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Abstract

We analyze a recently proposed method to estimate volatility and correlation when prices are observed at a high frequency rate. The method is based on Fourier analysis and does not require any data manipulation, leading to more robust estimates than the traditional methodologies proposed so far. In the first part of the paper, we evaluate the performance of the Fourier algorithm to reconstruct the time volatility of simulated univariate and bivariate models. In the second part, the Fourier method is used to investigate the volatility and correlation dynamics of futures markets over the Asian crisis period, with the purpose of detecting possible interdependencies and volatility transmissions across countries amid a period of financial turmoil.

Keywords: high frequency data, Fourier analysis, Asian crisis, volatility spillover.

1 Introduction

In recent years, high frequency data have become increasingly available for a wide range of securities allowing for a deeper understanding of complex intraday volatility and correlation dynamics. Within a high frequency domain the price formation is followed in real time, or tick-by-tick, resulting in a large amount of observed values and, therefore, in a virtually continuous process.

So far, temporal dependencies in financial markets have been mainly analyzed by means of parametric models such as GARCH (Bollerslev, Chou and Kroner, 1992; Bollerslev, Engle and Nelson, 1994; Shephard, 1996). High frequency data provide novel insights into the main features of these models. For instance, Andersen and Bollerslev (1998a) found that better ex-post measure of the underlying daily latent volatility factor, usually estimated by the daily (absolute or squared) return, can be computed by exploiting the entire sequence of absolute or square intraday returns. Andersen and Bollerslev (1998b), building on the continuous time stochastic volatility framework developed by Nelson (1990) and Drost and Werker (1996), applied the same idea to improve the forecasting performance of the popular GARCH(1,1), showing that the volatility forecasts closely correlate with the future latent daily volatility. Andersen, Bollerslev, Diebold and Labys (2001) extended and theoretically characterized this new measure, termed realized volatility. This methodology is nonetheless based on the strong assumption of regularly spaced data, whereas tick-by-tick quotes are, by nature, observed at uneven intervals over times. For implementation purposes, the observed series of prices are then synchronized or homogenized by imputation techniques such as linear interpolation and previous-tick interpolation. However, Barrucci and Renò (2002b) showed that the former, also used in Andersen *et. al.* (2001), induces a downward bias in the realized volatility estimator whose magnitude intensifies as the sampling frequency increases.

Regarding the analysis of volatility within the context of non synchronous quotes, several contributions can be found in the financial and econometric literature. Moving backward to the years preceding the large diffusion of high frequency databases, we can refer to the works of Scholes and Williams (1977) and Choen *at. al.* (1983); for more recent papers see Martens (2000), Oomen (2002), Barndorff-Nielsen and Shephard (2002), Brandt and Diebold (2003), Barndorff-Nielsen and Shephard (2004), among many others. Although very valid, all these approaches require, either directly or in a less explicit way, synchronization of the original data to be applied. An exception can be found in De Jong and Nijman (1997) where the cross products of returns are regressed on the number of common time units to these returns. Nevertheless, the estimation method is based on a discrete time model and yet no further developments have explored the possibility to extend it to a continuous time setting. Very recently, Hayashi and Yoshida (2005), also Hayashi and

Yoshida (2006), have proposed a new approach to estimate the covariance (correlation) of two diffusion processes when they are observed at discrete and asynchronous times. The method is an alternative, but similar in spirit, to the realized covariance. The resulting estimator is showed to be still unbiased and consistent as the observation interval shrinks to zero but it does not require any manipulation of the observed data.

In this paper, we will study an alternative, non parametric approach suggested in Malliavin and Mancino (2002) where the estimate of the variance-covariance matrix $\Sigma(t)$ of a multivariate process is computed via Fourier analysis. Being based on integration rather than on differentiation, the method well adapts to the inhomogeneous time structure of high frequency data. The procedure employs the observations in their original form as in Hayashi and Yoshida works but it also allows to recover the time evolution of $\Sigma(t)$ over a fixed window, an appealing feature that is not share, to our knowledge, by any other estimator in the field. Early recognition on the validity of the method can be found in Barrucci, Mancino and Renò (2000) where, by Monte Carlo experiments on equally spaced data, it was possible to estimate the volatility of a univariate process and the cross-volatilities of a multivariate process. Further applications have been evaluated in a more recent literature. Barrucci and Renò (2002a) measure ex-post volatility through the Fourier algorithm and found an improvement in the forecasting performance of the GARCH(1,1) model respect to the usual volatility measure given by the cumulative sum of square intraday returns. Renò (2003) exploits the Fourier approach to prove that the so-called Epps effect, due to Epps (1979), namely the tendency of correlation to decrease as sampling frequency increases, can be explained by asynchronous trading and lead-lags relationships. Precup and Iori (2005) show that the Fourier estimator generates more accurate results respect to interpolation based methods such as the standard Pearson coefficient and the co-volatility weighted measures proposed in Dacorogna *at. al.* (2001).

We initially use the Fourier estimator to reconstruct volatility trajectories of simulated univariate and bivariate models. Monte Carlo experiments are also performed to analyze the correlation behavior between two tick-by-tick asset prices as a function of the frequency scale. In a second stage of the study, we apply the method to the time series of three futures contracts continuously recorded from April to December 1997, which includes the Asia crisis. In particular, our dataset contains high frequency prices for two currency futures, the Australian dollar and the Japanese yen (both in terms of the US dollar) and for the S&P 500 index future. The currency futures were chosen because of the geographical proximity of these countries to the center of the East Asian Crisis and the index future because of the role of the underlying index as leading indicator of the US stock market performance. The objective is to extend the knowledge of price dynamics in futures markets by looking at possible volatility transmissions among currencies amid a period of financial turmoil.

