“An Empirical Analysis and Stochastic Modelling of Aggregate Demand Behaviour in a Spare Parts Inventory System.”

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Testing the Research Models

8.0 Introduction

We concluded chapter seven with the formation of the main theoretical models of aggregate item demand in terms of demand occasions and demand quantities. Through these models it was postulated that it is effectively the summation of the short term Log Series distribution of aggregate demand that leads to a discrete lognormal distribution as the stable long run model. Furthermore these aggregate LSD distributions themselves arise out of the Afwedson process, that operates in short time intervals across the family of parts. In this section we now turn to the issue of verifying these models against the empirical data. This is the process of retroduction as put forward by Simon. To test the theoretical models we have to demonstrate that the conditions for the Afwedson process can be shown to exist. That is the short term aggregate demand occasion is distributed as a Poisson distribution and that the short term aggregate demand quantity is distributed as the combined LSD - NBD distribution. Secondly we must show that these conditions can be seen in every successive time period and the summation of the outcomes lead to a lognormal form of demand volumes in the longer term.

8.1 Verification of the LSD distribution.

We showed in chapter six that the single period aggregate demand volume distribution for the first period 1979 was highly skewed, reverse 'J' shaped and it fitted the LSD distribution extremely well. We will now show that any subsequent single period following or preceding period one of 1979 also gives an empirical distribution of demand volume that is LSD distributed. And that a summation of any series of these single period distributions converges to a lognormal form. In the next chapter we will show by simulation that the summation of simulated LSD distributions also leads to the same integer lognormal distribution. In the table below is shown the aggregate distribution of demand for the first four periods 1979. The first period 1979 was shown in the previous chapter and we give the next four periods to show the close similarity and to verify that successive monthly aggregate demands are indeed LSD distributed.

Table 8.1

Single period demand frequencies 1979

demand quantity	first period	second period	third period	fourth period	fifth period
1-10	110	126	108	110	140
11-20	35	25	36	29	26
21-30	16	10	22	20	10
31-40	10	10	8	10	5
41-50	7	8	7	9	4
51-60	3	4	2	9	2
61-70	4	5	1	2	1
71-80	3	2	3	4	2
81-90	2	0	1	3	3
91+	0	3	0	1	2

The very close correspondence between the empirical distribution of the first period 1979 and the corresponding theoretical LSD distribution was shown in chapter six (tables 6.5 and 6.6 and figure 6.7). The Log Series distribution has the probability density function as previously shown namely-

$$P(r) = {}^{-q^x}/x\ln(1\text{-}q)$$

The parameter 'q' is best estimated as discussed in chapter seven by Ehrenberg's method using the mean of the empirical distribution 'w' ie-

$$\text{where } q = \frac{(w-1.4)}{(w-1.15)}$$

In the tables 8.2 and 8.3 below we show a more rigorous comparison of these single period empirical distributions with the LSD using periods two and five of 1979 as an example periods. The test used is the Kolmogorov Smirnov test discussed previously.

Table 8.2

LSD test for period two 1979

Actual frequency	theoretical frequency	actual cum. prop.*	theoretical cum. prop.*	difference
126	118	0.6631	0.6178	0.0452**
25	32	0.7947	0.7853	0.0094
10	16	0.8474	0.8691	-0.0217
10	9	0.9000	0.9162	-0.0162
8	6	0.9421	0.9476	-0.0055
4	4	0.9631	0.9686	-0.0055
5	3	0.9895	0.9842	0.0053
2	2	1.0000	0.9948	0.0052
0	1	1.0000	1.0000	0.0000

* These values were the cumulative frequencies of actual and theoretical calculated as proportions.

** This was the largest absolute value difference between the actual and theoretical cumulative proportions.

The theoretical maximum Dn values for the Kolomogorov-Smirnov test (with 'n' = 190) at the 1% and 5% levels are 0.0986 and 0.1182 respectively, compared to the actual maximum difference of

0.0452. Hence the result was highly significant and we can confidently accept the empirical distribution of period two as most likely a LSD distribution.

Applying the same procedure to period five 1979 we obtained the following results -

Table 8.3
LSD test for period five 1979

Actual frequency	theoretical frequency	actual cum. prop.*	theoretical cum. prop.*	difference
140	139	0.7239	0.7239	0.0000
26	24	0.8573	0.8489	0.0084
10	11	0.9086	0.9062	0.0024
5	6	0.9342	0.9375	-0.0033
4	4	0.9547	0.9583	-0.0036
2	3	0.9650	0.9739	-0.0089
1	2	0.9701	0.9843	-0.0142**
2	1	0.9804	0.9896	-0.0092
3	1	0.9957	0.9948	0.0009
2	1	1.0000	1.0000	0.0000

* These values were the cumulative frequencies of actual and theoretical calculated as proportions.

** This was the largest absolute value difference between the actual and theoretical cumulative proportions.

The maximum difference between the two cumulative distributions was again highly significant at both the 1% and 5% levels. Hence we can confidently conclude that in all probability the distribution of volumes in period five of 1979 is also LSD. Indeed all the single period aggregate demand volume distributions throughout 1979 gave similar highly significant results by the same test.

Now, as seen previously, it is the summation of these empirical distributions of 1979 that converge to an integer lognormal distribution of demand volumes. Therefore we now have very strong evidence that the summation of a series of similar, but independent, LSD distributions converge to an integer from of the lognormal distribution as the stable long run equilibrium distribution.

8.2 Verification of the NBD distribution.

The previous section verified that the aggregate demand quantities for all parts with a positive demand equal to or greater than unity in a single demand period is distributed as the LSD. ***Hence one of the three main conditions of the Afwedson model has been satisfied.*** We must now show that the distribution of demand volumes for all parts, whether there was a demand in the period or not for a particular spare, can be represented by the NBD model.

As shown previously through Poisson mixing the Negative Binomial distribution has the following 'gamma' form -

$$P(r) = (1+\frac{m}{k})^{-k} \frac{\Gamma(k+r)}{\Gamma(r+1)\Gamma(k)} \left(\frac{m}{m+k}\right)^{r}$$

for the probability of 'r' occurrences in fixed intervals of time. It exists for all positive integers 0, 1, 2, etc. and in general to 'r'. The two parameters of the distribution, mean 'm' and the exponent 'k', must be estimated when fitting empirical data. As shown before a parameter 'α' is sometimes used where 'α' = m/k. The best estimate of 'm' is the sample mean, which is unbiased and consistent, but the parameter 'k' must be estimated indirectly. The best and most efficient method is that given by Ehrenberg (1959, page 58) by equating the proportion of zero outcomes to the expected outcome, that is -

$$P(o) = (1 + m/k)^{-k}$$

This equation cannot be solved directly for '*k*', but values for the parameter can be readily obtained by iteration. This author developed simple iteration routines on a programmable calculator for this purpose. Thus to estimate '*k*' and $P(o)$ we must now take into account, in any appropriate sample, a representative proportion of spares parts with a zero demand in the period considered, but which are nevertheless still live demand spare parts. That is there must be a finite probability of demand in a reasonable period in the future. This raises a methodological problem of deciding what is and what is not still a live demand spare part. Some spares have such low demand probabilities that for all practical purposes they are dead items. To get over this problem this author arbitrarily decided that any spare part selected in a sample should have shown a demand of at least one unit in a following 12 month period, or a previous 12 month period. This then at least coincides with the period over which the overall model is ultimately being tested, ie annual period demand. For those parts with a demand probability that is substantially less than one per year it is difficult to determine if they are still effectively live or if the demand has really completely died away. It was considered that the criterion cut-off point used here was a rational standard to set. In theory any part on the parts listing could be demanded at some time, maybe just once in ten years, but this level of demand would for all practical purposes be regarded as a dead stock item.

There was also a sampling problem to resolve here because in general where sampling was used in data analysis 200 items were always sampled. If a 200 item sample was selected, including those live demand items, but with zero demand in that period taken into account, then only about 20% of items would show an actual demand value. The rest would be zero demands simply because the majority of the spares have very low demand volumes and most have a zero demand in any one single period chosen at random. Hence we could have ended up with a sample without much real value data in it. To get over this problem the samples used previously to test the LSD distribution were used giving about 200 positive values. Then, as shown below, the proportion of zero demands for live demand spares for the same period was determined so that the 200 positive values could have the correct proportion of zero demands added to them. The method of estimation of the parameters for the first period 1979 and the calculation of the proportion of zero demands are shown following as example-

From the ABC listing of 1979 the total number of live demand parts during the whole year was 9100 (i.e. those with a demand equal to or greater than 1 for the year). By counting, page by page, from a 'demand history' computer print out the number of parts with a live demand in period one of 1979 was 3,477 (from approximately 15,000 listed parts). Hence the total number of live demand parts, but showing zero demand in period one, was 5,623 (ie 9100 - 3,477); ie a proportion of 62% of the complete parts range. We also obtained a similar result by taking the total of parts known to have a live demand sometime in the year, ie 9,100, and expressing it as a proportion to the total listed parts, ie 15,000. This was a proportion of 0.61. From the empirical distribution of period one 1979, given previously, the total volume demanded was 3,180 for 193 different spare parts. But we knew from above that 62% of 'live' demands in period one were zero. Hence the 193 represented 38% of demands equal to or greater than one in the period. So 62% was equivalent to 315 zero demand parts in that period. Therefore we took our sample size for the first period to be 193 + 315 = 508. We then computed the mean for the period taking into account the zero demands.

Hence the mean = 3180/508 = 6.26.

Then $P(o) = 0.62$, the proportion of zero demands. Hence the parameter 'k 'was given by-

$$P(o) = (1 + m/_k)^{-k} \quad \text{i.e. } P(o) = (1 + 6.26/_k)^{-k}$$

from which it was calculated by iteration that $k = 0.12$. {One should note the small value of 'k ' which is one of the required conditions for the combined LSD/NBD model} Also the parameter 'α ' was therefore given as 6.26/0.12 = 52.16. Our next problem was to test the distribution of period one 1979 (including zero values) against a corresponding Negative Binomial distribution with the same parameter values. Developing NBD probabilities direct from the probability density function is very difficult. Fortunately a recurrence formula exists that can be used to generate the probability of 1, 2, 3, etc. from an NBD distribution if a value for $P(o)$ is available (see Ehrenberg 1972 page 59). The form of the recurrence formula is as shown below-

$$P_r = \left[\frac{a}{(1+a)}\right]\left[1 - \frac{(a-m)}{ar}\right]P_{r-1}$$

Hence we started with the calculated value of $P(o) = 0.62$ it was possible to determine any other value of Pr in succession. To achieve this a small programme was developed on a programmable calculator. (for later work an Excel spreadsheet was used) The calculated theoretical probabilities (using the recurrence formula) and associated theoretical frequencies are shown in the table following together with the empirical distribution for period one 1979-

Table 8.4
Empirical & theoretical distribution period 1 1979

Demand quantity	1st period 1979 actual	theoretical probability	theoretical frequency	difference
0	315	0.6546	332	17
1-10	110	0.2226	113	3
11-20	35	0.0505	26	-9
21-30	16	0.0262	13	-3
31-40	10	0.0159	8	-2
41-50	7	0.0105	5	-2
51-60	3	0.0073	4	1
61-70	4	0.0052	3	-1
71-80	3	0.0040	2	-1
81-90	2	0.0023	2	0
91-100	3	0.0012	1	-2

The close correspondence between the empirical distribution and the theoretical NBD can be readily seen from the above table. A Kolmogorov - Smirnov test between the theoretical and empirical distributions gave theoretical Dn values at 1% and 5% levels of significance of 0.0723 and 0.0603 respectively compared to an actual maximum Dn value of 0.0406, which is well within the acceptable range. Hence based on this test we can confidently regard the empirical distribution as being most likely NBD distributed. As with testing the LSD distribution the Chi Squared test was not used because it is just not suitable for such highly skewed distributions.

The same procedure as above was used to determine the number of zero demands in period five from which the actual NBD was then developed. This was then compared with the corresponding theoretical NBD with the same parameter values.

The mean for this distribution was calculated as -

$$3150/526 = 5.99$$

Thus with 0.62 as the proportion of zero demands the NBD parameter 'k' is given as before by iteration from-

$$P(o) = (1 + m/k)^{-k} \qquad \text{i.e. } P(o) = (1 + 5.99/k)^{-k}$$

from which it turned out that 'k' was 0.122 and therefore the parameter 'α' was given as :-

$$5.99/0.122 = 49.098$$

Thus using the recurrence formula as before and starting with $P(o) = 0.62$ the theoretical NBD frequencies were calculated as shown in the following table.

Table 8.5

Empirical & theoretical NBD frequencies for period 5 1979

Demand quantity	5th period 1979 actual	theoretical probability	theoretical frequency	difference
0	326	0.6380	332	-6
1-10	140	0.2370	123	17
11-20	26	0.0052	27	-1
21-30	10	0.0270	14	-4
31-40	5	0.0150	8	-3
41-50	4	0.0100	5	-1
51-60	2	0.0080	4	-2
61-70	1	0.0060	3	-2
71-80	2	0.0040	2	0
81-90	3	0.0020	1	2
91-100	2	0.0020	1	1

A Kolmogorov Smirnov test gave an actual Dn value of 0.019 against the 1% and 5% significance level Dn values of 0.0723 and 0.0603 respectively. Hence the result is, as in the previous case, highly significant and we must therefore conclude that the empirical distribution is most consistent with the Negative Binomial distribution. The closeness of fit was as good as shown in table 8.4, but the randomness of fit was more systematic due to the consecutive negative values in the last column.

These two example goodness of fit tests show us that aggregate demand distributions for four week periods in 1979 are very consistent with an NBD when the zero demands are taken into account. All the single demand periods for 1979 gave very similar results to those we have shown above. Furthermore we must recognise that each of these distributions also gave very close fits to LSD distributions. ***Therefore we can conclude that a second main condition of the Afwedson process model is now confirmed.***

8.4 Verification of the Poisson Occasions

In the following two subsections we principally address the question of testing the demand occasions model. The Afwedson compound model of demand quantity required that the underlying process of demand occasions should be Poisson. So also does the Poisson Gamma model with the additional requirement that the long run average of demand occasions be gamma distributed.

8.4 (a) single period demand occasions

The third condition that single period demand data must satisfy for the Afwedson model is that of Poisson demand occasion. That is in any single demand period the occurrence of demand across all items in the inventory range must be simple Poisson. Now we can regard this as

Poisson distributed in space whereas most Poisson distributions occur through time. In the latter situation it is time intervals that are chosen within which to capture events that will be tested for Poisson characteristics. The problem with determining Poisson events in space is to choose appropriate criteria for dividing the test area in suitable space segments within which to measure the occurrence of events. In observing a Poisson process taking place in time the time interval chosen must be small in relation to the total time being examined. Hence, if it were required to test if the arrival of customers to a supermarket was a Poisson process one would set a time interval of the order of minutes to study the arrival process over several hours. The botanist wishing to study if the distribution of certain wild plant species were Poisson distributed over a particular waste land he would divide the area up into many cells each of a few square feet in size to study a plot several hundred times larger.

Hence given some 9000 spare parts, each with the potential for an actual demand in any one period, the task was to decide on an appropriate method for selecting groups of parts upon which measurements could be taken to test for Poisson occurrence in any set period. It was arbitrarily decided to randomly select parts from the demand history report in groups of ten. This group size was considered to be very small in relation to the approximate number of live demand parts in the population (ie 9000). The only criteria used in this selection process for an individual part was to check that for any selected part number that there had been a demand for that part sometime in the following twelve periods to ensure that it was a spare part with a positive demand. This was considered sufficient to ensure that dead demand items were not selected.

Some 450 parts were randomly selected in this way from period one 1979 in groups of ten and the results are summarised below in table 8.6. The number 450 was arbitrarily chosen as a number large enough to be statistically significant, if the expected regularity existed, and to minimise the effects of random noise. From each member of each group of ten it was determined if there was a demand in that period or not. If there had been a demand it was regarded as a positive reading and given the value 1, (the actual value of the demand, eg 1, 2, 3, etc. was ignored). If there was no demand then it was given a zero value. The results are as shown in table 8.6 which follows.

Table 8.6
Number of Positive demand occasions

Value	0	1	2	3	4
Actual frequency	5	9	12	9	4
Theoretical frequency	4	10	12	9	3
difference	-1	1	0	0	-1

Value	5	6	7	8	9
Actual frequency	3	2	0	0	0
Theoretical frequency	3	1	0	0	0
difference	0	-1	0	0	0

The data in the above table then shows, for example, that from 45 groups of ten items sampled, three of the groups each showed five items with a positive demand, and two groups showed six items with a positive demand and so on. The very close correspondence between the actual frequencies and theoretical frequencies from a Poisson distribution with the same mean and variance is clearly evident. The mean of the actual distribution was 2.38 and the variance was also 2.38. Hence one of the main criteria for a Poisson distribution was satisfied ie the mean of the distribution is equal to its variance. The actual distribution also gave a highly significant Chi Squared test at the 1% level of significance. Hence one must conclude that demand occasions in a single time period are very definitely Poisson in nature. {note: although the distribution is skewed it is not markedly so and the Chi Square test can be validly used in this situation}.

The same process of checking for the Poisson occasion of demand was repeated for period five of 1979 with the results shown in table 8.7 which follows.

Table 8.7
Number of demand occasions

Value	**0**	**1**	**2**	**3**	**4**
Actual frequency	10	12	7	10	3
Theoretical frequency	7	13	12	8	4
difference	3	-1	-5	2	-1

Value	**5**	**6**	**7**	**8**	**9**
Actual frequency	2	1	0	0	0
Theoretical frequency	1	0	0	0	0
difference	1	1	0	0	0

The actual data gave a mean value of 1.87 with a variance of 1.94. (The mean was used to calculate the theoretical frequencies). Even though the variance is somewhat higher than the mean in this case the result is still significant at both the 1% and 5% levels as determined by a Chi Squared test against the corresponding theoretical Poisson distribution. The actual Chi Squared value was 5.24 against theoretical Chi Squared values of 13.28 and 9.49 respectively.

From the foregoing results we can be confident that the distribution of demand occasion across all parts in the DAF range during 1979 was simple Poisson. ***Thus the third main condition of the Afwedson process has been satisfied.***

From the foregoing analysis for the first and fifth periods of 1979 we have highly significant test results for the Afwedson process, namely

Poisson occurrence across all 'live' demand items, LSD distribution of demand quantities and an overall NBD distribution of all quantities including the zero demands for the period. Clearly a process entirely equivalent to the Afwedson process was operating in the first and fifth periods of 1979. Although not directly tested by all three tests, we can be totally confident that any single period in 1979 will yield the same results. (The Poisson demand occasion test was not repeated for all periods in 1979, but we were confident that the same results would have been obtained, especially as each single period in 1979 produced positive results for the LSD /NBD)

8.4 (b) multiperiod demand occasions.

Our model of aggregate demand occasions implies that the long run average of demand occasions across all parts should be Gamma distributed. To verify this we initially took groups of ten parts in clusters as above, but this time we determined the number of demand occasions that appeared in 15 consecutive demand periods instead of a single period. The mean and variance was measured for each cluster and in all cases the variance was found to be substantially larger than the mean. Thus for the single period case we found a simple Poisson process of demand occasion to be operating, but in the 15 period case clearly the simple Poisson was no longer appropriate. The observed increase in the variance of demand could only have come about as a result of mixing in the process, as we were observing demand occurrences in this analysis and not demand quantities. Hence there could not have been any increase in the variance due to compounding of demand quantities. Furthermore, the model we have developed in chapter seven shown in figure 7.5, predicts that the long run average mixing equation in the process should be a gamma variate.

In view of the increased variance results above we needed to test the proposition that long run Poisson mixing was occurring by a gamma variate. Hence 200 parts were selected at random and the average demand occurrence was measured for each over a 15 period duration. These averages were analysed as a data set and tabulated against a theoretical gamma distribution as follows-

Table 8.8
Gamma mixing test

demand occasion	Gamma distribution	empirical distribution	difference
1	21	28	-7
2	28	30	-2
3	29	26	3
4	26	23	5
5	22	17	-3
6	18	15	3
7	14	14	0
8	11	8	3
9	9	10	-1
10	7	6	1
11	5	7	-2
12	4	6	-2
13	3	4	-1
14	2	3	-1
15	1	4	-3
16	1	1	0
			Σ= −2

(In the above table all the empirical averages were multiplied by a constant 15 for convenience of data presentation). The empirical data in the above table gave a mean value of 5.2134 with a variance of 13.6006 The values of the corresponding gamma variate (integer values only) were determined from the gamma function as shown below:

$$f(x) = \left(\frac{x}{b}\right)^{c-1} \left[\frac{\exp(-x/b)}{b\Gamma(c)}\right]$$

The parameters '*c*' and '*b*' of this function were determined from the mean (x) and variance (σ^2) of the empirical data by the matching moment functions-

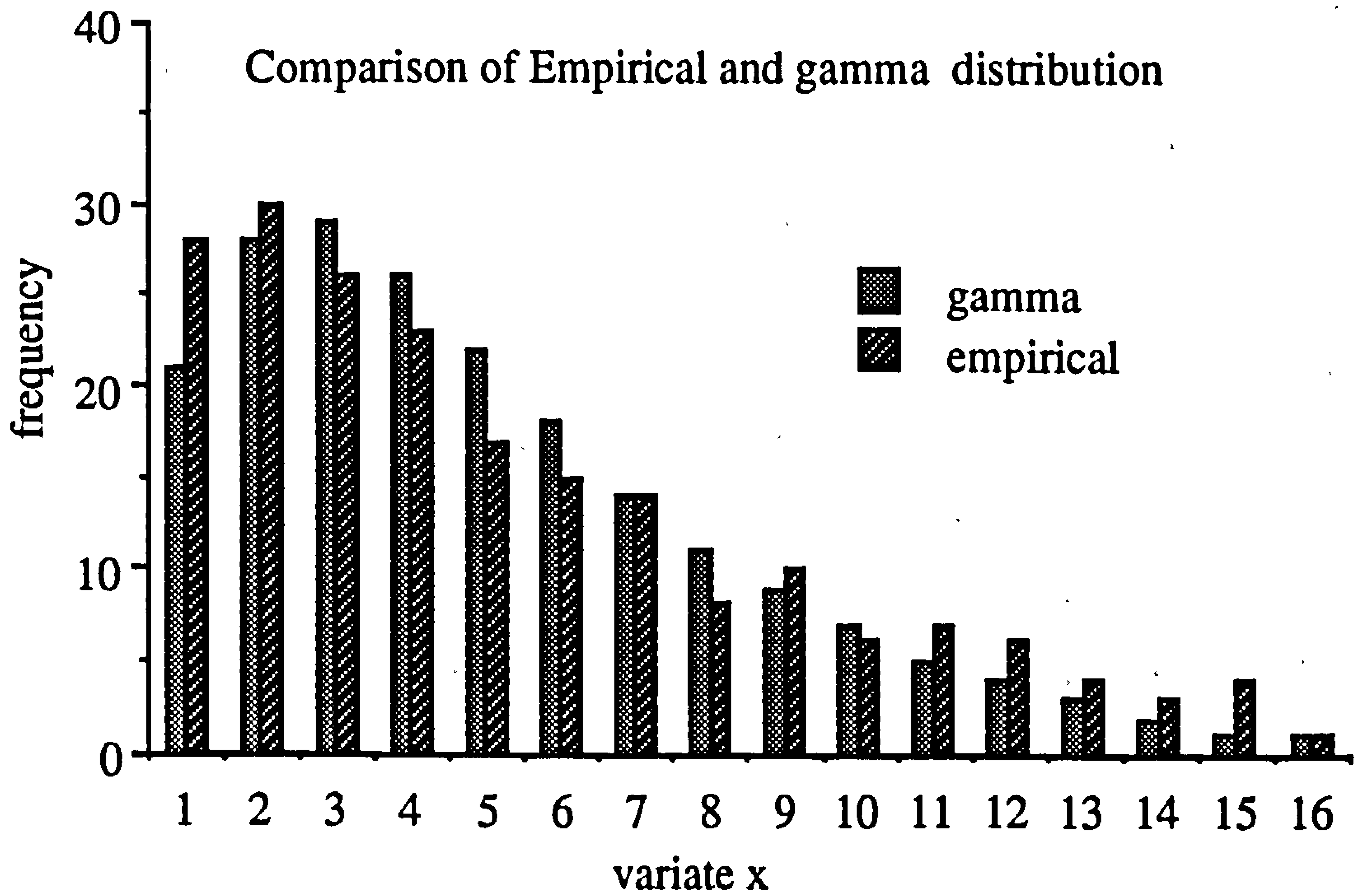
$$c = \left(\frac{\bar{x}}{\sigma}\right)^2 \quad \text{and} \quad b = \frac{\sigma^2}{\bar{x}}$$

from which we obtained '*c*' = 1.994 and '*b*' = 2.6088. These matching moments functions are those recommended by Hasting and Peacock (1974) as the most efficient estimators of '*c*' and '*b*'. Because we were only interested in integer values of the gamma variate and because the value of '*c*' was so close to unity we made use of the relationship shown :

$$\Gamma(c) = (c-1)! \qquad \text{when } c \text{ is integer}$$

A graphical comparison between the empirical and theoretical gamma distribution is shown below in figure 8.1.

Figure 8.1



Goodness of fit tests were not performed on the empirical data against the gamma variate because we used only integer values of the gamma function, which is essentially a continuous distribution. This did not invalidate the use of the function in this case, and the close correspondence between the aggregate means of demand occasions and a gamma function is readily seen from figure 8.1. The randomness of fit between the two distributions, as shown in table 8.8 was rather poor as can be judged by the successive positive and negative differences. The closeness of fit was quite good as given by the sum of differences at -2. However, perhaps the most important aspect is the fact that the empirical distribution is of the right general form (ie gamma) that the theory predicts should be seen, if gamma mixing is taking place in the aggregate demand occasions. And this is what all the empirical evidence has so far shown. Hence we should conclude that in all probability the long run average values of demand occasions aggregated across all parts are gamma distributed.

From all the foregoing analysis we can be sure that the combined Poisson Gamma process and the Afwedson process explain fully the regularity seen in demand volumes for any single (four week) period throughout 1979. We have shown that the combined LSD/NBD explains aggregate demand quantity in short time periods (four weeks in the DAF data), furthermore we have now also shown that when we consider demand occasion then the simple Poisson model explains the single period aggregate demand. In longer time periods (15) the gamma distribution is a strong candidate for the demand occasion. **All these findings are consistent with and are predicted by the theoretical models that we developed in chapter seven.** Our attention now turns to examining subsequent years to verify that 1979 was not a year in isolation.

8.5 Model testing in Years 1983 and 1985.

8.5 (a) tests for the Log Series distribution

We first consider single period empirical distributions for periods five through to seven of 1983, as example periods for that year. (In fact the periods were chosen quite arbitrarily). These are shown in the table below together with the theoretical LSD with a 'q' parameter value of 0.986.-

Table 8.9

Empirical and theoretical LSD for 1983

demand quantity	5th period	6th period	7th period	theoretical distribution
1-10	121	122	123	124
11-20	24	30	28	24
21-30	12	4	8	12
31-40	5	7	4	7
41-50	5	7	5	5
51-60	2	2	3	3
61-70	2	1	2	3
71-80	3	3	3	2
81-90	2	4	2	1
91-100	2	2	1	1
101-110	2	1	1	1
111-120	1	1	1	1
121-130	1	0	1	1
131-140	2	0	0	0
141-150	0	1	1	0
151-160	1	0	0	0
distribution mean	19.84	18.56	18.64	
empirical 'q' value	0.987	0.987	0.986	

By comparing the fifth period empirical LSD against the theoretical LSD we obtained a maximum Kolmogorov Smirnov value Dn actual at 0.0280 compared to theoretical values of $Dn_{0.01}$ and $Dn_{0.05}$ of 0.120 and 0.100 respectively. Periods six and seven were so close in parameter values to period five it was considered not necessary to test them directly against the same theoretical LSD.

Table 8.10 which follows shows the same analysis for 1985. [The periods one, four and nine of 1985 were arbitrarily chosen].

Table 8.10

Empirical and theoretical LSD for 1985

demand quantity	1st period	4th period	9th period	theoretical distribution
1-10	120	128	121	129
11-20	25	29	27	25
21-30	18	12	12	13
31-40	6	9	9	8
41-50	5	4	6	5
51-60	5	6	6	4
61-70	4	1	4	3
71-80	2	3	2	2
81-90	2	0	2	2
91-100	3	2	3	2
101-110	2	2	1	1
111-120	1	1	2	1
121-130	1	1	2	1
131-140	1	0	0	1
141-150	1	1	1	1
151+	1	2	2	2
distribution mean	19.031	16.457	19.293	
empirical 'q'	0.986	0.983	0.986	0.986

An examination of the empirical distributions in the table above against the corresponding theoretical LSD distribution reveals a very close fit that is consistent with the same analysis for earlier years. These empirical distributions in table 8.10 are also very close in form to the empirical distributions of 1983 in table 8.9. We can deduce from this that over the two year period the from 1983 to 1985 there has been very little change in the profile of the aggregate distribution of demand volumes. Intuitively we would expect to see this if the inventory range is stable.

8.5 (b) testing for the Negative Binomial distribution.

Using the methods described previously in section 8.1 we first made an estimate of the total number of individual parts demanded in 1983 from a print out of the demand history file for all parts (by scanning row by row page by page). The number was 10,627 different part numbers were demanded in the year, hence for any individual period we assumed that the number of live demands was this same value. Next we had to estimate how many actual demands there were in the target period (p5), by counting down the column for period five, and then subtract this number from the total demands to get an estimate of the proportion of zero BUT live demands in the period. This gave us $P(o)$ the probability of zero demands in the period. The value, surprisingly came out at 0.62, the same proportion as for 1979 ! Using this value we were then able to calculate the empirical NBD frequencies for period five 1983 as shown in the table 8.10, below together with the corresponding theoretical NBD frequencies. The latter were calculated form the recurrence formula given previously starting with $P(o) = 0.620$

Table 8.11

Empirical & theoretical NBD for period 5 1983

demand quantity	5th period actual	theoretical distribution	difference
0	298	332	17
1-10	121	113	3
11-20	24	26	-9
21-30	12	13	-3
31-40	5	8	-2
41-50	5	5	-2
51-60	2	4	1
61-70	2	3	-1
71-80	3	2	-1
81-90	2	2	0
91-100	2	1	2
	Σ=481	Σ=481	

The maximum Kolmogorov Smirnov Dn value obtained from a comparison of the empirical and theoretical distributions was 0.0287 compared to $Dn_{0.01}$ and $Dn_{0.05}$ of 0.0743 and 0.0620 respectively. Hence the result is highly significant and we have no grounds to reject the null hypothesis. In all probability the overall distribution of period five 1983 is Negative Binomial. We have not tested other periods in the same year, but based on the consistency of the data over periods five, six and seven and the very close similarity with 1979 we can be confident that the NBD would be obtained for any period throughout 1983.

This same process of testing for the NBD was repeated using period four of 1985 as the target single period, chosen at random from the three periods in table 8.10. Because of the stability of the LSD over several years we used the value of 0.62 for the proportion of zero demands in any given four week period in 1985, ie P(o) =0.62. (If this had turned out to be a poor judgement then it would have led to a worse fit of the NBD rather than better). On this basis we were able to calculate the mean demand volume for 1985 period four and subsequently the parameter 'k' of the NBD. Then using the recursive relationship used previously as shown in section 8.2 the theoretical values for the NBD were readily determined. These are shown in table 8.12 below with the empirical frequencies. We can readily see from this tabulation the very close fit between the empirical and theoretical distributions. The match is very good on both the basis of randomness of fit and closeness of fit. A Kolmogorov Smirnov test was not performed, because on visual evidence alone it was quite clear that a very significant result would have been obtained.

Table 8.12

Empirical & theoretical NBD for period 4 1985

demand quantity	empirical distribution	theoretical distribution	difference
0	325	338	-13
1-10	128	115	13
11-20	29	27	2
21-30	12	13	-1
31-40	9	8	1
41-50	4	5	-1
51-60	6	4	2
61-70	1	3	-2
71-80	3	2	1
81-90	0	2	-2
91-100	2	1	1
101-110	2	1	1
111-120	1	1	0
121-130	0	1	-1
131-140	1	1	0
141-150	0	0	0
	Σ=523	Σ=522	Σ= 1

8.6 Verification of the Law of Proportionate Effect.

Having established that the Afwedson process clearly operates in short time intervals we now show that the mechanism that governs growth and convergence to lognormality is the Law of Proportionate effect. We have seen empirically that by a simple summation of short period aggregate demands the empirical data did converge to lognormality as the stable long run distribution. We now look to the above law to explain the growth process. We first reconsider some of the important theoretical elements.

If the Law of Proportionate Effect governs the growth of a variate such as inventory usage values then, as previously seen, this can be shown in the form as shown -

$$(x_t - x_{t-1}) = (x_{t-1})\xi_{i,t}$$

Where x_t is the size of the variate at 't', x_{t-1} is the size in the previous period and $\xi_{i,t}$ is the set of randomising elements.

The above form of the law lends itself to a number of testable hypotheses so that empirical data can be tested for evidence that the law is governing the growth processes seen. Firstly the change in the size of a variate subject to this law should be a random proportion of the variate's size in the previous period. Secondly Singh and Whittington (1974) have shown that there should be no serial correlation in growth rates from one time period to the next. This also follows from the form of the law given earlier in chapter four, namely that-

$$x_n = x_0(1+\varepsilon_1)(1+\varepsilon_2) \text{ ---------- } (1+\varepsilon_n)$$

Therefore the size of the variate after 'n' stages is independent of the initial size and each step is a random independent increment. Furthermore Ijiri and Simon(1974, page 145) and others, have shown that if the Law is valid there should exist a relationship between the logarithms of the initial and final size such that-

$$\log_e x_n = \beta \log_e x_0 + \sum_{n=0}^{n=j} \varepsilon_n$$

this follows from the form below as previously given in chapter four.

$$\log_e x_n = \log_e x_0 + \varepsilon_1 + \varepsilon_2 + \text{-------} \ \varepsilon_n$$

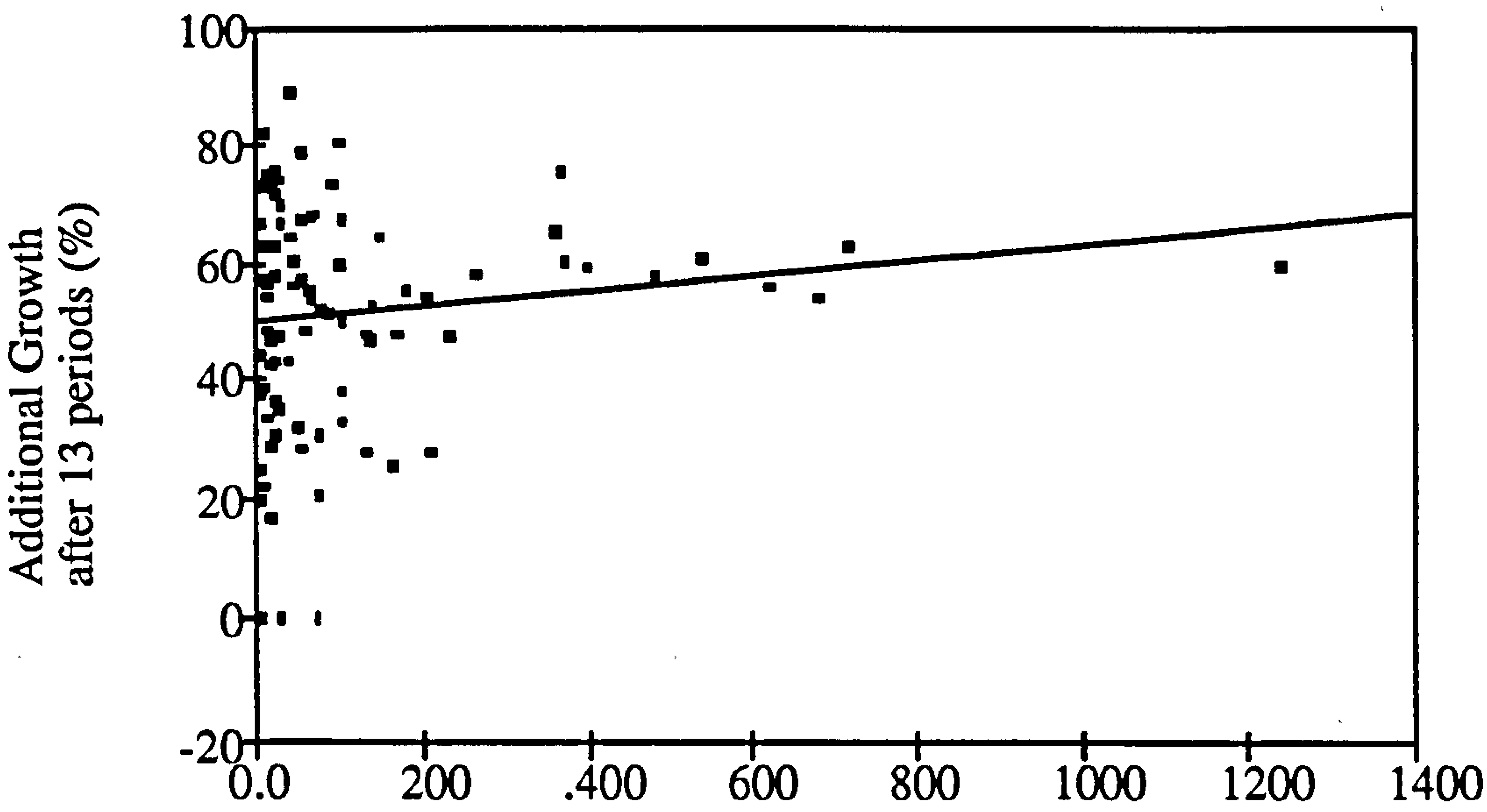
Hence a regression of the log size attained at time 't' should correlate strongly with the size attained in an earlier period say 't-1', with a regression coefficient of 1 and the residuals should be homoscedastic, that is to say the dispersion of the residuals about the regression line should be the same throughout the range of the data and they should have a zero mean.

8.6 (a) LPE growth test

The regression to test the first condition that there should be random growth was conducted on 100 randomly selected parts from the DAF data of 1979 and 1985. The cumulative demand after six months was taken for each part and then the additional growth over the next six months was expressed as a percentage of the growth at the six month point. This percentage growth rate was then regressed against the six month cumulative demand. The results from these regressions are shown below from which it can be seen that there was clearly no discernible relationship between growth and starting size in either year.

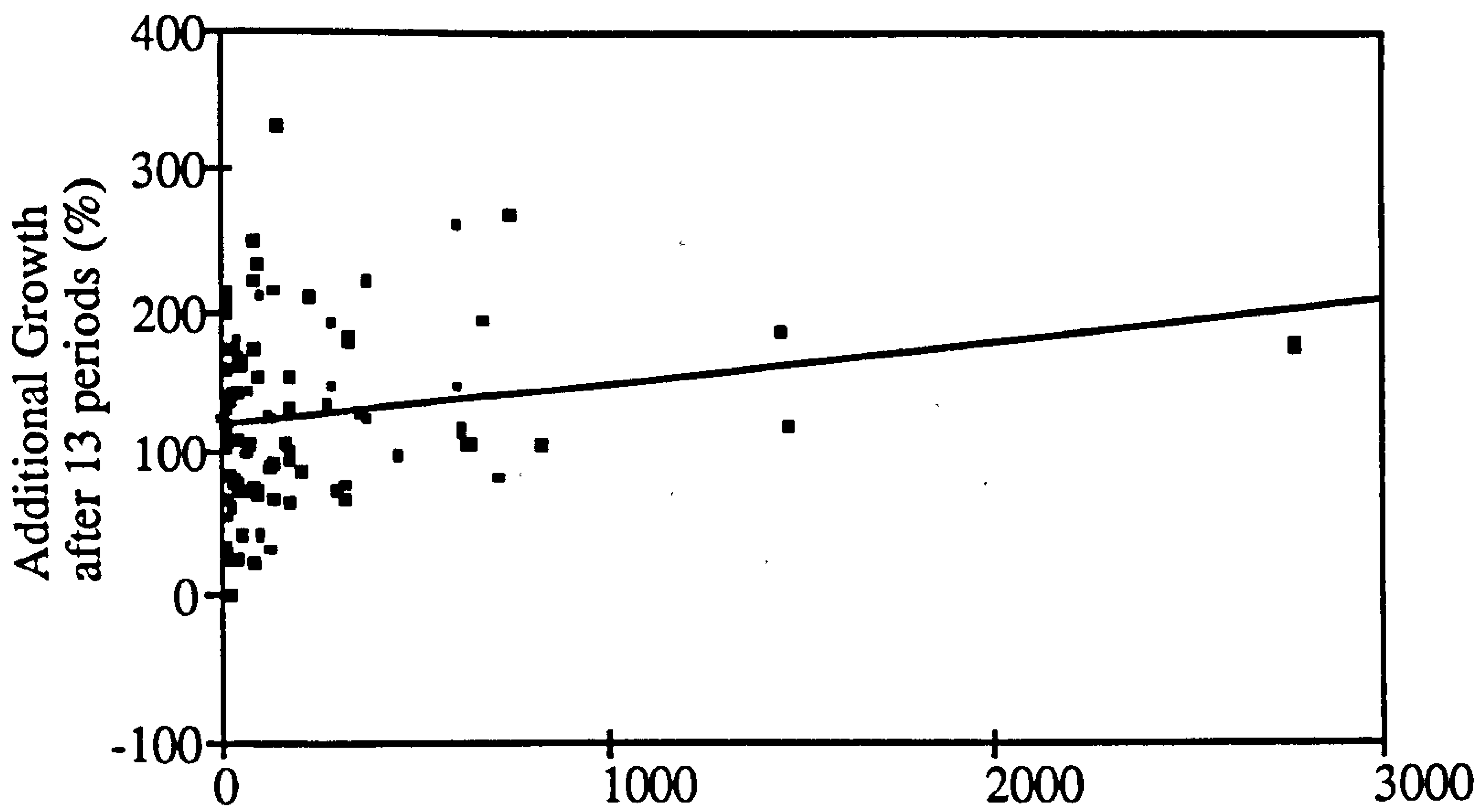
Figure 8.2

1979 LPE Growth Test



Cumulative Demand after six periods

1985 LPE Growth Test



Cumulative Demand after 6 periods

The analytical results of these regressions are as shown-

1979	**1985**
r = 0.134	r = 0.172
R^2 = 0.018	R^2 = 0.030
Standard error = 19.019	Standard error = 65.13

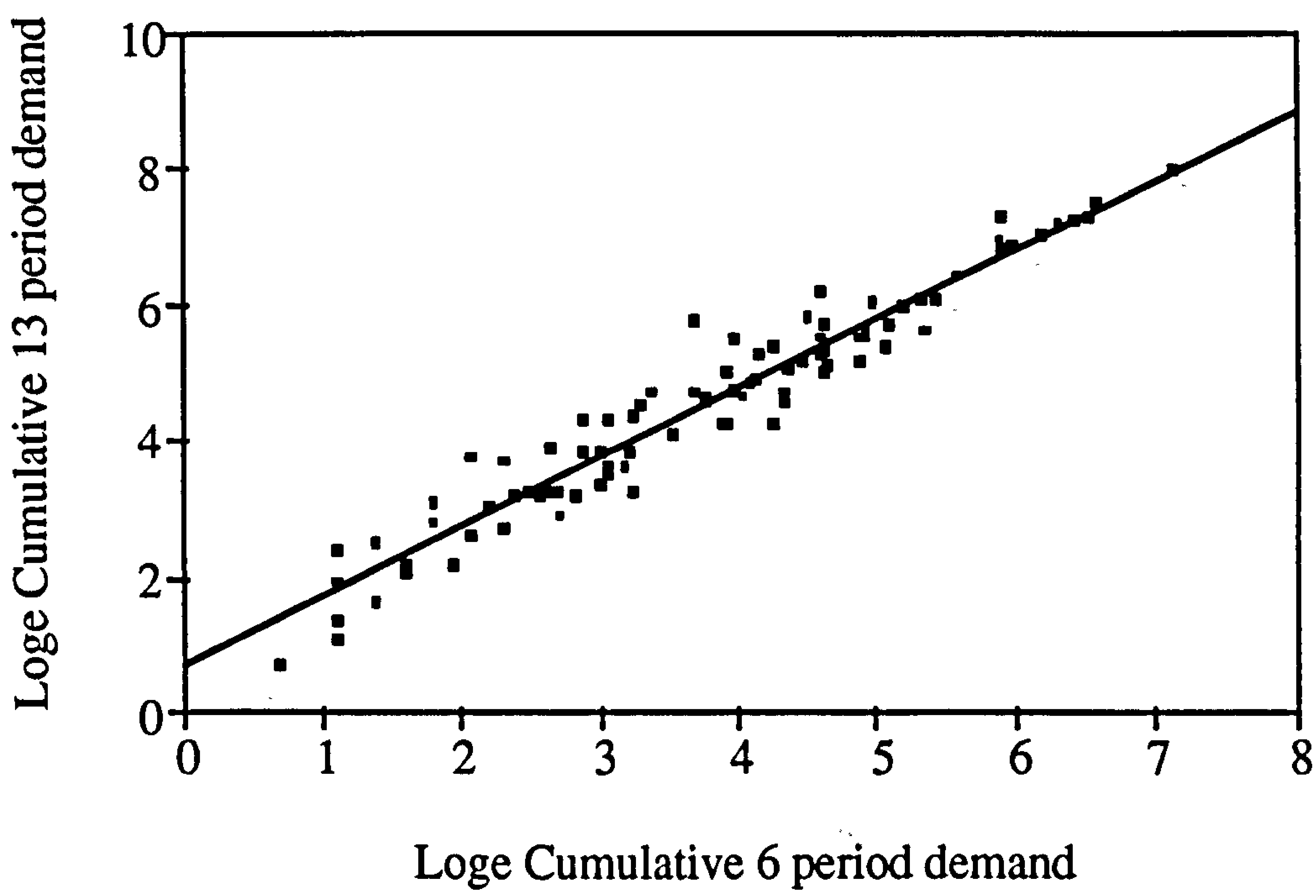
In both cases we can see from the above results that there is clearly no relationship between cumulative demand volume at six periods with the percentage demand growth over the next six periods. From this we can conclude that demand growth is truly random from period to period. Furthermore this being the situation then we should expect to see a regression relationship when the logarithms of volume growth at one period are related to the logarithm of additional or attained growth in a subsequent period. This is tested in the following section.

8.6 (b) regression of logarithms

To test the proposition that the logarithm of the size at time t is proportional to the logarithm of size at an earlier time say t-1 was tested by regressing the cumulative demand at 13 months with the cumulative demand at 6 months for the same two sets of 100 randomly selected parts as in the previous test. The results were as shown in the following figures :

Figure 8.3

Regression of the logarithms of Demand



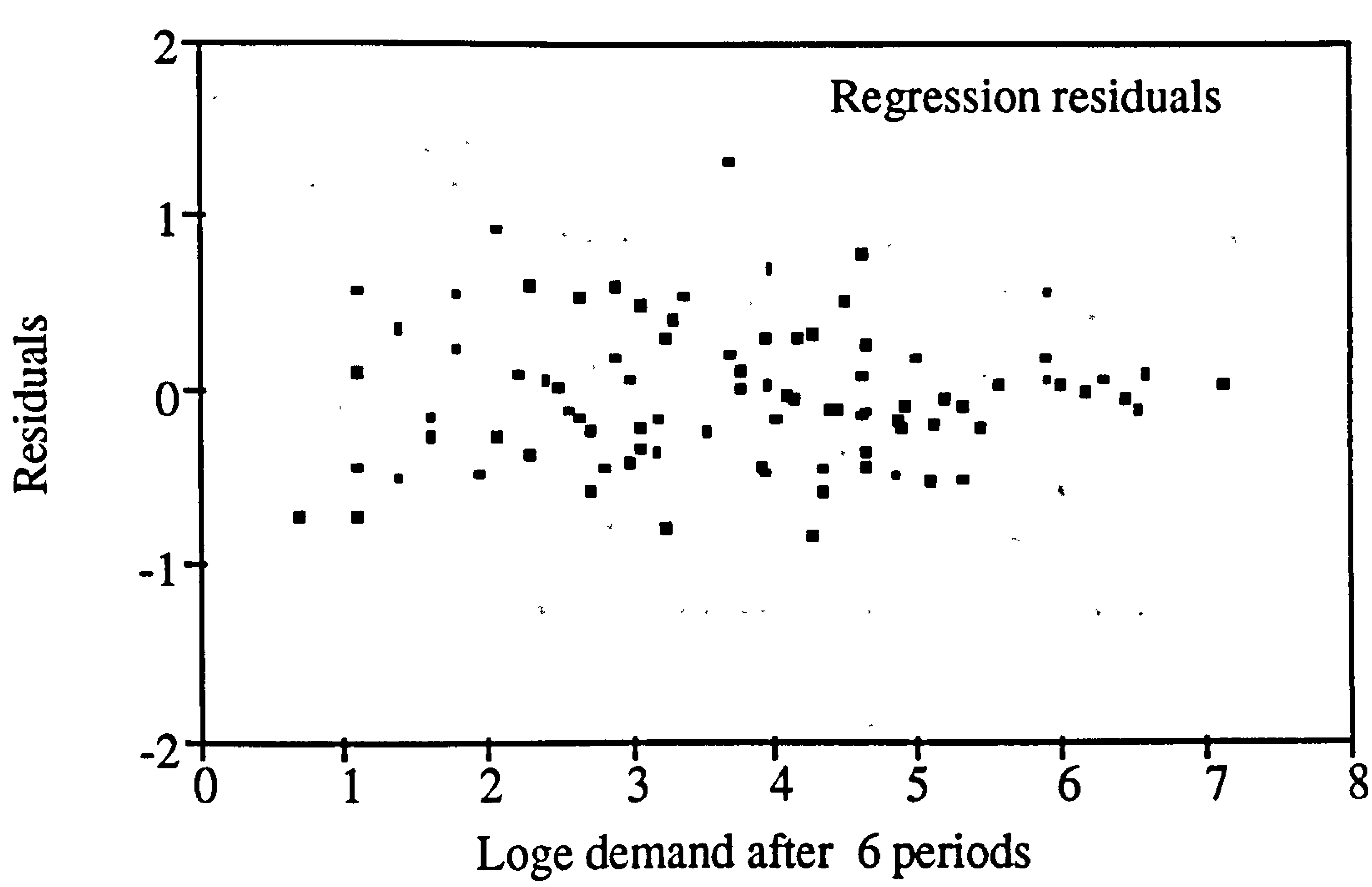
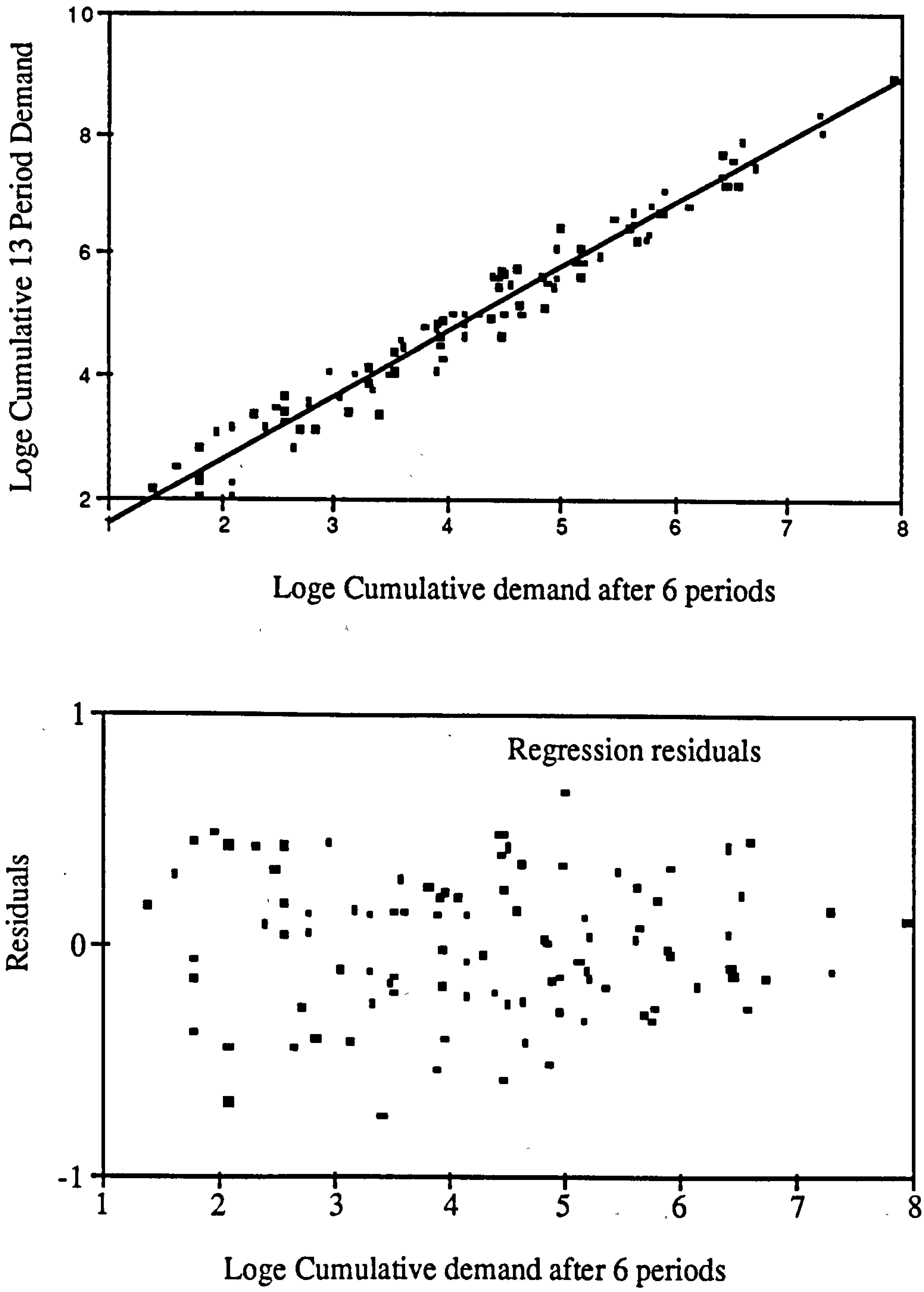


Figure 8.4

1985 Regression of the logarithms of Demand



It can be clearly seen from the above graphs that in both cases (1979 and 1985) the regression of the $\log_e$ of the cumulative demand at 13 periods is strongly related to the $\log_e$ of cumulative demand at 6 periods. The values of the correlation coefficient and coefficient of determination are highly significant as shown below. The regression residuals in both cases can also be seen to be randomly scattered around a zero mean and both sets visually pass as homoscedastic, (evenly scattered and distributed around a zero mean).

1979	**1985**
r = 0.966	r = 0.983
R^2 = 0.932	R^2 = 0.965
Standard error = 0.407	SE = 0.295
Durbin Watson statistic = 2.018	DW = 1.511

The regression equations produced by this analysis were as follows-

1979 - $$\log_e S_{i,t} = 0.698 + 1.026 \log_e S_{i,t-1}$$

1985- $$\log_e S_{i,t} = 0.591 + 1.042 \log_e S_{i,t-1}$$

Where $S_{i,t}$ is the size of element i in period t and $S_{i,t\text{-}1}$ is the size in the previous period.

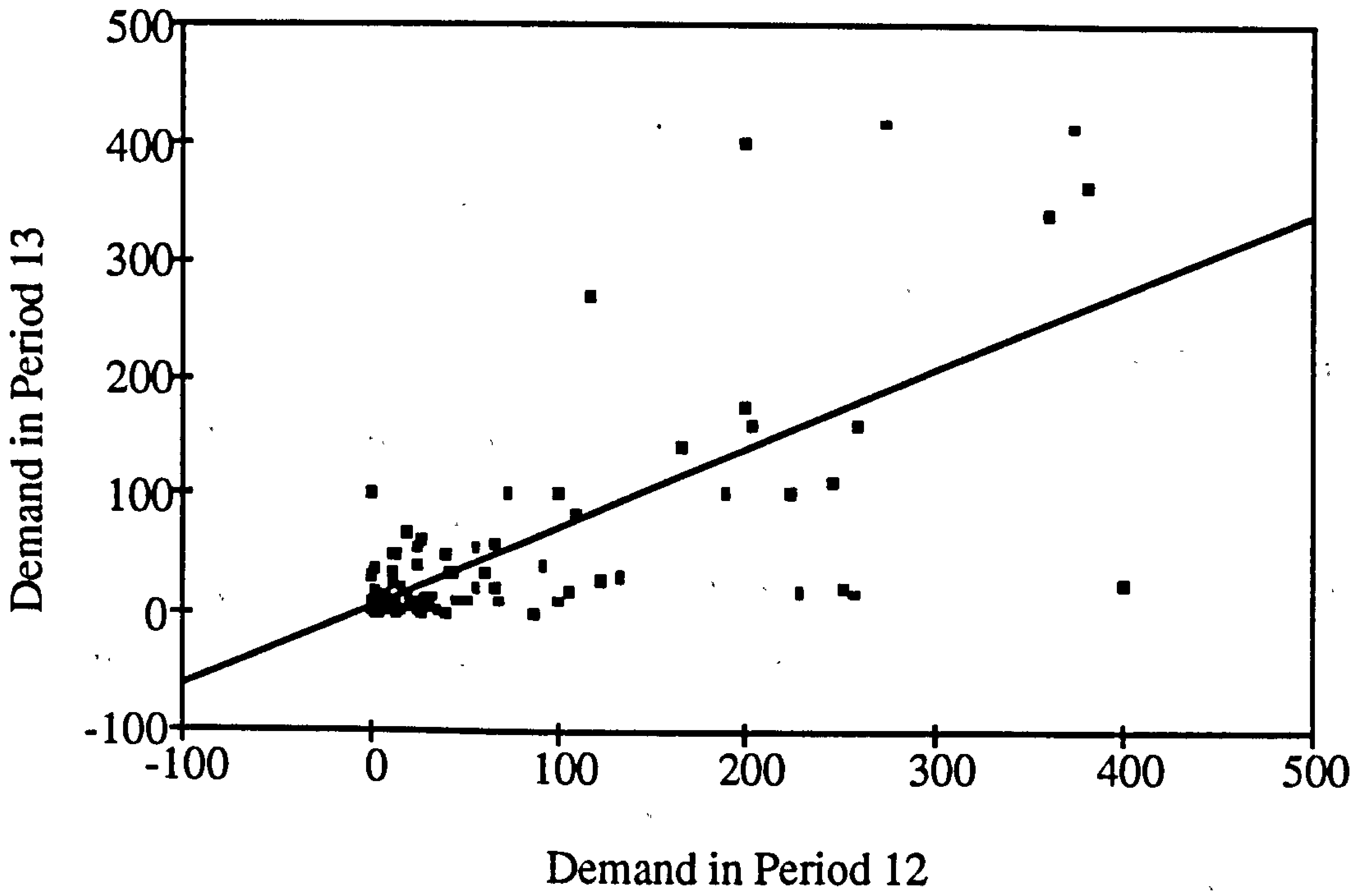
The regression coefficient 'b' in both cases was not significantly different from unity as measured by the 't' ratio test. Both correlation coefficients were also highly significant. Hence we can conclude from this analysis that the relationship between the two log sizes is highly significant and that the regression lines have a slope of unity as predicted by the Law of Proportionate Effect

8.6 (c) test for serial correlation

This test was conducted by regressing the demand in a randomly chosen four week period with the demand in the following four week period for 100 randomly selected parts. These results are shown -

figure 8.5

Test for Serial Correlation



This regression gave the following results

r = 0.713

R^2 = 0.503

Standard error =64.01

These values are statistically significant, although the explainable variation is only 50%. Hence there is a degree of serial correlation in the data. The strongest form of the Law of Proportionate Effect requires there should be <u>no serial correlation</u> in successive time periods. In view of the strength of the two previous tests we could not use this serial correlation test to reject the proposition that the law is valid and is operating in our DAF demand volume systems. We can conclude that based on prior theory, and the strong outcome of the growth correlation between the logarithms of demand volumes, that the Law of Proportionate effect is clearly applicable to the system, but not possibly in its most stringent form.

8.7 Conclusions.

We have demonstrated in this chapter that the Afwedson model clearly applies to aggregate item demands. This has been verified by showing that single period demand occasions are clearly Poisson, and when the actual quantity demanded is taken into account the distribution is LSD. Additionally when we take into account the zero demands in any period, but which are nevertheless still live demand items, then the overall distribution of short period demand quantities is that of the NBD. Additional evidence for the demand occasions model is given from the fact that mixing is shown to exist in the process, because the average values of the long run demand occasions are distributed in a form very close to a gamma distribution. Because the short period distribution of demand occasions was simple Poisson the increased variance which appears in the long run must have come about by mixing.

From the foregoing we can now confidently accept the hypotheses of chapter seven (a) through to (d) as correct and proven.

We have also demonstrated that the growth process which in stochastic terms accounts for the convergence to lognormality is by the Law of Proportionate Effect, although this may not be operating in its strongest form. The existence of a degree of serial correlation in the data for demand quantity in successive time periods does suggest the application of the law in a somewhat less stringent form. This does not however invalidate the Law of Proportionate Effect as the mechanism which still provides us with the most likely candidate to explain the growth to lognormality.

Hence we can now also accept that hypothesis (e) in chapter seven, that convergence to lognormality is by the Law of Proportionate Effect is most likely correct, but not in the most stringent form of the law.

At this stage if we reconsider our research model scheme given by figure 2.2 in chapter two, page 56, we can see that we have now satisfactorily utilised all three comparison points shown in the model to validate the various models and processes. In the next chapter we turn to simulation as a means of generating simulated data from our developed stochastic model.

Simulation Studies

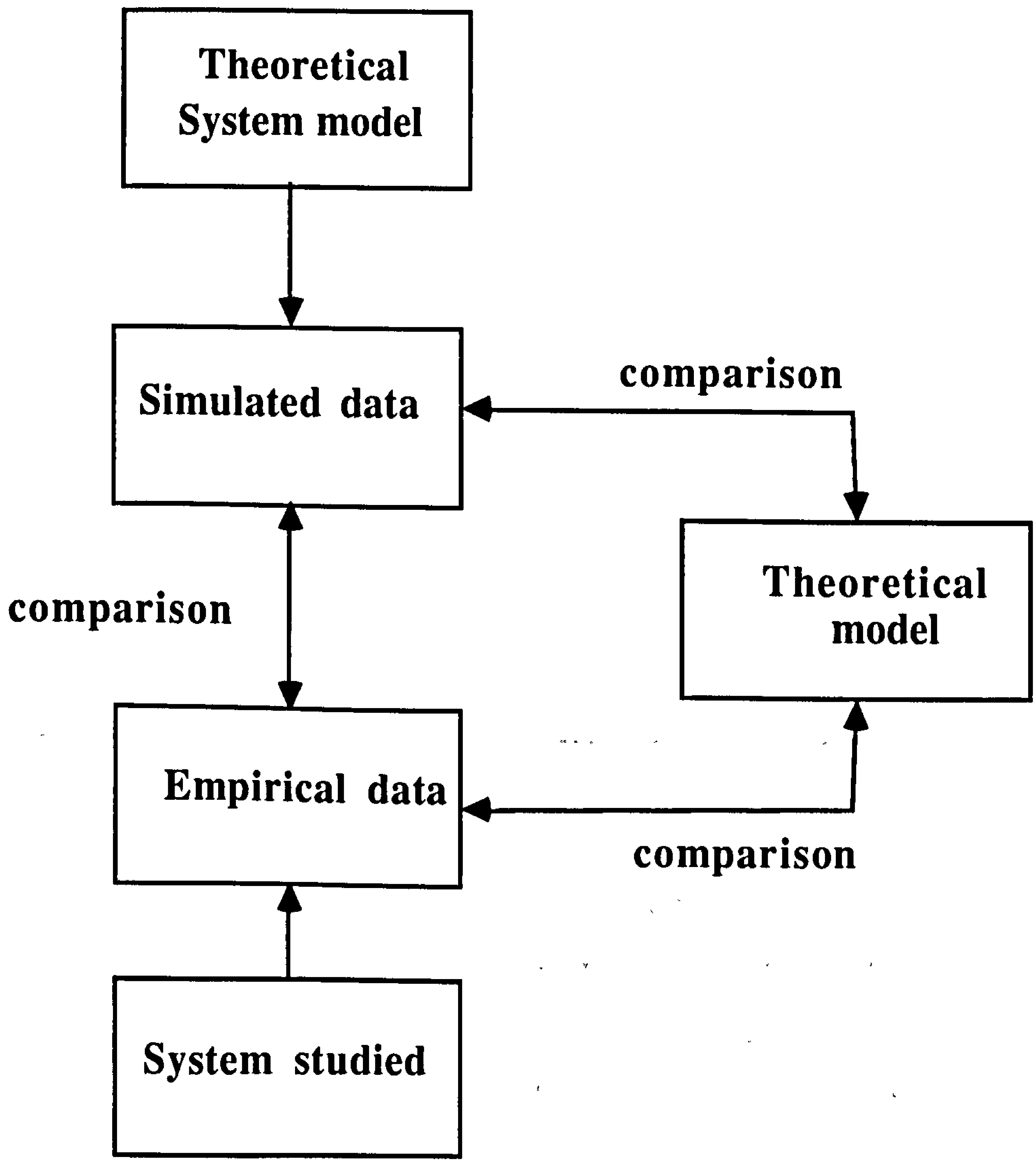
9.0 Introduction

The primary purpose of the simulation work described in this chapter was to validate the results obtained from the empirical analysis of chapter six and the model building and testing of chapters seven and eight. A secondary aim was to see what changes, if any , might be seen in the final distributions obtained by variations in the parameters of the system. It was not intended to explore all possible simulation combinations, this would have been potentially very large and not productive in the light of our purpose. Our objective in this chapter was to use just limited simulation studies as a validation and a check against previous results. It was also the intention to explore for insights to limiting conditions in the convergence processes to lognormality. Using this approach we were able to show that usage volume data almost identical with the empirical data seen in the DAF systems could be achieved. The distributions obtained had parameter values very close to those of the empirical distributions.

Replication and validation by simulation was considered an essential part of the methodology in this research. In particular this author was anxious to test the proposition that a sequence of Log Series distributions of aggregate demands will sum to a discrete form of the lognormal distribution using an appropriate model for the individual demand streams. The results so obtained were compared with the empirical data and appropriate theoretical models to see if the simulated models gave results close to those obtained from the empirical analysis. Two main thrusts were explored, firstly to see how the form of the distributions obtained varied with a change in LSD parameter 'q', and secondly to examine the effect of variations in the distribution of individual demand streams, principally the variance of demand.

The outline model of this part of the research, as presented and discussed in chapter two, is given again here to show schematically our aim -

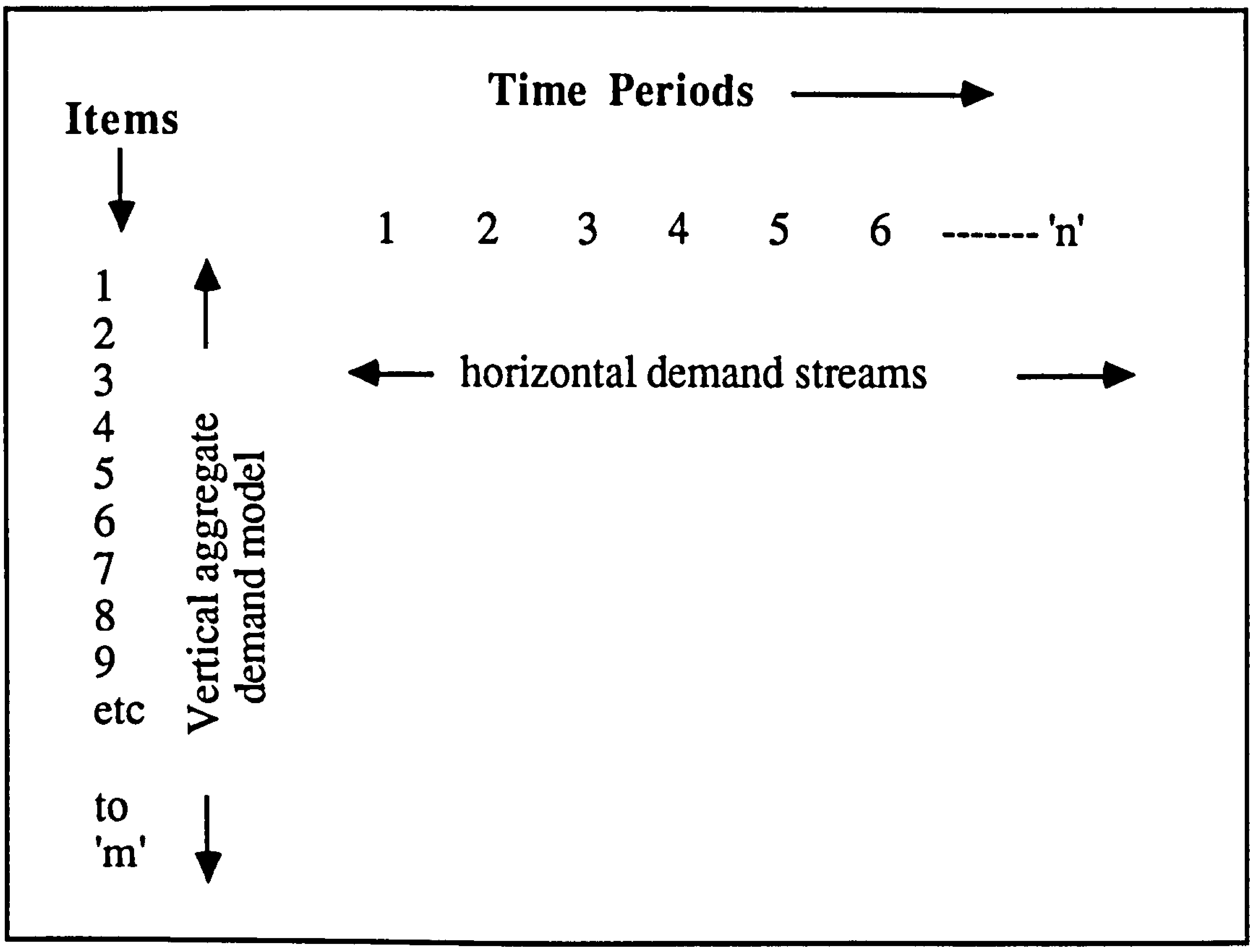
Figure 9.1
Scheme for Testing Empirical Data



The process of the simulation is shown in the following table-

Figure 9.2

Simulation table layout



Clearly from the work of chapters seven and eight the vertical distribution model of aggregate demand in the above table had to be the LSD. The problem to resolve was which distribution to use to model individual the horizontal demand streams. From empirical analysis of the DAF spares it was noted that the variance of demand was always greater than the mean, hence a simple Poisson stream would not be appropriate to replicate the empirical process. The literature previously reviewed on lumpy demand items, including spare parts, strongly supports the use of compound Poisson models to represent demand in unit time periods. As we are only concerned with demands in fixed interval of times we do not have the problem of considering demand volumes in any variable lead times. Hence, to use Bagchis' terminology (1987) we need only consider two main variables, namely the order intensity (OI) - the number of customers ordering in the period, and the order size (OS) - the amount ordered on each occasion. From our consideration in chapter five of the extensive literature on the modelling of independent demand processes, particularly that concerned with slow moving and lumpy demand items (two particular characteristics of spare parts demand) it is clear that compound distributions are the appropriate ones to use. In particular the 'Stuttering Poisson' by compounding the Poisson order intensity with geometric order size and the 'Negative Binomial' by compounding with a Log Series order size.

From work and research reported in the operational inventory literature the Stuttering Poisson has been a favoured compound Poisson model; whereas from reported work in the consumer purchase literature it is the NBD model which predominates. (But formulated by Poisson-gamma mixing to yield the gamma form). In appendix one of this work we report some limited analysis on period demands for selected DAF demand data and the results from this indicate that either the Negative Binomial or the Stuttering Poisson could be used. Both distributions gave extremely close and successful results in modelling DAF demand data. In chapter ten we consider the individual demands for a variety of spare parts from a retail car dealer DMC Ltd. These studies showed how close the geometric and log series distributions are as compounding distributions for automotive spare parts. As we discussed in chapter five, Sherbrooke (1968) has shown that the Stuttering Poisson and the NBD (produced by compounding) give very close results for a wide range of variance to mean ratios (q). Specifically it was reported that for values of 'q' up to three the two distributions give identical results, and the frequencies gradually diverge as 'q' exceeds three. In was not easy to test directly which compounding distribution might be appropriate in the DAF system, because all the DAF dealers are computerised and to gain sufficient data on individual order sizes, to measure compounding quantities, would have required working back through literally hundreds of invoices to obtain the required data. Our analysis in appendix one indicated that either the NBD or the Stuttering Poisson could be used and for a value of 'q' as high as seven the two distributions gave very close results. We eventually chose to use the integer form of the NBD for the simulation work in view of the closeness of the outcome from the two distributions and because the NBD is much easier to work with in practice, especially for larger values of the variate 'x'. The Stuttering Poisson requires the use of an awkward recursive formula, which becomes computationally very laborious to use for even modest sized demand values, because the number of calculations at each step increases in proportion to the value of 'x'. (For large demands, say 'x' approaching 100 per period on average, the Stuttering Poisson becomes prohibitive in calculation requirements). This is an area which requires a great deal more research and we refer to this issue again in chapter 13.

