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“2-Factor Models in Credit and Energy Markets”

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Abstract

This thesis is divided in two main parts. Part A is focusing on assessing the ability of structural – form framework to predict the spreads and the prices in two different market regimes before and during the credit crisis. In Part B a 2 – factor model with local volatility for oil market is developed.

For the first part three structural form models; Merton's (1974), Leland – Toft (1996) and Longstaff – Schwartz (1995); were implemented using different assumptions for volatility and debt maturity (i) exogenous volatility and actual bond maturity, (ii) exogenous volatility and adjusted maturity, (iii) model determined volatility and actual bond maturity and (iv) model determined volatility and adjusted maturity. To our knowledge it is the first time that the model is calibrated against such four alternatives. Another novel feature of our work is the usage of historical implied volatility was used for equity.

Results were in contrast with Lyden and Saraniti (2000) and Wei and Guo (1997) who argued that Merton's model dominates Longstaff and Schwartz in predictive accuracy as Longstaff and Schwartz model revealed a very good performance. The encouraging results during the first period (January 1998 - April 2006) led to a very critical element of this research – the implementation of the Longstaff and Schwartz (1995) model on 2007 – 2008 bond data. The assumption of simple capital structure is relaxed and a composite implied volatility is calculated. Again the model indicated very good performance in all cases proving an average predicted over actual credit spread ratio of 57%.

The second part of this research proposes a 2 – factor model with local volatility to price Oil Exotic Structures. The proposed approach utilizes the general multi – factor model framework and the interest rate modeling developments as described by Clewlow and Strickland (1999b) and Brigo and Mercurio (2006) respectively.

The model has the flexibility to generate different local volatility surfaces depending on the calibrated data. Moreover the model allows different correlation surface. The model is used to price a number of exotic structures – barrier options, Target Redemption Notes and European and Bermudan Swaptions – that are common in the oil market. Based on the results it is clear that being able to capture the smile dynamics is very important not only for valuation reasons but also for risk management purposes. The model can be calibrated directly and match market traded instruments such us swaptions and monthly strip options.

Chapter 1: OVERVIEW OF THE THESIS

This thesis is divided in two main parts. Part A is focusing on assessing the ability of structural – form framework to predict the spreads and the prices in two different market regimes before and during the credit crisis. In Part B a 2 – factor model with local volatility for oil market is developed.

Chapters 2 and 3 present the general framework for credit risk modeling and literature review. A very rich and useful source of information on the default risk of an obligor are the market prices of bonds and other defaultable securities that are issued by this obligor, and the prices of credit default swaps referencing this obligor's credit risk. Credit risk models can be divided into two main categories: (i) structural-form models, and (ii) reduced-form models. The theoretical framework for the following three structural form models of corporate bond pricing, Merton (1974), Leland and Toft (1996) and Longstaff and Schwartz (1995) is presented.

In Chapter 4 the results of the empirical application of Merton, Leland and Toft and Longstaff and Schwartz models are presented. While the Merton and Leland and Toft models perform on opposite directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads; Longstaff and Schwartz model reveals a very good performance. The use of equity implied volatility made a significant impact on the performance of the model. Also, in contrast with the existing literature we see that the model performed very well and in investment rated companies producing a median predicted over actual credit spread ratio greater than 35%. In addition the Longstaff and Schwatrz (1995) is implemented during the 2007 – 2008 credit crisis. The assumption of simple capital structure is relaxed and a composite implied volatility is calculated. Again the model indicated very good performance in all cases proving an average predicted over actual credit spread ratio of 57%. Interestingly though the average predicted credit spread was still estimated below the actual one in line with the previous implementation although the explanatory power of the model increased mainly driven by the higher market volatility. Finally, in Chapter 5 we utilize the structural form model framework for investment decision's between equity and credit.

In Chapter 6 a 2-factor model that accounts for volatility smile for oil markets is developed. There are two main approaches for the commodity futures price dynamics. The first approach developed by Gibson and Schwartz (1990), Schwartz (1997), Miltersen and Schwartz (1998) and Hilliard and Reis (1998) aims to capture the stochastic representation of the spot price and other factors such as convenience yield and interest rates. This modelling approach, although it allows a numerous set of dynamics for the commodity futures forward curve, has a number of problems driven by the fact that state variables can be unobservable. Also the convenience yield can be negative allowing arbitrage opportunities. Finally, the fact that spot prices and convenience yield have constant volatility and correlation is relative restrictive as they do not allow the variance of the spot and future prices and the correlation between them to vary on the level of the price or the convenience yield.

The second approach, developed by Clewlow and Strickland (1999a) and Clewlow and Strickland (1999b), focuses on the evolution of the forward curve. The development of the exchange traded futures resulted observable future prices up to various maturities depending on the underlying. The first near by contract is used to imply the convenience yield for the longer maturities. Multifactor models for commodity prices utilize the research on the interest rate term structure modeling.

The 2 – factor model with local volatility developed under this thesis, has the flexibility to generate different local volatility surfaces depending on the calibrated data. Moreover the model allows defining different correlation surface. The model is used to price a number of exotic structures – barrier options, Target Redemption Notes and European and Bermudan Swaptions – that are common in the oil market. Based on the results it is clear that being able to capture the smile dynamics is very important not only for valuation reasons but also for risk management purposes. The model can be calibrated directly and match market traded instruments such us swaptions and monthly strip options.

Finally, Chapter 7 summarizes the concluding remarks and Chapter 8 presents the future research challenges.

Chapter 2: CREDIT RISK MODELING: AN OVERVIEW

2.1. The components of credit risk

The most important components of the credit risk are the following:

Arrival risk is a term for the uncertainty whether a default will occur or not. To enable comparisons, it is specified with respect to a given time horizon, usually one year. The measure of arrival risk is the probability of default. The probability of default describes the distribution of the indicator variable default before the time horizon.

Timing risk refers to the uncertainty about the precise time of default. Knowledge about the time of default includes knowledge about the arrival risk for all possible time horizons, thus timing risk is more detailed and specific than arrival risk. The underlying unknown quantity (random variable) of timing risk is the time of default, and its risk is described by the probability distribution function of the time of default. If a default never happens, the time of default is set to infinity.

Recovery risk describes the uncertainty about the severity of the losses if a default has happened. In recovery risk, the uncertainty quantity is the actual payoff that a creditor receives after the default. It can be expressed in several ways. Market convention is to express the recovery rate of a bond or loan as the fraction of the notional value of the claim that is actually paid to the creditor. Recovery risk is described by the probability distribution of the recovery rate, i.e. the probabilities that the recovery rate is of a given magnitude. This probability distribution is a conditional distribution, conditional upon default.

Market risk describes a different kind of risk, the risk of changes in the market price of a defaultable asset, even if no default occurs. Apart from other market factors that also affect the prices of default-free claims, market risk is also driven by changes in timing and recovery risks, or at least changes in the market's perception of these risks. This risk might be called risk change risk.

Market risk models are dynamic models, thus they add an additional layer of complexity. To avoid arbitrage opportunities in the model, market risk must be modeled in consistency with time and recovery risk, and the changes in these risks, and in consistency with other market prices. For standard credit default swaps, changes in credit risk are in fact the dominant driver for market risk, while defaultable coupon bonds are also strongly influenced by changes in the default-free interest rates.

The impact of default risk can be affected by the behavior of other market variables like movements in default-free interest rates, exchange rates etc. These may influence the value of the defaultable claim, for instance if counterparty risk is considered in derivatives transactions. But it is also present in classical loans: for a given recovery rate, a default on a fixed-coupon loan in a high-interest-rate environment is less severe than a default on the same loan in a low-interest-rate environment, because the net present value of the lost claim is lower in the former case.

The term market price correlation risk covers this type of risk: the risk that defaults (and defaults likelihoods) are correlated with price movements of the defaultable asset.

While the arrival risk and timing risk are usually specific to one defaultable obligator, recovery risk, market risk and market price correlation risk are specific to a particular payment obligation of a given obligator, or at least to a particular class of payment obligations.

If the risk of joint defaults of several obligators is introduced, an additional component is introduced. Default correlation risk that describes the risk that several obligators default together. Again here we have joint arrival risk, which is described by the joint default probabilities over a given time horizon, and joint timing risk, which is described by the joint probability distribution function of the times of default.

From a theoretical point of view, it is desirable to include as many of the different faces of default risk as possible. This comes at the cost of additional complexity in the model, implementation problems and slower runtime. Therefore, the first question that should be answered is which risks should be included in the model. For example, dynamic models of market risk are necessary to risk-manage and mark-to-model credit derivatives and tradeable default bonds on a frequent basis. For a static book of loans this may be less important than having an accurate model of the default correlations. A second constraint is given by the available data. If there is no data to base a sophisticated model upon, a simpler version should be chosen that requires fewer inputs. Of course every simplification involves implicit assumptions about the risks that are modeled, and these assumptions may have consequences that are not always obvious.

2.2. Structural Approach

There are two main approaches to explaining the level of the credit spread; the first initiated by Black and Scholes (1973) and Merton (1974) considers corporate bonds in an option-pricing framework. The second is based on reduced-form models.

Over thirty years ago Black and Scholes (1973) and Merton (1974) initiated the modern analysis of corporate debt by pointing out that the holders of risky corporate bonds can be thought of as owners of risk-free bonds who have issued put options to the holders of the firm's equity. Models based on this approach are generally referred to as structural models. The equity can be considered a call option on the asset value of the firm with a strike price equal to the value of the liabilities. The value of the corporate debt can therefore be calculated as the default risk-free value of the debt minus the value of a default option. In other words, next to the risk-free rate, an investor in corporate bonds also demands a credit spread to compensate for the written call option. The strike price for this option equals to the face value of the debt and reflects the limited liability of equity holders in the event of bankruptcy. In the above classic approach the firm is financed by a zero – coupon bond with face value B and maturity T and the default can occur only at maturity of the debt.

The Merton's (1974) framework provided the base for the origination of extensions by adding features either to the process of the firm or the interest rates, or by relaxing some of the assumptions of the original framework (e.g. the default time). Black and Cox (1976) extended the model by allowing default prior to maturity (these types of models are often referred to as first passage time models) and including certain types of bond indenture provisions, such as safety covenants, subordination arrangements and restrictions on the financing of interest and dividend payments. An important difference in relation to Merton's framework is that is allows bondholders to bankrupt the firm anytime before maturity, as safety covenants give them the right to bankrupt or force a reorganization of the firm if it is doing poorly according to some standard. Regarding the value of the junior bonds they argued that can be derived from the price of senior bonds assuming that the holders of junior bonds will be paid after the holders of the senior bonds. Finally, the incorporation of interest and dividend payment restrictions, under certain circumstances, can alter the disadvantages that faced by the junior bondholders. Their findings concluded that the above provisions increase the value of bonds, and that they may have a quite significant effect on the behaviour of the firm's securities.

Table 2.1 – Summary of the main Structural Models I[1]

	Asset Value	Default Risk – Free Rate
Merton (1974)	$dV = \mu Vdt + \sigma Vdz$	$dr = rdt$
Black and Cox (1976)	$dV = (\mu - \delta)Vdt + \sigma Vdz$	$dr = rdt$
Leland (1994) and Leland and Toft (1996)	$dV = \left(\mu(V,t) - \delta\right)dt + \sigma Vdz$	$dr = rdt$
Kim, Ramaswamy and Sundaresan (1993)	$dV = (\mu - \gamma)Vdt + \sigma_1 Vdz_1$	$dr = \kappa(m - r)dt + \sigma_2 \sqrt{r} dz_2$
Longstaff and Schwartz (1995)	$dV = \mu Vdt + \sigma_1 Vdz_1$	$dr = (m - \kappa r)dt + \sigma_2 dz_2$
Briys and de Varenne (1997)	$dV = rVdt + \sigma_1 \left(\rho dz_2 + \sqrt{1-\rho^2} Vdz_1\right)$	$dr = \kappa(t)\left(m(t) - r\right)dt + \sigma_2(t)\sqrt{r} dz_2$
Zhou (1997)	$dV = (\mu - \lambda\delta)Vdt + \sigma_1 Vdz_1 + (\Pi - 1)dJ$	$dr = (m - \kappa r)dt + \sigma_2 dz_2$

V : is the value of firm's assets.

μ : is the instantaneous expected rate of return on firm's assets.

σ^2, σ_1^2 : is the instantaneous variance of the return on firm's assets.

$r(t)$: is the instantaneous risk – free rate.

$m(t)$: is the long – term rate mean.

$\kappa(t)$: is the speed of mean reversion.

σ_2^2 : is the instantaneous variance of the instantaneous risk – free rate.

δ : is the fraction of value paid out to security holders.

γV : is the net outflow from the firm resulting from optimal investment decisions.

dz : is a standard Wiener process.

dJ : is a Poisson process with intensity parameter λ and a jump amplitude equal to $\Pi > 0$. Note that $E[\Pi] = \delta + 1$.

ρ : is the correlation between two standard Wiener processes dz_1, dz_2 i.e. between the return on the firm's assets and the return on the market.

[1] Source: Bohn, Jeffrey, R., 2000, "A Survey of Contingent-Claims Approaches to Risky Debt Valuation", The Journal of Risk Finance, pp. 53-70.

Geske (1977) developed a new formula for evaluating subordinated debt using a compound option technique. Under his framework when a corporation has coupon bonds outstanding, the common stocks can be considered as a compound option. At every coupon date, the stockholders have the option of buying the next option by paying the coupon, else the firm defaults to bondholders. The final option is to repurchase the claim on the firm from the bondholders by paying off the principal at maturity.

Ho and Singer (1982) examined the effect of alternative bond indenture provisions on the risk of a firm's debt under the contingent – claim framework. They examined four indenture provisions the time to maturity, the promised payments schedule, financing restrictions and priority rules. A main difference in relation to relevant literature is that they used elasticity as a proxy for the risk and not the change in credit spreads. In contrast with Merton (1974) who argued that the change in the credit spread with respect to a change in maturity can be either sign, Ho and Singer, claimed that is an increasing function.

Kim, Ramaswamy and Sundaresan (1993) instead of considering the asset value, focused on the cash flow, arguing that the cash flow problem is the source that leads to bankruptcy. Furthermore, they incorporated Cox, Ingesoll and Ross (1985) framework as the stochastic process for the default – free interest rate. Under their model the default occurs when the firm's cash flow are not sufficient to repay the interest obligations. If that is the case the possibility that at the time of bankruptcy the value of the firm can be higher than the value of the remaining debt obligations is visible. Longstaff and Schwartz (1995) extended the Black and Cox (1976) model by allowing the short-term riskless rate of interest to be described by the dynamics as developed by Vasicek (1977). Furthermore, they assumed that if reorganization occurs during the life of a security, the security holder receives $1-w$ times the face value of the security at maturity. The factor w represents the percentage writedown on a security. If w=0 there is no writedown and the security holder is unimpaired. If w=1 the security holder receives nothing. A constrain about w is that the adding up settlements on all classes of claims cannot exceed K. So the recovery rate is considered like a boundary value, rather than exogenously determined. Briys and Varenne (1997) developed a model that is rooted in the Black and Cox (1976) and Longstaff and Schwartz (1995) framework by incorporating a stochastic barrier for default. When the barrier is being hit the bondholders receive an exogenously specified fraction of the remaining assets. This ensures that bondholders do not receive a payment greater than the firm value upon default. Moreover, captures the violation of the priority rule. Collin – Dufresne and Goldstein (2001) extended the Longstaff and Schwartz (1995) framework and they developed a model where firms adjust their capital structure to reflect changes in asset value. Collin – Dufresne and Goldstein pointed out that according to Longstaff and Schwartz framework the expected leverage ratio should decline exponentially over time. In Longstaff and Schwartz approach is specified a default boundary which was assumed constant. Since the firm value process follows a geometric Brownian motion, the expected firm value increases exponentially over time. Thus, if the debt level is assumed to be a monotonic function of the default boundary, it follows that it remains also constant over time, leading to leverage ratio that declines exponentially over time.

Leland (1994) and Leland and Toft (1996) developed their approach incorporating two more aspects. They considered both the optimal capital structure and the maturity of the debt in order to examine the debt value and derived endogenous conditions under which can bankruptcy will be declared. They endogenized taxes and bankruptcy costs, in determining the optimal asset value at which the firm should bankrupt. Furthermore in line with Longstaff and Schwartz framework they assumed that there is a boundary value V_B for the firm at which financial distress occurs. However, they have proven that this value is determined endogenously and shown to be constant in rational expectations equilibrium. One important difference from the Longstaff and Schwartz framework is that the riskless interest rate is assumed to be constant. Finally under their framework the firm continuously sells a constant principal amount of new debt with maturity of T years from issuance, which it will redeem at par upon maturity. Goldstein, Ju and Leland (2001) developed a dynamic model that gives the firm the option to issue additional debt at a specified upper boundary. Their main difference from previous models is that they did not model the assets value (Equity plus Debt) but the dynamics of the claim to earnings before interest and tax (EBIT). The main argument behind this approach is that the EBIT – generating machine is the source of firm's value and it runs independently of how this flow is distributed among the various groups (shareholders, bondholders, government).

Table 2.2 – Summary of the main Structural Models II[2]

	Default Barrier	Recovery
Merton (1974)	F	$V_{A(T)}$
Black and Cox (1976)	$LFe^{-r(T-t)}; AB$	$LFe^{-r(T-t)}$
Leland (1994) and Leland and Toft (1996)	$V^*_{A(T)}(\delta, T, \tau, a); AB$ [3]	$(1-L)V^*_{A(T)}$
Kim, Ramaswamy and Sundaresan (1993)	$c/\delta; AB$	$\min\left[(1-L(t))P(r,t,c), B_{A(t)}\right]$ [4]
Longstaff and Schwartz (1995)	$K; AB$	$(1-L)F$
Briys and de Varenne (1997)	$LFP(t,T)$ [5]$; AB$	$LFP(t,T)$
Zhou (1997)	$K; AB$	$(1-L)F$

F : is the face value of zero – coupon debt.

AB : is an absorbing barrier. The firm can enter into default prior to maturity if its assets value hits this barrier at any time up until maturity.

r : is the risk – free rate.

α : is the bankruptcy cost as a fraction of the value of the firm at the time of bankruptcy.

τ : is the firm's tax rate.

c : is the coupon to be paid to bondholders.

c/δ : is the default barrier.

T : is the maturity of the debt.

$P(t,T)$: is the price of a default – free discount bond with same tenor.

L : is the loss given default, so that the recovery amount given default is $(1-L)$.

K : is the boundary value for the firm at which default occurs.

[2] Source: Bohn, Jeffrey, R., 2000, "A Survey of Contingent-Claims Approaches to Risky Debt Valuation", The Journal of Risk Finance, pp. 53-70.

[3] The default barrier is endogenous.

[4] Upon default, debtholders receive the minimum between the total value of the firm and a fraction of an otherwise similar default risk – free bond.

[5] Designates the price of a risk – free bond based on the stochastic process behind the risk – free interest rates.

For the first time Zhou (2001) within the structural framework allowed the firm's value to follow a jump-diffusion process. According to this approach the observed spreads do not reflect only the credit risk that arises from the standard diffusion process of the firm's value, but also the credit risk that is linked with an unexpected drop in firm's value. Given the above, default can also occur unexpectedly because of a sudden drop in the firm's value. This assumption can explain the fact that the credit spreads on short term bonds are much larger than zero. The model appears to have two main drawbacks. First, as it is pointed out by Giesecke and Goldberg (2003), the model is extremely complex to be calibrated to market data. Second jumps in firm's value do not necessarily lead to default (see Giesecke (2001)).

2.3. Reduced Form Approach

An alternative approach to valuing corporate debt is the reduced form approach. Such models assume an exogenous stochastic process for the default probability and the recovery rate. If the assumption that a corporate bond sells for less than a similar Treasury bond because there is the possibility of default, holds, it follows that the value of a corporate bond is equal to the to the value of comparable Treasury bond, minus the present value of the cost of defaults. The latter depends on the probability of default and the loss in the case of default. By using this relationship to calculate the present value of the cost of defaults on a range of different bonds issued by the reference entity and making an assumption about recovery rates, the probability of the corporation defaulting at different future times can be estimated.

Reduced form models have their origins on Jarrow and Turnbull (1995). Their approach takes as a given a stochastic term structure of default – free interest rates and a stochastic term structure of credit spreads. Furthermore, it is assumed that the capital structure is irrelevant to the event of default. Under their framework default can occur at any time. They identify two classes of zero – coupon bonds. The first class is default – free zero – coupon bonds of all maturities and the second class is risky, subject to default, zero – coupon bonds. Then they decomposed the risky bonds into the product of two hypothetical securities, a zero – coupon bond denominated in a hypothetical currency, a promised of XYZ dollar called an XYZ, and a price in dollars of XYZs. If the bond is not in default, the exchange rate will be unity; else it will be less than one. The exchange rate analogy they used can be interpreted as the payoff ratio in case of default. For a simple two period model and under complete markets and arbitrage free economy they show that the risky zero – coupon bond payoffs at time 1 and 0 under the risk neutral probabilities are given by

$$\tilde{E}_1\left(e_1(2)\right) = \begin{cases} \delta \text{ if default at time 1} \\ \lambda\mu_1\delta + (1-\lambda\mu_1) \text{ else} \end{cases}$$

$$\tilde{E}_0\left(e_1(2)\right) = \lambda\mu_0\delta + (1-\lambda\mu_0)\left(\lambda\mu_1\delta + (1-\lambda\mu_1)\right)$$

$$\tilde{E}_0\left(e_1(2)\right) = \lambda\mu_0\delta + (1-\lambda\mu_0)$$

Where δ is the payoff per unit of face value on case of default and $\lambda\mu_i$ - $i = 0,1$, are the risk neutral probabilities for the two states economy.

Using the above they concluded that the risky zero – coupon bond price is its discounted expected payoff at time T, that is $\upsilon_1(t,T)=p_0(t,T)\tilde{E}_1(e_1(T))$. Similar approach is followed in valuing coupon bonds subject to default and options on defaultable bonds. Furthermore, the above framework was extended to a continuous time economy.

Jarrow, Lando and Turnbull (1997) extended the Jarrow and Turnbull (1995) model by assuming that the bankruptcy follows a discrete state space Markov chain process in credit ratings. Using this approach they incorporated the firm's credit rating as an indicator of the probability of default. According to their approach the state of a company can be specified in a K x K transition matrix Q.

$$Q=\begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1K} \\ q_{21} & q_{22} & \cdots & q_{2K} \\ \cdots & \cdots & \cdots & \cdots \\ q_{K-1,1} & q_{K-1,2} & \cdots & q_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

The q_{ij} represents the probability of going from state i to state j in one time step. The default is an absorbing state K. The probability of default, when standing at stage i, under the risk neutral probabilities was calculated as $\tilde{Q}_t^i(\tau^* \succ T)=\sum_{j\neq K}\tilde{q}_{ij}(t,T)=1-\tilde{q}_{iK}(t,T)$ (2.1), where τ^* is the time of default. Under that framework the price $\upsilon^i(t,T)$ of a risky zero – coupon bond that is issued by a company in credit class i is given by following equation $\upsilon^i(t,T)=p(t,T)\left(\delta+(1-\delta)\tilde{Q}_t^i(\tau^* \succ T)\right)$ (2.2).

So the credit spread is calculated as a function of the recovery rate δ and the risk neutral probabilities

$$f^i(t,T)=f(t,T)+1_{[\tau^*\succ T]}\log\left(\frac{\delta+(1-\delta)\tilde{Q}_t^i(\tau^*\succ T)}{\delta+(1-\delta)\tilde{Q}_t^i(\tau^*\succ T+1)}\right) \quad (2.3)$$

Also, they applied this framework in option valuation and in continuous time.

Duffie and Singleton (1999) proposed a different approach. They parameterize the losses at default in terms of the fractional reduction in market value that occurs at default. By defining, in a risk neutral environment, h_t to be the hazard rate of default at time t and L_t, the expected fractional loss in market value if default were to occur at t, so over a small, discrete interval Δt, a default will then occur with the risk-neutral probability $h_t \Delta t$.

The recovery market value is assumed to follow a stochastic process that is given by $E_s^Q(\varphi_{s+1}) = (1 - L_s) E_s^Q(V_{s+1})$,

where

E_s^Q :is the expectation under risk – neutral probability measure Q.

V :is the value of firm's assets.

φ_s :is the recovery in event of default.

L :is the loss given default, so that the recovery amount given default is $(1-L)$.

Duffie and Singleton (1999) provided analysis that allows the recovery market value to be correlated with the hazard rate, the loss process and the term structure of the default free interest rates.

2.4. Three Alternative Structural Models

2.4.1. Merton's Model

Merton (1974) initiated the modern analysis of corporate debt by pointing out that the holders of risky corporate bonds can be thought of as owners of risk-free bonds who have issued put options to the holders of the firm's equity. Models based on this approach are generally referred to as structural models. The equity can be considered a call option on the asset value of the firm with a strike price equal to the value of the liabilities. The value of the corporate debt can therefore be calculated as the default risk-free value of the debt minus the value of a default option. In other words, next to the risk-free rate, an investor in corporate bonds also demands a credit spread to compensate for the written call option. The strike price for this option equals to the face value of the debt and reflects the limited liability of equity holders in the event of bankruptcy.

Merton developed his formula for pricing corporate liabilities. In order to develop the model along the Black and Scholes lines the following assumptions were made:

1. There are no transaction costs, taxes, or problems of indivisibilities of assets.
2. There are a sufficient number of investors who can buy and sell as much of an asset he wants at the market price.
3. There is an exchange market for borrowing and lending at the same rate of interest.
4. Short-selling is allowed.
5. Trading in assets is continuous in time.
6. The Miller-Modigliani theorem that the value of the firm is invariant to its capital structure obtains.
7. The term-structure of interest rates is flat and known with certainty.
8. The dynamics for the value of the firm V through time can be described by the following stochastic differential equation $dV = \left(aV - C\right)dt + \sigma V dz$ where:
    a. α is the instantaneous expected rate of return on the firm per unit of time.
    b. If C>0 is the dollar payouts by the firm per unit time to either the shareholders or liabilities holders (eg dividends or interest payments).
    c. If C<0 it is the net dollars received by the firm from new financing.
    d. σ^2 is the instantaneous variance of the return on the firm per unit time.
    e. dz is a standard Gauss-Wiener process.

Assumption number 7 results that if r is the instantaneous riskless rate of interest is the same for all time so, the price of a riskless discount bond with payment one dollar at maturity τ equals $P(\tau) = \exp(-r\tau)$. Assumption number 8 requires that price movements are continuous and their returns are serially independent, which is consistent with the efficient market hypothesis.

Furthermore in order the application to be used for pricing corporate debt the additional assumptions were made:

9. The corporation has two classes of claims:
    a. A single, homogenous class of debt.
    b. A residual claim that is the equity.
10. The firm promises to pays to bond holders a total of B dollars on a specific calendar date T.
11. If at time T this payment is not met, the bondholders immediately take over the company.
12. The firm cannot issue any new senior neither equivalent rank debt nor can it pays dividends or do share repurchase prior to the maturity of debt.

Given the above assumptions on the maturity date T the firm must pay B to the bondholders or else the current equity will be valueless. If the value of the company V at time T is greater than the payment B, i.e. $\left(V(T) > B\right)$, the firm should pay the bondholders and the value of equity will be $V(T) - B$. If the value of the company at T is less or equal to the value of B, i.e. $\left(V(T) \leq B\right)$, then the firm cannot make the payment and it will default.

Merton developing an analysis on the above lines, which are identical to the Black and Scholes equations for a European call option on a non-dividend paying stock, concluded that the value of equity can be expressed by the following equation:

$$f(V,\tau) = V\Phi(x_1) - Be^{-r\tau}\Phi(x_2) \qquad (2.4)$$

V: the value of the firm

B: the value of the promised payment to bond holders

τ: the length of time until maturity

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}z^2\right] dz$: the normal distribution function

$$x_1 = \frac{\log\left[\frac{V}{B}\right] + \left(r + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

$x_2 = x_1 - \sigma\sqrt{\tau}$

σ^2: the instantaneous variance of the return on the firm per unit time

So the value of debt can be written as:

$$F(V,\tau) = Be^{-r\tau}\left\{\Phi\left[h_2\left(d,\sigma^2\tau\right)\right] + \frac{1}{d}\Phi\left[h_1\left(d,\sigma^2\tau\right)\right]\right\} \quad (2.5)$$

$d = \frac{Be^{-r\tau}}{V}$: the ratio of the present value of the promised payment to the current value of the firm

$$h_1\left(d,\sigma^2\tau\right) = -\frac{\frac{1}{2}\sigma^2\tau - \log(d)}{\sigma\sqrt{\tau}}$$

$$h_2\left(d,\sigma^2\tau\right) = -\frac{\frac{1}{2}\sigma^2\tau + \log(d)}{\sigma\sqrt{\tau}}$$

Merton rearranged the above equation and in terms of risk premium concluded that

$$R(\tau) - r = \frac{-1}{\tau}\log\left\{\Phi\left[h_2\left(d,\sigma^2\tau\right)\right] + \frac{1}{d}\Phi\left[h_1\left(d,\sigma^2\tau\right)\right]\right\} \quad (2.6)$$

$R(\tau)$: the yield to maturity on the risky debt provided that the firm doesnot default

$R(\tau) - r$: the risk premium between that yield to maturity and the riskless rate of return

Merton's analysis results that for a given maturity the risk premium is a function of the two following variables:

a. The variance of the firm's operations σ^2.
b. The ratio d that is the present value of the promised payment, discounted using the riskless rate r, to the current value of the firm.

From this point of view the value of debt has been written as a function of the value of the firm V, the promised payment at maturity B, and the time to maturity τ, the business risk of the firm σ^2 and the riskless rate of return r. Furthermore under this framework Merton has proven that the following relationships hold:

1. The value of debt is an increasing function of the current market value (V) of the firm.
2. The value of debt is an increasing function of the promised payment to maturity (B).
3. The value of debt is a decreasing function of the time to maturity (τ).
4. The value of debt is a decreasing function of the business risk of the firm (σ^2).
5. The value of debt is a decreasing function of the riskless rate of interest (r).

Going a step further and analyzing the implication of the relationship to credit spreads he concluded that the credit spreads as expressed by $R(\tau)-r$ are determined by the ratio d, the σ^2 and the τ and the relations are the followings:

1. The credit spread is an increasing function of the ratio of the present value of the promised payment to the current value of the firm (d).
2. The credit spread is an increasing function of the business risk of the firm (σ^2).
3. The change in the credit spread with respect to a change in maturity can be either sign.
4. The credit spread is a decreasing function of the riskless rate of interest (r).

2.4.2. Longstaff's and Schwartz's model

As it is clear from the above presentation one of the main drawback's of the Merton's approach initially and Geske's later was that default is occurred only when the firm exhausts its assets. Furthermore they assumed that interest rates are constant. Longstaff and Schwartz (1995) developed a new approach of valuing risky debt by incorporating both default risk and interest rate risk. The basic assumptions underlying their model are the following:

1. The total value of the assets of the firm V can be described by the following stochastic differential equation $dV = \mu V dt + \sigma V dZ_1$ where σ is a constant and Z_1 is a standard Wiener process.
2. The short-term riskless rate of interest r can be described by the following stochastic differential equation $dr = (\zeta - \beta r)\,dt + \eta dZ_2$ where ζ, β and η are constants and Z_2 is also a standard Wiener process. The instantaneous correlation between dZ_1 and dZ_2 is ρdt.
3. The Miller – Modigliani theorem holds.
4. There is a boundary value K for the firm at which financial distress occurs. As long as value V is greater than K, the firm continues to be able to meet its contractual obligations.

If V reaches K, the firm immediately enters to financial distress and defaults on all of its obligations. Furthermore some form of corporate restructuring takes place.

5. If reorganization occurs during the life of a security, the security holder receives $1-w$ times the face value of the security at maturity. The factor w represents the percentage writedown on a security. If w=0 there is no writedown and the security holder is unimpaired. If w=1 the security holder receives nothing. A constrain about w is that the adding up settlements on all classes of claims cannot exceed K.
6. Perfect markets are assumed in which securities are traded in continuous time.

Under the framework that is set on the above assumption they derived that the value of a risky discount bond is:

$$P(X,r,T)=D(r,T)-wD(r,T)Q(X,r,T) \quad (2.7)$$

P: the value of a risky discount bond

X: the ratio V/K

r: the riskless interest rate

T: the maturity date

$D(r,T)=\exp(A(T)-B(T)r)$: the value of a riskless discount bond Vasicek (1977)

$$A(T)=\left(\frac{\eta^2}{2\beta^2}-\frac{\alpha}{\beta}\right)T+\left(\frac{\eta^2}{\beta^3}-\frac{\alpha}{\beta^2}\right)(\exp(-\beta T)-1)-\left(\frac{\eta^2}{4\beta^3}\right)(\exp(-2\beta T)-1)$$

$$B(T)=\frac{1-\exp(-\beta T)}{\beta}$$

$$Q(X,r,T)=\lim_{n\to\infty}Q(X,r,T,n)$$

$$Q(X,r,T,n)=\sum_{i=1}^{n}q_i$$

$$q_1=N(a_1)$$

$$q_i=N(a_i)-\sum_{j=1}^{i-1}q_jN(b_{ij}),\ i=2,3,\ldots,n$$

$N(\cdot)$: the cumulative standard normal distribution function

$$a_i = \frac{-\ln X - M\left(iT/n, T\right)}{\sqrt{S\left(iT/n\right)}}$$

$$b_{ij} = \frac{M\left(jT/n, T\right) - M\left(iT/n, T\right)}{\sqrt{S\left(iT/n\right) - S\left(jT/n\right)}}$$

$$M\left(t, T\right) = \left(\frac{a - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2}\right)t + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3}\right)\exp\left(-\beta \mathrm{T}\right)\left(\exp(\beta \mathrm{T}) - 1\right) +$$

$$\left(\frac{r}{\beta} - \frac{a}{\beta^2} + \frac{\eta^2}{\beta^3}\right)\left(1 - \exp\left(-\beta t\right)\right) - \left(\frac{\eta^2}{2\beta^3}\right)\exp\left(-bT\right)\left(1 - \exp\left(-\beta t\right)\right)$$

$$S\left(t\right) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right)t - \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right)\left(1 - \exp(-\beta \mathrm{T})\right) + \left(\frac{\eta^2}{2\beta^3}\right)\left(1 - \exp\left(-2\beta t\right)\right)$$

The first term $D\left(r, T\right)$ of the equation represents the value the bond would have if it were riskless. The second term $wD\left(r, T\right)Q\left(X, r, T\right)$ represents a discount for the default risk of the bond. The first component $wD\left(r, T\right)$ is the present value of the writedown on the bond in the event of a default. The second component $Q\left(X, r, T\right)$ is the probability – under the risk-neutral measure – that a default occurs.

Longstaff's and Schwartz's framework leads to the following results:

- The price of a risky bond is an increasing function of the default-risk variable X, that is the ratio V/K.
- The bond's value is a decreasing value of the factor w.
- The bond's value is a decreasing function of the maturity date T.
- The bond's value is a decreasing function of riskless interest rate r.

Furthermore they extended the above model for valuing risky floating rate payments. They calculated that the value of a risky floating rate payment is given by the following equation:

$$F\left(X, r, \tau, T\right) = P\left(X, r, T\right)R\left(r, \tau, T\right) + wD\left(r, T\right)G\left(X, r, \tau, T\right) \quad (2.8)$$

F : the value of a risky floating rate payment

$$R(r,\tau,T) = r\exp(-\beta\tau) + \left(\frac{a}{\beta} - \frac{\eta^2}{\beta^2}\right)(1-\exp(-\beta\tau)) + \left(\frac{\eta^2}{2\beta^2}\right)\exp(-\beta T)\left(\exp(\beta\tau) - \exp(-\beta\tau)\right)$$

$$G(X,r,\tau,T) = \lim_{n\to\infty} G(X,r,\tau,T,n)$$

$$G(X,r,\tau,T,n) = \sum_{i=1}^{n} q_i \frac{C(\tau, iT/n)}{S(iT/n)} M(iT/n, T)$$

$$C(\tau,T) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2}\right)\exp(-\beta\tau)\left(\exp(\beta\min(\tau,t)) - 1\right)$$

$$-\left(\frac{\eta^2}{2\beta^2}\right)\exp(-\beta\tau)\exp(-\beta\tau)\left(\exp(2\beta\min(\tau,t)) - 1\right)$$

The first term, $P(X,r,T)R(r,\tau,T)$, of the equation is the price of a risky discount bond times the expected value of the value r at time τ under the risk-neutral process. The second term adjusts for the correlation between r and X, through the term $C(\tau,T)$ which is the covariance of the value of r at time τ with the value of lnX at the time t of its first passage to zero.

For the floating rate payments they concluded:

- The value of a floating rate coupon payment can be an increasing function of the maturity date T.
- The value of a floating rate coupon payment can be an increasing function of riskless interest rate r.

