatives. The role of yield spreads has become more important in the Euro zone fixed income markets since the launch of Euro as a single currency. The introduction of Euro has

[31] Although the terms *yield spread* and *credit spread* are used interchangeably in the literature, it should be noted that the yield spread reflects both credit risk and liquidity risk of corporate bonds, where as credit spread should only reflect the default risk of a bond.

resulted in the development of a deep and liquid bond market, which is promoted by: the once fragmented capital under the different currencies, the economic growth and development on a pan-European level under a unified legislative framework and allowing financial institutions to distribute credit risk on a wider base and in a more efficient manner by diversifying their liabilities away from the traditional country-specific load structures.

The literature on modelling credit spreads is extensive with theoretical studies concentrating on determination of credit spreads and empirical works attempting to find underlying variables which can explain the behaviour of credit spreads. Theoretical models are generally classified into reduced form and structural models, which differ in the way they deal with default process and consequently determination of credit spreads. Reduced form models describe the time a default occurs as a stochastic event governed by an intensity-based or hazard-rate process. As a result, they simplify the constraints of defining the causes of default (e.g. Jarrow et al., 1997, and Duffie and Singleton, 1999). On the other hand, structural models assume that the value of a firm follows a stochastic process and default occurs when the value of the firm falls below certain predetermined boundary (e.g. Black and Scholes, 1973, Longstaff and Schwartz, 1995, amongst others).

Other studies which concentrate on determinants of credit spreads include Delianedis and Geske (2001), Collin-Dufresne et al., 2001,Christiansen (2000), Brown (2001), Davies (2004), Pringent et al. (2001), and Duffie (1998) among others. For instance, Delianedis and Geske (2001) explain the components of credit risk using taxation, liquidity, recovery risk, volatility, and market factors and find that credit spreads are not primarily attributable to default.

Another branch of literature examines the time series behaviour of credit spreads and their moments including the time-varying dynamics of volatility of credit spreads. For instance, Weiss (1984) is the first study examining the time-varying volatility of bond yields using ARCH models and find significant ARCH effects in AAA corporate bond yields. Pedrosa and Roll (1998) analyses of US corporate credit spread indices reveal high level of persistence in volatility. Batchelor and Manzoni (2006) investigate the impact of rating revisions on volatility of sterling denominated Eurobond yield spreads and report asymmetry in the response of yield spread volatilities to changes in ratings.

While there is a large body of literature on modelling, formation, and determinants of credit spreads, as well as their time-varying volatilities, there has been little empirical investigation on the dynamic behaviour of higher moments of credit spreads, such as volatility, skewness and kurtosis. Having a good understanding about the nature and dynamics of credit spread moments is important when pricing credit derivatives, managing risk of bond portfolios, as well as asset allocation and investment strategies. For example, Tahani (2006) shows that a skewed GARCH model which incorporates higher moments of credit spreads can improve the valuation of credit spread options.

Furthermore, studies such as Bond (2001), Burns (2002), Angelidis, Benos and Degiannakis (2004) and Wilhelmsson (2007) show that GARCH models that account for higher moments provide significantly better VaR estimates compared to GARCH models with restricted second moment specifications. For instance, Bond (2001) finds that GARCH models using asymmetric non-normal distributions and specifically Hansen’s Skewed t-distribution provide better VaR forecasts when compared with the Gaussian or the mixed jump-diffusion distributions. Dahl and Yi (2005) find that leptokurtic distributions perform better in high volatility and that Hansen’s (1994) skewed t-distribution provides the best VaR forecast estimates. The findings of Angelidis, Benos and Degiannakis (2004) indicate that the mean process has no important role in the forecast of VaR estimators and leptokurtic distributions provide better VaR forecasts as they are able to capture extreme moves and fat tails. Finally, the study of Wilhelmson (2007), who proposes a Normal Inverse Gaussian (NIG) distribution that allows its parameters to vary over time and it is found to outperform the Gaussian GARCH model in terms of VaR estimators.

Therefore, the aim of this study is to investigate the nature and dynamics of credit spread moments in a set of iBoxx European Corporate Indices, and compare such behaviour across different credit ratings as well as maturities. To achieve this, we utilise a series models including simple asymmetric GARCH models, time-varying volatility, skewness and kurtosis models known as GARCH-SK, as well as variants of Markov Regime Switching GARCH models. The analyses allow us to assess and capture the dynamics of the shape of the distribution of credit spreads overtime. Furthermore, we examine the forecasting performance of these models in estimation of Value-at-Risk.

The remainder of this chapter is structured as follows. Section 2 presents different methodologies utilised for investigating the dynamics of moments of credit spreads. Section 3 describes the forecast evaluation techniques used for comparisons of different models. The empirical results and discussion on their significance are presented in section 4, while section 5 concludes.

5.2 Methodology

This section briefly presents the methodology applied in this chapter; for a detailed analysis refer to Chapter 3 in Section 3 entitled Univariate Volatility Models. Engle (1982), in his pioneering study, introduces a formal approach for modelling the variance of a time series by conditioning the variance of a time series on the square of lagged disturbances - error terms or shocks - in an autoregressive form known as Autoregressive Conditional Heteroskedasticity (ARCH) model. Bollerslev (1986) extends the ARCH model to generalised autoregressive conditional heteroskedasticity (GARCH) using lagged variance as an explanatory variable, which can reduce the dimension of the ARCH model significantly. Glosten *et al.* (1993) and Nelson (1991) propose the Threshold GARCH and Exponential GARCH specifications, respectively, to capture asymmetric response of volatility to positive and negative shocks. For instance, the GJR-GARCH model can be specified as (see Chapter 3, Section 3.3.1 and Equation 3.23):

$$\sigma_t^2 = \beta_o + \sum_{i=1}^{p} \beta_{1,i}\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_{2,j}\sigma_{t-j}^2 + \beta_3\varepsilon_{t-1}^2 I_{t-1}, \quad \varepsilon_t \sim IID\left(0,\sigma_t^2\right) \tag{5.1}$$

where ε_t represents independently and identically distributed (iid) error terms with zero mean and time-varying variance σ_t^2. The time-varying variance, σ_t^2, is then conditioned on its past values, past squared disturbances, and an indicator, I_{t-1} which takes value of one when the last period innovation term is negative and zero otherwise. Therefore, significance of the coefficient of the indicator term, β_3, measures the asymmetric response of variance to shocks.

However, the asymmetric GARCH models fail to capture the variation of higher moments over time. In this respect León, Rubio and Serna (2004) propose a model which accounts simultaneously for time-varying volatility, skewness and kurtosis. The SK-GARCH specification assuming a Gram-Charlier series expansion of the normal density function for the error term in the mean equation and has the following form (refer to Chapter 3, Section 3.3.3 and Equations 3.26, 3.27 and 3.28):

$$\begin{aligned}
&y_t = a_0 + \sum_{m=1}^{n} a_m y_{t-m} + \varepsilon_t, \quad \varepsilon_t \sim IID\left(0, \sigma_t^2\right) \\
&\varepsilon_t = \eta_t \sqrt{\sigma_t}, \quad \eta_t \sim \left(0,1\right) \\
&\sigma_t = \beta_0 + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, \\
&s_t = \gamma_0 + \sum_{i=1}^{q} \gamma_i \eta_{t-i}^3 + \sum_{j=1}^{q} \gamma_q s_{t-q}, \\
&k_t = \delta_0 + \sum_{i=1}^{q} \delta_i \eta_{t-i}^4 + \sum_{j=1}^{p} \delta_j k_{t-j},
\end{aligned} \tag{5.2}$$

where s_t and k_t are time-varying skewness and kurtosis which follow autoregressive their respective processes, while η_t represent standardised error terms. The log-likelihood function which is similar to the standardised normal with two additional terms accounting for time-varying skewness and kurtosis.

A different branch of literature focuses on market conditions and regime shifts and their impact on dynamics of moments of variables. For Lamourex and Lapstrapes (1990) show that the persistence of volatility can be due to the presence of structural breaks. Hamilton (1993, 1994), Cai (1994) and Hamilton and Susmel (1994) propose a regime-switching specification for capturing the structural changes in volatility of time series. Gray (1996), Susmel and Kalimipalli (2001), Klaassen (2002), Marcucci (2005), Perignon and Smith (2006) and Mitra, *et al*. (2007) illustrate that the forecasts of volatility improve if changes in regime is incorporated in the model, which allows the existence of two or more different volatility regimes characterized by a different levels as well as dynamics of volatility. Extending the above notion, Dueker (1997) showed how the shape parameters of asymmetric distributions vary across different regimes.

The simple GARCH(1,1) specification is extended into a two-state Markov regime switching GARCH[32] such that (see Chapter 3 Section 3.3.4 and Equations from 3.29 to 3.34):

$$\begin{aligned} r_t &= a_{0,s_t} + a_{1,s_t} r_{t-i,s_t} + a_{2,s_t} \varepsilon_{t-i,s_t} + \varepsilon_{t,s_t}, \quad \varepsilon_{t,s_t} \sim IID\left(0, \sigma^2_{t,s_t}\right) \\ \sigma^2_{i,t} &= \beta_{0,s_t} + \beta_{1,s_t} \sigma^2_{t-1,s_t} + \beta_{2,s_t} \varepsilon^2_{t-1,s_t} \end{aligned} \tag{5.3}$$

where s_t denotes the state in which the market is in and for example can take two values: $s_t = 1$ for a high-volatility regime and $s_t = 2$ for a low volatility regime.

5.3 Forecasting Performance

The performance of GARCH models with time-varying parametric distributions in estimation of Value-at-Risk of different assets and portfolios has been extensively examined. In order to compare the performance of different models in capturing the dynamics of volatility, skewness and kurtosis of credit spreads, we use different volatility forecast evaluation metrics such as the median absolute error (MAE), equation (5.4), and the median absolute percentage error (MAPE), equation (5.5) as well as Value at Risk (VaR) estimates.[33] The MAE and MAPE forecast evaluation metrics are based on the median of forecast errors because they are more robust than the mean error based metrics to heavy dispersion and skewed errors. Furthermore, since volatility is an unobserved variable, the MAE and the MAPE are both calculated based on the differences between forecasted variance and squared realised returns.

$$MAE = med(|\,\hat{\sigma}^2_{t+i} - r^2_{t+i}\,|) \qquad \text{for } i = t+1, \ldots, T \tag{5.4}$$

$$MAPE = med\left(\frac{|\,\hat{\sigma}^2_{t+i} - r^2_{t+i}\,|}{r^2_{t+i}}\right) \qquad \text{for } i = t+1, \ldots, T \tag{5.5}$$

---

[32] An increase in the number of regime sometimes results in over-parameterisation and difficulty in estimation due to regime-switching models being non-linear.

[33] Value-at-risk refers to a particular amount of money which is likely to be lost due to changes in the market, over a certain period of time and given some probability - known as confidence level. With the adoption of Basel II the number of exceptions is used to determine the levels of capital requirements. These exceptions are the number of occasions when the actual loss is larger by the predicted VaR model.

In the case of VaR estimates, the 1% estimates for different models considered are obtained using one and five period-ahead volatility forecasts and the conventional metrics suggested by Christoffersen (1998) are applied. These metrics are based on the proportion and sequence of periods where the realised change in the variable exceeds the VaR estimates. Therefore, an indicator function is defined as follows:

$$I_{t+1} = \begin{cases} 1 & \text{if} \quad r_{t+1} < VaR_{t+1} \mid \Omega_t \\ 0 & \text{if} \quad r_{t+1} \geq VaR_{t+1} \mid \Omega_t \end{cases} \tag{5.6}$$

where I_{t+1} is an indicator function and $VaR_{t+1} \mid \Omega_t$ is the inverse of the cumulative distribution function of the assumed distribution and it can be decomposed as:

$$VaR_{t+1} \mid \Omega_t = \hat{\mu}_{t+1} + F(\alpha)^{-1} \hat{\sigma}_{t+1} \tag{5.7}$$

where $\hat{\mu}_{t+1}$ is the conditional mean, $F(\alpha)^{-1}$ is the corresponding empirical quintile of the assumed distribution and $\hat{\sigma}_{t+1}$ is the *t+1* period forecast of the conditional standard deviation given the information at time t, Ω_t ,and α is probability of a change being greater than VaR (e.g. 1% or 5%).

The adequacy of the VaR estimates is examined by the application of the Christoffersen (1998) test for correct conditional coverage. This test can be divided into three sub-tests: **i)** test for correct unconditional coverage, **ii)** test for independence and, **iii)** test for the correct conditional coverage (see Chapter 3 Section 6.6 for more details). In this respect, the rejection of the model can be categorized as the unconditional coverage failure or the exception clustering or both. Christoffersen (1998) illustrated that a poor forecast may produce correct unconditional coverage but show signs of failure clustering and proposed a test for the correct conditional coverage. The best models are those that generate a coverage rate less than the nominal and a model is considered to be adequate for risk management when it is able to pass both the conditional and unconditional coverage tests.

5.4 Estimation Results

The data used in this chapter consist of credit spreads of corporate bond indices with different ratings and maturities published by Markit. The credit spreads are calculated as the difference between the yield on iBoxx Euro Corporate Indices and the yield on equivalent German government bonds. The selection of the German government bonds was based primarily on the grounds of their liquidity and, large size compared to other European and U.S. benchmark rates**,** and also on the fact that during the credit crisis period the German government market gained a safe-haven status in international financial marketst (see Ejsing and Sihvonen, 2009, and Schknecht, von Hagen, and Wolswijk, 2010). The inclusion of the iBoxx indices was decided on the premise that these indices provide accurate and high quality bond prices, are used as a proxy for the underlying market, and serve as a basis for derivative products and portfolio valuation. Their construction abides by the highest market standards as indicated by the EFFAS European Bond Commission Standardised Rules on Constructing and Calculating Bond Indices.

The first model to be examined is the GJR-GARCH of Equation (3.24) which is estimated over the period of 3rd January 2000 to 31st May 2007. The estimation results and diagnostics tests are presented in Table 5.1. The Schwarz Bayesian Information Criterion (SBIC) is used to select the appropriate number of lags in the mean model. The diagnostics tests including 1st and 10th order ARCH test and Ljung and Box test for autocorrelation indicate presence of autocorrelation and ARCH effects in standardised residuals of all models. However, increasing the number of lagged dependent variable in the mean equation as well as lagged squared error terms and variance in the variance equation could not eliminate autocorrelation and ARCH effects. This can suggest that ARIMA GJR-GARCH model with high orders may not be appropriate for modelling time-varying volatility of credit spread changes without taking into account the dynamics of higher moments.

The coefficients of the lagged squared error, β_1, lagged conditional variance, β_2, and coefficient of sign asymmetry, β_3, are significant in all models. The leverage or sign asymmetry coefficient, β_3, is negative and significant in all models except the model of the AAA index, which suggests that negative shocks imply a lower next period conditional variance in credit spread changes than positive shocks of the same sign, except for the AAA index.

A number of studies have found the leverage effect on volatility to be different across asset classes. For instance, while the leverage effects on volatility of equity markets and equity indices have found to be negative (see Glosten, Jagannathan, and Runkle, 1993, among others) volatility of commodity and commodity futures prices have shown to exhibit both positive and negative leverage effects (see McKenzie *et al.*, 2001). The economic theory that explains the leverage effect in the volatility of the credit spread changes relies explicitly on the relationship between credit spreads and interest rates. Longstaff and Schwartz (1995) and Annaert and De Ceuster (2000) find that credit spreads are negatively related to interest rates, and credit spreads narrow (widen) because the increase (decrease) of the interest rates is more significant than the movement of the bond yields. Hence, an unexpected drop in interest rate can cause credit spreads to widen and credit spread volatility to increase. This is mainly because interest rate declines can be considered as indication that the economy is contracting which in turn suggests that there might be greater uncertainty regarding the issuer’s cash flows and increase the probability of default of the issuer. The opposite relationship holds when there is a negative shock and credit spreads narrow. The negative shock can be due to an unexpected increase in interest rates causing the credit spread to narrow and credit spread volatility to decline. Again, interest rate increases might be considered as an indication that the economy is expanding which in turn means that there is less uncertainty regarding the issuer’s cash flow and probability of default.

In-sample volatility of credit spread changes for different rating indices and maturities are presented in Figures 5.1 and 5.2, respectively. The estimated volatilities display a consistent pattern across ratings and maturities; that is, lower ratings and long-term maturities have greater volatilities in line with their higher probability of downgrades and defaults. Furthermore, large variability with occasional spikes is observed during the introductory period of the Euro across all ratings and maturities. This could have been due to the market's reaction in restructuring the ways in which the Euro market functioned allowing the influx of huge amounts of capital from the fragmented currencies. Finally, from the middle of 2003 up until 2005, volatility declines as the market enters into a stable state with high economic growth with low volatility.

Table 5.1: Reports the estimation results of the GJR-GARCH model with Student-t Errors

This table reports the estimation results of the GJR-GARCH model with Student-t errors for the changes in credit spreads for different ratings. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. The GJR-GARCH model is given as:

$$y_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \quad , \quad \varepsilon_t \sim t_v(0, \sigma_t^2, v)$$

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 + \beta_3 I_{t-1} \varepsilon_{t-1}^2 \quad , \quad I_t = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t > 0 \end{cases}$$

where y_t is the change in credit spreads, $\boldsymbol{\sigma_t^2}$ is the conditional variance on day t, I_{t-1} is an indicator function that takes value of 1 when the ε_{t-1} is negative and 0 either wise. Additionally, the 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\boldsymbol{\chi^2(1)}$ 6.634 and 3.841, $\boldsymbol{\chi^2(10)}$ 23.209 and 18.307. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and the BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

Statistics	**AAA**	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
a_0	-0.001^{**}	-0.001^{***}	-0.001^{**}	-0.001^{***}	$-3 *10^{-05}$	$-2*10^{-04}$	$-2*10^{-04}$	$2*10^{-05}$
	(-2.696)	(-3.228)	(-2.514)	(-3.973)	(-0.159)	(-1.137)	(-1.173)	(0.152)
a_1	-0.369^{***}	-0.365^{***}	-0.304^{***}	-0.171^{***}	-0.307^{***}	-0.362^{***}	-0.349^{***}	-0.352^{***}
	(-19.877)	(-19.395)	(-16.058)	(-8.882)	(-16.392)	(-19.139)	(-17.951)	(-17.965)
β_0	$3*10^{-06***}$	$2*10^{-07}$	$2*10^{-07}$	$5*10^{-07}$	$7*10^{-07*}$	$4*10^{-07*}$	$4*10^{-07*}$	$5*10^{-07**}$
	(2.736)	(0.504)	(0.499)	(0.730)	(1.649)	(1.894)	(1.933)	(2.236)
β_1	0.167^{***}	0.053^{***}	0.053^{***}	0.088^{***}	0.089^{***}	0.146^{***}	0.159^{***}	0.160^{***}
	(7.007)	(5.221)	(5.499)	(7.163)	(6.628)	(7.627)	(7.950)	(7.828)
β_2	0.848^{***}	0.958^{***}	0.963^{***}	0.945^{***}	0.928^{***}	0.880^{***}	0.859^{***}	0.875^{***}
	(71.00)	(154.42)	(167.59)	(141.61)	(119.70)	(85.99)	(72.93)	(75.82)
β_3	-0.032	-0.021^{*}	-0.034^{***}	-0.064^{***}	-0.035^{**}	-0.051^{**}	-0.038	-0.069^{***}
	(-1.024)	(-1.698)	(-2.849)	(-4.579)	(-2.080)	(-2.214)	(-1.504)	(-2.810)
Dof	4.164^{***}	4.998^{***}	5.173^{***}	4.594^{***}	4.123^{***}	4.673^{***}	6.360^{***}	6.774^{***}
	(16.743)	(9.860)	(9.397)	(10.437)	(12.256)	(13.885)	(10.764)	(9.566)
Diagnostics								
Persistence	0.984	0.989	0.983	0.968	0.982	0.975	0.981	0.965
ARCH (1)	0.006	57.561	69.369	11.866	14.943	0.729	0.434	1.804
ARCH (10)	0.279	62.101	74.608	17.787	19.033	3.385	5.617	8.503
LB Q test (1)	0.160	3.504	2.756	0.004	0.006	0.573	1.929	1.399
LB Q test (10)	2.762	11.801	11.704	4.544	2.748	6.287	13.923	14.878
Log Likelihood	4065	4073	3963	3656	4053	4137	4314	4446
AIC	-8119	-8133	-7913	-7300	-8094	-8262	-8617	-8880
BIC	-8086	-8100	-7880	-7267	-8060	-8229	-8583	-8846

Figure 5.1 Presents the in-sample conditional variance of the rating indices [GJR-GARCH]

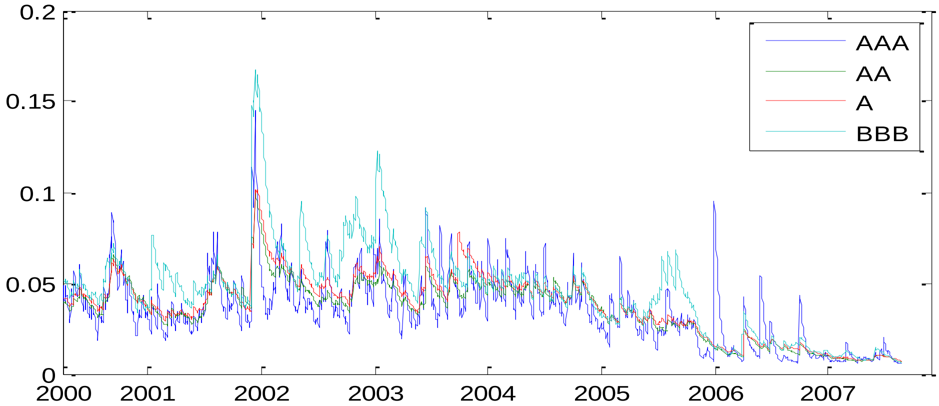
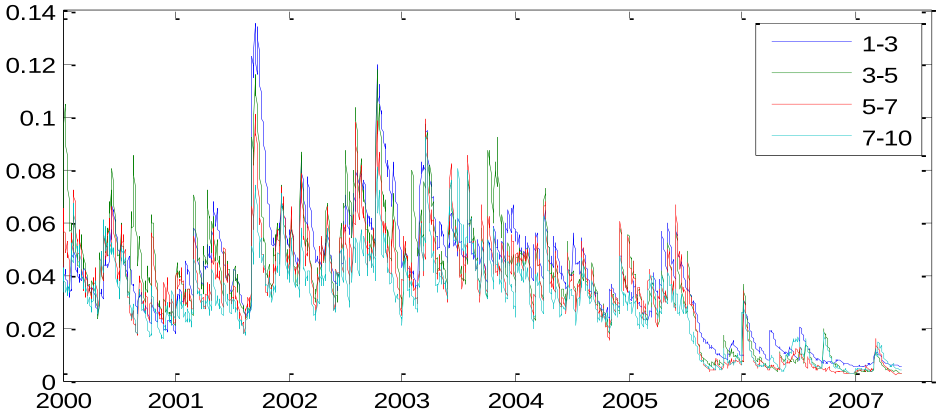


Figure 5.2 Presents the in-sample conditional variance of the maturity indices [GJR-GARCH]



The estimation results of the GRACH-SK model of Leon *et al.* (2004) for the changes in credit spreads of bond indices of different ratings and maturities are presented in Table 5.2. Once more, in all models the mean equation is considered to be a MA process of order one according to the AIC and SBIC. All models seem to be well specified with no ARCH effects and autocorrelation as indicated by 1st and 10th order ARCH and Ljung-Box tests.

The estimation results reveal that the coefficients of lagged square error terms, β_1, and lagged conditional variance, β_2, are significant in all models indicating that the GARCH(1,1) specification captures the dynamics of second moment of credit spread changes. The sum of coefficients of lagged error terms and lagged variance in each model, also presented in Table 5.2, indicate a moderate persistence level - between 0.839 and 0.864 - in time-varying variance of credit spread changes. The level of persistence in GARCH-SK models seem to be lower than GJR-GARCH models which suggests that considering the dynamics of higher moments (skewness and kurtosis) can reduce the persistence of estimated volatility of GARCH models.[34] The results are in line with Harvey and Siddique (1999) and Leon, Rubio and Serna (2004).

Moreover, coefficients of lagged cubic error terms, γ_1, and lagged conditional skewness, γ_2, are also highly significant indicating existence of time-varying skewness in all the changes credit spread models. Estimated coefficients of lagged error terms to the power of four, δ_1, and lagged conditional kurtosis, δ_2, are significant in all models suggesting presence of time-varying kurtosis in changes in credit spread models across ratings and maturities. The lowest coefficient of conditional skewness is observed for the model of the A index with a value of 0.425, while the largest for the model of the 7-10 index with a value of 0.770. The lowest coefficient of conditional kurtosis is observed for the model of the 7-10 index with a value of 0.886, while the largest for the model of the BBB index with a value of 0.906.

Persistence of time-varying estimates of higher moments of credit spread changes are also reported in Table 5.2; which is estimated as the sum of the appropriate coefficients of lagged square error terms, and lagged conditional higher moments (i.e. for example persistence of skewness is measured as the sum of the coefficients of the lagged cubic error terms, γ_1, and lagged conditional skewness, γ_2). Time-varying kurtosis tends to show a relatively higher degree of persistence in contrast to skewness. For instance, persistence of time-varying skewness of credit spreads range between 0.480 for A rating index and 0.798 for 7-10 maturity index, while persistence of time-varying kurtosis of credit spread changes is higher ranging between 0.974 for 7-10 year maturity index and 0.991 for the AAA rating index. The economic interpretation of this behaviour is that kurtosis is associated with sharp credit changes; whereas skewness reflects possible downgrades and credit spread widenings (see Anson, et al, 2004). These unexpected and rapid credit changes have a greater impact on credit spread changes than gradual and, to extent, anticipated downgrades and credit spread widening (see Anson, et al, 2004). These results are similar to the study of Leon, Rubio, and Serna (2004), who argue that periods of high (low) kurtosis are followed by periods of high (low) kurtosis since the coefficients of the lagged kurtosis are positive and significant.

[34] GARCH models are generally criticised for over estimation of persistence in volatility and consequently the time-varying volatility as well as prediction and forecasting of volatility (see Lamourex and Lastrapes, 1990).