9.1 Simulation model

The initial stage of the first simulation attempted was to replicate the attainment of the Log series distribution of the first period 1979 by the simulation of the demands of 200 individual items in such a way that their single period aggregated distribution was LSD. Then NBD demand streams were generated, using an appropriate software package, for the 200 items. In the first stage the demand series was simulated over 15 operating periods. (Actually 60 periods were simulated, but only the first 15 were examined in detail). The NBD's were chosen such that the mean value for each demand stream when considered in aggregate were in the same proportion as the frequencies of the LSD distribution to be simulated. The empirical LSD of the first period 1979 had a mean value of 18.26 and a distribution parameter 'q' = 0.985. For 200 items this would give a LSD with the following frequencies-

Table 9.1

LSD simulation base data

Variate value	LSD probability	frequency for n =200
1	0.2346	47
2	0.1155	23
3	0.7590	15
4	0.0560	11
5	0.0442	9
6	0.0362	8
7	0.0306	7
8	0.0264	6
9	0.0231	5
10	0.0205	4
11-20	0.1271	25
21-30	0.0650	12
31-40	0.0364	8
41-50	0.0285	6
51-60	0.0228	5
61-70	0.0170	4
71-80	0.0114	2
81-90	0.0100	2
91-100	0.0057	1
		total 200

Thus it can be seen from the table above that 47 demand streams were simulated from Negative Binomial distributions with a mean demand of between zero and one, 23 were simulated from NBD's with mean demand of two, 15 were simulated from NBDs with mean value of three etc. With the lower frequencies such has 51-60, 5 demand streams were simulated with NBD mean values randomly scattered in the range 51-60. This process was continued until the needed 200 such streams were produced. The method of generating Negative Binomial demands was through a standard computer routine for generating NBD distributions of specified parameters, from which individual demand values were drawn by a simple Monte Carlo random number selection technique.

The simulated aggregate distribution of the first period is shown below together with its natural log form of the same distribution. Because of problems with zero demands being produced by the sampling technique from the many runs at the low mean values, only 176 effective positive demands were ultimately simulated for the first period. These are shown in table 9.2 -

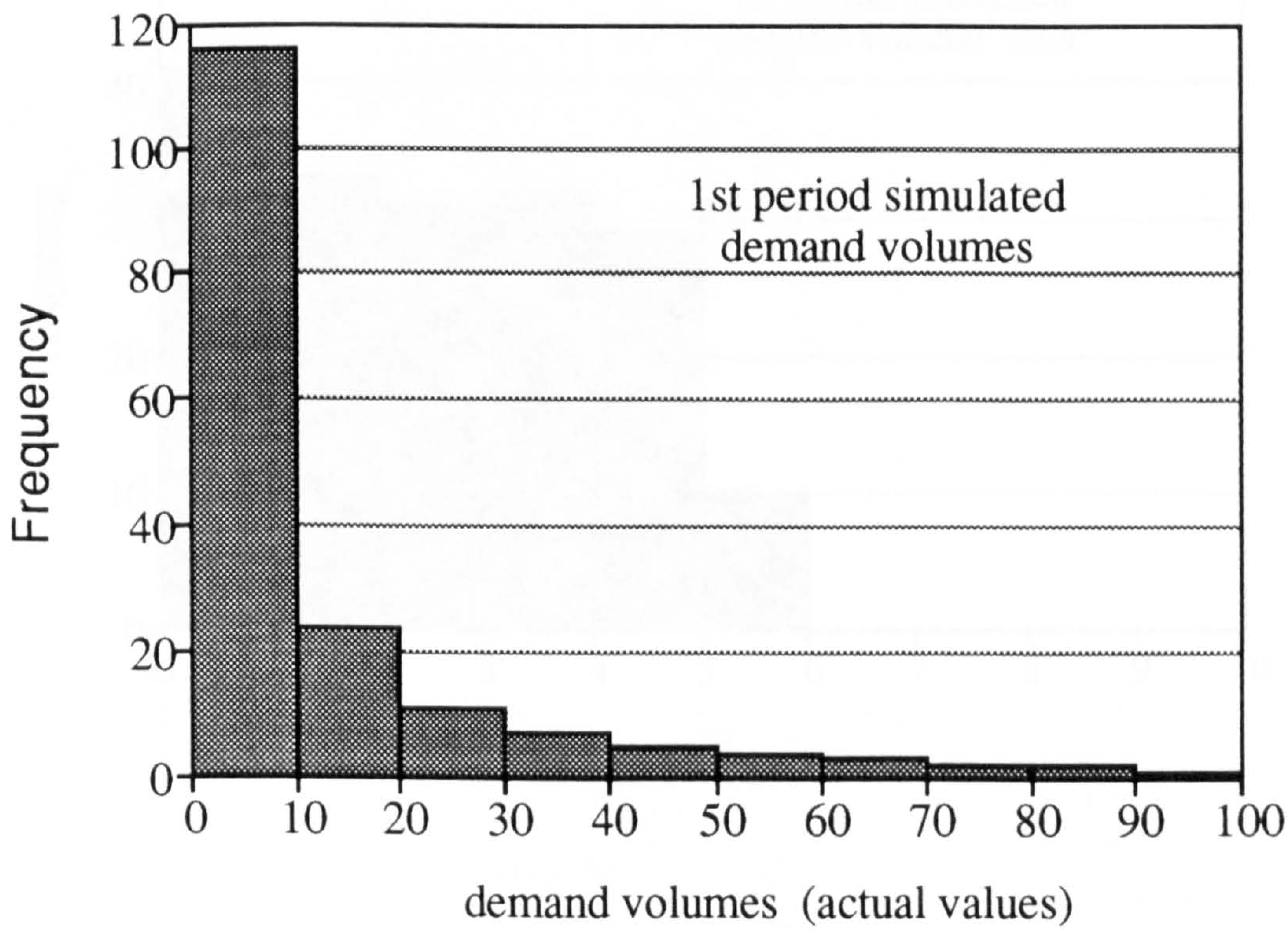
Table 9.2
First period simulation

Value band	simulated frequencies	1979 1st perid	Theoretical LSD
0-9	116	100	102
10-19	24	32	29
20-29	11	14	15
30-39	7	9	9
40-49	5	6	6
50-59	4	3	5
60-69	3	4	4
70-79	2	3	3
80-89	2	2	2
90-99	1	3	1
100-109	0	0	0
	Σ=176	Σ=176	Σ=176

This tabulation shows the close correspondence to both the actual frequency of the first period 1979 and the corresponding theoretical LSD with the same mean value. (all frequencies adjusted to a total of 176 for direct comparison). The simulated distribution shows perhaps too many values in the first cell and not enough in the second, however it was considered to be close enough to a LSD to be a valid starting point.

The histogram of the simulated first period of aggregate demand is shown below in figure 9.3 and the $\log_e$ form of the histogram is shown in figure 9.4.

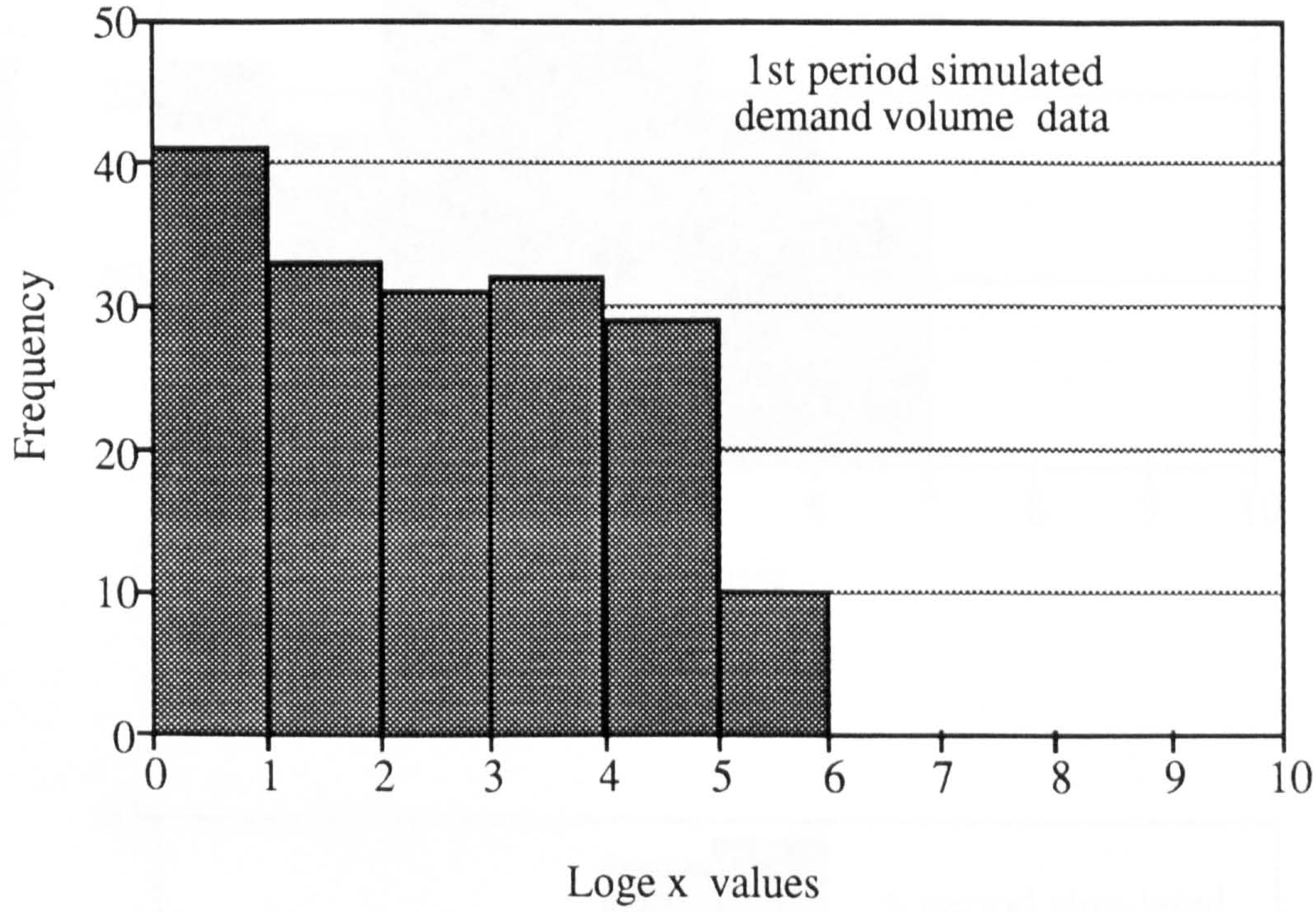
Figure 9.3

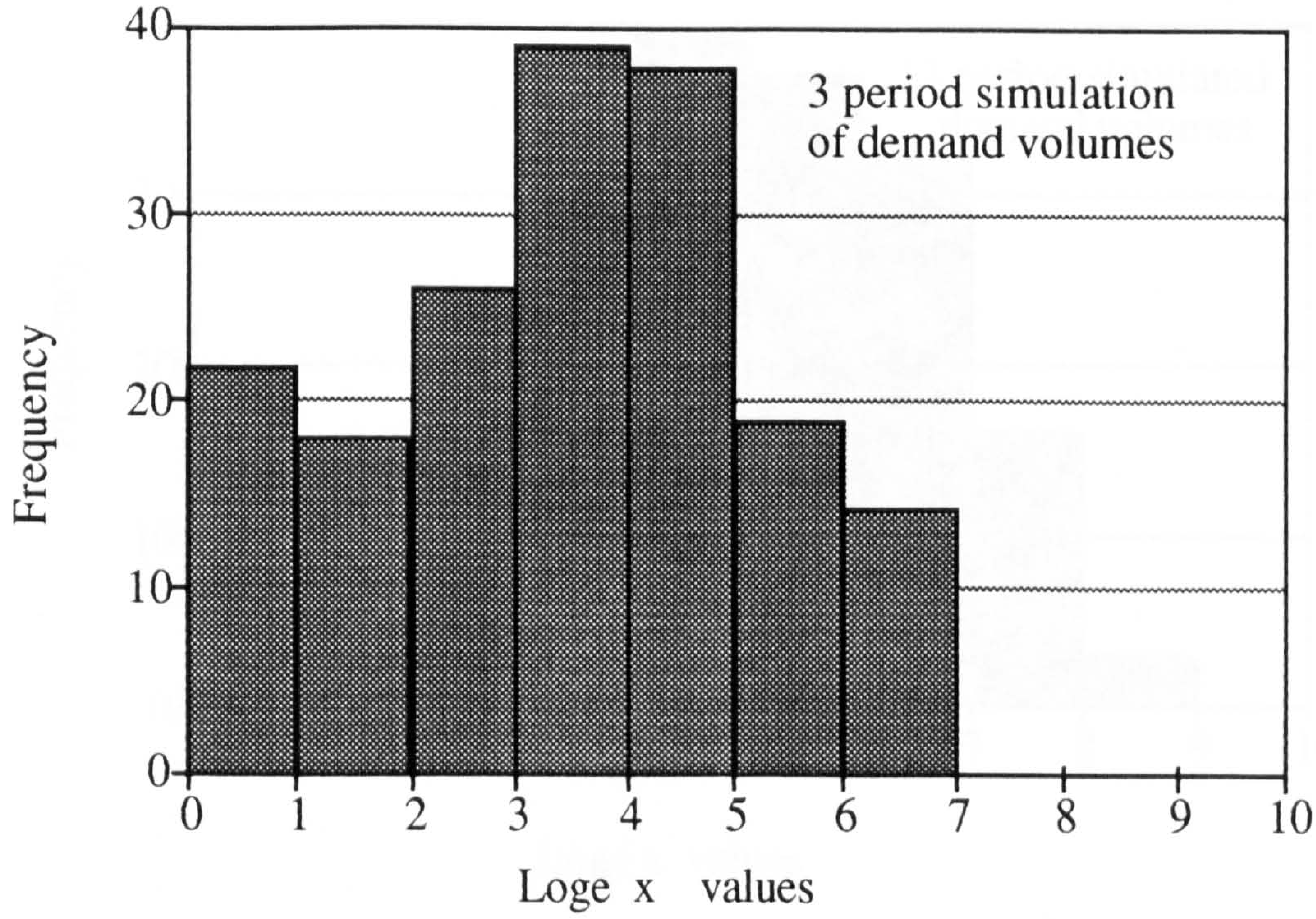


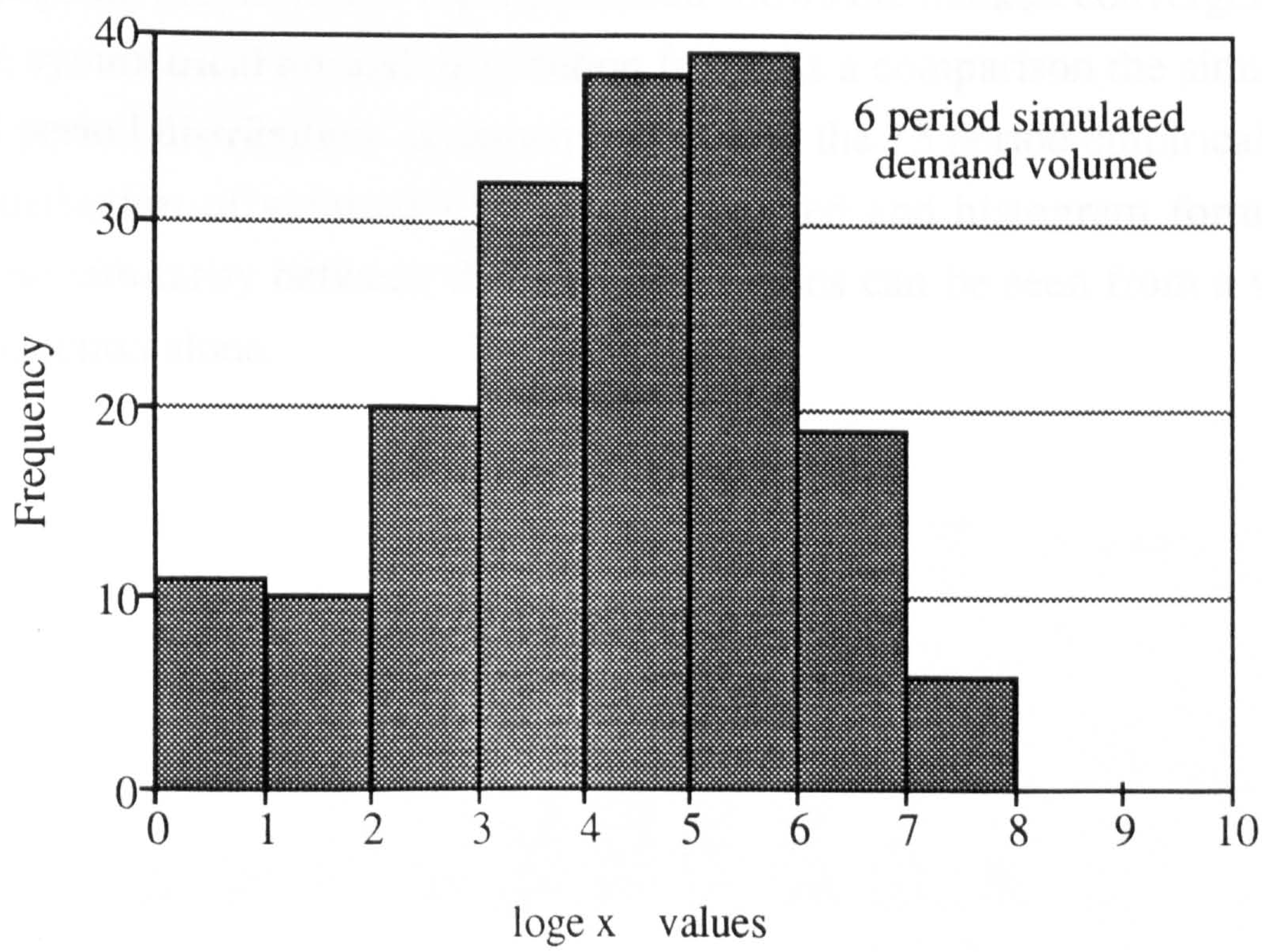
Demand streams were then simulated from a variety of NBD distributions for each simulated spare part using an NBD generation routine on a statistical package called 'statpack' to produce the required distributions. (In later simulation work an Excel spreadsheet model was used). A simple random number technique was used to generate individual demand values from the cumulative form of each NBD that was used. The next stage was to sum these individual simulated demand streams over 15 simulated operating periods to see what convergence was obtained at the aggregate level. The results of this are shown graphically in the following diagrams by reference to the natural log form of the aggregate distribution obtained at each successive three periods of the summation.

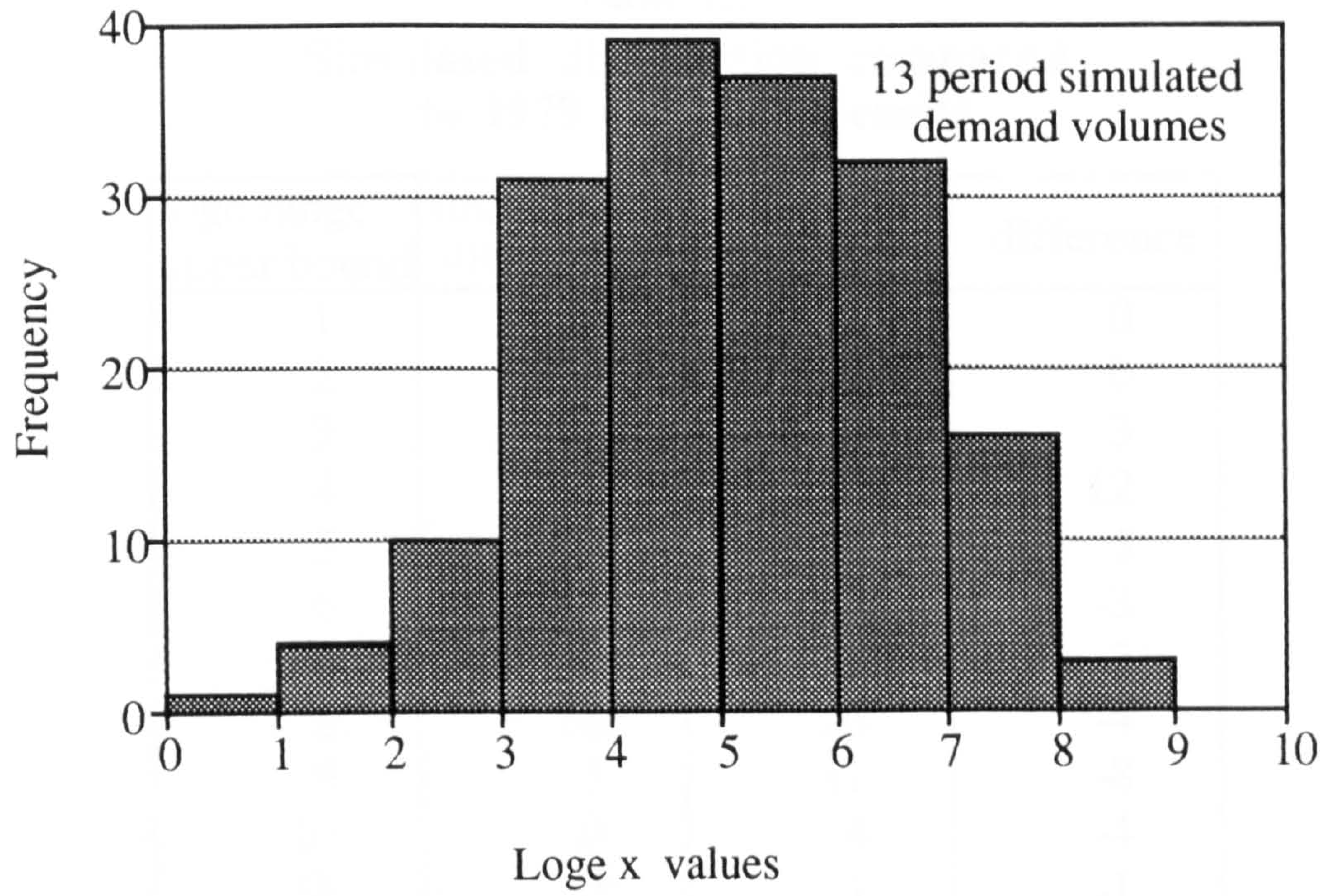
Figures 9.4 - 9.7

Log_e Simulated Demand Volume Histograms




3 period simulation
of demand volumes
Frequency
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
Loge x values


6 period simulated
demand volume
Frequency
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
loge x values



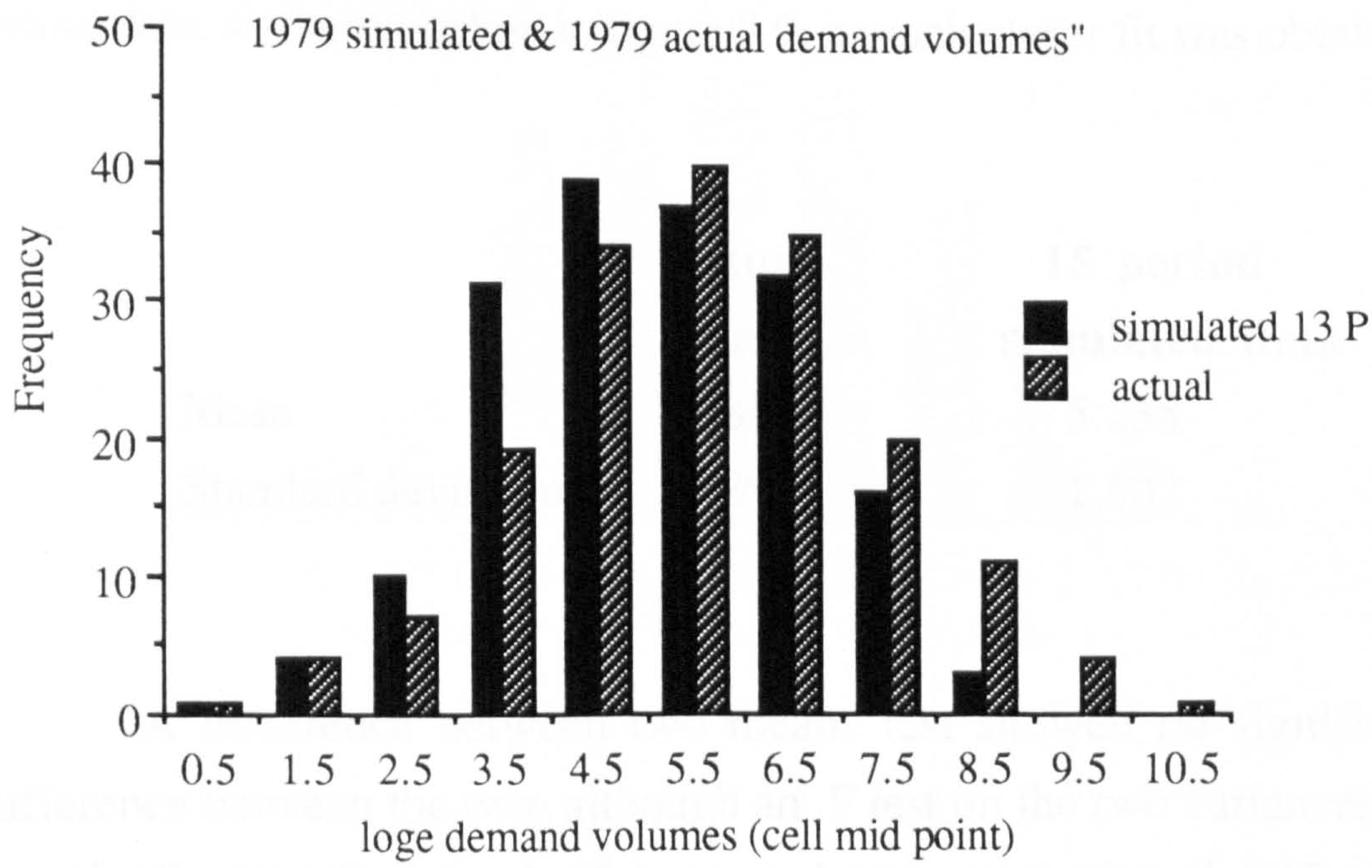
Clearly the convergence to lognormality is evident from the foregoing graphs as the log distribution shows the marked convergence to the symmetrical normal distribution form. As a comparison the simulated 13 period distribution is shown below with the 13 period empirical $\log_e$ distribution of volumes of 1979 in tabulated and histogram forms. A close similarity between the two distributions can be seen from a visual inspection alone.

Table 9.3

Simulated distribution compared to 1979 empirical results

loge range upper bound	simulated distribution	1979 (scaled) distribution	difference
1	1	1	0
2	4	4	0
3	10	7	3
4	31	19	12
5	39	34	5
6	37	40	-3
7	32	35	-3
8	16	20	-4
9	3	11	-8
10	0	4	-4
11	0	1	-1
12	0	0	0
	Σ=173	Σ=176	Σ=-3

Figure 9.8



The mean and standard deviation values between the actual and simulated data were as shown-

	Actual data	**13 period simulated data**
Mean	5.585	5.043
Standard deviation	1.789	1.574

A standard significance test on the difference between the above two means showed a significant difference between the two at both the 1% and 5% levels. A Chi Squared test gave an actual χ^2 value for the difference between the two distributions of 21.70, compared to theoretical values of $\chi^2_{0.05}$ at 12.59, and $\chi^2_{0.01}$ at 16.82. Hence, on statistical grounds, we must conclude that the two distributions are from significantly different populations.

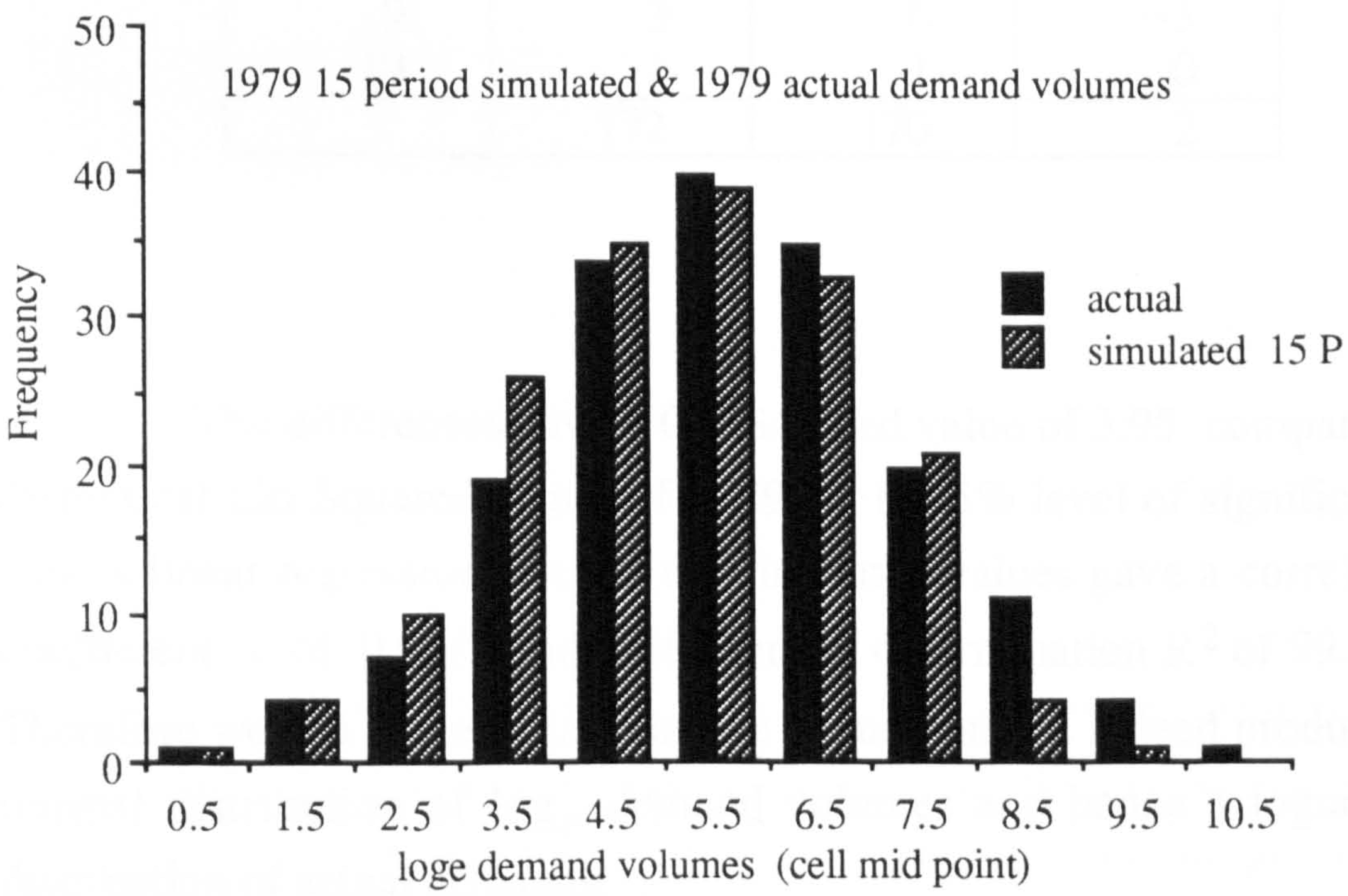
However, when we compared the 15 period simulated data with actual data, as shown below in figure 9.9, a much closer fit was obtained.

	Actual data	**15 period simulated data**
Mean	5.585	5.238
Standard deviation	1.789	1.602

A difference between two means test showed no significance difference between the two, although an F test on the two variances was marginally significant. A Chi squared test comparing the 15 period simulation with the 1979 distribution gave an actual χ^2 value of 13.3 which passes the Chi squared test at the 1% level, but fails at the 5% level.

Hence there is strong but not wholly conclusive evidence to show that the two distributions could be regarded as drawn from normal populations with very similar characteristics. However, whilst these results are not conclusive they are sufficiently close for us to be very confident that our cumulated aggregate Poisson-LSD/NBD demand model can reproduce the form of the empirical data. From the above results it can be seen that the simulated data appears to lag behind the actual data by some two to three periods; a phenomena that was observed in most of the simulations conducted in this research.

Figure 9.9



To verify that the simulated distribution can be regarded as a 'normal' distribution the frequencies obtained after 15 periods of simulation were compared with those of a theoretical normal distribution with the same mean and standard deviation, ie $m = 5.238$ and $s = 1.602$.

Table 9.4
Normality test for simulated results

loge range upper bound	simulated distributed	theoretical distribution	difference
1	0	1	-1
2	4	3	1
3	10	10	0
4	26	24	2
5	35	38	-3
6	39	40	-1
7	33	31	2
8	21	16	5
9	3	6	-3
10	1	1	0
	172	170	2

The differences gave a Chi Squared value of 3.95 compared to theoretical Chi Squared values of 12.95 at the 5% level of significance. Also a linear regression test on the simulated values gave a correlation coefficient 'r' of 0.9981 and coefficient of determination R^2 of 99.62%. Therefore we can be very sure that the simulation has indeed produced a normal distribution of $\log_e$ demand volumes and hence a lognormal distribution of actual demands.

To verify that we obtained a series of LSD distributions during the simulation, and that it was these that summed to lognormality, three separate single periods of the simulated data were compared with their corresponding theoretical LSD as shown in the following table :

Table 9.5

Simulated single period volumes

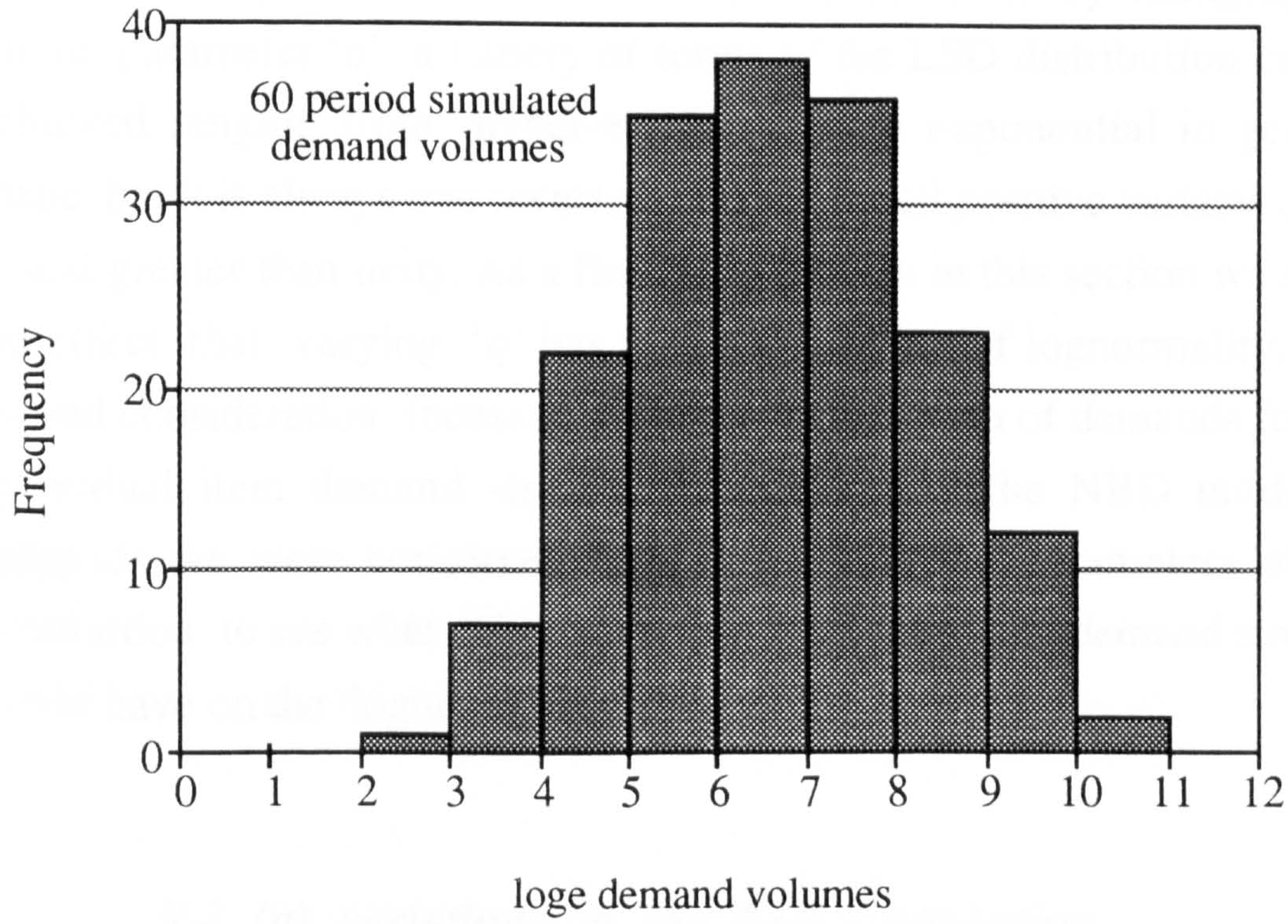
Frequency band	1st Period Sim.	1st Period Th.	6th period Sim.	6th period Th.	9th Period Sim.	9th Period Th.
1-10	116	121	86	96	87	95
11-20	24	22	24	18	25	18
21-30	11	11	11	9	6	9
31-40	7	6	4	5	5	5
41-50	5	4	4	3	4	4
51-60	4	3	4	2	4	3
61-70	3	2	3	2	2	2
71-80	2	2	3	1	2	2
81-90	2	1	0	1	3	1
91-100	1	1	1	1	1	1
101-110	1	1	1	1	1	1
	176	174	141	139	140	141
mean	14.371		15.234		16.34	
Dn max	0.036		0.07		0.056	

The critical values of the Kolmogorov Smirnov test for 141 observations are $Dn_{0.05} = 0.1145$ and $Dn_{0.01}=0.1372$, whilst the critical values for 175 observations are $Dn_{0.05} = 0.1031$ and $Dn_{0.01} = 0.1232$.

Hence from the above Dn_{max} values for periods 1, 6 and 9 we can see that all three distributions are clearly well within the accepted limits at both the 1% and 5% levels of significance. We have no grounds for rejecting the null hypothesis and therefore we can confidently accept that the simulated distributions are most likely LSD.

To see what the long term effects would be of continuing the simulation, a further 45 periods were simulated using the same NBD demand streams to give a total of 60 simulated periods. The $\log_e$ histogram obtained after this period is shown below in figure 9.10

Figure 9.10



It can be readily seen from the above that in the long run the distribution retains the characteristic shape of a normal distribution.

9.2 Simulation variations

From the foregoing we have shown that by using simulation and the models developed in chapter seven that almost identical results to the empirical data of 1979 can be obtained but with a lag of some two or three periods. This strongly supports the validity of the aggregate Afwedson model (Poisson-LSD/NBD) to explain the attainment of lognormality in demand volumes. Our attention now turns to a consideration of the parameters of the distributions used in the model and how variations in these affected the overall results obtained. A secondary purpose of the considerations here was to gain insight into any limiting conditions in the process at work.

Our process model starts with the LSD, but by changing the single parameter 'q' a variety of forms of the LSD distribution can be achieved ranging from hyper-exponential to exponential in general shape, but it is always true reverse 'J' shaped for all positive variates equal to and greater than unity. As a first consideration in this section we show the effect that varying 'q' has on the attainment of lognormality. Our second consideration focuses on the nature and form of demands for the individual item demand streams. In the case of the NBD model of demands we were concerned with changing the parameters of the distribution to see what effect the variance of individual demand streams would have on the lognormal distribution.

9.2 (a) variations in the LSD distribution.

In the simulation above we used an LSD distribution with parameter 'q' = 0.985 simply because the LSD that was fitted to the first period distribution of 1979 had that parameter value. In the following simulations we varied the value of 'q' but kept the demand streams constant in the sense that the same NBD distributions were used to generate individual demand streams. In this way the various outcomes were due to 'q' *and* not any variations in individual period demands.

The variations in 'q' used were as shown in the following table together with the parameters of the final normal distribution of $\log_e$ demands that was achieved :

Table 9.6
Variation in parameter *'q'*

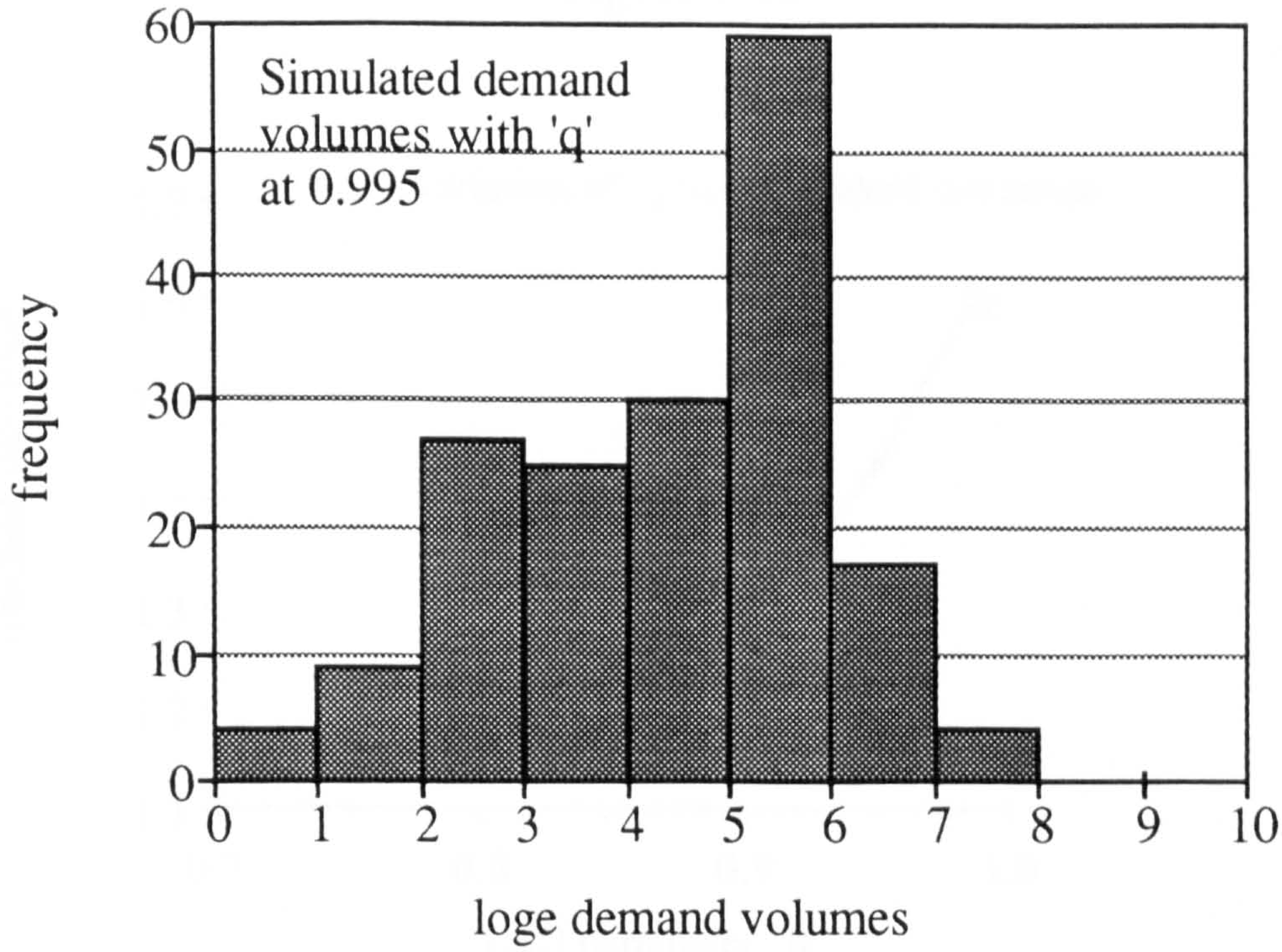
Parameter 'q'	0.750	0.800	0.850	0.900
mean of distribution	3.834	3.920	3.142	3.430
standard deviation	1.157	1.162	1.248	1.288

Parameter 'q'	0.950	0.980	0.990
mean of distribution	4.628	4.238	4.409
standard deviation	1.438	1.606	1.595

For simulation variations '*q*' = 0.75 through to '*q*' = 0.985 the process was consistently convergent to normality from period one to periods 15 with a form consistent with a normal distribution being obtained by around period 10 to 12. However, when '*q*' was set at 0.99 the process appeared to start to become chaotic in behaviour. Interestingly the final form obtained gave a distribution that passed the Kolmogorov Smirnov test for normality at both the 1% level and 5% levels of significance, but the convergence to normality was erratic and the form at 15 periods did not have the convincing symmetry of the the other simulation runs.

The rather irregular form of the distribution obtained after 15 periods when '*q*' was set at 0.995 is shown in figure 9.11 below -

figure 9.11



When one examines the parameters of the normal distribution obtained after each simulation the means do not appear to follow any discernible pattern, but there does appear to be a strong systematic variation in the variance of each distribution as shown in the following graphs :

Figure 9.12

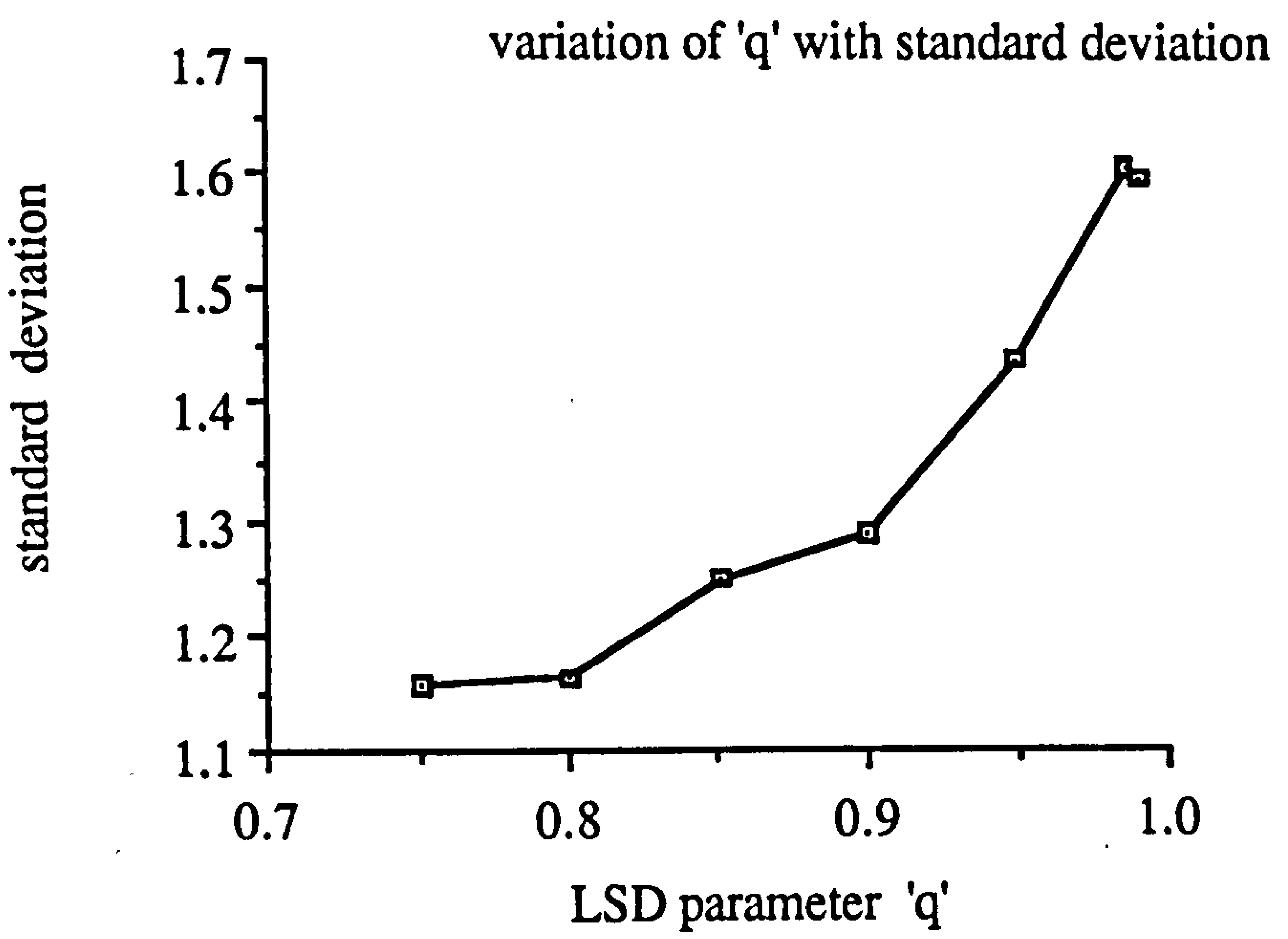
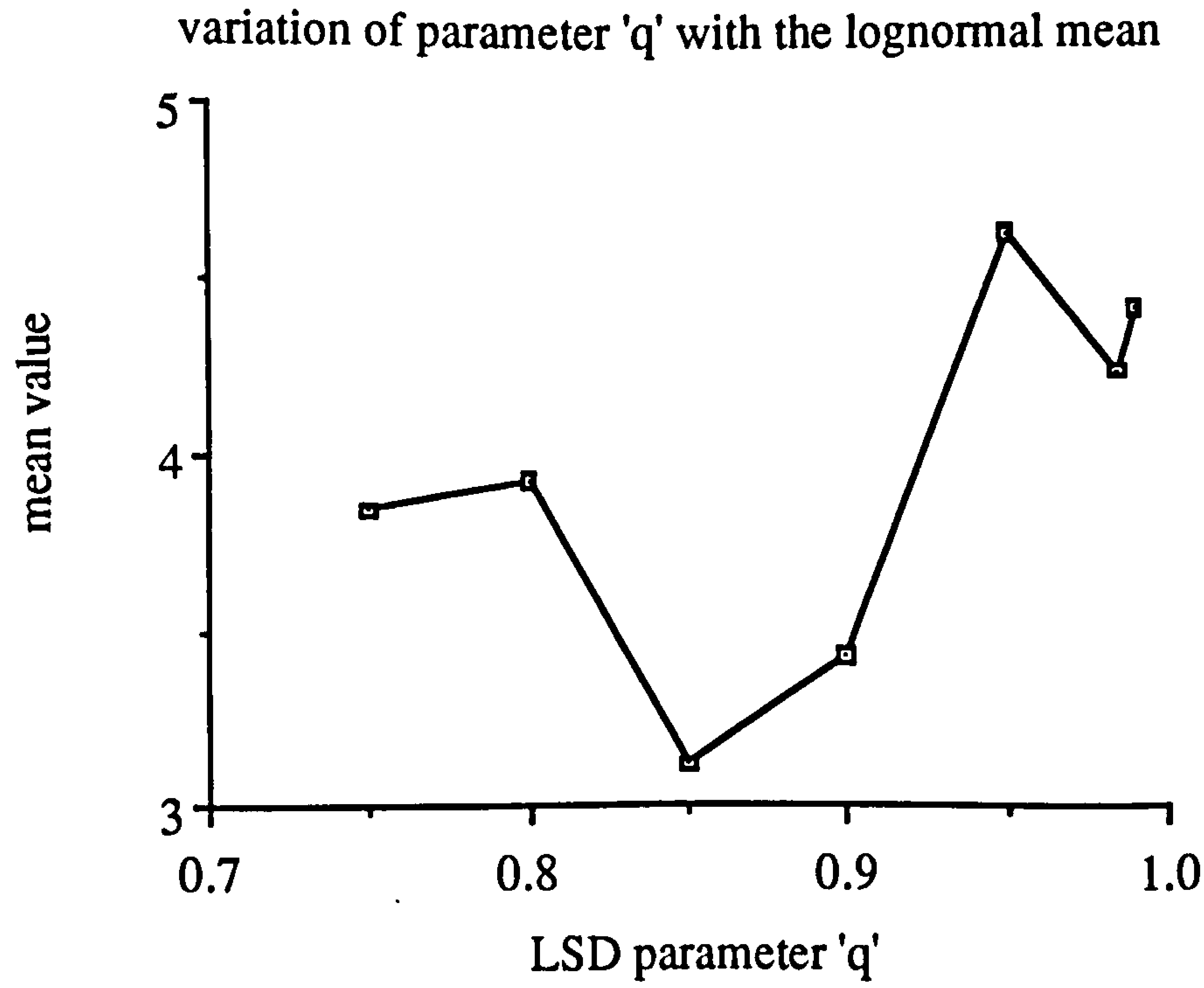


Figure 9.13



9.2 (b) variations in the demand streams.

We consider two main approaches here -firstly a simulation with the variance of the demand streams equal to the mean, and in the second the variance is set very high in relation to the mean demand.

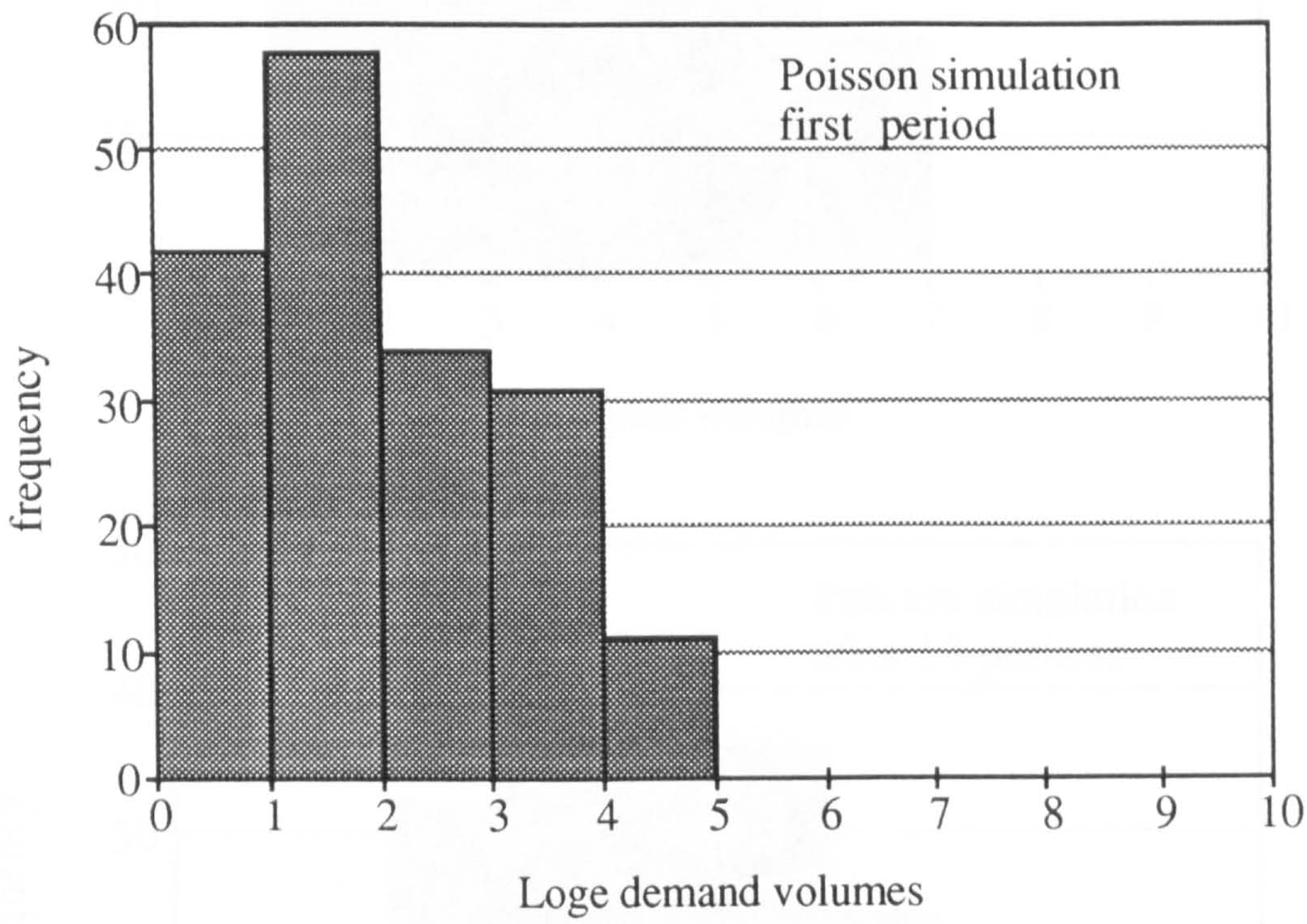
Given the overwhelming evidence for an underlying Poisson process of demand it seemed logical that the simple Poisson model should be used as a generating model for the first case. It was reasoned that this would also give the advantage of trying an alternative model of demand quantity whilst preserving the underlying Poisson behaviour and simultaneously reducing the variance to mean ratio of each demand stream to unity. In this simulation the LSD parameter was fixed at 0.985 so that the results would be directly comparable with previous empirical results and the simulation with the NBD model of demand. The frequencies used were as follows-

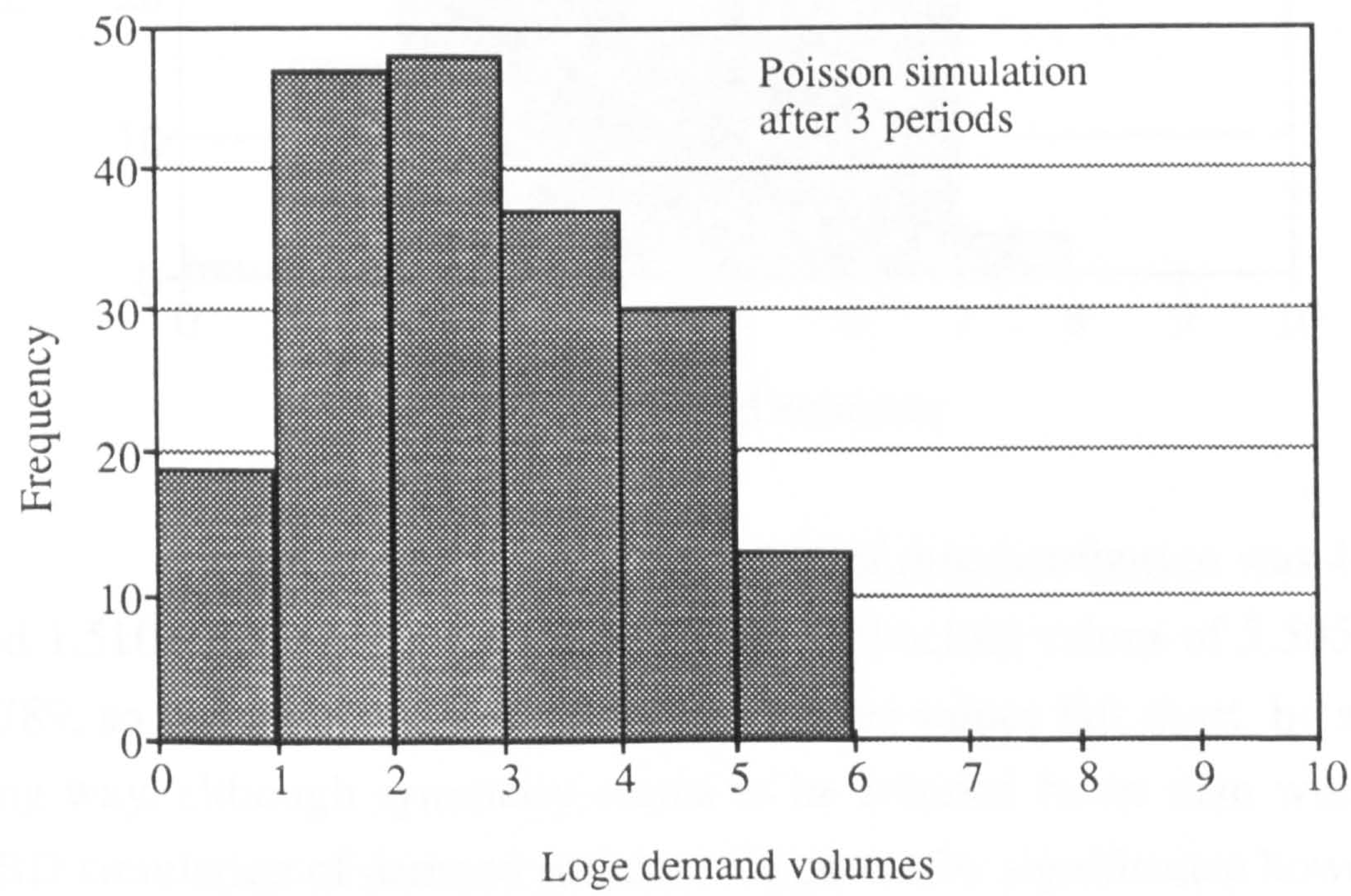
Table 9.7
Frequencies for simulation variations

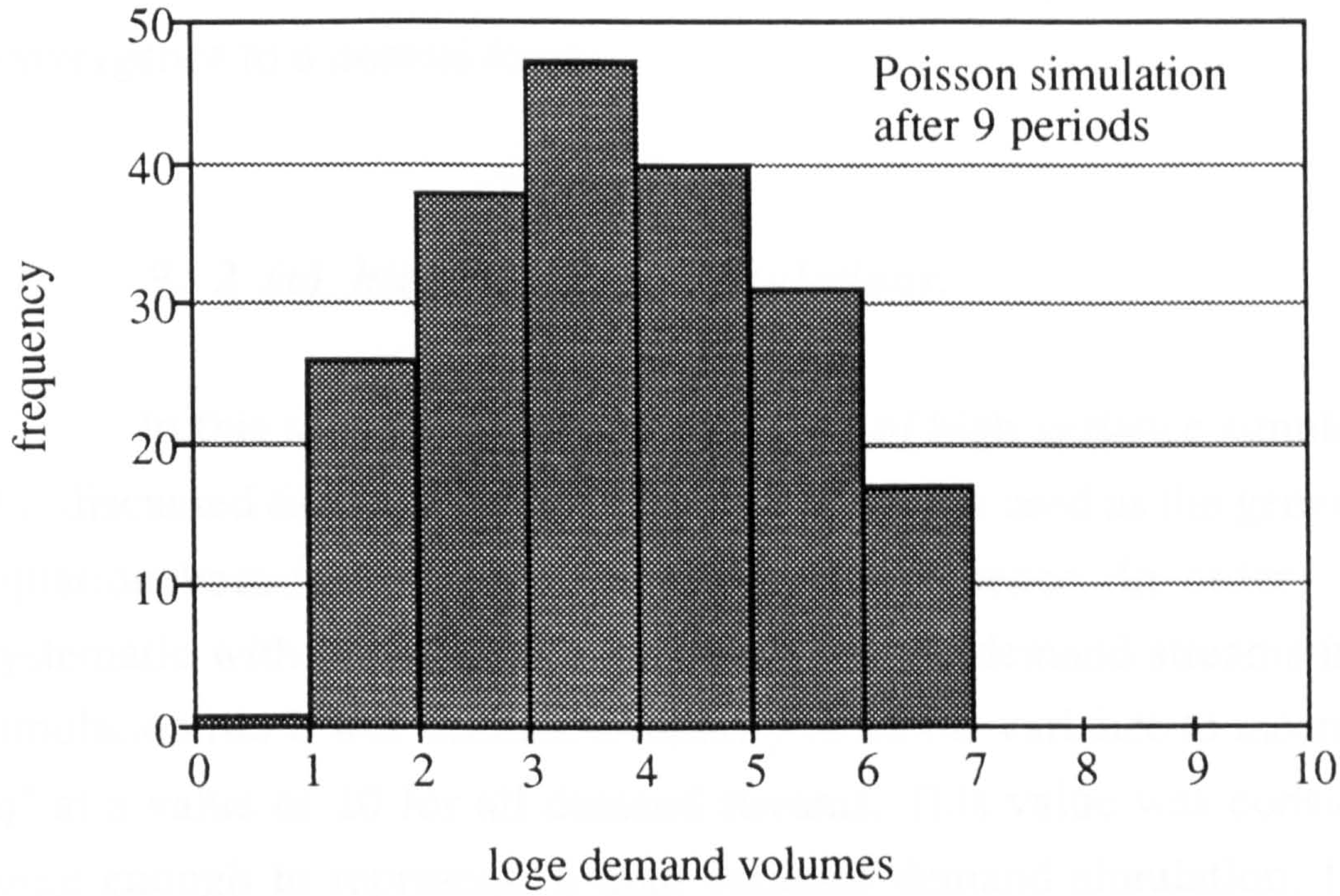
Value	frequency	value	frequency
1	47	11-20	25
2	23	21-30	12
3	15	31-40	8
4	11	41-50	6
5	9	51-60	5
6	8	61-70	4
7	7	71-80	2
8	6	81-90	2
9	5	91-100	1
10	4	100+	0
		Total frequency = 200	

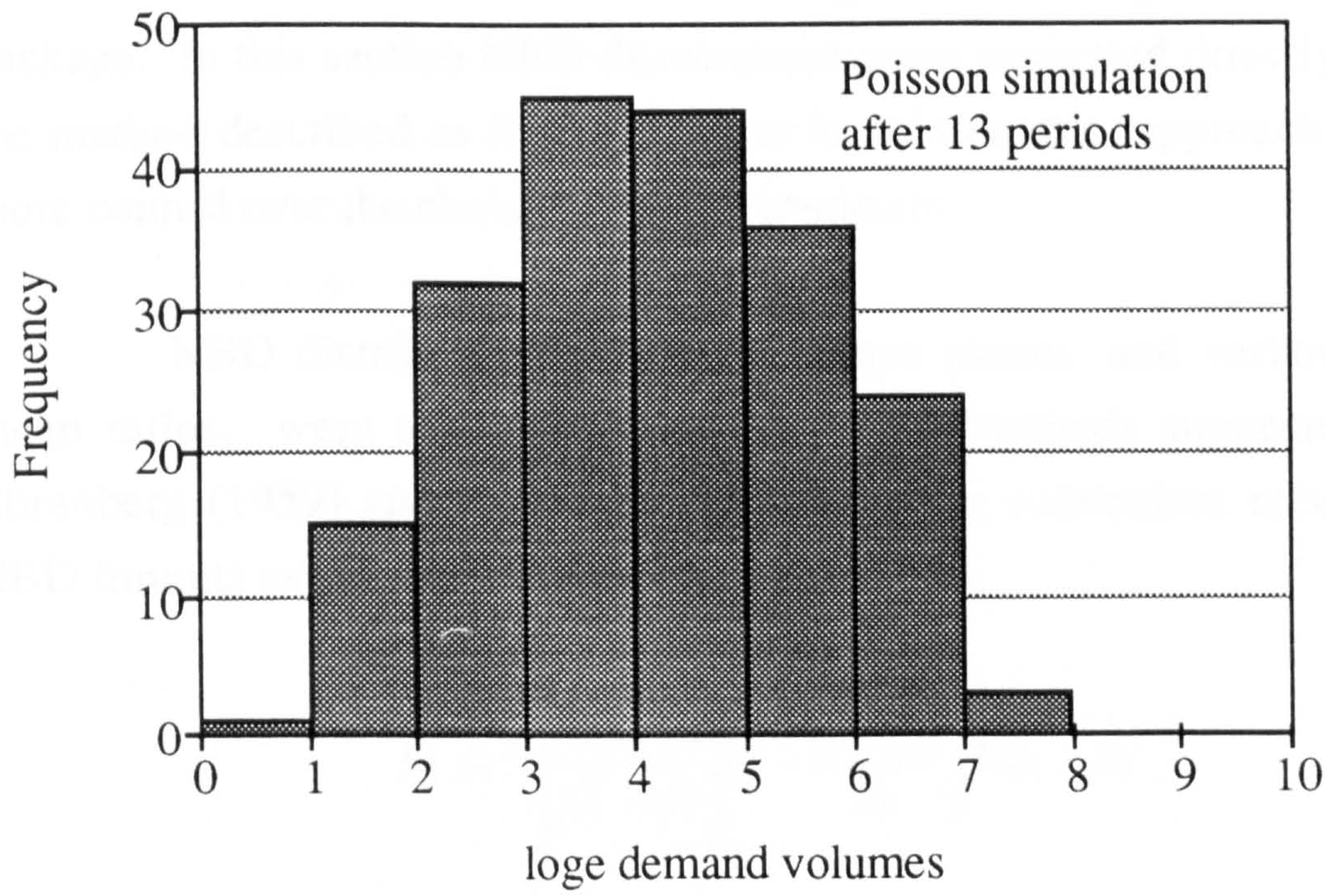
Using the above table of theoretical frequencies 47 Poisson streams were generated with a mean in the range 0.2 to 1, 23 streams were generated with a mean of 2 etc. The convergence process is seen in the following diagrams by reference to the log forms in each case. The 1st period LSD generated by the process is also shown.

Figures 9.14 - 9.17









The mean and standard deviation of this distribution was 4.157 and 1.510 respectively compared to the 1979 actual values of 5.505 and 1.789, so the simulated distribution parameter values fall short by some long way, although symmetry seems to be attained faster than with the NBD simulation of demand streams. Of particular significance however, is the fact that the Poisson distribution does produce a similar convergence to a normal form.

9. 2 (c) high variance simulations.

In this section we report the results of high variance simulation. As discussed earlier in this chapter, the NBD was used as the generating equation because of its computational convenience. In order to be systematic with respect to the variance of the demand streams in this simulation run it was decided arbitrarily to fix the variance to mean ratio 'q' at a value of 10 for all demand streams. This value was considered large enough to represent a high variance demand simulation. In our first simulation NBD distributions were generated using the Statpack package. In this section NBD distributions were generated directly by the method described as follows. It was found that this approach gave more control over the choice of NBD parameters.

NBD distributions of predetermined means, and variance to mean ratios, were easily generated using the methods suggested by Ehrenberg (1959) and Sherbrooke (1968), using convenient recursive NBD formula as shown-

$$P(x) = \left[\frac{a}{(1-a)}\right]\left[1 - \frac{(a-m)}{ax}\right]P(x-1)$$

We first determine $P(x{=}0)$ from the relationship given by Ehrenberg ie -

$$P(x{=}0) = (1 + m/_k)^{-k}$$

Where k is the Negative Binomial exponent.

Then subsequent probabilities for values $x = 1, 2, 3$, etc., for given means 'm' and variance to mean ratios 'q' were calculated by the above recursion formula. The values of 'k' and 'a' were calculated from the formula shown -

$$k = m/(q-1)$$

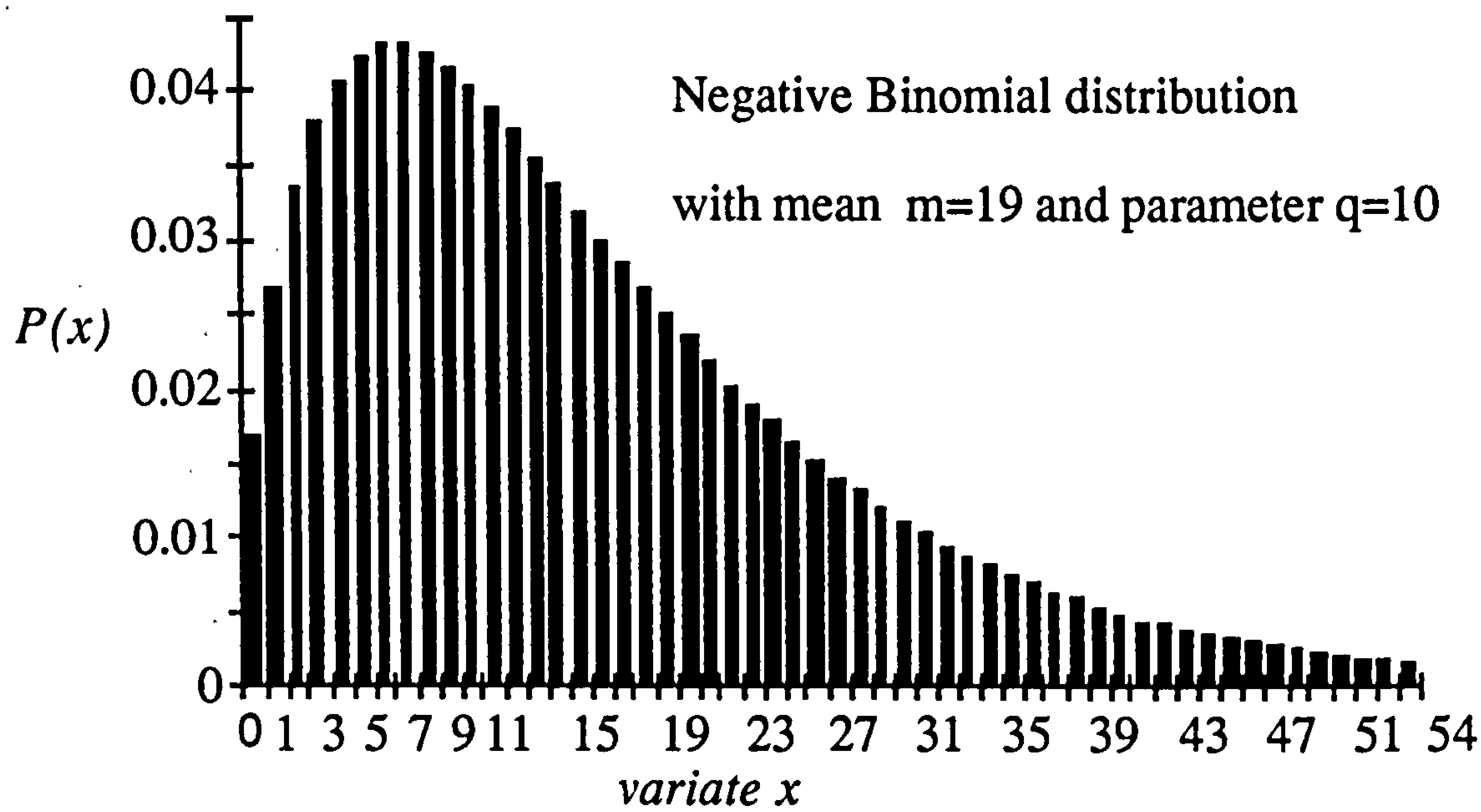
$$a = m/k$$

For example, when $m = 16$ and $q = 10$, we obtain $k = 1.7778$ and $a = 9$.

