The Application of Financial Theory to the Pricing of Upward Only Rent Reviews

by

P M Booth and D Walsh

Actuarial Research Paper No. 117

Department of Actuarial Science and Statistics
City University
London

November 1998

ISBN 1 901615 35 9

The Application of Financial Theory to the Pricing of Upward Only Rent Reviews

Philip Booth BA FIA FSS and Duncan Walsh PhD

Acknowledgements: The authors would like to thank the Institute of Actuaries' Research Committee for funding the work; additionally, we would like to thank the actuaries and property practitioner members of the Institute and Faculty of Actuaries' Property Investment sub-committee.

Keywords: property investment appraisal; option pricing theory; discounted cash flow

Abstract

Traditional and DCF methods of property appraisal do not deal adequately with the option nature of upward only rent reviews. DCF techniques typically value the higher of the expected market level of rents after future rent reviews and the current rent. However, the expected level of income after a review is higher than either of these quantities because, assuming no voids, the distribution of rents is truncated at the current rent until the lease comes to an end. This feature exhibits the same financial characteristics as a fixed income security with a call option to exchange the fixed income for the market rent if that is higher. The authors develop adjusted DCF methods for pricing this option and then develop an option pricing approach. The option pricing approach is an improvement on those previously published in the literature because it does not need to make the unrealistic assumption of perfect hedging. The DCF approach requires the input of a risk discount rate and the option pricing approach requires a parameter which has to be estimated from other investment market data.

1. Introduction

The traditional property lease in the UK involves an upward only rent review clause. This clause is an embedded option. When the rent is reviewed the holder of the freehold has the option of continuing to receive the higher of the market rent and the rent receivable at the time of the review. The value of this option depends on the probability that nominal rents will decrease. This depends critically on the expected growth rate of nominal rents and the variance of nominal rents. The former may depend on the rate of inflation if it is accepted that nominal rental growth and inflation are related.

Traditional approaches to property appraisal involve capitalising the income stream at an "all-risks yield". It is assumed that the income stream is constant although implicit allowance for future rental growth is taken into account by adjusting the all risks yield. These approaches are discussed, for example, in Baum and Crosby (1988). Discounted cash flow (DCF) approaches are also often used. These involve projecting cash flows explicitly and discounting at an appropriate rate of interest: see, for example, Adams, Booth and Venmore-Rowland (1993).

Neither of these approaches explicitly values the embedded upward only rent review option. It is possible that the methods have evolved to take account of it implicitly. This would involve lowering the all risks yield or lowering the discount rate in the traditional and discounted cash flow approaches respectively. Such methods, however, will be opaque with respect to the value of the option. If financial conditions change, for example if we move to a lower inflation environment, the value of the option will change but neither the traditional nor the DCF methods are sufficiently flexible to take account of such changes.

A literature has begun to develop which takes an option pricing approach to property appraisal. This was led by Ward and French, with the approaches being published in Ward and French (1997). This paper provides a critique of those approaches and develops, from first principles, an option pricing approach to valuing upward only rent reviews based upon more realistic assumptions. To do this, it is also necessary to develop a new framework of notation for appraisal using the DCF approach. The valuation issues are slightly different for property which is over rented: here the option has effectively already been enacted. The over-rented property literature will therefore also be considered. This paper only develops techniques for pricing property with one review. The approach to valuation when there are several reviews is the subject of further work. From a theoretical perspective, this latter situation is significantly more difficult than were just one review exists because the probability of the options being effected at each review will not be serially independent.

The format of the paper is as follows: in Section 2 we discuss contemporary property and DCF approaches to appraisal. In Section 3 we discuss the literature which recognised the option nature of the upward only rent review in the property domain. In Section 4 we develop a new notational framework and express DCF approaches using that framework. The errors from ignoring the embedded option are clear when this new framework is used. A corrected DCF method is then developed using the new framework. Section 5 develops an approach to pricing the option which uses derivative theory. Numerical examples are developed in Appendix One. Appendices Two, Three and Four look at related embedded options discussed in the financial literature.

2 Property Appraisal

2.1 Option Characteristics of Upward Only Rent Reviews

If market rents increase between reviews, the rent receivable after the next review will increase to reflect this. If market rents fall, the investor can continue to receive the current passing rent. This is an embedded option which has the typical option

characteristic of a non-linear pay off. If we consider the probability distribution of rents after the next review, there is a smooth distribution of probabilities above the current rent and then a significant probability that the rent will not change. Standard measures of risk, especially variance, do not capture the key features of this type of distribution well.

The rental income can be thought of as a combination of income from rents subject to either-way reviews together with options allowing the holder's income not to fall. The first part is more straightforward to value using discounted cash flow techniques because the distribution of possible cash flows will be continuous. This makes it easier to compare risk adjusted discount rates with those used for cash flows from other investments. Derivative mathematics can then be used to value the option component which, in this case, would be a "put" option. However, because either-way rental income is not a traded asset, an extra parameter needs to be introduced in order to obtain a value for the option. This is problematic but, because this parameter can be related to risk, it is not more difficult to choose a value for the parameter than it is to choose risk discount rates in general.

Another way of viewing rental income is as a combination of a level income (at the current rent) and an option to exchange this for the market rent at a future review date. This type of option is a "call" option. The first part of this package would just be valued as a bond. The second part would require derivative pricing techniques.

2.2 Over-rented Property

2.2.1 Appraisal Formulae

Over-rented property arises as a consequence of the upward only rent review clause being "activated". An option still exists on a property which is already over rented as, at the next review, the landlord can collect the higher of the passing rent and the market rent. Crosby et al (1997) discussed various approaches to the valuation of over-rented investment properties. This contribution is important because they develop models in the context of contemporary property theory but also incorporating historical property market conventions. The formula for using the "modified DCF" approach is as follows:

$$V_0 = f\, R_0\, a_{\overline{m+cn}|\,i} \;\; + R_0 \left(\frac{1+g}{1+i}\right)^{m+cn} a_{\overline{\infty}|\,y}$$

The formula has been changed slightly to reflect the notation used in this paper, which we define as follows:

V_0 is the value of the property

R_0 is the annual market rent

f^*R_0 is the annual passing rent (if $f > 1$, the property is over rented)

m is the number of years to the next rent review (4^*m is an integer)

n is the number of years between rent reviews

c is the crossover point

g_r is the estimated real growth rate of rents after the crossover point

g is the estimated nominal growth rate of rents before the crossover point

The crossover point is defined as the first value of c for which:

$$(1+g)^{m+c*n} > f > (1+g)^{m+(c-1)*n}$$

Thus the term rent is discounted at a given risk discount rate. Where the covenant strength is good, the risk discount rate would be based upon that used for an illiquid bond. The second term involves discounting a perpetuity, using an "all-risks yield".

Crosby et al (1997) discussed the determination of discount rates. Quoting French and Ward (1995), they suggest that a low-risk discount rate could be based on the redemption yield on corporate bonds, paying due regard to matching redemption dates to the unexpired term to rent revision and due regard to the strength of the tenant. They suggest issues which may lead to differences between corporate bond yields and the yields used to discount income from a particular tenant. For example, rent is a prior charge on company assets before bond interest and properties can be re-let in the event of a void. In general, the risk of different aspects of income from an over-rented property is different and different risk discount rates should be used. It may be necessary to divide the income into passing rent and market rent elements rather than dividing it into term and reversion elements. Adams and Booth (1996) consider this problem explicitly.

Adams and Booth reviewed traditional approaches to the appraisal of over-rented property, as well as proposing alternative "actuarial" approaches. The method described as the "conventional actuarial approach" involved the following formula which has been slightly adapted:

$$V_0 = f\, R_0\, \ddot{a}_{m+\overline{cn}|} + \frac{R_0\, \ddot{a}^{(4)}_{\overline{n}|}\, v^{m+cn} (1+g)^{m+cn}}{1-\left(\dfrac{1+g_r}{1+j}\right)}$$

A nominal interest rate is required for valuing the annuity making up the first term. The first term should be received with certainty, assuming no voids. A nominal interest rate is also needed to discount the rents received after the crossover point from the crossover point back to the valuation date. Finally, a real interest rate is needed to discount the rents received after the crossover point to the crossover point. The first interest rate could be derived from consideration of company bond yields of a company of similar standing to the tenant. The second and third interest rates are more difficult to rationalise but should be related to the rates used for valuing properties which are not over rented. The "actuarial approach" is similar to the "real value" approach in Crosby et al (1997).

