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# Freight options: price modelling and empirical analysis

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## **Abstract**

This paper discusses an extension of the traditional lognormal representation for the risk neutral spot freight rate dynamics to a diffusion model overlaid with jumps of random magnitude and arrival. Then, we develop a valuation framework for options on the average spot freight rate, which are commonly traded in the freight derivatives market. By exploiting the computational efficiency of the proposed pricing scheme, we calibrate the jump diffusion model using market quotes of options on the trip-charter route average Baltic Capesize, Panamax and Supramax Indices. We show that the jump-extended setting yields important model improvements over the basic lognormal setting.

*Keywords:* Shipping; Spot freight rates; Jump diffusion model; Forward start average options; Freight option price model

# 1 Introduction

The market for ocean-going shipping freight has undergone a fundamental transformation over the last decade from a service market, where freight rate was simply viewed as the cost of transporting raw materials by sea, to a market where freight rate can be bought and sold for investment purposes like any other financial asset or commodity. A number of factors have contributed to this transformation; the most prominent is the high level of freight rates, fuelled by an increase in the demand for raw materials from resource-poor emerging economies, coupled with prolonged periods of underinvestment in new capacity in the world shipping fleet in the 1980s and 1990s.

The recent years have been characterized by high volatility in the freight market and a corresponding growth in the derivatives market for freight. Traditionally, this market has been used by players in the physical freight market, such as shipowners, operators and trading houses, to hedge their risks, though this is now changing rapidly with the increasing participation of investment banks and hedge funds. Market participants trade forward contracts on shipping freight rates, known as forward freight agreements (FFAs). These are contracts to settle the average spot freight rate or charter hire rate over a specified period of time. FFAs proved to be an effective tool for hedging the physical exposure to the spot freight market and soon attracted the interest of practitioners, shipping market participants, and academics. Although these contracts allow traders to lock in a given freight rate over a period of time, they lack the flexibility that would enable their users to maintain the hedge in case market moved against them. Options, on the other hand, offer this flexibility. Freight options are generally negotiated over-the-counter (OTC) and subsequently cleared through a clearing house. The option market has gained popularity over the recent years, reaching an equivalent trading volume of 250 million tonnes for 2011 and an open interest of 150 million tonnes of cargo.

Freight options fall within the class of average options, also known as Asian options. These are contracts to settle the difference between the average value of some underlying asset and an agreed strike price (fixed strike option), or the difference between the value of the underlying at maturity and the average value (floating strike option). Depending on the duration of the averaging period, there exist variants of this option. Let  $[\tau, T]$ ,  $T > \tau \geq 0$  be the averaging period of the underlying with the option inception at  $t \in [0, T) \subseteq [\tau, T]$  and maturity at  $T$ . Then, we have three cases of Asian options: if  $t = \tau = 0$ , i.e., time to maturity and length of averaging period coincide, we have a *standard* option, if  $t = 0$  and the averaging starts at  $\tau > t$  we have a *forward start* option, whereas for the case  $\tau < t$  with  $\tau = 0$ , i.e., averaging is already started, we have an *in-progress* option.

In this research, we consider options on the spot Baltic Capesize Index (BCI), Baltic Pana-

max Index (BPI) and Baltic Supramax Index (BSI), which we define in detail in Section 2. The option payoff depends on the arithmetic average of the spot prices recorded at discrete time points over the averaging period. We concentrate on the most commonly traded forward start options. Hitherto, determining the fair price for such options has been a nontrivial task due to the unknown probability distribution of the arithmetic average. Based on the assumption of the spot rate evolving according to the lognormal process, Koekebakker et al. (2007) propose an approximation for the FFA rate dynamics and derive a price formula for the average option. Market practitioners also seem to favour the analytical formulae of Turnbull and Wakeman (1991) and Levy (1997), which are based on a lognormal approximation of the average spot rate distribution. Nevertheless, the empirical performance of the lognormal freight option models has not been properly assessed so far, and, given the increasing trading activity of freight options, this can be proved harmful. In addition, historic spot freight rates exhibit frequent upside jumps due to the inability of supply to immediately respond to increased demand for seaborne transportation, while downside movements are also remarkable during the recent market recession (September 2008 to February 2009).

Motivated by the previous discussion, for first time in the literature, we postulate a lognormal process overlaid with time-homogeneous Poisson jumps of normally distributed sizes (Merton, 1976) for the risk neutral spot freight dynamics and compare its performance against the basic lognormal diffusion. The contribution of this paper is twofold. On the computational side, we generalize previous work by Černý and Kyriakou (2011) to the exact valuation of *forward start* discrete arithmetic Asian options, with the *standard* Asian option as a special case. The proposed framework can then be implemented to infer the postulated risk neutral spot rate models using historic option data. On the empirical side, we fit the jump diffusion and lognormal option models to the market quotes of short-term, medium-term and long-term options on the BCI, BPI and BSI over the period January 2008 to July 2010. First, it is shown that high volatility and large jumps in the risk neutral spot BCI, BPI and BSI rates are mostly relevant to short-term contracts, however jumps are still important when pricing longer-term contracts. Second, our analysis of option price error statistics shows that the jump diffusion generates lower error than the lognormal model, especially for longer-term contracts in the panamax and supramax markets, and also reduces the level of underpricing or overpricing. Third, a regression analysis shows that superimposing jumps on the spot rate diffusion improves on the option pricing biases. Thus, we have sufficient evidence against the pure lognormal special case of the jump diffusion paradigm.

Finally, it is worth noting that the option model we examine in this paper can find further application in other areas of transportation research, including truckload options (see Tsai et al., 2011) which provide the right to buy truckload services on specific routes for fixed

prices. Variants of these options of the Asian type are also possible, in which case the proposed model provides an appropriate framework for their trading and pricing.

The outline of the paper is as follows. In Section 2, we introduce the market for freight options. In Section 3, we present the details of the postulated jump diffusion model for the risk neutral dynamics of the spot freight rates. In Section 4, we develop the valuation framework for forward start average options. The data is described in Section 5. The details of our empirical investigation and our findings are presented in Sections 6, 7 and 8. Section 9 concludes.

## 2 The market for freight options

Freight options belong to the wider family of Asian options. In general, Asian options provide a good defense against market manipulation of the underlying spot price prior to settlement, since the settlement price of the option is given by the average of the spot prices over the trading days of the settlement month. Further, the average value is less exposed to extreme movements at maturity resulting in option prices which are lower than the prices of, otherwise identical, plain vanilla options<sup>1</sup>. For these reasons Asian options are popular in thinly traded or highly volatile markets, such as the market for freight.

