Investment Policy for Defined Contribution Pension Scheme Members Close to Retirement

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1 Introduction

As the defined contribution pension market expands the issue of determining an appropriate investment policy will become more important. In the defined contribution scheme, the member takes the investment risk and, in the UK, will often have the option to determine the investment policy. Investment is likely to be in a number of unitised or mutual funds comprising, for example, UK Equities, conventional gilts or bonds, cash, overseas equities, property and index-linked gilts or bonds. Investors may also choose a managed fund (containing proportions of the above assets determined by investment managers) or they may choose a with-profit fund. In this work, it is assumed that the contributor has a number of unitised funds available and that he can switch between funds at any time or determine, at outset, a fund switching strategy which will automatically be undertaken by the provider.

There has been actuarial comment on the investment strategy an investor should follow in order to get the right mix of risk and return (for example, Khorasanee (1997), Khorasanee (1996), Booth (1995) and Knox (1993)). There has also been work published on the management and measurement of investment risk, with some applications to personal pension fund holders (for example, Gordon, Mitchell and Twinney ed. (1997) and Booth (1997)). Blake and Orszag (1997) have looked at some of the difficulties pensioners face when buying an annuity in the annuity market.

This paper looks specifically at appropriate investment strategies close to retirement. There has been some comment to the effect that, although an equity-based investment policy might be appropriate when an investor is at a relatively young age, a cash or bonds-based investment policy might be appropriate closer to retirement, in order to reduce the risk of a sudden equity market crash on the value of the fund. Some of this comment has been journalistic but is nevertheless influential. The comment was also made by Sze in the discussion of Knox (1993) and suggested as an area for further research.

Blake and Orszag (1997) have also commented that changes in annuity rates can create difficulties for pensioners who retire, when annuity rates are unfavourable. This comment ignores the likelihood of correlations between investment yields and annuity rates.

The Blake and Orszag comment, this would seem to be a fallacy under the following conditions:

(i) the contributor invested in a bond fund close to retirement.

(ii) The bond fund had the same duration as the cash flows from the annuity.

(iii) The annuity market was sufficiently efficient so that annuity rates reflected bond yields.

It is also the case that, if an investor holds a predominantly equity fund and equity yields are closely correlated with bond yields, the risk of investing in equities may be less than is commonly supposed.

However, if these three conditions do not hold, Blake and Orszag may well be correct in their hypothesis.

We will investigate condition (i) and, indirectly, condition (ii). We will also consider the risk of a predominantly equity-based investment strategy. The majority of defined contribution plan holders purchase a fixed annuity at retirement. As the market expands, index-linked annuities may become more common. We will therefore also consider the situation for an investor who wants to purchase an index-linked annuity. Investment in cash is less risky than investment in equities, if the investor is interested in the variability of the cash fund at retirement. However, if the investor is interested in the variability of the annuity to be received, the issue is more complex. If there is a tendency for investments in the fund to rise (fall) in value when annuity rates become less (more) favourable, investments to which there is capital-value risk attached might be more appropriate than investments which are guaranteed in capital value terms (such as cash). Furthermore, if equity yields are positively correlated with bond yields, it may be the case that changes in equity values will be offset by changes in annuity rates to some extent. This will mean that the annuity value which could be bought with the proceeds of an equity investment fund will not be as variable as the equity fund value itself. Thus a consideration of the variability of the underlying investments in the fund, without any consideration of their correlation with annuity rates, is simplistic.

2 Pre-Retirement Investment Risk: Methodology

2.1 Background to the Research Methodology

An individual who had accumulated a particular fund at a given point (reference point) before retirement was considered. The fund was defined in terms of salary at that point and future contributions were also defined in terms of salary. In the empirical part of the investigation, salary increases were assumed to be in line with national average salary changes (this would ignore scale increases close to retirement). The fund at the reference point, plus all later contributions were invested in various different investment funds. The returns from the investment funds were assumed to be in line with the returns from the BZW Equity-Gilt Study (1997) for the investment categories UK equities, conventional gilts, index-linked gilts and cash. US equity returns were taken from S&P 500 Composite Index converted into sterling terms.

In the first study, the authors looked at the accumulated cash fund and annuity value (both expressed as a proportion of salary) using five different investment strategies in the ten years before retirement (i.e. ten years was the reference point). All available post-war data was used so that the first retirements took place in 1955. In the second investigation, accumulated cash funds and index-linked annuities were considered.

Index-linked bonds and US equities were used as possible asset categories. The reference point was chosen as three years before retirement and the data period was limited to 1982 to 1997 because 1982 was the first year for which the BZW index-linked index was published. The second investigation was repeated using simulation and the Wilkie (1995) stochastic investment model.

2.2 Funding for Cash

For a male, it was assumed that a personal pension fund of 300% of salary had been accumulated by age 55. It was then assumed that a further 12% of salary would be invested in each of the remaining years until retirement. Retirement was at age 65. Five percent expenses were taken from each contribution and a 1% fund management charge was also subtracted from the fund each year. Five possible investment strategies were chosen from the reference point:

(a) the "inertia" strategy of continuing to invest to retirement in a diversified fund of 70% UK equities, 20% UK medium dated gilts and 10% cash: this will be described as the standard fund.

Four "lifestyle planning" strategies which involve moving investments into cash and gilts towards retirement. These were:

(b) the equity and cash components of the fund were switched into medium dated gilts over ten years. The amounts of the switch were such that the same proportions of the original equity and cash fund were switched each year and the whole of the fund was invested in medium dated gilts for the last year before retirement.

(c) The equity and gilt components of the fund were switched into cash over ten years. The amounts of the switch were such that the same proportions of the original equity and gilt funds were switched each year and the whole of the fund was invested in cash for the last year before retirement.

(d) The standard fund was used until three years before retirement and then the whole fund was switched into cash.

(e) The standard fund was used until three years before retirement and then the whole fund was switched into medium dated gilts.

The return on equities was determined by the total annual returns from the BZW Equity-Gilt study (1997). The return on cash was determined by the Treasury Bill returns from the same study. Returns from long-dated gilts were used as a proxy for returns on a managed gilt fund. This data was also taken from the BZW Equity-Gilt study.

At retirement the cash sum, relative to salary, from the different investment strategies was calculated for individuals starting their last ten years of investment before retirement in every year from 1945 to 1988 so that retirements took place from 1955 to the beginning of 1998. Salaries were projected using the average annual rate of increase of wages across several industrial series from Economic Trends (date). The expected return and risk from the various investment strategies was measured by considering the following:

1. The expected size of the cash sum relative to salary.

2. The variance of the cash sum relative to salary.

3. The semi-variance of the cash sum, relative to salary. The assumption here was that, as is discussed in Booth (1997) and Clarkson (1989), pensioners may well be more concerned about variability below a particular benchmark (for example a cash sum of less than 500% of salary) than about variability above that benchmark. An equity based fund, for example, may give more variability but that variability could be concentrated in observations above the benchmark cash sum. It should be noted that the semi-variance measure was calculated against a benchmark and not against the mean of the sample taken, in order to ensure consistency between the results from different investment strategies.

The relevant accumulation formula are given in the appendix.

2.3 Funding to Purchase a Fixed Annuity

Even if the standard fund appeared to be higher risk, according to the above three criteria, most holders of defined contribution pension plans will be more concerned about the income that their plan will generate than the lump sum. It is common for individuals to purchase annuities which are fixed in nominal terms or have fixed increases (rather than annuities which increase with prices or salaries). It was assumed that the accumulated funds under the various different investment strategies were used to purchase a level annuity. The following terms were used to determine the price of the annuity:

(i) The annuity was priced using the redemption yield from the BZW long gilt index, at the time of retirement.

(ii) Mortality was assumed to be: IM 80 C10 with a one year age deduction.

(iii) Expenses were assumed to be: 4.1% of the purchase price at outset plus 0.25% of each annuity payment.