An alternative to the conventional actuarial approach was described by Adams and Booth (1996) as the "convertible bond" approach. It used the traditional method of valuing convertible bonds. This method involves valuing the property as if it were let at open-market value and then valuing the "income advantage" (i.e. the difference between the passing rent and the market rent) up to the crossover point. Using the notation defined above, this gives rise to the following value:

$$V_0 = R_0\, \ddot{a}^{(4)}_{\overline{n}|} + f\, R_0\, \ddot{a}^{(4)}_{\overline{m+cn}|} - R_0\, \ddot{a}^{(4)}_{\overline{n}|} \left(\frac{1-\left(\frac{1+g}{1+i}\right)^{m+cn}}{1-\left(\frac{1+g}{1+i}\right)^{n}} \right)$$

To use this formula, requires *(m+c*n)/n* to be an integer, so that the valuation is taking place immediately after a rent review. The second and third term value the income advantage by valuing the whole of the passing rent before the crossover point and deducting the market rent received before the crossover point, which is already valued in the first term.

2.2.2 Option Characteristics of Over-rented Property

The holder of an over-rented property could be regarded as holding the right to a fixed income stream until the end of the lease. This could be valued at a rate of interest used for a company bond of similar standing to the tenant. The investor then has an out of the money call option to take the market rental stream if that becomes higher than the passing rent. This is how investors view convertible bonds: see Rutterford (1993). This rationalisation would lead us to choose one discount rate for the middle term of formula 2.2.1.3 and a different rate for the first and third terms. The middle term is only at risk if the tenant creates a void ("defaults" in corporate bond language). It would therefore be appropriate to value this fixed income at a rate related to the return from a similar corporate bond. A "risky" rate should be used for the first and third terms although it is not clear what risky investment would have similar characteristics to this call option.

Crosby, French and Ward (1997) and Baum and Crosby (1994) discuss the estimation of discount rates. Baum and Crosby do, in fact, recognise that the characteristics of an over-rented property are similar to those of a convertible bond. However, they do not apply that analogy when determining the appropriate discount rate. They suggest that the market rent is "doubly secured" (in the event of tenant default the property could be re-let at the market rent). They suggest that the market rent should therefore be valued at a lower rate of interest than that which would be used for a corporate bond. A higher risk premium is regarded as justified for the difference between the market rent and the passing rent because, if the tenant were to default, the property could be re-let only at the market rent. The problem with this approach is that there are no traded assets with which either of these payment streams can be compared.

3 Pricing the Embedded Option: Approaches in the Property Literature

Property pricing techniques may implicitly handle the fact that most rents in the UK are subject to upwards only rent reviews by using a low discount rate to reflect the guaranteed component of the income. However, when financial conditions change, the value of the upward only rent review option will change. Implicit techniques will not deal with this.

Ward and French (1997) have considered how to break down the value of a rental income stream into two components. One component represents the value of the income if the rent reviews could go either way and the other component is due to the upward only rent review structure in the lease. A decomposition such as this may be useful in comparing the value of different lease structures. It may also be helpful in identifying how valuable an upward only rent review rule is if inflationary conditions change. Ward and French (1997) made use of derivative pricing techniques. The holder of a call option has the right to buy an asset for an agreed price in the future. In the upward only rent review situation, Ward and French (1997) take the 'asset' to be the rental income subject to either-way reviews and take the 'price' to be the rental income prior to each review.

There are problems in applying the standard methods of derivative pricing to property. The standard pricing methods, which Ward and French (1997) use, are only applicable if the market value of the underlying asset behaves in a simple way. The underlying asset here is the market value of rents. Unfortunately, changes in rents in one quarter cannot be assumed to be independent of changes in rents in the previous quarter. A second problem is that derivatives are generally short term contracts. Prices of options can be calculated under the assumption that there will not be any fundamental changes to the market for the underlying asset. The longer the time period involved, the less secure is this assumption and the less reliable will be any model for the price of the asset. If the rents to be valued have reviews several years away, there must be some doubts about the validity of any model of rental income even if it could accurately describe historical data.

A third problem relates to 'hedging'. Hedging, in this context, means continually changing the amount of an asset which is held (and also changing the amount of cash held) in a particular manner. The theory behind standard derivative pricing techniques is only valid if there is an active market in small units of the underlying asset, which allows a hedged riskless portfolio to be created. This is not the case with property. The assumptions behind derivative pricing therefore become invalid. Also, there is no data on the behaviour of rents from properties subject to either-way reviews. It is unlikely that rental data based on upward only rent review rents would be a reliable guide to this behaviour.

Despite these problems with applying derivative pricing theory to property income, methods have been used in similar situations where there are long terms involved, no active markets and no reason to believe that a simple statistical model of prices is appropriate. These methods are discussed in Sections 4 and 5.

4 Corrected DCF Approaches to Property Appraisal

4.1 Framework of Notation

Traditional property approaches and DCF approaches to appraisal do not use notation and terminology which is appropriate for valuing implicit options. Probability notation is more useful as it enables us to understand explicitly the distribution of expected cash flows from the investment. In this section, we derive and apply appropriate notation to the upward only rent review appraisal.

We want to find expressions for the present value of the rental income in the five-year period following a review. All the equations in this section apply to the next review date only. The following notation will be used:

V	is the present value we are computing.
R	is the current annual rent (paid quarterly in advance).
$S(t)$	is the market rent at time t. It is unknown.
i	is the rate of interest appropriate for a corporate bond issued by a company of the same quality as the tenant (any liquidity issues will be ignored).
μ	is the expected force of growth in S which could be negative. This means that the expected value of $S(t)$ is given by

$$\mathrm{E}\left[S(t)\right] = S(0)\cdot e^{\mu\cdot t}$$

(The growth may alternatively be expressed in terms of the annual growth rate g, with $1+g=e^{\mu}$.)

In terms of this notation we are trying to find:

$$V\left[\mathrm{Max}\left(R\cdot\ddot{a}^{(4)}_{\overline{5}|i},\, S(t)\cdot\ddot{a}^{(4)}_{\overline{5}|i}\right)\right]$$

The annuity function represents the value of five years of level income after the review at time t. For $i = 8\%$, the value of the five-year annuity is $\ddot{a}^{(4)}_{\overline{5}|i} = 4.1904$. It can be taken outside the functions V and Max. To simplify some of the expressions that follow, the annuity term will therefore generally be left out. Hence we concentrate on finding the value:

$$V\left[\mathrm{Max}\left(R, S(t)\right)\right].$$

For some of the methods we will look at, a probability distribution is needed. A convenient assumption is that the logarithm of $S(t)$ has a normal distribution with variance $\sigma^2\cdot t$.

In Section 4.2, discounted cash flow methods of property appraisal are described in terms of the notation in Section 4.1; this enables us to understand the inaccuracies of these approaches more clearly. In Sections 4.3 and 4.4, we will consider how this inaccuracy can be corrected within a DCF framework. In Section 5 we will employ option pricing techniques to value the cash flows. All examples using these methods are in Appendix 1.

4.2. Expected Value Methods

4.2.1 Introduction

The general approach of the DCF methods is to take the value of the income stream as the value of the higher of the current rent and the rent which would be achieved if rents grow at the expected rate, i.e.:

$$V\left[\mathrm{Max}\left(R, S(t)\right)\right] = V\left[\mathrm{Max}\left(R, \mathrm{E}\left[S(t)\right]\right)\right].$$

$\mathrm{E}[\cdot]$ is the expected value. The DCF and contemporary property methods differ only in how the current rent, R, is split off from the rest of the rent. Two examples, which relate to over-rented property, will be given.

Let the crossover time t_C be defined so that the expected market rent overtakes the current rent at the crossover time.

4.2.2 "Risky plus Froth" Method

In this method, the rents are thought of as a continuously rising income plus some extra amount before the crossover time. The extra amount is the difference between the market rent and the current rent being received.

After crossover, the value is given by:

$$V = \frac{1}{(1+j)^t} \cdot \mathrm{E}\left[S(t)\right].$$

Before the crossover, the value is given by:

$$V = \frac{1}{(1+j)^t} \cdot \mathrm{E}\left[S(t)\right] + \frac{1}{(1+k)^t} \cdot \left\{R - \mathrm{E}\left[S(t)\right]\right\}.$$

A separate discount rate k is introduced to value the income above the market rent: see Section 2.2.2.