Freight options in the dry bulk market are traded on the Baltic Capesize Index (BCI), Baltic Panamax Index (BPI) and Baltic Supramax Index (BSI). These indices reflect freight movements in the 3 major classes of vessels used for the transportation of dry bulk commodities. Capesize vessels (172,000 metric tons (mt) deadweight (dwt)) transport iron ore mainly from South America and Australia to the Far East (primarily China), and coal from North America, Australia and South Africa to the Far East and North Europe. The name is attributed to the fact that this type of vessel is too large to transit the Panama canal, hence it has to navigate around Cape Horn. Panamax vessels (74,000 mt dwt) are used mainly to carry grains from North America, Argentina and Australia, and coal from North America, Australia and South Africa either to Europe or the Far East. This is the largest permissible vessel size that can transit the Panama canal fully laden. Finally, supramax vessels are smaller in size (52,454 mt dwt) and more versatile as they can call in more ports with smaller berths and shallow drafts. They are used to transport grains from North America, Argentina and Australia, steel and scrap and other minor bulk products (e.g., sugar, fertilizers, forest products, nonferrous metals and salt) virtually from all over the world. The indices published by the Baltic Exchange reflect the cost of hiring a vessel across a range of indicative shipping routes. As a benchmark for the level

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<sup>1</sup>An exception to that is when the Asian option is priced in the averaging period (in-progress option), in which case the Asian option price can naturally be higher than the vanilla option price, depending on the realization of the underlying asset price up to the valuation date (see Turnbull and Wakeman, 1991). We do not consider in-progress options in this study.

of freight rates, the market uses the trip-charter<sup>2</sup> route average, that is, 4TC for the capesize and panamax and 6TC for the supramax (see Table 1).

The Baltic indices are calculated on a daily basis by the Baltic Exchange based on data supplied by a panel of independent international shipbrokers, and are reported in the market at 13:00 hrs London time. This is an assessment market and the role of panellists is to assess and report a professional judgment of the prevailing open market level for routes defined by the Baltic Exchange. The use of panellists' assessments is motivated by the nature of the shipping markets where it is difficult to have a verifiable rate on a daily basis for every route. Freight options on a Baltic index settle the difference between the arithmetic average of the spot Baltic assessments over the trading days of the settlement month and an agreed strike price. The options are executed between two counterparties through a broker primarily as an OTC contract, though the majority of the trades are subsequently cleared through a clearing house.

### 3 The risk neutral spot freight rate model

Let  $(\Omega, \mathcal{F}, Q)$  be a complete probability space. We interpret  $Q$  as a risk neutral probability measure. Fix constant  $S_0 > 0$  and define the spot freight rate process

$$S_t = S_0 e^{L_t}, \quad (1)$$

where  $L_t, t > 0$  is assumed to be driven by the jump diffusion model

$$L_t = (r - \sigma^2/2 - \lambda E(e^X - 1)) t + \sigma W_t + \sum_{j=1}^{N_t} X_j \quad (2)$$

with  $L_0 \equiv 0$ ,  $r \geq 0$  the continuously compounded risk free rate of interest,  $W_t, t \geq 0$  a standard Brownian motion under  $Q$ ,  $\sigma \geq 0$  the diffusion coefficient, and  $N_t, t \geq 0$  a time-homogeneous  $Q$ -Poisson process with a constant arrival rate of  $\lambda > 0$  jumps per unit time, of size  $X$ ;  $X$  is modelled by a sequence of i.i.d. random variables with mean  $\mu_X := E(X) < \infty$ , variance  $\sigma_X^2 := \text{Var}(X) < \infty$  and characteristic function  $\varphi_X(u) := E(\exp(iuX))$ .  $W, N$  and  $X$  are assumed to be independent. From (1)–(2), we see that small spot movements are being accounted for by the diffusion term, whereas the jump terms model rarer large moves.

In this paper, we restrict our attention to the Merton model<sup>3</sup> (Merton, 1976) in which  $X$

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<sup>2</sup>A trip-charter contract is a shipping contract under which the charterer (shipper) agrees to hire the vessel from the shipowner for the duration of a specified trip.

<sup>3</sup>A possible alternative to the Merton model is the Kou model (Kou, 2002) with double exponentially distributed jump sizes. Both models lead to leptokurtic log-returns, with the Kou model providing explicit control on the likelihood of positive and negative jumps via an additional parameter, which makes the model harder to

follows the normal law with  $\varphi_X(u) = \exp(i\mu_X u - \sigma_X^2 u^2/2)$  and

$$E(e^{iu \ln(S_t/S_0)}) = e^{\Psi(u)t}, \quad (3)$$

where  $\Psi(u) := i(r - \sigma^2/2 - \lambda(\varphi_X(-i) - 1))u - \sigma^2 u^2/2 + \lambda(\varphi_X(u) - 1)$ . It follows by differentiation of the characteristic exponent that the first four cumulants of the Merton process are

$$\begin{aligned} c_1(\ln(S_t/S_0)) &:= E(\ln(S_t/S_0)) = (r - \sigma^2/2 - \lambda(\varphi_X(-i) - 1) + \lambda\mu_X)t, \\ c_2(\ln(S_t/S_0)) &:= \text{Var}(\ln(S_t/S_0)) = (\sigma^2 + \lambda(\mu_X^2 + \sigma_X^2))t, \\ c_3(\ln(S_t/S_0)) &:= E((\ln(S_t/S_0) - c_1)^3) = \lambda\mu_X(\mu_X^2 + 3\sigma_X^2)t, \\ c_4(\ln(S_t/S_0)) &:= E((\ln(S_t/S_0) - c_1)^4) - 3c_2^2 = \lambda(\mu_X^4 + 6\mu_X^2\sigma_X^2 + 3\sigma_X^4)t \end{aligned} \quad (4)$$

and the skewness coefficient and excess kurtosis are given respectively by

$$s(\ln(S_t/S_0)) := \frac{c_3(\ln(S_t/S_0))}{c_2(\ln(S_t/S_0))^{3/2}}, \quad (5)$$

$$\kappa(\ln(S_t/S_0)) := \frac{c_4(\ln(S_t/S_0))}{c_2(\ln(S_t/S_0))^2}. \quad (6)$$

## 4 Valuation framework for forward start Asian options

Assume an Asian option written on the spot freight rate  $S$  recorded over the period  $[\tau, T]$ ,  $T > \tau > 0$  at the following  $n > 1$  equidistant dates:  $t_0 = \tau, t_1 = \tau + \delta, \dots, t_k = \tau + k\delta, \dots, t_{n-1} = \tau + (n-1)\delta = T$ . The log-spot increment over any interval  $[t_{k-1}, t_k]$  is given by

$$Z_k := \ln \frac{S_{t_k}}{S_{t_{k-1}}} = L_{t_k} - L_{t_{k-1}},$$

where the increments  $\{L_{t_k} - L_{t_{k-1}}\}$  are independent and identically distributed under the model assumption (2).

In line with the market practice, we consider here a call option with fixed strike  $K > 0$  whose terminal payoff depends on the average of the past  $n$  spot prices

$$\left( \frac{1}{n} \sum_{k=0}^{n-1} S_{\tau+k\delta} - K \right)^+,$$

where  $x^+ := \max(x, 0)$ . In consistency with the framework presented in Večer (2002) and 

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calibrate though.