The expected value of the annuity, relative to projected salary, was calculated for each of the investment strategies and two measures of risk were used:

(i) The variance of the annuity relative to salary.
(ii) The semi-variance of the annuity relative to salary, using a benchmark of 50% of salary.

The formula used to calculate the amount of the annuity is given in the appendix.

2.4 Funding to Purchase an Index-linked Annuity

The basic state pension in the UK is indexed to the price level. It is not common for annuities bought using the proceeds of a defined contribution pension fund to be indexed to prices. However, as private provision displaces state provision and as investors become more aware of the risks they face in retirement, index-linked annuities may come to be regarded as more appropriate in retirement. This brings up two issues. Firstly, the appropriate matching asset for a prospective pensioner who required an index-linked annuity would be index-linked bonds and not cash or conventional gilts. Secondly, equities, as real assets, may be a closer to match to an index-linked annuity than to a fixed nominal annuity. In addition to this, conventional fixed interest gilts may become a poorer match (i.e. riskier) because their yields and hence capital values will be affected by changes in inflation expectations. Such changes in inflation expectations will not affect real yields used to calculate index-linked annuity values. There may therefore no-longer be a positive correlation between fund values and annuity prices, if nominal yields from conventional gilts change.

The methodology was as follows. It was assumed that a fund of 300% of salary was built up ten years before retirement (age 55). The standard fund from section 2.2 was then used until three years before retirement. The fund accumulated at that point was then switched into one of the four funds shown below. Contributions were again 12% of salary from age 55 until retirement. Expenses were the same as in the investigation in section 2.2.

The four funds were:

(a) a diversified strategy of: 50% UK equities; 15% US equities; 15% index-linked gilts; 10% conventional gilts; 10% cash.

(b) 100% cash.

(c) 100% conventional gilts.

(d) 100% index-linked gilts.

In section 3.3 these funds will be described as Funds A, B, C and D. Fund A is a reasonably diversified strategy similar to that which is likely to be used by many defined contribution scheme contributors, although only one overseas equity market has been included and property has not been included.

The accumulated funds were then used to purchase an annuity, linked to the retail price index, using the same mortality and expense assumptions as used to purchase the fixed nominal annuity. The real yield used to calculate the annuity price was the real yield from a theoretical ten year zero coupon index-linked bond taken from the Bank of England Statistical Abstract (1997). The relevant formulae are given in the appendix.

The expected value of the starting value of the annuity, relative to projected salary, was calculated for each of the investment strategies and two measures of risk were used:

(i) The variance of the starting value of the annuity relative to salary.
(ii) The semi-variance of the starting value of the annuity relative to salary, using a benchmark of 50% of salary.

This investigation could only be carried out using data from 1982 to 1997 inclusive. Given that the reference point was three years before retirement, this involved only 13 years of retirees. For this reason, an additional investigation was carried out. Investment returns were simulated using the Wilkie (1995) stochastic investment model. Accumulated funds were calculated and used to purchase annuities (both fixed nominal and price index linked) at retirement. This enabled us to obtain a probability distribution for the value of the annuity, for different investment strategies, which were based on the assumptions underlying the model. This is presented in section 3.4.

3. Cash Funds and Annuity Values from Different Pre-retirement Investment Strategies

3.1 Cash Funds

3.1.1 Accumulated Funds from Different Investment Strategies

This paper mainly considers the investment strategy of an individual funding for an annuity rather than funding for cash. However it is of interest to consider the accumulation of a cash fund under different investment strategies in order to compare the results with those for an individual funding for an annuity. Some of the fallacious comments about the risk of different investment strategies may arise from the confusion of capital value risk (from which cash funds can protect the investor) and income risk (with which a prospective pensioner may be most concerned and which is not hedged using a cash fund).

Summary results for the accumulation of a cash fund under five different investment strategies are shown in the table below. The figures shown relate to proportions of salary accumulated in the fund at retirement.

Table 3.1.1.1

	Fund A	Fund B	Fund C	Fund D	Fund E
Expected accumulation	5.668	4.398	4.495	4.950	4.927
standard deviation of accumulation	2.013	1.697	1.331	1.745	1.967
square root of semi-variance of accumulation	1.220	1.759	1.515	1.455	1.567

Notable results are as follows:

1. As might be expected, the mean accumulated fund is highest with Fund A, which predominantly contained equities.
2. Fund B, which was switched into gilts over a ten year period, had the lowest mean fund.
3. Fund A had the highest standard deviation of fund value.
4. Fund C, which was switched into cash over a ten year period, had the lowest standard deviation of fund value. Fund B was therefore mean-variance inefficient compared with Fund C.
5. The highest semi-variance of fund value was Fund B.
6. The lowest semi-variance of fund value was Fund A.

Thus, if standard deviation is the appropriate measure of risk, the investor who funds for cash appears to have a straightforward choice between moving into cash as retirement approaches and remaining with the fund predominantly invested in equities. The decision would be determined according to the investor's attitude towards risk. A fund such as Fund D (moving the whole fund into cash three years before retirement) or Fund E (moving the whole fund into gilts three years before retirement) offer a risk and return profile between that offered by Funds A and C.

If semi-variance is regarded as the appropriate measure of risk, Fund A would appear to provide the lowest risk as well as the highest expected accumulated fund. Switching the fund into cash for the last three years (Fund D) provides the second lowest semi-variance.

Semi-variance has the advantage of only considering poor outcomes, which could be regarded as having particularly adverse consequences for the pensioner. Fund A has a higher standard deviation about a higher mean than the other funds; in other words the distribution is more spread out but lies further to the right. Standard deviation does not, necessarily, give a good indication of risk. If the target fund is 500% of salary at retirement, there is a probability of greater than 0.5 of not meeting that target for all funds apart from Fund A. This is the reason why semi-variance suggests that Fund A is a low risk fund.

3.1.2 A Consideration of Particularly Adverse Outcomes

Continuing the discussion of downside risk and looking at individual data points, one observation is of interest. Cash performed very poorly relative to inflation and salaries in the mid to late 1970s. *The lowest accumulated fund from any fund arises from an individual retiring at the end of 1977 using Fund D.* This individual would have suffered from poor equity returns in 1973 and 1974. Then, in 1975 to 1977, cash would have provided a cumulative return of 34.9% whilst average salaries increased by 59.3%.

This particular observation, in fact, illustrates two implicit assumptions underlying the hypothesis that a movement into cash is a low risk investment strategy close to retirement. The first is that markets are efficient and that nothing can be gained from active asset management. This would imply that an investor should not attempt to adapt his strategy when equity markets fall to levels such as those seen in 1974. The second is that cash is a low risk investment the returns from which are not susceptible to real economic shocks. These implicit assumptions may or may not be valid. However our examination of the empirical evidence suggests that they should not go unchallenged.

3.1.3 Summary

Notwithstanding the comments made about downside risk, when a fund is accumulated for cash, without the target of buying an annuity, the empirical data suggests that the investor can trade risk and return by choosing between investment strategies which are predominantly equity based and those investment strategies which are concentrated in cash to a greater or lesser extent or which move into gilts over a short period before retirement. A so-called "lifestyle plan" which involves investment in gilts over a ten year period appears to produce an inefficient strategy. The reason for this is likely to be the inflation risk to which the investor is exposed over a relatively long period of time. For example, over the period 1974 to 1982 inclusive, which was a period of relatively high inflation, an individual who retired would have had an accumulated fund of less than 300% of final salary in all except two years (1978 and 1981).

The fact that risk and return can be traded in the investment decision is not a surprising result; neither is the fact that the cash funds give rise to the lowest risk as measured by standard deviation. However, the picture looks a little more ambiguous if we consider the result of buying an annuity with the fund.

3.2 Annuity Values

3.2.1 Annuity Values from Different Investment Strategies

The following table shows the risk and return summary for a male investing in the five different funds, where the investor bought a fixed annuity at retirement, using the accumulated fund. All figures are shown as a proportion of salary at retirement. Semi-variance is calculated using a benchmark annuity of 50% of salary.