Then $P(x=0) = 0.016681$

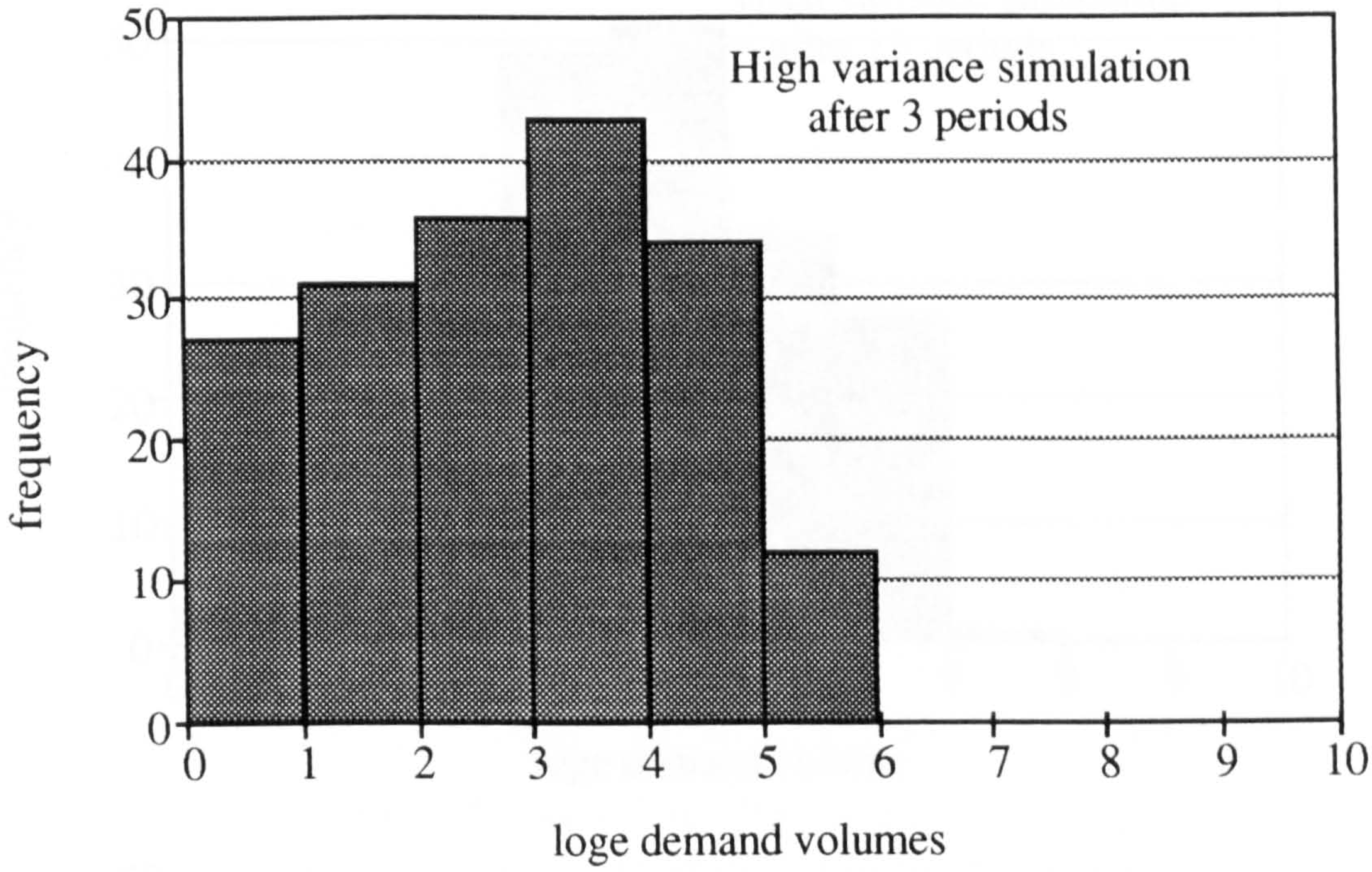
Using an Excel spreadsheet all subsequent probabilities were easily generated to yield the appropriate NBD probability density functions and their respective cumulative functions. Figure 9.18 shows a typical NBD generated this way-

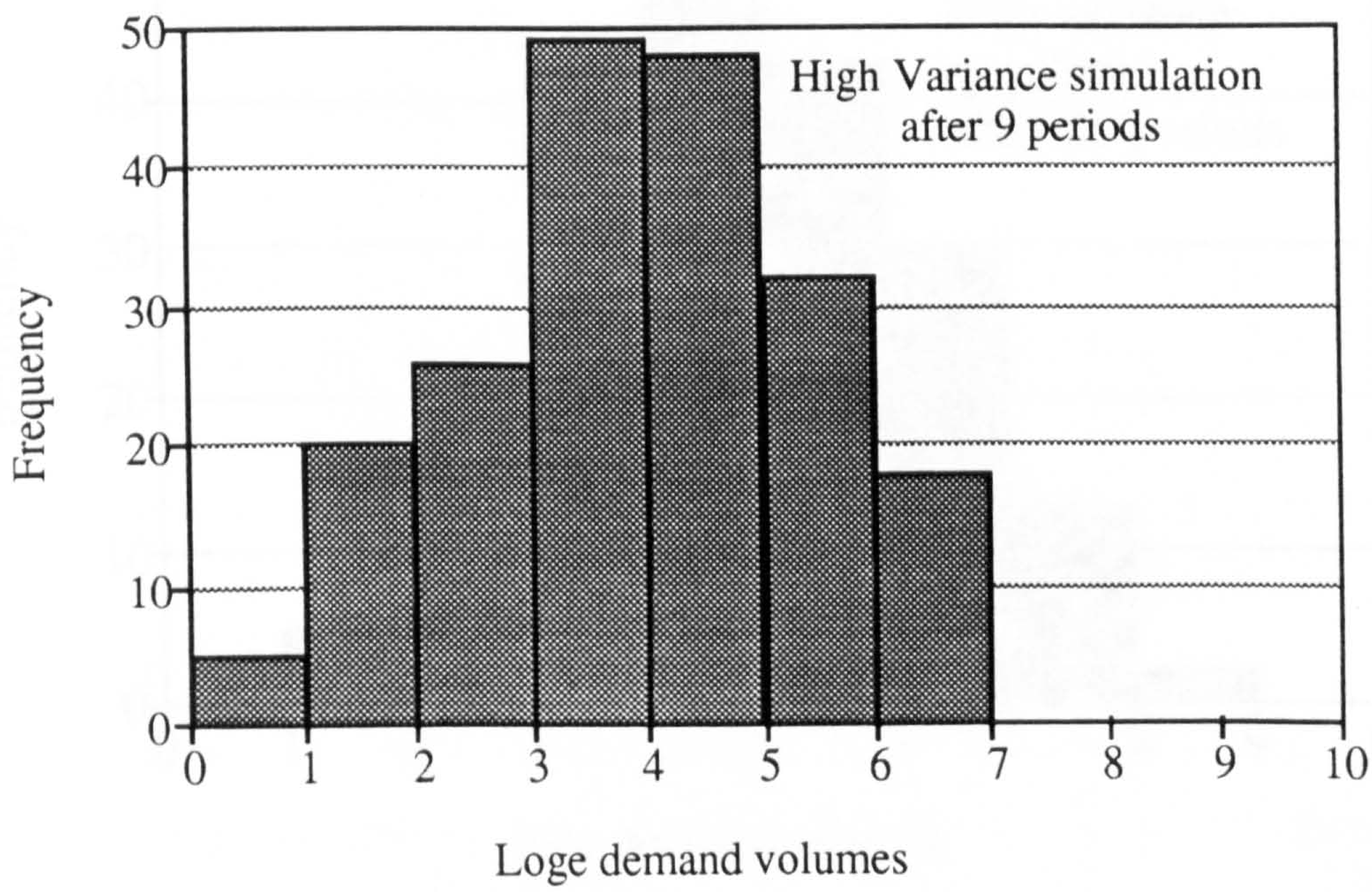
figure 9.18

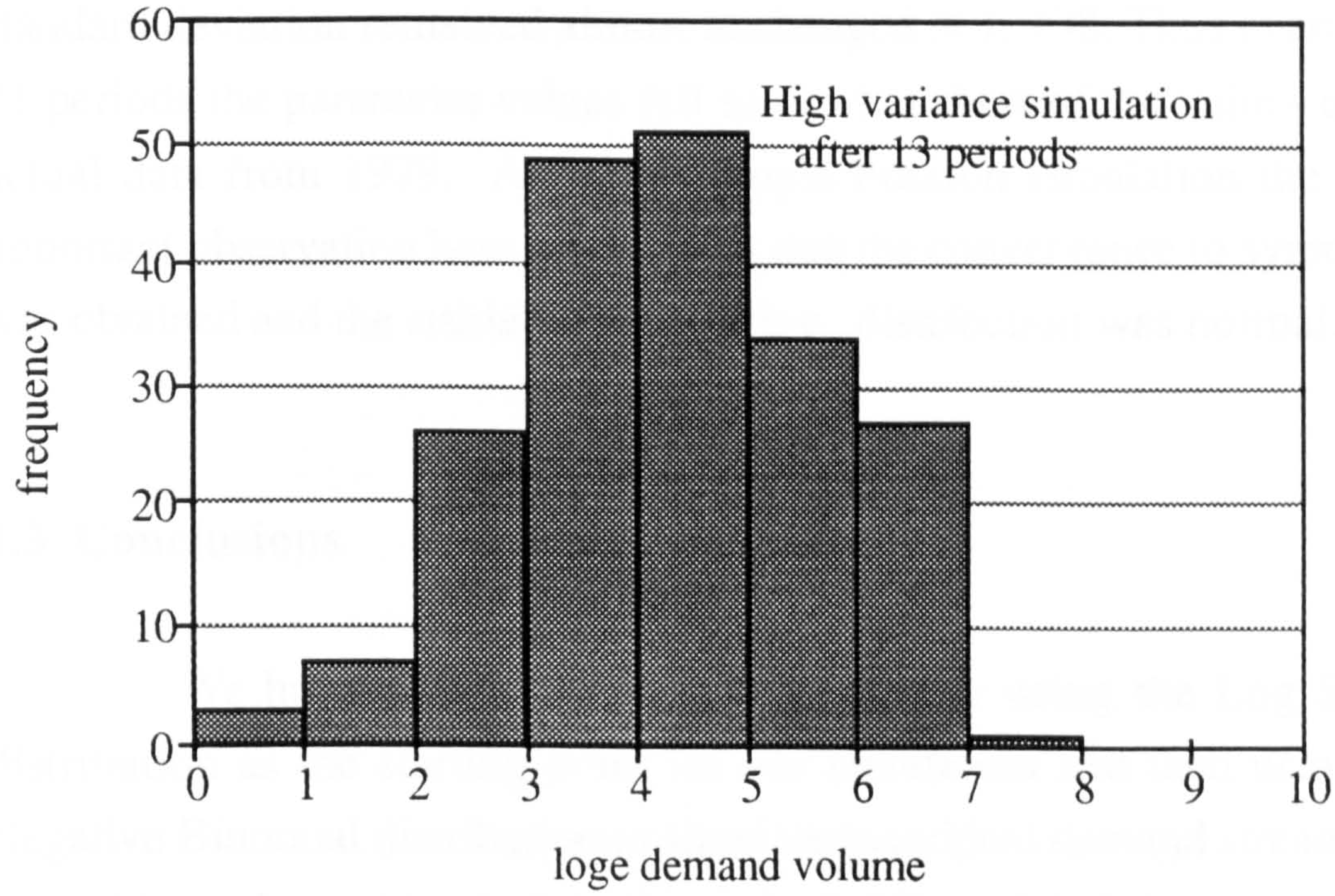


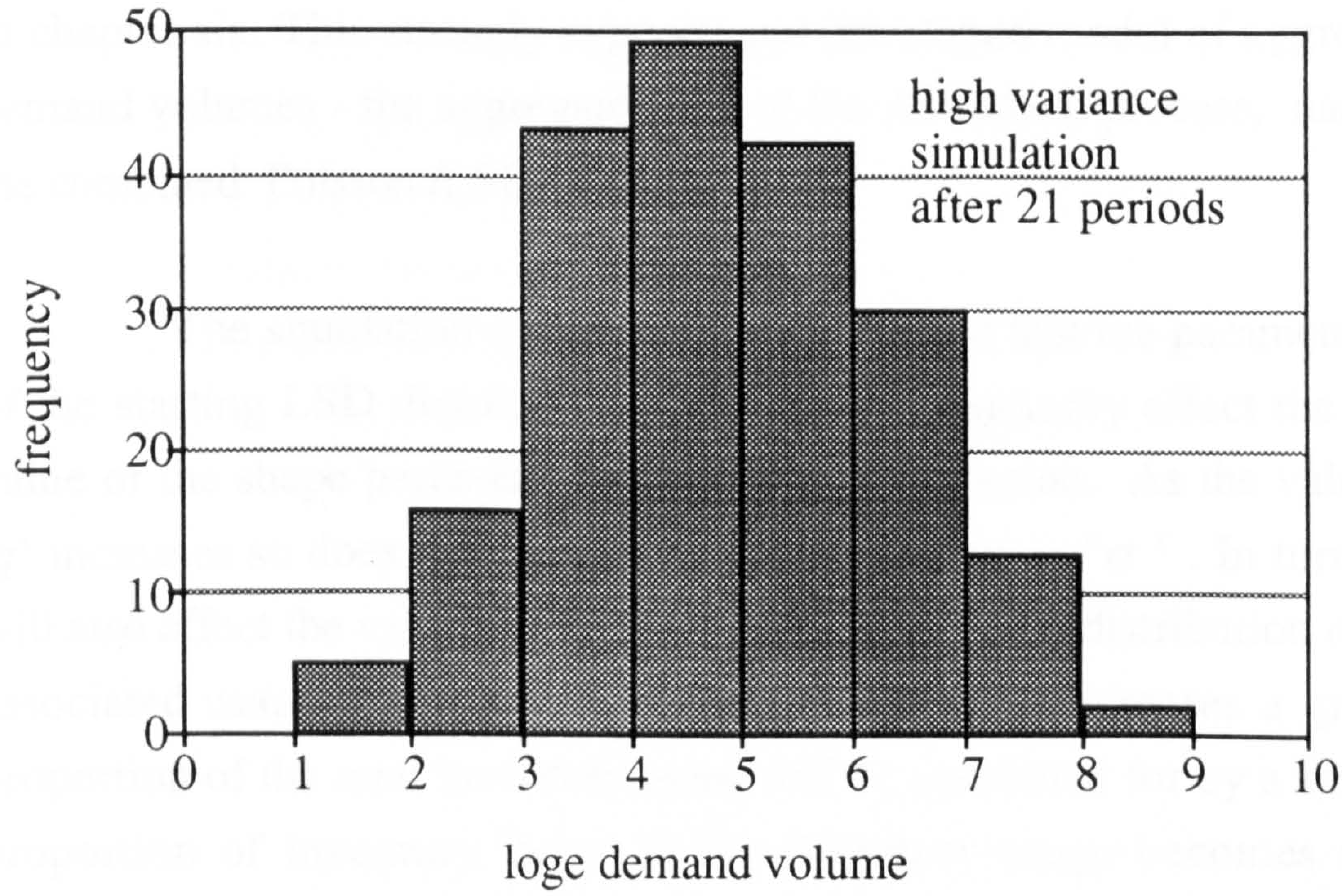
Random numbers were generated and then used to select values of 'x' randomly from the cumulative distributions. This process was repeated until 200 individual demand streams had been generated with their means in proportion to an aggregate LSD distribution with LSD parameter 'q' = 0.985, so that the results here would be directly comparable with previous simulations. The results are shown in the following histograms.

Figures 9.19 - 9.22









Interestingly it can be seen from this high variance simulation that we obtained a convergence process very similar to that using the simple Poisson model for demand streams. The mean and standard deviation obtained after 13 periods was 4.269 and 1.444 respectively, and after 21 periods of simulation the mean had grown to 4.890, but the standard deviation remained almost unchanged at 1. 438. Thus even after 21 periods the parameter values fell somewhat short of the values of the actual data from 1979. As in the simple Poisson simulation the most important observation here was the fact that the convergence to symmetry was obtained and the stable form of the $\log_e$ distribution was normal.

9.3 Conclusions

We have shown in this chapter that by using the Log Series distribution as the starting point for our simulation and then using the Negative Binomial distribution to simulate individual demand streams we can achieve almost identical results to the DAF empirical data sets shown in chapter six. This strongly supports our developed model of aggregate demand volumes - the aggregate form of the Afwedson process, namely the combined Poisson /LSD/NBD model.

The simulation variations clearly showed that the parameter 'q' of the starting LSD distribution does seem to markedly effect the final value of the shape parameter of lognormal distribution. As the value of 'q' increases so does the value of the shape parameter 'σ'. In turn this will also affect the value of 'σ' for the final lognormal distribution of the associated usage values which indicates that as 'q' increases a greater proportion of the total inventory value will be accounted for by a smaller proportion of inventory items, ie the inventory range becomes more concentrated. In general the only way that the starting value of 'q' can be increased for an actual inventory is for there to be a larger proportion of low volume items. These are most likely to be the important larger value items and these will have a large effect on the overall concentration of usage values in the total inventory.

The simulation variations also showed that the variance of individual item demand streams does not appear to affect the overall shape parameter 'σ ' of the final lognormal form obtained. Both the simple Poisson demand streams and the high variance NBD streams gave very similar results. This was a surprising finding because intuitively it seemed a fair bet that the demand stream variance would have a strong affect on the final lognormal distribution in some way. This would further suggest that the actual nature of the demand streams does not have a marked affect on the long run stable form of the system. Hence the demands could simple Poisson, NBD, Stuttering Poisson, or other modified Poisson forms.

Even though we only used limited simulations runs for each of the variations chosen the results were consistent and corresponded very closely to the forms of the empirical distributions of chapter six. Given the high degree of consistency we obtained the associated sampling error was not therefore a cause for concern. Hence we feel that given the purpose of simulation in this work, ie validation, then the considerable time investment required for more extensive simulation runs to achieve statistically significant results would not have been productive.

Supplementary Empirical Studies

10.0 Introduction

In this chapter we present some empirical studies on a variety of spare parts systems from different product markets and operating environments. The purpose of these limited investigations was to test the validity, or otherwise, of the features of the models developed in previous chapters. In particular because the previous empirical work and derived theory was based primarily on one company it was necessary to show empirically that the theory would transfer to other systems, even though the simulation studies showed that theoretically this should be so. The investigations are based on systems representing aircraft, lift gear, car, and bicycle spare parts. The general criteria set for the selection and use of any additional company data was it should be from a spare parts inventory, and the ultimate consumer demand for such items should be randomly generated from many independent sources; thus generally satisfying the broad conditions of a Poisson process.

10.1 Dan Air Spares Systems.

In 1984 Dan Air, the UK based airline company, were operating a fleet of approximately 50 aircraft to support its scheduled airline business. The majority of its fleet was Boeing aircraft comprising 707, 727 and 737 stock. Engineering services, including all spare support, was based at Lasham in Hampshire. The author visited the site in the summer of that year and collected data on spare parts usage for the current and previous four years: all data being held on stock cards. [The stock cards covered a five year period, so whilst data was being collected for the current year the demand for the previous four was also noted down. This proved to be of particular value as will be appreciated in the following sections]. The spares kept at Lasham were divided into two main groups -rotatable repair parts (approximately 15,000) and consumable spare parts (approximately 78,000). Because of the rather special nature of rotatable spares and the highly regularised aircraft maintenance schedules it was decided to concentrate on the consumable spares. It was felt that the nature of demand for rotatable parts would not be true Poisson in character, because most are withdrawn from service at preset intervals, and then serviced, long before failure is likely occur. Hence in the main they are not true failures in Poisson terms.

It was reasoned that consumable spares are more likely to reflect a random demand situation. In general they are replaced as needed, although the demand for many such items will correlate with rotatable spare replacements. Because of the vast number of stock cards involved a simple random number process was used to select a card from the many rows of cards in the card bins on the basis of location, ie by bin number and then position in the bin. This was a very laborious and time consuming process, but eventually some 165 usable parts data were selected and the demand values logged down. The parts price data was as given on each card and in theory was supposed to be the latest known price. If any card looked suspicious in this respect, ie. it looked like an old price not updated, then it was rejected. However, the consistency of the price data (ie all current prices) could not be guaranteed over all selections. Many cards also had to be rejected because there had been no demand movement for several years.

10.1 (a) testing for the log series distribution

In the initial analysis we tested the annual aggregate demand volumes for years 1980 to 1983 for lognormality. The histograms and frequency tables for all values > or =1 for each year are shown in table 10.1 below :

Table 10.1
DAN Air annual demands

Value range	1980	1981	1982	1983
1-10	89	93	98	112
11-20	14	16	16	8
21-30	7	5	5	8
31-40	3	3	5	3
41-50	1	2	4	3
51-60	2	2	2	3
61-70	1	2	1	2
71-80	1	1	0	2
81-90	3	1	1	0
91-100	0	0	2	1
100+	8	6	4	7

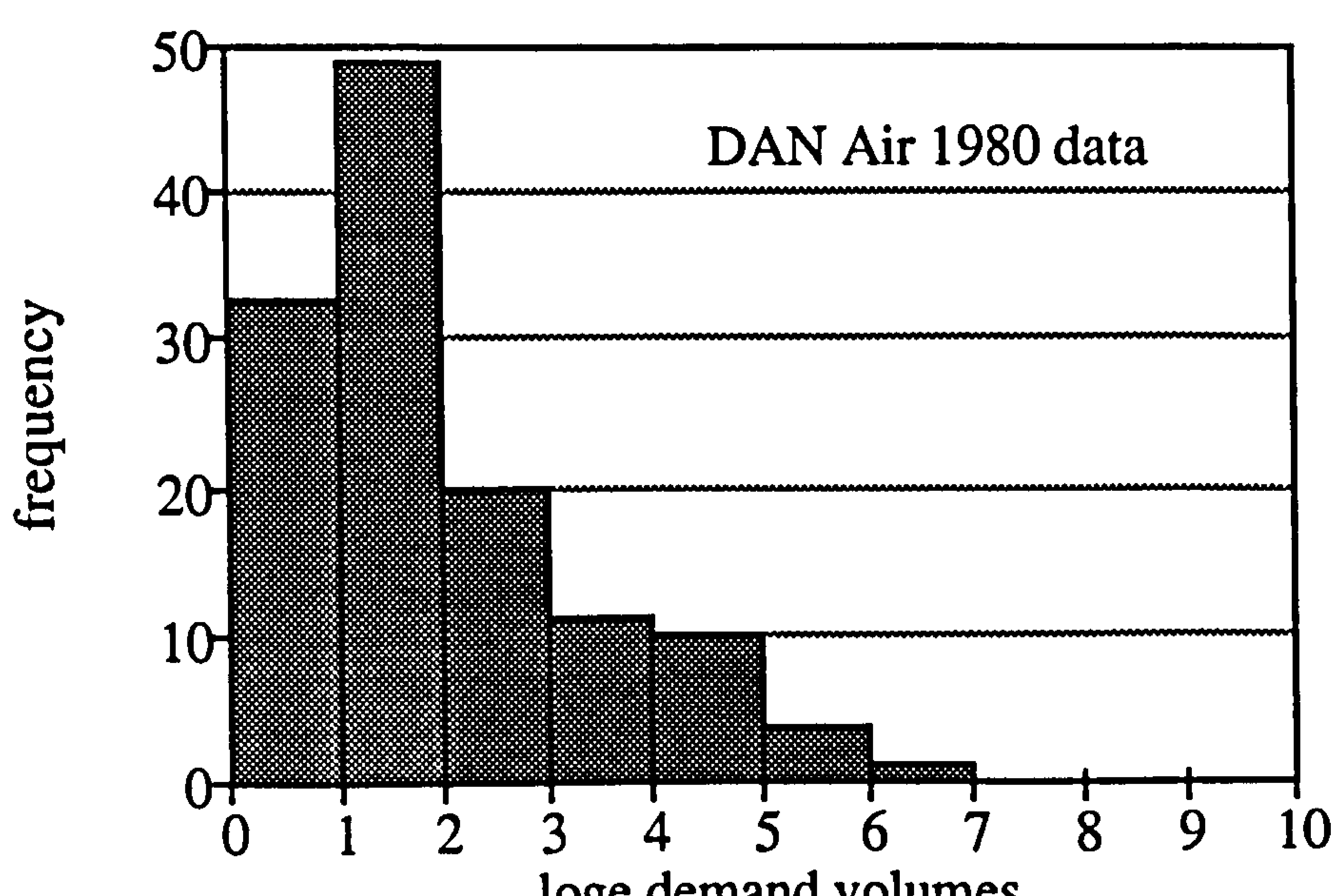
These four yearly data sets were not lognormal in form as the natural logarithmic values did not give anything like a normal distribution. This can be seen by examining figure 10.1 for 1980 which was typical of each year through to 1984. This was at first a disappointing and unexpected result. However, the yearly data did look very much like the single period DAF demand volume data, that was LSD distributed; so tests were conducted on years 1980 and 1981 against theoretical LSD, and these gave significant results at both 1% and 5% levels of significance by the Kolmogorov Smirnov test. The empirical and theoretical LSD frequencies for 1980 and 1981 are shown in table 10.2 :

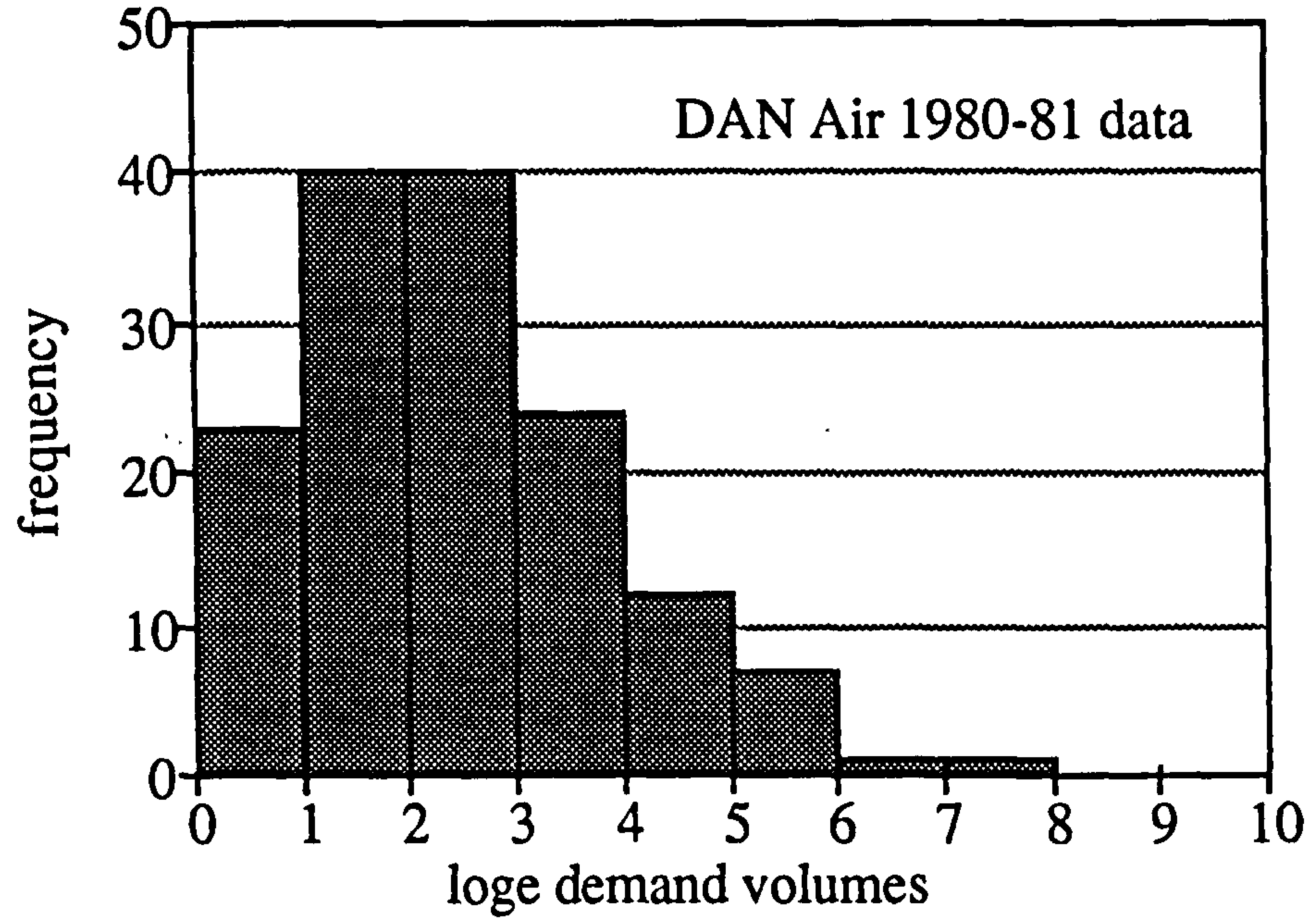
Table 10.2
DAN Air testing for the LSD

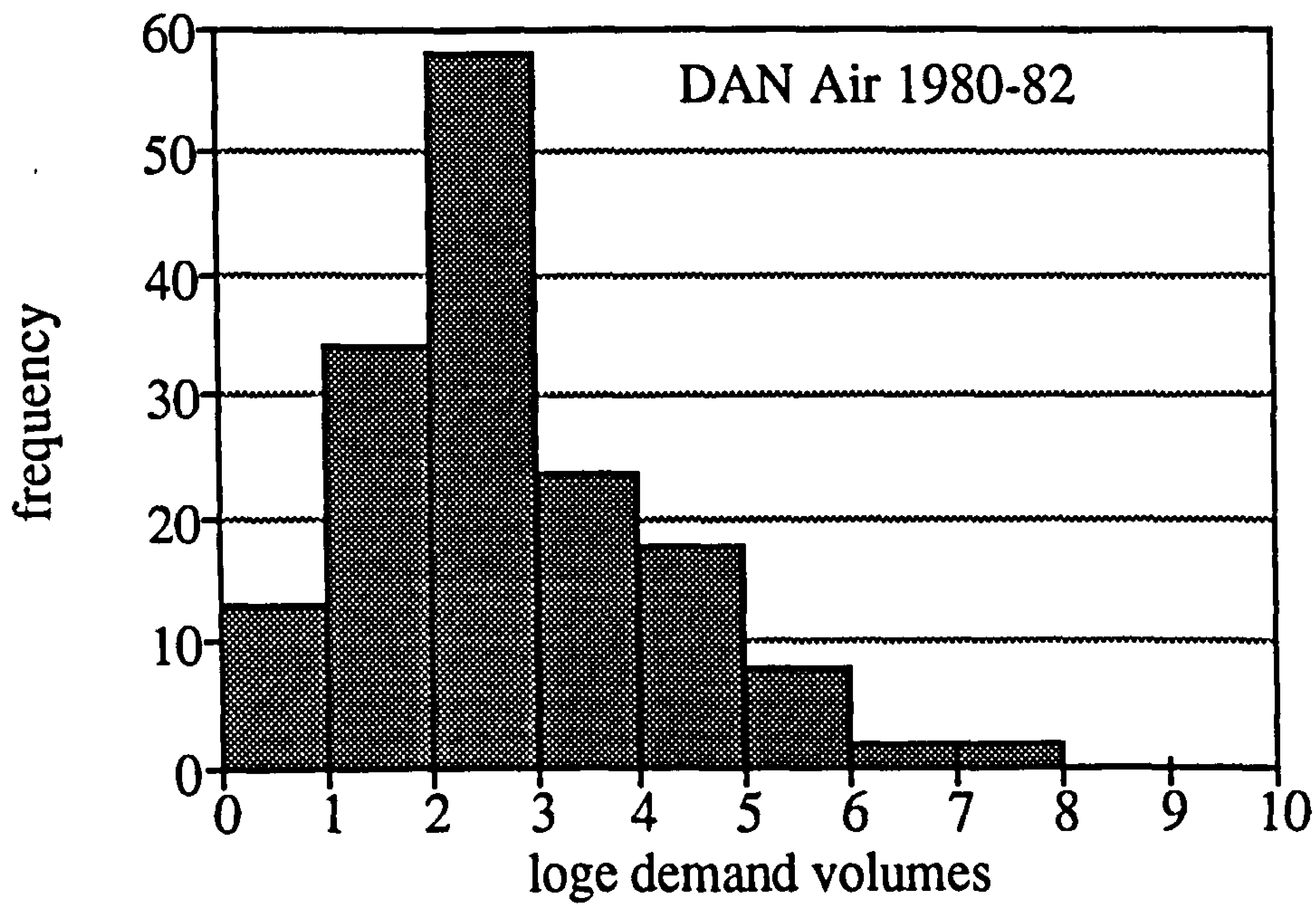
Value range	1980	LSD	1981	LSD
1-10	89	88	93	92
11-20	14	15	16	15
21-30	7	7	5	7
31-40	3	4	3	4
41-50	1	3	2	3
51-60	2	2	2	2
61-70	1	1	2	2
71-80	1	1	1	1
81-90	3	1	1	1
91-100	0	0	0	1

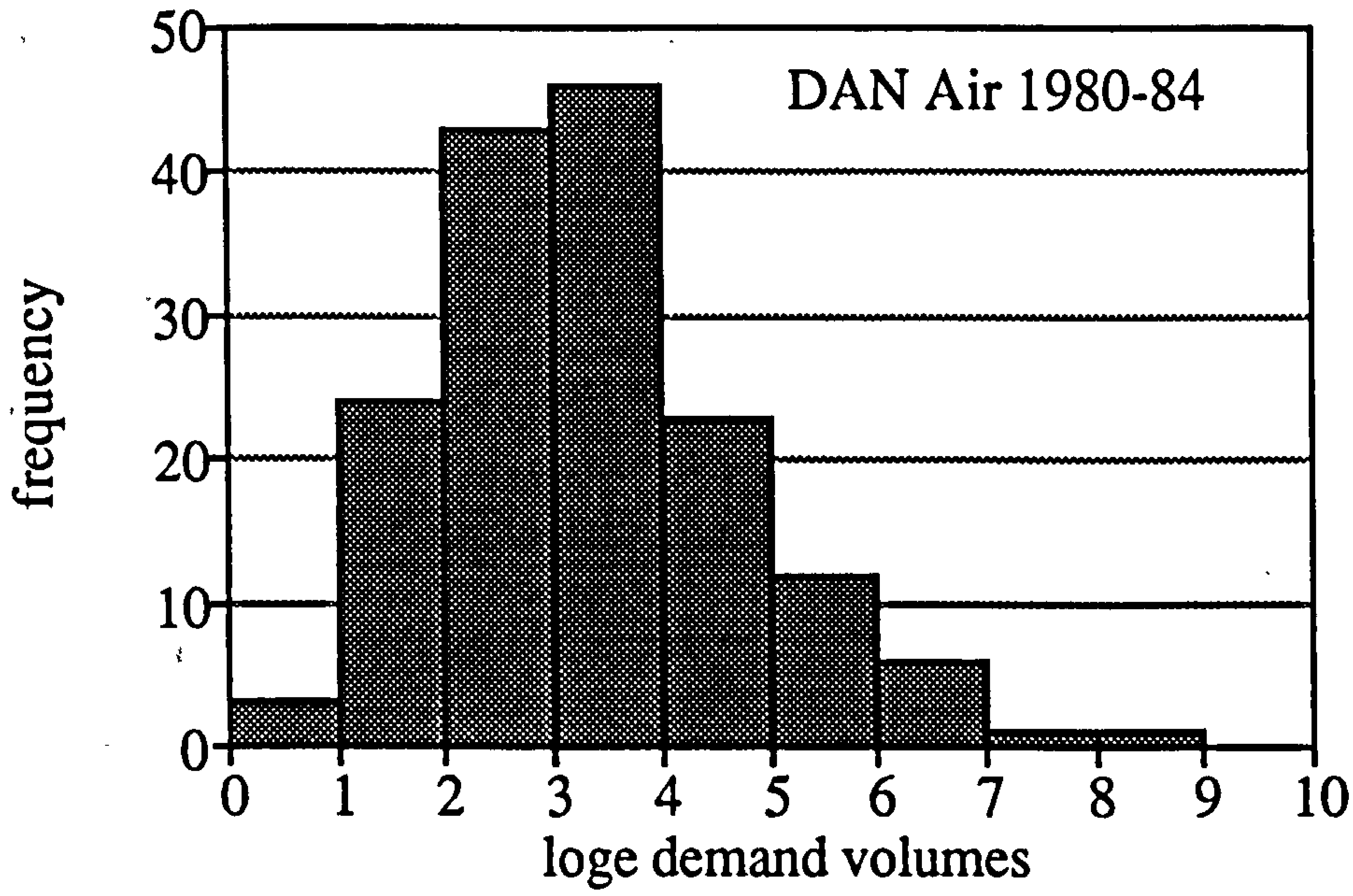
The very close correspondence between each empirical distribution in table 10.2 above and the LSD is clearly evident. Given the strong goodness of fit evidence for LSDs it was decided to progressively cumulate the aggregate demand volumes over the years 1980 to 1984 examining the aggregate $\log_e$ distribution obtained at each step. The histogram forms that were obtained are shown in the following diagrams-

figures 10.1 -10.4




DAN Air 1980-81 data
frequency
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
loge demand volumes


DAN Air 1980-82
frequency
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
loge demand volumes



It was very evident from the above histograms that the annual LSD demand volume distributions gradually converged to a lognormal distribution as shown by the normal form of the logarithms. When all the data was considered for the complete period 1980 to 1984 the $\log_e$ distribution showed a remarkable symmetry. This was tested for normality :

Table 10.3
Testing DAN Air data for normality

Loge demand upper cell value	1980-84 data	normal distribution	difference
0	0	1	-1
1	3	4	-1
2	24	17	7
3	42	38	4
4	47	46	1
5	22	33	-11
6	12	14	-2
7	6	3	3
8	1	1	0
9	1	0	1
	158	157	1

The theoretical frequency values were calculated from a normal distribution with mean 3.335 and standard deviation 1.4404. The actual Kolmogorov Smirnov statistic was Dn max = 0.071 against the theoretical KS test statistics of $Dn_{0.01}$ =0.127 and $Dn_{0.05}$ =0.108. The actual Chi Squared value was 11.11 against the test values $\chi^2_{0.01}$ of 18.475 and $\chi^2_{0.05}$ of 14.067. Hence we can be very confident that the empirical distribution of aggregate demand volumes is most likely lognormal in form. Furthermore this distribution was obtained by a summation of independent LSD distributions. This outcome is completely consistent with our aggregate Afwedson process model that was developed previously. This aggregate model clearly applies to the Dan Air demand volume data, but over a longer period of time than was seen in the DAF systems. The question here of course is why did the process take place over such a long period of time. The initial conclusion to this is that the average aggregate demand volumes are small compared to the DAF case, so that the rate of cumulation of demands over successive time periods is much slower.

10.1 (b) examination of DAN Air price data

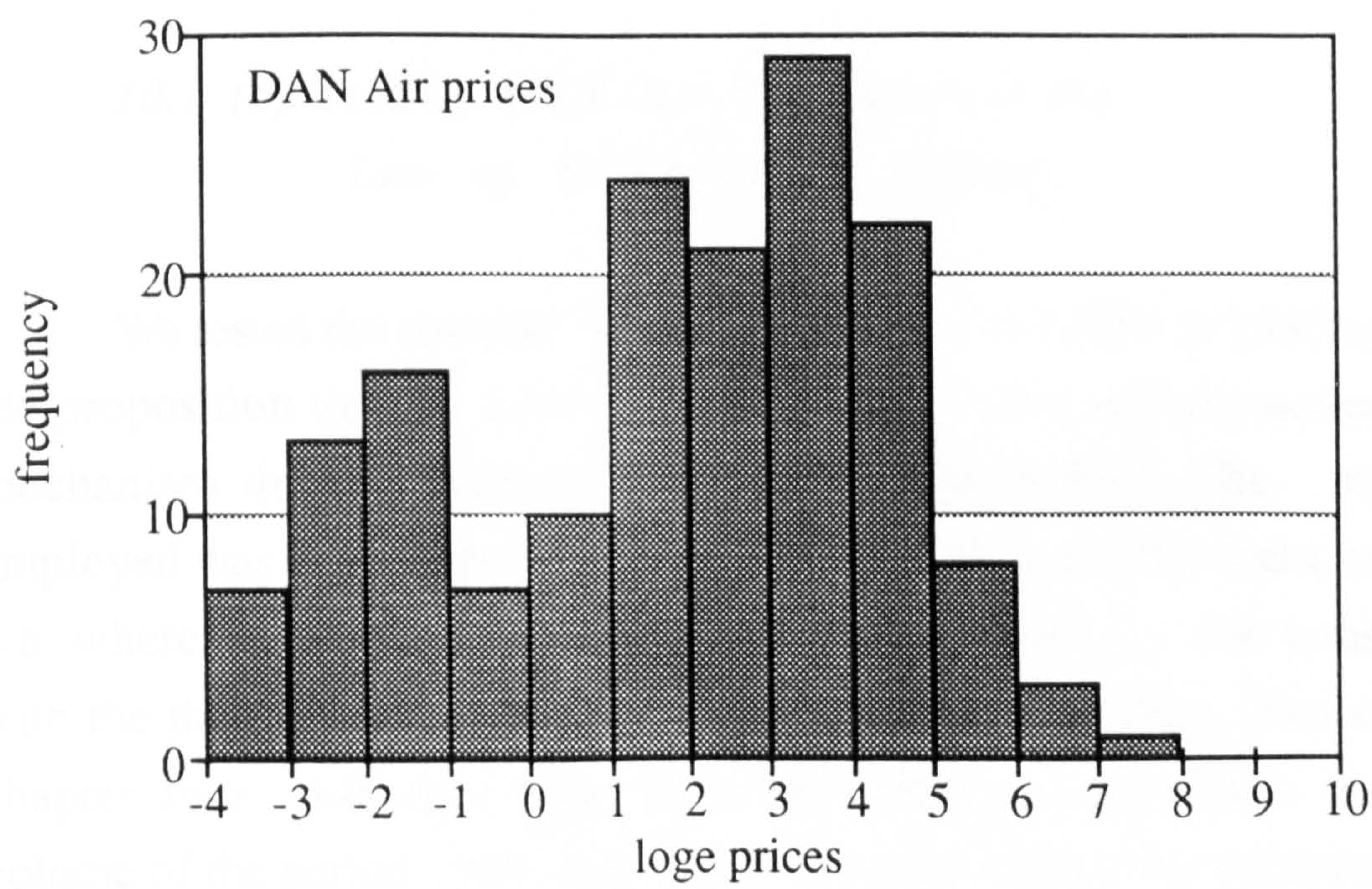
The purchase price data for all parts from the data set were also examined for lognormality. The results are tabulated below which show the frequency tabulation of the $\log_e$ price values together with the theoretical frequencies of the normal distribution with the same mean and standard deviation (ie mean = 1.7422 and standard deviation 2.657) -

Table 10.4

Testing DAN Air prices for normality

Loge price upper cell value	observed frequency	normal distribution	difference
-4	0	1	-1
-3	7	3	4
-2	13	7	6
-1	16	11	5
0	7	16	-9
1	10	22	-12
2	24	23	1
3	21	23	-2
4	29	19	10
5	22	14	8
6	8	9	-1
7	3	5	-2
8	2	2	0
9	0	1	-1

Figure 10.5



The actual Chi Squared value for this distribution was 33.407

compared to the $\chi^2_{0.01}$ and $\chi^2_{0.05}$ values of 20.091 and 15.507 respectively. The actual Kolmogorov Smirnov value was 0.0825 against theoretical test values $Dn_{0.01}$ and $Dn_{0.05}$ of 0.1261 and 0.1072 respectively. Hence on the basis of the Chi Squared test we should clearly reject the possibility of the distribution as being normal, although the Kolmogorov Smirnov test does not reject the null hypothesis. In view of the shape of the distribution, as seen from the histogram, and the high Chi Squared value we would wisely accept that the sample distribution is in all probability not normal. Hence consumable spare prices would therefore not be lognormally distributed. The bi-model nature of the price distribution suggests the sample may comprise readings from more than one underlying parent population.

From a usage value distribution point of view we could not use the lognormal distribution and theory to set aggregate inventory standards, especially as the annual demand volumes are not lognormal. The inventory estimates so produced by this process would have large errors.

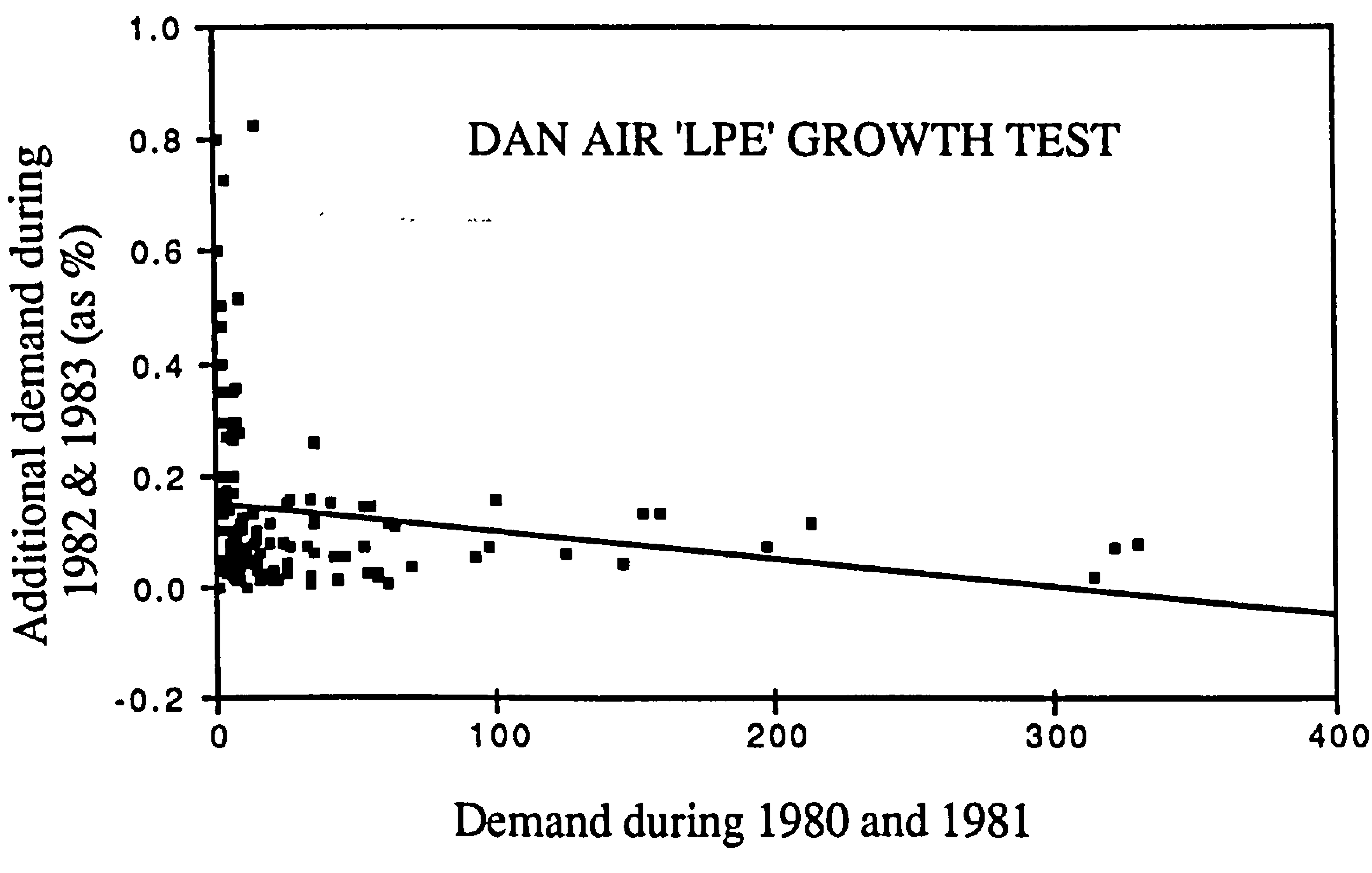
10.1 (c) Testing DAN Air data against the Law of Proportionate Effect

We tested the demand volumes over the period 1980 to 1983 against the proposition that the Law of Proportionate Effect was the underlying mechanism driving demand volumes to lognormality. The method employed was exactly the same process as shown in chapter eight, section 8.6, where we tested DAF data against the law. This was also consistent with the theory concerning the Law of Proportionate Effect discussed in chapter four. 146 data pairs from the DAN Air cumulative demand volume of the period 1980+1981 were regressed against the percentage of additional demand growth over the period 1982+1983. The regression results which follow clearly show that no relation exists between the size of demand at a period *t* to the growth in demand after period *t*+1. Hence growth is a random proportion of demand at any point as the law predicts.

correlation coefficient r = 0.183
coefficient of determination $R^2 = 0.033$

The plot is shown in figure 10.16 :

figure 10.6



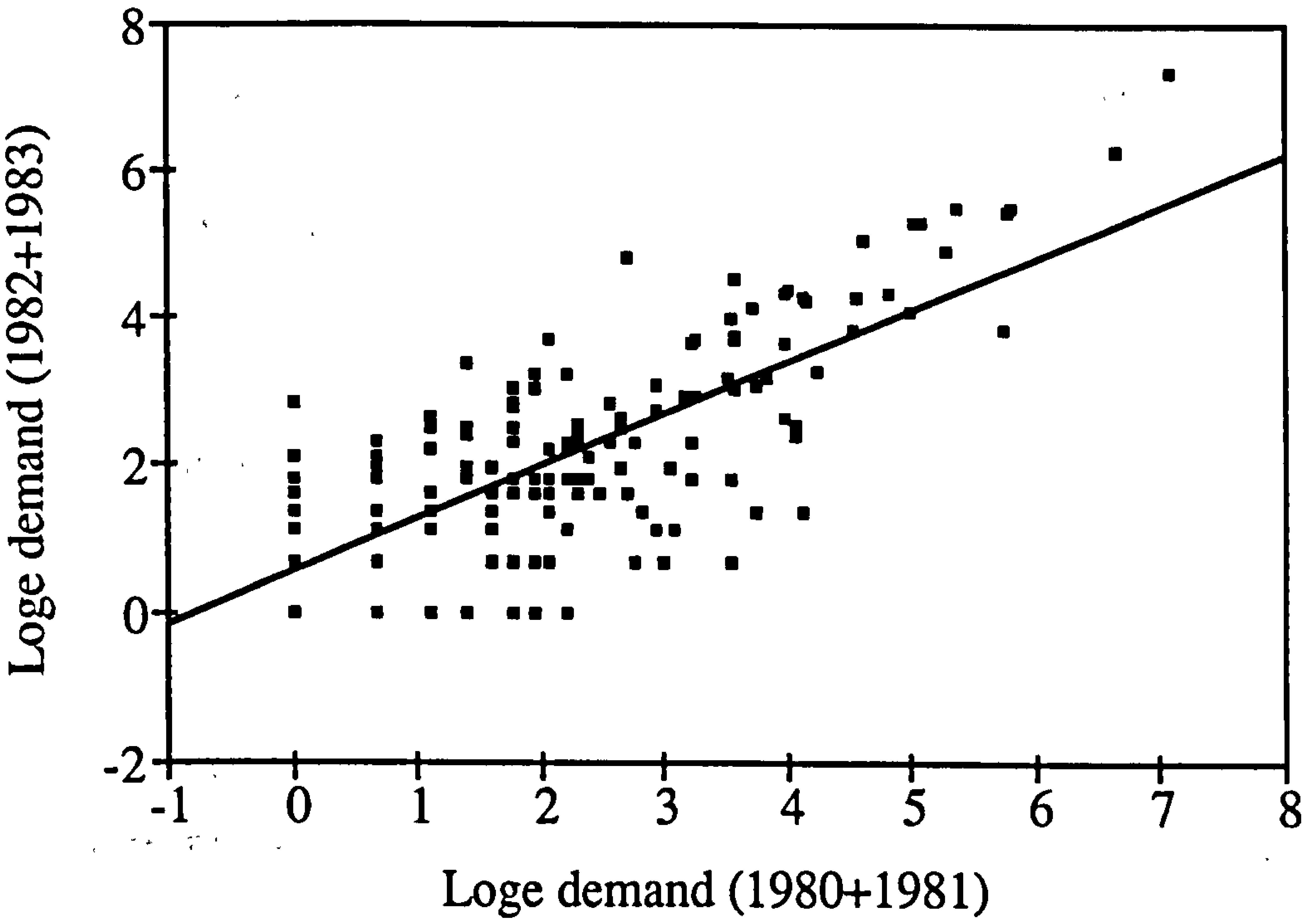
We next regressed the logarithm of cumulative demand for (1980+1981) against the logarithm of cumulative demand for (1982+1983). The regression results gave the following positive results.

correlation coefficient r = 0.731
coefficient of determination R^2
Durbin Watson test DW = 1.848
Standard error SE = 0.978

The regression plot is shown in figure 10.17 that follows :-

figure 10.7

Regression of the logarithms of demand



The positive regression equation produced from this was as follows:

$$\log_e S_{i,t} = -0.553 + 0.714 \log_e S_{i,t-1}$$

where $S_{i,t}$ is the size of demand of element i at period.

The regression coefficient '*b*' at 0.714 was significantly greater than 0 but was also significantly smaller than unity. This equation should be compared with those obtained for the DAF data on page 289 in chapter eight. We can conclude that we have obtained broadly similar results here to the DAF case, although the correlation is not as strong. We can however conclude that the evidence for the Law of Proportionate Effect driving demand volumes to lognormality in this case is also good and it certainly cannot be rejected on the basis of the evidence presented here.

10.2 Moore and Large Wholesalers

Moore and Large are a wholesaler of cycles and cycles spares and consumables supplying the cycle retail industry. In 1981 they were based at Luton in Bedfordshire supplying around 60 retails outlets. The range of cycle spares stocked at the time amounted to some 2,000 individual parts items. This was however, a very fluid number due to the quick obsolescence of some items and the company's policy of taking on new 'fashion' items fairly quickly. The spares ranged from consumable items such as tyres, tubes and brake blocks, replacement components such as gear mechanisms, wheels and fork sets, to impulse purchased items such as frame transfer sets, drinking bottles and horns. This latter group of items were not spare parts in the generally accepted sense of wear and repair items. However, they only constituted about 5% of the item range.

This author was given the opportunity to take a data sample from a computer print out listing the annual parts sales and purchased parts prices for the entire range of parts sold during 1981. The inventory range type and demand characteristics fitted the general criteria set for data selection, so a sample of 200 items were randomly selected for data analysis. This was a one off data sample and no further data was obtained from the company. In view of this annual demand volume, parts prices and annual parts usage values (sales turnover) were subjected to analysis for lognormality. The results of this are summarised below in both graphic and tabular forms.

10.2 (a) cycle spare prices distribution.

The form of the $\log_e$ distribution for cycles spares prices is shown in figure 10.8 which follows and table 10.5 from which the form of the normal distribution is evident.

figure 10.8

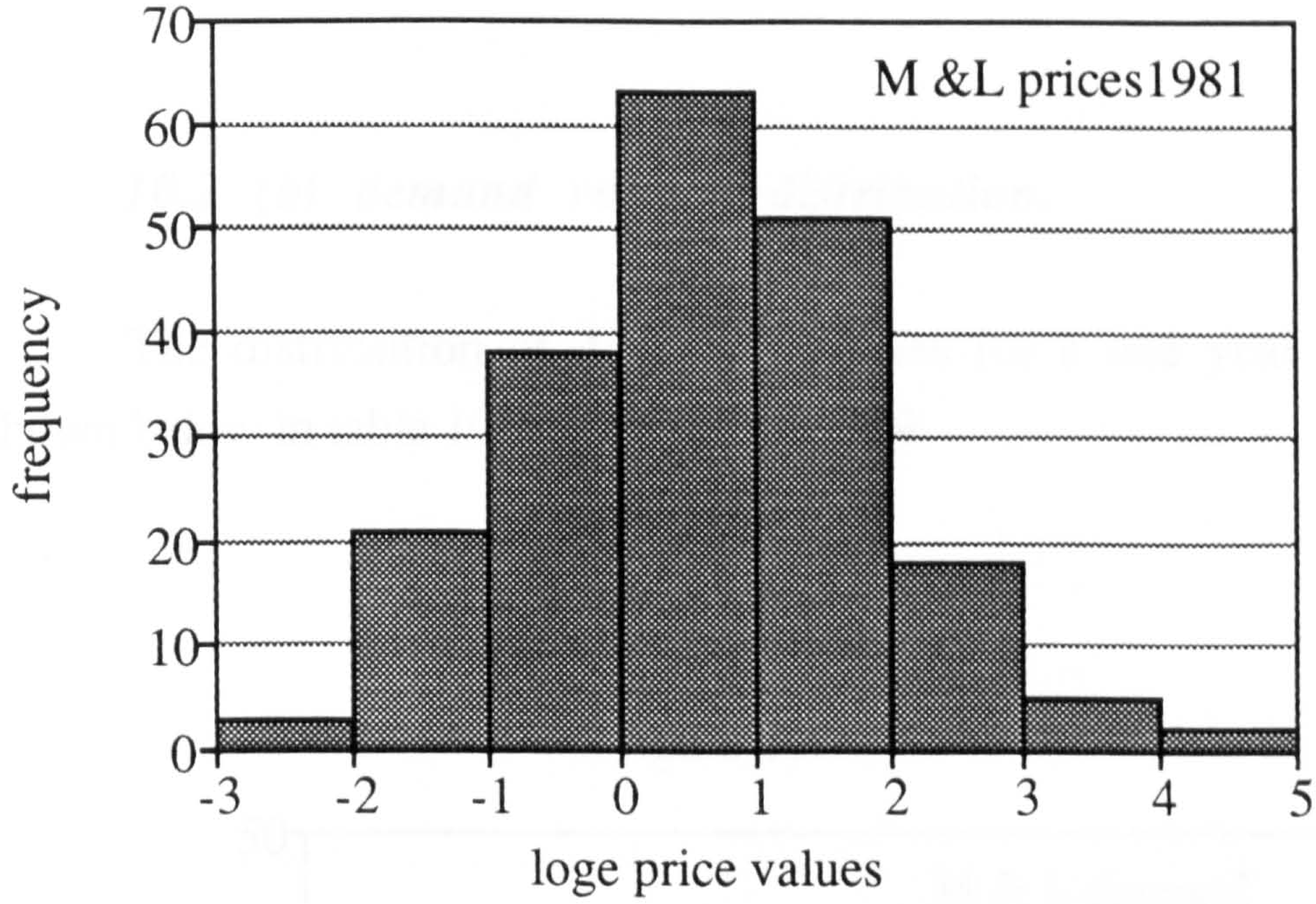


Table 10.5
Normality test on Cycle Parts prices

Loge price upper cell value	observed frequency	normal distribution	difference
-4	0	0	0
-3	1	0	1
-2	4	2	2
-1	17	22	-5
0	43	38	5
1	61	62	-1
2	48	52	-4
3	21	18	3
4	5	5	0
5	1	1	0
6	0	0	0

This distribution gave an actual Chi Square value of 4.767 compared to $\chi^2_{0.05}$ of 9.488, and $\chi^2_{0.01}$ of 13.277. This highly significant result together with the high degree of symmetry of the log histogram would strongly support the view that the parts prices are most likely to be lognormally distributed.

10.2 (b) demand volume distribution.

The distribution of demand volumes for a one year period are shown below in table 10.6 and in figure 10.9

figure 10.9

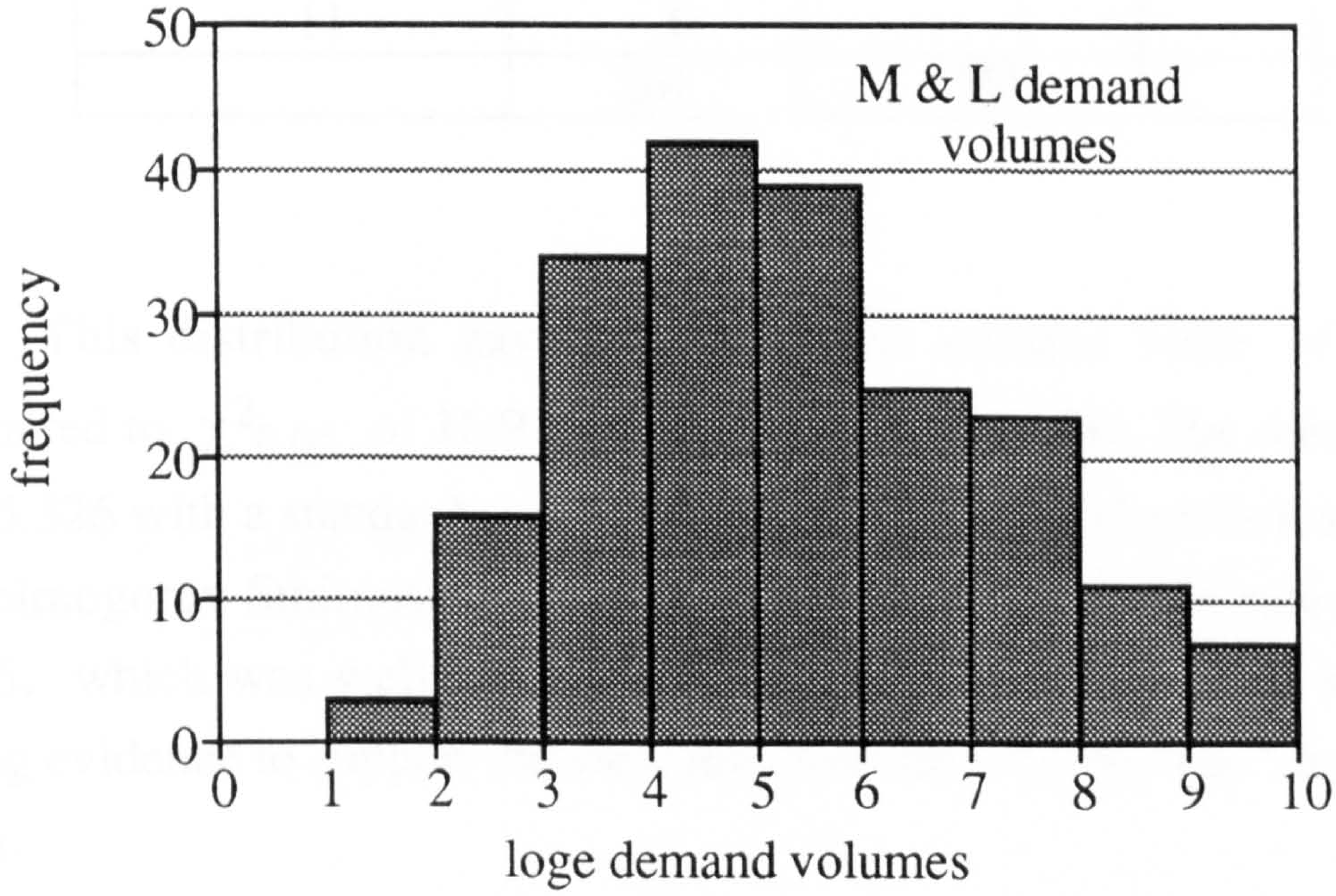


Table 10.6
Testing M & L demand volumes for normality

Loge volume upper cell value	observed frequency	normal distribution	difference
0	0	1	-1
1	0	2	-2
2	3	5	-2
3	17	13	4
4	34	26	8
5	42	39	3
6	39	43	-4
7	25	35	-10
8	23	21	2
9	12	10	2
10	6	3	3
11	0	1	-1
	201	199	2

This distribution gave an actual Chi squared value of 11.028 compared to $\chi^2_{0.05}$ of 16.919, and $\chi^2_{0.01}$ of 21.667. The mean value was 5.326 with a standard error of 0.128 and standard deviation at 1.814. A Kolmogorov Smirnov 'normality' test produced a significance level of 0.166, which was well within the 0.01 and 0.05 levels. Thus we have strong evidence to support the view that demand volumes are 'normal' in form.

10.2 (c) usage value distribution

Given the fact that the above evidence strongly supports lognormality for both prices and demand volumes then it follows that usage values should also be lognormal. However, this was put to the test as shown below to verify the theoretical prediction.

figure 10.10

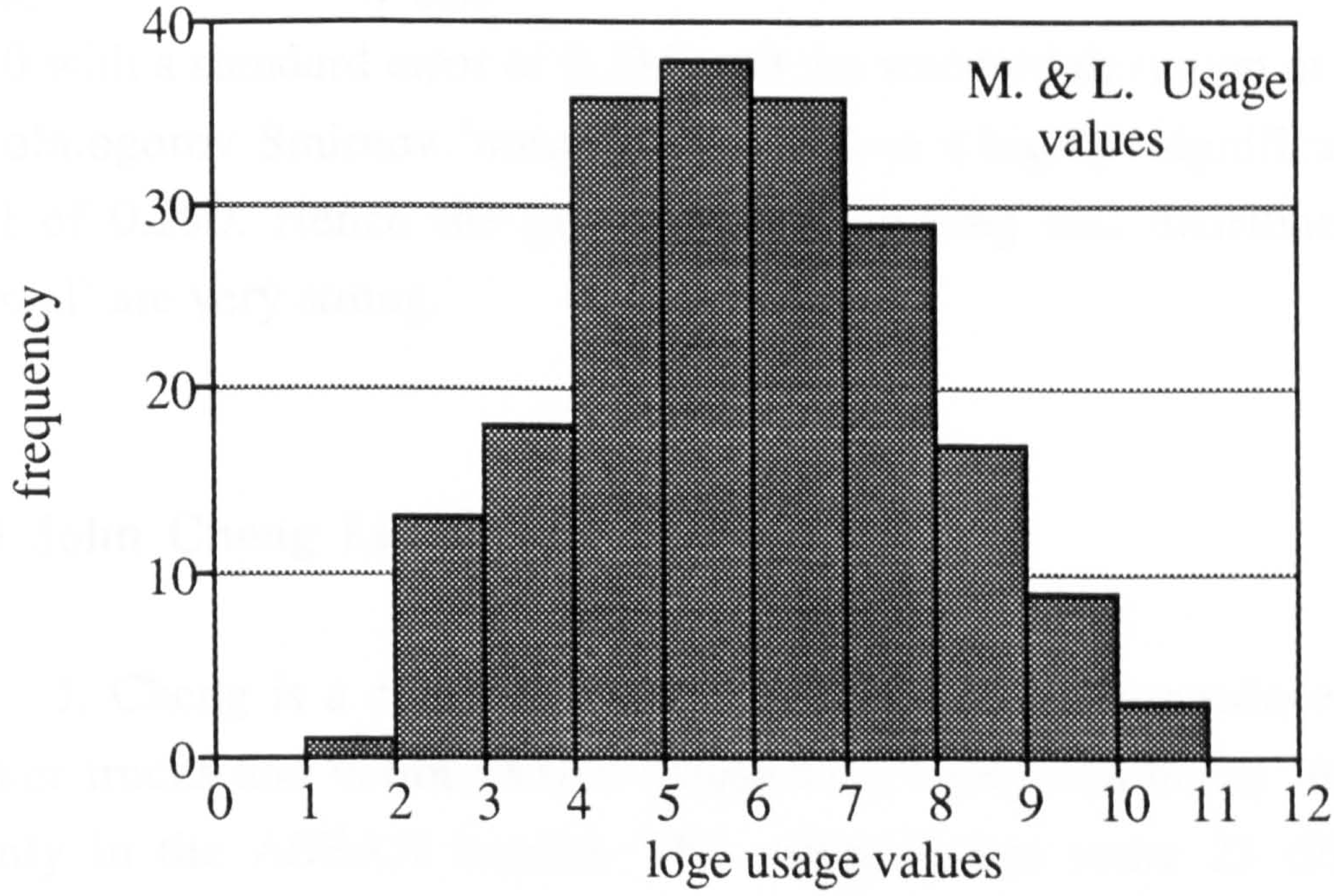


Table 10.7
Normality test on M & L Usage values

Loge usage value upper cell value	observed frequency	normal distribution	difference
1	0	1	-1
2	1	3	-2
3	13	8	5
4	18	19	-1
5	36	32	4
6	38	41	-3
7	36	40	-4
8	29	30	-1
9	17	16	1
10	9	7	2
11	3	2	1
12	0	1	-1
	200	200	0

This distribution gave an actual Chi squared of 2.001 compared to $\chi^2_{0.05}$ of 11.070 and $\chi^2_{0.01}$ of 15.086. The mean of the distribution was 5.910 with a standard error of 0.133, and the standard deviation at 1.874. A Kolmogorov Smirnov 'normality' test gave a highly significant test level of 0.290. Hence the grounds for accepting this distribution as 'normal' are very strong.

10.3 John Cheng Lift Gear.

J. Cheng is a company based in Singapore. They produce small stacker trucks and various small industrial lift gear and hoists for sales mainly in the ASEAN market. The company has some 25 different products in its full range and these are supported by a spare parts range of around 2000 items for the service and repair requirements of previously sold products. The data analysed here was provided to the author by one of his students, who conducted a study of the company's stock control policy in the summer of 1984, for his MSc thesis (Tan 1984).The inventory range and demand character satisfied our general criteria for data selection and analysis. 200 parts were selected at random from the spare parts range covering a demand period for the previous 12 months. The analysis of aggregate demand volumes, parts prices and usage values are shown below. In each case the data is shown in histogram form then tabulated against the corresponding normal distribution with the same mean and standard deviation. Chi Squared and Kolmogorov Smirnov 'normality' tests were carried out in each case.

10.3 (a) demand volume data.

The histogram for the demand volume data is shown in figure 10.11 :

Figure 10.11

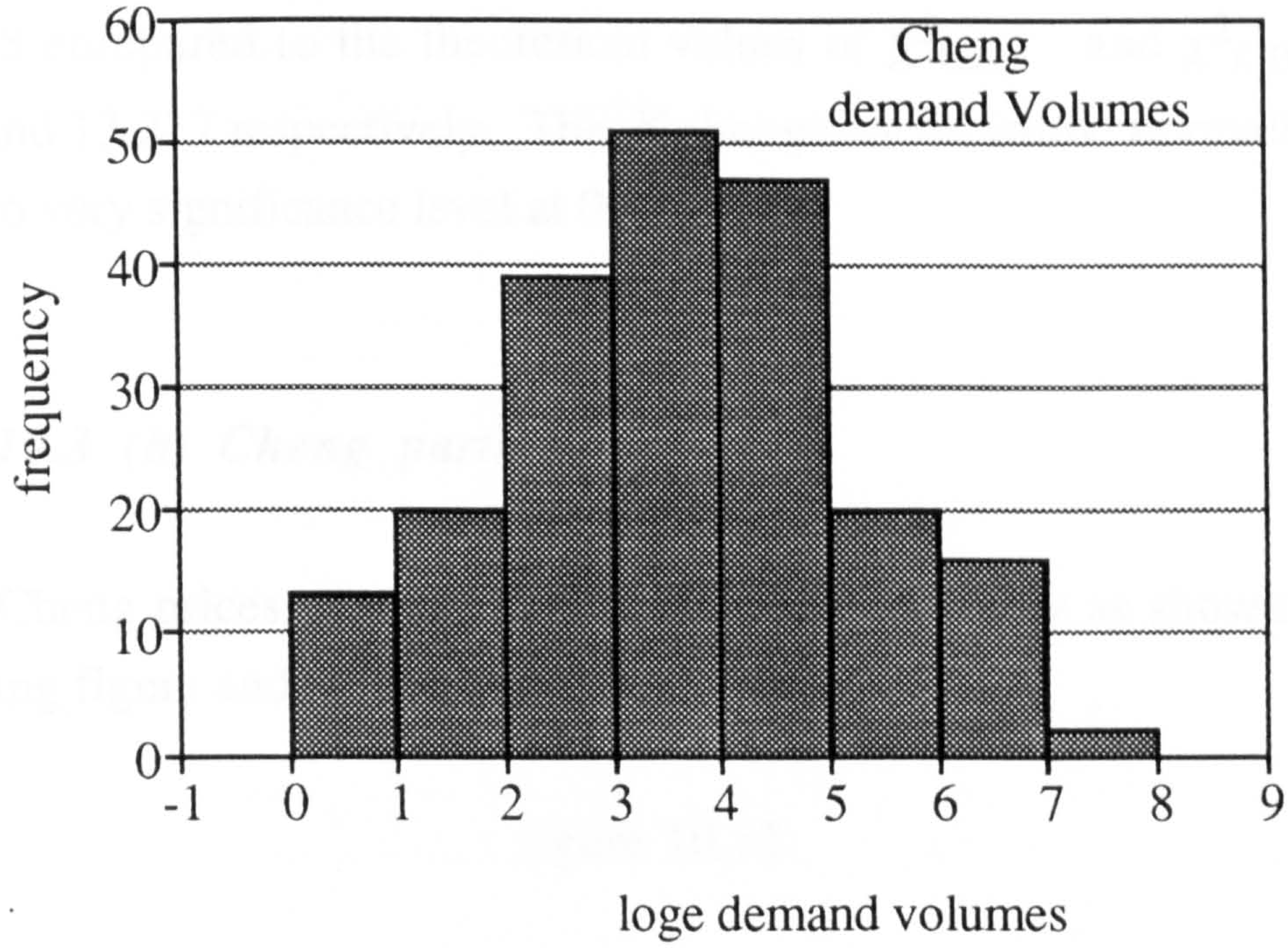


Table 10.8
Normality test on Cheng demand volumes

Loge demand vol. upper cell value	observed frequency	normal distribution	difference
-1	0	1	-1
0	0	2	-1
1	13	8	5
2	20	23	-3
3	39	39	0
4	51	48	3
5	47	49	-2
6	20	26	-6
7	16	10	6
8	2	3	-1
9	0	1	-1
10	0	0	0
	208	210	-1

This data gave a mean of 3.615 with standard error of 0.109, a standard deviation of 1.569. The actual Chi Squared value was very small at 4.868 compared to the theoretical values of $\chi^2_{0.05}$ and $\chi^2_{0.01}$ of 9.488 and 13.277 respectively. The Kolmogorov Smirnov normality test was also very significance level at 0.273.

