Predictions of changes in pore-water pressure around tunnels in clay

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ABSTRACT: Any underground construction causes changes to the stress state in the ground and this change generally causes the generation of excess pore-water pressures in saturated fine grained soils. Subsequent dissipation of these pressures can lead to settlements and potential damage and hence there is a need to understand and predict these changes in pore-water pressure. Simple plasticity and non-linear elastic solutions have been used to calculate pore-water pressure changes as a tunnel is constructed in clay. These are compared with previous centrifuge tests involving the simulation of tunnel excavation as well as new tests specifically designed to investigate the generation and subsequent dissipation behaviour of excess pore-water pressures. The paper reports on the new tests, presents the findings within the simple plasticity and non-linear elastic analysis framework.

1 INTRODUCTION

Modern tunnelling via Tunnel Boring Machine (TBM) will inevitably create a shape of cavity larger than the final product. The difference between these two volumes is described as 'volume loss' and usually expressed as a percentage of the excavated face. This volume loss results in movements that are apparent as settlements at the surface. When tunnelling is carried out through undrained soils there will be both some immediate ground movements and some excess pore pressures generated. Dissipation of these excess pore pressures over time will lead to further, long-term, settlements. Settlements of any kind have the potential to damage existing infrastructure both above and below ground.

Centrifuge modelling has provided valuable insight into this phenomenon. Mair (1979) placed a rubber bag within a preformed circular cavity. During the increase in acceleration the air within the rubber bag was pressurised equal to the soil overburden. Tunnelling-induced ground movements were simulated by reducing the pressure within this rubber bag.

Jacobsz (2002) developed apparatus capable of simulating the small strain movements around a tunnel in sand by replacing the air with water. The pressure within the rubber bag was maintained by a head of water pressure in a standpipe arrangement. Divall & Goodey (2012) adapted this method for clay. The volume of water removed from within the rubber bag was identical to the magnitude of volume loss in the ground since the experiments were performed in undrained clay and therefore constant volume.

Two centrifuge models of a single tunnel excavation were used to verify a new approach for estimating changes in pore pressure, the outcomes of which are presented in this paper.

2 MODEL TESTS

2.1 *Introduction*

Divall & Goodey (2012) outlined the apparatus used to model small strain tunnelling-induced ground movements in clay. The apparatus comprised three main elements (i) tunnelling system, (ii) support window and (iii) fluid control system. The single tunnel apparatus configuration featured in Divall & Goodey (2012) is identical to the one used by both Divall (2013) and for this study.

2.2 *Soil Preparation*

Speswhite clay slurry was prepared to a water content of 120% and placed within a strong box and one dimensional loaded to 500kPa and swelled to 250kPa over a seven-day period.

The front wall of the strong box was removed to gain access to the clay sample. In this study the moderately stiff clay was trimmed to 207mm high (C/D = 2) and a seamless stainless steel circular tube was used to bore a 50mm diameter cavity. The 'Tunnelling System' was placed within the cavity and connected to the 'Fluid Control system'. Prior to placement the rubber bag had been filled with water and de-aired. Finally, dyed blue Fraction E Leighton Buzzard sand was sprayed onto the front face to give a texture for later determination of ground movements using the geoPIV_RG software (Stanier *et al.*, 2015). The 'Support window' then replaced the front wall of the strong box. The model was sealed and placed on the centrifuge swing for testing.

2.3 *Instrumentation*

The support pressure within the rubber bag was monitored by a Tunnel Pressure Transducer (TPT) set at tunnel axis level. Pore Pressure Transducers (PPT) were installed mostly at the spring line on either side of the tunnel at varying radii. Figure 1 gives the layout of the centrifuge models and includes the positions of the PPTs from the test conducted for this paper and by Divall (2013) for completeness.