Given the equation for fixed rate debt the implications for the credit spreads are the following:

- Credit spread is an increasing function of the factor w.
- Credit spread is a decreasing function of the riskless interest rate r.
- Credit spreads are monotone increasing for high rated bonds and humped shaped for low rating bonds.
- The credit spreads are an increasing function of the correlation between the assets returns and the changes in interest rates.
- Credit spreads increase as the variance of the firm's asset σ^2 increase.

In order to support their framework provided the following evidence. They collected monthly data for Moody's industrial, utility and railroad corporate bond yield averages for the period

1977 to 1992. Also they collected the corresponding yields for 10-year and 30-year Treasury bonds. Credit spreads were computed by taking the average of the 10-year and30-year Treasury yields that matches the maturity of the corporate yield average for that month, and then subtracting the Treasury average from the corporate yield.

The empirical evidence showed the following:

1. Credit spreads increase in both absolute and relative terms as the credit rating of the bond decreases.
2. The same result is applied for the standard deviation of the credit spread.
3. Bonds with the same credit rating but from different industries or sectors are not necessary to have similar credit spreads.
4. Credit spreads narrow as interest rates increase.
5. Credit spreads are negatively related to returns on the firm's assets or equity.
6. If the ratio X is hold fixed, the interest-rate sensitivity of credit spreads increases with the value of the correlation ϱ between the assets returns and the changes in interest rates.

2.4.3. Leland's and Toft's model

Leland and Toft (1996) developed their approach incorporating two more aspects:

1. They considered both the optimal capital structure and the maturity of the debt in order to examine the debt value.
2. They derived endogenous conditions under which can bankruptcy will be declared.

The underlying assumption of the value of the firm V is in line with Merton (1974) and can be described by the following continuous diffusion process:

$$\frac{dV}{V} = \left[\mu(V,t) - \delta\right]dt + \sigma dz \quad (2.9)$$

V : the unleveraged value

σ : the proportional volatility

$\mu(V,t)$: the total expected rate of return on asset value V

δ : the constant fraction of the value paid out to security holders

dz : the increment of a standard Brownian motion

Furthermore in line with Longstaff's and Schwartz's framework they assumed that there is a boundary value V_B for the firm at which financial distress occurs. They have proven that this value is determined endogenously and shown to be constant in rational expectations equilibrium. One important difference from the Longstaff's and Schwartz's framework is that the riskless interest rate is assumed to be constant.

Finally the firm continuously sells a constant principal amount of new debt with maturity of T years from issuance, which it will redeem at par upon maturity. New bond principal is issued at a rate $p = P/T$ per year, where P is the total principal value of all outstanding bonds. The same amount of principal will be retired when the previously-issued bond matures. Bonds with principal p pay a constant coupon rate $c = C/T$ per year, implying the total coupon paid by all outstanding bonds is C per year. So the total debt service payments are time independent and equal to $C + P/T$ per year.

Given the above framework Leland and Toft proposed that the value of all outstanding bonds can be expressed by the following equation:

$$D(V;V_B,T) = \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-a)V_B - \frac{C}{r}\right)J(T) \quad (2.10)$$

$D(V;V_B,T)$: the total value of debt, when debt of maturity T is issued

a: the fraction of firm asset value lost in bankruptcy

$(1-a)V_B$: the value that is distributed to bond holders

$$I(T) = \frac{1}{T}\int_0^T e^{-rT} F(t)\,dt$$

$$F(t) = N\left[h_1(t)\right] + \left(\frac{V}{V_B}\right)^{-2a} N\left[h_2(t)\right]$$

$$J(T) = \frac{1}{T}\int_0^T G(t)\,dt$$

$$G(t) = \left(\frac{V}{V_B}\right)^{-a+z} N\left[q_1(t)\right] + \left(\frac{V}{V_B}\right)^{-a-z} N\left[q_2(t)\right]$$

$$a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}$$

$$b = \ln\left(\frac{V}{V_B}\right)$$

$$z = \frac{\left[\left(a\sigma^2\right)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}$$

$$h_1(t)=\frac{-b-a\sigma^2 t}{\sigma\sqrt{t}}$$
$$h_2(t)=\frac{-b+a\sigma^2 t}{\sigma\sqrt{t}}$$
$$q_1(t)=\frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}$$
$$q_2(t)=\frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}$$

$N(\cdot)$: the cumulative standard normal distribution

In order to test the validity of their approach they considered the following parameters:

- Riskless interest rate r=7.5%
- Corporate tax rate τ=35%
- Bankruptcy cost fraction α=50%
- Asset risk σ=20%
- The amount the firm pays to security holders is δV, with δ=7%
- Initial asset value V=100

Their "empirical" findings were the following:

- When leverage is low, for any given maturity, the value of debt falls as σ or the riskless rate of interest r increases. However, debt value increases with σ and r when leverage is high.
- Low and intermediate leverage shows curved market value and newly issued bond and about to be redeemed bonds sell at par. Bonds with remaining maturity between 0 and T sell above par.
- For very risky bonds, while newly issued bonds and about to be redeemed bonds sell at par, bonds with short remaining maturity sell substantially above par, while bonds of longer maturity less than T sell below par. This behaviour reflects the interplay between high coupon rates, the likelihood of bankruptcy and time remaining to maturity.
- At the optimal leverage credit spreads are negligible for issuance maturities of 2 years or less, and rise gradually for longer maturities.
- For high leverage levels credit spreads are high, but decrease as issuance maturity T increases beyond 1 year.

- For moderate to high leverage levels credit spreads are distinctly humped. That means that intermediate term debt offers higher yield than either very short or very long term debt.
- For the firms that have low leverage credit spreads are low and increase with issuance maturity T.
- The increase in volatility σ^2 leads to an increase in credit spreads with greater impact to the medium term maturity debt.
- Credit spreads are a decreasing function of riskless rate of interest. Furthermore for newly issued debt a rise in riskless rate will tilt credit spreads – will increase credit spreads of shorter term debt, but decrease spreads for long term debt.
- Credit spreads are an increasing function of bankruptcy costs. The impact is greater for medium maturity debt.

Chapter 3: AN OVERVIEW OF EMPIRICAL EVIDENCE

3.1. Empirical Evidence in the Literature

Although the line of theoretical research that followed the Merton's approach is very rich and managed to address various aspects of pricing credit risks, the empirical testing of these models is quite limited.

Sarig and Warga (1989) provided evidence that yields on pure discounts corporate bonds compared to the yields of similar maturity pure discount US government bond are actually in line with Merton's framework. They collected data for 119 US zero coupon governments bonds and 137 corporate issues representing 42 different sectors from February 1985 to September 1989. The data were filtered in order:

1. To avoid using price estimates (for bonds that have not been traded for days or months).
2. To eliminate all cases in which a rating change occurred in the sample period.
3. To eliminate all cases in which the reported price was below (above) the price of zero coupon bonds of shorter (longer) maturity issued by the same firm.
4. To eliminate all cases in which the bond was economically callable.

Their methodology was the following. For each month the yield of a zero coupon government bond was subtracted from the yield of each zero coupon corporate bond with identical maturity. If no government bond with identical maturity existed, the yields on the bonds with maturities more closely bounding the corporate bond were interpolated to obtain the appropriate riskless zero coupon bond yield. These spreads then were averaged across bonds in a given month and then across time.

The most considerable feature was the close resemblance of the spreads to the Merton's framework. In more detail they found that the term structure of credit spreads is downward sloping for firms with leverage ratio more than 1, humped for firms with medium leverage ratio and upward sloping for low leverage firms.

Furthermore their empirical results indicated than for AAA, AA and A bonds the term structure of credit spreads was upward sloping. For medium rating bonds (BBB and BB) the term structure was humped and for B/C rated bonds the credit spread was clearly a decreasing function of maturity (downward sloping curve).

The main drawback of their analysis is derived from the fact that the analysis is based on a relative limited time series sample. This is inhibitory for deriving accurate conclusions as some gaps appeared in the credit spreads curve created from this sample. However, it must not be ignored that they provide some empirical evidence for the Merton's framework validity in pure discount corporate bonds.

Batten et al. (2005) used a daily sample of non-callable Australian dollar denominated Eurobonds to study the empirical implications of the Longstaff – Schwartz's (1995) model to explain the behaviour of actual and relative credit spreads. The actual spread is the difference between the yields of risky corporate bonds and riskless bonds and the relative spread is the ratio. They used daily data from 2 January 1995 to 31 August 1998 (954 observations) for bonds of four different maturities and three different ratings. Their empirical results are much in line with the Longstaff – Schwartz's (1995) framework. The Australian credit spreads are a decreasing function of the credit rating and an increasing function of maturity. Also, both the actual and relative spreads became less volatile as maturity increased. Furthermore credit spreads, both actual and relative) are negatively related both with changes at the proxy for the asset factor (All Ordinaries Index) and with changes at the proxy for the interest arte factor (Australian Government bond). Batten et al. (2005) concluded to results consistent to the Longstaff – Schwartz's (1995) model and that the tests for the credit spreads must be restricted to one using actual rather than relative spreads, as relative spreads by construction bring about a stronger credit spread – interest rate factor relation.

Jones, Mason and Rosenfeld (1983, 1984), tested the Merton's framework. In their first paper at 1983, they used monthly data from January 1977 to January 1981 for 177 bonds issued by 15 firms. They did not limit their research in simple capital structures i.e. including only bonds with principal redeem at maturity but also included bonds with callable provisions and sinking funds. Their analysis concluded that the model overpriced the longer maturity bonds and riskier bonds, i.e. bonds of firms with high asset variance. Furthermore, they examined the performance under various parameters and suggested that the performance of the model differentiated according to coupon characteristics. The model underpriced high coupon bonds, more than 7%, and overpriced low coupon bonds. Regarding seniority the model overpriced junior debt, but provided, on average, good approximation of senior debt. Under a similar framework they expanded their research on a second paper (1984). Here their sample consisted by monthly data from January 1975 to January 1981 for 305 bonds from 27 firms.

Again they allowed callable bonds and sinking fund provisions. They concluded that the Merton's framework does not provide any information regarding investment grade bonds (BBB and above). However, the model performed better in valuing non-investment grade bonds (below BBB). Also they found negative relation between the model errors and both variance and maturity. Of course the fact that their sample consisted by bonds with callable optionalities and sinking fund provisions, as well as firms with multiple bond outstanding, can drive objections about the validity of their results, as actually was happened by Fisher (1984).
Huang and Huang (2003) tested the contingent claim framework under a different approach. As they argued credit risk is only one of the factors that are incorporated in the observable yield spread between corporate bond and Treasuries. Furthermore in some special economic situations credit risk premium can be potentially very high. So given the above, their analysis was primarily focused on examining how much of the spread is due to the credit risk using empirically reasonable parameters. Their study focused in a series of models including Longstaff's and Schwartz's (1995), Leland's and Toft's (1996) and Collin – Dufresne's and Goldstein's (2001). Also they developed two new approaches incorporating business cycles parameters and jump – diffusions process for firm's value. Their results showed that in Longstaff's and Schwartz's model for investment grade bonds credit risk accounts for a small proportion of yield spread, although increasing with maturity. For non investment grade bonds credit risk accounts for much greater fraction of yield spread. Also they showed that the model performed better assuming constant than stochastic interest rates. Although Leland's and Toft's (1996) model generated higher credit spreads for both investment and non – investment grade bonds, but this due to the fact that a perpetual bond is considered. The important result for Leland's and Toft's model was that the endogenous default boundaries were much lower than the bonds face value, which is absolutely in line with the historical average default recovery rate. Finally the Collin – Dufresne's and Goldstein's model produced results very close to Longstaff's and Schwartz's framework. Incorporating a factor to capture cyclical market risk premium Huang and Huang found that for investment grade bonds a larger fraction of the observed yield can be explained by credit risk, although that value is relative small (lower than 25%). Moreover, by allowing jumps in firm's value did not lead to significant better results in relation to the Longstaff's and Schwartz's framework.
Lyden and Saraniti (2000) tested the performance of Merton's and Longstaff – Schwartz's models and concluded that, although Merton's model underestimated systematically the credit

spreads, dominated the Longstaff – Schwartz's model in predictive accuracy. They used Bridge Information Systems corporate bond data base and filtered the data, so that the final sample compromised by bonds with the following characteristics:

1. The principal is retired at maturity. Callable, convertible, floating-rate or sinkable bonds were excluded.
2. The selected bond was the only one outstanding for the issuing company. Issuers of multiple bonds were excluded.
3. The issuing firm had publicly traded common stocks. Firms with outstanding preferred shares were excluded.
4. If the issuing firm merged with another firm or issued additional bonds the initial issue was dropped from the sample.
5. REITS and other trust-type vehicles were omitted.

Their final sample was 56 bonds with quarterly data from 1990 to June 1999.

They defined model error as the observed market spread to the U.S. Treasury curve minus the spread predicted by each model. Furthermore in order to capture the different seniorities of debt they examined three different possible structures. Their empirical results were quite interesting. Regarding the Merton's model they found that predictions are quite low in relation to the observed ones. The structure that produced the lower spread was the one assumed equal priority in the event of default. The mean absolute error in that case was calculated at 83.10 basis points (or 87.99 basis points on a per-bond basis) while in other structures was 85.09 basis points at the one where it was assumed that priority was given to short-term debt (91.33 basis points on a per-bond basis); and 106.97 basis point at the one where it was assumed that seniority to all other debt (109.98 basis points on a per-bond basis). This is a very important issue as it is not the case in "real markets" where various seniority structures are met, and definitely it is not assumed equal priority in the event of default between equity holders and debt holders. Furthermore their findings indicated that the market overprices the debt of large firms in relation to Merton's estimation. On the other hand the longer term and higher coupon debt were under priced relative to model's predictions. Here it is very critical to stress the fact that Merton's model was referring to extremely simple capital structure (equity and zero-coupon bond). Of course that case as pointed also from Lyden and Saraniti it is impossible to be met in any firm.

In testing the Longstaff-Schwartz's model very important results did emerge. Incorporating in the model early default, stochastic default free interest rates, correlation sensitivity between assets value and interest rates and industry recovery rates not only does not improve the performance but produces even smaller spreads. With recovery rate 47.7% and allowing early default the model's mean absolute error was 108.73 even larger from the worst estimation of the original Merton's model (where seniority is assumed for all other forms of debt except bond, 106.97 basis points). Even changing the default boundary does not improve the results. Adding stochastic interest rates produced even larger mean absolute errors, 113.90 basis points. The same results occur when assuming correlation of assets value with interest rates. In both cases, positive and negative correlation, the mean absolute error from 108.73 basis points is increased to 115.56 basis points and 111.11 basis points respectively. Finally differentiating the recovery rates depending on industry does not provided better results again, 129.03 basis points mean absolute error. Given all the above analysis Lyden and Saraniti concluded that Longstaff and Schwartz model tends to magnify the errors, producing lower spreads when Merton's predictions are low and higher spreads when Merton's predictions are high. That irregularity maybe is linked to that fact that the sample companies had only one bond outstanding, so allowing early default had no important effect on bonds value.

Teixeira (2005) tested the performance of three structural models Merton's (1974), Leland (1994) and Fan and Sundaresan (2000). Teixeira incorporated in his study the industry effect in the performance of these models. Furthermore he tried to solve the problem that emerged from the assumption of the simple capital structure at Merton's model and the complicated capital structures met in the market, using the duration of the bonds as a proxy for the maturity of Merton zero coupon bond debt. His sample consisted of quarterly observations from 2001 to 2004 for 50 bonds from 6 U.S. non-financial industries with publicly traded stocks. In addiction his selection criteria were quite similar to the ones followed by Lyden and Saraniti (2000). So in the sample were included only coupon bonds with all principal retired at maturity and excluded bonds with provisions like convertible, callable or putable, as well as floating-rate bonds or with sinking fund provisions. Furthermore were excluded bonds with time to maturity less than one year and bonds with the same quote for more than two months.

His findings regarding Merton's model are in line with the findings of Lyden and Saraniti (2000). Merton's predictions overestimate bonds prices and underestimate the credit spread. This conclusion is applied not only in the total sample but also for the industry averages.

Comparing these results with the results from Leland model concluded that Merton model tends to overestimate bond prices more than Leland's model. This underestimation tends to be less for high rating categories, and stronger among short maturity bonds for Merton's model. Moreover, Teixeira research provided two very interesting results. First, that Merton's and Leland's models performed better when they were applied at more risky firms, as he found extreme underestimation for low volatile firms. Secondly, the sector effect seems to have relative importance as models seem to perform better on some sectors and worse on others.

Trying to explain the spread errors, the following five firm variables that have systematic relationship with the Merton's model are recognized, leverage, asset volatility, market-to-book value ratio and stock returns. On the other hand Leland's spread error has relationship with the leverage, asset volatility, market-to-book value ratio and size. Especially for volatility seems that the assumption of constant volatility is critical for the prediction power of the models. Regarding the bond variables he found that only maturity and yield to maturity play a role in explaining the spread errors.

In contradiction with the above empirical results Gemmill (2002) provided evidence supportive of Merton's model. Gemmill's empirical test has an important advantage that is derived from the sample he used. In particular his sample consisted of zero-coupon bonds issued by closed-end funds in the UK over the period February 1992 and April 2001. Firstly, the fact that he dealt with zero-coupon bonds allows a direct implementation of Merton's model. Secondly, closed-end funds are companies with much simpler capital structures than the most corporations. Gemmill's result can be divided into three periods. The early one where market yields were significantly less than model yields. The period between 1994 and 1999 where the yields were rather similar and the period after the March 2000 where the market yields significantly exceeded model yields. Furthermore in accordance to previous empirical tests he found that the model tends to underestimate the spread for low leverage companies and as well as for bonds close to maturity. As far as it concerns the difference between market and model spreads he found that the difference is a decreasing function of market volatility and interest rates and an increasing function of the closed-end fund premium.

Duffee (1998) investigated the relation between treasury yields and corporate bond yield spreads, both callable and non-callable. As he points out this relation conveys information about the covariation between riskless rate of interest and the market's perception of default risk. He used monthly quoted prices on investment grade corporate bonds from January 1985 to March 1995 to examine how yield spreads vary with changes in the level and the slope of the treasury term structure. Regarding the corporate bonds with noncallable provisions the results indicated that an increase in the three month treasury yield corresponds to decline in yield spreads. This relation seems to hold for every combination of maturity and credit rating and strengthens as credit quality falls. Furthermore, the relation between yield spreads and the slope of the treasury term structure is also generally negative, especially for longer maturity bonds. Examining the effects of coupons Duffee pointed out that yield spreads on lower grade, long maturity bonds are strongly inversely related to the slope of the treasury yield curve, holding the short end of the curve constant. On the other hand, the spreads on lower grade, short maturity bonds are less strongly related to this slope. The same negative relation regarding spreads is applied on callable bonds, where appears stronger, and again strengthens for lower quality bonds. In contrast with noncallable bonds, the changes in yield spreads even for shorter maturity bonds are driven by the long end of the treasury curve. Moreover, the sensitivity of a callable bond's spread to changes in treasury yields is positively related to the bond's price. The above empirical findings are much in line with the contingent claim valuation framework of debt that has been developed by Merton (1974).

Leake (2003) explored the relationship between credit spreads on sterling corporate bonds and the term structure of UK interest rates. Leake used daily bond prices quotes from January 1990 to December 1998 for investment rating categories – Moody's rating Aa, A – which were divided to three duration categories (0 to 4 years, 4 to 8 years, 8 to 12 years). Bonds with call features or other embedded options were excluded. His findings were weaker than those found by Longstaff and Schwartz (1995) for US corporate bond credit spreads and term structure of US interest rates. In particular, the series exhibits that credit spreads are wider for lower-rated bonds and longer duration bonds. Furthermore, examining the volatility of the credit spreads during the second and third quarters of 1994, he concluded that the possible presence of stale prices could not account for the increase in credit spread volatility during the 1994 bear market. Instead, investor uncertainty during this period was likely the cause the increase in credit spreads volatility. Regarding the relation between credit spreads and term structure of UK interest rates it was suggested that there was a weak negative relationship between Aa-rated, short duration credit spreads and the slope of the term structure of interest rates and a weak negative relationship between Aa-rated, medium duration credit spreads and the level of the term structure of interest rates. Concluding, he pointed out that the low sensitivity of credit spreads to changes in the term structure of interest rates suggests that credit spreads on investment grade sterling corporate bonds have been driven by factors other than default risk. Queries can be raised about his findings as the fact that incorporated both UK and non-UK companies in his sample might have influence the results. Moreover, the prices were quotes rather than actual trading prices.

Wei and Guo (1997) provided an empirical study regarding the performance of Longstaff – Schwartz and Merton model for pricing corporate debt and the credit spread. They used Eurodollars as risky debt and U.S. Treasury bills as riskless debt. The advantage of choosing Eurodollars is that they are actively traded, so the prices are actual trading actions rather than quotes. They used weekly prices (Thursday's price) for each week in 1992 with maturities 7 days, 1 month, 3 months, 6 months and 1 year. Both models parameters were estimated from the observed term structure of riskless interest rates and credit structures of risky bonds. Their findings are much in line with most of the previous research as Merton's model, especially allowing changing volatility, indicated superior performance in relation to Longstaff – Schwartz framework. In particular by assuming constant volatility in Merton's models and testing against Longstaff - Schwartz they found that both models indicate similar performance, as Merton's model provided better estimations for 7 days maturity, while Longstaff – Schwartz for 6 months period. In all other cases both models performed similarly. However when they incorporated changing volatility in Merton's model the performance of the model increased substantially as performed better in four out of five cases. As they point out the fact that in Merton's model the credit term structure converges to a constant as time to maturity goes to infinity in relation to the fact that Longstaff – Schwartz credit term structure converges to zero, gives a relative advantage to Merton's model performance.

Delianedis and Geske (2001) modified Merton's (1974) model to include payouts, recovery and taxes in order to investigate the components of credit spreads on corporate bonds. They used monthly data for about 500 investment grade firms (AAA, AA, A and BBB) from November 1991 to December 1998. Bonds with options features were excluded from their sample. As it was expected they found that the corporate credit spreads tend to increase with duration and as the quality of the bond decreases. Also, the volatility of the credit spreads generally increases as the rating decreases. An important finding that is related to the fact that their sample period included the Asian crisis in 1998, is that spread level are less volatile over the time period up until the crisis. Regarding the proportion of the default spread they concluded that default spread accounts from 5% to 22% of the credit spread. This means that there are other major components that determine the remainder of the credit spread. They studied the role of recovery rates, taxes, jumps, liquidity and market risk factors. Although they found that default spread is a decreasing function of the recovery rate, this account very little to the overall credit spread as the probability of actual default is very small for these rating categories. The difference in tax treatment increases significantly the measures spread but still remains an important proportion to be explained. Regarding the jumps in the firms value the found it should be a down jump about 20% once a year (or 14% twice) in order the credit spread to be explained only by the default risk. This means that the stock volatility would have to increase by more than 100% over the actual observed volatility. Finally regarding the market risk factors they find that increases in liquidity reduce significantly the credit spread without significantly altering the default spread, leading to an increase in the explanatory proportion of the default risk. When stock market volatility increases the impact is greater on the default spread in relation to credit spread, leading again to an increase in the explanatory proportion of the default risk. On the other hand, an increase in stock market returns reduces the explanatory proportion of the default risk as the default spread is reduced relative to the credit spread. Finally, the changes in risk free interest rate are relatively insignificant for AA, A and BBB, while have impact on AAA bonds, as they might be considered as default free.

In a latter paper Delianedis and Geske (2003) provided important evidence that both Merton's (1974) and Geske's (1977) frameworks do possess significant early information about credit rating migrations. Although their research is not straightly linked with the specific one their findings are an important indicator about the validity and performance of the above models. Under the Merton's (1974) model they calculated the risk neutral default probability (RNDP) on a debt obligation with maturity date at T and under Geske's (1977) framework they calculated the following probabilities: a) the total probability of defaulting at both short term and long term debt, b) the short probability of only defaulting on the short term debt and c) the forward probability held today of defaulting on the long term debt, conditional on not defaulting on the short term debt. According to their point if prices and spreads contain information about expected credit migration of corporate bonds, then risk neutral probabilities should also contain this information. They used quarterly data for the period 1988 – 1999. The number of firms was on average 668 per year. Their findings were in favour of the models as both models performed well for investment grade firms, which have lower default probabilities as for non – investment grade firms, which have higher default probabilities, as well as, for rating upgrades and downgrades. Furthermore, and most important both models produce default probabilities that are able to forecast which firms are more likely to experience a future rate migration. Here the Geske's (1977) model because incorporates multiple default opportunities produces a term structure of default probabilities. In particular the probability of defaulting short term rises above the forward default probability two months before the actual default. The above findings indicate that beside the criticism on these models regarding their ability to produce credit spreads that are in accordance with the market, appear to capture important information regarding the risk neutral default probabilities.

One of the most comprehensive empirical research was performed by Eom, Helwege and Huang (2002), who implement the following five structural models, Merton's (1974), Geske's (1977), Leland's and Toft (1996), Longstaff's and Schwartz (1995) and Collin – Dufresne's and Goldstein (2001). They used 182 non callable fixed coupon bond prices on the last trading day of each December from 1986 to 1997. Their results show that although all of the models have substantial spread prediction errors, these differ among them in both the sign and the magnitude. In particular, Merton's model generated spreads that were too small. This tendency was stronger for high rated and shorter maturity bonds. Also in some cases of non investment grade and long term bonds overestimated spreads were produced. What worth noticing here is that the incorporation in the model the bond's price implied volatility seems to improve the performance of the model. The Geske's model indicated similar performance with Merton's one, although it performed relative better at investment grade and short maturity bonds. In contrast, with the above models Leland's and Toft model result indicated a tendency for over predicting bond spreads. This tendency appeared on every version of the model. A possible case is the fact that the assumption that he firm can continuously sell a constant principal amount of new debt increases substantially the probability of default. Regarding the Longstaff's and Schwartz model their results were supportive to the model in relation to Merton's and Geske's, as the predicted spreads were relative higher. On the other hand the absolute spread errors are almost double those of the above under the same recovery rate. The model performed better for longer maturity bonds. For shorter maturity, up to 10 years, tend to produce either very high or very low spreads in relation to actual ones. Finally, Collin – Dufresne's and Goldstein model show a strong tendency for overestimating spreads, and like the Longstaff and Schwartz model produced extreme spreads for riskiest bonds. Their findings were quite important to the extent that, on one hand confirms the empirical findings regarding the ability of the models to provide an accurate estimation about the credit spread. However, they showed that the underprediction problem maybe is posed by the fact that default is considered to be driven only by the high leverage ratio, the high asset volatility and high payout ratios. Furthermore they suggested that an alternative than the Vasicek model for the interest rates may produce more accurate results, eg higher spreads. Supportive to the above finding is Simon (2005) who show that allowing for a more general term structure dynamics under Collin – Dufresne's and Goldstein framework, can improve the yield spreads that produced by the models under the contingent claim framework. Although the estimated spreads are still lower than the actual ones, are higher than usually found in most other existing research.

Bohn (2000) through his research concluded that adjusting the spread over U.S. Treasuries with a factor that will capture the non-credit component of the spread, e.g. liquidity premium, can provide considerable support to the contingent-claim or Merton's framework. In particular he examined 600,000 observations from bonds issued by approximately 2,000 U.S. corporations between June 1992 and January 1999. Using so large sample Bohn overwhelmed the problems that are related with the relevant small sample sizes that are appeared in similar research. On the other hand, his sample definitely does not meet the requirements of the contingent-claim's framework as they were included bonds with various provisions. Although, these provisions were taken into account by changing the method the spread is calculated it is very difficult to estimate their impact as their value changes over time depending on factors such as the prevailing interest rate, the rating of the company etc. Bohn tested three different alternative versions of the model, concluding that while it is hard to estimate efficiently both the market Sharpe ratio and the time scaling parameter, keeping the Sharpe ratio constant over time allows the changes in credit spreads to be driven by changes in credit quality. Moreover, one of his important findings is that the non-credit component of the spread appears to have his own independent structure. Adjusting for this spread improves the quality of the Merton's framework fitting these data. The term structure of credit spreads implied by his data is in line with Merton's framework, as higher credit rating firms demonstrated positively sloped term structure and lower – credit quality firms is appeared humped or downward sloping.

Here it must be stated that Fons (1987) used Aaa/ AAA rated yields to represent the default risk free rate. His argument was on one hand that there was no default by bonds originally issued with this rating in the previous fifteen years. Furthermore these bonds had the same tax treatment as the other corporate bonds. Finally, by not applying the U.S. Treasury yield as the default free rate isolated the fact that yield differentials may reflect at some proportion the liquidity and marketability factor. Through his research he tried to establish the relationship between the risk premium required by holders of low rated corporate bonds and the actual default rate. His findings indicated that the default rates implied by corporate bonds returns exceed the actual ones, concluding that there is a reward for bearing default risk to the holders of these bonds. Although his research in not directly related with the specific one the above conclusions provide some very important aspects about the estimations of a models parameters.

Very relevant about the power of the general contingent-claim framework to explain the credit spread changes are the findings of Collin-Dufresne, Goldstein and Martin (2001). Using as a starting point that structural models generate predictions for what the theoretical determinants of credit spreads (spot rate, yield curve, leverage, volatility, downward jump in firms' value, business climate) should be then the changes in these determinants should provide explanation about the observed changes in credit spreads. The used monthly observations of 688 bonds with no callable or puttable provisions and maturity more than four years issued by 261 different issuers, from July 1988 to December 1997. Although, all the variables are found both economically and statistically significant in explaining variations in individual firms' credit spread, they capture only around 25% of the variation. This implies that credit spreads contain a large systematic component that lies outside of the structural model framework. In order to empower their result the expand their regression by including additional explanatory variables (changes in liquidity, proxy for firm value process, non-linear effects, equity return systematic factors, economic state variables, leading effects of stocks on bonds). Again their results indicated that even with the addition of these variables the explanatory power of the model was increased to 34%, implying that credit spread changes of individual bonds are mostly driven by an aggregate factor. These findings are not in line with the structural model's framework, initiated by Merton, which support a relationship that credit spreads can be explained in relation to factors such as leverage, volatility and interest rates.

Anderson and Sundaresan (2000) introduced an extended model with the framework of Merton's (1974), Leland's (1994), Anderson and Sundaresan (1996) and Mella – Barral and Perraudin (1997). Their findings were supportive to the contingent – claim models as they concluded that incorporating endogenous default barriers can improve the performance of the

models. They used data for AAA, A, and BBB (S&P ratings) from August 1970 to December 1996. The models were tested under the BBB parameters. Regarding the Merton's model their result indicated the model in order to produce corporate yields that are consistent with the market implies extremely high volatility levels. Furthermore, the credit risk component of the spread accounts for only four basis points of the observed yield spread. The results referring to Leland's model and AST model (they named AST the new model that is a special case of Anderson and Sundaresan and Mella – Barral and Perraudin models), indicated that by allowing for the endogenous determination of the default barrier can lead to an improvement of the structural model. Both the above mentioned models produced spreads that correlate more highly, although the difference is not very large, with observed spreads in relation to Merton's model and under more realistic volatility parameter (especially for the AST model).
Arora, Bohn and Zhu (2005) performed an empirical test of two structural models and one reduced-form model. In particular they tested the original Merton's model, the Vasicek – Kealhofer model – that is the base for the Moody's KMV model – and the Hull and White model. As Bohn (2000) they argued that using U.S. treasury curve for risk-free interest rate might be a wrong benchmark and result the under-predicted spreads that found in other research. Instead they argued that the appropriate corporate default risk-free curve is close to the U.S. swap curve. They used daily price quotes of bonds that being issued from 542 firms from October 2000 to June 2004. As our research is referred to structural models the relevant results are discussed. Regarding the Merton's model their results were consistent with other research as indicated that the model systematically underpredicted the actual CDS spread. Furthermore, it has been shown that even using CDS spread, instead of U.S. treasury; the model underestimates the credit risk. The above tendency is present independently of the how risky is the firm (although the measured the risk on the basis of CDS spread, it can be seen as the analogy of the rating). Another worth mentioning result is that the model's performance worsens across larger firms. The Vasicek – Kealhofer model is far more realistic in relation to the original Merton's framework as it accommodates short-term liabilities, long-term liabilities, convertible debt, preferred equity and common equity. The results indicate that incorporating in the model more realistic parameters that are related primarily with the ability to capture the differences in various capital structures can produce more consistent results. The Vasicek – Kealhofer model outperformed Merton's model in all cases.

The fact that U.S. treasuries do not provide the appropriate default risk-free interest rate when testing structural models for corporate bonds is supported also by the findings of Ericsson, Reneby and Wang (2005). They argued that the bond yield spreads compensate not only for credit risk but also for liquidity or marketability risk. So it should not be surprising that structural models overprice corporate bonds or equivalently underpredict credit spreads. They used three structural models, Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000), and provided results that show that this is not the case when the models are applied for Credit Default Swap spread. In particular, their result regarding the performance of the models when they estimate bond spreads were in line with relevant research. All the three models underestimated the bond spreads in relation to the actual ones by more than 30%; Leland's estimated mean spread was 60 bps, Fan and Sundaresan 77 bps and Leland's and Toft 112 bps, while the actual mean spread was 168 bps. On the other hand, the underestimations were significantly reduced when the models were applied to CDS premia for Leland and Fan and Sundaresan, while Leland's and Toft model overestimated the CDS premia by 8 bps. Furthermore one of the most interesting of their results is that all structural models residual spreads are consistently 60 bps higher than residual CDS premia. This is supportive to the hypothesis that credit default swap contain less of the non-default component of the spread, suggesting that a possible shortcoming of the structural models is that they cannot take into account the illiquidity risk.

Regarding the validity of the structural models in predicting the changes in credit spreads strong evidence in favour of the models is presented by Avramov, Jostova and Philipov (2005). They used data for 2,375 U.S. fixed rate, with no equity or derivative features corporate bonds from September 1990 to January 2003. Their result indicate that structural model's variables can explain 67%, 54% and 35% of the total variation in credit spread changes in low – rated, middle – rated and high – rated bonds respectively. The above results are in contrast with the results of Jones, Mason and Rosenfeld (1984) who argued that actually structural model variables explain only a small fraction of credit risk.

Stein (2005) provided some very interesting insight on the incompleteness of Merton's framework. Although its study regarded the ability of Merton-type structural models to explain and predict default events, his finds can be very useful both as a general evaluation of Merton's framework but furthermore provide very interesting guidance for how this framework can improve its performance. As Stein argued Merton-based approach is based on two strong assumptions. Firstly, it is assumed that equity markets, on average, contain complete information about the credit quality of the firm and do not contain non-credit related information. Secondly, the Merton's framework is the correct one with which to complete decode the market information and translate it into credit evaluations. Stein suggested that additional variables increase the predictive power of the single factor Merton-based model. Even implementing graphical analysis he proved that adding a single financial ratio, such as ROA, can differentiate estimation about the default probabilities. Stein performed a cross-sectional analysis using 20 years data, from 1980 to 1999. Then he took 5-year cohorts of data starting in 1980 and estimated a series logistic regressions in which the Merton variable and ROA were regressed against a one year flag. This flag was set 1 if a company defaulted within one year of the observed variables 0 if it did not. That procedure produced a series of 16 cohorts on which regression equations were estimated. The results provided strong evidence that ROA adds significantly to the explanation of default, even in the presence of Merton variable. This result provides empirical support for the assertion that the multifactor model explains more the default behaviour than the pure Merton model. Furthermore it provides an intuition about how the performance of this model can be improved by adding another variable.

3.2. Contribution of the thesis to the literature

Although the line of theoretical research that followed the Merton's approach is very rich and managed to address various aspects of pricing credit risks, the empirical testing of these models is quite limited. The most important studies are Wei and Guo (1997), Lyden and Saraniti (2000), Teixeira (2005) and Eom, Helwege and Huang (2002).

The process we followed to determine the sample, is similar to the ones that were followed by Lyden and Saraniti (2000), Teixeira (2005) and Eom, Helwege and Huang (2002). To our knowledge it is the first time that the model is calibrated against these four alternatives. Furthermore it is important to state the fact that the historical implied volatility was used for equity.

The results from out study were in contrast with Lyden and Saraniti (2000) and Wei and Guo (1997) who argued that Merton's model dominates Longstaff and Schwartz in predictive accuracy; as Longstaff and Schwartz model revealed a very good performance.. Merton's and

Leland and Toft models perform on different directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads. Longstaff and Schwartz model predictive power is reflected to the predicted over actual credit spread ration that was greater than 35% for the majority of the companies.

