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Abstract

The literature on North-South trade has explored conditions under which international trade might be a factor magnifying income disparities between the advanced North and the backward South. Little attention has yet been placed on the effect of trade on countries that do not display substantial dissimilarities concerning capital endowments. We show that even when no single country is technologically more advanced than any other one and productivity changes are uniform and identical in all countries, international trade may still be a source of income divergence. Income divergence will be experienced when comparative advantages induce patterns of specialisation that, although optimal for each country at some initial point in time, do not offer the same scope for improvements in terms of subsequent quality upgrading of final products.

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1 Introduction

In the past two decades a number of articles on international trade have started to acknowledge the importance of using non-homothetic preferences to capture some relevant features of North-South trade – e.g., Matsuyama (2000), Flam and Helpman (1987), and Stokey (1991). These papers have developed tractable models that predict patterns of specialisation where richer countries produce and export goods with high income demand elasticity. One of the main predictions of those models is that the impact of international trade on growth may be uneven across countries which are at different stages in the process of development. More precisely, trade would tend to be more beneficial to developed economies, and it may even be detrimental to underdeveloped countries. The key mechanism at work is the one originally argued by Prebisch (1950) and Singer (1950): as the world income rises, world aggregate demand deviates towards the goods produced by richer economies (the North), improving their terms of trade and, thereby, magnifying initial income disparities between South and North.

The papers mentioned above thus restrict the attention to a world economy where some countries (the North) have somehow historically accumulated larger amounts of human and physical capital than others (the South), and show conditions under which trade magnifies initial income disparities resulting from those capital differences. However, the pattern of international specialisation and trade might also be the source of income differentials between countries that do not display any substantial dissimilarity regarding their levels of human and physical capital. In this paper, we look at economies that start off with similar capital endowments, and propose a theory of uneven growth induced by trade, based on non-homothetic preferences and productive specialisation driven by comparative advantages.

Our theory rests on five fundamental elements. First, there exists a large variety of consumption goods in the economy. Second, each variety of consumption goods is present in several degrees of quality, with higher qualities being increasingly costly to produce. Third, some varieties offer larger scope for quality upgrading than others, in the sense that it is easier to increase their quality. Fourth, individuals care about the quality of the goods they consume and, moreover, their willingness to pay for higher quality of consumption increases with their income. Fifth, countries which are similar in terms of their average productivities specialise in the production of different varieties of goods according to their comparative advantages.

The first four elements above give room for non-homothetic demand schedules, where the income demand elasticity of different varieties of goods is tied to the specific degree of quality in which each particular variety is (optimally) traded in the market. The last element yields patterns of regional specialisation that, combined with non-homothetic demand schedules, may lead to divergent dynamics among countries that are initially similar in terms of capital endowments. In such a framework, we show that international trade may induce income divergence across countries characterised by similar initial income levels and with no *absolute* advantages over one another. In particular, income divergence will be experienced when comparative advantages dictate patterns of specialisation that, although optimal for each specific country at a given point in time, do not offer the same scope for technological improvements in terms of subsequent quality upgrading of final goods.

To convey some preliminary intuition of how non-homothetic demand schedules arise as an equilibrium result of our model, it is worth discussing in further detail some of the specificities of the commodity space. In that respect, we follow the quality ladder structure featured in Grossman and Helpman (1991) – that is, in a continuum of horizontally differentiated varieties of goods, an infinite number of qualities for each variety are available in the market. Our commodity space is thus bidimensional, with the horizontal axis indexing the variety of the good and the vertical axis indexing the quality level of a specific variety. Unlike Grossman-Helpman, however, in our framework the optimal expenditure shares across varieties do not remain constant as income changes. In particular, we postulate that the additional utility the individual derives from a marginal increase in the quality of the goods he consumes increases with the quantity of consumption, hence with the individual's income (in other words, the individual's *taste for quality* increases with his income). As a result, as individuals become richer they will optimally shift resources towards those varieties whose quality can be set at relatively higher levels. The budget constraint, in turn, implies that the extent by which quality can be raised for any given variety is related to its specific cost of quality upgrading. Thus, the distribution of quality upgrading across varieties results from the interaction between the underlying technological structure and the response of the consumers' *taste for quality* to income variations. If the cost of quality upgrading differs across varieties, then the shift towards higher-quality goods with rising income will (optimally) occur at different speeds across varieties. More precisely, the lower the cost of quality upgrading for a specific variety, the faster the speed of quality upgrading for this variety. This uneven climbing-up-the-quality-ladder will in turn lead to non-homothetic demand schedules, where the fraction of income spent in different varieties depends on the level of income.

When we introduce this interaction between quality upgrading and comparative advantages into a general equilibrium model of international trade, we show that it may lead to income divergence through its effect on the terms of trade. To briefly characterise this mechanism, take some hypothetical country (call it country Z) that specialises in the production of a variety v, which exhibits high cost of quality upgrading.[1] According to the mechanism proposed in this paper, quality upgrading for variety v will be relatively slow as world income grows. Hence, the world expenditure share on v will decrease over time. As a result, as the world income rises, Z will experience a decline in its terms of trade, because the types of goods it produces display low income demand elasticity.

Our model predicts that the richer economies are those that specialise in the production of varieties of goods that exhibit larger scope for quality upgrading (in turn, implying that they specialise in the production of higher-quality goods). This prediction is confirmed by the data, as shown in Khandelwal (2008). That paper estimates the length of quality ladders for different industries, showing that import penetration from poorer economies in the US is lower in industries that exhibit longer quality ladders (hence larger scope for quality upgrading), while exports to the US originating from other developed economies tend to belong precisely to those industries and, in particular, to the upper spectrum of their respective (long) quality ladders.[2]

The paper is organised as follows. Section 2 illustrates an historical example. Section 3 describes the set-up of the model. Section 4 solves the consumer's problem in a partial equilibrium set-up, illustrating the specificities of the non-homotheticity of demand in our model. Section 5 computes the general equilibrium in the world economy, and analyses the effects of uniform aggregate productivity growth. Section 6 extends the comparative statics exercises to account for changes in the individual skills heterogeneity and population growth. Section 7 concludes. The appendices contain the omitted proofs and some additional algebraic derivations used in the main text.

[1] This specialisation decision might be related, for instance, to geographical conditions influencing the technology available in Z. The example in the next section will illustrate this point in further detail.

[2] Schott (2004) also shows that the quality dimension within varieties of goods (measured in his paper by unit values) is key to understand trade patterns in the world economy. In particular, he provides evidence that US import unit values correlate positively with the exporter's GDP per head. Moreover, this positive correlation tends to be more pronounced for varieties that exhibit larger scope for quality upgrading (e.g., manufactured goods) compared to more homogeneous goods (e.g., natural resources goods). See also Hallak (2006) for related evidence.

2 An Illustrative Historical Example – colonial Jamaica and pre-industrial Argentina

Situations where the mechanism proposed in this paper may have played an important role include the cases of economies for which exogenous initial geographical conditions greatly influenced their specialisation in the world economy during some period in history. As an illustrative example, we take the case of colonial Jamaica and compare it to the one of pre-industrial Argentina.

From the second half of the XVII century until the first half of the XIX century, the Jamaican economy grew mainly based on the production and export of sugar from sugarcane. This is not surprising given the excellent climatic conditions this tropical island offered for that type of crop. By 1805, Jamaica was the largest sugar exporter in the world (Higman, 2005). Given the value attributed to sugar by European consumers, during that period Jamaica was deemed probably the most important British colony in the Americas (Hall, 1959). Although sugar was indeed a very valuable consumption good at that time, it clearly was a type of good with very limited scope for undergoing subsequent improvements in quality. As such, according to our model, sugar was bound to eventually lose its status of luxury among consumers. In fact, by the second half of the XIX century, sugar began to lose its economic preeminence in the world markets and started experiencing a long phase of declining prices.

In Argentina, geographical conditions made this country exceptionally apt for the breeding of cattle and growing cereals, which constituted the main engines of its economy until 1920. The commercial production of cattle started in the late second half of the XVIII century with the appearance of the *saladeros* – slaughterhouses where meat would be cured by drying and salting. Salt-cured beef was a rather unsophisticated product that was mostly exported to Cuba and Brazil to feed slaves. In fact, the industry of the *saladeros* did not mean a big push to the Argentinean economy, which was at that time still a very marginal country within the world economy.

The *big boom* for the cattle industry in Argentina came much later, at the end of the XIX century. Unlike the sugar industry, the cattle industry had some scope for quality upgrading, in the form of chilled and frozen beef. The market size for this product, certainly more appreciated by consumers than salt-cured beef, was however initially quite limited, since the transportation cost induced a huge differential in the prices of the two goods. Yet, in Europe, income had been continuously rising during the XIX century, thanks to the massive technological advancement that followed the advent of the Industrial Revolution. The availability of a higher-quality good in the cattle industry eventually attracted well-to-do European consumers, whose demand induced Argentinean firms to export large amounts of chilled and frozen beef to Europe.[3] During this period, Argentina grew on average at rate of 5% yearly, attracted millions of immigrants from Europe and became one of the richest countries in the world.[4] The exportation of chilled and frozen beef was undoubtedly one of the main activities that spurred this phase of fast and steady economic growth in Argentina between 1880-1920.

This example illustrates how exogenous geographical conditions greatly influenced the path of GDP growth in Jamaica and in Argentina via the evolution of their exports, in the way our model would predict. Jamaica was comparatively efficient at producing sugar, while Argentina enjoyed a comparative advantage in beef production. Sugar offered very limited scope for quality improvements, which is analogous to assuming that the cost of quality upgrading for sugar products is extremely high. On the contrary, beef did offer a lot more scope for quality upgrading than sugar, which materialised in the switch from salt-cured beef production (lower-quality good) to chilled and frozen beef (higher-quality good). As predicted by our model, sugar exports initially sustained high growth in Jamaica, until rising income in the world shifted aggregate demand towards varieties of goods which could be offered in higher quality degrees, such as chilled and frozen beef from Argentina.

The previous example also points out an important feature of our model. Quality upgrading is a phenomenon that occurs *within* industries, and (possibly) heterogeneously *across* different industries. This feature, which contrasts with models that take luxuries as an exogenous category (e.g., models with hierarchical or "0/1" preferences), allows our model to account for both the initial difference in GDP per capita between the two countries *and* the following catching-up (and overtaking) by Argentina relative to Jamaica.[5]

[3] The main market for Argentinean chilled and frozen beef at that time was by far the prosperous Great Britain of end of XIX and beginning of XX century (in 1914, 83.5% of the total Argentinean exports of chilled and frozen beef was sent to the UK). See Rapaport (1988).