Several studies have focused on the properties of futures returns and volatility and temporal relationships between spot and futures markets. For example, Kawaller et al. (1987) have shown that S&P 500 index futures returns lead S&P 500 spot returns by up to forty minutes, while the spot market rarely leads the futures market beyond one minute, in accordance with the hypothesis that investors with better market-wide information prefer to trade in stock index futures. Chan and Karolyi (1991), Abhyankar (1995), Tse (1999) and Min (1999) report that unlike a lead-lag relation, there is a bi-directional or contemporaneous relationship among the spot and the futures markets volatility, with innovations in either market spilling on the other. However, while contagion has been so far investigated by analyzing the behavior of several asset classes such as stocks, bonds and exchange rates, most of the studies have focused on spot rather than on futures markets. A notable exception is the paper by Najand *at. al.* (1992) where the authors study volatility spillover in five daily currency futures prices over the period January 1980 and December 1989. They find that ARCH and spillover effects are both present but tend to alternate over time. Tai (2003) looks at contagion effects in both conditional means and volatilities among British pound, Canadian dollar, Deutsche mark, and Swiss franc futures markets detecting spillover in coincidence of the 1992 ERM crisis.

The reminder of the paper is organized as follow. Section 2 introduces the Fourier algorithm and briefly outlines its implementation. Numerical experiments are performed in Section 3 to test the reliability of the method. Section 4 presents a literature review on the most common methods used to test for contagion during the Asian crisis. Results are discussed in Section 5 while section 6 concludes.

2 The Fourier method: theory and implementation

The estimator proposed in Malliavin and Mancino (2002) is a fully non parametric method to compute the time-varying correlation matrix between two or more assets. It is based on the only *a priori* assumption the log-price $p_i(t) = \log S_i(t)$, of an asset i at time t, follows a diffusion process of the form

$$dp_i(t) = \sum_{j=1}^{d} \sigma_{ij}(t) dW_j(t) + \mu_i(t)dt, \quad i = 1, \ldots, d, \tag{1}$$

where $\sigma(t)$ and $\mu(t)$ are time dependent random functions and $W(t)$ are independent Brownian motions. For this kind of models, we define the volatility matrix as

$$\Sigma_{ij}(t) = \sum_{k=1}^{d} \sigma_{ik}(t)\sigma_{kj}(t) \tag{2}$$

typically estimated through the well-known pathwise formula, due to Norber Wiener (see, for instance, Karatzas and Shreve, 1991)

$$\langle p_i, p_j \rangle_t = \int_0^t \Sigma_{ij}(u)du, \tag{3}$$

where $\langle p_i, p_j \rangle_t$ is the quadratic covariation of the process. Although the latter is an unbiased estimator for $\Sigma_{ij}(t)$, it also requires a differentiation procedure that results to be quite unstable when observations are missed at the selected mesh points, as it might be the case working with tick-by-tick data. On the other side, the linear and previous tick interpolation methods then adopted to synchronized the series have the drawback to reduce the number of observations and to induce spurious autocorrelations among the returns. The Fourier algorithm instead is based on integration and can be directly applied to the tick-by-tick data, including all the observations, two important features that highlight its natural adaptability to the high frequency framework.

The volatility matrix $\Sigma_{ij}(t)$ is reconstructed on a fixed time window through the Fourier coefficients of dp_i defined as

$$\begin{aligned} a_0(dp_i) &= \frac{1}{2\pi}\int_0^{2\pi} dp_i \\ a_k(dp_i) &= \frac{1}{\pi}\int_0^{2\pi} \cos(kt)dp_i \\ b_k(dp_i) &= \frac{1}{\pi}\int_0^{2\pi} \sin(kt)dp_i. \end{aligned}$$

Note that by changing the origin and rescale the unit of time we can always reduce the observed time window $[0, T]$ to $[0, 2\pi]$. Malliavin and Mancino (2002) derived a mathematical expression for the Fourier coefficients of Σ_{ij} based upon the coefficients of dp_i. The result is reported below without proof.

Theorem 2.1. *For a fixed integer $n_0 > 0$, the Fourier coefficients of the volatility matrix are given by:*

$$a_0(\Sigma_{ij}) = \lim_{N\to\infty} \frac{\pi}{N+1-n_0} \sum_{s=n_0}^{N} [a_s(dp_i)a_s(dp_j) + b_s(dp_i)b_s(dp_j)] \tag{4}$$

$$a_k(\Sigma_{ij}) = \lim_{N\to\infty} \frac{\pi}{N+1-n_0} \sum_{s=n_0}^{N} [a_s(dp_i)a_{s+k}(dp_j) + b_s(dp_j)b_{s+k}(dp_i)] \tag{5}$$

$$b_k(\Sigma_{ij}) = \lim_{N\to\infty} \frac{\pi}{N+1-n_0} \sum_{s=n_0}^{N} [a_s(dp_i)b_{s+k}(dp_j) - b_s(dp_i)a_{s+k}(dp_j)]\,. \tag{6}$$

The integer n_0 represents the number of coefficients it is advisable to omit, should they be affected by the drift term in equation (1). By the Fourier-Féjer inversion formula, $\Sigma_{ij}(t)$ can be then obtained pointwise as

$$\Sigma_{ij}(t) = \lim_{M \to \infty} \sum_{k=0}^{M} \left[a_k(\Sigma_{ij}) \cos(kt) + b_k(\Sigma_{ij}) \sin(kt) \right]. \tag{7}$$

The use of this formula allows us to keep the characteristics of the volatility matrix, i.e. the partial sums on the right-hand side in the above expression are still symmetric positive definite matrices.