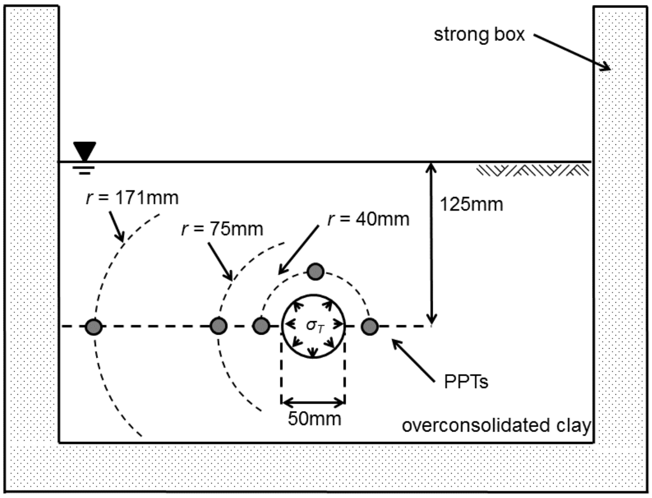

Figure 1. Schematic of models tested and PPT positions

2.4 *Testing*

The centrifuge acceleration was increased to 100g with the 'Tunnel support pressure standpipe' maintaining the pressure within the rubber bag with a preset static head of water. At this stage the model was left approximately 24 hours for the model to reach equilibrium. The phreatic surface was set at ground surface level.

The test began by removing either 3% or 5% (Test 1 or Test 2) of the supporting fluid from within the 'Tunnelling System' at a rate that was sufficient to give an undrained response (the required volume of fluid was removed from the tunnel in under 90 seconds). Once the short-term settlements had stabilised the test was deemed to have finished and the centrifuge was stopped.

3 CIRCULAR TUNNEL CAVITY IDEALISED AS CAVITY CONTRACTION IN AN ELASTIC-PERFECTLY PLASTIC MATERIAL

A relatively straightforward analytical approach to analysing the test data is to use a thick cylinder idealisation of cavity contraction in undrained soil. This was used successfully by Mair & Taylor (1993) for interpreting ground movements and pore pressure changes near tunnel excavations in clay. That paper also highlighted the significance of non-linear elasticity when looking at changes in pore pressure and a simple non-linear constitutive model demonstrated that changes in pore pressure would be predicted even in the elastically straining region.

The changes in pore-pressure in the elastic zone could, therefore, be given by:

$$\frac{\Delta u}{S_u} = -\frac{R_{pnl}}{r} \tag{1}$$

The radius to the plastic zone, R_{pnl}, is given by:

$$\frac{R_{pnl}}{a} = exp\left(\frac{N}{2} - 1\right) \tag{2}$$

and

$$N = \frac{\gamma b - \sigma_T}{S_u} \tag{3}$$

Where:

a = inner radius of the tunnel
b = distance from tunnel axis-level to surface
γ = bulk unit weight of soil
S_u = undrained shear strength
σ_T = internal tunnel support pressure.

Further generalisation can be achieved by adopting the power law function adopted by Bolton & Whittle (1999) in the analysis of cavity expansion (pressuremeter) data. This relates shear stress, τ to shear strain ε_γ as:

$$\tau = \alpha \varepsilon_\gamma^\beta \tag{4}$$

α and β are constants that can be found by backfitting to experimental data. Figure 2, after Bolton & Whittle (1999) shows the form of the constitutive model adopted for the undrained clay.

In analysing the tunnel (cavity contraction) problem, several assumptions need to be made. As illustrated in Figure 3, the tunnel is assumed to be circular with radius $a = D/2$ where D is tunnel diameter and is the central cavity in a weightless cylindrical soil of outer radius $b = (C + D/2)$ where C is the soil cover above the tunnel.