[4] By 1913, the GDP per head in Argentina was similar to that of France and Germany – see Blanchard and Perez Enrri (2002).

[5] Further details on hierarchical preferences can be found in Bertola, Foellmi and Zweimuller (2006, pp. 302-320). The "0/1" specification of preferences is due to Foellmi, Hepenstrick and Zweimuller (2008).

3 Structure of the Model

We consider a world composed by two countries: the *Home* country and the *Foreign* country. For brevity, hereafter we refer to the former as H and to the latter as F. These two economies share a common commodity space, defined along two distinct dimensions: *horizontal* and *vertical*. The first dimension (*horizontal*) designates the *variety* of the good – e.g., fruit products, TV, etc. Different varieties are indexed by the letter v along the variety space $\mathbb{V} \subset \mathbb{R} : v \in [0,1]$. The second dimension (*vertical*) refers to the intrinsic *quality* of the good of a particular variety v – e.g., organic vs. non-organic fruit products, LCD TV vs. cathode ray tube TV, etc. Within each variety $v \in \mathbb{V}$, commodities are *vertically* ordered by the quality-index q belonging to the set $\mathbb{Q} \subset \mathbb{R} : q \in [1, \infty)$, where a higher q denotes a higher quality level. The commodity space is then given by the set $\mathbb{V} \times \mathbb{Q} = [0,1] \times [1, \infty)$, and each commodity is identified by a pair $(v,q) \in \mathbb{V} \times \mathbb{Q}$.[6]

We assume that all commodities are tradable. Additionally, we assume there are no transport cost and no tariffs affecting international trade.

3.1 Preferences and Budget Constraint

Both H and F are inhabited by a continuum of individuals with identical preferences defined over the commodity space $\mathbb{V} \times \mathbb{Q}$. Whenever it proves needed, hereafter we adopt the following notation: unstarred symbols refer to H, starred ones to F.

Denote by $x_{vq} \in \mathbb{R}_+$ the consumed *quantity* of commodity $(v,q) \in \mathbb{V} \times \mathbb{Q}$ (i.e., the consumed quantity of variety v in quality q) by a representative individual from H. His preferences are summarised by the following utility function:

$$U = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} C_{vq} dq \right] dv$$
$$\text{with } \; C_{vq} = \begin{cases} x_{vq} & \text{if } \; x_{vq} < 1 \\ (x_{vq})^q & \text{if } \; x_{vq} \geq 1 \end{cases} \tag{1}$$

where C_{vq} represents a quality-adjusted consumption index.

[6] In our setup, different varieties should be then understood as groups of commodities that aim at satisfying different *needs*. On the other hand, different qualities for a particular variety refer to the *extent* (or *degree*) in which the *need* is actually satisfied by the commodity. In that regard, food satisfies a different need when compared to TVs (physiological nutrition vs. visual entertainment), but an LCD TV satisfies the need for visual entertainment (objectively!) *better* than a cathode ray tube TV.

The utility function captures the notion that quality is a desirable feature. To this aim, (1) is specified in such a way that quality is never bad. In particular, quality magnifies the utility derived from (physical) consumption only when $x_{vq} > 1$. This last property of (1) intends to capture the idea that individuals first seek to satisfy their basic consumption needs, and just after these basic needs are met, do they start paying attention to the quality dimension of the goods they consume.

Some additional properties about the utility function specified in (1) are worth noting. First, within each variety v, marginal utility is unbounded above as consumption approaches zero, implying that all varieties will be actively consumed in an optimum. Second, convexity in quantities of the inner integrals of U means that individuals will optimally consume only *one* type of quality for each variety v. Third, considering two different levels of the quality-index $\underline{q} < \overline{q}$ for the same variety v, the marginal rate of substitution of $x_{v\overline{q}}$ for $x_{v\underline{q}}$ is non-decreasing along a *proportional expansion path* of $x_{v\overline{q}}$ and $x_{v\underline{q}}$.[7] This last property of (1) will allow demand functions to display non-homothetic behaviour, where the rich spend a larger fraction of their income in high-qualities than the poor.[8]

Each individual is endowed with one unit of *effective* labour, which is supplied inelastically. Labour is immobile across countries. As a result, each individual in H supplies his entire labour endowment to domestic firms in return of a wage $w \in \mathbb{R}_{++}$ (hereafter, all prices are measured in a common *numeraire*). This wage represents the only source of income for the individual. Therefore, his budget constraint reads as follows:

$$\int_{\mathbb{V}} \int_{\mathbb{Q}} p_{vq}\, x_{vq}\, dq\, dv \leq w \tag{2}$$

where $p_{vq} \in \mathbb{R}_{++}$ denotes the (international) price of each unit of commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$.

[7] To see this, note the $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is defined by $(\partial U / \partial x_{v\overline{q}}) \diagup (\partial U / \partial x_{v\underline{q}})$, and along a proportional expansion path $x_{v\overline{q}} = k\, x_{v\underline{q}}$, where $k > 0$. Then, for $x_{v\underline{q}}, x_{v\overline{q}} > 1$:

$$MRS(k\, x_{v\underline{q}}, x_{v\underline{q}}) = \frac{\overline{q}}{\underline{q}} k^{\overline{q}-1} \left(x_{v\underline{q}}\right)^{\overline{q}-\underline{q}},$$

from where it is clear that, along the *ray* $x_{v\overline{q}} = k\, x_{v\underline{q}}$, $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is increasing in $x_{v\underline{q}}$.

[8] A crucial assumption for our model is that quality upgrading is increasingly appreciated by the consumer as he turns richer, as captured by (1). An alternative utility specification that would still exhibit that feature and work for our model would have the quality-adjusted consumption index: $C_{vq} = (1 + x_{vq})^{1+q}$, where $x_{vq} \geq 0$ and $q \geq 0$. In particular, the main partial equilibrium result, that is Corollary 1 ahead in the text, would still hold under this specification (a formal proof is available from the authors upon request). In turn, all the general equilibrium results presented later on in Sections 5 and 6 would hold as well. The ultimate reason for choosing the specification in (1) is essentially motivated by algebraic tractability.

We define $\beta_v \equiv w^{-1} \int_{\mathbb{Q}} p_{vq}\, x_{vq}\, dq$ as the *demand intensity* of variety $v \in \mathbb{V}$.[9] In the optimum, given the specification in (1), the budget constraint (2) will naturally bind. It is thus straightforward to notice that demand intensities will sum up to one across varieties (i.e., $\int_{\mathbb{V}} \beta_v dv = 1$).

All individuals in the world face the same prices for the reproducible commodities. As a result, the analogous expressions in (1) and (2) corresponding to F read, respectively, as follows: $U^* = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max \left\{ x^*_{vq}, \left(x^*_{vq} \right)^q \right\} dq \right] dv$ and $\int_{\mathbb{V}} \int_{\mathbb{Q}} p_{vq} x^*_{vq} dq dv \leq w^*$; where w^* denotes the wage in F in terms of the common *numeraire* (clearly, since labour is immobile, w and w^* need not be equal).

3.2 Technology

In both countries competitive firms produce commodities based on linear production functions in which labour represents their only input. We let unit labour requirements vary both across varieties and across qualities of each variety. Also, we let unit labour requirements differ across countries. In particular, in H the unit labour requirement for commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ is given by $c_{vq} = a(v)\, q^{\eta(v)}/\kappa$, while in F is given by $c^*_{vq} = a^*(v)\, q^{\eta(v)}/\kappa$; where $\kappa > 0$ denotes a *world aggregate-productivity* parameter, $a(v)$ and $a^*(v)$ represent *variety-specific* technological parameters which may differ between countries, and $\eta(v)$ summarises the *cost elasticity of quality upgrading* for each variety v which is assumed to be the same for H and F. We suppose that $a(v) : [0, 1] \to \mathbb{R}_{++}$, where $a'(\cdot) \geq 0$; analogously, $a^*(v) : [0, 1] \to \mathbb{R}_{++}$, where $a^{*\prime}(\cdot) \geq 0$. We also assume that $\eta(v) : [0, 1] \to \mathbb{R}_{++}$, where $\eta'(\cdot) > 0$ and $\eta(0) > 1$.[10]

The next assumption dictates the pattern of *comparative* advantages across countries.

Assumption 1 *Let $A(v) \equiv a^*(v)/a(v)$. We suppose: $A(0.5) = 1$ and $A'(v) < 0$.*

Assumption 1 represents the *only* source of heterogeneity across countries in our model. In particular, this last assumption implies that H enjoys a comparative advantage in the production

[9] Demand intensities are the continuous counterpart of the discrete-case expenditure shares. Their relationship is analogous to that between densities and discrete probabilities. We borrow this nomenclature from Horvath (2000).

[10] From the labour requirements functions it is apparent that qualitative upgrade is costly, which seems a natural assumption to make. Additionally, from our assumptions it follows that $\eta(v) > 1$ for all $v \in \mathbb{V}$, which implies that the marginal cost of improving quality is, for each variety, increasing along the quality space. In that sense, this assumption also seems quite natural, as it reflects the fact that subsequent quality improvements become increasingly costly. Finally, note that $\eta'(\cdot) > 0$ –coupled with $a'(\cdot) \geq 0$– implies that varieties are sorted by their cost of quality upgrading.

of lower-indexed commodities, while F has a comparative advantage in the production of upper-indexed commodities.[11]

Note that given the cost functions c_{vq} and c^*_{vq} specified above, we are allowing countries to possibly display identical income per head in equilibrium, since we are not imposing any direct source of *absolute* advantage in the model. Furthermore, notice that because $\eta(v)$ is the same for H and F, the nature of the comparative advantages does not change as we move up in the quality ladder. In that sense, in the model comparative advantages always refer to particular varieties of goods, irrespective of the quality at which this variety is actually produced (for example, a country that has a comparative advantage in producing fruit products, will have this advantage both in organic and in non-organic fruit products).

In our world economy, each country will naturally specialise in those commodities which they can produce more cheaply. As a result, the *international* price of each commodity will be given by $p_{vq} = \min\left\{c_{vq}w, c^*_{vq}w^*\right\}$. Given Assumptions 1, we can write the international price of each commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ as follows:

$$p_{vq} = \kappa^{-1}\alpha(v)\, q^{\eta(v)}, \tag{3}$$

where $\alpha(v) \equiv \min\{a(v)\, w, a^*(v)\, w^*\}$. In addition, from (3), the marginal variety m (that is, the variety that can be produced by both countries at the same cost) satisfies:

$$A(m) = w/w^*. \tag{4}$$

Equation (4) implies that, *given* the relative wage w/w^*, H will produce all the varieties in the interval $[0, m]$ and F will produce all the varieties within $[m, 1]$.