Table 5.2: Reports the estimation results of the GARCH-SK model

This table reports the estimation results of the GARCH-SK model for the changes in credit spreads for different ratings. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. The GARCH-SK model is given as:

$$\varepsilon_t = \eta_t \sigma_t \quad , \quad \eta_t \sim iid(0,1)$$

$$y_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \ , \quad \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2$$

$$s_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1}, \quad k_t = \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 k_{t-1}$$

where y_t is the change in credit spreads, σ_t^2 is the conditional variance on day t, s_t is the conditional skewness on day t and k_t the conditional kurtosis on day t. Additionally, the 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and the BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

Statistics	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
a_0	-0.001***	-0.001***	-0.001***	-0.001***	-4.0*10^-04***	-1.5*10^-04***	-4.1*10^-04***	-0.001***
	(-38.813)	(-41.111)	(-42.809)	(-31.319)	(-20.653)	(-12.362)	(-33.108)	-29.290)
a_1	-0.394***	-0.396***	-0.355***	-0.192***	-0.267***	-0.360***	-0.403***	-0.416***
	(-387.70)	(-636.10)	(-512.64)	(-179.79)	(-446.81)	(-459.63)	(-586.94)	(-361.92)
β_0	1.1*10^-04***	9.6*10^-05***	1.0*10^-04***	9.7*10^-05***	1.0*10^-04***	9.8*10^-05***	8.6*10^-05***	1.3*10^-04***
	(120.42)	(243.79)	(135.99)	(100.94)	(138.98)	(145.47)	(204.05)	(56.76)
β_1	0.051***	0.050***	0.048***	0.050***	0.050***	0.048***	0.043***	0.048***
	(178.47)	(243.79)	(129.28)	(184.05)	(164.16)	(238.30)	(166.52)	(107.09)
β_2	0.788***	0.802***	0.797***	0.800***	0.794***	0.802***	0.821***	0.766***
	(659.5)	(1295.8)	(696.5)	(732.9)	(844.4)	(946.6)	(1176.4)	(278.3)
γ_0	-0.002	-0.019***	-0.040***	-0.015**	-0.039***	0.003	0.058***	-1.7*10^-08***
	(-0.412)	(-3.439)	(-4.644)	(-2.135)	(-7.216)	(0.925)	(14.491)	(-1.4*10^-06)
γ_1	0.026***	0.036***	0.055***	0.037***	0.030***	0.052***	0.051***	0.028***
	(10.823)	(30.441)	(20.115)	(13.948)	(16.220)	(26.279)	(35.897)	(17.817)
γ_2	0.639***	0.590***	0.425***	0.569***	0.616***	0.573***	0.542***	0.770***
	(27.153)	(42.061)	(31.149)	(22.597)	(30.828)	(50.479)	(41.251)	(53.516)
δ_0	0.088***	0.080***	0.145***	0.112***	0.181***	0.111***	0.152***	0.231***
	(25.786)	(28.588)	(32.888)	(40.487)	(81.344)	(66.721)	(73.838)	(43.309)
δ_1	0.093***	0.088***	0.084***	0.074***	0.079***	0.093***	0.076***	0.088***
	(129.920)	(162.536)	(89.210)	(98.962)	(104.176)	(101.78)	(196.46)	(83.96)
δ_2	0.898***	0.901***	0.897***	0.906***	0.899***	0.895***	0.899***	0.886***
	(3068.9)	(3207.4)	(1676.8)	(1732.8)	(1697.7)	(1509.4)	(3887.7)	(1007.5)
Diagnostics								
Persistence								
Volatility	0.839	0.852	0.845	0.851	0.844	0.851	0.864	0.813
Skewness	0.665	0.626	0.480	0.606	0.646	0.625	0.593	0.798
Kurtosis	0.991	0.989	0.981	0.980	0.979	0.988	0.975	0.974
ARCH(1)	0.252	1.651	0.909	0.211	5.335	4.321	6.461	7.296
ARCH(10)	3.551	5.161	3.768	2.769	6.517	5.885	16.070	18.692
LB Q test(10)	5.745	6.345	3.266	2.355	5.216	5.606	5.899	6.299
Log Likelihood	12.076	9.564	9.198	10.652	8.597	9.842	13.918	20.085
AIC	3789	3957	3866	3587	3854	3897	4185	4645
BIC	-7556	-7892	-7710	-7152	-7686	-7772	-8348	-9267

The in-sample conditional variance, skewness and kurtosis of the changes in credit spreads for different ratings and maturity indices are presented in Figures 5.5, 5.6, 5.7 and 5.8. There seem to be a consistent pattern in time-varying moments of the changes in credit spreads both across ratings and maturities. Time-varying skewness of credit spread changes for lower ratings (BBB and A indices) and long-term maturities (5-7 and 7-10 year indices) seem to fluctuate more that those for higher ratings (AAA and AA indices) and shorter maturities (1-3 and 3-5 year indices). This can be explained by a larger probability of credit upgrades or downgrades and defaults of low rated and long maturity bonds with higher unconditional credit risk (see Anson *et al.*, 2004).

Another characteristic of the estimated time-varying skewness is the negative spikes observed for the A, BBB and 5-7 indices during high volatility periods and the positive spikes of the AAA and 1-3 indices during lower volatility periods. For instance, during the high volatility period credit spreads widen and interest rates decrease suggesting a deteriorating economy and therefore a negative effect on the credit-worthiness of the lower ratings and long-term maturities. On the other hand, during the low volatility period when the economy is expanding, credit spreads narrow because uncertainty about firms' cash flow decreases and interest rates increase. This combined effect in turn suggests that the issuers are able to honour their contractual obligations and has a positive effect on their credit-worthiness.

Kurtosis exhibits similar behaviour to those of time-varying volatility and skewness. The plot of estimated coefficient of time-varying kurtosis is above three for the lower rating (A and BBB rating indices) and longer-term maturities (5-7 and 7-10 year maturity Indices). Higher kurtosis values for lower rating and longer maturities suggest that there are more extreme movement in the changes in credit spreads of these indices. On the other hand, coefficients of time-varying kurtosis below three for the higher-rating and short-term maturities, suggest that there is no extreme movement in credit spread changes for these assets and there distributions are close to a normal distribution.

Figure 5.3 Presents the in-sample conditional variance of the rating indices

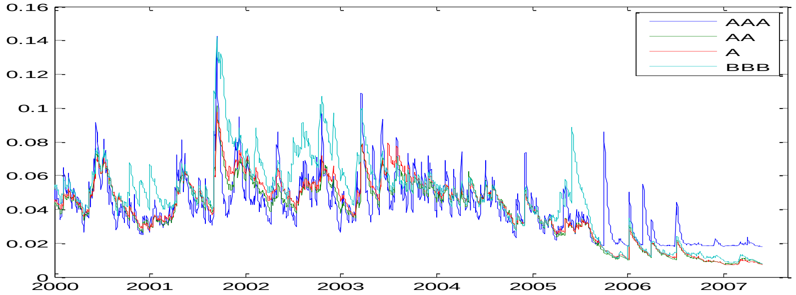


Figure 5.4 Presents the in-sample conditional variance of the maturity indices

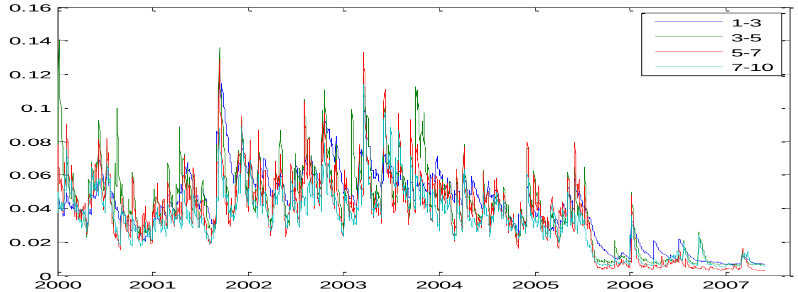


Figure 5.5 Presents the in-sample conditional variance of the rating indices [SKGARCH]

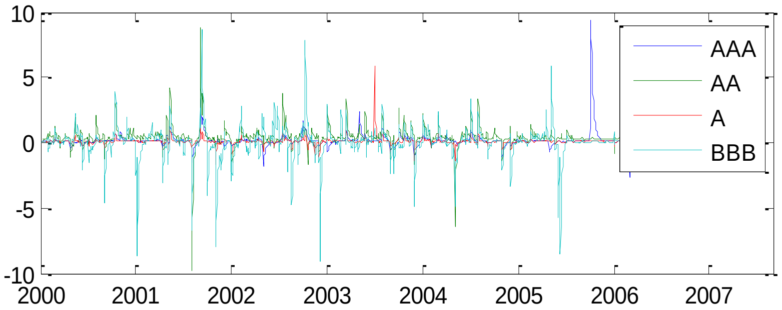


Figure 5.6 Presents the in-sample conditional variance of the maturities indices [SKGARCH]

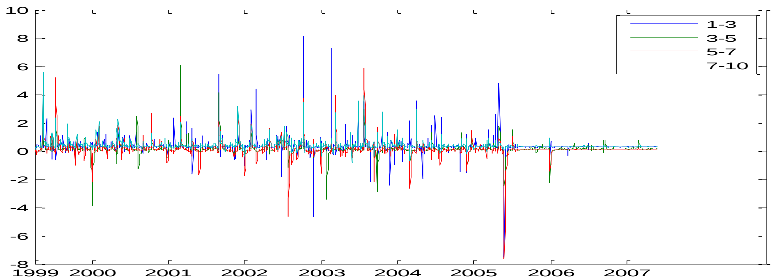


Figure 5.7 Presents the in-sample conditional variance of the ratings indices [SKGARCH]

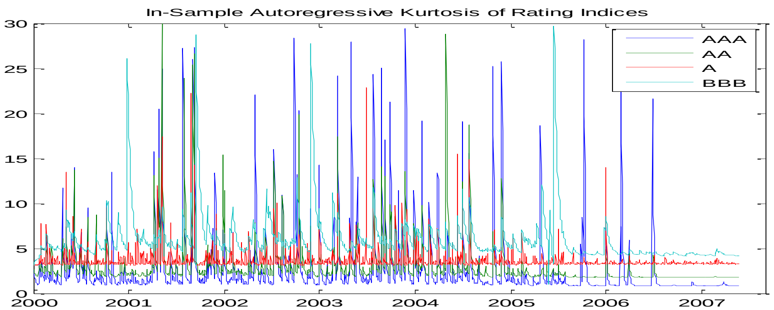
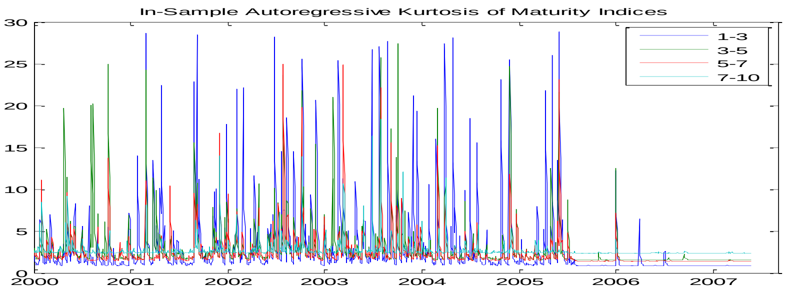


Figure 5.8 Presents the in-sample conditional variance of the maturities indices [SKGARCH]



However, during periods of high volatility, the estimated time-varying kurtosis for the higher-rating and short-term maturities seems to spike several times which could be due to sharp changes in the credit market conditions and an increase in the probability of default. These extreme movements which can be picked by time-varying skewness and kurtosis models are very important for risk management purposes and VaR calculations because their prediction can improve the VaR estimates.

Table 5.3 presents the estimation results of the Markov Regime Switching GARCH model with switching degrees-of-freedom (MRS-GARCH-tv)[35] as suggested by Klaassen (2002). Results of diagnostics tests indicate that all models are well specified. For instance, Engle's ARCH tests reject the presence of 1st and 10th order ARCH effects in standardised residuals of all models and time horizons, while the results of the Ljung-Box Q-test reject the presence of 1st and 10th order autocorrelation in standardised residuals of all models at the 5% significance level. The estimated transition probabilities, p_{11} and p_{22}, are significant in all models with magnitude of more than 99% indicating high degree of persistence in each regime.

The estimated MRS-GARCH-tv model allows the degrees of freedom, v_{st}, to switch between the two regimes, which can be considered as an indirect parameterisation of time-varying kurtosis. The estimated coefficients of the degrees-of-freedom for the first regime (v_1) are larger than four, the largest value observed for the 7-10 index with a value of 12.714, while the lowest of the 1-3 index with a value of 4.353. The estimated coefficients of the degrees-of-freedom (v_2) in the second regime are smaller than 4 indicating significant excess kurtosis and fat-tails in the distributions of credit spread changes in the second regime with high volatility. Similar results were presented in the study of Marcucci (2009).

Estimated coefficients of lagged squared errors in variance equations, β_1^1 and β_1^2 ,are significant at the 10% level in regime one (low volatility regime), and at the 5% level in regime two (high volatility regime). Estimated coefficients of lagged conditional variance, β_2^1 and β_2^2 , are also significant in both regimes in all models across rating

[35] The Markov Regime Switching GARCH model with Normal Distribution was also estimated, but was omitted from the study as it did not offer any forecasting improvement over the MRS-GARCH-tv model.

and maturity indices. In this model, the first regime is characterized by low volatility persistence levels as indicated by the estimated coefficients, whereas the second regime is characterized by high volatility level and persistence.

Hansen (1992) argues that it is not possible to compare the single-regime GARCH with the regime-switching GARCH models, since standard econometric tests for model specification are not appropriate. This is mainly because under the null some parameters are unidentified.

Table 5.3: **Reports the estimation results of the MRS-GARCH-tv model**

This table reports the estimation results of the MRS-GARCH-tv model for the changes in credit spreads for different ratings. The estimation is performed by the method of quasi maximum likelihood using the BFGS algorithm in Matlab 7.8 software package. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. The MRS-GARCH-tv model is given as:

$$y_t = \alpha_{0,st} + \alpha_{1,st}\varepsilon_{t-1} + \varepsilon_{t,st} \qquad , \qquad \varepsilon_{t,st} \sim iid\,(0, \sigma^2_{t,st}, v_{st})$$

$$\sigma^2_{t,st} = \beta_{0,st} + \beta_{1,st}\varepsilon^2_{t-1} + \beta_{2,st}\sigma^2_{t-1}$$

where y_t is the change in credit spreads, $\sigma^2_{t,st}$ is the conditional variance on day t and in state s_t, and v_{st} the degrees-of-freedom of the Student-t distribution at state s_t. Additionally, the 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and the BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

Statistics	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
$a_0^{(1)}$	-0.001	-0.001	-0.001	-0.001	$9.4*10^{-05}$	$3.9*10^{-05}$	$8.2*10^{-05}$	$1.1*10^{-04}$
	(-0.494)	(-0.190)	(-0.305)	(-1.330)	(0.109)	(0.615)	(0.033)	(0.219)
$a_0^{(2)}$	-0.004***	-0.004	$2*10^{-05***}$	$-4*10^{-04}$	-0.001	$-3*10^{-04***}$	-0.001***	-0.001
	(-6.032)	(-0.079)	(9.145)	(-5.315)	(-0.059)	(-17.626)	(-3.441)	(-0.221)
$a_1^{(1)}$	-0.002**	0.032	0.078**	0.117	-0.045	-0.008	0.044	0.101
	(-2.089)	(0.217)	(1.761)	(0.263)	(-0.111)	(-0.360)	(0.197)	(0.137)
$a_1^{(2)}$	-0.392***	-0.400	-0.369***	-0.230***	-0.299***	-0.357	-0.410	-0.420***
	(-6.015)	(-1.291)	(-6.201)	(-17.204)	(-9.463)	(-1.080)	(-1.233)	(-3.459)
$\beta_0^{(1)}$	0.002***	0.002***	0.002	0.002	0.002	0.002	0.011	0.001
	(8.778)	(10.643)	(9.730)	(7.881)	(9.105)	(9.814)	(9.985)	(10.099)
$\beta_0^{(2)}$	$2*10^{-04**}$	$7*10^{-05***}$	$8.6*10^{-05}$	$6.8*10^{-05}$	$6.9*10^{-05}$	$3.9*10^{-05}$	$6.0*10^{-05}$	$1.4*10^{-05}$
	(2.145)	(4.428)	(3.708)	(2.625)	(2.875)	(2.889)	(2.925)	(3.524)
$\beta_1^{(1)}$	0.065*	0.069*	0.071*	0.075	0.024	0.022*	0.030*	0.027*
	(1.899)	(1.798)	(1.787)	(1.556)	(1.612)	(1.742)	(1.779)	(1.896)
$\beta_1^{(2)}$	0.042**	0.067**	0.102**	0.212**	0.110**	0.252*	0.201**	0.257**
	(2.271)	(2.194)	(2.221)	(2.398)	(2.304)	(1.847)	(2.521)	(2.544)
$\beta_2^{(1)}$	0.423***	0.394***	0.412***	0.368***	0.490***	0.491***	0.458***	0.393***
	(5.626)	(5.902)	(5.823)	(5.153)	(5.524)	(6.087)	(6.497)	(6.507)
$\beta_2^{(2)}$	0.901**	0.789**	0.769**	0.664*	0.621*	0.644***	0.755**	0.685***
	(2.557)	(2.365)	(2.169)	(1.784)	(1.775)	(2.770)	(2.551)	(3.108)
P_{11}	0.997***	0.998***	0.997***	0.999***	0.996***	0.997***	0.997***	0.998***
	(2794.4)	(2523.2)	(3413.6)	(2446.5)	(1930.2)	(1985.3)	(1975.1)	(1934.0)
P_{22}	0.994***	0.998***	0.998***	0.997***	0.995***	0.996***	0.995***	0.995***
	(410.7)	(412.2)	(493.4)	(463.8)	(701.5)	(725.9)	(788.1)	(800.2)
$v^{(1)}$	5.556***	6.416***	5.951***	4.607***	4.354***	5.578***	12.672***	12.714***
	(15.392)	(5.965)	(6.580)	(7.759)	(9.453)	(20.285)	(3.545)	(3.686)
$v^{(2)}$	2.165***	3.321***	3.312***	3.163***	3.133***	2.634***	3.147***	3.249***
	(6.788)	(6.589)	(6.554)	(6.014)	(21.023)	(12.474)	(11.783)	(6.893)

Table continues at the next page

Diagnostics								
Vol. Pers.								
Regime 1	0.488	0.463	0.483	0.443	0.514	0.513	0.488	0.420
Regime 2	0.943	0.856	0.871	0.876	0.731	0.896	0.956	0.942
ARCH(1)	0.013	0.332	0.145	0.089	0.042	0.002	2.054	0.002
ARCH(10)	0.163	8.586	10.808	9.309	3.294	2.246	6.463	0.100
LB Q test(1)	0.572	0.715	0.488	1.652	5.395	0.214	0.324	1.215
LB Q test(10)	15.825	21.374	21.735	11.755	20.351	18.614	20.196	22.446
Log Likelihood	4124	4034	3670	3662	4015	4118	4444	4438
AIC	-8224	-8044	-7316	-7299	-8006	-8213	-8864	-8852
BIC	-8157	-7977	-7249	-7233	-7939	-8146	-8797	-8786

Figure 5.9 and Figure 5.10 present the in-sample estimated MRSGARCH volatility of rating and maturity indices, respectively. It can be seen that volatility of all indices reduces significantly from early 2006 to mid 2007, compared to the earlier period, as the market enters into a relatively calm period with low volatility and high growth.

Figure 5.9 Presents the estimated conditional variance of the rating indices [MRS-GARCH-tv]

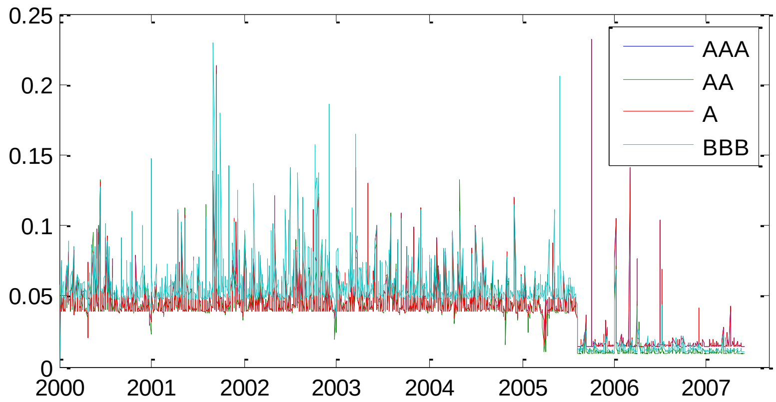
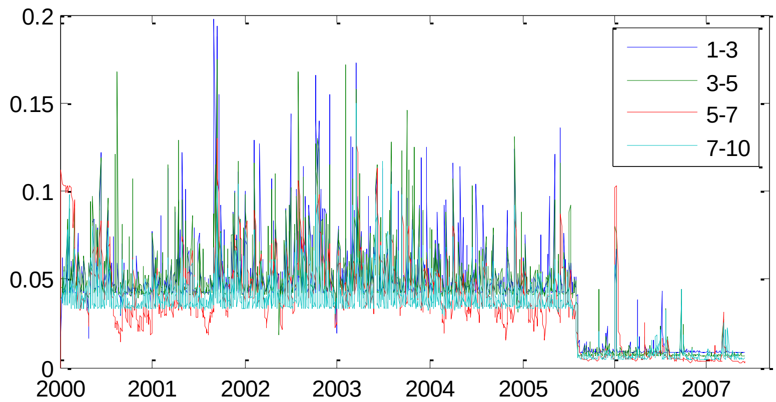


Figure 5.10 Presents the estimated conditional variance of the maturity indices [MRS-GARCH-tv]



5.4.1 Out-of-Sample Performance of Models

In order to further investigate the appropriateness of the proposed models in capturing the dynamics of higher moments of credit spread changes, out-of-sample forecasting performance of the models, in terms of prediction of the changes in credit spread volatility, are compared using different statistics and VaR analysis. The out-of-sample forecast exercise is performed over the period from 1st June 2007 to 30th April 2009, which covers the credit crisis period. The statistics used for the comparison of volatility prediction are median absolute error (MAE) and median absolute percentage error (MAPE), while Christoffersen's independence, unconditional coverage and conditional coverage Likelihood Ratio tests are performed on estimated VaR from proposed GJR-GARCH, GARCK-SK, MRS-GARCH models, as well and RiskMetrics, Historical Simulation (HS) and Filtered Historical Simulation (FHS) methods.

Table 5.4 and Table 5.5 present the MAE and MAPE statistics which are calculated based on the differences between forecasted variance and squared realised returns. These statistics are based on the median of forecast errors as they are more robust than the mean error based forecast evaluation statistics to heavy dispersion and skewed forecast errors. The MAE for the 1-day forecast horizon reveals that the GARCH-SK yields the best volatility forecast for the changes in credit spread of rating indices (AAA, AA, A and BBB) and the GJR-GARCH for the volatility of changes in credit spread of maturity indices (1-3, 3-5 and 5-7). This is followed by the MRS-GARCH-tv which performs similar to GARCH-SK in forecasting volatility of changes in credit spread of AA and A indices as well as 7-10 maturity index. The MAE statistics for comparison of volatility forecast over a 5-day horizon reveals that overall GJR-GARCH provides the most accurate forecast for volatility of credit spread changes amongst competing models.

The MAPE statistics for 1-day-ahead volatility forecast reveals that GARCH-SK out-performs other models in the case of the rating indices (AAA, AA, A and BBB), while the MRS-GARCH-tv produces forecast with similar accuracy for volatility of credit spread changes of AA and A indices. The GJR-GARCH provides the most accurate volatility forecast for the maturity indices (1-3, 3-5 and 5-7), while the MRS-GARCH-tv model seems to yield the most accurate forecast for credit spread changes of the 7-

10 year maturity index. Overall, it seems that in general the GJR-GARCH outperforms the other models in forecasting 1-day-ahead volatility of credit spread changes of maturity indices and 5-day-ahead for rating indices. On the other hand, GARCH-SK seems to produce more accurate 1-day-ahead volatility forecasts for credit spread changes of rating indices and 5-day-ahead maturity indices. The Regime Switching volatility model seems to have poor forecasting performance overall, similar results are presented in Marcucci (2009) among others.

Table 5.4: Reports the median absolute error of volatility forecasts

The table reports the median absolute error (MAE) metric which is based on the median of forecast errors. Since volatility is an unobserved variable the forecast error is calculated based on the differences between the forecasted variance and squared realized returns. The back-testing sample period is from 1st June 2007 - 30th April 2009, a total of 500 observations. The MAE is:

$$MAE = med(|\hat{\sigma}^2_{t+i} - r^2_{t+i}|) \quad \text{for } i = t+1,\dots,T$$

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
				1day				
GJR-GARCH	0.0008	0.0012	0.0013	0.0013	0.0005	0.0007	0.0006	0.0010
GARCH-SK	0.0003	0.0003	0.0003	0.0012	0.0011	0.0013	0.0013	0.0010
MRSGARCHtv	0.0128	0.0003	0.0003	0.0019	0.0018	0.0026	0.0025	0.0009
				5day				
GJR-GARCH	0.003	0.003	0.004	0.004	0.002	0.003	0.003	0.004
GARCH-SK	0.003	0.004	0.005	0.004	0.002	0.002	0.003	0.003
MRSGARCHtv	0.055	0.004	0.005	0.013	0.010	0.014	0.013	0.006

Table 5.5: Reports the median absolute percentage error of volatility forecasts

The table reports the median absolute percentage error (MAE) metric which is based on the median of forecast errors. Since volatility is an unobserved variable the forecast error is calculated based on the differences between the forecasted variance and squared realized returns. The back-testing sample period is from 1st June 2007 - 30th April 2009, a total of 500 observations. The MAE is:

$$MAPE = med\left(\frac{|\hat{\sigma}^2_{t+i} - r^2_{t+i}|}{r^2_{t+i}}\right) \quad \text{for } i = t+1,\dots,T$$

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
				1day				
GJR-GARCH	3.025	4.600	3.926	3.799	3.842	4.197	2.996	4.941
GARCH-SK	1.000	1.000	1.000	2.259	5.839	7.938	6.361	3.860
MRSGARCHtv	30.426	1.000	1.000	5.266	8.296	10.443	8.479	2.675
				5day				
GJR-GARCH	0.904	0.873	0.897	0.919	0.869	0.913	0.930	0.912
GARCH-SK	1.000	1.000	1.000	0.937	0.901	0.905	0.924	0.925
MRSGARCHtv	16.280	1.000	1.000	1.577	2.383	2.392	1.669	0.883

5.4.2 Value-at-Risk Estimation

In this section the performance of different models in terms of estimation of 1% Value-at-Risk over 1-day and 5-day horizons for long and short positions is investigated. The Value-at-Risk analysis is focused on the 1% level as it is the level used by Basel II to evaluate models in terms of their performance in a back-testing exercise. Back-testing performance of different models is examined over the period from 1st June 2007 to 30th April 2009, using four statistics, namely: percentage proportion of failures (PF%), unconditional coverage (UC), independence (I), and conditional coverage (CC) log-likelihood tests.

Starting with the percentage proportion of failures for 1-day VaR estimates of long and short position presented in Table 5.6, it can be seen that the MRS-GARCH, GJR-GARCH and Extreme Value Theory tend to exhibit the lowest average PF for long and short positions. Other models seem to show PF of above 1% consistently. For short positions, the MRS-GARCH-tv exhibits 1% VaR violations for the A and BBB ratings and the GJR-GARCH for the AA and A rating indices, followed by EVT which exhibits the lowest percentage of failures. In the case of 5-day 1% VaR estimates, it can be seen that MRS-GARCH-tv, GJR-GARCH and EVT perform better than other models for long positions, followed by SK-GARCH, RiskMetrics, FHS and HS for long and short positions. The HS and RiskMetrics seem to show the highest VaR violations over a 5-day horizon for long and short positions. The reason for observing a lower percentage of VaR violations in long positions during the back-testing period is because during this period credit spreads were generally increasing, due to the credit crisis.

In order to test whether the proportion of VaR failures correspond statistically to level of α considered for VaR estimation, we perform a likelihood ratio test proposed by Christoffersen (1998), known as unconditional coverage test (see Chapter 3, Section 6.6 for more details on back-testing VaR) and the tolerance level α is considered to be 1%. The Likelihood Ratio test for unconditional coverage of VaR estimates over the back-testing period is presented in Table 5.7. The unconditional coverage test is not rejected at a 5% significance level for 1-day VaR for long position for the majority of models, except for the GARCH-SK for the AAA and 7-10 index, HS for all indices, and FHS for AA and BBB indices. The unconditional coverage test is rejected at a 5%

significance level for 1-day VaR for short positions in the case of HS and Risk Metrics for all indices, and in the case of GARCH-SK for the AAA and BBB indices and FHS for the AAA, BBB and 1-3 indices.

Table 5.6: Percentage of Failure of VaR estimates of different models

The table reports the percentage of failure of the different specifications over the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations.