Večer and Xu (2004), the option payoff is equivalently given by

$$\left(\sum_{k=0}^{n-1} \alpha_k S_{\tau+k\delta}\right)^+$$

with  $\alpha_0 := 1/n - K/S_\tau$  and  $\alpha_k := 1/n > 0$  for all  $0 < k \leq n-1$ . Note here that  $\tau > 0$  represents some future time after the inception of the contract, therefore both  $S_\tau$  and  $\alpha_0$  are random.

Define the reverse filtration  $\mathbb{G} = \{\mathcal{G}_k\}_{k=1}^{n-1}$

$$\mathcal{G}_k = \sigma\{Z_{n-1}, Z_{n-2}, \dots, Z_{n-k}\}$$

and the process

$$Y_k = \ln(e^{Y_{k-1}} + \alpha_{n-k}) + Z_{n-k}, \quad 1 < k \leq n-1, \quad (7)$$

$$Y_1 = \ln \alpha_{n-1} + Z_{n-1}. \quad (8)$$

On evaluating  $Y_k$  recursively using (7)–(8), it is straightforward to show that

$$\left(\sum_{k=0}^{n-1} \alpha_k S_{\tau+k\delta}\right)^+ = S_\tau(e^{Y_{n-1}} + \alpha_0)^+ = (S_\tau(e^{Y_{n-1}} + 1/n) - K)^+. \quad (9)$$

Result (9) is known as the Carverhill-Clewlow-Hodges factorization (see Carverhill and Clewlow, 1990). Define the function  $g : \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}$  as

$$g(s, y) = (s(e^y + 1/n) - K)^+. \quad (10)$$

Then, from (9), computing

$$e^{-rT} E[g(S_\tau, Y_{n-1})]$$

under the risk neutral measure  $Q$  yields the time-0 price of the forward start Asian option.

**Lemma 1** *Consider the function  $g$  as in (10). Further, define  $c(S_0, \kappa(Y_{n-1}), \tau) = E[(S_\tau - \kappa(Y_{n-1}))^+ | Y_{n-1}]$  for some function  $\kappa : \mathbb{R} \rightarrow \mathbb{R}$  where  $S$  follows the dynamics (1) and  $Y_{n-1}$  is given by (7)–(8). The following equality holds:*

$$E[g(S_\tau, Y_{n-1})] = E[(e^{Y_{n-1}} + 1/n)c(S_0, K(e^{Y_{n-1}} + 1/n)^{-1}, \tau)].$$

**Proof.** From (10), we have that

$$\begin{aligned}
E[g(S_\tau, Y_{n-1})] &= E[E[(S_\tau(e^{Y_{n-1}} + 1/n) - K)^+ | S_\tau]] \\
&= E[(e^{Y_{n-1}} + 1/n)E[(S_\tau - K(e^{Y_{n-1}} + 1/n)^{-1})^+ | Y_{n-1}]] \\
&= E[(e^{Y_{n-1}} + 1/n)c(S_0, K(e^{Y_{n-1}} + 1/n)^{-1}, \tau)],
\end{aligned}$$

where, by piecewise continuity of the function  $g(s, y) = (s(e^y + 1/n) - K)^+$ , the second equality follows from changing the order of integration. ■

Lemma 1 provides us with a flexible means to discard explicit dependence of the option payoff on  $S_\tau$  and write the option price as the expected value of a new function of  $Y_{n-1}$  only:

$$e^{-rT} E[g(S_\tau, Y_{n-1})] = e^{-rT} E[(e^{Y_{n-1}} + 1/n)c(S_0, K(e^{Y_{n-1}} + 1/n)^{-1}, \tau)]. \quad (11)$$

Since the process  $Y$  is Markov in the filtration  $\mathbb{G}$ , expectation (11) can be computed recursively on a one-dimensional grid.

**Theorem 2** *Assume that for all  $0 < k \leq n - 1$  the random variables  $Z_{n-k}$  have density functions  $f_k$ . Consider the positive constants  $\alpha_k = 1/n$  for  $0 < k \leq n - 1$ . Define the functions  $p_k : \mathbb{R} \rightarrow \mathbb{R}$ ,  $0 < k \leq n - 1$ ,  $q_k : \mathbb{R} \rightarrow \mathbb{R}$ ,  $0 \leq k < n - 1$  and  $h_k : \mathbb{R} \rightarrow \mathbb{R}$ ,  $0 < k < n - 1$  as follows*

$$p_{n-1}(y) = (e^y + 1/n)c(S_0, K(e^y + 1/n)^{-1}, \tau), \quad (12)$$

$$q_{k-1}(x) = \int_{-\infty}^{\infty} p_k(x+z)f_k(z)dz, \quad 0 < k \leq n - 1, \quad (13)$$

$$h_{k-1}(y) = \ln(e^y + \alpha_{n-k}), \quad 1 < k \leq n - 1,$$

$$p_{k-1}(y) = q_{k-1}(h_{k-1}(y)), \quad 1 < k \leq n - 1.$$

*Then, the time-0 price of the forward start Asian call option with fixed strike is given by*

$$e^{-rT} q_0(\ln \alpha_{n-1}).$$

**Proof.** Eq. (12) follows from Lemma 1. For the remaining, see the proof of Theorem 3.1 in Černý and Kyriakou (2011). ■

The recursive pricing scheme presented in Theorem 2 can be computed by Fourier transform. In particular, by its definition,  $c(S_0, K(e^y + 1/n)^{-1}, \tau)$  in (12) corresponds to the (forward) price of a hypothetical European vanilla option on  $S$  having initial value  $S_0$ , with strike price  $K(e^y + 1/n)^{-1}$  and time to maturity  $\tau$ . This can be computed accurately by Fourier transform on a grid of strikes as in Carr and Madan (1999). The iterative backward convolution in (13)

can also be computed by Fourier transform. Details about the existence of the Fourier transform of the convolution, including an efficient and highly accurate numerical implementation using the so-called chirp  $z$ -transform, can be found in Černý and Kyriakou (2011). Note that in the proposed setup we require that the density of the log-spot increments is known only indirectly via its characteristic function, e.g., for the Merton model, see (3).

**Remark 3** Given the price of the fixed strike call, the price of a floating strike put option can be obtained using a symmetry relationship derived in Eberlein and Papapantoleon (2005), while the prices of fixed strike put and floating strike call options can be obtained via standard put-call parity. Finally, as pointed out by Večeř (2002), in-progress Asian options can be rewritten in terms of standard Asians, which are themselves a special case of forward start Asians.