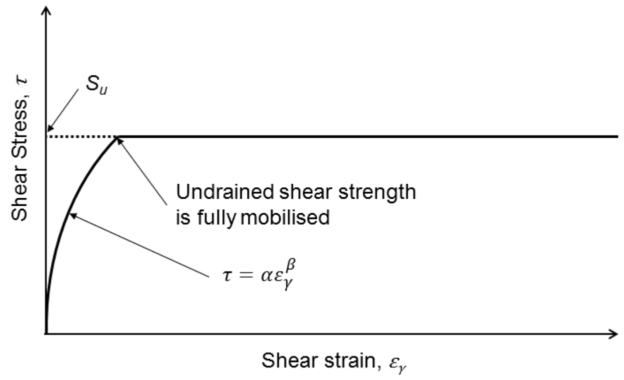

Figure 2. Shear stress: shear strain response for ideal non-linear elastic/perfectly plastic soil (after Bolton & Whittle, 1999)

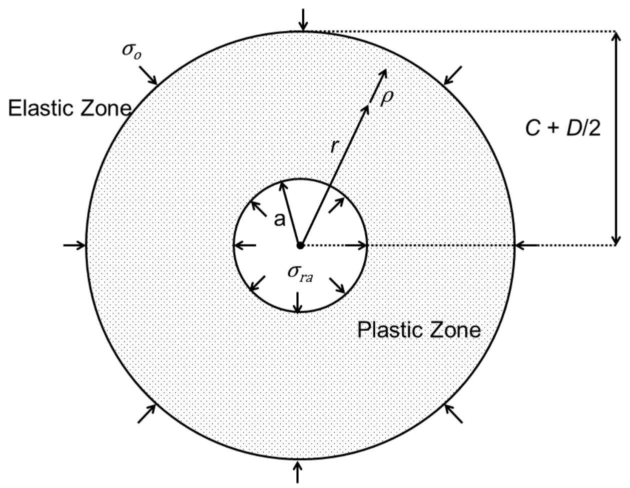

Figure 3. Depiction of a circular tunnel cavity idealised as a cavity contraction

In the following derivation, b will be assumed to be large relative to a. The effect of soil weight is simulated by applying a radial compressive stress, σ_o, at the outer radius of the cylinder equal to the normal vertical overburden stress at the tunnel axis level i.e. $\sigma_o = \gamma(C + D/2)$ where γ is the bulk unit weight of soil. It should be noted that a compression positive sign convention has been adopted. Any support to the tunnel cavity, for example from a pressure support (slurry) or tunnel lining, is simulated by a radial compressive stress, σ_{ra}, at the inner boundary of the cavity. In the initial stage before tunnelling commences, the soil will be in a uniformly isotropic stress state corresponding to σ_o.

Radial displacement, ρ, at radius, r, leads to circumferential strain $\varepsilon_\theta = -\rho/r$ and since there is constant volume straining in this undrained analysis, the radial strain $\varepsilon_r = \rho/r = -\varepsilon_\theta$. The shear strain is then $\varepsilon_\gamma = \varepsilon_r - \varepsilon_\theta = 2\rho/r$. Again, for constant volume straining, $\rho \times 2\pi r = \rho_a \times 2\pi a$ where ρ_a is the radial movement at the tunnel intrados of radius a. This leads to:

$$\varepsilon_\gamma = 2\rho_a a/r^2 \tag{5}$$

The radial and circumferential stresses, σ_r and σ_θ respectively, are related by the equation of radial equilibrium:

$$\sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr} \tag{6}$$

In the tunnel excavation (cavity contraction) problem and with a compression positive sign convention, the shear stress is given by $\tau = (\sigma_\theta - \sigma_r)/2$. Thus:

$$r\frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r = 2\tau = 2\alpha\varepsilon_\gamma^\beta = 2\alpha\left(\frac{2\rho_a a}{r^2}\right)^\beta \tag{7}$$

Integration along with the boundary condition $\sigma_r = \sigma_0$ at the outer radius, b, which is assumed large relative to a leads to:

$$\sigma_r = \sigma_o - \frac{\alpha}{\beta}\frac{(2\rho_a a)^\beta}{r^{2\beta}} \tag{8}$$