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
				1 Day Long Position				
GJR-GARCH	0.000	0.006	0.006	0.002	0.004	0.002	0.002	0.002
GARCH-SK	0.032	0.022	0.020	0.022	0.018	0.024	0.026	0.038
MRS-GARCH-tv	0.008	0.008	0.008	0.002	0.006	0.006	0.006	0.006
HS	0.044	0.034	0.048	0.034	0.032	0.034	0.042	0.030
FHS	0.036	0.020	0.030	0.022	0.034	0.056	0.054	0.064
RiskMetrics	0.018	0.016	0.016	0.008	0.008	0.010	0.008	0.008
EVT	0.014	0.010	0.008	0.008	0.008	0.006	0.008	0.008
				1 Day Short Position				
GJR-GARCH	0.000	0.008	0.006	0.000	0.000	0.000	0.000	0.000
GARCH-SK	0.036	0.024	0.014	0.036	0.012	0.006	0.008	0.008
MRS-GARCH-tv	0.000	0.000	0.002	0.002	0.000	0.000	0.000	0.000
HS	0.042	0.036	0.046	0.048	0.042	0.028	0.042	0.036
FHS	0.026	0.018	0.024	0.030	0.028	0.020	0.020	0.018
RiskMetrics	0.036	0.032	0.042	0.050	0.038	0.032	0.038	0.036
EVT	0.006	0.008	0.006	0.006	0.002	0.004	0.008	0.008
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				5 Day Long Position				
GJR-GARCH	0.022	0.008	0.010	0.000	0.000	0.020	0.028	0.022
GARCH-SK	0.060	0.068	0.036	0.062	0.040	0.048	0.064	0.062
MRS-GARCH-tv	0.016	0.002	0.002	0.000	0.010	0.022	0.026	0.012
HS	0.058	0.052	0.064	0.042	0.054	0.086	0.092	0.086
FHS	0.036	0.020	0.030	0.022	0.034	0.056	0.054	0.064
RiskMetrics	0.024	0.032	0.032	0.018	0.016	0.042	0.048	0.036
EVT	0.020	0.022	0.020	0.012	0.018	0.026	0.040	0.036
				5 Day Short Position				
GJR-GARCH	0.024	0.026	0.034	0.030	0.020	0.034	0.058	0.054
GARCH-SK	0.076	0.070	0.042	0.092	0.032	0.034	0.074	0.064
MRS-GARCH-tv	0.022	0.034	0.020	0.018	0.052	0.030	0.038	0.030
HS	0.082	0.090	0.140	0.132	0.134	0.132	0.164	0.138
FHS	0.060	0.046	0.078	0.092	0.080	0.096	0.098	0.060
RiskMetrics	0.086	0.096	0.128	0.138	0.128	0.176	0.172	0.138
EVT	0.032	0.036	0.032	0.030	0.014	0.030	0.054	0.030

In the case of 1% 5-day VaR estimates, the results of unconditional coverage tests seem to be mixed. For instance, the unconditional coverage test at 5% significance level for long positions is not rejected for GJR-GARCH, MRS-GARCH-tv, FHS for the AA and BBB indices, RiskMetrics for AAA, BBB and 1-3 indices and EVT, except for the GJR-GARCH and MRS-GARCH-tv for the 5-7 index and EVT for the 5-7 and 7-10 indices, whereas the unconditional coverage test at 5% significance level for short positions is rejected for the majority of models except for the GJR-GARCH for AAA and 1-3 indices, MRS-GARCH-tv for the AAA, A and BBB indices and EVT for 1-3 index.

Table 5.7: Estimation of the Likelihood Ratio Test of the Unconditional Coverage

The table reports Christoffersen's (1998) likelihood ratio test of the unconditional coverage during the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations. The likelihood ratio test of the unconditional coverage is chi-square asymptotic with one degrees-of-freedom. The 1% and 5% critical values for $\chi^2(1)$ are 6.634 and 3.841.

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				1 Day Long Position				
GJR-GARCH	-	0.410	0.410	2.090	1.022	2.090	2.090	2.090
GARCH-SK	6.717	2.353	1.700	2.353	1.135	3.088	3.897	10.045
MRS-GARCH-tv	0.094	0.094	0.094	2.090	0.410	0.410	0.410	0.410
HS	13.802	7.775	16.517	7.775	6.717	7.775	12.506	5.716
FHS	3.088	3.088	5.716	3.897	1.135	2.353	0.668	3.897
RiskMetrics	1.135	0.668	0.668	0.094	0.094	0.000	0.094	0.094
EVT	0.312	0.000	0.094	0.094	0.094	0.410	0.094	0.094
				1 Day Short Position				
GJR-GARCH	-	0.094	0.410	-	-	-	-	-
GARCH-SK	8.885	3.088	0.312	8.885	0.082	0.410	0.094	0.094
MRS-GARCH-tv	-	-	2.090	2.090	-	-	-	-
HS	12.506	8.885	15.140	16.517	12.506	4.775	12.506	8.885
FHS	3.897	1.135	3.088	5.716	4.775	1.700	1.700	1.135
RiskMetrics	8.885	6.717	12.506	17.932	10.045	6.717	10.045	8.885
EVT	0.410	0.094	0.410	0.410	2.090	1.022	0.094	0.094
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				5 Day Long Position				
GJR-GARCH	2.418	0.084	-	-	-	1.753	4.871	2.418
GARCH-SK	25.804	32.491	9.025	27.432	11.415	16.723	29.090	27.432
MRS-GARCH-tv	0.700	2.049	2.049	-	0.000	2.418	3.983	0.093
HS	24.207	19.612	29.090	12.679	21.110	49.080	55.027	49.080
FHS	9.025	1.753	5.823	2.418	7.904	22.642	21.110	29.090
RiskMetrics	3.163	6.835	6.835	1.177	0.700	12.679	16.723	9.025
EVT	1.700	2.353	1.700	0.082	1.135	3.897	11.253	8.885
				5 Day Short Position				
GJR-GARCH	3.163	3.983	7.904	5.823	1.753	7.904	24.207	21.110
GARCH-SK	39.618	34.233	12.679	55.027	6.835	7.904	37.798	29.090
MRS-GARCH-tv	2.418	7.904	1.753	1.177	19.612	5.823	10.196	5.823
HS	44.829	52.581	107.880	98.340	100.702	98.340	137.848	105.473
FHS	25.532	15.140	41.090	54.574	42.948	58.620	60.674	25.532
RiskMetrics	48.661	58.620	93.659	105.473	93.659	153.531	148.254	105.473
EVT	6.717	8.885	6.717	5.716	0.312	5.716	20.871	5.716

The next test in our back-testing process is the independence test of VaR estimates, which ignores the unconditional coverage and tests the clustering of VaR violations. The results of the independence test for VaR estimates of different models over the back-testing period are reported in Table 5.8.

The results of the independence tests generally indicate that all models pass the tests at a 5% significance level in the case of 1-day VaR for both short and long positions, with the exceptions of HS VaR estimates for 3-5, 5-7, and 7-10 maturity indices, FHS for the 7-10 index for long positions, GARCH-SK for the 7-10 index, HS for all indices, FHS for the AAA, AA, , 3-5, 5-7 and 7-10 indices and RiskMetrics for the AAA, AA, 3-5, 5-7 and 7-10 indices, for short positions. However, the results of the likelihood ratio test for independence of 5-day VaR estimates indicate that all models fail the test at a 5% significance level for both long and short positions, with the exceptions of the GJR-GARCH for the BBB and 1-3 indices and MRS-GARCH-tv for the AA, A and BBB rating indices for long positions. This result is important and suggests that not all models can be relied on for long horizon VaR estimation, perhaps because of clustering of large credit spread changes movements.

A correct unconditional coverage may have limited accuracy conditionally, while the test of independence does not take into account correct coverage. Consequently, the conditional coverage likelihood ratio test is equivalent to testing the joint null hypothesis of an independent failure process with failure probability α against the alternative of a first order Markov failure process. This is performed through the likelihood ratio test for conditional coverage, and the results over the back-testing period are presented in Table 5.9.

Table 5.8: Estimation of the Likelihood Ratio Test of Independence

The table reports Christoffersen's (1998) likelihood ratio test of the independence during the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations. The likelihood ratio test of the independence is chi-square asymptotic with one degrees-of-freedom. The 1% and 5% critical values for $\chi^2(1)$ are 6.634 and 3.841.

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				Long Positions				
				1 Day Long Position				
GJR-GARCH	-	-	-	-	-	-	-	-
GARCH-SK	2.819	-	2.746	-	0.925	-	3.908	1.995
MRS-GARCH-tv	-	-	2.374	-	-	-	-	-
HS	0.410	0.119	2.181	1.033	0.171	9.274	9.127	7.914
FHS	-	-	0.234	0.398	0.925	2.409	3.574	6.441
RiskMetrics	-	-	1.116	-	-	-	-	-
EVT	-	-	-	-	-	-	-	-
				1 Day Short Position				
GJR-GARCH	-	2.374	2.955	-	-	-	-	-
GARCH-SK	0.077	0.502	-	-	-	-	-	6.516
MRS-GARCH-tv	-	-	-	-	-	-	-	-
HS	6.813	8.586	3.991	1.026	6.813	3.509	9.127	6.151
FHS	12.731	9.540	4.351	3.148	0.310	8.608	8.608	6.041
RiskMetrics	4.025	7.275	1.555	1.948	0.741	5.973	6.670	6.151
EVT	-	2.374	2.955	-	-	-	-	2.374
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				5 Day Long Position				
GJR-GARCH	11.167	11.992	10.059	-	-	16.504	18.999	11.167
GARCH-SK	25.697	21.111	21.108	34.278	18.573	20.412	29.403	16.200
MRS-GARCH-tv	20.359	-	-	-	5.475	19.421	25.146	4.698
HS	37.685	39.371	47.902	24.280	33.530	64.928	70.350	56.660
FHS	21.108	21.322	25.931	24.362	22.553	39.558	45.972	52.240
RiskMetrics	27.350	24.149	33.444	13.557	10.589	28.091	39.184	29.207
EVT	16.567	24.446	16.567	13.608	13.609	20.625	29.948	29.333
				5 Day Short Position				
GJR-GARCH	22.302	34.858	26.691	21.585	21.322	15.271	30.362	29.860
GARCH-SK	27.675	17.594	6.755	31.713	20.097	22.553	23.288	23.287
MRS-GARCH-tv	24.362	22.553	8.567	9.498	39.371	25.931	23.451	17.627
HS	70.573	49.242	87.762	94.591	83.518	99.073	103.768	78.620
FHS	57.626	46.448	66.688	74.080	64.050	78.851	80.628	48.266
RiskMetrics	74.387	64.867	94.729	70.557	71.587	95.300	96.725	78.620
EVT	28.697	29.333	28.697	21.679	11.947	30.833	41.768	21.679

In the case of 1% 1-day VaR for long positions, the conditional coverage test is not rejected at a 5% significant level for VaR estimates by the GJR-GARCH, GARCH-SK for the AA, BBB, 1-3 and 3-5 indices, MRS-GARCH-tv, FHS for AAA, AA and 1-3 indices, RM and EVT; whereas for short positions the conditional coverage test is not rejects at a 5% significant level for VaR estimates by the GJR-GARCH model, GARCH-SK, MRS-GARCH-tv and EVT, with the exception of AAA and BBB indices of the GARCH-SK model. In the case of 1% 5-day VaR estimates, the conditional coverage test is rejected at a 5% significant for all models for both positions, with the exceptions of the GJR-GARCH for BBB and 1-3 indices, and MRS-GARCH-tv for the AA, A and BBB indices. The VaR estimates of econometric models and EVT performed well for long positions over the back-testing period because large movements in credit spread changes were in the form of credit spread increases.

Table 5.9: Estimation of the Likelihood Ratio Test of the Conditional Coverage

The table reports Christoffersen's (1998) likelihood ratio test of the conditional coverage during the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations. The likelihood ratio test of the conditional coverage is chi-square asymptotic with two degrees-of-freedom. The 1% and 5% critical values for $\chi^2(1)$ are 9.21 and 5.99.

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
				1 Day Long Position				
GJR-GARCH	-	0.410	0.410	2.090	1.022	2.090	2.090	2.090
GARCH-SK	9.536	2.353	4.446	2.353	1.135	3.088	7.805	12.040
MRS-GARCH-tv	0.094	0.094	0.094	2.090	0.410	0.410	0.410	0.410
HS	14.212	7.894	18.698	8.808	6.889	17.049	21.633	13.630
FHS	3.088	3.088	5.951	4.295	2.060	4.763	4.242	10.338
RiskMetrics	1.135	0.668	1.784	0.094	0.094	0.000	0.094	0.094
EVT	0.312	-	0.094	0.094	0.094	0.410	0.094	0.094
				1 Day Short Position				
GJR-GARCH	-	2.468	3.365	-	-	-	-	-
GARCH-SK	8.962	3.590	0.312	8.885	0.082	0.410	0.094	6.610
MRS-GARCH-tv	-	-	2.090	2.090	-	-	-	-
HS	19.319	17.471	19.131	17.543	19.319	8.283	21.633	15.035
FHS	16.628	10.674	7.439	8.864	5.084	10.308	10.308	7.176
RiskMetrics	12.910	13.993	14.061	19.880	10.786	12.690	16.716	15.035
EVT	0.410	2.468	3.365	0.410	2.090	1.022	0.094	2.468
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				5 Day Long Position				
GJR-GARCH	13.585	12.076	10.059	-	-	18.257	23.870	13.585
GARCH-SK	51.501	53.603	30.133	61.710	29.987	37.135	58.492	43.632
MRS-GARCH-tv	21.059	2.049	2.049	-	5.475	21.839	29.129	4.792
HS	61.892	58.983	76.991	36.959	54.640	114.008	125.377	105.741
FHS	30.133	23.075	31.755	26.779	30.457	62.200	67.082	81.330
RiskMetrics	30.513	30.985	40.279	14.734	11.289	40.770	55.907	38.232
EVT	18.267	26.799	18.267	13.691	14.744	24.522	41.201	38.218
				5 Day Short Position				
GJR-GARCH	25.465	38.841	34.594	27.408	23.075	23.175	54.570	50.970
GARCH-SK	67.293	51.827	19.433	86.741	26.933	30.457	61.086	52.377
MRS-GARCH-tv	26.779	30.457	10.320	10.675	58.983	31.755	33.646	23.451
HS	115.403	101.824	195.642	192.931	184.221	197.413	241.616	184.093
FHS	83.158	61.588	107.777	128.653	106.998	137.471	141.301	73.798
RiskMetrics	123.049	123.487	188.389	176.030	165.246	248.831	244.979	184.093
EVT	35.414	38.218	35.414	27.395	12.259	36.549	62.640	27.395

In general, the back-testing exercise revealed mixed results regarding the appropriateness and accuracy of models in VaR estimation, even when higher moments were incorporated. One reason for obtaining such inconclusive results on the accuracy VaR estimates could be the back-testing period, which has coincided with the most turbulent period for financial markets. The second reason for the failure of models which take into account asymmetry in the dynamics of distribution of credit spreads (e.g. GARCH-SK) to produce superior VaR estimates could be the fact that by considering the dynamics of higher moments (skewness and kurtosis) the persistence of the estimated and forecasted volatility is reduced. A third reason could be that errors in forecasting simultaneously variance, skewness and kurtosis are reflected in the Value-at-Risk.

Furthermore, we subject our models to the Basel II test to examine and compare the appropriateness of different models with respect to compliance with Basel II. According to Basel II, models are grouped into three categories: green, yellow and red depending on the number of 1% VaR violations. A model is classified as green when there is more than 99.99% probability that it's estimated 1% VaR violations fall within the theoretical (1%) number of VaR violations. By the same token a model is classified as yellow when there is more than 95% probability that its 1% VaR violations fall within the theoretical (1%) number of exceptions, and finally, a model is classified as red if there is less than 95% probability that the number of realised 1% VaR violations exceed the number of theoretical VaR violations. Therefore, in the case of 1-day VaR estimates, models whose percentage of violations (%RF) is below 1.732% $(0.01+1.6449\sqrt{(0.01)(0.99)/500})$ are in the green zone, models between 1.732% and 2.655% $(0.01+3.7190\sqrt{(0.01)(0.99)/500})$ are in the yellow zone, while those that lie above 2.655% are within the red limit.

For 1-day VaR and long position, MRS-GARCH-tv, GJR-GARCH, EVT, and RiskMetrics, with the exception of the AAA rating, seem to fall within the green zone, while HS is the worst performing model and GARCH-SK falls mainly in the yellow acceptance region. For 1-day VaR and short positions, MRS-GARCH-tv, GJR-GARCH, GARCH-SK with the exception of AAA, AA and BBB indices, and EVT perform well, while HS, FHS and RiskMetrics fall in the red acceptance region. This is surprising for the GARCH-SK model, since by taking into account dynamics of higher moments of credit spread changes, this model should have produced superior VaR estimates.

In the case of 5-day VaR estimates, in both long and short positions, all models seem to fail the Basel test with the exceptions of GJR-GARCH for the AA, A, BBB and 1-3 indices, by falling in the green limit, MRS-GARCH-tv by falling in the green and yellow limits, RiskMetrics for 1-3 index and EVT for the BBB index by falling in the green limits, for long positions; the GJR-GARCH for the AAA, AA and 1-3 indices and the MRS-GARCH-tv for the AAA, A and BBB indices by falling within the yellow limit for short positions. The BASEL II compliance back-testing exercise revealed that econometric models performed well for the 5-day 1% VaR long positions, since large movements in credit spread changes were in the form of large credit spread increases during the credit crisis period.

Finally, Figures 5.11 to 5.18 present the 1-day 1% VaR estimates of GJR-GARCH, GARCH-SK, and MRS-GARCH-tv models, for both long and short positions over the back-testing period for rating and maturity indices. The Value-at-Risk for the GARCH-SK model appears to have asymmetric shape for long and short positions as expected, but it seems to behave relatively erratically compared to the other formulations. This occurs because the inverse cumulative function of the Gram-Charlier distribution which is required in computing the Value-at-Risk estimates (see equation 3.68) contains estimates of skewness and kurtosis. In addition, the out-of-sample period in which the models are tested and the Value-at-Risk estimates are computed is the recent credit crisis period. During this period as can be seen from Figures 5.9, 5.10, 5.11 and 5.12, skewness and kurtosis increase significantly and exhibit frequent spikes, thus affecting the inverse CDF at each step of the forecast.

Table 5.10: BASEL II Model Performance Categorization

This table reports the BASELL II model categorization. Models are grouped into three categories: green, yellow and red depending on the number of 1% VaR violations. The general formula for specifying the BASEL limits is defined as: $0.01 - \Phi^{-1}(p)\sqrt{(0.01 \cdot 0.99)/N}$, where Φ^{-1} denotes the inverse of the standard normal cumulative distribution, and N the total number of forecasts. The back-testing period is from 1st June 2007 - 30th April 2009, and has a total of 500 observations.

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				1 Day Long Position				
GJR-GARCH	Green	Green	Green	Green	Green	Green	Green	Green
GARCH-SK	Red	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Red
MRS-GARCH-tv	Green	Green	Green	Green	Green	Green	Green	Green
HS	Red	Red	Red	Red	Red	Red	Red	Red
FHS	Red	Yellow	Red	Yellow	Red	Red	Red	Red
RiskMetrics	Yellow	Green	Green	Green	Green	Green	Green	Green
EVT	Green	Green	Green	Green	Green	Green	Green	Green
				1 Day Short Position				
GJR-GARCH	Green	Green	Green	Green	Green	Green	Green	Green
GARCH-SK	Red	Yellow	Green	Red	Green	Green	Green	Green
MRS-GARCH-tv	Green	Green	Green	Green	Green	Green	Green	Green
HS	Red	Red	Red	Red	Red	Red	Red	Red
FHS	Yellow	Yellow	Yellow	Red	Red	Yellow	Yellow	Yellow
RiskMetrics	Red	Red	Red	Red	Red	Red	Red	Red
EVT	Green	Green	Green	Green	Green	Green	Green	Green
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
				5 Day Long Position				
GJR-GARCH	Yellow	Green	Green	Green	Green	Yellow	Red	Yellow
GARCH-SK	Red	Red	Red	Red	Red	Red	Red	Red
MRS-GARCH-tv	Green	Green	Green	Green	Green	Yellow	Yellow	Green
HS	Red	Red	Red	Red	Red	Red	Red	Red
FHS	Red	Yellow	Red	Yellow	Red	Red	Red	Red
RiskMetrics	Yellow	Red	Red	Yellow	Green	Red	Red	Red
EVT	Yellow	Yellow	Yellow	Green	Yellow	Yellow	Red	Red
				5 Day Short Position				
GJR-GARCH	Yellow	Yellow	Red	Red	Yellow	Red	Red	Red
GARCH-SK	Red	Red	Red	Red	Red	Red	Red	Red
MRS-GARCH-tv	Yellow	Red	Yellow	Yellow	Red	Red	Red	Red
HS	Red	Red	Red	Red	Red	Red	Red	Red
FHS	Red	Red	Red	Red	Red	Red	Red	Red
RiskMetrics	Red	Red	Red	Red	Red	Red	Red	Red
EVT	Red	Red	Red	Red	Red	Red	Red	Red

Figure 5.11: Presents the of out-of-sample 1% VaR estimates of the AAA Index

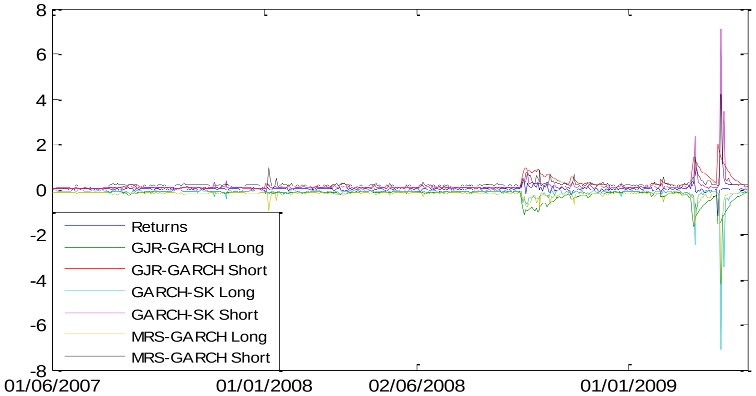


Figure 5.12 Presents the of out-of-sample 1% VaR estimates of the AA Index

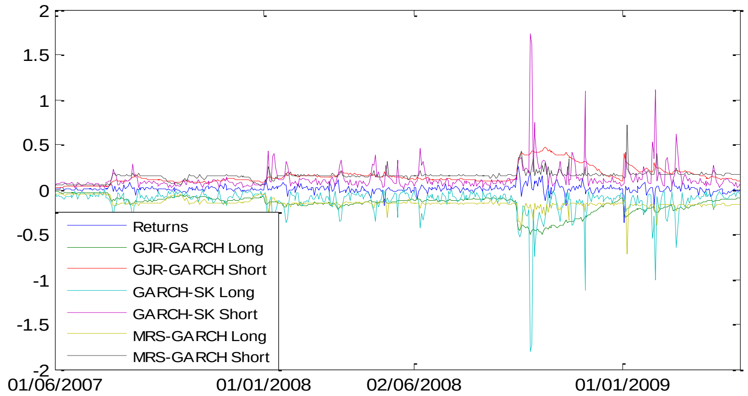


Figure 5.13 Presents the of out-of-sample 1% VaR estimates of the A Index

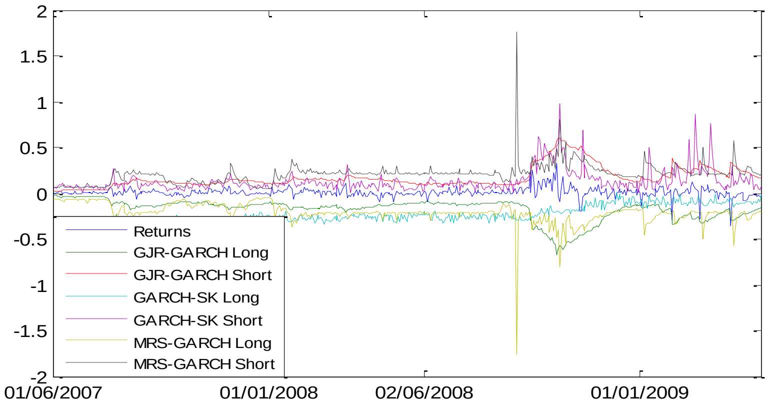


Figure 5.14 Presents the of out-of-sample 1% VaR estimates of the BBB Index

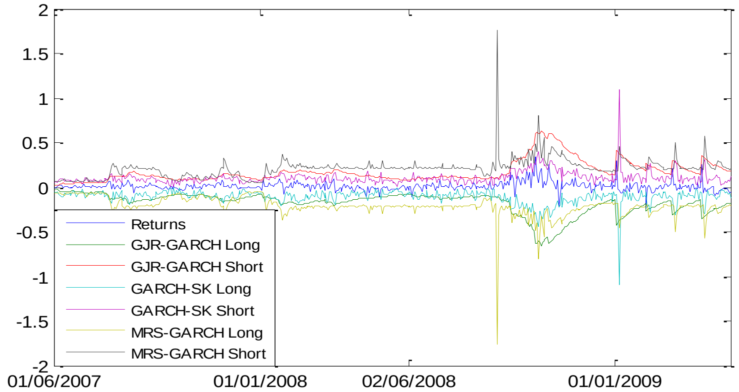


Figure 5.15: Presents the of out-of-sample 1% VaR estimates of the 1-3 year Index

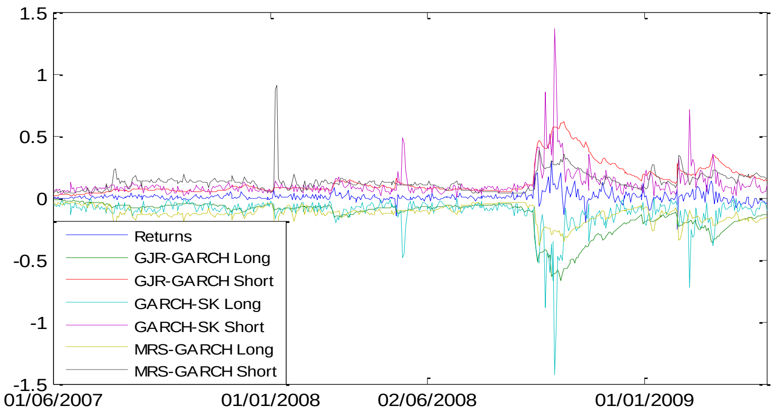


Figure 5.16 Presents the of out-of-sample 1% VaR estimates of the 3-5 year Index

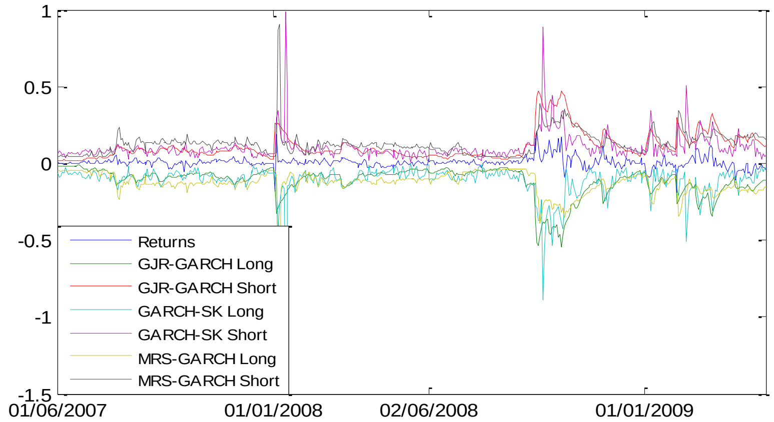


Figure 5.17 Presents the of out-of-sample 1% VaR estimates of the 5-7 year Index

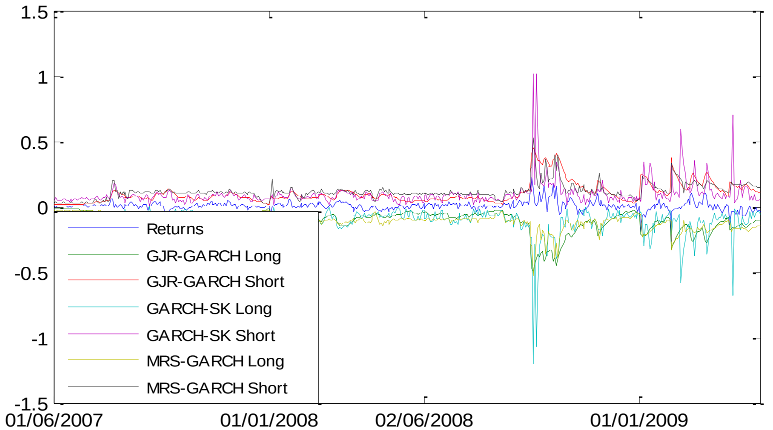
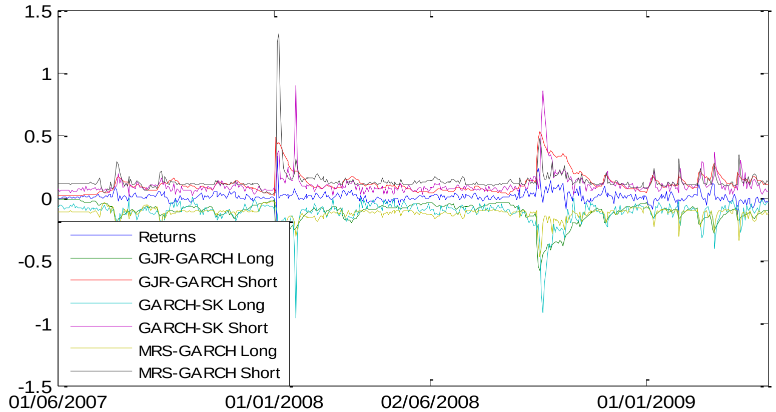


Figure 5.18 Presents the of out-of-sample 1% VaR estimates of the 7-10 year Index



5.4 Conclusions

The aim of this chapter was to investigate and model the time-varying dynamics of credit spread moments of European corporate bond indices. The examination of the dynamics of volatility and the conditional higher moments was undertaken by the application and comparison of a set of models, which to the best of our knowledge had not been applied previously in the European fixed income market. The models estimated were the GJR-GARCH, GARCH-SK, MRS-GARCH-tv models and their forecasting performance of volatility and Value-at-Risk estimates was examined.