The implementation is carried out by computing the Fourier coefficients of dp_i via integration by parts as follow

$$a_k(dp) = \frac{1}{\pi} \int_0^{2\pi} \cos(kt) dp = \frac{p(2\pi) - p(0)}{\pi} - \frac{k}{\pi} \int_0^{2\pi} \sin(kt) p(t) dt. \tag{8}$$

However, a price does not evolve continuously but, instead, it is observed at unevenly intervals in the form of tick-by-tick quotes $p(t_i)$, $i = 1 \ldots, n$, where n corresponds to the number of observations in the re-scaled interval $[0, 2\pi]$. Therefore, to implement the method and, in particular the integration, we need an assumption on the way data are connected. A possible option is to set $p(t_i) = p(t_{i+1})$, in other words, to consider piecewise constant prices over the interval $[t_i, t_{i+1}]$. Under this assumption, the integral in (8) becomes

$$\frac{k}{\pi} \int_{t_i}^{t_{i+1}} \sin(kt) p(t) dt = p(t_i) \frac{k}{\pi} \int_{t_i}^{t_{i+1}} \sin(kt) dt = p(t_i) \frac{1}{\pi} \left(\cos(kt_i) - \cos(kt_{i+1}) \right).$$

In the integration by parts formula (8), the constant term $\frac{p(2\pi)-p(0)}{\pi}$ can be set to zero by adding a drift term in the diffusion equation (1) so that

$$\tilde{p}(t) = p(t) - \frac{p(2\pi) - p(0)}{\pi} t.$$

This change of variable will not have any effect on the final estimates and it will also remove a possible source of bias.

Another aspect of the computation is related with the choice of a convenient frequency at which to stop the expansions (4)-(6). The smallest Fourier wavelength that can be evaluated to avoid aliasing is twice the smallest distance between two consecutive prices, here denote with τ. It can also be seen as the minimum frequency rate at which tick-by-tick data are sampled, i.e. τ usually equal to 1 second. In the frequency domain, this correspond to the highest frequency $\frac{n}{2\tau}$ also known as Nyquist frequency (see Priestley, 1979), where n is the number of observations. We can then conclude that a reasonable value for N is given by $\frac{n}{2}$.

3 Numerical Analysis

The performance of the Fourier algorithm as volatility estimator was initially tested on the short interest rate model introduced in Chan *et. al.* (1992). This is a broad class of processes that includes the mean reverting version of the Ornstein-Uhlenbeck process proposed by Vasicek (1977) and the one-factor general equilibrium model developed in Cox, Ingersoll and Ross (CIR) (1985). It is defined as the solution of the following stochastic differential equation (SDE)

$$dr(t) = \beta(\alpha - r(t))dt + \eta r^{\gamma}(t)dW(t). \tag{9}$$

To simulate the process, we have used the parameters estimated in Jiang (1998) on the 3-month Treasury bill rates via an indirect inference approach and given by $\hat{\alpha} = 0.079(0.044)$, $\hat{\beta} = 0.093(0.100)$, $\hat{\gamma} = 1.474(0.008)$, and $\hat{\eta} = 0.794(0.019)$, where the numbers in the parenthesis are the standard deviations of the estimates.

In order to estimate the volatility path, we have adopted the technique suggested in Malliavin and Thalmaier (2005) to obtain positive volatility terms despite only a finite number of coefficients is employed in the summation (7). It follows that the volatility is estimated by taking instead

$$\sigma^2(t) = \sum_{k=0}^{M} \varphi(\delta k) \left[a_k(\sigma^2)\cos(kt) + b_k(\sigma^2)\sin(kt) \right], \tag{10}$$

with $\sigma^2 = \Sigma_{ii}$ in equations (4)-(6) and where $\varphi(\cdot)$ is a variant of the Fejer kernel (Priestley, 1979) defined as

$$\varphi(x) = \frac{\sin^2(x)}{x^2}, \quad \varphi(0) = 0.$$

The original Fejer kernel is a smoother method responsible for eliminating a well-know problem in Fourier analysis, namely the Gibbs phenomenon[1].

To mirror the inherent non homogenous nature of the high frequency data, we have simulated an unevenly series of ticks by extracting the transaction times between contiguous trades, the so called durations, from an Exponential(λ) with $\lambda = 10$. The choice of the Exponential was motivated by a simple statistical analysis of the differences $t_i - t_{i-1}$ for the 3-month futures contracts used in Section 5. In all cases the empirical distribution is

[1]The Gibbs phenomenon arises when a piecewise continuously differentiable function is approximate by a Fourier series. It can be shown, for instance by applying the method to a simple sawtooth function, that the series displays an overshoot in the left-hand side interval of the discontinuity and a symmetric undershoot in the right-hand sided of the interval. The overshoot does not vanish as the frequency increases but, instead, approaches a finite limit, i.e. the height of the overshoot does not decrease by increasing the number of terms in the series.

well approximated by an Exponential, despite the different level of liquidity of the three contracts. Figure 1 illustrates the temporal behavior of the diffusion coefficient $\sigma^2(t) = \hat{\eta}^2 r^{2\hat{\gamma}}(t)$ on 10 days of trading of 8 hours each with a time step of 1 second. The estimate is consistent with the simulated path leading to a good reconstruction of the trajectory with $\delta = \frac{1}{50}$. It is important to note that the trajectory was estimated directly from the generated values of the interest rates and not from the simulated volatility time series.