## 5 Option data and estimation methodology

In this study we focus on the capesize, panamax and supramax sectors of the dry bulk market with highest liquidity and activity in freight options. The period under investigation includes every Friday (if not available, then Thursday) from January 04, 2008 to July 02, 2010, i.e., a total of 131 weeks. We consider the Baltic Option Assessments (BOAs) published by the Baltic Exchange; these are assessments of implied volatilities for at-the-money options, i.e., options with strikes equal to the prevailing FFA rates, which are submitted to the Baltic Exchange by freight option brokers and are published daily. In line with the market practice, we use the approximate Asian option price formula of Turnbull and Wakeman (1991) and Levy (1997) to retrieve the market quotes from the implied volatilities, for each sector of the dry bulk market and each week in the sample period. The market quotes are for forward start freight call options on the BCI, BPI and BSI for the next four quarters (+1Q, +2Q, +3Q and +4Q) and the next two calendar years (+1CAL and +2CAL), which are the most commonly used contract maturities.

Each quarterly contract consists of 3 options that expire at the end of each month in the quarter of interest, whereas a calendar contract is a strip of 12 monthly options. This amounts to a total of 36 option prices available on each week in the sample for each sector. We illustrate the structure of the contract by means of an example: assume that on January 04, 2008 an investor holds the BCI +1Q contract. This contract comprises 3 freight options which settle at the end of April 2008, May 2008, and June 2008. The settlement prices of each of these options are given by the average of the BCI spot rates over the trading days of the respective settlement month. Time to the first monitoring date (i.e., the first trading day of the settlement month) and time to maturity (i.e., the last trading day of the settlement month) are expressed as fractions of a year.

We perform the calibration exercise on 4 sets of data for each sector. First, we use the whole set of option prices, which we call “All options”. Then, we partition the entire set into 3 subsets comprising short-term, medium-term and long-term options: the first subset is the +Q which encompasses the quarterly contracts altogether, i.e., +1Q, +2Q, +3Q, +4Q; the second subset includes the +1CAL options; and the third subset the +2CAL options. We then perform a separate calibration for each subset.

To calibrate the risk neutral spot rate model, we resort to standard practice for extracting parameter estimates from observed option prices. More specifically, let  $P^M$  be the market option prices and  $P^\theta$  the option prices for a spot model with parameters  $\theta$ . For the computation of the model option prices, we use the algorithm presented in Theorem 2. For the Merton model (see Eq. 2),  $\theta = \{\mu_X, \sigma_X, \lambda, \sigma\}$ ; in the absence of the jump part, (2) reduces to the normal model with  $\theta = \sigma$ . We obtain the parameter estimates of the risk neutral distribution by minimizing the quadratic pricing error (e.g., see Andersen and Andreasen, 2000; Bates, 1996) at each date  $w$  in our option data (sub-)set

$$\mathcal{D}(\theta_w) := \sum_{i=1}^m |P_{i,w}^{\theta_w} - P_{i,w}^M|^2,$$

where  $m$  is the size of the (sub-)set at the particular date. In fact, we solve for

$$\theta_w^* = \arg \min_{\theta_w \in \Theta_w} \mathcal{D}(\theta_w),$$

where  $\Theta_w$  is the set of all parameters  $\theta_w$  such that the discounted price model  $S_t e^{-rt}$  parameterized by  $\theta_w$  is a martingale under the risk neutral probability measure  $Q$ . We say that the model with parameters  $\theta_w = \theta_w^*$  reproduces the market option prices in a least squares sense. The parameters  $\theta_w^*$  are estimated weekly for  $w = 1, \dots, 131$  weeks.

As a proxy for the risk free rate in our computations, we use the average 3-month US T-bill rate throughout the sample period.

## 6 Empirical performance of the Merton spot model

We discuss the estimation results of the risk neutral Merton model for the BCI, BPI and BSI spot rates for each of the “All options”, “+Q”, “+1CAL” and “+2CAL” sets defined in Section 5. In Table 2, we report the averages of the implied parameter values obtained across the weeks in the sample period, along with their standard errors, for the 3 Baltic indices.

We begin the analysis with the calibration results for the “All options” set. For the BCI, BPI and BSI, the estimated annual jump arrival rates,  $\lambda$ , are 0.5231, 0.8014, 0.4551, respectively.

The mean jump sizes,  $\mu_X$ , are -80.08%, -41.26% for the BCI and BPI and 35.6% for the BSI. From Eq. (4), the volatilities of the 1-year ahead log-increments of the spot indices,  $c_2(\ln(S_1/S_0))^{1/2}$ , as computed using the average implied parameter estimates are 94%, 85% and 66% for the capesize, panamax and supramax sectors. In addition, the percentage contributions of the jump component to the total variance of the fitted log-spot rate model,  $(\sigma^2/(\mu_X^2 + \sigma_X^2) + 1)^{-1}$ , are 80%, 74% and 57% for the BCI, BPI and BSI. The observed positive relationship between spot freight volatility and vessel size is not surprising: smaller vessels are more flexible to operate in different routes reducing, thus, the spot volatility, whereas the operation of larger vessels is more rigid. It is common for smaller vessels to do better than larger vessels in poor market conditions due to their versatility, which can also explain the reported positive BSI mean jump size as opposed to the negative ones found for the BCI and BPI. Abrupt upside changes in the spot freight markets are common due to the shape of the supply and demand curves for ocean shipping; when the supply and demand schedules are tight and the fleet is fully utilized, a positive demand shock leads to a large upward price move. A clear exception to this is observed during the second half of 2008 for the BCI and BPI which are characterized by frequent and large negative jumps. On the contrary, the effect of the 2008 recession is less perceptible on the BSI since smaller vessels manage to secure employment more easily.

We analyze further by considering the estimation results for the short-term (+Q), medium-term (+1CAL) and long-term (+2CAL) option subsets. For all sectors, the average estimate for  $\lambda$  is highest for the +Q contracts, while this reduces markedly for the +1CAL and +2CAL contracts. The average estimate for  $\mu_X$  becomes less negative (or even positive) for longer-term contracts: for the BCI,  $\mu_X$  is negative for all contract maturities due to stronger recession effects on larger vessels as noted earlier, whereas for the BPI (BSI) the average estimate for  $\mu_X$  is positive for the +2CAL (+1CAL and +2CAL) contracts implying a lower impact on smaller vessels and distant maturities.

The volatilities of the 1-year ahead log-increments of the BCI, BPI and BSI spot rates are respectively 173%, 136% and 113% for the +Q contracts, 79%, 64% and 62% for the +1CAL contracts, whereas for the +2CAL contracts volatilities drop to 59% for all sectors. The observed decaying term structure of volatility is consistent with the well-known Samuelson (1965) effect which is common in the commodities markets (short-end forward commodities curves are more sensitive to information flow). A similar decaying pattern is evident in the diffusion parameter,  $\sigma$ , while the standard deviation of the jump size,  $\sigma_X$ , is higher for nearby maturities. In addition, using expression (6) the excess kurtosis of the 1-year ahead log-increments of the spot indices,  $\kappa(\ln(S_1/S_0))$ , is higher for longer-term contracts. This is consistent with the increasing percentage attribution of the total variance to the jump component. It is also confirmed by the historical log-FFA rate distribution which exhibits fat tails for distant maturities due to

fluctuations in liquidity, given that the volumes of these contracts are small.