10.3 (b) Cheng parts prices

Cheng prices were also checked for lognormality as shown in the following figure and table 10.9.

figure 10.12

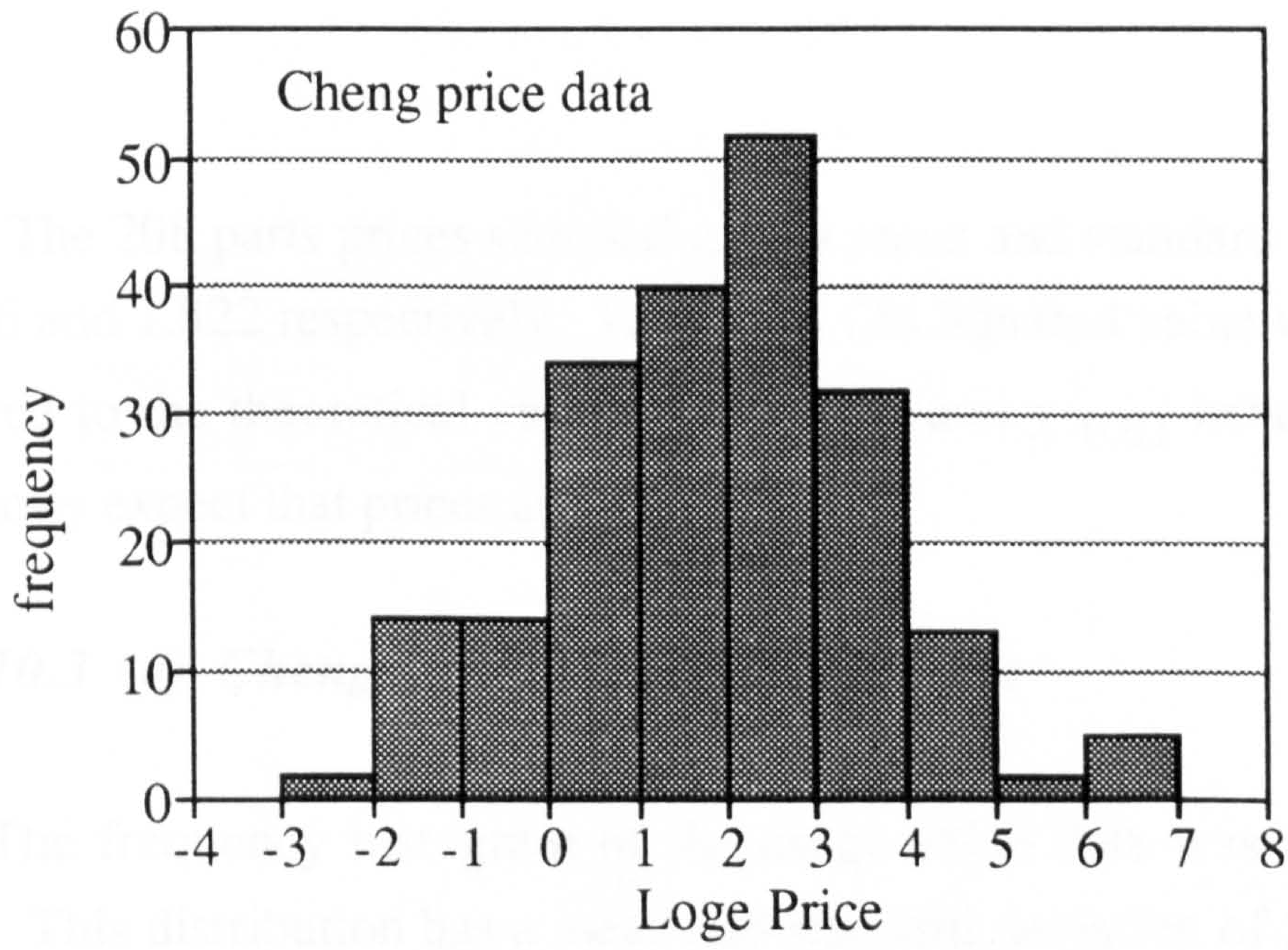


Table 10.9
Normality test on Cheng parts prices

Loge prices upper cell value	observed frequency	normal distribution	difference
-2	2	2	0
-1	14	8	-1
0	14	19	-5
1	34	35	-1
2	40	44	-4
3	52	43	9
4	32	31	1
5	13	17	-4
6	2	7	-5
7	5	2	3
8	0	0	0
	208	208	-7

The 208 parts prices sampled gave a mean and standard deviation of 1.916 and 1.822 respectively. The actual Chi Squared value was 9.942 compared to the theoretical values of $\chi^2_{0.05}$ and $\chi^2_{0.01}$ hence we can confidently expect that prices are lognormal

10.3 (c) Cheng usage value distribution

The frequency histogram of the usage value data was as shown below. This distribution has a mean and standard deviation of 5.531 and 1.809 respectively. The theoretical frequencies used in table 10.10 were calculated using these same mean and standard deviation values.

figure 10.13

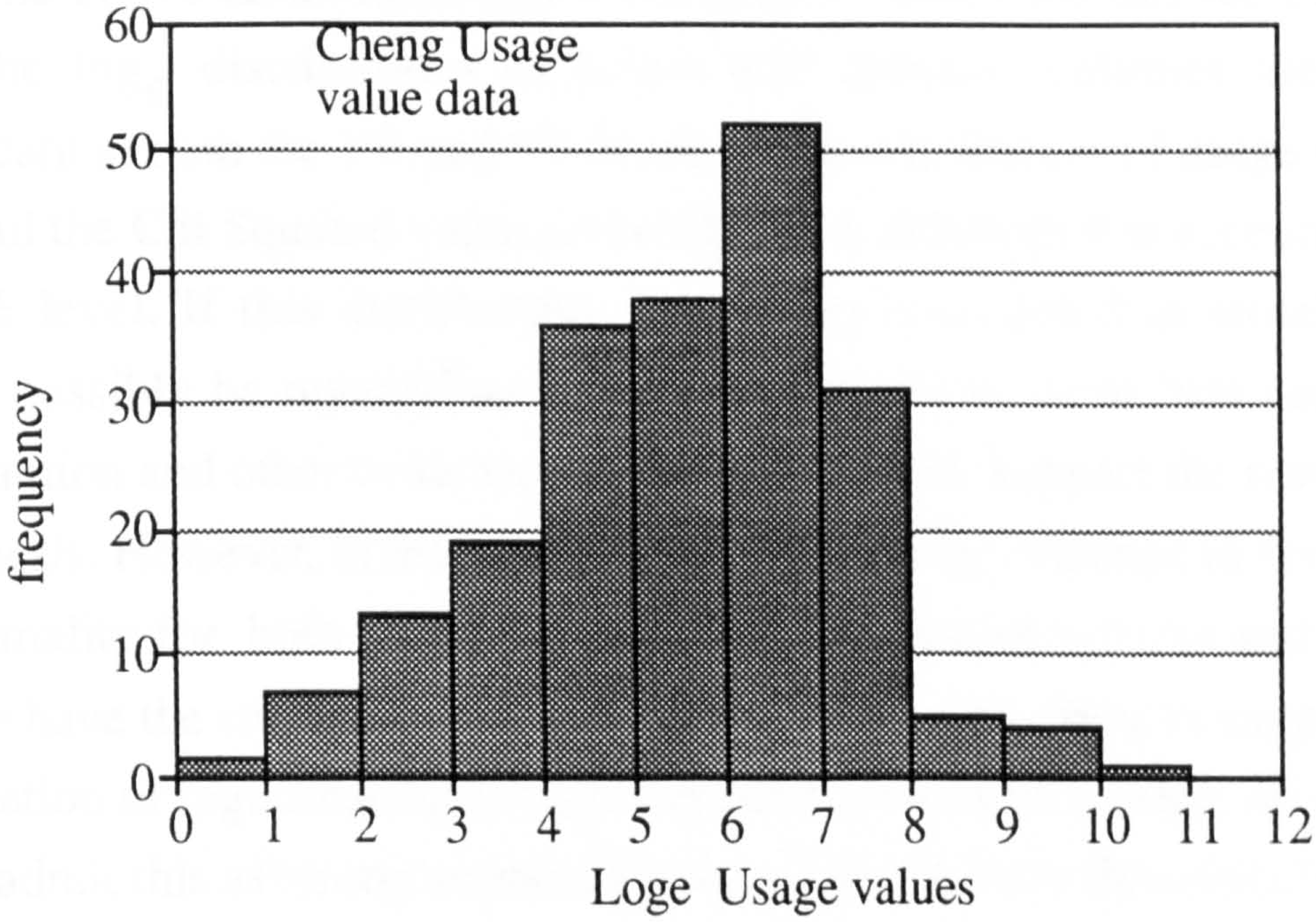


Table 10.10

Normality test on Cheng Usage value data

Loge prices upper cell value	observed frequency	normal distribution	difference
0	0	0	0
1	2	1	-1
2	7	4	3
3	13	12	1
4	19	24	-5
5	36	37	-1
6	38	48	-10
7	52	39	13
8	31	25	6
9	5	12	-7
10	3	4	-1
11	1	1	0
12	0	0	0
	207	207	-2

The actual Chi Squared value was 15.497 compared to the theoretical values of $\chi^2_{0.05}$ and $\chi^2_{0.01}$ of 12.592 and 16.812 respectively. From the above tabulations and Chi Squared values we can see that the both the $\log_e$ distributions of prices and demand volumes are very significant at both the 1% and 5% levels. The distribution of usage values does fail the Chi Squared value at the 5% level, although it is acceptable at the 1% level. If this distribution were being considered in isolation it would possibly be rejected as a normal distribution, or at best accepted with caution and other evidence would be sought to support the normality hypothesis. However, in the case here there is strong evidence in favour of lognormality for both the component parts of demand volume and prices and we have the stochastic evidence of the previous chapters to support the expectation of lognormality in such a system. [Although in truth we cannot really admit this as strong support, because it is this very theoretical model we are seeking additional support for by this analysis]. So we must be content here to say that the product of two lognormal distributions must itself also be lognormal; then we can accept that in all probability the distribution of usage values in the case of the J. Cheng data is also lognormal.

10.4 DMC Limited.

In 1982 DMC Ltd. was a small Cyprus based dealer selling Lancia and Honda cars. They also provided full spares and service support for all models previously sold in the Cyprus market. The company maintained a stock level of some 6,000 different parts for the Honda cars and in addition some 8,000 parts lines for the Lancia models. This company and its inventory data was particularly valuable to the present study because it operated such crude stock control methods and records. Very rudimentary stock control was essentially all done manually using stock record cards giving access to individual parts usage data over several years of operation. This is quite unusual in the motor industry in the 80's as some form of computer control is now almost universal in a spare parts environment. The access to data on stock cards gave the valuable opportunity to study demand rates and particularly times between demands for selected parts, and hence an opportunity to test for Poisson occurrence of demand and compounding of demand.

Given the crude methods of controlling stock not surprisingly the company was very overstocked on spares holding. Annual sales of all parts in 1981 was £110,000 and the value of the parts stock at year end was £135,000 giving a stock turnover ratio less than one.

A 200 item sample of annual sales data was extracted from annual sales records for the purpose of examining the annual usage distribution. The rest of the analysis was confined to examining individual item demand rates.

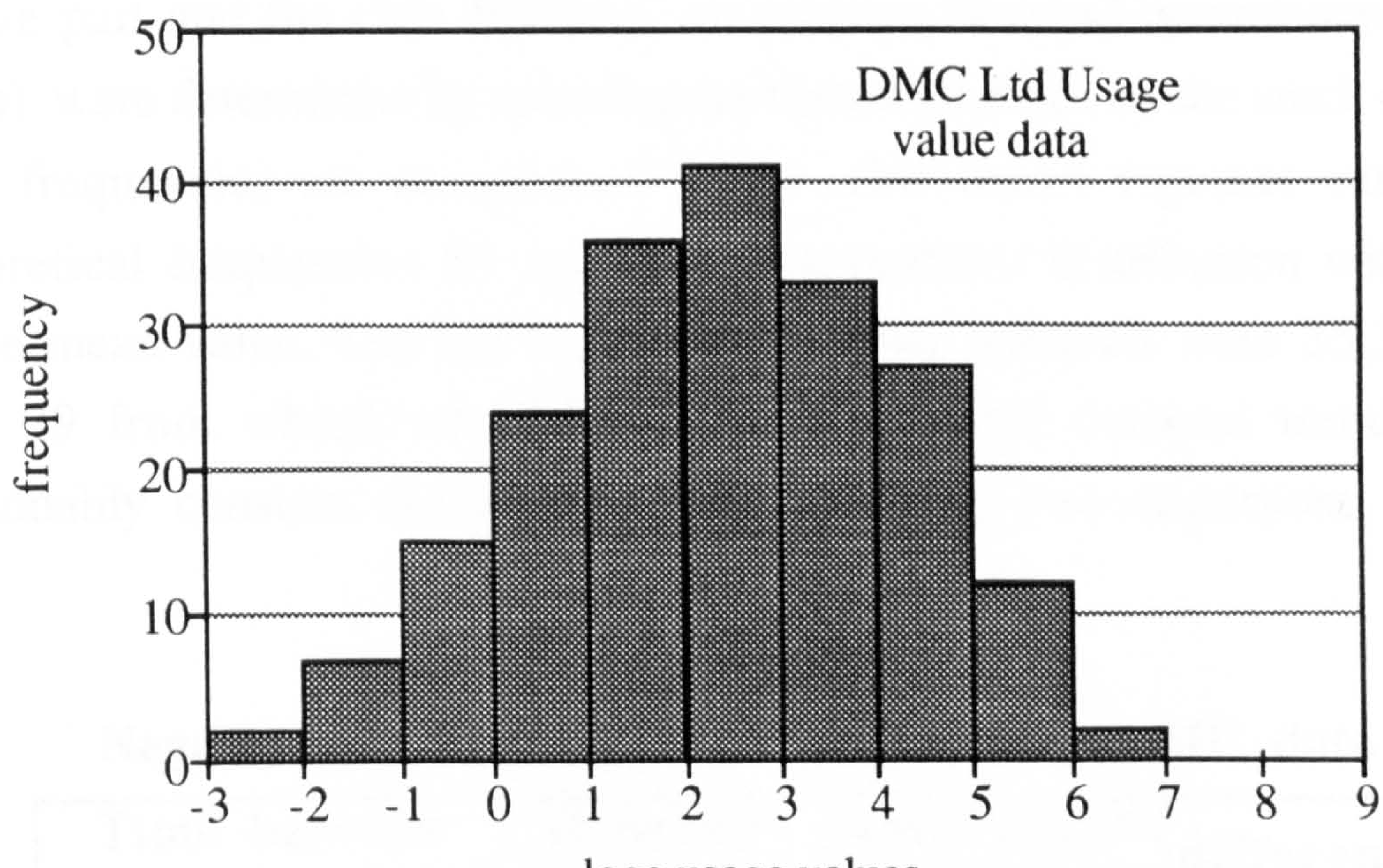
10.4 (a) usage value distribution

Table 10.11
Normality test on DMC Usage value data

Loge usage values upper cell value	observed frequency	normal distribution	difference
-2	2	1	1
-1	7	5	-1
0	14	14	0
1	25	27	-2
2	35	38	-3
3	42	42	0
4	32	35	-3
5	28	22	6
6	12	10	2
7	2	4	-2
8	0	1	-1
	199	199	-3

The theoretical frequencies were calculated from a normal distribution with the same mean and standard deviation as the empirical distribution (ie x = 2.310 and s = 1.861). The actual Chi Squared value was 5.145 compared to theoretical values of 12.592 and 16.812 at 5% and 1% levels of significance respectively. The distribution also passed the Kolmogorov Smirnov test at the 1% and 5% levels of significance. Given these test results and the high degree of symmetry of the $\log_e$ histogram as shown below, then we can be confident in accepting these usage values as most likely lognormal.

figure 10.14



10.4(b) demand analysis.

This analysis was carried out to examine the occurrence of demand by measuring the time between demands, and the distribution of the quantity of demands. We were anxious to see if the Poisson distribution was appropriate for motor component spares and to see what compounding distribution might be affecting demand quantities. To achieve this analysis care had to be taken when selecting appropriate spare parts items. A sufficiently long demand period was required with very little trend evident in the sales, as far as could be ascertained from the stock cards. Also, it was necessary to remove the effects of multiple customer purchases on any one day; so in general it was the slow moving parts that met the necessary requirements due to the very low demand rates that could be measured from the cards.

(i) part no. 2280053 Clutch unit (expensive low volume wear part)

The stock cards over a four year period were examined for the above part and the time between consecutive demand occurrences (in days) were determined by counting days between dates on the stock cards. The frequencies are summarised in the table below together with the theoretical frequencies for a negative exponential distribution with the same mean value. The last four complete year demands were 35,38,18, and 29 from which we deduce that the overall demand trend was reasonably constant, at least sufficiently so for our present purpose.

Table 10.12

Negative exponential distribution test on DMC data

Time between demands (days)	observed distribution	theoretical distribution	difference
1-5	70	64	6
6-10	28	30	-1
11-15	14	14	0
16-20	1	7	-6
21-25	3	3	0
26-30	1	1	0
30+	0	2	-2
	117	121	-3

The very close correspondence between the theoretical distribution and the empirical distribution is very evident from the above tabulation. This was a most re-assuring finding after counting through four years worth of stock card data. Using a Kolmogorov Smirnov test we obtained significant test results at both 1% and 5% levels of significance. The mean of the distribution was as shown at 6.632 from which we can derive the mean of the associated Poisson mean of demand occurrences. The mean of a negative exponential interevent distribution is the reciprocal of the Poisson occurrence mean (basic theory of Poisson processes, see chapter five). Hence the mean occurrence of demand is therefore 1/6.632 =0.151 occurrences per day.

Next we considered the distribution of the number of demands on each demand occasion for this same part number. This was as follows in table 10.13 :

Table 10.13

Testing for demand compounding

Number of demands	frequency
1	122
2	6
3	3
4	1

As would be expected for demands at the retail level the amount of compounding is very small, most of the demands are for a single unit. From the theoretical discussion of chapter five we saw that the favoured models for compounding, in the lumpy demand environment, were the geometric distribution (leading to the Stuttering Poisson distribution) and the Log Series distribution (leading to the integer NBD model). The data in the table was tested against both distributions to see if either could explain compounding. The fitted theoretical and empirical frequencies are shown below in table 10.14 for a LSD parameter value '*q*' of 0.15 and a distribution mean of 1.11. The geometric parameter 'P' was calculated from the empirical mean from P = 1/mean as shown chapter six.

Table 10.14

Fitting the LSD & Geometric distributions to DMC data

number of demand	empirical frequencies	LSD frequencies	Geometric frequencies
1	122	122	116
2	6	8	14
3	3	1	2
4	1	0	0

It can be seen that an almost exact fit was obtained between the empirical data and the LSD, whereas the Geometric distribution gave an inferior closeness of fit. This is a limited data set so one could not draw very strong conclusions regarding the efficiency of one or the other of the two distributions, but on the evidence so far the LSD looks the better candidate. Hence from the foregoing we have very promising but limited evidence that the Afwedson model (Poisson occurrence and LSD compounding of demand) is a very good fit to the above demand data over the four year period for the clutch unit, where-as the Stuttering Poisson might well be a candidate with different compounding parameter values.

We next attempted to fit a NBD to all demands in the four year period including days of zero demand. We calculated $P(0)$ for the NBD first by assuming 1,040 working days in the four year period, then knowing that there had been 132 days on which a demand occurred, we could deduce that the days of zero demand were 908.

$$\text{Hence } P(0) = 908/_{1040} = 0.8731$$

Subsequent probabilities were calculated as shown in chapter eight using the recursive NBD formula :

$$P(r)=\left[\frac{a}{(1-a)}\right]\left[1-\frac{(a-m)}{ar}\right]P_{r-1}$$

where 'm' is the mean of the distribution.
The exponent 'k' was calculated from

$$k= {}^{m}/_{(q\text{-}1)}$$

where $q =$ variance /mean ratio and $a = {}^{m}/_{k}$

Now from the calculated mean of 0.1413 and variance 0.1599 the theoretical NBD frequencies were determined and compared with the empirical frequencies as shown in table 10.15 :

Table 10.15

NBD test of DMC demand data

Number of demands	empirical distribution	NBD distribution
0	908	908
1	122	128
2	6	17
3	3	2
4	1	1
5	0	0

The very close correspondence between the distributions is evident. We can deduce from this, so far, that certainly for this particular spare part demand occasions are Poisson, the individual demand quantities are LSD and the overall distribution of demand quantity, including the zero demand occasions is NBD. Hence these are all the conditions for the

Poisson LSD -NBD model, ie Afwedson model, but applied to a single item.

(ii) Other parts interevent distributions

We determined the interval between demands for two other parts- a water pump (medium price, high volume), a gasket set (moderate volume, low price). In both cases there was almost no compounding of demand as each demand occasion observed from the stock cards was for single units. The distribution of days between demands was determined as before by counting days between demand dates on the stock cards. Both distributions are summarised together with the theoretical frequencies from the corresponding exponential distributions-

(a) gasket set

Table 10.16

Testing for the exponential distribution

Time between demands (days)	observed distribution	exponential distribution	difference
1-10	28	29	-1
11-20	21	20	-1
21-30	16	12	4
31-40	5	7	-2
41-50	2	4	-2
51-60	3	2	1
60+	2	3	-1
	77	77	-2

(b) water pump

The tabulation follows in table 10.17

Table 10.17

Testing for the exponential distribution

Time between demands (days)	empirical distribution	exponential distribution	difference
1-5	70	60	10
6-10	28	28	0
11-15	14	14	0
16-20	1	7	-6
21-25	3	5	-2
26-30	1	3	-1
31+	4	4	0

In both cases of the above parts the evidence for a negative exponential interevent distribution is good and hence the counting distribution of demand occasions in a fixed interval will be simple Poisson.

(c) additional compound distributions

We consider here two additional parts (an oil seal and a dust cover) both of which showed considerable compounding in the units demanded. Using the stock cards we determined the quantity demanded at each demand occasion for both parts over a four year period. The data was then examined to see which of our two theoretical distributions might give the superior fit to the empirical observations.

Table 10.18

Validating LSD &Geometric distributions

Number of demands	observed distribution	Geometric distribution	difference	LSD distribution	difference
1	249	241	8	249	0
2	35	50	-15	42	-7
3	12	10	2	10	2
4	6	2	4	2	4
5	2	2	0	0	2
6	0	0	0	0	0
	304	305	-1	303	1

The theoretical distribution parameters for the above tabulation were - LSD parameter 'q' = 0.85 and Geometric parameter 'p' 0.7938 =

Table 10.19

Validating the LSD and Geometric distributions against the dust cover demand data

Number of demands	observed distribution	LSD distribution	difference	Geometric distribution	difference
1	142	143	-1	162	-20
2	96	61	35	86	10
3	35	35	0	45	-10
4	23	22	1	13	10
5	10	15	-5	6	4
6	8	11	-3	3	5
7	0	8	-8	2	-2
8	4	6	-2	1	3
9	0	4	-4	1	-1
10	1	3	-2	1	0
11	1	3	-2	0	1
12	0	2	-2	0	0
13	0	1	-1	0	0
	320	314	6	320	0

The theoretical distribution parameters for the above tabulation were - LSD parameter 'q' = 0.34 and Geometric parameter 'P' = 0.4692

In table 10.19 the data fit favours the Geometric distribution over the LSD in this particular case on the basis of the randomness of fit and the overall closeness of fit. The systematic run of negative differences in the case of the LSD fit is not a good sign. This generally indicates a wrongly fitted model in the first place. In table 10.18 the data looks a marginally better fit to the LSD distribution because the actual magnitude of the departures are smaller than with the Geometric fit, although the overall closeness of fit is the same in both cases.

10.5 Conclusions

The data sets examined here have enabled us to at least verify a number of aspects of the work of previous chapters. In particular the Dan Air data has shown all the characteristics of the aggregate Afwedson process that was seen in the DAF system, but the empirical demand volume data only converged to this model over a several year period. This was almost certainly due to the much lower overall level of demand volumes in the DAN Air case compared to DAF Trucks. We must also take into account that the Dan Air data is from just one operator holding spares for their aircraft own fleet consumption. In contrast DAF is the distributor level for a system of retailers selling spares on to a vast number of operators. The results of the analysis of the Dan Air data show that the lognormal distribution could not be used to set aggregate inventory standards in a short period, or even annually, because of the poor fit of annual demand volume data to the lognormal. The price distribution was also a very poor fit to the lognormal. We suspected that the price data was very out of date, and comprised values drawn effectively spanning several years. In table 10.20 which follows we give an estimates of the lognormal parameters for DAN Air for comparison with the other systems described in this chapter. To produce these estimates we made a very crude estimation of the likely distribution of DAN Air prices if they had all been from the same time period. The estimation is based on the normal curve drawn against the DAN Air price histogram as shown in figure 10.15 on page 360.

The Moore and Large and J.Cheng data sets confirmed the expectation that usage values in such systems were lognormal. We were in fact somewhat surprised to find the cycles spares data to be such a good fit. It was assumed prior to the analysis that such a spares environment would not behave like traditional replacement parts due to the likelihood of major differences in the way and manner that demand for cycle spares tend to be generated. That is by whim or fashion depending often on the attitudes of parents and children to cycle repairs and upgrading. The products are also sold into a traditional consumer market.

The DMC data sets were very valuable simply because the company operated such a crude stock control system. This gave access to rather unique data on stock cards, that showed the individual demands day by day over a four year period. Hence it enabled the analysis to be undertaken on the distribution of individual order quantities and thus to examine the form of the compounding distributions. On the basis of this analysis the LSD looked a superior fit to the Geometric distribution and this strongly supported the use of the NBD as the appropriate compounding model. However, because the analysis was limited to just a small number of parts we could not draw any strong conclusions regarding which of the two distributions would be appropriate in a given situation. We did draw sufficient confidence that either the sP or the NBD would suffice in our simulation studies reported in chapter nine. This is an area that requires further research to discern the conditions that favour one or the other distributions. The DMC data also gave an opportunity to examine for the existence of the negative exponential distribution for the time between orders and we were able to show a good fit of the data to this distribution hence supporting the existence of a Poisson process and the occurrence of Poisson demand in unit time periods.

In table 10.20 below we summarise some key parameters from the usage value analysis of the five spare part systems we have investigated. The degree of concentration in these systems is measured by the value of σ but we have also shown the proportion of activity accounted for by the top 20% of items. When the equipment is very complex, such as with aircraft and trucks, the range of spares to support the service requirements are commensurately very large. Typically such ranges include a very large number of small value items, usually a few pence, and a small number of very high value items, often thousands of pounds. In these circumstances, and due to this effect, the concentration is often very high as we have seen with DAF and DAN Air. On this basis we were somewhat surprised to see the Moore and Large cycles spares concentration to be as high as 1.874, although on reflection we could see that it was really the very high volumes of moderately priced items that added to the concentration in this case. A further generalisation that one can make is that as one moves from the manufacturing level to the wholesale level then the degree of concentration tends to decrease. [This was observed by RG Brown way back in 1959]. We have seen this effect in DAF Trucks when in 1980 the concentration as measured by σ at the Distributor level was 2.501 whilst the average dealer level was around 2.250 at the same time.

The single main conclusion we draw from the additional empirical work presented in this chapter is that these datas show DAF parts demand are in no way special, in the sense of displaying unique stochastic features not seen elsewhere. The features and characteristics we have seen and presented from the DAF analysis can be seen in these additional studies in varying degrees from system to system. In particular the short period demand volume being LSD/NBD distributed. Individual demand volume streams appearing either NBD or Stuttering Poisson distributed. Usage values, prices and volumes (conditional on the time period) all lognormally distributed. We are therefore confident that all our theory development and modelling work from the DAF environment is applicable to a very wide variety of spare parts environments.

Table 10.20

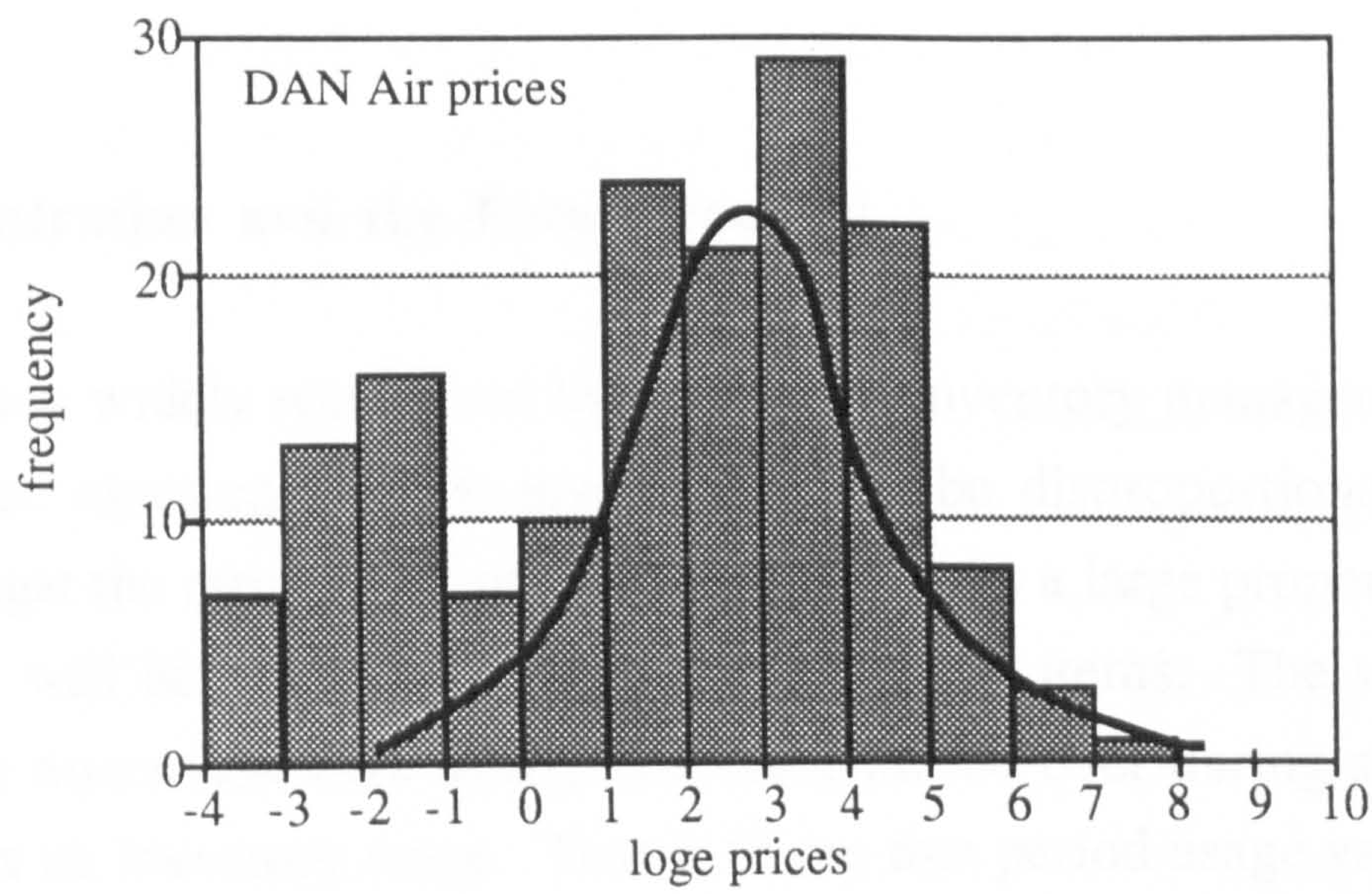
Summary of parameters for systems studied

System	type of spares	location parameter	shape parameter	concentration ratio (top 20% items activity)	number of spares in range
DMC ltd	Cars	2.310	1.861	82 %	14000
John Cheng	Lift Gear	5.531	1.809	80 %	2000
DAN Air	Aircraft	5.835	2.305	93 %	78000
M and L ltd	Cycles	5.910	1.874	84 %	2000
DAF Trucks	Truck	5.203	2.313	93 %	12500

Year to which the data relates -

DAF Trucks	1985
Moore and Large	1981
John Cheng	1984
DMC ltd	1982
DAN Air	1984

figure 10.15



Pareto Concepts, Usage Values and Concentration of Econometric Variates

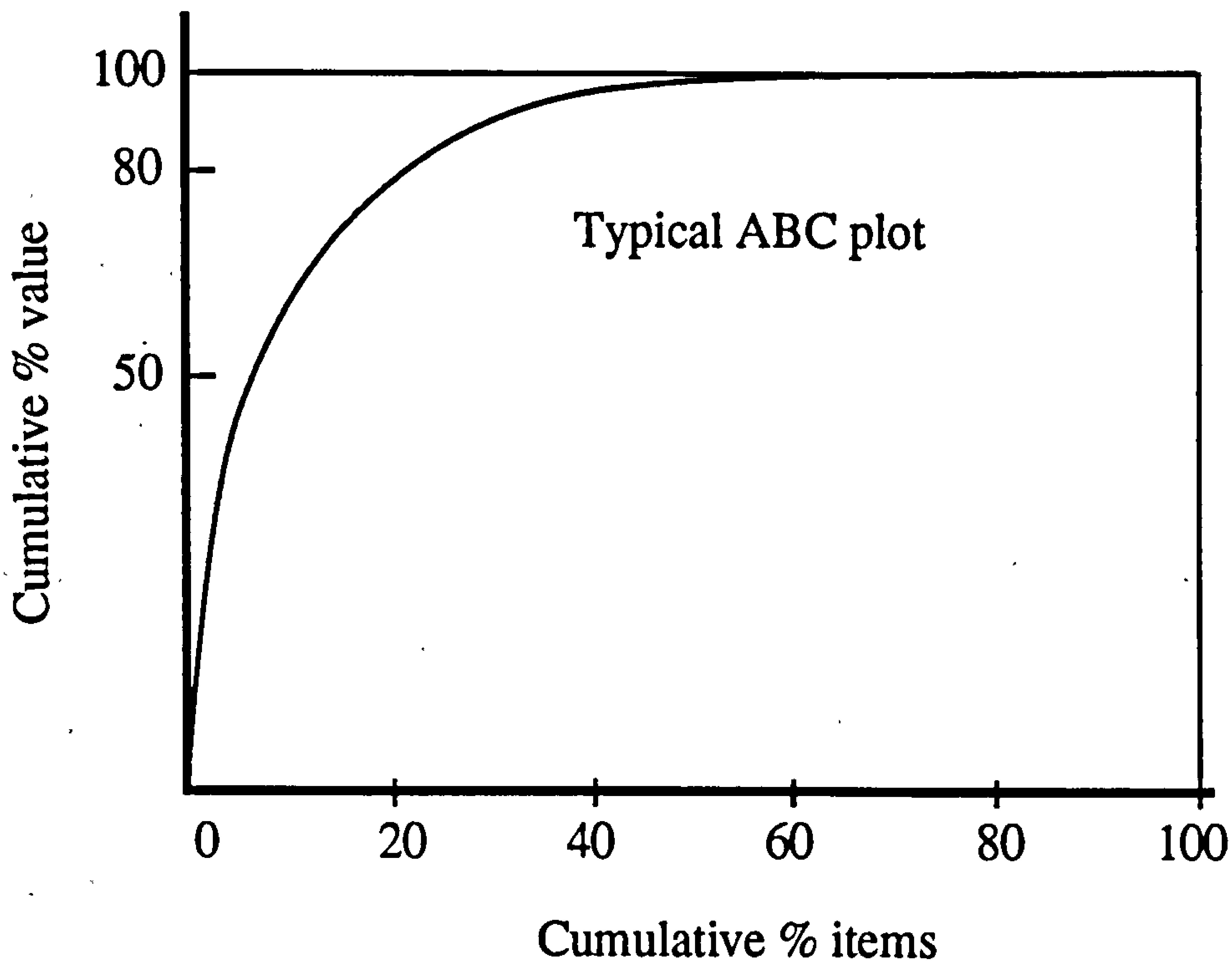
11.0 Introduction.

This chapter examines the origin and development of the well known Pareto concept and traces the misapplication of this name to inventory categorisation procedures. The concept of concentration of value in certain econometric variates is also examined together with a critical consideration of the so called link of Pareto analysis, as applied to inventory systems, with the log-normal distribution of inventory item usage values. We then consider the form and occurrence of Lorenz curves and show that any skewed data set can form such distributions. This chapter has partly been a reflection and development from what has been presented so far; it also provides valuable literature reviews and concepts that underpin some of the development work of the next chapter. This is largely the reason for position of this chapter in the thesis.

11.1 Concentration and the Pareto concept

It is now widely recognised in the field of inventory management that the value invested in most inventories will be disproportionately spread amongst the range of items. In almost all cases a large proportion of the value will be vested in a small portion of the items. The same phenomenon arises when we look at the value turned over during some period across an inventory range. That is to say that period usage values are frequently disproportional in the sense that a few items account for the majority of the value turned over in a given period. Almost every modern text on Production or Inventory Management now discusses this phenomena and its practical value to inventory management. The process of presenting inventory usage data in the cumulative curve form is frequently referred to as ABC analysis, 80/20 analysis, or Pareto analysis. A typical ABC plot is shown in figure 11.1 shown below.

figure 11.1



The use of this methodology for the categorisation of inventory ranges into specified value groups -often arbitrarily three named A, B and C groups- is now an established part of inventory management practice. The term 80/20 analysis stems from the often observed fact that 20% of the items account for 80% of the value. Discussions of this technique are to be found in most standard Operations management texts, for example, Lockyer (1972), Constable and New (1976), Loomba (1978), Wild (1979), Lewis (1975, 1981), Peterson and Silver (1979), and Bestwick and Lockyer (1982); and more recently Krajewski and Ritzman (1987), Bennett et. al.(1988), Schroeder (1989), Chase and Aquilano (1989), and Nahmias (1989). Indeed no standard Operations text would be complete without some reference to the methodology. Unfortunately most, in the main, just repeat the technique in its original form , with discussions on the criteria for choosing categories and methods of presenting the information. A small number of authors who have attempted to develop and extend the basic ABC principle have referred to the fact that inventory period usage values can be satisfactorily represented by a lognormal distribution. The value of this development and the link with lognormal distributions, if one does exist, is on the grounds that the properties of the lognormal distribution (as discussed in Chapter two) enable valuable aggregate inventory calculations to be made. The extension of the ABC concept to link with the lognormal distributions has been mentioned by Lockyer (1972),Van Hees and Monhemius (1972), Bestwick and Lockyer (1982) and Peterson and Silver (1979 and 1985). The views expressed in these references stem from Brown's original work and claims (1959 &1963) that period usage values are lognormal distributions. In his various works Brown (op cit) presented empirical evidence of inventory period usage values that can be modelled by lognormal distributions. However, what seems to have gone unnoticed by some of the authors who followed Brown was that his published empirical studies were all on spare parts inventories. As we have seen from our earlier discussions the demand processes for spare parts are almost always independently derived from a vast number of independent customers. This is one of the main requirements for a Poisson process to be in operation. We now argue that this is a critical factor in any direct link between the lognormal distribution and the Pareto distribution. In fact, as will be shown here whereas almost any inventory range can be described in cumulative curve forms of the classical Pareto type only certain kinds of inventory can be modelled by a lognormal distribution, ie those for whom the underlying demand is Poisson in nature. Hence the link between the two distributions is not a direct one and therefore the occurrence of one distribution in an inventory is not a direct consequence of the other distribution as implied in some sources

The precise nature of the systems measured by Brown are not given much prominence and the casual reader might well assume from Brown's work that the use of lognormal distributions can be applied to many types of inventory items. In the text references of Van Hees and Monhemius, Peterson and Silver and Lockyer and Bestwick, previously cited, it is not at all clear if the inventories considered are production/purchased inventories, in-process inventories or finished stock inventories. More specifically it is not clear if they are inventories for which the derived demand is Poisson in character. Lockyer and Bestwick cite the use of the Pareto concept in terms of a variety of production management applications and go on to say:-

> *"The lognormal distribution adequately represents the Pareto curve and therefore lognormal probability paper may be used to good effect in Pareto analysis exercises"*

In their text Magee and Boodman (1967) say :

> *"The Pareto curve as applied to inventories is a lognormal curve".*

It would be easy to assume from these works that the many quoted uses of the Pareto concept to production inventories could be extended also to the use of the lognormal distribution in these cases. This author contends, based on our work and conclusions here, that this is only so if the inventory in question has been generated to meet the requirements of an independent demand system, such as that produced for spare parts; ie where a Poisson process is operating. Strong evidence for this stems from the work of a number of authors. Heron, for example, has published a number of articles on the use of the lognormal distribution in estimating aggregate inventory standards. In his publications of (1974,1978, and 1981 [in Wild ed]) he based his work and practical applications of the use of the lognormal distribution on the Warmdot inventory given by R G

Brown (1963 & 1967). This is an inventory of 35,000 spare parts held for heating and ventilation equipment. In his article (1976) Heron refers to three inventory ranges, the Warmdot inventory, the Transpo inventory (which comprises 29,000 spare parts for transportation equipment) and the Sureship Wholesale inventory (11,000 items of unspecified type). These are no doubt fictitious company names, but the data is certainly from real companies, who Brown and Heron have had contact with for research or consulting purposes. In the publications by Schary and Howard (1970) and (1971) in which the lognormal distribution is again applied to various practical calculations on aggregate inventory estimates the precise nature of the relevant inventories are not specified, but the clear implication is that the work is presented in the context of finished goods inventories for which it is reasonable to assume that the demand is independently derived from many customers.

It is clear that the disproportionality principle applies to most if not all inventory situations and from previous work it is apparent that usage values can, in some cases, be modelled by lognormal functions. What seems to have gone unanswered is the form of the distribution of usage values in those cases where lognormal functions are not appropriate. Furthermore there is no reported research in the inventory literature into the cause of the underlying stochastic nature of the disproportionality of inventory usage values, nor into the stability of the distributions obtained. This is quite contrary to the work that has gone on in other fields into the disproportionality effects of many economic variates.

11.2 Historical development of the concentration principle.

The disproportionality, or concentration, of a number of other economic variates, such as the distribution of firm sizes, the number of employees in firms within an industry and the distribution of wealth and income, has been recognised for many years and seems to stem from the pioneering work on income distributions by Vilfredo Pareto (1897), where he showed that in many economies the wealth of the nation was concentrated in the hands of a relatively small proportion of the population. Following Pareto's work a number of authors have shown that income distributions can also be modelled by lognormal models Kapteyn (1916), Kapteyn and Van Uven (1916), Gibrat (1931) and more recently Lydall (1959) and Thatcher (1968).

Parallelling the work on income distributions, but starting much later, is the recognition that the sizes of business enterprises in most industries are disproportionate; that is there exists a small number of very large firms and a large number of comparatively small ones. The literature is rich in the study of the growth and concentration of firm sizes and a particularly fertile period occurred between the early 1950's until the mid 70's. The major efforts during this period seemed to be to discover the underlying stochastic mechanisms providing the impetus for growth and concentration; and with finding theoretical distributions that satisfactorily fit the observed empirical firm size distributions found in various industries. In particular, valuable work has been published by Adelman (1958), Steindle (1965), Simon H.A. (1955), Ijiri and Simon (1964), Hyman and Pashignan (1962), Ijiri and Simon (1967), Samuels (1965), Quandt (1966), Mansfield (1962), Simon and Bonini (1968), Shorrocks (1975), and Singh and Whittington (1974).

A common conclusion with most of these authors is that the Gibrat assumption (the Law of Proportionate Effect) is the most plausible stochastic process explaining the growth mechanism of such economic systems. There does not appear however, much agreement about the nature of the equilibrium distributions attained by such growth processes and as we discussed earlier in this work, the Pareto, lognormal LSD, and in Simon's case the Yule distribution have all been put forward as candidates to explain the finals forms obtained in various systems. Additionally some authors have addressed the problem of measuring and placing a meaning to the actual concentrations values found in the variates they were studying. In particular, Hart (1957), Blair (1956), Hart and Prais (1956), Nelson (1963), Quandt (1966), Silberman (1967). The econometricians, statisticians and economists, who have studied these fields have, rightly so, posed questions and sought answers regarding the nature of the underlying distributions giving the disproportionality so found, and with discovering the plausible stochastic processes that resulted in the distributions found.

This approach has led to an appreciation of the nature and stability of such processes and the distributions found. In those cases where the empirical data was not a clear fit to a particular distribution then evidence from the underlying stochastic processes was often sought to suggest the form of the equilibrium distribution appropriate to the system. A further important point arising out of this field of work is the appreciation of the fact that the parameters of the equilibrium distributions may well be changing over time due to underlying stochastic and economic effects. This awareness has been particularly noted and researched in the study of firm sizes and the distribution of incomes, as discussed by Hart (1957), Hart and Prais (1956) and Blair (1956). On a macro scale it is clearly very important (for policy making) to know the underlying processes, equilibrium distributions, and the process stability of the key economic variates. If the distribution of firm sizes for example, in a particular industry, is becoming more concentrated then government agencies, policy makers and those concerned with the relevant markets and market forces would wish to try and understand why it is so. Such an increase in concentration could arise because the larger firms are gaining business at the expense of the smaller ones or, the industry is going through a period of mergers or, perhaps because many more smaller firms are coming into the industry. Similar requirements exist to understand better the fundamental nature of the distribution of incomes. If the concentration of wealth in society is changing then it is a prime requirement of politicians, economists and government advisers to know why.

The wealth of fundamental research work that has been conducted in the studies of the growth and dispersion of wealth and the distribution of firm sizes has no parallel in the inventory field. Yet the basic nature of the issues are similar - that is, the growth, spread and concentration of econometric variates governed, almost certainly, as we have seen in this work, by underlying stochastic processes.

11.3 The concentration of inventory usage values

The earliest reference regarding the concentration of value in inventory systems comes from the second world war. An incomplete reference reported in Kulvanich (1976) quoted the US Armed Forces during the second world war when investigations into logistics problems revealed that between 80% and 90% of the fiscal value in inventory was accounted for by between 10% and 20% of the items stocked. In 1951 H. F. Dickie published his milestone article (in inventory management terms at least), "***ABC Analysis, shoot for dollars***", which was based on his work and analysis of inventories at the General Electric Company. In his article he clearly and simply showed that by plotting cumulative percent items against cumulative percent value, the classical ABC inventory curve is obtained. In the same article, Dickie went on to show how normative decision rules regarding inventory item control can be based on suitable categories. He arbitrarily segmented his item range into those items (8%) which accounted for 75% of the cost, those items (25%) which accounted for 25% of the cost and those items (67%) which accounted for the remaining 5% of the cost. He referred to these groups as A, B and C respectively.

This pioneering approach and methodology by Dickie was probably the origin of the name ABC Analysis. It is important to note that Dickie did not refer to his work as Pareto analysis, nor to the form of the curve that he obtained as a Pareto curve. The process of applying control methods according to such a simple categorisation method is now a common and well tried practice in inventory management. Over the period 1951 to the present time the same basic methodology as that presented by Dickie has been presented many times in the management literature with various modifications and applications. Most of the variants are concerned with choice of the number of groups in the categorisation, the criteria for choosing groups and the choice of control methods to apply to each group. It is not considered important in this work to review the basic variations, however, the following journal references give a wide coverage to most of the permutations that have been discussed in the literature. Brown (1963), Norbon(1973), BIM Report (1975), Zimmerman (1975), Claycombe and Sullivan (1975), Reuter (1976, 1976a, 1978), Jageti (1976), Conroy (1977) and Rivers (1982). What is important from these references, and from the production managements texts previously quoted, is the fact that the disproportionality principle in inventory values either as stock held or stock turned over in a given period, is a universal phenomena that holds in almost any inventory range of reasonable size.

11.4 The erroneous use of the Pareto name

The origin of the name Pareto analysis as applied to inventory systems seems to rest with Juran (1964), who, with extensive examples, draws attention to the fact that many industrial situations exist where a vital few members of various assortments account for most of the effect of interest. Amongst the many examples given by Juran are the value of inventories tied up in a small portion of the range. In cost analysis approximately 20% of the factory parts contain 80% of the factory costs. In quality control the bulk of the in service failures, machine shop scrap, rework and sorting costs are traceable to a vital few in service failure modes, shop defects, products, components, processes, vendor designs and operators etc. What, according to Juran (1964) that runs through all these phenomena is the principle of the "***vital few and trivial many***".

According to Juran in 1974 he claims that it was he in the late 1940's who named this universal phenomena the '*Pareto Principle*' and the name had endured, especially in industrial management applications. However, by his own admission, (Juran 1974) it was a mistake to name the phenomena the Pareto Principle. The following quote is taken directly from the Quality Control Handbook (3rd Edition) 1974 edited by J M Juran:-

> *"Vilfredo Pareto, an Italian economist (1848- 1923), had studied the distribution of wealth and had quantified the extent of inequality and nonuniformity of this distribution. However, he had not generalised this concept of unequal distribution to other fields. To make matters worse the cumulative Pareto curves first published in the quality control handbook should have been identified with M.O. Lorenz, who had used such curves to depict the concentration of wealth in a graphic form." (M.O. Lorenz, Methods of Measuring the Concentration of Wealth. American Statistical Association Publication, Vol. 9, pages 200- 219, 1904).*

Dr Juran (1974) then goes on to set the matter straight-

> *"Numerous men over the centuries have observed the existence of the phenomena of the 'vital few and the trivial many' as it applied to their local sphere of activity. Pareto observed this phenomena as applied to the distribution of wealth and advanced*

the theory of a logarithmic law of income distribution to fit the phenomena. Lorenz developed a form of cumulative curve to depict the distribution of wealth graphically". Juran was (seemingly) the first to identify the phenomena of the vital few and trivial many as a universal applicable to many fields. Juran applied the name the Pareto Principle to this universal and also coined the phrase 'vital few and trivial many'.

Juran then applied the use of Lorenz curves to depict this universal in graphic forms. The cumulative curves depicting percentage value against percentage items as used in inventory control are therefore Lorenz curves, not Pareto curves, but the name Pareto analysis or Pareto principle has now, it would seem, passed into the management jargon and is very likely to stick. This is unfortunate for M.O. Lorenz for not getting justifiable credit, and also for Vilfredo Pareto, because to some extent it clouds the issue over the real nature of Pareto's distributions, of which three were developed for economic analysis as shown below.

11.5 The form of Pareto Curves

In his work on income distribution Pareto put forward three general equations (probability distribution functions) to summarise and model the observed empirical data on income distributions (none of which, in fact, give the precise form of the so called Pareto curve as applied and named in industrial applications). Easton (1974) shows the form of the true Pareto distributions -

The Pareto distribution of the first kind-

$$F(x) = kx^{-\alpha}$$

where 'k' and 'a' are constants (α = scale parameter)

The second and third Pareto forms are as follows-

$$F(x) = \frac{k}{(x+c)^{\alpha}}$$

$$F(x) = \frac{ke^{-b\alpha}}{x^{\alpha}}$$ 'c' and 'b' are both constant

The last Pareto form is also known as the Champernowne distribution (Easton 1974).. The Pareto curve as developed and applied in industrial situations is obtained analytically as shown in the following section.

11.6 The analytical form of the ABC principle

Consider an inventory range of 'n' items and period usage values of each item given by x. If the probability density function of the period usage values is given by $f(x)$ then the total value of the inventory turnover is given by the function:

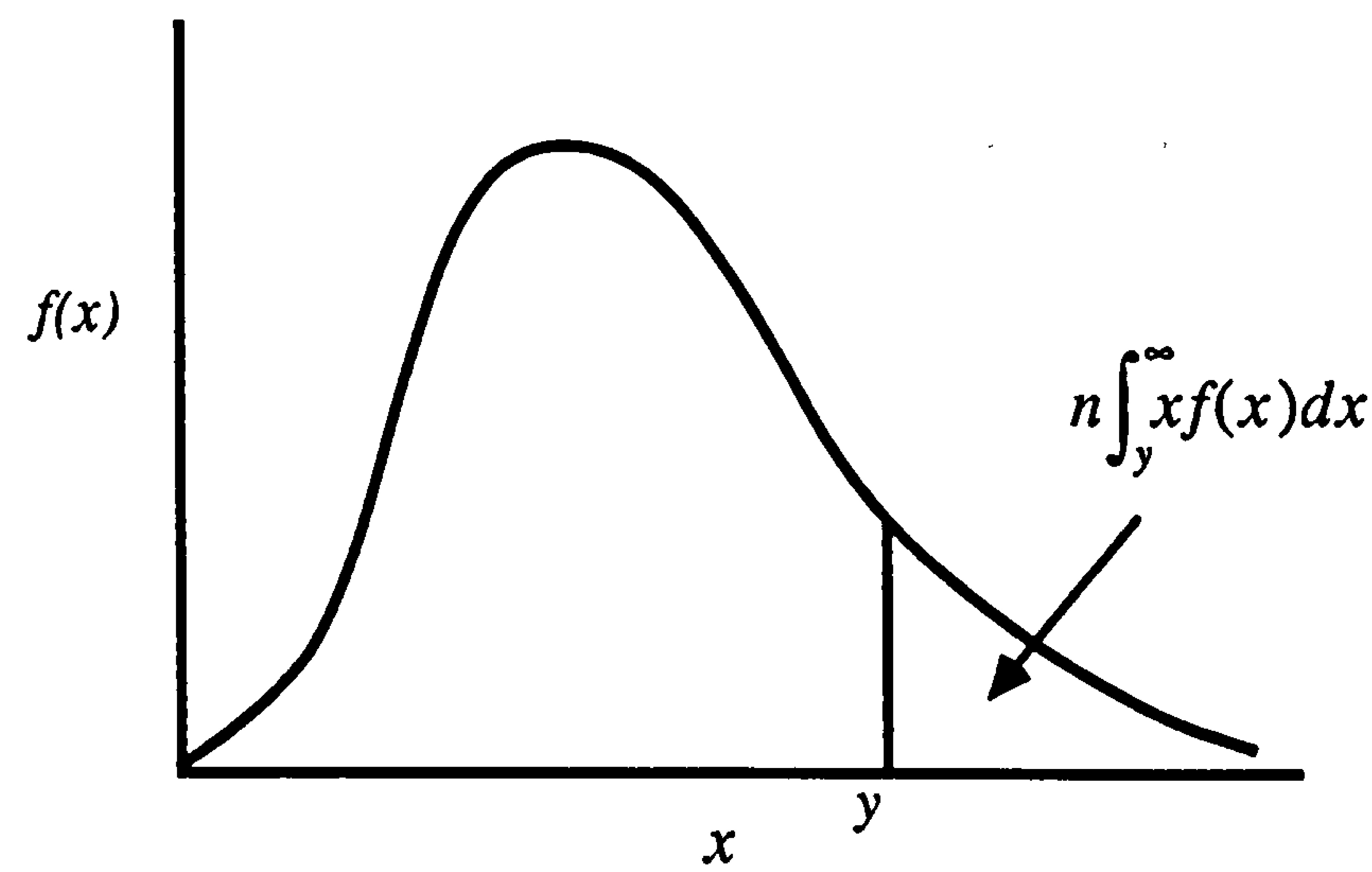
$$n\int_0^{\infty} xf(x)dx$$

The proportion of value (usage value) given by items whose individual values exceed a particular value 'y' is given by -

$$n\int_y^{\infty} xf(x)dx$$

or graphically it is shown as shown below :-

figure 11.2



The usual representation of the distribution of usage values of inventories is to plot the ratio (v) (Cumulative proportion of value) against (w) (the cumulative proportion of items) as shown by Van Hees and Monhemius (1972). Thus :-

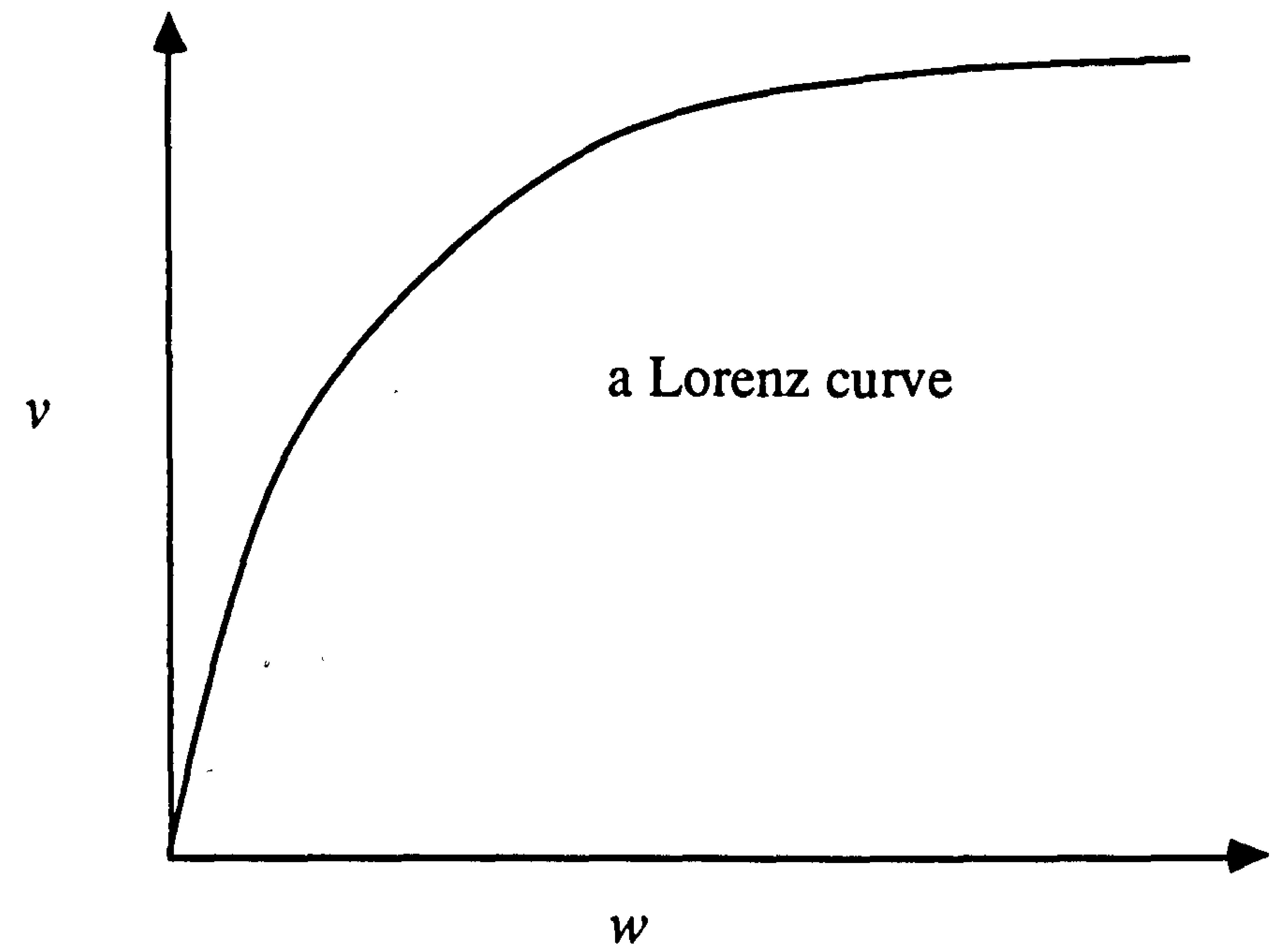
$$v = \frac{\int_y^\infty xf(x)dx}{\int_0^\infty xf(x)dx}$$

and also -

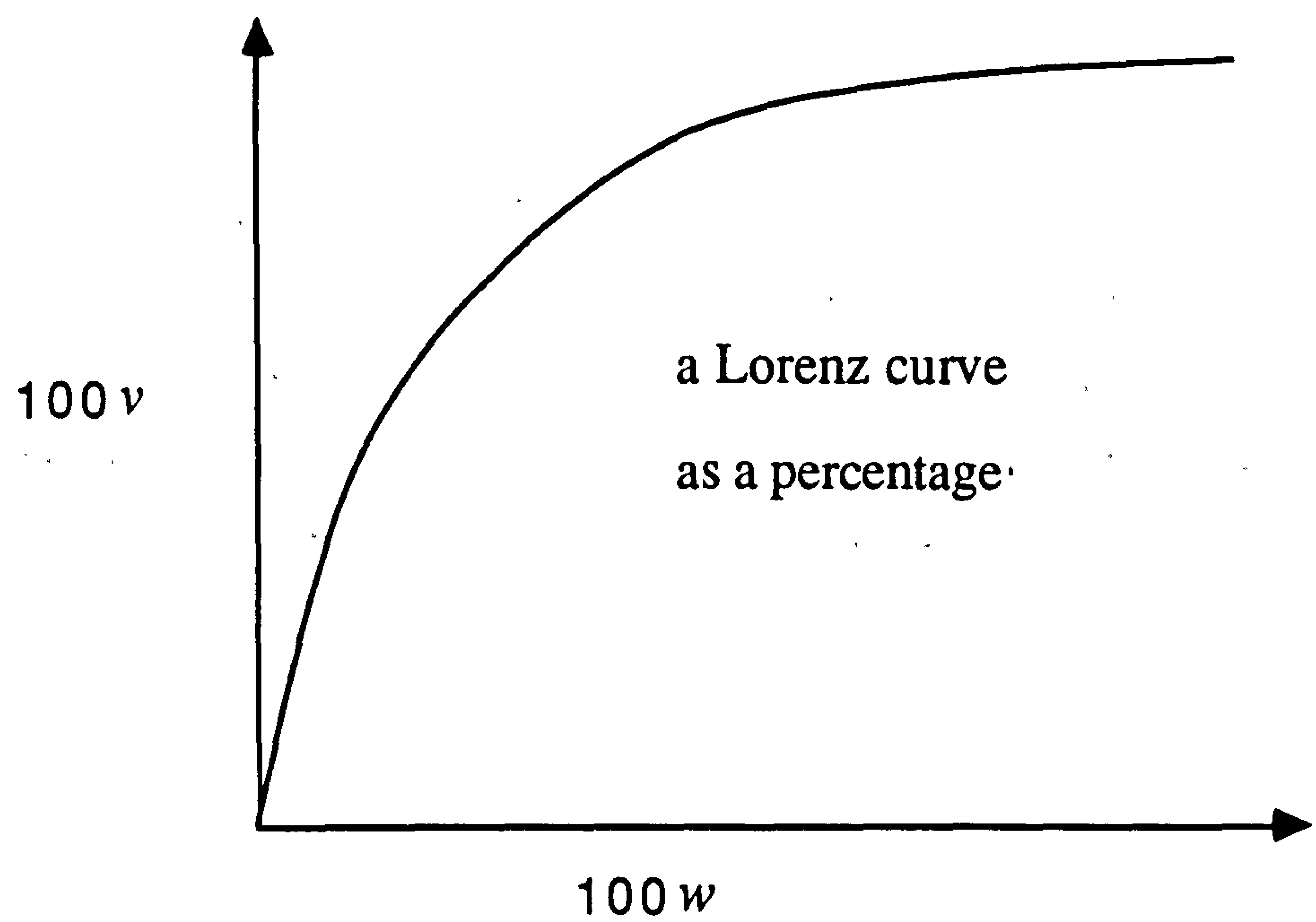
$$w = \int_y^\infty f(x)dx = [1 - F(x)]$$

If v is plotted against w we then obtain the Lorenz curve as shown:-

figure 11.3



or as a percentage to give an ABC from -



At this stage it is now possible to demonstrate analytically that data such as inventory period usage values do not necessarily give rise to a lognormal distribution.

If $f(x)$ is a lognormal probability density function as given by-

$$d\Lambda = \frac{1}{z\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(\log_e x - \mu)^2\right] dx$$

substitute $d\Lambda$ for $f(x)$ we get -

$$v = \frac{\int_y^\infty x d\Lambda(x : \mu, \sigma^2)}{\int_0^\infty x d\Lambda(x : \mu, \sigma^2)}$$

and

$$w = \int_y^\infty x d\Lambda(x : \mu, \sigma^2)$$

We can plot v against w and obtain the cumulative Lorenz curve as before. However, $f(x)$ does not need to be lognormal, it can be almost any skew statistical distribution and we will still get a cumulative Lorenz curve when v is plotted against w. Such curves will of course change shape some what, but they will still follow the same general form, provided the effect being measured is disproportionately spread amongst the range of items.

