Claims reserving in the presence of excess-of-loss reinsurance using micro models based on aggregate data

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Abstract

This paper addresses a new problem in the literature, which is how to consider reserving issues for a portfolio of general insurance policies when there is excess-of-loss reinsurance. This is very important for pricing considerations and for decision making regarding capital issues. The paper sets out how this is currently often tackled in practice and provides an alternative approach using recent developments in stochastic claims reserving. These alternative approaches are illustrated and compared in an example using real data. The stochastic modelling framework used in this paper is Double Chain Ladder, but other approaches would also be possible. The paper sets out an approach which could be explored further and built on in future research.

Keywords: General Insurance; Claims Reserving; Risk

1 Introduction

The subject of this paper is the distribution of outstanding claims for a portfolio of general insurance policies in the presence of excess-of-loss reinsurance protection. This is a subject area which, to the best knowledge of the authors, has not previously appeared in the actuarial literature. It is a very important subject from a practical point of view, and there have been many papers on the estimation of outstanding claims and reinsurance separately to develop both the theory and practical tools for actuaries. To date, they have not been considered together.

This is perhaps surprising because the estimation of outstanding claims net of reinsurance for such a portfolio is very commonly needed, for example when a reinsurance underwriter is pricing either a retrospective loss portfolio transfer treaty or a prospective proportional quota share on the retention. These are both actively used for solvency capital management and it would therefore be desirable to have estimates of both the expected net outstanding claims and the uncertainty around these. Better still would be estimates of the distribution of net outstanding claims. This paper develops methods to address all of these issues for excess-of-loss reinsurance and compares the results with what is often done in the practical context using existing reserving methods.

With the advances in stochastic reserving methodology, it is now possible to develop coherent theoretical frameworks for the estimation of the distribution of outstanding claims net of excess-of-loss reinsurance. It is important to note that the most commonly used stochastic claims reserving methods, such as bootstrapping the over-dispersed Poisson model (England and Verrall, 1999, 2002), will be of limited value in this context. The fundamental issue that needs to be addressed is how to consider the net outstanding claims such that the effect of the excess-of-loss reinsurance contract can be accurately taken into account. The only way to do this is to use a model which considers individual claims, or at least one which simulates future claims individually rather than aggregated.

Individual claims reserving, or reserving based on granular data, has been the subject of increased attention in actuarial literature. See for example Antonio and Plat (2014). The majority of the methods which have been developed operate entirely at the level of individual claims and this can perhaps make them appear to be overly complex to implement and use in a practical context. In contrast, a series of papers beginning with Verrall, Nielsen and Jessen (2010) and continuing with Martínez-Miranda, Nielsen, Nielsen and Verrall (2011), Martínez-Miranda, Nielsen and Verrall (2012), Martínez-Miranda, Nielsen and Verrall (2013b) and Martínez-Miranda, Nielsen, Verrall and Wüthrich (2015) has developed a hybrid approach which uses data aggregated in the standard way into triangles in order to estimate models for claims at the individual level. We believe that this makes it easier to apply the fundamental advantages of stochastic reserving for individual claims using the theory which has recently been developed to more complex practical issues such as excess-of-loss reinsurance. Of course, it would be possible to investigate these practical issues using other individual claims reserving methods, and we anticipate that this may be done in the future by other authors.

In this paper, we bring together all the methodology developed in the papers above based around the Double Chain Ladder (DCL) method. It is clear that in the practical context it is important to have stable estimates of all parameters if practically useful simulations of future claims are to be generated. This means that it is important to use the full range of methods available within the framework of DCL, paying particular attention to the way claims increase with accident period.

The paper is set out as follows. Section 2 outlines the approach which is commonly used in practice when considering reserves with reinsurance. Section 3 summarises the theoretical model which we will use in this paper, DCL. Section 4 revisits the Bayesian DCL method of Martínez-Miranda, Nielsen and Verrall (2013b) and the basic assumptions of a Bornhuetter-Ferguson approach giving a modification to DCL of Martínez-Miranda, Nielsen and Verrall (2012), which we call Bornhuetter-Ferguson Double Chain Ladder prior (BDCL prior). In section 5 we describe how the data are usually prepared in practice in order to analyse the claims net of reinsurance (and the reinsurers claims). In section 6 we show how this can be done in a more coherent way within the framework of DCL and BDCL prior. Sections 5 and 6 also contain illustrations and comparison of the practical approach and the new approach. Section 7 contains the conclusions.

2 The practical approach

In general insurance or casualty portfolios (including general third party liability, motor third party liability, employer's liability, medical malpractice) insurance companies commonly seek excess-of-loss reinsurance protection on an occurrence year basis. This means that the insurer's exposure to any individual loss occurring in any given year is limited to a predefined amount called the retention or priority. The retention is usually chosen taking into account the volatility of claims which are likely to arise from the portfolio, the insurer's risk appetite and solvency position. And in practice it is also driven by past experience of claims from the portfolio and the available price in the market. Typically, these reinsurance treaties have a one year duration and are renegotiated every year so that the retention level may change from year to year. There may be clauses in the treaties which affect the actual retention on claims each year: for example, an indexation clause. Thus, whenever data are considered over a period of years for such portfolios, the insurer's retained amounts for any individual loss will be dependent on the year in which the loss occurred.

The estimation of the ultimate net incurred claims in order to set the net total unpaid reserve for such a portfolio is a common actuarial task for reinsurance underwriters when asked to price either a retrospective loss portfolio transfer (LPT) treaty or a prospective proportional quota share (QS) on the retention. In the case of a LPT, the cession to the LPT reinsurer can be either on a gross basis, in which case the cedant will transfer the right of recoveries from excess-of-loss reinsurers to the LPT reinsurer, or a net basis, which means that only the retained loss portfolio is ceded. However, irrespective of the cession basis (assuming an acceptable counterparty rating of the excess-of-loss reinsurers) the evaluation of the reserves is to be done on the loss portfolio net of historical inuring excess-of-loss recoveries. In the case of the prospective QS, the actual historical excess-of-loss retentions are ignored as the estimation of the net outstanding claims is carried out on an 'as-if' basis using a common historical retention equal to that of a prospective excess-of-loss treaty. From a theoretical point of view, QS is a simpler subcase of what would be the more generalized case of the LPT where instead of one common excess-of-loss retention for all years there can be different historical retentions depending on the conditions of each year's excess-of-loss treaty. In this paper, we will consider the QS case, thereby assuming one common retention for all occurrence years.

Typically, the kind of data the reinsurer receives for the purpose of pricing these treaties may come in various formats. If the systems of the insurer are set to account for the existence of excess-of-loss reinsurance, it is possible to receive triangular data with incurred losses already capped at the historical retention. In short, these are known as net triangles. In addition to this, most insurers should be able to query their databases to produce net triangles at a given common retention. In practice, however, the insurance company will either supply gross triangles plus the recoveries triangles, or in the case of QS, gross triangles plus the triangulations of large individual claims, for example with incurred amount at 50% of the prospective retention or above. This is the typical threshold that an excess-of-loss reinsurer sets for the claims data requirement. If sufficient data about individual claims are available (particularly large claims), the QS reinsurer will be able to construct the recoveries triangles and price the treaty at different levels of prospective excess-of-loss retention.

In practice, the reinsurance underwriter or actuary will estimate the net outstanding claims (in the case of an LPT) or the ultimate claims (in the case of QS) for each accident year by applying traditional actuarial reserving methods on the net triangles which result from subtracting the reinsurance recovery triangle from the gross triangle.

The problem with this approach is that although actuarial reserving methods can be applied to the resulting net triangle in the same way as they are applied to gross triangles, reinsurance recoveries for potential future development of individual claims or newly reported claims are not taken into account because the recoveries triangle construction is limited to the development period already observed. In other words, the recoveries triangle is constructed on the basis of the incurred value at the given valuation date and not on the basis of the ultimate cost of each claim. This presents many issues for the reinsurer to consider. Not only is the ultimate incurred value of a claim unknown, just the incurred value at the particular development point in time, but also the observation period for each of these claims depends on when they were reported. This typically results in there being no recoveries observed in recent accident years. In addition to this, different accident years may have different reporting lags. Ultimately this is a problem of incorrect sampling of the recoveries triangle and this leads to problems with the net triangle to which actuarial reserving methods are applied. As a result of this, it is not clear whether estimating the net reserve using the net triangles constructed in this way leads to reasonable point estimates.

As reinsurance is a very competitive business, price is the principal factor for an insurer in deciding whether to cede the portfolio to one particular reinsurer or another. In the case of capital motivated reinsurance transactions, reinsurance competes with other forms of capital such as subordinated debt, and the pricing implications of the estimation of net outstanding claims can also lead to a decision not to cede at all if the cost of reinsurance is directly compared to the cost of the capital relief such a transaction achieves. For these reasons, having more information about the accuracy of the estimation would be very desirable.