It was found that the estimated volatilities, skewness and kurtosis display a consistent pattern across ratings and maturities. Lower ratings and long-term maturities have greater conditional volatilities and kurtosis. This behaviour reflects the higher probability of downgrades and defaults of lower rating and long-term maturity indices. Conditional skewness was found to fluctuate more for lower ratings and long-term maturities and displayed occasional spikes. This meant that during high volatility periods credit spreads widen and interest rates decrease suggesting a deteriorating economy and therefore the negative effect of the credit-worthiness of lower ratings and long-term maturities is reflected as negative spikes in the conditional skewness. Finally, conditional kurtosis exhibits spikes during high volatility periods for higher-ratings and short-term maturities reflecting sharp changes in their credit quality.

The evaluation of the aforementioned formulation's performance was carried out by comparing the 1-day and 5-day ahead volatility forecasts and VaR estimators. It was shown that GARCH-SK produced the most accurate 1-day volatility forecasts while GJR-GARCH yielded the most accurate 5-day volatility forecasts. The adequacy of VaR estimates was examined by the application of the Christoffersen (1998) back-testing procedure. The back-testing exercise revealed mixed results on the appropriateness and accuracy of the examined models, since the back-testing period coincided with the credit crisis. However when the results are examined in combination then econometric models seem to perform well for longer horizons. These results are in line with those reported by Brooks and Persand (2003), Dacco and Satchell (1999) and Marcucci (2009), who do not find a uniformly accurate model for all time horizons either.

Chapter 6

Modelling the time-varying correlation of the European corporate credit spreads

6.1 Introduction

Correlation between asset prices is the most important parameter in portfolio selection, asset allocation, the pricing of derivatives, risk management, as well as trading and hedging activities. Fixed income instruments have a leading role in institutional investors' asset allocation, due to their correlation with liability structures. They were originally characterized by simple cash flows, and have now transformed into securities with increased and complex cash flow structures that appeal to a broader investor base. Therefore, understanding the powers that control bond markets and portfolio credit risk, as well as their dynamics and risk management of these complex securities, is essential for one to make effective use of portfolio strategies.

Existing literature has shown that correlation between different asset classes varies over time (e.g. Engel and Rosenberg, 1995, Bollerslev, 1990, amongst others) and has an important role in the pricing of derivatives, portfolio selection, trading and hedging and risk management. Modelling and estimating time-varying correlation and covariance matrices has a profound effect within all asset classes. This has been highlighted in a number of financial studies. For instance, Alexander (1998) and Alexander and Leigh (1997) highlight the importance of accurate covariance matrices in the estimation of portfolios Value-at-Risk. In other financial applications, Kroner and Ng (1998) show the importance of covariance matrices in determination of hedge ratio and dynamic portfolio optimization. Similar results are presented by Engel and Rosenberg (1995) for option pricing. The comparison of accuracy of different models and specifications for covariance estimation has been the subject of many studies. Takayuki and Yoshinori (2008) test a variety of multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models and find that the DCC-GARCH provides the best VaR estimators in intraday data of the Tokyo Stock Exchange. Alexander (1998) also suggests that the O-GARCH model provides better volatility forecasts in highly-correlated systems. Finally, the Therese (2008) study proves that the Engle and Sheppard (2001) DCC-GARCH formulation provides better short-run volatility forecasts compared with other DCC-GARCH specifications.

However, the literature on estimation of time-varying correlation in the area of credit spreads, and specifically in credit portfolio hedging and trading is limited. Berndt, Ritchken and Sun (2009) highlight the importance of credit spread correlation modelling by incorporating interest rate and credit spread correlation in the pricing of credit derivatives. Roscovan (2008) constructs a hedging strategy by relating bond portfolio returns to changes in credit spreads. Friewarld and Pichler (2008) propose a spread based model to price credit derivatives that incorporates the correlation of credit spreads. They compare their model with other conventional approaches and find that their model is superior during market turbulences. Bobey (2009) investigates the relationship between systematic default correlation and corporate bond credit spreads and finds that credit spreads are positively related to the Collateral Debt Obligation (CDO) market implied default correlation.

As the literature in the field of time-varying correlation of credit spreads is limited, this study aims to fill the gap by examining and comparing the properties and performance of the different multivariate GARCH models in capturing the dynamics of correlation between the credit spread indices. The models examined within this study are the Orthogonal-GARCH, the Constant and Dynamic Correlation GARCH models, Risk Metrics and Diagonal-BEKK formulation. The performance of these models is examined by determining whether they produce accurate VaR estimates over the most turbulent market period, the resent credit crisis. This chapter is organised as follows: Section 6.2 presents the methodology, Section 6.3 outlines the estimation results, and Sections 6.4 concludes.

6.2 Methodology

This section briefly presents the methodology applied in this chapter; for a detailed analysis refer to Chapter 3 in Section 4 entitled Multivariate Volatility Models. The pioneering study of Bollerslev *et al.* (1988) generalizes the univariate models into a general multivariate volatility representation MGARCH. The later studies of Bollerslev (1990), Engle and Shephard (2001), Alexander (2002) among others have extended the MGARCH model to capture more acurately the time-varying covariances and in extention time-varying correlations. All these formulations have a wide application in the financial world, ranging from asset and option pricing, hedging and risk management, as well as portfolio construction and asset allocation. This section generalizes the univariate models to multivariate models which capture the dynamic relationships between the volatility and correlation of the credit spread indices.

Engle and Kroner (1995) propose the BEKK model which guarantees the conditional covariance matrix to be positive-definite as (see Chapter 3 Section 4.1):

$$\mathbf{H_t} = \mathbf{CC'} + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{A_i}\left(\boldsymbol{\varepsilon}_{\mathbf{t-i}}\boldsymbol{\varepsilon}'_{\mathbf{t-i}}\right)\mathbf{A}'_{\mathbf{i}} + \sum_{\mathbf{j=1}}^{\mathbf{q}} \mathbf{B_j}\mathbf{H_{t-j}}\mathbf{B}'_{\mathbf{j}} \tag{6.1}$$

where $\mathbf{A_i}$, $\mathbf{B_j}$ and **C** are N x N parameter matrices and **C** is a lower triangular matrix. However, this formulation has a number of disadvantages. The first drawback is the difficulty of interpreting the parameters $\mathbf{A_i}$ and $\mathbf{B_j}$ and, secondly, the estimated number of parameters of the model increases as the number of series and dimensions of p and q increase $\left[n^2\left(p+q\right)+n\left(n+1\right)/2\right]$. However, the Diagonal BEKK addresses the problem of an increased number of parameters, by imposing the $\mathbf{A_i}$ and $\mathbf{B_j}$ matrices to be diagonal matrices.

Bollerslev (1990) proposes another approach in reducing the number of parameters, the Constant Correlation MGARCH model in which the conditional correlations are constant and the covariance matrix is then defined as (see Chapter 3 Section 4.3):

$$\mathbf{H_t} = \mathbf{D_t R D_t} = \left(\rho_{ij}\sqrt{\sigma_{iit}\sigma_{jjt}}\right) \tag{6.2}$$

where $\mathbf{D_t}$ is a NxN diagonal matrix of time varying standard deviations from a univariate GARCH model with $\sqrt{\sigma_{iit}}$ for the i^{th} diagonal element and $\mathbf{R}$ is the constant correlation matrix which is a symmetric positive definite matrix with $\rho_{ii} = 1 \quad \forall \quad i$.

While the assumption of the constant correlation ensures positive definiteness and provides computational simplicity and estimation, it may not be appropriate and sometimes too restrictive in real life applications. Engle and Shephard (2001) propose the Dynamic Conditional Correlation Multivariate GARCH model and the dynamic covariance matrix is given as (see Chapter 3 Section 4.3):

$$\mathbf{H_t} = \left(\mathbf{1} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{a_m} - \sum_{\mathbf{j=1}}^{\mathbf{q}} \boldsymbol{\beta}_\mathbf{n}\right)\bar{\mathbf{H}} + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{a_i}\left(\boldsymbol{\varepsilon}_\mathbf{t-i}\boldsymbol{\varepsilon}'_\mathbf{t-i}\right) + \sum_{\mathbf{j=1}}^{\mathbf{q}} \boldsymbol{\beta}_\mathbf{j}\mathbf{H_{t-j}} \tag{6.3}$$

where $\boldsymbol{\varepsilon}_\mathbf{t} \sim \mathbf{N}\left(\mathbf{0}, \mathbf{H_t}\right)$ and $\bar{\mathbf{H}}$ is the unconditional covariance (for further information in the estimation procedure, please, refer to Chapter 3 section 4.3, Equations 3.53, 3.54 and 3.55).

Another approach in simplifying the dynamic structure of a multivariate volatility process is the application of factor models (see Chapter 3 Section 4.2). One such model is the orthogonal GARCH model which allows N x N GARCH covariance matrices to be estimated from just *M* univariate GARCH models; where *N* are the number of random variables and *M* the number of principal components (see Section 5 of Chapter 3 for more details regarding principal component analysis). The time-varying covariance matrix $\mathbf{H_t}$ is given as:

$$\mathbf{H_t} = \mathbf{A D_t A'} \tag{6.4}$$

where $\mathbf{A} = \left(\omega_{ij}^{*}\right)$ is the matrix of the normalised factor weights, $\mathbf{D_t}$ is the diagonal matrix of variances of the principal components estimated by a GARCH model and requires the estimation of N(N+5)/2 parameters.

6.3 Estimation Results

This section begins by presenting the descriptive statistics of two equally weighted credit portfolios[36]: the rating and maturity. The section then presents the unconditional correlation between credit spread changes indices, and the Engle and Shepphard (2001) test for constant correlation between the indices (refer to Chapter 3 Section 4.4). It then continues to the estimation of PCA and O-GARCH model, as well as CC-GARCH, DCC-GARCH and Diagonal BEKK formulations. Finally, it presents the out-of-sample performance of the models in computing accurate Value-at-Risk estimates for minimum variance and equally weighted portfolios. It should be observed that this study does not consider an overall credit portfolio, as the estimation of time-varying correlations would have led to spurious results. This happens because an overall portfolio would have been comprised by both rating and maturity indices, but the rating indices are included in the maturity indices, and the maturity indices are included in the rating indices.

Table 6.1 presents the descriptive statistics of the two portfolios. The descriptive statistics of the credit spread and the annualized first differences in credit spread portfolios reveal that the overall coefficients of volatility and skewness have decreased compared to the individual indices. Furthermore, the credit spread portfolios are normally distributed according to the Jarque-Bera test, in contrast with the changes in credit spread portfolios which are not normally distributed at 1% significant level.

[36] The equally weighted portfolios are estimated in order to present the descriptive statistics of a benchmark portfolio. However, during the out-of-sample examination, the minimum variance portfolios are estimated.

Table 6.1 Descriptive statistics of the equally weighted credit spread portfolios

The table reports the descriptive statistics of the equally weighted portfolio for the in-sample period from 03/01/2000 to 31/05/2007. The 1% and 5% critical values for the Jarque-Bera ($\chi^2(2)$) are 9.21 and 5.99.

	Credit spreads		*First Differences in of credit spreads*	
Statistics	**Rating**	**Maturity**	**Rating**	**Maturity**
Mean	1.180	1.160	-0.105	-0.080
Std. Dev.	0.327	0.368	0.690	0.698
Skewness	0.038	0.046	0.074	0.067
Kurtosis	2.683	2.788	7.424	7.486
Jarque-Bera	8.534	4.302	1577	1622

Table 6.2 reports the unconditional correlation matrix of the changes in credit spread indices. The results reveal that all credit spread changes indices are positively correlated while correlation decreases as ratings decline and maturities increase. The highest correlation is observed between the AA and A indices (0.926), followed by the 5-7 and 7-10 indices (0.876). On the other hand, the lowest correlation is observed between 1-3 and 5-7 indices (0.730). These results are in line with the studies of Das *et al.* (2006) and Byrne, Fazio and Fiess (2010). The former argues that firms of the highest credit quality have the highest correlations suggesting that these firms are exposed to an economy wide factor. The latter argue that high correlation in long-term maturities is due to financial integration, and in this case European financial integration, while the lower correlation between short-term and long-term rate is attributed to the fact that short-term rates are more susceptible to shocks and monetary policies.

Table 6.2 Unconditional correlation matrix of the credit spread changes for the period from 03/01/2000 to 30/04/2009

Indices	**AAA**	**AA**	**A**	**BBB**
AAA	1.000	0.765	0.807	0.601
AA		1.000	0.926	0.781
A			1.000	0.793
BBB				1.000
Indices	**1-3**	**3-5**	**5-7**	**7-10**
1-3	1.000	0.771	0.730	0.747
3-5		1.000	0.871	0.828
5-7			1.000	0.876
7-10				1.000

Table 6.3 presents the Engle and Shepphard (2001) test of the null hypothesis of the constant correlation against an alternative of dynamic condition correlation. The null hypothesis of a null correlation is rejected for all of the indices in favour of a dynamic structure. Similar results were presented in the study of Engle and Shepphard (2001)

Table 6.3 Engle and Sheppard (2001) constant correlation test

The table reports the estimation results of the constant correlation test proposed by Engle and Sheppard (2001). The critical values are estimated from a χ^2 distribution given the probability the correlation is constant with degrees-of-freedom equal to the number of lags plus 1.

Lags	1-Day		5-Day		15-Day		30-Day		60-Day	
	Statistic	χ^2	*Statistic*	χ^2	*Statistic*	χ^2	*Statistic*	χ^2	*Statistic*	χ^2
					Rating Indices					
AAA-AA	1.952	0.3769	3.025	0.806	4.843	0.996	50.808	0.014	94.175	0.004
AAA-A	2.990	0.2242	26.133	0.000	29.124	0.023	38.216	0.174	52.830	0.763
AAA-BBB	2.137	0.3436	2.458	0.873	4.727	0.997	223.360	0.000	336.067	0.000
AA-A	1.022	0.5998	2.285	0.892	6.299	0.985	143.151	0.000	209.430	0.000
AA-BBB	0.668	0.7161	1.361	0.968	1.947	1.000	5.585	1.000	58.845	0.554
A-BBB	0.575	0.7502	2.168	0.904	4.092	0.999	1379.699	0.000	2207.681	0.000
					Maturity Indices					
1-3-3-5	1.644	0.440	3.682	0.720	7.066	0.972	15.805	0.989	30.612	1.000
1-3-5-7	38.960	0.000	43.047	0.000	49.427	0.000	165.201	0.000	181.839	0.000
1-3-7-10	8.603	0.014	9.066	0.170	11.087	0.804	17.179	0.979	23.134	1.000
3-5-5-7	130.277	0.000	146.046	0.000	166.861	0.000	197.208	0.000	233.113	0.000
3-5-7-10	18.442	0.000	23.120	0.001	27.454	0.037	37.014	0.211	570.426	0.000
5-7-7-10	115.944	0.000	116.248	0.000	118.532	0.000	120.060	0.000	128.734	0.000
					Portfolios					
Rating	9.704	$7.8*10^{-03}$	13.608	$3.4*10^{-02}$	16.997	0.391	95.587	$1.7*10^{-08}$	179.110	$1.5*10^{-13}$
Maturity	70.699	$4.4**10^{-16}$	72.859	$1.1*10^{-13}$	80.059	$1.6*10^{-10}$	89.727	$1.3*10^{-07}$	155.490	$3.4*10^{-10}$

The first frameworks to be examined are the PCA and the O-GARCH(1,1) specifications. Table 6.4 presents the PCA for the rating and maturity changes in credit spread portfolios. The first principal component is able to explain 89.9% of the total variation in the rating portfolio and 86.3% in the maturity portfolio. This component is called the level or trend component as its factor weights suggest that an upward movement in the first principal component will induce a parallel shift in the credit spread curve. The second component is called the slope or tilt and it is able to explain 6.5% and 7.7% of variation for the corresponding portfolios. Its factor weights suggest that an upward movement in the second principal components will induce such a change in the slope that when ratings deteriorate the credit spread curve decreases, when the 1-3 maturity increases the credit spread curve increases while decreases for other maturities. The third component is called the curvature and explains up to 2.4% and 3.5%, respectively. Finally, the last 1.3% and 2.5% of the variability is explained by the fourth component.

The study focuses on the first two components which can explain up to 96% of the total variability and Table 6.5 presents the O-GARCH(1,1) models of these components. Alexander (2000) argues that all the variation stems from the second principal component, because if only one component is considered then all variables are assumed to be perfectly correlated. The coefficients on the lagged squared error β_1 and the lagged conditional variance β_2 coefficients are statistically significant. Alexander (2000) presents similar results for the UK yield curve.

Table 6.4 Reports the eigenvectors and factor weights of the principal component analysis for the period from 03/01/2000 to 31/05/2007.

		Rating Portfolio						
PC	**Eigenvalues**	**Explained Variance**	**Cumulative R^2**	**Factor Weights**				
1	0.008	0.899	0.899	0.501	0.510	0.512	0.476
2	0.001	0.065	0.964	0.416	0.257	0.138	-0.861
3	0.000	0.024	0.987	-0.752	0.381	0.511	-0.167
4	0.000	0.013	1.000	0.104	-0.727	0.677	-0.058
		Maturity Portfolio					
	Eigenvalues	**Explained Variance**	**Cumulative R^2**	**Factor Weights**			
1	0.006	0.863	0.863	0.478	0.502	0.510	0.509
2	0.001	0.077	0.940	0.861	-0.131	-0.410	-0.269
3	0.000	0.035	0.975	0.119	-0.820	0.160	0.537
4	0.000	0.025	1.000	0.122	-0.242	0.739	-0.617

Table 6.5 Estimation results of the Orthogonal-GARCH model

The table reports the estimation results of the O-GARCH model. The estimation is performed by the method of quasi maximum likelihood using the BGFS algorithm in Matlab 7.8 software package. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. The GARCH model is given as:

$$\sigma_t^2 = \beta_o + \beta_1\varepsilon_{t-1}^2 + \beta_2\sigma_{t-1}^2, \quad \varepsilon_t \sim \left(0, \sigma_t^2\right)$$

where σ_t^2 is the conditional variance on day *t*. Additionally, the 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307.

	PC of the Rating Portfolio		**PC of Maturity Portfolio**	
	Factor 1	***Factor 2***	***Factor 1***	***Factor 2***
β_o	0	7.58*10^{-05}	0.004	0
	(0.000)	(85.91)	(1006)	(0.000)
β_1	0.029	0.032	0.202	0.014
	(6913)	(279.16)	(297.95)	(5106)
β_2	0.971	0.968	0.798	0.986
	(4260)	(1959)	(2077)	(1389)
Diagnostics:				
ARCH(1)	0.036	0.085	2.155	14.634
ARCH(10)	0.748	0.521	3.525	11.679

The estimation of the CC-GARCH and DCC-GARCH[37] models was undertaken into two stages following Engle and Sheppard (2001) methodology. The first stage estimates the univariate GJR-GARCH specification for the changes in credit spreads and the second stage uses the transformed by their standard deviation residuals of the first stage to estimate the parameters of the dynamic correlation conditioned on the parameters estimated by the first stage. Table 6.6 presents the CC-GARCH and DCC-GARCH estimated parameters. Panel A presents the estimated coefficients of the GJR-GARCH. Panel B presents the coefficients of the conditional correlation estimated by the CC-GARCH specification. These coefficients are higher than those estimated by the unconditional correlation and are highly statistically significant. The conditional correlations are between 0.645 and 0.967, the largest is observed between the AA and A indices and the lowest between the 1-3 and 5-7 indices. Panel C presents the estimated coefficients of the DCC-GARCH model which are highly significant, suggesting that correlation is highly persistent for both European corporate changes in credit spread portfolios. Finally, Engle's heteroskedasticity test and Ljung-Box

[37] The multivariate Normal distribution is applied in the estimation of the Multivariate GARCH models, since convergence issues arose when the multivariate Student t-distribution or Gram-Charlier expansion series were considered.

autocorrelation Q-test for one and ten lags reveal that the model is well specified as there is no presence of heteroskedasticity or autocorrelation.

Table 6.6 Estimation results of the CC-GARCH and DCC-GARCH models

The table reports the estimation results of the CC-GARCH and DCC-GARCH models. The estimation is performed by the method of quasi maximum likelihood using the BGFS algorithm in Matlab 7.8 software package. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. Panel A: For the first stage estimation the GJR-GARCH model is given as:

$$y_{i,t} = \alpha_{0,i} + \alpha_{1,i}\varepsilon_{t-1,i} + \varepsilon_{t,i}, \qquad \boldsymbol{\varepsilon}_{\mathbf{t}} \sim iid\left(0, \mathbf{H}_{\mathbf{t}}\right)$$

$$\sigma_{i,t} = \beta_{0,i} + \beta_{1,i}\varepsilon_{t-1,i}^2 + \beta_{2,i}\sigma_{t-1,i}^2 + \beta_{3,i}\varepsilon_{t-1,i}^2 I_{t-1,i}, \quad i = 1,2,3,4$$

where y_t is the change in credit spreads, σ_t^2 is the conditional variance on day t, I_{t-1} is an indicator function that takes value of 1 when the ε_{t-1} is negative and 0 either wise and i, the different rating and maturity classes. Panel B: The CC-GARCH model is given as:

$$\mathbf{H}_{\mathbf{t}} = \mathbf{D}_{\mathbf{t}}\mathbf{R}\mathbf{D}_{\mathbf{t}}$$

where $\mathbf{D}_{\mathbf{t}}$ is a NxN diagonal matrix of time varying standard deviations and $\mathbf{R}$ is the constant correlation matrix Panel C: The DCC-GARCH model is given as:

$$\mathbf{H}_{\mathbf{t}} = \left(1-\gamma_0-\gamma_1\right)\bar{\mathbf{H}} + \gamma_0\boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}} + \gamma_1\mathbf{H}_{\mathbf{t-1}}$$

Additionally, the 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and the BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

Panel A: First Stage Univariate GJR-GARCH Estimation

	AAA	*AA*	*A*	*BBB*	*1-3*	*3-5*	*5-7*	*7-10*
$\alpha_{0,i}$	0.000	-0.001	-0.001	-0.001	-3.4*10^{-04}	1.9*10^{-04}	-4.3*10^{-04}	-2.7*10^{-04}
	(-0.278)	(-2.379)	(-1.657)	(-3.339)	(-1.280)	(0.947)	(-3.374)	(-1.533)
$\alpha_{1,i}$	-0.393	-0.415	-0.344	-0.178	-0.324	-0.430	-0.380	-0.396
	(-24.989)	(-23.778)	(-20.866)	(-10.970)	(-23.727)	(-28.672)	(-21.738)	(-24.934)
$\beta_{0,i}$	2.0*10^{-07}	2.0*10^{-07}	2.0*10^{-07}	7.9*10^{-07}	3.6*10^{-07}	2.0*10^{-07}	7.1*10^{-07}	1.0*10^{-07}
	(0.975)	(0.987)	(0.876)	(2.390)	(1.491)	(3.541)	(4.825)	(4.939)
$\beta_{1,i}$	0.009	0.035	0.038	0.081	0.074	0.039	0.160	0.148
	(9.574)	(11.140)	(12.614)	(17.625)	(18.339)	(17.195)	(15.112)	(11.331)
$\beta_{2,i}$	0.984	0.970	0.976	0.959	0.956	0.977	0.879	0.902
	(1187.454)	(438.078)	(526.761)	(464.675)	(519.831)	(1013.956)	(149.005)	(136.059)
$\beta_{3,i}$	0.012	-0.011	-0.029	-0.081	-0.060	-0.035	-0.078	-0.100
	(5.248)	(-2.616)	(-7.183)	(-12.566)	(-11.632)	(-9.839)	(-5.831)	(-6.911)

Table continues at the next page

Panel B: CC-GARCH Estimation								
Correlation Matrix, R, for the rating portfolio								
		Indices	**AAA**	**AA**	**A**	**BBB**		
		AAA	1.000	0.843	0.842	0.724		
				(71.61)	(83.47)	(46.44)		
		AA		1.000	0.967	0.839		
					(11644)	(330.58)		
		A			1.000	0.864		
						(438.14)		
		BBB				1.000		
Correlation Matrix, R, for the maturity portfolio								
		Indices	**1-3**	**3-5**	**5-7**	**7-10**		
		1-3	1.000	0.723	0.645	0.664		
				(33.67)	(18.23)	(5.27)		
		3-5		1.000	0.805	0.742		
					(145.90)	(6.84)		
		5-7			1.000	0.792		
						(9.53)		
		7-10				1.000		
Panel C: Second Stage DCC-GARCH Estimation								
γ_0	0.055				0.025			
	(20.940)				(30.003)			
γ_1	0.834				0.955			
	(37.774)				(51.413)			
Diagnostics								
ARCH(1)	0.027	0.018	0.008	0.744	0.472	0.052	0.003	0.205
ARCH(10)	0.667	3.341	0.453	3.777	3.312	1.892	5.342	2.149
LB Q test(1)	0.001	0.000	0.018	0.001	0.391	0.028	0.809	0.299
LB Q test(10)	11.776	8.882	11.430	4.817	2.549	11.181	13.857	13.919
	CC-GARCH	**DCC-GARCH**			**CC-GARCH**	**DCC-GARCH**		
Log Likelihood	19867	20318			18653	19103		
AIC	-39722	-40620			-37294	-38190		
BIC	-39714	-40616			-37286	-38186		

Table 6.7 presents the Diagonal-BEKK estimation results. The coefficients of **C, A** and **B** vectors are highly significant except for the BBB, 5-7 and 7-10 of the **C** vector. This means that the covariance vector and in extention the correlation matrix is affected by the lagged squared errors – shocks and the lagged covariance vector. This result may provide possible insights regarding the effects of shocks on the long-term dynamics of correlation, which are of great interest in the macroeconomic environment.

Engle and Sheppard (2001) argue that while it is not possible to directly compare the dynamic and the constant conditional correlation multivariate models using the LR statistics, the information criteria can be deployed as a comparison measurement. According to the Akaike and Bayesian information criteria the Diagonal BEKK formulation is the best model to describe the correlation structure between the changes in credit spread indices.