Table 3.2.1.1

	Fund A	Fund B	Fund C	Fund D	Fund E
Expected annuity	0.605	0.476	0.486	0.534	0.530
standard deviation of annuity	0.219	0.190	0.151	0.195	0.214
square root of semi-variance of annuity	0.129	0.195	0.173	0.163	0.171

Notable results are as follows:

1. Fund A, predominantly invested in equities, produced the highest expected annuity value.
2. Fund B, gradually switched into gilts over ten years, produced the lowest expected annuity value.
3. Fund A had the highest standard deviation of annuity value.
4. Fund C, gradually switched into cash over ten years, had the lowest standard deviation of annuity value.
5. The highest semi-variance arose from Fund B.
6. The lowest semi-variance arose from Fund A.

Despite some obvious similarities, there are some important differences between the results where an annuity is purchased and the results where the investor only considers the accumulation of a cash fund. The ratio of the standard deviation measures for Fund E, (investing in gilts from three years before retirement) and Fund D (investing in cash from three years before retirement) is smaller when we consider the annuity than when the accumulated funds were considered. The reason for this is that investing in gilts is a closer match to the income stream required from the annuity than it is for the investor accumulating a fund for cash at retirement. Also, Fund E now has the third lowest semi-variance; Fund C's semi-variance being higher than Fund E's when the annuity purchase is considered.

Further inspection of the data is necessary to understand the relative risks of cash and gilts as investment strategies close to retirement, for an investor expecting to buy an annuity. If we take the three years from 1991 to 1993 (inclusive), interest rates fell along the whole yield curve over the period. Annuity prices rose from £7.9557 per £1 of annuity (1st January 1991) to £10.4393 per £1 (1st January 1994). Fund D (invest in cash from three years before retirement) produced an annuity of 0.5692 (relative to salary) on 1st January 1994), reduced from 0.9373 on 1st January 1991. Fund E (invest in gilts three years before retirement) produced an annuity value of 0.7946 which was an increase from 0.7792. The issues here are subtle. Clearly cash has too short a duration to match an annuity fixed in nominal terms. An individual is exposed to the risk of falling interest rates. However, gilts appear to be a reasonable match for the annuity income stream required. The problem with gilts is that most gilt funds probably have a higher duration than the annuity value when an investor is very close to retirement: this may change with improving life expectancy. A period of rising yields in the last year before retirement could be damaging for an investor. For example, an investor using Fund D and retiring on 1st January 1995 would have had a pension of 0.6796 of salary (a fall of nearly 13% as compared with 1st January 1994). This fall is caused by the rise in interest rates over the year. The investor in Fund E would have seen his pension increase by nearly 9%.

If lifestyle plans are not to leave investors with hidden risks they will have to be sophisticated. In particular, a low risk strategy where the investor purchases a fixed nominal annuity would involve investing in gilts a few years from retirement. Then as the duration of the income the pensioner wishes to receive from the fund becomes shorter, the investor should switch a proportion into cash such that the duration of the mix of cash and gilts is approximately the same as the duration of the ultimate income stream required from the annuity.

The standard deviation from Fund A is a lower proportion of that from Fund C when an annuity is purchased than when the accumulated fund is considered. The standard deviation from Funds B and C are closer than when the accumulated fund is considered but Fund C (gradual movement into cash over ten years) still gives a lower standard deviation of annuity relative to salary than Fund B (gradual movement into gilts over ten years). Funds A and C seem to be closer together, from a risk perspective, when the annuities are considered than when the cash funds are considered because of the correlation between conventional gilt yields (which are inversely related to annuity prices) and equity yields (see section 4). It does not follow that, because cash is less sensitive than equities from the capital value point of view, it is therefore significantly less risky than equities when the investor requires an annuity.

3.2.2 A Consideration of Particularly Adverse Outcomes

The lowest semi-variance of annuity value (using 50% of salary as a benchmark) is found from Fund A. There could be two explanations for this. Firstly, it could be the case that the semi-variance benchmark has been set too high relative to the mean so that those funds with the lowest mean return fail to meet the benchmark too often. This would imply either that the contributor is willing to have a higher contribution rate than is indicated by our model or that he is willing to accept a lower mean fund or annuity value without adverse consequences arising. Secondly, it could be the case that Fund A avoids severely adverse outcomes, which other strategies, such as that used in Fund C, suffer. It is difficult to investigate the first hypothesis without proposing a specific utility function for the investor which is beyond the scope of this work. However, by inspection of particular outcomes, we can examine the second hypothesis.

There are ten years when investing in the cash fund close to retirement provides a pension of less than 40% of final salary. This compares with only six years where the predominantly equity fund is used. There are a number of years when the cash fund significantly under performs wages so that the value of the fund relative to average salary drops in the three years close to retirement. Indeed there are six years where an individual could have retired and found that the value of the cash fund dropped relative to average salary in *each* of the three years before retirement. There was only one year (1962) for which this was the case with an equity fund.

It is of particular interest to consider the relationship between equity returns and the yields on which annuities would be purchased. There are 17 years where equities produce negative returns relative to wages in the period 1955 to 1997; however, changes in gilt yields were such that annuity prices fell in all but one of these 17 years. Thus the correlation between gilt yields (and hence annuity rates) and equity yields (and hence equity values) is an important factor which should be taken into account when considering the risk of an investment strategy involving equities. Equities are interest rate sensitive and therefore some of the inherent risk in equity investment is offset by the fact that they produce a long-term income stream which provides some match (although not a perfect one) for the required annuity.

3.2.3 Further Consideration of the Downside

Variance of the annuity value can be an inappropriate measure of investment risk in some circumstances. In particular, if the equity fund produces a high variance of the annuity value but around a high expected annuity value, variance may not indicate effectively the probability of an adverse outcome. Semi-variance can be used but, in turn, it ignores any variability above the benchmark used to calculate semi-variance. In financial economics, a concept known as "stochastic dominance": see Hand and Jakka ed. (1998), is often used. First order stochastic dominance of investment strategy X by investment strategy Y requires that the cumulative probability distribution function of outcomes from investment strategy Y lies always to the right or underneath that of investment strategy X. This means that for any level of annuity, the probability of achieving less than that value of the annuity is greater for strategy X. It is generally accepted that all rational investors would prefer strategy Y in those circumstances.

We will look at the stochastic dominance characteristics of different pairs of investment strategies.

The following table shows the probability of achieving an annuity value less than various benchmarks up to 80% of salary. This does not allow complete consideration of the concept of stochastic dominance as only benchmark points and not the full distribution is illustrated.

Table 3.2.3.1

Cumulative frequency %	Fund A	Fund B	Fund C	Fund D	Fund E
< 0.2	0	2.3	0	2.3	0
< 0.3	4.7	11.6	11.6	9.3	9.3
< 0.4	14.0	44.2	23.3	27.9	32.6
< 0.5	44.2	67.4	65.1	48.8	53.5
< 0.6	51.2	69.8	81.4	69.8	65.1
< 0.7	69.8	83.7	88.4	83.7	79.1
< 0.8	81.4	95.3	97.7	86.0	90.7

Fund C (move into cash over ten years) is stochastically dominated by the predominantly equity fund (Fund A). For any proportion of salary, the investor has a higher probability of achieving a pension lower than that level for Fund C than for Fund A. In fact, Fund A stochastically dominates all the other investment strategies, which suggests that it is unambiguously the best strategy. Indeed, further inspection of the data demonstrates that there are only three years (1974, 1975 and 1977) in which Fund C would have produced a higher annuity than Fund A.

3.2.4 Summary

When the purchase of an annuity is considered, the issue of risk is less clear cut. A cash fund does not match the duration of the required annuity payments so that investors are susceptible to a rise in annuity rates not matched by a corresponding increase in returns from the fund. A fixed interest bond fund is likely to be too volatile for the pensioner who requires an annuity in about three years time. When we look at downside measures of risk, the equity fund seems to perform better than the other funds. There are a number of particularly adverse outcomes arising from investment in a cash fund, close to retirement, at times when salary growth was higher than cash interest rates and long-term interest rates fell.