The ideal solution to this problem would be to estimate the ultimate incurred for each individual claim. This could be done by modelling the individual aspects of each claim, which could include (for example) loss of income, dependants, future inflation, medical expenses etc. This is the aim of claims adjusters, and it has to be recognised that their estimates can be quite volatile. An actuarial approach would be to simulate from the individual claims so that to estimate the ultimate recoveries per accident year. While there have been considerable advances in the consideration of individual claims data in recent years, the application of the methods would probably still present challenges in practical settings. For this reason, the approach in this paper is to use methods which use aggregated data for the estimation but which are designed in order to allow inferences and simulation to be carried out at the level of individual claims. The framework we use is Double Chain Ladder and its extensions, which are set out in the next section.

3 Double Chain Ladder

This section summarises the Double Chain Ladder (DCL) method developed in Verrall et al. (2010) and Martínez-Miranda et al. (2011 and 2012). The formulation of DCL and related models in Martínez-Miranda et al. (2011, 2012, 2013a,b) allows us to estimate the settlement delay and therefore to predict Reported But Not Settled (RBNS) and Incurred But Not Reported (IBNR) reserves separately. In contrast with other approaches (for example Antonio and Plat (2014)) which are also based on individual claims (micro models), our aim is not to perform the estimation using the individual claims data. It would be possible to use such an approach, but we believe that DCL offers a simpler procedure which should be easier to use in the practical context described in section 2 since DCL and the related models are estimated using only data in the aggregated triangles which are usually available in practice.

The approach of DCL is based on a model defined at the level of individual claims (a micro model) but estimated using data in aggregated triangles. We first describe the micro model, and then show how this can be estimated using conventional triangles of data. The micro model is constructed from three components: the settlement delay, the individual payments and the reported numbers of claims (known as the counts).

This section sets out the model assumptions from Martínez-Miranda et al. (2015), which pointed out that it would be possible to use more general modelling assumptions if the only question of interest was the mean or best estimate. For the more general case, Martínez-Miranda et al. (2015) generalized the original assumptions of DCL in order to add prior knowledge. Therefore, we will refer to this model as DCL prior.

DCL makes use of both the data and expert knowledge extracted from incurred data. The information required to apply DCL are the aggregated incurred counts (data), aggregated payments (data) and aggregated incurred payments (expert knowledge). All of these three objects will have the same structural form, and without loss of generality they are assumed to consist of the usual triangles defined on $\mathcal{I}$, where

$$\mathcal{I} = \{(i, j) : i = 1, \ldots, m, j = 0, \ldots, m-1;\ i + j \leq m\}.$$

Here, $m > 0$ is the number of underwriting or accident years observed. It will be assumed that the reporting delay (the time from the occurance of a claim until it is reported), and the settlement delay (the time between the report of a claim and its settlement) are both bounded by m. This, in contrast to the classical CLM, will make it possible to also get estimates in the "tail" where the reporting delay plus the settlement delay is greater than m. The information required is as follows.

Aggregated incurred counts: $N_{\mathcal{I}} = \{N_{ik} : (i,k) \in \mathcal{I}\}$, with N_{ik} being the total number of claims which were incurred in year i and reported in year $i+k$ (i.e. a reporting delay of k). Note that each of these N_{ik} reported claims is assumed to generate a number of payments, i.e. a claims payment cash flow.

Aggregated payments: $X_{\mathcal{I}} = \{X_{ij} : (i,j) \in \mathcal{I}\}$, with X_{ij} being the total payments from claims incurred in year i and paid with j periods delay from year i.

Note that the meaning of the second suffix of triangle $\mathcal{I}$ varies between the two different sets of data. In the counts triangle it represents the reporting delay and in the payments triangle it represents the development delay, which is reporting delay plus settlement delay. For the aggregated incurred payments, some theory at the level of individual claims is required.

Let N_{ikl}^{paid} denote the number of the future payments originating from the N_{ik} reported claims, which are paid with a delay of $k+l$, where $l = 0, \ldots, m-1$.
Also, let $Y_{ikl}^{(h)}$ denote the individual settled payments which arise from N_{ikl}^{paid}, $h = 1, \ldots, N_{ikl}^{paid}$.
Finally, define X_{ikl} to be the aggregate claims originating from underwriting year i, which are reported after a delay of k and paid with an overall delay of $k+l$. Then

$$X_{ikl} = \sum_{h=1}^{N_{ikl}^{paid}} Y_{ikl}^{(h)}, \quad (i,k) \in \mathcal{I},\ l = 0, \ldots, m-1,$$

The observed aggregated payments can be written as

$$X_{ij} = \sum_{l=0}^{j} X_{i,j-l,l} = \sum_{l=0}^{j} \sum_{h=1}^{N_{i,j-l,l}^{paid}} Y_{i,j-l,l}^{(h)}.$$

With these definitions, we make the following distributional assumptions.

A1. The numbers of reported claims, N_{ik}, are independent random variables for all (i,k) and have a Poisson distribution with cross-classified mean $\mathrm{E}[N_{ik}] = \alpha_i \beta_k$ and identification $\sum_{k=0}^{m-1} \beta_k = 1$.

A2. Given N_{ik}, the numbers of paid claims follow a multinomial distribution, so that the random vector $(N_{i,k,0}^{paid}, \ldots, N_{i,k,m-1}^{paid}) \sim \mathrm{Multi}(N_{ik}; p_0, \ldots, p_{m-1})$, for each (i,k), where $m-1$ is the assumed maximum delay and $(p_0, \ldots, p_{m-1})$ denote the delay probabilities such that $\sum_{l=0}^{m-1} p_l = 1$ and $0 \le p_l \le 1, \forall l = 0, \ldots, m-1$.

A3. The individual payments $Y^{(h)}_{i,j-l,l}$ are independent and have a mixed type distribution with Q_i being the probability of a "zero-claim" i.e. $\mathrm{P}\left\{Y^{(h)}_{i,j-l,l} = 0\right\} = Q_i$. We assume that $Y^{(h)}_{i,j-l,l} | Y^{(h)}_{i,j-l,l} > 0$ has a distribution with conditional mean μ_{ij} and conditional variance σ^2_{ij}, for each $i = 1, \ldots, m, j = 0, \ldots, m-1$. We also assume that the mean depends on the accident year and payment year such that $\mu_{ij} = \mu\gamma_i\delta_j$. Here, μ is a common mean factor and δ_j and γ_i can be interpreted as being the inflation in the payment year and the accident year, respectively. The variance follows a similar structure, with $\sigma^2_{ij} = \sigma^2\gamma_i^2\delta_j^2$, where σ^2 is a common variance factor.

A4. *Independence*: We assume that settled payments, $Y^{(h)}_{ikl}$ are independent of the numbers of reported claims, N_{ik}.

Assumption A1 is, apart from the distribution, the classical chain ladder assumption applied to the counts triangle, with the main point being the multiplicativity between underwriting year and reporting delay. Assumptions A2-A4 are necessary to connect reporting delay, settlement delay and development delay - the main idea of DCL. A3 also acknowledges the fact that reported claims can be closed without a payment being made - the so-called zero-claims.

This is the model which was set out in Martínez-Miranda et al. (2015) and it is a more general situation than Martínez-Miranda et al. (2012) since it assumes that the distribution depends on the accident year and the development year and also allows for zero-claims. Under these assumptions, the first two moments of the unconditional distribution of $Y^{(h)}_{i,j-l,l}$ are given by:

$$\begin{aligned} \mathrm{E}[Y^{(h)}_{i,j-l,l}] &= \gamma_i\delta_j(1-Q_i)\mu && (1)\\ \mathrm{V}(Y^{(h)}_{i,j-l,l}) &= \gamma_i^2\delta_j^2(1-Q_i)\left(\sigma^2 + Q_i\mu^2\right) && (2) \end{aligned}$$

Following the similar calculations as Martínez-Miranda et al. (2012), it can be shown that under the above assumptions the unconditional mean of X_{ij} can be written as

$$\mathrm{E}[X_{ij}] = \gamma_i(1-Q_i)\mu\alpha_i\delta_j\sum_{l=0}^{j}\beta_{j-l}p_l = \widetilde{\alpha}_i\widetilde{\beta}_j, \qquad (3)$$

where

$$\widetilde{\alpha}_i = \gamma_i(1-Q_i)\mu\alpha_i$$

and

$$\widetilde{\beta}_j = \delta_j\sum_{l=0}^{j}\beta_{j-l}p_l.$$

Equation (3) is the key in deriving the outstanding loss liabilities.

Note that when $Q_i = 0 \; \forall i = 1, ..., m$ and $\delta_j = 1 \; \forall j = 0, \ldots, m-1$, the situation reverts back to the DCL model as set out Martínez-Miranda et al. (2012).

4 Parameter estimation for the DCL method

In this section we first set $\delta_j = 1 \quad \forall j$ and $Q_i = 0 \quad \forall i$ and show how to estimate the remaining parameters in DCL. The approach to incorporating the development inflation and the zero-claims probability will be described in the next section.