Figs. 1, 2 and 3 illustrate the weekly evolutions of the estimates for  $\lambda$ ,  $\mu_X$  and the volatility of the BCI, BPI and BSI spot rates, as obtained from the whole and maturity-partitioned datasets. There is strong evidence suggesting that for longer-term contracts the estimates for  $\lambda$ ,  $\mu_X$  and the volatility for all indices are more stable and close to their regular levels, i.e., 1 expected jump per year of mean size 100% and standard deviation 100%. A clear exception is noted during the period of transient structural change (September 2008 to February 2009) when  $\lambda$  and the volatility attain their peak levels, whereas  $\mu_X$  attains its lowest negative level.

A collective view of the results so far suggests that the jump terms introduced in the risk neutral spot freight model are able to describe flexibly abrupt changes in the 3 sectors of the dry bulk market. Although elevated volatilities and large jumps are more relevant to shorter-term contracts, jumps are still important when pricing longer-term contracts. The market is mainly characterized by positive jumps during the period of investigation, aside from the sub-period of the market recession when jumps are mostly negative, especially for the capesize sector due to limited vessel employment.

## 7 Performance of the option model

In this section we test the performance of the spot jump diffusion (JD) model in freight option pricing using a battery of statistics which measure the tracking error from the market quotes as well as the level of under- or over-pricing. As the benchmark to the JD model, we consider the lognormal (LogN) model which we calibrate using the same datasets. The comparison is made on the basis of the mean and median percentage errors (MPE and MdPE), the mean and median absolute percentage errors (MAPE and MdAPE), the root mean square error (RMSE) and relative RMSE. More specifically, we calculate for each week  $w$  in the sample period

$$\text{MPE}_w := \frac{1}{m} \sum_{i=1}^m \frac{P_{i,w}^{\theta^*} - P_{i,w}^M}{P_{i,w}^M}, \quad (14)$$

$$\text{MAPE}_w := \frac{1}{m} \sum_{i=1}^m \frac{|P_{i,w}^{\theta^*} - P_{i,w}^M|}{P_{i,w}^M}, \quad (15)$$

$$\text{RMSE}_w := \sqrt{\frac{1}{m} \sum_{i=1}^m |P_{i,w}^{\theta^*} - P_{i,w}^M|^2}, \quad (16)$$

$$\text{RRMSE}_w := \sqrt{\frac{1}{m} \sum_{i=1}^m \left| \frac{P_{i,w}^{\theta^*} - P_{i,w}^M}{P_{i,w}^M} \right|^2}. \quad (17)$$

Table 3 presents the averages of the metrics (14)–(17) obtained across the weeks in the sample period for each Baltic index, for the whole and maturity-partitioned datasets.

To test the null hypothesis that the MPE and MdPE are equal to zero, we employ the stationary bootstrap of Politis and Romano (1994). We find that, when significant, the estimated metrics are smaller (in absolute value) for the JD model. The negative MPE and MdPE for the BCI and BPI options suggest that both models tend to underprice these, whereas the results for the BSI options are mixed. Additional tests have indicated that the observed underpricing is mainly attributed to the recession period<sup>4</sup>.

We apply the reality check of White (2000) and the stationary bootstrap of Politis and Romano (1994) to test whether the jump diffusion model assumption for the spot rates yields statistically significant improvements in the performance measures<sup>5</sup>, as these are prone to data snooping bias. The MAPE, MdAPE, \$ RMSE and RRMSE metrics show that the LogN model across all sectors and datasets generates higher error than the JD model at 1% significance level. For the BCI, BPI and BSI, the RRMSE lies in the range 6.40%–26.47%, 6.82%–25.75% and 3.51%–3.63% for the JD, and 23.55%–50.77%, 25.13%–54.51% and 25.37%–77.62% for the LogN model. In addition, Fig. 4 illustrates the surface of the time series of the JD model MAPE for all traded contracts (+1Q, +2Q, +3Q, +4Q, +1CAL, +2CAL)<sup>6</sup> in each sector. Differences in the pricing error levels then become evident across sectors and contract maturities: firstly, the more volatile capesize sector with larger vessels is associated with higher error especially during the recession months; secondly, longer-term contracts exhibit lower error.

Finally, we test for possible differences between pricing errors when the model overprices or underprices options. We inspect the proportion and magnitude of positive and negative pricing errors for each model using an adapted version of the mixed mean error (MME) statistics of Brailsford and Faff (1996). Let O and U be the instances when each model overstates and

<sup>4</sup>A simple regression of the MPE on a constant and a dummy variable for the period September 2008 to February 2009 has shown significant underpricing (negative dummy coefficient), while the constant term in the regression is not significant, i.e., the MPE is on average zero if we exclude the recession months.

<sup>5</sup>Implementation of the reality check is based on a bootstrap resampling procedure. First, we define loss function LF differentials between the LogN and JD models based on the MAPE, MdAPE, \$ RMSE, RRMSE, see Eqs. (15)–(17), also MME(O) and MME(U), see Eqs. (18)–(19); for example,  $LF_w = RMSE_{LogN,w} - RMSE_{JD,w}$ . Second, using 10,000 bootstrap simulations, we estimate the mean loss differentials  $E(LF_w)$ . Finally, we formulate the null hypothesis that the benchmark LogN is not outperformed by the JD, i.e.,  $H_0: E(LF_w) \leq 0$ . For more details, we refer to White (2000); a thorough description of the algorithm can also be found in Appendix C of Sullivan et al. (1999).

<sup>6</sup>The JD model option prices, and subsequently the pricing errors, are for the parameter estimates obtained from the calibration based on the maturity-partitioned datasets “+Q”, “+1CAL” and “+2CAL”, see Section 5 for a description. Note that the model prices of the options included in the +1Q, +2Q, +3Q and +4Q contracts are based on the parameter estimates from the “+Q” calibration.

understates the option prices; the relevant metrics are

$$\text{MME}_w(\text{O}) := \frac{1}{m} \left( \sum_{i=1}^{\text{U}} |P_{i,w}^{\theta^*} - P_{i,w}^M| + \sum_{i=1}^{\text{O}} \sqrt{|P_{i,w}^{\theta^*} - P_{i,w}^M|} \right), \quad (18)$$

$$\text{MME}_w(\text{U}) := \frac{1}{m} \left( \sum_{i=1}^{\text{U}} \sqrt{|P_{i,w}^{\theta^*} - P_{i,w}^M|} + \sum_{i=1}^{\text{O}} |P_{i,w}^{\theta^*} - P_{i,w}^M| \right), \quad (19)$$

for each week  $w$  in the sample. The averages of the error statistics across weeks reported in Table 4 suggest that the frequency of over- or under-prediction by both models across all sectors and contract maturities lies in the range 35%–65%. The MME statistics are lower for the JD model in all cases at conventional significance levels. It is found that the JD model reduces the level of overpricing (underpricing) of the BCI, BPI, BSI options induced by the LogN model by factors of 1.91 (2.77), 3.11 (3.93) and 7.57 (6.74) on average. It is further shown that the level of over- or under-pricing is high for short-term contracts, while this decreases for longer-term contracts.