4 The Individual's Optimal Consumption Choice

In this section we present the optimal consumption choice of a representative individual from H, given the set of prices in the world economy. The results so obtained can be easily extended to an individual from F, which is done in Appendix B.

An individual from H chooses the quantities $x_{vq} \in \mathbb{R}_+$ to consume of each commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ to solve the following problem:

[11] Assumption 1 is standard in the international trade literature of Ricardian tradition. Letting $\eta(\cdot)$ vary across countries change in a similar fashion as $a(\cdot)$ would not qualitatively alter the results of the paper – in fact, adding heterogeneity on $\eta(\cdot)$, on top of that on $a(\cdot)$, would *reinforce* our findings.

$$\begin{aligned}
\max_{\{x_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}} \quad & U = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max \left\{ x_{vq}, (x_{vq})^{q} \right\} dq \right] dv, \\
\text{subject to:} \quad & \textstyle\int_{\mathbb{V}} \beta_v dv = 1; \\
& \beta_v = w^{-1} \int_{\mathbb{Q}} p_{vq} x_{vq} dq, \quad \forall v \in \mathbb{V} \\
& p_{vq} = \kappa^{-1} \alpha(v)\, q^{\eta(v)}, \quad \forall (v,q) \in \mathbb{V} \times \mathbb{Q}.
\end{aligned} \tag{5}$$

In order to solve the consumer's optimisation problem (5), it proves convenient to state the following preliminary results.

Lemma 1 (Preliminary Results)

(i) For each variety $v \in \mathbb{V}$, at most one quality, denoted henceforth by $q_v \in \mathbb{Q}$, is consumed in strictly positive amount in an optimum; formally: $x_{vq_v} \geq 0$, $x_{vq} = 0$, $\forall q \neq q_v$.

(ii) Take x_{vq_v}, $\forall v \in \mathbb{V}$. Then: $q_v > 1 \Rightarrow x_{vq_v} > 1$.

Proof. See Appendix C. ■

From Lemma 1, Part (i), it immediately follows that the income devoted to purchasing commodities of variety v is entirely spent on quality q_v. Hence, for each $v \in \mathbb{V}$ the consumed quantity of the optimal quality q_v, is given by $x_{vq_v} = \beta_v w / p_{vq_v}$. In addition, if we also consider Part (ii), it follows that we may replace the inner integral $\int_{\mathbb{Q}} \max \left\{ x_{vq}, \left(x^*_{vq}\right)^{q} \right\} dq$ in (5) by the simpler expression $\left(x^*_{vq_v}\right)^{q_v}$, without altering any of the final results of that problem.[12]

Given Lemma 1 the individual's optimisation problem in (5) can be thus restated in a simpler form in terms of two sets of control variables $\{\beta_v, q_v\}_{v\in\mathbb{V}}$ replacing the set of physical quantities $\{x_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}$. In particular, (5) can be restated in the following *reduced-form*:

$$\begin{aligned}
\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad & U = \int_{\mathbb{V}} q_v \ln \left(\frac{w\beta_v}{p_{vq_v}} \right) dv, \\
\textit{subject to:} \quad & \int_{\mathbb{V}} \beta_v dv = 1, \\
& q_v \geq 1, \ \forall v \in \mathbb{V}, \\
& p_{vq_v} = \kappa^{-1} \alpha(v)\, q_v^{\eta(v)}, \ \forall v \in \mathbb{V}.
\end{aligned} \tag{6}$$

[12] To see this more clearly, notice that (keeping in mind the physical constraint $x_{vq} > 0$) $x_{vq_v} < (x_{vq_v})^{q_v}$ if and only if $x_{vq_v} < 1$ and $q_v > 1$, which according to Lemma 1, Part (ii), cannot be true.

The first-order conditions corresponding to (6) are stated in the Appendix A. From those first-order conditions we may obtain the following expression for each β_v in the optimum:

$$\beta_v = \frac{q_v}{Q}, \ \forall v \in \mathbb{V}. \tag{7}$$

where $Q \equiv \int_{\mathbb{V}} q_z dz$ can be regarded as an aggregate index measuring the optimal consumption bundle's *average quality.* Notice that, according to (7), the fraction of income spent on variety v is determined by its optimal quality relative to the average quality of consumption. In that regard, if all varieties were optimally consumed at identical quality degrees (i.e., if $q_v = Q$, $\forall v \in \mathbb{V}$), then $\beta_v = 1$ would hold for all $v \in \mathbb{V}$, and our model would behave exactly as the one by Dornbusch, Fischer and Samuelson (1977).

4.1 Distribution of Qualities and Demand Intensities across Varieties

Given the technology in the world economy –summarised by κ, $\alpha(\cdot)$ and $\eta(\cdot)$– it is possible to characterise the distribution of the optimal qualities across varieties according to their position within the set $\mathbb{V}$. Lemma 2 provides the first result in that direction.

Lemma 2

Consider two varieties $\underline{v}, \bar{v} \in \mathbb{V}$, such that $\underline{v} < \bar{v}$. Then: $q_{\underline{v}} \geq q_{\bar{v}}$, with strict inequality if and only if $q_{\underline{v}} > 1$.

Proof. See Appendix C. ■

Lemma 2, implies that the consumed quality q_v is non-increasing in the variety-index v. The underlying intuition for Lemma 2 is straightforward – those varieties which can be more cheaply upgraded tend to be optimally consumed in higher quality levels.

The monotonicity of q_v implied by Lemma 2 allows us to split the variety space in two disjoint subsets. The first subset containing varieties that are bound to be consumed at the baseline quality level (i.e. with $q_v = 1$) – these are the higher-indexed varieties. The second one comprising the varieties for which the constraint $q_v \geq 1$ in (6) does not bind – these are the lower-indexed varieties. Let us denote by $\mathbb{L} \subseteq \mathbb{V}$ the latter subset.

Definition 1 *Let $\mathbb{L} = \{v \in \mathbb{V} : \lambda_v = 0\}$, where λ_v is the Lagrange multiplier associated to the constraint $q_v \geq 1$.*

Remark 1 *Both $\mathbb{L} = \emptyset$ and $\mathbb{L} = \mathbb{V}$ are in principle possible. In fact, $\mathbb{L} = \emptyset$ will hold if κ is sufficiently small, while $\mathbb{L} = \mathbb{V}$ will hold if κ is sufficiently large. (See Lemma 3 ahead.)*

Lastly, regarding the distribution of the demand intensities, from the condition in (7) we can observe that, in the optimum, demand intensities are set proportional to the optimal qualities. As a result, the distribution of β_v across varieties will qualitatively mirror that one of q_v.

4.2 Effects of Aggregate Productivity Shocks on Demand

When the technology is subject to changes, both *substitution-effects* (due to adjustments in relative prices) and *income-effects* (due to the overall effect of variations in productivity) arise. Here we focus our attention solely on income-effects. In order to isolate income-effects from substitution-effects, we let the parameter κ vary, while we keep constant the functions $a(\cdot)$, $a^*(\cdot)$ and $\eta(\cdot)$.

Lemma 3

Let $\underline{\kappa} \equiv a(0)\exp[\eta(0)]$*. Then:*

(i) for all $\kappa \in (0, \underline{\kappa}) : \mathbb{L} = \emptyset$*;*

(ii) for all $\kappa \geq \underline{\kappa} : \mathbb{L} = [0, \tilde{v}(\kappa)]$*, where* $\tilde{v}(\kappa) : [\underline{\kappa}, \infty) \rightarrow [0, 1]$*,* $\tilde{v}(\underline{\kappa}) = 0$*, and* $\tilde{v}'(\kappa) > 0$ *whenever* $\tilde{v}(\kappa) < 1$*.*

Proof. See Appendix C. ■

In short, Lemma 3 implies that the subset of varieties consumed at the *baseline* quality level initially comprises the entire set $\mathbb{V}$, and eventually starts narrowing as world aggregate productivity increases beyond the threshold $\underline{\kappa}$. The next lemma describes in further detail how optimal qualities evolve as the parameter κ changes.

Lemma 4

i) If $\kappa \in (0, \underline{\kappa})$*:* $\partial q_v / \partial \kappa = 0$ *for all* $v \in \mathbb{V}$*;*

ii) If $\kappa \geq \underline{\kappa}$*: a) for all* $v \in \mathbb{L}$*,* $\partial q_v / \partial \kappa > 0$*; b) for all* $v \notin \mathbb{L}$*,* $\partial q_v / \partial \kappa = 0$*; c) for all* $\underline{v}, \overline{v} \in \mathbb{V}$*, such that* $\underline{v} < \overline{v}$*,* $\partial q_{\underline{v}} / \partial \kappa \geq \partial q_{\overline{v}} / \partial \kappa$*, with strict inequality if and only if* $\underline{v} \in \mathbb{L}$*.*

Proof. See Appendix C. ■

Lemma 4 shows that, for all varieties belonging to $\mathbb{L}$, quality increases when the aggregate productivity in the world rises. Furthermore, this effect is stronger for those varieties whose quality can be more cheaply upgraded – i.e., those varieties carrying a lower $\eta(v)$. On the other hand, we can observe that the optimal quality of varieties that do not belong to $\mathbb{L}$ does not respond to (infinitesimal!) changes in κ.

Based on Lemma 3 and Lemma 4, we can accordingly identify two distinct regimes depending on the level of κ that prevails. First, we refer to an economy such that $\kappa \leq \underline{\kappa}$ as a *subsistence economy* – in a subsistence economy all varieties are consumed at the baseline quality level. Second, we refer to an economy with $\kappa > \underline{\kappa}$ as a *modern economy* – in a modern economy some varieties (and possibly all of them) are consumed strictly above the baseline quality level. In what follows we proceed to further characterise these two regimes.

Subsistence Economy: $\kappa \leq \underline{\kappa}$

In this regime $q_v = 1$ holds for all $v \in \mathbb{V}$. This in turn means that $Q = 1$ and $\beta_v = 1$ must hold for all $v \in \mathbb{V}$ as well. Thus, in a subsistence economy demand intensities remain constant and equal to one for all varieties as κ increases.[13] In that regard, a subsistence economy displays analogous behaviour to the economy discussed in Dornbusch *et al* (1977), where demand schedules are homothetic.