Substitution of Equations (7) and (8) into the equilibrium Equation (6) gives:

$$\sigma_\theta = \sigma_o + \alpha\left(2 - \frac{1}{\beta}\right)\frac{(2\rho_a a)^\beta}{r^{2\beta}} \tag{9}$$

At the radius of the plastically deforming zone, $r = c$, the mobilised shear stress is equal to the undrained shear strength, s_u. Therefore:

$$s_u = \frac{(\sigma_\theta - \sigma_r)}{2} = \alpha\frac{(2\rho_a a)^\beta}{c^{2\beta}} \tag{10}$$

which leads to

$$c^{2\beta} = \frac{\alpha}{s_u}(2\rho_a a)^\beta \tag{11}$$

The radial stress at the interface between the elastically and plastically deforming regions, σ_{rc}, is then determined by substituting Equation (11) into Equation (8):

$$\sigma_{rc} = \sigma_o - \frac{s_u}{\beta} \tag{12}$$

Throughout the plastically deforming zone, the undrained strength is fully mobilised and $\sigma_\theta - \sigma_r = 2s_u$. Substitution into Equation (6) and noting that $\sigma_r = \sigma_{rc}$ at $r = c$ gives:

$$\sigma_r = \sigma_o - \frac{s_u}{\beta} - 2s_u \ln\left(\frac{c}{r}\right) \quad (13)$$

The stability ratio, N, relates the difference between the outer stress, σ_o, and inner support stress, σ_{ra}, to the undrained strength and so using Equations (3) and (13) gives:

$$N = \frac{\sigma_o - \sigma_{ra}}{s_u} = 2\ln\left(\frac{c}{a}\right) + \frac{1}{\beta} \quad (14)$$

This can be rewritten as:

$$c^2 = a^2 \exp\left(N - \frac{1}{\beta}\right) \quad (15)$$

4 PREDICTING CHANGES IN PORE PRESSURE

The change in pore pressure, Δu, due to tunnel excavation can be determined by assuming there is constant mean effective stress during elastic and perfectly plastic shear strain development (see Bolton & Whittle, 1999; Mair & Taylor, 1993). Therefore, the change in pore pressure corresponds to the change in mean normal total stress. For this plane strain problem, the relevant mean stress is:

$$s = \frac{(\sigma_r + \sigma_\theta)}{2} \quad (16)$$

Therefore, the change in pore pressure is given by:

$$\Delta u = \frac{(\Delta\sigma_r + \Delta\sigma_\theta)}{2} = \frac{(\sigma_r - \sigma_o)}{2} + \frac{(\sigma_\theta - \sigma_o)}{2}$$

$$\text{i.e. } \Delta u = \frac{(\sigma_r + \sigma_\theta)}{2} - \sigma_o \quad (17)$$

Thus, using Equations (8) and (9), in the elastic region Equation (17) becomes,

$$\Delta u = \alpha\left(1 - \frac{1}{\beta}\right)\frac{(2\rho_a a)^\beta}{r^{2\beta}} = s_u\left(1 - \frac{1}{\beta}\right)\frac{c^{2\beta}}{r^{2\beta}} \quad (18)$$

The radius of the interface between the elastically and plastically deforming regions, c, is a function of the constitutive model and varies with the exponent, β and stability ratio N.

It is useful to make use of this interface radius for the special case of a linear elastic model as this will allow data to be presented with due account being taken of the stability ratio at which measurements were made. The linear elastic model corresponds to $\beta = 1$ and the interface radius, c_{le}, can then be obtained from the expression for stability ratio as:

$$c_{le}^2 = a^2 \exp(N - 1) \quad (19)$$

The ratio of radii of the plastically straining regions for the non-linear and linear elastic models is then given by:

$$\frac{c^2}{c_{le}^2} = \exp\left(1 - \frac{1}{\beta}\right) \quad (20)$$

The expression for change in pore pressure in the elastically deforming zone can then be re-written as:

$$\frac{\Delta u}{s_u} = \left(\frac{c_{le}}{r}\right)^{2\beta}\left(1 - \frac{1}{\beta}\right)\exp(\beta - 1) \quad (21)$$