4.2.3 "Risk-Free plus Extra" Method

The value with this method is:

$$V = \frac{1}{(1+i)^t} \cdot R + \frac{1}{(1+j)^t} \cdot \mathrm{Max}\left(\mathrm{E}\left[S(t)\right] - R, 0\right).$$

This method is effectively the same as the "convertible bond" approach of Adams and Booth (1996) reviewed in Section 2.2.1. Although this notation has a general application to properties regardless of whether they are over rented.

4.3 Discounting the Expected Value

Before proceeding, it is important to appreciate the fundamental problem of the conventional DCF approaches implied by the above formulae. A highly simplified example helps illustrate this point for readers not familiar with probability theory.

Example

The current level of rents is 10 (the units are irrelevant). The level of market rents can take any of the values 5, 6, 7, ..., 14 each with probability 0.1 (i.e. a uniform distribution). Each of the methods in Section 4.2 would calculate $E[S(t)] = 9.5$ and compare with $R = 10$ and discount whichever is bigger (in this case R).

If the level of $S(t)$ were 9.5 with certainty (i.e. a point distribution), exactly the same present value would be discounted.

A corrected DCF approach or an option pricing approach would not discount the higher of the two expected values but would discount the expected amount of the actual income stream. In the case of the uniform distribution of rents, the expected value of the income stream is:

$$0.1*(11 + 12 + 13 + 14) + 0.6*10 = 11$$

(if $S(t) < 10$, the rent will be 10: the probability of this is 0.6, thus there is a probability weight of 0.6 at a rental level of 10)

In the case of the point distribution of market rents, the expected income stream is 10.

Thus the contemporary DCF approach does not take account of the probability distribution of future rental outcomes in a way which allows properly for the non-linear pay offs implied by the upward only rent review option.

Another example of the weakness of the above formula occurs of $E[S(t)] < R$. Assume $R = 10$ and $S(t) \sim N(x,1)$. If $x = 9.99$, Pr *(income receivable* $> 10) = 0.5$; if $x = 5$, Pr *(income receivable* $> 10) = 0$. However, an income stream of 10 would be valued in both cases. Mathematically this problem with most published valuation approaches arises from taking the maximum of two expected pay offs, rather than the expected value of all possible pay offs. This is the fundamental weakness of the DCF expected present value approach.

4.4 A Revised DCF Approach

If we correct for this by using a distribution for $S(t)$ it leads to the following formula for the value of the future income, based on the "Risk Free plus Extra" method:

$$V = \frac{1}{(1+i)^t} \cdot R + \frac{1}{(1+j)^t} \cdot \mathrm{E}\left[\mathrm{Max}\left(S(t) - R, 0\right)\right].$$

The mathematical adjustment is that the expectation is now outside the square bracket. For certain simplified distributions for *S(t)* we can obtain precise valuation formulae. If we choose a lognormal distribution for *S(t)* with:

$$\mathrm{E}\left[S(t)\right] = S(0) \cdot e^{\mu \cdot t}$$

and

$$\mathrm{Var}\left[\log\left(S(t)\right)\right] = \sigma^2 \cdot t$$

it leads to:

$$\mathrm{E}\left[\mathrm{Max}\left(S(t) - R, 0\right)\right] = \mathrm{E}\left[S(t)\right] \cdot N(a_1) - R \cdot N(a_2)$$

with

$$a_1 = \frac{\log\left(\mathrm{E}\left[S(t)\right] / R\right) + \left(\sigma^2 / 2\right) \cdot t}{\sigma \cdot \sqrt{t}}$$

and

$$a_2 = \frac{\log\left(\mathrm{E}\left[S(t)\right] / R\right) - \left(\sigma^2 / 2\right) \cdot t}{\sigma \cdot \sqrt{t}}$$

Where $N(\cdot)$ is the cumulative normal distribution. When E[*S*(*t*)] is large compared with *R*, $N(a_1)$ and $N(a_2)$ are both just below 1. When E[*S*(*t*)] is small then they are both just above 0. Putting the equations together leads to:

$$V = \frac{1}{(1+i)^t} \cdot R + \frac{1}{(1+j)^t} \cdot \left\{\mathrm{E}\left[S(t)\right] \cdot N(a_1) - R \cdot N(a_2)\right\}.$$

This expression for *V* is theoretically correct in that it gives values which change smoothly with changes in the expected value of and variance of future rents. It has allowed completely for the option characteristics of the property freehold. Using numerical methods it could be applied to a property using any assumptions regarding future distributions of market rents. It is compatible with the approach to option pricing of Pemberton (1997).

The discount rate *i* is used to discount the fixed stream. For ease of reference this will be described in this section, Section 5 and Appendix 1 as the "risk free rate", as this income stream is known with certainty, assuming no voids. In fact, this rate should be related to corporate bond yields which could be expected to be paid by the tenant (or a company of similar credit rating). This should be related to the yields on secured rather than unsecured lending, although there should be an adjustment for the illiquidity of property.

However, whilst this method might be used where closed form derivative solutions are inappropriate because the necessary assumptions do not hold, it has not solved the valuation problem. The difficulty lies in choosing the risk discount rate *j*. It might be thought that a single rate *j* could be chosen to be appropriate for discounting any

quantity which involves the unknown element $S(t)$. However, this is not possible as the following argument demonstrates.

Suppose j could be used to value the expected value of the three unknown quantities $S(t)$, Max($R - S(t)$, 0) and Max($S(t) - R$, 0). The values may be found by finding the mean of the probability distributions for each and discounting. A combination of one of each of the first two items and minus one of the third has a value of $R/(1+j)^t$. However, this particular combination would provide the known amount R in all circumstances and its value should therefore be $R/(1+i)^t$. Thus j cannot be used for valuing all these items and combinations of them.

This does not mean that the equation for V is wrong, just that the discount rate j cannot be applied generally to all amounts involving $S(t)$. This is inconvenient. It would have been useful to be able to take j from a "familiar" situation such as continuously rising rental income and use the same discount rate for valuations in less standard circumstances.

Of course, difficulty in determining the risk discount rate is not an unfamiliar problem in property appraisal, the same problem exists in contemporary DCF approaches: it is necessary to determine the risk discount rate subjectively. The revised DCF approach at least discounts the correct expected value and we believe it to be a useful approach where option pricing approaches are inappropriate because the necessary assumptions do not hold. However, an analytical derivatives approach may deal with this problem of selecting a discount rate in particular circumstances.

5. The Development of Derivative Techniques to Price Upward Only Rent Reviews

5.1 General Issues in Option Pricing

This section considers the basic elements in the approach to pricing derivatives. Derivatives are based on some underlying quantity such as the price of an equity share, the level of a stock market index or a measure of inflation. If analogies are to be drawn between the value of rental income and the pricing of derivatives, the underlying quantity most likely to be relevant would be the level of market rents. These rents may be the actual level of rack rents (as with the call option mentioned above) or a notional quantity relating to properties with either-way reviews (as with the put option, or the option considered by Ward and French, 1997). A crucial element in assigning a price to the derivative is a statistical model of the behaviour of the underlying quantity.

A statistical model is required which describes changes to the underlying quantity over time. If this statistical model is of a suitable form (described below) and if the underlying quantity is traded (i.e. there is an underlying *security*) the value of the derivative can be found by applying a "no arbitrage" rule. A risk-free rate of interest is then used in discounting. For other statistical models, derivative pricing techniques are still valid, but they lead to equations for the value of the derivative which are more difficult to solve. It is possible to extend the techniques to cover derivatives based on non-traded quantities, although a parameter relating to risk must be introduced.

As a starting point to the valuation of financial derivatives it is helpful to consider a relatively simple situation where the derivative is based on underlying securities for which the price S changes according to:

$$dS / S = \mu \cdot dt + \sigma \cdot dZ$$

where t is time, μ is the average growth rate of S, σ is the volatility of the movements in logarithm of price and dZ is a normally distributed random variable with mean 0 and variance dt (see, for example, Hull, 1997 and Wilmott et al., 1995). This process is a lognormal random walk. Although it is too simple to describe some asset price patterns, the values of derivatives relating to assets with different behaviours can often be related to the value of derivatives based on assets which do follow a lognormal random walk.