Modern Economy: $\kappa > \underline{\kappa}$

This regime is characterised by $q_v > 1$ for all $v \in [0, \tilde{v}(\kappa))$. Hence, the average quality can be written as $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z \, dz$, from where it follows that $\partial Q / \partial \kappa = \int_0^{\tilde{v}(\kappa)} (\partial q_z / \partial \kappa) \, dz > 0$. Since $\partial q_v / \partial \kappa = 0$ for all $v \notin \mathbb{L}$, then because of (7), $\partial \beta_v / \partial \kappa < 0$ must hold for all $v \notin \mathbb{L}$. As a result, given that $\int_{\mathbb{V}} \beta_v \, dv = 1$, it must thus be the case that the demand intensities of some (and possibly all) $v \in \mathbb{L}$ will increase as κ rises. Let $\mathbb{J} \subset \mathbb{V}$ denote the subset of $\mathbb{V}$ comprising all those varieties for which $\partial \beta_v / \partial \kappa > 0$.

Definition 2 *Let* $\mathbb{J} = \{v \in \mathbb{V} : \partial \beta_v / \partial \kappa > 0\}$.

In a subsistence economy $\mathbb{J} = \emptyset$, while in a modern economy $\mathbb{J} \neq \emptyset$. In other words, in a modern economy the homotheticity of demand intensities no longer holds, as a subset of varieties whose income demand elasticity is larger than one shows up. Notice finally that $\mathbb{J} \subset \mathbb{L}$, since $\partial q_v / \partial \kappa > 0$ is a necessary condition for $\partial \beta_v / \partial \kappa > 0$ to hold.

The next proposition further characterises the behaviour of the demand intensity β_v of a generic variety v, in relation to those of the other varieties, as κ rises.

[13] It must be noted that this result applies only if $\kappa \leq \underline{\kappa}$ holds *after* performing the comparative statics exercise.

Proposition 1

Consider any two varieties $\underline{v}, \overline{v} \in \mathbb{V}$, such that $\underline{v} < \overline{v}$. Then:

i) If $\underline{v} \in \mathbb{J}$: $\partial\beta_{\underline{v}}/\partial\kappa > \partial\beta_{\overline{v}}/\partial\kappa$;

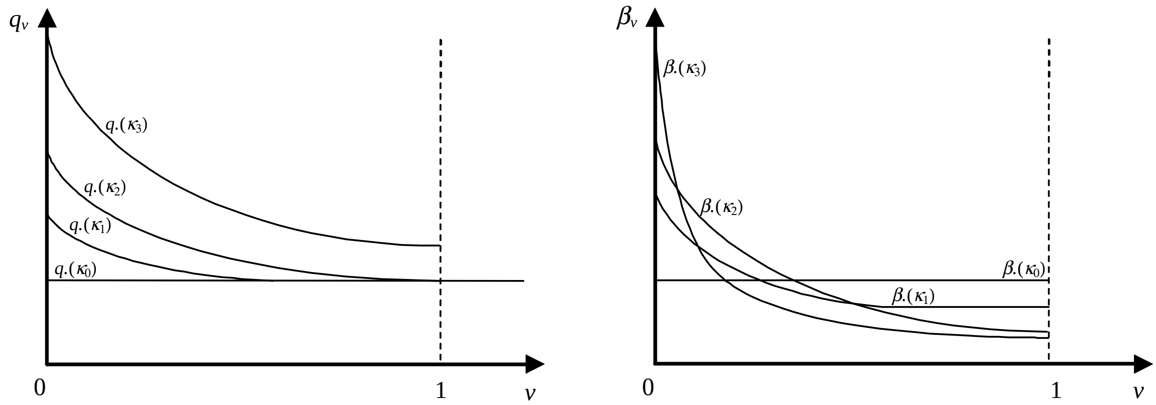
ii) If $\underline{v} \notin \mathbb{J}$: $\partial\beta_{\overline{v}}/\partial\kappa \leq 0$.

Proof. See Appendix C. ■

To interpret our previous results more clearly, notice that $\mathbb{J}$ may be understood as the set of *luxury goods*, where by luxury goods we refer to those varieties whose income demand elasticity is larger than 1. Since the set $\mathbb{J}$ always comprises lower-indexed varieties, the luxury goods are exactly those varieties whose quality degree q_v is relatively high compared to the average quality Q. In that sense, in our model it is the (relative) *quality* that determines whether or not a particular variety is *luxurious*.

Figure 1 illustrates this feature graphically. The distributions of qualities and demand intensities across varieties are drawn for four different levels of worldwide aggregate productivity ($\kappa_0 \leq \underline{\kappa} < \kappa_1 < \kappa_2 < \kappa_3$). When individuals are still poor (i.e., for a level of productivity $\kappa_0 \leq \underline{\kappa}$), satisfying all basic needs constitutes their main goal, leading them to keep the quality of all goods at the baseline level and setting accordingly equal demand intensities for all varieties. As individuals become richer, some varieties – for a level of productivity $\kappa_1 \in (\kappa_0, \kappa_2)$ – and eventually all varieties – for a level of productivity $\kappa_3 \geq \kappa_2$ – are consumed in higher quality degrees. As a result, for those three levels of κ, a subset of varieties with $\beta_v > 1$ appears in the

FIGURE 1 – Changes in the distribution of qualities and demand intensities as aggregate productivity increases



lower spectrum of the (unit) variety set. Additionally, the varieties whose quality degree is relatively higher attract increasingly larger income shares, as given the preference specification in (1) individuals tend to *value* high-quality commodities relatively more as they become wealthier. This last point is formalised in the following corollary.

Corollary 1

Let $\vartheta(v) \equiv \int_0^v \beta_z \, dz$. *Then:*

(i) If $\kappa < \underline{\kappa} : \partial\vartheta(v)/\partial\kappa = 0, \forall v \in \mathbb{V};$

(ii) If $\kappa \geq \underline{\kappa} : \partial\vartheta(v)/\partial\kappa > 0, \forall v \in [0,1).$

Proof. See Appendix C. ■

Corollary 1 synthesizes the eventual non-homothetic behaviour of the demand schedules implied by our model. In particular, whenever $\kappa < \underline{\kappa}$, demand schedules are homothetic across varieties. However, when κ lies above the threshold $\underline{\kappa}$, income starts being spent in growing proportion on lower-indexed varieties.

5 General Equilibrium in the World Economy

In Section 4, we have studied the optimal consumption choice of an individual from H, taking the wages in H and in F, w and w^*, as exogenously given. (In Appendix B, we do the same for the case of an individual from F.) These wages in turn determine the prices of all reproducible commodities in the world economy through equation (3). Our former analysis has therefore yielded only partial equilibrium results.

The present section computes the general equilibrium in this world economy. This requires endogenising wages and, thereby, the prices of all reproducible commodities. Given that in a general equilibrium only relative prices are determined, we henceforth take the wage in F as the *numeraire*, by setting $w^* = 1$.

In order to disregard the effects of heterogeneous population size in different countries, we suppose that both H and F are inhabited by a continuum of individuals with identical mass, which we normalise to one. (We explore the general equilibrium effects of heterogenous population size and growth later on in Section 6).

A representative individual from H will then solve:

$$\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad U=\int_0^m q_v \ln\left(\frac{\beta_v\kappa}{a(v)\,q_v^{\eta(v)}}\right)dv+\int_m^1 q_v\ln\left(\frac{\beta_v\kappa\, w}{a^*(v)\,q_v^{\eta(v)}}\right)dv,$$
$$\text{subject to:} \quad \int_{\mathbb{V}}\beta_v\,dv=1;\ \text{and } q_v\geq 1,\ \forall v\in\mathbb{V}. \tag{8}$$

On the other hand, a representative individual from F solves:

$$\max_{\{q_v^*,\beta_v^*\}_{v\in\mathbb{V}}} \quad U^*=\int_0^m q_v^* \ln\left(\frac{\beta_v^*\kappa}{a(v)\,q_v^{\eta(v)}w}\right)dv+\int_m^1 q_v^*\ln\left(\frac{\beta_v^*\kappa}{a^*(v)\,q_v^{\eta(v)}}\right)dv,$$
$$\text{subject to:} \quad \int_{\mathbb{V}}\beta_v^*\,dv=1;\ \text{and } q_v^*\geq 1,\ \forall v\in\mathbb{V}. \tag{9}$$

The solution of (8) and (9) yields the demand functions of each variety $v\in\mathbb{V}$ by H and F, respectively. By using $\vartheta(v)\equiv\int_0^v\beta_z\,dz$ – as defined in Corollary 1 – and $\vartheta^*(v)\equiv\int_0^v\beta_z^*dz$ – see Corollary 1 (Foreign) in Appendix B – we can write the equilibrium condition for the market of goods produced in H as follows:

$$\vartheta(m)\,w+\vartheta^*(m)=w, \tag{10}$$

where m is the marginal variety as defined by (4). Condition (10) essentially says that the aggregate amount of income spent by the world in goods produced in H must be equal to the aggregate income of H. This condition can also be understood as the equilibrium condition for the labour market in H.[14]

The world economy general equilibrium is determined by (4), (8), (9), and (10). We will henceforth focus our attention on the equilibrium values of w and m, and on how these two variables respond to some simple comparative statics exercises.

Worldwide Uniform Aggregate Productivity Growth

In this subsection, we look at the impact of changes in κ on the equilibrium values of w and m. We can split the results in two different cases.

Subsistence economies: $\kappa\leq\underline{\kappa}$

From our previous discussion, we can observe that when $\kappa\leq\underline{\kappa}$, the optimal demand intensities are set at $\beta_v=\beta_v^*=1$ for all $v\in\mathbb{V}$. This result in turn implies that $\vartheta(m)=\vartheta^*(m)=m$.

[14] Because of the Walras' Law, an analogous condition can be derived for the equilibrium in the labour market in F.

Therefore, (10) simplifies to:

$$w = m/(1-m). \tag{11}$$

Combining then (4) with (11), leads to $m/(1-m) = A(m)$, from where it follows that, for all $\kappa \leq \underline{\kappa}$, in equilibrium: $w = 1$ and $m = 0.5$. That is, H and F exhibit the *same* level of income, and the pattern of regional specialisation is accordingly dictated by the "natural" comparative advantage of each country without relative-wage bias (i.e., the comparative advantage that derives purely from the heterogeneity in the technological structure implied by Assumption 1).[15]

Modern economies: $\kappa > \underline{\kappa}$

When aggregate productivity is sufficiently high, the income equality between H and F no longer holds. In particular, as κ rises above the threshold $\underline{\kappa}$, the terms of trade start moving in favour of H, and thus H becomes relatively richer than F. Furthermore, the income disparity between H and F increases as κ keeps rising.

Proposition 2

Suppose Assumption 1 holds. In addition, suppose $\kappa > \underline{\kappa}$. *Then, in equilibrium:*

(i) $w > 1$ *and* $m < 0.5$.