In the model (9) employed so far, the randomness of the volatility component is generated by the state variable itself with the parameter γ measuring the degree of dependence of the variance from the interest rate level. A natural step into the analysis now consists in applying the estimation methodology to a more complex structure, where the volatility is characterized by its own latent stochastic process. In particular, we suppose that a stock price S and its variance v satisfied the following SDEs

$$dS(t) = \mu(t)S(t)dt + \sqrt{v(t)}S(t)dW_1 \tag{11}$$

and

$$dv(t) = \lambda(\bar{v} - v(t))dt + \eta\sqrt{v(t)}dW_2 \tag{12}$$

with

$$\langle dW_1, dW_2 \rangle = \rho\, dt,$$

where λ is the speed of reversion of $v(t)$ to its long term mean $\bar{v}$. This process is well-known in finance as Heston model (Heston, 1993). The process followed by $v(t)$ may be recognized as belonging to the class (9) introduced at the beginning of the section when $\gamma = \frac{1}{2}$.

We have simulated 10 days of trading of 8 hours each with a time step of 1 second. Figure 2 plots the trajectory of the historical volatility with exponential sampling. Although the Heston framework is quite complex, the obtained reconstruction well follows the dynamics of the original process (12) with a good representation of both the abrupt changes and the more regular sections in the volatility path. The value of the smoothing parameter was set to $\frac{1}{25}$.

Another noteworthy application of the Fourier algorithm consists in estimating the integrated volatility matrix Σ_{ij}

$$\hat{\sigma}^2_{ij} = \frac{1}{2\pi}\int_0^{2\pi} \Sigma_{ij}(s)ds,$$

over a fixed time window $[0, 2\pi]$. With a minimum computational effort, it can be proved that

$$\hat{\sigma}^2_{ij} = 2\pi a_0(\Sigma_{ij}) \quad \Rightarrow \quad \rho_{ij} = \frac{\hat{\sigma}^2_{ij}}{\hat{\sigma}_{ii}\hat{\sigma}_{jj}},$$

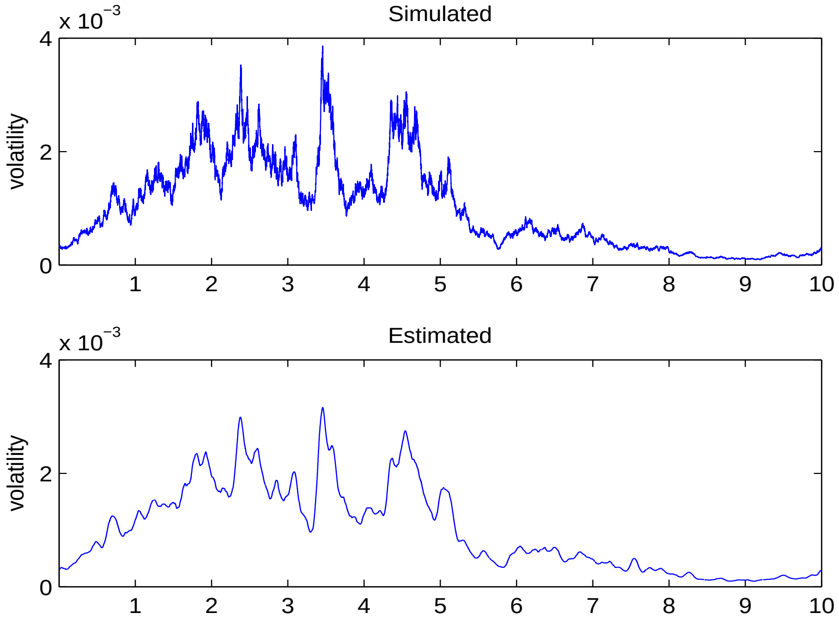

Figure 1: Top panel: squared diffusion coefficient $\hat{\eta}^2 r^{2\hat{\gamma}}(t)$ as simulated by model (9). Bottom panel: estimated trajectory via Fourier method (unevenly high frequency data).

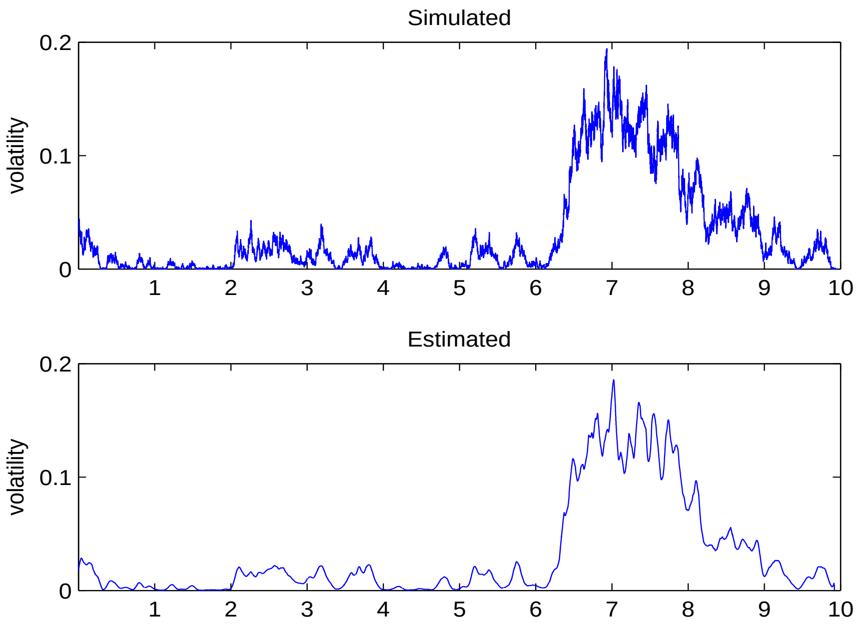

Figure 2: Top panel: one simulated path of $\sigma^2(t)$ by model (12). Bottom panel: volatility reconstruction with Fourier method (unevenly high frequency data).