One of the most comprehensive empirical research was performed by Eom, Helwege and Huang (2002), who implement the following five structural models, Merton's (1974), Geske's (1977), Leland's and Toft (1996), Longstaff's and Schwartz (1995) and Collin – Dufresne's and Goldstein (2001). They used 182 non callable fixed coupon bond prices on the last trading day of each December from 1986 to 1997. Their results show that although all of the models have substantial spread prediction errors, these differ among them in both the sign and the magnitude. In particular, Merton's model generated spreads that were too small. This tendency was stronger for high rated and shorter maturity bonds. Also in some cases of non investment grade and long term bonds overestimated spreads were produced. What worth noticing here is that the incorporation in the model the bond's price implied volatility seems to improve the performance of the model. The Geske's model indicated similar performance with Merton's one, although it performed relative better at investment grade and short maturity bonds. In contrast, with the above models Leland's and Toft model result indicated a tendency for over predicting bond spreads. This tendency appeared on every version of the model. A possible case is the fact that the assumption that he firm can continuously sell a constant principal amount of new debt increases substantially the probability of default. Regarding the Longstaff's and Schwartz model their results were supportive to the model in relation to Merton's and Geske's, as the predicted spreads were relative higher. On the other hand the absolute spread errors are almost double those of the above under the same recovery rate. The model performed better for longer maturity bonds. For shorter maturity, up to 10 years, tend to produce either very high or very low spreads in relation to actual ones. Finally, Collin – Dufresne's and Goldstein model show a strong tendency for overestimating spreads, and like the Longstaff and Schwartz model produced extreme spreads for riskiest bonds. Their findings were quite important to the extent that, on one hand confirms the empirical findings regarding the ability of the models to provide an accurate estimation about the credit spread. However, they showed that the underprediction problem maybe is posed by the fact that default is considered to be driven only by the high leverage ratio, the high asset volatility and high payout ratios. Furthermore they suggested that an alternative than the Vasicek model for the interest rates may produce more accurate results, eg higher spreads. Supportive to the above finding is Simon (2005) who show that allowing for a more general term structure dynamics under Collin – Dufresne's and Goldstein framework, can improve the yield spreads that produced by the models under the contingent claim framework. Although the estimated spreads are still lower than the actual ones, are higher than usually found in most other existing research.

Wei and Guo (1997) provided an empirical study regarding the performance of Longstaff – Schwartz and Merton model for pricing corporate debt and the credit spread. They used Eurodollars as risky debt and U.S. Treasury bills as riskless debt. The advantage of choosing Eurodollars is that they are actively traded, so the prices are actual trading actions rather than quotes. They used weekly prices (Thursday's price) for each week in 1992 with maturities 7 days, 1 month, 3 months, 6 months and 1 year. Both models parameters were estimated from the observed term structure of riskless interest rates and credit structures of risky bonds. Their findings are much in line with most of the previous research as Merton's model, especially allowing changing volatility, indicated superior performance in relation to Longstaff – Schwartz framework. In particular by assuming constant volatility in Merton's models and testing against Longstaff - Schwartz they found that both models indicate similar performance, as Merton's model provided better estimations for 7 days maturity, while Longstaff – Schwartz for 6 months period. In all other cases both models performed similarly. However when they incorporated changing volatility in Merton's model the performance of the model increased substantially as performed better in four out of five cases. As they point out the fact that in Merton's model the credit term structure converges to a constant as time to maturity goes to infinity in relation to the fact that Longstaff – Schwartz credit term structure converges to zero, gives a relative advantage to Merton's model performance.

Lyden and Saraniti (2000) tested the performance of Merton's and Longstaff – Schwartz's models and concluded that, although Merton's model underestimated systematically the credit spreads, dominated the Longstaff – Schwartz's model in predictive accuracy. Regarding the Merton's model they found that predictions are quite low in relation to the observed ones. Furthermore their findings indicated that the market overprices the debt of large firms in relation to Merton's estimation. On the other hand the longer term and higher coupon debt were under priced relative to model's predictions. Here it is very critical to stress the fact that Merton's model was referring to extremely simple capital structure (equity and zero-coupon bond). In testing the Longstaff-Schwartz's model important results did emerge. Incorporating in the model early default, stochastic default free interest rates, correlation sensitivity between assets value and interest rates and industry recovery rates not only does not improve the performance but produces even smaller spreads. Even changing the default boundary does not improve the results. Adding stochastic interest rates produced even larger mean absolute errors. The same results occur when assuming correlation of assets value with interest rates. Finally differentiating the recovery rates depending on industry does not provided better results again. Given all the above analysis Lyden and Saraniti concluded that Longstaff and Schwartz model tends to magnify the errors, producing lower spreads when Merton's predictions are low and higher spreads when Merton's predictions are high. That irregularity maybe is linked to that fact that the sample companies had only one bond outstanding, so allowing early default had no important effect on bonds value.

Teixeira (2005) tested the performance of three structural models Merton's (1974), Leland (1994) and Fan and Sundaresan (2000). Teixeira incorporated in his study the industry effect in the performance of these models. Furthermore he tried to solve the problem that emerged from the assumption of the simple capital structure at Merton's model and the complicated capital structures met in the market, using the duration of the bonds as a proxy for the maturity of Merton zero coupon bond debt. His sample consisted of quarterly observations from 2001 to 2004 for 50 bonds from 6 U.S. non-financial industries with publicly traded stocks. In addiction his selection criteria were quite similar to the ones followed by Lyden and Saraniti (2000). So in the sample were included only coupon bonds with all principal retired at maturity and excluded bonds with provisions like convertible, callable or putable, as well as floating-rate bonds or with sinking fund provisions. Furthermore were excluded bonds with time to maturity less than one year and bonds with the same quote for more than two months.

His findings regarding Merton's model are in line with the findings of Lyden and Saraniti (2000). Merton's predictions overestimate bonds prices and underestimate the credit spread. This conclusion is applied not only in the total sample but also for the industry averages. Comparing these results with the results from Leland model concluded that Merton model tends to overestimate bond prices more than Leland's model. This underestimation tends to be less for high rating categories, and stronger among short maturity bonds for Merton's model. Moreover, Teixeira research provided two very interesting results. First, that Merton's and Leland's models performed better when they were applied at more risky firms, as he found extreme underestimation for low volatile firms. Secondly, the sector effect seems to have relative importance as models seem to perform better on some sectors and worse on others.

Trying to explain the spread errors, the following five firm variables that have systematic relationship with the Merton's model are recognized, leverage, asset volatility, market-to-book value ratio and stock returns. On the other hand Leland's spread error has relationship with the leverage, asset volatility, market-to-book value ratio and size. Especially for volatility seems that the assumption of constant volatility is critical for the prediction power of the models. Regarding the bond variables he found that only maturity and yield to maturity play a role in explaining the spread errors.

The process we followed to determine the sample, is similar to the ones that were followed by Lyden and Saraniti (2000), Teixeira (2005) and Eom, Helwege and Huang (2002). To our knowledge it is the first time that the model is calibrated against these four alternatives. Furthermore it is important to state the fact that the historical implied volatility was used for equity.

The results from out study were in contrast with Lyden and Saraniti (2000) and Wei and Guo (1997) who argued that Merton's model dominates Longstaff and Schwartz in predictive accuracy; as Longstaff and Schwartz model revealed a very good performance.. Merton's and Leland and Toft models perform on different directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads. Longstaff and Schwartz model predictive power is reflected to the predicted over actual credit spread ration that was greater than 35% for the majority of the companies.

Chapter 4: AN EMPIRICAL COMPARISON OF MERTON, LELAND AND TOFT AND LONGSTAFF AND SCHWARTZ MODELS FOR CREDIT RISK

4.1. Data

The choice of structural models is based on the fact that this class of models assumes that the knowledge of information that relates to firm's assets and liabilities. In most situations, this knowledge leads to a predictable default time. In contrast, reduced form models assume that the market already has some complete or incomplete knowledge of the firm's condition. In most cases, this imperfect knowledge leads to an inaccessible default time. As such structural models require more primary information (that is available through Annual Reports and other Regulatory Fillings) and assume no assessment requirement of this information by the market.

The ideal implementation of Merton, Leland and Toft and Longstaff and Schwartz model requires zero coupon bonds that have been issued by corporations that have only one single class of debt outstanding. Unfortunately it is impossible to find corporations that satisfy the above restrictions, so comparisons were made using bonds that have reliable prices and straightforward cashflows. Furthermore the bonds are issued by corporations with relative simple capital structures. Corporations with complex capital structure raise doubts whether the pricing errors are due to the assumptions of the model or to their inefficiency to price the debt of corporations with complicated capital structure.

A Bloomberg search was performed using the following criteria for the creating of this sample: 1) US non – financial corporations, 2) only fixed or zero coupon bonds, 3) all the principal is retired at maturity (bullet bonds) 4) bonds with embedded optionalities like callable, convertible, putable are excluded, 5) floating-rate or sinkable bonds are excluded as well.

As a result, a list of 3,714 fixed or zero coupon bullet bonds in US Dollars that have been issued by US non-financial corporations[6] was produced. Next, the specific sample was filtered in order to include listed corporations with relative simple capital structure and traded bonds. As such, we aim for corporations that do not have no more than two bonds outstanding. The traded bonds were identified through the Trade Reporting and Compliance (TRAC) system.

[6] The sectors that have been included are Basic Materials, Communications, Consumer Cyclical, Consumer Non – Cyclical, Energy, Industrial, Technology and Utilities.

TRAC uses the Trade Reporting and Compliance Engine (TRACE) to research corporate trade data. TRACE data is disseminated to the public via the Bond Trade Dissemination Service (BTDS) data feed product. The Securities and Exchange Commission (SEC) had approved proposed rules that require National Association of Securities Dealers (NASD) members to report secondary market transactions in eligible fixed income securities to the NASD, and subject certain transaction reports to dissemination. TRACE enables regulators to oversee the corporate debt market and better detect misconduct while improving investor confidence in this market. The above filtration resulted a final sample of 22 firms with 27 bonds outstanding for the period from 1st January 1998 until 13th April 2006. Note that no data were available before 1998 while 2006 is the year before the recent credit crisis, so market liquidity and bid – offer spreads were relative tight. Table 4.1 lists such companies and the number of bullet bonds outstanding.

Table 4.1 – Final Sample January 1998 – April 2006

A/A	Company Name	Num. of Bullet Bonds Outstanding
1	GREAT ATLANTIC & PACIFIC	1
2	HUMANA INC	1
3	MILLIPORE CORP	1
4	POPE & TALBOT	2
5	SPRINT CORP	1
6	HARMAN INTL	1
7	NORDSTROM INC	2
8	NVR INC	1
9	OFFICE MAX INC	1
10	STAPLES INC	1
11	SEITEL INC	1
12	CARLISLE COS INC	2
13	CRANE CO.	1
14	INTL SHIPHOLDING	1
15	JLG INDUSTRIES	1
16	WORTHINGTON INDS	1
17	REYNOLDS & REYN	1
18	TEXAS INSTRUMENT	1
19	CLECO CORP	1
20	NICOR GAS	2
21	NISOURCE INC	1
22	SOUTHERN UNION	2

Data on bond features, prices and yields are taken from Bloomberg. The balance sheet and equity historical data for the above sample were provided from Datastream. Interest rate data are from Constant Maturity Treasury series as provided by the Federal Reserve. Table 4.2 presents the key bond features.

Table 4.2 – Bond Features January 1998 – April 2006

Name	Coupon	Issue Date	Maturity	Moody's Rating	S & P Rating	Amount Issued
GREAT ATLA & PAC	7.75	10/07/1997	15/04/2007	Caa1	B-	300,000,000
HUMANA INC	6.3	05/08/2003	01/08/2018	Baa3	BBB	300,000,000
MILLIPORE CORP	7.5	01/04/1997	01/04/2007	Baa3	BBB	100,000,000
POPE & TALBOT	8.375	02/06/1993	01/06/2013	Caa1	CCC+	75,000,000
POPE & TALBOT	8.375	02/10/2002	01/06/2013	Caa1	CCC+	60,000,000
SPRINT CORP	9.25	15/04/1992	15/04/2022	Baa2	A-	200,000,000
HARMAN INTL	7.32	01/07/1997	01/07/2007	Baa2	BBB+	150,000,000
NORDSTROM INC	5.625	20/01/1999	15/01/2009	Baa1	A	250,000,000
NORDSTROM INC	6.95	16/03/1998	15/03/2028	Baa1	A	300,000,000
NVR INC	5	17/06/2003	15/06/2010	Baa3	BBB-	200,000,000
OFFICE MAX INC	8.25	29/03/1999	15/03/2019	Ba2	B+	5,000,000
STAPLES INC	7.125	12/08/1997	15/08/2007	Baa2	BBB	200,000,000
SEITEL INC	11.75	08/02/2005	15/07/2011	B3	NA	193,000,000
CARLISLE COS INC	7.25	28/01/1997	15/01/2007	Baa2	BBB	150,000,000
CRANE CO.	6.75	21/09/1998	01/10/2006	Baa2	BBB	100,000,000
INTL SHIPHOLDING	7.75	27/03/1998	15/10/2007	B1	B-	110,000,000
JLG INDUSTRIES	8.25	08/09/2003	01/05/2008	B2	BB	125,000,000
WORTHINGTON INDS	7.125	24/05/1996	15/05/2006	Baa2	BBB	200,000,000
REYNOLDS & REYN	7	18/12/1996	15/12/2006	Ba1	BBB	100,000,000
TEXAS INSTRUMENT	8.75	01/04/1992	01/04/2007	A2	A	150,000,000
CLECO CORP	6.52	07/05/1999	15/05/2009	Baa1	BBB	50,000,000
NICOR GAS	6.58	25/02/1998	15/02/2028	Aa3	AA	50,000,000
NICOR GAS	6.58	25/02/1998	15/02/2028	Aaa	AAA	50,000,000
NISOURCE INC	3.628	01/11/2004	01/11/2006	Baa3	BBB	80,623,000
SOUTHERN UNION	7.6	31/01/1994	01/02/2024	Baa3	BBB	475,000,000
SOUTHERN UNION	8.25	03/11/1999	15/11/2029	Baa3	BBB	300,000,000

4.2. Parameter Estimation

4.2.1. Merton's Model

The next step is to determine and compute the relevant parameters. These parameters can be divided into three groups. The first group is related to firm specific factors. The second group is referred to bond - debt characteristics and the third group of parameters produce the default – free term structure.

Regarding the first group, the following parameters should be estimated: the value of the firm's assets, the leverage ratio, the payout ratio and the volatility.

The asset value at time t is estimated by the market value of equity plus the book value of the long term debt.

The leverage ratio l is calculated as

$$l = \frac{\text{Book Value of Debt}}{\text{Market Value of Equity} + \text{Book Value of Debt}}$$

The book value of debt is adjusted to take into account the Merton's zero – coupon bond face value.

Regarding the payout ratio, the dividend yield is used. That provides a good proxy as the coupon payments are incorporated to Merton's zero – coupon bond value.

A very important parameter in a structural form model is the asset return volatility that cannot be observed. In order to calculate the volatility of the assets two approaches have been applied. Under the first approach, the volatility is determined exogenously and is calculated as a function of the following parameters: the leverage ratio l, the implied volatility from call options[7], a proxy for the volatility of the debt and the correlation of returns between debt and equity. Here it must be stated that for four companies (NVR INC, SEITEL INC, INTL SHIPHOLDING, JLG INDUSTRIES) there was no data available regarding the historical implied volatility. In the above cases a 90 – day's window of historical volatility was used. Given the fact there are no listed traded options for the sample bonds, the historical volatility

[7] The data regarding the historical implied volatility based on call options were provided from Bloomberg. The historical implied volatility for each day is calculated as a weighted average of the three calls with strike price closest to the at – the – money strike.

of the traded debt was used as a proxy for the volatility of the debt. Also the correlation between the debt and the equity was calculated on historical returns. The volatility of the assets is estimated as follows

$$\sigma_{Assets} = \sqrt{(1-l)^2 \sigma_{Equity}^2 + l^2 \sigma_{Debt}^2 + 2\rho_{Equity,Debt}(1-l)l\sigma_{Equity}\sigma_{Debt}} \quad (4.1).$$

σ_{Assets} : is the volatility of the company assets.

σ_{Equity} : is the volatility of the company equity.

σ_{Debt} : is the volatility of company debt.

$\rho_{Equity,Debt}$: is the correlation between debt and equity.

l : is the leverage ratio.

The motivation to use this approach is based on the fact that if the firm's assets are funded both by equity and debt their volatility should depend upon both as well as on their correlation.

The second approach calculates the volatility by solving Merton's model. In the Merton's model the value of the equity is a call option on the firm's asset value with strike price equal to the face value of debt. The pay-off at maturity is $E_T = \max(V_T - B, 0)$, where V_T is the value of the firm at time T and B is the value of payment to bond holders. Using the Black – Scholes formula gives the value of the equity today as

$$E = Ve^{-i\tau}N(d_1) - Be^{-r\tau}N(d_2) \quad (4.2) \text{ where}$$

$$d_1 = \frac{\ln\left(\frac{V}{B}\right) + \left(r - i + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}$$

V : is the value of the firm.

B : is the value of the promised payment to bond holders.

τ : is the time until maturity.

r : is the instantaneous risk – free rate.

i : are any dividend payments.

σ_A : is the volatility of on the firm assets.

$N(\bullet)$: the cumulative standard normal distribution.

From Ito's lemma we know that the relationship between the equity volatility and the asset volatility is the following:

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_A V_0 \Leftrightarrow \sigma_E = \frac{\partial E}{\partial V}\frac{V_0}{E_0}\sigma_A \Leftrightarrow \sigma_E = N(d_1)\frac{V_0}{E_0}\sigma_A \quad (4.3)$$

The above equation can be solved numerically since all other parameters except σ_A are known. When we solve the above equation, σ_E is set to be equal to the equity implied volatility.

The second group of parameters is comprised by bond - debt characteristics, which include the face value of debt and its maturity. One of the basic assumptions under Merton's framework is that the company has a single homogenous class of debt outstanding and promises to pay to bond holders a total of B dollars on the terminal time T. However, all the companies in the sample, even they have as simple a capital structure as possible, have several kinds of debt. Furthermore the traded debt i.e. bonds, has coupons with all principal retired at maturity. In order to move from the available coupon bullet bonds to Merton's zero – coupon bonds the following methodology was followed. As it is already mentioned, most companies in our sample have one coupon bond outstanding. For these companies all the coupons and the principal are discounted to time zero, i.e. the issue date, using the relevant risk – free rate. Then the sum of the present values is compounded to the duration of the bond. The duration is used as a proxy for the maturity of the Merton's zero – coupon bond's debt. Given the definition of the duration, which takes into account the weighted average of the maturity of each coupon and the principal, this is quite reasonable[8]. For the companies that have two bonds outstanding the same process was applied but instead of duration it has been used the weighted average duration.

Having calculated the Merton's zero – coupon bond's value the face value of debt is calculated as the sum of the face value of the traded debt plus the non – traded debt. Here it should be stated that the non – traded debt is calculated as the sum of short – term debt plus the long – term debt and excludes accounts payable, minority interests and any other liabilities that appear under total liabilities on balance sheet. Above it was explained how the duration of the traded debt is used as a proxy for the maturity of the Merton's zero – coupon bond. As far as the maturity for the total debt is concerned and since it is very hard to identify the maturity of each individual category of debt outstanding, two different assumptions were applied. The first assumes that whole debt has average maturity equal to the maturity of the Merton's zero – coupon bond. The second approach divides the debt into traded debt, short – term debt and long – term debt. Regarding the traded debt the duration is used as a proxy for the maturity. For the short – term debt it is assumed that matures in one years' time. Finally, a proxy should be determined for the weighted average maturity of the long – term debt. It is assumed that the long – term debt has a weighted average maturity equal to the corporate bond average maturity as it is published by Thomson Financial[9]. Having established a proxy for the maturity of each category of debt the maturity of the debt is calculated as the weighted average maturity of the three categories of debt.

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8 Duration is defined as

$$D=\frac{\frac{C}{(1+Y)}+\frac{2C}{(1+Y)^2}+\ldots+\frac{(n-1)C}{(1+Y)^{n-1}}+\frac{n(C+100)}{(1+Y)^n}}{\frac{C}{(1+Y)}+\frac{C}{(1+Y)^2}+\ldots+\frac{C}{(1+Y)^{n-1}}+\frac{(C+100)}{(1+Y)^n}}\Rightarrow D=\frac{\frac{C}{(1+Y)}+\frac{2C}{(1+Y)^2}+\ldots+\frac{(n-1)C}{(1+Y)^{n-1}}+\frac{n(C+100)}{(1+Y)^n}}{P}$$

. (Professor's John Hatgioannides Notes on Fixed Income)

9 Corporate Bond Average Maturity includes all non – convertible corporate debt, MTNs and Yankee bonds, but excludes all issues with maturities of one year or less, CDs and federal and agency debt. It is provided by Thomson Financial.

Once all the necessary parameters have been estimated the implementation of Merton's model is straight forward. The value of defaultable debt is given by the solution of the following differential equation:

$$\frac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 D}{\partial V^2} + rV\frac{\partial D}{\partial V} - rD - \frac{\partial D}{\partial \tau} = 0$$

where $D = \min[V, B]$ and the boundary conditions $D[0,\tau] = E[0,\tau] = 0$ and $\frac{D[V,\tau]}{V} \leq 1$.

Under this framework equity is a call option on the value of the firm with strike price the face value of debt. The calculation of the equity and debt value can be calculated by the Black – Scholes – Merton (1973) formulae:

$$\begin{aligned} E &= Ve^{-i\tau}N(d_1) - Be^{-r\tau}N(d_2) \\ D &= Ve^{-i\tau}N(-d_1) + Be^{-r\tau}N(d_2) \end{aligned} \qquad (4.4)$$

where

V: the value of the firm
B: the value of the promised payment to bond holders
σ^2: the instantaneous variance of the return on the firm per unit time

$$d_1 = \frac{\ln\left(\frac{V}{B}\right) + \left(r - i + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$N(\bullet)$ the cumulative standard normal distribution.

Once the value of the debt is calculated the yield to maturity is computed as

$$ytm = -\frac{\ln\left(D/B\right)}{T},$$

where D is the debt value and B is the value of the promised payments to bond holders.

The credit spread is computed by subtracting the predicting bond yield from the yield of the risk – free bond with the same maturity.

4.2.2. Leland and Toft Model

Leland and Toft (1996) considered both the optimal capital structure and the maturity of the debt in order to examine the debt value. Furthermore, they derived endogenous conditions under which bankruptcy will be declared. Under their framework, the firm continuously sells a constant principal amount of new debt with maturity of T years from issuance, which it will redeem at par upon maturity. New bond principal is issued at a rate $p = P/T$ per year, where P is the total principal value of all outstanding bonds. The same amount of principal will be retired when the previously-issued bond matures. Bonds with principal p pay a constant coupon rate $c = C/T$ per year, implying the total coupon paid by all outstanding bonds is C per year. So the total debt service payments are time independent and equal to $C + P/T$ per year. If default occurs bondholders receive only a fraction of the firm's asset value.

The implementation of the Leland and Toft model assumes additional assumptions for the coupon payment, the payout ratio, the corporate tax rate and the bankruptcy cost. For the volatility estimation of the firm's asset we use the same values that were used for the implementation of Merton's model. Regarding the coupon payments the model is implemented under two different assumptions. The first is the implementation of the model assuming that the coupon is issued over a face value of debt that equals to $p = P/T$, where P is the total principal value of all outstanding debt, considering non traded debt and traded debt together. That approach requires determining the value of the perpetual coupon payments. Initially we calculate the 30 year annuity rate by discounting the average coupon of all bonds in the sample ($7.369) and using principal at maturity $100, using the constant maturity Treasury series. That method is also used in Teixeira (2005). Once this rate is calculated the perpetual coupon payment is calculated by solving the equation:

$Be^{-rt} = \dfrac{c(t)}{r_{annuity}}$ (4.5), where B is the total debt of the firm.

In order to implement this approach the same proxy – the weighted average maturity of short – term, long – term and traded debt – that was used in the implementation of the Merton's model is used.

The second assumption is that the face value of debt is equal to the face value of traded debt and the coupon payments are equal to the coupon payments of the outstanding bond.

The payout ratio measures the firm's payments to equity holders and bond holders. In order to obtain a good proxy for that parameter the weighted average of the bond's coupon payment and the firm's equity payout ratio is calculated. The weights are determined using the leverage ratio, as it was calculated in Merton's model.

The bankruptcy cost is determined as $(1-\text{recovery rate})$. The recovery rates were obtained from the Moody's Report "Default and Recovery Rates of Corporate Bond Issuers, 1920 – 2005". Each year the recovery rate used is the Annual Average Default Bond Recovery Rate for all corporate bonds. The corporate tax rate is assumed to be flat all years at 35%.

Given the above assumptions and in order to calculate the value of the unleveraged firm the following equation is solved numerically:

$$E(t)=\upsilon(t)-D(t) \quad (4.6)$$

where

$\upsilon(t)$ is the total market value of the firm

$D(t)$ is the total value of debt.

The analytic formulas for the calculation of the total market value of firm and the total value of debt are the equations (7) and (8) on the original paper of Leland and Toft (1996). The value of outstanding bonds is:

$$D(V;V_B,T)=\frac{C}{r}+\left(P-\frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT}-I(T)\right)+\left((1-a)V_B-\frac{C}{r}\right)J(T) \quad (4.7)$$

$D(V;V_B,T)$: the value of all outstanding bonds.

P : the total principal value of all outstanding bonds.

C : the value of total coupon paid by all outstanding bonds.

V_B : the endogenously – determined bankruptcy asset level.

r : is the instantaneous risk – free rate.

α : is the fraction of firm asset value lost in bankruptcy.

T : maturity of the debt.

$$I(T)=\frac{1}{T}\int_0^T e^{-rt}F(t)dt$$

$$J(T)=\frac{1}{T}\int_0^T G(t)dt$$

$$F(t) = N[h_1(t)] + \left(\frac{V}{V_B}\right)^{-2a} N[h_2(t)]$$

$$G(t) = \left(\frac{V}{V_B}\right)^{-a+z} N[q_1(t)] + \left(\frac{V}{V_B}\right)^{-a-z} N[q_2(t)]$$

$$q_1(t) = \frac{(-b - z\sigma^2 t)}{\sigma\sqrt{t}}; q_2(t) = \frac{(-b + z\sigma^2 t)}{\sigma\sqrt{t}}$$

$$h_1(t) = \frac{(-b - a\sigma^2 t)}{\sigma\sqrt{t}}; h_2(t) = \frac{(-b + a\sigma^2 t)}{\sigma\sqrt{t}}$$

$$a = \frac{(r - \delta - (\sigma^2/2))}{\sigma^2}; b = \ln\left(\frac{V}{V_B}\right); z = \frac{\left[(a\sigma^2)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}$$

$N(\bullet)$: the cumulative standard normal distribution.

The total value of the firm equals the asset value plus the value of tax benefits, less the value of bankruptcy costs, over the infinite horizon. As a result total firm value will be given by:

$$\upsilon(V;V_B) = V + \frac{\tau C}{r}\left(1 - \left(\frac{V}{V_B}\right)^{-x}\right) - aV_B\left(\frac{V}{V_B}\right)^{-x} \quad (4.8)$$

υ: total market value of the firm.

τ : is the corporate tax rate.

C : the value of total coupon paid by all outstanding bonds.

V_B : the endogenously – determined bankruptcy asset level.

r : is the instantaneous risk – free rate.

α : is the fraction of firm asset value lost in bankruptcy.

$x = a + z$.

$$z = \frac{\left[(a\sigma^2)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}$$

σ^2 : is the volatility of firm's assets.

The value of the defaultable bond is given by the following equation:

$$d(t)=\frac{c(t)}{r}+e^{-rt}\left[p(t)-\frac{c(t)}{r}\right][1-F(t)]+\left[\rho(t)V_B-\frac{c(t)}{r}\right]G(t) \quad (4.9)$$

$p(t)$: is the principal of the bond.

$c(t)$: is the constant coupon flow.

V_B : the endogenously – determined bankruptcy asset level.

$\rho(t)$: is the fraction of asset value V_B which debt of maturity t receives in the event of bankruptcy.

r : is the instantaneous risk – free rate.

$$F(t)=N[h_1(t)]+\left(\frac{V}{V_B}\right)^{-2a}N[h_2(t)]$$

$$G(t)=\left(\frac{V}{V_B}\right)^{-a+z}N[q_1(t)]+\left(\frac{V}{V_B}\right)^{-a-z}N[q_2(t)]$$

$$q_1(t)=\frac{\left(-b-z\sigma^2 t\right)}{\sigma\sqrt{t}};\ q_2(t)=\frac{\left(-b+z\sigma^2 t\right)}{\sigma\sqrt{t}}$$

$$h_1(t)=\frac{\left(-b-a\sigma^2 t\right)}{\sigma\sqrt{t}};\ h_2(t)=\frac{\left(-b+a\sigma^2 t\right)}{\sigma\sqrt{t}}$$

$$a=\frac{\left(r-\delta-\left(\sigma^2/2\right)\right)}{\sigma^2}$$

$$b=\ln\left(\frac{V}{V_B}\right)$$

$$z=\frac{\left[\left(a\sigma^2\right)^2+2r\sigma^2\right]^{1/2}}{\sigma^2}$$

σ^2: is the volatility of firm's assets.

δ: is the constant proportional cash flow generated by the assets and distributed to security holders.

The equilibrium default – triggering asset value V_B is given by solving the equation $\left.\frac{\partial E}{\partial V}\right|_{V=V_B}=0$ where E is the value of equity.

4.2.3. Longstaff and Schwartz Model

The Longstaff and Schwartz model considers the valuation of the corporate bonds under stochastic interest rates. The interest rate dynamics are described by the Vasicek (1977) model. Under their framework there is a boundary value K for the firm at which financial distress occurs. As long as value V is greater than K, the firm continues to be able to meet its contractual obligations. If V reaches K, the firm immediately enters to financial distress and defaults on all of its obligations.

Similarly to Merton's model the value of the assets is calculated as the sum the market value of equity plus the book value of the long term debt. As in Leland and Toft, for the volatility estimation of the firm's asset we use the same values that were used for the implementation of Merton's model.

Longstaff and Schwartz use a default-risk variable X that is the ratio V/K, where V is the value of the assets and K is the face value of debt. The face value of debt is calculated as the sum of long term and short term debt.

The recovery rates, as in Leland and Toft, were obtained from Moody's Report "Default and Recovery Rates of Corporate Bond Issuers, 1920 – 2005". Each year the recovery rate is used is the Annual Average Default Bond Recovery Rate for all corporate bonds.

As it is stated above the specific model incorporates stochastic interest rates, which follow the dynamics of the Vasicek (1977) model. In order to estimate the parameters the model is calibrated every quarter on the treasury curve. The treasury curve was constructed using constant maturity Treasury series that is being published on daily basis by the Federal Reserve and provides the yields for following maturities; 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year . Cubic spline interpolation was used to interpolate between the fixed points.

Another parameter that needs to be determined is the instantaneous correlation between assets and interest rates. As this is not an observable parameter a proxy should be determined. The instantaneous correlation is calculated using the daily returns of the 6 month treasury rate and equity prices over a rolling window of 180 days.

Once the parameters are estimated the value of a risky discount bond is calculated as:

$$P(X,r,T) = D(r,T) - wD(r,T)Q(X,r,T) \quad (4.10)$$

P: the value of a risky discount bond

X: the ratio V/K

r: the riskless interest rate

T: the maturity date

$D(r,T) = \exp(A(T) - B(T)r)$: the value of a riskless discount bond in Vasicek (1977)

$$A(T) = \left(\frac{\eta^2}{2\beta^2} - \frac{\alpha}{\beta}\right)T + \left(\frac{\eta^2}{\beta^3} - \frac{\alpha}{\beta^2}\right)(\exp(-\beta T) - 1) - \left(\frac{\eta^2}{4\beta^3}\right)(\exp(-2\beta T) - 1)$$

$$B(T) = \frac{1 - \exp(-\beta T)}{\beta}$$

$$Q(X,r,T) = \lim_{n\to\infty} Q(X,r,T,n)$$

$$Q(X,r,T,n) = \sum_{i=1}^{n} q_i$$

$$q_1 = N(a_1)$$

$$q_i = N(a_i) - \sum_{j=1}^{i-1} q_j N(b_{ij}), i = 2,3,\ldots,n$$

$N(\cdot)$: the cumulative standard normal distribution function

r: is the short – term risk – free rate.

σ^2: is the volatility of firm's assets.

ζ, β, η: are constants.

ρ: is the instantaneous correlation between assets and interest rates.

$$a_i = \frac{-\ln X - M(iT/n, T)}{\sqrt{S(iT/n)}}$$

$$b_{ij} = \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n) - S(jT/n)}}$$

$$M(t,T) = \left(\frac{a - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2}\right)t + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3}\right)\exp(-\beta \mathrm{T})(\exp(\beta \mathrm{T}) - 1) +$$

$$\left(\frac{r}{\beta} - \frac{a}{\beta^2} + \frac{\eta^2}{\beta^3}\right)(1 - \exp(-\beta t)) - \left(\frac{\eta^2}{2\beta^3}\right)\exp(-bT)(1 - \exp(-\beta t))$$

$$S(t) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right)t - \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right)(1 - \exp(-\beta \mathrm{T})) + \left(\frac{\eta^2}{2\beta^3}\right)(1 - \exp(-2\beta t))$$

The first term $D(r,T)$ in equation (4.10) represents the value that the bond would have if it were riskless. The second term $wD(r,T)Q(X,r,T)$ represents a discount for the default risk of the bond. The first component $wD(r,T)$ is the present value of the writedown on the bond in the event of a default. The second component $Q(X,r,T)$ is the probability – under the risk-neutral measure – that a default occurs.

The third group of parameters is the default – free term structure. In order to estimate the default – free interest rate curve constant maturity Treasury yield data were used. The constant maturity Treasury series is being published on a daily basis by the Federal Reserve and provides the yields for following maturities; 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year . Cubic spline interpolation was used to construct the daily default – free interest rate curve.

4.3. Regression Model

In order to understand the performance of the models we run a regression with the spread as the dependent variable. The following five independent variables where used: (i) the ratio V/K, where V is the value of the assets and K is the face value of debt, (ii) the duration of the debt, (iii) the implied volatility of the equity, (iv) the overnight FED rate, and (v) the S&P 500 Index. Clearly by using the V/K ratio we are able to capture both the value of the assets and the leverage ratio. The reason that the equity implied volatility was used instead of the asset volatility is related to the fact that equity implied volatility is an observable parameter whether the volatility of the assets is an unobservable parameter and the methodology of calculation might lead to different results. The overnight FED rate is included in order to capture the level of the default – free rate curve. The S & P 500 is used capture any non – systematic factors that related to the business cycle of the economy. The results are presented in Tables 4.7 and 4.8.