The case when $\delta_j = 1 \quad \forall j$ and $Q_i = 0 \quad \forall i$ was proposed in Martínez-Miranda et al. (2012) which developed the DCL method to estimate the parameters and a summary of this is provided in this section. The DCL method considers the simple chain-ladder algorithm applied to the triangles of paid claims, $X_{\mathcal{I}}$, and incurred counts, $N_{\mathcal{I}}$. Therefore, as implied by the name Double Chain Ladder, the classical chain-ladder model (CLM) is applied twice and from this everything needed to estimate the outstanding claims is available. It was also shown that this estimation procedure can give identical results as the CLM for paid data when the observed counts are replaced by their fitted values.

An appealing feature of the DCL estimation method is that it uses the estimates of the chain ladder parameters from the triangle of counts and the triangle of payments. Assumption A1 in Section 3 defined a standard chain-ladder model for the counts data, N_{ij}. A similar model can be defined for the triangle of paid data, X_{ij}, with parameters $\widetilde{\alpha}_i$ and $\widetilde{\beta}_j$. We denote the estimates of the parameters, using the chain-ladder model on each triangle, by $(\widehat{\alpha}_i, \widehat{\beta}_j)$ and $(\widehat{\widetilde{\alpha}}_i, \widehat{\widetilde{\beta}}_j)$, respectively, for $i = 1, \ldots, m, j = 0, \ldots, m-1$. Note that it is straightforward to obtain these estimates using the development factors provided by the chain ladder algorithm, as follows.

Consider the counts triangle (a similar approach can be used for the parameters of the paid triangle) and denote by $\widehat{\lambda}_j$, $j = 1, 2, \ldots, m-1$, the corresponding estimated development factors. Then the estimates of β_j for $j = 0, \ldots, m-1$ can be calculated by

$$\widehat{\beta}_0 = \frac{1}{\prod_{l=1}^{m-1} \widehat{\lambda}_l} \tag{4}$$

and

$$\widehat{\beta}_j = \frac{\widehat{\lambda}_j - 1}{\prod_{l=j}^{m-1} \widehat{\lambda}_l} \tag{5}$$

for $j = 1, \ldots, m-1$. The estimates of the parameters for the accident years can be derived from the latest cumulative entry in each row through the formula:

$$\widehat{\alpha}_i = \sum_{j=0}^{m-i} N_{ij} \prod_{j=m-i+1}^{m-1} \widehat{\lambda}_j. \tag{6}$$

The same procedure can be used to produce $(\widehat{\widetilde{\alpha}}_i, \widehat{\widetilde{\beta}}_j)$ from the triangle of paid data, and the DCL method estimates the rest of the parameters in the model formulated in A1-A4 using just the above estimates. Specifically, the reporting delay probabilities $\{p_0, \ldots, p_{m-1}\}$ can be estimated by solving the linear system given below to obtain estimates of $\{\pi_0, \ldots, \pi_{m-1}\}$.

$$\begin{pmatrix} \widehat{\widetilde{\beta}}_0 \\ \vdots \\ \vdots \\ \widehat{\widetilde{\beta}_{m-1}} \end{pmatrix} = \begin{pmatrix} \widehat{\beta}_0 & 0 & \cdots & 0 \\ \widehat{\beta}_1 & \widehat{\beta}_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \widehat{\beta_{m-1}} & \cdots & \widehat{\beta}_1 & \widehat{\beta}_0 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \vdots \\ \vdots \\ \pi_{m-1} \end{pmatrix}. \tag{7}$$

Once the solution $\{\widehat{\pi}_0, \ldots, \widehat{\pi}_{m-1}\}$ is obtained, these preliminary delay parameters are adjusted to have the desired probability vector, $(\widehat{p}_0, \ldots, \widehat{p}_{m-1})$ which satisfies the restrictions that $0 \leq \widehat{p}_l < 1$ and $\sum_{l=0}^{m-1} \widehat{p}_l = 1$. For more details of this estimation procedure, see Martínez-Miranda et al. (2012).

For the mean and variance of the distribution of individual payments DCL estimates the inflation parameters, $\gamma = \{\gamma_i : i = 1, \ldots, m\}$, and the mean factor, μ, through the expression:

$$\widehat{\gamma}_i = \frac{\widehat{\widetilde{\alpha}}_i}{\widehat{\alpha}_i \widehat{\mu}} \quad i = 1, \ldots, m. \tag{8}$$

To ensure identifiability DCL sets $\gamma_1 = 1$, so that μ can be estimated by

$$\widehat{\mu} = \frac{\widehat{\widetilde{\alpha}}_1}{\widehat{\alpha}_1}. \tag{9}$$

The inflation parameters, $\widehat{\gamma}_i$, are estimated by substituting $\widehat{\mu}$ into equation (8). It only remains to adjust the final $\widehat{\mu}$ according to the estimates $\widehat{p}_l$ and in order to ensure that $\sum_{k=0}^{m-1} \widetilde{\beta}_k = 1$. This is done by dividing $\widehat{\mu}$ by κ, where $\kappa = \sum_{j=0}^{m-1} \sum_{l=0}^{j} \widehat{\beta}_{j-l} \widehat{p}_l$. Hereafter, in a slight abuse of notation, we will retain the notation $\widehat{\mu}$ for the corrected estimator of μ.

The estimate of outstanding claims is obtained by substituting the above estimates into the expression for the unconditional mean. In doing this, it is useful to split it into the Reported But Not Settled (RBNS) and Incurred But Not Reported

(IBNR) components by considering payments on already reported claims and claims which will be reported in the future. For $i+j>m$, we define

$$\widehat{X}_{ij}^{rbns} = \sum_{l=i-m+j}^{j} \widehat{N}_{i,j-l}\widehat{p}_l\widehat{\mu}\widehat{\gamma}_i \tag{10}$$

and

$$\widehat{X}_{ij}^{ibnr} = \sum_{l=\max(0,j-m+1)}^{i-m+j-1} \widehat{N}_{i,j-l}\widehat{p}_l\widehat{\mu}\widehat{\gamma}_i, \tag{11}$$

respectively, where $\widehat{N}_{ij} = \widehat{\alpha}_i\widehat{\beta}_j$.

The estimate of total outstanding claims is calculated by adding the RBNS and IBNR components i.e. $\widehat{X}_{ij}^{DCL} = \widehat{X}_{ij}^{rbns} + \widehat{X}_{ij}^{ibnr}$. This is equivalent to the aim of the standard CLM in just the lower triangle (ignoring any tail effects), i.e. for $(i,j) \in \mathcal{J}_1 = \{i = 2,\ldots,m; j = 0,\ldots,m-1 \text{ so } i+j = m+1,\ldots,2m-1\}$. For the DCL, the estimates of outstanding claims extend further to provide tail estimates by considering $i = 1,\ldots,m$ and $j = m,\ldots,2m-1$.

Finally to provide the full cash flow the predictive distribution can be approximated using parametric bootstrap methods as Martínez-Miranda et al. (2011) described. In order to do this, it is necessary to estimate the variances, σ_i^2 ($i = 1,\ldots,m$). Verrall et al. (2010) showed that assumptions similar to A1–A4 can be used to show that the conditional variance of X_{ij} is approximately proportional to its mean. Using this result, it is straightforward to estimate the variance using over-dispersed Poisson distributions with common over-dispersion parameter, φ.
As in Verrall et al. (2010), the over-dispersion parameter φ can be estimated by

$$\widehat{\varphi} = \frac{1}{n-m}\sum_{i,j\in\mathcal{I}} \frac{(X_{ij} - \widehat{X}_{ij}^{DCL})^2}{\widehat{X}_{ij}^{DCL}\widehat{\gamma}_i}, \tag{12}$$

where $n = m(m+1)/2$ and $\widehat{X}_{ij}^{DCL} = \sum_{l=0}^{j} N_{i,j-l}\widehat{p}_l\widehat{\mu}\widehat{\gamma}_i$. Then the variance factor of individual payments can be estimated by

$$\widehat{\sigma}_i^2 = \widehat{\sigma}^2\widehat{\gamma}_i^2 \tag{13}$$

for each $i = 1,\ldots,m$, where $\widehat{\sigma}^2 = \widehat{\mu}\widehat{\varphi} - \widehat{\mu}^2$.

4.1 The BDCL prior method

This section summarises the methodology developed in Martínez-Miranda et al. (2013b) and Martínez-Miranda et al. (2015) using the DCL method as set out in section 4 and incorporating information about inflation in the severity of individual claims and the number of zero claims. The approach is to first assume that the parameters for this are known, and note that they cannot be estimated using the triangles $X_{\mathcal{I}}$ and $N_{\mathcal{I}}$. The estimation of these parameters requires some extra data, which is described at the end of this section.

As in the DCL prior method in Martínez-Miranda et al. (2015), the payments triangle X_{ij} is first adjusted by dividing by the development inflation δ_j and the zero-claims probability Q_i. This gives a triangle of adjusted payments:

$$\widetilde{X}_{ij} = \frac{X_{ij}}{\delta_j(1-Q_i)}.$$

It is easy to verify that the triangle $\{\widetilde{X}_{ij}; (i,j) \in \mathcal{I}\}$ together with the counts triangle $N_{\mathcal{I}}$ follow model assumptions A1-A4 with $Q_i = 0 \quad \forall i$ and $\delta_j = 1 \quad \forall j$. The DCL method is applied to the adjusted payments triangle and the reported counts triangle as usual and all the DCL parameters are estimated.