3.2.5 Some Notes of Caution

An investigation using one data set does not necessarily prove that a predominantly equity fund is the lowest risk strategy. The following additional points should be considered:

1. Other strategies have not been included (for example a mixed portfolio of cash and bonds with the proportions determined so that the duration of the portfolio is the same as the duration of the annuity required at retirement). The authors are not aware of this degree of sophistication in the funds offered by the so-called lifestyle plans.
2. The period that has been considered has been a relatively successful period for equity returns. It includes a period when equities may have been "re-rated" against bonds because equities were regarded as having good inflation hedging

characteristics as compared with bonds. Nevertheless, the investigation does use forty six years of data.

3. The period under consideration was subject to real shocks in the cash market (i.e. cash under performed inflation dramatically due to economic conditions and institutional structures). This period is unlikely to repeat itself. However, once again, the authors would point to the long data series which has been used.
4. The equity fund suffers from not being as diversified as it could be. For example, index-linked gilts, property and overseas equities are excluded. Their exclusion is as a result of a lack of available data. US equities and index-linked gilts have been included in section 3.3.

The conclusion we would reach is that there is no *prima facie* case for using gilt or cash funds close to retirement if a pensioner wishes to purchase an annuity. Remaining in a fund which is predominantly equity based appears to give the best results from a risk and return point of view.

3.3 Price Index-Linked Annuities

3.3.1 Real Annuity Values from Different Investment Strategies

One of the possible causes of a change in conventional gilt yields close to retirement is a change in inflation expectations. If the investor is intending to buy an annuity which is fixed in nominal terms, a change in conventional gilt yields should be offset by a change in annuity rates. A matched portfolio of conventional gilts may therefore be an appropriate risk minimisation strategy (although, as has been stated, the duration of most managed portfolios of conventional gilts may be too great). However, if an index-linked annuity is required, an increase in inflation expectations close to retirement (which would reduce the value of a conventional gilt fund) will not be offset by a change in index-linked gilt yields which are used to calculate index-linked annuity prices. Indeed, conventional gilts may prove to be the most risky investment. If there is a change in inflation expectations but no change in real yields, index-linked annuity prices should be unchanged and the value of equity, index-linked and cash funds would also be unchanged, if they are unaffected, in real terms, by a change in nominal variables. On the other hand, equity yields and index-linked gilt yields may be quite closely correlated because both are real yields. This would mean that falls in equity values would be closely correlated with falls in annuity prices, reducing the risk of investing in equities close to retirement, relative to the risk of conventional gilts or cash. The relationship between equity yields, index-linked gilt yields and conventional gilt yields is investigated in Section 4.

There is limited data which can be used to investigate the hypothesis that conventional gilts are an inappropriate investment close to retirement for an investor who wishes to purchase an annuity which is linked to a price index. Index-linked gilts have only been available in the UK for 15 years. In the first few years they were relatively unmarketable. The available data were used to investigate the pension an individual would be able to purchase from four different investment strategies. The details of the four investment strategies used are given in Section 2.4 and formulae are given in the Appendix.

The following table shows the risk and return summary for a male investing in the four funds. The annuity value is calculated by taking the initial level of the annuity as a proportion of the salary at retirement.

Table 3.3.1.1

	Fund A	Fund B	Fund C	Fund D
Expected Annuity	0.563	0.501	0.529	0.469
standard deviation of annuity	0.074	0.075	0.080	0.070
semi-variance of annuity	0.011	0.045	0.025	0.069

The results are of interest but clearly require interpretation. Particularly notable features are:

1. The expected annuity is highest from the diversified fund, Fund A.
2. The fund invested in index-linked gilts, Fund D, produced the lowest expected annuity.
3. The lowest standard deviation of annuity was from Fund D.
4. The highest standard deviation of annuity was from Fund C, invested in gilts.
5. The lowest semi-variance of the annuity was from Fund A.
6. The highest semi-variance of annuity was from Fund D.

Notwithstanding certain comments which will be made below, the index-linked fund could be regarded, *a priori*, as being the lowest risk fund. It matches the (index-linked) payments that will be received from the annuity. It is not surprising that this fund has the lowest variance of real annuity value. It is also not surprising that it produces the lowest expected annuity.

The standard deviation of the annuity produced by Fund D may not be as small as would be expected *a priori.* This can be explained if we consider more closely the extent to which Fund D matches the ultimate annuity payments. The duration of the bonds in the index-linked gilt index is much higher than the duration of the annuity payments. The payments from an index-linked bond are weighted heavily towards their redemption time whereas the payments comprising the life annuity decline over time. It is worthwhile illustrating this by looking at a particular year. During 1994, real yields from index-linked gilts increased from 2.81% to 3.86%. The index-linked gilt fund produced a total return of -7.9% with the capital value falling by 12%(the total return was approximately -11% when allowing for wage inflation). The annuity rate declined to £14.00 per £ of annuity from £12.79 per £ of annuity. Thus the annuity cost fell by 8.6% whilst the capital value of the fund fell by about 12%. Thus, whilst index-linked gilts may be good matching assets, as far as their inherent characteristics are concerned, if an index-linked fund is used for investment close to retirement in a "lifestyle" plan, the fund should be managed so that the bonds in the fund are matched, by duration, to the annuity to be purchased at retirement. Alternatively, a mix of the cash and index-linked funds should be used so that the duration of the overall fund is equal to that of the required ultimate payments. The authors are not aware that this is generally the case.

The effect described above will affect the variance of the index-linked gilt fund. The semi-variance will be affected by that factor and two other factors. Firstly, the general drift upwards of index-linked gilt yields, during the investigation period, means that the above effect produces a number of low outcomes for the investor who uses the index-linked gilt fund. Also the mean annuity value is very close to the semi-variance benchmark with only four years producing an annuity with a starting value greater than 50% of salary at retirement.

The low semi-variance and relatively high mean fund value from Fund C is explained by the particularly strong performance of conventional gilts during the period under consideration. The short data period used for this investigation does not bring out fully the risks of conventional gilts. The period was one of continually declining inflation expectations and gilt yields. Using the simple measure of market expectations of inflation, derived from conventional and index-linked gilt yields (see Deacon and Derry, 1994), inflation expectations fell from 6.7% to 3.3% from 1983 to 1997.

When the results of this section are considered in conjunction with those in sections 3.2 and 3.4, we do not believe that there is convincing evidence to suggest that conventional gilts are a low risk investment. We would caution against the view that conventional gilts are appropriate in a low inflation environment but not in a high inflation environment. It is not the level of inflation which is important (as long as the expected future level of inflation is reflected in gilt yields) but any potential shocks to inflation. By their nature shocks are unforeseeable. It is legitimate to say that the institutional and monetary background is such that inflation is unlikely to increase significantly over a given three year period. However, lifestyle investment plans can be arranged several years in advance for funds to be switched close to retirement. This arrangement may not be reviewed before the switch automatically takes place. Index-linked gilts, rather than conventional gilts would better protect the investor from inflation shocks in these circumstances. In fact, even during the period examined, which was one of continually falling inflation expectations, the gilt fund had the highest variance of the real annuity value.

The diversified fund shows the highest mean annuity value and the second lowest standard deviation of annuity value. There appears not to be strong evidence for moving out of a diversified fund close to retirement. Two qualifications need to be made to this point. Firstly, the period chosen is one of strongly performing equity markets (although gilt markets also performed strongly and there were no "real shocks" in the cash market). On the other hand, risk could have been reduced further by diversifying into other overseas equity markets and into property.

Figures for the mean and variance of the accumulated fund (equivalent to section 3.1.) are not shown for this investigation. However, it is of interest to note that, when the accumulated funds are considered over this data period, cash (Fund C) appears to be the lowest risk investment. However, when the real annuities are considered cash is the second highest risk investment, according to the variance measure. This is compatible with *a priori* reasoning.