Table 4.3 – Adjusted R-Square, Standard Error and ANOVA statistic

A/A	Company Name	Adjusted R^2	Standard Error	F - statistic	F-significance
1	CARLISLE COS INC	0.525	0.281	3.879	0.044
2	CLECO CORP	0.751	0.148	1079.511	0.000
3	CRANE CO.	N/A	N/A	N/A	N/A
4	GREAT ATLANTIC & PACIFIC	0.630	0.318	106.568	0.000
5	HARMAN INTL	0.943	0.116	4306.031	0.000
6	HUMANA INC	0.682	0.072	57.170	0.000
7	INTL SHIPHOLDING	0.127	0.219	2.046	0.100
8	JLG INDUSTRIES	0.777	0.147	27.438	0.000
9	MILLIPORE CORP	N/A	N/A	N/A	N/A
10	NICOR GAS	0.832	0.113	890.566	0.000
11	NISOURCE INC	0.094	0.399	1.685	0.171
12	NORDSTROM INC	0.419	0.105	29.998	0.000
13	NVR INC	0.127	0.131	5.233	0.000
14	OFFICE MAX INC	0.520	0.161	161.853	0.000
15	POPE & TALBOT	0.914	0.122	178.453	0.000
16	REYNOLDS & REYN	N/A	N/A	N/A	N/A
17	SEITEL INC	0.865	0.059	104.621	0.000
18	SOUTHERN UNION	0.554	0.077	23.096	0.000
19	SPRINT CORP	0.336	0.101	10.807	0.000
20	STAPLES INC	0.046	0.489	2.090	0.072
21	TEXAS INSTRUMENT	0.899	0.163	3374.293	0.000
22	WORTHINGTON INDS	0.053	0.615	1.535	0.199

Table 4.4 – Regression Results[10]

A/A	Company Name	Intercept	V/K =X	Implied Volatility	O/N Rate	S&P 500	Duration
1	CARLISLE COS INC	29.309	-3.234	2.653	1.395	-4.862	-0.372
		50.462	2.139	1.312	0.867	6.660	1.367
		0.581	-1.512	2.022	1.609	-0.730	-0.272
2	CLECO CORP	4.522	-0.297	0.255	-0.013	-0.195	0.815
		0.392	0.027	0.019	0.010	0.057	0.015
		11.533	-10.873	13.637	-1.220	-3.412	54.925
3	CRANE CO.	N/A	N/A	N/A	N/A	N/A	N/A
4	GREAT ATLANTIC & PACIFIC	40.978	-1.038	-0.190	-2.053	-2.916	-2.533
		9.395	0.208	0.239	0.367	1.161	0.313
		4.362	-4.999	-0.794	-5.596	-2.512	-8.102
5	HARMAN INTL	15.391	-0.368	0.128	0.143	-1.604	0.523
		0.256	0.023	0.024	0.012	0.042	0.025
		60.105	-15.753	5.386	12.142	-38.039	20.718
6	HUMANA INC	15.548	0.112	0.021	-0.634	-0.101	-2.933
		3.953	0.097	0.083	0.066	0.463	0.605
		3.933	1.158	0.255	-9.580	-0.217	-4.846
7	INTL SHIPHOLDING	56.453	-1.577	0.415	0.483	-7.506	-1.168
		20.257	2.325	0.180	1.163	2.756	1.512
		2.787	-0.678	2.307	0.415	-2.723	-0.772
8	JLG INDUSTRIES	27.039	0.310	0.994	1.710	-5.473	2.787
		12.544	0.206	0.168	0.596	1.560	0.997
		2.155	1.504	5.910	2.869	-3.508	2.794
9	MILLIPORE CORP	N/A	N/A	N/A	N/A	N/A	N/A
10	NICOR GAS	24.885	0.298	0.297	-0.230	-1.006	-5.087
		0.844	0.056	0.030	0.010	0.067	0.227
		29.470	5.338	9.946	-23.611	-14.956	-22.397
11	NISOURCE INC	14.282	1.125	1.167	0.481	-2.416	0.380
		38.092	5.532	0.809	2.150	4.464	1.820
		0.375	0.203	1.442	0.224	-0.541	0.209
12	NORDSTROM INC	40.363	0.161	0.115	0.030	-3.372	-5.705
		4.211	0.056	0.083	0.067	0.455	0.658
		9.584	2.856	1.383	0.444	-7.411	-8.665
13	NVR INC	15.773	0.152	-0.029	-0.357	-0.870	-1.951
		4.236	0.068	0.065	0.227	0.609	0.814
		3.724	2.233	-0.449	-1.574	-1.428	-2.396
14	OFFICE MAX INC	10.382	-1.071	0.133	-0.096	-1.380	2.765
		0.776	0.096	0.046	0.024	0.142	0.220
		13.381	-11.170	2.858	-3.970	-9.721	12.560
15	POPE & TALBOT	5.489	-0.425	0.008	0.077	1.196	-4.621
		4.965	0.375	0.039	0.338	0.685	0.683
		1.105	-1.133	0.201	0.229	1.747	-6.766
16	REYNOLDS & REYN	N/A	N/A	N/A	N/A	N/A	N/A
17	SEITEL INC	5.000	-1.168	0.524	0.221	0.214	-1.667
		4.325	0.140	0.118	0.182	0.501	0.430
		1.156	-8.355	4.425	1.216	0.426	-3.873
18	SOUTHERN UNION	14.468	-0.436	-0.033	-0.111	0.105	-3.717
		3.444	0.140	0.075	0.096	0.431	0.853
		4.201	-3.116	-0.433	-1.167	0.243	-4.357
19	SPRINT CORP	19.568	-0.291	-0.030	-0.250	-0.585	-3.602
		4.181	0.170	0.068	0.088	0.504	0.805
		4.681	-1.713	-0.441	-2.852	-1.161	-4.477
20	STAPLES INC	8.083	-0.545	-1.004	-1.201	1.277	-0.834
		22.272	0.402	0.593	0.784	3.027	1.171
		0.363	-1.355	-1.693	-1.531	0.422	-0.713
21	TEXAS INSTRUMENT	2.737	-0.355	0.304	0.118	0.091	0.728
		0.407	0.012	0.024	0.012	0.066	0.014
		6.721	-29.525	12.889	10.017	1.363	51.783
22	WORTHINGTON INDS	34.218	2.661	-3.264	0.778	-3.820	-0.007
		36.828	1.161	1.852	0.996	4.950	0.349
		0.929	2.292	-1.763	0.781	-0.772	-0.020

[10] The first number is the coefficient, the second number is the standard error and the third number is the t – statistic.

In most cases, the five independent variables provide a very good explanation of the behaviour of the credit spread, although the significance of each parameter does vary.

4.4. Empirical Results

The aim of this section is to present and discuss the credit spread predicted by Merton, Leland and Toft and Longstaff and Schwartz models. Tables 4.3 – 4.6 show the results found under each one of the four cases that have been examined for each model (exogenous and model determined asset volatility, actual maturity and adjusted maturity).

Table 4.5 – Exogenous Volatility and Adjusted Maturity

		MERTON'S CS			ACTUAL CS			M CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	0.011	0.011	0.011	112.564	69.798	99.900	0.014%	0.007%
2	CLECO CORP	68.283	91.030	29.303	159.040	55.425	150.600	40.922%	18.846%
3	CRANE CO.	267.315	138.079	318.171	66.771	12.700	69.400	422.735%	478.453%
4	GREAT ATLANTIC & PACIFIC	264.258	196.147	162.455	400.192	213.144	376.100	63.277%	64.209%
5	HARMAN INTL	20.532	41.013	0.302	122.326	83.567	81.600	9.564%	0.468%
6	HUMANA INC	3.033	6.082	1.406	171.363	33.744	155.000	1.578%	0.885%
7	INTL SHIPHOLDING	547.154	36.271	547.883	299.165	73.434	290.900	192.299%	188.697%
8	JLG INDUSTRIES	22.430	34.107	0.287	172.664	54.000	159.600	10.257%	0.200%
9	MILLIPORE CORP	1.090	1.541	0.543	143.900	134.591	87.450	1.434%	0.534%
10	NICOR GAS	47.604	44.688	30.267	261.626	87.962	282.325	17.645%	13.427%
11	NISOURCE INC	351.796	36.314	357.725	46.318	61.883	37.850	816.084%	526.850%
12	NORDSTROM INC	3.197	5.618	0.930	144.326	17.891	144.084	2.160%	0.650%
13	NVR INC	0.058	0.111	0.000	147.937	20.885	143.300	0.043%	0.000%
14	OFFICE MAX INC	36.095	22.697	30.652	352.183	90.620	354.000	11.138%	8.254%
15	POPE & TALBOT	402.086	187.707	385.309	801.859	304.854	820.180	54.702%	49.927%
16	REYNOLDS & REYN	0.167	0.337	0.006	115.792	70.811	95.050	0.241%	0.004%
17	SEITEL INC	16.507	10.377	13.673	513.579	80.645	512.950	3.104%	3.608%
18	SOUTHERN UNION	13.533	9.669	10.478	218.010	32.283	208.829	6.142%	5.231%
19	SPRINT CORP	8.314	10.123	6.494	244.580	36.447	241.950	3.219%	2.498%
20	STAPLES INC	0.001	0.002	0.000	73.761	56.152	57.600	0.001%	0.000%
21	TEXAS INSTRUMENT	15.433	52.610	0.068	95.512	61.932	118.900	11.272%	0.062%
22	WORTHINGTON INDS	0.643	0.928	0.263	61.839	50.557	50.200	2.124%	0.500%

(a) Merton's models predicts zero or close to zero spreads for CARLISLE COS INC, HARMAN INTL, JLG INDUSTRIES, MILLIPORE CORP, NVR INC, REYNOLDS & REYNOLDS, STAPLES INC, TEXAS INSTRUMENT, WORTHINGTON INDS. All these companies are investment rated except JLG INDUSTRIES. From the above companies HARMAN INTL, JLG INDUSTRIES, NVR INC, REYNOLDS & REYNOLDS, STAPLES INC and TEXAS INSTRUMENT have a very low leverage ratio (average leverage ratio 11.0%, 8.8%, 10.6%, 11.4% 3.8% and 2.1%, respectively). Regarding CARLISLE COS INC, MILLIPORE CORP and WORTHINGTON INDS although they have higher leverage ratios (17.8%, 17.5% and 25.3% respectively), their bonds are relatively close to maturity.

		LELAND & TOFT CS			ACTUAL CS			LT CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	360.362	79.438	388.874	112.564	69.798	99.900	462.134%	404.334%
2	CLECO CORP	288.062	68.868	301.123	159.040	55.425	150.600	212.224%	159.878%
3	CRANE CO.	243.794	3.621	243.191	66.771	12.700	69.400	377.412%	354.064%
4	GREAT ATLANTIC & PACIFIC	274.459	36.544	265.870	400.192	213.144	376.100	91.760%	70.714%
5	HARMAN INTL	482.724	63.139	486.614	122.326	83.567	81.600	702.916%	493.150%
6	HUMANA INC	290.074	52.295	292.171	171.363	33.744	155.000	170.626%	169.083%
7	INTL SHIPHOLDING	165.107	34.338	163.886	299.165	73.434	290.900	58.266%	53.289%
8	JLG INDUSTRIES	243.864	43.016	242.372	172.664	54.000	159.600	149.384%	143.173%
9	MILLIPORE CORP	496.234	30.046	505.660	143.900	134.591	87.450	602.669%	581.369%
10	NICOR GAS	367.616	57.177	380.575	261.626	87.962	282.325	153.257%	137.080%
11	NISOURCE INC	126.757	37.438	146.935	46.318	61.883	37.850	1007.490%	364.481%
12	NORDSTROM INC	425.984	24.683	430.305	144.326	17.891	144.084	299.486%	293.152%
13	NVR INC	30.025	115.618	1.992	147.937	20.885	143.300	16.393%	1.355%
14	OFFICE MAX INC	243.128	67.102	257.010	352.183	90.620	354.000	70.652%	66.024%
15	POPE & TALBOT	49.211	67.182	35.091	801.859	304.854	820.180	8.907%	4.858%
16	REYNOLDS & REYN	448.814	58.737	458.382	115.792	70.811	95.050	555.491%	489.396%
17	SEITEL INC	150.811	20.343	155.148	513.579	80.645	512.950	29.570%	29.128%
18	SOUTHERN UNION	427.865	21.036	428.455	218.010	32.283	208.829	199.935%	204.809%
19	SPRINT CORP	695.111	535.083	561.077	244.580	36.447	241.950	271.496%	237.273%
20	STAPLES INC	452.683	38.652	462.932	73.761	56.152	57.600	1311.289%	721.713%
21	TEXAS INSTRUMENT	446.034	107.061	454.041	95.512	61.932	118.900	466.992%	364.003%
22	WORTHINGTON INDS	26.901	134.037	33.116	61.839	50.557	50.200	43.501%	62.665%

(b) The Leland and Toft model produces a substantial overestimation of credit spread, which is in line with the findings of Eom, Helwege and Huang (2002).

		LONGSTAFF & SCHWARTZ CS			ACTUAL CS			LS CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	63.969	23.129	60.062	112.564	69.798	99.900	83.974%	82.045%
2	CLECO CORP	279.679	269.775	186.130	159.040	55.425	150.600	168.587%	113.256%
3	CRANE CO.	378.104	138.516	438.012	66.771	12.700	69.400	579.112%	624.979%
4	GREAT ATLANTIC & PACIFIC	640.292	372.745	487.198	400.192	213.144	376.100	166.479%	170.515%
5	HARMAN INTL	125.512	115.750	83.875	122.326	83.567	81.600	116.354%	100.522%
6	HUMANA INC	66.542	22.876	65.410	171.363	33.744	155.000	38.920%	37.716%
7	INTL SHIPHOLDING	965.336	69.684	973.831	299.165	73.434	290.900	340.492%	336.705%
8	JLG INDUSTRIES	96.424	67.723	63.813	172.664	54.000	159.600	52.437%	43.925%
9	MILLIPORE CORP	69.191	24.437	60.977	143.900	134.591	87.450	88.035%	81.064%
10	NICOR GAS	156.472	124.560	112.357	261.626	87.962	282.325	56.712%	49.460%
11	NISOURCE INC	593.005	41.800	599.469	46.318	61.883	37.850	1280.301%	901.623%
12	NORDSTROM INC	70.457	20.779	68.132	144.326	17.891	144.084	48.677%	46.169%
13	NVR INC	53.881	21.240	49.557	147.937	20.885	143.300	36.481%	35.913%
14	OFFICE MAX INC	241.740	170.505	158.672	352.183	90.620	354.000	74.037%	50.964%
15	POPE & TALBOT	972.871	218.403	1005.667	801.859	304.854	820.180	131.954%	129.226%
16	REYNOLDS & REYN	63.902	18.596	57.708	115.792	70.811	95.050	77.916%	64.334%
17	SEITEL INC	128.096	58.322	107.635	513.579	80.645	512.950	24.219%	22.804%
18	SOUTHERN UNION	140.371	66.701	125.444	218.010	32.283	208.829	62.507%	60.751%
19	SPRINT CORP	85.203	24.780	81.986	244.580	36.447	241.950	34.971%	35.157%
20	STAPLES INC	70.098	21.623	71.717	73.761	56.152	57.600	188.608%	105.206%
21	TEXAS INSTRUMENT	135.284	96.451	128.526	95.512	61.932	118.900	41.283%	109.445%
22	WORTHINGTON INDS	29.770	19.467	25.657	61.839	50.557	50.200	48.142%	48.386%

(c) The Longstaff and Schwartz model performed better versus both Merton and Leland and Toft models. Under exogenous volatility and adjusted maturity model was able to explain more than 35% of the credit spread in 12 cases (CARLISLE COS INC, HUMANA INC, JLG INDUSTRIES, MILLIPORE CORP, NICOR GAS, NORDSTROM INC, NVR INC, OFFICE MAX, REYNOLD & REYN, SOUTHERN UNION, SPRINT CORP, and WORTHINGTON INDS).

Table 4.6 – Exogenous Volatility and Actual Bond Maturity

		MERTON'S CS			ACTUAL CS			M CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	0.000	0.000	0.000	130.536	63.485	103.400	0.000%	0.000%
2	CLECO CORP	66.028	92.130	18.566	163.556	45.549	154.800	34.660%	13.878%
3	CRANE CO.	1.127	0.953	1.157	53.886	12.363	56.300	2.117%	2.056%
4	GREAT ATLANTIC & PACIFIC	281.490	290.440	103.316	424.795	228.365	392.100	54.488%	48.085%
5	HARMAN INTL	16.145	33.553	0.011	174.239	83.064	145.700	6.466%	0.008%
6	HUMANA INC	7.588	7.747	5.639	150.877	20.173	145.100	4.793%	3.768%
7	INTL SHIPHOLDING	588.279	44.377	592.985	303.973	72.681	295.200	203.801%	196.682%
8	JLG INDUSTRIES	8.393	13.046	0.003	176.954	59.181	158.300	3.552%	0.002%
9	MILLIPORE CORP	0.005	0.015	0.000	158.817	135.135	100.250	0.004%	0.000%
10	NICOR GAS	71.699	43.051	56.011	158.755	40.752	161.050	43.316%	40.051%
11	NISOURCE INC	196.197	95.680	179.402	84.774	55.118	73.000	262.906%	239.740%
12	NORDSTROM INC	4.045	6.619	1.345	137.632	18.665	138.255	2.803%	1.007%
13	NVR INC	0.562	1.037	0.000	141.748	19.579	139.700	0.403%	0.000%
14	OFFICE MAX INC	41.055	23.867	35.294	335.971	78.323	321.650	12.858%	10.485%
15	POPE & TALBOT	397.814	186.370	382.173	801.158	305.726	819.980	54.108%	49.680%
16	REYNOLDS & REYN	0.000	0.001	0.000	144.283	74.959	125.100	0.000%	0.000%
17	SEITEL INC	16.615	10.391	13.822	513.293	80.449	512.750	3.127%	3.660%
18	SOUTHERN UNION	10.956	5.989	9.344	196.968	22.448	197.241	5.468%	5.024%
19	SPRINT CORP	14.544	13.256	12.881	223.590	27.694	224.400	6.401%	5.722%
20	STAPLES INC	0.000	0.000	0.000	90.637	53.200	74.550	0.000%	0.000%
21	TEXAS INSTRUMENT	11.849	47.695	0.098	112.245	49.036	124.250	7.715%	0.083%
22	WORTHINGTON INDS	0.029	0.053	0.003	80.918	50.116	76.800	0.050%	0.005%

(a) Merton's model predicts zero or close to zero spreads for most of the cases under exogenous volatility and actual bond maturity.

		LELAND & TOFT CS			ACTUAL CS			LT CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	310.240	55.998	323.946	130.536	63.485	103.400	285.793%	286.297%
2	CLECO CORP	157.038	92.289	173.693	163.556	45.549	154.800	106.803%	105.947%
3	CRANE CO.	176.092	0.217	176.157	36.200	12.166	36.700	560.442%	479.993%
4	GREAT ATLANTIC & PACIFIC	346.830	32.529	357.791	424.795	228.365	392.100	103.045%	87.405%
5	HARMAN INTL	319.293	71.068	330.402	174.239	83.064	145.700	231.881%	262.734%
6	HUMANA INC	43.238	34.401	50.580	150.877	20.173	145.100	31.058%	36.491%
7	INTL SHIPHOLDING	317.176	10.269	316.274	303.973	72.681	295.200	109.793%	107.526%
8	JLG INDUSTRIES	332.720	24.858	330.954	176.954	59.181	158.300	203.440%	202.887%
9	MILLIPORE CORP	310.965	34.677	317.545	158.817	135.135	100.250	317.739%	302.977%
10	NICOR GAS	-43.858	35.154	-46.594	158.755	40.752	161.050	-31.853%	-34.557%
11	NISOURCE INC	-11.617	74.086	12.659	84.774	55.118	73.000	-30.945%	10.582%
12	NORDSTROM INC	245.702	49.139	244.412	137.632	18.665	138.255	182.791%	181.725%
13	NVR INC	80.558	30.168	82.657	141.748	19.579	139.700	57.411%	61.160%
14	OFFICE MAX INC	181.658	47.457	201.685	335.971	78.323	321.650	56.679%	54.654%
15	POPE & TALBOT	305.742	114.059	295.916	801.158	305.726	819.980	43.075%	37.152%
16	REYNOLDS & REYN	295.888	58.555	294.083	144.283	74.959	125.100	260.067%	246.427%
17	SEITEL INC	231.380	39.048	227.601	513.293	80.449	512.750	47.067%	39.395%
18	SOUTHERN UNION	117.436	26.935	116.916	196.968	22.448	197.241	60.309%	61.815%
19	SPRINT CORP	236.258	8.519	237.400	223.590	27.694	224.400	107.086%	105.223%
20	STAPLES INC	280.637	40.553	284.104	90.637	53.200	74.550	390.084%	373.488%
21	TEXAS INSTRUMENT	339.544	130.677	338.589	112.245	49.036	124.250	421.432%	223.283%
22	WORTHINGTON INDS	309.483	47.973	309.297	80.918	50.116	76.800	567.068%	433.564%

(b) Under exogenous volatility and actual bond maturity Leland and Toft model performed relative well, with predicted versus actual credit spread ratio greater than 35%, for eight companies (GREAT ATLANTIC & PACIFIC, HUMANA INC, INTL SHIPHOLDING, NVR INC, OFFICE MAX INC, POPE & TALBOT, SEITEL INC, and SOUTHERN UNION). The median predicted versus actual credit spread ratio is between 36.491% and 107.526%.

		LONGSTAFF & SCHWARTZ CS			ACTUAL CS			LS CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	37.232	25.479	36.545	130.536	63.485	103.400	37.109%	35.757%
2	CLECO CORP	282.925	275.535	196.077	163.556	45.549	154.800	158.219%	113.426%
3	CRANE CO.	-1.141	3.831	0.232	53.886	12.363	56.300	-3.627%	0.343%
4	GREAT ATLANTIC & PACIFIC	799.180	747.383	364.203	424.795	228.365	392.100	161.765%	152.991%
5	HARMAN INTL	127.614	108.352	96.023	174.239	83.064	145.700	68.955%	61.318%
6	HUMANA INC	76.454	22.261	77.696	150.877	20.173	145.100	50.974%	50.541%
7	INTL SHIPHOLDING	1208.781	110.922	1205.089	303.973	72.681	295.200	418.683%	406.780%
8	JLG INDUSTRIES	56.471	37.967	41.892	176.954	59.181	158.300	29.901%	31.172%
9	MILLIPORE CORP	27.409	17.766	20.545	158.817	135.135	100.250	27.622%	28.319%
10	NICOR GAS	106.688	79.298	86.101	158.755	40.752	161.050	66.242%	64.926%
11	NISOURCE INC	708.372	146.185	683.635	84.774	55.118	73.000	981.082%	964.630%
12	NORDSTROM INC	69.564	21.261	67.068	137.632	18.665	138.255	50.385%	47.418%
13	NVR INC	69.900	18.896	70.222	141.748	19.579	139.700	49.827%	49.872%
14	OFFICE MAX INC	229.056	165.763	144.556	335.971	78.323	321.650	70.987%	51.854%
15	POPE & TALBOT	951.813	216.891	993.734	801.158	305.726	819.980	129.068%	126.276%
16	REYNOLDS & REYN	34.216	27.603	25.477	144.283	74.959	125.100	25.200%	20.693%
17	SEITEL INC	129.089	58.362	108.796	513.293	80.449	512.750	24.432%	23.014%
18	SOUTHERN UNION	123.494	41.046	121.434	196.968	22.448	197.241	62.105%	63.497%
19	SPRINT CORP	86.201	23.942	85.764	223.590	27.694	224.400	38.949%	38.355%
20	STAPLES INC	43.007	21.050	39.115	90.637	53.200	74.550	56.379%	51.188%
21	TEXAS INSTRUMENT	130.682	93.052	130.395	112.245	49.036	124.250	117.228%	108.994%
22	WORTHINGTON INDS	14.496	20.282	11.057	80.918	50.116	76.800	23.507%	14.478%

(c) The Longstaff and Schwartz model performed better versus both Merton and Leland and Toft models, Not only is able to explain more than 35% of the spread in majority of the companies but also overestimates the spread in only 2 cases (INTL SHIPHOLDING and NISOURCE INC).

One obvious result is that Merton's model clearly overestimates corporate bond prices, although under the exogenous determined and model determined volatility, Merton's model performed extremely well on the following cases GREAT ATLANTIC & PACIFIC, NICOR GAS and POPE & TALBOT explaining more than 35% of the credit spread. Also it is important to state that NICOR GAS is investment rated (Aaa by Moody's and AAA by S & P for the one issue and Aa3 and AA for the other issue) and Merton's model performs really well in predicting the yield when the actual maturity is used. The fact that the weighted maturity of the debt is more that 10 years seems in favour of using the actual maturity, rather than the adjusted one.

The next group of companies is made by pulling together the ones for which the predicted credit spread is only a small fraction of the actual. These companies are: CLECO CORP, HUMANA INC, NORDSTROM INC, OFFICE MAX INC, SEITEL INC, SOUTHERN UNION and SPRINT CORP. All the above companies are investment rated except OFFICE MAX INC and SEITEL INC.

The model predicts zero or very close to zero credit spread under all four cases for the following companies CARLISLE COS INC, HARMAN INTL, JLG INDUSTRIES, MILLIPORE CORP, NVR INC, REYNOLDS & REYNOLDS, STAPLES INC, TEXAS INSTRUMENT, WORTHINGTON INDS. All these companies are again investment rated except JLG INDUSTRIES. From the above companies HARMAN INTL, JLG INDUSTRIES, NVR INC, REYNOLDS & REYNOLDS, STAPLES INC and TEXAS INSTRUMENT have a very low leverage ratio (average leverage ratio 11.0%, 8.8%, 10.6%, 11.4% 3.8% and 2.1%, respectively). Regarding CARLISLE COS INC, MILLIPORE CORP and WORTHINGTON INDS although they have higher leverage ratios (17.8%, 17.5% and 25.3% respectively), their bonds are relatively close to maturity.

Finally the model overpredicts the credit spread for INTL SHIPHOLDING and NISOURCE INC under exogenous determined volatility on both adjusted and bond's actual maturity.

The performance of the model is totally different when it is calibrated against model determined volatility. For INTL SHIPHOLDING the predicted credit spread is zero under both cases of maturity. Regarding NISOURCE INC, when the bonds' actual maturity is used the credit spread is found to be 0.180 bps, leading to a ratio 0.288%. In contrast, in the adjusted maturity case the model provides a median ratio of 33.821%, as the calculated median credit spread is 24.323 bps and the actual is 37.850 bps.

In contrast with Merton's model that clearly underpredicts the credit spreads, the Leland and Toft model produces a substantial overestimation of credit spread, which is in line with the findings of Eom, Helwege and Huang (2002). Moreover, as it is clear from the Tables 4.3 – 4.6, the use of adjusted or actual bond maturity has a strong impact on the performance of the model. Given our results, our focus is when the model is implemented under the actual bond maturity. As it is pointed by Eom, Helwege and Huang (2002), the fact that the model assumes that the firm continuously sells a constant principal amount of new debt with principal $p = P/T$ per year, and pays a constant coupon rate $c = C/T$ per year, can result in an increase in default probabilities. The average overestimation is more than twice, compared with the results of Eom, Helwege and Huang (2002). The model overestimated credit spreads in thirteen cases when the exogenous volatility is used and in twelve cases when the model determined volatility is used. The highest median predicted spread is 176.157 bps against the actual 36.700 bps (for CRANE CO. both under exogenous and model determined volatility). Also it is important to observe that the majority of the companies where the model overpredicts the spread are investment rated (the only one that is not investment rated is JLG INDUSTRIES).

Under actual bond maturity the model performed very well, with predicted versus actual credit spread ratio greater than 35%, for eight companies (GREAT ATLANTIC & PACIFIC, HUMANA INC, INTL SHIPHOLDING, NVR INC, OFFICE MAX INC, POPE & TALBOT, SEITEL INC, and SOUTHERN UNION). For the above companies when the exogenous volatility is used the median predicted versus actual credit spread ratio is between 36.491% and 107.526%; while when the model is implemented using the model determined volatility the ratio is between 25.004% and 99.836%.

Table 4.7 – Model Determined Volatility and Adjusted Maturity

		MERTON'S CS			ACTUAL CS			M CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	0.000	0.000	0.000	115.271	67.857	100.750	0.000%	0.000%
2	CLECO CORP	102.596	151.293	31.445	159.173	55.005	150.500	59.195%	22.970%
3	CRANE CO.	282.356	147.277	336.395	64.486	12.724	66.500	446.426%	505.858%
4	GREAT ATLANTIC & PACIFIC	105.728	60.965	100.273	404.540	215.804	383.000	28.891%	27.954%
5	HARMAN INTL	20.381	41.618	0.243	130.456	82.256	83.800	9.474%	0.373%
6	HUMANA INC	5.731	8.846	3.339	166.741	29.495	153.200	3.062%	2.008%
7	INTL SHIPHOLDING	0.012	0.043	0.000	300.332	73.199	290.700	0.004%	0.000%
8	JLG INDUSTRIES	24.209	36.855	0.298	173.518	54.820	159.500	11.066%	0.209%
9	MILLIPORE CORP	0.101	0.140	0.040	144.992	134.564	87.000	0.131%	0.059%
10	NICOR GAS	141.761	137.683	86.068	240.438	74.287	257.725	52.926%	39.965%
11	NISOURCE INC	28.642	15.432	24.323	46.344	61.867	37.850	-88.240%	33.821%
12	NORDSTROM INC	3.467	6.174	0.983	142.000	18.105	141.843	2.955%	1.061%
13	NVR INC	0.146	0.275	0.000	147.276	20.405	143.400	0.107%	0.000%
14	OFFICE MAX INC	41.678	39.649	28.805	352.106	90.569	353.850	12.405%	8.294%
15	POPE & TALBOT	253.133	509.405	153.513	801.636	305.132	820.080	32.749%	18.150%
16	REYNOLDS & REYN	0.087	0.169	0.004	122.250	72.198	98.650	0.121%	0.003%
17	SEITEL INC	12.894	8.504	13.349	513.460	80.561	512.850	2.444%	2.946%
18	SOUTHERN UNION	8.075	7.413	6.304	210.897	27.736	205.989	3.759%	2.913%
19	SPRINT CORP	5.911	9.441	3.111	244.153	36.143	241.750	2.349%	1.297%
20	STAPLES INC	0.000	0.002	0.000	76.884	55.476	59.200	0.001%	0.000%
21	TEXAS INSTRUMENT	15.832	53.700	0.069	99.818	55.863	122.600	11.571%	0.064%
22	WORTHINGTON INDS	0.104	0.243	0.024	68.096	50.669	63.900	0.306%	0.034%

(a) Similar to other cases Merton's model predicts zero or close to zero spreads for most of the companies.

		LELAND & TOFT CS			ACTUAL CS			LT CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	360.372	79.443	388.882	112.564	69.798	99.900	462.148%	404.341%
2	CLECO CORP	295.803	71.604	309.238	159.040	55.425	150.600	217.179%	173.900%
3	CRANE CO.	243.513	3.516	242.640	66.771	12.700	69.400	377.004%	353.244%
4	GREAT ATLANTIC & PACIFIC	261.690	64.199	273.093	400.192	213.144	376.100	91.243%	70.648%
5	HARMAN INTL	482.556	63.532	485.664	122.326	83.567	81.600	702.935%	493.151%
6	HUMANA INC	289.625	52.135	291.743	171.363	33.744	155.000	170.387%	168.509%
7	INTL SHIPHOLDING	231.607	37.724	243.280	299.165	73.434	290.900	82.111%	78.280%
8	JLG INDUSTRIES	243.928	43.075	242.392	172.664	54.000	159.600	149.417%	143.176%
9	MILLIPORE CORP	499.006	32.226	507.142	143.900	134.591	87.450	606.478%	586.260%
10	NICOR GAS	355.870	66.137	364.445	261.626	87.962	282.325	146.962%	131.586%
11	NISOURCE INC	207.759	30.835	215.579	46.318	61.883	37.850	448.552%	245.304%
12	NORDSTROM INC	427.177	24.189	431.335	144.326	17.891	144.084	300.291%	294.366%
13	NVR INC	30.024	115.619	1.992	147.937	20.885	143.300	16.393%	1.355%
14	OFFICE MAX INC	247.131	68.821	257.662	352.183	90.620	354.000	71.902%	68.074%
15	POPE & TALBOT	28.557	115.685	2.056	801.859	304.854	820.180	6.200%	0.312%
16	REYNOLDS & REYN	449.036	58.971	458.405	115.792	70.811	95.050	555.871%	489.444%
17	SEITEL INC	169.005	25.879	174.930	513.579	80.645	512.950	33.006%	32.658%
18	SOUTHERN UNION	449.508	24.921	449.947	218.010	32.283	208.829	209.567%	214.006%
19	SPRINT CORP	834.067	675.376	640.120	244.580	36.447	241.950	324.944%	272.267%
20	STAPLES INC	452.683	38.652	462.932	73.761	56.152	57.600	1311.289%	721.713%
21	TEXAS INSTRUMENT	446.015	107.083	453.916	95.512	61.932	118.900	466.971%	363.812%
22	WORTHINGTON INDS	26.926	134.125	33.085	61.839	50.557	50.200	43.543%	62.666%

(b) For Leland and Toft model results under model determined volatility and adjusted maturity are very close to exogenous volatility and adjusted maturity, with model over-predicting spreads for majority of the companies.

		LONGSTAFF & SCHWARTZ CS			ACTUAL CS			LS CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	63.954	23.127	60.029	112.564	69.798	99.900	83.957%	82.018%
2	CLECO CORP	360.760	406.119	197.302	159.040	55.425	150.600	210.221%	119.945%
3	CRANE CO.	371.100	139.675	431.279	66.771	12.700	69.400	568.146%	615.946%
4	GREAT ATLANTIC & PACIFIC	437.730	270.087	377.218	400.192	213.144	376.100	113.520%	116.873%
5	HARMAN INTL	117.916	105.592	82.410	122.326	83.567	81.600	112.685%	95.712%
6	HUMANA INC	68.472	23.695	66.355	171.363	33.744	155.000	39.965%	39.692%
7	INTL SHIPHOLDING	55.567	14.512	52.333	299.165	73.434	290.900	19.057%	18.171%
8	JLG INDUSTRIES	95.564	66.607	63.765	172.664	54.000	159.600	52.014%	43.875%
9	MILLIPORE CORP	66.779	22.744	59.914	143.900	134.591	87.450	84.772%	79.705%
10	NICOR GAS	327.211	308.476	178.261	261.626	87.962	282.325	113.395%	88.866%
11	NISOURCE INC	118.154	38.417	112.106	46.318	61.883	37.850	255.095%	141.339%
12	NORDSTROM INC	69.284	20.621	66.387	144.326	17.891	144.084	47.890%	45.022%
13	NVR INC	53.979	21.201	49.939	147.937	20.885	143.300	36.554%	35.930%
14	OFFICE MAX INC	262.576	220.949	148.918	352.183	90.620	354.000	78.927%	46.800%
15	POPE & TALBOT	700.143	304.810	736.247	801.859	304.854	820.180	89.426%	80.846%
16	REYNOLDS & REYN	63.671	18.425	57.580	115.792	70.811	95.050	77.542%	64.261%
17	SEITEL INC	75.724	28.900	68.345	513.579	80.645	512.950	14.533%	14.253%
18	SOUTHERN UNION	105.455	51.406	98.089	218.010	32.283	208.829	47.282%	47.205%
19	SPRINT CORP	80.158	24.072	76.901	244.580	36.447	241.950	33.122%	32.033%
20	STAPLES INC	70.098	21.622	71.717	73.761	56.152	57.600	188.607%	105.206%
21	TEXAS INSTRUMENT	135.177	96.384	128.433	95.512	61.932	118.900	41.211%	109.489%
22	WORTHINGTON INDS	29.280	19.099	25.438	61.839	50.557	50.200	237.754%	48.373%

(c) Similarly, to the other cases the Longstaff and Schwartz model out-performed versus both Merton and Leland and Toft models. Its explanatory power was more than 35% of the spread in 13 cases of the companies. Also the overestimation of the spread was limited to only one case.

Table 4.8 – Model Determined Volatility and Actual Bond Maturity

		MERTON'S CS			ACTUAL CS			M CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	0.000	0.000	0.000	130.536	63.485	103.400	0.000%	0.000%
2	CLECO CORP	100.656	152.516	26.698	163.556	45.549	154.800	52.051%	15.607%
3	CRANE CO.	1.116	0.946	1.145	53.886	12.363	56.300	2.098%	2.034%
4	GREAT ATLANTIC & PACIFIC	94.455	101.675	54.111	424.795	228.365	392.100	18.858%	16.657%
5	HARMAN INTL	16.020	33.894	0.008	174.239	83.064	145.700	6.427%	0.006%
6	HUMANA INC	13.638	11.679	11.234	150.877	20.173	145.100	8.719%	7.728%
7	INTL SHIPHOLDING	0.006	0.021	0.000	303.973	72.681	295.200	0.002%	0.000%
8	JLG INDUSTRIES	9.014	14.023	0.003	176.954	59.181	158.300	3.814%	0.002%
9	MILLIPORE CORP	0.000	0.000	0.000	158.817	135.135	100.250	0.000%	0.000%
10	NICOR GAS	170.511	106.745	133.275	158.755	40.752	161.050	103.164%	96.192%
11	NISOURCE INC	1.711	2.951	0.180	84.774	55.118	73.000	1.963%	0.288%
12	NORDSTROM INC	4.377	7.265	1.422	137.632	18.665	138.255	3.701%	1.504%
13	NVR INC	1.102	2.008	0.000	141.748	19.579	139.700	0.791%	0.000%
14	OFFICE MAX INC	46.897	39.672	34.240	335.971	78.323	321.650	14.254%	10.719%
15	POPE & TALBOT	249.811	503.543	151.502	801.158	305.726	819.980	32.314%	17.777%
16	REYNOLDS & REYN	0.000	0.000	0.000	144.283	74.959	125.100	0.000%	0.000%
17	SEITEL INC	13.000	8.534	13.555	513.293	80.449	512.750	2.467%	2.977%
18	SOUTHERN UNION	6.387	4.956	5.016	196.968	22.448	197.241	3.258%	2.472%
19	SPRINT CORP	11.209	13.392	7.335	223.590	27.694	224.400	5.008%	3.118%
20	STAPLES INC	0.000	0.000	0.000	90.637	53.200	74.550	0.000%	0.000%
21	TEXAS INSTRUMENT	12.164	48.550	0.102	112.245	49.036	124.250	7.921%	0.084%
22	WORTHINGTON INDS	0.002	0.006	0.000	80.918	50.116	76.800	0.004%	0.000%

(a) In line with the other cases, Merton's mode predicts zero or close to zero credit spread for majority of the cases.