Table 6.7 Estimation results of Diagonal-BEKK model

The table reports the estimation results of the Diagonal BEKK model. The estimation is performed by the method of quasi maximum likelihood using the BGFS algorithm in Matlab 7.8 software package. The sample period is from 3rd January 2000 to 31st May 2007, a total of 1933 observations. The numbers in parentheses are the t-stats. The t-stats reported are very sensitive to the software and the algorithm employed. The D-BEKK is given as:

$$\mathbf{H_t} = \mathbf{CC'} + \mathbf{A}\left(\boldsymbol{\varepsilon}_{\mathbf{t-1}}\boldsymbol{\varepsilon}'_{\mathbf{t-1}}\right)\mathbf{A'} + \mathbf{BH_{t-1}B'}$$

The 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and the BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	Rating Portfolio				**Maturity Portfolio**			
	AAA	***AA***	***A***	***BBB***	***1-3***	***3-5***	***5-7***	***7-10***
C	0.005				0.002			
	(11226)				(1196)			
	0.001	0.002			0.001	0.001		
	(2328)	(3702)			(297.024)	(770.492)		
	0.002	0.002	$5.043*10^{-5}$		0.001	0.000	0.001	
	(18740)	(2472)	(2076)		(517.687)	(-3.348)	(212.535)	
	0.002	0.002	0.001	$-7.3*10^{-8}$	$3.8*10^{-5}$	0.002	$5.1*10^{-5}$	$-1.2*10^{-10}$
	(2084)	(7879)	(2098)	(-0.112)	(7.800)	(3333)	(0.861)	(0.000)
A	0.275				0.293			
	(81.387)				(498.016)			
		0.270				0.191		
		(76.998)				(797.408)		
			0.328				0.203	
			(53.263)				(1386.502)	
				0.304				0.216
				(72.263)				(603.724)
B	0.961				0.961			
	(337648)				(1187796)			
		0.951				0.984		
		(152614)				(1320773)		
			0.966				0.981	
			(529350)				(4253782)	
				0.959				0.977
				(152289)				(2906232)
Diagnostics								
ARCH(1)	0.026	0.129	0.040	0.897	0.331	8.457	3.738	11.347
ARCH(10)	0.598	2.086	0.494	2.377	2.847	5.082	6.956	21.621
Log Likelihood	20670				19103			
AIC	-41303				-38868			
BIC	-41203				-38767			

Figures 6.1 to 6.12 present the in-sample correlation dynamics between the different credit spread changes indices. The correlations for the OGARCH and Diagonal BEKK models are estimated at any time *t* by $\rho_{t,ij} = \text{cov}\left(\sigma_{t,i}\sigma_{t,j}\right) / \sigma_{t,i}\sigma_{t,j}$. Correlation estimates between the maturity indices appear to be more volatile compared to the correlation estimates of the rating indices. Overall, for the rating indices, correlation estimates are more erratic for the DCC-GARCH model followed by the Diagonal BEKK formulation. For the maturity indices most fluctuations in correlation are exhibited by the O-GARCH model, followed by the DCC-GARCH and Diagonal-BEKK. The O-GARCH fairs good during the 2000 to 2005 period, which is a period of high correlation. However, the O-GARCH is unable to provide accurate correlation estimates during the lower correlation period, which is observed after the middle of 2005. Alexander (2001) and can der Weide (2002) argue that the O-GARCH specification underestimates correlation when the data exhibits weak dependence.

Friewald and Pichler (2008) argue that there is a direct relationship between Credit Default Swap spreads correlation and default correlation. Another branch of the literature finds that default correlation is directly linked to the business cycle and changes over time (see Giesecke, 2004 and Das *et al.* 2006). The estimated correlations presented in Figures 6.1 to 6.12 reveal that there may be a direct relationship between credit spread correlation and default correlation. During periods of market turbulence, when the economy is contracting and credit spreads widen, the correlation coefficients between credit spreads are high, suggesting higher default probabilities and correlation. However, correlation coefficients between credit spreads changes decline from 2005 up until 2007 when the market enters a more stable state and experiences a high economic growth, suggesting a decrease in default probabilities and correlation.

Figure 6.1 Presents the in-sample conditional correlation between AAA-AA Indices

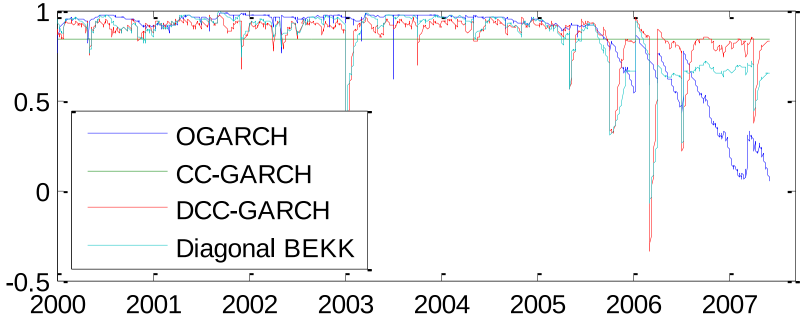


Figure 6.2 Presents the in-sample conditional correlation between AAA-A Indices

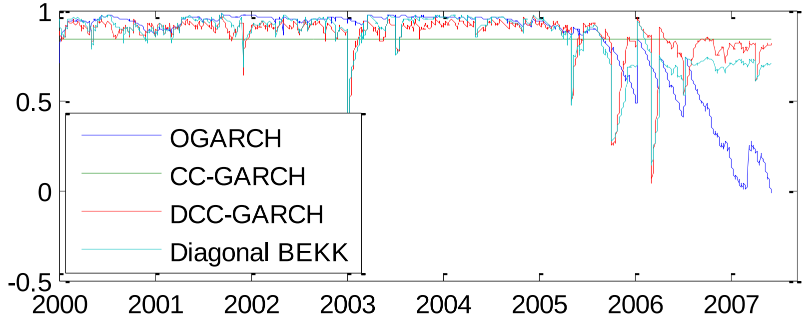


Figure 6.3 Presents the in-sample conditional correlation between AAA-BBB Indices

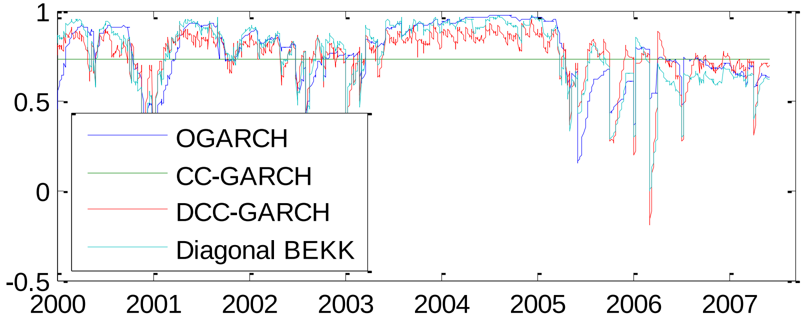


Figure 6.4 In-Sample Conditional Correlation between AA-A Indices

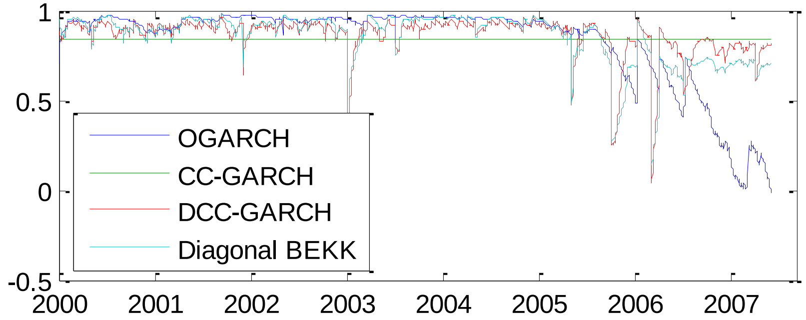


Figure 6.5 Presents the in-sample conditional correlation between AA-BBB Indices

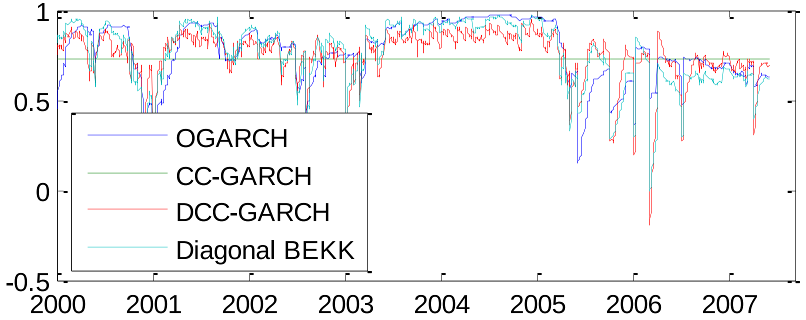


Figure 6.6 Presents the in-sample conditional correlation between A-BBB Indices

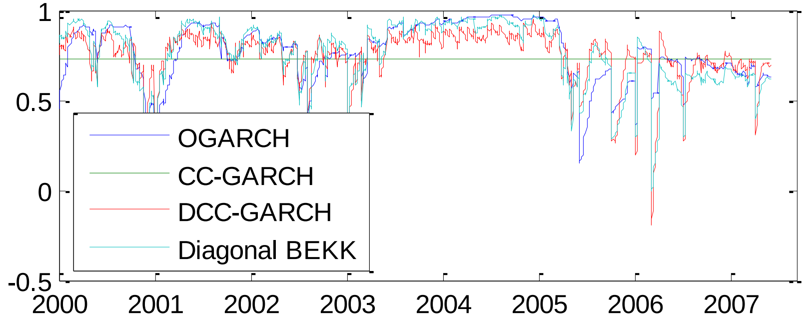


Figure 6.7 Presents the in-sample conditional correlation between 1-3-3-5 Indices

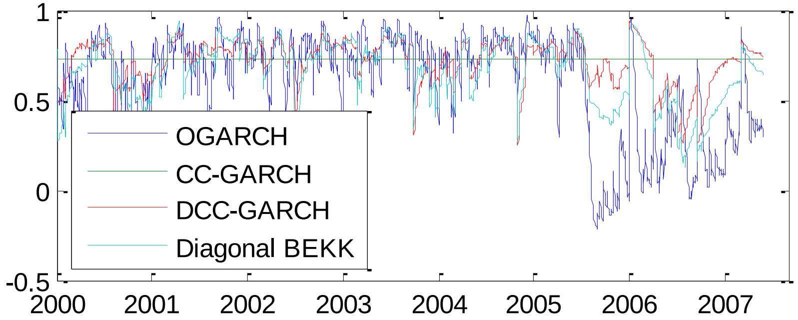


Figure 6.8 Presents the in-sample conditional correlation between 1-3-5-7 Indices

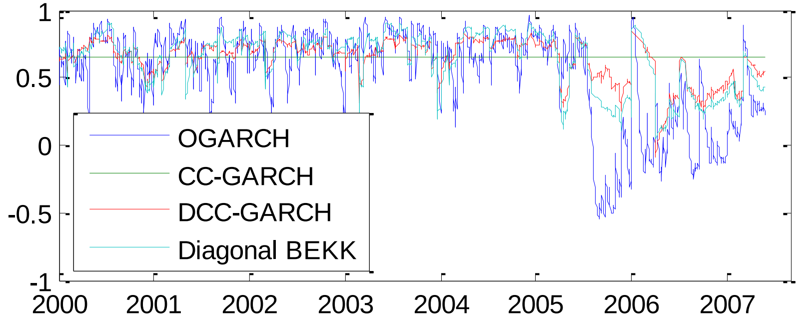


Figure 6.9 Presents the in-sample conditional correlation between 1-3-7-10 Indices

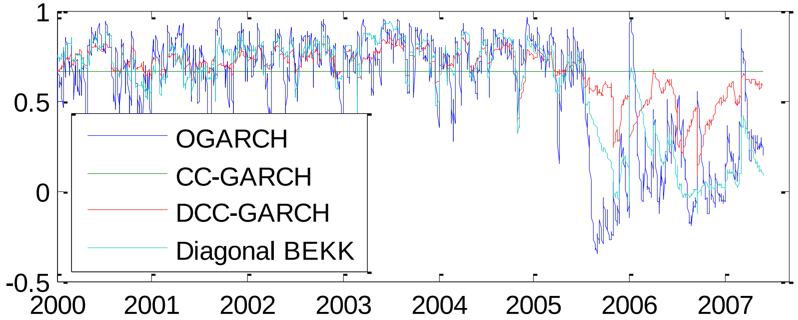


Figure 6.10 Presents the in-sample conditional correlation between 3-5-5-7 Indices

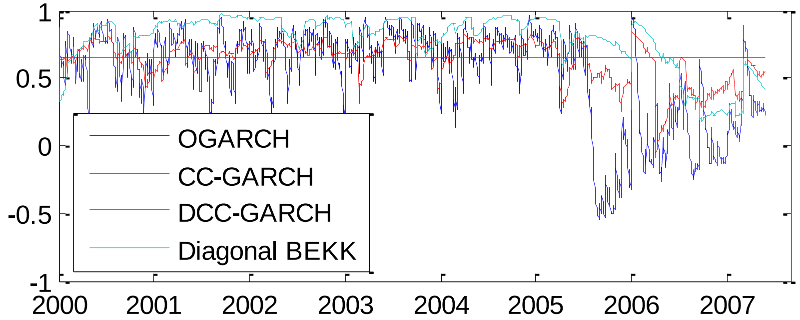


Figure 6.11 In-Sample Conditional Correlation between 3-5-7-10 Indices

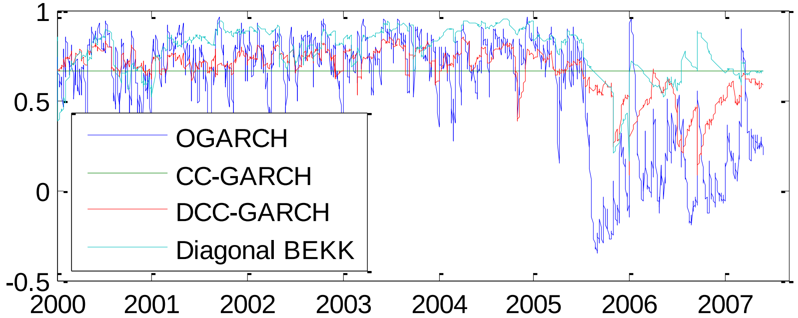
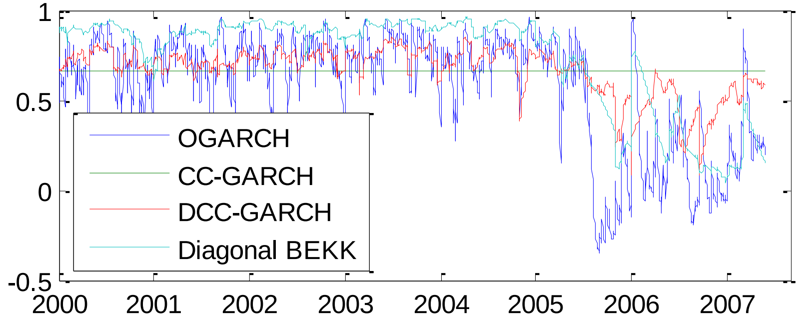


Figure 6.12 In-Sample Conditional Correlation between 5-7-710 Indices



6.3.1 Value-at-Risk Estimation

The performance of multivariate models in producing accurate VaR estimates for credit spread changes is examined using the minimum variance portfolio which is estimated by the forecasted variance-covariance matrix under each specification and two equally weighed portfolios, based on the rating and the maturity indices. The back-testing performance of different models is examined over the period from 1st June 2007 to 30th April 2009 and the Christoffersen (1998) test for the conditional coverage is applied to examine the accuracy of these estimates. The Value-at-Risk analysis focuses on the 1% 1-day VaR level, as it is the level used by Basel (2002) to evaluate models. Table 6.9 presents four statistics: percentage proportion of failures (PF%); and the unconditional coverage, independence and conditional coverage likelihood ratio tests for both minimum variance and equal weighted portfolios and for long and short positions.

The minimum variance portfolio is of particular interest as the weights on the changes in credit spread indices are determined by the estimated variance-covariance matrix and the average minimum variance weights are presented in Table 6.8. The average weights show that all specifications attribute larger weights to the higher rating indices, and O-GARCH and CC-GARCH attribute larger weights to long-term maturity indices, while DCC-GARCH, Diagonal-BEKK and RiskMetrics attribute larger weights for the short- and long-term maturity indices.

Table 6.8 Reports the average weights of the minimum variance portfolios for the period from 1st June 2007 - 30th April 2009

	Ratings				**Maturities**			
	AAA	**AA**	**A**	**BBB**	**1-3**	**3-5**	**5-7**	**7-10**
O-GARCH	0.478	0.484	0.006	0.032	0.120	0	0.002	0.878
CC-GARCH	0.329	0.644	0.007	0.002	0.105	0	0.137	0.758
DCC-GARCH	0.290	0.385	0.184	0.141	0.330	0.174	0.313	0.183
D-BEKK	0.230	0.529	0.169	0.070	0.320	0.180	0.215	0.285
RiskMetrics	0.387	0.382	0.121	0.111	0.388	0.204	0.190	0.218

Starting with percentage proportion of failures for 1-day VaR estimates of long and short positions, it can be seen that the models of the minimum variance portfolio exhibit the lowest PF. This occurs because the VaR estimates are compared with the corresponding portfolios, for instance, the VaR estimates computed by the minimum variance portfolios, are then compared with the returns of the minimum variance portfolios, whose overall variability is reduced compared to the equally weighted portfolios. Overall, the O-GARCH, CC-GARCH and Diagonal-BEKK exhibit the lowest PF for both long and short positions.

The first likelihood ratio test proposed by Christoffersen (1998) is known as the test of unconditional coverage. This test is not rejected at a 5% significance level for either minimum variance rating or maturity portfolios for long positions. On the other hand, the test is rejected at a 5% significance level for both minimum variance and equally weighted portfolio for short positions, except for the models in the minimum variance rating portfolio, and Diagonal-BEKK in the equally weighted portfolios. The next test is the independence test of VaR estimates, which ignores the unconditional coverage and tests the clustering of VaR violations. The test is rejected at a 5% significance level for example for the O-GARCH and CC-GARCH models for the minimum variance rating portfolio for long positions. The independence test of VaR estimates indicates that the violations are clustered.

A correct unconditional coverage may have limited accuracy conditionally, while the test of independence does not take into account correct coverage. Therefore, the conditional coverage likelihood ratio test is equivalent to testing the joint null hypothesis of first order Markov failure process. The conditional coverage likelihood test is rejected at a 5% significance level in the case of the DCC-GARCH model in the equally weighted maturity portfolio for long positions, and in minimum variance rating portfolio for short positions. Additionally, the conditional coverage likelihood test is rejected at a 5% significance level by the O-GARCH, CC-GARCH in the minimum variance maturity portfolio for long positions, O-GARCH and RiskMetrics in the equally weighted portfolios for short positions, and DCC-GARCH in the equally weighted maturity portfolio for short positions.

Basel II groups models into three categories - green, yellow and red - depending on the number of 1% VaR violations. A model lies within the green limit when there is a 99.99% probability that its estimated 1% VaR violations will fall within the theoretical number of VaR violations, within the yellow limit when there is more than 95% probability that its 1% VaR violation will fall within the theoretical (1%) number of exceptions, and finally, a model is classified as red if there is less than 95% probability that the number of realised 1% VaR violations will exceed the number of theoretical VaR violations. Models whose %RF is below 1.732% $(0.01+1.6449\sqrt{(0.01)(0.99)/500})$ are in the green zone, models between 2.655% $(0.01+3.7190\sqrt{(0.01)(0.99)/500})$ and 1.732% are in the yellow zone and all the others lie in the red zone.

According to Basel II, all the models lie within the green limit in the minimum variance and equally weighted portfolios for both long and short positions, with the exception of the DCC-GARCH model of the minimum variance maturity portfolio for long positions. Furthermore, all models of the minimum variance portfoliso and for short positions lie in the green limit, whereas the CC-GARCH and DCC-GARCH models as well as the DCC-GARCH and Diagonal-BEKK models of the equally weighted maturity portfolio for short positions lie in the yellow limit. Finally, in the red limit lie for short positions the O-GARCH and CC-GARCH of the minimum variance maturity portfolio, and O-GARCH and RiskMetrics of the equally weighted portfolios.

The VaR loss function reported in Table 6.10 tests the ability of the VaR models to forecast the portfolio losses and the lowest the reported statistic the accurate the VaR model. The lowest statistic for both long and short positions and portfolios are reported for the RiskMetrics and DCC-GARCH models, followed by the CC-GARCH, O-GARCH and D-BEKK.

The back-testing exercise revealed mixed results regarding the appropriateness of the proposed models in VaR estimation. Although, the D-BEKK and DCC-GARCH pass Christoffersen's (1998) and BASEL II tests for both long and short positions and both portfolios, the VaR loss function reveals that the RiskMetrics is the model of choise in forecasting the losses of the portfolios. One possible reason for obtaining such results could be the choise of the multivariate distribution (i.e. the multivariate normal distribution, since neither the multivariate Student t-distribution nor the multivariate Gram-Charlier expansion series were able to converge during the recent financial crisis), which may not capture the series characterictics during the resent financial crisis.

Table 6.9 Reports the Value-at-Risk analytics for the long and short positions of the minimum variance and equally weighted portfolios

The table reports the Christoffersen's (1998) likelihood ratio test of the unconditional coverage, independence, and conditional coverage during the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations. The two former tests are chi-square asymptotic with one degrees-of-freedom and the 1% and 5% critical values for $\chi^2(1)$ are 6.634 and 3.841. The latter is chi-square asymptotic with two degrees-of-freedom and the 1% and 5% critical values for $\chi^2(1)$ are 9.21 and 5.99. This table also reports the BASELL II model categorization. Models are grouped into three categories: green, yellow and red depending on the number of 1% VaR violations. The general formula for specifying the BASEL limits is defined as: $0.01 - \Phi^{-1}(p)\sqrt{(0.01 \cdot 0.99)/N}$, where Φ^{-1} denotes the inverse of the standard normal cumulative distribution, and N the total number of forecasts.

	PF%		Basel		LRUC		LRIND		LRCC	
Long Positions										
Minimum Variance Portfolios										
	Rating	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*
O-GARCH	0.40%	1.60%	Green	Green	1.027	0.668	-	6.779	1.027	7.447
CC-GARCH	0.40%	1.60%	Green	Green	1.027	0.663	-	6.784	1.027	7.447
DCC-GARCH	1.00%	1.80%	Green	Yellow	0.000	1.128	-	0.927	0.000	2.054
D-BEKK	0.80%	1.00%	Green	Green	0.096	0.001	-	-	0.096	0.001
RiskMetrics	1.40%	1.60%	Green	Green	0.309	0.663	1.344	1.118	1.652	1.780
Equal Weighted Portfolios										
O-GARCH	1.40%	1.00%	Green	Green	0.309	0.000	1.344	-	1.652	0.000
CC-GARCH	1.20%	0.80%	Green	Green	0.081	0.096	1.615	-	1.696	0.096
DCC-GARCH	1.60%	1.00%	Green	Green	0.663	0.000	3.577	-	4.240	0.000
D-BEKK	1.20%	0.80%	Green	Green	0.413	0.413	1.615	-	-	0.413
RiskMetrics	1.20%	0.60%	Green	Green	0.081	0.413	1.615	-	1.696	0.413
Short Positions										
Minimum Variance Portfolios										
	PF%		**Basel**		**LRUC**		**LRIND**		**LRCC**	
	Rating	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*	*Rating*	*Maturity*
O-GARCH	0.60%	3.39%	Green	Red	0.413	7.775	-	15.354	0.413	7.775
CC-GARCH	0.60%	3.79%	Green	Red	0.413	10.045	-	13.380	0.413	10.045
DCC-GARCH	1.40%	1.60%	Green	Green	0.309	0.668	4.098	3.574	4.406	0.668
D-BEKK	1.00%	2.59%	Green	Yellow	0.000	3.897	1.949	6.441	1.949	3.897
RiskMetrics	0.60%	1.40%	Green	Green	0.413	0.309	-	4.098	0.413	4.406
Equal Weighted Portfolios										
O-GARCH	3.79%	2.99%	Red	Red	10.020	5.698	3.661	1.399	13.682	7.097
CC-GARCH	2.59%	2.20%	Yellow	Yellow	3.883	2.343	1.849	2.412	5.732	4.755
DCC-GARCH	2.59%	2.79%	Yellow	Red	3.883	4.759	0.399	1.612	4.282	6.370
D-BEKK	1.20%	1.80%	Green	Yellow	0.081	1.128	1.615	-	1.696	1.128
RiskMetrics	2.79%	3.19%	Red	Red	4.759	6.698	3.514	4.871	8.282	11.568

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Table 6.10 VaR Loss Function

The table reports the VaR loss function during the back-testing period from 1st June 2007 - 30th April 2009, with a total of 500 observations. This loss functions measures the ability of the proposed VaR models to forecast the portfolio losses and it is given as:

$$RMSE_i = \sqrt{\frac{\sum\left(VaR_j - y_i\right)^2}{N}}$$

where y_i are the portfolio changes in credit spreads (i.e. i = rating and maturity portfolios), VaR_j are the 99% VaR estimates of the different models (i.e. j = O-GARCH, CC-GARCH, DCC-GARCH and D-BEKK) and N are the number of forecasted observations.

	O-GARCH	CC-GARCH	DCC-GARCH	D-BEKK	RiskMetrics
			Long Positions		
			Rating Portfolios		
Equally Weighted	0.127	0.136	0.131	0.137	0.125
Minimum Variance	0.109	0.114	0.094	0.101	0.095
			Maturity Portfolios		
Equally Weighted	0.097	0.098	0.095	0.105	0.095
Minimum Variance	0.082	0.075	0.073	0.092	0.070
			Short Positions		
			Rating Portfolios		
Equally Weighted	0.108	0.117	0.111	0.114	0.107
Minimum Variance	0.101	0.109	0.087	0.095	0.088
			Maturity Portfolios		
Equally Weighted	0.078	0.080	0.075	0.083	0.077
Minimum Variance	0.076	0.070	0.067	0.085	0.070

Figures 6.13, 6.14, 6.15 and 6.16 present the excessive losses for 99% VaR of both minimum variance and equally weighted portfolios for long and short positions over the back-testing period. The Value-at-Risk appear smoother for the Diagonal-BEKK for the equally weighted rating portfolio and DCC-GARCH for the equally weighted maturity portfolio, followed by the O-GARCH, while the CC-GARCH and DCC-GARCH formulations, which exhibit the most erratic VaR estimates. The Value-at-Risk estimates for the minimum variance portfolios appear more erratic for the O-GARCH model, followed by the Diagonal-BEKK and CC-GARCH and DCC-GARCH models.