There are several aspects of this random walk which might not be suitable for describing changes in rents. Two of these will be discussed here. First, a constant volatility may be inappropriate. There have been periods when rents have oscillated strongly and other periods when changes have been much weaker. For example, 1986 to 1990 and 1994 to 1998 were, respectively, periods with high volatility and low volatility according to the IPD monthly index of office rents. Second, changes in rents may be serially correlated with each other. For example, a change in rent might be more likely to exceed the mean growth rate if the previous change had also exceeded the average than if the previous change had been below average. Both of these aspects, changing volatility and serial correlations, can be addressed by developing derivative pricing techniques further. Whilst the derivatives method in this paper is limited to the log normal random walk case, it is worth discussing approaches which can be used in more general cases.

A European call option gives the holder of the option the right to buy a particular asset for an agreed price on a specified future date. This type of option is analogous to the one embedded in an upward only rent review contract. The value of such a call option relating to an asset with a price which follows a lognormal random walk is given by:

$$C = S \cdot N(a_1) - E \cdot \exp\left[-r \cdot (T - t_0)\right] \cdot N(a_2)$$

with

$$a_1 = \frac{\log(S / E) + \left(r + \frac{\sigma^2}{2}\right) \cdot (T - t_0)}{\sigma \cdot \sqrt{T - t_0}}$$

$$a_2 = \frac{\log(S / E) + \left(r - \frac{\sigma^2}{2}\right) \cdot (T - t_0)}{\sigma \cdot \sqrt{T - t_0}}$$

Here, E is the price to be paid at time T if the option holder chooses to buy the asset then; t_0 is the present time and r is the rate of interest earned by cash. $N(\cdot)$ is the cumulative distribution function for a standardised normal random variable. This

expression for C is the Black Scholes formula for a European call option: see Adams et al (1993) for a derivation. This formula can actually be used for some assets which do not follow a straightforward lognormal random walk. In these circumstances, the volatility (σ) must generally be interpreted to mean something other than a constant variance in (the logarithm of) the underlying asset price.

If the lognormal random walk is not a good description of changes in the value of the underlying variable, adjustments can be made. If σ were to change in a known way over time, i.e. $\sigma(t)$ were known, the Black Scholes formula can still be used to give the value of a European call option so long as σ^2 is taken to be the average variance over the lifetime of the option (see, for example, Section 6.5 of Wilmott et al. 1995).

Allowing for such changes in variability of rents is unlikely to be sufficient as future changes to the volatility will not be known accurately in advance. A more useful treatment would recognise that volatility might also depend on the underlying asset value, S. If this is the case, the Black Scholes formula cannot generally be used, even in a revised form (see Section 8.1 of Kemp, 1997). Where σ does depend on S it is possible to express the price of an option in terms of a differential equation, the solution to which would depend on the form of $\sigma(S)$.

An alternative is to use a stochastic model for the variance. This means that there is a random element to the changes in the volatility of the underlying asset. This does allow for a wider range of possible patterns in rental income. Methods for pricing derivatives in such circumstances are discussed in chapter 19 of Hull (1997).

The lognormal random walk assumes that changes in the logarithm of price do not depend on the preceding changes in price. This is a useful assumption for traded securities because, if such dependencies did exist, it would be possible to devise low risk trading strategies which would be expected to significantly outperform the market (see Kemp 1996). For property rents, such serial dependencies could exist for an extended period because of the absence of a sufficiently liquid market for the rental income.

There has been some work on pricing derivatives where successive changes in the value of the underlying asset are not independent of each other. One process, described by Shimko (1992) is the mean reverting process:

$$dS = \alpha \cdot \left(\bar{S} - S\right) \cdot dt + \sigma \cdot S^{\gamma} \cdot dZ$$

If the parameter $\gamma = ½$ an exact expression can be found for the value of a call option based on an index following this process. If $\gamma = 1$, any derivative based on an index following this process would have the same value as a derivative based on an asset following a lognormal random walk. In this case, the mean reverting nature is actually irrelevant to the value of the option.

The derivative pricing methodologies mentioned above apply when the quantity underlying the derivative is a traded security. This would not be the case with an option relating to market rents. However, the same methodologies can be applied, with modifications, when the underlying is not a traded security (Hull, 1997, chapter 13). Without a traded security, it is impossible to form an exact hedged portfolio for a derivative, hence the value of the derivative must depend on some measure of risk.

It is still necessary to have a stochastic model for the underlying quantity. If the stochastic model is the lognormal random walk with parameters μ and σ, the formula for the value of a derivative based on a non-traded quantity would generally be similar to the formula for the value of a derivative based on a traded asset. The differences relate to the risk free interest rate. If the value of a derivative based on a traded asset were of the form:

$$V = \exp\left[-r \cdot (T - t_0)\right] \cdot g(S, E, r, \sigma, T - t_0)$$

the value of a derivative based on the non-traded quantity would be the same except that in g the interest rate r would be replaced by $\mu - \lambda \cdot \sigma$. The number λ is related to investment risk, and is called the "market price of risk". Different investors may place different values on λ and therefore assign different values to these derivatives. λ is similar to the quantity β which appears in the capital asset pricing model, both are linked to the excess return an investor needs above the risk free rate when holding a risky asset.

We will approach the problem by using the method of Hull (1997) for a non-traded quantity. We will, however, assume a log normal random walk for *S(t)*. For more complex distributions of *S(t),* it may be better to apply the revised DCF approach of Section 4.4 using simulation and numerical techniques.

5.2 Formulae for the Derivatives Approach

Because continuous hedging assumptions do not apply, the parameter λ, is introduced to represent risk (see Section 5.1). It replaces the risk discount rate j used in the revised DCF method of Section 4.1. If rental income were securitised and a market existed in the securities there would be no need for this risk parameter, as perfect hedging would be possible. The value for the greater of the current rent and the market rent at time t is given by the expression:

$$V = \frac{1}{(1+i)^t} \cdot \left\{R + \mathrm{E}\left[\mathrm{Max}\left(S(t) \cdot e^{-\lambda \cdot \sigma \cdot t} - R, 0\right)\right]\right\}.$$

i.e. using the corporate bond discount rate but reducing S(t) using the parameter λ.

Using the distributional assumptions of 4.4 and taking the expected value leads to:

$$V = \frac{1}{(1+i)^t} \cdot \left\{R + \mathrm{E}\left[S(t)\right] \cdot e^{-\lambda \cdot \sigma \cdot t} \cdot N(b_1) - R \cdot N(b_2)\right\},$$

with

$$b_1 = \frac{\log\left(\mathrm{E}\left[S(t)\right] \cdot e^{-\lambda \cdot \sigma \cdot t} / R\right) + \left(\sigma^2 / 2\right) \cdot t}{\sigma \cdot \sqrt{t}}$$

and

$$b_2 = \frac{\log\left(\mathrm{E}\left[S(t)\right] \cdot e^{-\lambda \cdot \sigma \cdot t} / R\right) - \left(\sigma^2 / 2\right) \cdot t}{\sigma \cdot \sqrt{t}}$$

This is very similar to the equation in Section 4. The changes are that $S(t)$ is reduced by using the risk parameter λ before any values are found. This is offset by discounting at the risk free rate of interest. Despite the similarities, the two approaches are conceptually different. It is impossible to switch from one form to the other simply by an appropriate choice of j and λ. As is shown by the numerical examples in Appendix one, if j and λ are chosen to equalise the present value at one value for the expected rate of rental growth, the present values will be different at another expected rate of rental growth.

5.3 An Interpretation of λ

In the context of derivative theory the parameter λ is called the "market price of risk". The concept is developed in financial texts such as Lentz and Tse (1995), Hull (1997) and Shimko (1992). One way of choosing a value for λ is by comparing the value of a quantity under the derivatives approach with one obtained using risk discount rates, for a particular set of parameter inputs (see Appendix one).