with the Fourier correlation coefficient ρ_{ij} on the right-hand side. Borrowing the idea in Renò (2003), we performed a numerical test on the bivariate stochastic model

$$\begin{aligned} dp_1(t) &= \sigma_1(t)dW_1(t), \\ dp_2(t) &= \sigma_2(t)dW_2(t), \\ d\sigma_1^2(t) &= \lambda_1[\omega_1 - \sigma_1^2(t)]dt + \sqrt{2\lambda_1\theta_1}\sigma_1^2(t)dW_3(t) \\ d\sigma_2^2(t) &= \lambda_2[\omega_2 - \sigma_2^2(t)]dt + \sqrt{2\lambda_2\theta_2}\sigma_2^2(t)dW_4(t) \end{aligned} \tag{13}$$

with $\langle dW_1, dW_2 \rangle = \rho$. This is the bivariate continuous time limit of the well-known GARCH(1,1) model, whose parameter were estimated in Andersen and Bollerslev (1998) on the daily return times series of the DEM-USD and JPY-USD exchange rates as follow

$$\begin{aligned} \theta_1 &= 0.035 & \theta_2 &= 0.054 \\ \omega_1 &= 0.636 & \omega_2 &= 0.476 \\ \lambda_1 &= 0.296 & \lambda_2 &= 0.480. \end{aligned}$$

The value for the correlation was set to $\rho = 0.35$. To gain a deeper insight into the accuracy of the estimates provided by the Fourier algorithm, we have compared the method with the well-known Pearson correlation estimator. The results are illustrate below.

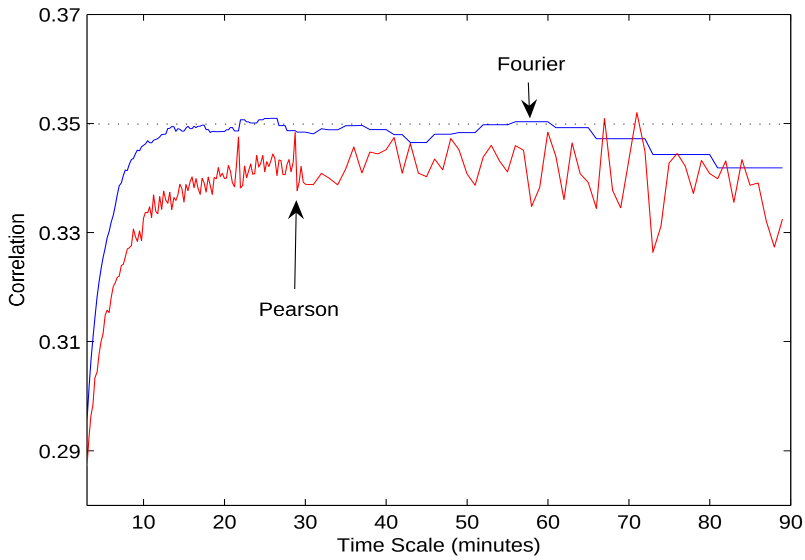

Figure 3: Fourier correlation and Pearson correlation between two simulated asset prices according to model 13. The dotted line represents the true correlation level set to 0.35.

Model (13) was run $1,000$ times with a time step of 1 second over a period of 24 hours (86400 seconds) trading. As before, the durations are extracted from an exponential distribution with $\lambda_1 = 15$ and $\lambda_2 = 40$. Up to 30 minutes, we have sampled every 25 seconds, instead of every minute, to ensure a more detailed representation. From the plot, we observe that at short time scales (less then 10 minutes), which correspond to high frequencies , the correlation is well out the benchmark to converge very fast towards it as the frequency decreases. It can be inferred that the frequency scale N in equations (4)-(6) must be chosen with care in order to obtain a stable correlation spectra. It is also apparent as the Fourier method, differently from the Pearson estimate, is able to provide a much smoother correlation trajectory. Moreover, the Pearson approach can only be applied to homogeneous and synchronous series whereas the Fourier correlation is directly calculated from the observed prices without any previous data manipulation.

4 Asian Crisis and Contagion

The last decade has witnessed dramatic movements in financial markets starting with the ERM breakdown in 1992, followed by the 1994-5 Mexican peso crisis, which spawned the so-called "tequila effect", the East Asian crisis in 1997 and the Russian virus of 1998. However, it is since the financial turbulence in Asia that policymakers and economists have engaged in considerable research to identify and analyze the causes of financial contagion.