Figure 6.13 Presents the excessive losses for 99% VaR of the equally weighted rating portfolio

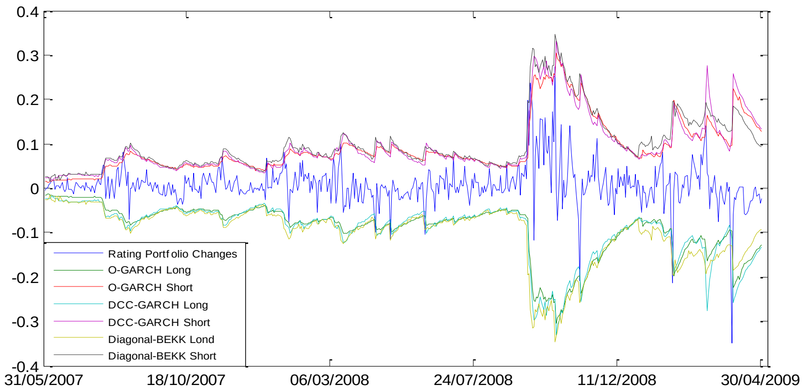


Figure 6.14 Presents the excessive losses for 99% VaR of the equally weighted maturity portfolio

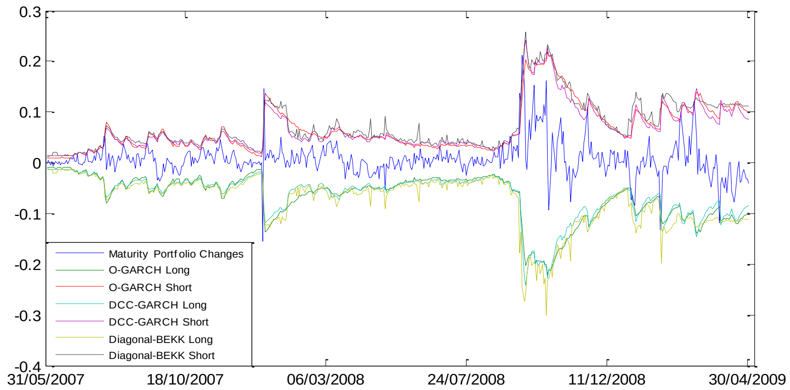


Figure 6.15 Presents the excessive losses for 99% VaR of the minimum variance rating portfolio

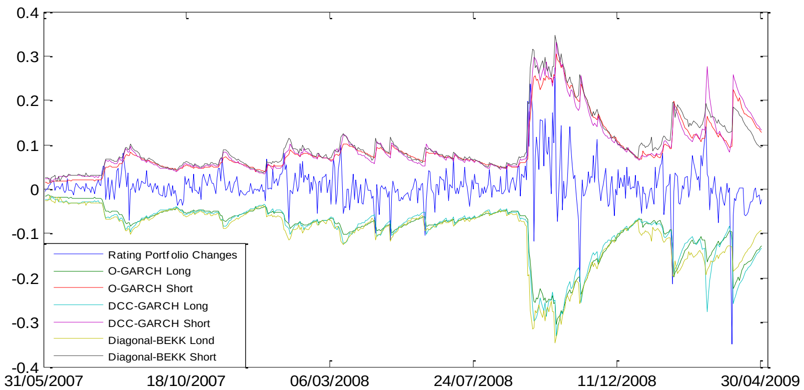
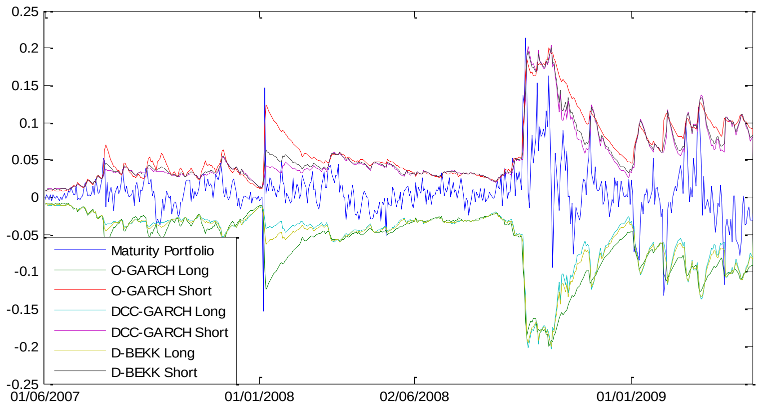


Figure 6.16 Presents the excessive losses for 99% VaR of the minimum variance maturity portfolio



6.4 Conclusions

This chapter examined and compared the properties and performance of the different multivariate GARCH models in capturing the correlation between credit spread changes. The significance of modelling the time-varying correlation of credit spread changes has been illustrated in risk management, portfolio selection and hedging, as well as in the pricing and hedging of derivatives products (e.g. Mahoney, 1995, Lumsdaine, 2009, Friewarld and Pichler, 2008, and Roscovan, 2008). Furthermore, it was found that the correlation of credit spread changes is time-varying and affected by market conditions. During periods of market turbulence, when the economy is contracting and credit spreads widen, the correlation coefficients between credit spread changes are high, suggesting higher default probabilities and correlation, while when the market enters a more stable state, experiences a high economic growth, correlation coefficients between credit spread changes decline suggesting a decrease in default probabilities and correlation.

This chapter also examined which formulation produced the best Value-at-Risk estimates based on risk management's loss functions. The adequacy of the different formulations in computing VaR estimates is examined by the application of the Christoffersen (1998) test for the conditional coverage, BASEL II and an VaR loss function which measures how well the VaR model forecasts the changes in credit spreads portfolio losses. The empirical results reveal that Diagonal-BEKK out-performs the other multivariate GARCH formulations in terms of in-sample goodness-of-fit statistics and with DCC-GARCH and RiskMetrics passed Christoffersen's conditional coverage test, showing good-out-of sample performance, for both minimum variance and equally weighted portfolios for long and short VaR positions. However, RiskMetrics according to the VaR loss function is the best model in forecasting the portfolios losses, compared with the other more complex formulations such as the DCC-GARCH and D-BEKK models.

Chapter 7

Determinants of Credit Spreads of European Corporate Bond Indices

7.1 Introduction

Credit spreads are defined as the difference between the yield to maturity of a corporate bond and the yield to maturity of a comparable government bond. Credit spreads reflect the likelihood of failure of an issuer to honour his obligation. For example, during a contracting economy credit spreads widen, as the cash flows of an issuer are reduced, thus increasing the likelihood of him failing to honour his contractual obligations. The opposite relationship is true in an expanding economy. Credit spreads are an important financial variable, since they are used as indicators of economic progression, investment decisions, trading and hedging, as well as pricing credit derivatives. Their role has become increasingly significant in the European fixed income markets since the introduction of the Euro as a single currency. This introduction provided the means for a pan-European economic growth and cross-border development - a development that has reshaped the mechanics of the financial environment and has introduced a liberalization of the once fragmented capital movements and products under different currencies, and allowed the dispersion of credit risk over a wider base and in a more efficient manner (Fabozzi and Choudhry, 2004).

Over the last few years, a number of theoretical studies and empirical works have been devoted to the investigation of the behaviour of corporate bond prices as well as the determination of the drivers behind credit spreads. The theoretical literature of valuing risky claims can be separated into two categories: structural and reduced form models. Structural models spring from the work of Black and Scholes (1973), who explain how equity owners hold a call option on the firm, and Merton (1973 and 1974), who extended the Black and Scholes framework. Merton (1974) assumes that the value of the firm follows a stochastic process and default occurs when the value of a firm falls below a predetermined boundary. On the other hand, reduced form models spring from the studies of Jarrow and Turnbull (1995) and Duffie and Singleton (1999), who treat default as a pure jump process following an intensity-based or a hazard-rate process.

The determinants that influence the Black, Scholes and Mertons' formula[38] are the risk-free rate, the underlying value of the security and its variance. However, the yield curve slope is also considered as a determinant because it can be regarded as a proxy for the future interest rate movements. For instance, an increase in the risk-free rate could lead to a decrease in the default probability and, consequently, a reduction in the credit spreads. A steep yield curve may imply a future increase in the interest rates which in turn may lead to a tightening of credit spreads, while a declining yield curve slope may imply a deterioration in the overall economy which may induce a widening in credit spreads. Although both the firm’s value and volatility are unobservable quantities, the equity level and estimated (implied) volatility are used as their approximations. This means that a firm’s positive stock market return increases the firm’s values, reduces the firm’s leverage, and by extension the probability of default which in turn reduces credit spreads.

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[38] Black and Scholes (1973) formula is $C=f(s,x,\sigma,r,T)$

In contrast to the characteristics of structural models, empirical studies find evidence of the opposite relationship between the risk-free interest rate and credit spreads. The studies of Delianedis and Geske (2001) and Dufresne, et al. (2001) are the first to approach the determinants of credit spreads in a more empirically-orientated framework, by using a variety of econometric models and a set of macroeconomic and financial factors as determinants.

Christiansen (2000) found a negative correlation between credit spreads and the term structure in days not belonging to macroeconomic announcements. Delianedis and Geske (2001), Dufresne, et al. (2001), Brown (2001) and Huang (2003**)** used a variety of factors such as taxation, liquidity, recovery risk, VIX -an implied volatility index-, term structure, and other macroeconomic indicators as determinants. Studies such as those of Pedrosa and Roll (1998), Morris, Neale and Rolph (1998) employ cointegration techniques and all argue that there exists a systematic risk in credit spreads. Furthermore, Gray (1996), Christiansen (2002), Davies (2004), Perignon and Smith (2006), and Alexander and Kaesk (2007) employ regime switching techniques in modelling the determinants of credit spreads and credit default swap spreads and have found that regimes are able to explain the underlying dynamics. Specifically, Davies (2004) applies a regime switching model and finds that the risk-free rate has a negative effect only over low volatility periods and that other determinants have significant explanatory power for the short-term dynamic path of the corporate bond spread and, especially, on the high volatility regime.

Table 7.1 summarizes the effect of the determinants on credit spreads. For instance, an increase in the risk-free rate increases the risk-neutral drift of the firm's value process which decreases the default probability and consequently, lowers credit spreads (see Longstaff and Schwartz, 1995).

Table 7.1 Reports the expected sign of the explanatory variables based on the literature

Variables	Description	Expected Sign
ΔL_t	Change in the level of the risk-free rate	-
ΔS_t	Change in the slope of yield curve	-
ΔE_t	Return on the MSCI Berra Pan-Euro Index	-
ΔV_t	Change in the EURO STOXX 50 Volatility Index	+
ΔI_t	Change in the EuroMTS Inflation Index	+
ΔC_t	Change in the Goldman Sachs S&P Commodity Index	+

The aim of this chapter is to model the impact of the risk-free rate and other important determinants, such as inflation (measured as the EuroMTS inflation index) and commodity prices (measured as the Goldman Sachs Commodity Index), which have not been previously considered as macroeconomic drivers, on credit spreads. The economic interpretation regarding the inclusion of these two determinants is made on the basis that positive values of inflation and commodity prices may suggest deterioration in the wider macroeconomic outlook, which may affect the default probability and recovery rates, and as a result widen credit spreads.

Since this study focuses on the impact of a set of determinants on European corpotate credit spreads, which raise questions regarding the optimality of a currency area in the Euro zone. The theory of Optimum Currency Area is developed by Mundell (1961), McKinnon (1963) among others which describes the cost-benefit of monetary integration and define the currency area as an optimal geographic domain of different currencies with fixed exchange rates.[39] The overall results presented in this study will not be affected by assumptions of the Optimal Currency Area, since the proposed determinants are based on aggregate indices that capture the pan-European characteristics.

[39] There are cases when individual countries lose control of their monetary policy to be more susceptible to asymmetric shocks. The Optimum Currency Area is concerned with evaluating the asymmetrical shocks within optimal economic areas and evaluating the efficiency of possible absorption mechanisms such as: labour mobility, fiscal transfers among others (see Marjan, 2007, among others).

This chapter also aims to investigate the nonlinear effects of the determinants on credit spread changes and whether their influence varies during different market conditions. In order to achieve these goals, two statistical techniques, namely the Markov regime switching regression model and feed-forward neural network, are applied. The Markov regime switching regression model is able to capture structural breaks in the time series, which may be due to changes in government policy, market microstructure, seasonality, and business cycles, to name but a few. This model springs from the work of Goldfield and Quandt (1973) and Hamilton (1989). They are the first to model the structural breaks and are extended by the studies of Hamilton (1993 and 1994) and Hamilton and Susmel (1994), who propose that the influence of explanatory variables should be allowed to be state-dependent.

Artificial neural networks[40] (ANN) is a statistical approach which has not been previously applied in the modelling of the non-linear relationship between determinants and credit spreads. From the various neural network specifications the feed-forward neural networks have been successfully applied in finance (see Ripley, 1993). Feed-forward networks with one hidden layer can approximate to any desired degree of accuracy any continuous nonlinear function, if a sufficiently large number of nodes are used (see Cybenko, 1989, Hornic, Stinchombe and White, 1989, Hornik and Stinchombe, 1992). Neural networks model nonlinear relationships between input and output layers, while the information moves in only one direction, from the input layer, through the hidden layer and, finally, to the output layer. Neural Networks are able to capture the non-linear relationship of the impact of the determinants on credit spreads and along with the Markov regime switching model could provide better out-of-sample predictions compared to a set of benchmark models such as OLS, ARIMA and VAR.

The chapter is organised as follows: Sections 2 and 3 present the methodology and estimation results respectively, Section 4 denotes the forecasting performance and Section 5 concludes.

[40] Neural networks emerged in the late-1800s as an attempt to describe the processing behaviour of the human mind; since then they have been expanded and applied in many scientific areas with varying degrees of success (see Ripley, 1993 and Cheng and Titterington, 1994, for a list of financial applications).

7.2 Methodology

Under the Markov regime switching approach proposed by Hamilton (1993, 1994) and Hamilton and Susmel (1994), the influence of explanatory variables can be allowed to be state-dependent with the universe of possible occurrences being categorised by s_t different states (see Chapter 3 Section 2.4 for further information on Markov regime switching regression model). The model for the mean equation is described by the following equation:

$$\Delta y_t = a_{s_t,0} + a_{s_t,1}\Delta y_{t-1} + a_{s_t,2}\Delta L_t + a_{s_t,3}\Delta S_t + a_{s_t,4}RE_t + a_{s_t,5}\Delta V_t + a_{s_t,6}\Delta I_t + a_{s_t,7}\Delta C_t + \varepsilon_{s_t,t}, \quad \varepsilon_{s_t,t} \sim iid\left(0, \sigma_{s_t}{}^2\right) \tag{7.1}$$

where Δy_{t-1} is the lagged change in credit spreads, ΔL_t is the change in the level of the risk-free rate which is the 5-year German government bond yield, ΔS_t is the change in the slope of yield curve which is the difference between the 10-year and 2-year yields on German government bonds, $R\mathrm{E}_t$ denotes the returns of the MSCI Berra Pan-Euro Index, ΔV_t is the change in the EURO STOXX 50 Volatility Index, ΔI_t is the change in the EuroMTS Inflation Index and ΔC_t is the change of the Goldman Sachs S&P Commodity Index (refer to Chapter 4 Section 2.2 for a review of the determinants).[41]

Another branch of the literature that models the non-linear dynamics of time series is the feed-forward neural networks. They do so by modelling the complex relationships between the input and output layers while the information moves in only one direction, from the input layer to the output layer. The application of the feed-forward neural network in forecasting credit spread changes entails three main steps. The first step involves specifying the input nodes (i.e. the determinants), the number of hidden layers and hidden nodes, and the output node (i.e. the credit spread changes). Determining the number of hidden layers follows the studies of Cybenko (1989), Hornic, Stinchombe and White (1989) and Hornik and Stinchombe (1992), who argue that one hidden layer can approximate to any desired degree of accuracy any continuous nonlinear function.

[41] Hausman's test for endogeneity is not rejected for the null of no endogeneity at a 5% confidence level.

Furthermore, there have been some studies to suggest a number of rules[42] that can be applied in order to determine the number of nodes in the hidden layer. Tan (1997) argues that it is very difficult to derive a good network topology from these rules. Therefore, the nodes in the hidden layer are determined in a heuristic approach. This means that a number of neural networks with different number of nodes are estimated and the best performing network among them is selected (see Priddy and Keller, 2005).

After specifying the input nodes, the nodes in the hidden layer and output nodes, the next step is to partition the input and output vector into two disjoint sets: *a training,* and *a validation set*. During the second step, the neural network is trained and the generalization ability of the network is tested in the *validation set*. The final step entails the estimation of mean forecasts for the back-testing period.

7.3 Estimation Results

Overall, the variables investigated in this study have some ability to explain changes in credit spread changes and the sign of the estimated coefficients generally agree with the existing literature and expected theory. The results of the linear regression analysis are presented in Table 7.2 and the determinants are able to explain from 11.8% up to 26.4% of the changes in credit spreads, according to the R-bar squared goodness-of-fit statistic. Specifically, the lagged terms in credit spread changes are negative and highly significant, suggesting the existence of short-term mean reversion (see Bierens, Huang and Kong, 2003).

The coeffiecients of the risk-free rate are negative and significant for all indices except for the model with the BBB rating. This means that an increase in the risk-free rate lowers the credit spread, a result consistent with the empirical findings of Longstaff and Schwartz (1995), Dufresne, *et al.* (2001), Davies (2004) amongst others. In addition, the sensitivity of credit spreads to the risk-free rate increases as credit quality improves, a result similar to Dufresne, *et al.* (2001). The coeffiecients of the slope are

[42] For a complete list of the rules refer to Tan, C., (1997), "An Artificial Neural Network Primer with Financial Applications Examples in Financial Distress Prediction and Foreign Exchange Hybrid Trading System" Bond University, School of Information Technology, Australia.

positive and significant for the models of the rating indices and the short-term maturity (1-3) index, and negative and significant for the long-term (7-10) index. Although, the results differ from what is presented in the relevant theoretical literature, a number of empirical studies such as Dufresne, *et al.* (2001) and Davies (2004) have reported similar coefficients for the slope. However, the positive coefficient of slope can be interpreted as follows: a positive slope reflects future expectations that the economy will grow in the future, and this growth will be associated with a greater expectation that inflation will rise. Therefore investors demand a premium associated with the future uncertainty of the inflation’s movement and the risk imposed to the future value of cash flows (see also Risa, 2001 and Avramov, Jostova and Philipov, 2007 among others).

The coeffiecients of the equity index are significant only for the models with the low rating (BBB) and low- to medium-term maturities (1-3, 3-5 and 5-7). The coefficients of the volatility index are positive and significant only for the models with ratings AAA, AA and A and long term maturity (7-10). Both of these results are consistent with the literature.
The coeffiecients of the inflation index are positive, statistically and economically significant for all indices. This means that an increase in the inflation may increase the uncertainty of future market conditions and future value of cash flows. As uncertainty about the purchasing power increases and the real value of money and other monetary instruments decrease, these conditions adversely affect the expected recovery rate (see Altman and Kishore, 1996) increasing the default probability and, thus, inducing a widening in credit spreads. Finally, the coefficients of the commodity index are statistically and economically insignificant during the examined period.

The model with one lag in credit spread changes exhibited autocorrelation according to the Breusch-Godfrey test and, therefore, a second lag in credit spread changes was introduced. Consequently, after the introduction of the second lag, the model is well behaved according to both White’s (1980) and Breusch-Godfrey tests. White’s (1980) general test provides the necessary means to test for heteroskedasticity. The null hypothesis that errors are homoscedastic is not rejected at a 1% level for all indices, which means that the variance of the error terms is constant. The Breusch-Godfrey serial correlation test is a more general and robust test than the standard Durbin and

Watson statistic. The null hypothesis that there is no serial correlation of order three is marginally rejected only for the models with AAA and AA ratings.

Before continuing to the Markov regime switching regression model, this study further investigates the linear impact of the determinants on credit changes over different time periods. Figures 4.6 and 4.7 presented in Chapter 4 illustrate the presence of two separate states during the in-sample period. The first one covers the period from 03/01/2000 to 31/08/2005 and is characterized by high volatility, while the second one covers the period from 01/09/2005 to 29/05/2007 and is characterized by a lower volatility.

Table 7.2 Reports the linear regression results for the period 03/01/2000 to 29/05/2007

This table reports the estimation results of the linear regression for the period from 3rd January 2000 to 31st May 2007, a total of 1933 observations and the specification is given as:

$$\Delta y_t = a_0 + a_1 \Delta y_{t-1} + a_2 \Delta L_t + a_3 \Delta S_t + a_4 RE_t + a_5 \Delta V_t + a_6 \Delta I_t + a_7 \Delta C_t + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma^2\right)$$

where Δy_{t-1} is the lagged change in credit spreads, ΔL_t is the change in the level of the risk-free rate which is the 5-year German government bond yield, ΔS_t is the change in the slope of yield curve which is the difference between the 10-year and 2-year yields on German government bonds, RE_t denotes the returns of the MSCI Berra Pan-Euro Index, ΔV_t is the change in the VSTOXX Index, ΔI_t is the change in the EuroMTS Inflation Index and ΔC_t is the change of the Goldman Sachs S&P Commodity Index.

The numbers in parentheses are the t-stats and *,**,*** indicate the significance at the 10%, 5% and 1% levels. The F-statistic tests the null hypothesis that all coefficients, except the constant, are zero. The critical values of the F(8, 1925) at 1% and 5% are 2.52 and 1.94. This null hypothesis is rejected, implying that the determinants are jointly significant. The White Heteroscedasticity, Breusch-Godfrey and ARCH tests are asymptotically χ^2 distributed under their respective null hypotheses. For the White test the degrees-of-freedom equal the number of slope coefficients excluding the constant. Thus the 1% and 5% $\chi^2(8)$ critical values are 20.09 and 15.50 respectively. For the Breusch-Godfrey the 1% and 5% $\chi^2(3)$ critical values are 11.34 and 7.81. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
a_0	-0.001	-0.001	-0.001	-0.001	$-3.7*10^{-04}$	$-1.9*10^{-04}$	$-4.3*10^{-04}$	$-3.6*10^{-04}$
	(-1.174)	(-1.394)	(-1.063)	(-1.110)	(-0.529)	(-0.330)	(-0.818)	(-0.842)
Δy_{t-1}	-0.523***	-0.582***	-0.517***	-0.242***	-0.345***	-0.504***	-0.489***	-0.544***
	(-19.880)	(-22.095)	(-19.577)	(-9.452)	(-13.583)	(-18.034)	(-19.655)	(-21.457)
Δy_{t-2}	-0.149***	-0.173***	-0.116***	0.025	-0.041*	-0.145***	-0.131***	-0.179***
	(-6.332)	(-7.236)	(-4.891)	(1.102)	(-1.786)	(-6.269)	(-5.697)	(-7.748)
ΔL_t	-0.076***	-0.110***	-0.099***	-0.001	-0.058**	-0.147***	-0.063***	-0.080***
	(-3.284)	(-4.976)	(-4.129)	(-0.027)	(-2.122)	(-5.102)	(-2.770)	(-4.040)
ΔS_t	0.204***	0.245***	0.241***	0.149***	0.110***	0.003	-0.049*	-0.069***
	(7.225)	(9.026)	(8.106)	(3.694)	(3.223)	(0.086)	(-1.731)	(-2.870)
RE_t	0.061	0.051	-0.146	-0.634***	-0.487***	-0.351***	-0.301***	-0.179**
	(0.591)	(0.529)	(-1.365)	(-4.258)	(-3.874)	(-3.037)	(-2.910)	(-2.065)
ΔV_t	0.003***	0.003***	0.003***	0.003**	0.001	0.001	0.002**	0.002***
	(3.493)	(4.154)	(3.380)	(2.356)	(1.465)	(1.328)	(2.043)	(2.998)
ΔI_t	0.016***	0.011***	0.013***	0.022***	0.019***	0.009***	0.012***	0.010***
	(5.418)	(4.022)	(4.409)	(5.502)	(5.640)	(2.504)	(4.228)	(3.995)
ΔC_t	$-1.4*10^{-04}$	$-8.1*10^{-05}$	$-8.8*10^{-05}$	$-7.3*10^{-05}$	$-3.4*10^{-05}$	$-1.8*10^{-04}$	$-1.7*10^{-04}$	$-1.5*10^{-05}$
	(-0.808)	(-0.500)	(-0.491)	(-0.299)	(-0.163)	(-0.932)	(-0.970)	(-0.106)
Diagnostics								
R-bar squared	0.237	0.260	0.232	0.118	0.149	0.207	0.234	0.264
F-statistic	75.789	85.551	73.895	33.335	43.206	63.794	74.548	87.419
White	8.654	9.186	7.976	5.283	5.489	6.914	7.861	7.384
BG	11.703	15.176	2.296	1.136	0.510	1.748	5.756	6.641
Log	3609	3664	3546	3096	3365	3410	3630	3912
AIC	-7201	-7310	-7074	-6174	-6712	-6802	-7242	-7807
BIC	-7189	-7298	-7062	-6163	-6701	-6791	-7230	-7795

Tables 7.3 and 7.4 present the linear regression results for the two sub-periods. For the first period from 03/01/2000 to 31/08/2005 the sign and significance of the estimated coefficients is similar to the whole in-sample period. Specifically, the estimated coefficients of the lagged terms in credit spread changes, the risk-free rate and inflation continue to be highly economically and statistically significant. The coeffiecients of the lagged credit spread changes are negative and significant, suggesting short-term mean reversion. The coefficients of the risk-free rate are negative and significant except for the model of the BBB index. This means that an increase in the risk-free rate increases the risk-neutral drift rate of the firm's value process which in turn decreases the default probability and, therefore, lowers credit spreads. The coefficients of inflation are positive and significant, suggesting that an increase in inflation may have an adverse impact on the overall economy and could negatively affect an issuer's ability to service his obligations resulting in increasing his default probabilities and leading to the widening of credit spreads.

However, during the second sub-period where volatility of credit spreads is relatively low, the coefficients of determination of models (adjusted R-square), ranging between 2.3% and 13.6%, indicate a low explanatory power. This means that changes in credit spread cannot be explained by the economic factors included in the model. This finding is in accordance with Alexander and Kaesk (2007) and Davies (2004) who argue that during high volatility periods the economic variables are able to explain a larger portion of the credit spread changes than during low ones. Finally, White's heteroskedasticity test and the Breusch-Godrey autocorrelation test reveal that the error terms of all models are homoscedastic and do not present any serial correlation.