## 8 Regression analysis of pricing errors

Following Bakshi et al. (1997) and other authors, in this section we evaluate the pricing biases of the JD and LogN models. To this end, we perform a regression analysis on the relative percentage pricing errors  $\text{PE} = (P^{\theta^*} - P^M)/P^M$ :

$$\text{PE} = \beta_0 + \beta_1 T + \beta_2 T^2 + \beta_3 \hat{\sigma}_S + \beta_4 TS + \varepsilon,$$

where  $T$  is the option time to maturity (option-specific),  $\hat{\sigma}_S$  the annualized 6-month rolling standard deviation of the spot freight rate  $S$  (market-specific) and  $TS$  the yield differential between the 30-year and 3-month US T-bill rates (general market conditions). We further employ  $T^2$  as an explanatory variable to account for a nonlinear time-to-maturity effect.  $\varepsilon$  denotes a white noise term.

The regression results are reported in Table 5. In general, the estimated coefficients are higher (in absolute value) for the LogN model implying a more significant impact of the examined factors on the pricing error. The coefficients  $\beta_1$  for  $T$  (when significant) are positive for the JD and negative for the LogN model, whereas the coefficients  $\beta_2$  for  $T^2$  have opposite signs. Therefore, with the exception of the “+2CAL” dataset, this implies a concave (convex) relationship of the error induced by the JD (LogN) option model with  $T$ . The negative sign of the  $\beta_3$  coefficients (when significant) implies that the model option prices tend to understate the market prices as  $\hat{\sigma}_S$  increases. Finally, the yield differential movements appear to have a

rather small effect on the pricing error.

The pricing biases, as measured by the  $R^2$ , are always lower for the JD model. For example, the  $R^2$  of the BCI, BPI and BSI “All options” OLS are 6.6%, 9.7%, 4.0% for the JD against 12.3%, 19.5%, 27.2% for the LogN model. The  $F$  statistics further suggest that the JD model can reduce the bias effect, which remains significant though. Succinctly, the inclusion of jumps in the risk neutral spot freight dynamics appears to deliver model option prices that are least prone to systematic errors.

## 9 Summary and conclusions

The contribution of the paper to the literature is an exact valuation framework for freight options whose payoff corresponds to that of a forward start option written on the arithmetic average of the spot freight rate. For first time in the literature, we assume that the risk neutral spot rate dynamics are characterized by a Merton jump diffusion which generalizes the traditionally used lognormal diffusion by adding a jump component. We calibrate the risk neutral jump diffusion process, and the lognormal special case using short-term, medium-term and long-term option data on the Baltic Capesize, Panamax and Supramax Indices. Furthermore, our period of investigation (January 2008 to July 2010) encompasses the sub-period of the market recession of 2008 which allows us to study its effects on the option prices.

We provide evidence that the presence of jump terms in the spot freight process can flexibly describe extreme movements in the capesize, panamax and supramax sectors of the dry bulk market. Analysis on maturity-partitioned datasets shows that, although high volatilities and large jumps are mostly relevant to short-term contracts, jumps are still important when pricing longer-term contracts. The market is characterized mainly by upward jumps, except during the recession months with large downside jumps affecting mostly the capesize sector due to limited vessel employment. We further analyze the improvement in the freight option pricing performance brought by the jump diffusion model by computing a number of error statistics. It is found that the jump diffusion generates lower error than the lognormal model and also reduces the level of underpricing or overpricing. Highest pricing error is observed for short-term options traded in the capesize sector with largest vessels. Finally, a regression analysis shows that the jump diffusion generates pricing errors with least option-specific, market-specific and general market conditions-related biases, confirming its superior performance.

The implications of this research are critical for market participants as for them the main consideration is that the fair price for the freight option truly reflects the risks in the market.

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Table 1: Composition of the trip-charter routes of the Baltic indices.

Route	Route Description	Wt. (%)
Baltic Capesize Index		
C8	Delivery Gibraltar–Hamburg range for a Trans-Atlantic round voyage	25%
C9	Delivery ARA–Mediterranean range for a trip to the Far East, redelivery China–Japan range	25%
C10	Delivery China–Japan for a Pacific round voyage, redelivery China–Japan range	25%
C11	Delivery China–Japan range for a trip to ARA or the Mediterranean	25%
Baltic Panamax Index		
P1A	Delivery Gibraltar–Hamburg range for a Trans-Atlantic round voyage	25%
P2A	Delivery Cape Skaw–Gibraltar range for a trip to the Far East (Japan–S. Korea range) via US Gulf	25%
P3A	Delivery Japan–S. Korea for a Trans-Pacific round voyage	25%
P4A	Delivery Far East for a trip to Europe (Cape Skaw–Cape Passero) via North Pacific or Australia	25%
Baltic Supramax Index		
S1A	Delivery Antwerp–Skaw range for a trip to the Far East	12.5%
S1B	Delivery Mediterranean for a trip to the Far East	12.5%
S2	Delivery Japan–S. Korea range for a round voyage via North Pacific or Australia	25%
S3	Delivery Japan–S. Korea for a trip to Gibraltar–Cape Skaw range	25%
S4A	Delivery US Gulf for a trip to Europe (Cape Skaw–Cape Passero range)	12.5%
S4B	Delivery Cape Skaw–Cape Passero range for a trip to US Gulf	12.5%

The table presents the routes composing the Baltic Capesize, Panamax and Supramax Indices as of June 2011. BCI routes C8–C11 are for a standard 172,000 metric tons (mt) deadweight (dwt) capesize vessel; BPI routes P1A–P4A are for a standard 74,000 mt dwt panamax vessel; BSI routes S1A–S4B are for a 52,454 mt dwt Tesea 52 type vessel. Note that ARA refers to the Amsterdam–Rotterdam–Antwerp range. For more details, see the Baltic Exchange website ([www.balticexchange.com](http://www.balticexchange.com)).

Table 2: Implied risk neutral Merton jump diffusion parameters:  $\lambda$ ,  $\mu_X$ ,  $\sigma_X$ ,  $\sigma$ .