3.3.2 A Consideration of Particularly Adverse Outcomes

As has been mentioned, Fund D has the highest semi-variance. It is worth asking whether this is because of persistently mediocre returns or whether there were some very poor outcomes. In fact, the only two observations which produced initial pension levels below 40% of salary are in respect of Fund D, in 1986 and in 1988. It is worth looking at these outcomes in more detail and asking why the investment conditions at the time did not produce such adverse outcomes for the other funds.

In the period 1984 to 1986 index-linked gilts had a cumulative performance of 12.9% (nominal); wage inflation was 24.1%. The cause of the under performance of the index-linked gilt fund was a rise in real yields from 3.24% to 3.68%. Annuity rates did fall; but because of the low duration of the annuity, relative to that of the gilts in the index-linked gilt index, annuity prices fell by only 3.7%. Thus the index-linked gilt fund suffers from a low average return but, because of the mismatching by duration, also suffers from shocks such as changes in real investment yields. The diversified fund is protected from such shocks to individual investment markets. In only one year (1994) did all of the investments in the diversified fund produce a return of less than 10%. There were no years in which all the investments in the diversified fund under performed wage inflation. In contrast, the index-linked gilt fund under performed wage inflation in seven out of fourteen years.

3.3.3 Further Consideration of the Downside

It is worth considering the concept of stochastic dominance again. Table 3.3.3.1 shows the probability of the different funds producing annuity values less than particular benchmark levels.

Table 3.3.3.1

Cumulative frequency %	Fund A	Fund B	Fund C	Fund D
< 0.2	0	0	0	0
< 0.3	0	0	0	0
< 0.4	0	0	0	15
< 0.5	15	54	54	69
< 0.6	69	92	85	100
< 0.7	92	100	92	100
< 0.8	100	100	100	100

Once again, Fund A appears to stochastically dominate all other funds, suggesting that it is unambiguously the best strategy. If we compare individual outcomes for Fund A with those for Fund D (the lowest risk fund by the standard deviation measure) we find that there were no occasions when Fund D provided a higher annuity than Fund A.

3.3.4 Summary

In conclusion, it can be said that there is no strong evidence to suggest that a "lifestyle" investment policy is desirable and that a reasonably diversified fund is not appropriate until retirement. Conventional gilts suffer from inflation risk, even though it was not evident in this data period. Cash suffers from not being matched by duration to the annuity which is to be purchased. Index-linked gilts suffer from a lower mean return and the problem that index-linked gilt funds are not necessarily managed to match the term of the annuity which is to be purchased. Any lifestyle plan which is designed to minimise risk should ensure that the term of bonds in an index-linked bond fund is appropriate or that index-linked bond funds are mixed with cash funds in the appropriate proportions.

3.4 A Simulation Approach

3.4.1 Simulations Using the Wilkie Asset Model

The investigation using historical investment market data is of interest because it indicates the actual distribution of outcomes an investor would have achieved if different investment strategies had been used in actual investment conditions. However, there are weaknesses of this approach. In particular, lifestyle investment strategies are recommended to prospective pensioners at the current time. The fact that they were not suitable at a particular time in history (for example when cash rates were significantly below the rate of inflation) does not mean that they are not suitable at the present time. The Wilkie stochastic investment model is fitted using historical data but judgement is used in the development process so that short term features of the data would not necessarily affect the parameters of the model or the chosen model structure: see Wilkie (1995). The model was also fitted using the longest possible data period for each asset class; this contrasts with the investigation using empirical data where the data set for all asset classes is restricted to the period relating to the shortest data period available for a particular asset class. Economic judgement is also used to impose certain constraints on the Wilkie model. These constraints are not necessarily features which are clear from the data but *a priori* reasoning suggests that they are reasonable. For example, any increase in the price level is automatically fed through into an increase in dividends, albeit with a considerable time lag.

The fund built-up at the reference point, the contribution rate and the investment strategies chosen immediately before retirement (Funds A to D) were exactly the same as those used for the investigations in section 3.3, as described in section 2.4. There are two differences in detail. Consols (UK gilts which are effectively perpetuities) were used instead of a mixed fund of gilts and the returns calculated from the index-linked fund are equivalent to calculating the returns from a theoretical index-linked perpetuity. This reflects the structure of the Wilkie model from which the returns were simulated. One thousand simulations were carried out to obtain a probability distribution of the accumulated cash fund, the nominal annuity and the real annuity from each investment strategy. Details of the Wilkie model are given in Wilkie (1995). The formulae used to calculate total returns from the different asset classes are given in the Appendix.

3.4.2 Cash Funds from Different Investment Strategies

Table 3.4.2.1 shows the expected accumulated fund and standard deviation of accumulated fund from different investment strategies.

Table 3.4.2.1

	Fund A	Fund B	Fund C	Fund D
Expected cash fund	5.922	5.282	5.539	5.617
standard deviation of cash fund	1.205	1.248	1.556	1.263

The diversified but predominantly equity fund, Fund A, produces the highest expected fund and the lowest standard deviation of fund. This result provides further evidence for the argument against changing investment strategy close to retirement. Although the figures are not shown here, the authors found that the diversified fund stochastically dominated all the other funds.

The highest standard deviation arises from Fund C, predominantly invested in consols. This is not surprising given the structure of the model. Model users often reduce the standard deviation on the consols model so that it more closely reflects what common-sense would suggest is reasonable. In our investigation, we used a standard deviation of half that proposed by Wilkie (1995). Nevertheless, the consols fund still produces a higher standard deviation of return than either index-linked gilts, cash fund or the diversified fund.

It may be surprising that the cash fund does not give the lowest standard deviation of fund value. However, it should be noted that the main determinant of the standard deviation is not the variability of returns but the variability of returns relative to average salary increases. We have seen from sections 3.1 to 3.3 that cash is far from immune from real economic shocks when its return is compared with average salary inflation.

3.4.3 Annuity Values from Different Investment Strategies

The results for an investor who purchases an annuity at retirement are shown in the table below. The annuity was assumed to be purchased using the long-term gilt rate, at the time of retirement, derived from each specific simulation

Table 3.4.3.1

	Fund A	Fund B	Fund C	Fund D
Expected annuity value	0.586	0.522	0.545	0.558
standard deviation of annuity value	0.094	0.101	0.123	0.113

There is little that needs to be said about this table. The ordering of the funds by expected annuity and standard deviation of annuity are the same as when the accumulated fund is considered. One notable observation is that the ratios of the standard deviations from Fund A relative to Fund B (cash) and that of Fund C (gilts) relative to Fund B are lower when annuity values are considered than when the cash funds are considered. This is consistent with prior reasoning. Fund A and Fund C give "protection" from fluctuating annuity rates because of the relationship between long gilt yields, equity yields and annuity prices. However, the duration of the gilts modelled in the Wilkie model (consols are effectively perpetuities) is greater than the duration of the annuity purchased at retirement so that, in fact, Fund C still provides the highest standard deviation of annuity.

It should be noted that Fund A, once again, stochastically dominated all other investment strategies.

3.4.4 Index-linked Annuity Values from Different Investment Strategies

In this part of the analysis, the accumulated fund was used to buy an annuity linked to the retail price index. The real yield used to determine the price of the annuity was the real yield from index-linked stocks, at the time of retirement, from each specific simulation.