		LELAND & TOFT CS			ACTUAL CS			LT CS/ACTUAL CS	
A/A	**Company Name**	**Mean**	**St. Dev.**	**Median**	**Mean**	**St. Dev.**	**Median**	**Mean**	**Median**
1	CARLISLE COS INC	310.240	55.998	323.946	130.536	63.485	103.400	285.793%	286.297%
2	CLECO CORP	154.828	90.835	172.129	163.556	45.549	154.800	105.687%	101.908%
3	CRANE CO.	176.092	0.217	176.157	36.200	12.166	36.700	560.442%	479.993%
4	GREAT ATLANTIC & PACIFIC	337.224	26.366	345.009	424.795	228.365	392.100	101.475%	86.361%
5	HARMAN INTL	318.659	71.819	330.277	174.239	83.064	145.700	231.624%	262.736%
6	HUMANA INC	33.969	33.602	39.738	150.877	20.173	145.100	24.740%	28.749%
7	INTL SHIPHOLDING	296.414	15.629	295.076	303.973	72.681	295.200	102.725%	99.836%
8	JLG INDUSTRIES	332.711	24.846	330.954	176.954	59.181	158.300	203.437%	202.887%
9	MILLIPORE CORP	310.965	34.677	317.545	158.817	135.135	100.250	317.739%	302.977%
10	NICOR GAS	-33.764	53.386	-50.504	158.755	40.752	161.050	-28.029%	-37.781%
11	NISOURCE INC	-11.617	74.086	12.659	84.774	55.118	73.000	-30.945%	10.582%
12	NORDSTROM INC	256.657	47.738	258.288	137.632	18.665	138.255	190.914%	191.537%
13	NVR INC	80.285	30.481	82.641	141.748	19.579	139.700	57.214%	61.159%
14	OFFICE MAX INC	181.652	47.461	201.685	335.971	78.323	321.650	56.678%	54.654%
15	POPE & TALBOT	228.341	205.563	188.705	801.158	305.726	819.980	31.585%	25.004%
16	REYNOLDS & REYN	295.888	58.555	294.083	144.283	74.959	125.100	260.067%	246.427%
17	SEITEL INC	221.477	50.074	207.161	513.293	80.449	512.750	45.449%	36.482%
18	SOUTHERN UNION	176.677	44.407	179.695	196.968	22.448	197.241	90.250%	90.431%
19	SPRINT CORP	236.283	8.530	237.412	223.590	27.694	224.400	107.096%	105.230%
20	STAPLES INC	280.637	40.553	284.104	90.637	53.200	74.550	390.084%	373.488%
21	TEXAS INSTRUMENT	339.536	130.687	338.591	112.245	49.036	124.250	421.428%	223.293%
22	WORTHINGTON INDS	309.483	47.973	309.297	80.918	50.116	76.800	567.068%	433.564%

(b) Under model-determined volatility and actual bond maturity Leland and Toft model performed relative well and results are similar to exogenous volatility and actual bond maturity.

		LONGSTAFF & SCHWARTZ CS			ACTUAL CS			LS CS/ACTUAL CS	
A/A	Company Name	Mean	St. Dev.	Median	Mean	St. Dev.	Median	Mean	Median
1	CARLISLE COS INC	37.232	25.479	36.545	130.536	63.485	103.400	37.109%	35.757%
2	CLECO CORP	365.447	411.759	211.709	163.556	45.549	154.800	198.764%	122.870%
3	CRANE CO.	-1.352	3.796	-0.007	53.886	12.363	56.300	-4.021%	-0.010%
4	GREAT ATLANTIC & PACIFIC	450.362	498.589	212.249	424.795	228.365	392.100	85.861%	70.114%
5	HARMAN INTL	120.955	98.416	95.427	174.239	83.064	145.700	66.320%	60.753%
6	HUMANA INC	82.150	24.030	82.655	150.877	20.173	145.100	54.749%	55.362%
7	INTL SHIPHOLDING	35.714	12.856	33.314	303.973	72.681	295.200	11.939%	11.801%
8	JLG INDUSTRIES	55.486	36.591	41.892	176.954	59.181	158.300	29.479%	31.165%
9	MILLIPORE CORP	27.399	17.752	20.545	158.817	135.135	100.250	27.613%	28.319%
10	NICOR GAS	230.603	164.042	167.731	158.755	40.752	161.050	139.610%	131.035%
11	NISOURCE INC	40.261	34.328	39.845	84.774	55.118	73.000	48.851%	36.492%
12	NORDSTROM INC	67.984	20.905	65.345	137.632	18.665	138.255	49.280%	46.126%
13	NVR INC	70.703	18.896	72.357	141.748	19.579	139.700	50.404%	51.426%
14	OFFICE MAX INC	246.483	203.131	137.864	335.971	78.323	321.650	75.244%	48.227%
15	POPE & TALBOT	687.248	298.769	717.321	801.158	305.726	819.980	87.843%	79.366%
16	REYNOLDS & REYN	34.216	27.603	25.477	144.283	74.959	125.100	25.200%	20.693%
17	SEITEL INC	76.369	29.048	68.748	513.293	80.449	512.750	14.666%	14.348%
18	SOUTHERN UNION	92.745	33.159	92.068	196.968	22.448	197.241	46.836%	46.506%
19	SPRINT CORP	80.349	26.842	77.227	223.590	27.694	224.400	36.485%	35.180%
20	STAPLES INC	43.007	21.050	39.115	90.637	53.200	74.550	56.379%	51.188%
21	TEXAS INSTRUMENT	130.592	92.943	130.417	112.245	49.036	124.250	117.181%	108.863%
22	WORTHINGTON INDS	14.490	20.275	11.057	80.918	50.116	76.800	23.497%	14.478%

(c) Under model-determined volatility and actual bond maturity, Longstaff and Schwartz model out-performed versus both Merton and Leland and Toft models. Its explanatory power was more than 35% of the spread in 12 cases and in contrast with the previous cases there was no major overestimation error.

While Merton and Leland and Toft model perform on opposite directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads, Longstaff and Schwartz model reveals a very good performance. The model is able to produce a ratio of the predicted over actual credit spread that is greater than 35% for thirteen companies (CARLISLE COS INC, HARMAN INTL, HUMANA INC, JLG INDUSTRIES, MILLIPORE CORP, NICOR GAS, NORDSTROM INC, NVR INC, OFFICE MAX INC, POPE & TALBOT, REYNOLDS & REYNOLDS, SOUTHERN UNION and WORTHINGTON INDS) when it is implemented using model determined volatility and adjusted maturity. Moreover, although it underestimates bond prices in six cases, if we exclude CRANE CO., that is very close to maturity and the credit spread is expected to be overestimated if the model is implemented using the adjusted maturity, the overestimation as it is clear from the predicted over actual credit median ratio is never more than 50%.

Longstaff and Schwartz model indicated very good performance in all other three cases. The overestimation appears to be larger when the model is implemented under exogenous volatility and adjusted maturity. An interesting result is that for NISOURCE INC the median predicted credit spread is overestimated more than nine times when the model is implemented under exogenous volatility. In contrast, when it is implemented under model determined volatility the results are much better.

4.5. The Performance of the Longstaff and Schwartz Model during the Credit Crisis

A Bloomberg search was performed using the following criteria: 1) the sample included US non – financial corporations, 2) consider only fixed or zero coupon bonds, 3) all the principal is retired at maturity (bullet bonds), 4) bonds with embedded optionalities like callable, convertible, putable were excluded, 5) the sample excluded floating-rate or sinkable bonds.

As a result 1,853 fixed or zero coupon bullet bonds in US Dollars that have been issued US non-financial corporations[11] are selected. This number is significantly smaller compared to our original investigation as market conditions lead a lot of companies to refinance and restructure their debt. The specific sample was filtered in order to include listed corporations. At the same time the assumption of the simple capital structured is relaxed. The traded bonds were identified through executable prices and the TRAC system. TRAC uses the Trade Reporting and Compliance Engine (TRACE) to research corporate trade data. TRACE data is disseminated to the public via the Bond Trade Dissemination Service (BTDS) data feed product. The SEC had approved proposed rules that require NASD members to report secondary market transactions in eligible fixed income securities to the NASD, and subject certain transaction reports to dissemination. TRACE enables regulators to oversee the corporate debt market and better detect misconduct while improving investor confidence in this market. From the above sample 20 randomly selected bonds determined our final sample. Table 4.10 summarizes the key bond features of the selected bonds for the period from 25th July 2007 until 25th July 2008. The reason that the following period was selected as after July the liquidity deteriorated significantly leading to Lehman bankruptcy in September and AIG rescue by FED.

Data on bond features, prices and yields, as well as balance and equity historical data are taken from Bloomberg. Interest rate data are from Constant Maturity Treasury series as provided by Federal Reserve.

[11] The sectors that have been included are Basic Materials, Communications, Consumer Cyclical, Consumer Non – Cyclical, Energy, Industrial, Technology and Utilities.

Table 4.9 – Bond Features July 2007 – July 2008

Name	Coupon	Issue Date	Maturity	S & P Rating	Amount Issued
BOWATER INC	9	09/08/1989	01/08/2009	CCC	300,000,000
ARROW ELEC INC	7.5	22/01/1997	15/01/2027	BBB-	200,000,000
WITCO CORP	6.875	12/02/1996	01/02/2026	BB	150,000,000
CAMPBELL SOUP CO	8.875	09/05/1991	01/05/2021	A	200,000,000
EASTMAN KODAK CO	7.25	10/10/2003	15/11/2013	B	500,000,000
CORNING INC	6.85	03/03/1999	01/03/2029	BBB+	150,000,000
KOHLS CORP	7.375	15/10/1996	15/10/2011	BBB+	100,000,000
3M CO	5.125	08/11/2006	06/11/2009	AA	400,000,000
NSTAR ELECTRIC	7.8	17/05/1995	15/05/2010	A+	125,000,000
PULTE CORP	8.125	26/02/2001	01/03/2011	BB	200,000,000
SAKS INC	9.875	19/02/2002	01/10/2011	B+	141,000,000
SONOCO PRODUCTS	9.2	12/08/1991	01/08/2021	BBB	100,000,000
PROCTER & GAMBLE	5.5	27/01/2004	01/02/2034	AA-	500,000,000
STEEL DYNAMICS	7.375	12/10/2007	01/11/2012	BB	700,000,000
STANLEY WORKS	5	20/03/2007	15/03/2010	A	200,000,000
DEERE & CO	6.95	17/04/2002	25/04/2014	A	700,000,000
UNISYS CORP	6.875	17/03/2003	15/03/2010	B+	300,000,000
WENDY'S INTL	7	19/12/1995	15/12/2025	BB-	100,000,000
WAL-MART STORES	4.5	09/06/2005	01/07/2015	AA	750,000,000
XEROX CORP	7.2	28/03/1996	01/04/2016	BBB	250,000,000

The next step is to determine and compute the relevant parameters. These parameters can be divided into three groups. The first group is related to firm specific factors. The second group is referred to bond - debt characteristics and the third group of parameters define the default – free term structure.

The Longstaff and Schwartz model considers the valuation of the corporate bonds under stochastic interest rates. The interest rate dynamics are described by the Vasicek (1977) model. In this framework there is a boundary value K for the firm at which financial distress occurs. As long as value V is greater than K, the firm continues to be able to meet its contractual obligations. If V reaches K, the firm immediately enters to financial distress and defaults on all of its obligations.

The value of the assets is calculated as the sum the market value of equity plus the book value of the long term debt. The volatility is calculated by solving Merton's model. In the Merton's model the value of the equity is a call option on the firm's asset value with strike price equal to the face value of debt. The pay-off at maturity is $E_T = \max(V_T - B,0)$, where V_T is the value of the firm at time T and B is the value of payment to bond holders. Using the Black – Scholes formula gives the value of the equity today as

$$E = Ve^{-i\tau}N(d_1) - Be^{-r\tau}N(d_2) \text{ where}$$

$$d_1 = \frac{\ln\left(\frac{V}{B}\right) + \left(r - i + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}$$

V : is the value of the firm.

B : is the value of the promised payment to bond holders.

τ : is the time until maturity.

r : is the instantaneous risk – free rate.

i : are any dividend payments.

σ_A : is the volatility of on the firm assets.

$N(\bullet)$: the cumulative standard normal distribution.

From Ito's lemma we know that the relationship between the equity volatility and the asset volatility is the following:

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_A V_0 \Leftrightarrow \sigma_E = \frac{\partial E}{\partial V}\frac{V_0}{E_0}\sigma_A \Leftrightarrow \sigma_E = N(d_1)\frac{V_0}{E_0}\sigma_A$$

The above equation can be solved numerically since all other parameters except σ_A are known. When we solve the above equation, σ_E is set to be equal to the equity implied volatility. For the equity implied volatility we use the composite implied volatility for the stock. The composite implied volatility is calculated by taking a suitably weighted average of the individual implied volatilities. The weights are calculated according to its trading volume and the moneyness. Heaviest weighting is applied to those with the highest volumes and strike closest to the current share price. The use of a composite volatility has primary importance as captures the market expectation around the value of the equity price and as a result the value of the assets. For example if market expectation is that the value of equity will fall will require larger premium for put vs. call options.

Longstaff and Schwartz use a default-risk variable X that is the ratio V/K, where V is the value of the assets and K is the face value of debt. The face value of debt is calculated as the sum of long term and short term debt.

As there is no market information available for the period on recovery rates, we assume a 30% recovery rate which is very popular.

The parameter estimation of Longstaff and Schwartz is presented in detail in Paragraph 4.2.3.

The third group of parameters is the default – free term structure. In order to estimate the default – free interest rate curve constant maturity Treasury yield data were used. The constant maturity Treasury series is being published on daily basis by the Federal Reserve and provides the yields for following maturities; 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year . Cubic spline interpolation was used to construct the daily default – free interest rate curve.

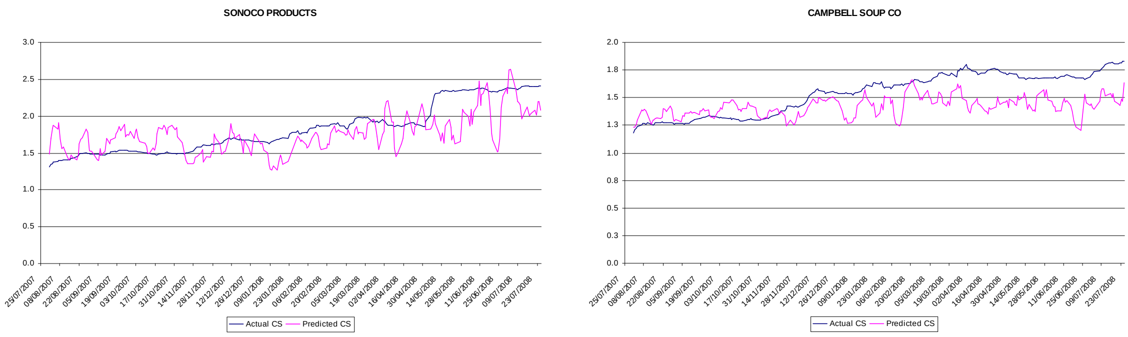
Longstaff and Schwartz model indicated very good performance in all cases proving an average predicted over actual credit spread ratio of 57%. The results are presented in Table 4.11. For eight companies BOWATER INC, CAMPBELL SOUP CO, WITCO CORP,

STEEL DYNAMICS, SONOCO PRODUCTS, SAKS INC, KOHLS CORP and UNISYS CORP the predicted over actual credit spread ratio was greater than 50%. Five of these companies are non investment graded. Importantly the high median ratios are estimated for two invested graded companies the SONOCO PRODUCTS and CAMPBELL SOUP CO with median ratio is 98.50% and 93.23% respectively. The median predicted credit spread for SONOCO PRODUCTS is 172.19 bps versus actual 175.85 bps and for CAMPBELL SOUP CO is 141.14 bps against 159.85 bps. Both bonds are paying high coupons and are long dated with maturity within 2021. Graph 4.2 presents the results for SONOCO PRODUCTS and CAMPBELL SOUP CO.

Table 4.10 – Longstaff and Schwartz Model Results

A/A	Company Name	LONGSTAFF & SCHWARTZ CS Mean	St. Dev.	Median	ACTUAL CS Mean	St. Dev.	Median	LS CS/ACTUAL CS Mean	Median
1	BOWATER INC	1130.69	51.81	844.68	1300.44	58.41	900.90	87.35%	86.03%
2	ARROW ELEC INC	76.44	8.87	78.23	253.77	6.69	245.90	28.36%	37.89%
3	3M CO	13.19	0.33	12.85	106.87	1.73	111.70	12.28%	12.28%
4	CAMPBELL SOUP CO	141.97	1.12	141.14	154.28	1.22	159.85	93.09%	93.23%
5	CORNING INC	59.64	2.35	58.61	181.21	1.42	188.95	32.97%	33.65%
6	WAL-MART STORES	15.05	0.26	14.94	135.02	1.78	128.60	11.18%	11.13%
7	EASTMAN KODAK CO	671.97	10.51	642.58	520.36	5.96	557.65	129.67%	128.74%
8	PROCTER & GAMBLE	19.39	0.62	19.01	104.45	1.35	102.20	18.53%	18.21%
9	WITCO CORP	421.49	6.31	407.67	443.87	4.26	445.10	95.64%	95.14%
10	DEERE & CO	66.44	1.41	63.86	149.73	1.94	152.60	44.53%	44.37%
11	STEEL DYNAMICS	336.88	4.29	333.69	415.87	3.60	419.75	81.64%	80.22%
12	SONOCO PRODUCTS	175.30	2.77	172.19	182.98	2.13	175.85	97.39%	98.50%
13	SAKS INC	423.05	6.82	413.73	473.42	5.85	468.95	88.87%	89.28%
14	PULTE CORP	341.80	9.71	360.70	313.90	7.90	347.35	109.88%	109.99%
15	KOHLS CORP	107.32	2.68	106.19	201.74	4.02	234.75	53.29%	53.77%
16	XEROX CORP	48.00	1.58	45.47	194.50	2.43	197.35	24.84%	25.29%
17	WENDY'S INTL	85.15	2.56	83.01	460.56	4.70	460.25	18.42%	18.94%
18	UNISYS CORP	311.38	5.70	306.91	388.73	6.08	425.10	80.21%	80.34%
19	STANLEY WORKS	27.23	0.79	25.42	117.08	1.62	126.85	23.21%	22.94%
20	NSTAR ELECTRIC	24.05	0.70	23.88	137.55	2.12	149.20	17.49%	17.54%

Graph 4.1 – Actual vs. Predicted Credit Spread for Sonoco and Campbell



For the five non investment grade the estimated predicted over actual credit spreads are in all cases greater then 80%. Especially for the companies BOWATER INC and UNISYS CORP – see Graph 4.3 – that their bonds are relative close to maturity the predicted over actual credit spread median ratio is 86.03% and 80.34%. The explanatory power of the model in these two cases is increased by the increased equity volatility that these companies indicate. For the BOWATER INC the average implied volatility during that period was 106% and while for UNISYS CORP 58% jumping to 67% for the 2008 period.

Graph 4.2 – Actual vs. Predicted Credit Spread for Bowater and Unisys

Finally for EASTMAN KODAK CO and PULTE CORP the median predicted spread is 643 bps and 361 bps respectively versus actual 558 bps and 347 bps.

4.6. Conclusion

This Chapter tests alternative structural models for pricing corporate debt. Three models, Merton's, Leland and Toft and Longstaff and Schwartz were examined under four different assumptions of volatility and debt maturity (i) exogenous volatility and actual bond maturity, (ii) exogenous volatility and adjusted maturity, (iii) model determined volatility and actual bond maturity and finally (iv) model determined volatility and adjusted maturity. The sample includes only companies with relative simple capital structure and maximum two bonds outstanding. The process to determine the sample is similar to the ones that were followed by Lyden and Saraniti (2000), Teixeira (2005) and Eom, Helwege and Huang (2002). To our knowledge, it is the first time that the models are calibrated against these four alternatives. Furthermore it is important to state the fact that for the first time in the literature the historical implied volatility was used for equity.

For the sample 1998 – 2006, Merton's and Leland and Toft models perform on different directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads. On the other hand Longstaff and Schwartz model reveals a very good performance. The model is able to produce a ratio of the predicted over the actual credit spread that is greater than 35% for the majority of the companies. Furthermore, even when there was an overprediction error that was on limited magnitude and definitely much less compared to Leland and Toft. The above results are in contrast with Lyden and Saraniti (2000) and Wei and Guo (1997) who argued that Merton's model dominates Longstaff and Schwartz in predictive accuracy.

The Longstaff and Schwartz (1995) model is applied also on 2007 – 2008 bond data. The assumption of simple capital structure is relaxed and a composite implied volatility is calculated. The use of a composite volatility has primary importance as captures the market expectation around the value of the equity price and as a result the value of the assets. For example if market expectation is that the value of equity will fall will require larger premium for put vs. call options. Again the model indicated very good performance in all cases proving an average predicted over actual credit spread ratio of 57%. Interestingly though the average predicted credit spread was still estimated below the actual one in line with our earlier implementation, although the explanatory power of the model has increased; this is mainly driven by the higher market volatility observed during the crisis.

Chapter 5: USING THE THEORY OF CONTINGENT CLAIMS FOR CORPORATE DEBT IN CREDIT – EQUITY INVESTING

We perform an empirical application of the theory of contingent claims for corporate debt in credit – equity investing decisions. The relationship between Credit and Equity markets is an important signal for investment decisions but not simple to capture. We utilize Longstaff and Schwartz (1995) model in order to get signals about the future performance of the equity and determine a medium term investment strategy.

The analysis of the credit – equity relation for any company has its theoretical origins over thirty years ago, when Black and Scholes (1973) and Merton (1974) initiated the modern analysis of corporate debt by pointing out that the holders of risky corporate bonds can be thought of as owners of risk-free bonds who have issued put options to the holders of the firm's equity. Models based on this approach are generally referred to as structural models. The equity can be considered a call option on the asset value of the firm with a strike price equal to the value of the liabilities. The Merton's (1974) framework provided the base for the origination of extensions by adding features either to the process of the firm or the interest rates, or by relaxing some of the assumptions of the original framework (e.g. the default time).
Fundamentally, equity valuation and credit risk are both driven in part by a company's financial condition. The relationship between equity and credit markets is an important signal but not simple to model or capture. If a company's financial condition improves, its equity price should rise, all else being equal, and credit spread should tighten. This means that information on a company's financials should be reflected in both the equity and credit markets. This should allow us to spot valuation anomalies between these markets and determine a trading strategy. Here we use the Longstaff and Schwartz (1995) model with a kernel smoothing function in order to provide medium term estimation about the performance of the equity.
The methodology is focused on the utilization of market input parameters, credit spread and equity implied option volatilities in order to determine the medium term expected value of the equity. Under Longstaff and Schwartz (1995) framework the total value of the assets of the firm V can be described by the following stochastic differential equation $dV = \mu V dt + \sigma V dZ_1$ where μ is the drift, σ is a constant volatility and Z_1 is a standard Wiener

process. The asset value at time t is estimated by the market value of equity plus the book value of the long term debt. Furthermore, a default-risk variable X is included, defined as the ratio V/K, where V is the value of the assets and K is the face value of debt. The face value of debt is calculated as the sum of long term and short term debt.

The recovery rates were obtained from Moody's Report "Default and Recovery Rates of Corporate Bond Issuers, 1920 – 2005". Each year for the period January 1998 – April 2006, the recovery rate is used is the Annual Average Default Bond Recovery Rate for all corporate bonds.

The specific model incorporates stochastic interest rates, which follow the dynamics of the Vasicek (1977) model, $dr = (\zeta - \beta r)dt + \eta dZ_2$; where ζ, β and η are constants and Z_2 is also a standard Wiener process. In order to estimate the parameters the model is calibrated every quarter on the treasury curve. The treasury curve was constructed using constant maturity Treasury series that is being published on daily basis by the Federal Reserve and provides the yields for following maturities; 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year . Cubic spline interpolation was used to interpolate between the fixed points.

Another parameter that needs to be determined is the instantaneous correlation between assets and interest rates. As this is not an observable parameter a proxy should be determined. The instantaneous correlation is calculated using the daily returns of the 6 month treasury rate and equity prices over a rolling window of 180 days.

Once the parameters are estimated the value of a risky discount bond is calculated as:

$$P(X,r,T) = D(r,T) - wD(r,T)Q(X,r,T)$$

P: the value of a risky discount bond

X: the ratio V/K

r: the riskless interest rate

T: the maturity date

$D(r,T) = \exp(A(T) - B(T)r)$: the value of a riskless discount bond Vasicek (1977)

$$A(T) = \left(\frac{\eta^2}{2\beta^2} - \frac{\alpha}{\beta}\right)T + \left(\frac{\eta^2}{\beta^3} - \frac{\alpha}{\beta^2}\right)(\exp(-\beta T) - 1) - \left(\frac{\eta^2}{4\beta^3}\right)(\exp(-2\beta T) - 1)$$

$$B(T) = \frac{1 - \exp(-\beta T)}{\beta}$$

$$Q(X,r,T) = \lim_{n\to\infty} Q(X,r,T,n)$$

$$Q(X,r,T,n) = \sum_{i=1}^{n} q_i$$

$$q_1 = N(a_1)$$

$$q_i = N(a_i) - \sum_{j=1}^{i-1} q_j N(b_{ij}), i = 2,3,\ldots,n$$

$N(\cdot)$: the cumulative standard normal distribution function

$$a_i = \frac{-\ln X - M(iT/n,T)}{\sqrt{S(iT/n)}}$$

$$b_{ij} = \frac{M(jT/n,T) - M(iT/n,T)}{\sqrt{S(iT/n) - S(jT/n)}}$$

$$M(t,T) = \left(\frac{a - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2}\right)t + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3}\right)\exp(-\beta \mathrm{T})(\exp(\beta \mathrm{T}) - 1) +$$
$$\left(\frac{r}{\beta} - \frac{a}{\beta^2} + \frac{\eta^2}{\beta^3}\right)(1 - \exp(-\beta t)) - \left(\frac{\eta^2}{2\beta^3}\right)\exp(-bT)(1 - \exp(-\beta t))$$

$$S(t) = \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right)t - \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right)(1 - \exp(-\beta \mathrm{T})) + \left(\frac{\eta^2}{2\beta^3}\right)(1 - \exp(-2\beta t))$$

The first term $D(r,T)$ of the equation represents the value the bond would have if it were riskless. The second term $wD(r,T)Q(X,r,T)$ represents a discount for the default risk of the bond. The first component $wD(r,T)$ is the present value of the writedown on the bond in the event of a default. The second component $Q(X,r,T)$ is the probability – under the risk-neutral measure – that a default occurs.

The above equation is solved numerically for the implied, using the market observed credit spread, V. That give us a sequence V_1, V_2, …, V_n from a continuous random variable V with a probability density function f. We apply Kernel smoothing in order to find an estimate of f and determine the medium term expected value of the equity.

A conceptually simple approach to represent the weight sequence $\{W_{ni}(x)\}_{i=1}^{n}$ is to describe the shape of the weight function $W_{ni}(x)$ by a density function with a scale parameter that adjusts the size and the form of the weights near x. It is quite common to refer to this shape function as a kernel K. The kernel is a continuous, bounded and symmetric real function K which integrates to one, i.e. $\int K(u)du = 1$.

For one-dimensional x the weight sequence for kernel smoothers is defined by

$$W_{ni}(x) = K_{h_n}(x - X_i) / \hat{f}_{h_n}(x) \quad (5.1)$$

where $\hat{f}_{h_n}(x) = n^{-1}\sum_{i=1}^{n} K_{h_n}(x - X_i)$ and $K_{h_n}(u) = h_n^{-1}K(u/h_n)$ is the kernel with scale factor h_n.

Suppressing the dependence $h = h_n$ of on the sample size n, the kernel weight sequence is conveniently abbreviated as $\{W_{hi}(x)\}_{i=1}^{n}$. The function $\hat{f}_h(\bullet)$ is the Rosenblatt-Parzen kernel density estimator (Rosenblatt (1956); Parzen (1962)) of the (marginal) density of X. The form of kernel weights $W_{hi}(x)$ has been proposed by Nadaraya (1964) and Watson (1964):

$$\hat{m}_h(x) = \frac{n^{-1}\sum_{i=1}^{n} K_h(x - X_i)Y_i}{n^{-1}\sum_{i=1}^{n} K_h(x - X_i)} \quad (5.2)$$

is often called the Nadaraya-Watson estimator.

The shape of the kernel weights is determined by K, whereas the size of the weights is parameterized by h, which is called the bandwidth. The normalization of the weights $\hat{f}_h(x)$ makes it possible to adapt to the local intensity of the X – variables and, in addition, guarantees that the weights sum to 1.

There is a similarity between local polynomial fitting and kernel smoothing. For fixed x, the kernel estimator $\hat{m}_h(x)$ with positive weights $W_{hi}(x)$ is the solution to the following minimization problem

$$\min_t \sum_{i=1}^{n} K_h(x - X_i)(Y_i - t)^2 = \sum_{i=1}^{n} K_h(x - X_i)(Y_i - \hat{m}_h(x))^2 \quad (5.3).$$

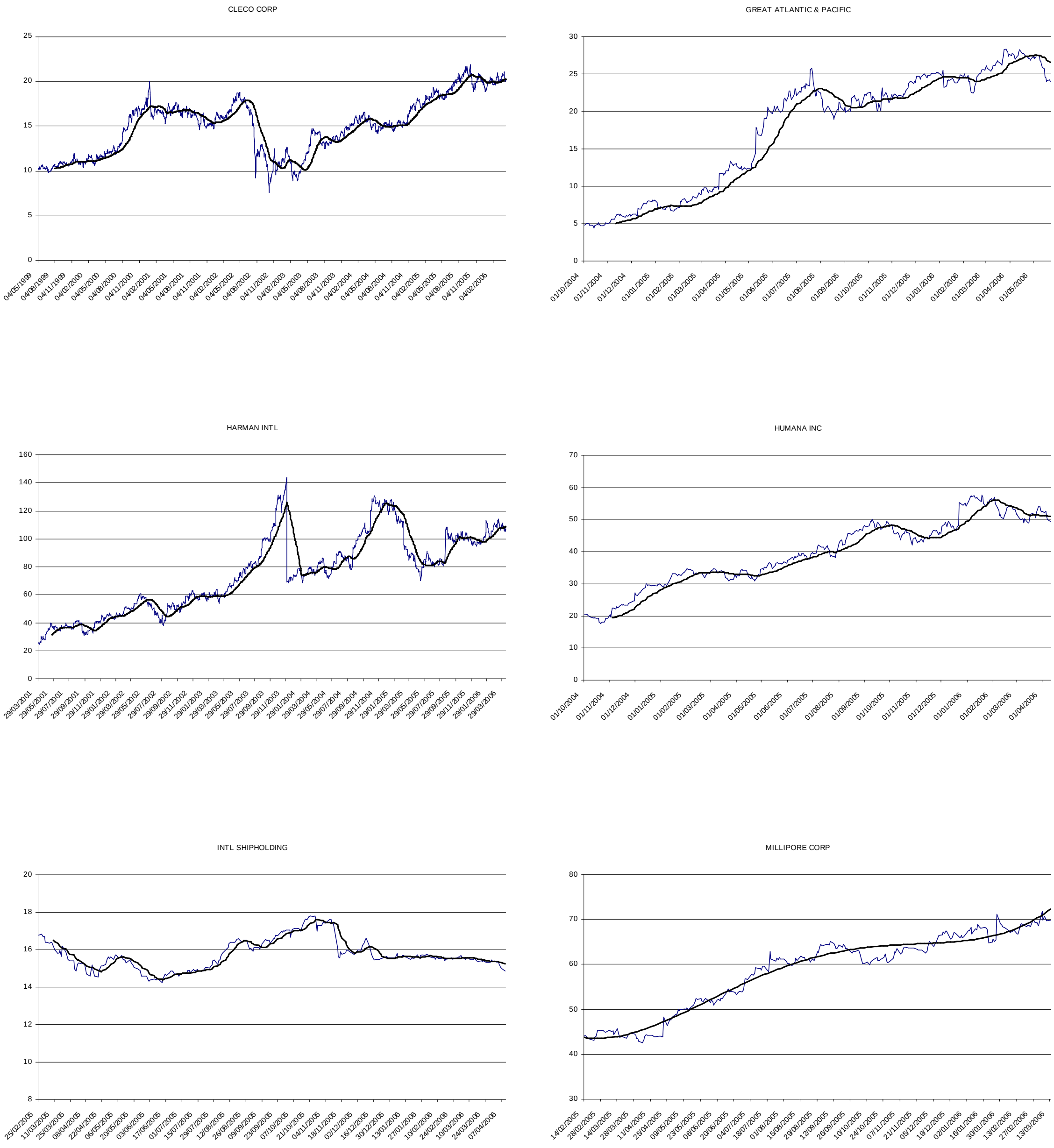
In this sense, the kernel smoother can be understood as a local constant polynomial fit; it minimizes, in a neighbourhood around x – determined in a shape and span by the sequence K_h - the sum of squared residuals.

Table 4.9 presents the expected versus realized return for each name. As it is clear the methodology is able to capture the directional movement of the equity in 18 out of 21 cases. Furthermore as it is clear from the Graph 4.1 it captures very well the medium term trend of the return.

Table 5.1 – Expected versus Realized Daily Return

A/A	Company Name	Expected Return	Realized Return
1	CARLISLE COS INC	-3.656%	2.459%
2	CLECO CORP	-0.058%	-0.021%
3	CRANE CO.	N/A	N/A
4	GREAT ATLANTIC & PACIFIC	0.155%	0.559%
5	HARMAN INTL	0.182%	0.112%
6	HUMANA INC	1.021%	0.675%
7	INTL SHIPHOLDING	-2.268%	-0.339%
8	JLG INDUSTRIES	-1.889%	0.818%
9	MILLIPORE CORP	1.196%	4.205%
10	NICOR GAS	0.357%	0.043%
11	NISOURCE INC	-0.714%	-0.140%
12	NORDSTROM INC	0.182%	0.010%
13	NVR INC	0.407%	0.252%
14	OFFICE MAX INC	0.006%	0.020%
15	POPE & TALBOT	-0.673%	-1.486%
16	REYNOLDS & REYN	-1.365%	0.943%
17	SEITEL INC	1.251%	1.275%
18	SOUTHERN UNION	0.171%	0.194%
19	SPRINT CORP	0.348%	0.282%
20	STAPLES INC	-0.839%	-0.150%
21	TEXAS INSTRUMENT	-0.033%	-0.018%
22	WORTHINGTON INDS	-0.422%	-0.028%

Graph 5.1 – Expected versus Realized Daily Return



Chapter 6: A 2 – FACTOR MODEL WITH LOCAL VOLATILITY TO PRICE EXOTIC DERIVATIVES IN OIL MARKET

6.1. Motivation

Commodity markets cover physical assets such as precious metals, base metals, energy (oil, electricity), wheat, cotton, and weather. Most of the trading is done using futures. However, over the last few years, an OTC market has also been growing as an increasing number of market participants are trading in exotic options. Market participants range from airline companies, refineries, producers, electricity companies, banks and hedge funds. The rationale for trading in commodities markets varies depending on the participants but in general is:

- Hedging against price fluctuations: Producers, refiners and consumers would look to it. For example, a producer, that is a participant who wants to sell the physical commodity, will hedge his selling price. On the other hand, a consumer will try to hedge his buying price.

- Speculation: trading OTC derivatives, as compared to spot assets, presents many advantages, as futures:

    - are more leveraged than the spot instruments because of the low margin requirement,

    - are cheaper in terms of transaction costs

    - and finally do not require storage during the lifetime of the contract.

- Arbitrage between spot and futures markets: for commodities, the cash and carry arbitrage is more difficult to realize because of storage and delivery costs.

The motivation to develop to proposed modeling framework is driven by the fact that the developments of the OTC market, as well as, the complexity of the products required lead the need for more sophisticated models both for valuation and risk management. The work is extending the Brigo and Mercurio (2006) interest rate modelling framework to commodities, developing an approach that captures the de-correlation between the futures contracts and the smile dynamics.

6.2. Literature Review

The development of commodities market not only through exchanges but also through OTC products has supported the development of various exotics structures. The construction of the futures curve is very critical in commodity markets as they provide information about the future expectations of the market participants and views around the future demand and supply. Under the Rational Expectations Hypothesis, forward price is the forecast of the spot price in the future. That assumption does not hold for commodity markets. Crude Oil curves typically exhibit one of the following shapes a) backwardation when futures with shorter maturity are more expensive than those maturing later and b) contango when longer maturities are more expensive versus shorter dated futures. The model that captures the dynamics of the forward curve should be able to match the future prices at t_0 and generate futures prices containing empirically observed features.

There are two main approaches for the commodity futures price dynamics. The first aims to capture the stochastic representation of the spot price and other factors such as convenience yield and interest rates. Gibson and Schwartz (1990) presented a two – factor model with constant volatility. They assume that spot price and convenience yield follow a constant correlation joint stochastic process. Convenience yield is similar to the dividend yield and follows a mean reverting Ornstein – Uhlenbeck process. The spot price is assumed to follow a Geometric Brownian motion. In a later paper Schwartz (1997) introduced stochastic interest rates as a third factor. The addition of stochastic interest rates is presented also in Miltersen and Schwartz (1998) and Hilliard and Reis (1998) although does not have any significant impact in the construction of the forward curve. This modelling approach, although allows model that generate a numerous set of dynamics for the commodity futures forward curve, has a number of problems driven by the fact that state variables can be unobservable. Also the convenience yield can be negative allowing arbitrage opportunities. Finally, the fact that spot prices and convenience yield have constant volatility and correlation is relative restrictive as they do not allow the variance of the spot and future prices and the correlation between them to vary on the level of the price or the convenience yield.