Dataset	$\lambda$	$\mu_X$	$\sigma_X$	$\sigma$	$c_2^{1/2}$	$s$	$\kappa$	% Contr. Jump to Var.
Baltic Capesize Index								
All options	0.5231	-0.8008	0.8494	0.4184	0.942	-1.405	3.151	0.803
	[0.053]	[0.297]	[0.053]	[0.024]				
+Q	1.1738	-1.3634	0.7402	0.4122	1.731	-1.081	1.370	0.943
	[0.100]	[0.290]	[0.043]	[0.027]				
+1CAL	0.6106	-0.7541	0.6108	0.2061	0.789	-1.602	3.225	0.931
	[0.084]	[0.288]	[0.039]	[0.017]				
+2CAL	0.6047	-0.4401	0.6033	0.1139	0.592	-1.651	4.231	0.963
	[0.055]	[0.173]	[0.039]	[0.011]				
Baltic Panamax Index								
All options	0.8014	-0.4126	0.7010	0.4344	0.847	-0.926	1.946	0.738
	[0.082]	[0.266]	[0.048]	[0.027]				
+Q	1.2079	-0.6851	0.8541	0.6364	1.361	-0.872	1.362	0.781
	[0.122]	[0.276]	[0.050]	[0.030]				
+1CAL	0.5571	-0.3530	0.7144	0.2411	0.642	-1.231	3.871	0.859
	[0.076]	[0.277]	[0.036]	[0.018]				
+2CAL	0.5769	0.1378	0.7550	0.0992	0.591	0.665	4.907	0.972
	[0.047]	[0.148]	[0.030]	[0.008]				
Baltic Supramax Index								
All options	0.4551	0.3560	0.6457	0.4353	0.661	0.773	2.037	0.566
	[0.052]	[0.275]	[0.043]	[0.026]				
+Q	0.8157	-0.1344	0.9951	0.6726	1.129	-0.228	1.530	0.645
	[0.083]	[0.262]	[0.045]	[0.031]				
+1CAL	0.4135	0.2537	0.8349	0.2693	0.622	0.938	4.770	0.813
	[0.049]	[0.218]	[0.036]	[0.028]				
+2CAL	0.3331	0.5245	0.8542	0.1120	0.589	2.104	7.951	0.964
	[0.041]	[0.128]	[0.041]	[0.007]				

Estimation has been performed on a weekly basis over the period January 04, 2008 to July 02, 2010 (i.e., 131 weeks) for each of the “All options”, “+Q”, “+1CAL” and “+2CAL” datasets, see Section 5, for the BCI, BPI and BSI. The table presents the averages (across weeks) of the annual parameter estimates of the fitted risk neutral jump diffusion spot price processes, along with their standard errors in [ ]. In addition, we present the resulting volatility  $c_2(\ln(S_1/S_0))^{1/2}$ , see Eq. (4), skewness coef.  $s(\ln(S_1/S_0))$ , see Eq. (5), and excess kurt.  $\kappa(\ln(S_1/S_0))$ , see Eq. (6), of the 1-year ahead log-increments of the spot indices, and the % contribution of the jump part to the total variance of the fitted log-spot price model given by  $(\sigma^2/(\mu_X^2 + \sigma_X^2) + 1)^{-1}$ .

Table 3: Analysis of option pricing errors.

Model	Dataset	MPE	MdPE	MAPE	MdAPE	\$ RMSE	RRMSE
Baltic Capesize Index							
Jump diffusion	All options	-0.0684 <sup>a</sup>	-0.0770 <sup>a</sup>	0.1698 <sup>a</sup>	0.1462 <sup>a</sup>	1,246 <sup>a</sup>	0.2091 <sup>a</sup>
	+Q	-0.1421 <sup>a</sup>	-0.1409 <sup>a</sup>	0.2193 <sup>a</sup>	0.1922 <sup>a</sup>	1,044 <sup>a</sup>	0.2647 <sup>a</sup>
	+1CAL	-0.0615 <sup>a</sup>	-0.0658 <sup>a</sup>	0.1249 <sup>a</sup>	0.1106 <sup>a</sup>	390.0 <sup>a</sup>	0.1500 <sup>a</sup>
	+2CAL	-0.0109 <sup>c</sup>	-0.0106 <sup>b</sup>	0.0553 <sup>a</sup>	0.0504 <sup>a</sup>	192.5 <sup>a</sup>	0.0640 <sup>a</sup>
Lognormal	All options	-0.0688 <sup>b</sup>	-0.1191 <sup>a</sup>	0.3499	0.3096	3,644	0.4367
	+Q	-0.2463 <sup>a</sup>	-0.3099 <sup>a</sup>	0.4413	0.4212	2,904	0.5077
	+1CAL	-0.1345 <sup>b</sup>	-0.2135 <sup>a</sup>	0.3823	0.3598	3,139	0.4401
	+2CAL	-0.0972 <sup>a</sup>	-0.1120 <sup>a</sup>	0.2163	0.2152	1,695	0.2355
Baltic Panamax Index							
Jump diffusion	All options	-0.0550 <sup>c</sup>	-0.0603 <sup>a</sup>	0.1952 <sup>a</sup>	0.1573 <sup>a</sup>	1,017 <sup>a</sup>	0.2575 <sup>a</sup>
	+Q	-0.0207 <sup>b</sup>	-0.0391 <sup>a</sup>	0.1210 <sup>a</sup>	0.1028 <sup>a</sup>	496.2 <sup>a</sup>	0.1506 <sup>a</sup>
	+1CAL	-0.0256 <sup>c</sup>	-0.0171 <sup>c</sup>	0.0538 <sup>a</sup>	0.0400 <sup>a</sup>	83.02 <sup>a</sup>	0.0712 <sup>a</sup>
	+2CAL	-0.0038 <sup>b</sup>	-0.0059 <sup>a</sup>	0.0571 <sup>a</sup>	0.0530 <sup>a</sup>	196.9 <sup>a</sup>	0.0682 <sup>a</sup>
Lognormal	All options	-0.0609 <sup>c</sup>	-0.1573 <sup>a</sup>	0.4511	0.4192	2,397	0.5451
	+Q	-0.0398 <sup>c</sup>	-0.0547 <sup>c</sup>	0.4107	0.3750	1,843	0.4908
	+1CAL	-0.0355 <sup>c</sup>	-0.0429 <sup>c</sup>	0.4115	0.3851	1,783	0.4662
	+2CAL	-0.0819 <sup>b</sup>	-0.1019 <sup>a</sup>	0.2274	0.2274	1,099	0.2513
Baltic Supramax Index							
Jump diffusion	All options	-0.0194	-0.0263 <sup>c</sup>	0.1708 <sup>a</sup>	0.1337 <sup>a</sup>	716.9 <sup>a</sup>	0.2363 <sup>a</sup>
	+Q	0.0014	-0.0011	0.0945 <sup>a</sup>	0.0786 <sup>a</sup>	306.9 <sup>a</sup>	0.1233 <sup>a</sup>
	+1CAL	-0.0061	-0.0033	0.0504 <sup>a</sup>	0.0413 <sup>a</sup>	62.40 <sup>a</sup>	0.0648 <sup>a</sup>
	+2CAL	0.0036 <sup>a</sup>	0.0043 <sup>a</sup>	0.0283 <sup>a</sup>	0.0240 <sup>a</sup>	58.99 <sup>a</sup>	0.0351 <sup>a</sup>
Lognormal	All options	-0.0159	-0.1734 <sup>a</sup>	0.5479	0.5134	2,309	0.6800
	+Q	0.3916 <sup>a</sup>	0.3441 <sup>a</sup>	0.6869	0.6682	2,083	0.7762
	+1CAL	0.2768 <sup>b</sup>	0.1912	0.5524	0.5085	1,591	0.6133
	+2CAL	-0.0543 <sup>b</sup>	-0.0792 <sup>a</sup>	0.2256	0.2270	906.6	0.2537