The term contagion has been an evolving concept in the academic literature but there is still disagreement on its precise meaning. The most common definition splits contagion into two categories: fundamental based contagion and *pure* contagion (see Calvo and Reinhart, 1996). The first category refers to the transmission of shocks between countries or markets routes through real links such as trade, macroeconomic similarities and financial connections. Under the second category, contagion arises when comovements cannot be explained in terms of fundamentals, and common shocks and all channels of potential interconnection are either not present or controlled for. Three major approaches have been applied by researchers in order to identify contagion: correlation of asset prices, transmission of volatility changes, and conditional probability of currency crises (Dornbusch, Park and Claessens, 2000; Pericoli and Sbracia, 2003). The estimation of correlation coefficients among stock returns is the most common method used to uncover contagion effects. While high correlations among countries are not necessarily evidence of contagion, but purely a reflection of cross-country dependence, a significant increase in the correlation coefficient after a shock to one country is usually interpreted as a sign of contagion. King and Wadhwani (1990) were among the first to define contagion as a significant increase in the correlation between assets returns and, by implementing this measure, offered supporting evidence for contagion during the October 1987 crash. Baig and Goldfajn (1998) show that the cross-country correlations among currencies and sovereign spreads of Indonesia, Korea, Malaysia, the Philippines and Thailand significantly increased during the East Asian crisis period compared to other periods. They also provide evidence there is cross-border contagion in the currency market after employing dummy variables to control for own-country news and other fundamentals. However, Forbes and Rigobon (2001, 2002) prove that the correlation coefficient underlying traditional test for contagion is biased upward during period of market turmoil. Indeed, being conditional on the variance of one of the two markets under analysis, an increase in the market volatility, which is likely to happen after a crisis, can lead to incorrectly accept that cross-market correlations has also increased and, therefore, that contagion has occurred. They suggest a simple way to account for this effect and show that, once the so adjusted coefficient is applied to the 1997 East Asian crisis, the 1994 Mexican peso collapse, and the 1987 US stock market crash data, the correlation across multi-country returns is no longer significant. In a recent paper, Arestis et. al. (2005), after correcting for the heteroskedasticity bias, extend this study by employing the sequential dummy test proposed in Caporale *et. al.* (2005), which is based on a set of less restrictive over-identifying assumptions then the one used by Forbes and Rigobon (2002) to test for contagion. They find some evidence of contagion from Indonesia to the UK and from Korea and Thailand to France during the Asian crisis, mainly concentrated in the second semester of 1997. Several studies have also attempted to explore how, amid a crisis period, changes of volatility in one market preceded changes of volatility in another, a phenomenon refereed to as volatility spillover. A methodology commonly used to asses such changes is based on the estimation of multivariate GARCH models. Park and Song (1998) apply a GARCH framework to provide empirical evidence of volatility spillover among foreign exchange markets in East Asian countries during the crisis period. They find that the effects of the crisis in Indonesia and Thailand were transmitted to the Korean foreign exchange market, while the Korean crisis was not contagious to the two Southeast Asian countries. Dungey, Fry and Martin (2003) use a bivariate GARCH model between the Asian equity markets and the Australian equity market to show that the former poorly contributed to the total volatility in the Australian market over the period leading to a little significant evidence of contagion. The last approach aims to estimate the probability that one country is reached by the crisis given that other countries have already experienced it. Eichengreen, Rose and Wypolsz (1996), by means of a panel of quarterly macroeconomic and political data covering 20 industrial economies from 1959 to 1993, relate the probability of a crisis to a set of explanatory variables through a probit model across countries and prove that contagion appears to spread more easily to countries which are closely tied by international trade linkages than to countries in similar macroeconomic circumstances. Using a similar approach, Caramazza, Ricci and Salgado (1999), investigate the Mexican, Asian, and Russian crises. The results indicate that fundamentals such as trade spillovers, common creditor and financial fragility are highly significant in explaining the three crises, while exchange rate regimes and capital controls do not seem to matter.

5 Data Analysis

As an application of the Fourier method, we have investigated return correlations and volatility dynamics of futures contracts over the Asian crisis period. Our analysis is based on the 3-month S&P 500 index futures, the 3-month JPY-USD futures and the 3-month AUD-USD futures, all observed tick-by-tick over the period April-December 1997. In particular, the futures on the S&P 500 is the most liquid contract with $702,165$ tick prices followed by the AUD-USD currency futures with $226,360$ and the JPY-USD contract with $19,027$ available quotes. The Australian market is therefore characterized by the smaller turnover of transactions. We analyze only near-to-maturity contracts, which are the most liquid ones and apply a rolling-over mechanism to construct the actively traded times series shown in Figure 4 (left-hand panels) together with their relative log returns series (right-hand panels). The effect of the Hong Kong stock market crash, also known as "mini-crash", caused by the Asian crisis on October 27 is evident and translates into a clear price drop for both the AUD-USD and the S&P 500 index futures. On the other side, by looking at the graphs for the JPY-USD contract, it appears that the Asian events did not have a remarkable effect on the Japanese market.

In Figure 5 we plot the volatility estimates for the three contracts in hand over the period under consideration. Note as the trajectory for the Australian futures (middle panel) is thinner compared to the other contracts due to the low liquidity of the asset. We observe that the volatilities of the AUD-USD and the S&P 500 futures follow each other rather closely between July and November 1997 and in particular around the "mini-crash" of October 27. A peak on May 21, following Thailand announced (on May 15) of wide-ranging capital controls, is detected on the AUD-USD futures but not on the other two contracts. The JPY-USD futures also presents a spike on October 27, but not a persistently high volatility after the event. The volatility of the JPY-USD is highest during the end of May and mid June 1997, before the crisis properly started, and cluster of high volatility are also detected around June 10. A second period of increased volatility is observed from August to September 1997. The Japanese economy was only marginally affected by the 1997 turmoil (Dungey, Fry and Martin, 2004). Milton Friedman (1999) argued that the severe period of recession and stagnation Japan was going through actually predated and transcended the Asian crisis. Ellis and Lewis (2001), by analyzing daily market-close data for stock prices, bond futures prices and exchange rates, found that developments in the US market generally had a much greater influence on price movements and volatility than