Since estimating the underwriting year inflation, γ_i, in DCL is a weak point because it might be estimated with significant uncertainty (see Martínez-Miranda et al. (2013b)), the underwriting year inflation is estimated from the less volatile incurred data.

The model for the incurred triangle, which is technically based on expert knowledge and not actual data is as follows.

Aggregated incurred payments: $I_{\mathcal{I}} = \{I_{ik} : (i,k) \in \mathcal{I}\}$, where

$$I_{ik} = \sum_{s=0}^{k}\sum_{l=0}^{m-1} E[X_{isl}|\ \mathcal{F}_{i+k}] - \sum_{s=0}^{k-1}\sum_{l=0}^{m-1} E[X_{isl}|\ \mathcal{F}_{i+k-1}],$$

and $\mathcal{F}_h$ is an increasing filtration illustrating the expert knowledge at time point h.

Note that this definition is slightly different from Martínez-Miranda et al. (2013b). This definition is used to emphasise that the incurred triangle is cumulative and should be constructed in such a way that for every additional year an additional diagonal is added to the bottom of the current incurred triangle while the rest of the triangle stays the same as for the year before.

The incurred data are also adjusted in the following way

$$\widetilde{I}_{ik} = \frac{I_{ik}}{(1-Q_i)}.$$

so that the adjusted payments triangle $\widetilde{X}_{ij}$ and the adjusted incurred triangle $\widetilde{I}_{ik}$ have the same underwriting year inflation.

We provide a theoretical justification in the appendix for replacing the unstable estimates of the severity inflation from the payments by the more stable estimates from the incured data. The appendix shows that

$$E\left[\widetilde{I}_{ik}\right] = E\left[\frac{I_{ik}}{(1-Q_i)}\right] = \alpha_i \gamma_i \mu \beta_k \delta_k,$$

which shows that $\widetilde{X}_{ij}$ and $\widetilde{I}_{ik}$ have the same underwriting year parameters. This justifies that we replace $\widehat{\gamma}_i$ which we obtained by applying DCL on $\widetilde{X}_{ij}$ by the accident year inflation $\widehat{\gamma}_i^I$ we got by applying DCL on the adjusted incurred triangle $\widetilde{I}_{ik}$.

The next step is based on the so called BDCL method in Martínez-Miranda et al. (2013b), which allows us to use the underwriting year inflation of the incurred data. Since the expected value of $\widetilde{I}_{ik}$ has the same multiplicative structure as the expected value of X_{ik} with $Q_i = 0$, the DCL estimation method may be applied to the triangles of reported counts, N_{ik}, and the adjusted aggregated incurred claims, $\widetilde{I}_{ik}$. The parameters $\{\gamma_i : i = 1, \ldots, m\}$ are estimated exactly as described in Section 4, except that the triangle of aggregate paid claims is replaced by the triangle of adjusted aggregated incurred claims. Then we can replace the DCL underwriting year inflation estimates by those obtained from the adjusted incurred data.

The last step is now to multiply the estimates of the outstanding liabilities we obtained in this procedure by the development inflation δ_j and the zero-claims probability $(1-Q_i)$ again. Let $\widetilde{X}_{ij}^{BDCL}$ be the predicted value of $\widetilde{X}_{ij}$ by using the above described BDCL method. This is obtained exactly as described in the DCL method above by adding the RBNS and IBNR, but with the replaced underwriting year inflation using the adjusted incurred triangle. Then the predicted value of X_{ij} including the prior information will be given by $\widetilde{X}_{ij}^{new} = \delta_j(1-Q_i)\widetilde{X}_{ij}^{BDCL}$, for $(i,j) \in \mathcal{J}_1$. This way it is possible to generate the distribution of future values incorporating the prior information.

The method then consists of the following six-step procedure:

- *Step 1: Payments triangle adjustment.*
  Divide the payments triangle by the development inflation δ_j and the zero-claims probability $(1-Q_i)$ to get the adjusted payments triangle $\widetilde{X}_{ij} = \frac{X_{ij}}{\delta_j(1-Q_i)}$ to attain to the DCL framework.

- *Step 2: Incurred data adjustment.*
  Divide the aggregated incurred data by the zero-claims probability $(1-Q_i)$ to get the adjusted incurred triangle $\widetilde{I}_{ik} = \frac{I_{ik}}{(1-Q_i)}$ so that the estimate of the underwriting year inflation doesn't change. Note that I_{ik} are incurred claims for accident year i and development period k.

- *Step 3: Parameter estimation.*

  Estimate the model parameters using DCL for the data in the triangles $N_{\mathcal{I}}$ and $\widetilde{X}_{\mathcal{I}}$ and denote the parameter estimates by $(\widehat{p}_0, \ldots, \widehat{p}_{m-1})$, $\widehat{\mu}$, $\widehat{\sigma}^2$ and $\{\widehat{\gamma}_i : i = 1, \ldots, m\}$.

  Repeat this estimation using DCL but replacing the adjusted triangle of paid claims by the adjusted triangle of incurred data: $\widetilde{I}_{\mathcal{I}} = \{\widetilde{I}_{ik} : (i,k) \in \mathcal{I}\}$. Keep only the resulting estimated inflation parameters, denoted by $\{\widehat{\gamma}_i^I : i = 1, \ldots, m\}$.

- *Step 4: Bornhuetter-Ferguson adjustment.*

  Replace the inflation parameters $\{\widehat{\gamma}_i : i = 1, \ldots, m\}$ from the adjusted paid data by the estimates from the adjusted incurred triangle, $\{\widehat{\gamma}_i^I : i = 1, \ldots, m\}$.

- *Step 5: DCL prediction.*

  Get the prediction of the outstanding liabilities using DCL, more precisely the RBNS and IBNR estimates as in (10) and (11).

- *Step 6: Prediction readjustment.*

  Readjust the RBNS and IBNR estimates by multiplying them with the development inflation δ_j and the zero-claims probability $(1 - Q_i)$ and sum them up to get the final estimate of the total outstanding claims in our original framework.

While Martínez-Miranda et al. (2015) did assume prior knowledge on severity development inflation and zero-claims, it did not take advantage of prior knowledge of accident year inflation that often could be extracted from incurred data, see Martínez-Miranda et al. (2013b).

Finally, we give an example showing how additional data can be used to provide prior information in practice. As in Martínez-Miranda et al. (2015), the development inflation and the zero-claims probability can be estimated by using a new run-off triangle. Specifically we observe the total number of non-zero payments in accounting year $i + j$ from claims with accident year i and denote this by R_{ij}. The corresponding triangle is denoted by $\mathcal{R}_{\mathcal{I}} = \{R_{ij} : (i,j) \in \mathcal{I}\}$. Note that the variables R_{ij} have cross-classified mean $\mathrm{E}[R_{ij}] = \alpha_i^R \beta_j^R$ for all (i,j). Therefore, we can use the three triangles $(N_I, \mathcal{R}_I, X_I)$ simultaneously and simply apply the chain ladder algorithm three times:

$$
\begin{aligned}
&N_{\mathcal{I}} \text{ provides the chain ladder estimators } \widehat{\alpha}_i \text{ and } \widehat{\beta}_j \text{ for } \alpha_i \text{ and } \beta_j,\\
&\mathcal{R}_{\mathcal{I}} \text{ provides the chain ladder estimators } \widehat{\alpha}_i^R \text{ and } \widehat{\beta}_j^R \text{ for } \alpha_i^R \text{ and } \beta_j^R,\\
&X_{\mathcal{I}} \text{ provides the chain ladder estimators } \widehat{\widetilde{\alpha}}_i \text{ and } \widehat{\widetilde{\beta}}_j \text{ for } \widetilde{\alpha}_i \text{ and } \widetilde{\beta}_j.
\end{aligned}
$$

Now the probability of zero-claims in the underwriting year, Q_i, can be estimated from the expression

$$\widehat{Q}_i = 1 - \frac{\widehat{\alpha}_i^R}{\widehat{\alpha}_i}. \tag{14}$$

Furthermore, the development inflation parameters can be estimated by

$$\widehat{\delta}_j = \frac{\widehat{\widetilde{\beta}}_j}{\sum_{l=0}^{j} \widehat{\beta}_{j-l}\widehat{\pi}_l} = \frac{\widehat{\widetilde{\beta}}_j}{\widehat{\beta}_j^R}. \tag{15}$$

5 Classical chain ladder split

This section explains how the reinsurance split is done in practice. For the purposes of this paper we have used motor third party liability bodily injury loss data from a medium-sized Greek insurer. The triangular data provided were on an accident year basis with yearly development periods for accident periods from 2000 to 2014 and included all gross bodily injury claims incurred during the period and reported by 31 December 2014, which is the valuation date. The total portfolio exposure measured as earned vehicle years was by 2014 around 400,000 in comparison to around 100,000 in 2000. In addition to the incurred and paid triangles, we also received the corresponding reported (non-zero) counts triangle and the open claims counts triangles. Individual claim triangulations were received for claims above EUR 200,000. For this illustration, we have applied a common priority of EUR 500,000. In order to create the recoveries triangles, we first identify all individual claims for which the excess-of-loss reinsurer has a participation, i.e. for losses whose incurred value based on cumulative payments and the case reserve as at the valuation date exceeds the priority threshold of EUR 500,000. Then the reinsurance recoveries triangles for those individual claims is constructed and aggregated on an accident year basis to match the gross triangles. Finally to construct the net triangles we subtracted the recoveries triangles from the gross triangles.