Table 3.4.4.1

	Fund A	Fund B	Fund C	Fund D
Expected index-linked annuity value	0.445	0.397	0.416	0.421
standard deviation of index-linked annuity value	0.090	0.094	0.117	0.091

Once again, the ordering of the expected index-linked annuity values are the same. Fund A still provides the highest expected annuity value and the lowest standard deviation. The most notable additional observations arise from the relationship between the standard deviation of annuity value arising from the cash fund with that arising from the index-linked gilt fund. The index-linked gilt fund now provides the second lowest standard deviation. A change in the relative level of the standard deviation of these two funds is consistent with prior reasoning. Any fall (rise) in index-linked gilt yields close to retirement will be reflected in a lower (higher) index-linked annuity at retirement, if the cash fund is used. If the index-linked gilt fund is used, any change in real interest rates will be offset by a change in annuity rates. However, the duration of the index-linked gilt fund will not match the duration of the pension. This will lead to residual variation in the index-linked annuity when the index-linked fund is used. This highlights two issues. Firstly, it is important to determine the criteria an investor uses to define risk when determining the appropriate investment policy: a cash fund could be regarded as risky if an index-linked annuity is required because the cash fund is mismatched by term and investment type. Secondly, if the investor wishes to minimise risk, it is important to ensure that the appropriate fund, which matches the required index-linked pension, is matched by duration to the pension. The index-linked gilt fund implied by the Wilkie model does not match the pension by duration as the method for calculating total returns described in Wilkie (1995) is equivalent to calculating the total return on a hypothetical index-linked perpetuity. If it is believed that lifestyle investment strategies should be used to decrease risk, they should ensure that matching by duration takes place or that the investment policy is split appropriately between cash and bonds to manage the duration.

4 The Relationship between Equity Yields, Conventional Gilt Yields and Index-Linked Gilt Yields

4.1 The Importance of Matching

We have found that it is by no means clear that an investment strategy using funds containing a mixture of investments but predominantly equities are more risky than funds containing cash, when an investor is close to retirement. We have also found that the relative risk of the predominantly equity fund seems to decrease when an annuity is purchased and decrease further when an index-linked annuity is purchased. Furthermore, the relative risk of index-linked gilts, as compared with cash or conventional gilts, seems to decrease when an index-linked annuity is purchased.

These results can be rationalised in terms of traditional actuarial principles of cash flow matching. The annuity (conventional or index-linked) should be regarded as the investor's liability. The value (or price) of the annuity has a particular sensitivity to interest rates. Cash is a poor match for either a conventional or index-linked annuity. Conventional gilts may be a good match for a conventional annuity but most conventional gilt funds will have a duration that is too long to match the payment from an annuity. The index-linked annuity is best matched by index-linked gilts but, again, most index-linked gilt funds have a duration which is too long to match the payments from an index-linked annuity.

We have seen that, empirically, equity funds may have a relatively low risk when they are used to match an annuity. This low level of risk may be partly attributable to the fact that equities derive their value from a future income stream and that value is itself therefore interest rate sensitive. Because equities are real assets, we would expect this phenomenon to be most apparent when the relationship between index-linked gilt yields and equity values is considered. In this section we look at the extent to which changes in equity yields (and therefore equity values) can be explained by changes in index-linked gilt yields (which would give rise to changes in annuity prices which may partially offset the effect on equity values). This should give theoretical underpinning to our empirical observations.

4.2 Empirical Relationships

Figure 4.2.1 shows the evolution of conventional gilt, index-linked gilt and equity yields over time since UK index-linked gilts were first issued in 1981. From inspecting this plot a number of issues are clear. There is a tendency for yields to move together. There are occasions (for example late 1993) when all three series produce local troughs. However, the index-linked gilt and equity yields series do not move closely until 1986 and appear to have moved very closely since 1991. This might suggest that the conclusions we have reached about the ability of an equity fund value to fluctuate in a way which offsets changes in real annuity yields could have been even stronger had we just used data taken from the later period. It is possible that index-linked gilt and equity yields have been more closely correlated since 1986 because index-linked gilts have gradually become more marketable since they were first issued.

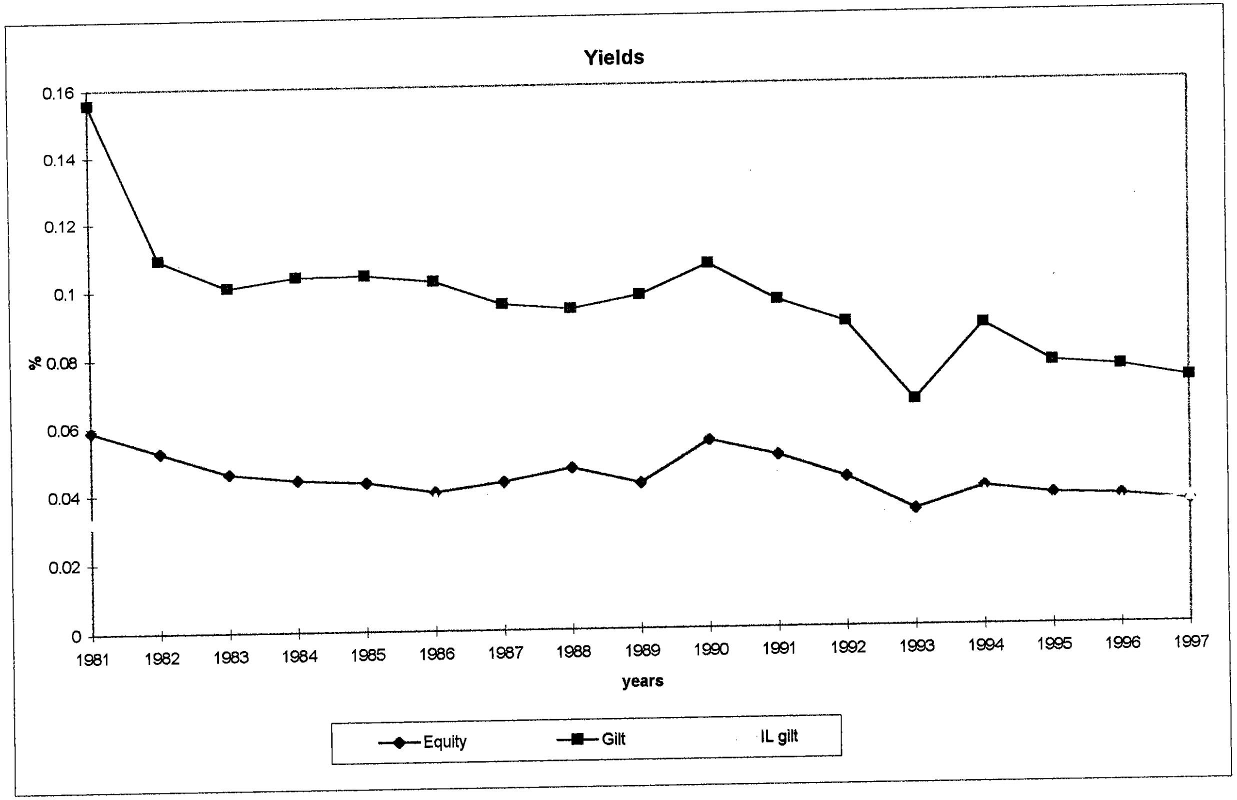
Yields
%
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997
years
Equity
Gilt
IL gilt

Figure 4.2-1

4.3 Correlation Analysis

An analysis of the correlation between index-linked gilt, conventional gilt and equity yields was undertaken using annual and quarterly data over different time periods. The trends in the quarterly data appear the same as the trends in the annual data so only the former will be discussed. The following tables illustrate the relationships:

Table 4.3.1 Correlation between Yields, 1981-1997

1981 - 1997	**Equity yields**	**Conventional gilt yields**
Conventional gilt yields	0.8361	
Index-linked gilt yields	-0.1111	-0.2278

Table 4.3.2 Correlation between Yields, 1985-1997

1985 - 1997	**Equity yields**	**Conventional gilt yields**
Conventional gilt yields	0.6868	
Index-linked gilt yields	0.6343	0.5783

Table 4.3.3 Correlation between Yields, 1990-1997

1990 - 1997	**Equity yields**	**Conventional gilt yields**
Conventional gilt yields	0.9094	
Index-linked gilt yields	0.8559	0.8188

Within tables 4.3.2 and 4.3.3, the correlation coefficients between the different investments are not significantly different from each other. Looking at the whole period, in table 4.3.1, we see that equity and conventional gilt yields are strongly correlated but index-linked gilt yields and equity yields and conventional gilt yields and index-linked gilt yields are not.