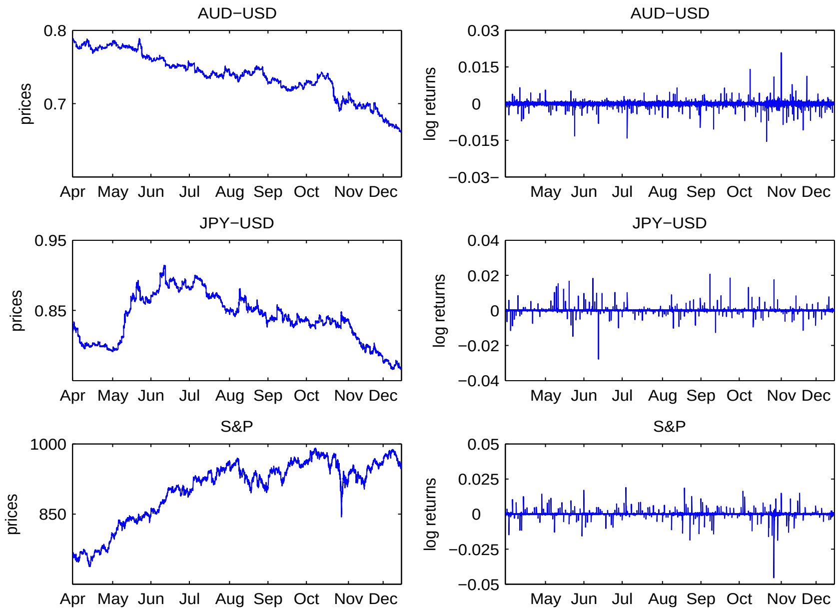

Figure 4: Times series of the tick prices (left-hand panels) and of the log returns (right-hand panels) for the future contracts under study.

cross-market shocks originating in the Asian crisis economies. They also provide evidence that stock markets reacted to the developments in Asia after the United States did, instead of responding directly to the news itself. Volatility of both the Australian dollar and the New Zealand dollar exchange rates against the US dollar increased remarkably during the Asian crisis, building towards the end of the period, and remained high into the world crisis period. Our results seem to strengthen the analysis of Ellis and Lewis and show that the volatility of the AUD-USD futures follows closely the volatility of the S&P 500 index futures. This conclusion is well supported by the clear, large spike in the volatility trajectory of the Australian futures which occurs soon after a similar level of volatility is detected for the futures on the index.

Finally, we have derived the integrated weekly correlations across the three contracts for the period April-November 1997. Figure 6 illustrates the obtained trajectories. The frequency scale was set to a value corresponding to 30 minutes in the time domain. This value is a trade-off between short and long time scales where non synchronicity and lack of statistics respectively, may generate downward biases in the estimated correlations (see Figure 3). We also checked that the correlations patterns reconstructed are stable when changing the frequency scale around the chosen value. Figure 4 and Figure 6 are consistent with each other. Starting from mid July, both the currency futures curves are downward sloping, whereas the S&P 500 index performance is on overall positive. By looking at the correlation time series, the trajectory relative to the currency futures shows an increasing level of mutual dependence. An opposite pattern can instead be observed between the JPY-USD futures and the S&P 500 index futures, with the lowest correlation value reached towards the end of October. There is no clear evidence that a spike in the correlation spectra may be due to structural changes affecting cross-market linkages, and as such to contagion. Indeed, periods of high volatility, that characterized crucial stages of the Asian crisis, do not seem to play a significant role in driving the correlation up or down. For instance, the largest correlation value between the Australian and US market is detected around mid September, during a relative calm period for the S&P 500 index futures, and not soon after the October"mini-cras", as expected. Similarly, the futures on the AUD-USD and JPY-USD exchange rates show a high degree of correlation around mid June when the volatilities of the two contracts are relatively low.

6 Conclusions

In this paper we have implemented the Fourier methodology proposed by Malliavin and Mancino (2002) to compute historical volatility and correlation. Whilst classical methods require equally spaced observations, the Fourier estimator, being based on integration rather than on differentiation, naturally exploits the inhomogeneous time structure of the high frequency prices without any prior data manipulation. We have first tested the performance of the method through numerical experiments obtaining volatility reconstructions that very well match the simulated volatility dynamics. Further evidence is provided by computing the integrated correlation of a bivariate diffusion process. We have then applied the estimator to futures time series, observed at a high frequency level, during the East Asian crisis of 1997. Our results are coherent with the exiting literature. We have found that the Australian economy was not impacted by the Asian events directly but its reaction was rather driven by the developments in the US market. Also, we have observed that the Asian turmoil did not have a noticeable effect on the Japanese market.

We believe that our analysis would benefit from a more complete dataset including pre-crisis and post-crisis samples to gain a deeper insight into the contagion problem. Our correlation reconstruction considers only a symmetric contemporaneous relationship across markets, whereas a wider range of data would allow for a better outline of the inherently non symmetric nature of contagion by capturing the lead-lag relationship between returns or volatilities across markets. Nonetheless, our method based on time varying estimates has the advantage to look at contagion from a more dynamical prospective respect to the existing approaches.