Tables 1, 2 and 3 show the gross payments triangle as well as the net payments triangle and the recoveries triangle after the split.

345115	550155	1165973	1703483	1890192	1165754	1112124	1172613	424153	401842	147602	171353	79772	281345	266548
350771	1307179	1843726	2746214	2587467	1844121	768873	307287	600012	37169	955466	209065	273838	25515	
495836	1698146	2426735	4139259	3010904	3245849	2424486	2040852	722079	1836805	885848	226836	619320		
618012	1711657	2560425	3288447	2973974	2834974	1397271	766589	1327613	692792	525813	159328			
948012	2809693	3788681	3993582	2964559	3255971	812468	1953830	810337	624285	128553				
953333	2325650	4075835	5574008	5441890	4169716	2838634	2334831	1134118	542453					
1266559	3436062	3840499	5820558	5346293	5101668	1912277	2327070	1806402						
1590238	3794768	4470900	4917158	5652933	4753303	3542325	2247495							
1484722	2726065	3958068	4602047	4100856	3243477	1490074								
1832755	2174965	4105838	3803571	3609703	4316400									
1829793	2603989	3768836	4089502	2826510										
1569902	2820043	4116023	2578524											
1391136	3576140	1997271												
2260336	3695117													
981728														

Table 1: Gross payments in Euro.

345115	550155	1165973	1703483	1890192	1165754	1112124	1172613	269840	397567	142807	161728	79772	281345	266548
350771	1307179	1843726	2746214	2566190	1844121	768873	307287	600012	37170	917356	209065	273838	25515	
495836	1698146	2425159	4051087	3010904	3131364	2212115	1751729	717586	1558083	612638	217397	499180		
618012	1711657	2560425	3258378	2903194	2741042	1392506	761189	1125816	607190	523861	158426			
948012	2744998	3782286	3954751	2872013	3096314	805336	1945795	789209	604091	96978				
953333	2325650	4075835	5444170	5235030	3869351	2793398	2224029	1040856	516568					
1266559	3436062	3840500	5669342	5345112	4861694	1807798	2244928	1802414						
1590238	3794768	4470899	4917158	5613373	4488985	3438274	2246031							
1484722	2726066	3958068	4590839	4100856	3243477	1490074								
1832755	2174965	4105838	3785096	3605612	4314345									
1829793	2603989	3768836	4037457	2416537										
1569902	2820044	4116022	2578524											
1391136	3576141	1997270												
2260336	3695117													
981728														

Table 2: Net payments in Euro.

We can now apply different reserving methods to all three of these triangles and compare the results. First, for the gross payments triangle, the classical CLM as well as DCL (with the adjustments set out in Martínez-Miranda et al., 2012) give a reserve of EUR 138,952,059. In a practical context, the appropriateness of all estimators should be considered. However, for the purposes of the illustration of the methods in this paper, we will assume that this is an appropriate estimate and are therefore trying to get a similar result. While the BDCL method of Martínez-Miranda et al. (2013b) with $\delta_j = 1 \quad \forall j$ and $Q_i = 0 \quad \forall i$ (EUR 168,124,495) as well as the DCL prior method of Martínez-Miranda et al. (2015) (EUR 160,468,905) give a much higher reserve, the BDCL prior method has a reserve of EUR 139,434,204, which is very close to the CLM result and may provide some justification for the new modification.

Applied to the net payments triangle, the CLM and DCL provide a reserve of EUR 133,200,160. Similar to the results for the gross payments, the BDCL (EUR 161,432,377) and the DCL prior method (EUR 153,728,832) have a much higher reserve, whereas the BDCL prior method comes to a reserve of EUR 133,724,859, which is again very close to the CLM reserve.

While it may not always be suitable to appply chain ladder models to recoveries triangles, it is interesting in the context of this paper to examine what the results

0	0	0	0	0	0	0	0	154313	4275	4795	9625	0	0	0
0	0	0	0	21277	0	0	0	0	0	38110	0	0	0	
0	0	1576	88172	0	114485	212371	289123	4493	278722	273209	9440	120140		
0	0	0	30069	70780	93931	4765	5400	201797	85601	1953	902			
0	64695	6396	38830	92545	159658	7132	8035	21128	20196	31574				
0	0	0	129838	206860	300365	45235	110803	93262	25886					
0	0	0	151216	1180	239975	104479	82142	3988						
0	0	0	0	39561	264317	104051	1465							
0	0	0	11209	0	0	0								
0	0	0	18475	4091	2055									
0	0	0	52045	409973										
0	0	0	0.00											
0	0	0												
0	0													
0														

Table 3: Recoveries in Euro.

show. For these data, the results for the recoveries triangle are a bit different. As predicted in section 2, the CLM and DCL reserves are slightly overestimated at EUR 3,667,605. While the BDCL method gives an even bigger reserve of EUR 4,010,271, the DCL prior reserve appears to be better estimated with a value of EUR 2,437,866. Again, the BDCL prior method calculates a reserve which appears to be more appropriate at EUR 2,502,285.

Given these results, the conclusion for this practical approach is that we should apply a bootstrap method based on the BDCL prior method described in section 4.1. For simplicity we do not include the option for zero-claims probability in the bootstrap results in this paper. Figure 1 shows the results of the BDCL prior bootstrap method applied to the gross payments triangle. This shows the cash flow on the left hand side, and the reserve on the right hand side, split into IBNR and RBNS as well as the total reserve, all in Euros. The corresponding estimates, such as the mean for the total reserve of EUR 135,826,096, can be found in Table 4 in the Appendix.

The more interesting results for this bootstrap can be found in Figure 2, which shows the reserves for the net triangle on the left side together with the recoveries triangle on the right side. These results will be compared to the split done using the BDCL prior method and simulation of individual claims in the following section. The corresponding estimates can be found in Table 5 for the net triangle (mean total reserve = EUR 130,115,416) as well as Table 6 for the recoveries triangle (mean total reserve= EUR 2,346,690).

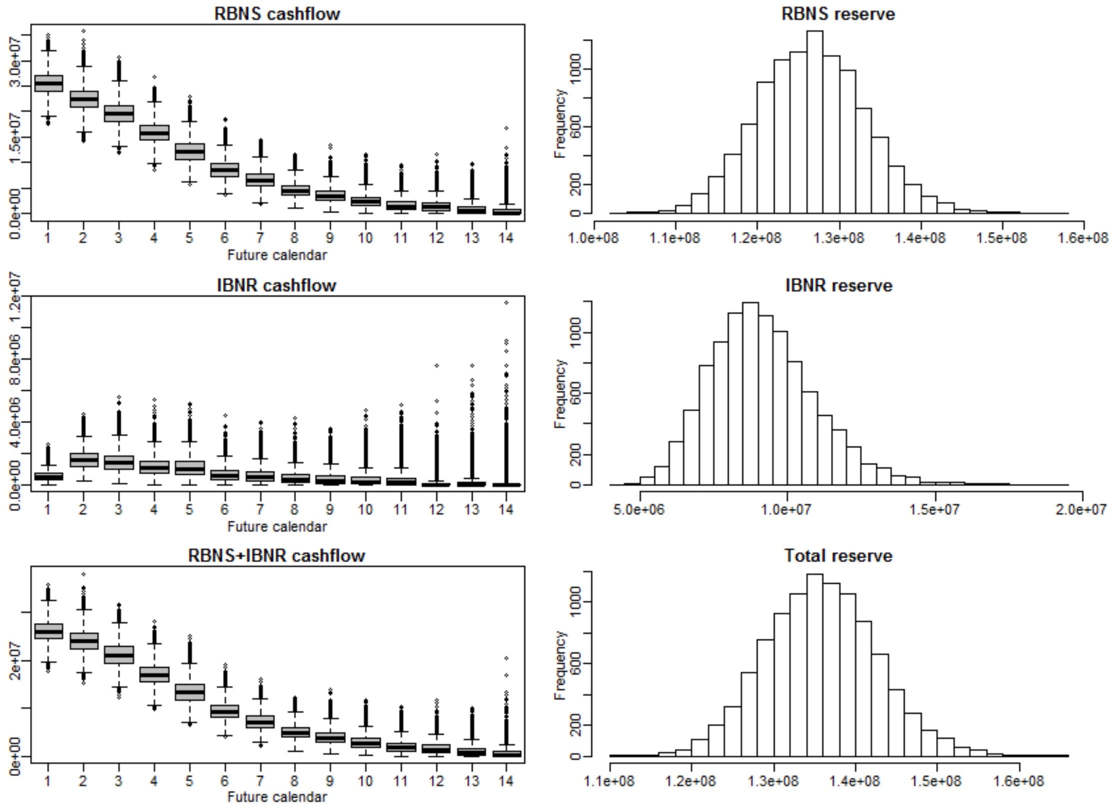

Figure 1: BDCL prior bootstrap applied on the gross payments triangle, 10,000 times. The cash flow as well as the reserves in Euros. The results are given for the IBNR, RBNS and the total.