(ii) $\partial w/\partial \kappa > 0$ *and* $\partial m/\partial \kappa < 0$.

Proof.

Part (i). When $\kappa > \underline{\kappa}$, from Corollary 1 it follows that $\vartheta(m) > m$ and $\vartheta^*(m) > m$. As a result, by using (10), we can obtain:

$$w = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m}. \tag{12}$$

Combining next (12) with (4), and recalling Assumption 1 leads to:

$$A(m) = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m} \quad \Leftrightarrow \quad m < 0.5.$$

Finally, since $m < 0.5$, (4) implies that $w > 1$.

[15] Notice that, since $w = 1$ for all $\kappa \leq \underline{\kappa}$, in fact $\underline{\kappa} = \underline{\kappa}^*$ (that is, the threshold on κ that divides a subsistence-economy from a modern economy happens to be the same for both H and F). As a consequence, we can refer to *both* thresholds simply as $\underline{\kappa}$.

Part (ii). Next, to study how w and m vary as κ keeps rising above $\underline{\kappa}$, we differentiate the equilibrium conditions (4) and (10). This leads to:

$$\frac{\partial w}{\partial \kappa} = A'(m)\frac{\partial m}{\partial \kappa} \tag{13}$$

and

$$(w\beta_m + \beta_m^*)\,\frac{\partial m}{\partial \kappa} + \left(w\frac{\partial \vartheta(m)}{\partial w} + \vartheta(m) + \frac{\partial \vartheta^*(m)}{\partial w}\right)\frac{\partial w}{\partial \kappa} + \left(\frac{\partial \vartheta(m)}{\partial \kappa} + \frac{\partial \vartheta^*(m)}{\partial \kappa}\right) = \frac{\partial w}{\partial \kappa}, \tag{14}$$

where the first term in (14) uses the fact that $\partial\vartheta(m)/\partial m = \beta_m$ and $\partial\vartheta^*(m)/\partial m = \beta_m^*$. Plugging (13) into (14), we can obtain:

$$\frac{\partial m}{\partial \kappa} = \frac{\partial\vartheta(m)/\partial\kappa + \partial\vartheta^*(m)/\partial\kappa}{\left[1 - \vartheta(m) - w\,\partial\vartheta(m)/\partial w - \partial\vartheta^*(m)/\partial w\right] A'(m) - (w\beta_m + \beta_m^*)}. \tag{15}$$

For determining the sign of (15), we can use the following two results: first, Corollary 1 states that both $\partial\vartheta(m)/\partial\kappa > 0$ and $\partial\vartheta^*(m)/\partial\kappa > 0$; second, as shown in Appendix D, $\partial\vartheta(m)/\partial w \leq 0$ and $\partial\vartheta^*(m)/\partial w < 0$. Therefore, since $1 - \vartheta(m) > 0$ and $A'(m) < 0$, then $\partial m/\partial\kappa < 0$ obtains from the right-hand side of (15). Finally, from (13) it then follows that $\partial w/\partial\kappa > 0$. ■

Proposition 2 shows that as the worldwide productivity parameter, κ, increases, the income in H eventually begins diverging away from the income in F. The reason for the divergence rests on the fact that H enjoys a comparative advantage in lower-indexed varieties, which tend to be consumed in relatively higher quality levels and display accordingly higher income demand elasticity. As a consequence, as the world economy grows *uniformly* above $\underline{\kappa}$, aggregate world expenditure shifts towards the set of commodities produced by H. The ensuing excess demand for commodities produced in H causes excess labour demand in H and w thus goes up. In turn, as w rises, the marginal variety moves to the left (i.e., m falls), and some of the varieties that used to be produced by H start being now produced by F, restoring the equilibrium in the labour markets.

The endogenous emergence of income disparities in the absence of absolute advantages in this two-country world economy represents the main result and novelty of our paper. Initially, H and F display the same level of income per head. Although sectorial productivities differ across the two countries and govern the patterns of regional specialisation, their heterogeneity is not enough to warrant income disparities between H and F. This is because at low levels of worldwide aggregate productivity the willingness to pay for high-quality commodities is not large enough for tilting aggregate demand disproportionally towards the varieties produced by H. However, in a context where worldwide aggregate productivity rises leading to higher incomes in both H and F, varieties exhibiting larger scope for quality upgrading become increasingly appreciated by the consumer and, thus, start absorbing larger budget shares. Within a general equilibrium framework, this mechanism implies that aggregate demand shifts towards H, inducing faster income growth in H relative to F, via the secular tendency to improve H's terms of trade.

6 Some Other Comparative Statics Exercises

This subsection briefly studies the results of two other comparative statics experiments commonly explored by the previous literature on international trade with non-homothetic preferences. Firstly, we analyse the effects of uneven population growth across countries, and show that the country in which population grows faster tends to experience a decline in its terms of trade and relative income. Secondly, we look at the case of income inequality within countries, and show that inequality within countries tends to improve the terms of trade and the relative income of the economy that specialises in varieties that display higher income demand elasticity.

6.1 Introducing Uneven Population Growth

So far we have assumed that each country is identical regarding all relevant features for our model. In this subsection we let the population size in F differ from that in H. Let the total mass of individuals in F equal $L > 1$. Thus, the labour market equilibrium condition in H will be given by:

$$\vartheta(m)w + L\vartheta^*(m) = w. \tag{16}$$

Visual inspection on (16) and (10) –combined with (4)– immediately implies that the equilibrium value of w that is delivered by (16) will be strictly larger than that yielded by (10). In particular, in equilibrium $w > 1$, regardless of the value of κ. The next proposition shows that this source of income divergence between F and H is stronger the larger the value of L.

Proposition 3

The relative wage in H is increasing in the size of the population in F.

Proof. Totally differentiating (16), and bearing in mind that, from (4), $\partial w/\partial L = A'(m)\ (\partial m/\partial L)$ must hold in equilibrium, leads to:

$$\frac{\partial w}{\partial L} = \frac{w}{L} \frac{1 - \vartheta(m)}{\left[1 - \vartheta(m) - w\frac{\partial \vartheta(m)}{\partial w} - L\frac{\partial \vartheta^*(m)}{\partial w} - \frac{1}{A'(m)}\left(\beta_m w + \beta_m^* L\right)\right]} > 0. \tag{17}$$

where the positive in (17) obtains from $\partial\vartheta(m)/\partial w \leq 0$, $\partial\vartheta^*(m)/\partial w \leq 0$, and $A'(m) < 0$. ■

When the population in F increases, the relative wage w must go up so as to accommodate the excess supply of labour in F. More precisely, a larger L requires more goods to be produced by F in order to keep full employment there; this is accomplished by letting w go up, which in turn shifts the marginal variety m to the left, helping restore the equilibrium in the labour markets.

The result that the H relative wage rises as the relative population of F increases is in line with the models in Flam and Helpman (1987), Stokey (1991) and Matsuyama (2000). However, some interesting differences are also present. In Flam-Helpman and Stokey, although the optimal bundle of goods traded in the market changes, no new goods actually appear in the world economy as w rises due to uneven population growth in the world. In Matsuyama, new goods start being produced in the country whose population grows slower (i.e., in H). In our model, this feature is present too. Furthermore, new goods start being produced *also* by F. More precisely, as individuals in H become richer when w increases, they will start demanding higher qualities for the varieties produced in F, as well as for those produced domestically. This last result is absent in Matsuyama, where F does not begin producing new goods, but only takes on the production of (some) varieties that are abandoned by H as w increases.[16]

In our model, the equilibrium adjustments triggered by heterogeneity in population growth rates across countries generates thus two types of product cycle phenomena. First, like in Matsuyama, we have an *international product cycle* phenomenon involving both countries simultaneously, similar to those discussed by Linder (1961) and Vernon (1966), where over time the production of lower-quality goods moves from H to F, while H specialises in more sophisticated higher-quality goods. The second phenomenon, which is novel to our model, occurs within each variety and could be called *regional product cycle*, as it involves single countries individually: rising income in H leads both H *and* F to abandon the manufacturing of lower-quality goods and replace them with the production of goods of higher quality standards.

[16] For example, our model then predicts that Africa will start to produce organic bananas to sell in Europe, as increasingly richer European consumers begin desiring to purchase higher-quality fruit products. This type of adjustment in the production structure of the economies whose population grows faster and relative income declines is absent in papers cited above in the main text.

6.2 Introducing Income Inequality within each Country

In this subsection we examine the general equilibrium consequences of introducing some degree of income heterogeneity within countries.

To keep the analysis short and concise, we introduce income inequality only in F (H), while we maintain the assumption that the population in H (F) is homogeneous. In particular, we assume that F (H) is inhabited by two types of individuals: p and r, where the p stands for *poor* and r stands for *rich.* Each sub-group of foreigners has mass equal to 0.5. A type p is endowed with $1 - \iota$ units of effective labour, while a type r is endowed with $1 + \iota$ units of it; where $\iota \in (0, 1)$. On the other hand, in H (F) everyone is endowed with 1 unit of effective labour.

Introducing income inequality in the model leads to interesting results when the types p are so poor that, in equilibrium, they consume all varieties at the baseline quality level, whereas in contrast the types r can afford consuming some of the varieties strictly above that level. To focus on such case, we accordingly set $\kappa = \underline{\kappa}$.

Proposition 4

Suppose that the population in F (H) is split in two groups, p and r, of equal mass. Individuals in p are endowed with $1 - \iota$ units of effective labour and individuals in r are endowed with $1 + \iota$ units of it, where $\iota \in (0, 1)$. The population in H (F) is homogeneously endowed with one unit of effective labour. Additionally, suppose that $\kappa = \underline{\kappa}$. Then, in equilibrium:

(i) $w > 1$ and $m < 0.5$.

(ii) $\partial w/\partial \iota > 0$ and $\partial m/\partial \iota < 0$.

Proof. See Appendix C. ■

When $\kappa = \underline{\kappa}$, introducing income inequality in F raises the relative wage in H. This is owing to the non-homotheticity of the demand schedules of the rich foreigners. More precisely, increasing ι transfers income from the poor foreigners, who spend a fraction m of it in goods from H, to the rich foreigners, who spend a fraction $\vartheta_r^*(m) > m$ of their income on those commodities. As a result, aggregate demand for goods produced in H rises leading to higher w.

Similarly, we can observe that incorporating inequality in H carries similar consequences on w and m. This is the case because the rich locals would tend to shift aggregate demand towards the goods produced in H, exactly the same shift induced by the presence of rich foreigners.

7 Conclusion

We have proposed a model of international trade with non-homothetic preferences based on comparative advantages that are *unrelated* to the stage in the process of development in which countries are. This last feature represents the main point of departure with respect to the past literature on North-South trade, where comparative advantages originate from the fact that some countries (the North) have historically accumulated larger amounts of capital than others (the South).