The second approach focuses on the evolution of the forward curve. The development of the exchange traded futures resulted observable future prices up to various maturities depending on the underlying. The first near by contract is used to imply the convenience yield for the longer maturities. Multifactor models for commodity prices utilize the research on the interest rate term structure modelling. In particular the framework set for interest rates by Heath, Jarrow and Morton (1992) model can be used to model forward futures energy prices. At earlier stages Clewlow and Strickland (1999a) presented a one – factor model that is used to derive analytical pricing formulae for standard options and can be used to price exotic energy derivatives using trinomial tree consistently with the forward curve and the volatility structure. In a later paper Clewlow and Strickland (1999b) described the general framework with a multi – factor model that is consistent not only with observable futures prices but also the volatilities and the correlations of the futures prices. Under that general framework the following representation is considered

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t,T) dZ_i(t),$$

where n is the number of risk factors, $\sigma_i(t,T)$ are the volatilities of the risk factors and $Z_i(t)$ are Brownian motions. This approach focuses on the martingale property $\{F(t,T)\}_{t \le T}$ under measure Q.

Term structure models of commodity prices aim to reproduce as accurately as possible the future prices observed in the market. They also provide a mean for the discovery of futures prices for horizons beyond exchange – traded maturities. The main term structure models, from the simplest one – factor models to the more sophisticated, multi – factor models, borrow the analysis developed for interest rate models. The term structure models of commodity prices share three assumptions:

1. The market of assets is free of frictions, taxes and transaction costs.
2. Trading takes place continuously.

3. Lending and borrowing rates are equal so there are no short sale constraints.

Then, the same method as the one developed in the context of interest rates is used to construct the model. First, the state variables are defined and their dynamic is specified. Then, knowing that the price of a future's contract is a expressed as a function of the state variables, the time and the contract's expiration date, it is possible to obtain the dynamic behaviour of the future's price. The transportation of the theoretical framework developed for interest rates in the case of commodities is not straightforward. The reasoning is based on the assumption that the market is complete and in such a market, a derivative asset can be duplicated by a combination of other existing assets. If the latter are sufficiently traded to be arbitrage free, they can constitute a ledge portfolio whose behaviour replicates the derivative behaviour. Their contribution is fixed such as there are no arbitrage opportunities and the strategy is risk – free. Then, in equilibrium, the return of the portfolio must be the risk – free rate. The valuation is made in a risk neutral world. The transposition problem arises from the fact that commodity markets are not complete. Real markets are far from being free of arbitrage opportunities.

There have been several one – factor models in the literature on commodity prices. A futures price is often defined as the expectation, conditionally to the available information at a date t, of the future spot price. Indeed, the spot price is the main determinant of the futures price. Thus, most one-factor models rely on the spot price. There have been several one-factor models in the literature on commodity prices. These models can be separated in step with the dynamic behaviour that is retained for the spot price. Brennan and Schwartz (1985), Gibson and Schwartz (1989 and 1990), Brennan (1991), Gabillon (1992 and 1995) use a geometric Brownian motion, whereas Schwartz (1997), Cortazar and Schwartz (1997), Routledge, Seppi and Spatt (2000) refer themselves to a mean reverting process. Moreover, the models can be distinguished in step with the assumption they retain concerning the convenience yield. Among the different one-factor models with a geometric Brownian motion, Brennan and Schwartz's model (1985) is the most famous. It has been extensively used in subsequent research on commodity prices (Schwartz (1998), Schwartz and Smith (2000), Nowman and Wang (2001), Cortazar, Schwartz and Casassus (2001)).

The Geometric Brownian motion is a dynamic commonly used to represent the behaviour of stock prices. When applied to commodities, the spot price's dynamic is the following:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \text{ (6.1)}$$

where:

S_t : is the spot price

μ : is the drift of the spot price

σ : is the spot price volatility

dW_t : is an increment to a standard Brownian motion.

The use of this representation implies that the variation of the spot price at t is independent of the previous variations and the drift μ conducts the price's evolution. An arbitrage free argument and the construction of a hedging portfolio leads to the fundamental valuation equation of the future's price:

$$\frac{1}{2}\sigma^2 S^2 F_{SS} + (r - c)SF_S - F_\tau = 0 \text{ (6.2)}$$

where:

F : is the future's price

S : is the spot price

μ : is the drift of the spot price

σ : is the spot price volatility

c : is the convenience yield

r : is the interest rate

The terminal boundary condition associated with this equation is:

$$F(S,T,T) = S(T) \text{ (6.3)}$$

It represents the convergence of the future and the spot process at the contract's expiration. This convergence is due to the possibility to deliver the commodity at maturity. It is insured by arbitrage operations between the physical and the financial markets.

The solution of the model expresses the relationship at time t between an observable future's price F for delivery in T and the state variable S and is the following:

$$F(S,t,T) = Se^{(r-c)\tau} \quad (6.4)$$

where:

F : is the future's price

S : is the spot price

c : is the convenience yield

r : is the interest rate

τ : is the T-t

Brennan and Schwartz (1985) also fix the definition of the convenience yield. The convenience yield is the flow of services that accrues to an owner of the physical commodity but not to the owner of a contract for future delivery of the commodity. Recognizing the time lost and the costs incurred in transporting a commodity from one location to another, the convenience yield may be thought of as the value of being able to profit from temporary local shortages of the commodity through ownership of the physical commodity. The profit may arise either from local price variations or from the ability to maintain a production process as a result of ownership of an inventory of raw material.

Although, Brennan and Schwartz's model is probably the most simple term structure model of commodity prices, the geometric Brownian motion is probably not the best approach to represent the price dynamic. Indeed, the storage theory and the Samuelson effect show that the mean reverting process is probably more relevant.

Among the different one-factor models retaining the mean reverting process, Schwartz's model (1997), inspired by Ross (1995), is probably the most commonly used. In that case, the dynamic of the spot price is the following:

$$dS_t = S_t \kappa(\mu - \ln S_t)dt + \sigma S_t dW_t \quad (6.5)$$

where:

S_t : is the spot price

κ : is the speed of adjustment of the spot price

μ : is the long run mean log price

σ : is the spot price volatility

dW_t : is an increment to a standard Brownian motion.

In this situation, the spot price fluctuates around its long run mean. The presence of a speed of adjustment insures that the state variable will always return to its long run mean μ. Therefore, two factors influence the spot price behaviour. First, it has a propensity to return to its long run mean. Second, it is simultaneously volatile and random shocks can move it away from μ.

The use of a mean reversion process for the spot price makes it possible to take into account the behaviour of the operators in the physical market. When the spot price is lower than its long run mean, the industrials, expecting a rise in the spot price, reconstitute their stocks, whereas the producers reduce their production rate. The increasing demand on the spot market and the simultaneous reduction of supply have a rising influence on the spot price. Conversely, when the spot price is higher than its long run mean, industrials try to reduce their surplus stocks and producers increase their production rate, pushing thus the spot price to lower levels.

This formulation of the spot price behaviour is preferable to the geometric Brownian motion, but it is not perfect. For example, the mean reverting process does not exclude that the state variable become negative. The same critic was addressed to this stochastic process in the case of interest rates. Moreover, the storage theory shows that in commodity markets, the basis does not behave similarly in backwardation and in contango. The mean reverting process previously presented does not allow taking into account that characteristic.

The mean reverting process was also used by Cortazar and Schwartz (1997), in a more sophisticated model. Indeed, the authors introduce a variable convenience yield that depends on the deviation of the spot price to a long-term average price.

The homogeneity in the choice of the state variables disappears when a second stochastic variable is introduced in term structure models of commodity prices. Most of the time, the second state variable is the convenience yield. However, models based on long-term price or on volatility of the spot price have also been developed. In all these models, the introduction of a second state variable allows obtaining richer shapes of curves than one-factor models (especially for long term maturities) and richer volatility structures. This improvement is rather costly, because two-factor models are more complex.

Schwartz's model (1997) was used as a reference to develop several models that are more sophisticated (Schwartz (1998), Schwartz and Smith (2000), Yan (2002)). It is inspired by the one proposed by Gibson and Schwartz (1990). Compared with its former version, the latest model is more tractable because it has an analytical solution.

The two – factor model assumes that the spot price S and the convenience yield C can explain the behaviour of the future's price F. The dynamics are defined as:

$$\begin{aligned} dS_t &= (\mu - C_t)S_t dt + \sigma_S S_t dW_t \\ dC_t &= k(a - C_t)dt + \sigma_C dZ_t \end{aligned} \quad (6.6)$$

where:

S_t : is the spot price

C_t : is the convenience yield

μ : is the drift of the spot price

σ_S : is the spot price volatility

α : is the long run mean of the convenience yield

κ : is the speed of adjustment of the convenience yield

σ_C : is the volatility of the convenience yield

dW_t : is an increment to a standard Brownian motion associated with S.

dZ_t : is an increment to a standard Brownian motion associated with C.

$\kappa, \sigma_S, \sigma_C \succ 0$

In this model, the convenience yield is mean reverting and it intervenes in the spot price dynamic. The Ornstein – Uhlenbeck process relies on the hypothesis that there is a regeneration property of inventories, namely that there is a level of stocks, which satisfies the needs of industry under normal conditions. The behaviour of the operators in the physical market guarantees the existence of this normal level. When the convenience yield is low, the stocks are abundant and the operators sustain a high storage cost compared with the benefits related to holding the raw materials. Therefore, if they are rational, they try to reduce these surplus stocks. Conversely, when the stocks are rare the operators tend to reconstitute them. Moreover, as the storage theory showed, the two state variables are correlated. Both the spot price and the convenience yield are indeed an inverse function of the inventories level. Nevertheless, as Gibson and Schwartz (1990) demonstrated, the correlation between these two variables is not perfect. Therefore, the increments to standard Brownian motions are correlated, with:

$$E[dW \times dZ] = \rho dt$$

where ρ is the correlation coefficient.

Using the arbitrage free argument and the construction of a hedging portfolio leads to the solution of the model. It expresses the relationship at t between an observable futures price F for delivery in T and the state variables S and C.

$$F(S_t, C_t, t, T) = S_t e^{-C_t \frac{1-e^{-\kappa\tau}}{\kappa} + B_\tau} \quad (6.7)$$

where:

$$B_\tau = \left[\left(r - \widehat{\alpha} + \frac{\sigma_C^2}{2\kappa^2} - \frac{\sigma_S \sigma_C \rho}{\kappa}\right)\tau\right]\left[\frac{\sigma_C^2}{4}\frac{1-e^{-2\kappa\tau}}{\kappa^3}\right] + \left[\left(\widehat{\alpha}\kappa + \sigma_S\sigma_C\rho - \frac{\sigma_C^2}{\kappa}\right)\left(\frac{1-e^{-\kappa\tau}}{\kappa^2}\right)\right]$$

$$\widehat{\alpha} = \alpha - \left(\frac{\lambda}{\kappa}\right)$$

F : is the future's price

S_t : is the spot price

C_t : is the convenience yield

μ : is the drift of the spot price

σ_S : is the spot price volatility

α : is the long run mean of the convenience yield

κ : is the speed of adjustment of the convenience yield

σ_C : is the volatility of the convenience yield

ρ : is the correlation coefficient.

r : is the constant risk – free interest rate

λ : is the market price of convenience yield risk

$\tau = T - t$ is the maturity of the future's contract

This formulation represents a limit as it ignores that in commodity markets, price volatility is positively correlated with the degree of backwardation. This phenomenon has been widely commented and reported (Williams and Wright (1991), Ng and Pirrong (1994), Litzenberg and Rabinowitz (1995)) and it can be explained by the examination of arbitrage relationships between the physical and the futures markets. Such a study shows that the basis has an asymmetrical behavior: in contango, its level is limited to storage costs. This is not the case in backwardation arbitrage can always be relied upon to prevent the forward price from exceeding the spot price by more than net carrying cost, but can not be equally effective in preventing the forward price from exceeding the spot price by less than net carrying cost.

Furthermore, the basis is stable in contango, and volatile in backwardation. This phenomenon leads sometimes to consider that the convenience yield is an option (Heinkel, Howe and Hughes (1990), Milonas and Tomadakis (1997), Milonas and Henker (2001)) or that it has an asymmetrical behaviour. This assumption has been introduced in term structure models by Brennan (1991), Routledge, Seppi, and Spatt (2000), and Lautier and Galli (2001).

Brennan (1991) introduces an asymmetric convenience yield in his model because he takes into account a non-negativity constraint on inventory. However, he supposes that the convenience yield is deterministic. In the model presented by Routledge et al (2000), the asymmetry in the behaviour of the convenience yield is introduced in the correlation between the spot price and the convenience yield. This correlation is higher in backwardation than in contango. In this model, the convenience yield is an endogenous variable, determined by the storage process. However, it is stochastic. The two factors are the spot price and exogenous transitory shocks affecting supply and demand. Lautier and Galli (2001) propose a two-factor model inspired by Schwartz's model (1997), where the convenience yield is also mean reverting and acts as a continuous dividend. An asymmetry is however introduced in the convenience yield dynamic, it is high and volatile in backwardation, when inventories are rare. It is conversely low and stable when inventories are abundant. The asymmetry is measured by the parameter β. When the latter is set to zero, the asymmetrical model reduces to Schwartz's model.

Another approach of the term structure of commodity prices consists in considering the decreasing pattern of volatilities along the prices curve. In that situation, it is possible to infer that the two state variables are the extremities of the prices curve, namely the spot price and the long-term price. This kind of approach was followed by Gabillon (1992) and Schwartz and Smith (2000).

Gabillon (1992) uses the spot and the long-term prices as state variables. In this model, the convenience yield is an endogenous variable, which depends on the two factors. The use of the long-term price as a second state variable is justified by the fact that the long-term price can be influenced by elements that are exogenous to the physical market, such as expected inflation, interest rates, or prices for renewable energies. Thus, the spot and the long-term prices reassemble all the factors allowing the description of the term structure movements. The author retains a geometric Brownian motion to represent the behaviour of the long-term price. Moreover, the two state-variable are assumed positively correlated.

Schwartz and Smith (2000) propose a two-factor model that allows mean reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. Those factors are not directly observables, but they are estimated from spot and futures prices. Movements in prices for long-maturity futures contracts provide information about the equilibrium price level, and differences between the prices for short and long-term contracts provide information about short-term variations in prices. This model does not explicitly consider changes in convenience yields over time, but it is equivalent to the two-factor model proposed by Gibson and Schwartz (1990), in that the state variables in one of the models can be expressed as linear combinations of the state variables in the other model. The spot price is decomposed into two stochastic factors:

$$\ln S_t = \chi_t + \xi_t \quad (6.8)$$

where:

S_t : is the spot price

χ_t : is the short – term deviation in prices

ξ_t : is the equilibrium price level

The short-term deviation is assumed to revert to zero, following an Ornstein-Uhlenbeck process, and the equilibrium level is assumed to follow a Brownian motion process. The dynamics of these two state variables are the following:

$$
\begin{aligned}
d\chi_t &= -\kappa\chi_t dt + \sigma_\chi dz_\chi \\
d\xi_t &= \mu dt + \sigma_\xi dz_\xi
\end{aligned}
\qquad (6.9)
$$

where:

χ_t : is the short – term deviation in prices

ξ_t : is the equilibrium price level

κ : is the speed of adjustment of the short – term deviation

σ_χ : is the volatility of the short – term prices

dz_χ : is an increment to a standard Brownian motion associated with χ_t

μ : is the drift to the equilibrium price level

σ_ξ : is the volatility of the equilibrium price level

dz_ξ : is an increment to a standard Brownian motion associated with ξ_t

Changes in the short-term deviation represent temporary changes in prices (e.g. caused by abrupt weather alteration, supply disruption, etc) and are not expected to persist. They are tempered by the ability of market participants to adjust inventory levels in response to changing market conditions. Changes in the long-term level represent fundamental modifications, which are expected to persist. The latter are due to changes in the number of producers in the industry, and the long-term equilibrium is also determined by expectations of exhausting supply, improving technology for the production and discovery of the commodity, inflation, as well as political and regulatory effects.

6.3. General Framework

The characteristics of commodity prices are different than other prices in the financial markets to extend that are driven by supply and demand levels, can exhibit seasonal effects and mean reverting behaviour. In addition forward price exhibit a different behaviour depending on time to maturity as contracts get closer to their maturity date the volatility usually increases.

As a consequence of these characteristics one factor models are not suitable for pricing exotics structures within energy markets as:

- They are not able to generate all the commodity curve shapes observed in market.
- They are not able to generate all types of commodity curve changes observed in market.
- The changes over small time periods of any two commodity prices dependent variables will be perfectly correlated.

In line with the developments in the interest rate modeling energy markets model the forward price instead of the spot price. Under this framework future prices are viewed as a single point on the forward curves and the movement of the entire curve is modeled. Multi – factor models are more flexible and are able to generate additional commodity curve shapes and curve movements in relation to one – factor model. In addition, multi – factor models allow non – perfect correlations between different commodity variables.

In a general multi – factor diffusion model the fundamental assumption is that the future price can be represented by an n – dimensional vector process of state variables following the diffusion process

$$dX_{(i,t)} = \alpha_i(X_t, t)dt + \sum_{j=1}^{n} \beta_{ij}(X_t, t)dW_{(j,t)} \quad (6.10)$$

where α is a function from $\left[A \subseteq R^n\right] \times R_+$ into R^n and β is a function from $\left[A \subseteq R^n\right] \times R_+$ into a set of $n \times n$ of real numbers.

The absence of arbitrage implies that the future price process will be the following

$$dF_t = F_t\left(\mu(X_t,t)dt + \sum_{j=1}^{n}\sigma_j(X_t,t)dW_{(j,t)}\right) \quad (6.11).$$

Under the risk neutral probability measure Q can be written:

$$dX_{(i,t)} = \overline{\alpha}(X_t,t)dt + \sum_{j=1}^{n}\beta_{ij}(X_t,t)dW^{Q}_{(j,t)}\text{, where } \overline{\alpha}_i(X,t) = \alpha_i(X,t) - \sum_{j=1}^{n}\beta_{ij}(X,t)\lambda_j(X,t).$$

Using the above general framework and the Brigo and Mercurio (2006) specification the price at time t for the future contract expiring at time T $F(t,T)$ is given by the following diffusion process:

$$\frac{dF(t,T)}{F(t,T)} = A(t,T)dW_S(t) + B(t,T)dW_L(t) \quad (6.12)$$

Here it should be stated that the Brigo and Mercurio (2006) framework is referred to interest rate modelling, rather than commodities. The de-correlation between future contracts has much stronger impact in commodity than interest rate market.

The following two general stochastic processes in terms of two independent Brownian motions are considered:

$$\begin{aligned} dX_t &= -aX_t dt + x\sigma^{X}_{S,t}dW_S(t) \\ dY_t &= -bY_t dt + \eta\rho\sigma^{X}_{S,t}dW_S(t) + \eta\sigma^{Y}_{L,t}\sqrt{1-\rho^2}\,dW_L(t) \end{aligned} \quad (6.13)$$

X_t, Y_t : are general stochastic processes.

$\sigma^{X}_{S,t}$: is the volatility of $W_S(t)$.

$\sigma^{Y}_{L,t}$: is the volatility of $W_L(t)$.

a, b, x, ρ : are constants.

$W_S(t), W_L(t)$: are Brownian motions.

Assuming there is a relationship between $W_S(t), W_L(t)$ volatilities, the futures price at time t can be written as

$$\frac{dF(t,T)}{F(t,T)} = \sigma_t^F \left(1 + xe^{-\alpha(T-t)}\right)\rho_{(i,T-t)} dW_S(t) + \sqrt{1-\rho_{(i,T-t)}^2}\, v\sigma_t^F dW_L(t). \quad (6.14)$$

$F(t,T)$: is the future price as time t.

σ_t^F : is the instantaneous local volatility.

a, b, v, ρ : are constants.

$W_S(t), W_L(t)$: are Brownian motions.

As described by Derman, Kani and Zou (1996) if implied volatility of an option is the market's estimate of the average future volatility during the life of the option, the local volatility can be viewed as the market's estimate of volatility at a particular future time and market level. Local volatility surface can be estimated at a particular future time t and market level F using the implied volatility surface of standard European options. Derman and Kanin (1994) and Dupire (1994) have shown that local volatility is unique and can be calculated using the price of standard European options. The asset value is assumed to follow a random walk with the returns being normally distributed

$$\frac{dS}{S} = \mu dt + \sigma_t^S dZ\ ,$$

where S is the asset value and μ is the drift. The instantaneous local volatility is a deterministic function of the asset value and time and uniquely determined from the volatility smile by construction of an implied binomial tree. In the continuous limit the formula for σ_t^S becomes

$$\left(\sigma_t^S\right)^2 = 2\frac{\left(\dfrac{\partial C_{Kt}}{\partial t} + \mu K\dfrac{\partial C_{Kt}}{\partial t} + rC_{Kt}\right)}{K^2\dfrac{\partial^2 C_{Kt}}{\partial K^2}} \quad (6.15),$$

C_{Kt} is the market value of an option with strike price K and maturity t.

For the model developed at each node (n,i,j) where $n \geq 0$ is the time step and $0 \leq i \leq n$, $0 \leq j \leq n$ are the states, the price is assumed to be determined by the value of the factors X_t and Y_t:

$$\begin{aligned} X_{t+s} &= X_t e^{-\alpha s} + \int_t^{t+s} e^{-\alpha(t+s-u)} x \sigma_u^F dW_S(u) \\ Y_{t+s} &= Y_t + \int_t^{t+s} \rho \sigma_u^F dW_S(u) + \int_t^{t+s} v\sqrt{1-\rho^2}\sigma_u^F dW_L(u) \end{aligned} \tag{6.16}$$

From the above it is obtained the following:

$$\begin{aligned} X(t_{n+1}) &\approx X(t_n)(1-\alpha\Delta t_n) + A_0(n)\sqrt{\Delta t_n} Z_S \\ Y(t_{n+1}) &\approx Y(t_n) + A_1(n)\sqrt{\Delta t_n} Z_S + A_2(n)\sqrt{\Delta t_n} Z_L \end{aligned} \tag{6.17}$$

Clearly building multi – dimensional trees is not a trivial exercise. For the case of a two factor model a three – dimensional three is required.

The local volatility $\sigma_{n,i,j}$ then can be determined as the annualized standard deviation at (n,i,j). Hull and White (1994 and 1996) describe the process of building a 2 – factor tree. The first step is to fix time step Δt that will be used to build both processes. The variables X and Y will be defined as:

$$\begin{aligned} X(t) &= iJ_0(t-1) \\ Y(t) &= iJ_1(t-1) + jJ_2(t-1) \end{aligned} \tag{6.18}$$

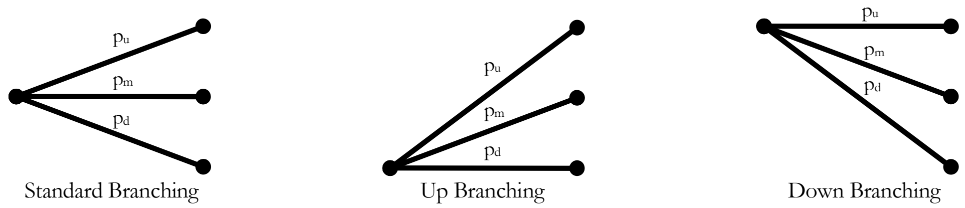
where $J_0(t) = A_0(t)\sqrt{3}\sqrt{\Delta t}$

Next step requires determining the maximum and minimum node index $J_{\max}$, $J_{\min}$ and which branching method to use:

a) At the top node $J_{\max}$

b) At intermediate node

c) At the bottom node $J_{\min}$



Starting with X at a specific node $iJ_0(t-1)$ can move to three possible states: $(i+1)J_0(t),(i-1)J_0(t),(i)J_0(t)$ with probabilities p_u, p_d and p_m respectively.

The probabilities are calculated by matching the first two moments of the three dynamics to the first and the second moments of the continuous time dynamics. It is clear that the condition $p_u + p_d + p_m = 1$ needs to be satisfied. The equations are:

$$\begin{aligned}
&iJ_0(t-1) - aiJ_0(t-1)dt = p_m iJ_0(t) + p_u(i+1)J_0(t) + p_d(i-1)J_0(t) \\
&A_0^2 dt = (p_u - p_d)^2 J_0^2 + J_0^2(p_u + p_d) - 2J_0^2((p_u - p_d)p_u - (p_u - p_d)p_d) \quad (6.19) \\
&p_u + p_d + p_m = 1
\end{aligned}$$

Having determined the p probabilities, a certain grid point $iJ_1(t-1) + jJ_2(t-1)$ can move to $iJ_1(t)+(j+1)J_2(t), iJ_1(t)+(j-1)J_2(t), iJ_1(t)+jJ_2(t)$ with probabilities $p_u^{'}, p_d^{'}, p_m^{'}$ respectively.

For independent processes X and Y $(\rho = 0)$ the transition probabilities are given below.

Table 6.1 – Tree Transition Probabilities

Y – move	X – move		
	Down	Middle	Up
Up	$p_u^{'} p_d$	$p_u^{'} p_m$	$p_u^{'} p_u$
Middle	$p_m^{'} p_d$	$p_m^{'} p_m$	$p_m^{'} p_u$
Down	$p_d^{'} p_d$	$p_d^{'} p_m$	$p_d^{'} p_u$

For non – zero correlation transition probabilities need to be modified to recover the correct univariate moments.

6.4. Barrier Options

Barrier options are one of the simplest and most commonly traded exotic options. These are the simplest path dependent options as are ordinary calls and puts that their pay off is contingent on another knock-in or knock-out event. The most basic type of barrier option is the single barrier that comes in four different types:

1. Down & In
2. Down & Out
3. Up & In
4. Up & Out

The terms "In" – "Out" barrier imply whether the option is activated or "dies" once the barrier is crossed. The terms "Down" – "Up" determine the barrier is crossed from below or above.

As described by Derman, Kani, Ergener and Bardhan (1995) pricing barriers using binomial or trinomial trees results in slow convergence especially when the barrier is close to spot. They acknowledge two types of errors. The first type of error is caused by the unavoidable existence of the tree itself, which "quantizes" the asset price and the instants in time at which it can be observed. The second type of error occurs because of the inability of the lattice to accurately represent the terms of the option i.e. for a chosen tree the available asset prices are fixed. In order to overcome these inaccuracies Derman, Kani, Ergener and Bardhan (1995) consider the application of an effective barrier and a modified barrier, between which lies the true barrier. Because the specified barrier lies between two sets of nodes on the tree, the correct option value is regarded as the one obtained by interpolating the two option values corresponding to moving the barrier up to the effective barrier and moving the barrier down to the modified barrier.

The following WTI Call barrier options are priced under the above model:

Table 6.2 – Barrier Structures

	Type	Maturity	Strike	Barrier
Structure E	Knock-Out	19-Aug-10	50	70
Structure F	Knock-In	19-Aug-10	50	70
Structure G	Knock-Out	19-Dec-11	85	100
Structure H	Knock-In	19-Dec-11	85	100
Structure I	Knock-Out	19-Dec-14	75	85
Structure J	Knock-In	19-Dec-14	75	85

6.5. Target Redemption Notes

The above model is used to estimate for valuation of Target Redemption Notes (TARN) in WTI NYMEX. Target Redemption notes are index linked notes that provide a sum of coupons until the accumulated amount of coupons has reached a pre-specified level. Once target is reached the note will be terminated with final payment of the par.

The note value is calculated as the sum of present values of the coupon payments and the par. As discussed by Brigo and Mercurio (2006) assuming the TARN has a set of payment times $T_1, T_2, \ldots, T_N$, a coupon c, a trigger level A and an overall sum of coupons S. The actual coupon $C(T_i)$ paid at time T_i is given by

$$C(T_i) = \begin{cases} c & i = 1,2,\ldots,k \\ \min\left(c, \left(S - \sum_{j=1}^{i-1} C(T_j) \right)^+ \right) 1\{\omega L(T_{i-1}, T_i) \leq \omega A\} & k < i < N \\ \left(S - \sum_{j=1}^{N-1} C(T_j) \right)^+ & i = N \end{cases} \tag{6.20}$$

where $\omega = 1 or (-1)$ is used to determine upper (lower) trigger level and where no trigger condition for the last payment so that $\sum_{j=1}^{N} C(T_j) = S$.

The last non – zero coupon payment occurs at random time $T_{i=\tau}$, that is also the actual maturity of the note as the notional is paid at par, where $\tau := \min\left\{ \tau : \sum_{j=1}^{\tau} C(T_j) = S \right\}$.

It is clear that the WTI NYMEX volatility generates uncertainty in the coupon payments paid on the coupon dates and uncertainty in the redemption date of the note.

The following payout TARN structures between Party A and Party B are examined.

a) Assuming that the target has not been reached Part A will pay $(K_{Put} - Monthly\Pr ice_i)^+$ and $(Monthly\Pr ice_i - K_{CallA})^+$

b) Party B will pay $\left(Monthly\Pr ice_i - K_{CallB}\right)^+$

c) The target event is occurred when on the first month that the accumulated coupon paid is greater or equal to a predetermined amount S.

$Monthly\Pr ice_i$ is the average WTI price over the month i. Clearly $K_{CallB} < K_{CallA}$ and graphically the above Payout is presented to the following Graph.

Graph 6.1 – Monthly Target Redemption Note Payoff Function

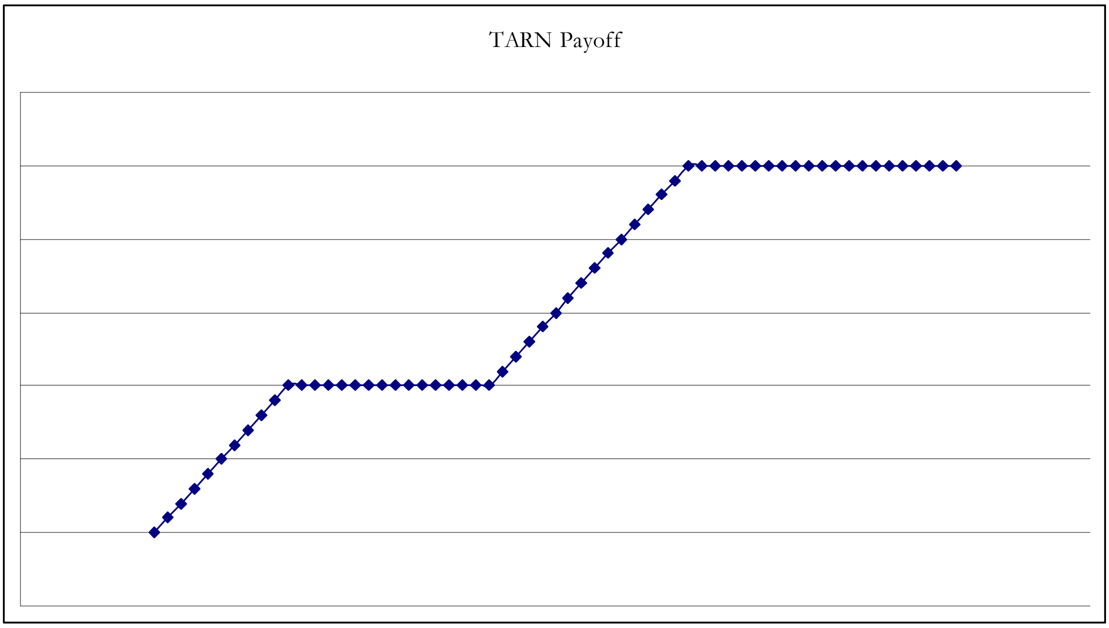


Table 6.3 depicts the assumed strike structures:

Table 6.3 – Target Redemption Notes Structures

	K_{Put}	K_{CallA}	K_{CallB}
Structure A	30	55	75
Structure B	40	55	65
Structure C	50	65	80
Structure D	60	70	90

For computational speed we assume that all structures mature in 2.6 years (i.e. end of September 2011).

6.6. European and Bermudan Swaptions

A typical exotic cost structure in oil market is one that offers buyers lower exposure to high oil prices by buying a strip of monthly capped swaps with low strikes and in return they sell the option to sell a similar structure on a future expiry date. A more advanced structure can involve call spreads and may include additional features such that after executing the trade a party can cancel the remaining cash flows after one or more pre-agreed expiry dates.

Clearly extendible and cancellable swaps are like a portfolio of plain vanilla swaps plus swaptions. Based on the above the following parity relationships should hold:

- Extendible Pay = Plain Vanilla Pay + Payer Swaption
- Extendible Receive = Plain Vanilla Receive + Receiver Swaption
- Cancelable Pay = Plain Vanilla Pay + Receiver Swaption
- Cancelable Receive = Plain Vanilla Receive + Payer Swaption

While European swaptions are straight forward exercise Bermudan swaptions are more complicated as one has to assess the value of the option to exercise versus the value to postpone the exercise for the future.

As described by Brigo and Mercurio (2006), consider a swap with first payment in T_α and paying at $T_{\alpha+1},...,T_\beta$ and assume that one has the right to enter the swap at any of the following times $T_h, T_{h+1},...,T_k$ with $T_\alpha \le T_h < T_k \le T_\beta$. For each l, select a set of times t^l as follows $T_l = t_1^l < t_2^l < ... < t_{d(l)}^l = T_{l+1}$. Set $l+1=\beta$ and position in a new time interval $[T_l, T_{l+1}]$ and add the remaining cash flows. While going backwards from time t_{i+1}^l to t_i^l, propagate backwards the vector of the portfolio prices as follows $P_{i,j}^l(T_s) = e^{-r_{i,j}^l(t_{i+1}^l - t_i^l)}\left[p_u P_{i+1,k+1}^l(T_s) + p_m P_{i+1,k}^l(T_s) + p_d P_{i+1,k-1}^l(T_s)\right]$ (6.21) for all $s = l+1,...,\beta$.

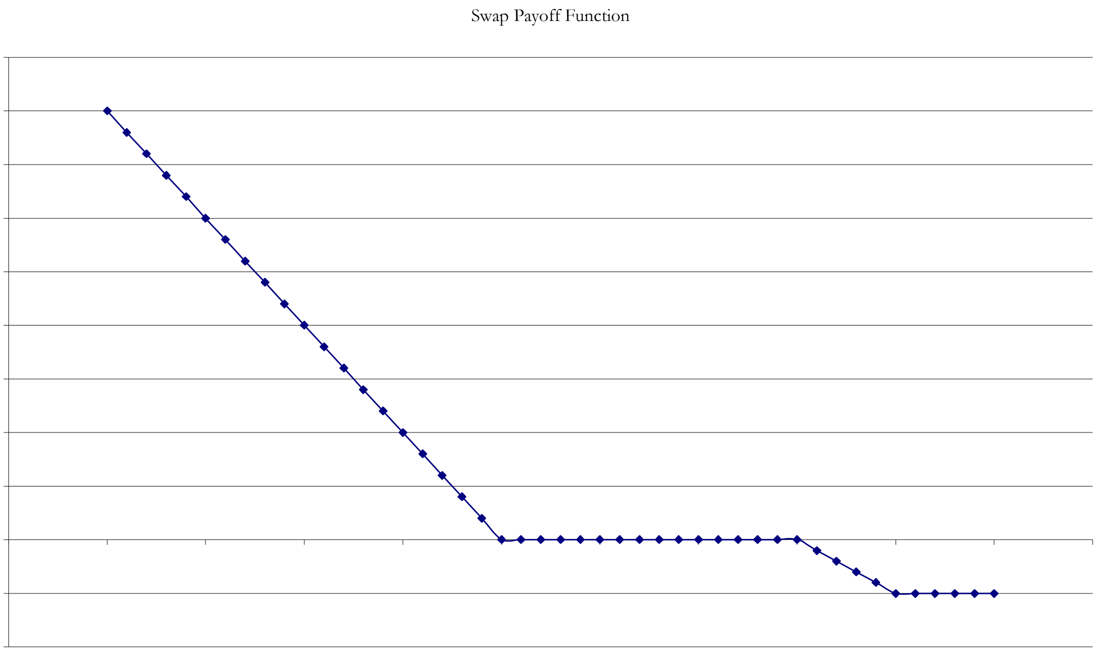
If $i > 1$ then decrease i by one and go back until $i = 1$, that implies $t_1^l = T_l$. The above process is repeated until $l = k$ that is the last point in time that the option can be exercised. For each level j in the current column of the tree the value of the underlying portfolio is calculated. The backwardly – Cumulated value from Continuation of the Bermudan swaption is defined as the value of portfolio in each node j of the current time level in the tree. Then set $i + 1 = d(l)$ and calculate backwards from time t_{i+1}^l to t_i^l the vector of the swap portfolio and the backwardly – Cumulated value from Continuation of the Bermudan swaption. The value of the underlying swap portfolio is given by the same as above formula. The backwardly – Cumulated value from Continuation of the Bermudan swaption (CC) is given by $CC_{i,j}^l = e^{-r_{i,j}^l\left(t_{i+1}^l - t_i^l\right)}\left[p_u CC_{i+1,k+1}^l + p_m CC_{i+1,k}^l + p_d CC_{i+1,k-1}^l\right]$ (6.22). Similarly if $i > 1$ then decrease i by one and go back until $i = 1$, that implies $t_1^l = T_l$. The process is repeated until we reach the first allowed exercise time T_h and there are no exercise options left as time moves backward. The current backwardly – Cumulated value from Continuation is rolled backwards until time 0.