6 The prior knowledge double chain ladder split

In this section we consider an alternative method for the split into net payments triangle and recoveries triangle to that used in practice and outlined in the previous section. This method simulates individual claims using the model in section 3 which requires just the aggregated data and then splits the simulated individual claims using a given retention. When these individual claims are split into a net and a recoveries part, they will be aggregated again and a bootstrap method will be applied. In the following, this simulation method will be explained in detail.

First, the development inflation needs to be extracted using the approach of Martinez-Miranda et al. (2015). Then, following the approach of this paper, the development inflation is divided out of the payments, so that the original DCL type of model can be applied. The BDCL method is applied and we obtain the corresponding parameters, which are used to calculate the mean and variance which are required for the simulation of individual claims. The individual claims are simulated with a Gamma distribution, using the counts which are simulated with a Poisson

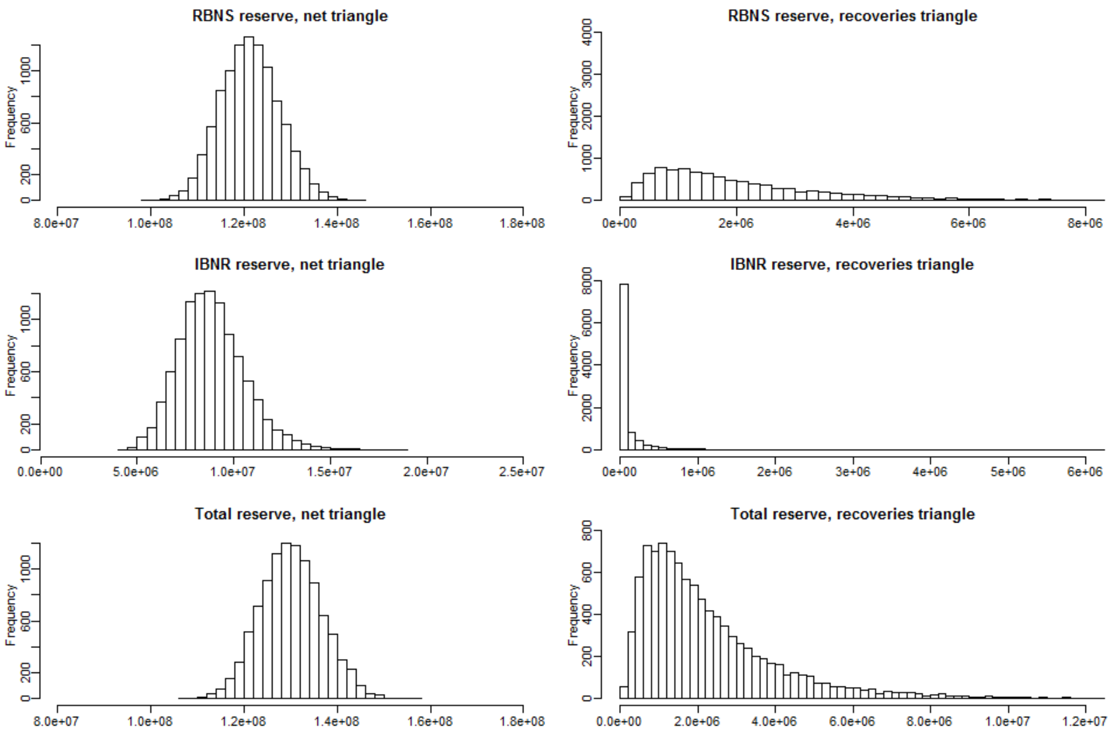

Figure 2: BDCL prior bootstrap results for the reserves given in Euros, separated into IBNR and RBNS together with the Total reserves. On the left hand side the bootstrap method was applied on the net payments triangle and on the right hand side on the recoveries triangle. The bootstrap was done 10,000 times.

distribution in line with the DCL framework.

In contrast with the practical approach outlined in the previous section, the split between insurance and reinsurance can be done for each individual claim, where it is simply decided whether the value of the claim is smaller than a predetermined value given by the reinsurance assumptions. If the claim is smaller, it is added to the insurance triangle. If it is bigger, the predetermined value is added to the insurance triangle and the excess is added to the reinsurance triangle.

This process is done separately for IBNR and RBNS claims. Then, the individual claims can be added up to the usual aggregated IBNR and RBNS triangles. This procedure is repeated multiple times. Finally, we multiply the development inflation back to the results and calculate any required statistics such as the mean, quantiles etc.

The simulation method consists of the following procedure, following along the steps of section 4.1:

- *Step 1: Payments triangle adjustment.*

Divide the payments triangle by the development inflation δ_j and the zero-claims probability $(1-Q_i)$ to get the adjusted payments triangle $\widetilde{X}_{ij} = \frac{X_{ij}}{\delta_j(1-Q_i)}$ to attain to the DCL framework.

- *Step 2: Incurred data adjustment.*
  Divide the aggregated incurred data by the zero-claims probability $(1 - Q_i)$ to get the adjusted incurred triangle $\widetilde{I}_{ik} = \frac{I_{ik}}{(1-Q_i)}$ so that the estimate of the underwriting year inflation doesn't change. Note that I_{ik} are incurred claims for accident year i and development period k.

- *Step 3: Parameter estimation.*
  Estimate the model parameters using DCL for the data in the triangles $N_{\mathcal{I}}$ and $\widetilde{X}_{\mathcal{I}}$ and denote the parameter estimates by $(\widehat{p}_0, \dots, \widehat{p}_{m-1})$, $\widehat{\mu}$, $\widehat{\sigma}^2$ and $\{\widehat{\gamma}_i : i = 1, \dots, m\}$.
  Repeat this estimation using DCL but replacing the adjusted triangle of paid claims by the adjusted triangle of incurred data: $\widetilde{I}_{\mathcal{I}} = \{\widetilde{I}_{ik} : (i, k) \in \mathcal{I}\}$. Keep only the resulting estimated inflation parameters, denoted by $\{\widehat{\gamma}_i^I : i = 1, \dots, m\}$.

- *Step 4: Bornhuetter-Ferguson adjustment.*
  Replace the inflation parameters $\{\widehat{\gamma}_i : i = 1, \dots, m\}$ from the adjusted paid data by the estimates from the adjusted incurred triangle, $\{\widehat{\gamma}_i^I : i = 1, \dots, m\}$.

- *Step 5: Mean and variance.*
  Calculate estimates of $\gamma_i\mu$ and $\gamma_i^2\sigma^2$.

- *Step 6: Simulate counts.*
  Simulate IBNR counts, N_{IBNR}, and RBNS counts, N_{RBNS}, separately using $pois(\alpha_i\beta_j)$.

- *Step 7: Simulate individual payments.*
  Simulate individual payments (IBNR and RBNS separately) using $gamma(E^2/Var, Var/E)$ N_{IBNR}-times and N_{RBNS}-times, respectively.

- *Step 8: Split.*
  The individual claims are split into a net part for the insurance company as well as a recoveries part for the reinsurance company. The split is done by comparing the values of the individual claims to a given retention value for each accident year.

- *Step 9: Aggregate data.*
  The split claims are aggregated into triangles.

- *Step 10: Repeat multiple times.*
  The process is repeated multiple times.

- *Step 11: Prediction readjustment.*
  Readjust the RBNS and IBNR estimates by multiplying them with the development inflation δ_j and the zero-claims probability $(1 - Q_i)$ and sum them up to get the final estimate of the total outstanding claims in our original framework.

- *Step 12: Calculate mean, quantiles etc.*
  Finally, statistical values such as the mean and variance can be calculated.

Figure 3 shows this simulation applied to the gross payments data triangle given in Table 1 with 10,000 repetitions. This gives the reserves in Euros, split into IBNR and RBNS together with the total reserves. On the left hand side we have the plots for the net triangle resulting from the simulated split, and on the right hand side the results for the recoveries triangle.