Suppose that the risk discount rate for $S(t)$ can be agreed to be j. Then the value of $S(t)$ can be written as:

$$V_1 = \frac{1}{(1+j)^t} \cdot \mathrm{E}\left[S(t)\right].$$

The value according to the derivatives approach is:

$$V_2 = \frac{1}{(1+i)^t} \cdot \mathrm{E}\left[S(t)\right] \cdot e^{-\lambda \cdot \sigma \cdot t}$$

The two values will be equal when:

$$\lambda \cdot \sigma = \log\left[(1+j)/(1+i)\right] \approx j - i$$

A different meaning may be attached to λ by comparing values relating to $S(t)$ with values of traded assets. Suppose a portfolio of assets is constructed with a behaviour that closely matches that of rental income. (The portfolio might consist of a mixture of bonds and shares, including property company shares.) Denote the expected return on this portfolio by μ_P and the variance in the logarithm of the portfolio value by σ_P. Let ρ be the correlation between changes in S and changes in the value of the portfolio. The portfolio should be chosen so that ρ is close to 1. The risks associated with S may then be priced by using the relationship:

$$\lambda = \rho \cdot \frac{\mu_P - \log(1+i)}{\sigma_P}$$

This resembles the relationship between returns on risky assets in the Capital Asset Pricing Model. However, there is no need to take the portfolio to be the whole market. If a portfolio can be found showing a high degree of correlation with rent levels, then most of the risk relating to uncertainties in the rental income will be allowed for in λ.

Whilst it is possible to deal with the problem that perfect hedging is not possible by using the adjustment factor before discounting, it is not possible to deal with the more complex distributional assumptions for $S(t)$ which would be more realistic than a log normal random walk in a straightforward way. This and the extension of the methodology to cover a series of reviews is the subject of further work. The revised DCF approach of Section 4.4 can easily be extended to include more complex distributions for $S(t)$ if numerical calculation methods are used.

6 Conclusion

This paper has reviewed the literature on property pricing and upward only rent reviews. It finds that options embedded in upward only rent reviews are not properly priced using DCF or traditional approaches. Ward and French (1997) have developed techniques for valuing the option but the assumptions necessary for those techniques to work do not hold.

A new notational framework is developed and the DCF approach to valuation described within that framework. That framework allows us to see clearly why the DCF approach to valuation may mis-price property. In Section 4.4 a revised DCF approach to valuation is developed. This takes account of the embedded option and is used to obtain a formula for the value of the option for simple assumptions regarding the pattern of rental growth. Numerical methods could be used where rental growth assumptions are more complex. A risk discount rate has to be chosen if this method is used. In Section 5, a pure derivatives approach to pricing the embedded option is developed. A risk discount rate does not have to be chosen and an analytical framework for determining the necessary risk parameter can easily be established. The disadvantage of this approach is that it requires rather restrictive assumptions to hold.

References

Adams A. T. and Booth P. M. (1996), The appraisal of over-rented property, *Journal of Property Finance*, Volume 7, No. 3, pp 7-22.

Adams A. T., Booth P. M. and Venmore-Rowland P., (1993), Theoretical Volatility Measures for Freehold Property Investments, *Journal of Property Research*, Vol. 10, No. 3, pp 153-66

Adams A. T., Bloomfield D. S. F., Booth P. M. and England P. D. (1993), *Investment Mathematics and Statistics*, Graham and Trotman, UK.

Baum A. and Crosby N., (1995), Over-rented properties: bond or equity? A case study of market value, investment worth and actual price, *Journal of Property Valuation and Investment,* Volume 13, No. 2, pp 31-40.

Baum A. and Crosby N. (1988), *Property Investment Appraisal*, Routledge & Keegan Paul, London.

Beenstock M. and Brasse V. (1986), Using Options to Price Maturity Guarantees, *Journal of the Institute of Actuaries*, Volume 113, Part 1 pp 151-166.

Crosby N., French N. S., and Ward C. W. R. (1997), Contemporary UK market valuation methods for over-rented properties: a framework for risk adjustment, *Journal of Property Research*, Volume 14, pp 99-115.

Daykin C. D., Pentikainen T. and Pesonen M. (1994), *Practical Risk Theory for Actuaries*, Chapman and Hall, London.

Dodhia M. and Sheldon T. (1994), *Guaranteed Equity Products*, Staple Inn Actuarial Society.

French N. S. and Ward C. W. R. (1995), Valuation and arbitrage, *Journal of Property Research*, Volume 12, pp 1-11.

Hull J. C. (1997), *Options, Futures and Other Derivative Securities* (third edition), Prentice-Hall International, London.

Kemp M. H. D. (1997), Actuaries and derivatives, *British Actuarial Journal*, Volume 3, pp 51-162.

Kemp M. H. D. (1996), Asset liability modelling for pension funds, *Proceedings of the 1996 Investment Conference*, Vol. 1, pp 115-139.

Lentz G. H. and Tse K. S. M. (1995), An option pricing approach to the valuation of real estate contaminated with hazardous materials, *Journal of Real Estate Finance and Economics,* Volume 10, pp 121-144.

Pemberton J. M. (1997), *Equity Option Valuation Made Simple- and More Reliable*, Staple Inn Actuarial Society.

Riddiough T. J. (1995), Replicating and Hedging Real Estate Risk, *Real Estate Finance,* Volume 12, pp 88-95.

Rutterford J., (1993), *Introduction to Stock Exchange Investment*, Macmillan, Basingstoke.

Shimko D. C. (1992), *Finance in Continuous Time: A Primer*, Kolb, Miami, Florida.

Ward C. W. R. and French N. S. (1997), The valuation of upwards only rent reviews: an option pricing approach, *Journal of Property Valuation and Investment*, Volume 15.

Wilkie A. D. (1995), More on a stochastic investment model for actuarial use, *British Actuarial Journal*, Volume 1, pp 777-964.

Wilmott P., Howison S. D., and Dewynne J. D., (1985), *The Mathematics of Financial Derivatives*, Cambridge University Press.

Appendix One: Numerical Examples

In this section methods described in this paper are used to produce values for the rental income in excess of the current rent in various circumstances. We only look at the rent up to the second review from the purchase point.

In the examples, the current rent receivable is 1, the next review date is 4 years away and the "risk-free" (or corporate bond) interest rate is 6%. The value of unit income from a property with no reviews would be: $\ddot{a}^{(4)}_{\overline{9}|i} = 7.0549$. The tables give the additional present value of rents arising after the next review, due to the upward only rent review, calculated by the three different methods. Methods 2 and 3 allow for the option nature in some way; method 1 simply calculates the higher of the two expected values: the current rent receivable or the market rent based on anticipated rental growth. The total present value of the next 9 years rent is found by adding 7.0549 to the value in the tables.

The quantities which are varied are the current market rent ($S(0)$ = 1 and 0.9): the latter value implying over renting; the expected growth rate of rents (μ = 0%, 1%, 2%, 3%, 4% and 5%) and, where it is needed, the standard deviation or rents (σ = 2% and 10%). 10% is the standard deviation estimated from the data, the 2% rate is used for comparison, given the importance of this parameter in determining the value of the upward only rent review. Values are calculated for two risk discount rates (j = 8% and 10%). For each $S(0)$ two values of λ are used for the option pricing method. These values are chosen so that when μ = 3% the second and third methods agree on the value of the extra rental income for the two different risk discount rates used in the second method.

Table 1
Value of Extra Rental Income when $S(0) = R$, σ = 10%

Method	Risk, j or λ	Expected growth rate, μ					
		0%	1%	2%	3%	4%	5%
1	8%	0	0.1311	0.2675	0.4095	0.5572	0.7110
1	10%	0	0.1218	0.2285	0.3805	0.5178	0.6607
2	8%	0.2558	0.3317	0.4212	**0.5242**	0.6406	0.7700
2	10%	0.2377	0.3083	0.3914	**0.4871**	0.5953	0.7155
3	0.035	0.2504	0.3272	0.4185	**0.5244**	0.6448	0.7794
3	0.069	0.2274	0.2995	0.3858	**0.4867**	0.6022	0.7321

Table 2
Value of Extra Rental Income when $S(0) = R$, , $\sigma = 2\%$

Method	Risk, j or λ	Expected growth rate, μ					
		0%	1%	2%	3%	4%	5%
1	8%	0	0.1311	0.2675	0.4095	0.5572	0.7110
1	10%	0	0.1218	0.2485	0.3805	0.5178	0.6607
2	8%	0.0512	0.1420	0.2686	**0.4095**	0.5572	0.7110
2	10%	0.0476	0.1319	0.2496	**0.3805**	0.5178	0.6607
3	0.035	0.0421	0.1290	0.2598	**0.4096**	0.5675	0.7319
3	0.069	0.0319	0.1083	0.2329	**0.3804**	0.5370	0.7001