The option to extend the following monthly payout structures between Party A and Party B are examined:

a) Part A will pay $\left(Monthly\Pr ice_i - K_{CallB}\right)^+$ and $2x\left(K_{Put} - Monthly\Pr ice_i\right)^+$.

b) Party B will pay $\left(Monthly\Pr ice_i - K_{CallA}\right)^+$.

Graph 6.2 – Monthly Swap Payoff Function



Structures K, L and M represent Bermudan extendibles that can be exercised quarterly up to the last exercise date that is the one below, while N, O and P are European extendibles with the relevant exercise date. Table 5.4 shows the assumed strike structures:

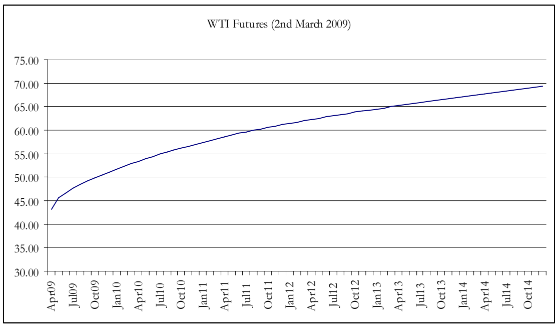
Table 6.4 – European and Bermudan Swaptions Structures

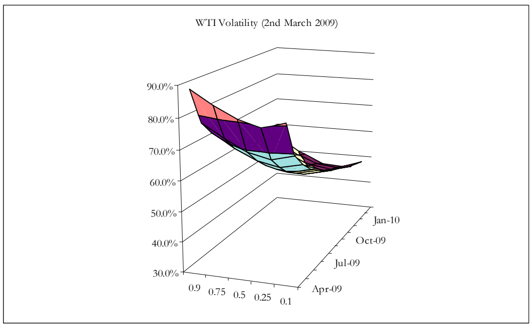
	Exercise	Maturity	K_{Put}	K_{CallA}	K_{CallB}
Structure K	30-Sep-10	31-Dec-10	40	55	60
Structure L	30-Sep-10	31-Dec-10	90	105	115
Structure M	30-Jun-10	30-Sep-10	70	80	85
Structure N	31-Dec-09	31-Dec-14	50	65	75
Structure O	30-Jun-10	31-Dec-20	80	85	90
Structure P	31-Dec-11	31-Dec-16	70	75	80

6.7. Data

Market environment is constructed using NYMEX data on 2nd March 2009 and presented on the graphs below.

Graph 6.3 – WTI Futures Curve and Volatility Surface as of 2nd March 2009

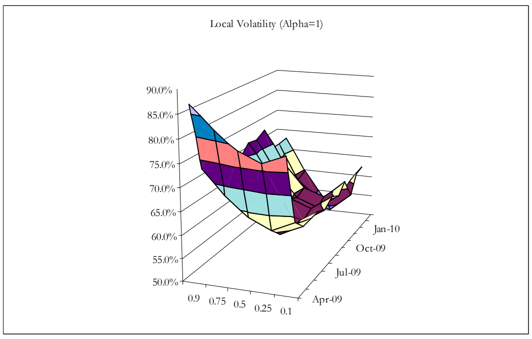


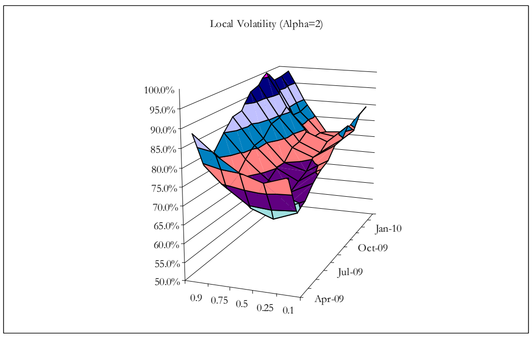


6.8. Results

The approach described above has the flexibility to generate different local volatility surfaces depending on the calibrated data. The impact of change in alpha to local volatility is shown on the graphs below.

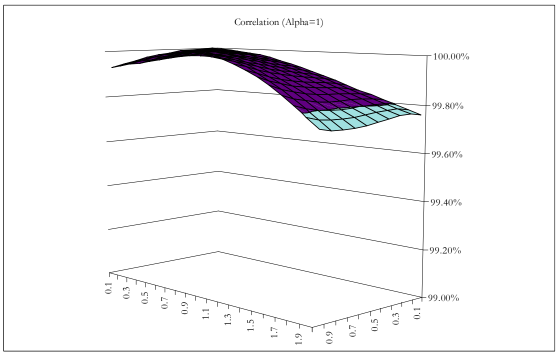
Graph 6.4 – Local Volatility Surfaces for Alpha=1 and Alpha=2

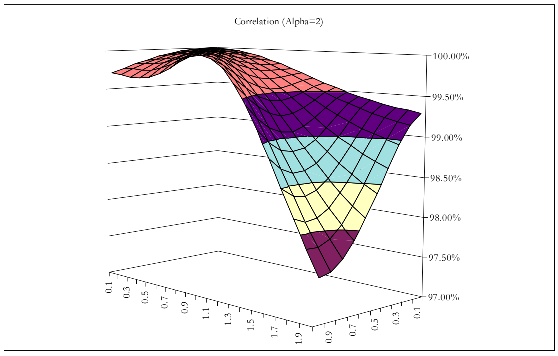




Moreover the model allows for de-correlation between the futures contract depending on the relationship between the volatilities and the level of alpha parameter.

Graph 6.5 – Correlation Surfaces for Alpha=1 and Alpha=2





As indicated in Table 5.1 capturing the smile is very important especially to value barrier options especially when future price is close to the barrier. For structures E and I that Knock-Out at \$70 and \$85 and the future prices at t_0 are \$55.28 and \$69.59 – August 2010 and January 2015 contracts respectively – the smile value is significantly higher in all cases. Another interesting case is Structure G with strike \$85 and upper Knock-Out barrier at \$100, while the futures price for the January 2012 contract is \$61.43 the smile value becomes significant as future price moves closer to strike.

Table 6.5 – Barriers Smile Value in relation to price

Structure F

	-5	-1	0	1	5	10	15
Smile Valuation	8.647	11.401	12.156	12.934	16.239	20.619	25.082
Non-Smile Valuation	9.165	11.739	12.423	13.121	16.049	19.975	24.148
Smile Value	-0.518	-0.339	-0.267	-0.187	0.190	0.644	0.934

Structure G

	-5	-1	0	1	5	10	15
Smile Valuation	0.100	0.119	0.125	0.130	0.153	0.187	0.216
Non-Smile Valuation	0.097	0.103	0.103	0.103	0.101	0.095	0.086
Smile Value	0.003	0.017	0.021	0.027	0.052	0.092	0.130

Structure H

	-5	-1	0	1	5	10	15
Smile Valuation	4.391	5.494	5.795	6.108	7.471	9.451	11.779
Non-Smile Valuation	4.135	5.378	5.718	6.068	7.583	9.714	12.101
Smile Value	0.256	0.116	0.078	0.040	-0.112	-0.262	-0.323

Structure I

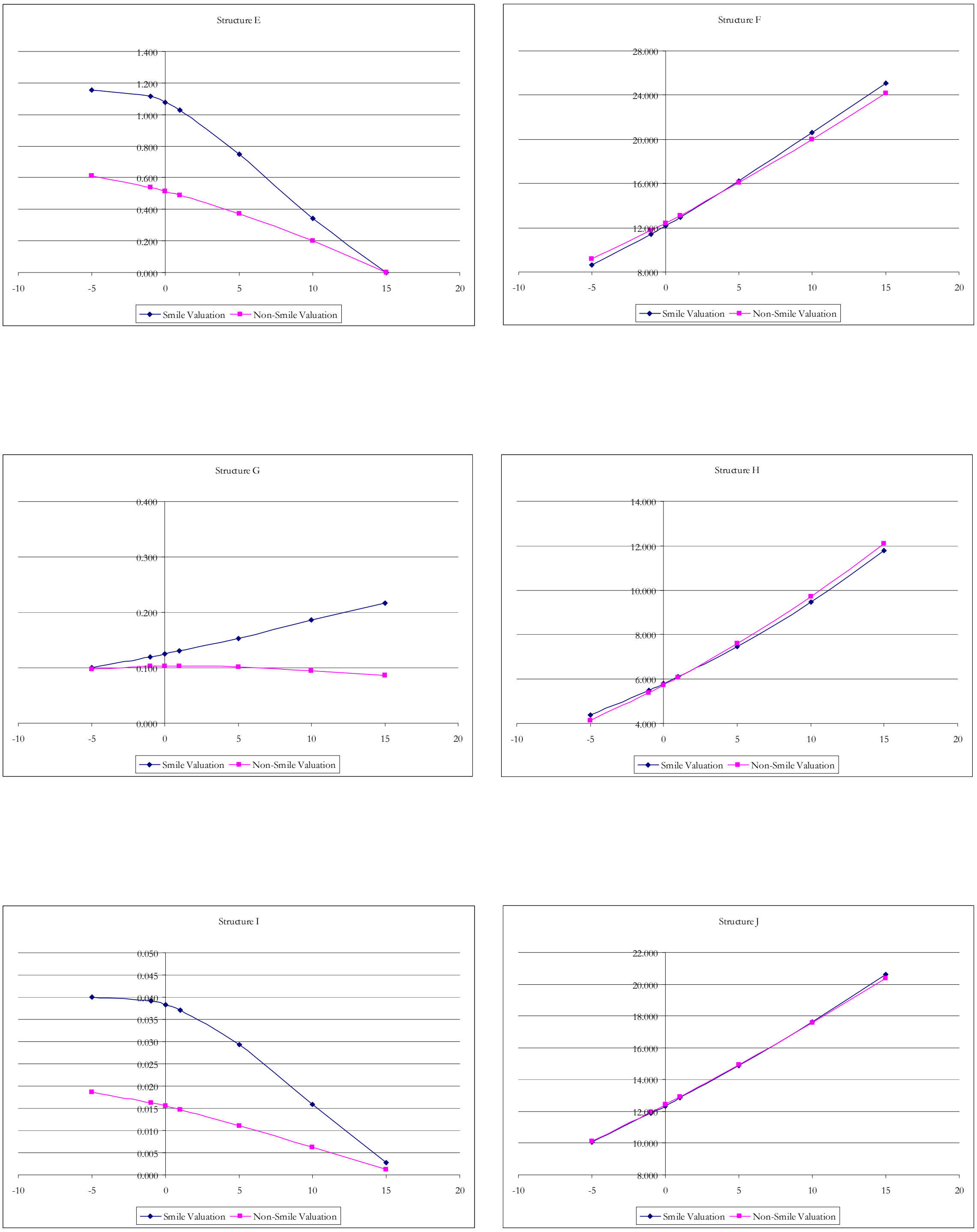
	-5	-1	0	1	5	10	15
Smile Valuation	0.040	0.039	0.038	0.037	0.029	0.016	0.003
Non-Smile Valuation	0.019	0.016	0.015	0.015	0.011	0.006	0.001
Smile Value	0.021	0.023	0.023	0.022	0.018	0.010	0.002

Structure J

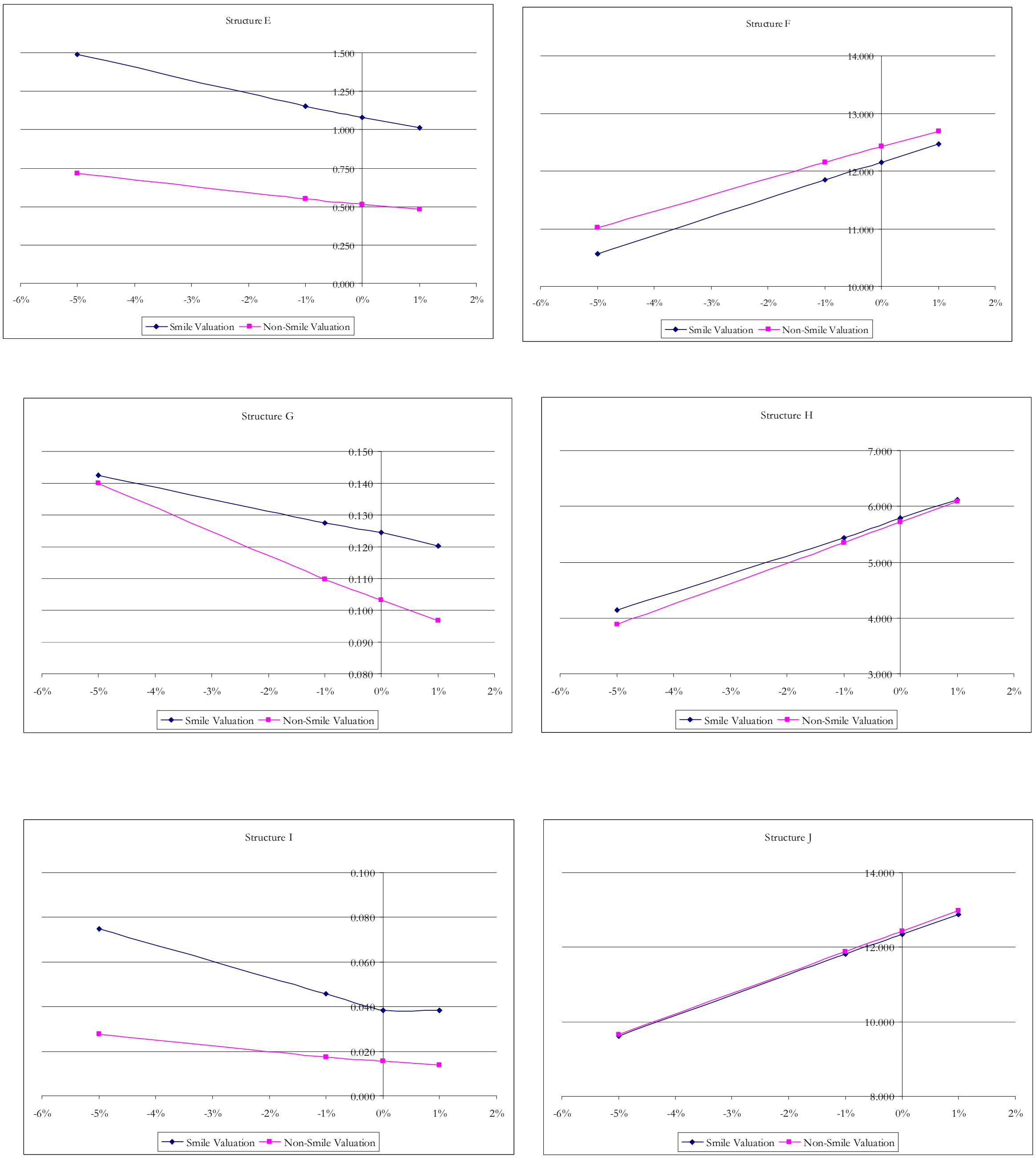
	-5	-1	0	1	5	10	15
Smile Valuation	10.082	11.876	12.351	12.837	14.877	17.634	20.593
Non-Smile Valuation	10.135	11.956	12.431	12.914	14.918	17.574	20.386
Smile Value	-0.053	-0.080	-0.080	-0.077	-0.041	0.060	0.207

Another important aspect of the smile value is related to the hedging on the structure. It is clear for the Graph 5.1 below that the change of the Call Knock – Out option changes more rapidly when is valued under smile. This behaviour is closer to reality as in practice for barrier options that price is moving closer to the barrier a more dynamic hedging strategy is required.

Graph 6.6 – Impact of Price Change for Barriers



Graph 6.7 – Impact of Volatility Change for Barriers



Regarding TARNs structures as indicated in Table 5.2 the smile impact is very significant for TARNs Structures A and B that the Call A strike is at $55.

Table 6.6 – TARS Valuation

	Smile Valuation	St. Dev.	Non-Smile Valuation	St. Dev.	Smile Value
Structure A	7.542	0.362	-5.679	0.243	13.221
Structure B	75.152	0.621	58.611	0.482	16.540
Structure C	211.486	0.739	204.220	0.557	7.266
Structure D	398.860	0.689	402.287	0.465	-3.427

Table 6.7 – TARNs Smile Value in relation to price

Structure A

	-5	-1	0	1	5	10	15
Smile Valuation	23.864	10.458	7.542	4.780	-4.902	-14.566	-22.245
Non-Smile Valuation	9.019	-3.112	-5.679	-8.078	-16.320	-24.295	-30.334
Smile Value	14.845	13.570	13.221	12.858	11.418	9.729	8.089

Structure B

	-5	-1	0	1	5	10	15
Smile Valuation	109.503	81.253	75.152	69.393	49.054	28.537	12.033
Non-Smile Valuation	96.893	65.338	58.611	52.323	30.803	10.355	-4.959
Smile Value	12.610	15.915	16.540	17.069	18.251	18.182	16.992

Structure C

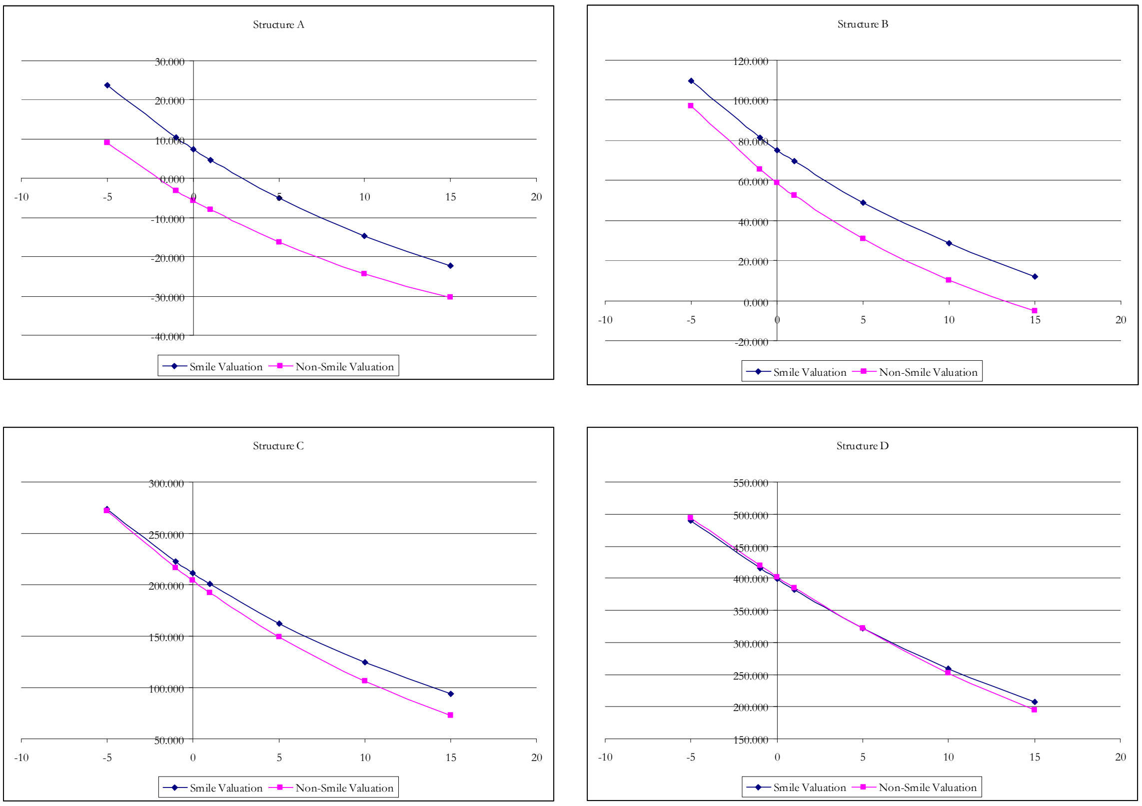
	-5	-1	0	1	5	10	15
Smile Valuation	273.247	222.697	211.486	200.721	162.636	124.259	93.588
Non-Smile Valuation	271.853	216.709	204.220	192.262	149.423	106.379	73.094
Smile Value	1.393	5.988	7.266	8.459	13.214	17.881	20.493

Structure D

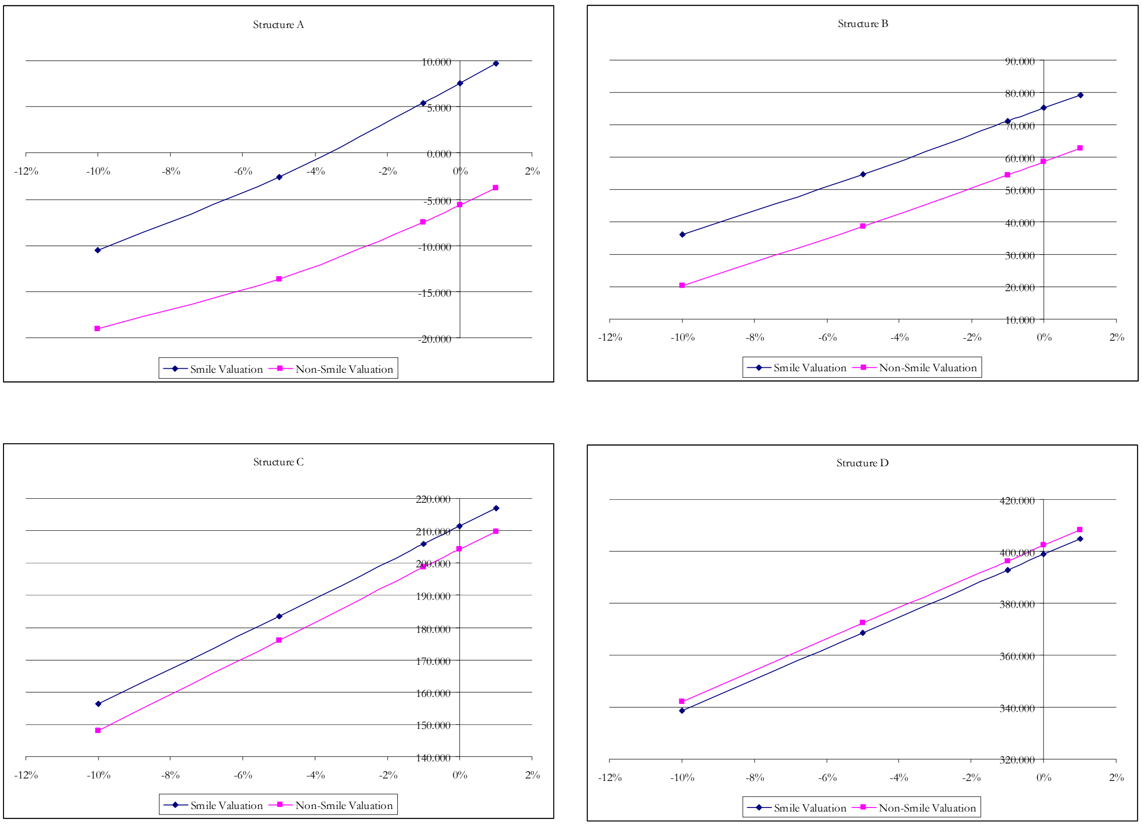
	-5	-1	0	1	5	10	15
Smile Valuation	489.183	415.762	398.860	382.482	322.590	259.321	207.002
Non-Smile Valuation	494.564	419.788	402.287	385.233	321.548	252.351	194.351
Smile Value	-5.381	-4.026	-3.427	-2.750	1.042	6.970	12.652

The importance of the model that incorporates the smile is clear on the Graph 5.1 below that presents the change of the value of the structure in relation to price. The smile valuation is clearly capturing better the market dynamics. On the other hand, as expected parallel, movements of volatility do not have significant impact on smile value (i.e. the difference between the smile and non – smile valuation).

Graph 6.8 – Impact of Price Change for TARNs



Graph 6.9 – Impact of Volatility Change for TARNs



The valuation of the European and Bermudan extendible structures is presented on Table 5.4 below. The absence of smile results a significant valuation error on the optionality to extend the structures described (the average valuation error is greater than 30%). Non – smile valuation is in favour of Party B as the specific structures offer protection against high WTI prices but the value to cancel the structure is more in the money for Party A[12] . In addition as it is clear for the Graph 5.5 the non – smile valuation can create greater hedging costs against the price movements as price changes are relative more steep.

12 The best way to view it is that Party A has the option to cancel the structure.

Table 6.8 – European and Bermudan Extendibles Smile Value in relation to price

Structure K

	-5	-1	0	1	5	10	15
Smile Valuation	14.991	19.210	20.215	21.366	25.653	31.054	35.896
Non-Smile Valuation	17.249	21.755	22.906	24.156	29.162	35.821	42.937
Smile Value	-2.258	-2.545	-2.691	-2.790	-3.509	-4.767	-7.041

Structure L

	-5	-1	0	1	5	10	15
Smile Valuation	1.740	2.396	2.564	2.735	3.627	4.998	6.681
Non-Smile Valuation	1.683	2.501	2.736	2.975	4.165	6.036	8.343
Smile Value	0.056	-0.105	-0.172	-0.240	-0.538	-1.038	-1.661

Structure M

	-5	-1	0	1	5	10	15
Smile Valuation	1.657	2.286	2.455	2.639	3.516	4.854	6.474
Non-Smile Valuation	2.038	2.935	3.160	3.409	4.592	6.332	8.355
Smile Value	-0.381	-0.649	-0.705	-0.770	-1.077	-1.478	-1.880

Structure N

	-5	-1	0	1	5	10	15
Smile Valuation	32.456	45.249	48.898	51.872	64.104	85.288	109.984
Non-Smile Valuation	47.629	64.386	68.851	73.598	93.624	120.643	149.417
Smile Value	-15.173	-19.136	-19.954	-21.726	-29.520	-35.355	-39.433

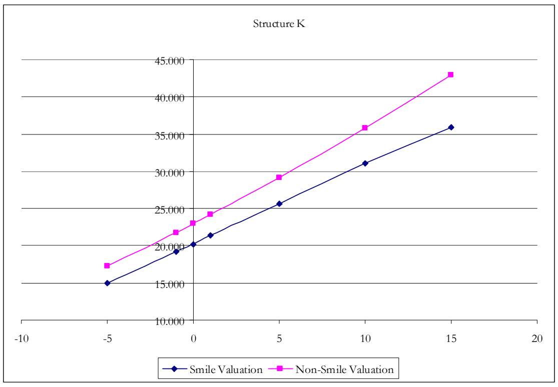
Structure O

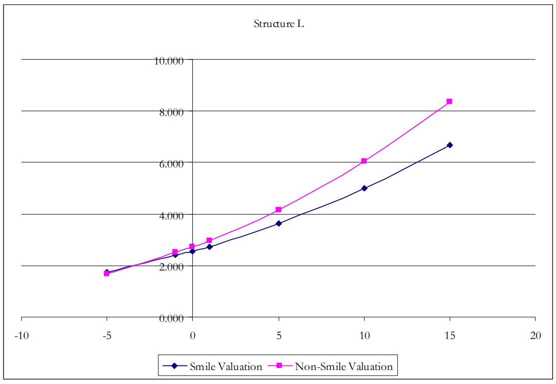
	-5	-1	0	1	5	10	15
Smile Valuation	0.833	1.205	1.310	1.440	1.984	2.820	3.920
Non-Smile Valuation	2.389	3.516	3.837	4.234	5.919	8.556	12.073
Smile Value	-1.556	-2.311	-2.526	-2.794	-3.935	-5.736	-8.153

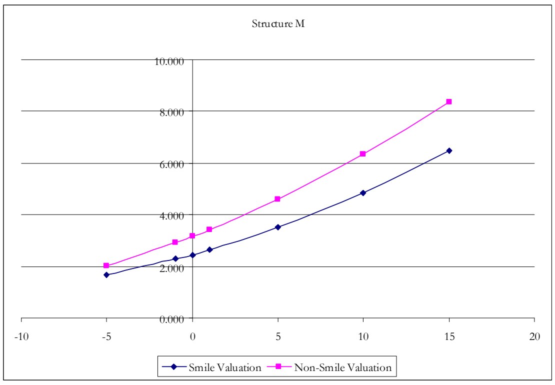
Structure P

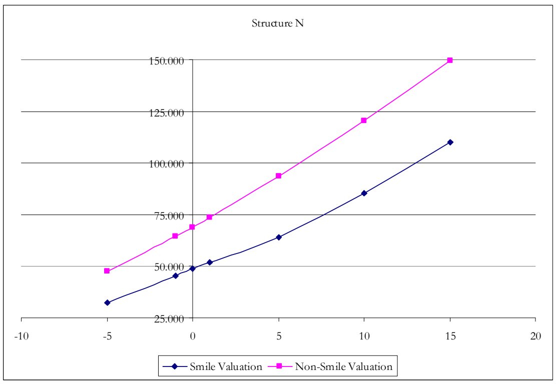
	-5	-1	0	1	5	10	15
Smile Valuation	10.621	12.846	13.427	14.081	16.827	20.922	25.843
Non-Smile Valuation	18.181	22.678	23.824	25.086	30.177	37.117	44.574
Smile Value	-7.560	-9.832	-10.397	-11.005	-13.350	-16.195	-18.731

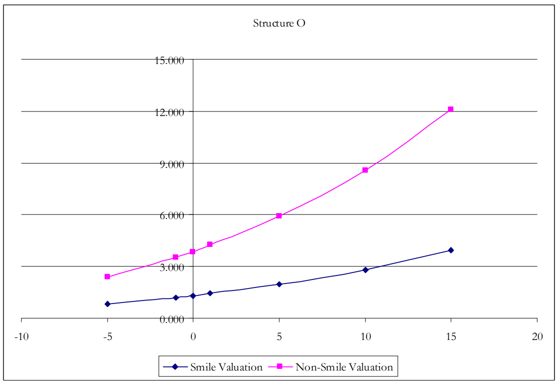
Graph 6.10 – Impact of Price Change for Extendibles

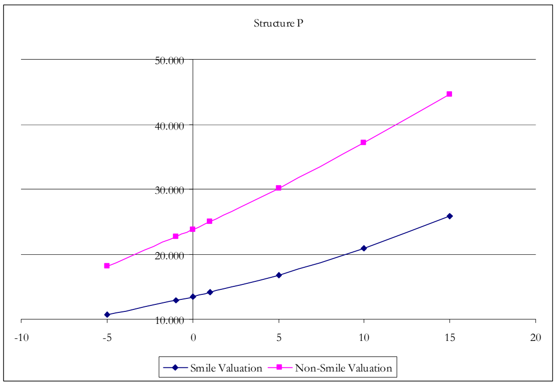




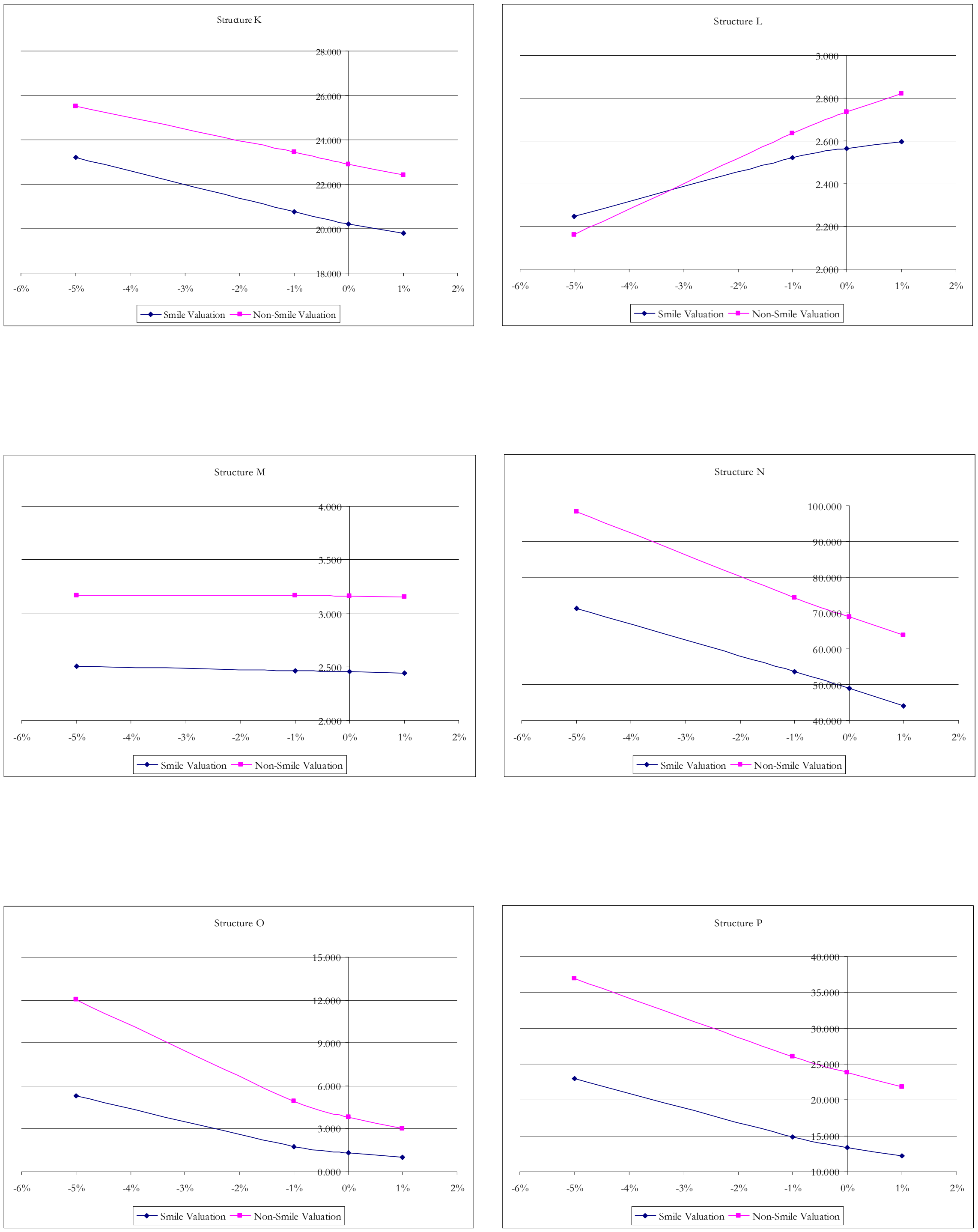








Graph 6.11 – Impact of Volatility Change for Extendibles



6.9. Hedging

The implication of the smile is important not only for the pricing but also for the hedging. The structures used to investigate the implication of the smile in hedging are the following:

Table 6.9 – Barrier Option Structure for Hedging

	Type	Maturity	Strike	Barrier
Structure G	Knock-Out	19-Dec-11	85	100

Table 6.10 – Extendible Structure for Hedging

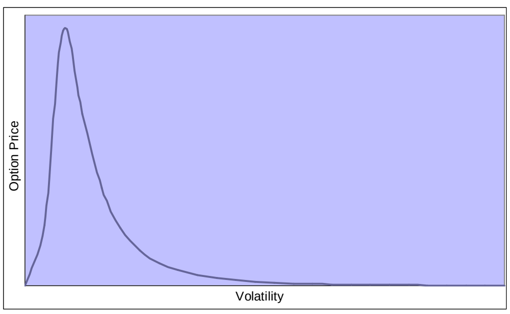
	Exercise	Maturity	K_{Put}	K_{CallA}	K_{CallB}
Structure K	30-Sep-10	31-Dec-10	40	55	60

The first part of the analysis is referred to barrier options and the second to the extenbible structures.

6.9.3. Barrier Option Hedging

The discontinuity of the barrier option payoff complicates the hedging, especially in cases where the barrier is in the money region such as up and out call. The importance of reflecting the proper volatility dynamics within the model is clear from the Graph 5.7 below:

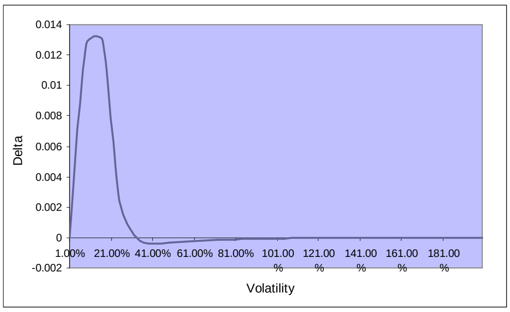
Graph 6.12 – Up and Out Call Price vs. Volatility

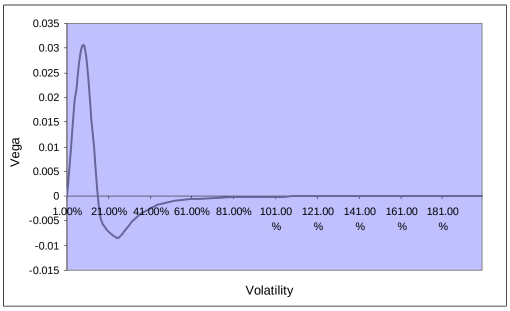


As expected volatility brings the barrier closer in a non-linear way so the slope of the skew should be taken into account.