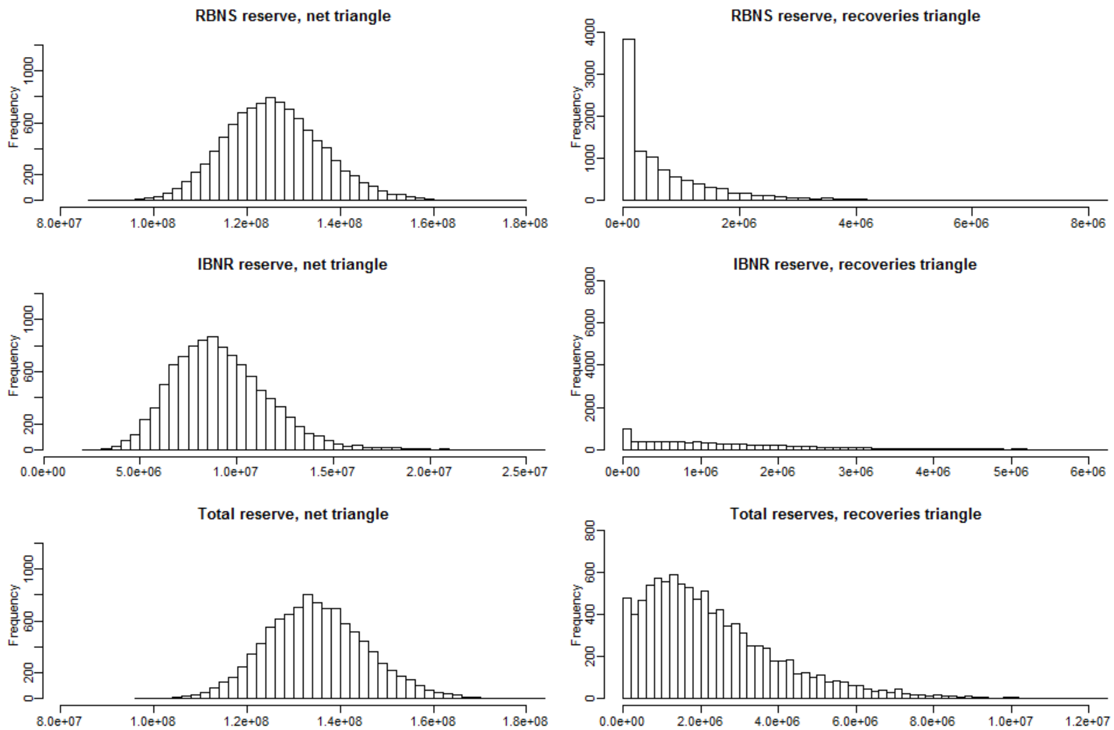

Figure 3: Results of the simualtions for the reserves given in Euros and seperated into IBNR and RBNS, together with the Total reserves. On the left hand side the net payments triangle is shown, where the split was done for the individually simulated claims. On the right hand side the recoveries triangle is presented. The simulation was done 10,000 times.

The results of the new method can be applied to those of the practical approach in the previous section. This is summarised in Figure 4.

The left side shows the Total reserves for the net triangles. While the graph for the BDCL prior bootstrap in clearly higher, the tail for the simulation method is clearly bigger on both sides. The results for the Total reserves for the recoveries triangles are similar. The graph for the BDCL prior bootstrap starts lower around zero, but the rest of the two graphs is very similar. However, the distributions of the two different approaches have some similarities but need to have greater consideration in a practial context.

7 Conclusions

As presented in Figure 4, the results of our new simulation split method can be compared to the results given by the method used in practice. For the new method, the split via simulation presented in Chapter 6 is carried out by estimating individual

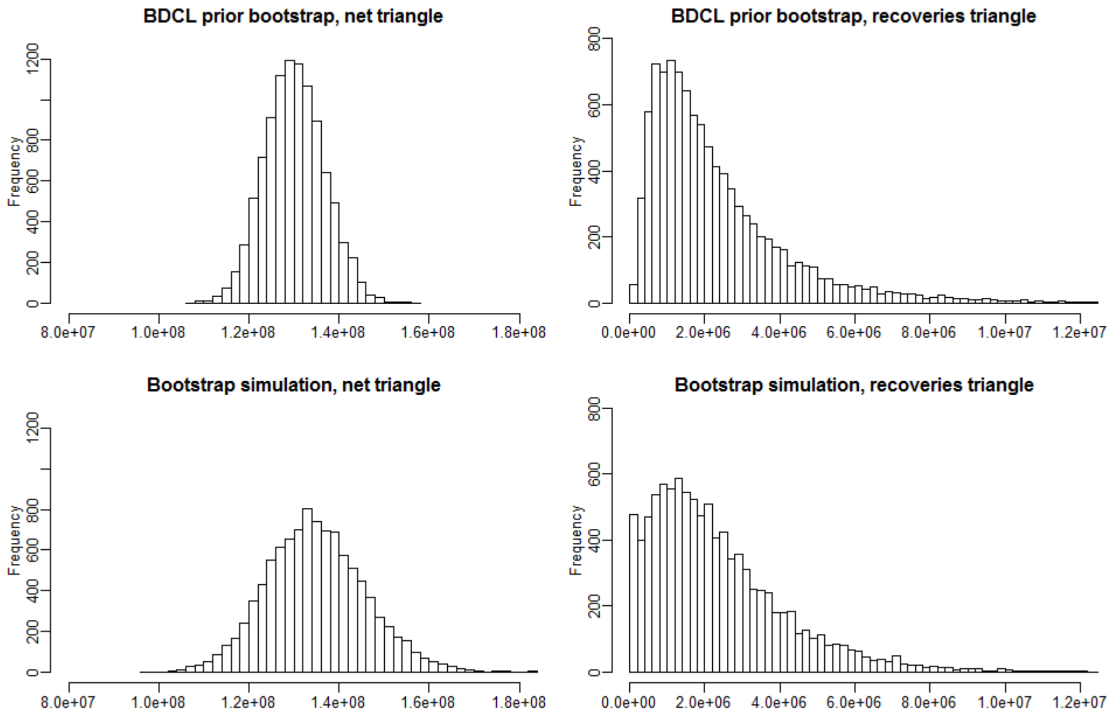

Figure 4: Total reserves for the BDCL prior bootstrap, applied on the split data from Chapter 5, in comparison to the Total reserves from the bootstrap simulation from Chapter 6

claims from aggregate data and applying a bootstrap method afterwards. This gives a more credible, coherent and flexible framework and it would be very interesting to test this in a practical context where the retention level varies by year. It would also be useful to assess the usefulness of this approach by considering a range of different types of data and to compare with other individual reserving methods. We believe that the framework of DCL is probably easier to use in practice and yet still has enough flexibility for a more coherent consideration of the effects of reinsurance on reserves.

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A Appendix

To justify the BDCL method introduced in chapter 4.1, we need to calculate the expectation of the adjusted incurred triangle.

First, we use the definition of I_{ik}, the tower property and the definition of X_{ikl}

to get

$$
\begin{aligned}
E[I_{ik}] &= E\left[\sum_{s=0}^{k}\sum_{l=0}^{m-1} E[X_{isl}|\ \mathcal{F}_{i+k}] - \sum_{s=0}^{k-1}\sum_{l=0}^{m-1} E[X_{isl}|\ \mathcal{F}_{i+k-1}]\right] \\
&= \sum_{s=0}^{k}\sum_{l=0}^{m-1} E[X_{isl}] - \sum_{s=0}^{k-1}\sum_{l=0}^{m-1} E[X_{isl}] \\
&= \sum_{l=0}^{m-1} E[X_{ikl}] \\
&= \sum_{l=0}^{m-1} E\left[\sum_{h=0}^{N_{ikl}^{paid}} Y_{ikl}^{(h)}\right].
\end{aligned}
$$

Now, using (A4), we can apply Wald's equation and use the tower property again. Hence, we obtain

$$
\begin{aligned}
E[I_{ik}] &= \sum_{l=0}^{m-1} E\left[N_{ikl}^{paid}\right] E\left[Y_{ikl}^{(h)}\right] \\
&= \sum_{l=0}^{m-1} E\left[E\left[N_{ikl}^{paid}|N_{ik}\right]\right] E\left[Y_{ikl}^{(h)}\right].
\end{aligned}
$$

Therefore, using (A2), (A1), (A3), and (1), we conclude the following unconditional mean for the incurred claims

$$
\begin{aligned}
E[I_{ik}] &= \sum_{l=0}^{m-1} E[N_{ik}p_l] E\left[Y_{ikl}^{(h)}\right] \\
&= \sum_{l=0}^{m-1} \alpha_i\beta_k p_l \gamma_i \delta_k (1-Q_i)\mu \\
&= \alpha_i\gamma_i(1-Q_i)\mu\beta_k\delta_k \sum_{l=0}^{m-1} p_l \\
&= \alpha_i\gamma_i(1-Q_i)\mu\beta_k\delta_k.
\end{aligned}
$$

Finally, we consider the expectation of $\widetilde{I}_{ik}$, which is the incurred claims triangle where we Divided out the zero-claims probability.

$$
\begin{aligned}
E\left[\widetilde{I}_{ik}\right] &= E\left[\frac{I_{ik}}{(1-Q_i)}\right] \\
&= \alpha_i\gamma_i\mu\beta_k\delta_k.
\end{aligned}
$$

This means that $\widetilde{X}_{ij}$ and $\widetilde{I}_{ik}$ have the same underwriting year parameters. Therefore, using either one of these triangles, we estimate the same DCL parameters, including the accident year inflation parameter γ_i. That justifies that we replace $\widehat{\gamma}_i$ which we obtained by applying DCL on $\widetilde{X}_{ij}$ by the accident year inflation $\widehat{\gamma}_i^{\,I}$ we got by applying DCL on the adjusted incurred triangle $\widetilde{I}_{ik}$.