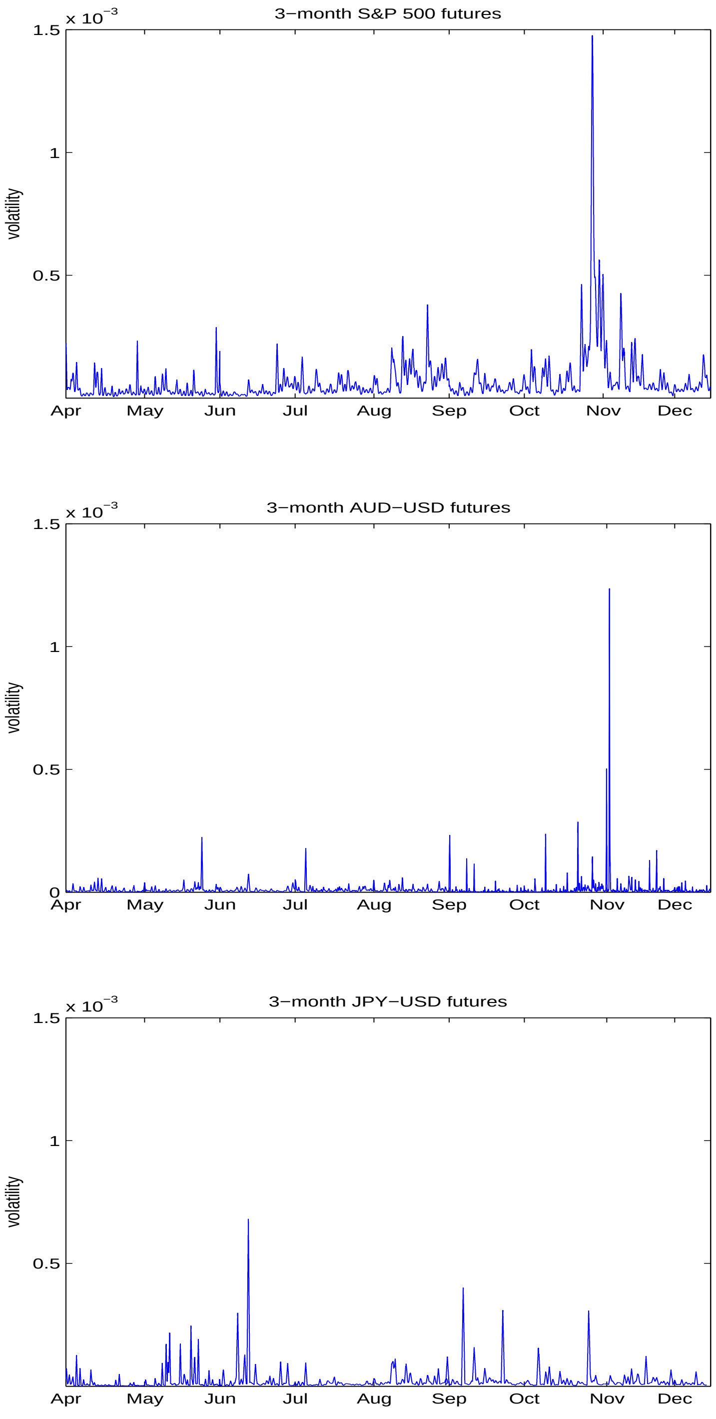

Figure 5: Estimated volatility via Fourier method for the S&P 500 (top panel), the AUD-USD (middle panel) and the JPY-USD (bottom panel) futures over the period Apr-Dec1997.

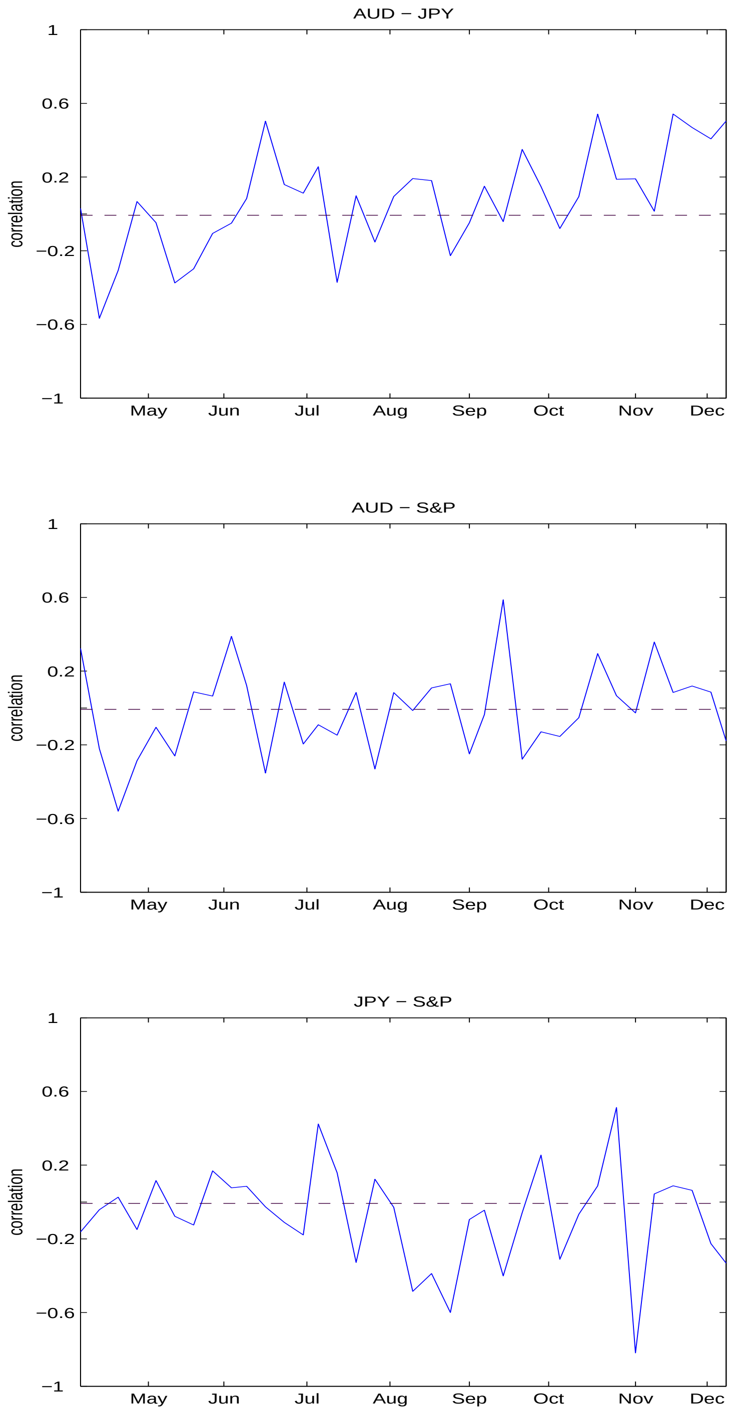

Figure 6: Weekly correlation estimated via Fourier method over the period Apr-Dec 1997: AUD-JPY (top panel), AUD-S&P (middle panel) and JPY-S&P (bottom panel). The time scale was set to 30 min.

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