The key novel finding of our model is observing that even when no single country enjoys a clear absolute advantage over any other country and productivity changes are uniform and identical in all countries, international trade may still be the source of income divergence in the world economy. In particular, countries' incomes will diverge when comparative advantages induce patterns of specialisation that, although optimal for each country at early stages in the process of development, do not offer the same scope for improvements in terms of quality upgrading of final products in the long run.

Our model also points out that heterogeneity in population growth rates across countries generates two distinct types of product cycle phenomena. The first is an *international product cycle* phenomenon *à la* Linder-Vernon, where over time one economy takes on the production of goods previously produced by another economy. The second – which is novel to our model – is a *regional product cycle* that occurs within each variety and within each economy: rising income in *one* particular economy makes *all* economies engage in the production of goods of higher quality standards (so as to satisfy the increasing demand for high qualities by the economy that is becoming wealthier).

In this paper, we have focused on illustrating how our theory may shed light on historical cases where comparative advantages emerged as a result of heterogeneous geographical conditions. Other issues, including what determines the relative successes or failures of economies with similar comparative advantages, and why richer countries tend to trade among themselves more than countries with substantial income disparities do, are the subject of ongoing research.

Appendices

A First-Order Conditions for Consumption Choice in H

The optimisation problem in (6) yields the following first-order conditions (where μ represents the Lagrange multiplier associated to the budget constraint and $\{\lambda_v\}_{v\in\mathbb{V}}$ denote those associated to the constraints $\{q_v \geq 1\}_{v\in\mathbb{V}}$):

$$\ln\left(\frac{\beta_v w}{\kappa^{-1}\alpha(v)\, q_v^{\eta(v)}}\right) - \eta(v) + \lambda_v = 0,\ \forall v \in \mathbb{V}; \tag{18}$$

$$\frac{q_v}{\beta_v} - \mu = 0,\ \forall v \in \mathbb{V}; \tag{19}$$

$$q_v - 1 \geq 0,\ \lambda_v \geq 0,\ \text{and}\ \lambda_v (q_v - 1) = 0,\ \forall v \in \mathbb{V}; \tag{20}$$

$$1 - \int_{\mathbb{V}} \beta_v \, dv = 0. \tag{21}$$

From (19), it follows that $\beta_v = q_v/\mu$. Then, replacing this last expression into (21) leads to $\int_{\mathbb{V}} q_z \, dz = \mu$, from where the condition (7) immediately obtains by using again (19).

By using the condition (7), we can rewrite (18) as:

$$\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln\kappa + \ln Q + [\eta(v) - 1]\ln q_v. \tag{22}$$

The expression in (22) will be used in many of the following proofs.

B Optimal Consumption Choice in F

Bearing in mind Assumption 1, we can write down the optimisation problem faced by a representative individual from F as follows:

$$\max_{\{x^*_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}} \quad U^* = \int_{\mathbb{V}} \ln\left[\int_{\mathbb{Q}} \max\left\{x^*_{vq}, \left(x^*_{vq}\right)^q\right\} dq\right] dv,$$

$$\text{subject to:} \quad \textstyle\int_{\mathbb{V}} \beta^*_v dv = 1;$$

$$\beta^*_v = (w^*)^{-1} \int_{\mathbb{Q}} p_{vq} x^*_{vq} dq, \quad \forall v \in \mathbb{V};$$

$$p_{vq} = \kappa^{-1} q^{\eta(v)} \alpha(v), \quad \forall (v, q) \in \mathbb{V} \times \mathbb{Q}.$$

Lemma 1 holds for x^*_{vq} in a similar fashion as for x_{vq}. Hence, we can re-state the problem specified above in terms of q^*_v and β^*_v, as it was previously done for H (where q^*_v now denotes the quality of variety v consumed, in the optimum, in F). This way, we can obtain the following first-order conditions, which constitute the analogous versions for F of (7) and (22), respectively:

$$\beta_v^* = \frac{q_v^*}{\int_{\mathbb{V}} q_z^* \, dz}, \; \forall v \in \mathbb{V}, \tag{23}$$

$$\lambda_v^* = \eta(v) + \ln[\alpha(v)/w^*] - \ln\kappa + \ln Q^* + [\eta(v) - 1]\ln q_v^*. \tag{24}$$

Given the first-order conditions in (23) and (24), all the ensuing results found in Section 4 follow through in qualitative terms. In particular, we can derive functions $\{q_v^*\}_{v\in\mathbb{V}}$ and $\{\beta_v^*\}_{v\in\mathbb{V}}$ displaying identical qualitative properties as their *counterparts* in H, that is $\{q_v\}_{v\in\mathbb{V}}$ and $\{\beta_v\}_{v\in\mathbb{V}}$, in terms of Lemmas 2 - 4 and Proposition 1. Furthermore, we can similarly find the threshold $\underline{\kappa}^*$ for the worldwide aggregate-productivity parameter, which splits F in the regimes of *subsistence-economy* and *modern-economy*; both exhibiting analogous properties as described for H.[17] Finally, likewise for H in Corollary 1, for F the following holds:

Corollary 2

Let $\vartheta^*(v) \equiv \int_0^v \beta_z^* \, dz$. *Then:*

(i) If $\kappa < \underline{\kappa}^* : \partial\vartheta^*(v)/\partial\kappa = 0, \; \forall\, v \in \mathbb{V}$*;*

(ii) If $\kappa \geq \underline{\kappa}^* : \partial\vartheta^*(v)/\partial\kappa > 0, \; \forall\, v \in [0,1)$*.*

C Omitted Proofs

Proof of Lemma 1.

Part (i). First, notice that utility is given by an additive function over logarithms. Optimization can thus be split in two stages: (a) maximise U with respect to the logarithms; (b) maximise those logarithms with respect to x_{vq}. About (b), notice that the logarithms are defined on the integral over convex functions of x_{vq}, and therefore are themselves convex functions. It follows that (b) optimally requires corner solutions, so the result claimed obtains.

Part (ii). The proof follows immediately from noting that, for all $v \in \mathbb{V}$, utility derived from consuming $x_{vq} \in (0,1]$ is independent of the quality-index q, while according to (3) the price of commodity $(v,q) \in \mathbb{V} \times \mathbb{Q}$ is strictly increasing along the quality space. ■

Proof of Lemma 2.

[17] From Section 5, it is straightforward to observe that, given Assumption 1, $\underline{\kappa}^* = \underline{\kappa}$.

First, suppose $q_{\underline{v}} < q_{\overline{v}}$. Since $q_{\underline{v}} \geq 1$, then $q_{\overline{v}} > 1$, hence (22) paired with (20) yield: $\eta(\underline{v}) + \ln[\alpha(\underline{v})/w] - \ln(\kappa/Q) \geq 0$, while $\eta(\overline{v}) + \ln[\alpha(\overline{v})/w] - \ln(\kappa/Q) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} = 0$. Thus:

$$\eta(\underline{v}) + \ln\alpha(\underline{v}) \geq \eta(\overline{v}) + \ln\alpha(\overline{v}) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}}.$$

This last equality in turn leads to:

$$[\eta(\overline{v}) - \eta(\underline{v})] + \ln[\alpha(\overline{v})/\alpha(\underline{v})] + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} \leq 0,$$

which cannot possibly hold if $q_{\overline{v}} > 1$, as its left-hand side would then be strictly positive. Therefore, it must necessarily be the case that $q_{\underline{v}} \geq q_{\overline{v}}$.

Next, suppose $q_{\underline{v}} = q_{\overline{v}} > 1$. In this case, (22) in conjunction with (20) yield:

$$\eta(\underline{v}) + \ln\alpha(\underline{v}) + [\eta(\underline{v}) - 1]\ln q_{\overline{v}} = \eta(\overline{v}) + \ln\alpha(\overline{v}) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} = 0.$$

This last equality in turn leads to:

$$-[\eta(\overline{v}) - \eta(\underline{v})](1 + \ln q_{\overline{v}}) = \ln[\alpha(\overline{v})/\alpha(\underline{v})].$$

However, this last equality cannot possibly hold since its right-hand side is strictly positive, while the left-hand side is negative. As a result, $q_{\underline{v}} > q_{\overline{v}}$ must necessarily hold when $q_{\underline{v}} > 1$. ■

Proof of Lemma 3.

In order to prove this lemma it proves convenient to state first the following result:

Claim 1 The optimal quality q_v of any variety $v \in \mathbb{V}$ can be written as follows:

$$q_v = \max\left\{\Phi_{0,v}(q_0)^{\Upsilon_{0,v}}, 1\right\}; \tag{25}$$

where:

$$\Phi_{0,v} \equiv \left[\frac{e^{\eta(0)}\alpha(0)}{e^{\eta(v)}\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} > 0, \text{ and } \Upsilon_{0,v} \equiv \frac{\eta(0) - 1}{\eta(v) - 1} > 0.$$

Proof. See Appendix D.

Next, notice that, from (25), $\partial\Phi_{0,v}(v)/\partial v < 0$ and $\partial\Upsilon_{0,v}(v)/\partial v < 0$ since $a'(v) \geq 0$ and $\eta'(v) > 0$, hence the set $\mathbb{L} \subseteq \mathbb{V}$ comprises the lower-indexed varieties in $\mathbb{V}$, with $\tilde{v}(\underline{\kappa})$ representing its upper bound.

Part (i). When $\kappa \in (0, \underline{\kappa})$, conditions stipulated in (20) and (22) applied on $v = 0$ entail that: $q_0 = 1$ and $\lambda_0 > 0$. As a result, from Lemma 2 it follows that $q_v = 1$, $\forall v \in \mathbb{V}$. Therefore, since $a'(v) \geq 0$ and $\eta'(v) > 0$, again from (22), $\lambda_v > 0$ for all $v \in \mathbb{V}$ obtains, and thus $\mathbb{L} = \emptyset$.

Part (ii). Note that (22) applied on $v = 0$, in conjunction Lemma 2, implies that when $\kappa = \underline{\kappa}$, then $\lambda_0 = 0$ and $q_0 = 1$. Then, Lemma 2 implies $Q = 1$. Using these results in (22) yields: $\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln \underline{\kappa}$, implying that $\lambda_v > 0$ for all $v \in (0, 1]$. As a result, the set $\mathbb{L} = 0$, meaning that $\tilde{v}(\underline{\kappa}) = 0$.

Claim 2 If $\tilde{v}(\kappa) < 1$, then $q_{\tilde{v}(\kappa)} = 1$.

Proof. See Appendix D.