Table 3
Value of Extra Rental Income when $S(0) = 0.9*R$, $\sigma = 10\%$

Method	Risk, j or λ	Expected growth rate, μ					
		0%	1%	2%	3%	4%	5%
1	8%	0	0	0	0.0474	0.1804	0.3188
1	10%	0	0	0	0.0440	0.1676	0.2962
2	8%	0.1153	0.1592	0.2144	**0.2821**	0.3629	0.4573
2	10%	0.1071	0.1479	0.1992	**0.2621**	0.3372	0.4249
3	0.035	0.1103	0.1537	0.2088	**0.2769**	0.3589	0.4555
3	0.069	0.0979	0.1377	0.1886	**0.2522**	0.3294	0.4210

Table 4
Value of Extra Rental Income when $S(0) = 0.9*R$, $\sigma = 2\%$

Method	Risk, j or λ	Expected growth rate, μ					
		0%	1%	2%	3%	4%	5%
1	8%	0	0	0	0.0474	0.1804	0.3188
1	10%	0	0	0	0.0440	0.1676	0.2962
2	8%	0.0002	0.0027	0.0202	**0.0787**	0.1856	0.3192
2	10%	0.0001	0.0025	0.0188	**0.0732**	0.1724	0.2966
3	0.035	0.0001	0.0017	0.1530	**0.0674**	0.1732	0.3134
3	0.069	0	0.0010	0.0107	**0.0533**	0.1496	0.2854

Commentary on the Tables

Clearly a significant number of comparisons between the tables could be drawn. We will focus on the issues relating to the rationale for developing this paper. Intuition suggests that the value of the option will be high when there is a high chance of a

property becoming over rented. This will happen at high values of rental standard deviation and low values of rental growth. 10% is the value for standard deviation of rents which has been estimated from the data. The concern of the authors is that, as we move to an inflation environment which implies lower nominal rental growth, DCF and traditional methods of valuation will not take account of the changed financial conditions. Nominal rental growth of 0% and 5% would be reasonable for low and medium inflation environments. Thus we will look at results of different valuation methods with expected rental growth of 0% and 5%. We will make comparisons for standard deviation of rental growth of 10% and then look at the effect of assuming standard deviation of rental growth of 2%.

The value of the level rent for nine years is 7.0549 at a 6% "risk free" rate. The pricing inaccuracy from using method 1 rather than one of the technically correct methods 2 or 3 could be defined as:

100(value of additional rent after review using method 2 or 3 - value of additional rent after review using method 1)/(value of additional rent after review using method 1 + 7.0549) %*

In other words, the percentage increase in the value of the property from using the correct method; the value of the property includes the value of the constant unit rent. For expected rental growth of 0% and $j = 8\%$ this value would be: 3.6% for method 2 and not significantly different for method 3. For expected rental growth of 5% this mispricing would be 0.76%. Thus the increase in value of the upward only rent review clause due to a fall in expected rental growth from 5% to 0% is 2.8%. This is a change arising only from the increased value of the option and is independent of any effects caused by a change in the yield basis or changes in real rental growth. If this additional value has not been priced into the market over the period during which inflation expectations have fallen, due to the pricing techniques used by property practitioners, property prices will be 2.8% below their value when an allowance is made for the increased value of the upward only rent review. Reflecting that 2.8% rise in value over a five year period would have increased performance by 0.55% per year. The figure of 2.8% can also be used as an estimate of the increased cost of the upward only rent review clause to tenants and the rise in equilibrium rents which could take place which would exactly compensate for the removal of the clause.

By way of comparison, it can be seen from table 2 that the difference between methods 1, 2 and 3 is negligible at all rates of expected rental growth when the standard deviation of rents is 2%. It can be seen from all the tables that, when expected rental growth is high, the difference in the value calculated from methods 2 and 3 is also very small. This arises because there is a lower probability of negative rental growth the higher is expected rental growth.

Appendix Two: Real Estate Development Options

The owner of a property often has the option to develop the property in the future. Such real estate development options are long term and are related to assets which are not traded securities. Although this type of option is very different from financial derivative options, there have been papers which apply derivative approaches to these "real" options.

There are overlaps between the valuation of these options and the valuation of rental income in general, including the complications such as upward only rent reviews and over-renting which we have been considering. Here, we outline very briefly the approach taken by Lentz and Tse (1995); their paper also contains several references to the literature on development options.

- The aim is to value a series of uncertain cash flows related to the property
- These cash flows are assumed, without any attempt at justification, to follow a lognormal random walk
- In order to derive a value for these cash flows, a portfolio of traded securities is constructed. The behaviour of this portfolio is also taken to be a lognormal random walk and should be similar to the development of the cash flows: Riddiough (1995) explains how such a portfolio may be found.
- The value of the cash flows then depends on the expected growth of the cash flows, the expected growth of the hedging portfolio, the variances of these two items, the correlation between the stochastic elements of the two processes, a risk free interest rate and the size of current cash flows

Shimko (1992) considers this same method for valuing non-traded assets, although not specifically in the context of property related income. For the case where the cash flows are rental income, the results would be the same as those discussed in the previous section. The approach gives a particular meaning to the quantity λ which featured in the values in Section 5, it may be re-written as

$$\lambda = \rho \cdot \frac{\alpha_H - r}{\sigma_H}$$

where α_H and σ_H are the mean and variance for the lognormal random walk followed by the hedge portfolio and r is the risk-free rate of interest. The correlation between the cash flows and the hedge portfolio are described by ρ. This is defined as follows: if the random element for the cash flows is dZ and the random element for the hedge portfolio is dZ_H then $\rho \cdot dt = dZ \cdot dZ_H$. Under this definition $-1 \le \rho \le 1$.

This form for the price of risk is similar to that which arises in the capital asset pricing model (CAPM). As with the CAPM, a low or negative correlation leads to a higher value for a particular asset than if the correlation were more strongly positive. Also, any uncertainty which is not related to the correlation is not priced. Note, however, that there is no reason why the hedge portfolio should be the whole "market". In fact, the most appropriate hedge portfolio might be one designed to produce a value of ρ close to 1. In this case most of the uncertainty would be reflected in the value assigned to the cash flows. Where ρ is large, λ is large and the value assigned to the cash flows is reduced.

Appendix Three: With-Profit Insurance Policies

In looking for ways to value the upward only rent review option for property, one could turn to the techniques of analysing with-profit liabilities within insurance companies. A with-profit contract guarantees a particular sum assured for the policy. Reversionary and terminal bonuses are then declared from the profits of the company. Terminal bonuses are not paid until the policy matures. The analogy with property is

as follows. The term of the policy is analogous to the term of the lease (although property, unlike the insurance policy has a value after the lease has expired). The guaranteed sum assured is analogous to the market level of rents. The reversionary bonus which, once added, cannot be taken away is analogous to the market level of rents increasing at a rent review.

There are at least two differences of principle between with-profit policies and upward only rent reviews. Firstly, the initial market level of rent is not in the control of the property freeholder but the insurance company can set a guaranteed sum assured well below the level which could be achieved from investment in guaranteed fixed interest investment: in other words, the insurer could set the guarantee so that it is significantly "out of the money" and choose an investment policy so that the guarantee can be achieved with near certainty. An analogy here would be if the upward only rent review clause had the floor on rents set at, say, x% of the market level of rents negotiable between the landlord and the tenant, where $x<100$. Secondly, the office often chooses to "smooth" policyholder's returns. It can therefore transfer wealth from policyholders who have held their policies when investment returns have been high to those who have held them when investment returns have been low. There is an element of discretion in the management of the fund which does not exist with freehold property investment.

The most common practical approaches to bonus setting and valuation do not use option pricing theory. Daykin et al (1994) illustrate how the bonus may be set. The bonus would be set in such a way that the reserves the insurance company need to hold to pay guaranteed sums assured plus bonuses already declared would be significantly less than the value of the total investment fund available to pay bonuses. At maturity, the surplus of the investment fund over the guaranteed sum assured plus declared bonuses is used to pay a terminal bonus.

Wilkie (1987) proposed an option pricing approach to bonus setting. However, this was more of a device to ensure equity between policyholders than a device to price or value the cost of the guarantees. Wilkie proposed looking at the combination of equities and put options which would have been necessary to produce a particular level of guaranteed bonuses. The rationale was that, when bonus guarantees were added, the insurance company was implicitly selling put options to the policyholder at prices which were known retrospectively.