There has clearly been a significant change in the relationship between index-linked gilt yields and equity yields. Soon after issue, index-linked gilt yields and equity yields were uncorrelated. Indeed, figure 4.2.1 gives the impression of negative correlation in the early years and table 4.3.1 confirms this. However, as index-linked gilts became more marketable, it would appear that index-linked gilt and equity yields became strongly correlated. This gives credence to the conclusion in section 3 that a predominantly equity-based diversified fund may provide an investor with reasonable protection close to retirement. Equity capital values and index-linked annuity prices should move inversely.

It is, of course, not possible to repeat the analysis of section 3.4 only using data from 1990. Results based on only seven years would be meaningless. However, it is of interest to note that, if the relationship that has developed in recent years between index-linked gilt yields and equity yields continues, it is likely that equities will appear a relatively lower risk investments than our study has indicated. The relationship between equity capital values and the yields which underlie index-linked annuity prices appears to have strengthened.

Brief comments will be made about the relationships between the other yield series. Conventional gilt yields and index-linked gilt yields and conventional gilt yields and equity yields appear to be strongly correlated from 1990. Equity yields and conventional gilt yields were also strongly correlated before this period. The whole period from 1981 has been one of falling inflationary expectations and a relatively stable economic environment. It is therefore not surprising that investors have seen changes in nominal yields from conventional gilts to be associated with changes in the real component rather than in the inflation component. This does not mean that conventional gilts are intrinsically appropriate investments to match real liabilities such as index-linked annuities.

5 Conclusion

It is tempting to think that an appropriate investment policy for a defined contribution pension scheme involves investing predominantly in equity funds until the prospective pensioner is close to retirement and then, in the last few years, gradually changing the investment strategy to one based on investments such as cash, conventional gilts or index-linked gilts. Many so-called "lifestyle" investment plans use such an approach. This work has investigated this hypothesis using two data sets and a simulation model and found that there is no evidence to support it. From a theoretical and practical point of view, the analysis which leads to the acceptance of that hypothesis is incomplete. A number of points are not considered. In particular, the following:

1. Cash itself is not immune from economic shocks, when its performance is compared with salary growth. Some of the lowest annuities arise when a cash fund was used close to retirement at a time when cash under performed salary growth significantly.
2. Cash is not a matching asset if the investor requires a pension. The investor will be susceptible to increases in annuity prices which will not be compensated by a rise in the fund value. This is true for both index-linked and fixed nominal pensions.
3. Conventional gilts are not matching assets if an investor wants an index-linked pension. If inflation expectations rise and long-term nominal interest rates rise there would be a fall in the value of the conventional gilt fund close to retirement

which would not be compensated by a change in the rates for index-linked annuities.

4. If investment policy is to be changed close to retirement to minimise risk, it is important that the duration of the investment funds match the duration of the required annuity, with index-linked bonds being used if an index-linked annuity is to be purchased and conventional bonds being used if a fixed nominal annuity is to be purchased. However, it would be difficult to achieve this on an individual basis. One solution for the individual would be to develop a lifestyle investment strategy which invested particular proportions in a cash fund and either an index-linked fund or a conventional gilt fund (depending on the form of annuity that is required). The proportions in cash and gilts should then be adjusted to ensure that the duration of the overall fund was approximately matched to the duration of ultimate expected cash flows from the annuity at the various ages. This would involve increasing the cash proportion (at the expense of gilts) close to retirement. This may give a low risk strategy but would also have a lower expected return than a predominantly equity fund. Whether this strategy was appropriate would depend on the risk/return preferences of the investor. This could be an area for further research.

Two things are clear from this analysis. Firstly, the higher expected returns from equities should be taken into account when looking at the risk of under performing a particular benchmark in terms of the pension required. Largely because of the higher expected return of equities a diversified but predominantly equity-based fund appears to stochastically dominate a cash, index-linked gilt or conventional gilt fund in all five different investigations, using two different data sets. Secondly, the relationship between equity yields and yields from the asset which is appropriate for matching the ultimate pension that is required should be taken into account when determining the risk of the investment strategy.

It has been assumed throughout this analysis that annuity rates adjust quickly to changes in conventional or index-linked gilt yields. This is a useful working assumption because it separates the investment risk issues from other issues such as lack of competition in the annuity market. The responsiveness of annuity prices to changes in investment yields is worthy of separate investigation.

It should also be noted that the authors do not advocate an equity-based investment strategy up to retirement at all times. The conclusion is a weaker one that there is no evidence to suggest that a gilts-based strategy is necessarily less risky. The quity market could fall for reasons not connected with real investment yields at any time. However, we would contend that the arguments used to justify a gilts-based investment strategy close to retirement ignore the relationship between equity and index-linked gilt yields and ignore the fact that higher expected returns can give the investor a lower probability of achieving an adverse result.

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Appendix

Accumulation of the Fund

The formula for the accumulation of the fund which is used to provide a cash sum (section 2.2), to purchase a fixed nominal annuity (section 2.3) or to purchase an index-linked annuity (section 2.4), per unit of salary is:

$$F_t = [F_{t-1} + 0.12*e(t)/e(1)]*[1 + w_{1(t)}*equity(t) + w_{2(t)}*cash(t) + w_{3(t)}*gilts(t) + w_{4(t)}*index\text{-}linked\ gilts + w_{5(t)}*U.S.\ Equities]*0.99$$

where

F_t = fund at end of year t
F_{t-1} = fund at end of year $t-1$
$e(t)$ = index of earnings at beginning of year t.
$w_{1(t)}$ = proportion of fund invested in equities at the beginning of year t
$w_{2(t)}$ = proportion invested in cash at the beginning of year t
$w_{3(t)}$ = proportion invested in gilts at beginning of year t.
$w_{4(t)}$ = proportion invested in index-linked gilts at the beginning of year t.
$w_{5(t)}$ = proportion invested in U.S. equities at the beginning of year t.
(N.B. throughout the investigations in section 2.2, $w_{4(t)}$ and $w_{5(t)}$ would be zero).
$F_0 = 3.0*S$

equity(t), cash(t), gilts(t), index-linked gilts(t) and *U.S. equities(t)* are the returns from equities, cash, gilts, index-linked gilts and U.S. equities respectively during year t. The returns from all asset classes except U.S. equities are taken from published sources (see main body of the text). The returns from U.S. equities are calculated as follows:

$$(1 + i_t) = ER(t - 1)/ ER(t)[(P(t)/P(t - 1)) + Y(t)]$$

where:

i_t = return from US equities in the year to 31st December
$ER(t)$ = exchange rate on 31st December of year t
$P(t)$ = *S&P 500 Composite Price Index* on 31st December of year t
$Y(t)$ = Dividend yield on *S&P 500 Composite Price Index* on 31st December of year t

The formula is an approximate formula and may understate returns slightly.

The final fund value at retirement, F_T, would then be expressed per unit salary at outset. It is then multiplied by $e(1)/e(T)$, where T is the time of retirement so that it is expressed in terms of salary at retirement.

Purchase of the Fixed Nominal Annuity

The level of the annuity for somebody retiring in a given year is:

$$AnnR(year) = [0.959*F_T*e(1)/e(T)]/(p.v.\ of\ a\ life\ annuity\ of\ 1\ per\ annum\ (year))$$

$$\text{p.v. of a life annuity of 1 per annum (year)} = \text{sum of } {}_tp_x v^t(1 + 0.0025*1.04^t)$$

The present value of the annuity at a given retirement time is determined by the level of long-term conventional gilt yields at the time of retirement. The value of the annuity is automatically expressed as a proportion of salary at retirement. Mortality is assumed not to vary with the time the annuity is taken out (in order that the investment risk effect is isolated from any change in mortality). The annuity is assumed to be whole life and payable annually in advance. Expenses are assumed to be 4.1% of the purchase price and, initially, 0.25% of the annuity amount, rising by 4% per annum.