Given Claim 2 and Lemma 2, the aggregate quality index can be written as follows: $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v \, dv$. Furthermore, observe that, whenever $\tilde{v}(\kappa) < 1$, $\ln(\kappa/Q) = \eta(\tilde{v}(\kappa)) + \ln[\alpha(\tilde{v}(\kappa))/w]$ must hold in equilibrium. This last condition yields, after some simple algebra, $Q = \kappa w \exp[-\eta(\tilde{v})]/\alpha(\tilde{v})$. In addition to that, because of Lemma 2, in equilibrium, $[\eta(v) - 1]\ln q_v = \ln(\kappa/Q) - \eta(v) - \ln[\alpha(v)/w]$ must hold for any $v \leq \tilde{v}(\kappa)$. By using the former in the latter, after some algebra, we may obtain:

$$q_v = q_v(\tilde{v}(\kappa)) \equiv \left[\frac{\alpha(\tilde{v}(\kappa))}{\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} \exp\left[\frac{\eta(\tilde{v}(\kappa)) - \eta(v)}{\eta(v) - 1}\right], \quad \forall v \in [0, \tilde{v}(\kappa)]. \tag{26}$$

In equilibrium, it must be the case that:

$$\kappa w \exp[-\eta(\tilde{v}(\kappa))]/a(\tilde{v}(\kappa)) = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v(\tilde{v}(\kappa)) \, dv, \tag{27}$$

where the right hand-side of (27) uses (26). Computing the total differentiation of (27), yields after some algebra:[18]

$$\frac{Q}{\kappa} d\kappa = \left[\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right] \left[Q + \int_0^{\tilde{v}(\kappa)} \frac{q_v}{\eta(v) - 1} dv\right] d\tilde{v},$$

leading finally to:

$$\frac{d\tilde{v}}{d\kappa} = \left[\frac{\kappa}{Q}\left(\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right)\left(1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} \frac{\eta(v)}{\eta(v) - 1} q_v dv\right)\right]^{-1} > 0.$$

where the last inequality follows from the properties of the functions $\alpha(\cdot)$ and $\eta(\cdot)$ ■

Proof of Lemma 4.

Part (i). Proof follows immediately from noting that Lemma 3 implies that, whenever $\kappa \in$

[18] One subtle caveat applies here. Even if both $wa(v)$ and $w^*a^*(v)$ are differentiable functions over the whole domain of v, the envelope function $\alpha(v)$ will not necessarily be so. In particular, $\alpha(v)$ may not be differentiable at the point $v = m$. As a result, if $\tilde{v} = m$, $\alpha'(\tilde{v})$ may not exist. In the very specific case where this "anomaly" holds, we take that $\alpha'(v) = \lim_{\triangle v \to 0^+} \frac{\triangle \alpha(v)}{\triangle v}$.

$(0, \underline{\kappa})$, $q_v = 1$ must hold for all $v \in \mathbb{V}$. Thus, whenever $\kappa \in (0, \underline{\kappa})$, $\partial q_v / \partial \kappa = 0$ for all $v \in \mathbb{V}$.

Part (ii.a). Differentiating (25), computed for any $v \in \mathbb{L}$, with respect to κ yields:

$$\frac{dq_v}{d\kappa} = \frac{\eta(0) - 1}{\eta(v) - 1} \left[\frac{e^{\eta(0)} \alpha(0)}{e^{\eta(v)} \alpha(v)} \right]^{\frac{1}{\eta(v)-1}} (q_0)^{\frac{\eta(0)-\eta(v)}{\eta(v)-1}} \frac{dq_0}{d\kappa}, \ \forall v \in \mathbb{L}.$$

Using again (25), the equation above can be written:

$$\frac{dq_v}{d\kappa} = \frac{\eta(0) - 1}{\eta(v) - 1} \frac{q_v}{q_0} \frac{dq_0}{d\kappa}, \ \forall v \in \mathbb{L}. \tag{28}$$

(Since $\eta(\cdot) > 1$, notice that $dq_v / d\kappa$ and $dq_0 / d\kappa$ must then share the same sign, for all $v \in \mathbb{L}$). Given that $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z \, dz$, it follows that:

$$\frac{dQ}{d\kappa} = \int_0^{\tilde{v}(\kappa)} \frac{dq_z}{d\kappa} dz = \frac{1}{q_0} \left(\int_0^{\tilde{v}(\kappa)} \frac{\eta(0) - 1}{\eta(z) - 1} q_z dz \right) \frac{dq_0}{d\kappa}.$$

Applying (22) to $v = 0$ when $\lambda_0 = 0$ yields: $q_0 = \left[a(0) e^{\eta(0)} Q\right]^{-\frac{1}{\eta(0)-1}} \kappa^{\frac{1}{\eta(0)-1}}$. Thus:

$$\frac{dq_0}{d\kappa} = \frac{q_0}{\eta(0) - 1} \frac{Q}{\kappa} \left(1 - \tilde{v} + \int_0^{\tilde{v}(\kappa)} \frac{\eta(z)}{\eta(z) - 1} q_z \, dz \right)^{-1} > 0.$$

Therefore, from (28) it follows that $dq_v / d\kappa > 0, \ \forall v \in \mathbb{L}$ must also hold.

Part (ii.b). Since $q_v = 1$ must hold for all $v \notin \mathbb{L}$. Proof is analogous to that of Part (i) of this Proposition.

Part (ii.c). Part (ii.a) and (ii.b) of this Proposition, taken together, imply that $dq_{\underline{v}} / d\kappa = dq_{\overline{v}} / d\kappa = 0$ if $\underline{v}, \overline{v} \notin \mathbb{L}$, and $dq_{\underline{v}} / d\kappa > dq_{\overline{v}} / d\kappa = 0$ if $\underline{v} \in \mathbb{L}$ and $\overline{v} \notin \mathbb{L}$. For $\underline{v}, \overline{v} \in \mathbb{L}$, such that $\underline{v} < \overline{v}$, (28) leads to:

$$\frac{dq_{\underline{v}}}{d\kappa} = \frac{\eta(0) - 1}{\eta(\underline{v}) - 1} \frac{q_{\underline{v}}}{q_0} \frac{dq_0}{d\kappa} > \frac{\eta(0) - 1}{\eta(\overline{v}) - 1} \frac{q_{\overline{v}}}{q_0} \frac{dq_0}{d\kappa} = \frac{dq_{\overline{v}}}{d\kappa},$$

since by assumption $\eta(\underline{v}) < \eta(\overline{v})$ and, from Lemma 2, $q_{\underline{v}} > q_{\overline{v}}$. ■

Proof of Proposition 1.

Firstly, considering the definition of average quality, taking logarithms and differentiating (7) with respect to κ yields: $(d\beta_v / d\kappa) / \beta_v = (dq_v / d\kappa) / q_v - (dQ / d\kappa) / Q$. Using (28), we can write:

$$\frac{dq_{\underline{v}}}{d\kappa} \frac{1}{q_{\underline{v}}} = \frac{\eta(0) - 1}{\eta(\underline{v}) - 1} \frac{dq_0}{d\kappa} \frac{1}{q_0} > \frac{\eta(0) - 1}{\eta(\overline{v}) - 1} \frac{dq_0}{d\kappa} \frac{1}{q_0} = \frac{dq_{\overline{v}}}{d\kappa} \frac{1}{q_{\overline{v}}}.$$

Hence:

$$\frac{d\beta_{\underline{v}}}{d\kappa} \frac{1}{\beta_{\underline{v}}} > \frac{d\beta_{\overline{v}}}{d\kappa} \frac{1}{\beta_{\overline{v}}}. \tag{29}$$

Part (i). Using (29), the claim trivially follows by noting that, from Lemma 2 in conjunction with (7), $\beta_{\underline{v}} > \beta_{\overline{v}}$ must always hold.

Part (ii). Suppose instead that $\partial\beta_{\overline{v}}/\partial\kappa > 0$ when $\partial\beta_{\underline{v}}/\partial\kappa \leq 0$. Using (29), it follows that:

$$\frac{d\beta_{\overline{v}}}{d\kappa} < \frac{\beta_{\overline{v}}}{\beta_{\underline{v}}}\frac{d\beta_{\underline{v}}}{d\kappa} \leq 0;$$

which contradicts the fact that $\partial\beta_{\overline{v}}/\partial\kappa > 0$ when $\partial\beta_{\underline{v}}/\partial\kappa \leq 0$. As a result, if $\underline{v} \notin \mathbb{J}$, then $\partial\beta_{\overline{v}}/\partial\kappa \leq 0$ must hold. ∎

Proof of Corollary 1.

Preliminarily, recall $\int_{z\in\mathbb{V}} \beta_z \, dz = 1$, which implies $\int_0^1 (\partial\beta_z/\partial\kappa)\, dz = 0$.[19]

Part (i). Claim immediately follows since, whenever $\kappa < \underline{\kappa}$, $\partial\beta_z/\partial\kappa = 0$ for all $z \in \mathbb{V}$.

Part (ii). Note first that when $\kappa \geq \underline{\kappa}$, the set $\mathbb{J} \neq \emptyset$. As a result, from Proposition 1, Part (i), it follows that $\int_0^v (\partial\beta_z/\partial\kappa)\, dz > \int_v^1 (\partial\beta_z/\partial\kappa)\, dz$. Then, since $\int_0^v (\partial\beta_z/\partial\kappa)\, dz + \int_v^1 (\partial\beta_z/\partial\kappa)\, dz = 0$, we must necessarily have that $\int_0^v (\partial\beta_z/\partial\kappa)\, dz > 0$. ∎

Proof of Proposition 4. Suppose that the heterogeneity in the effective labour endowments is in country F.

Part (i). When $\kappa = \underline{\kappa}$, in F, $\vartheta_p^*(m) = m$ and $\vartheta_r^*(m) > m$; where $\vartheta_j^*(m)$ denotes the fraction of income that types $j \in \{p, r\}$ spend on varieties belonging to $[0, m)$. On the other hand, in H, $\vartheta(m) = m$; since when $\kappa = \underline{\kappa}$ all individuals from H optimally consume all varieties at the baseline level.[20] As a result, from (10), the equilibrium condition in the labour market in H reads as follows:

$$m\,w + \tfrac{1}{2}\left[(1-\iota)\,m + (1+\iota)\vartheta_r^*(m)\right] = w. \tag{30}$$

From (30), (4) and Assumption 1, since $\vartheta_r^*(m) > m$, it immediately follows that $w > 1$, which in turn implies $m < 0.5$.