In addition the delta and the vega of the Up and Out call will take negative values as the value of the option will jump down to zero when is in – the – money. As market approaches the strike, up to a specific point option delivers more and more delta as the probability of exercise increases. Beyond this point delta becomes negative reflecting the fact that the value of the option can be zero with some probability, such some of the delta accumulated needs to be sold – off. The Delta and Vega profile as function of volatility for Structure G (Up and Out Call) is presented below:

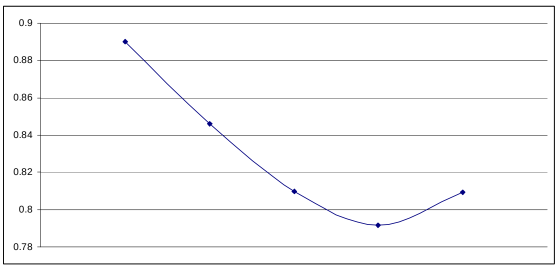
Graph 6.13 – Impact of Volatility for Up and Out Call





At the point of pricing market was indicating negative skew i.e. the volatility is higher for puts vs. the calls with direct impact on the hedging strategy as described later in the section.

Graph 6.14 – Market Skew as of 2nd March 2009



The payout of the up and out call $C_{F,t}$ is given by the boundary:

1) $C_{F,2.85} = (F - 85)^{+}$ if $F \leq 100$

2) $C_{100,t} = 0$ if $0 \leq t \leq 2.85$

A natural decision for the first condition is a European call option with strike 85. In order to match the 2nd boundary condition the following process is followed:

1) The life of the option is divided into N steps with length Δt.

2) A European call option with strike 100 and maturity 2.85 to match the boundary at the point $[100, (N-1)\Delta t]$.

3) Choose a European call option with strike 100 and maturity $(N-1)\Delta t$ to match the boundary at the point $[100, (N-2)\Delta t]$ etc.

Note that options are chosen in such a way that their value is zero on the parts of the boundary that is matched by the earlier option. The option described at point 2) above has zero value on the boundary matched by the option with strike 100 that is used for the first boundary condition.

For the Structure G above time is divided into 14 intervals and 15 vanilla options are used. At time 0.38 the portfolio consists of a European call option with strike 85 and 14 options with strike 100 and maturities 0.38 up to 2.85 years. The replication portfolio position is the following:

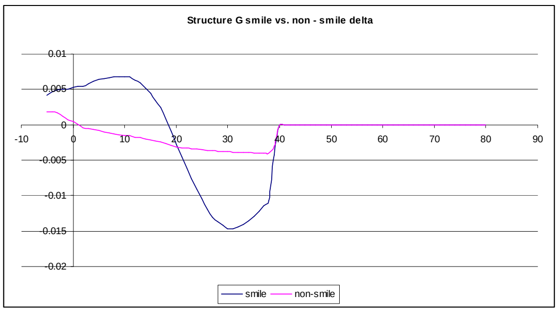
Table 6.11 – Hedging Portfolio for Structure G

Type	Position	K	T to maturity
Call16	1.000	85	2.85
Call15	-3.224	100	2.85
Call14	1.183	100	2.66
Call13	0.364	100	2.47
Call12	0.171	100	2.28
Call11	0.099	100	2.09
Call10	0.065	100	1.90
Call9	0.047	100	1.71
Call8	0.036	100	1.52
Call7	0.028	100	1.33
Call6	0.022	100	1.14
Call5	0.019	100	0.95
Call4	0.016	100	0.76
Call3	0.014	100	0.57
Call2	0.012	100	0.38
Call1	0.001	100	0.19

The value of the position is - 0.123.

The existence of negative skew in the specific cases results that delta enters to negative region in higher price under smile dynamics vs. non – smile.

Graph 6.15 – Smile vs. non – smile delta for Structure G



The hedging portfolio delivers delta that more consistent with the delta delivered under smile valuation. To hedge the structure G the above vanilla portfolio should be shorted and obviously unwound when any of the boundary conditions is reached. In addition depending on the market movement some re-balancing might be required. Assuming no re-balancing the delta delivered for a relative wide range of price movements is presented on the Table 6.12.

Table 6.12 – Delta delivered Smile, Non-smile and Hedging portfolio

	Smile Delta	Non-smile Delta	Hedge Delta
-5	0.0042	0.0019	0.0054
-3	0.0049	0.0016	0.0057
-1	0.0051	0.0007	0.0059
0	0.0053	0.0004	0.0059
1	0.0055	0.0001	0.0059
3	0.0058	-0.0006	0.0058
5	0.0064	-0.0008	0.0055
7	0.0066	-0.0012	0.0051
9	0.0068	-0.0014	0.0046
11	0.0068	-0.0015	0.0040

The portfolio the performing also to hedge the vega exposure of the barrier option. Clearly the vega hedging requires more dynamic trading of the hedging portfolio in order to be more efficient. Here it should be noted that the vega delivered by the smile approach is less vs. the non – smile valuation as the market for the specific period was indicating negative skew as described above.

Table 6.13 – Vega delivered Smile, Non-smile and Hedging portfolio

	Smile Vega	Non-smile Vega	Hedge Vega
-5%	-0.0036	-0.0086	-0.0015
-3%	-0.0039	-0.0075	-0.0023
0	-0.0043	-0.0065	-0.0066
2%	-0.0049	-0.0059	-0.0053
4%	-0.0047	-0.0050	-0.0050

The hedging strategy described above is examined also in a dynamic framework. The following portfolio is assumed:

$$P_t = G_t + H_t \text{ (6.23)}$$

where:

P_t :is the value of the portfolio at time t.

G_t :is the value of the Barrier Option G at time t.

H_t :is the value of the hedging position H at time t.

As described by Fusai and Roncoroni (2008) the aim is to estimate the expected value $\theta = \mathrm{E}(X)$ of a random variable X with distribution P_X , on the underlying probability space (Ω, F, P). A sample mean of this variable is any random average given by the following equation:

$$\hat{\theta}_n(X) = \frac{1}{n}\sum_{i=1}^{n} X^{(i)} \text{ (6.24),}$$

where $X = \left(X^{(1)},\ldots,X^{(n)}\right)$ is a random vector with independent and identically distributed components with common distribution P_X . If $x = (x_1,\ldots,x_n)$ is a sample of this vector, then $\hat{\theta}_n$ can be taken as an approximation to the target quantity θ for at least two reasons. First, this quantity has mean θ and variance $Var(X)/n$. This suggests that for n sufficiently large, the estimation $\hat{\theta}_n$ converges to the target quantity, as the law of large numbers states that this is the case. Second, according to the central limit theorem a normalized centered sample means converge in distribution to a standard normal variable i.e.

$$z_n = \frac{\hat{\theta}_n(X) - \theta}{\hat{\sigma}_n/\sqrt{n}} \xrightarrow{d} N(0,1) \text{ as } n \to \infty \text{ (6.25).}$$

The above expression means that the cumulative distribution function of the random variable z_n converges pointwise to the cumulative distribution function of a Gaussian variable with zero mean and variance once. The normalization is performed by using the unbiased estimator of the mean square error

$$\hat{\sigma}_n(X)=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\hat{\theta}_n(X)\right)^2} \quad (6.26).$$

The estimation error $\hat{\theta}_n(X)-\theta$ is approximately distributed as a normal $N(0,\hat{\sigma}_n/n)$. Given the above the simulation algorithm to value a derivative with pay-off at time $T, Q(X_{T,T})$ is the following:

1. Fix n "large".
2. Generate n independent paths $x_{t,T}^1,\ldots,x_{t,T}^n$ and $y_{t,T}^1,\ldots,y_{t,T}^n$, of processes X and Y on $[t,T]$.
3. Compute the discount factor and the pay – off over each path $\left[x_{t,T}^{(i)},y_{t,T}^{(i)}\right]$.
4. The present value of the pay – off over each path is given by $V^{(i)}=\exp\left(-\int_t^T r(u)du\right)\times Q\left(x_{t,T}^{(i)},y_{t,T}^{(i)}\right)$ (6.27).
5. Return the sum $V^{(1)},\ldots,V^{(n)}$ divided by n.

In order to examine the performance of the portfolio a number of 100,000 simulations are performed between two points in time and the results for specific point are returned and presented. The time intervals are the following:

- 2nd March 2009 – 24th July 2009
- 24th July 2009 – 23rd February 2010
- 23rd February 2010 – 23rd October 2010

- 23rd October 2010 – 23rd February 2011

- 23rd February 2011 – 23rd September 2011

The rational that was followed to determine the time intervals is based on the market practice that the strategy for the exotic trades hedging is reviewed usually after 4 or more months. In addition the hedging position is re – balanced if the difference between the barrier option and the hedging portfolio is greater than 10%.

The results for each time interval are presented below. For the first simulation for the time interval between 2nd March 2009 and 24th July 2009 the value of the barrier option and the value of each option of the hedging portfolio, with the initial positions are presented under Table 6.14. Note that the value of options 1 – 2 is 0 as they have expired, while the value option 3 is close to 0 as it is deep out – of – the – money.

Table 6.14 – Barrier Option Value and Hedging Portfolio 24th July 2009

Type	Position	Value
Call16	1.000	9.68
Call15	-3.224	-20.99
Call14	1.183	7.26
Call13	0.364	2.12
Call12	0.171	0.89
Call11	0.099	0.46
Call10	0.065	0.24
Call9	0.047	0.16
Call8	0.036	0.10
Call7	0.028	0.07
Call6	0.022	0.03
Call5	0.019	0.02
Call4	0.016	0.01
Call3	0.014	0.00
Structure G		0.22

Based on the formula (5.14) it is apparent that the hedging positions needs to be re – balanced, resulting a gain of 0.27 from selling some of the options. The updated positions are presented under Table 6.15. The value of the re – balanced position is – 0.213. Note that Call option 3 also is closed.

Table 6.15 – Re – Balanced Hedging Position 24th July 2009

Type	Position	Value
Call16	1.000	9.6826
Call15	-3.223	-20.9992
Call14	1.183	7.2592
Call13	0.364	2.1193
Call12	0.161	0.8380
Call11	0.090	0.4136
Call10	0.055	0.2057
Call9	0.037	0.1217
Call8	0.026	0.0734
Call7	0.018	0.0427
Call6	0.012	0.0188
Call5	0.009	0.0085
Call4	0.006	0.0026
Call3	0.000	0.0000
Structure G		0.2227

The next date that the hedging strategy will be reviewed is on 23rd February 2010. Following up the same simulation process as above, the value of the hedging portfolio and the barrier position is presented in Table 6.16. Again options 3 – 5 have expired and option 6 has a value close to 0.

Table 6.16 – Barrier Option Value and Hedging Portfolio 23rd February 2010

Type	Value
Call16	9.3135
Call15	-19.0092
Call14	6.3925
Call13	1.7933
Call12	0.6648
Call11	0.3174
Call10	0.1450
Call9	0.0716
Call8	0.0336
Call7	0.0110
Call6	0.0000
Structure G	0.2452

So as of 23rd February 2010 the position is writing loses of – 0.031. Given that the difference between the position and the hedging portfolio is not material – 0.021 no re – balancing of the hedging portfolio is required.

Next simulation is performed between 23rd February 2010 and 23rd October 2010. Based on the simulation for this time interval the estimated Profit and Loss is – 0.094. In addition as it is indicated by Table 6.17 a re – balancing of the portfolio is required. Table 6.18 represents the hedging portfolio after re-balancing.

Table 6.17 – Barrier Option Value and Hedging Portfolio 23rd October 2010

Type	Value
Call16	3.1022
Call15	-5.8172
Call14	1.7965
Call13	0.4661
Call12	0.1307
Call11	0.0431
Call10	0.0027
Structure G	0.1606

Table 6.18 – Re – Balanced Hedging Position 23rd October 2010

Type	Position	Value
Call16	1.0000	3.1022
Call15	-3.2236	-5.8172
Call14	1.2583	1.9118
Call13	0.3643	0.4661
Call12	0.1610	0.1307
Call11	0.0897	0.0431
Call10	0.0551	0.0027
Structure G		0.1606

The next to the last simulation is on the 23rd February 2011 and the value of the Barrier Option and the Hedging Portfolio are given on Table 5.19. So as of 23rd February 2011 the position is writing gains of 0.016 and in addition the difference between the position and the hedging portfolio is not material so no re – balancing of the hedging portfolio is required.

Table 6.19 – Barrier Option Value and Hedging Portfolio 23rd February 2011

Type	Value
Call16	4.2419
Call15	-7.0930
Call14	2.1770
Call13	0.4356
Call12	0.0448
Call11	0.0001
Structure G	0.2092

Final simulation is performed between 23rd February 2011 and 23rd September 2011. Based on the simulation for this final time interval the estimated Profit and Loss is – 0.011. In addition as it is indicated by Table 6.20 the values of the position and the hedging portfolio are in line so again no re – balancing is required.

Table 6.20 – Barrier Option Value and Hedging Portfolio 23rd September 2011

Type	Value
Call16	1.7743
Call15	-2.0809
Call14	0.0570
Structure G	0.2539

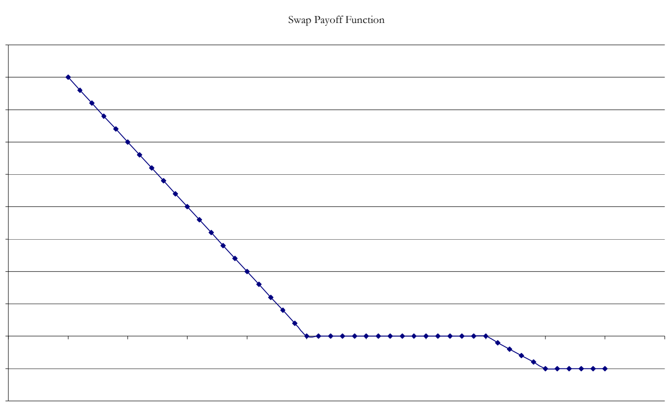
As indicted by the analysis above the Total Profit and Loss (including both portfolio management and re-hedging positions) is 0.028.

6.9.3. Bermudan Extendible Structure Hedging

The hedging of the Bermudan Extendible structures is one of the most challenging problems within energy market. These structures are used quite extensively by airlines companies and oil distillers. The structures create exposure to forward volatility and forward smile and proper risk management is essential in order to minimize unnecessary Profit and Loss fluctuations and costs. The approach below will hedge the vega exposure using liquid hedging instruments.

A natural assumption is to use vanilla swaptions, caps and / or floors with different exercise dates that generate the same payoff.

Graph 6.16 – Bermudan Extendible Monthly Payoff



The general framework that is used to setup the hedging strategy is the following:

1) Identify n number of $h_i, 1 \le i \le n$ hedging instruments that will be used to hedge the Bermudan structure that has q extension dates with $n \prec q$.

2) Construct the following portfolio consisting of the Bermudan structure and its hedging instruments, $Bermudan - \sum_{i=1}^{n} w_i h_i$ where w_i represents the amount – weight of the hedging i selected.

3) The vega of the portfolio is the following $\frac{\partial Bermudan}{\partial \sigma_j} - \sum_{i=1}^{n} w_i \frac{\partial h_i}{\partial \sigma_i}$ (6.28), where σ_i is the market volatility for each hedging instrument and $\sigma_j, 1 \le j \le q$ is the risk of the Bermudan.

4) The weights w_i of the hedging instruments are chosen in order to minimize the vega exposure of the Bermudan structure. The exercise is a straight forward linear algebra application:

   a. Define matrix M where $M_{jk} = \frac{\partial h_k}{\partial \sigma_j}$ and vectors U, w such $U_j = \frac{\partial Bermudan}{\partial \sigma_i}$ and $w = (w_1, w_2, \ldots, w_n)^T$.

   b. The vega exposure of the portfolio is $U - Mw$.

   c. Since $n \prec q$ it is not expected to eliminate q with n vanilla instruments so the problem is to minimize the sum of squares of the vega risks i.e. $\min(U - Mw)^T (U - Mw)$ (6.29).

   d. The solution of the above assuming $n \prec q$ is $w = \left(M^T M\right)^{-1} MU$.

Structure K provides the option to extend the following monthly payout structures between Party A and Party B:

a) Part A will pay $(Monthly\Pr ice_i - K_{CallB})^+$ and $2x(K_{Put} - Monthly\Pr ice_i)^+$.

b) Party B will pay $(Monthly\Pr ice_i - K_{CallA})^+$.

The strikes are the following $K_{Put} = 40$, $K_{CallA} = 55$ and $K_{CallB} = 60$. The final maturity of the trade is Dec – 10 and the exercise frequency is quarterly with first exercise Mar – 09 and last exercise Sep – 10 i.e. the structure has 7 possible extension dates.

The vanilla products that are natural hedging instruments for the specific structure, are caps with strikes 55 and 60 and floors with strike 40. Viewing the hedging problem from Party's B perspective the hedging position is $-Floor_{40} + Cap_{55} - Cap_{60}$. The dates we choose for the hedging instrument are Jun – 09, Mar – 10 and Sep – 10, so the hedging portfolio becomes

$$
\begin{aligned}
&w_{Jun-09}(-Floor_{40} + Cap_{55} - Cap_{60}) + \\
&w_{Mar-10}(-Floor_{40} + Cap_{55} - Cap_{60}) + \\
&w_{Sep-10}(-Floor_{40} + Cap_{55} - Cap_{60})
\end{aligned}
$$

Using the algorithm described above results are the following:

$$
\begin{aligned}
w_{Jun-09} &= 0.20 \\
w_{Mar-10} &= 0.96 \\
w_{Sep-10} &= 0.65
\end{aligned}
$$

The Table 6.21 below represents the vega delivered by the Bermudan structure and its hedges as the current level of volatility

Table 6.21 – Vega delivered by Bermudan structure and Hedging position

	Vega
Bermudan Structure	**-0.472**
Hedge Jun-09	0.083
Hedge Mar-10	0.333
Hedge Sep-10	0.055
Total Hedges	**0.471**

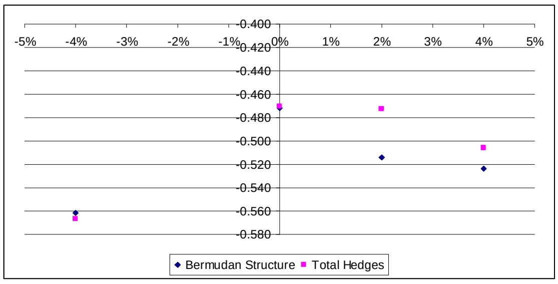
As presented in Table 6.22 the hedging strategy delivers also a delta position that can be used to hedge the delta of the Bermudan structure. The remaining Delta can be hedged directly in the WTI futures market.

Table 6.22 – Delta delivered by Bermudan structure and Hedging position

	Delta
Bermudan Structure	**0.976**
Hedge Jun-09	-0.091
Hedge Mar-10	-0.588
Hedge Sep-10	-0.169
Total Hedges	**-0.849**

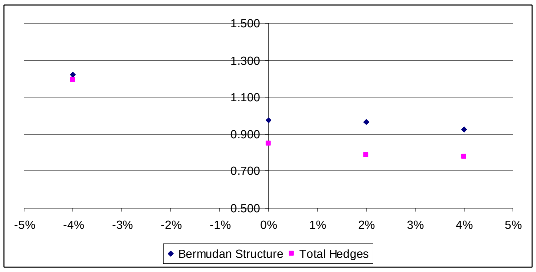
One of the advantages of the above strategy is that provides a relative good hedge for the second order risks such as $\partial Vega / \partial Volatility$ and $\partial Delta / \partial Volatility$. Graph 5.17 represents how the vega exposure of the Bermudan structure changes with respect to volatility. For a better comparison vega exposure of the hedges is presented on the same sign with the Bermudan structure.

Graph 6.17 – Vega Exposure change with respect of volatility



Similarly the delta delivered by the hedges when volatility changes can be used to partly hedge the delta exposure of the structure to the changes in volatility, with the remaining delta being hedged directly in the futures market. As above for comparison same sign is applied for the structure and the hedges.

Graph 6.18 – Delta Exposure change with respect of volatility



Using the same methodology as described in Paragraph 6.8.1 the hedging strategy above is examined also in a dynamic framework. Similarly in order to examine the performance of the portfolio a number of 100,000 simulations are performed between two points in time and the results for specific point are returned and presented. The main reason that time intervals are re – defined is that the maturity of the trade is different. The time intervals are the following:

- 2nd March 2009 – 24th May 2009
- 24th May 2009 – 24rd September 2009
- 24rd September 2009 – 24rd January 2010

As of 2nd March 2009 the value of the Bermudan structure and the hedging position is as follows.

Table 6.23 – Bermudan Structure and Hedging Portfolio 2nd March 2009

Type	Value
Hedge 3	-5.843
Hedge 2	-11.712
Hedge 1	-2.665
Structure K	20.215

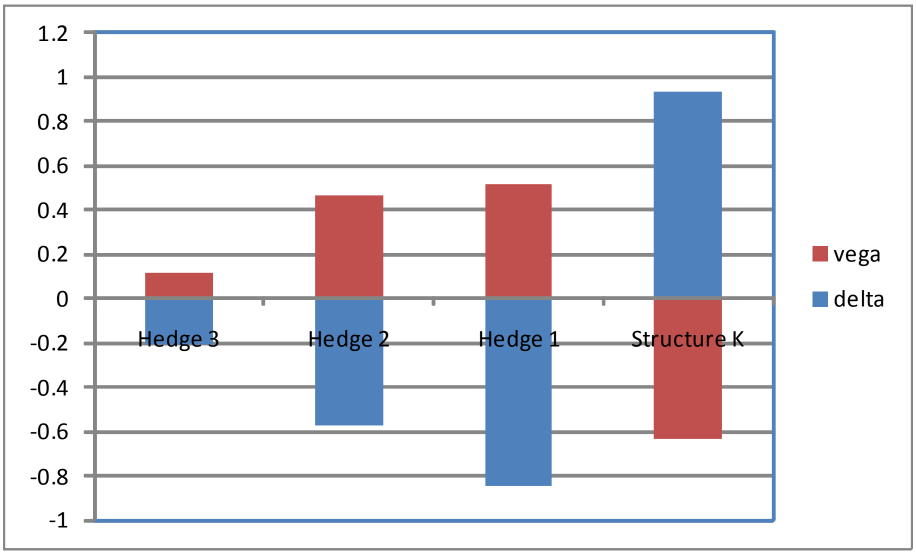
For the first simulation for the time interval between 2nd March 2009 and 24th May 2009 the value of the Bermudan structure and the hedging portfolio are presented under Table 6.24.

Table 6.24 – Bermudan Structure and Hedging Portfolio 24th May 2009

Type	Value
Hedge 3	-6.806
Hedge 2	-13.933
Hedge 1	-1.287
Structure K	22.102

Based on the simulation for this final time interval the estimated Profit and Loss is 0.082. In addition as it is indicated by Table 6.24 above the values of the position and the hedging portfolio are in line so no re – balancing is required. Moreover it is interesting to examine the vega and delta that are delivered.

Graph 6.19 – Vega and Delta Exposure as of 24th May 2009



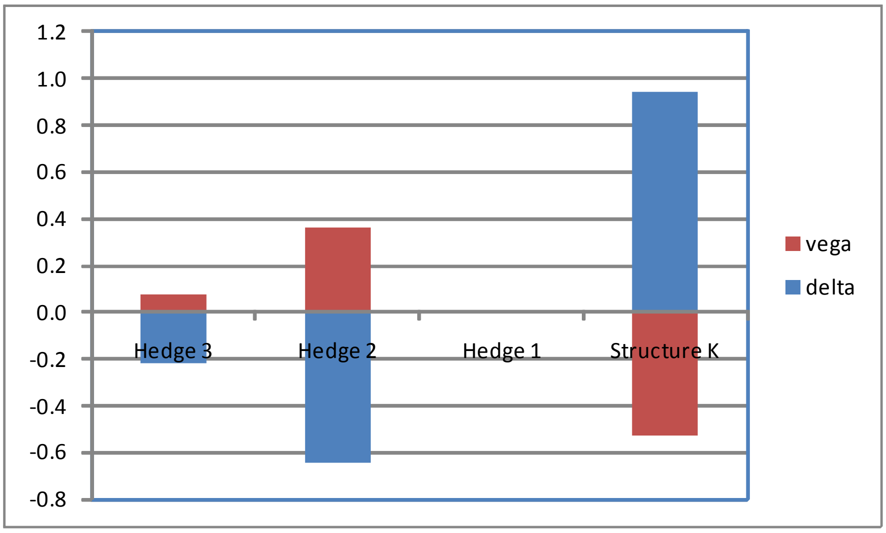
Next simulation is performed between 24th May 2009 and 24rd September 2009. Based on the simulation for this time interval the estimated Profit and Loss is – 0.633. In addition as it is indicated by Table 6.25 no re – balancing of the portfolio is required. Obviously Hedge 1 position is closed.

Table 6.25 – Bermudan Structure and Hedging Portfolio 24th September 2009

Type	Value
Hedge 3	-6.320
Hedge 2	-15.520
Hedge 1	-
Structure K	21.283

Again it is important to state that the vega and delta delivered. The vega delivered by the Bermudan structure is – 0.527, while the hedging positions is 0.436. Similarly, the delta of the Bermudan structure is 0.944 against – 0.864 of the hedging positions.

Graph 6.20 – Vega and Delta Exposure as of 24th September 2009



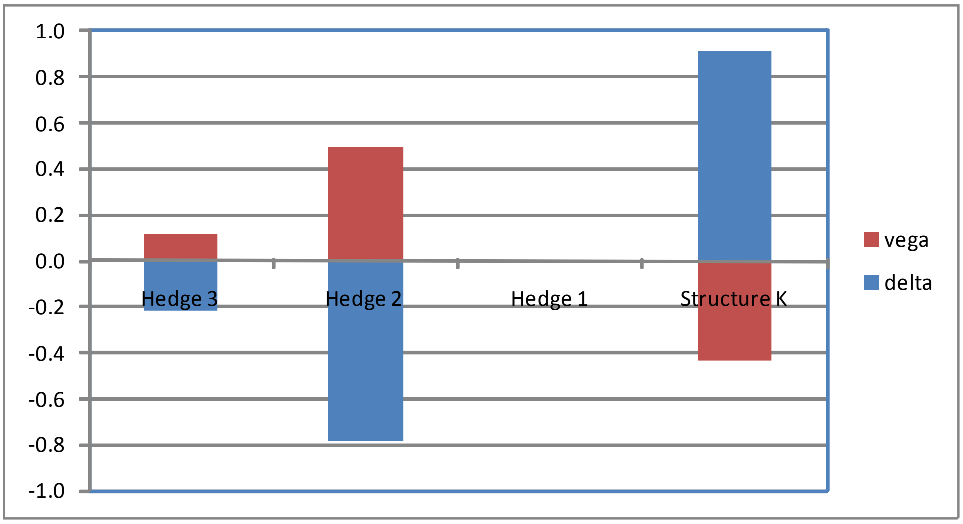
The final simulation is performed between 24th September 2009 and 24rd January 2010. The estimated Profit and Loss is 0.581 and results are under Table 6.26. The value of the Bermudan structure is reduced 2.582, while the hedging positions is generating a positive Profit of 3.163.

Table 6.26 – Bermudan Structure and Hedging Portfolio 24th January 2010

Type	Value
Hedge 3	-7.357
Hedge 2	-11.319
Hedge 1	-
Structure K	18.701

Similar to previous simulations the net first order risks delivered by the Bermudan position and the hedging portfolio are in line as presented in Graph 6.21. This supports the reasonableness of the hedge and that no need of re – balancing is required.

Graph 6.21 – Vega and Delta Exposure as of 24th January 2010



As indicted by the analysis above the Total Profit and Loss (including both portfolio management and re-hedging positions) is 0.030. The value generated from the Bermudan structure across all simulation is – 1.514 and the hedging portfolio is generating 1.544. One of the advantages on the strategy is that as indicated from the simulations above no re – balancing required.

6.10. Conclusion

A two – factor model with local volatility for oil products is proposed. The model utilizes as basis interest rate modelling developments and applies them into a very dynamic market in which the smile impact can be significant. The dynamics of the commodity market are significantly different from interest rates e.g. in commodities futures contracts are traded individually up until a specific date and the final expiration for an option is pre-determined – May 2010 contract is traded up until 20th April 2010 and the option on the contract traded in NYMEX expiries on 15th April 2010. Given the dynamics of oil market the model should be able to capture accurately the de-correlation between the futures contracts and the smile dynamics.

The approach described has the flexibility to generate different local volatility surfaces depending on the calibrated data. Moreover the model allows for different correlation surfaces. The model is used to price a number of exotic structures that are common in the oil market. Based on the results it is clear that being able to capture the smile dynamics is very important not only for valuation reasons but also for risk management purposes against a model that does not reflect the smile dynamics. The model can be calibrated directly and match market traded instruments such us swaptions and monthly strip options.

Chapter 7: CONCLUDING REMARKS

7.1. Application of Merton's, Leland and Toft and Longstaff and Schwartz Valuation Framework for Corporate Debt

The specific research was conducted during one of the most volatile periods in recent history with special significant events like the collapse of major financial institutions and the sharp decline in market liquidity. For the first part three structural form models were studied Merton's, Leland – Toft and Longstaff – Schwartz. These specifications were implemented using different assumptions for volatility and debt maturity such as (i) exogenous volatility and actual bond maturity, (ii) exogenous volatility and adjusted maturity, (iii) model determined volatility and actual bond maturity and (iv) model determined volatility and adjusted maturity. The process to determine the sample, using the following criteria 1) US non – financial corporations, 2) only fixed or zero coupon bonds, 3) all the principal is retired at maturity (bullet bonds) 4) bonds with embedded optionalities like callable, convertible, putable are excluded, 5) floating-rate or sinkable bonds are excluded as well, is similar to the ones that were followed by Lyden and Saraniti (2000), Teixeira (2005) and Eom, Helwege and Huang (2002). To our knowledge it is the first time that the model is calibrated against these four alternatives. Furthermore it is important to state the fact that the historical implied volatility was used for equity. At this first stage just prior the credit crunch only companies with relative simple capital structure and maximum of two bonds were included. The period covered is January 1998 until April 2006.

Results were in contrast with Lyden and Saraniti (2000) and Wei and Guo (1997) who argued that Merton's model dominates Longstaff and Schwartz in predictive accuracy; as Longstaff and Schwartz model revealed a very good performance.. Merton's and Leland and Toft models perform on different directions, namely Merton underestimates credit spreads, while Leland and Toft overestimates credit spreads. Longstaff and Schwartz model predictive power is reflected to the predicted over actual credit spread ration that was greater than 35% for the majority of the companies. The model was able to produce a ratio predicted over actual credit spread that is greater than 35% for thirteen companies (CARLISLE COS INC, HARMAN INTL, HUMANA INC, JLG INDUSTRIES, MILLIPORE CORP, NICOR GAS, NORDSTROM INC, NVR INC, OFFICE MAX INC, POPE & TALBOT, REYNOLDS &

REYNOLDS, SOUTHERN UNION and WORTHINGTON INDS) when it is implemented against model determined volatility and adjusted maturity. Furthermore, even when there was an overprediction error that was on limited magnitude and definitely much less compared to Leland and Toft. Also the overestimation is not systematic due to the whole period of each bond but it appears in some intervals and still the model appears to capture the shape of the credit spread very well.

The encouraging results during the 1998 – 2006 led to a very critical element of this research – the application of the Longstaff and Schwartz (1995) model is applied also on 2007 – 2008 bond data. The assumption of simple capital structure is relaxed and a composite implied volatility is calculated. Again the model indicated very good performance in all cases proving an average predicted over actual credit spread ratio of 57%. Interestingly though the average predicted credit spread was still estimated below the actual one in line with the previous implementation although the explanatory power of the model increased mainly driven by the higher market volatility.

For eight companies BOWATER INC, CAMPBELL SOUP CO, WITCO CORP, STEEL DYNAMICS, SONOCO PRODUCTS, SAKS INC, KOHLS CORP and UNISYS CORP the predicted over actual credit spread ratio was greater that 50%. Five of these companies are non investment graded. For the five non investment grade the estimated predicted over actual credit spreads are in all cases greater then 80%. Importantly the high median ratios are estimated for two invested graded companies the SONOCO PRODUCTS and CAMPBELL SOUP CO with median ratio is 98.50% and 93.23% respectively.

7.2. A 2 – Factor Model with Local Volatility to Price Exotic Derivatives in Oils' Market

The second part of this research proposes a 2 – factor model with local volatility to price Oil Exotic Structures. The proposed approach utilizes the general multi – factor model framework and the interest rate modeling developments as described by Clewlow and Strickland (1999b) and Brigo and Mercurio (2006) respectively. The world has witnessed the oil prices display sharp volatility throughout the year of 2006 – 2009. Reaching a record high of over \$147 per barrel during the early part of 2008 and then falling sharply below \$40 per barrel. The implied volatility of the prompt month contract reached to levels greater than 100% and smile effect became a significant element of valuation and risk management.

Multi – factor models are more flexible and are able to generate additional commodity curve shapes and curve movements in relation to one – factor model. In addition, multi – factor models allow non – perfect correlations between different commodity variables a characteristic that is very important in Oil Market. Finally the dynamics of the commodity market are significantly different from interest rates e.g. in commodities futures contracts are traded individually up until a specific date and the final expiration for an option is pre-determined – May 2010 contract is traded up until 20th April 2010 and the option on the contract traded in NYMEX expiries on 15th April 2010.

The following two general stochastic processes in terms of two independent Brownian motions are considered:

$$dX_t = -aX_t dt + x\sigma_{S,t}^X dW_S(t)$$
$$dY_t = -bY_t dt + \eta\rho\sigma_{S,t}^X dW_S(t) + \eta\sigma_{L,t}^Y\sqrt{1-\rho^2}\,dW_L(t)$$

Assuming there is a relationship between $W_S(t), W_L(t)$ volatilities, the futures price at time t can be written as

$$\frac{dF(t,T)}{F(t,T)} = \sigma_t^F\left(1 + xe^{-\alpha(T-t)}\right)\rho_{(i,T-t)}dW_S(t) + \sqrt{1-\rho_{(i,T-t)}^2}\,\nu\sigma_t^F dW_L(t).$$

The approach described has the flexibility to generate different local volatility surfaces depending on the calibrated data. Moreover the model allows defining different correlation surface. The model is used to price a number of exotic structures – barrier options, Target Redemption Notes and European and Bermudan Swaptions – that are common in the oil market. Based on the results it is clear that being able to capture the smile dynamics is very important not only for valuation reasons but also for risk management purposes. The model can be calibrated directly and match market traded instruments such us swaptions and monthly strip options.

Chapter 8: FUTURE RESEARCH

In light of the conclusions drawn above, there is a need for further research around the impact of the liquidity to the price of credit risk. The specific research was conducted during one of the most volatility periods in recent history with special significant events like the collapse of major financial institutions and the sharp decline in market liquidity. Liquidity risk has been thought to be an important factor affecting bond pricing. However, measuring and tracking liquidity spreads remains an elusive task. One of the major obstacles is that liquidity risk is often confounded with effects of other factors (e.g., default, information and market risks), which are difficult to disentangle empirically. Liquidity is also a broad concept, which may be referred to as ease of accessing funds or trading assets, or a state factor that systematically affects asset pricing. Different concepts could lead to very different liquidity metrics. Unless it is properly defined, measuring and comparing liquidity effects can be a very challenging task.

A specific attention, investigation and analysis need to be given to the impact of macro – factors to the credit spread levels. Under this, there is a need to investigate and identify approaches to decompose the liquidity element to the spread and the credit risk. In addition there is a need to analyze the features and characteristics of concentration within market participants that allows the fair price of credit risk. Furthermore, future research should question the impact of the government intervention to the credit markets.

Last but not least on there is a need to identify and understand the relationship between the equity volatility and the credit spread. Under the specific research using a composite volatility calculated by taking a suitably weighted average of the individual implied volatilities the predicted credit spread is improved considerably. Therefore empirical evidence and future research should try to understand the impact of options trading and short selling to credit spreads.

Regarding the second area for the specific research, the development of a 2 – factor model for oil structured products, future research can work to extend the specific approach to support time – varying portfolios. The backward integration approach calibrates to swaptions on underlying swaps that run from extension date to final maturity. The model therefore only works for underlyings that run from extension date to final maturity. This holds for all components of the underlying portfolio. In other words, the model does not support time – varying portfolios. In addition the model does not take into account settlement delay for the extension strike payments, so the strike payment date must be equal to the corresponding extension date.

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