Table 7.3 Reports the linear regression results for the period 03/01/2000 to 31/08/2005

This table reports the estimation results of the linear regression for the period from 3rd January 2000 to 31st August 2007, a total of 1479 observations and the specification is given as:

$$\Delta y_t = a_0 + a_1 \Delta y_{t-1} + a_2 \Delta L_t + a_3 \Delta S_t + a_4 RE_t + a_5 \Delta V_t + a_6 \Delta I_t + a_7 \Delta C_t + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma^2\right)$$

where Δy_{t-1} is the lagged change in credit spreads, ΔL_t is the change in the level of the risk-free rate which is the 5-year German government bond yield, ΔS_t is the change in the slope of yield curve which is the difference between the 10-year and 2-year yields on German government bonds, RE_t denotes the returns of the MSCI Berra Pan-Euro Index, ΔV_t is the change in the VSTOXX Index, ΔI_t is the change in the EuroMTS Inflation Index and ΔC_t is the change of the Goldman Sachs S&P Commodity Index

The numbers in parentheses are the t-stats and *,**,*** indicate the significance at the 10%, 5% and 1% levels. The F-statistic tests the null hypothesis that all coefficients, except the constant, are zero. The critical values of the F(8, 1471) at 1% and 5% are 2.65 and 2.02. The White Heteroskedasticity, Breusch-Godfrey and ARCH tests are asymptotically χ^2 distributed under their respective null hypotheses. For the White test the degrees-of-freedom equal the number of slope coefficients excluding the constant. Thus the 1% and 5% $\chi^2(8)$ critical values are 20.09 and 15.50 respectively. For the Breusch-Godfrey the 1% and 5% $\chi^2(3)$ critical values are 11.34 and 7.81. Finally, the information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
a_0	-0.001*	-0.001*	-0.001	-0.002	-0.001	-0.001	-0.001	-0.001*
	(-1.866)	(-1.716)	(-1.433)	(-1.292)	(-1.170)	(-1.042)	(-1.526)	(-1.813)
Δy_{t-1}	-0.553***	-0.590***	-0.524***	-0.241***	-0.344***	-0.513***	-0.494***	-0.558***
	(-18.234)	(-19.431)	(-17.188)	(-8.193)	(-11.760)	(-15.887)	(-17.244)	(-19.142)
Δy_{t-2}	-0.161***	-0.173***	-0.114***	0.027	-0.038	-0.148***	-0.131***	-0.181***
	(-5.946)	(-6.316)	(-4.203)	(1.019)	(-1.428)	(-5.553)	(-4.977)	(-6.858)
ΔL_t	-0.097***	-0.120***	-0.109***	-0.001	-0.059**	-0.164***	-0.073***	-0.095***
	(-3.612)	(-4.612)	(-3.866)	(-0.028)	(-1.833)	(-4.766)	(-2.674)	(-4.077)
ΔS_t	0.195***	0.235***	0.229***	0.135***	0.112***	-0.009	-0.063*	-0.084***
	(6.010)	(7.348)	(6.535)	(2.799)	(2.749)	(-0.232)	(-1.867)	(-2.950)
RE_t	0.086	0.113	-0.094	-0.634***	-0.493***	-0.339***	-0.274***	-0.149
	(0.734)	(0.985)	(-0.746)	(-3.589)	(-3.303)	(-2.482)	(-2.233)	(-1.464)
ΔV_t	0.003***	0.004***	0.003***	0.003***	0.002	0.001	0.002**	0.002***
	(3.430)	(4.039)	(3.345)	(2.276)	(1.417)	(1.345)	(2.018)	(2.802)
ΔI_t	0.019***	0.016***	0.018***	0.028***	0.025***	0.013***	0.018***	0.014***
	(5.293)	(4.422)	(4.700)	(5.428)	(5.746)	(2.879)	(4.709)	(4.445)
ΔC_t	$-1.6*10^{-04}$	$-1.8*10^{-04}$	$-1.6*10^{-04}$	$-1.1*10^{-04}$	$-8.7*10^{-05}$	$-3.6*10^{-04}$	$-3.1*10^{-04}$	$-6.6*10^{-05}$
	(-0.641)	(-0.757)	(-0.609)	(-0.283)	(-0.279)	(-1.254)	(-1.184)	(-0.306)
Diagnostics								
R-bar squared	0.262	0.272	0.245	0.128	0.160	0.216	0.246	0.280
F-statistic	66.537	69.915	60.863	27.948	36.115	51.887	61.187	72.747
White	6.881	6.290	5.289	4.573	4.092	4.721	5.886	5.217
BG	9.201	10.304	1.193	0.875	0.492	1.304	3.627	4.123
Log	2618	2631	2541	2190	2396	2428	2597	2820
AIC	-5217	-5244	-5064	-4363	-4775	-4837	-5175	-5623
BIC	-5207	-5234	-5053	-4352	-4764	-4827	-5165	-5612

Table 7.4 Reports the linear regression results for the period 01/09/2005 to 29/05/2007

This table reports the estimation results of the linear regression for the period from 1[rd]September 2005 to 29[st] April 2007, a total of 454 observations and the specification is given as:

$$\Delta y_t = a_0 + a_1\Delta y_{t-1} + a_2\Delta L_t + a_3\Delta S_t + a_4 RE_t + a_5\Delta V_t + a_6\Delta I_t + a_7\Delta C_t + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0,\sigma^2\right)$$

where Δy_{t-1} is the lagged change in credit spreads, ΔL_t is the change in the level of the risk-free rate which is the 5-year German government bond yield, ΔS_t is the change in the slope of yield curve which is the difference between the 10-year and 2-year yields on German government bonds, RE_t denotes the returns of the MSCI Berra Pan-Euro Index, ΔV_t is the change in the VSTOXX Index, ΔI_t is the change in the EuroMTS Inflation Index and ΔC_t is the change of the Goldman Sachs S&P Commodity Index

The numbers in parentheses are the t-stats and *,**,*** indicate the significance at the 10%, 5% and 1% levels. The F-statistic tests the null hypothesis that all coefficients, except the constant, are zero. The critical values of the F(7, 447) at 1% and 5% are 2.68 and 2.03. The White Heteroskedasticity, Breusch-Godfrey and ARCH tests are asymptotically χ^2 distributed under their respective null hypotheses. For the White test the degrees-of-freedom equal the number of slope coefficients excluding the constant. Thus the 1% and 5% $\chi^2(7)$ critical values are 18.47 and 14.06 respectively. For the Breusch-Godfrey the 1% and 5% $\chi^2(2)$ critical values are 9.21 and 5.99. Finally, the information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
a_0	1.2*10^{-04}	-3.7*10^{-04}	-3.7*10^{-04}	-5.6*10^{-04}	2.6*10^{-04}	5.1*10^{-04}	4.7*10^{-04}	6.6*10^{-04}
	(0.165)	(-0.876)	(-0.778)	(-0.966)	(0.000)	(1.592)	(1.665)	(1.898)
Δy_{t-1}	-0.115***	-0.330***	-0.252***	-0.134***	-0.361	-0.386***	-0.267***	-0.150***
	(-2.228)	(-6.283)	(-4.476)	(-2.308)	(0.050)	(-7.990)	(-5.686)	(-3.083)
ΔL_t	0.032	-0.006	0.004	-0.002	-0.024	-0.015	0.020	0.036**
	(0.952)	(-0.259)	(0.182)	(-0.079)	(0.022)	(-0.794)	(1.373)	(2.213)
ΔS_t	0.103**	0.178***	0.163***	0.127***	-0.025	-0.009	-0.021	-0.014
	(1.961)	(5.165)	(4.190)	(2.904)	(0.028)	(-0.364)	(-1.047)	(-0.588)
RE_t	-0.115	-0.296**	-0.348***	-0.374**	-0.149	-0.284***	-0.296***	-0.177*
	(-0.590)	(-2.380)	(-2.590)	(-2.459)	(0.116)	(-2.909)	(-3.652)	(-1.903)
ΔV_t	-0.003*	-0.002**	-0.003**	-0.003*	-0.001	-0.002**	-0.002**	0.000
	(-1.796)	(-1.998)	(-2.191)	(-1.910)	(0.001)	(-2.201)	(-2.415)	(-0.289)
ΔI_t	0.007**	0.004*	0.005**	0.005*	0.002	0.002	0.002	0.004***
	(2.175)	(1.937)	(2.107)	(1.880)	(0.002)	(0.974)	(1.543)	(2.689)
ΔC_t	-1.6*10^{-04}	-5.0*10^{-05}	-1.1*10^{-04}	-1.3*10^{-04}	-2.0*10^{-07}	3.4*10^{-06}	-5.2*10^{-05}	-1.5*10^{-05}
	(-1.182)	(-0.594)	(-1.194)	(-1.251)	(0.000)	(0.051)	(-0.944)	(-0.237)
Diagnostics								
R-bar squared	0.023	0.080	0.050	0.024	0.098	0.136	0.086	0.038
F-statistic	2.546	6.618	4.398	2.575	7.998	11.156	7.105	3.563
White	0.378	1.520	1.400	1.517	0.707	0.880	0.456	0.709
BG	1.513	8.909	3.163	0.452	8.729	7.334	4.170	3.036
Log	1205	1396	1370	1321	1426	1497	1596	1540
AIC	-2394	-2775	-2724	-2626	-2837	-2979	-3176	-3064
BIC	-2389	-2770	-2719	-2621	-2831	-2974	-3171	-3059

Table 7.5 presents the Markov regime switching [MRS] regression results. The annualized standard deviations reveal the presence of two regimes. The first regime corresponds to the low volatility period, and the second regime to high period of volatility. The highest annualized volatility is exhibited in the MRS model of the BBB index with a value of 0.804 and the lowest in the MRS model of the 3-5 index with a value of 0.049, followed by the model of the AAA rating with a value of 0.050. The average probabilities over the sample period suggest that the process of the credit spread changes spends on average more time in the high volatility regime than in the low volatility. The unconditional probabilities are large for both regimes, for example for the AAA index the p_{11} is 0.977 and p_{22} is 0.993. Similar results are presented in Alexander and Kaesk (2007) and occur because of persistent volatility in both regimes.

The Markov regime switching regression model reveals that the variables are regime switching and are able to explain a larger portion of the changes in credit spreads, compared with the linear models discussed previously. The highest adjusted R^2 is observed for the model of the AA index with a value of 39.4% followed by the model of the AAA with a value of 35.9%, while the lowest is seen for the model of the BBB index value of 26.5%. Overall, the sign of the estimated coefficients agrees with the literature, empirical studies and suggested theory. However, during the low volatility regime, the estimated coefficients have a weak statistical and economic significance. This finding is in accordance with Davies (2004) and Alexander and Kaesk (2007), who find that only one regime plays a significant role.

In the first regime, the estimated coefficients of the lagged credit spread changes are positive and significant for the models of the low rating index (BBB). The estimated coefficients of the risk-free rate are insignificant for all indices, while the estimated coefficients of the slope are positive and significant only for the model of the A and BBB rating indices. The estimated coefficients of the equity, volatility and inflation are insignificant for all indices. The estimated coefficients of the commodities are negative and significant only for the model of the BBB index.

In the second regime, the estimated coefficients of the lagged credit spread changes and risk-free rate are negative and significant for all models. This result is new to the literature, as Davies (2004) finds that the level is negative and significant only during the low volatility regime and low ratings. The estimated coefficients of the slope are positive and significant for the models with the higher ratings (AAA,AA, A and BBB) and low-term maturity (1-3) and negative and significant for the models with the medium to long-term maturity indices (3-5, 5-7 and 7-10). The estimated coefficients of equity are negative and significant only for the models of the BBB, 1-3 and 7-10 indices. The estimated coefficients of volatility are positive and significant for the models of the AAA, A and 7-10 indices. The estimated coefficients of inflation are positive and significant for all models and the coefficients of commodity are insignificant. This means that during the high volatility period an increase in inflation may increase the uncertainty of future market conditions and the future value of cash flows which adversely affects the expected recovery, thus increasing the default probability and inducing a widening in credit spreads.

Engle's heteroskedasticity test and Ljung-Box autocorrelation Q-test reveal that the error terms are homoscedastic and do not present any degree of autocorrelation. Finally, Figures 7.2 and 7.3 present the smooth state probabilities of the Markov regime switching models of the AAA and 1-3 indices. They reveal that during the period from 2000 to the end of 2005 the credit spread changes are characterized by high volatility while during the period from 2006 to 2007, the credit spreads are characterized by low volatility.

Table 7.5 Reports the estimation results of the Markov Regime Switching Model [MRS]

This table reports the estimation results of the Markov regime switching regression model for the period from 3rd January 2000 to 31st May 2007, a total of 1933 observations and the specification is given as:

$$\Delta y_t = a_{s_t,0} + a_{s_t,1}\Delta y_{t-1} + a_{s_t,2}\Delta L_t + a_{s_t,3}\Delta S_t + a_{s_t,4}RE_t + a_{s_t,5}\Delta V_t + a_{s_t,6}\Delta I_t + a_{s_t,7}\Delta C_t + \varepsilon_{s_t,t}, \quad \varepsilon_{s_t,t} \sim iid\left(0, \sigma_{s_t}^{\ 2}\right)$$

where Δy_{t-1} is the lagged change in credit spreads, ΔL_t is the change in the level of the risk-free rate which is the 5-year German government bond yield, ΔS_t is the change in the slope of yield curve which is the difference between the 10-year and 2-year yields on German government bonds, RE_t denotes the returns of the MSCI Berra Pan-Euro Index, ΔV_t is the change in the VSTOXX Index, ΔI_t is the change in the EuroMTS Inflation Index, ΔC_t is the change of the Goldman Sachs S&P Commodity Index and s_t the state.

The numbers in parentheses are the t-stats and *,**,*** indicate the significance at the 10%, 5% and 1% levels. The probabilities (regime 1 and regime 2) are the diagonal elements of the unconditional probability matrix (for further information refer to Chapter 3 Sections 2.4 and 3.4). APR1 and APR2 refer to the average probabilities in being in regime 1 and regime 2, respectively. The 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. Finally, the information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
a_0								
Regime 1	$1.5*10^{-04}$	$1.6*10^{-04}$	$4.2*10^{-04}$	$4.7*10^{-04}$	$6.2*10^{-04}$	$3.4*10^{-04}$	$7.7*10^{-04}$	$5.0*10^{-05}$
	(0.540)	(0.114)	(0.685)	(0.182)	(0.028)	(0.300)	(0.172)	(0.419)
Regime 2	-0.002*	-0.002	-0.002**	-0.002***	-0.001	-0.002	-0.002	-0.001
	(-1.872)	(-0.049)	(-2.077)	(-5.319)	(-0.065)	(-0.033)	(-0.468)	(-0.514)
Δy_{t-1}								
Regime 1	-0.001	0.005	0.037	0.064***	-0.045	-0.009	0.064	0.095
	(-0.055)	(0.039)	(0.849)	(2.264)	(-0.212)	(-0.178)	(0.265)	(0.029)
Regime 2	-0.220***	-0.230	-0.200***	-0.059***	-0.109	-0.105**	-0.260***	-0.260***
	(-5.417)	(-1.131)	(-12.594)	(-13.458)	(-0.694)	(-1.794)	(-5.654)	(-7.783)
ΔL_t								
Regime 1	-0.041	-0.047	-0.041	-0.079	-0.131	-0.067	0.026	0.014
	(-0.053)	(-0.005)	(-0.368)	(-0.517)	(-1.244)	(-1.019)	(0.435)	(0.574)
Regime 2	-0.330***	-0.326***	-0.339***	-0.416***	-0.414***	-0.602***	-0.272***	-0.266***
	(-14.587)	(-3.759)	(-18.693)	(-3.056)	(-12.246)	(-17.117)	(-9.657)	(-5.622)
ΔS_t								
Regime 1	0.301	0.432	0.451	0.445	-0.068	-0.097	-0.092	-0.094
	(1.395)	(2.627)	(2.445)	(3.336)	(-0.092)	(-0.616)	(-0.367)	(-0.586)
Regime 2	0.314	0.377	0.380	0.338	0.108	-0.049	-0.047	-0.167
	(15.423)	(0.928)	(4.232)	(20.498)	(3.334)	(-0.297)	(-0.131)	(-6.315)
RE_t								
Regime 1	-0.060	-0.112	-0.191	-0.208	-0.043	-0.051	-0.104	-0.153
	(-0.544)	(-0.292)	(-0.304)	(-1.314)	(-0.217)	(-0.146)	(-0.098)	(-0.432)
Regime 2	0.068**	0.048	-0.117	-0.496	-0.382***	-0.240	-0.232	-0.113**
	(2.206)	(0.035)	(-1.366)	(-1.428)	(-6.033)	(-0.338)	(-0.892)	(-2.041)
ΔV_t								
Regime 1	$-7.4*10^{-06}$	-0.001	-0.001	-0.001**	7.2E-05	$-4.4*10^{-05}$	$-1.9*10^{-04}$	$-4.6*10^{-04}$
	(-0.037)	(-0.482)	(-0.771)	(-1.827)	(0.071)	(-0.198)	(-0.077)	(-0.016)
Regime 2	0.002***	0.002	0.002***	0.002	0.001	0.001	0.001	0.002*

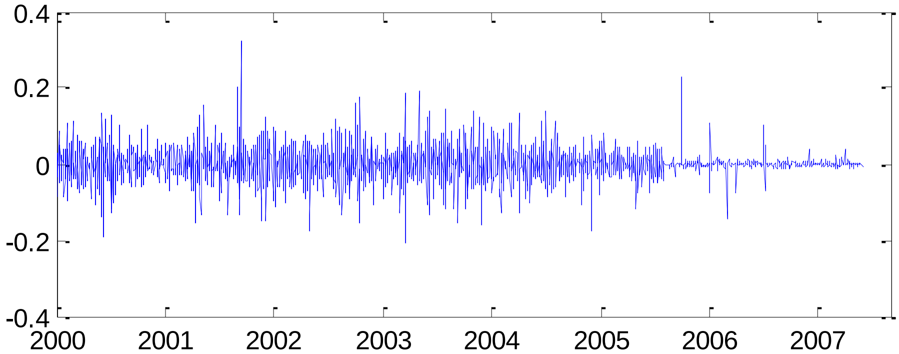
	(8.690)	(0.402)	(3.598)	(0.164)	(1.028)	(0.795)	(0.113)	(1.702)

Table continues at the next page

ΔI_t								
Regime 1	$-1.2*10^{-04}$	$-1.6*10^{-04}$	$4.0*10^{-05}$	0.001	$-3.5*10^{-04}$	-0.001	$-7.3*10^{-05}$	$3.5*10^{-05}$
	(-0.064)	(-0.005)	(0.089)	(0.421)	(-0.667)	(-0.027)	(-0.037)	(0.014)
Regime 2	0.024***	0.021	0.023***	0.018***	0.019***	0.012***	0.020***	0.018***
	(4.741)	(0.729)	(4.265)	(25.539)	(4.400)	(2.693)	(2.628)	(4.201)
ΔC_t								
Regime 1	$-2.7*10^{-05}$	$-2.9*10^{-05}$	-1.6E-05	$-4.3*10^{-05}$***	$-1.2*10^{-05}$	$-2.4*10^{-05}$	$-3.3*10^{-05}$	$1.6*10^{-05}$
	(-0.889)	(-0.188)	(-0.988)	(-3.953)	(-0.144)	(-1.311)	(-0.644)	(1.255)
Regime 2	$-2.6*10^{-04}$***	$-1.3*10^{-04}$	$-6.6*10^{-05}$	$-1.4*10^{-04}$	$-9.5*10^{-05}$	$-3.0*10^{-04}$	$-1.9*10^{-04}$	$-4.2*10^{-05}$
	(-6.774)	(-0.033)	(-0.149)	(-0.272)	(-0.280)	(-0.655)	(-0.092)	(-0.351)
Diagnostics								
Annualised SD								
Regime 1	0.050	0.051	0.069	0.099	0.065	0.049	0.053	0.056
Regime 2	0.620	0.595	0.630	0.804	0.712	0.647	0.650	0.551
Probabilities								
Regime 1	0.977	0.964	0.979	0.932	0.936	0.921	0.994	0.984
Regime 2	0.993	0.915	0.982	0.901	0.976	0.976	0.815	0.955
APR1	0.317	0.287	0.291	0.301	0.365	0.328	0.226	0.213
APR2	0.683	0.713	0.709	0.699	0.635	0.672	0.775	0.787
R-bar squared	0.359	0.394	0.376	0.265	0.276	0.423	0.267	0.307
ARCH (1)	0.001	0.002	0.001	0.015	0.007	0.001	0.055	0.003
ARCH (10)	0.009	0.018	0.005	0.151	0.094	0.009	0.692	0.032
LB Q test (1)	0.000	0.011	0.001	0.007	0.001	0.010	0.000	0.000
LB Q test (10)	2.789	0.409	0.005	0.144	5.391	0.035	6.612	0.221
Log Likelihood	4562	4613	4419	3901	4283	4532	4470	4621
AIC	-9094	-9203	-8821	-7787	-8542	-9036	-8934	-9223
BIC	-9074	-9182	-8801	-7766	-8521	-9016	-8913	-9202

Figure 7.1 Presents the first difference in credit spreads and the smoothed state probabilities for the AAA index

Panel A: Credit spread changes of the AAA index



Panel B: Smoothed state probabilities of the first regime

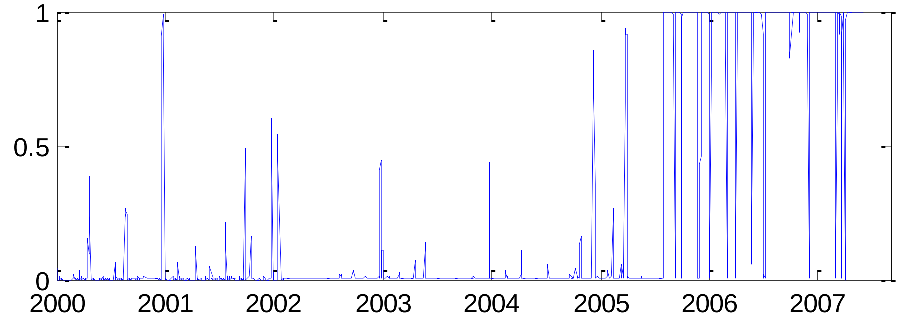
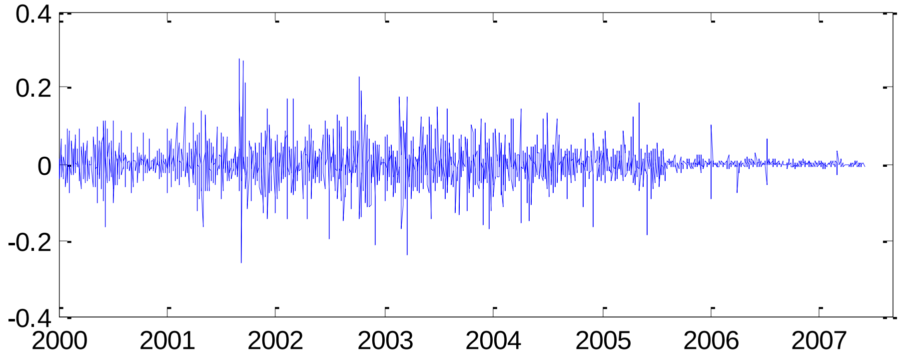
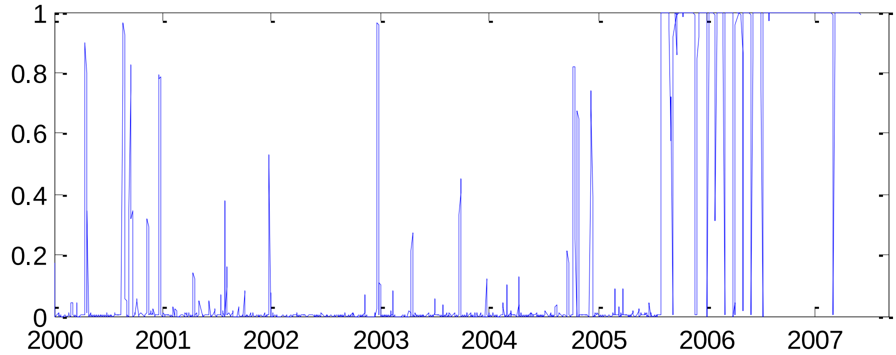


Figure 7.2 Presents the first difference in credit spreads and the smoothed state probabilities for the 1-3 index

Panel A: Credit spread changes of the 1-3 index



Panel B: Smoothed state probabilities of the first regime



As described in Chapter 3 Section 2.5, there are three main steps in forecasting credit spread changes using the feed-forward neural network. In the first step the input nodes (i.e. the determinants), the number of nodes in the hidden layer, and the output node (i.e. the credit spread changes) are specified. The number of nodes in the hidden layer is determined by estimating a number of neural networks with different number of nodes and the best performing network among them is selected. The selection of the nodes is based on the estimation of the root mean square error (RMSE) for the training and validation sets[43] and the results are presented in Table 7.6. The period the test is performed is from 03/01/2000 to 31/05/2007, with a total of 1933 observations, and the size of training and validations are 1450 and 483 observations, respectivelly.

It is found that 25 nodes for the majority of the indices in the hidden layer minimize the root mean square error in the training and validation sets. After specifying the input nodes, the nodes in the hidden layer and output nodes, which lead to a 8-25-1 feed-forward neural network, the next step is to partition the input and output vector into two disjoint sets: *training,* and *validation test.* During the second step the neural network is trained and the generalization ability of the network is tested in the *validation set*, and the final step estimates the mean forecasts for the back-testing period.

Table 7.6 Reports the root mean square error for the training and validation sets for different number of nodes in the hidden layer

Nodes	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
				Training Set				
5	0.043	0.041	0.045	0.057	0.049	0.048	0.043	0.037
10	0.042	0.042	0.048	0.064	0.047	0.047	0.043	0.037
15	0.042	0.043	0.044	0.054	0.047	0.048	0.042	0.036
20	0.042	0.042	0.045	0.055	0.048	0.050	0.042	0.036
25	0.041	0.041	0.044	0.053	0.046	0.047	0.042	0.036
30	0.044	0.041	0.051	0.056	0.049	0.046	0.041	0.036
35	0.041	0.041	0.043	0.054	0.050	0.047	0.044	0.037
40	0.041	0.041	0.044	0.052	0.057	0.047	0.041	0.036
45	0.041	0.041	0.047	0.056	0.050	0.047	0.044	0.036

Table continues at the next page

Validation Set

[43] The test set in this study is not applied in the conventional NN fitting approach, but it is used to forecast mean estimates given the information at time t.

5	0.019	0.014	0.015	0.016	0.012	0.012	0.012	0.011
10	0.019	0.015	0.016	0.016	0.013	0.013	0.011	0.012
15	0.020	0.015	0.016	0.015	0.012	0.013	0.012	0.012
20	0.020	0.015	0.016	0.020	0.013	0.014	0.012	0.012
25	0.019	0.015	0.015	0.015	0.012	0.012	0.011	0.012
30	0.019	0.015	0.016	0.017	0.014	0.014	0.011	0.012
35	0.019	0.015	0.015	0.016	0.015	0.012	0.012	0.012
40	0.019	0.016	0.014	0.016	0.014	0.013	0.012	0.012
45	0.019	0.015	0.015	0.015	0.014	0.013	0.012	0.012

The final models to be examined are the ARIMA(1,1) and VAR(1) which are considered as the benchmark models. These models are considered as the base models since they capture the dynamic structure of a financial time series, and are not designed to capture the influence of the different determinants of credit spread changes. Table 7.7 reports the estimation results of the ARIMA(1,1) model; whereas Table 7.8 reports the results of the VAR(1) model.

Table 7.7 Estimation results of the ARIMA(1,1)

This table reports the estimation results of the ARIMA(1,1) for the period from 3rd January 2000 to 31st May 2007, a total of 1933 observations and the specification is given as:

$$y_t = a_0 + a_1 y_{t-1} + a_2 \varepsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2\right)$$

where y_t are the changes in credit spreads. The numbers in parentheses are the t-stats. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
a_0	-2*10^{-05}	-5*10^{-05}	-3*10^{-05}	-7*10^{-05}	1*10^{-05}	-4*10^{-05}	-1*10^{-05}	-4*10^{-05}
	(-0.599)	(-0948)	(-0.534)	(-0.541)	(0.116)	(-0.735)	(-0.218)	(-0.102)
a_1	0.009	-0.060	-0.155	-0.346	-0.125	-0.065	-0.008	-0.032
	(0.261)	(-1.607)	(-3.557)	(-4.125)	(-1.952)	(-1.637)	(-2.213)	(-0.959)
a_2	-0.462	-0.391	-0.239	0.163	-0.145	-0.339	-0.379	-0.472
	(-13.183)	(-10.761)	(-5.379)	(1.934)	(-2.323)	(-8.865)	(-10.606)	(-14.852)
Diagnostics								
LogLikelihood	3529	3557	3435	3011	3281	3316	3558	3836
AIC	-3192	-3248	-3004	-2156	-2696	-2767	-3249	-3805
BIC	-4934	-4991	-4747	-3898	-4439	-4509	-4991	-5547

Table 7.8 reveals a feedback relationship between credit spread changes, according to the coefficients of the **A** matrix. For example, the coefficient $a_{1,2} = -0.243$ expresses the conditional effect of $y_{AA,t-1}$ on $y_{AAA,t}$ given $\left(y_{AAA,t-1}, y_{A,t-1}, y_{BBB,t-1}\right)$. The coefficient

$a_{1,2} = -0.243 \neq 0$ in the first equation and the coefficient $a_{2,1}$ is statistically insignificant in the second equation, indicating the existence of an unidirectional relationship from the AAA index to the AA index. Similar inferences can be made for all the indices.

Table 7.8 Estimation results of the Vector Autoregressive model VAR(1)

This table reports the estimation results of the VAR(1) for the period from 3rd January 2000 to 31st May 2007, a total of 1933 observations and the specification is given as:

$$\mathbf{y_t} = \mathbf{C} + \mathbf{A}\mathbf{y_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t}, \quad \boldsymbol{\varepsilon}_\mathbf{t} \sim N\left(0, \mathbf{H_t}\right)$$

where y_t are the changes in credit spreads. The numbers in parentheses are the t-stats. The information criteria are calculated as follows: AIC = -2*LLF + 2*(Number of Parameters) and BIC = -2*LLF + (Number of Parameters)*log(Number of Observations).