Part (ii). Totally differentiating (30), and using the fact that, from (4), $\partial w/\partial\iota = A'(m)\,(\partial m/\partial\iota)$ must be verified in equilibrium, leads to:

$$\frac{\partial w}{\partial \iota} = \frac{(1+\iota)\,(\partial\vartheta_r^*/\partial\iota) + (\vartheta_r^*(m) - m)}{2(1-m) - (1+\iota)\,(\partial\vartheta_r^*/\partial w) - \left[2w + (1-\iota) + (1+\iota)\beta_{r,m}^*\right]/A'(m)} > 0. \tag{31}$$

[19] Note that it is then trivial to observe that $\partial\vartheta(1)/\partial\kappa = 0$, $\forall\kappa > 0$.

[20] Recall from Lemma 3 that $\underline{\kappa} \equiv a\,(0)\exp\left[\eta\,(0)\right]$, which is independent of w/w^*.

The positive sign in (31) stems from the fact that $\partial\vartheta_r^*(m)/\partial\iota > 0$, $\partial\vartheta_r^*(m)/\partial w < 0$, and $A'(m) < 0$.

An analogous proof holds for the case in which the heterogeneity in the effective labour endowments is in country H. See Appendix C. ■

D Auxiliary Derivations and Proofs

Proof of Claim 1

Recall that $q_v = 1$, $\forall v \notin \mathbb{L}$. For all other varieties, (22) in conjunction with (20) yield:

$$\eta(v) + \ln\alpha(v) + [\eta(v) - 1]\ln q_v = \eta(0) + \ln\alpha(0) + [\eta(0) - 1]\ln q_0,\ \forall v \in \mathbb{L}.$$

Isolating $[\eta(v) - 1]\ln q_v$, and applying exponentials to both sides gives:

$$(q_v)^{\eta(v)-1} = \frac{e^{\eta(0)}}{e^{\eta(v)}}\frac{\alpha(0)}{\alpha(v)}(q_0)^{\eta(0)-1},\ \forall v \in \mathbb{L}.$$

Finally, raising both sides to the power $[\eta(v) - 1]^{-1}$, and considering Lemma 2, (25) obtains.

Proof of Claim 2

By definition of $\mathbb{L}$, $\lambda_{\tilde{v}(\kappa)} = 0$. Thus, the condition (22) applied on $\tilde{v}(\kappa)$ yields:

$$\eta(\tilde{v}(\kappa)) + \ln[\alpha(\tilde{v}(\kappa))/w] - \ln\kappa + \ln Q = -[\eta(\tilde{v}(\kappa)) - 1]\ln q_{\tilde{v}(\kappa)} \tag{32}$$

Suppose now that $q_{\tilde{v}(\kappa)} > 1$, and take some $\varepsilon \in (0, 1 - \tilde{v}(\kappa)]$. Then, since $v = \tilde{v}(\kappa) + \varepsilon \notin \mathbb{L}$, it must be the case that:

$$\eta(\tilde{v}(\kappa) + \varepsilon) + \ln[\alpha(\tilde{v}(\kappa) + \varepsilon)/w] - \ln\kappa + \ln Q = \lambda_{\tilde{v}(\kappa)+\varepsilon}. \tag{33}$$

Then, by continuity of $\eta(\cdot)$ and $\alpha(\cdot)$, and using the result in (32), we must have:

$$\lim_{\varepsilon\to 0}\{\eta(\tilde{v}(\kappa) + \varepsilon) + \ln[\alpha(\tilde{v}(\kappa) + \varepsilon)/w] - \ln\kappa + \ln Q\} = -[\eta(\tilde{v}(\kappa)) - 1]\ln q_{\tilde{v}(\kappa)} < 0.$$

Hence, $q_{\tilde{v}(\kappa)} > 1$ cannot possibly hold when $\tilde{v}(\kappa) < 1$ as it would imply that $\lambda_{\tilde{v}(\kappa)+\varepsilon} < 0$ in (33) for $\varepsilon \to 0$, violating (20).

Proof of $\partial\vartheta(m)/\partial w \leq 0$.

Suppose first that $\tilde{v} < m$. Then, $\mathbb{L} \subset [0, m)$. Differentiating (22) with respect to w yields:

$$\frac{\eta(v) - 1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = 0,\ \forall v \in \mathbb{L}. \tag{34}$$

Furthermore, from (25) it follows that:

$$\frac{\partial q_v}{\partial w}=\frac{\eta\left(0\right)-1}{\eta\left(v\right)-1}\frac{q_v}{q_0}\frac{\partial q_0}{\partial w},\ \forall v\in\mathbb{L}. \tag{35}$$

Since $\partial Q/\partial w=\int_0^{\tilde{v}}\left(\partial q_z/\partial w\right)dz$, combining (34) and (35) yields:

$$\left(1-\tilde{v}+\int_0^{\tilde{v}}\frac{\eta\left(z\right)}{\eta\left(z\right)-1}q_z dz\right)\frac{\eta\left(0\right)-1}{q_0}\frac{1}{Q}\frac{\partial q_0}{\partial w}=0\quad\Rightarrow\quad\frac{\partial q_0}{\partial w}=0,$$

Therefore, using again (35), $\partial q_v/\partial w=0$ for all $v\in[0,\tilde{v}]$ obtains. In addition, because of Lemma 2, it must thus be the case that $\partial q_v/\partial w=0$ holds as well for all $v\in(\tilde{v},1]$. Finally, recalling (7) it then follows that $\partial\beta_v/\partial w=0$ for all $v\in\mathbb{V}$, which in turn implies that $\partial\vartheta\left(m\right)/\partial w=0$.

Suppose now that $\tilde{v}\geq m$. Differentiating (22) with respect to w now yields:

$$\frac{\eta\left(v\right)-1}{q_v}\frac{\partial q_v}{\partial w}+\frac{1}{Q}\frac{\partial Q}{\partial w}=\begin{cases}0, & \forall v\in[0,m)\\ 1/w, & \forall v\in[m,\tilde{v}]\end{cases} \tag{36}$$

From (36) it follows that a necessary condition for $\partial\vartheta\left(m\right)/\partial w>0$ to hold is that $\partial Q/\partial w<0$.[21] However, (36) means that if $\partial Q/\partial w<0$, then $\partial q_v/\partial w>0$ should hold for all $v\in[m,\tilde{v}]$. If $\tilde{v}=1$, it must be straightforward to observe that $\partial Q/\partial w<0$ cannot thus hold. Alternatively, if $\tilde{v}<1$, then $\partial Q/\partial w<0$ would require that $\partial q_v/\partial w<0$ prevails for some $v\in(\tilde{v},1]$ which is not feasible either since it would lead to violating the constraint $q_v\leq 1$. As a result, $\partial Q/\partial w\geq 0$ must hold, which in turn implies $\partial\vartheta\left(m\right)/\partial w\leq 0$. ■

Proof of $\partial\vartheta^{*}\left(m\right)/\partial w<0$.

Suppose first that $\tilde{v}^{*}<m$. Then, $\mathbb{L}^{*}\subset[0,m)$. Differentiating (22) – adjusted for representing an individual from F – with respect to w yields:

$$\frac{\eta\left(v\right)-1}{q_v^{*}}\frac{\partial q_v^{*}}{\partial w}+\frac{1}{Q^{*}}\frac{\partial Q^{*}}{\partial w}=-\frac{1}{w},\ \forall v\in\mathbb{L}^{*}. \tag{37}$$

In addition, from (25) it follows that:

$$\frac{\partial q_v^{*}}{\partial w}=\frac{\eta\left(0\right)-1}{\eta\left(v\right)-1}\frac{q_v^{*}}{q_0^{*}}\frac{\partial q_0^{*}}{\partial w},\ \forall v\in\mathbb{L}^{*}. \tag{38}$$

Combining (37) and (38) leads to:

$$\left(1-\tilde{v}^{*}+\int_0^{\tilde{v}^{*}}\frac{\eta\left(z\right)}{\eta\left(z\right)-1}q_z dz\right)\frac{\eta\left(0\right)-1}{q_0^{*}}\frac{1}{Q^{*}}\frac{\partial q_0^{*}}{\partial w}=-\frac{1}{w}\quad\Rightarrow\quad\frac{\partial q_0^{*}}{\partial w}<0.$$

[21] Otherwise, if $\partial Q/\partial w\geq 0$, (36) would imply that $\partial q_v/\partial w\leq 0$ for all $v\in[0,m)$. Recalling (7), it is then straightforward to observe that $\partial Q/\partial w\geq 0$ would mean $\partial\beta_v/\partial w\leq 0$ for all $v\in[0,m)$, which in turn leads to $\partial\vartheta\left(m\right)/\partial m\leq 0$.

Hence, using again (38), $\partial q_v^*/\partial w < 0$ for all $v \in [0, \tilde{v}^*]$ obtains, which in turn implies $\partial Q^*/\partial w < 0$. Next, since for all $v \geq \tilde{v}^*$ the constraint $q_v^* \geq 1$ is binding, it must be the case that $\partial q_v^*/\partial w \geq 0$, $\forall v \in (\tilde{v}^*, 1]$. As a result, because of (7), $\partial \beta_v^*/\partial w > 0$ for all $v \in [m, 1]$ follows, which in turn implies $\partial \vartheta^*(m)/\partial w < 0$.

Suppose now $\tilde{v}^* \geq m$. Differentiating (22) with respect to w now yields:

$$\frac{\eta(v) - 1}{q_v^*} \frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*} \frac{\partial Q^*}{\partial w} = \begin{cases} -1/w, & \forall v \in [0, m) \\ 0, & \forall v \in [m, \tilde{v}^*] \end{cases} \tag{39}$$

Suppose $\partial Q^*/\partial w \geq 0$. From (39) it follows that $\partial q_v^*/\partial w < 0$ for all $v \in [0, \tilde{v}^*)$. Furthermore, Lemma 2 then implies that $\partial q_v^*/\partial w \leq 0$ for all $v \in [\tilde{v}^*, 1]$; as a result, $\partial Q^*/\partial w < 0$ must necessarily hold. Now, notice that if $\partial Q^*/\partial w < 0$, then (39) implies $\partial q_v^*/\partial w > 0$ for all $v \in [m, \tilde{v}^*]$. Moreover, in case $\tilde{v}^* < 1$, since $\forall v \in (\tilde{v}^*, 1]$ the constraint $q_v^* \geq 1$ is binding, $\partial q_v^*/\partial w \geq 0$ must necessarily hold for all $v \in (\tilde{v}^*, 1]$. As a result, if $\partial Q^*/\partial w < 0$, then $\partial \beta_v^*/\partial w > 0$ for all $v \in [m, 1]$, which in turn leads to $\partial \vartheta^*(m)/\partial w < 0$. ■

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