11.7 Empirical evidence and support

The foregoing analytical reasoning is easily borne out by empirical analysis. As discussed above the function $f(x)$ does not need to be lognormal it can be almost any skewed function and the characteristic Lorenz curve can be obtained. We can demonstrate this by using a simple skewed function and we take the Log series distribution as example. This distribution is discussed and used in model building in chapter eight. As we have seen the log series function has the form-

$$f(x) = {}^{-q}/_{x \log(1 - q)} \quad \text{for parameter } q.$$

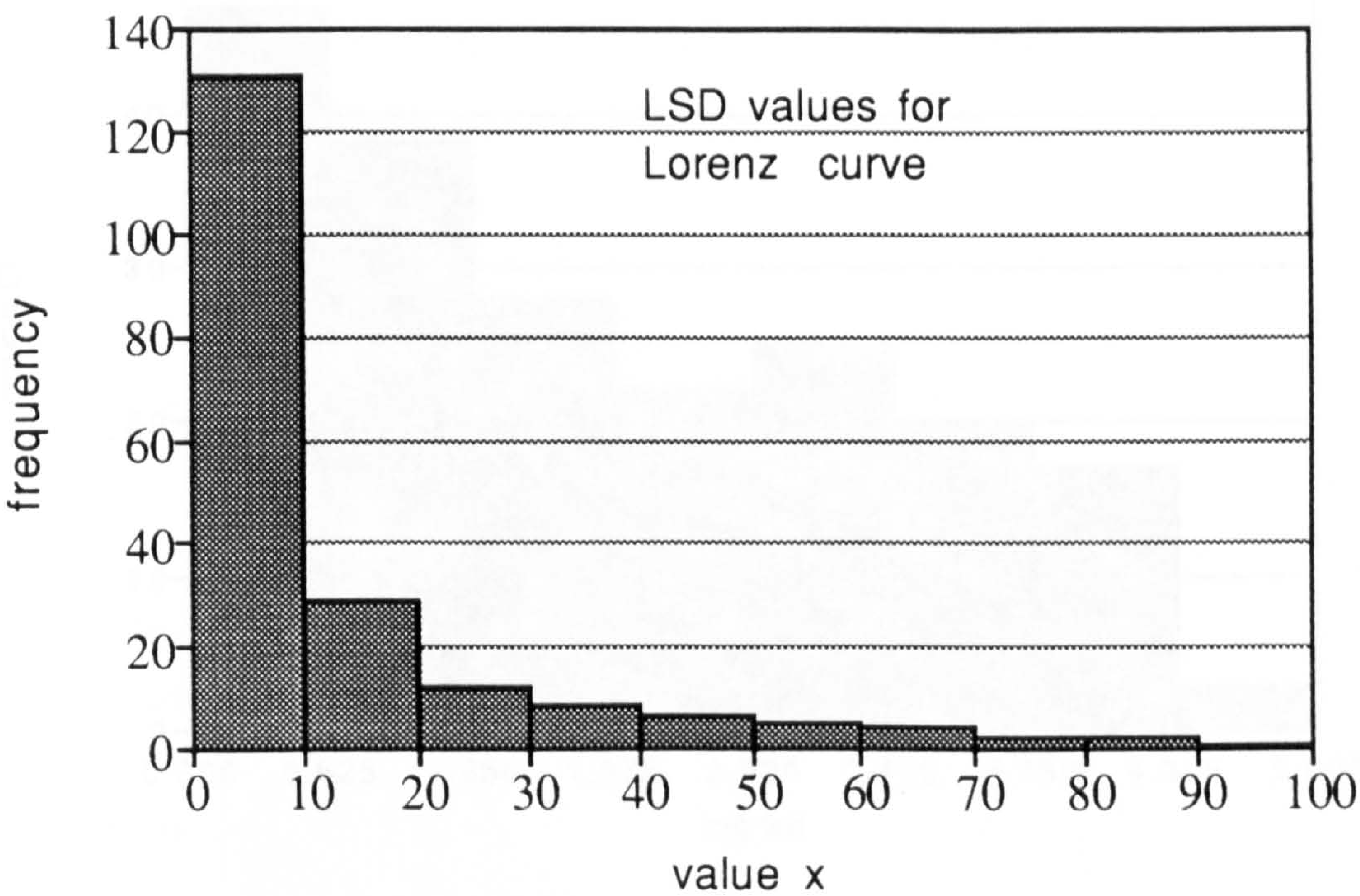
If we set $q = 0.985$ then we can obtain the following set of probabilities and the numerical outcomes for a sample data set of 200 items as shown in table 11.1 :-

Table 11.1
LSD frequencies

Value	Probability	frequency for 200 items
1	0.2346	47
2	0.1155	23
3	0.0758	15
4	0.0560	11
5	0.0441	9
6	0.0362	8
7	0.0306	7
8	0.0264	6
9	0.0231	5
10	0.0205	4
11-20	0.1271	25
21-30	0.0650	12
31-40	0.0364	8
41-50	0.0285	6
51-60	0.0228	5
61-70	0.0170	4
71-80	0.0114	2
81-90	0.0100	2
91-100	0.0060	1

The characteristic LSD distribution produced by this process is seen in the figure below-

figure 11.4



To produce a Pareto (Lorenz) curve from this data we just simply cumulate the distribution starting with the largest values first. Then for convenience we take percentage values of each incremental point on the cumulation and plot this against the cumulative percentage of items. The values are shown in the tabulation 11.1 and the typical Lorenz curve follows in figure 11.6. Just to show that we do not have a lognormal curve a histogram of the log values is also shown in figure 11.5. If the data (distribution) had been lognormal the histogram would have shown the characteristic symmetrical shape of the normal curve.

figure 11.5

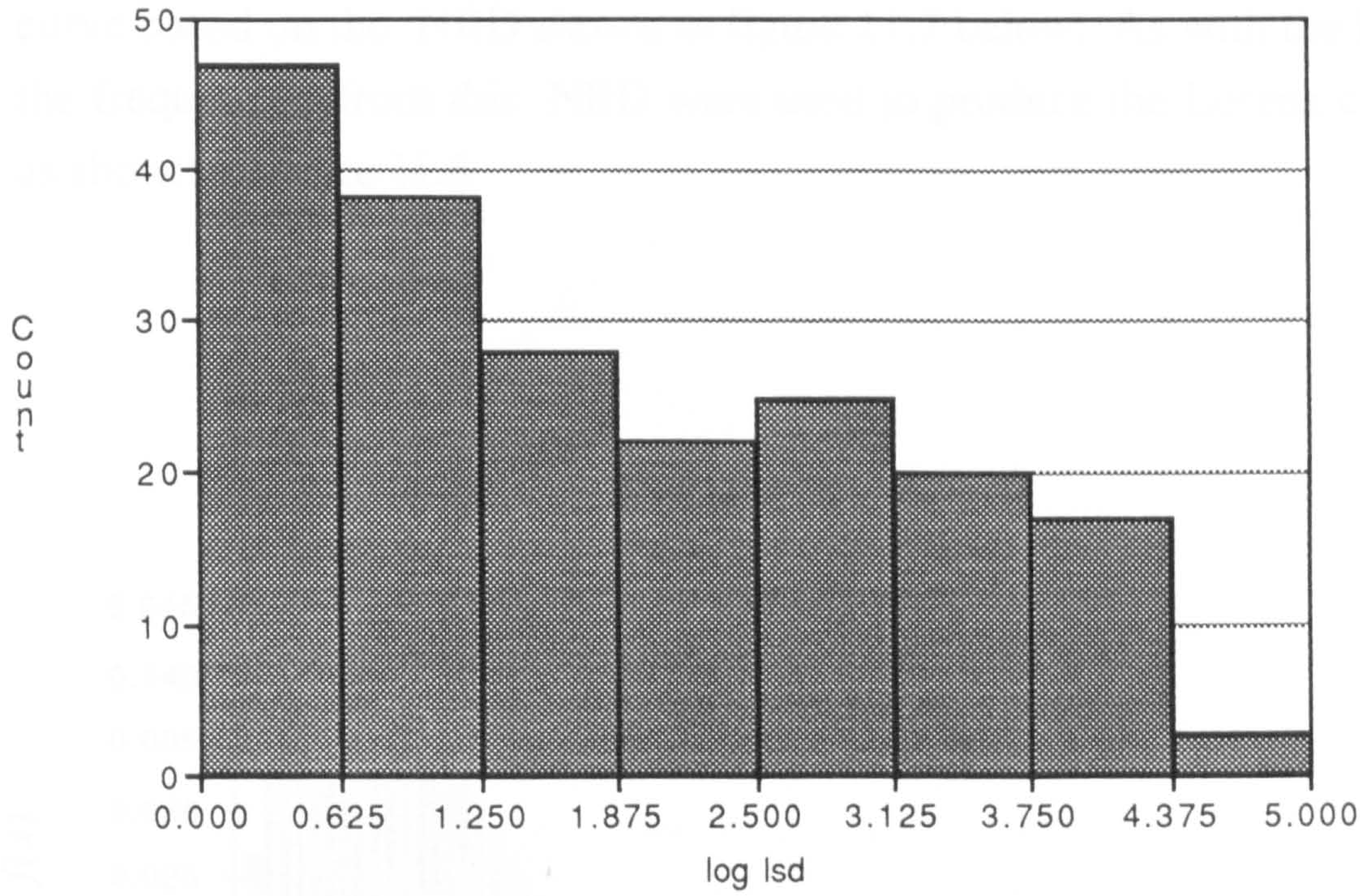
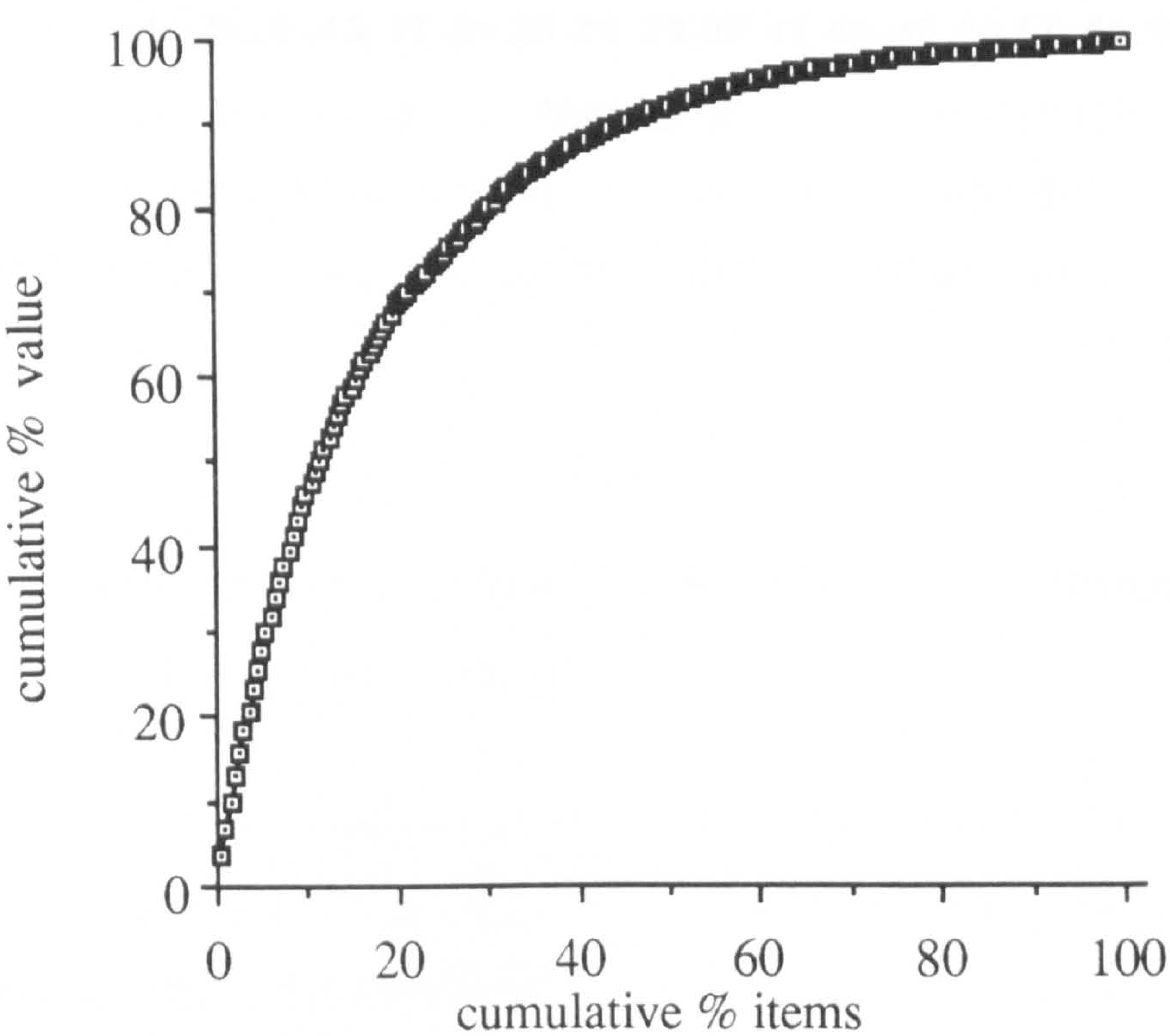


figure 11.6

Lorenz curve produced from an LSD distribution



We also used a Negative Binomial distribution to produce a Lorenz curve based on the NBD shown in figure 11.7 below. As with the LSD the frequencies from this NBD were used to produce the Lorenz curve as shown in figure 11.8

figure 11.7

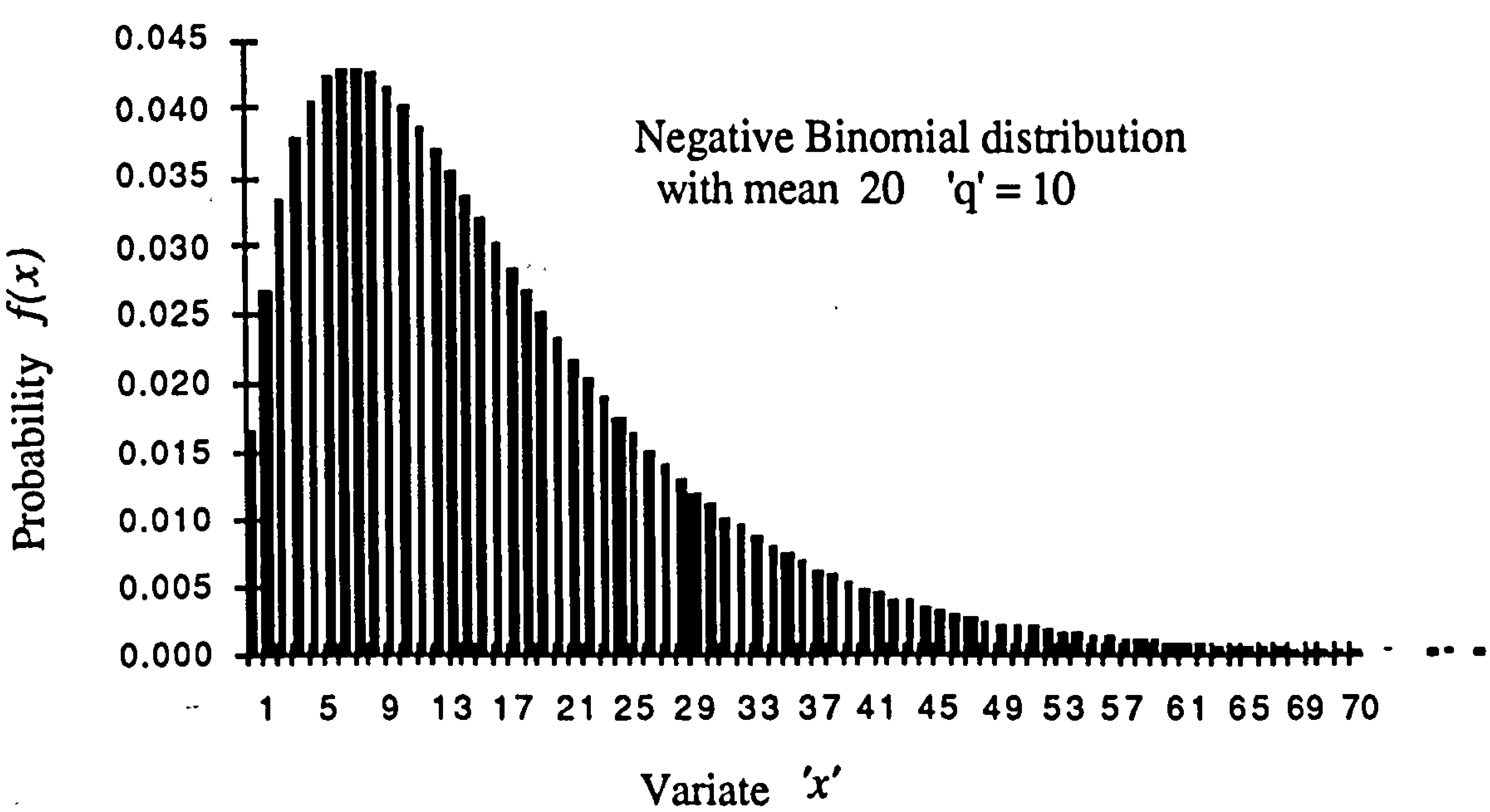
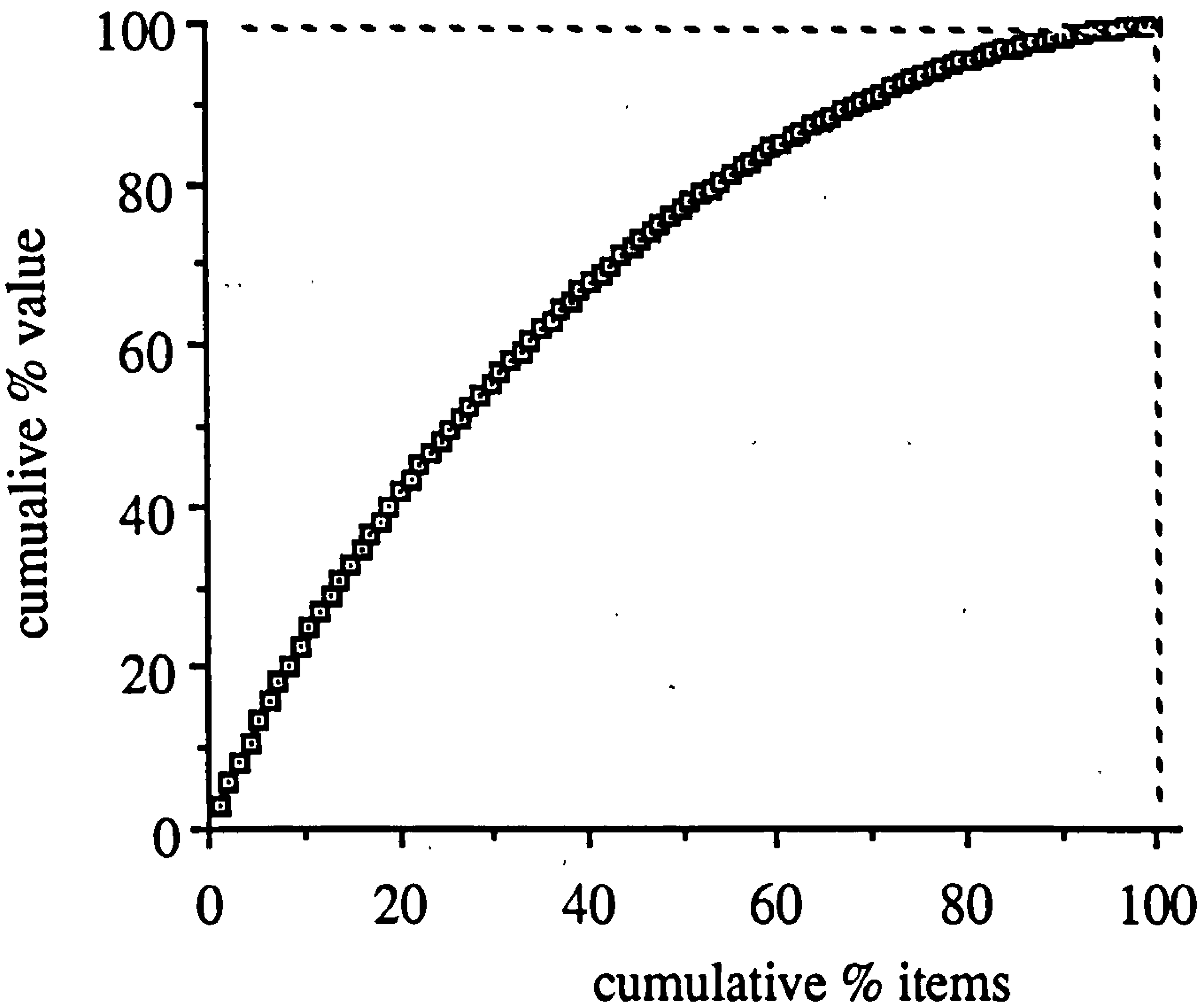


figure 11.8

Lorenz curve produced from NBD data



We can see from the Lorenz curve of figure 11.8 above that it is not as curved (concentrated), as in the LSD generated case, but it is still a Lorenz curve and produced by a skewed distribution that is not lognormal, nor is it as skewed as most lognormal functions. A much more skewed NBD would have produced a more concentrated Lorenz curve.

11.8 A relationship between the lognormal distribution and the Pareto(ABC) curve

Although we have shown in this chapter that the classical Pareto (ABC) curve need not necessarily be based on a lognormal variate, the latter can always be presented in the cumulative ABC form. For example, all our DAF usage value data in this work is lognormal distributed and because this variate is skewed it can be presented in the traditional ABC curve form. In these cases it is possible to derive useful relationships between the parameters of the lognormal curve and particular measures of interest from the ABC curve.

It is of interest to management to know what proportion of items in an inventory range account for a particular measure of interest, usually the turnover per annum. Such measures are of significant interest to Operational management due to the fact that they highlight and focus attention onto those items which have the greatest disproportionate effect and hence demand particular attention for control purposes. In the DAF case the top 20% items account for around 95% of the total usage value in a given period. For lognormally distributed variates this can be readily deduced by just a knowledge of the shape parameter of the particular lognormal distribution Lockyer (1982) shows the proportion (or percentage) of a particular measure that can be accounted for by the top 20% of the items in an assortment. For example, the proportion of the total usage value for an inventory range that can be accounted for by the top 20% items.

We use Lockyer's terminology and development here to show the link between the lognormal distribution and the ABC curve, and to show how we can readily determine what proportion of activity is accounted for by a given proportion of items.

"If y_x represents the activity of interest above which x% of the items occur, the proportion of the total activity measure represented by these x% of the items is $\phi(z^*\{y_x\})$, where $z^*\{y_x\}$ can be shown to be $z\{y_x\}-\log_e\rho$. Thus the proportion of the total activity measure represented by the top x% of the items is a function only of the standard ratio ρ." [note :where $\rho = e^{\sigma}$].

Hence for example, if x is set at 20% (the top 20% items) then $z(y_x)$ =0.84. That is the normal ordinate z = 0.84 for the 20% of the area in the right hand tail of the normal distribution. Therefore $z^*(y_x)$ =0.84 - $\log_e \rho$, from which the proportion $\phi(z^*\{y_x\})$ of the total activity measure represented by the top 20% of items can be read directly from normal curve tables. Using this approach Lockyer (op cit) presents the following table for various values of the standard ratio ρ.

table 11.2

Total value in the top 20% items

ρ	$\ln \rho$	$0.84 - \ln \rho$	$\phi\{z^*(y_x)\}$
1	0.000	0.840	20.00
2	0.696	0.247	40.00
3	1.099	-0.259	60.00
4	1.386	-0.546	70.60
5	1.609	-0.769	77.00
6	1.792	-0.952	83.00
7	1.946	-1.106	86.70
8	2.079	-1.239	89.30
9	2.197	-1.357	91.30
10	2.303	-1.463	92.80
11	2.398	-1.558	94.20
12	2.485	-1.645	95.00
13	2.565	-1.725	95.70
14	2.639	-1.799	96.30
15	2.708	-1.868	96.90

The form of the relationship between the standard ratio and the activity accounted for by the top 20% of items is readily seen from the graph of figure 11.9 below. Van Hees and Monhemius (1972) present a similar table to Lockyer, but in terms of different values of the percent of items as shown in table 11.3 below.

figure 11.9

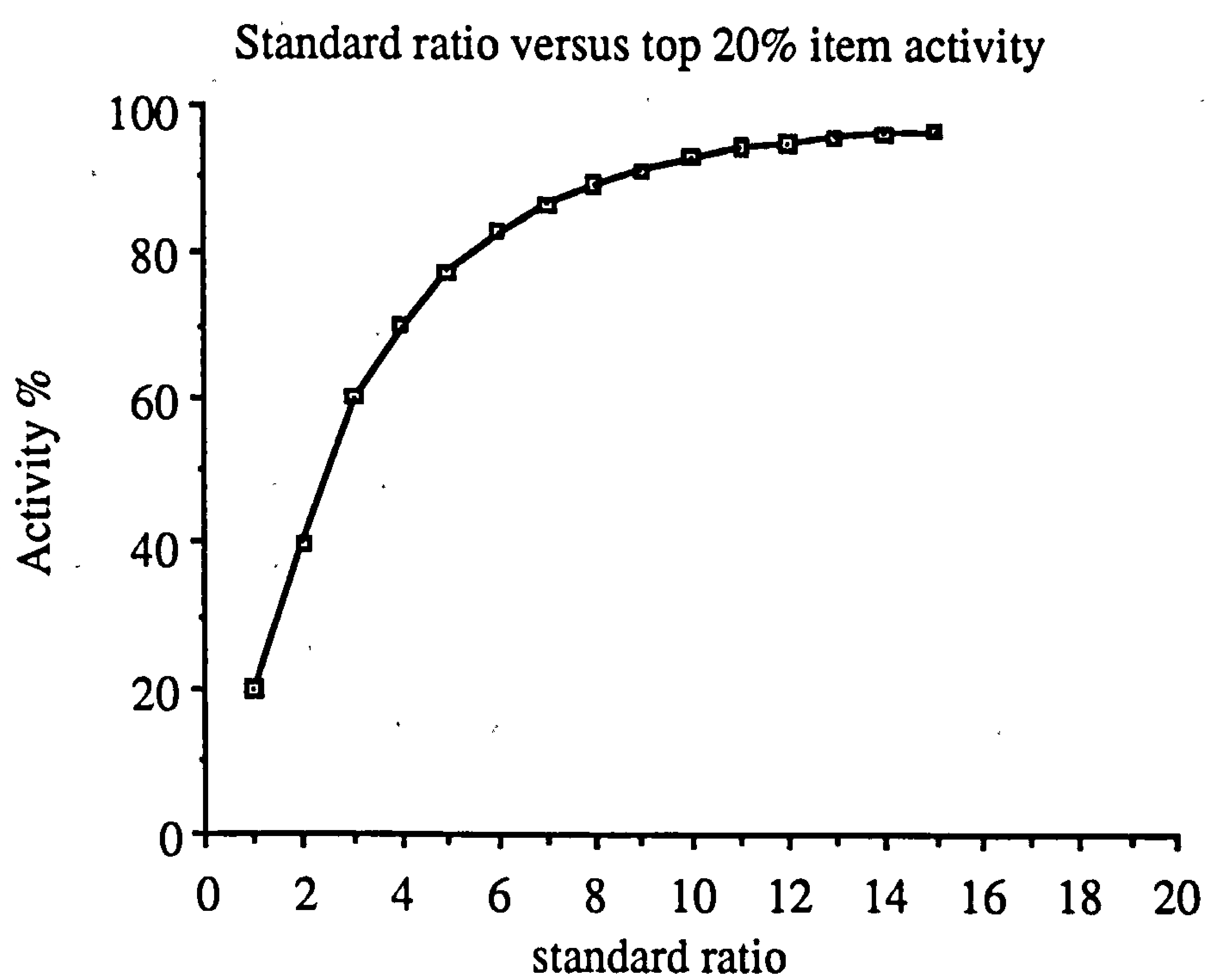


table 11.3

	% value		
% items	$\sigma = 1$	$\sigma = 1.684$	$\sigma = 2.3$
1	9.20	26.00	49.00
5	25.90	51.60	74.40
10	38.90	65.60	84.60
20	56.30	80.00	92.80
50	84.10	95.40	98.90

Clearly this approach can be extended and it is possible to determine for any proportion of items the value of the corresponding proportion activity accounted for by those items. Tables relating proportion of items to proportion of activity are not generally available, but the above formulation can be used to calculate the values of interest given either σ or ρ of the lognormal distribution. We reconsider this theory again in chapter 12 where we present a novel approach and method to the problem of monitoring the value tied up in the stock invested in an inventory.

Using a somewhat different approach to the question of the disproportionality effect of the item range Aitchison and Brown (1957) give a table in their appendix, (page 154), which shows the proportion of items that do not exceed the mean value of the lognormal variate. Whilst this may be of interest for some applications it is not as useful for inventory management purposes as Lockyer's development above.

11.9 Conclusions

We have attempted in this chapter to correct some misbeliefs concerning the disproportionality principle in inventories. Namely that the Pareto name has been wrongly applied and this erroneous use appears to stem from Juran, although the term Pareto analysis has become so ingrained in management terminology it is likely to stick. Secondly and far more importantly there is no direct link between inventory ABC curves and lognormal distributions of the same variate. Any highly skewed distribution will produce the typical form of ABC curves of varying concentration if the data is presented correctly. On the other hand lognormally distributed usage values can always be presented in an ABC curve form as can any other highly skew data set that may be distributed according to some other skewed probability model.

We have seen from this chapter and from the work of earlier chapters that inventory usage values are invariably disproportionately concentrated. In those cases where the inventory variate is lognormally distributed then the degree of concentration is likely to be very high especially, so if we are considering spare parts systems. By utilising the principle of concentration of inventory usage values we have presented a very important relationship between the lognormal distribution and the proportion of value accruing to a given proportion of items in the population that has the lognormal variate. Using the methods described by Lockyer it is possible to determine for any value of the lognormal shape parameter what the corresponding proportion of items will be that fall in any given fractile of the item range. This valuable relationship and some of the other ideas of presented in this chapter form the basis of some novel methods of monitoring inventory parameters that we present in the next chapter.

Aggregate inventory perspectives

12.0 Introduction

After a lengthy journey examining the lognormal distribution from a theoretical, modelling and validation point of view we now turn to some practical inventory perspectives which call on the underlying properties of this distribution to yield deeper perspectives of independent demand inventories. No direct attempt has been made here to develop particular inventory decision rules, rather our primary purpose is to focus on some fundamental relationships which could underpin the development of inventory norms, although we indicate several fruitful areas for optimisation of particular situations.

12.1 The distribution of inventory variates

We have seen from several US studies that inventory usage values for independent demand items are very likely to be lognormal. From the work here we can say that if the inventory is concerned with replacement spare parts for capital equipment then the usage rates, if calculated from a sufficiently long period, will almost certainly be lognormal. Furthermore we have shown empirically that both usage volume (depending on the time period) and unit price will both be lognormal. This is consistent with the theory of Aitchison and Brown as discussed in chapter three, namely that if two variates, 'A' and 'B', are both lognormal then the product AB is also lognormal, but with different parameters values of μ and σ. Likewise if 'A' and 'B' are two independent positive variates such that the product AB is lognormal then both A and B are lognormal. (Or as a special case if one of the variates is constant then the other is lognormal). Furthermore Aitchison and Brown also showed that if 'A' is lognormal with parameters μ and σ, then, if α and β are constants, the power function αA^{β} is also lognormal with parameters $(\alpha + \beta\mu, \beta^2\sigma^2)$. This has great utility in inventory theory because certain functions of inventory variates can be expressed in terms of other inventory variates. The whole basis on which the theory of the lognormal distribution rests as a tool to set aggregate inventory standards depends on such relationships. This was discussed at some length in chapter three.

We showed earlier in this work that when usage values are lognormal then so too are usage volumes provided the time period is sufficiently long. Hence any inventory variate that can be formulated as a power function of usage volumes will also be lognormal. Furthermore, as any lognormal variate times a constant will be lognormal then the mean demand for a range of items will also be lognormal, as the mean is the total of demand volumes times the inverse of the number of items in the range. In the following we show that when volumes are lognormal then a number of related inventory functions are also lognormal.

For example, the standard deviation of item demand volumes can be formulated in terms of average item sales volume by the function shown below :

$a\bar{x}^{\beta}$ where $\bar{x}$ is the mean demand ----------------12.1

From 100 randomly selected DAF parts items the log of the mean demand was regressed against standard deviation (σ) of demand (see appendix three) and this gave the regression model shown :

$$\log_e \sigma = \log_e \alpha + \beta \log_e \bar{x}$$

from which it was found that $\log_e \alpha = 0.253$ and $\beta = 0.814$ with a correlation coefficient r = 0.975 and $R^2 = 0.951$. The Durbin Watson statistic was 1.86 (well within the acceptable range for no autocorrelation with a sample of 100 items). Hence the standard deviation of demand can be very confidently related to the mean demand by -

$$\sigma = 1.278(\bar{x})^{0.814}$$

Now as safety stocks are usually expressed as some rational function of the standard deviation of demand, for example :-

$$z\sigma(R+L)^{0.5}$$

Where z is the safety stock factor to set the appropriate service level for the relevant demand distribution. R and L are the review and lead time periods respectively. (R will be 0 for a reorder policy system).

If $(R + L)$ is constant and σ is replaced by the function - $a\bar{x}^{\beta}$ then we can write -

$$z\alpha(\bar{x})^{\beta}(R+L)^{0.5}$$

and this function will be lognormally distributed provided that all the mean values $\bar{x}_i$ are lognormally distributed for a range of items $i = 1$ to m. Now as the cycle stock formulation in most computer controlled inventory situations is based on a Wilson EOQ or variation we can write -

$$EOQ(Q) = (EOQfactor)(V_{ai} / P_i)^{0.5}$$

where V_{ai} is the annual demand volume of item i
and P_i is the price of item i

If usage volumes V_{ai} for items $i = 1$ to m are lognormal then so to will be the cycle stock function above because it is equivalent to equation 12.1. Now the total stock volume invested in an inventory is the sum of safety stock plus cycle stock so we can write -

$$\text{Total stock item}_i = (EOQ\,factor)(V_{ai} / P_i)^{0.5} + z\alpha(\bar{x})^{\beta}(R + L)^{0.5}$$

which in fact is the volume of stock at the peak of each cycle, so average stock held will be one half the cycle stock plus safety stock ie -

$$\text{Average stock item}_i = 1/2(EOQ\,factor)(V_{ai} / P_i)^{0.5} + z\alpha(\bar{x})^{\beta}(R + L)^{0.5}$$

From this function we have that average stock for a range of items i=1 to m is the sum of two lognormal functions, and from the work of previous chapters it was shown empirically that the sum of lognormal functions tends also to be lognormal. Hence the above average stock formulation is most likely to be a lognormal function. Furthermore, if we multiply the function by the item price (also a lognormal variate in the spare parts case), then the monetary investments in stock volumes will be lognormal. Now as period usage volumes and usage values are both lognormal (subject to a sufficient time period), then it also follows that the turnover ratios for individual items in a parts range will be lognormally distributed, ie the function -

$$\frac{\text{Annual Usage Volume}}{\left[1/2(EOQfactor)(V_{ai} / P_i)^{0.5} + z\alpha(\bar{x})^{\beta}(R + L)^{0.5}\right]} \quad \text{-----------12.2}$$

will be lognormal across the range of items for i = 1 to m. This is a significant conclusion because we can deduce from this that the range of values we can expect to see in individual item turnover rates will be very wide simply because the lognormal distribution is in most cases so very highly skewed. This very fact then calls into question the use of the classical inventory turnover ratio to meaningfully measure the overall speed of turnover of an inventory range of spare parts. Before we give consideration to this issue we first turn to an empirical test of the hypothesis that cycle stocks, safety stocks and hence both total and average stocks and turnover ratios will all be lognormal, if the individual item mean values are lognormal.

The reader is referred to appendix five where we show the results of the tests for lognormality on simulated safety stocks, cycle stocks and average stocks for 200 simulated lognormal item demand volumes and prices. The results clearly show that each of these functions do indeed look very much like lognormal functions and on the basis of the tests reported in appendix five we can conclude that in probability, as we predicted, these functions can be regarded as drawn from lognormal populations.

12.2 The form of turnover ratios.

The classical inventory turnover ratio is simply calculated from total sales divided by total investment in inventory as shown-

$$\text{(Total sales)}/\text{(investment in stock)}$$

Which in truth only gives the simple turnover ratio of the average inventory item and it does not give any indication of just how fast or how slow the extreme items in the inventory range may actually be moving. Indeed when usage values and volumes are highly skewed the value of the classical ratio is not anywhere near the value of the average of the individual turnover ratios. This result is entirely due to the skewness of the usage distribution. We can show this empirically by a simple example. The simulation tabulation in appendix five showed the simulated inventory parameters we derived using lognormally distributed demand volumes and demand prices. The individual turnover ratios were determined for each item using function 12.2 shown above, and, as can be seen from appendix five, figure A5.6, this function does indeed turn out to be lognormal; which is what one would expect to find given that it is the ratio of two lognormal functions. Now as we can see from the table in appendix five the overall 'classical' inventory turnover ratio was calculated at 17 times per annum; an impressive value by most yardsticks of inventory turnover. However, when we examined individual turnover rates a very different picture was seen as the ratios ranged from the highest at 23.6 to the lowest at 0.2 with a mean of 4.90, far lower than the calculated classical ratio. This difference between the overall classical ratio and the average of individual ratios was entirely due to the highly skewed nature of demand volumes and investment stock volumes, and the fact that the average of the individual turnover ratios was not nearly so influenced by the extreme skewness of the usage value data as is the classical ratio. This same phenomena will also apply to other simple average based ratios due to the skewness in the distribution of usage volumes and usage values. Both the ‘stock to sales’ ratio and the ‘day sales in stock’ ratio will, if calculated in the classical way for an inventory as a whole, be vastly different from the average of the same ratios calculated for individual items. In the case of highly skewed distributions the average of the individual item ratios is a much more realistic value to measure and represent the behaviour of all items in aggregate, but far less convenient to calculate.

Now most inventory managers and shop keepers know, by experience and intuition, that the faster turnover rate inventory items in a range of items are more profitable than slower turnover rate items. The more times you take the margin in a given period the more profitable will be the item concerned. But this is ultimately subject to the costs of reordering and stock out risks and associated costs. The smaller the cycle stock the faster will be the turnover rate, but ordering costs begin to rise rapidly and so too does the risk of stock out because of the greater frequency of cycle stock depletion down to the safety stock level. From the formulation 12.2 for the turnover of each item in an inventory range we can develop a relationship between item turnover and profitability.

If the margin earned by an item '*i*' is M_i (as a proportion of price) and the cost of holding an item '*i*' in stock is C_h (also as a proportion of price as we defined in chapter three), then the actual margin earned by item *i*, with a turnover rate of '*k*' times per period, will be $k(M_i)$. (*k* being defined by function 12.2)

Now the total gross margin earned will be $P_i k(M_i) V_{hi}$

where V_{hi} is the average volume of stock held for item *i*

and P_i is the item price.

Then total stock holding cost will be $P_i C_h V_{hi}$

And total ordering cost will be proportional to $C_o k$

Where C_o is the cost of placing a single order and covers receiving, checking and possibly expediting costs.

NOTE: we use the function $C_o k$ to express order costs rather than the more conventional function - (Annual demand/1/2{EOQ})C_o because the latter can yield quite unrealistic values for the number of reorder cycles when usage values across the inventory range are very highly skewed as in the case of the lognormal. We regard the inventory turnover ratio to be a more realistic reflection and measure of the most likely number of orders placed per annum in many inventory situations.

From the foregoing we can now formulate the real (net) margin, as measured by return R_i by an item i as follows :-

$$\text{Net Margin}_i = kM_iP_iV_{hi} - [C_hP_iV_{hi} + C_ok] \quad \text{------------12.3}$$

Now if the annual demand volume for item i is V_{ai}

then we can set the turnover rate k as $k = \frac{V_{ai}}{V_{hi}}$

(which is equivalent to function 12.2).

After substitution of this function in equation 12.3 above followed by further simplification we obtain the function 12.4 below:

$$\text{Net margin item } i \quad R_i = V_{ai}P_i\left\{M_i - \frac{C_h}{k} - \frac{C_ok}{P_iV_{ai}}\right\} \quad \text{-----------12.4}$$

But $V_{ai}P_i$ is the annual sales rate (S_i) for item i hence we can further simplify function 12.4 to yield function 12.5 below:

$$\text{Net margin item } i \quad R_i = S_i\left\{M_i - \frac{C_h}{k} - \frac{C_ok}{S_i}\right\} \quad \text{------------12.5}$$

This formula contains three variables S_i M_i and k which are all lognormal functions and hence it becomes amenable to aggregate analysis for a whole range of items using the basic theory we have shown in previous chapters.

In this form there is no penalty for lost sales which is the appropriate model for the DAF case as the company is essentially in a captive demand situation. This formula is much simpler and certainly more flexible to use than a formulation given by Heron (1976), who used a similar process reasoning to arrive at an expression for the real net margin R_i (or rate of return) on the investment for individual items. Heron's model formulated R_i in terms of powers of the sales rate S_i. [Each term in Heron's formula for order costs, holding costs and stock out costs were expressed in powers of the sales]. For specified values of order cost, holding cost, and stock out cost Heron produced the following-

$$R_i = 0.32S_i - 1.805*10^{-6}S_i^{1.438} - 9.152*106S_i^{-4.312} - 0.0793S_i^{0.7555} - 5 - 1.871S_i^{0.5} - 0.1533S_i^{0.938} + 1.625*10^6S_i^{-3.787} + 0.2953S_i^{0.8045}$$

Heron clearly went to a great deal of trouble to develop this formula in order that the real margin could be measured from just a knowledge of the expected sales. However, we feel it is unnecessarily complicated for general use, despite the simplification of only one variable S_i. The major problem is that it must be reformulated for each and every change in the value of any input variable. We believe this would prove irksome to the average inventory manager. Our formula 12.5 above is far simpler, has greater flexibility for general use and it should have more appeal to operational management. It is also capable of manipulation in several ways to give insight to the items and inventory being examined. For example, if we plot real return R_i against the turnover rate 'k' then we can see the effect that 'k' has on the real net margin earned. This is shown in figure 12.1 for a variety of values of 'k' and for fixed values of C_o C_h S_i and M_i. The maximising effect of 'k' can be readily seen in figure 12.1.

figure 12.1

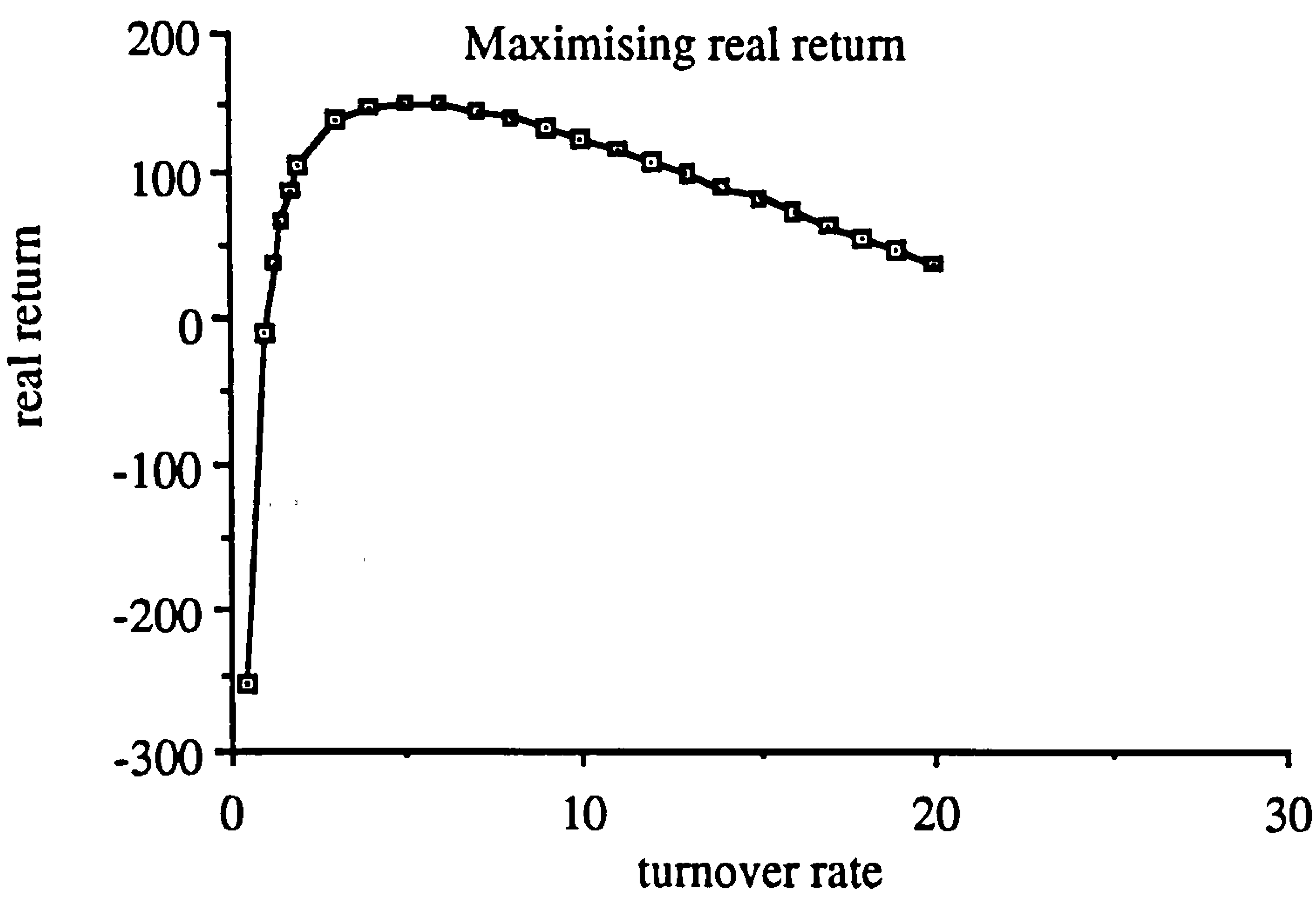
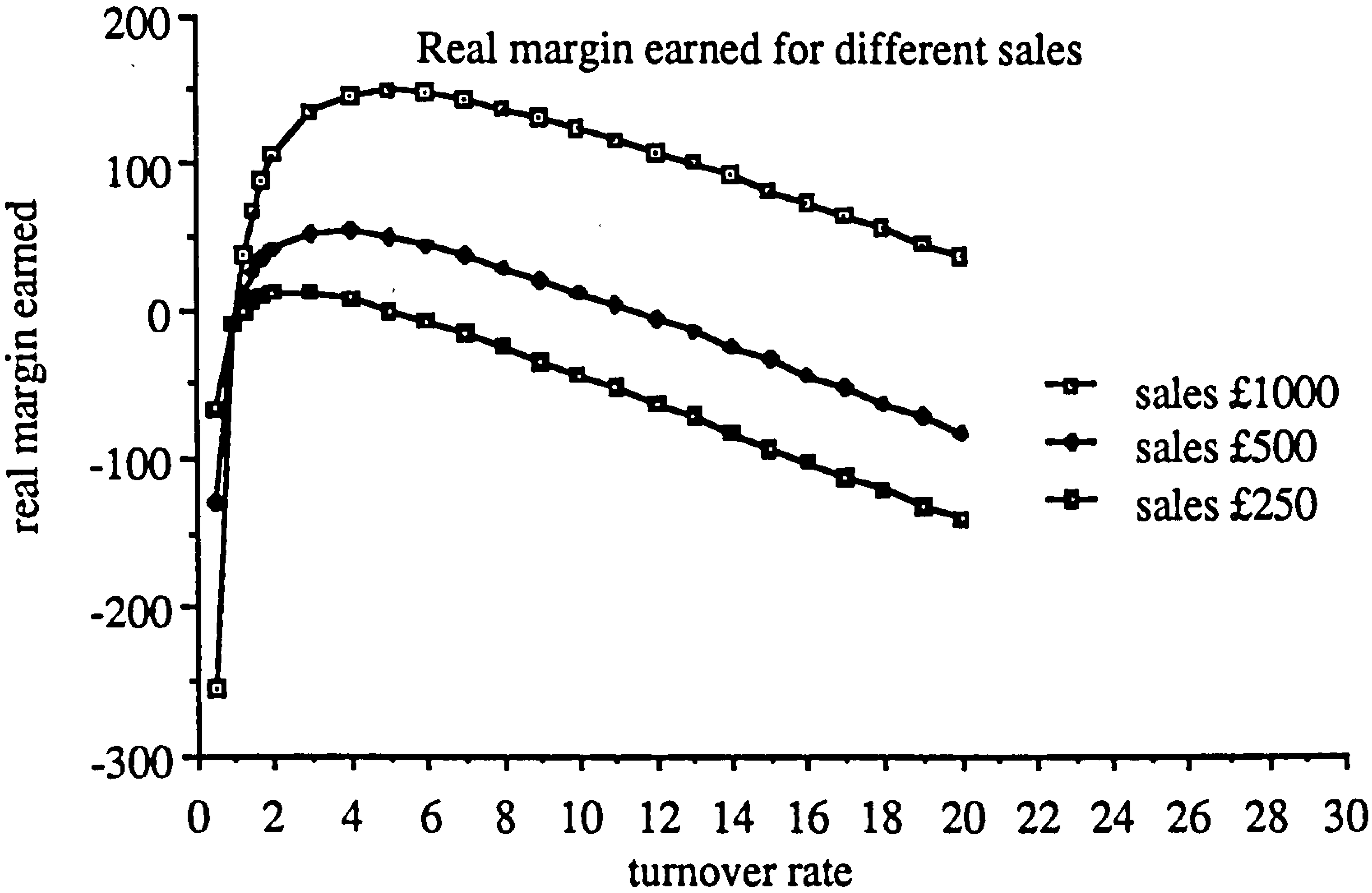


Figure 12.2 below shows the same presentation for a variety of sales rates

figure 12.2



The concept of a profit maximising turnover rate can be formulated analytically as follows. From the formulation 12.5 above we can write

$$R_i = S_i M_i - \frac{S_i C_h}{k} - C_o k$$

Now we take the first differential of R_i with respect to k thus -

$$\frac{dR_i}{dk} = \frac{C_h S_i}{k^2} - C_o$$

For a maximum we equate this differential function to zero and on rearranging we can express 'k' in terms of the sales-

$$k = \sqrt{\frac{C_h S_i}{C_0}} \qquad \text{---------------------12.6}$$

This function gives us the optimum value of 'k' which produces the maximum value of the real rate of net return R_i given values for C_o, C_h and S_i. The change of optimum 'k' values with sales is shown below in figure 12.3 below. Figure 12.4 shows the same relationship using log scales from which it is easier to relate 'k' to sales.

figure 12.3

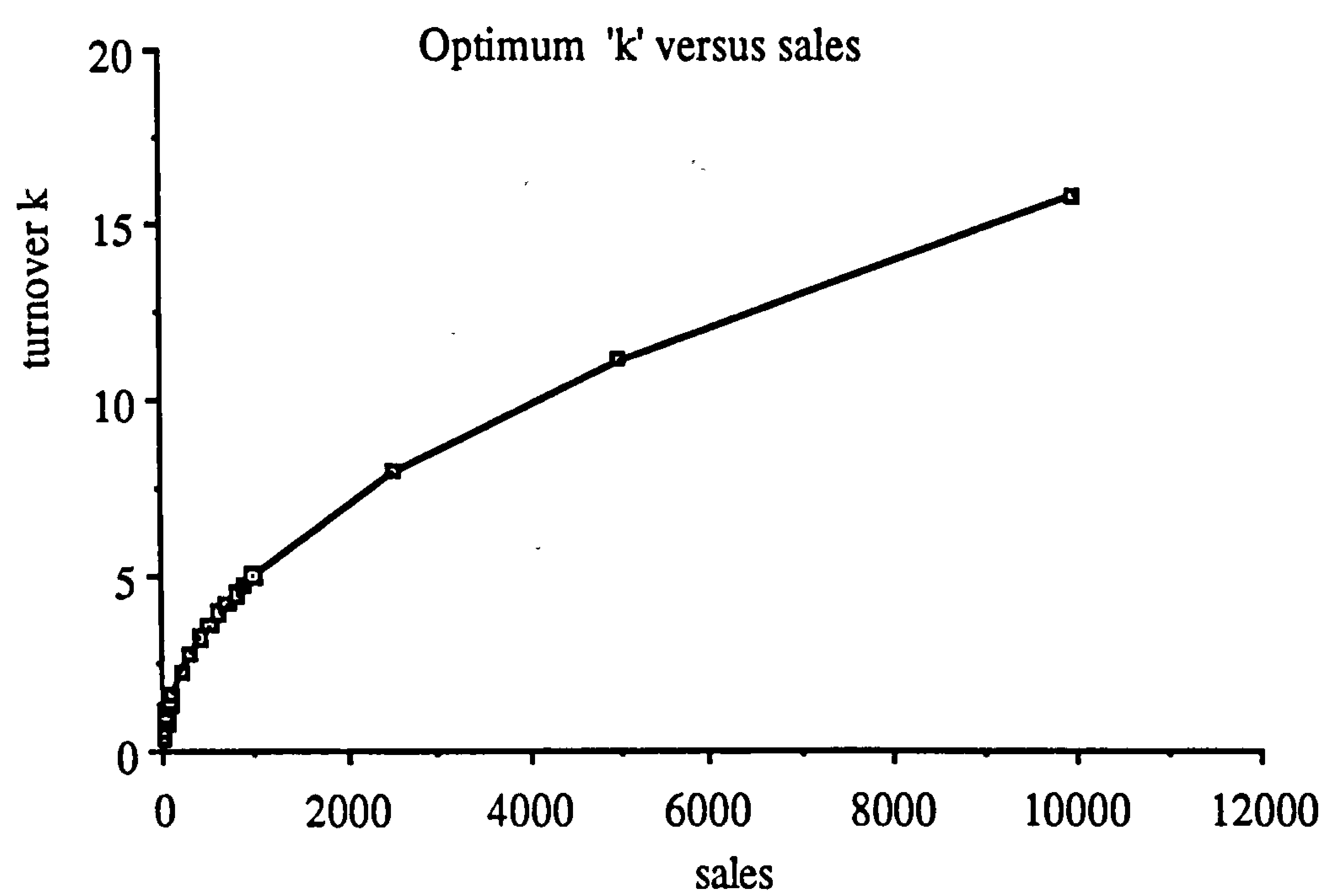
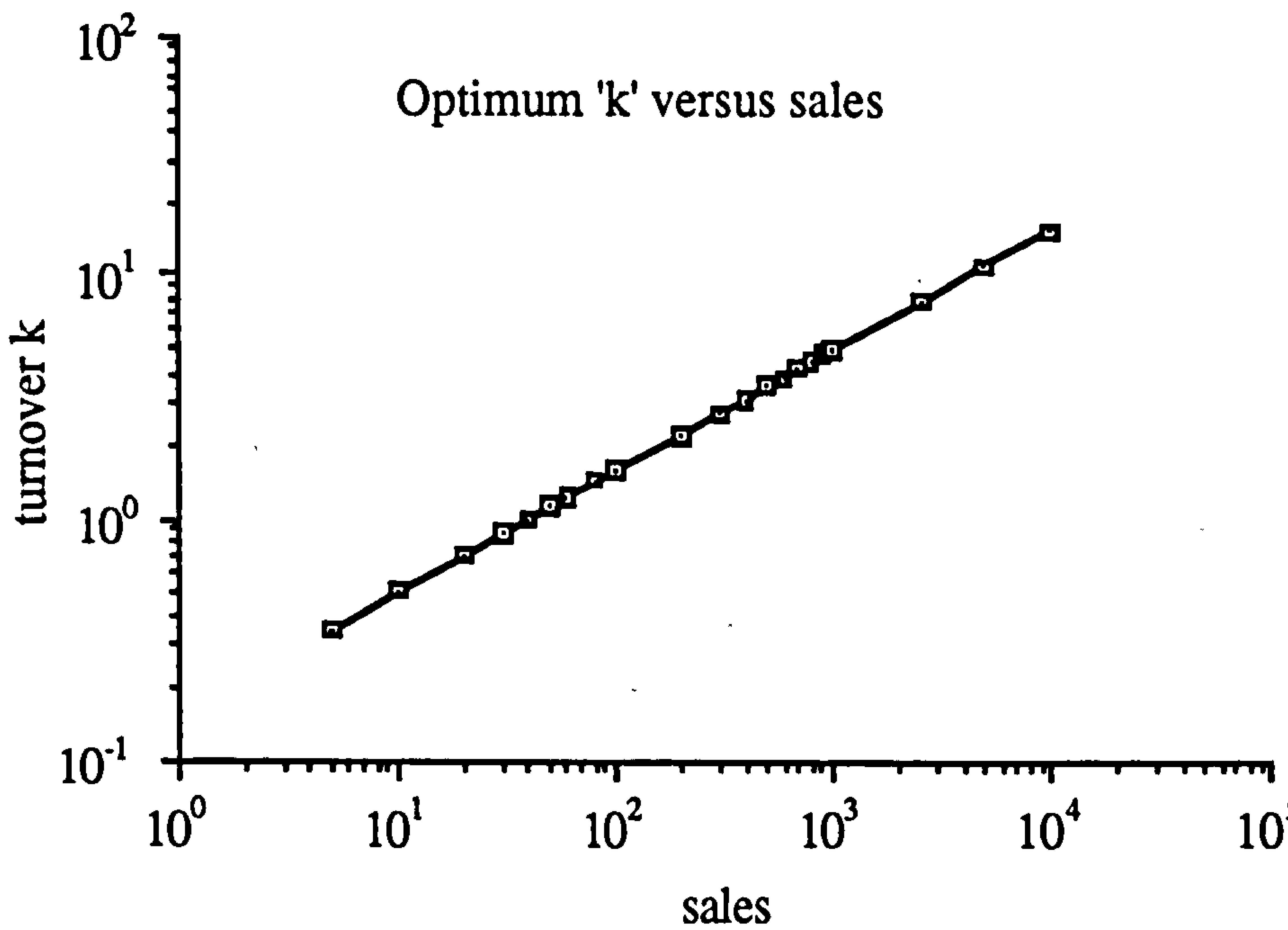


figure 12.4



Now formula 12.6 from above can be rewritten as -

$$k = \sqrt{\frac{C_h}{C_o}}\ [S_i]^{0.5}$$

and this is of the general form :-

$$k = \alpha[S_i]^{\beta}$$

and if all the S_i values are lognormally distributed then all the optimum k values will be lognormally distributed with parameter values given by -

$$\mu_* = (\alpha + \beta\mu) \quad \text{and} \quad \sigma_*^2 = \beta^2\sigma^2$$

Where μ_* and σ_*^2 are the parameter values of the lognormal distribution of 'k' whilst μ and σ^2 are the parameter values of S_i. This is consistent with our earlier prediction that the *k* values will be lognormal.

In figure 12.5 below we show the aggregate picture of plotting the real net return R_i as a percentage of sales for all 200 items from the tabulation in appendix five, against the item turnover rate '*k*'. The graph clearly shows the negative values of the net return at low turnover rates, and also the maximising effect as the rate of turnover increases much beyond a value of ten times in the period. Figure 12.6 shows the effect more closely in the critical elbow region of the graph between a turnover rate '*k*' of three and ten. Beyond a turnover rate of ten the real net margin, as a percent of sales, cannot be improved further and there would be little point in trying to turn the stock over any faster than about fifteen times in the period in question.

figure 12.5

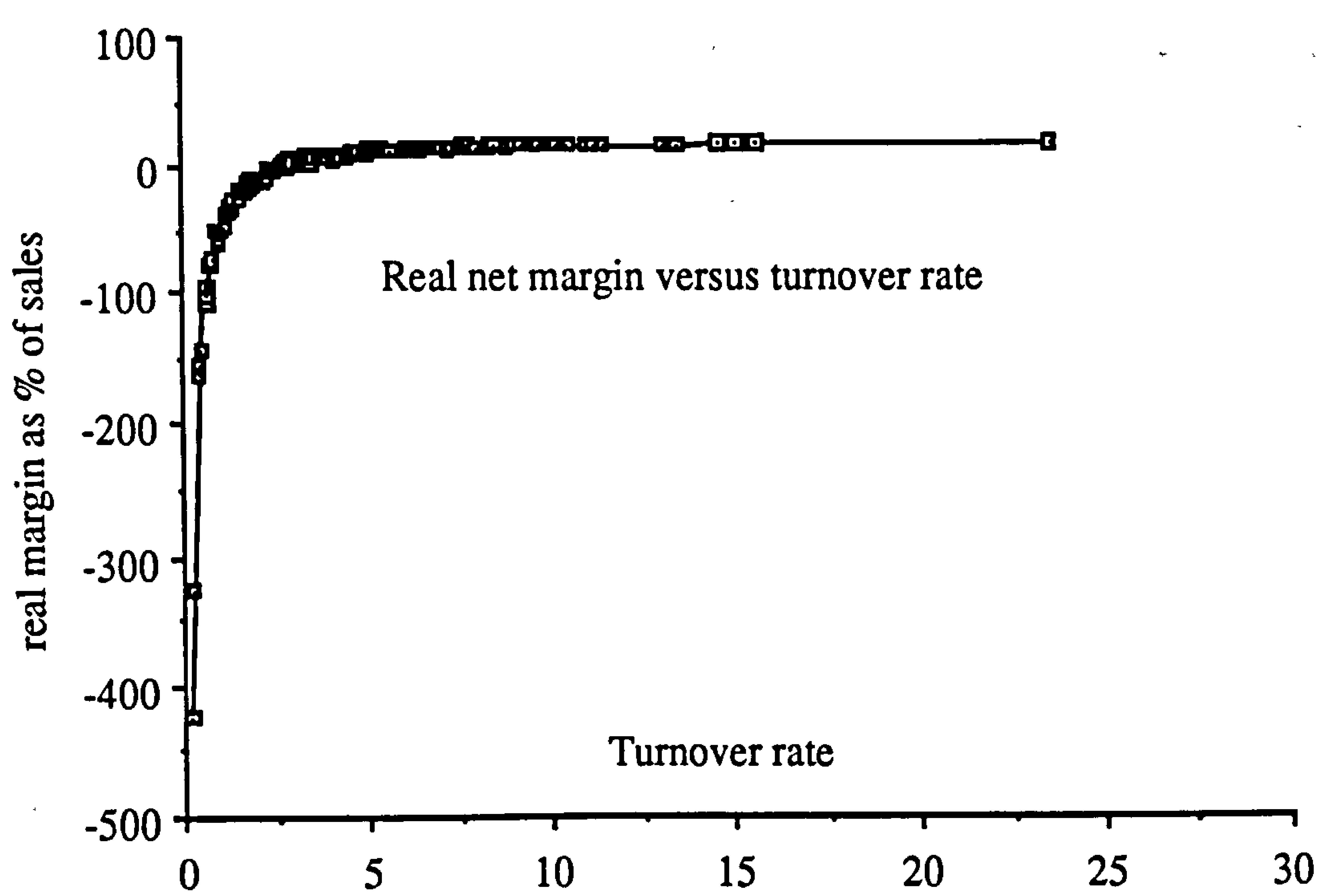
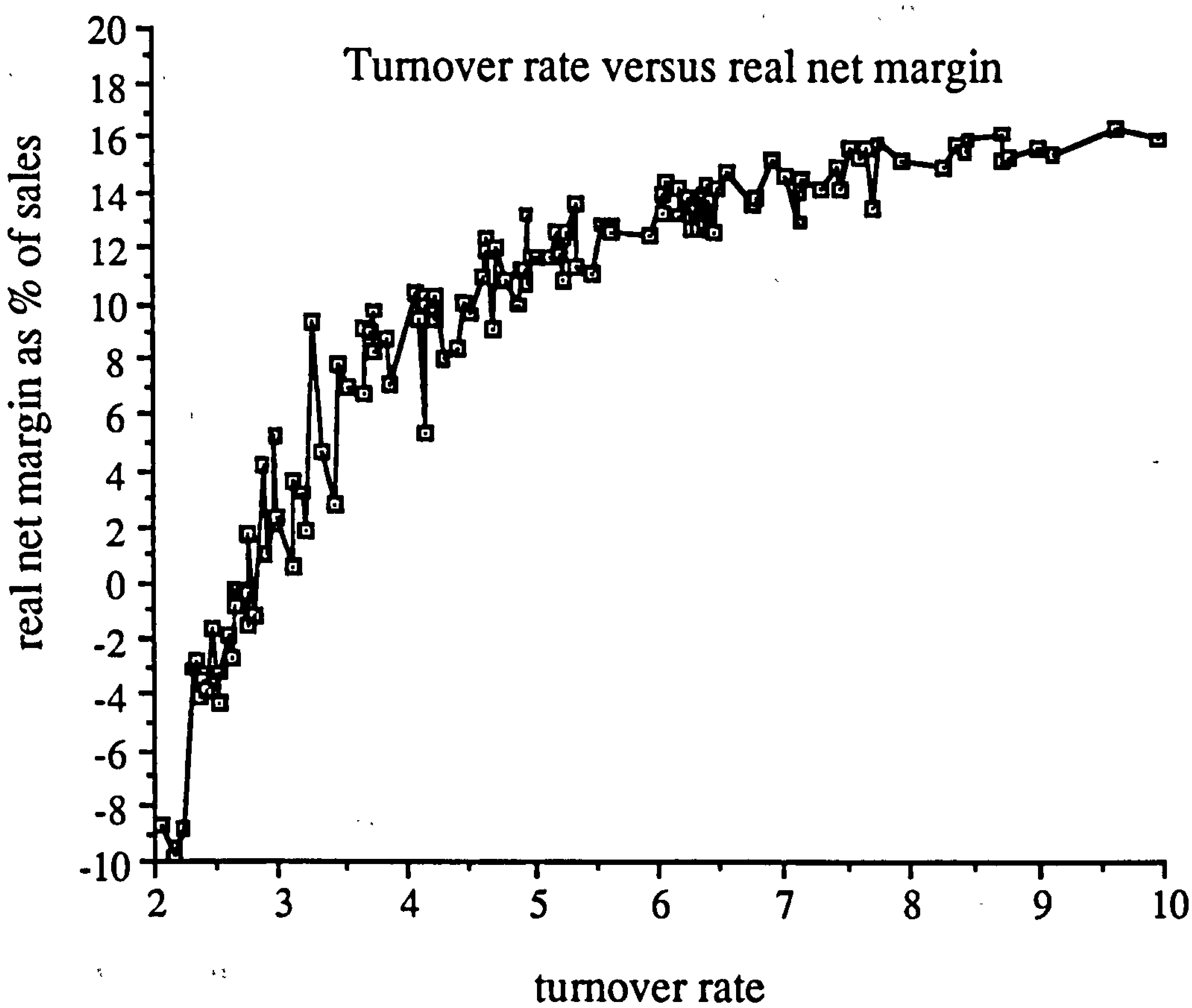


figure 12.6



The flexibility of our function 12.4 can also be seen by using it to determine the margin M_i that should be applied to prices to achieve a target real return R_i. By rearranging 12.5 we can express M_i in terms of the remaining factors and variables as shown below :

$$M_i = S_i \left\{ R_i + \frac{C_h}{kS_i} + C_o k \right\} \qquad \text{--------} \quad 12.7$$

The function 12.5 can also be readily modified to admit a stockout cost as we show below. Stockout costs are not insignificant, even in a captive demand situation such as the DAF system, but it can be very difficult to assign an appropriate value to C_S the single cost incurred in ordering, expediting and receiving of an out of stock item. Assuming management can assign a meaningful value to C_S then the proportion α of 'stock in stock out' cycles which may be at risk of a stock out will be directly related to 'k' the stock turnover rate. (1-α) is the service we can expect from the system in terms of the proportion cycles where demand is covered by the stock held. Hence we argue that the stockout cost is incurred in αk cycles, and the total stockout cost incurred will be $\alpha k C_S$. Therefore our function 12.5 can be modified as follows :

$$R_i = S_i\left\{M_i - \frac{C_h}{k} - \frac{C_o k}{S_i} - \frac{\alpha C_s k}{S_i}\right\} \qquad \text{---------------12.8}$$

After differentiating R_i with respect to k and rearranging function 12.8 above leads to a modified version of our optimum turnover rate model which we show below :

$$k = \sqrt{\frac{C_h S_i}{[C_o + \alpha C_s]}} \qquad \text{---------12.9}$$

This function 12.9 leads to the same curves as seen in figures 12.3 and 12.4 except for the fact that the optimum value of k is reduced marginally to take into account the additional cost factor αC_S. If the stockout penalties are proportional to the actual number of units out of stock then the partial expectation function, as shown by Lewis (1981, page 74), can be used to derive a stockout cost function and then used in place of C_S above.

The models we have developed above can provide Operational managers with valuable aggregate perspectives of the inventory in question which can then aid rational inventory management decisions. For example, because the individual turnover ratios are the 'correct' ones, based as they are on the theoretical values of cycle stock and safety stock, then an objective view can be taken on the profit margins produced. Any adverse values of the profit margin percent, such as negative values or unacceptably low values, indicate stock items that should (a) be eliminated from the product line, or (b) be moved backwards in the distribution channel ie from retail level to wholesale level, (c) have the unit prices increased until the profit margin is acceptable, or (d) at least evaluated in terms of the real cost of carrying such an items in the range. Some of these decisions are clearly related to marketing policy. For example, in some operational systems it may be considered prudent sales policy to hold certain items in stock even though they produce negative returns. Under such circumstances then management at least know what the real cost of holding the item will be, and, if market conditions allow, what price to charge per item to cover the negative return. Clearly this is just one example of using the above aggregate relationship and others can be developed.

12.3 Distribution Channel Decisions

One of the more difficult problems to resolve in a distribution channel embracing say factory level to wholesale level to retail level, is how best to deploy the inventory across the whole range to effect (a) a competitive market service in terms of price and availability, and (b) to minimise the cost of holding stock in the total system. DAF trucks, for example, operate at all three levels, although in the UK most of the appointed dealers are independent businesses. However, the company is well aware that the marketing success of its truck products depends very heavily on the perceived level of service an operator can expect to get from spares support. No operator wants his truck out of service any longer than absolutely necessary and certainly not for want of a crucial spare part. To ensure a high level of service from spares support the company could deploy appropriate stock levels of the entire stock range at each local dealer then replenish these stocks from the central warehouse on a routine basis. Even so such a policy would not guarantee a 100 % service at the dealer level because of the stochastic nature of demand. So to balance ordering and holding costs, in particular, service levels must be set for each item to give a coverage at, for example, typical service levels of around 90% to 96% depending on the criticality of particular items. This approach still generally implies a heavy carrying cost penalty in the typical spare parts environment due to the very large item range and diversity of demand behaviours. Also, because usage values are lognormally distributed the tail of the distribution is very long and many very high value low demand items would have to be held in the system. The associated high variance of demand for such items makes the demand pattern particularly lumpy and these can be expensive items to hold in any spares system.

By the use of effective replenishment logistics it makes sense not to carry all items at the retail level. Significant savings can be made by pushing some items back up the distribution chain and then meeting dealer demand by express delivery systems from distributor stock. This works and is generally acceptable from the ultimate customers point of view for many slow moving very expensive parts. However, there are many parts which are moderately fast moving and moderately expensive where the right policy is not obvious. The policy at DAF has been to recommend to dealers to carry between 60% and 70% of the spares range (and set a 90 to 95 % service level on these items), but this has been based purely on the level of annual sales value and not on profitability analysis. For slower moving items still retained as a stock item at the dealer level there is always the possibility that the margin earned will not cover the holding costs if the item is in stock beyond a certain point. Such situations can be revealed by the use of our concept of a real net margin.

From our previous model relating real net profit margin to turnover rates we have a rational starting point from which to make decisions on which items to move backwards in the chain. The model gives us the point at which returns became negative, and hence the cut off point indicating those items which should be moved backwards in the distribution chain. In the DAF case analysis of turnover rates shows that the policy of 70% on sales level alone leaves many negative return items at the retail level, whilst many profitable items are moved backwards in the chain. The same analysis can be carried out at the wholesale level to show which items should be moved back to the factory level. From the simulation analysis shown in appendix five, where we used EOQ and safety stock formulations equivalent to those used at DAF, it can be seen from the simulation tabulation that from 200 items 75 of them show a negative real return. These items plus those with only small positive real net margins are candidates for moving backwards in the distribution channel. The service reputation in DAF, for items moved backwards in the channel, can be maintained by efficient information flow from each level to the next and by efficient transport logistics. In 1986 DAF had a 24 hour guaranteed delivery from the wholesale level to the dealer for 'VOR' items (vehicle off the road) and 48 hours from factory level to wholesale level.

The approach to move at least all negative return items backwards in the chain could be very cost effective in the DAF system because in 1986 the company had a captive demand population for most of its spares product range. Hence all sales in a given period at the retail level would ultimately be sales in the same or the following period at the wholesale level. Very few items in the parts range were obtainable from competing spare parts agents, although the situation was expected to gradually change as the market becomes large enough to attract spare parts copies. Hence there is very little penalty in the short run from lost sales and provided the at a very high level the customer experiences a very efficient service even for out of stock items. What we must take account here is the marginal costs involved, because as items are moved backwards in the distribution chain the fixed cost component of holding costs must be recovered by a smaller number of items. Hence more items may, as a result, show a negative return and the process becomes iterative. The alternative approach is to ensure that C_h (the holding cost factor) only represents the variable cost component of holding cost. This will be the direct cost of capital to support each unit of value invested in a stock item.

An alternative approach to the negative return items would be to use equation 12.7 and calculate the value of M_i (the actual margin applied to the price) for each item i for optimum turnover value k to achieve an acceptable real net margin R_i.

12.4 Monitoring Dealer Inventory Performance.

It is vitally important to any distributor such as DAF, who sell capital equipment, that the customers receive a high level of after sales support. When dealerships are appointed to serve a given geographical market area the distributor has a responsibility to ensure that the local dealers has properly trained staff and appropriate systems in place to provide all the professional support required to enable the dealer and distributor to be competitive in the market place. DAF's competitive position in the market place depends heavily on the dealer support for existing and prospective customers. Inventory service is just one area where the distributor must monitor the dealers ongoing performance. We now consider an approach to this problem.

One of the significant results that has come out of the research here is the remarkable stability of the lognormality of inventory usage values.

The changes year to year over a 12 year period at the wholesale level of the DAF system have shown only very small changes in the shape parameter of the annual usage rate distributions. The practical conclusion of this is that inventory aggregates themselves are very stable year to year. Furthermore the retail level usage value distributions must therefore also show the same stability year to year. Usage values at the distributor level are the sum of the usage values for each individual dealer, and analysis by this author has shown that the lognormal distributions of usage values at the dealer level reveals distributions that are very similar in terms of the shape parameter from dealer to dealer. As investment stocks at the dealer level are set consistently based on rational formula for safety stocks and cycle stocks which in turn dictate what the profile of turnover ratios will be; then there exists a rational means to monitor dealer performance in aggregate inventory terms.

The mean value of any lognormal variate can, as we have shown previously, be written as follows from the theory of chapter three-

$$\alpha = e^{\mu + 1/2\sigma^2}$$

Now the global annual sales of all parts at the distributor level can be written as -

$$\alpha = ne^{\mu + 1/2\sigma^2}$$

where 'n' is the number of parts in the active stock range and μ and σ are the mean and standard deviation of the $\log_e$ distribution of parts usage values, ie the parameters of the lognormal distribution. In a totally captive market all the dealers parts sales are translated into total sales at the wholesale level. Also the value of the lognormal shape parameter of usage values σ will be a strong reflection of each local market being a characteristic of the market sales volume and the local aggregate pricing level. If V_i is the volume demanded of item 'i' and P_i is the price of item 'i' then the usage rate P_iV_i values are lognormal with parameters

$$(\mu_v + \mu_p, \quad \sigma_v^2 + \sigma_p^2)$$

where μ_v and $\sigma^2{}_v$ are the parameters of the volume distribution and μ_p and $\sigma^2{}_p$ are the parameters of the price distribution.

Now $(\sigma^2{}_v + \sigma^2{}_p)$, the shape parameter of the usage value distribution, is known from empirical analysis and it will be characteristic of the local market level. Furthermore it will be approximately the same for all dealers at the same market level. The location parameter $(\mu_v + \mu_p)$ of the same distribution will vary from dealer to dealer because it depends directly on the general level of parts sales for each dealer. Now assuming that all dealers use rational methods to set safety stocks and cycle stocks then it is possible to know what each dealer should be carrying in stock to support its local parts market

For any individual part 'i' the average investment will be -

$$\text{Total stock } = P_i\left\{(EOQfactor)(\bar{x}_i / P_i)^{0.5} + z\alpha(\bar{x}_i)^{\beta}(R+L)^{0.5}\right\}$$

It would not be difficult for the wholesaler, (DAF HQ distributor in our case), to establish what the local dealer EOQ and demand variance factors are. (R+L) in the DAF case were always 2 weeks (in 1986) for routine stock replenishment. Hence it is possible to predict at any time what stock level the local dealer should be carrying for any item 'i' and what turnover rate 'k' he should be achieving for the same item. However, the wholesaler will generally not be too interested in the performance of individual items. He will be much interested in aggregate inventory factors such as total investment in stock, inventory turnover ratios and the service levels achieved. By use of these factors the wholesaler will be able to monitor if the appointed dealer is providing the right market and service support to local customers.

By a complete enumeration of the above formula it would be possible to estimate these same factors for the entire stock range. With same 20,000 plus items in the range however, one could easily use the lognormal aggregate inventory approach. For example, the lognormal estimate of investment in dealer stocks will be the sum of cycle and safety stocks as shown-

$$\text{Total Investment } = n\left\{(EOQfactor)(\bar{x}_r)^{0.5}(Fcs) + z\alpha(\bar{x}_r)^{\beta}(Fss)\right\}$$

where Fcs is the lognormal cycle stock factor $\{ \exp[j(j-1)(\sigma^2/2)] \}$ with $j = 0.5$, and Fss is the lognormal safety stock factor given by $\{\exp[j(j-1)(\sigma^2/2)]\}$ where now $j = \beta$.
and $\bar{x}_r$ is the mean sales for the entire range of items.

Clearly this same formulation for all dealers in the dealership (30 in DAF's case in 1986) will give the entire stock in the system at any one time. If any individual dealership has substantially more or less than the values from the above formulation, then it deserves investigation by the wholesaler. Low values can mean the local dealer is not giving the appropriate level of service. If levels are too high it could mean the dealers stock systems are not functioning correctly and that he is losing profitability in this area. It is in the distributor's best interest that all the dealers in the network achieve a satisfactory level of profit to ensure efficient and continued operations in all areas of the business. A dealer in financial difficulty is a liability to himself and the distributor and possibly to the detriment of the product image in the local and wider market.

12.6 A Novel Approach to Monitoring Stock Investment Levels

One of the major ongoing concerns in inventory management is to monitor the service given to the customer and to monitor the amount of money invested in stock. With good computer controlled systems it is possible to monitor both very effectively on an item by item basis. However, there can be serious problems with very large inventories because the surveillance of the inventory range using item by item control does not always reveal the overall performance and behaviour of the range as a whole. One of the classic problems that can occur is the inventory range starts to become unbalanced. It is quite possible for an inventory range to have approximately the correct investment, as measured perhaps by an overall (classical) turnover ratio and yet within the range there may well be a growing incidence of stocks outs with some items whilst other items are overstocked. What can be happening from an overall investment point of view is that the over investment in one group of items is being counterbalanced by the under investment in another group. The reason why this can happen, even in situations where appropriate software is use to track demand, calculate reorder points and safety stocks, is, in the main, because of the human intervention. There are a multiplicity of reasons why stock levels become unbalanced. A few of the common ones are given below and are based on many of the situations which have arisen in the DAF spares environment.

(a) Management or supervisors may override the stock mechanisms, in anticipation of new demand trends which automatic forecasting of historical data patterns will not reveal. This can be a frequent problem in a spares environment when new items are added in to stock, which are intended to replace old ones.

(b) Management may not have complete trust in the computer models used and may override reorder suggestions and adjust upwards or downwards based on their best judgment of the prevailing situation.

(c) Many errors occur due to defective information processing at various points in the system. For example- Returns from dealers may not be booked into stock correctly causing stock reporting errors. Picking errors in the warehouse and incorrect deliveries to the dealers, although recorded as the correct item part numbers, will cause serious stock errors. Defective or incorrect parts being returned back through the channel, but not properly recorded, at departure or receipt points. Incorrect quantities being booked into stock. Stock theft of some easily removed items. (including off the back of the lorry problems).

One potentially very effective way to track and control such problems is to use the properties of the lognormal model. We know from empirical research reported in chapter six that the parameters of the appropriate lognormal models are very stable. The shape parameter of usage values change very slowly from year to year. Furthermore we saw in chapter 11 section 11.8 that there is a useful link between the lognormal distribution of usage values and the cumulative (Lorenz) ABC curve of usage values. Namely, that for a given shape parameter σ direct relationships between a given proportion of items and the proportion of activity they account for can be readily determined. Additionally we have seen from the work earlier in this chapter that the average investment values tied up in individual stock items are lognormally distributed, and therefore there will be a standard deviation and standard ratio of the associated log values of these investment values.

In a system where the cycle stocks and the safety stocks are rational functions of the usage values then there will be a value of the standard deviation (and standard ratio) of the usage values that is the one and only correct one to use for aggregate calculations. Furthermore because the particular value of the standard deviation of the log usage values is sensitive to the distribution of value amongst the item range any significant change in the value of the standard deviation (or standard ratio) is indicative of an underlying change in the inventory. If the range remains fixed in the number of different parts then an increase in the standard deviation indicates a range that is becoming more concentrated. That is proportionately more value is tied up in the faster moving items, and less so in the slower ones, yet overall the the total investment in stock will not necessarily reflect this. It depends whether there has been a value shift in the range. If an overstocking has occurred in one group that is offset by an understocking value in another group, then overall there may be nothing to suggest a problem is occurring in investment terms. The total value in the stock will be the same. However, the slower moving items may well start to experience stock outs more frequent than the level permitted by the given service level. And because such items are slow moving the increase in stock out incidence may not be apparent for some time. The additional penalty for overstocking on faster moving lines may also not be apparent for some time because they will be experiencing less than the permitted stockouts and nobody complains about good service.