B Appendix: results

	mean.total	sd.total	Q1.total	Q5.total	Q50.total	Q95.total	Q99.total
1	26097618.95	2392931.78	20886771.49	22324140.69	26032394.84	30174384.25	32079872.39
2	24158342.00	2521628.99	18748036.29	20192400.26	24091427.19	28492022.04	30484948.59
3	21139107.92	2530340.00	15801701.38	17228688.02	21014715.81	25525694.20	27528586.39
4	17091987.23	2379023.40	12055839.23	13347033.36	16983395.41	21181064.02	23252390.09
5	13372286.22	2323742.40	8653948.82	9808879.04	13216264.91	17435683.34	19528445.78
6	9353233.12	1938764.78	5505334.88	6405189.05	9234879.15	12725025.87	14589337.48
7	7270948.52	1782991.56	3830245.99	4640998.13	7126495.58	10452121.69	12142565.81
8	5092876.23	1565939.62	2152866.47	2815683.11	4930607.05	7959667.88	9439412.66
9	3962208.40	1526824.45	1300943.34	1856061.17	3763333.34	6731605.89	8528455.08
10	2834724.63	1403185.60	604038.64	986673.29	2617255.21	5409836.47	7127125.04
11	1966353.27	1273631.19	183392.94	412683.67	1713562.13	4430254.95	6110966.12
12	1665933.68	1263099.74	55024.65	211665.11	1369893.70	4069701.90	5879908.19
13	1081440.15	1211645.98	0.00	3929.37	693181.94	3515330.99	5457305.15
14	739036.15	1258684.81	0.00	0.00	197604.85	3293106.90	5580981.11
Tot.	135826096.47	6838341.10	120590726.18	124742962.72	135733838.59	147245419.76	152650623.32

Table 4: Results from the BDCL prior bootstrap of Chapter 5 for the gross payments triangle in Euro. These are the total reserves, so the sum of the IBNR and RBNS. The bootstrap was done 10,000 times.

	mean.total	sd.total	Q1.total	Q5.total	Q50.total	Q95.total	Q99.total
1	25375456.64	2309316.00	20410202.99	21812817.43	25272033.15	29356647.09	31145553.62
2	23367871.35	2391471.13	18264214.15	19600152.45	23304682.05	27451256.58	29394697.61
3	20374818.94	2435560.18	15175869.33	16509581.78	20282202.40	24608492.86	26523551.82
4	16413241.22	2287170.69	11487626.17	12906855.53	16295219.87	20434716.77	22162714.95
5	12717529.98	2223454.61	8145667.34	9350835.16	12539153.03	16613696.28	18518427.48
6	8838253.04	1808325.93	5274691.37	6130174.30	8692292.16	12008506.92	13727742.86
7	6778492.54	1711673.76	3445007.42	4273133.05	6610866.19	9837135.77	11560686.36
8	4737099.84	1455547.35	2077801.13	2629966.98	4581030.90	7373478.63	8819808.31
9	3615560.12	1405362.20	1198578.44	1685708.15	3406266.55	6214069.17	7716290.78
10	2627966.69	1307648.92	552692.38	926132.07	2411894.66	5038921.62	6740394.75
11	1895212.46	1236013.10	154656.55	378060.29	1651657.82	4237719.26	5827700.68
12	1580611.59	1196904.29	51467.78	211778.91	1291175.73	3880311.90	5580858.46
13	1074318.33	1203949.11	0.00	4932.29	679491.51	3577721.41	5309385.83
14	718983.45	1174810.48	0.00	0.00	197268.01	3101279.38	5554770.14
Tot.	130115416.19	6562266.35	115379290.79	119627422.38	129969539.83	141239104.59	145533609.00

Table 5: Results from the BDCL prior bootstrap of Chapter 5 for the net payments triangle in Euro. These are the total reserves, so the sum of the IBNR and RBNS. The bootstrap was done 10,000 times.

	mean.total	sd.total	Q1.total	Q5.total	Q50.total	Q95.total	Q99.total
1	585872.24	941073.48	3137.43	9692.16	234273.07	2324506.56	4640738.44
2	293379.56	516258.50	282.59	1565.84	99755.80	1255333.65	2582023.85
3	327232.59	652097.26	325.07	3114.14	108482.10	1371200.64	3107224.35
4	295013.89	623676.31	52.34	903.10	82677.89	1370850.44	3250473.11
5	285731.58	693697.94	0.09	22.14	50689.04	1307049.38	3516983.09
6	143036.48	593527.17	0.00	0.09	6699.26	590149.50	2798207.32
7	82530.96	242632.80	0.00	0.00	3369.64	437610.08	1128328.50
8	216566.46	976588.86	0.00	0.00	539.53	996581.57	4397546.27
9	35267.81	188761.48	0.00	0.00	0.00	144058.40	948118.52
10	33907.97	239799.97	0.00	0.00	0.00	86469.40	1014379.78
11	6000.67	47366.28	0.00	0.00	0.00	6537.46	168103.36
12	42150.26	277582.67	0.00	0.00	0.00	135258.32	1185302.35
Tot.	2346690.46	1988645.51	243455.77	447488.14	1791080.87	6151492.72	9678591.63

Table 6: Results from the BDCL prior bootstrap of Chapter 5 for the recoveries triangle in Euro. These are the total reserves, so the sum of the IBNR and RBNS. The bootstrap was done 10,000 times.

	mean.total_net	sd.total_net	Q1.total_net	Q5.total_net	Q50.total_net	Q95.total_net	Q99.total_net
1	25905055.39	3727872.86	18100170.70	20132476.50	25711255.14	32201375.33	35508312.24
2	23932228.36	3965650.38	15820465.78	17946494.20	23679016.53	30885342.55	34376799.12
3	20917492.41	3926735.56	12793883.25	14877169.70	20690675.61	27788991.01	31208214.31
4	17049799.96	3786929.48	9634375.59	11338190.75	16765247.54	23598688.19	27331568.40
5	13240383.34	3585513.43	6547856.46	8031280.04	12866692.70	19726623.28	23318292.49
6	9248605.85	2926938.34	3933173.83	5117001.41	8882062.82	14567816.43	17615450.70
7	7219015.05	2807373.23	2490929.63	3427338.94	6835029.89	12397890.85	15656106.74
8	5114767.79	2446660.80	1175107.61	1911161.06	4723691.15	9712334.84	12595022.88
9	3928749.50	2327558.23	561106.50	1058269.55	3510452.03	8218138.60	11571565.67
10	2908164.48	2210888.76	179192.80	463185.85	2362591.36	7073530.70	10415142.51
11	1947509.09	1916662.98	12757.13	95797.52	1397202.77	5713283.57	9201440.39
12	1665008.85	1940642.31	1420.44	22588.46	1015936.26	5613686.03	9156401.85
13	1098807.18	1928443.07	0.00	0.81	294000.20	4877308.49	9461457.99
14	758933.18	1923947.33	0.00	0.00	13448.57	4233172.23	9580323.43
Tot.	134934520.44	10884256.09	111265162.53	117982934.21	134411921.90	153765855.98	162619948.01

Table 7: Results for split via the simulation of Chapter 6. Presented are the resulting net payments in Euro. These are the total reserves, so the sum of the IBNR and RBNS. The simulation was done 10,000 times.

	mean.total_Ex	sd.total_Ex	Q1.total_Ex	Q5.total_Ex	Q50.total_Ex	Q95.total_Ex	Q99.total_Ex
1	263869.44	490083.99	0.00	0.00	0.00	1262238.30	2279729.73
2	305291.26	572326.32	0.00	0.00	12446.89	1395779.60	2519679.49
3	316386.93	602782.84	0.00	0.00	0.00	1519499.66	2759573.25
4	301615.74	621647.67	0.00	0.00	0.00	1527947.84	2883860.62
5	283558.21	643016.92	0.00	0.00	0.00	1479181.86	2963271.38
6	198625.05	516634.95	0.00	0.00	0.00	1183692.09	2368634.91
7	161994.67	495942.87	0.00	0.00	0.00	1034987.24	2399881.13
8	124559.56	443331.70	0.00	0.00	0.00	855657.13	2190795.45
9	121863.37	519594.03	0.00	0.00	0.00	793234.00	2456611.84
10	92518.12	436308.94	0.00	0.00	0.00	582338.07	2281324.28
11	78236.23	493011.67	0.00	0.00	0.00	222062.24	2175314.15
12	50266.04	412240.24	0.00	0.00	0.00	0.00	1572289.27
13	43666.00	414266.48	0.00	0.00	0.00	0.00	1526001.39
14	33882.15	460701.52	0.00	0.00	0.00	0.00	164083.49
Tot.	2376332.76	1933288.21	0.00	213633.08	1947229.02	5954247.89	8892712.70

Table 8: Results for split via the simulation of Chapter 6. Presented are the resulting recoveries in Euro. These are the total reserves, so the sum of the IBNR and RBNS. The simulation was done 10,000 times.