In the plastically deforming region, the change in pore pressure is given by:

$$\Delta u = \frac{(\sigma_r + \sigma_\theta)}{2} - \sigma_o = \sigma_r + s_u - \sigma_o \quad (22)$$

This leads to:

$$\frac{\Delta u}{s_u} = 1 - \frac{1}{\beta} - \ln\left(\frac{c}{r}\right)^2 = -2\ln\left(\frac{c_{le}}{r}\right) \quad (23)$$

The analysis presented previously by Mair and Taylor (1993) can be recovered from the above by setting $\beta = 0.5$. The new approach allows greater variability in constitutive model and the use of c_{le} for normalising radius allows the important effect of operational stability ratio on pore pressure change to be taken into account when presenting data.

5 COMPARISON WITH EXPERIMENTAL PORE PRESSURE DATA

The changes in pore pressure from the centrifuge tests are presented in Figure 4 along with a line which is described by Equations (21) and (23). The value of β in these equations is equal to 0.55. This information has been presented in the dimensionless axes of change in pore pressure divided by undrained shear strength against linear elastic interface radius divided by radius to account for different tunnel geometry and magnitudes of volume loss in each of the tests. It should be noted that increasing values of c_{le}/r imply positions closer to the tunnel. Also included in Figure 4 are values from Mair & Taylor (1993) and from Divall (2013).

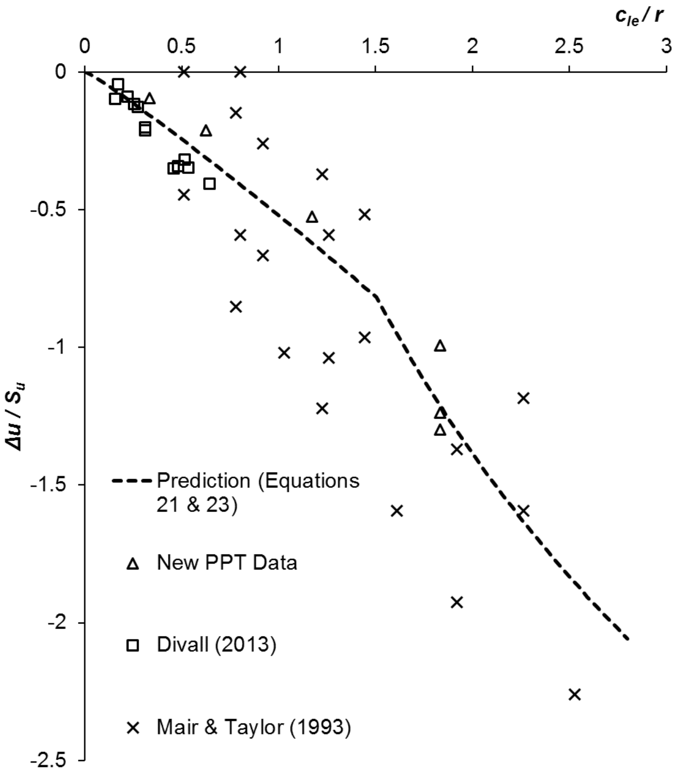

Figure 4. Pore pressure data against the new approach for estimating changes in pore pressure from a tunnel excavation in fine grained soils.

6 DISCUSSION & CONCLUSION

The data from the two centrifuge tests have been shown to verify the new approach for predicting pore pressure changes at various radii around a tunnel excavation in fine grained soils. Data from previous studies also shows broad agreement with the proposed method. The new method accounts for non-linear elastic behaviour of the soil and improves on the predictions presented previously by Mair & Taylor (1993). The design line shown in Figure 4 allows predictions to be made for different geometry and magnitudes of volume losses.

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