Alternatively if there is a shift downwards in the standard deviation of the usage values then the opposite effects will occur; the inventory will become less concentrated. Whether this becomes apparent to management will depend if there has been any significant value changes within the range and what counterbalancing has taken place if any. It will also depend on the methods management use to monitor stock levels. By using our lognormal approach and knowing the correct and theoretical value of the shape parameter we can readily determine from the theory given in chapter 11 what the correct proportion of value should exist in each percentile of the item range. If we find an actual value of the stock investment that is significantly different from the correct value then management attention is needed. We argue here that using this lognormal parameter monitoring approach is the most effective way in which to track and monitor the behaviour of the investment in an inventory range as a whole entity.

Based on our 200 item simulation in appendix five we have calculated the correct value of the standard deviation of the lognormally distributed average stock investment values by two methods. First we determined it by simply calculating the standard deviation directly from the calculated individual values of stock held. This value was 1.96. Second we determined a relationship between the theoretical stock held and the usage values by a regression method. The 200 usage values were regressed against the 200 average stock invested values from which we deduced the relationship -

$$\log(\text{Stock held}) = \log\alpha + \beta\,(\log \text{usage values})$$

which gave the following regression results-

$\beta = 0.687$ and $\alpha = 1.885$
$R^2 = 0.998$
$r = 0.994$
Durbin Watson test value = 0.968

Hence there was a highly significant and very close correlation between the two variables. A degree of autocorrelation was present as shown by the value of the Durbin Watson test value and a scatter plot of the regression residuals, but it was very small in absolute value terms. From this regression we deduced that the the value of stock held could be expressed usefully in terms of the corresponding usage values in the formula shown -

$$\text{stock held} = 1.885(\text{usage values})^{0.678} \quad \text{------------}12.10$$

Now we are able to deduce from the theory of the lognormal distribution (reproductive property (i) given in chapter three, section 3.2) that the standard deviation of inventory usage values, say (σ_{us}), will be related to the standard deviation of the stock held, say ($\sigma_{stk.held}$), by the relationship -

$$\sigma_{stk.held} = \sqrt{\beta^2 \sigma^2_{us.val.}} \quad \text{------------------}12.11$$

where β is the regression coefficient

This approach gave a value of $\sigma_{stk.held}$ of 1.90, which is in close agreement with the first measure of 1.96 obtained directly from the simulation. Now since the lognormal distribution of usage value is a very stable parameter year to year with only small changes in $\sigma_{us.val.}$ then the lognormal distribution of the stock held will likewise be very stable as measured by $\sigma_{stk.held}$. The very close fitting regression model 12.10 above is the most effective way to measure $\sigma_{stk.held}$ for the average stock held. Now from this measure of inventory concentration and the theory presented in section 11.8 of the previous chapter we can determine the correct proportions of investment value in chosen percentile item groupings. The appropriate part of the theory, due to Lockyer (1982), (repeated here from chapter 11 for continuity), is as follows :

If y_x represents the activity of interest above which x% of the items occur, the proportion of the total activity measure represented by these x% of the items is $\phi(z^*\{y_x\})$, where $z^*\{y_x\}$

can be shown to be $z\{y_x\}$*-log*$_e\rho$*. Thus the proportion of the total activity measure represented by the top x% of the items is a function only of the standard ratio* ρ*.* [where $\rho = e^{\sigma}$ and σ is the standard deviation of the appropriate lognormal variate]

Now for example, if we arbitrarily choose decile groupings of inventory items in rank order of investment value in stock from our 200 item simulation of appendix five, then the proportion of total investment in each decile group are as shown in table 12.1 below.

table 12.1

Activity in top 20% versus standard ratio

percent items	normal ordinate (z)	z-lnρ	activity percentage
top 10%	1.28	-0.62	73.24
2nd 10%	0.84	-1.06	12.30
3rd 10%	0.52	-1.38	6.08
4th 10%	0.25	-1.65	3.43
5th 10%	0.00	-1.90	2.08
6th 10%	-0.25	-2.15	1.29
7th 10%	-0.52	-2.42	0.80
8th 10%	-0.84	-2.74	0.47
9th 10%	-1.28	-3.18	0.21
10th 10%			0.10

We can readily see the disproportionality of the item range from this tabulation. The top 10% of items account for 73% of the total monetary investment in the inventory, whereas the bottom 10% of items accounts for only 0.1% of the value. Now these proportions are the theoretical values that we would expect to see as they are effectively derived from the chosen formula for determining cycle stocks and safety stocks, which in turn determine the theoretical values of the '*stock held*' stocks, which in turn determine the theoretical values of the '*stock held*' for each item. The lognormal distribution of this variate then gives us the theoretical value of the lognormal shape parameter $\sigma_{stk.held}$ which can be measured from a simulation, or from the relationships given in formulas 12.10 and 12.11 above.

Now if we monitor the logarithms of the actual values of stock held for an appropriate sample of items and we find that the value of $\sigma_{stk.held}$ so measured is significantly different to the theoretical value then we can be sure that there has been a fundamental change in the profile of the inventory. For example; suppose that we find the actual value of $\sigma_{stk.held}$ to be 10% lower than the theoretical value , ie 1.71 instead of 1.90 then the proportionate change in the distribution of the total stock value amongst our decile groupings will be as shown in table 12.2 below.

table 12.2

Activity proportion for different σ values

σ =1.90	σ =1.71	
activity percentage	activity percentage	difference
66.64	73.24	-6.60
14.14	12.30	1.84
7.52	6.08	1.44
4.49	3.43	1.06
2.75	2.08	0.67
1.96	1.29	0.67
1.21	0.80	0.41
0.75	0.47	0.28
0.40	0.21	0.19
0.14	0.10	0.04

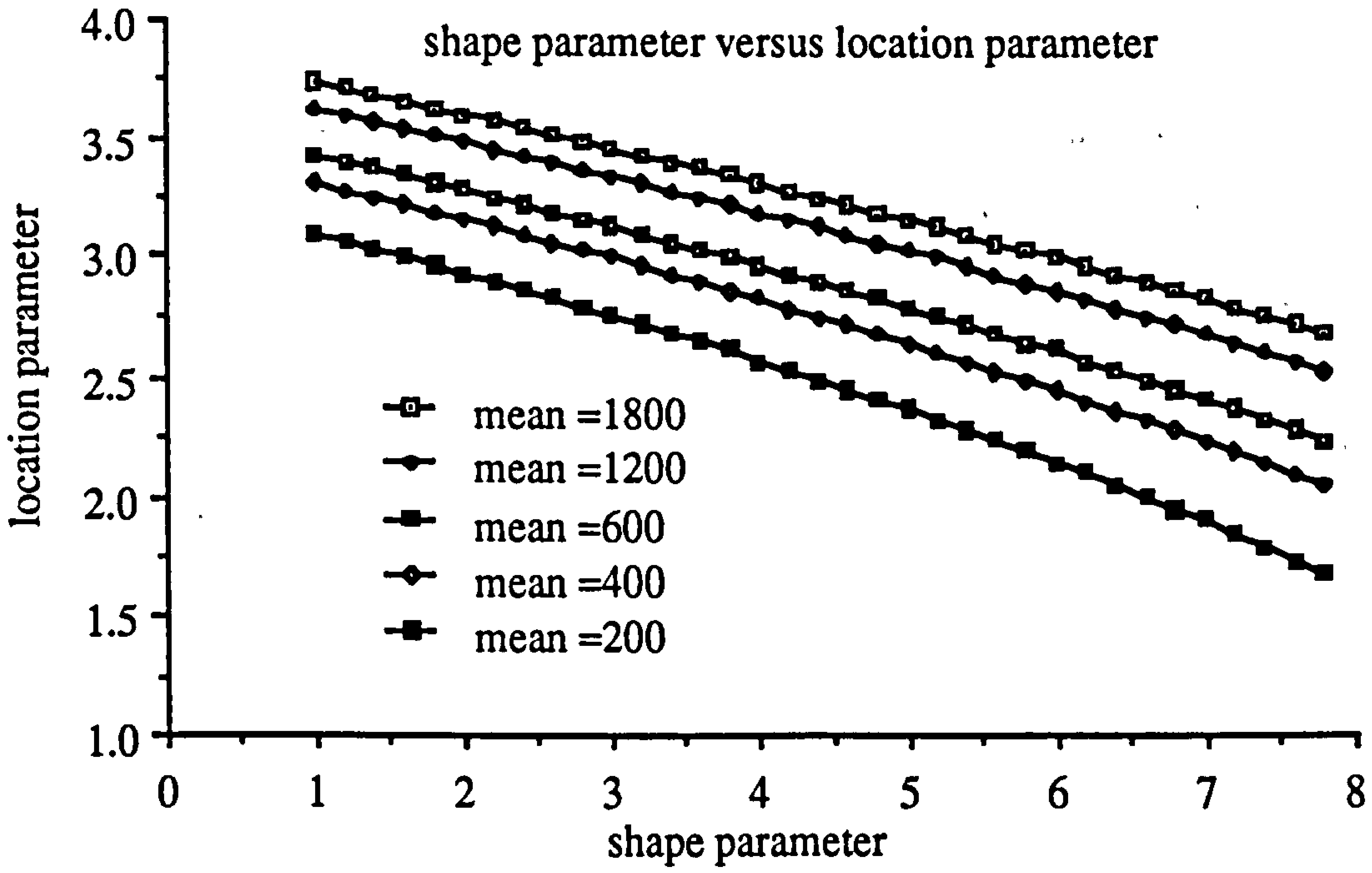
We can see from this table that there has been a 6.6% reduction in the investment in the top 10% items and the remaining groups have each remained unchanged then the difference in the σ values indicates a value shift within the range of items held in stock. Furthermore this value shift will have been to the detriment of the service level for the top 10% of items. Furthermore for the total stock investment to have remained unchanged then the location parameter μ must have increased to compensate for the shift in the value of σ because of the relationship between these parameter as shown by the mean value function of a lognormal variate shown in chapter three ie-

$$\alpha = e^{\mu + 1/2\sigma^2}$$

It is however, quite possible for the total investment (and hence also the mean value if 'n' the number of items remains fixed) to have increased in the above example and yet the value of σ may still have decreased as shown. In such cases to make sense of what is happening to the stock profile we make use of the location parameter of the lognormal distribution. In such cases there will have been a value shift in the stock profile as well as an overall increase in the money invested. It is possible to envisage many practical situations where various combinations of management action and undesirable manipulation of stock levels change the stock range profile and the amount invested. In general if σ remains fixed, but μ changes then there will have been an overall investment change, but the proportion of the value amongst the range will have remained constant. If σ changes but μ remains fixed then there will have been a shift in the proportion of value amongst the range.

Figure 12.6 shows just a few of the possible combinations between these parameter values for a range of fixed values of the mean α to show the overall effect on investment values in stock. The various values of the shape parameter σ should also be related the proportion of value distributed amongst the inventory range as indicated by table 12.1

figure 12.7



From all the foregoing we can see that by just having the theoretical values of σ and μ for the stock investment, and n the number of parts in the range, then we have a very effective and accurate method of monitoring the stock profile. By using the relationship between σ and μ and the function for the overall mean as previously shown ie :

$$\alpha = e^{\mu + 1/2\sigma^2}$$

then any shift in the value invested, or a change in inventory range concentration can be readily evaluated by comparing the theoretical parameter values to the actual measured values of σ and μ.

12.7 Conclusions.

We have seen in this chapter a number of possible novel applications of the lognormal distribution to particular aggregate inventory management issues and potential problem areas. We have only indicated some of the possibilities here and additional research is needed on empirical data to demonstrate the efficacy of such methods. We feel in particular that the novel methods described in the last section for monitoring stock levels and the dynamic changes that can occur in an inventory hold considerable promise as effective management tools for aggregate inventory management. These methods are the subject of continuing research by this author.

Research Conclusions

13.0 Introduction

In this final chapter we drawn together the diversity of findings and conclusions that have come out of the extensive analysis we have undertaken over many years, much of it based on data from DAF Trucks spare parts inventory. We also highlight where this research has provided added value to the knowledge that was already known from previously reported work. Finally we consider the areas where fruitful work could be continued. There are two parts to this; namely those areas which we feel naturally follow on from this work, and those areas which we have encountered on the way which, although not directly related to our quest, have nevertheless been revealed as areas worthy of investigation.

13.1 General findings

Our initial question in this research was effectively 'could the lognormal distribution represent the distribution of usage values for spare parts'. We have beyond any doubt shown this to be the case in the DAF system. Our additional empirical studies have shown that spare parts usage values in other organisations, which hold and sell spare parts to support capital equipment previously sold (or in own operational use), are also lognormal. These studies have confirmed in a European context what was claimed many years ago in the United States, and so the findings were in no sense surprising. We would in fact have been surprised to find that usage values were not lognormal for large spare parts inventories given the US based work of Brown and Heron. What we did not expect to find however, was such extremely close fits of usage values to lognormal distributions, especially those in the DAF Trucks case. This observation alone was strong evidence of a very stable underlying process at work governing the form of usage values. A further significant, although simple finding, was the fact that, with qualification, both spare parts prices and usage volumes were also lognormally distributed. Furthermore we have shown from the DAF case that the distributions for usage values have parameters that were very stable from year to year over the period 1975 to 1985. The shape parameter σ has been remarkably stable, whilst the location parameter μ showed a gradually increasing value from year to year, which is a reflection of the gradual increase in prices and to some extent a gradual increase in overall demand volumes from year to year.

We have also demonstrated in this work that usage values remain consistently lognormal over long periods of time; 11 years in DAF's case. Additionally we have shown that it is lognormal forms of spare parts prices and demand volumes that are essential to the lognormality of usage values. We have also been able to explain the underlying processes at work which account for this lognormality and this has been supported by simulation studies and model testing using the process of retroduction.

13.1(a) The distribution of prices.

The lognormal forms of the various price distributions that we extensively tested have only been reported as an empirical finding in this work. We have been unable to discover any testable stochastic reasons why parts prices should be lognormal in form. Throughout the entire analysis period DAF inventory prices remained consistently lognormal as we demonstrated by a variety of statistical tests. We also found spare prices to be lognormal in the other spares systems studied apart from the DAN air data. In this case we had a strong suspicion that the data for all items in the set was not from the same time period. The most likely stochastic process to explain lognormality of prices is a form of the Law of Proportionate Effect expressed in terms of the 'theory of breakage'. As an empirical finding we are very confident that spare parts prices for complex capital equipment are lognormal, but other than a rather simplistic physical analogy, using the theory of breakage we have no other explanations why.

13.1(b) The distribution of aggregate demand volumes

We have shown beyond doubt that for very short time periods the aggregate distribution of demand volumes in the DAF case is not lognormal, but was found to be consistently distributed as the combined LSD/NBD model. The distribution is LSD when only the positive demands are considered and NBD when the zero (but still live demand items) are also considered. We have also shown that the pattern of aggregate demand occasions, or incidence, in short periods is fully described by a simple Poisson model. These three models, Poisson demand occurrence, LSD demand quantity and overall NBD demand quantity, are the necessary and complete conditions that satisfy the Afwedson Poisson process of aggregate item behaviour. Furthermore we saw this process to be consistently in operation in the DAF case from 1978 through to 1986, although we only reported full year data up to 1985 as the last year. [It was not verified for the period 1975 to 1977 because period demand data was not available]. We also saw the same process to be in operation in the DAN Air data, although we were not able to test it so extensively as in the DAF case. Our definition of short (or single) period in the DAF systems was four weeks, whereas in the DAN Air case we had to consider one year. We attribute this difference of degree to be due to the fact that in DAN Air the volumes demanded overall across all parts were so much lower than those in the DAF case. The important point is however, that although the time scale between the two systems was different the underlying process was the same.

The second major finding in our work was the fact that the short period aggregate demand volumes could be cumulated period by period and the overall distribution that was obtained was consistently lognormal. The convergence in the DAF case required something in the order of nine periods (some nine months) before we could be sure that lognormality was obtained, whereas in the DAN Air case nearly four years were required. Furthermore we found that once the system had obtained a lognormal form this remained the stable long run distribution of the system. This was in the face of the findings of Ijiri and Simon who maintained that the Yule distribution is to be expected from systems driven by the Law of Proportionate Effect and where item entries and exits occur in the system. What must be borne in mind is that the work of these two authors was based largely on the distribution of firm sizes in specified industries. We can certainly regard the number of firms in such industries as being small compared with say the number of different parts in an inventory range. Hence it is very likely that the proportion of firm entries and exits to these industries were large compared to the number of firms in each corresponding industry. In such cases it is conceivable that Ijiri and Simon did observe long term effects that we have not seen in our spare parts systems, where the entries and exits (per annum) have been comparatively small in relation to the total size of the parts range. (There was something in the order of 300 parts per year net change over the period 1978 to 1986 for an inventory range varying from 9,000 to 12,000 active parts). We can only conclude here that we are not necessarily in disagreement with Ijiri and Simon, because there may well be much wider boundary conditions of entries and exits that would permit the attainment of the Yule distribution, but these were certainly not observable in the systems we studied.

The Afwedson process model we have applied to aggregate demand is in affect a Poisson compound distribution, with the LSD being the compounding function that generates the overall NBD of aggregate demand behaviour. This is equivalent to the compounding models that have been applied to single item demands, but in our work applied to heterogeneous populations. Also the Afwedson model as applied to spare parts aggregate demand is analogous to the early research work on heterogeneous populations presented or discussed in the Biometrics literature principally by Fisher (1943), Anscombe (1950) and Quenouille (1949). Interestingly, and as a side issue, we found strong evidence that the period by period demand for many individual items in the DAF inventory were also demand compounded and that the NBD model fitted our data sets extremely well. We also found that the Stuttering Poisson (sP) model fitted the data very closely. In fact from the limited comparisons we carried out it was very difficult to differentiate between these two distributions.

Our simulation studies were conducted to verify and support the empirical findings and to provide additional support to our proposition that the cumulation, period by period, of an Afwedson model of short term aggregate demand yields a lognormal distribution as the stable long run model of the system. This was a very positive outcome and we were able to demonstrate a strong empirical relationship between the starting LSD parameter 'q' of the model and the value of the lognormal shape parameter σ when a stable lognormal model was obtained.

We also showed by simulation that the convergence of short period aggregate LSD/NBD demand is independent of the variance of the demand streams. Simple Poisson, NBD and high variance NBD demand streams all produced lognormal models with means and variances very close to those of the empirical distributions. We did not use the Stuttering Poisson distribution as the demand stream model because of the computational difficulties with large values of the variate being simulated. We are however, confident that an sP model would also have produced a lognormal distribution because of its Poisson character and a form that is very close to the NBD. We conclude therefore that the aggregate lognormal distribution of demand volumes is dependent on the parameters of the starting LSD/NBD aggregate distribution, but independent of the variance of individual demand streams, providing the underlying character is Poisson.

In the model testing of chapter eight we were able to verify that it is the Law of Proportionate Effect that drives the convergence to lognormality, although there was some evidence to suggest that the law may not be operating in its most stringent form. In reality this is of academic interest only, as far as our quest for a deeper understanding of the aggregate demand behaviour is concerned, but it is nonetheless a significant finding that has not been reported before in the context of inventory systems. What is of significant value concerning the convergence is the speed or time it takes to achieve a stable lognormal distribution. This is related to the general level of aggregate demand volumes. It appears that the higher is the general level of aggregate demand volumes then the faster is the convergence to lognormality.

When we looked at aggregate demand occasions in the DAF case we found evidence of mixing taking place in the long run, because the long run empirical aggregate demand occasions distribution could be fitted to a Gamma distribution and the value of the variance was greater than the mean. In the short run there was no evidence of this with single period demand occasions being described adequately by the simple Poisson model. The increased variance could only come from long run mixing processes. We conclude from this that whilst the evidence for Afwedson model was very strong there was evidence that Poisson mixing is also in operation. One process does not preclude the other. Theoretically either could exist separately, or both could co-exist in the system. From a practical perspective this finding has little value because from an inventory management point of view we are far more interested in demand quantities and values, both in the short term and long term, rather than demand occasion or incidence.

13.1 (c) Acceptance of the research hypotheses

All the findings and conclusions in this chapter summarise original work that has not been reported previously in the literature. These conclusions also fully support our working hypotheses of chapter five (iv) and (v) and the more refined hypotheses of chapter seven (a) though to (e). They are listed again below for convenience. These hypotheses were developed from the theoretical reviews of chapters four and five, the empirical analysis of chapter six and the new theory development of chapter seven.

(iv) In the long run aggregate inventory item usage rates are lognormally distributed as the stable long run equilibrium distribution. The convergence of usage rates to lognormality is governed by the Law of Proportionate Effect.

(v) Furthermore, as usage values are the product of item prices and item demand volumes, then these two factors are also lognormally distributed. This is providing that the period over which the demand is measured is sufficiently long for the process to have converged.

Both hypotheses (iv) and (v) have been proven correct from our work and results given chapter six.

(a) In the case of prices the inventory range must be large and complex in the sense that it must comprise many small value items in addition to very high value items as typically found in spare parts inventories for complex capital equipment such as commercial vehicles, aircraft, tractors etc. In the case of demand volumes the period must be comparatively long and it is a discrete form of the lognormal distribution that is attained as the stable long run distribution by the summation of short period demand volumes.

The phenomena of demand quantities being lognormal has been proved and we have shown that this is the result of the cumulation of short period aggregate demands. The item range complexity question is still somewhat tentative; we are not sure from our work just how limited the item range could become before the conditions for lognormality are no longer applicable. We are doubtful if an inventory comprising just a few hundred items would achieve lognormality of demand volumes; this is an area that requires additional research to verify any limiting lower boundary conditions that may exist.

(b) In comparatively short time periods the aggregate distribution of demand quantity its fully described and modelled by the Log Series distribution of R.A.Fisher. This distribution is itself a special case of the Negative Binomial distribution when the proportion of very low demands in the population is high. In the same time period the aggregate distribution of demand occasions is described by the simple Poisson process.

(c) The underlying stochastic process that explains the occurrence of the Log Series distribution of aggregate inventory item demand quantity is the 'Afwedson Compound Poisson Process' as previously developed and discussed.

(d) The Log Series distribution of aggregate demand will, if cumulated over successive time periods, gradually converge to a distribution, which is discrete and has all the characteristics of the integer form of the lognormal distribution known as the distribution of counts.

Hypotheses (b), (c) and (d) have all been proven from our empirical analysis of chapter six, the retroductive model testing and validation of chapter eight, and then further supported by our simulation work of chapter nine.

(e) The stochastic process that governs the convergence of demand volumes, and hence also of usage values, to the lognormal distribution of counts is the the Law of Proportionate Effect.

We have shown the Law of Proportionate Effect to be operating in both the DAF and DAN Air cases during the cumulation of demand volumes. However, because of a similar degree of serial correlation in the regression test in both cases we doubt that the law may be operating in its most stringent form.

13.2 Factors which control lognormal parameter values

One of the prime concerns of this research was to understand the underlying basis for lognormality of inventory usage values. This was pursued on the premise that achieving a deeper understanding of the processes and mechanisms at work would provide additional opportunities for aggregate inventory management decision making and control tools. From our foregoing work and conclusions we are able to develop a scheme that explains those factors that govern the parameters of the lognormal distribution of usage values. As we have already noted both in this chapter and earlier ones it is the shape parameter in particular that is of prime interest to inventory management, although a consideration of what controls the shape parameter 'σ' also indicates what controls 'μ' the location parameter.

The respective contributions to the parameters of the usage value distribution comes equally from the constituent price and volume distributions as can be seen from the functions for the lognormal mean and variance first shown in chapter three. If we let α and β be the mean and variance respectively of the usage value distribution then we can write the function as follows :

$$\text{distribution mean } \alpha = e^{\mu+1/2\sigma^2}$$

$$\text{distribution variance } \beta^2 = e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

Where σ and μ are the parameters of the distribution.

Then we can write -

$$\alpha = e^{(\mu^2{}_{price}+\mu^2{}_{vol})+1/2(\sigma^2{}_{price}+\sigma^2{}_{vol})}$$

$$\beta^2 = e^{2(\mu_{price}+\mu_{vol})+(\sigma^2{}_{price}+\sigma^2{}_{vol})}\left(e^{(\sigma^2{}_{price}+\sigma^2{}_{vol})}-1\right)$$

this arises out of the basic theory of lognormal function given in chapter three because :-

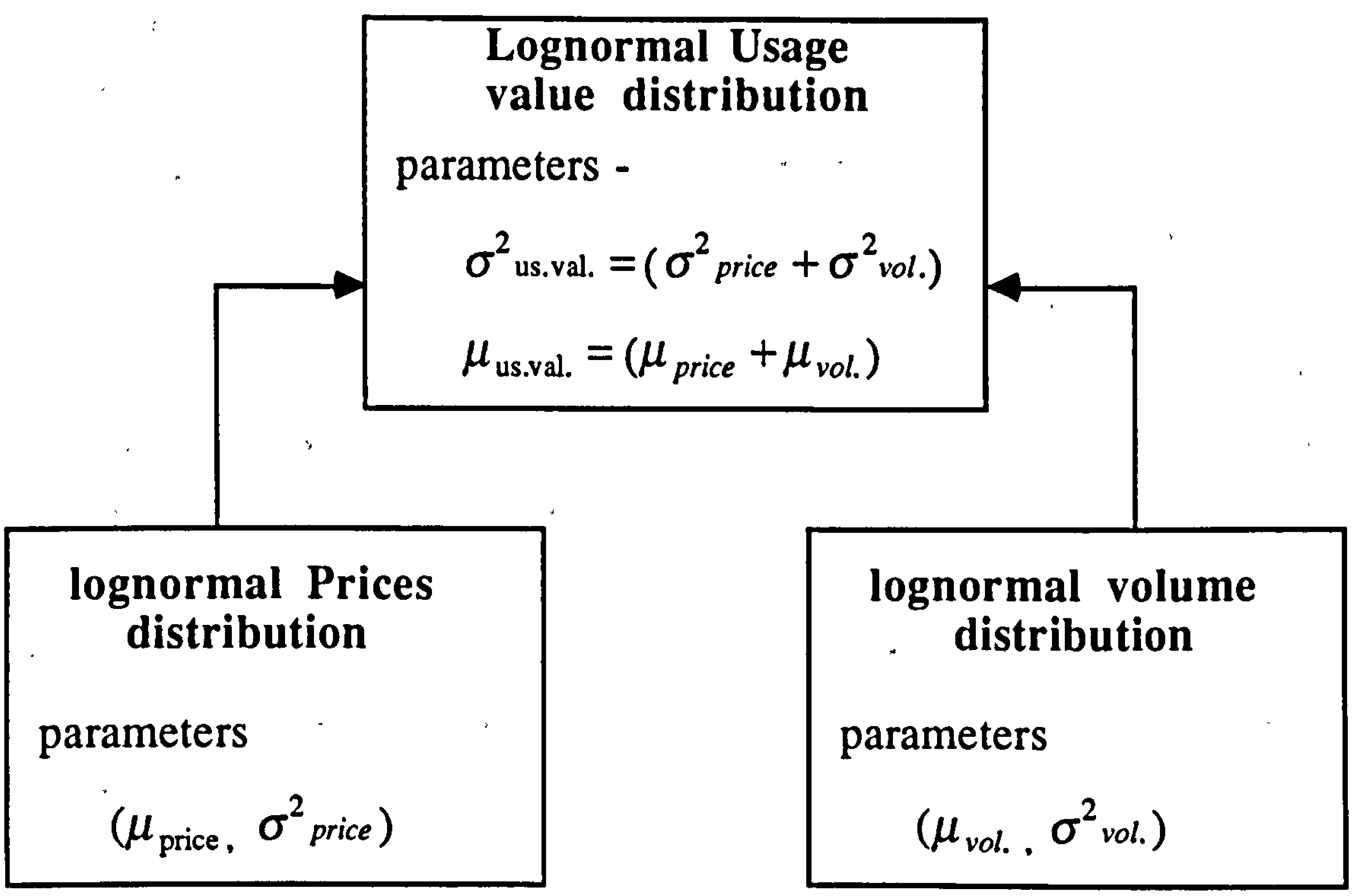
$$\sigma^2{}_{us.val.} = \sigma^2{}_{price} + \sigma^2{}_{vol.}$$

and $$\mu_{us.val.} = \mu_{price} + \mu_{vol.}$$

When the value of the shape parameter σ is of the order two then the contribution to the distribution mean is approximately equal between the two parameters. As a lognormal distribution becomes very skewed then the shape parameter has greater influence on the mean and variance of the distribution.

From this stage we can now develop an overall scheme showing those factors that explain the values of the parameter values of the lognormal usage value distribution. This scheme can also indicate what direction these values are likely to move in when underlying changes take place. As a first step we start the development as shown below in figure 13.1 below :

figure 13.1



This shows that the parameter values of the usage value distribution are governed equally by the values of the parameters of the separate price and volume distributions. However, each distribution is independent of the other and for example a change in the parameter values of the price distribution in no way affect the parameter values of the volume distribution.

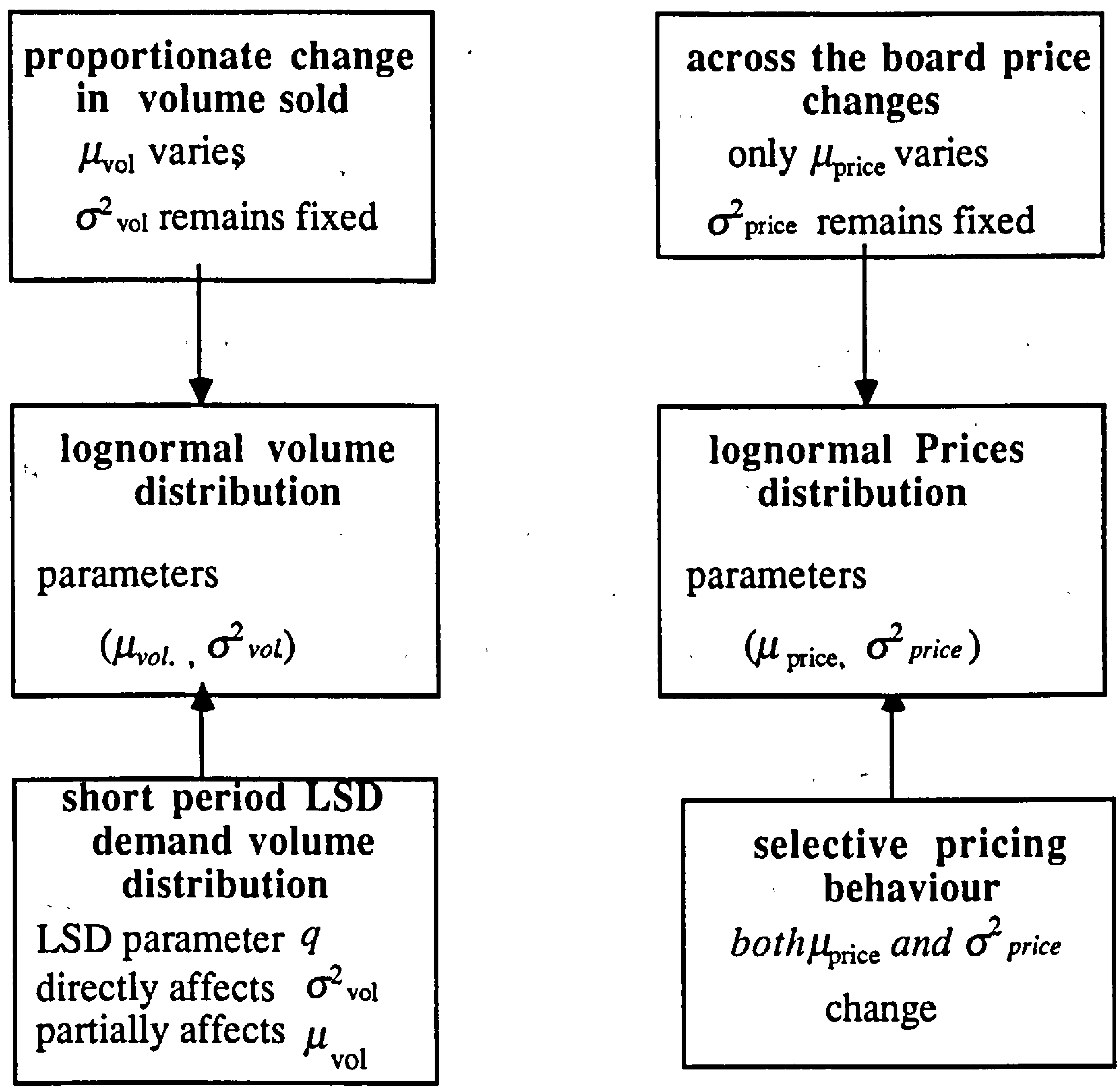
As a second stage we need to consider what factors influence and control the parameters values of the price distribution and the volume distribution. As we have already discussed the price distribution can only be specified as an empirical finding. However, unlike the volume distribution, which is determined ultimately by failure processes and customer service policies and behaviour which in turn determine demand volumes, the price distribution is a function of product structure and management pricing policy. The product structure is in a sense fixed, certainly in the short run, because a truck comprises a certain number and type of components each with a propensity to wear and failure. Management determine the value to be assigned to a given component part to reflect the costs of either the purchase of that part from outside suppliers, or the value (cost) accrued during its production. The part is then distributed down a distribution chain and at each stage markups are added. Hence managers do affect considerably what the price will be at each stage, that, when considered over several thousand part numbers, will affect to a significant degree the parameters of the lognormal distribution of prices. In particular the location parameter will strongly reflect the value added at each stage. The shape parameter will reflect management behaviour in terms of selective pricing across the range. A policy of equal markups on all parts at each stage will increase the location parameter, but the shape parameter will remain constant. Selective management pricing on certain parts will alter both the shape parameter σ and location parameter μ jointly. This was the basis of our reasoning in chapter 12 section 12.6. where we discussed the possible use of the parameters as a unique way to monitor inventory performance.

When we consider the volume distribution we know that the short period process that affects lognormality is the Afwedson process and the parameter 'q' of the LSD distribution relates to σ the lognormal shape parameter. As 'q' increases the so too does σ, but not linearly. The LSD parameter 'q' is determined ultimately by the wear and failure process of individual parts and the servicing behaviour of the truck operators. It is a measure of the aggregate profile of demand volumes in very short time periods. At the distributor level '*q*' will, to a degree, also be affected by the short run stock replenishment decisions of the dealers.

As a next stage in our scheme development we show the principal factors that affect price and volume distribution parameters in figure 13.2 shown on the next page. Figure 13.1 and 13.2 indicate the principal factors that ultimately affect the overall parameter values of the lognormal distribution of usage values. The fact that we have observed considerable stability in the parameter values in the DAF system over the period 1975 to 1985 can be interpreted with the help of the above two figures. As far as the overall shape parameter of usage values are concerned we observed very little change in the value of this parameter over the period, yet when we examine the factors that are principally responsible for its value we see it is controlled principally by σ_{prices} and $\sigma_{volumes}$. What has happened over the period is that there have been small and counter balancing changes in both parameters year to year. As $\sigma_{volumes}$ has decreased so σ_{prices} has increased and the overall affect has been very little change in the value of $\sigma_{us.values}$. This is not to say that $\sigma_{us.values}$ would always be very stable. For example, if management at DAF Trucks decided upon a policy of intensive selective pricing on substantial numbers of parts, in closely related price groups, then the value of σ_{prices} would change significantly.

This change could be an increase or a decrease depending on which group of parts were affected by the policy. If there were selective price increases on high value group items then σ_{prices} would increase, ie. prices would become even more concentrated. Conversely if selective price increases were aimed at low value item groups then σ_{prices} would decrease. Furthermore we can reason that selective price decreases in corresponding groups would have the opposite effects to the foregoing.

figure 13.2



So far our models above assume a constant parts population in terms of the number of different items in the range. Another factor that could potentially affect the lognormal distribution parameter values in the longer term is the addition (and subtraction) of active part numbers to the parts range. What the overall effect turns out to be depends on two factors, the prices of the added part numbers and their sales volume. In general the addition of high price high volume parts would increase both price concentration and demand volume concentration. The addition of low value low demand volume parts would have the opposite effect. High price but low volume parts would increase price concentration, but decrease volume concentration, and low price high volume will have the complementary effect. Over the period of study on the DAF systems the parts range has gradually increased year by year. The major trend has been to add high volume low value parts. Hence the overall effect on the lognormal parameter values has tended to be an increase in volume concentration and a decrease in price concentration.

In chapter four section 4.4 we discussed the work of Hart and Prais(1956) and Hart (1957) on industry size and concentration. Hart in particular considered the effect of entries and exits on the variance of the equilibrium distributions and formulated an equation (eq. 4.4) that related the industry firm size variance to the mean and variance of the survivors and the mean and variance of the new entrants. They concluded that new entrants decrease the variance whilst exists increase the variance. It is possible to rationalise this with firms in industries because the level of business once enjoyed by those firms that leave the industry will be shared by the remaining firms. Conversely new firms will compete for existing business with existing firms, and so reduce industry concentration. Unfortunately the same similar reasoning cannot be applied to new parts entering a spare parts range, the situation is far too complex to draw any comparatively simplified assumptions. We can only recognise empirically the general direction of change that entries and exits will have as discussed in our schemes above.

13.3 Aggregate Inventory standards

As previously discussed the use of the lognormal distribution to set aggregate inventory standards is due to the pioneering work of RG Brown (1959) who then used the basic principle of using the parameters of the usage value distribution to set inventory decision rules. Brown's work was followed by a period when the lognormal distribution received a modest amount of attention in the inventory literature and amongst the more prominent authors were Brown (op cit) and (1963,1963a, 1967), Heron (1968,1974, 1976,1978 and 1981) and to as lesser extent Schary and Howard (1970 and 1971), and Bestwick and Lockyer (1982), as we discussed in chapter three. According to Heron, and others (eg Schary and Howard) IBM utilised the basic theory in their distribution software system 'IBM Wholesale Impact', although this author has not had the opportunity to see this system or its documentation. Apart from IBM no other reported commercial applications has been referenced in the literature. From around 1983 onwards very little has been published on the application of the distribution to inventory issues. In fact one might conclude that the general opinion might have been that a valuable technique has been taken as far as it could be in terms of practical applications. Furthermore it might have been argued that with the advent of faster and smaller computers and increasingly flexible software then the need to set inventory standards using lognormal aggregate methods was no longer needed because one could just as easily run through the entire inventory range with whatever calculation was needed. This might well have become true, accept for the fact that we have considered spare parts inventories in this research and these have item ranges that are often very large.

In general spare parts inventories are the largest and most complex of all inventories that one can encounter. DAF Holland with just 60,000 part numbers is really quite modest in its range of items. Ford UK carry over 1 million different part numbers. Within the automotive industry, the construction equipment industry and the aircraft industry, hundreds of thousands of part numbers in a range are the order of the day. We argue that in inventory systems similar to these situations the use of the lognormal distribution to set and evaluate aggregate inventory standards is still a most valuable tool to use as the methodology is very accurate and cost effective compared to complete enumerations; and certainly superior in accuracy terms to audits by sampling.

However, our concern here is not to try and justify such use, we have taken the theory a stage further and our findings and models have opened new areas of application. Because spare parts usage volumes are lognormally distributed, subject to the time period, and so too are parts prices, then a number of other inventory factors that derive from these are also lognormally distributed. We have seen from chapter 12 that the average volume held in stock will be lognormal if calculated on the basis of functions that can be expressed directly in terms of the sales volume. As parts prices are lognormal then so too will stock values be lognormal. From these facts we have also seen that individual turnover rates across the inventory range are lognormally distributed, and the individual variation in turnover rate from item to item is as a consequence very wide. We have argued and shown in chapter 12 that because of this phenomena the use of the classical aggregate measures of inventory performance are not truly reflective of what is really happening in the inventory. Lognormal distributions are almost always very highly skewed and in such cases a simple average measure can be very misleading. Hence the broad based aggregate ratios such as 'stock turnover rates', 'day sales in stock', and 'stock to sales' when calculated in the classical way are not very helpful to Operational management. If an average ratio is to be used then it should be based on the calculation of the average of the individual ratios. This is a much truer representation of the behaviour of the range of items. We feel that the inventory factors and relationships we have shown and discussed in chapter 12 can be used to develop a range of new tools for aggregate inventory performance measurements not possible with previous knowledge. We have not developed such tools in this work, that was never the intention, but we hope this current work will enable others to build on these ideas.

13.4 The Integration of Knowledge

In the execution of this research we have had recourse to call on existing knowledge from a variety of fields of enquiry. In consequence we can claim to have integrated concepts and theories from several knowledge areas. Indeed the development of the theory underpinning our empirical observations, and model development has only been possible by the integration of theory and empirical observations from several diverse fields of enquiry. We have drawn on work from fields as diverse as biometrics, consumer purchase theory, theoretical statistics, applied statistics, industrial economics and inventory theory and practice.

From the theory of the distribution of firm sizes we have drawn important concepts from the stochastic processes that govern both the growth of economic variates, such as firm sizes, and the form of the equilibrium distributions obtained. This area has also provided valuable ideas regarding the concept of concentration of economic variates. These ideas were discussed in chapter four and to a lesser extent in chapter eleven.

Our early search for various applications of the lognormal distribution lead us, via Geoff Easton's (1975) work, to the literature on consumer purchase theory. Here we found valuable theory on repeat purchases and the NBD/LSD models of Andrew Ehrenberg. Ehrenberg's Poisson Gamma model proved to be of direct value in our modelling work and enabled us to consider the important distinction between purchase incidence(or occasion) and purchase quantity. Ehrenberg and Easton also provided valuable stochastic models that we have been able to adapt to our work in chapter five.

The field of applied statistics has provided a considerable amount of theory that has underpinned our research development. In particular the theory of recurrent event processes in terms of compound and mixing Poisson processes proved to be of immeasurable value. The application of compound models in inventory theory provided a starting point to see what bridges could be made across the gap from single item behaviour to the behaviour of a whole class of items in aggregate. As we have seen it is the Afwedson model of Poisson compounding that provides the underlying explanation of the form of aggregate demand volumes. However, we could not have made the necessary deductions and inferences regarding the application of the Afwedson model to heterogeneous spare parts systems without the evidence of heterogeneous item behaviour drawn from work in the Biometrics field. This was a fortuitous finding, and in particular the work of Fisher(1943), and Jones and Mollison (1948), and then also the work of Anscombe (1950) and Quenouille (1949), discussed in chapter seven, proved to be a turning point in our work. This gave us considerable confidence to move forward along the path to heterogeneous compounding for aggregate spare parts parts demand quantity in short time periods.

13.5 Further Research

There are a number of areas where we feel that further research is justified. We have been unable in this work to examine the underlying processes at work which account for parts prices being lognormally distributed. From an inventory management point of view it is sufficient to accept it as a proved empirical fact. However, from a marketing point of view we think there should be merit in trying to understand this phenomena at a more fundamental level. The process may well have great utility in terms of pricing theory. We also feel there is mileage in the possibility of using the parameters of the price lognormal distribution to monitor competitor prices. It is always possible to watch competitor price moves item by item, but we feel that watching them move in aggregate by the change in lognormal parameter values will give a deeper insight to the competitor's pricing behaviour not possible by other means especially if the product range is large.

All our work in this thesis has been focused on spare parts systems. All the underlying theory rests on the fact that in such systems, and in the environment that generates the demand for such items, the underlying demand character is Poisson in nature. There are however other product fields that should yield similar market demand and product profile characteristics and hence quite possibly the same underlying theory will hold. An area of promise are food products inventories of the type typically encountered in supermarkets. These inventories are often comparatively large and of the order of several thousand of items. Furthermore the demand generation is from a large independently buying population. The requirements for an underlying Poisson process may well exist in such environments. This author had the opportunity at once time to examine inventory problems in a confectionery wholesale warehouse operation (Trebor Group Distribution). Although it was not part of the project inventory usage values were subjected to lognormal analysis, and they were found to fit very well to a lognormal distribution! We were not too surprised by this finding but it was not pursued any further. If the theory we have developed in this thesis could be applied to such product fields it could ultimately have greater utility than in the spare parts case. For one thing it is very likely that the lognormal distribution will fit usage values of such inventories in very short time periods simply because of the typically very high demand volumes involved in such systems. Far higher than in our DAF systems for example. We recommend that the lognormal theory be investigated in other product fields and we are confident that all we have developed in this work can be applied to a number of product fields providing the underlying demand process is Poisson.

In the case of fast moving inventories of the food merchandise type we feel there should be significant merit in examining the demand volumes very carefully to see if the Yule distribution is a more accurate description of the aggregate demand. Such inventories are likely to display a vastly greater degree of item entries and exits to the system than we have seen in spare parts systems. Also the item range will be much more limited in size than spare parts, being typically of the order of three or four thousand in an average supermarket operation. Hence the overall effect of system entries and exits may well be significantly larger than we have observed in spares systems.

At one point in the general investigation carried out by this author on DAF data we attempted to test the hypothesis that the shape parameter of the lognormal distribution would be closely related to a measure of the ageing vehicle fleet in the field. What DAF executives referred to as the 'Truck Park'. Intuitively it seemed that as the vehicle fleet aged, as it did steadily from 1975 until 1986, then this phenomena should be reflected in some way in the changing value of the shape parameter of the lognormal distribution of usage values. The rational for this was on the basis of the fact that as trucks age they generally have a greater call on the more expensive parts, eg engines, gear boxes, differentials etc. If such a process is taking place then it is argued that it will have an effect on the profile of the usage value distribution. If such a relationship can be established then the changing average truck age may be used as an indicator of the future shape parameter of the associated lognormal distribution. Preliminary analysis failed to reveal any meaningful measure of truck age which we could relate to the changing value of σ year by year. However, this author contends that such a relationship should exist and additional research in this area is worth pursuing.

Although it was not a direct focus of our work we had need to consider the nature of period demands for individual spare parts. As we discussed in chapter five and then again from an empirical point of view in appendix one we concluded that compound Poisson models are the appropriate ones to use for spare parts fixed interval demand, although for low demand volumes the simple Poisson seems appropriate. In our limited work in this area of investigation the case for the NBD and the Stuttering Poisson distributions was extremely strong. Furthermore we feel these two distributions should be examined closely to ascertain the conditions under which they apply. We were not able to differentiate between them, but from an inventory theory point of view it would be a significant value to know the conditions in which they apply, the circumstances where they give similar results and the conditions when they diverge. Sherbrooke (1968) has done a limited amount of work here, but not nearly enough to be really helpful. According to Sherbrooke (op cit) it was claimed that for variance to mean ratios (q) up to a value of three, the two distributions give almost identical results (ie probabilities of the variate x), but begin to diverge thereafter. We found that for values of q up to seven the probabilities were very close, but the NBD produces a slightly longer tail. This fact alone would will have importance for setting reorder levels. We also found certain errors in Sherbooke's tabulated probability values that puts some of his results in question. Ultimately from an inventory management point of view it is the mean and variance of demand in the replenishment lead time that is important for setting inventory parameters, but one must know the nature of the appropriate models in fixed intervals first and then combine these with an appropriate choice of lead time distribution.

We also consider there is merit in researching the possibility of the so called Stuttering Erlang process as an appropriate model to represent spare parts demand between retail and wholesale points in a distribution chain. The degree of demand regularity introduced due to the retail restocking effects may exhibit underlying behaviour that is better modelled by an Erlang process than by a Poisson process.

Determination of the Distribution of DAF period demands

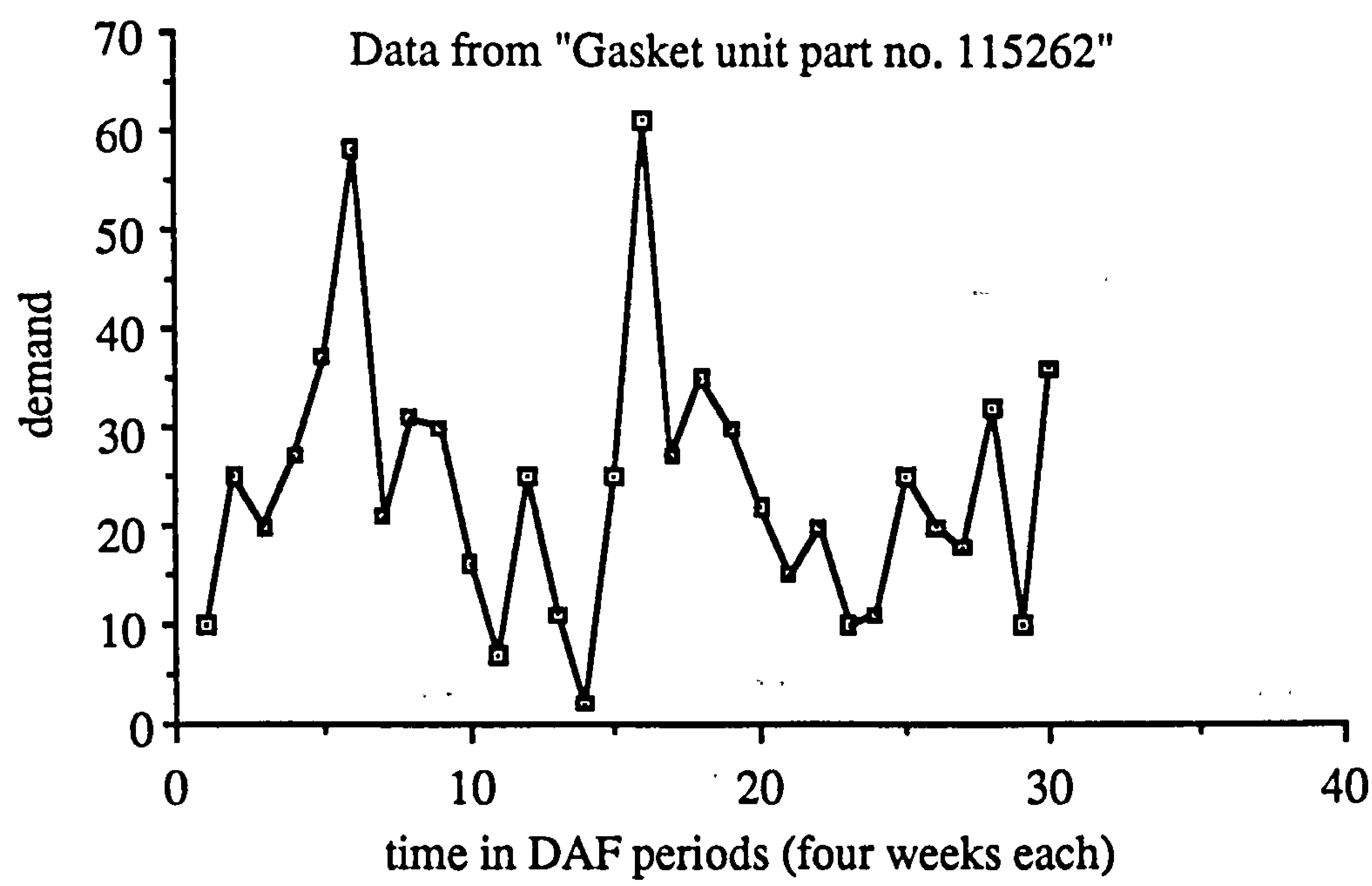
1.0 Introduction

In this section we show the calculations undertaken to demonstrate that the period demands in the DAF case are almost certainly compound Poisson distributed with either the Negative binomial distribution (NBD) or the Stuttering Poisson (sP) distribution as very strong candidates. Four parts were chosen randomly after checking that the demand over a 30 period time span showed no significant trend in each case. The mean and variance of demand of each empirical demand stream were used to calculate the probabilities of the corresponding NBD and Stuttering Poisson distributions, which in turn were used to generate theoretical demands. The empirical demands were then tested against the theoretical values for goodness of fit.

1.1 Gasket set

The 30 demand values for the Gasket set (part number 115262) are shown in the graph below, where it can be seen that no significant trend is evident in the chosen data set. This was an important characteristic because any significant trend in the data would interfere with an objective test against an appropriate (and stationary) NBD or sP model.

figure A1.1



The parameter values of the empirical data were as follows-

Mean $m = 23.900$
variance $s^2 = 176.079$
Variance to mean ratio 'q' =7.369

These values were used to determine the form of the appropriate NBD. Firstly we determined $P(x=0)$ for the NBD from -

$$P(x=0)=\left(1+\frac{m}{k}\right)^{-k}$$

where $k = m/(q-1)$

Subsequent probabilities were calculated as shown in chapter eight using the recursive NBD relationship :-

$$P(x) = \left(\frac{\alpha}{1+\alpha}\right)\left(1 - \frac{\alpha - m}{\alpha x}\right)P_{x-1}$$

where $\alpha = m/k$

The form of the particular NBD so generated is shown in figure A1.2 below and table A1.2 shows the theoretical demand values generated from the NBD, together with the relative frequencies and cumulative relative frequencies. The frequencies for the sP distribution were calculated using the recursion formula that was shown in chapter five, namely :-

$$R_n = \frac{(1-\rho)\lambda t}{n} \sum_{j=1}^{j=m} j\rho^{j-1} R_{n-j}$$

To use this model we determined the values of the parameters ρ and λ by the following relationships given by Ward (1978) :-

$$\rho = \frac{(q-1)}{(q+1)}$$

$$\lambda = \frac{2m}{(q+1)}$$

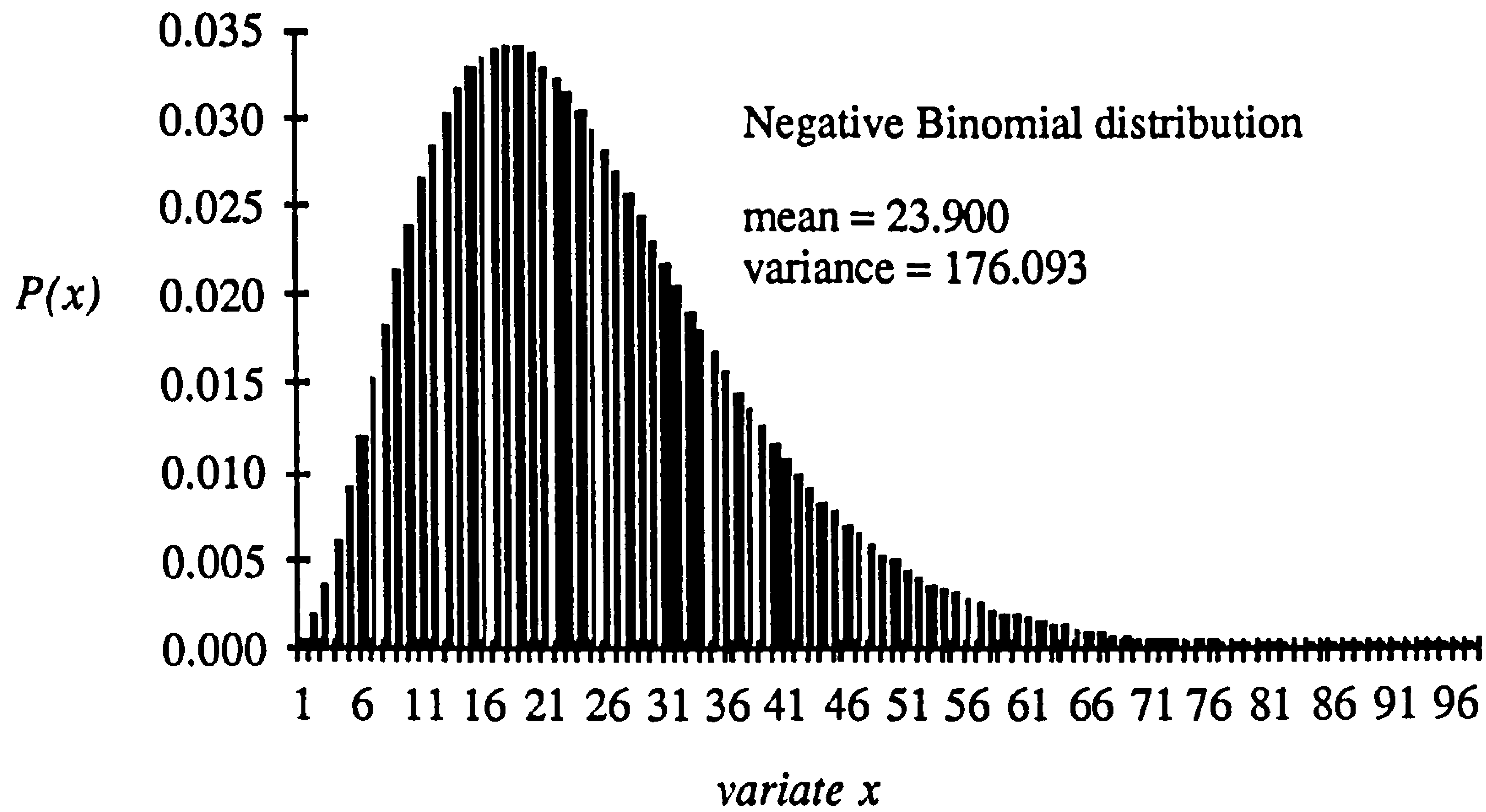
where 'm' is the distribution mean and 'q' is the variance to mean ratio.

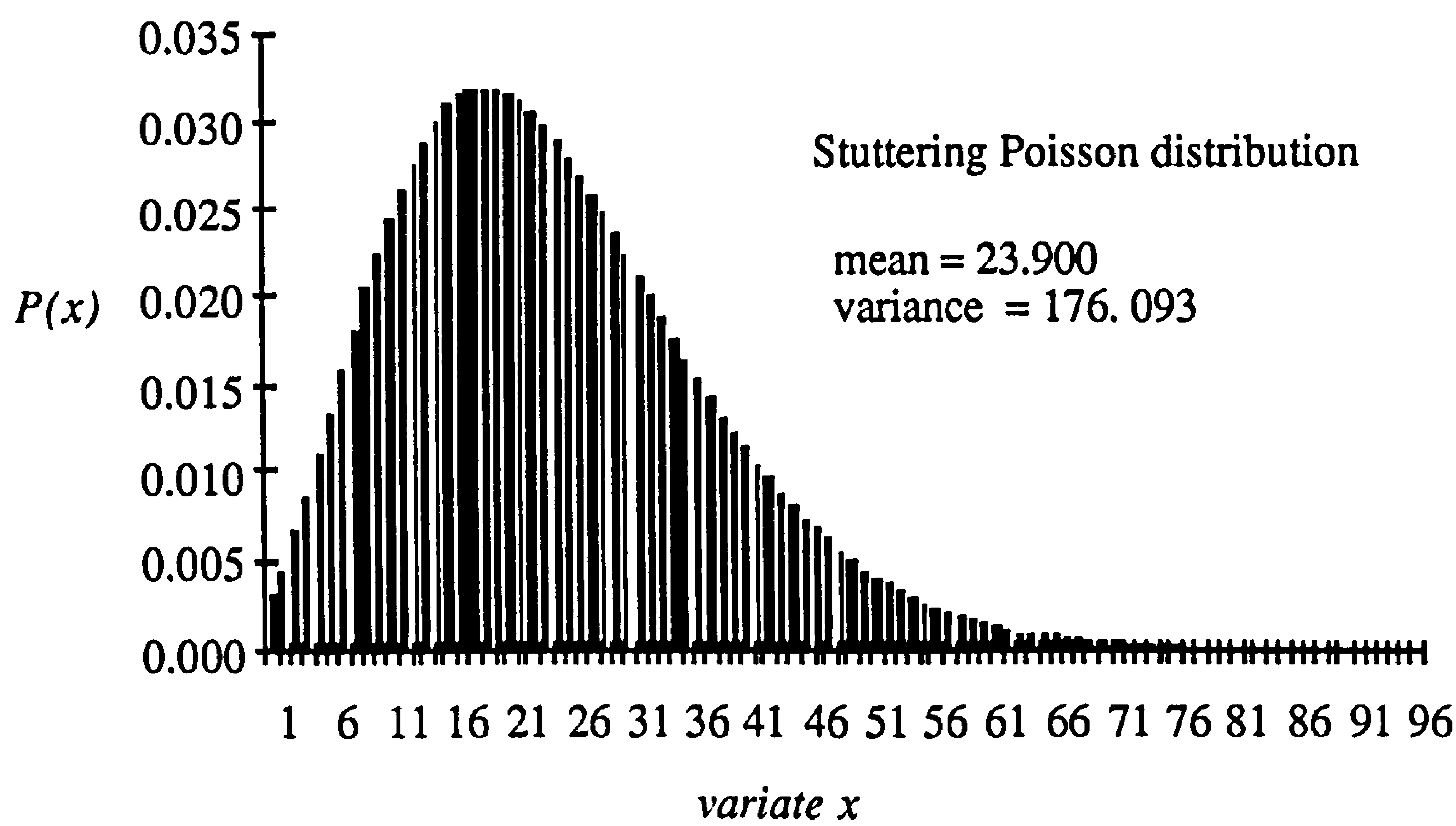
An Excel spreadsheet model was developed to calculate the sP frequencies for values up to 100. Because the number of calculations involved are proportional to the value of 'x' the sP distribution soon becomes very laborious to use, and would require a very high powered system for large values of 'x'. (An sP distribution with mean value equal to 100, or greater, would require main frame capacity. We used a pc system model based on an 'Excel' spreadsheet and this could cope with values from distributions with means up to 50 and comparatively long tails)

Table A1.1 shows the empirical gasket data compared to the corresponding sP distribution, whilst table A1.2 shows the same data compared to the NBD distribution.

In both cases of the NBD and the sP the cumulative frequencies were used to determine a goodness of fit using the Kolmogorov Smirnov test. It can be seen from tables A1.1 and A1.2 that the maximum Dn values for the difference between the two cumulative distributions in each test was very small at 0.0233 and 0.0300 respectively. These compare extremely favourable with the theoretical Kolmogorov Smirnov Dn test values of 0.24 and 0.27 respectively for a sample size of 30. (Dn values provided in Kendall - Advanced theory of Statistics, vol. five) Hence on this basis the empirical data could easily pass as either NBD or sP distributed; both are highly significant. If there is any difference at all, based on the KS test, then it is marginally in favour of the NBD, although one could not claim the difference to be significant; it is far too small and could be explained away due to sampling error.

figures A1.2 and A1.3





Figures A1.4 and A1.5 show the cumulative frequencies as line graphs from which can be seen the very close correspondence between the empirical data and the theoretical data generated from both the NBD and the sP distributions.

figure A1.4
Negative Binomial Distribution Test

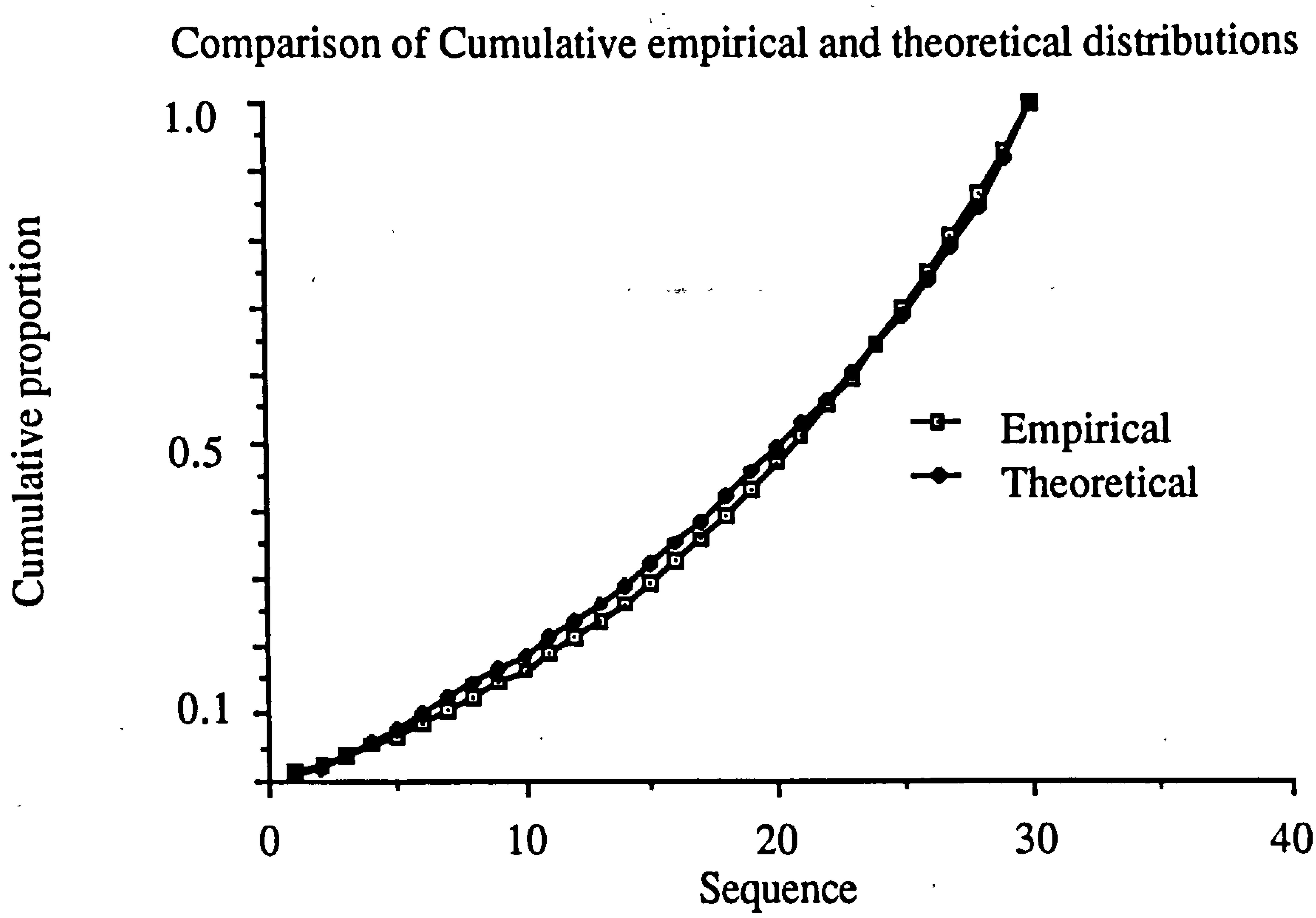
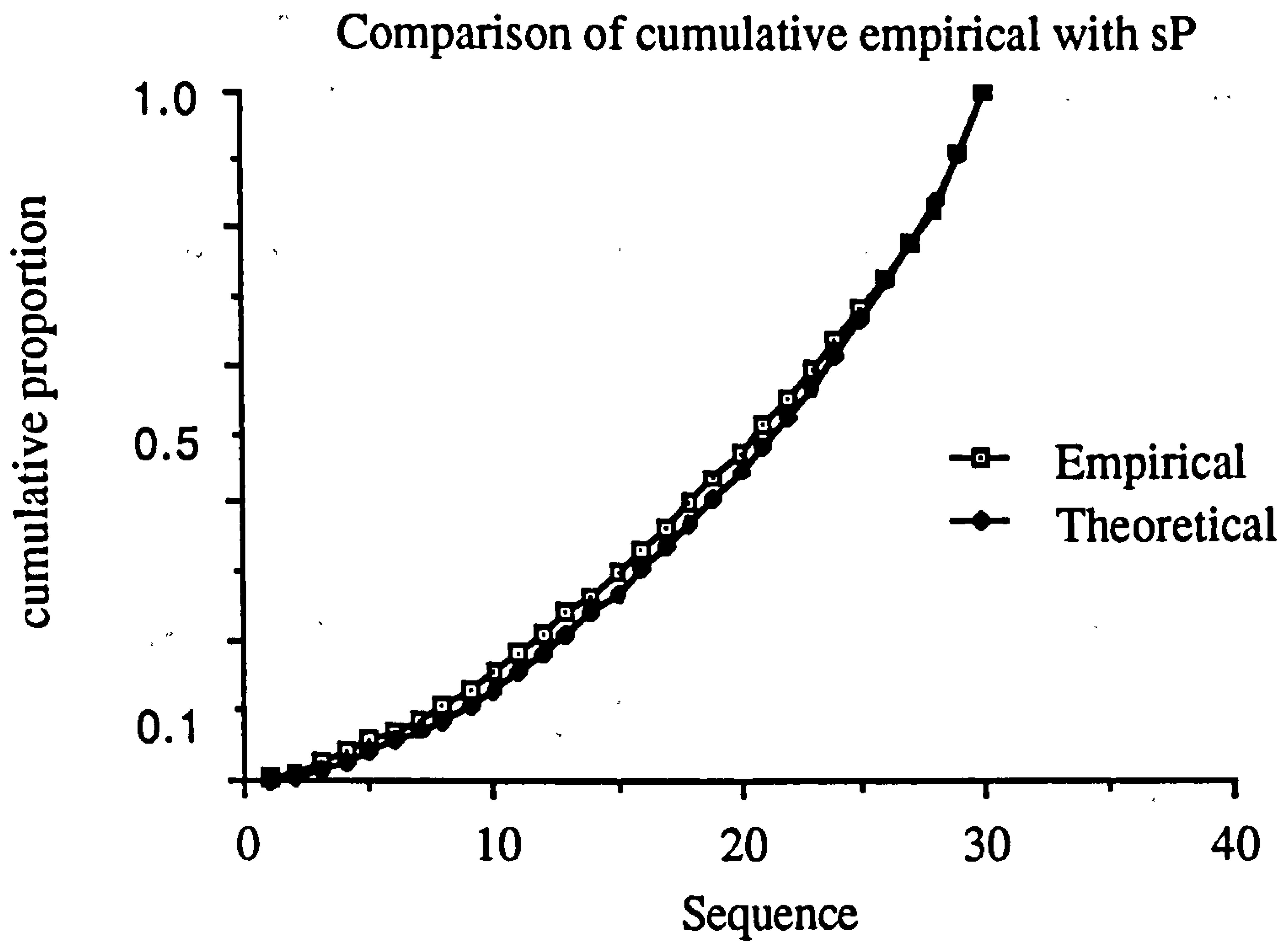


figure A1.5
Stuttering Poisson Distribution Test



From the above results we have very strong evidence to suggest that the period demands for the Gasket set are distributed according to a compound Poisson distribution and it could well be either as a Negative Binomial distribution or a Stuttering Poisson distribution. Figure A1.9 and A1.10 in section A1.3 below show that the two distributions are so close that it is very difficult to separate them in terms of frequencies.

A1.2 Additional parts

The three other parts that were subjected to the same NBD comparison analysis as the gasket set were-

Part No. 103820 'King Pin' - a wear out item
Part No. 229963 'Temperature indicator' -electrical item
Part No. R241787 'Starter motor' -expensive repairable part

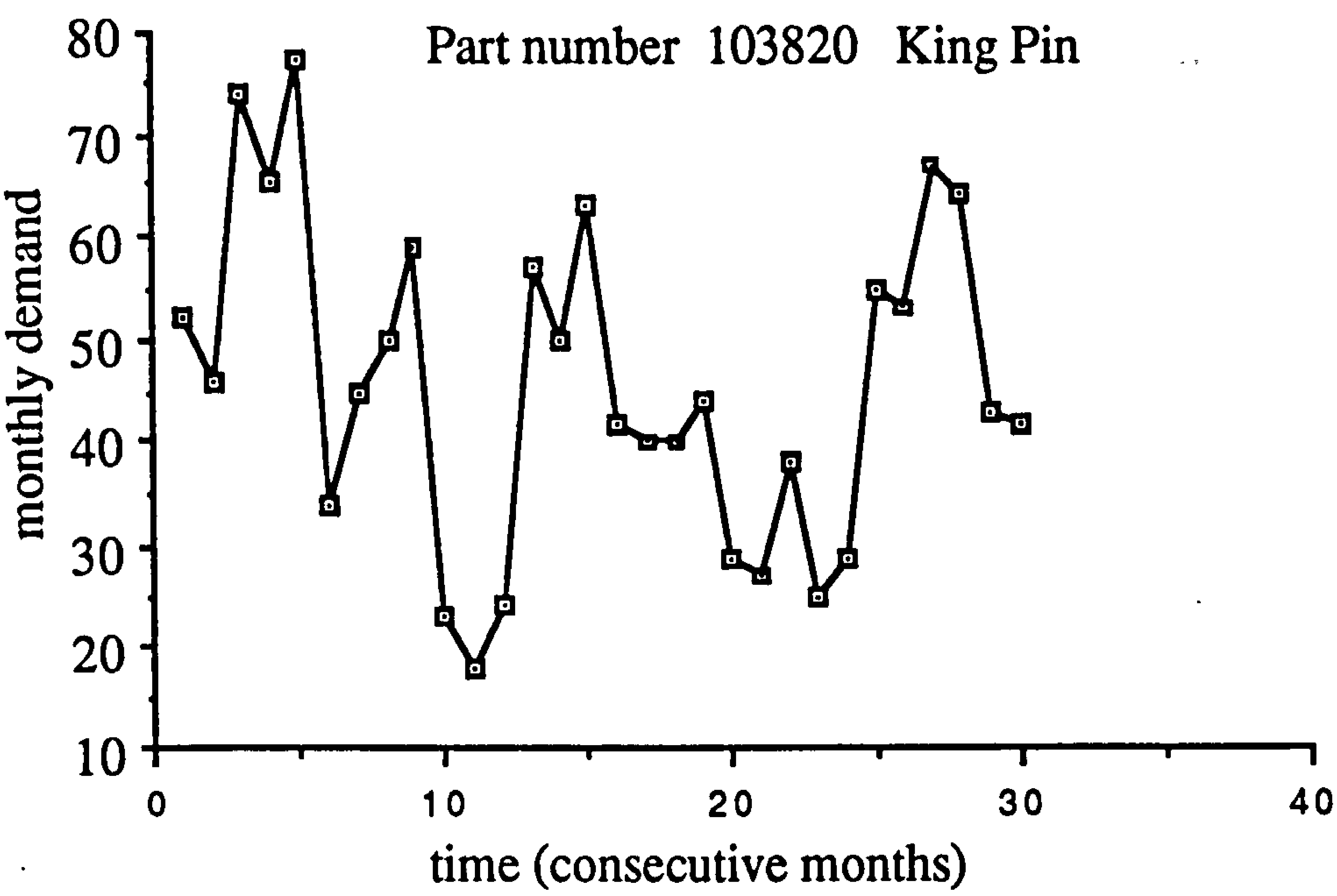
The analysis on each one produced very similar results to the Gasket set in terms of the highly significant Kolmogorov Smirnov test.