		Rating credit spread changes			
	C	$y_{AAA,t-1}$	$y_{AA,t-1}$	$y_{A,t-1}$	$y_{BBB,t-1}$
$y_{AAA,t}$	0.001	0.122	-0.243	-0.022	-0.074
	(0.001)	(3.574)	(-3.598)	(-0.319)	(-2.162)
$y_{AA,t}$	0.001	-0.022	-0.314	0.166	-0.063
	(0.001)	(-0.801)	(-5.854)	(2.999)	(-2.294)
$y_{A,t}$	0.002	-0.026	-0.432	0.246	-0.009
	(0.001	(-0.878)	(-7.390)	(4.059)	(-0.293)
$y_{BBB,t}$	0.002	-0.084	-0.598	0.369	0.098
	(0.001)	(-2.543)	(-9.158)	(5.462)	(2.931)
Diagnostics					
Log Likelihood	20774				
AIC	-41539				
BIC	-41516				
		Maturity credit spread changes			
	C	$y_{1-3,t-1}$	$y_{3-5,t-1}$	$y_{5-7,t-1}$	$y_{7-10,t-1}$
$y_{1-3,t}$	0.002	0.002	0.012	-0.136	-0.102
	(0.001)	(0.050)	(0.269)	(-2.533)	(-1.946)
$y_{3-5,t}$	0.002	0.149	-0.241	-0.038	-0.116
	(0.001)	(4.620)	(-5.470)	(-0.717)	(-2.250)
$y_{5-7,t}$	0.001	0.084	0.009	-0.298	-0.021
	(0.001)	(2.832)	(0.228)	(-6.164)	(-0.448))
$y_{7-10,t}$	0.002	0.135	-0.022	-0.038	-0.304
	(0.001)	(4.960)	(-0.583)	(-0.849)	(-6.989)
Diagnostics					
Log Likelihood	22054				
AIC	-44100				
BIC	-44077				

7.4 Forecasting Performance

The performance of the different models in capturing the market dynamics of the credit spread changes is assessed by the different mean forecast metrics. In order to forecast the credit spreads the conditional expectation of its future value needs to be estimated by:

$$E\left(y_t \mid \Omega_{t-1}\right)=a_{s_t,0}+a_{s_t,1}\Delta y_{t-1}+a_{s_t,2}\Delta L_{t-1}+a_{s_t,3}\Delta S_{t-1}+a_{s_t,4}RE_{t-1}+a_{s_t,5}\Delta V_{t-1}+a_{s_t,6}I\Delta_{t-1}+a_{s_t,7}\Delta C_{t-1}+\varepsilon_{s_t,t-1} \quad (7.2)$$

The accuracy of out-of-sample mean forecasts will be assessed by the mean squared errors (MSE), mean absolute error (MAE), Theil's U-Statistic (1966), Root mean square error (RMSE) and the percentage correct sign predictions (PCSP). The mean absolute percentage error (MAPE) and adjusted mean absolute percentage error (AMAPE) are not reliable in the case of credit spreads, as the absolute values in some indices are less than one. The lowest the values of the MSE, MAE and RMSE the more accurate are the predictions of the different models. In addition, Theil's U- Statistic is bounded between zero and one. The more the statistic approaches zero, the greater the model's forecasting accuracy. The correct sign prediction measure evaluates a model's ability to accurately forecast the sign of the future return series.

The different criteria are given as:

$$MSE=\frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s}-\hat{y}_{t,s}\right)^2 \quad (7.3)$$

$$MAE=\frac{1}{n}\sum_{t=1}^{n}\left|y_{t+s}-\hat{y}_{t,s}\right| \quad (7.4)$$

$$RMSE=\sqrt{\frac{\sum\left(y_{t+s}-\hat{y}_{t,s}\right)^2}{n}} \quad (7.5)$$

$$U=\frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s}-\hat{y}_{t,s}\right)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s}\right)^2}+\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\hat{y}_{t,s}\right)^2}} \quad (7.6)$$

$$PCSP=\frac{1}{n}\sum_{t=1}^{n}z_{t+s}, \qquad z_{t+s}=\begin{cases}1, & y_{t+s}\hat{y}_{t,s}>0\\ 0, & otherwise\end{cases} \quad (7.7)$$

The out-of-sample forecast exercise is performed over the period from 1st June 2007 to 30th April 2009 and the estimation returns of the different measures and for the following models: Ordinary Least Square (OLS); ARIMA(1,1); VAR(1); Markov Regime Switching Model (MRS); and Neural Networks (NN), are reported in Table 7.8.

The mean square error reveals that the feed-forward neural network fares the best for the AAA, A, AA, BBB, 1-3 and 7-10 indices, while the Markov regime switching model out-performs the other models for the 3-5 and 5-7 models, followed by ARIMA(1,1), OLS and VAR(1). The mean absolute error shows that feed-forward neural network out-performs the other models in the AAA, AA, A, BBB and 3-5 and 5-7 indices, followed by the Markov regime switching model ARIMA(1,1), OLS and VAR(1). The root mean square error statistic reveals the feed-forward neural network to out-perform the other formulations in the AAA, AA, A, BBB, 1-3, and 7-10 indices, followed by the Markov regime switching model for the 3-5 and 5-7 indices, the ARIMA(1,1), OLS and VAR(1). The Theil's U-Statistic is bounded between zero and one and the more the statistic approaches zero, the greater the model's forecasting accuracy. This test reveals that the feed-forward neural network out-performs the other models in all the indices, followed by the Markov regime switching model except for the AAA index where the OLS fares betters, followed by the ARIMA(1,1) and VAR(1). The percentage correct sign prediction shows that the feed-forward neural network out-performs all other specification except for the 1-3 maturity index where the Markov regime switching model fares better, both of these models are able to predict the correct sign in some of the indices up to 65% of the time, followed by the OLS, ARIMA(1,1) and VAR(1).

According to all loss functions the feed-forward neural network followed by the Markov regime switching model, estimate the most accurate out-of-sample mean forecasts. The poor performance of the OLS, ARIMA(1,1) and VAR(1) models may be attributed to the models' lack of capturing the determinants' impact on credit spreads during the credit crisis period.

Table 7.9 Estimation results of the different forecasting loss functions

The table reports the mean square error (MSE), mean absolute error (MAE), root mean square error (RMSE), Theil's U-Statistic and percentage correct sign predictions (PCSP). The back-testing sample period is from 1st June 2007 - 30th April 2009, a total of 500 observations. The rankings of the forecast statistics given an index are denoted as (A) for the best and (E) for the worst.

Mean square error

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s} - \hat{y}_{t,s}\right)^2$$

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
OLS	0.0074 (D)	0.0025 (D)	0.0036 (D)	0.0029 (D)	0.0020 (D)	0.0018 (D)	0.0018 (C)	0.0022 (C)
ARIMA(1,1)	0.0074 (C)	0.0024 (C)	0.0034 (C)	0.0028 (C)	0.0020 (C)	0.0018 (C)	0.0018 (D)	0.0023 (E)
VAR(1)	0.0076 (E)	0.0027 (E)	0.0037 (E)	0.0035 (E)	0.0024 (E)	0.0019 (E)	0.0020 (E)	0.0022 (D)
MRS	0.0067 (B)	0.0023 (B)	0.0028 (B)	0.0025 (B)	0.0017 (B)	0.0014 (A)	0.0013 (A)	0.0022 (B)
NN	0.0054 (A)	0.0020 (A)	0.0024 (A)	0.0023 (A)	0.0013 (A)	0.0016 (B)	0.0013 (B)	0.0021 (A)

Mean absolute error

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|y_{t+s} - \hat{y}_{t,s}\right|$$

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
OLS	0.038 (D)	0.031 (D)	0.037 (D)	0.034 (D)	0.026 (D)	0.027 (D)	0.027 (C)	0.030 (C)
ARIMA(1,1)	0.038 (C)	0.030 (C)	0.036 (C)	0.033 (C)	0.025 (C)	0.027 (C)	0.028 (D)	0.031 (E)
VAR(1)	0.039 (E)	0.031 (E)	0.037 (E)	0.037 (E)	0.028 (E)	0.028 (E)	0.028 (E)	0.030 (D)
MRS	0.035 (B)	0.031 (B)	0.031 (A)	0.031 (A)	0.024 (B)	0.022 (A)	0.022 (A)	0.029 (B)
NN	0.031 (A)	0.031 (A)	0.032 (B)	0.033 (B)	0.023 (A)	0.026 (B)	0.024 (B)	0.027 (A)

Root mean square errors

$$RMSE = \sqrt{\sum\left(y_{t+s} - \hat{y}_{t,s}\right)^2 / n}$$

	AAA	AA	A	BBB	1-3Y	3-5Y	5-7Y	7-10Y
OLS	0.0862 (D)	0.0496 (D)	0.0601 (D)	0.0539 (D)	0.0453 (D)	0.0426 (D)	0.0419 (C)	0.0471 (D)
ARIMA(1,1)	0.0862 (C)	0.0486 (C)	0.0584 (C)	0.0533 (C)	0.0447 (C)	0.0423 (C)	0.0425 (D)	0.0476 (E)
VAR(1)	0.0873 (E)	0.0517 (E)	0.0611 (E)	0.0590 (E)	0.0487 (E)	0.0438 (E)	0.0442 (E)	0.0467 (C)
MRS	0.0819 (B)	0.0465 (B)	0.0527 (B)	0.0503 (B)	0.0413 (B)	0.0381 (A)	0.0356 (A)	0.0469 (B)
NN	0.0738 (A)	0.0450 (A)	0.0490 (A)	0.0482 (A)	0.0366 (A)	0.0406 (B)	0.0372 (B)	0.0457 (A)

Table continues at the next page

Theil's U-Statistic								
$$\frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s}-\hat{y}_{t,s}\right)^{2}}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_{t+s}\right)^{2}}+\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\hat{y}_{t,s}\right)^{2}}}$$								
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
OLS	0.861 (B)	0.797 (C)	0.833 (C)	0.836 (D)	0.808 (C)	0.795 (C)	0.813 (C)	0.804 (C)
ARIMA(1,1)	0.868 (C)	0.819 (D)	0.872 (E)	0.926 (E)	0.901 (E)	0.873 (E)	0.875 (D)	0.834 (E)
VAR(1)	0.853 (D)	0.850 (E)	0.836 (D)	0.753 (C)	0.897 (D)	0.862 (D)	0.893 (E)	0.834 (D)
MRS	0.888 (E)	0.793 (B)	0.806 (B)	0.787 (B)	0.807 (B)	0.790 (B)	0.799 (B)	0.817 (B)
NN	0.567 (A)	0.618 (A)	0.541 (A)	0.520 (A)	0.470 (A)	0.595 (A)	0.601 (A)	0.585 (A)
Percentage correct sign predictions								
$$PCSP=\frac{1}{n}\sum_{t=1}^{n}z_{t+s},\quad z_{t+s}=\begin{cases}1, & y_{t+s}\hat{y}_{t,s}>0\\ 0, & otherwise\end{cases}$$								
	AAA	**AA**	**A**	**BBB**	**1-3Y**	**3-5Y**	**5-7Y**	**7-10Y**
OLS	0.460 (C)	0.450 (C)	0.444 (C)	0.460 (C)	0.516 (C)	0.496 (D)	0.466 (C)	0.420 (B)
ARIMA(1,1)	0.440 (D)	0.440 (D)	0.416 (D)	0.426 (E)	0.432 (D)	0.352 (C)	0.326 (D)	0.340 (D)
VAR(1)	0.390 (E)	0.374 (E)	0.406 (E)	0.448 (D)	0.404 (E)	0.342 (E)	0.326 (D)	0.324 (E)
MRS	0.550 (B)	0.586 (B)	0.586 (B)	0.602 (A)	0.580 (A)	0.590 (B)	0.650 (B)	0.382 (C)
NN	0.590 (A)	0.596 (A)	0.622 (A)	0.580 (B)	0.560 (B)	0.618 (A)	0.678 (A)	0.628 (A)

7.5 Conclusions

This chapter examined the influence of determinants on the daily changes of credit spread indices and introduced two new determinants that have not been previously considered: the inflation; and commodity indices. However, the study revealed that only inflation was statistically and economically significant. In addition, this chapter investigated whether the influence of the determinants on credit spread changes is regime dependent in addition to examining the nonlinear effect of the determinants on credit spread changes through the application of two statistical techniques: the Markov regime switching regression model and the feed-forward neural network.

The Markov regime switching regression model revealed that the effect of the determinants on credit spread changes is regime dependent. Overall, the determinants are able to explain a portion of the credit spread variation and the signs of the estimated coefficients are in line with the literature and suggested theory. However, the Markov regime switching regression model offers new evidence regarding the effects of the determinants on credit spreads as well as the economic interpretation of the signs of the estimated coefficients. The Markov regime switching regression model reveals that during the low volatility regime the estimated coefficients have a weak statistical and economic significance. However, during the high volatility regime most of the estimated coefficients are statistical significant. This suggests that during the high volatility period the inflation and the expected future volatility increase, suggesting a future uncertainty of market conditions and future value of cash flows which may adversely affect the overall economy. At the same time, governments adopt monetary policies such as reducing interest rates in order to reduce inflation and market uncertainty. Therefore, the combination of these interest rate cuts and increases in inflation accompanied by the higher yield demanded by investors widens credit spreads.

Finally, both the feed-forward neural networks and Markov regime switching models are able to capture the non-linear relationship between the determinants and credit spreads. These two specifications are able to estimate the most accurate out-of-sample mean forecasts and correct sign predictions compared to OLS, ARIMA(1,1) and VAR(1).

Chapter 8

Conclusions and suggestions for future research

8.1 Introduction

This chapter summarises the econometric and statistical approaches applied as well as the findings and main conclusions of each empirical chapter, and offers suggestions for future research. The main subject of the thesis was to provide further insight into and enhance our understanding of the dynamics of European corporate credit spreads during the credit crisis period. This is because credit spreads are important financial tools, since they are used as indicators of economic progression, investment decisions, trading and hedging, as well as pricing credit derivatives. Their role has increasingly become more significant for the Euro Fixed Income markets since the introduction of the Euro, which reshaped the mechanics of the financial environment. The introduction of single currency in Europe provided the means for a pan-European economic growth and cross-border development, liberalized a vast inflow of capital which was once fragmented into different currencies, and provided the dynamics of cross-border investments around a unified legislative framework.

The empirical research has been formulated to cover three important areas: modelling the dynamics of higher moments and regime shifts across different credit ratings and maturities, modelling the time-varying correlation between credit spread indices and examine the effect of a number of determinants on credit spreads during different market conditions. These areas have been investigated in detail in order to help enhance our understanding about the behaviour and dynamics of European corporate credit spreads and provided new evidence and insights of the behaviour of credit spreads.

The first area of interest allowed the assessment of the dynamics of the shape of the distribution of credit spreads overtime while the second allowed the investigation of the dynamics and behaviour of correlation overtime. The models that were applied in these areas were also compared in order to establish which model produces the best volatility and VaR estimates based on risk management loss functions. The final area provided new evidence regarding the effects of the determinants on credit spreads and applied a statistical model that had not been previously employed in capturing the non-linear relationship between the determinants and credit spread changes.

The structure of this chapter is as follows: Section 8.2 offers a summary of the thesis and discusses the conclusions of each empirical chapter as well as the implications of the findings, while Section 8.3 discusses the limitations of this study and proposes suggestions for future research.

8.2 Summary of the thesis, Conclusions and Implications of our findings

Chapter one provided a brief historical description of the events that led to the creation of the European Union and described how the introduction of the single currency was the catalyst for reshaping the mechanics of the European financial markets. In Chapter 1, a debt security and the different types of risk that affect this instrument were also introduced and it was highlighted that one of the most important types of risk to which market participants are exposed is credit risk. The main areas of research which needed further investigation were identified and the objectives of the thesis were presented.

Chapter 2 presented credit risk as the most important risk posed to market participants and reviewed the different methodologies that are applied to measure credit risk. Chapter 2 also discussed the importance of modelling the dynamics of higher moments and time-varying correlation in risk management, hedging, portfolio allocation and option pricing and presented the empirical findings of the behaviour of corporate bond prices as well as the determination of the drivers behind credit spreads. Finally, it discussed the shortcomings of the literature and highlighted the area that needed further investigation.

Chapter 3 discussed in detail the econometric and statistical methodologies that are applied throughout the thesis. It introduced the models for investigating the univariate properties of time series, including stationarity and unit root tests and presented multivariate and non-linear time series analysis such as VAR and Markov regime switching models. Chapter 3 also introduced the ARCH and GARCH models, which are applied in modelling the time-varying volatilities of time series as well as their extensions to capture the dynamics of higher moments and changes in the dynamics of volatility of financial time series. There was also a discussion of multivariate volatility models such as BEKK, O-GARCH, CC-GARCH and DCC-GARCH as well as the principal component analysis. Finally, Chapter 3 presented the risk management loss functions and back testing techniques depicted by Christoffersen (1998) to measure the efficiency of the different VaR estimates.

Chapter 4 introduced the data set used for empirical analysis, reviewed the statistical properties of the different variables and examined the univariate properties of the series such as stationarity and unit roots. Furthermore, it described how the credit spreads were estimated and discussed the reasons that led to the selection of the German government bonds as benchmark instruments. Having explained the research theme in the first chapter, reviewed the relevant literature in the second chapter and discussed the methodology in the third chapter and data in the fourth, the subsequent sections are devoted to the discussion of the conclusions and their implications.

8.2.1 Dynamics of Credit Spreads of European Corporate Bond Indices

Chapter 5 investigated the nature and dynamics of credit spreads moments of the European corporate bond indices, and compare such behaviour across different credit ratings and maturities. The examination of the dynamics of volatility and the conditional higher moments was performed by the application of a series of models including the GJR-GARCH, GARCH-SK and variants of Markov Regime Switching GARCH. It was found that the estimated volatilities, skewness and kurtosis display a consistent pattern across ratings and maturities. Lower ratings and long-term maturities have greater conditional volatilities and kurtosis. This behaviour reflects the higher probability of downgrades and defaults of lower rating and long-term maturity indices. Conditional skewness was found to fluctuate more for lower ratings and long-term maturities and displayed occasional spikes. This meant that during high volatility periods credit spreads widen and interest rates decrease suggesting a deteriorating economy and therefore the negative effect of the credit-worthiness of lower ratings and long-term maturities is reflected as negative spikes in the conditional skewness.

Chapter 5 also examined the appropriateness of the proposed models in capturing the dynamics of higher moments in credit spreads in terms of forecasting volatility and producing Value-at-Risk estimates. The adequacy of VaR estimates was examined by the application of the Christoffersen (1998) back-testing procedure. The back-testing revealed mixed results on the appropriateness and accuracy of models, results which are in line with those reported by Brooks and Persand (2003), Dacco and Satchell (1999) and Marcucci (2009), who do not find a uniformly accurate model for all time horizons either.

Chapter 5 illustrated that modelling the dynamics of higher moments and regime shifts of financial time series was important not only for risk management, as this study showed, but for derivative pricing, investment decisions, as well as trading and hedging strategies. The GARCH-SK model used has a number of important applications. First, the GARCH-SK model may better describe the distributional properties of financial asset returns. Second, the estimated coefficients of the GARCH-SK model may be used to obtain separate estimates of the market-required risk premium associated with variance, skewness and kurtosis risk. Third, the GARCH-SK model could then be applied in portfolio allocation to assess whether the trade-off between the mean, variance, skewness and kurtosis that was estimated from the series of returns for the chosen portfolio was an optimal one given the market-required returns for each type of risk. Fourth, the GARCH-SK model may be useful in estimating future coefficients of variance, skewness and kurtosis which are unknown parameters in option pricing models that account for skewness and kurtosis and may be useful in testing the information content of option implied coefficients of variance, skewness and kurtosis. Finally, the Markov regime switching GARCH model can be applied in option pricing and in estimating regime dependent hedge ratios that change as market conditions change.

8.2.2 Modelling the Time-varying Correlation of Credit Spreads

Chapter 6 investigated the behaviour of time-varying correlation in credit spreads and compared the properties and performances of the different multivariate GARCH models. The models examined are the Orthogonal-GARCH, the Constant and Dynamic Correlation GARCH and Diagonal-BEKK. The results revealed that the correlation of credit spreads is time-varying and is affected by market conditions. This means that during periods of expansion where firms expand and diversify, and experience an increase in their cash flows, the likelihood that the issuers might be unable to honour their contractual obligations reduces. During this time the correlation between credit spreads of different ratings and maturities reduces and so do credit spreads. This may suggest that the overall default risk is reduced. On the other hand, during a contracting economy, firm’s experience a decline in cash flows, increasing the likelihood that the issuers might be unable to service their debt obligations. Consequently, correlation between credit spreads increases and so do credit spreads.

Chapter 6 also examined the performance of the different models by determining whether they produce accurate VaR estimates based on risk managements’ loss functions. Even thought all models failed to reject the three likelihood ratio tests showing good-out-of sample performance, Diagonal-BEKK and RiskMetrics out-performed the other multivariate GARCH specifications in terms of the lowest percentage of failures observed for both long and short Value-at-Risk positions.

Chapter 6 showed that correlation of credit spreads is time-varying and is affected by market conditions. These findings have important implications in risk management as well as in the pricing of credit derivatives that assume correlation of credit spreads to be constant, in investigating the relationship between systematic default correlation and credit spreads, in credit portfolio allocation in estimation of the optimal portfolio weights and hedge ratios and finally, in the trading and hedging of default risk.

8.2.3 Regime Switching Determinants of European Corporate Credit Spreads

Chapter 7 examined the influence of the risk-free rate and important macroeconomic determinants on the daily changes of credit spread indices. In Chapter 7 two new determinants were used that had not been previously considered: the inflation and commodity price indices. The results of this study showed that only inflation was statistically and economically significant.

Moreover, Chapter 7 also investigated whether the influence of the determinants on credit spread changes is regime dependent by applying a Markov regime switching regression model. The Markov regime switching regression model revealed that the effect of the determinants on credit spread changes is regime dependent and the determinants are able to explain a portion of the credit spread variation. Although, the sign of the estimated coefficients agrees with the previous literature and theory, the Markov regime switching model offered new evidence regarding the effects of the determinants on credit spreads. It revealed that only during the high volatility period the estimated coefficients are statistically and economically significant. During this period, inflation and the expected future volatility increase, suggesting a future uncertainty of market conditions and future cash flows. However, in order to reduce inflation and market uncertainty, governments adopt monetary policies, such as reducing interest rates, which consequently widens credit spreads.

In addition, Chapter 7 introduced a feed-forward neural network, an approach which had not been previously applied in modelling the non-linear relationship between determinants and credit spreads. This models the nonlinear relationship between input and output layers while the information moves in only one direction, from the input layer (i.e. the determinants) through the hidden layer and finally to the output layer (i.e. credit spread changes). Finally, in Chapter 7 it was revealed that both the Markov regime switching model and Neural Networks were able to estimate the most accurate out-of-sample mean forecasts and correct sign predictions compared to OLS, ARIMA(1,1) and VAR(1) models.

The results presented in Chapter seven can be applied in estimating regime dependent hedge ratios and formulating trading strategies that change as market conditions change. In addition, they may be applied in the pricing of credit spread options and other credit derivative instruments.

8.3 Limitations of the study and suggestions for future research

The aim of this section is to highlight limitations in the econometric approaches or availability of data investigated in this thesis and suggests possible directions in which future research can be undertaken to improve and enhance our understanding in the area of credit spreads and credit modelling in general.

The theme of the research was to examine the dynamics of European corporate credit spread indices for the AAA, AA, A and BBB ratings and 1-3, 3-5, 5-7 and 7-10 maturity indices. This by itself poses a limitation in the availability of data, as it would be of great interest if the rating indices could be further broken down into maturities (i.e. AAA with maturities of 1-3, 3-5, 5-7 and 7-10) so as to examine and discuss in greater detail the dynamics of these indices as well as the effects of the determinants on them.

We investigated the nature and dynamics of credit spread moments by applying an autoregressive conditional variance, skewness and kurtosis model, the GARCH-SK. Future research may be conducted to examine, independently, the impact of time-varying skewness and kurtosis in computing volatility forecasts and VaR estimates. In addition, the Markov regime switching model, presented in Chapter 7, was not extended to capture the dynamics of conditional volatility, as these types of models are highly non-linear and it would have resulted in over-parameterization as well as unreliability of the estimated parameters.

We investigated the behaviour of time-varying correlation in credit spreads and compared the properties and performances of the different multivariate GARCH models. It would be of great interest to estimate the multivariate version of the GARCH-SK model, which would shed light on the relationship between the moments of each series employed in the portfolio. However, some issues arose during the estimation of MGARCH models with multivariate Student t-Distribution and Gram-Charlier expansion series. Furthermore, we estimated the constant correlation CC-GARCH-SK model and we found that it did not offer any benefit over the CC-GARCH model. Finally, the multivariate Gram-Charlier does not provide further insight in the cross dynamics of the higher-moments (see Appendix 8.A), which might be an area for future investigation.

Finally, the results presented in Chapter 7 offer some insight into the dynamics of the European corporate credit spreads and demonstrate how different determinants affect credit spreads under different market conditions. It might be of interest to examine the dynamics of other markets such as those in the US and Asia, and to establish whether similar behaviour is exhibited during the credit crisis period.

APPENDIX 8.A

Multivariate Gram-Charlier Distribution

The multivariate density function of the Gram-Charlier expansion series presented in this section ensures positive definiteness, given a vector $\mathbf{X}' = \left(x_{1,t}, x_{2,t}, \ldots, x_{n,t}\right)$ the density is defined as:

$$F_T\left(\mathbf{X}\right) = \frac{1}{n+1} G\left(\mathbf{X}\right) + \frac{1}{n+1} \prod_{i=1}^{n} g\left(x_i\right) \sum_{i=1}^{n} \frac{1}{k_i} \left[1 + \sum_{s=1}^{q} d_{si} H_s\left(x_i\right)\right]^2 \qquad (8A.1)$$

where:

$$G\left(\mathbf{X}\right) = \left(2\pi\right)^{-n/2} \left|\boldsymbol{\Sigma}^{-1/2}\right| e^{-\frac{1}{2}\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X}} \qquad (8A.2)$$

$$k_i = \int g\left(x_i\right) \left[1 + \sum_{s=1}^{q} d_{si} H_s\left(x_i\right)\right]^2 dx_i = 1 + \sum_{s=1}^{q} d_{si}^2 s! \qquad (8A.3)$$

where $G\left(X\right)$ denotes the multivariate normal distribution with zero mean and variance matrix Σ, whose marginal densities are the univariate normal, $g\left(x_i\right)$, d_s denotes the s$^{\text{th}}$ order Hermite polynomial parameters corresponding to the individual variables, $\left\{H\left(x\right)\right\}$ represents the Hermite polynomials such that for $i \geq 2$ they hold the following recurrence relation:

$$H_i\left(x\right) = \left(xH_{i-1}\left(x\right) - \sqrt{i-1}H_{i-2}\left(x\right)\right) \Big/ \sqrt{i} \qquad (8A.4)$$

and they satisfy the following conditions:

$$\begin{aligned} &\int_{-\infty}^{\infty} H_i^2\left(x\right)\varphi\left(x\right) dx = 1, \quad \forall i \\ &\int_{-\infty}^{\infty} H_i\left(x\right) H_j\left(x\right) \varphi\left(x\right) dx = 0, \quad \forall i \neq j \end{aligned} \qquad (8A.5)$$

In the bivariate case for variables x_t and y_t the density becomes:

$$F_T\left(x_t, y_t\right) = \frac{1}{3} G\left(x_t, y_t\right) + \frac{1}{3} g\left(x_t\right) g\left(y_t\right) \left\{ \frac{1}{k_x}\left[1 + \sum_{s=1}^{q} d_{sx} H_s\left(x_t\right)\right]^2 + \frac{1}{k_y}\left[1 + \sum_{s=1}^{q} d_{sy} H_s\left(y_t\right)\right]^2 \right\} \qquad (8A.6)$$

where

$$k_x = \int g(x_t)\left[1+\sum_{s=1}^{q} d_{sx} H_s(x_t)\right]^2 dx_t = 1+\sum_{s=1}^{q} d_{sx} s!$$
$$k_y = \int g(y_t)\left[1+\sum_{s=1}^{q} d_{sy} H_s(y_t)\right]^2 dy_t = 1+\sum_{s=1}^{q} d_{sy} s! \qquad (8A.7)$$

The multivariate density integrates to one and its marginal densities are also univariate density functions. For simplicity the proof is shown for the bivariate case:

$$\int F_T(x_t, y_t)\, dy_t = \frac{1}{3}\int G(x_t, y_t) dy_t + \frac{1}{3k_x} g(x_t)\left\{\left[1+\sum_{s=1}^{q} d_{sx} H_s(x_t)\right]^2\right\}\int g(y_t)\, dy_t +$$
$$+\frac{1}{3k_y} g(x_t)\int\left[1+\sum_{s=1}^{q} d_{sy} H_s(y_t)\right]^2 g(y_t)\, dy_t = \qquad (8A.8)$$
$$= \frac{2}{3} g(x_t) + \frac{1}{3k_x} g(x_t)\left\{\left[1+\sum_{s=1}^{q} d_{sx} H_s(x_t)\right]^2\right\} = f_T(x_t),$$

Thus:

$$\int\int F_T(x_t, y_t)\, dy_t = \frac{2}{3}\int g(x_t)\, dx_t + \frac{1}{3k_x}\int g(x_t)\left\{\left[1+\sum_{s=1}^{q} d_{sx} H_s(x_t)\right]^2\right\} = \frac{2}{3}+\frac{1}{3} = 1 \qquad (8A.9)$$

Therefore, the proposed density function is a true density as it integrates to one.

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