Other work on guarantees in life insurance policies, for example Beenstock and Brasse (1986) and Dodhia and Sheldon (1994) have concentrated on maturity guarantees. These involve a more simple structure than a with-profit policy. A guaranteed minimum maturity value is paid which is equivalent to a long-term put option. Dodhia and Sheldon refer to "lock-in guarantees". Here gains in a relevant investment index are "locked in" to the contract. For example, if the FTSE 100 index rises to 25% above its level at the beginning of the contract, the higher level becomes the guarantee. The authors have not found any specifically actuarial literature on these products. Under certain assumptions, they can be priced using derivatives theory or cash-flow techniques which are discussed in the literature and used for valuing upward only rent reviews in Sections 4.4 and 5.

We conclude this section by commenting that with-profit policies are a useful analogy with upward only rent reviews. However, there is little literature on which the authors can usefully draw to develop valuation techniques for upward only rent reviews.

Appendix Four Convertible Bonds

In the simplest form, a convertible bond provides the holder with the income and redemption proceeds of a bond together with the right, at any time up to the redemption date, to exchange the bond for shares in the issuing company.

There are clear similarities between this type of asset and over-rented property: there is a period of level income followed, potentially, by a period of rising, but uncertain, income. For property, the value of the income might be given by the general form

V = PV(fixed income until the end of the lease @ rate i) + v^t × PV(extra income @ rate k)

The extra income will be evaluated from time t, the crossover point, onward. This cross-over time would depend on the expected growth rate of market rents. The interest rate i would be lower than the nominal rate k and the discount factor v might correspond to an interest rate between i and k.

Convertible bonds can be thought of as being similar to call options, with the "exercise price" being the fixed income forgone by exchanging the bond for shares. Derivative techniques can be used to value convertible bonds. A key feature of simple convertible bonds, as far as derivative pricing is concerned, is that the option may be exercised at any time. Such options are called American options in contrast to the European options mentioned in Section 5. Exact formulae cannot be found for the values of American options. Instead, differential equations are obtained which link the value of the option to other quantities. These equations must be solved numerically. The "other quantities" include, for a convertible bond, the coupon paid on the bond, the share price, the share price volatility, the level of dividend payments and the bond yield (see Wilmott et al. 1995, chapter 18).

The items effecting the value of convertible bonds have counterparts in the context of the income from over-rented property. The coupons are equivalent to the rent being received now, the dividends are equivalent to the income if market rents were being charged. The share price cannot be interpreted so directly: the counterpart to the share price is the present value of future income, at market levels of rent, in perpetuity. The volatility of this quantity would replace the volatility of share price if this approach were to be used to value properties.

There is one aspect which applies to rental income which does not apply to convertible bonds. One of these is that "conversion" cannot take place at all times, it can only happen on review dates. This aspect can be dealt with in the derivative pricing framework by treating the option as European although, if there is more than one review date, it would have to be a sequence of European call options. The fact that the options are European makes them simpler to value.

CITY University

DEPARTMENT OF ACTUARIAL SCIENCE AND STATISTICS

Actuarial Research Papers since 1992

32. England P.D. and Verrall R.J. Dynamic Estimation for Models of Excess Mortality. January 1992. 19 pages. ISBN 1 874 770 32 8

33. Verrall R.J. A State Space Formulation of Whittaker-Henderson Graduation, with Extensions. January 1992. 13 pages. ISBN 1 874 770 33 6

34. Verrall R.J. Graduation by Dynamic Regression Methods. January 1992. 37 pages. ISBN 1 874 770 34 4

35. Gerrard R.G. and Haberman S. Stability of Pension Systems when Gains/Losses are Amortized and Rates of Return are Autoregressive. March 1992. 12 pages. ISBN 1 874 770 35 2

36. Haberman S. HIV, AIDS and the Approximate Calculation of Life Insurance Functions, Annuities and Net Premiums. April 1992. 28 pages. ISBN 1 874 770 36 0

37. Cooper D.R. Savings and Loans: An Historical Perspective. May 1992. 29 pages. ISBN 1 874 770 37 9

38. Verrall R.J. Dynamic Bayes Estimation for AIDS Delay Tables. May 1992. 16 pages. ISBN 1 874 770 38 7

39. Kaye G.D. Data Sources Relating to Life Assurance Expenses. May 1992. 39 pages. Presented to the Staple Inn Actuarial Society and Royal Statistical Society - February 1992. ISBN 1 874 770 39 5

40. Renshaw A.E., and Haberman S. On the Graduation Associated with a Multiple State Model for Permanent Health Insurance. May 1992. ISBN 1 874 770 40 9

41. England P.D. Statistical Modelling of Excess Mortality Number 3. June 1992. 163 pages. ISBN 1 874 770 41 7

42. Bloomfield D.S.F. and Haberman S. Male Social Class Mortality Differences Around 1981: An Extension to Include Childhood Ages. 21 pages. June 1992. ISBN 1 874 770 42 5

43. Berg M.P and Haberman S. Trend Analysis and Prediction Procedures for Time Nonhomogeneous Claim Processes. 33 pages. June 1992. ISBN 1 874 770 43 3

44. Booth P.M. The Single Market for Insurance, Free Capital Movements and attempts to Fix Exchange Rates. October 1992. 28 pages. ISBN 1 874 770 44 1

45. Verrall R.J. Chain Ladder with Varying Run-off Evolutions. February 1993. 15 pages. ISBN 1 874 770 45 X

46. Gavin J., Haberman S. and Verrall R.J. Moving Weighted Average Graduation using Kernel Estimation. November 1992. 14 pages. ISBN 1 874 770 46 8

47. Gavin J., Haberman S. and Verrall R.J. On the Choice of Bandwidth for Kernel Graduation. November 1992. 21 pages. ISBN 1 874 770 47 6

48. S. Haberman. Pension Funding with Time Delays and the Optimal Spread Period. May 1993. 13 pages. ISBN 1 874 770 48 4

49. S. Haberman. Stochastic Investment Returns and the Present Value of Future Contributions in a Defined Benefit Pension Scheme. May 1993. 22 pages. ISBN 1 874 770 49 2

50. A. Zimbidis and S. Haberman. Delay, Feedback and Variability of Pension Contributions and Fund Levels. May 1993. 19 pages. ISBN 1 874 770 50 6

51. S. Haberman. Pension Funding: The Effect of Changing The Frequency Valuations. June 1993. 19 pages. ISBN 1 874 770 51 4

52. S Haberman. HIV, AIDS Markov Chains and PHI. June 1993. 15 pages. ISBN 1 874 770 52 2

53. S Haberman. A Consideration of Pension Credit and Termination Insurance. June 1993. 22 pages. ISBN 1 874 770 53 0

54. M Z Khorasanee. Survey of Actuarial Practice in the Funding of UK Defined Benefit Pension Schemes. July 1993. 19 pages. ISBN 1 874 770 54 9

55. P M Booth, R G Chadburn and A S K Ong. A Utility Maximisation Approach to Asset Allocation. September 1993. 40 pages. ISBN 1 874 770 55 7

56. R G Chadburn. Bias in Select Mortality Investigations. August 1993. 62 pages. ISBN 1 874 770 56 5

57. M Z Khorasanee. A Comparison of Pension Funding Strategies by means of Simulations for a Model Scheme. August 1993. 43 pages. ISBN 1 874 770 57 3

58. A E Renshaw, P Hatzopolous and S Haberman. Recent Mortality Trends in Male Assured Lives. June 1993. 23 pages. ISBN 1 874 770 58 1

59. E Pitacco. Disability Risk Models: Towards A Unifying Approach. September 1993. 33 pages. ISBN 1 874 770 59 X

60. M Boskov and R J Verrall. Premium Rating by Geographic Area Using Spatial Models. September 1993. 14 pages. ISBN 1 874 770 60 3

61. R G Chadburn. Managing Mutual Life Offices: The Use of an Expense Ratio in New Business Decision Making and Expense Control. October 1993. 21 pages. ISBN 1 874 770 61 1

62. Haberman S. Pension Funding Modelling and Stochastic Investment Returns. 56 pages. March 1994. ISBN 1 874 770 62 X

63. Renshaw A E and Verrall R J. The Stochastic Model Underlying the Chain-Ladder Technique. 25 pages. April 1994. ISBN 1 874 770 63 8

64. Haberman S and Sung J-H. Dynamic Approaches to Pension Funding. 22 pages. April 1994. ISBN 1 874 770 64 6

65. Renshaw A.E. On the Second Moment Properties and the Implementation of Certain GLIM Based Stochastic Claims Reserving Models. 36 pages. September 1994. ISBN 1 874 770 65 4