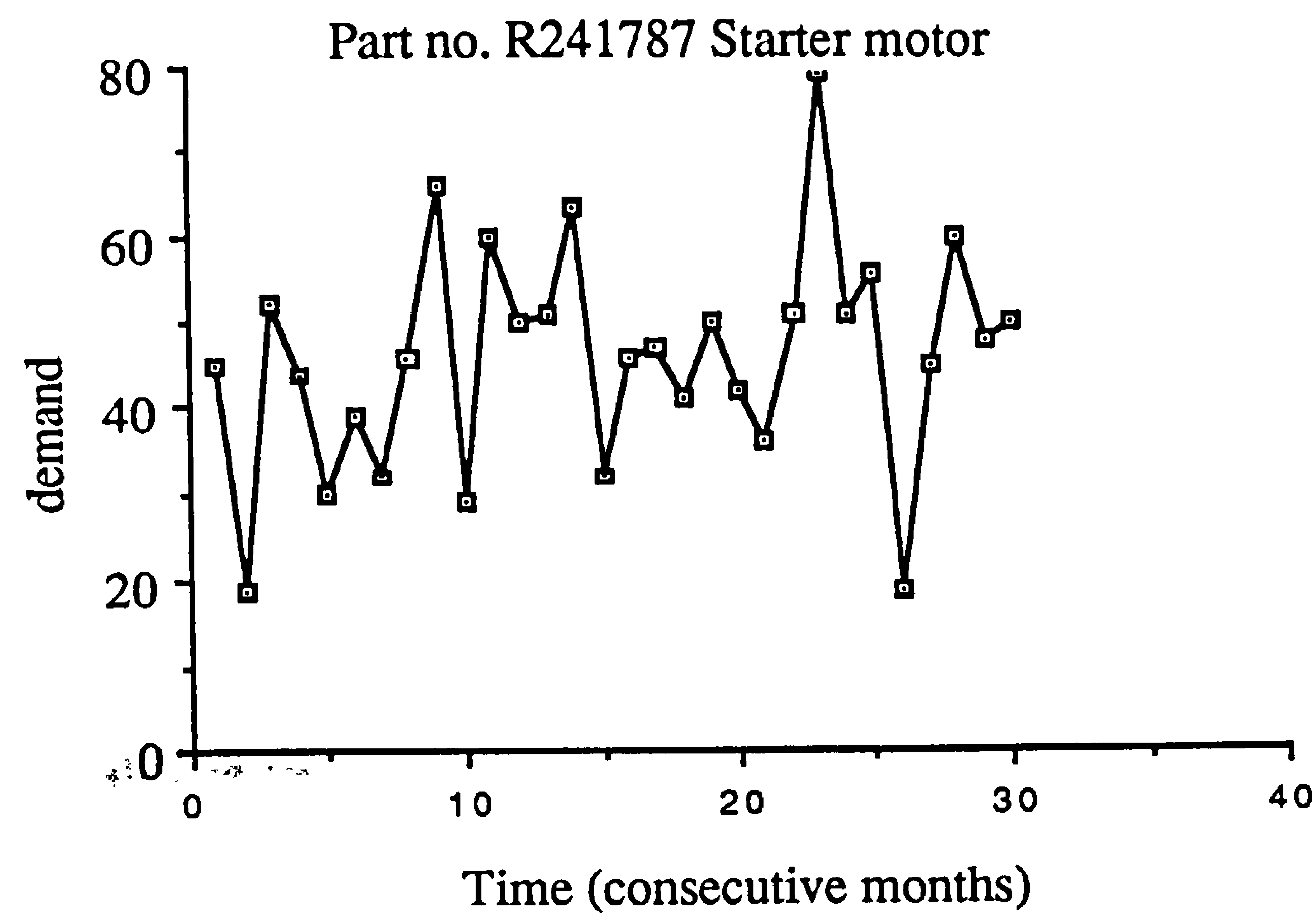
	Mean	variance	Max observed Dn
Part No. 103820	45.833	246.557	0.0255
Part No. 229963	11.533	33.292	0.0295
Part No. R241787	45.966	173.257	0.0226

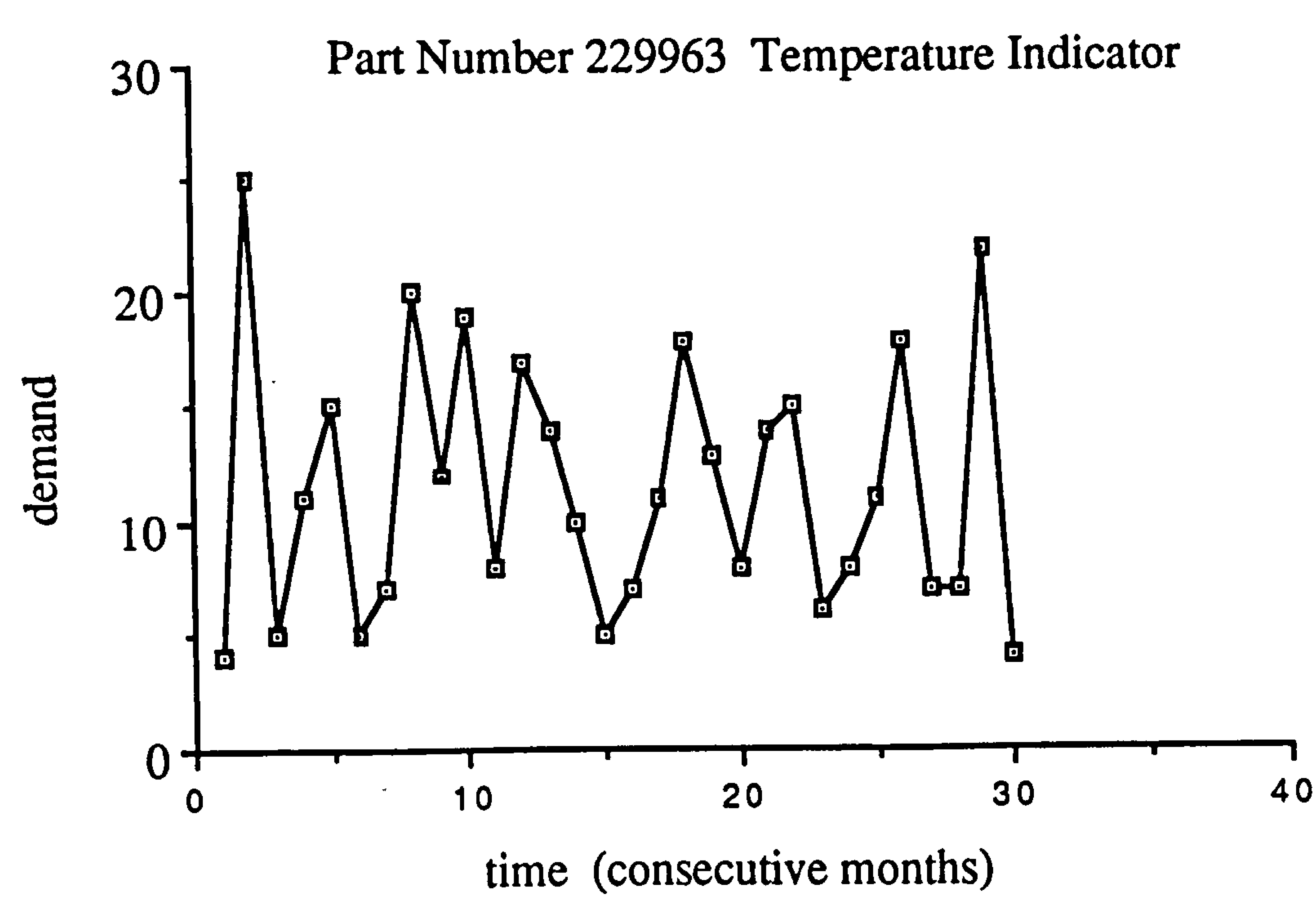
Each of the above Dn_{max} values were compared to the theoretical values of $Dn_{0.01}$ and $Dn_{0.05}$ at 0.27 and 0.24 respectively. Hence, as with the gasket set they are very significant results and are strongly indicative that the demands for these parts are distributed as compound distributions, that are very likely NBD; although we have not proved it to be any more efficient than the corresponding sP. This detailed analysis has been limited to just these four randomly chosen parts, but we are confident from the results that the NBD is a satisfactory distribution to model period demands for a wide variety of parts in the DAF case. Hence we were also confident that it was a valid choice to model demand streams in the simulation work of chapter nine.

The actual demand patterns for each of these three parts are shown in the following figures where it can be see that each had a reasonably stable demand pattern over the time span chosen. (A 30 period time span from early 1983 until 1985). It was considered better to choose parts that had this characteristic rather than try to remove any trend effects.

figures A1.6 to A1.8




Part no. R241787 Starter motor
80
60
40
20
0
demand
0
10
20
30
40
Time (consecutive months)

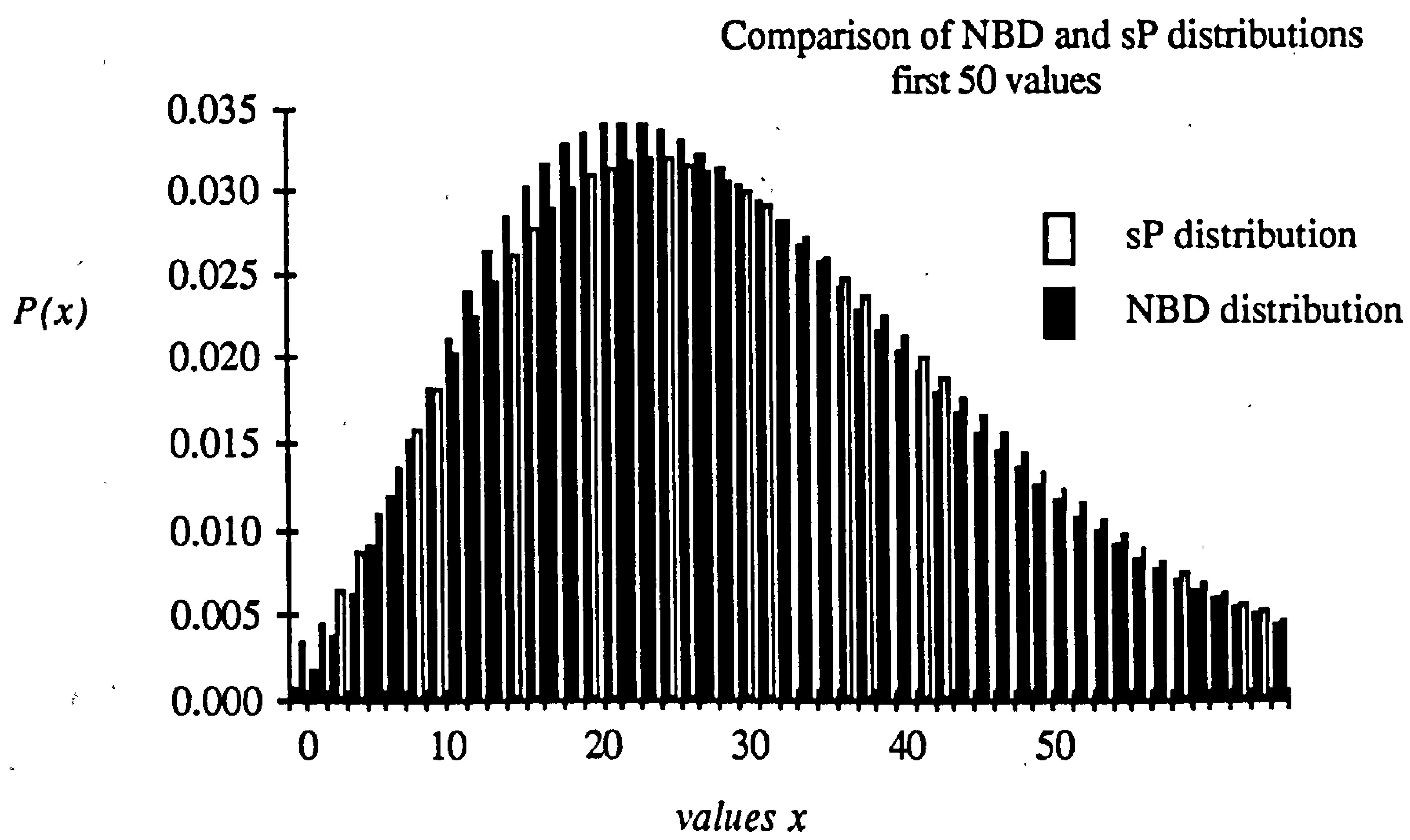

Part Number 229963 Temperature Indicator
30
20
10
0
demand
0
10
20
30
40
time (consecutive months)

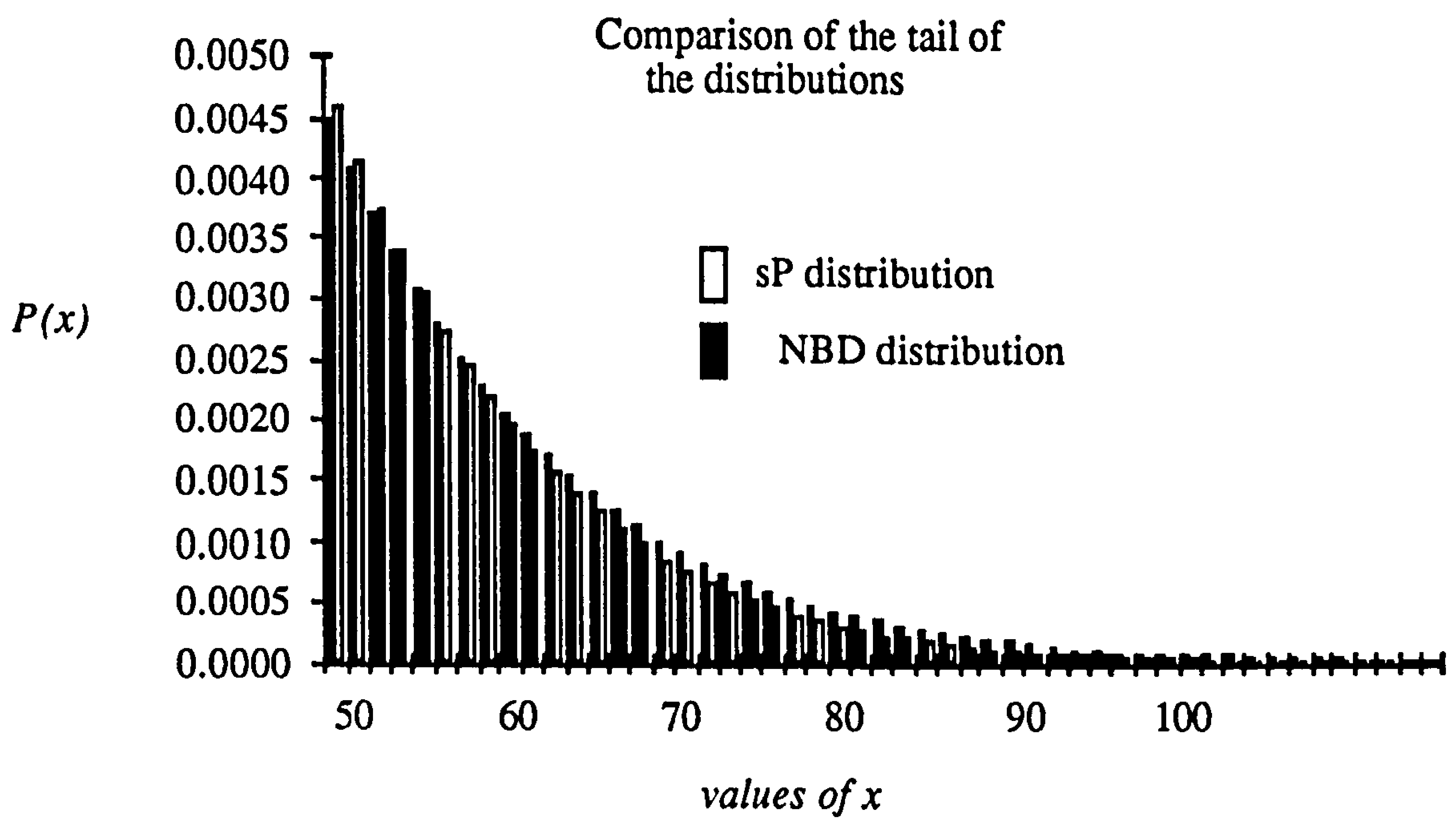
A1.4 Comparison of the NBD and sP distributions

In figures A1.9 and A1.10 we show a detailed comparison, frequency by frequency, of the NBD and sP distributions used in the gasket analysis. Whilst detailed comparisons of NBDs and sPs have no central role in our research it is of passing interest to see the comparison on this particular data set.

By examining the frequencies in the way shown it can be seen where the two distributions correspond and where they depart, from each other. It can also be seen that there are several cross-over points (three in fact). The frequencies in the long tail seem to correspond very well.

figure A1.9 and A1.10





A1.4 Conclusions

We can conclude form the work in this appendix that we are quite confident in regarding the period demands for individual items as being compounded and that the NBD provides a very good fit to the data. The Stuttering Poisson also provides a good fit over the ranges of parameter values we have explored. Certainly from the gasket set data the two distributions are very close indeed, even with a variance to mean ratio 'q' of 7.367. According to Sherbrooke(1968), in his limited work with the two distributions, he reported a gradual divergence in the frequencies as 'q' exceeded a value of three. In our work here 'q' was equal to 7 in the gasket data and there was a very close correspondence between the two distributions as shown by figures A1.9 and A1.10. This is clearly an area that deserves further research to establish more precisely the range and nature of the conditions over which the two distributions correspond and then diverge.

Appendix 1

Table A1.1

Gasket set data testing aginst the Stuttering Poisson distribution

	Empirical frequency	Empirical f(x)	Empirical F(x)	Theoretical frequency	Theoretical f'(x)	Theoretical F'(x)	K.Smirnov F(x)-F'(x)
	2	0.0028	0.0028	1	0.0015	0.0015	0.0013
	7	0.0098	0.0126	3	0.0044	0.0059	0.0067
	10	0.0139	0.0265	6	0.0088	0.0147	0.0118
	10	0.0139	0.0405	7	0.0102	0.0249	0.0155
	10	0.0139	0.0544	10	0.0146	0.0396	0.0148
	11	0.0153	0.0697	10	0.0146	0.0542	0.0155
	11	0.0153	0.0851	10	0.0146	0.0688	0.0162
	15	0.0209	0.1060	12	0.0176	0.0864	0.0196
	16	0.0223	0.1283	14	0.0205	0.1069	0.0214
	18	0.0251	0.1534	15	0.0220	0.1289	0.0245
	20	0.0279	0.1813	17	0.0249	0.1538	0.0276
	20	0.0279	0.2092	19	0.0278	0.1816	0.0276
	20	0.0279	0.2371	19	0.0278	0.2094	0.0277
	21	0.0293	0.2664	19	0.0278	0.2372	0.0292
	22	0.0307	0.2971	22	0.0322	0.2694	0.0276
	25	0.0349	0.3319	23	0.0337	0.3031	0.0288
	25	0.0349	0.3668	23	0.0337	0.3368	0.0300
	25	0.0349	0.4017	24	0.0351	0.3719	0.0298
	25	0.0349	0.4366	25	0.0366	0.4085	0.0280
	27	0.0377	0.4742	26	0.0381	0.4466	0.0276
	27	0.0377	0.5119	26	0.0381	0.4847	0.0272
	30	0.0418	0.5537	27	0.0395	0.5242	0.0295
	30	0.0418	0.5955	30	0.0439	0.5681	0.0274
	31	0.0432	0.6388	33	0.0483	0.6164	0.0223
	32	0.0446	0.6834	36	0.0527	0.6691	0.0143
	35	0.0488	0.7322	38	0.0556	0.7248	0.0074
	36	0.0502	0.7824	39	0.0571	0.7819	0.0006
	37	0.0516	0.8340	40	0.0586	0.8404	-0.0064
	58	0.0809	0.9149	47	0.0688	0.9093	0.0057
	61	0.0851	1.0000	62	0.0908	1.0000	0.0000
sum	717			683			
mean	23.9			22.76667			
stdev	13.27001			13.78326			
var	176.0931			189.9782			
var/mean	7.367912			8.344575			

Table A1.2

Gasket set data testing against the NBD distribution.

	Empirical frequency	Empirical f(x)	Empirical F(x)	Theoretical frequency	Theoretical f'(x)	Theoretical F'(x)	K.Smirnov F(x)-F'(x)
	2	0.0028	0.0028	5	0.0075	0.0075	-0.0047
	7	0.0098	0.0126	7	0.0105	0.0180	-0.0054
	10	0.0139	0.0265	8	0.0120	0.0299	-0.0034
	10	0.0139	0.0405	11	0.0164	0.0464	-0.0059
	10	0.0139	0.0544	11	0.0164	0.0628	-0.0084
	11	0.0153	0.0697	11	0.0164	0.0792	-0.0095
	11	0.0153	0.0851	12	0.0179	0.0972	-0.0121
	15	0.0209	0.1060	14	0.0209	0.1181	-0.0121
	16	0.0223	0.1283	16	0.0239	0.1420	-0.0137
	18	0.0251	0.1534	17	0.0254	0.1674	-0.0140
	20	0.0279	0.1813	18	0.0269	0.1943	-0.0130
	20	0.0279	0.2092	18	0.0269	0.2213	-0.0120
	20	0.0279	0.2371	21	0.0314	0.2526	-0.0155
	21	0.0293	0.2664	21	0.0314	0.2840	-0.0176
	22	0.0307	0.2971	22	0.0329	0.3169	-0.0198
	25	0.0349	0.3319	23	0.0344	0.3513	-0.0193
	25	0.0349	0.3668	23	0.0344	0.3857	-0.0189
	25	0.0349	0.4017	23	0.0344	0.4201	-0.0184
	25	0.0349	0.4366	24	0.0359	0.4559	-0.0194
	27	0.0377	0.4742	26	0.0389	0.4948	-0.0206
	27	0.0377	0.5119	27	0.0404	0.5352	-0.0233
	30	0.0418	0.5537	27	0.0404	0.5755	-0.0218
	30	0.0418	0.5955	27	0.0404	0.6159	-0.0203
	31	0.0432	0.6388	30	0.0448	0.6607	-0.0219
	32	0.0446	0.6834	30	0.0448	0.7056	-0.0221
	35	0.0488	0.7322	30	0.0448	0.7504	-0.0182
	36	0.0502	0.7824	37	0.0553	0.8057	-0.0233
	37	0.0516	0.8340	38	0.0568	0.8625	-0.0285
	58	0.0809	0.9149	44	0.0658	0.9283	-0.0133
	61	0.0851	1.0000	48	0.0717	1.0000	0.0000
sum	717			669			
mean	24			22			
stdev	13.27001			10.60302			
var	176.0931			112.4241			
var/mean	7.367912			5.041441			

Determination of Sample Sizes to Estimate Standard Deviations

1.0 General considerations

In much of the empirical analysis undertaken in this work we had to resort to taking samples, and hence the question of the sample size had to be given careful consideration. In the main our sampling needs were to determine the standard deviation, and hence variance, of logarithmically transformed data from demand volumes, demand prices, or usage values. Hence we were examining in most cases data that was normally distributed or closely normal. In most cases we used sample sizes of 200 and the rational for this is developed below.

From statistical theory, Spiegel (1961) and Kendall(1963), it is known that provided the parent population is approximately normally distributed, then the sampling distribution of 's' for large samples is normally distributed with mean s and standard deviation $s/\sqrt{2n}$. Hence we can assert with a probability of (1-α) that -

$$s - z_{\alpha/2}\frac{\sigma}{\sqrt{2n}} < \sigma < s + z_{\alpha/2}\frac{\sigma}{\sqrt{2n}}$$

Where $z_{\alpha/2}$ is the standard normal deviate at significance level α.

Now we can equate the confidence interval function to the required accuracy (β) as a percentage of 'σ' the true population standard deviation as follows -

$$z_{\alpha/2}\frac{s}{\sqrt{2n}} = \beta\%$$

Now for large samples we can replace 'σ' by 's' the sample standard deviation with only a small loss of accuracy, hence we can write :

$$z_{\alpha/2}\frac{s}{\sqrt{2n}}=\frac{s\beta}{100}$$

from which we can write -

$$\frac{z_{\alpha/2}}{\sqrt{2n}}=\beta$$

where β is now the accuracy required as a proportion of σ. After rearranging we can express the function in terms of n -

$$n=1/2\left[\frac{z_{\alpha/2}}{\beta}\right]^2$$

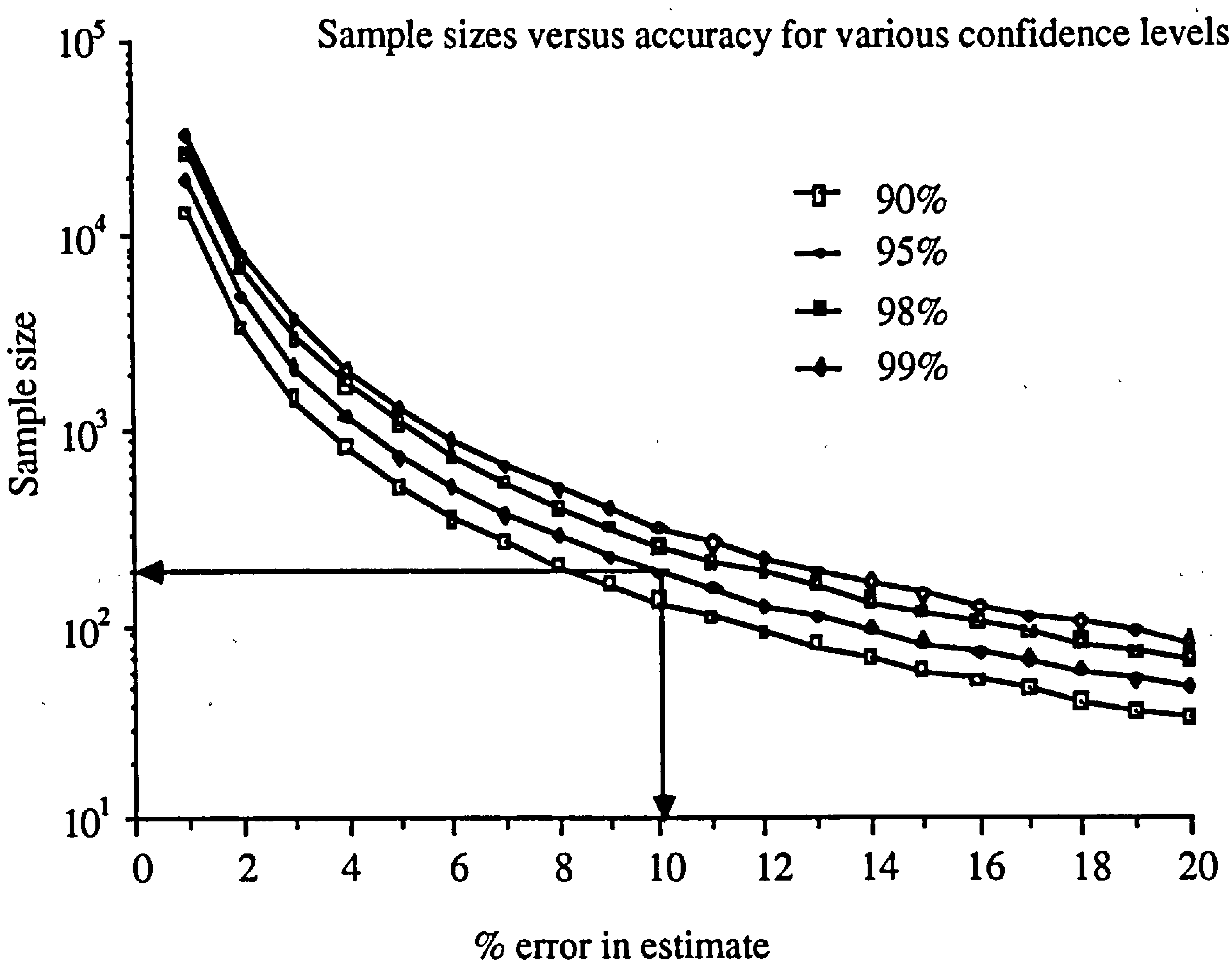
Thus for a 95% confidence level, which sets the normal ordinate at 1.96, and an accuracy of 10% we can determine n as follows :

$$n=1/2\left[\frac{1.96}{0.1}\right]^2$$

from which n is given as 192. In considering the trade off between sample size, required accuracy and confidence levels, in our work in previous chapters, we chose n to be 200, which was considered to be a reasonable trade off between sample size and accuracy. Thus it can be seen from the above development that when n=200 this gave a 95% confidence of obtaining estimates of σ within 10% of its true value.

Higher values of confidence, or smaller accuracy limits, demand very large values of n as can be seen from the following tabulation of n against the estimate accuracy β for different confidence levels.

Figure A2.1



It can be clearly seen from the above diagram that the sample size required begins to rise very fast for accuracy levels much below 10%, and at 5% and less the sampling requirements are really prohibitive.

A relationship between mean demand and standard deviation of demand

1.0 General considerations

In this appendix we verify the proposition that the mean demand of item i can be related to its standard deviation of demand by the general function -

$$\sigma = \alpha\left(\overline{x_i}\right)^{\beta}$$

where α and β are constants and σ is that standard deviation of demand for item i.

100 items were selected at random from a DAF parts history file and the mean and standard deviation were calculated from a 15 period time duration during 1984/1985. The regression of $\log_e$standard deviation against the $\log_e$mean produced the following results-

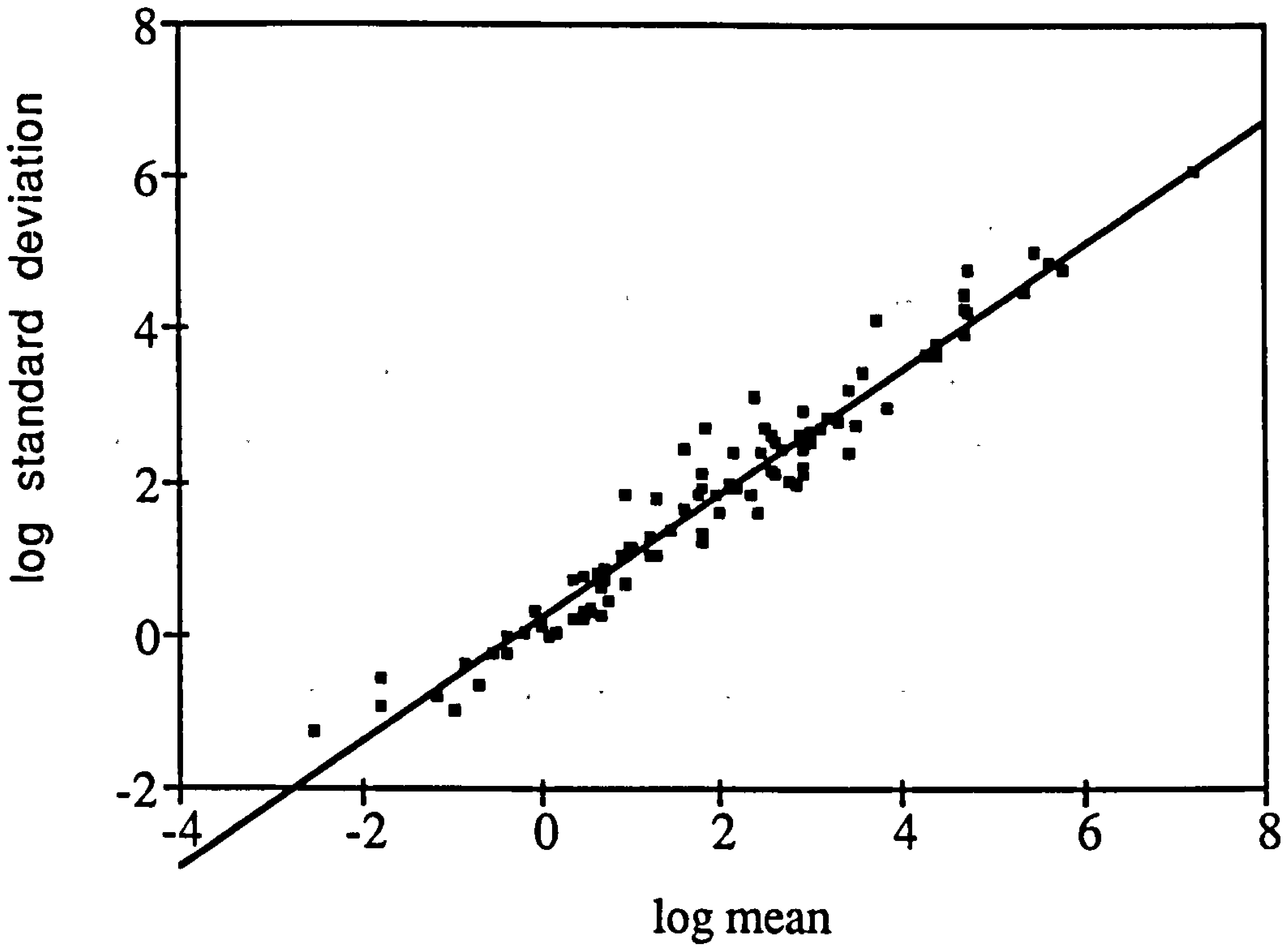
$$\log_e \sigma = 0.253 + 0.814 \log_e \overline{x}$$

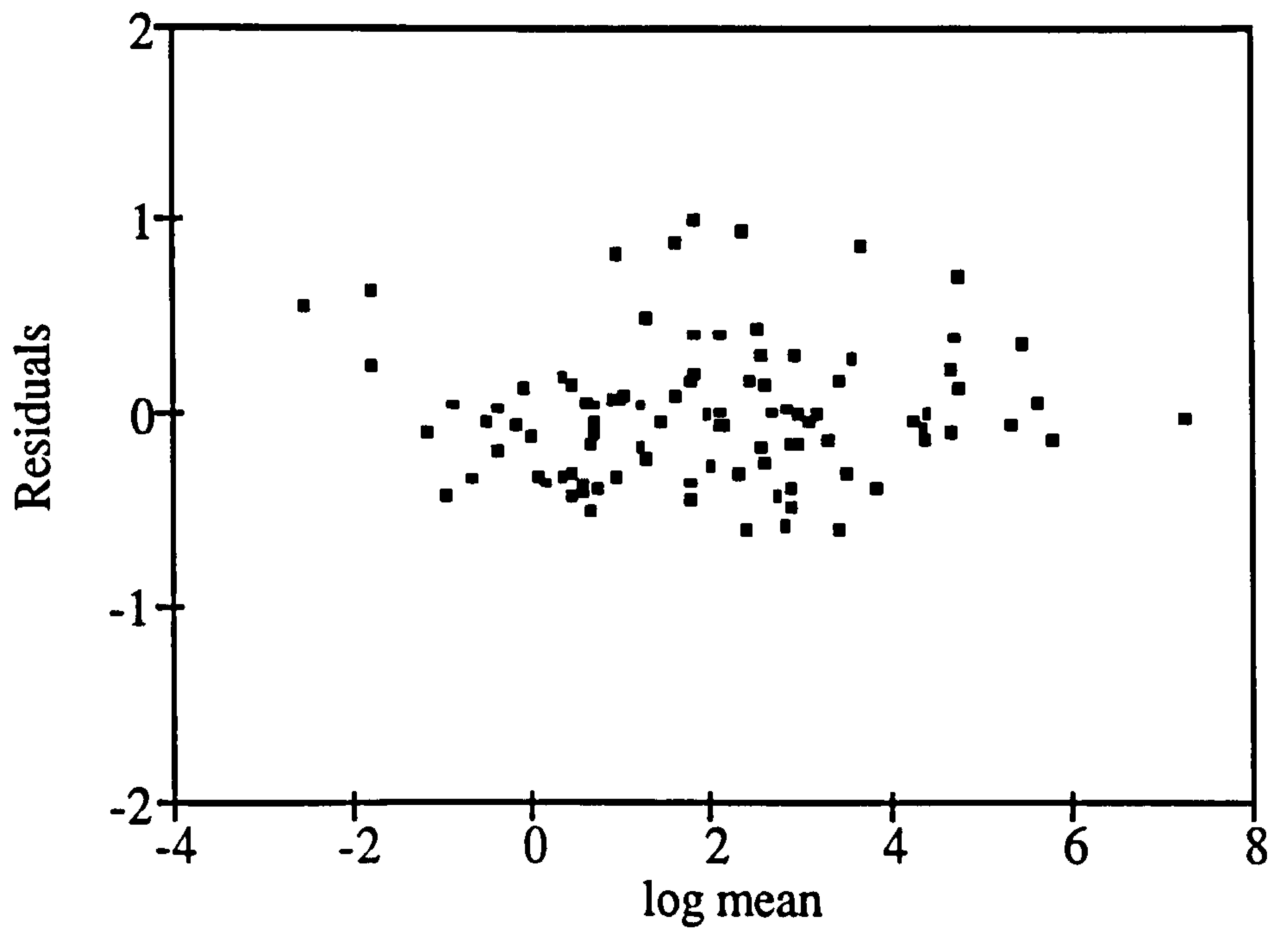
from which it was determined that α = 1.287 and therefore

$$\sigma = 1.287(\overline{x})^{0.814}$$

correlation coefficient r = 0.975
coefficient of determination $R^2 = 0.951$
standard error = 0.355
Durbin Watson test = 1.860

figure A3.1





It can be clearly seen from the above results that the standard deviation does indeed regress very closely the mean of demand, and we can be very confident in the function -

$$\sigma = 1.287(\bar{x})^{0.814}$$

to be a very god representation of the standard deviation of demand in place of σ. The Durbin Watson test gave a good value of the test statistic 'd' at 1.86 indicating no serial correlation, and the scatter plot of the residuals shows them to be randomly spread around zero, and they would certainly pass as not exhibiting any heteroscedasticity.

A Regression Test for Lognormal Data

1.0 General considerations

The lognormal distribution has quantiles of any order as shown by Aitchison and Brown (1957) and there exists a relationship between the quantiles of order 'q' of the lognormal distribution and those also of order 'q' of the corresponding normal distribution. This can be expressed in the following formula -

$$\xi_q = e^{\mu + v_q \sigma}$$

where ξ_q are the quantiles of the lognormal distribution

and where v_q are the quantiles of the normal distribution

In the above formula μ and σ are the shape and location parameters respectively of the lognormal distribution and also the mean and standard deviation of the transformed natural log data, such that if x is the lognormal variate then $\log_e x$ is the corresponding normal variate.

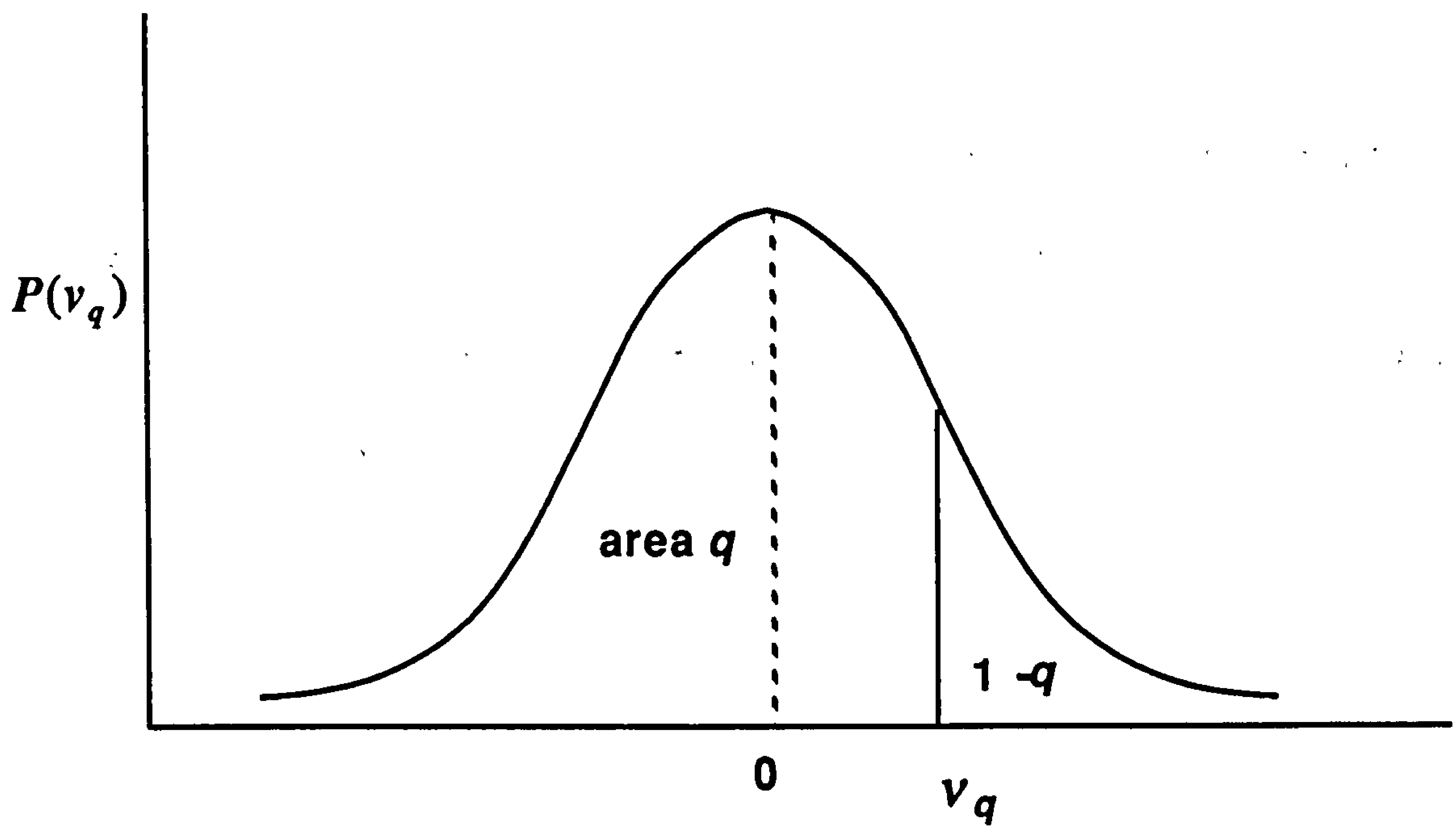
From this we can write -

$$\log_e \xi_q = \sigma v_q + \mu$$

hence the locus of v against $\log_e \xi_q$ is a straight line with slope equal to σ and intercept μ.

The quantile v_q is the value of the standard normal deviate [measured on the normal ordinate scale $N(x)$], that gives the area under the curve equal to q as shown in the diagram below :

figure A4.1



Now if $F(x_i)$ denotes the sample distribution function, so that $F(x_i)$ is the proportion or percentage of sample values that are equal to or less than x, then if we write:-

$$q_i = F(x_i)$$

and $y_i = \log_e x_i$ then we can write: -

$$\log_e x_i = \sigma v_{qi} + \mu$$

or $y_i = \sigma v_{qi} + \mu$

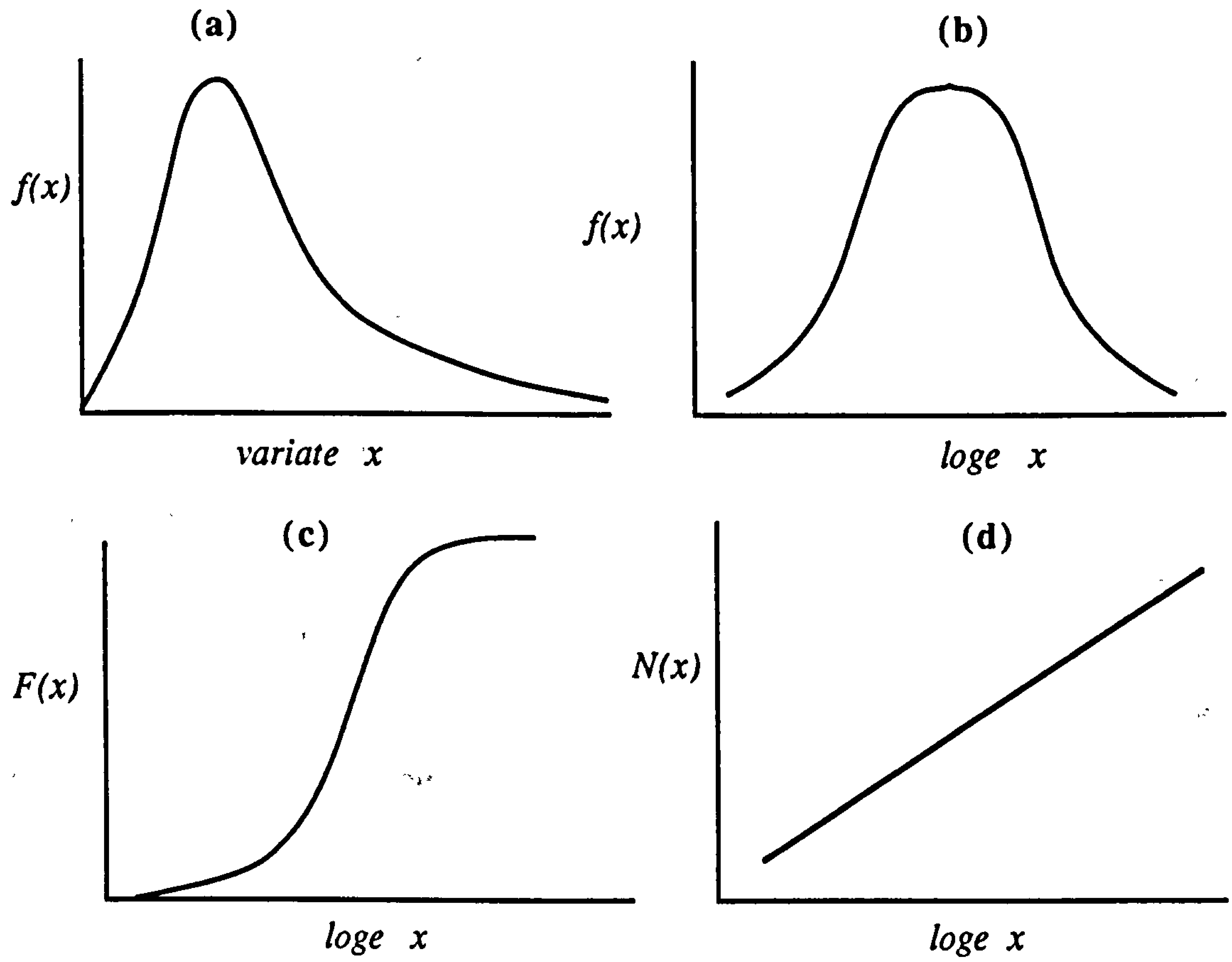
If the sample distribution is lognormal the points (v_{qi}, y_i) will lie on a straight line. Hence a regression test can be applied by regressing $\log_e x$ against the normal ordinate v_{qi} from which the goodness of fit of empirical data to a lognormal distribution can be judged by the usual criteria for judging regression models.

To apply the test in practice the sample data is cumulated to produce the empirical sample distribution $F(x)$ and the corresponding sample normal ordinate for each step in the cumulation is determined from Schmeiser's (1979) approximation as follows -

$$v_{qi} = \frac{F(x)^{0.135} - [1 - F(x)]^{0.135}}{0.1975}$$

This approximation produces values of the normal ordinate with less than one half percent error. From the above developments $\log_e x_i$ is then regressed against v_{qi}. The process is shown graphically in figure A4.2 below, which is identical to figure 6.3 first shown in chapter six page 170.

Figure A4.2



All the regression tests applied in the early chapters of this work were conducted using an Excel spreadsheet model, that required the sample frequency to be entered in equal logarithmic bands. The model then cumulated $f(x)$ to give $F(x)$ from which the normal ordinate was calculated. The mid cell log value of the sample groupings were then regressed against the Schmeiser calculated normal ordinates, thus reproducing figure (d) from above.

Regression Tests on Various Inventory Functions

1.0 General considerations

In this appendix we present the results from a simulated experiment to enable us to see the distribution form of various inventory parameters, namely safety stock, cycle stock, average stock held and the form of individual item inventory turnover ratios. The process was achieved by simulating 200 item demand volumes from a lognormal distribution with the same mean and standard deviation parameters as volumes from 1979 and then for each item we simulated 200 lognormal prices also with the same parameter values from the price distribution of 1979. Hence we were assured that we were starting with both lognormally distributed volumes and prices, and therefore lognormal usage values.

Safety stocks were calculated from the relationship developed in appendix three, namely the function : -

$$\sigma = \alpha\left(\overline{x_i}\right)^{\beta}$$

with alpha at 1.287 and beta 0.814. The level of service was determined by setting the normal deviate 'k' at 1.96 for a so called 95% service level, (assuming a fixed lead time of one period).

Cycles stocks were determined from a simple Wilson EOQ model as follows :-

$$EOQ = \frac{5.284}{2}\sqrt{\frac{V_i}{P_i}}$$

where 5.284 was the EOQ factor used by DAF in 1979/80.

The average stock held was simply the safety stock plus half cycle stock. The individual item turnover was determined by dividing the annual sales volume by the average stock held. The tabulation for the 200 items is shown at the back of this appendix. The following charts show the factors of interest in $\log_e$ form.

figure A5.1

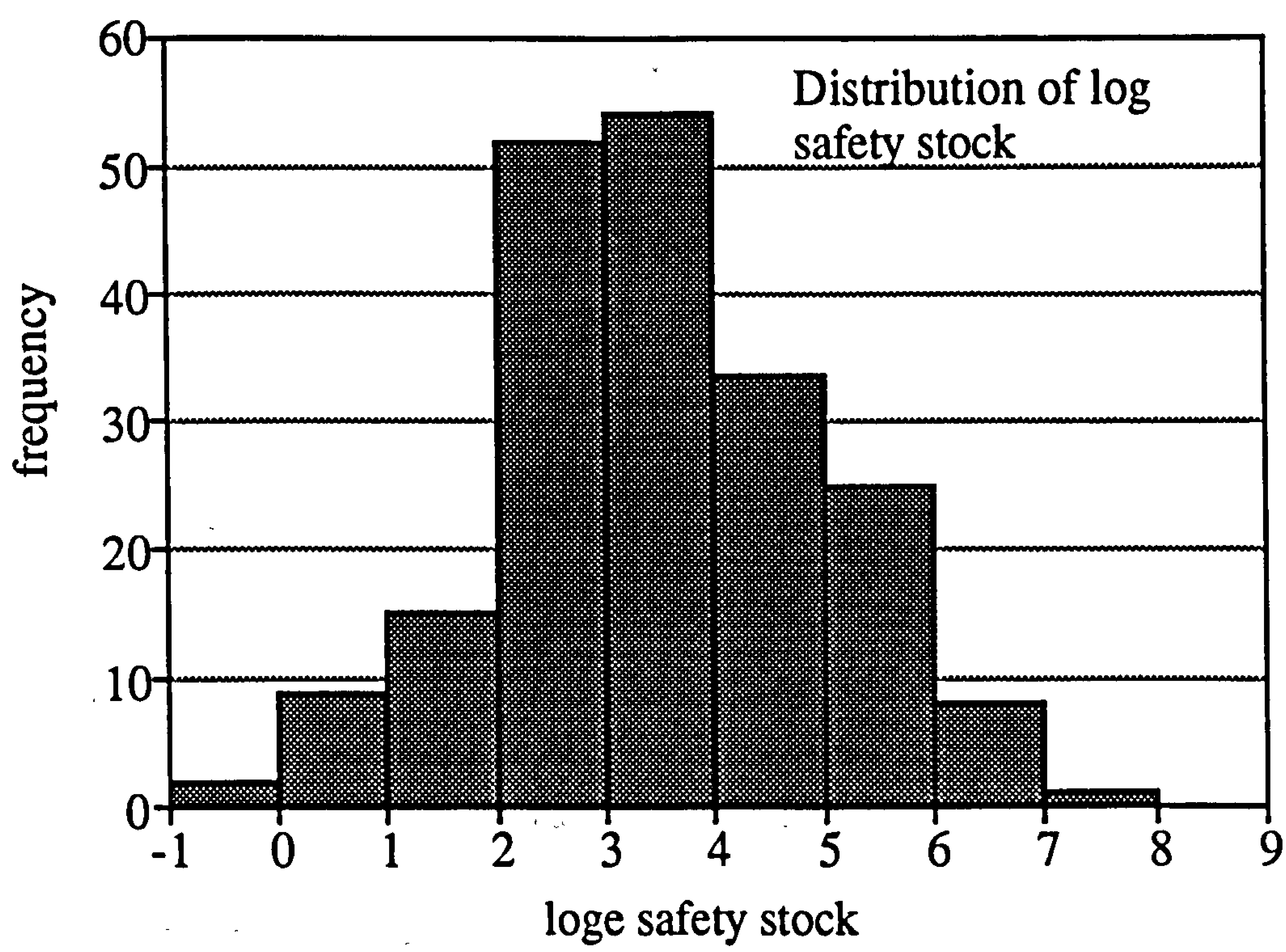


figure A5.2

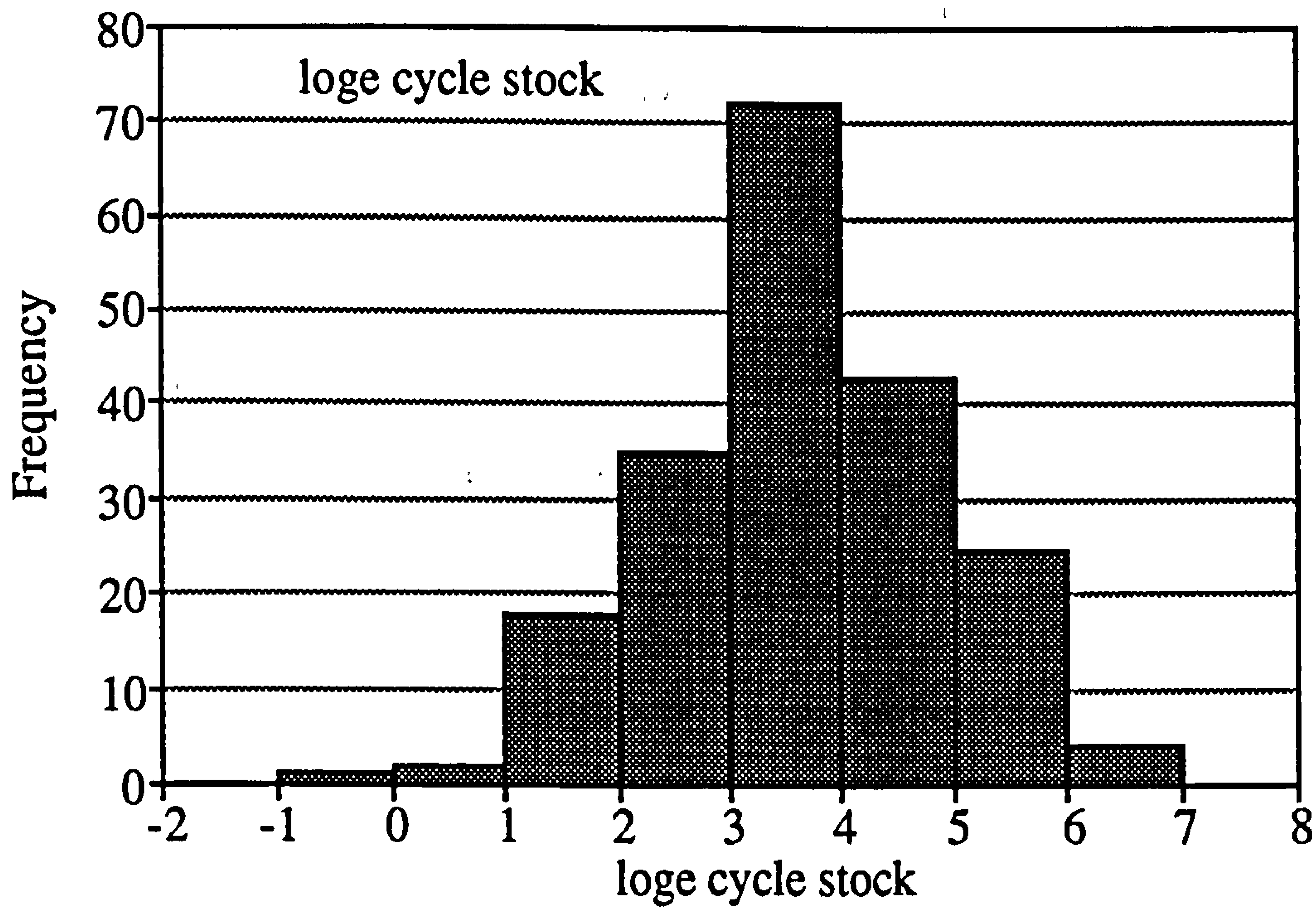
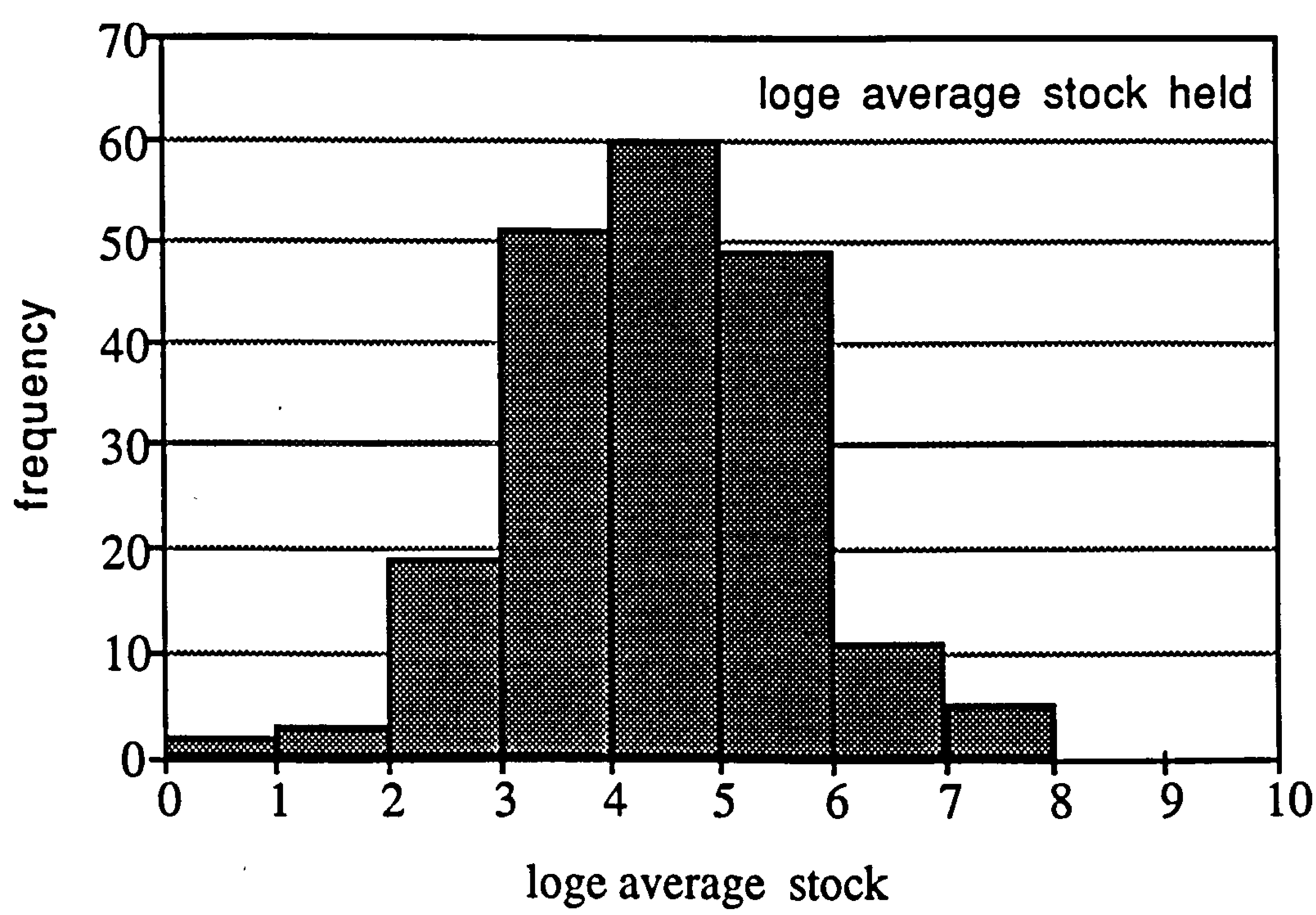


figure A5.3

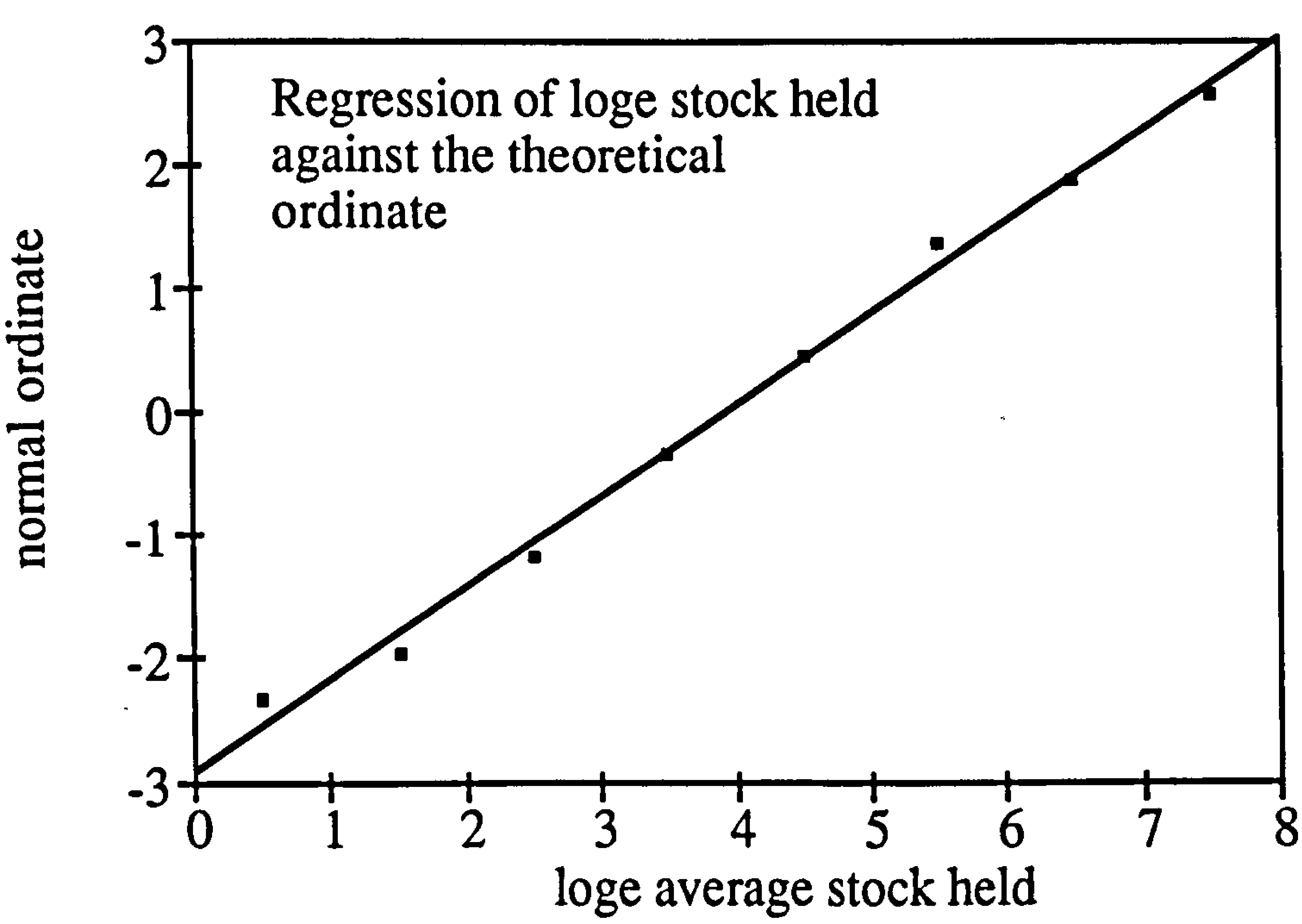


The average stock held was subjected to a regression test for normality as explained in appendix four with the following good results-

correlation coefficient $r = 0.997$
coefficient of determination $R^2 = 0.994$
Durbin Watson test = 2.530

The plot of the regression line is shown in figure A5.4 and the plot of the residuals is shown in figure A5.5. The results confirm what was predicted from previous work in this thesis that the average stock held is lognormal because both safety stocks and cycle stocks are themselves lognormal functions.

figure A5.4



The correlation coefficient is statistically significant and the Durbin Watson test does not indicate any strong auto correlation.

figure A5.5

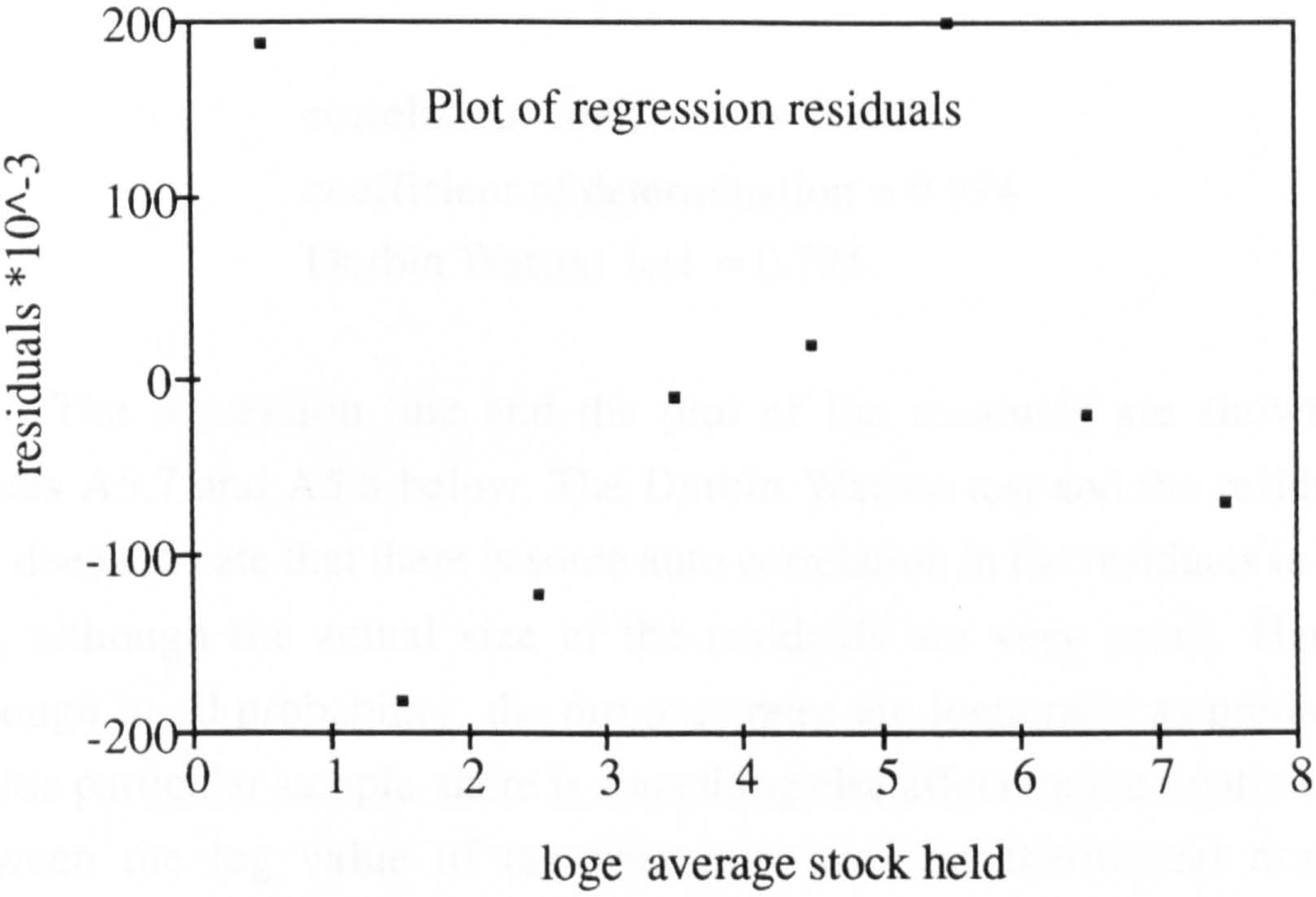
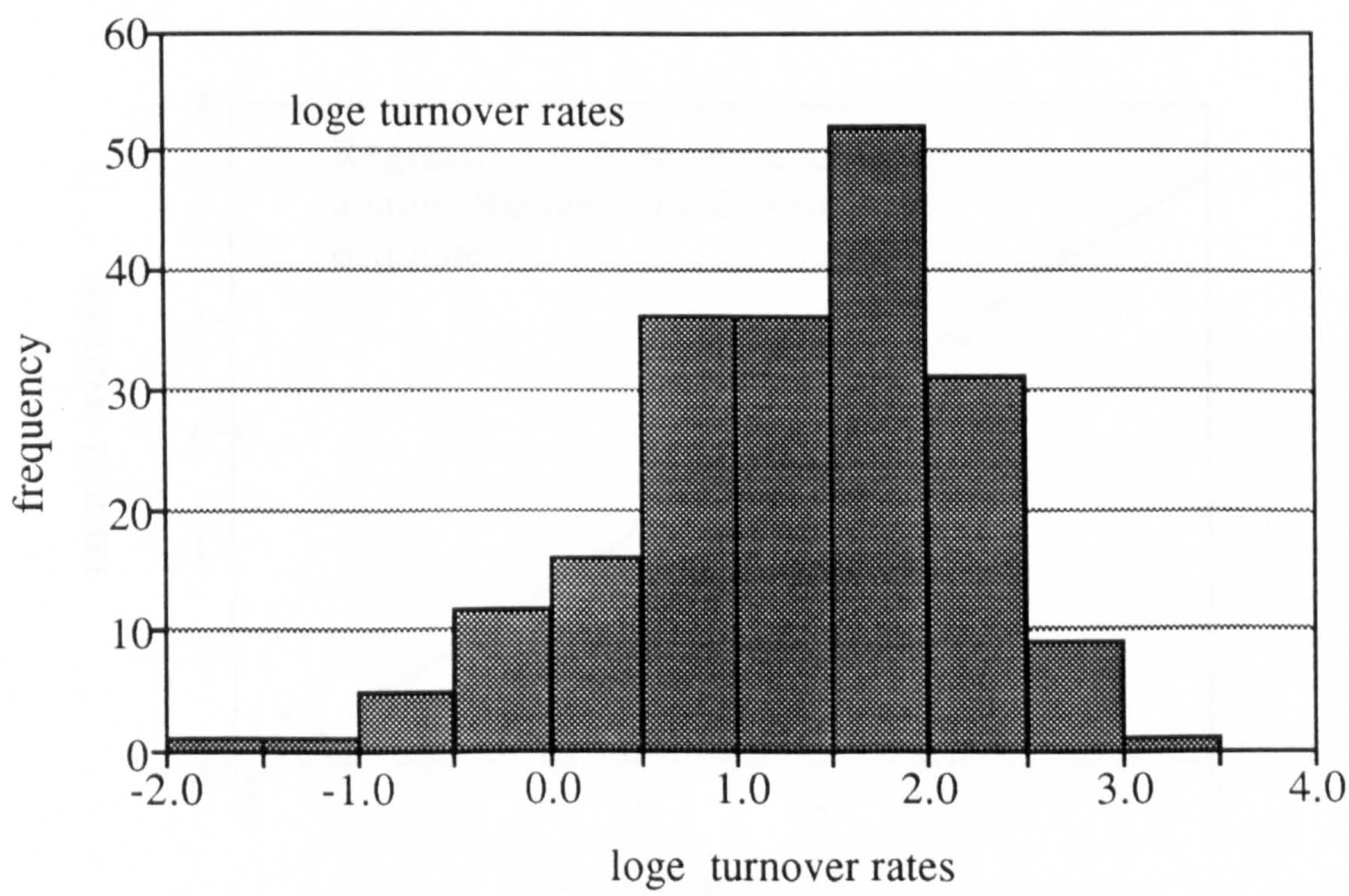


figure A5.6



The $\log_e$ turnover rates were also subjected to a regression test for lognormality with the following results-

correlation coefficient = 0.988
coefficient of determination = 0.974
Durbin Watson test = 0.795

The regression line and the plot of the residuals are shown in figures A5.7 and A5.8 below. The Durbin Watson test and the residuals plot does indicate that there is some auto correlation in the residuals in this test, although the actual size of the residuals are very small. Hence, although in all probability the turnover rates are lognormal as predicted in this particular sample there is something else affecting the relationship between the log value of turnover rates and the theoretical normal ordinate.