Purchase of the Index-Linked Annuity

The index-linked annuity is level in real terms. The formula is therefore identical to that used for the present value of the fixed nominal annuity except for the following differences in detail:

(a) The annuity is fixed in real terms rather than in nominal terms (this does not change the formula, it merely changes the currency units of the amounts being discounted).
(b) Real yields from index-linked gilts are used for discounting the real payments: specifically, the real ten year zero coupon yields are used, as published in the Bank of England Statistical Abstract, 1997.
(c) Expenses are assumed to increase in line with prices (and are therefore constant in real terms); this is in place of assuming 4% escalation.

The level of index-linked annuity for somebody retiring in a given year is:

$$AnnR(year) = [0.959*F_T*e(1)/e(T)]/(p.v.\ of\ a\ life\ annuity\ of\ 1\ per\ annum\ (year))$$

$$\text{p.v. of a life annuity of 1 per annum (year)} = \text{sum of } {}_tp_x v^t(1 + 0.0025)$$

Application of the Wilkie Stochastic Investment Model

The Wilkie (1995) model was used. In particular the series were required for UK price inflation, UK wage inflation, UK equity returns, cash returns, long gilt (consol) returns, index-linked gilt returns (in fact the returns from a hypothetical index-linked perpetuity are generated from the Wilkie model) and US equity returns. The model is described in detail below, with particular features relevant to this investigation described in greater detail. If a more detailed description or critique is required, the reader should refer to Wilkie (1995) and the discussion contained therein.

The model for the RPI index $Q(t)$ at time t is:

$$Q(t) = Q(t\text{-}1)exp(I(t))$$

where $I(t)$ is the force of price inflation over the year $(t\text{-}1,t)$ and is modelled as:

$$I(t) = QMU + QA(I(t\text{-}1)\text{-}QMU) + QSD.QZ(t)$$

where $QZ(t)$ is iid $N(0,1)$

The parameter values are:

QMU = 0.0473 (equivalent to a mean rate of inflation of 4.73%)
QA = 0.5773
QSD = 0.0427

The Model for the Wage Index, $W(t)$, at time t is:

$$W(t) = W(t\text{-}1)exp(J(t))$$

where $J(t)$ is the wage inflation over the year $(t,t\text{-}1)$ and is modelled as:

$$J(t) = WW1.I(t) + WW2.I(t\text{-}1) + WMU + WSD.WZ(t)$$

where $WZ(t)$ are i.i.d. $N(0,1)$.

The parameter values used are:

$WW1$ = 0.6021
$WW2$ = 0.2671
WMU = 0.0214
WSD = 0.0233

This can be interpreted as wage inflation depending upon price inflation during the current and previous years; a constant term which represents real wage inflation and a random term.

This model is described as "Model (iii)" in Wilkie (1995). It does not have "unit gain" from wages to prices. Average wage inflation is:

$$(WW1 + WW2).QMU + WMU = 6.25\%$$

This represents real wage inflation of 1.52%. This ignores any possibility of scale increases. It is felt by the authors that this is reasonable given that the work concentrates on investment policy in the time leading up to retirement which for individuals around average income is likely to be a period during which scale increases are limited. This model has higher real wage increases in times of low inflation. This might imply a degree of money illusion; on the other hand, it may be reasonable to assume that, although increases in the price level caused by a monetary shock may leave real wages unaffected, an increase in the price level caused by a supply shock or "real" shock may lead to real wages falling. This does not contradict monetary theories of inflation which suggest that real shocks can give rise to once and for all changes in the *price level* (which would reduce real wages) but not lead to sustained *inflation.* The details of the model are, of course, open to debate (see Wilkie, 1995).

The model for the value of the UK equity total return index $PR(t)$ at time t is:

$$PR(t) = PR(t-1).(P(t) + D(t))/P(t-1)$$

where

$P(t)$ is the value of the price index at time t; $P(t) = D(t)/Y(t)$
$Y(t)$ is the dividend yield on ordinary shares at time t and
$D(t)$ is the dividend index at time t

The share price index is found by dividing the dividend index by the yield index. The total return accumulation factor, $PR(t)/PR(t-1)$ is found by adding the price index to the dividend yield and dividing by the price index at time $(t-1)$.

$Y(t)$ is modelled as:

$$Y(t) = exp(YW.I(t) + YN(t))$$

where

$$YN(t) = lnYMU + YA(YN(t-1) - lnYMU) + YE(t)$$
and
$$YE(t) = YSD.YZ(t)$$

where

$YZ(t)$ are iid $N(0,1)$

The parameters are:

$YW = 1.7940$
$YA = 0.5492$
$YMU = 0.0377$
$YSD = 0.1552$

Thus share dividend yields depend on their mean, inflation and the deviation of the past level of yields from the mean and a random term.

and $D(t)$ is modelled as:

$$D(t) = D(t-1).exp\{DW.DM(t) + (1-DW).I(t) + DMU + DY.YE(t-1) + DB.DE(t-1) + DE(t)\}$$

where

$DM(t) = DD.I(t) + (1\text{-}DD).DM(t\text{-}1)$
$DE(t) = DSD.DZ(t)$ where $DZ(t)$ are iid $N(0,1)$.

The parameters are:

$DW = 0.5793$
$DD = 0.1344$
$DMU = 0.0157$
$DY = -0.1761$
$DB = 0.5733$
$DSD = 0.0671$

The dividend index depends on inflation and previous values of inflation; a mean level of growth; a random term; the previous error term from the dividend index model and the previous error term from the dividend yield model. Two important aspects are the mean level of dividend growth which is estimated at 1.57% and the fact that a 1% increase in the price level completely feeds through to dividend growth, with a lag.

The nominal return from index-linked gilts at time t, $RR(t)$ is found from:

$$RR(t) = RR(t\text{-}1).\{1/R(t) + 1\}.R(t\text{-}1).\{Q(t)/Q(t\text{-}1)\}$$

where $R(t)$ is the real yield, modelled as:

$$R(t) = RMU.exp(RA.(lnR(t\text{-}1) - lnRMU) + RSD.RZ(t)$$

where

$RZ(t)$ are iid $N(0,1)$ and
$RMU = 0.0386$
$RA = 0.4936$
$RSD = 0.0731$

The real yield is modelled as an auto regressive process. An important aspect of the return calculation is that that the real yield is used in such a way that the real yield from a theoretical perpetuity is calculated first and the price index is then applied to obtain a nominal yield. *RMU*, the mean real yield is 3.86%.

The yield, $C(t)$ from conventional gilts is modelled as follows:

$$C(t) = CW.CM(t) + CMU.exp(CN(t))$$

where

$$CM(t) = CD.I(t) + (1 - CD).CM(t\text{-}1)$$
$$CN(t) = CA1.CN(t\text{-}1) + CY.YE(t) + CSD.CZ(t)$$

and

$CZ(t)$ are *iid* $N(0,1)$
$CW = 1$
$CD = 0.045$
$CMU = 0.0305$
$CA1 = 0.9$
$CY = 0.34$
$CSD = 0.185$

The nominal return at time t, $CR(t)$, is:

$$CR(t) = CR(t\text{-}1).(1/C(t) + 1).C(t\text{-}1).$$

This model provides a mean real return from gilts of 3.05%. The nominal return also depends on lagged values of inflation with unit gain, the previous year's residual and the residual from the dividend yield model. The variance of the model used has been reduced from that value suggested in Wilkie (1995); the higher variance does not appear to give results which have intuitive justification. The return from gilts is calculated as the return from a hypothetical perpetuity.

Cash is regarded as a one year bond and the return from cash, $B(t)$ is modelled as:

$$B(t) = C(t).exp\{-BD(t)\}$$

where

$$BD(t) = BMU + BA.\{BD(t\text{-}1) - BMU\} + BSD.BZ(t)$$

where

$BMU = 0.23$
$BA = 0.74$
$BSD = 0.18$

Thus cash yields are modelled as a function of consols yields. They are always below consols yields; the extent to which they are below depends on a constant term, an auto-regressive process and a random error.

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