66. Booth P.M., J.N. Allan, and J.W. Jang. An Evaluation of the UK Life Insurance Mismatching Test. September 1994. ISBN 1 874 770 66 2

67. Booth P.M. and Stroinski K. Insurance and Investment Markets in Poland. September 1994. 35 pages. ISBN 1 874 770 67 0

68. Ong A. A Stochastic Model for Treasury-Bills: An Extension to Wilkie's Model. September 1994. 12 pages. ISBN 1 874 770 68 9

69. Bloomfield D.S.F. Moving on from Undergraduate Exams to Professional Exams: Actuaries. November 1994. 22 pages. ISBN 1 874 770 69 7

70. Huber P. A Review of Wilkie's Stochastic Investment Model. January 1995. 22 pages. ISBN 1 874 770 70 0

71. Renshaw A.E. On the Graduation of `Amounts'. January 1995. 24 pages. ISBN 1 874 770 71 9

72. Renshaw A.E. Claims Reserving by Joint Modelling. December 1994. 26 pages. ISBN 1 874 770 72 7

73. Renshaw A.E. Graduation and Generalised Linear Models: An Overview. February 1995. 40 pages. ISBN 1 874 770 73 5

74. Khorasanee M.Z. Simulation of Investment Returns for a Money Purchase Fund. June 1995. 20 pages. ISBN 1 874 770 74 3

75. Owadally M.I. and Haberman S. Finite-time Pension Fund Dynamics with Random Rates of Return. June 1995. 28 pages. ISBN 1 874 770 75 1

76. Owadally M.I. and Haberman S. Stochastic Investment Modelling and Optimal Funding Strategies. June 1995. 25 pages. ISBN 1 874 770 76 X

77. Khorasanee M.Z Applying the Defined Benefit Principle to a Defined Contribution Scheme. August 1995. 30 pages. ISBN 1 874 770 77 8

78. Sebastiani P. and Settimi R. Experimental Design for Non-Linear Problems. September 1995. 13 pages. ISBN 1 874 770 78 6

79. Verrall R.J. Whittaker Graduation and Parametric State Space Models. November 1995. 23 pages. ISBN 1 874 770 79 4

80. Verrall R.J. Claims Reserving and Generalised Additive Models. November 1995. 17 pages. ISBN 1 874 770 80 8

81. Nelder J.A. and Verrall R.J. Credibility Theory and Generalized Linear Models. November 1995. 15 pages. ISBN 1 874 770 81 6

82. Renshaw A.E., Haberman S. and Hatzopoulos P. On The Duality of Assumptions Underpinning The Construction of Life Tables. December 1995. 17 Pages. ISBN 1 874 770 82 4

83. Chadburn R.G. Use of a Parametric Risk Measure in Assessing Risk Based Capital and Insolvency Constraints for With Profits Life Insurance. March 1996. 17 Pages. ISBN 1 874 770 84 0

84. Haberman S. Landmarks in the History of Actuarial Science (up to 1919). March 1996. 62 Pages. ISBN 1 874 770 85 9

85. Renshaw A.E. and Haberman S. Dual Modelling and Select Mortality. March 1996. 30 Pages. ISBN 1 874 770 88 3

86. Booth P.M. Long-Term Care for the Elderly: A Review of Policy Options. April 1996. 45 Pages. ISBN 1 874 770 89 1

87. Huber P.P. A Note on the Jump-Equilibrium Model. April 1996. 17 Pages. ISBN 1 874 770 90 5

88. Haberman S and Wong L.Y.P. Moving Average Rates of Return and the Variability of Pension Contributions and Fund Levels for a Defined Benefit Pension Scheme. May 1996. 51 Pages. ISBN 1 874 770 91 3

89. Cooper D.R. Providing Pensions for Employees with Varied Working Lives. June 1996. 25 Pages. ISBN 1 874 770 93 X

90. Khorasanee M.Z. Annuity Choices for Pensioners. August 1996. 25 Pages. ISBN 1 874 770 94 8

91. Verrall R.J. A Unified Framework for Graduation. November 1996. 25 Pages. ISBN 1 874 770 99 9

92. Haberman S. and Renshaw A.E. A Different Perspective on UK Assured Lives Select Mortality. November 1996. 61 Pages. ISBN 1 874 770 00 X

93. Booth P.M. The Analysis of Actuarial Investment Risk. March 1997. 43 Pages. ISBN 1 901615 03 0

94. Booth P.M., Chadburn R.G. and Ong A.S.K. Utility-Maximisation and the Control of Solvency for Life Insurance Funds. April 1997. 39 Pages. ISBN 1 901615 04 9

95. Chadburn R.G. The Use of Capital, Bonus Policy and Investment Policy in the Control of Solvency for With-Profits Life Insurance Companies in the UK. April 1997. 29 Pages. ISBN 1 901615 05 7

96. Renshaw A.E. and Haberman S. A Simple Graphical Method for the Comparison of Two Mortality Experiences. April 1997. 32 Pages. ISBN 1 901615 06 5

97. Wong C.F.W. and Haberman S. A Short Note on Arma (1, 1) Investment Rates of Return and Pension Funding. April 1997. 14 Pages. ISBN 1 901615 07 3

98. Puzey A S. A General Theory of Mortality Rate Estimators. June 1997. 26 Pages. ISBN 1 901615 08 1

99. Puzey A S. On the Bias of the Conventional Actuarial Estimator of q_x. June 1997. 14 Pages. ISBN 1 901615 09 X

100. Walsh D. and Booth P.M. Actuarial Techniques in Pricing for Risk in Bank Lending. June 1997. 55 Pages. ISBN 1 901615 12 X

101. Haberman S. and Walsh D. Analysis of Trends in PHI Claim Inception Data. July 1997. 51 Pages. ISBN 1 901615 16 2

102. Haberman S. and Smith D. Stochastic Investment Modelling and Pension Funding: A Simulation Based Analysis. November 1997. 91 Pages. ISBN 1 901615 19 7

103. Rickayzen B.D. A Sensitivity Analysis of the Parameters used in a PHI Multiple State Model. December 1997. 18 Pages. ISBN 1 901615 20 0

104. Verrall R.J. and Yakoubov Y.H. A Fuzzy Approach to Grouping by Policyholder Age in General Insurance. January 1998. 18 Pages. ISBN 1 901615 22 7

105. Yakoubov Y.H. and Haberman S. Review of Actuarial Applications of Fuzzy Set Theory. February 1998. 88 Pages. ISBN 1 901615 23 5

106. Haberman S. Stochastic Modelling of Pension Scheme Dynamics. February 1998. 41 Pages. ISBN 1 901615 24 3

107. Cooper D.R. A Re-appraisal of the Revalued Career Average Benefit Design for Occupational Pension Schemes. February 1998. 12 Pages. ISBN 1 901615 25 1

108. Wright I.D. A Stochastic Asset Model using Vector Auto-regression. February 1998. 59 Pages. ISBN 1 901615 26 X

109. Huber P.P. and Verrall R.J. The Need for Theory in Actuarial Economic Models. March 1998. 15 Pages. ISBN 1 901615 27 8

110. Booth P.M. and Yakoubov Y. Investment Policy for Defined Contribution Pension Scheme Members Close to Retirement. May 1998. 32 Pages ISBN 1 901615 28 6

111. Chadburn R.G. A Genetic Approach to the Modelling of Sickness Rates, with Application to Life Insurance Risk Classification. May 1998. 17 Pages. ISBN 1 901615 29 4

112. Wright I.D. A Stochastic Approach to Pension Scheme Funding. June 1998. 24 Pages. ISBN 1 901615 30 8

113. Renshaw A.E. and Haberman S. Modelling the Recent Time Trends in UK Permanent Health Insurance Recovery, Mortality and Claim Inception Transition Intensities. June 1998. 57 Pages. ISBN 1 901615 31 6

114. Megaloudi C. and Haberman S. Contribution and Solvency Risk in a Defined Benefit Pension Scheme. July 1998. 39 Pages ISBN 1 901615 32 4

115. Chadburn R.G. Controlling Solvency and Maximising Policyholders' Returns: A Comparison of Management Strategies for Accumulating With-Profits Long-Term Insurance Business. August 1998. 29 Pages ISBN 1 901615 33 2