figure A5.7

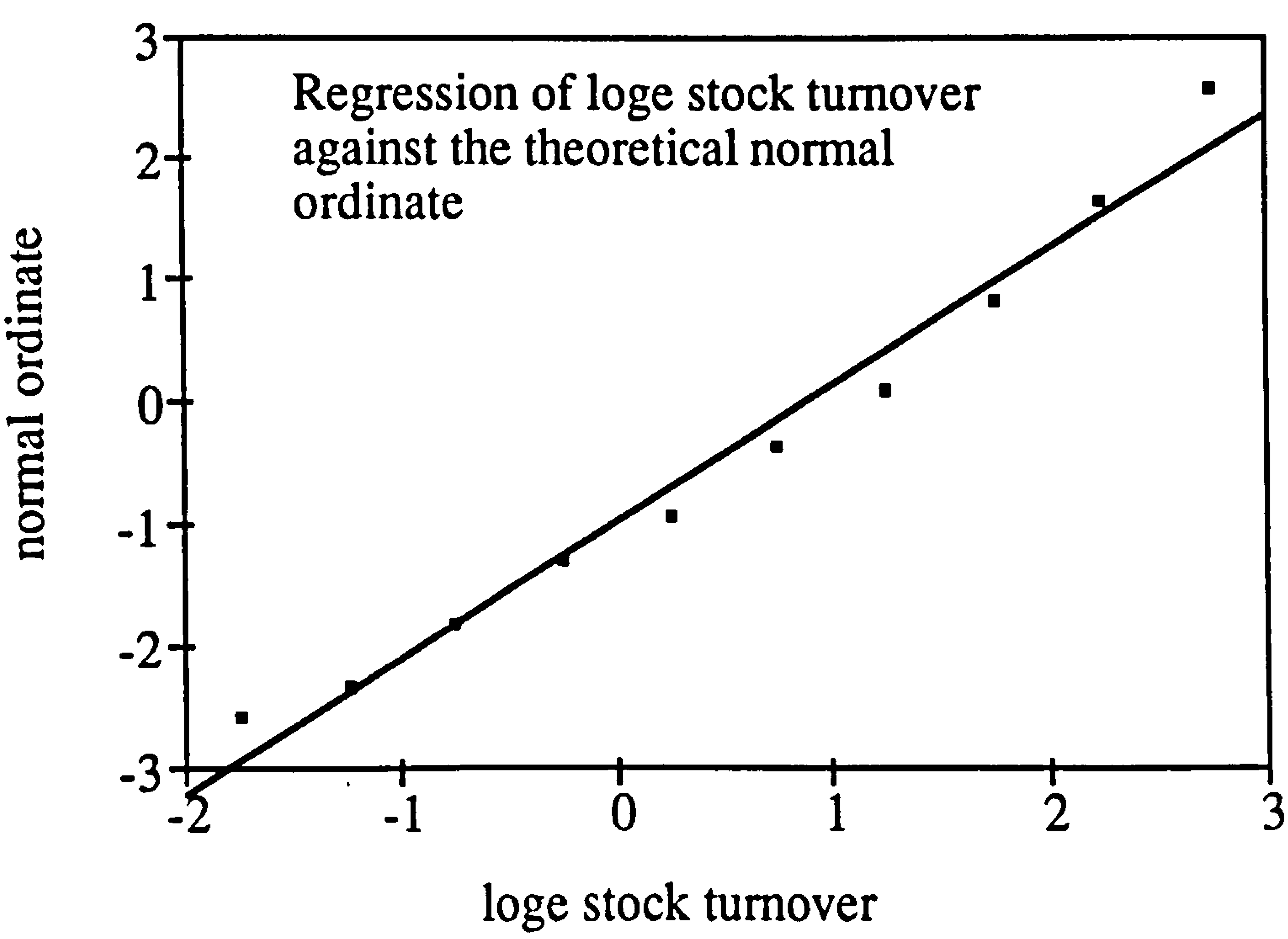
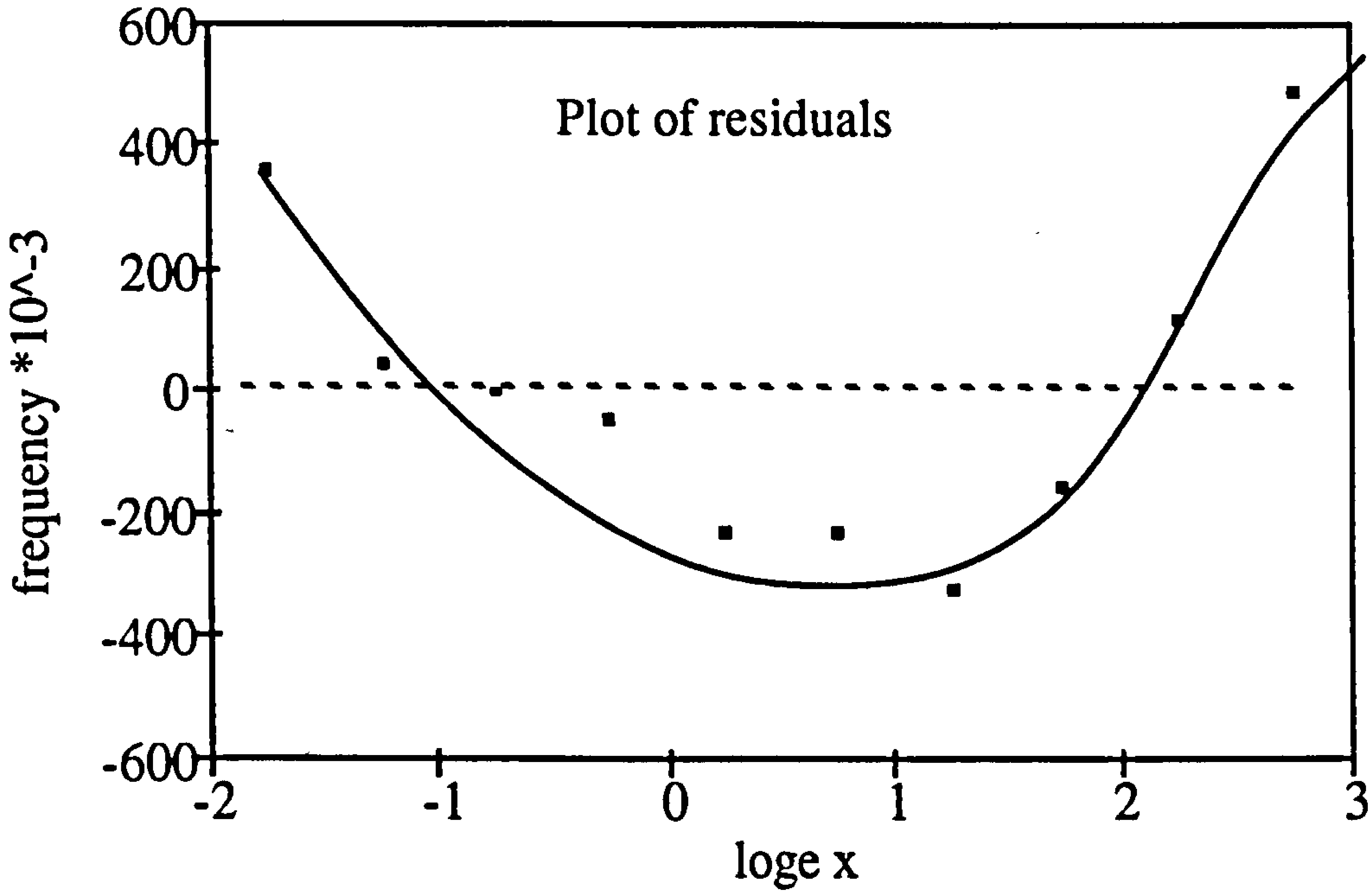


figure A5.8



The results of the 200 item simulation are shown in the following tables :

DEMAND VOLUME	PRICE	USAGE VALUE	SAFETY STOCK	1/2 CYCLE STOCK	AVERAGE STOCK HELD	VALUE AVE. STOCK HELD	TURNOVER RATES	REAL MARGIN	REAL MARGIN AS % US. VAL.	Rank no.
13	0.02	0.27	2.53	67.03	69.57	1.41	0.19	-1.12	-423.15	1
17	0.03	0.43	3.11	67.54	70.64	1.81	0.24	-1.41	-326.63	2
50	0.03	1.53	7.50	105.81	113.31	3.50	0.44	-2.49	-162.76	3
31	0.05	1.53	5.16	66.92	72.08	3.52	0.43	-2.49	-162.84	4
7	0.23	1.63	1.53	14.52	16.05	3.73	0.44	-2.54	-155.88	5
2	0.80	1.89	0.63	4.53	5.16	4.14	0.46	-2.69	-142.25	6
152	0.02	3.25	18.63	222.31	240.94	5.16	0.63	-3.42	-105.19	7
210	0.02	3.46	24.31	298.58	322.90	5.32	0.65	-3.51	-101.31	8
6	0.56	3.40	1.35	8.68	10.03	5.63	0.60	-3.42	-100.77	9
10	0.36	3.72	2.08	14.03	16.10	5.85	0.64	-3.56	-95.49	10
7	0.75	5.01	1.46	7.85	9.31	7.01	0.71	-3.96	-79.06	11
111	0.05	5.62	14.50	124.17	138.67	7.00	0.80	-4.19	-74.50	12
199	0.04	8.04	23.29	185.79	209.07	8.43	0.95	-4.74	-58.89	13
290	0.03	8.26	31.60	266.68	298.28	8.49	0.97	-4.79	-57.93	14
43	0.21	8.89	6.71	38.31	45.02	9.26	0.96	-4.84	-54.45	15
65	0.14	9.18	9.35	56.70	66.05	9.32	0.98	-4.90	-53.38	16
49	0.19	9.39	7.37	41.86	49.23	9.52	0.99	-4.92	-52.43	17
105	0.09	9.75	13.84	89.09	102.94	9.53	1.02	-5.00	-51.31	18
22	0.43	9.63	3.92	19.01	22.92	9.89	0.97	-4.94	-51.26	19
42	0.24	9.93	6.57	35.35	41.92	9.87	1.01	-5.00	-50.33	20
299	0.04	10.62	32.38	242.39	274.77	9.76	1.09	-5.16	-48.55	21
2	4.60	9.38	0.56	1.76	2.32	10.66	0.88	-4.84	-51.61	22
39	0.36	13.97	6.19	27.68	33.87	12.08	1.16	-5.46	-39.06	23
57	0.26	14.76	8.46	39.54	48.00	12.32	1.20	-5.54	-37.51	24
92	0.19	17.35	12.38	58.22	70.60	13.35	1.30	-5.73	-33.05	25
43	0.40	16.91	6.62	27.34	33.96	13.49	1.25	-5.68	-33.59	26

200 item simulation

281	0.08	23.56	30.76	152.80	183.56	15.41	1.53	-6.03	-25.58	27
12	1.81	22.46	2.43	6.93	9.37	16.91	1.33	-5.89	-26.24	28
9	2.55	23.35	1.90	5.01	6.90	17.60	1.33	-5.92	-25.34	29
63	0.41	25.91	9.17	32.94	42.11	17.19	1.51	-6.00	-23.17	30
299	0.10	29.48	32.38	145.49	177.87	17.54	1.68	-6.09	-20.66	31
94	0.31	29.04	12.68	46.33	59.01	18.13	1.60	-6.04	-20.79	32
82	0.35	28.98	11.31	40.32	51.63	18.21	1.59	-6.03	-20.82	33
91	0.34	30.53	12.27	43.41	55.68	18.72	1.63	-6.03	-19.76	34
267	0.12	32.35	29.49	123.80	153.28	18.61	1.74	-6.07	-18.75	35
93	0.36	33.47	12.52	42.51	55.03	19.79	1.69	-6.01	-17.95	36
432	0.08	36.11	43.69	189.92	233.60	19.53	1.85	-6.03	-16.71	37
53	0.63	33.14	7.91	24.28	32.19	20.16	1.64	-6.00	-18.09	38
384	0.10	37.25	39.69	166.19	205.87	19.98	1.86	-6.00	-16.11	39
103	0.37	37.85	13.59	44.18	57.77	21.25	1.78	-5.93	-15.67	40
11	3.23	34.39	2.14	4.80	6.94	22.42	1.53	-5.98	-17.40	41
61	0.63	38.04	8.84	25.99	34.83	21.84	1.74	-5.91	-15.54	42
172	0.24	40.35	20.61	71.39	92.00	21.63	1.87	-5.88	-14.57	43
2242	0.02	48.02	166.93	854.86	1021.79	21.88	2.19	-5.74	-11.95	44
58	0.72	41.87	8.57	23.85	32.42	23.24	1.80	-5.80	-13.86	45
603	0.08	46.79	57.29	232.77	290.06	22.52	2.08	-5.71	-12.20	46
17	2.36	39.59	3.10	7.04	10.14	23.95	1.65	-5.88	-14.85	47
40	1.06	42.85	6.34	16.27	22.61	24.03	1.78	-5.77	-13.47	48
109	0.43	47.26	14.25	41.94	56.20	24.34	1.94	-5.62	-11.89	49
247	0.20	49.23	27.72	93.01	120.72	24.06	2.05	-5.56	-11.29	50
58	0.80	47.07	8.58	22.52	31.10	25.03	1.88	-5.62	-11.93	51
67	0.73	48.65	9.53	25.20	34.73	25.39	1.92	-5.55	-11.41	52
110	0.45	50.13	14.38	41.17	55.55	25.24	1.99	-5.49	-10.95	53
310	0.18	54.36	33.34	111.07	144.41	25.33	2.15	-5.31	-9.77	54

200 item simulation

357	0.16	55.46	37.43	126.74	164.17	25.48	2.18	-5.26	-9.48	55
363	0.16	58.30	37.95	125.73	163.68	26.26	2.22	-5.10	-8.75	56
76	0.77	58.53	10.63	26.29	36.93	28.39	2.06	-5.06	-8.64	57
359	0.23	81.09	37.55	105.23	142.78	32.28	2.51	-3.51	-4.33	58
101	0.82	82.64	13.41	29.42	42.83	34.97	2.36	-3.42	-4.13	59
128	0.66	84.06	16.24	36.90	53.14	34.88	2.41	-3.29	-3.92	60
177	0.50	89.35	21.15	49.54	70.69	35.64	2.51	-2.85	-3.19	61
95	0.92	87.14	12.73	26.88	39.61	36.34	2.40	-3.07	-3.52	62
302	0.31	93.18	32.62	82.57	115.19	35.58	2.62	-2.51	-2.70	63
50	1.83	90.88	7.49	13.73	21.22	38.94	2.33	-2.84	-3.13	64
363	0.29	103.70	37.89	94.08	131.97	37.74	2.75	-1.57	-1.52	65
49	1.92	93.41	7.39	13.31	20.70	39.71	2.35	-2.64	-2.83	66
156	0.64	100.74	19.09	41.13	60.22	38.83	2.59	-1.88	-1.86	67
374	0.28	106.54	38.89	95.84	134.73	38.34	2.78	-1.31	-1.23	68
58	1.79	103.81	8.52	15.04	23.56	42.17	2.46	-1.74	-1.67	69
113	0.98	111.44	14.71	28.40	43.11	42.34	2.63	-0.94	-0.85	70
177	0.66	116.46	21.10	43.26	64.36	42.42	2.75	-0.42	-0.36	71
916	0.14	127.93	80.57	214.02	294.59	41.13	3.11	0.81	0.63	72
92	1.28	118.22	12.43	22.42	34.85	44.66	2.65	-0.34	-0.29	73
87	1.35	117.86	11.86	21.19	33.05	44.74	2.63	-0.39	-0.33	74
104	1.15	119.16	13.70	25.15	38.85	44.55	2.67	-0.23	-0.19	75
673	0.22	145.28	62.70	147.60	210.31	45.37	3.20	2.64	1.81	76
170	0.79	134.89	20.48	38.74	59.21	46.91	2.88	1.40	1.04	77
1127	0.14	161.98	95.35	233.94	329.29	47.33	3.42	4.51	2.78	78
151	1.01	152.40	18.56	32.29	50.85	51.36	2.97	3.20	2.10	79
163	0.95	154.85	19.72	34.53	54.25	51.66	3.00	3.48	2.25	80
57	2.63	149.38	8.37	12.27	20.64	54.33	2.75	2.58	1.73	81
240	0.72	172.00	27.05	48.29	75.34	54.06	3.18	5.45	3.17	82

200 item simulation

4638	0.05	223.51	301.65	819.70	1121.36	54.03	4.14	11.95	5.34	83
128	1.45	186.13	16.25	24.82	41.07	59.64	3.12	6.85	3.68	84
184	1.15	211.34	21.84	33.49	55.33	63.45	3.33	9.91	4.69	85
28	7.77	216.06	4.68	4.99	9.68	75.23	2.87	9.15	4.24	86
25	10.23	255.55	4.29	4.13	8.42	86.15	2.97	13.40	5.24	87
194	1.49	289.86	22.76	30.09	52.85	79.00	3.67	19.60	6.76	88
326	0.93	304.12	34.73	49.36	84.09	78.49	3.87	21.78	7.16	89
733	0.47	347.72	67.20	103.87	171.07	81.14	4.29	28.06	8.07	90
95	3.31	313.06	12.70	14.14	26.84	88.73	3.53	21.88	6.99	91
857	0.43	368.85	76.29	117.88	194.17	83.58	4.41	31.04	8.42	92
1238	0.33	413.38	102.93	160.87	263.81	88.09	4.69	37.48	9.07	93
97	4.10	396.78	12.94	12.85	25.79	105.62	3.76	32.64	8.23	94
41	9.48	391.65	6.47	5.52	11.99	113.57	3.45	30.47	7.78	95
1063	0.46	494.41	90.96	126.36	217.32	101.03	4.89	49.00	9.91	96
92	4.84	444.69	12.39	11.51	23.90	115.69	3.84	38.86	8.74	97
419	1.16	486.56	42.63	50.21	92.84	107.76	4.52	46.92	9.64	98
189	2.57	486.07	22.32	22.69	45.01	115.56	4.21	45.72	9.41	99
54	9.08	489.49	8.03	6.44	14.47	131.35	3.73	43.59	8.90	100
127	3.98	506.92	16.17	14.95	31.12	123.82	4.09	47.86	9.44	101
261	2.11	550.79	29.01	29.41	58.42	123.17	4.47	55.32	10.04	102
147	3.69	540.96	18.12	16.65	34.77	128.34	4.22	52.83	9.77	103
43	12.01	521.83	6.74	5.03	11.76	141.25	3.69	47.21	9.05	104
1865	0.34	632.28	143.71	196.00	339.71	115.14	5.49	69.95	11.06	105
1142	0.55	622.24	96.37	120.92	217.29	118.43	5.25	67.90	10.91	106
585	1.07	627.87	55.91	61.66	117.57	126.22	4.97	67.81	10.80	107
104	5.68	588.95	13.68	11.29	24.96	141.78	4.15	58.64	9.96	108
991	0.71	705.79	85.85	98.51	184.36	131.36	5.37	80.26	11.37	109
310	2.19	676.77	33.32	31.45	64.76	141.55	4.78	73.76	10.90	110

200 item simulation

98	6.73	657.42	13.02	10.06	23.09	155.41	4.23	67.94	10.33	111
34	19.33	648.28	5.46	3.48	8.94	172.73	3.75	62.82	9.69	112
380	1.88	716.46	39.40	37.55	76.95	144.91	4.94	80.04	11.17	113
75	9.09	677.44	10.45	7.57	18.02	163.75	4.14	69.81	10.31	114
340	2.15	730.20	35.93	33.22	69.16	148.61	4.91	81.80	11.20	115
177	4.14	732.39	21.10	17.25	38.35	158.97	4.61	80.36	10.97	116
55	13.49	745.69	8.20	5.35	13.55	182.68	4.08	78.01	10.46	117
428	1.90	813.47	43.33	39.62	82.95	157.77	5.16	94.74	11.65	118
3925	0.23	910.77	263.32	343.62	606.93	140.83	6.47	114.04	12.52	119
458	1.84	843.28	45.80	41.65	87.45	161.09	5.23	99.39	11.79	120
7	114.72	842.44	1.58	0.67	2.25	258.47	3.26	77.91	9.25	121
313	2.68	839.08	33.59	28.53	62.11	166.64	5.04	97.68	11.64	122
273	3.13	855.17	30.09	24.69	54.78	171.44	4.99	99.65	11.65	123
8884	0.12	1023.80	512.00	733.59	1245.59	143.54	7.13	133.17	13.01	124
1317	0.71	938.51	108.26	113.60	221.86	158.08	5.94	116.53	12.42	125
2220	0.44	969.02	165.57	188.40	353.98	154.52	6.27	122.36	12.63	126
2253	0.44	996.53	167.61	188.59	356.20	157.52	6.33	126.74	12.72	127
2037	0.52	1061.33	154.38	165.19	319.57	166.51	6.37	136.82	12.89	128
605	1.71	1033.06	57.47	49.74	107.21	183.05	5.64	129.12	12.50	129
12721	0.10	1213.50	685.75	964.80	1650.56	157.45	7.71	164.64	13.57	130
283	3.83	1084.93	30.99	22.72	53.71	205.70	5.27	134.18	12.37	131
90	12.12	1094.74	12.22	7.21	19.44	235.53	4.65	129.70	11.85	132
91	12.93	1177.64	12.30	7.01	19.31	249.76	4.72	141.74	12.04	133
231	5.32	1230.61	26.27	17.42	43.68	232.48	5.29	155.20	12.61	134
203	6.08	1238.01	23.67	15.28	38.95	236.98	5.22	155.61	12.57	135
908	1.42	1285.96	79.98	66.90	146.88	208.01	6.18	170.06	13.22	136
387	3.24	1254.21	39.98	28.90	68.88	223.00	5.62	161.44	12.87	137
320	4.08	1306.69	34.22	23.39	57.61	235.24	5.55	168.55	12.90	138

200 item simulation

1869	0.75	1398.63	143.94	132.04	275.99	206.52	6.77	190.68	13.63	139
564	2.59	1458.62	54.29	39.03	93.32	241.27	6.05	195.16	13.38	140
55	28.44	1550.35	8.10	3.66	11.76	334.44	4.64	191.19	12.33	141
1355	1.21	1639.64	110.76	88.38	199.14	241.05	6.80	228.41	13.93	142
777	2.11	1641.28	70.44	50.66	121.11	255.85	6.41	225.84	13.76	143
1926	0.90	1734.59	147.50	122.18	269.68	242.88	7.14	245.49	14.15	144
2797	0.63	1772.64	199.82	175.49	375.32	237.89	7.45	253.35	14.29	145
2308	0.76	1760.27	170.91	145.35	316.26	241.19	7.30	250.50	14.23	146
587	3.10	1821.15	56.09	36.35	92.44	286.70	6.35	252.81	13.88	147
520	3.68	1912.60	50.79	31.40	82.18	302.43	6.32	266.49	13.93	148
431	4.69	2020.72	43.58	25.31	68.89	323.24	6.25	282.17	13.96	149
473	4.41	2086.43	47.01	27.34	74.35	328.18	6.36	293.40	14.06	150
1151	1.98	2273.78	97.00	63.77	160.76	317.62	7.16	330.83	14.55	151
49	57.32	2791.06	7.39	2.44	9.83	563.20	4.96	369.43	13.24	152
459	5.23	2402.33	45.91	24.75	70.66	369.71	6.50	343.56	14.30	153
249	9.82	2445.96	27.90	13.30	41.20	404.72	6.04	343.60	14.05	154
5631	0.46	2610.21	353.25	291.21	644.46	298.72	8.74	397.47	15.23	155
3338	0.77	2586.82	230.79	173.40	404.19	313.22	8.26	390.36	15.09	156
81	36.58	2951.80	11.15	3.92	15.07	551.40	5.35	403.53	13.67	157
775	3.36	2602.39	70.33	40.16	110.48	370.83	7.02	381.16	14.65	158
324	8.67	2807.86	34.56	16.15	50.71	439.59	6.39	404.15	14.39	159
5096	0.55	2827.58	325.64	253.17	578.81	321.19	8.80	433.94	15.35	160
234	12.54	2940.02	26.57	11.43	37.99	476.37	6.17	420.41	14.30	161
6452	0.46	2993.41	394.60	311.54	706.14	327.63	9.14	463.85	15.50	162
992	3.08	3050.83	85.92	47.43	133.35	410.28	7.44	457.34	14.99	163
157	24.45	3847.37	19.20	6.70	25.90	633.37	6.07	555.16	14.43	164
1503	2.26	3391.94	120.54	68.18	188.73	425.90	7.96	518.76	15.29	165
2041	1.89	3855.07	154.63	86.85	241.48	456.11	8.45	600.37	15.57	166

200 item simulation

3409	1.15	3929.01	234.77	143.68	378.45	436.19	9.01	618.91	15.75	167
235	19.51	4578.50	26.59	9.17	35.76	697.43	6.56	680.21	14.86	168
793	5.20	4126.30	71.66	32.63	104.29	542.40	7.61	632.11	15.32	169
7541	0.56	4188.66	448.02	307.83	755.85	419.85	9.98	671.87	16.04	170
247	27.10	6681.43	27.67	7.97	35.64	965.97	6.92	1018.83	15.25	171
1180	4.70	5541.97	98.99	41.88	140.87	661.60	8.38	876.41	15.81	172
424	19.14	8119.85	43.05	12.44	55.49	1062.00	7.65	1274.79	15.70	173
374	22.48	8409.04	38.86	10.78	49.63	1115.78	7.54	1316.93	15.66	174
942	7.88	7428.47	82.43	28.88	111.31	877.52	8.47	1188.58	16.00	175
298	55.62	16564.09	32.27	6.11	38.39	2135.19	7.76	2641.23	15.95	176
782	15.59	12185.77	70.79	18.71	89.49	1395.33	8.73	1983.62	16.28	177
1586	7.39	11731.45	125.96	38.70	164.65	1217.61	9.63	1942.47	16.56	178
2523	5.06	12772.12	183.76	58.98	242.74	1228.82	10.39	2144.20	16.79	179
2589	5.09	13174.21	187.69	59.60	247.29	1258.14	10.47	2215.52	16.82	180
1808	7.84	14171.79	140.13	40.14	180.27	1412.64	10.03	2370.44	16.73	181
898	33.91	30452.69	79.27	13.60	92.86	3148.81	9.67	5107.21	16.77	182
3569	5.25	18750.58	243.72	68.87	312.59	1642.09	11.42	3211.81	17.13	183
1201	23.70	28479.40	100.45	18.81	119.26	2827.04	10.07	4807.47	16.88	184
647	72.28	46764.75	60.70	7.90	68.60	4958.43	9.43	7827.70	16.74	185
8805	2.15	18973.12	508.26	168.88	677.15	1459.15	13.00	3304.87	17.42	186
2064	13.73	28337.38	156.02	32.39	188.41	2587.26	10.95	4847.49	17.11	187
953	55.07	52463.50	83.17	10.99	94.16	5185.06	10.12	8896.71	16.96	188
2256	13.83	31192.31	167.75	33.74	201.49	2786.27	11.20	5357.80	17.18	189
2644	11.08	29288.01	190.92	40.82	231.74	2566.72	11.41	5041.94	17.22	190
2112	21.95	46345.80	158.97	25.91	184.89	4057.90	11.42	8006.11	17.27	191
21872	1.36	29849.74	1065.99	334.47	1400.46	1911.27	15.62	5334.10	17.87	192
2692	109.78	295579.70	193.74	13.08	206.82	22705.72	13.02	52252.15	17.68	193
3726	42.85	159652.95	252.40	24.64	277.04	11870.14	13.45	28315.75	17.74	194

200 item simulation

7488	11.25	84273.60	445.48	68.15	513.63	5780.49	14.58	15062.26	17.87	195
13004	5.06	65765.43	698.14	133.97	832.12	4208.26	15.63	11828.10	17.99	196
8284	13.09	108423.24	483.64	66.47	550.11	7200.13	15.06	19464.38	17.95	197
5873	53.49	314169.35	365.54	27.68	393.22	21035.16	14.94	56463.58	17.97	198
6046	137.21	829506.96	374.27	17.54	391.80	53758.12	15.43	149712.24	18.05	199
55540	64.49	3582011.58	2276.13	77.53	2353.66	151796.34	23.60	670769.02	18.73	200
	SUM=	6044582.87			SUM =	346808.57				
		RATIO =	17.43			RATIO	4.90			

The 'classical' turnover ratio

Average of all the individual turnover ratios

200 item simulation

Parameter estimation Methods

1.0 General considerations

As we have shown in chapter three the lognormal distribution has two parameters namely σ the shape parameter, and μ the scale or location parameter. A third parameter ρ is sometimes referred to in some inventory calculations and has been called the the STANDARD RATIO by R.G. Brown (1959). However, ρ is in fact directly related to the shape parameter as follows-

$$\rho = e^{\sigma}$$

The parameter estimation methods for the lognormal distribution are discussed in depth in by Aitchison and Brown (1957). From the extensive work by these authors on the efficiency of the parameter estimation methods they conclude that the maximum likelihood method provides the most accurate values. This approach provides estimates that are unbiased and consistent, whether for grouped or ungrouped data. Therefore all the estimates on distribution parameters that we have used in this work are based on the maximum likelihood function.

The maximum likelihood formula for 'σ' and 'μ' are as follows-

$$\sigma^2_{\ln x} = \frac{n\sum f_i(\ln x_i)^2 - \left[\sum f_i(\ln x_i)\right]^2}{n(n-1)}$$

for $i = 1$ to n

where $\sigma^2_{\ln x}$ is the maximum likelihood estimator for σ^2.

The likelihood estimator for the scale parameter μ is obtained from -

$$\bar{\mu} = \ln x_i = \sum \frac{f_i(\ln x_i}{n}$$

for $i = 1$ to n

Usage Value Data Sets

1.0 General considerations

In this section we present ten example pages from one of the computer prints from the DAF Parts department computer system. This data is from the 1979 ABC Usage Value distribution and it was used to produce the lognormal plot on log normal graph paper shown in chapter one. We give the first ten pages that shows the cumulative distribution up to the 75% percentile level. It was not feasible to include any more pages because of the long tail of the distribution. The complete print out was nearly 300 pages long. (A photocopy of the complete printout can be made available to other researchers on request to the author at City University, as can examples of demand history printouts for various years.). A nominal charge would be made to cover photocopy costs.

SP520. 25/01/80

DAF TRUCKS (GB) LTD

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANDEY.T.D. TURNOVER	PCNT OF TOTAL	DIS CDE	PCK QTY	EXPORT PRICE	CUMULATIVE TURNOVER
1	R627841	161	1250.55	201338.55	3.84	3	1	1191.00	201338.55
2	631086	2336	23.47	54825.92	4.89	4	1	.00	256164.47
3	614092	250	146.19	49047.50	5.82	4	1	.00	305211.97
4	629242	17	2601.72	44229.24	6.67	3	1	.00	349441.21
5	611049	17824	2.46	43847.04	7.50	0	1	.00	393288.25
6	R440875	20	2081.75	41635.00	8.30	2	1	.00	434923.25
7	R435026	20	2054.09	41081.80	9.08	2	1	.00	476005.05
8	629166	1723	23.77	40955.71	9.86	1	1	.00	516960.76
9	247138	17195	2.26	38860.70	10.61	0	1	.00	555821.46
10	R165497	1011	38.19	38610.09	11.34	1	1	36.37	594431.55
11	9009920	5327	7.04	37502.08	12.06	1	4	6.70	631933.63
12	114786	16342	2.26	36932.92	12.76	0	1	.00	668866.55
13	R161854	523	63.44	33179.12	13.40	1	1	60.42	702045.67
14	9009921	4180	7.40	30932.00	13.99	1	4	7.05	732977.67
15	R624702	397	75.60	30013.20	14.56	1	1	72.00	762990.87
16	265045	2681	11.05	29625.05	15.12	0	1	.00	792615.92
17	R634176	737	40.16	29597.92	15.69	1	1	38.25	822213.84
18	R504640	471	61.43	28933.53	16.24	1	1	58.50	851147.37
19	259930	242	117.36	28401.12	16.78	4	1	.00	879548.49
20	R627861	20	1415.14	28302.80	17.32		1	1347.75	907851.29
21	117351	597	46.64	27844.08	17.86	4	1	.00	935695.37
22	515193	8	3394.59	27156.72	18.37	3	1	.00	962852.09
23	R442163	13	2054.09	26703.17	18.88	2	1	.00	989555.26
24	621303	20	1322.06	26441.20	19.39	3	1	.00	1015996.46
25	240474	1622	16.25	26357.50	19.89	4	1	.00	1042353.96
26	621506	13	2022.74	26295.62	20.39	3	1	.00	1068649.58
27	R441032	10	2561.15	25611.50	20.88	2	1	.00	1094261.08
28	167679	1010	24.67	24916.70	21.36	4	1	.00	1119177.78
29	R440589	12	2054.09	24649.08	21.83	2	1	.00	1143826.86
30	617743	282	83.05	23420.10	22.27	4	1	.00	1167246.96
31	9007192	14	1640.76	22970.64	22.71	0	1	1562.63	1190217.60
32	118086	346	66.20	22905.20	23.15	4	1	.00	1213122.80
33	633328	361	63.15	22797.15	23.58	3	1	.00	1235919.95
34	615785	46	488.68	22479.28	24.01	1	1	.00	1258399.23
35	9009969	247	90.93	22459.71	24.44	6	1	86.60	1280858.94
36	629240	10	2242.16	22421.60	24.87	3	1	.00	1303280.54
37	R241787	229	94.50	21640.50	25.28	1	1	90.00	1324921.04
38	531355	542	38.82	21040.44	25.68	1	1	.00	1345961.48
39	227450	18	1161.65	20909.70	26.08	3	1	.00	1366871.18
40	241720	104	198.60	20654.40	26.48	1	1	.00	1387525.58
41	9009998	620	30.56	18947.20	26.84	5	1	29.10	1406472.78
42	R117379	283	65.53	18544.99	27.19	1	1	62.41	1425017.77
43	618262	7	2557.47	17902.29	27.53	3	1	.00	1442920.06
44	R602228	46	378.00	17388.00	27.87	3	1	360.00	1460308.06
45	263990	529	32.80	17351.20	28.20	4	1	.00	1477659.26
46	R504129	142	120.75	17146.50	28.52	3	1	115.00	1494805.76
47	252331	120	139.62	16754.40	28.84	4	1	.00	1511560.16
48	240596	166	98.70	16384.20	29.16	4	1	.00	1527944.36
49	161518	350	46.64	16324.00	29.47	4	1	.00	1544268.36
50	631075	4666	3.49	16284.34	29.78	4	1	.00	1560552.70
51	628471	9	1751.69	15765.21	30.08	3	1	.00	1576317.91
52	191173	345	45.14	15573.30	30.38	4	1	.00	1591891.21
53	253596	454	33.70	15299.80	30.67	1	1	.00	1607191.01

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANDEY.T.D. TURNOVER	PCNT OF TOTAL	DIS CDE	PCK QTY	EXPORT PRICE	CUMULATIVE TURNOVER
54	R623348	12	1251.51	15018.12	30.96	3	1	1191.91	1622209.13
55	633329	211	70.99	14978.89	31.24	3	1	.00	1637188.02
56	615410	542	26.78	14514.76	31.52	4	1	.00	1651702.78
57	9009939	1953	7.40	14452.20	31.79	1	4	7.05	1666154.98
58	632922	1182	12.04	14231.28	32.07	1	1	.00	1680386.26
59	515365	504	27.98	14101.92	32.33	4	1	.00	1694488.18
60	505367	112	122.77	13750.24	32.60	4	1	.00	1708238.42
61	9009940	1937	7.04	13636.48	32.66	1	4	6.70	1721874.90
62	9009957	428	31.80	13610.40	33.12	5	1	30.29	1735485.30
63	229978	250	53.86	13465.00	33.37	1	1	.00	1748950.30
64	632284	13	1032.58	13423.54	33.63	3	1	.00	1762373.84
65	R252262	140	95.55	13377.00	33.89	3	1	91.00	1775750.84
66	260401	147	90.27	13269.69	34.14	4	1	.00	1789020.53
67	261489	10	1322.06	13220.60	34.39	3	1	.00	1802241.13
68	624993	117	112.54	13167.18	34.64	1	1	.00	1815408.31
69	632923	1040	12.64	13145.60	34.89	1	1	.00	1828553.91
70	R609386	33	397.69	13123.77	35.14	3	1	378.75	1841677.68
71	264120	2682	4.87	13061.34	35.39	4	1	.00	1854739.02
72	167053	355	36.71	13032.05	35.64	1	1	.00	1867771.07
73	501576	4134	3.13	12939.42	35.89	4	2	.00	1880710.49
74	503366	277	46.64	12919.28	36.13	1	1	.00	1893629.77
75	R436029	5	2561.15	12805.75	36.38	2	1	.00	1906435.52
76	112294	1039	12.30	12779.70	36.62	0	1	.00	1919215.22
77	R505763	14	893.47	12508.58	36.86		1	850.92	1931723.80
78	R613097	306	40.45	12377.70	37.10	1	1	38.52	1944101.50
79	619600	390	30.99	12086.10	37.33	1	1	.00	1956187.60
80	170835	582	20.76	12082.32	37.56	4	1	.00	1968269.92
81	172287	1258	9.51	11963.58	37.79	1	1	.00	1980233.50
82	179426	75	157.68	11826.00	38.01	4	1	.00	1992059.50
83	501531	3658	3.13	11449.54	38.23	4	1	.00	2003509.04
84	117302	17	666.82	11335.94	38.45	4	1	.00	2014844.98
85	255982	306	37.01	11325.06	38.66	4	1	.00	2026170.04
86	9050406	120	94.34	11320.80	38.88	6	1	89.85	2037490.84
87	260204	735	15.35	11282.25	39.10	1	1	.00	2048773.09
88	145450	3260	3.43	11181.80	39.31	4	1	.00	2059954.89
89	9009924	363	30.56	11093.28	39.52	5	1	29.10	2071048.17
90	R261981	9	1230.86	11077.74	39.73	3	1	1172.25	2082125.91
91	502772	161	68.61	11046.21	39.94	4	1	.00	2093172.12
92	608263	95	115.55	10977.25	40.15	1	1	.00	2104149.37
93	631229	213	51.46	10960.98	40.36	1	1	.00	2115110.35
94	632257	140	78.24	10953.60	40.57	4	1	.00	2126063.95
95	9009917	1347	7.97	10735.59	40.78	1	4	7.59	2136799.54
96	241054	553	19.26	10650.78	40.98	4	1	.00	2147450.32
97	660006	3	3394.59	10183.77	41.17	3	1	.00	2157634.09
98	506975	3	3394.59	10183.77	41.37	3	1	.00	2167817.86
99	512797	7	1454.82	10183.74	41.56	3	1	.00	2178001.60
100	573731	73	137.22	10017.06	41.75	4	1	.00	2188018.66
101	257530	51	196.19	10005.69	41.94	4	1	.00	2198024.35
102	627861	5	1974.80	9874.00	42.13	3	1	.00	2207898.35
103	9009916	1310	7.53	9864.30	42.32	1	4	7.17	2217762.65
104	R618253	8	1230.86	9846.88	42.51		1	1172.25	2227609.53
105	110301	127	75.83	9630.41	42.69	4	1	.00	2237239.94
106	116144	274	34.91	9565.34	42.87	4	1	.00	2246805.28

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANDEY.T.D. TURNOVER	PCNT OF TOTAL	CIS CDE	PCK QTY	EXPCRT PRICE	CUMULATIVE TURNOVER
107	694052	2686	3.55	9535.30	43.06	4	1	.00	2256340.58
108	259523	1778	5.36	9530.08	43.24	4	1	.00	2265870.66
109	R117405	148	63.63	9417.24	43.42	1	1	60.60	2275287.90
110	R518694	7	1329.30	9305.10	43.60		1	1266.00	2284593.00
111	618254	4	2301.17	9204.68	43.77	3	1	.00	2293797.68
112	619359	2194	4.15	9105.10	43.94	5	1	.00	2302902.78
113	514433	79	114.35	9033.65	44.12	4	1	.00	2311936.43
114	614868	94	95.69	8994.86	44.29	4	1	.00	2320931.29
115	615932	73	122.77	8962.21	44.46	1	1	.00	2329893.50
116	620462	17	519.98	8839.66	44.63	4	1	.00	2338733.16
117	516109	202	43.63	8813.26	44.80	4	1	.00	2347546.42
118	515194	3	2909.65	8728.95	44.96	3	1	.00	2356275.37
119	R629305	151	57.66	8706.66	45.13	1	1	54.91	2364982.03
120	511632	874	9.87	8626.38	45.29	4	1	.00	2373608.41
121	238670	4466	1.93	8619.38	45.46	4	2	1.70	2382227.79
122	R172034	7	1229.29	8605.03	45.62	3	1	1170.75	2390832.82
123	618554	1084	7.82	8476.88	45.78	5	1	.00	2399309.70
124	262990	66	127.59	8420.94	45.95	4	1	.00	2407730.64
125	9009956	264	31.80	8395.20	46.11	5	1	30.29	2416125.84
126	171316	421	19.86	8361.06	46.26	4	1	.00	2424486.90
127	626966	826	10.11	8350.86	46.42	4	1	.00	2432837.76
128	R440763	4	2081.75	8327.00	46.58	3	1	.00	2441164.76
129	504352	3385	2.44	8259.40	46.74	1	1	.00	2449424.16
130	637023	3602	2.29	8248.58	46.90	4	1	.00	2457672.74
131	262996	1461	5.54	8093.94	47.05	1	1	.00	2465766.68
132	302535	1308	6.14	8031.12	47.21	1	1	.00	2473797.80
133	519146	138	58.08	8015.04	47.36	3	1	.00	2481812.84
134	639060	1195	6.62	7910.90	47.51	1	1	.00	2489723.74
135	R512181	59	131.25	7743.75	47.66		1	125.00	2497467.49
136	505835	547	14.14	7734.58	47.81	1	1	.00	2505202.07
137	548588	152	50.55	7683.60	47.95	4	1	.00	2512885.67
138	9009996	251	30.56	7670.56	48.10	5	1	29.10	2520556.23
139	616315	182	41.53	7558.46	48.24	4	1	.00	2528114.69
140	252809	446	16.55	7381.30	48.38	1	1	.00	2535495.99
141	632009	178	40.92	7283.76	48.52	4	1	.00	2542779.75
142	238770	4605	1.58	7275.90	48.66	4	2	1.50	2550055.65
143	618959	228	31.90	7273.20	48.80	4	1	.00	2557328.85
144	628291	205	34.91	7156.55	48.94	4	1	.00	2564485.40
145	R618261	5	1428.25	7141.25	49.07	3	1	1360.24	2571626.65
146	502706	98	72.82	7136.36	49.21	4	1	.00	2578763.01
147	9009874	675	10.48	7074.00	49.34	0	1	9.98	2585837.01
148	508473	243	28.89	7020.27	49.48	1	1	.00	2592857.28
149	331960	30	232.30	6969.00	49.61	4	1	.00	2599826.28
150	9009876	621	11.08	6880.68	49.74	0	1	10.55	2606706.96
151	330076	346	19.56	6767.76	49.87	1	1	.00	2613474.72
152	R108966	17	397.69	6760.73	50.00	3	1	378.75	2620235.45
153	613966	1275	5.30	6757.50	50.13	4	2	.00	2626992.95
154	627792	3	2240.32	6720.96	50.26	3	1	.00	2633713.91
155	106295	1738	3.85	6691.30	50.39	1	1	.00	2640405.21
156	166681	3362	1.99	6690.38	50.51	4	2	.00	2647095.59
157	596563	173	38.52	6663.96	50.64	4	1	.00	2653759.55
158	9007222	90	73.72	6634.80	50.77	0	1	70.21	2660394.35
159	113947	1149	5.76	6616.24	50.89	1	1	5.49	2667012.59

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANDEY.T.D. TURNOVER	PCNT OF TOTAL	DIS CDE	PCK QTY	EXPORT PRICE	CUMULATIVE TURNOVER
160	636020	118	55.97	6604.46	51.02	4	1	.00	2673617.05
161	505388	163410	.04	6536.40	51.14	4	100	.00	2680153.45
162	218692	163	40.02	6523.26	51.27	4	1	.00	2686676.71
163	512181	36	180.55	6499.80	51.39	1	1	.00	2693176.51
164	608130	1143	5.66	6469.38	51.52	1	1	.00	2699645.89
165	590783	1032	6.26	6460.32	51.64	1	1	.00	2706106.21
166	264733	239	26.78	6400.42	51.76	4	1	.00	2712506.63
167	627190	4	1596.80	6387.20	51.88	3	1	.00	2718893.83
168	507454	347	18.36	6370.92	52.00	1	1	.00	2725264.75
169	256273	295	21.36	6301.20	52.12	1	1	.00	2731565.95
170	616422	930	6.74	6268.20	52.24	1	1	.00	2737834.15
171	109402	1702	3.67	6246.34	52.36	1	1	.00	2744080.49
172	629122	5	1246.47	6232.35	52.48	3	1	.00	2750312.84
173	P440200	3	2054.09	6162.27	52.60	3	1	.00	2756475.11
174	R440609	3	2054.09	6162.27	52.72	2	1	.00	2762637.38
175	106321	242	25.28	6117.76	52.83	1	1	.00	2768755.14
176	241721	242	24.98	6045.16	52.95	4	1	.00	2774800.30
177	240524	1355	4.45	6029.75	53.06	4	1	.00	2780830.05
178	212523	3796	1.56	5921.76	53.18	4	2	1.49	2786751.81
179	100644	7502	.78	5851.56	53.29	4	2	.00	2792603.37
180	508483	107	54.46	5827.22	53.40	4	1	.00	2798430.59
181	213342	5400	1.06	5724.00	53.51	0	1	.00	2804154.59
182	616542	95	60.18	5717.10	53.62	1	1	.00	2809871.69
183	R117380	90	63.08	5677.20	53.73	1	1	60.08	2815548.89
184	622249	200	28.29	5658.00	53.84	4	1	.00	2821206.89
185	9009873	534	10.48	5596.32	53.94	0	1	9.98	2826803.21
186	175402	1975	2.80	5530.00	54.05	4	1	.00	2832333.21
187	597794	82	67.40	5526.80	54.15	4	1	.00	2837860.01
188	638044	5582	.99	5526.18	54.26	4	1	.00	2843386.19
189	617744	553	9.99	5524.47	54.36	4	1	.00	2848910.66
190	621091	1173	4.69	5501.37	54.47	4	1	.00	2854412.03
191	680030	48	113.14	5430.72	54.57	4	1	.00	2859842.75
192	165974	762	7.10	5410.20	54.68	1	1	.00	2865252.95
193	501219	395	13.54	5348.30	54.78	4	1	.00	2870601.25
194	331841	653	8.18	5341.54	54.88	1	1	.00	2875942.79
195	100587	172	30.69	5278.68	54.98	1	1	.00	2881221.47
196	263625	26	202.21	5257.46	55.08	1	1	.00	2886478.93
197	621092	1102	4.69	5168.38	55.18	4	1	.00	2891647.31
198	617179	3	1720.34	5161.02	55.28	3	1	.00	2896808.33
199	165153	1643	3.13	5142.59	55.38	4	1	.00	2901950.92
200	9007019	5	1028.48	5142.40	55.47	0	1	979.50	2907093.32
201	R442398	2	2561.15	5122.30	55.57	2	1	.00	2912215.62
202	509960	135	37.91	5117.85	55.67	4	1	.00	2917333.47
203	R115958	4	1276.64	5106.56	55.77	3	1	1215.85	2922440.03
204	9009980	105	48.14	5054.70	55.86	4	1	45.85	2927494.73
205	174456	1151	4.39	5052.89	55.96	5	1	.00	2932547.62
206	581285	373	13.54	5050.42	56.06	4	1	.00	2937598.04
207	117307	57	88.47	5042.79	56.15	4	1	.00	2942640.83
208	9007022	127	39.69	5040.63	56.25	0	1	37.80	2947681.46
209	504298	2987	1.68	5018.16	56.34	4	1	1.60	2952699.62
210	171215	109	45.74	4985.66	56.44	1	1	.00	2957685.28
211	632616	1047	4.75	4973.25	56.53	1	1	.00	2962658.53
212	261488	5	992.01	4960.05	56.63	3	1	.00	2967618.58

A B C ANALYSIS BY TURNOVER

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213	166495	562	8.79	4939.98	56.72	1	1	.00	2972558.56
214	194778	346	14.14	4892.44	56.82	1	1	.00	2977451.00
215	519234	40	121.70	4868.00	56.91	3	1	.00	2982319.00
216	229348	5572	.87	4847.64	57.00	0	1	.00	2987166.64
217	R518685	4	1209.58	4838.32	57.09	3	1	1151.98	2992004.96
218	509144	499	9.63	4805.37	57.19	4	1	.00	2996810.33
219	629038	4	1191.15	4764.60	57.28	3	1	.00	3001574.93
220	114574	52	91.48	4756.96	57.37	4	1	.00	3006331.89
221	109996	1825	2.59	4726.75	57.46	5	1	.00	3011058.64
222	330621	1075	4.39	4719.25	57.55	1	1	.00	3015777.89
223	263997	2907	1.62	4709.34	57.64	4	1	.00	3020487.23
224	9009872	447	10.48	4684.56	57.73	0	1	9.98	3025171.79
225	R117425	46	101.51	4669.46	57.82	1	1	96.68	3029841.25
226	104159	1379	3.37	4647.23	57.91	4	1	.00	3034488.48
227	R115959	7	663.23	4642.61	57.99		1	631.65	3039131.09
228	506834	1330	3.49	4641.70	58.08	5	1	.00	3043772.79
229	240180	6553	.70	4587.10	58.17	4	12	.67	3048359.89
230	R602230	7	655.04	4585.28	58.26		1	623.85	3052945.17
231	R615932	75	60.90	4567.50	58.34		1	58.00	3057512.67
232	R616542	130	34.65	4504.50	58.43		1	33.00	3062017.17
233	517174	44	101.71	4475.24	58.52	4	1	.00	3066492.41
234	R240440	66	67.73	4470.18	58.60	1	1	64.50	3070962.59
235	171305	3098	1.44	4461.12	58.69	4	1	.00	3075423.71
236	9007183	1180	3.78	4460.40	58.77	0	1	3.60	3079884.11
237	R161852	70	63.63	4454.10	58.86	1	1	60.60	3084338.21
238	610058	2	2220.04	4440.08	58.94	3	1	.00	3088778.29
239	615748	16	276.84	4429.44	59.03	4	1	.00	3093207.73
240	9009982	204	21.69	4424.76	59.11	4	1	20.66	3097632.49
241	9007264	78	56.18	4382.04	59.19	0	1	53.50	3102014.53
242	632744	46	95.09	4374.14	59.28	4	1	.00	3106388.67
243	R412447	3	1452.98	4358.94	59.36	2	1	.00	3110747.61
244	R440817	3	1452.98	4358.94	59.44	2	1	.00	3115106.55
245	R241720	33	131.25	4331.25	59.53		1	125.00	3119437.80
246	596572	186	23.17	4309.62	59.61	4	1	.00	3123747.42
247	104992	90	47.84	4305.60	59.69	1	1	.00	3128053.02
248	R629949	88	48.83	4297.04	59.77	1	1	46.50	3132350.06
249	626756	250	17.15	4287.50	59.85	4	1	.00	3136637.56
250	596573	185	23.17	4286.45	59.94	4	1	.00	3140924.01
251	140783	1002	4.27	4278.54	60.02	1	1	.00	3145202.55
252	109740	302	14.14	4270.28	60.10	1	1	.00	3149472.83
253	9009862	926	4.58	4241.08	60.18	0	1	4.36	3153713.91
254	799005	10839	.39	4227.21	60.26	4	10	.00	3157941.12
255	113365	216	19.56	4224.96	60.34	1	1	.00	3162166.08
256	532895	482	8.67	4178.94	60.42	4	1	.00	3166345.02
257	597793	62	67.40	4178.80	60.50	4	1	.00	3170523.82
258	9009881	1004	4.16	4176.64	60.58	0	1	3.96	3174700.46
259	117246	1100	3.79	4169.00	60.66	4	1	.00	3178869.46
260	610957	961	4.33	4161.13	60.74	1	1	.00	3183030.59
261	169144	2	2079.90	4159.80	60.82	3	1	.00	3187190.39
262	9009875	374	11.08	4143.92	60.90	0	1	10.55	3191334.31
263	179416	1512	2.71	4097.52	60.98	1	1	.00	3195431.83
264	646167	457	8.85	4058.16	61.05	4	1	.00	3199489.99
265	597031	151	26.78	4043.78	61.13	4	1	.00	3203533.77

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANDEY.T.D. TURNOVER	PCNT OF TOTAL	DIS CDE	PCK QTY	EXPORT PRICE	CUMULATIVE TURNOVER
266	615612	2974	1.35	4014.90	61.21	4	6	.00	3207548.67
267	516659	38	105.32	4002.16	61.28	4	1	.00	3211550.83
268	330636	831	4.81	3997.11	61.36	4	1	.00	3215547.94
269	596066	187	21.36	3994.32	61.44	4	1	.00	3219542.26
270	516409	79	50.55	3993.45	61.51	4	1	.00	3223535.71
271	596065	186	21.36	3972.96	61.59	4	1	.00	3227508.67
272	615823	159	24.98	3971.82	61.66	4	1	.00	3231480.49
273	646166	446	8.88	3960.48	61.74	4	1	.00	3235440.97
274	9007109	1569	2.52	3953.88	61.82	0	1	2.40	3239394.85
275	9071500	153	25.79	3945.87	61.89	0	1	24.56	3243340.72
276	R623275	3	1314.34	3943.02	61.97		1	1251.75	3247283.74
277	622042	332	11.80	3917.60	62.04	4	1	.00	3251201.34
278	264734	146	26.78	3909.68	62.12	4	1	.00	3255111.22
279	112305	410	9.51	3899.10	62.19	1	1	.00	3259010.32
280	117383	2701	1.44	3889.44	62.26	4	2	.00	3262899.76
281	632432	183	21.06	3853.98	62.34	1	1	.00	3266753.74
282	114042	12	318.97	3827.64	62.41	4	1	.00	3270581.38
283	257445	263	14.44	3797.72	62.48	1	1	.00	3274379.10
284	613965	757	5.00	3785.00	62.56	4	2	.00	3278164.10
285	242199	582	6.50	3783.00	62.63	4	2	.00	3281947.10
286	R106408	94	40.16	3775.04	62.70	1	1	38.25	3285722.14
287	249551	49	77.03	3774.47	62.77	1	1	.00	3289496.61
288	260242	304	12.34	3751.36	62.84	1	1	.00	3293247.97
289	229071	1113	3.37	3750.81	62.91	4	1	.00	3296998.78
290	331961	16	232.30	3716.80	62.99	4	1	.00	3300715.58
291	799053	732	5.06	3703.92	63.06	4	1	.00	3304419.50
292	509623	1101	3.35	3688.35	63.13	0	1	.00	3308107.85
293	R009970	60	61.43	3685.80	63.20	0	1	58.50	3311793.65
294	596786	172	21.36	3673.92	63.27	4	1	.00	3315467.57
295	259931	3241	1.13	3662.33	63.34	4	1	.00	3319129.90
296	518432	270	13.54	3655.80	63.41	4	1	.00	3322785.70
297	R611093	39	93.45	3644.55	63.48	1	1	89.00	3326430.25
298	252220	170	21.36	3631.20	63.55	1	1	.00	3330061.45
299	263910	22	164.90	3627.80	63.61	1	1	.00	3333689.25
300	R513141	9	400.84	3607.56	63.68		1	381.75	3337296.81
301	609958	575	6.26	3599.50	63.75	1	1	.00	3340896.31
302	532298	139	25.88	3597.32	63.82	4	1	.00	3344493.63
303	615808	364	9.87	3592.68	63.89	4	1	.00	3348086.31
304	118729	265	13.54	3588.10	63.96	4	1	.00	3351674.41
305	241955	1300	2.74	3562.00	64.03	4	1	.00	3355236.41
306	179415	1424	2.50	3560.00	64.09	1	1	.00	3358796.41
307	605826	365	9.75	3558.75	64.16	1	1	.00	3362355.16
308	501207	166	21.36	3545.76	64.23	4	1	.00	3365900.92
309	9009896	449	7.85	3524.65	64.30	0	1	7.48	3369425.57
310	330141	179	19.56	3501.24	64.36	4	1	.00	3372926.81
311	205547	40	87.26	3490.40	64.43	4	1	.00	3376417.21
312	608131	714	4.87	3477.18	64.50	1	1	.00	3379894.39
313	248180	632	5.48	3463.36	64.56	1	1	.00	3383357.75
314	217264	74	46.64	3451.36	64.63	4	1	.00	3386809.11
315	638056	13271	.26	3450.46	64.69	4	1	.00	3390259.57
316	116107	16	215.45	3447.20	64.76	4	1	.00	3393706.77
317	680132	2	1720.34	3440.68	64.83	3	1	.00	3397147.45
318	626965	920	3.73	3431.60	64.89	4	1	.00	3400579.05

A B C ANALYSIS BY TURNOVER

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319	609957	606	5.66	3429.96	64.96	1	1	.00	3404009.01
320	646165	461	7.40	3411.40	65.02	4	1	.00	3407420.41
321	512774	423	8.06	3409.38	65.09	1	1	6.19	3410829.79
322	119144	56	60.85	3407.60	65.15	3	1	.00	3414237.39
323	627066	18	188.97	3401.46	65.22	4	1	.00	3417638.85
324	R165495	66	51.24	3381.84	65.28	1	1	48.60	3421020.69
325	R108967	8	420.53	3364.24	65.35	3	1	400.50	3424384.93
326	241795	34	98.70	3355.80	65.41	4	1	.00	3427740.73
327	505285	16758	.20	3351.60	65.47	4	20	.00	3431092.33
328	597002	125	26.78	3347.50	65.54	4	1	.00	3434439.83
329	118366	125	26.78	3347.50	65.60	4	1	.00	3437787.33
330	249474	44	75.83	3336.52	65.66	4	1	.00	3441123.85
331	249447	135	24.67	3330.45	65.73	4	1	.00	3444454.30
332	116039	251	13.24	3323.24	65.79	4	1	.00	3447777.54
333	694229	86	38.52	3312.72	65.85	5	1	.00	3451090.26
334	179627	1765	1.87	3300.55	65.92	4	1	.00	3454390.81
335	108626	215	15.35	3300.25	65.98	4	1	.00	3457691.06
336	R612545	33	99.75	3291.75	66.04	3	1	95.00	3460982.81
337	9007033	65	50.40	3276.00	66.11		1	48.00	3464258.81
338	586969	135	24.07	3249.45	66.17	1	1	.00	3467508.26
339	R166102	34	95.55	3248.70	66.23	3	1	91.00	3470756.96
340	631110	130	24.98	3247.40	66.29	4	1	.00	3474004.36
341	629034	4	811.31	3245.24	66.35	3	1	.00	3477249.60
342	694072	341	9.51	3242.91	66.42	1	1	.00	3480492.51
343	617738	121	26.78	3240.38	66.48	4	1	.00	3483732.89
344	609564	33	98.10	3237.30	66.54	1	1	.00	3486970.19
345	330631	477	6.74	3214.98	66.60	1	1	.00	3490185.17
346	9071613	25	127.96	3199.00	66.66	0	1	121.87	3493384.17
347	240693	1140	2.80	3192.00	66.72	4	1	.00	3496576.17
348	113911	746	4.27	3185.42	66.78	4	1	.00	3499761.59
349	9009922	540	5.88	3175.20	66.84	1	4	5.60	3502936.79
350	179703	412	7.70	3172.40	66.90	1	1	.00	3506109.19
351	166909	30	105.32	3159.60	66.97	4	1	.00	3509268.79
352	240564	4211	.75	3158.25	67.03	4	2	.00	3512427.04
353	170985	57	55.37	3156.09	67.09	1	1	.00	3515583.13
354	9007309	88	35.79	3149.52	67.15		1	34.09	3518732.65
355	104368	1005	3.13	3145.65	67.21	4	1	.00	3521878.30
356	144922	11219	.28	3141.32	67.27	4	25	.00	3525019.62
357	164447	347	9.03	3133.41	67.33	4	2	.00	3528153.03
358	173294	104	30.09	3129.36	67.39	4	1	.00	3531282.39
359	259393	120	25.88	3105.60	67.44	1	1	.00	3534387.99
360	619576	542	5.72	3100.24	67.50	4	1	.00	3537488.23
361	117343	48	64.40	3091.20	67.56	4	1	.00	3540579.43
362	R105057	244	12.60	3074.40	67.62	4	1	12.00	3543653.83
363	117244	157	19.56	3070.92	67.68	4	1	.00	3546724.75
364	523331	126	24.37	3070.62	67.74	1	1	.00	3549795.37
365	262633	194	15.65	3036.10	67.80	1	1	.00	3552831.47
366	104099	121	24.98	3022.58	67.85	4	1	.00	3555854.05
367	9007008	36	83.92	3021.12	67.91	0	1	79.92	3558875.17
368	R609780	8	377.23	3017.84	67.97	3	1	359.27	3561893.01
369	9009981	69	43.68	3013.92	68.03	4	1	41.60	3564906.93
370	513140	5	597.01	2985.05	68.08	1	1	.00	3567891.98
371	606923	91	32.80	2984.80	68.14	1	1	.00	3570876.78

A B C ANALYSIS BY TURNOVER

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372	504640	29	102.31	2966.99	68.20	1	1	.00	3573843.77
373	639394	367	8.06	2958.02	68.25	1	1	.00	3576801.79
374	9007108	4	737.10	2948.40	68.31	0	1	702.00	3579750.19
375	508991	2448	1.19	2913.12	68.37	4	1	.00	3582663.31
376	262108	78	37.31	2910.18	68.42	1	1	.00	3585573.49
377	621994	116	24.98	2897.68	68.48	4	1	.00	3588471.17
378	179664	130	22.27	2895.10	68.53	4	1	.00	3591366.27
379	507650	62	46.64	2891.68	68.59	4	1	.00	3594257.95
380	R259814	75	38.29	2871.75	68.64	1	1	36.47	3597129.70
381	172500	212	13.54	2870.48	68.70	1	1	.00	3600000.18
382	508336	65	43.63	2835.95	68.75	4	1	.00	3602836.13
383	632401	60	46.94	2816.40	68.80	1	1	.00	3605652.53
384	242253	433	6.50	2814.50	68.86	4	2	.00	3608467.03
385	259524	792	3.55	2811.60	68.91	4	2	.00	3611278.63
386	R624147	222	12.60	2797.20	68.97	4	1	12.00	3614075.83
387	R108965	7	397.69	2783.83	69.02	3	1	378.75	3616859.66
388	636284	220	12.64	2780.80	69.07	4	1	.00	3619640.46
389	117326	2010	1.38	2773.80	69.12	4	1	.00	3622414.26
390	612564	1022	2.71	2769.62	69.18	1	1	.00	3625183.88
391	105587	364	7.58	2759.12	69.23	4	1	.00	3627943.00
392	9009871	263	10.48	2756.24	69.28	0	1	9.98	3630699.24
393	109083	263	10.47	2753.61	69.33	1	1	.00	3633452.85
394	620199	51	53.86	2746.86	69.39	4	1	.00	3636199.71
395	641131	43	63.79	2742.97	69.44	1	1	.00	3638942.68
396	103820	230	11.92	2741.60	69.49	4	1	.00	3641684.28
397	247540	47	58.08	2729.76	69.54	1	1	.00	3644414.04
398	606922	83	32.80	2722.40	69.60	1	1	.00	3647136.44
399	179442	98	27.68	2712.64	69.65	4	1	.00	3649849.08
400	615585	18	150.46	2708.28	69.70	4	1	.00	3652557.36
401	109421	692	3.91	2705.72	69.75	4	1	.00	3655263.08
402	229061	20	134.81	2696.20	69.80	4	1	.00	3657959.28
403	9009883	582	4.58	2665.56	69.85	0	1	4.36	3660624.84
404	171309	230	11.56	2658.80	69.90	4	1	.00	3663283.64
405	541123	138	19.26	2657.88	69.95	1	1	.00	3665941.52
406	516570	152	17.45	2652.40	70.01	4	1	.00	3668593.92
407	9009880	634	4.16	2637.44	70.06	0	1	3.96	3671231.36
408	171041	85	30.99	2634.15	70.11	4	1	.00	3673865.51
409	637351	270	9.75	2632.50	70.16	4	1	.00	3676498.01
410	9009988	690	3.78	2608.20	70.21	5	2	3.60	3679106.21
411	602231	3	861.09	2583.27	70.26	3	1	.00	3681689.48
412	R617173	3	861.00	2583.00	70.30		1	820.00	3684272.48
413	261009	2483	1.04	2582.32	70.35	4	1	.00	3686854.80
414	514026	44	58.68	2581.92	70.40	4	1	.00	3689436.72
415	622759	36	71.62	2576.32	70.45	4	1	.00	3692015.04
416	511535	1238	2.08	2575.04	70.50	1	1	.00	3694590.08
417	174069	43	59.88	2574.84	70.55	4	1	.00	3697164.92
418	507428	144	17.75	2556.00	70.60	4	1	.00	3699720.92
419	9009923	421	6.04	2542.84	70.65	1	4	5.75	3702263.76
420	115122	88	28.89	2542.32	70.70	4	1	.00	3704806.08
421	629869	148	17.15	2536.20	70.74	4	1	.00	3707344.28
422	206137	55	46.04	2532.20	70.79	4	1	.00	3709876.48
423	622247	868	2.92	2528.72	70.84	4	1	.00	3712405.20
424	115007	1590	1.59	2528.10	70.89	4	1	.00	3714933.30

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425	241103	3822	.66	2522.52	70.94	4	10	.00	3717455.82
426	216295	36	70.07	2522.52	70.99	3	1	.00	3719978.34
427	247526	238	10.59	2520.42	71.03	1	1	.00	3722498.76
428	544862	23	106.93	2505.39	71.08	4	1	.00	3725004.15
429	259022	99	25.28	2502.72	71.13	1	1	.00	3727506.87
430	190069	1151	2.17	2497.67	71.18	4	1	.00	3730004.54
431	9007184	891	2.80	2494.80	71.22	0	1	2.67	3732499.34
432	R422045	1	2491.09	2491.09	71.27	2	1	.00	3734990.43
433	R440739	1	2491.09	2491.09	71.32	3	1	.00	3737481.52
434	R422062	1	2491.09	2491.09	71.37	2	1	.00	3739972.61
435	614943	429	5.78	2479.62	71.41	1	1	.00	3742452.23
436	535705	38	65.21	2477.98	71.46	0	1	.00	3744930.21
437	694288	241	10.23	2465.43	71.51	1	1	.00	3747395.64
438	261990	101	24.37	2461.37	71.56	1	1	.00	3749857.01
439	101519	905	2.71	2452.55	71.60	4	1	.00	3752309.56
440	171199	783	3.13	2450.79	71.65	4	1	.00	3754760.35
441	228412	83	29.49	2447.67	71.70	4	1	.00	3757208.02
442	247513	209	11.68	2441.12	71.74	1	1	.00	3759649.14
443	615271	20	121.57	2431.40	71.79	4	1	.00	3762080.54
444	9001050	121	19.87	2404.27	71.84	0	1	.00	3764484.81
445	615279	25	95.69	2392.25	71.88	4	1	.00	3766877.06
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	71.93	4	1	.00	3769251.12
447	610698	39	60.78	2370.42	71.97	4	1	.00	3771621.54
448	622979	1206	1.96	2363.76	72.02	4	6	.00	3773985.30
449	619565	581	4.03	2341.43	72.06	4	1	.00	3776326.73
450	217266	2067	1.13	2335.71	72.11	4	10	.00	3778662.44
451	109401	900	2.59	2331.00	72.15	1	1	.00	3780993.44
452	R431236	1	2330.67	2330.67	72.19	2	1	.00	3783324.11
453	9001093	117	19.87	2324.79	72.24	0	1	.00	3785648.90
454	118159	1375	1.69	2323.75	72.28	5	1	.00	3787972.65
455	631650	5	464.61	2323.05	72.33	1	1	.00	3790295.70
456	680105	4	576.55	2306.20	72.37	1	1	.00	3792601.90
457	171327	2155	1.07	2305.65	72.42	4	10	.00	3794907.75
458	102123	225	10.23	2301.75	72.46	4	1	.00	3797209.50
459	638548	13	176.94	2300.22	72.50	4	1	.00	3799509.72
460	629085	26	88.47	2300.22	72.55	4	1	.00	3801809.94
461	541453	1031	2.23	2299.13	72.59	4	1	.00	3804109.07
462	229525	2166	1.06	2295.96	72.64	1	1	.00	3806405.03
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	72.68	1	1	.00	3808700.27
464	609570	61	37.61	2294.21	72.72	1	1	.00	3810994.48
465	680107	4	572.94	2291.76	72.77	1	1	.00	3813286.24
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	72.81	4	1	.00	3815577.28
467	325235	20800	.11	2288.00	72.85	4	50	.00	3817865.28
468	518088	52	43.93	2284.36	72.90	4	1	.00	3820149.64
469	212275	165	13.84	2283.60	72.94	4	1	.00	3822433.24
470	103429	20	113.14	2262.80	72.98	4	1	.00	3824696.04
471	9009999	842	2.68	2256.56	73.03	1	1	2.55	3826952.60
472	547750	125	18.05	2256.25	73.07	1	1	.00	3829208.85
473	331815	52	43.33	2253.16	73.11	1	1	.00	3831462.01
474	514313	34	66.20	2250.80	73.16	4	1	.00	3833712.81
475	179444	81	27.68	2242.08	73.20	4	1	.00	3835954.89
476	507765	16	139.62	2233.92	73.24	1	1	.00	3838188.81
477	546611	273	8.18	2233.14	73.26	1	1	.00	3840421.95

A B C ANALYSIS BY TURNOVER

SEQ NO	PART NUMBER	QUANTITY	Y.T.D. COST PRICE	LANCEY.T.D. TURNOVER	PCNT OF TOTAL	DIS CDE	PCK QTY	EXPORT PRICE	CUMULATIVE TURNOVER
478	511756	112	19.86	2224.32	73.33	1	1	.00	3842646.27
479	171419	139	15.95	2217.05	73.37	1	1	.00	3844863.32
480	191175	2011	1.10	2212.10	73.41	4	1	.00	3847075.42
481	R602388	3	735.00	2205.00	73.45		1	700.00	3849280.42
482	116070	320	6.86	2195.20	73.50	1	1	.00	3851475.62
483	R517097	174	12.60	2192.40	73.54	4	1	12.00	3853668.02
484	146142	19846	.11	2183.06	73.58	4	50	.00	3855851.08
485	799006	2266	.96	2175.36	73.62	4	10	.00	3858026.44
486	513392	14	155.27	2173.78	73.66	4	1	.00	3860200.22
487	535625	10	217.36	2173.60	73.70	0	1	.00	3862373.82
488	140363	896	2.41	2159.36	73.74	1	1	.00	3864533.18
489	229062	16	134.81	2156.96	73.79	4	1	.00	3866690.14
490	9007078	1365	1.58	2156.70	73.83	0	1	1.50	3868846.84
491	631111	86	24.98	2148.28	73.87	4	1	.00	3870995.12
492	642826	241	8.88	2140.08	73.91	4	1	.00	3873135.20
493	638043	2415	.88	2125.20	73.95	4	1	.00	3875260.40
494	617465	1	2124.15	2124.15	73.99	3	1	.00	3877384.55
495	R252683	57	37.11	2115.27	74.03	1	1	35.34	3879499.82
496	618432	59	35.81	2112.79	74.07	4	1	.00	3881612.61
497	179443	72	29.19	2101.68	74.11	4	1	.00	3883714.29
498	609541	48	43.63	2094.24	74.15	1	1	.00	3885808.53
499	9009979	1260	1.66	2091.60	74.19	1	1	1.58	3887900.13
500	R118273	4	521.49	2085.96	74.23	8	1	496.66	3889986.09
501	581284	235	8.79	2065.65	74.27	4	1	.00	3892051.74
502	253965	264	7.82	2064.48	74.31	4	1	.00	3894116.22
503	605950	1004	2.05	2058.20	74.35	4	1	1.95	3896174.42
504	140880	51	40.32	2056.32	74.39	1	1	.00	3898230.74
505	R441092	1	2054.09	2054.09	74.43	2	1	.00	3900284.83
506	R441113	1	2054.09	2054.09	74.47	2	1	.00	3902338.92
507	R441268	1	2054.09	2054.09	74.51	2	1	.00	3904393.01
508	R440695	1	2054.09	2054.09	74.54	2	1	.00	3906447.10
509	140362	852	2.41	2053.32	74.58	1	1	.00	3908500.42
510	535901	4	511.73	2046.92	74.62	0	1	.00	3910547.34
511	260388	17	119.76	2035.92	74.66	4	1	.00	3912583.26
512	620161	263	7.70	2025.10	74.70	4	1	.00	3914608.36
513	623275	1	2022.74	2022.74	74.74	3	1	.00	3916631.10
514	330190	140	14.44	2021.60	74.78	4	1	.00	3918652.70
515	622978	2402	.84	2017.68	74.82	4	6	.00	3920670.38
516	9009915	381	5.29	2015.49	74.85	1	4	5.04	3922685.87
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	74.89	4	1	.00	3924679.05
518	597151	72	27.68	1992.96	74.93	4	1	.00	3926672.01
519	619650	176	11.31	1990.56	74.97	1	1	.00	3928662.57
520	114578	21	94.49	1984.29	75.01	4	1	.00	3930646.86
521	611240	166	11.92	1978.72	75.04	1	1	.00	3932625.58
522	511578	415	4.75	1971.25	75.08	4	1	.00	3934596.83
523	115713	23	85.46	1965.58	75.12	1	1	.00	3936562.41
524	R116529	21	93.45	1962.45	75.16	1	1	89.00	3938524.86
525	596737	56	34.91	1954.96	75.19	4	1	.00	3940479.82
526	9001015	98	19.87	1947.26	75.23	0	1	.00	3942427.08
527	513506	342	5.66	1935.72	75.27	1	1	.00	3944362.80
528	115249	44	43.93	1932.92	75.30	1	1	.00	3946295.72
529	503443	52	37.01	1924.52	75.34	4	1	.00	3948220.24
530	265229	3	639.14	1917.42	75.38	1	1	.00	3950137.66

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