The table reports the averages, obtained across the weeks in the sample period, of the error statistics MPE, MdPE, MAPE, MdAPE, \$ RMSE and RRMSE, see Eqs. (14)–(17). The option model prices used in the computation of the error statistics for each week follow from the parameter estimates for the Merton jump diffusion spot price model and the benchmark lognormal model for the same datasets (see notes in Table 2). To test the null hypothesis that the MPE and MdPE are equal to zero, we employ the stationary bootstrap of Politis and Romano (1994) with 10,000 bootstrap simulations and a smoothing parameter of 0.1. In addition, we consider the MAPE, MdAPE, \$ RMSE and RRMSE estimated statistics to test the null hypothesis of the Merton jump diffusion model not performing better than the lognormal model by employing the reality check of White (2000) and the stationary bootstrap of Politis and Romano (1994). Superscripts *a*, *b*, *c* indicate significance at 1%, 5%, 10% levels.

Table 4: Analysis of option pricing errors cont'd.

Dataset	Jump diffusion model				Lognormal model			
	%O	MME(O)	%U	MME(U)	%O	MME(O)	%U	MME(U)
Baltic Capesize Index								
All options	0.35	0.1505 <sup>c</sup>	0.65	0.1048 <sup>a</sup>	0.53	0.2265	0.47	0.2460
+Q	0.37	0.2442 <sup>a</sup>	0.63	0.1639 <sup>a</sup>	0.37	0.4114	0.63	0.3828
+1CAL	0.33	0.1311 <sup>a</sup>	0.67	0.0873 <sup>a</sup>	0.52	0.2536	0.48	0.2583
+2CAL	0.45	0.0590 <sup>b</sup>	0.55	0.0473 <sup>b</sup>	0.69	0.1485	0.31	0.1622
Baltic Panamax Index								
All options	0.42	0.1539 <sup>a</sup>	0.58	0.1264 <sup>a</sup>	0.47	0.2991	0.53	0.2784
+Q	0.49	0.1111 <sup>a</sup>	0.51	0.0822 <sup>a</sup>	0.46	0.3316	0.54	0.3126
+1CAL	0.45	0.0726 <sup>a</sup>	0.55	0.0461 <sup>a</sup>	0.49	0.2930	0.51	0.2579
+2CAL	0.50	0.0385 <sup>c</sup>	0.50	0.0311 <sup>c</sup>	0.57	0.1333	0.43	0.1280
Baltic Supramax Index								
All options	0.59	0.1088 <sup>a</sup>	0.41	0.1155 <sup>a</sup>	0.52	0.3537	0.48	0.3497
+Q	0.47	0.0686 <sup>a</sup>	0.53	0.0574 <sup>a</sup>	0.68	0.6169	0.32	0.5770
+1CAL	0.43	0.0390 <sup>a</sup>	0.57	0.0351 <sup>a</sup>	0.55	0.4374	0.45	0.3584
+2CAL	0.64	0.0152 <sup>c</sup>	0.36	0.0319 <sup>c</sup>	0.52	0.1039	0.48	0.0927

The table reports the averages, obtained across the weeks in the sample period, of the mean overprediction and underprediction statistics, see Eqs. (18)–(19), as adapted from Brailsford and Faff (1996). %O and %U are the proportional instances of over- and under-pricing, respectively. Superscripts *a*, *b*, *c* indicate significance at 1%, 5%, 10% levels.

Table 5: Results on regression of pricing errors.

Model	Dataset	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$R^2$	$F$ stat.
Baltic Capesize Index								
Jump diffusion	All options	-0.316 <sup>a</sup>	0.344 <sup>a</sup>	-0.110 <sup>a</sup>	-0.061 <sup>a</sup>	0.036 <sup>a</sup>	0.066	81.9 <sup>a</sup>
	+Q	-1.206 <sup>a</sup>	0.895 <sup>a</sup>	-0.553 <sup>b</sup>	0.104 <sup>a</sup>	0.167 <sup>a</sup>	0.225	105.5 <sup>a</sup>
	+1CAL	-0.602 <sup>a</sup>	0.799 <sup>a</sup>	-0.330 <sup>a</sup>	0.005	0.029 <sup>b</sup>	0.093	41.4 <sup>a</sup>
	+2CAL	0.019	0.045	-0.021	-0.027 <sup>a</sup>	0.003	0.043	18.7 <sup>a</sup>
Lognormal	All options	0.645 <sup>a</sup>	-0.644 <sup>a</sup>	0.149 <sup>a</sup>	-0.054 <sup>a</sup>	-0.035 <sup>b</sup>	0.123	161.2 <sup>a</sup>
	+Q	-1.166 <sup>a</sup>	-1.412 <sup>b</sup>	0.740 <sup>c</sup>	0.175 <sup>a</sup>	0.334 <sup>a</sup>	0.342	188.8 <sup>a</sup>
	+1CAL	0.160	-1.101	0.363	-0.061 <sup>a</sup>	0.141 <sup>a</sup>	0.131	60.0 <sup>a</sup>
	+2CAL	-0.290	0.314	-0.103	-0.176	0.071 <sup>a</sup>	0.179	86.9 <sup>a</sup>
Baltic Panamax Index								
Jump diffusion	All options	-0.327 <sup>a</sup>	0.337 <sup>a</sup>	-0.112 <sup>a</sup>	-0.149 <sup>a</sup>	0.061 <sup>a</sup>	0.097	123.8 <sup>a</sup>
	+Q	-0.397 <sup>a</sup>	0.789 <sup>a</sup>	-0.582 <sup>a</sup>	-0.075 <sup>a</sup>	0.057 <sup>a</sup>	0.103	42.3 <sup>a</sup>
	+1CAL	-0.375 <sup>a</sup>	0.556 <sup>a</sup>	-0.223 <sup>a</sup>	-0.048 <sup>a</sup>	0.022 <sup>a</sup>	0.165	78.4 <sup>a</sup>
	+2CAL	-0.053	0.158	-0.059	-0.002	-0.003	0.137	63.4 <sup>a</sup>
Lognormal	All options	0.622 <sup>a</sup>	-0.944 <sup>a</sup>	0.208 <sup>a</sup>	-0.136 <sup>a</sup>	0.061 <sup>b</sup>	0.195	279.0 <sup>a</sup>
	+Q	0.411 <sup>c</sup>	-2.120 <sup>a</sup>	1.149 <sup>b</sup>	-0.054	0.126 <sup>a</sup>	0.134	57.0 <sup>a</sup>
	+1CAL	0.765 <sup>c</sup>	-1.299 <sup>c</sup>	0.280	-0.232 <sup>a</sup>	0.151 <sup>a</sup>	0.256	136.1 <sup>a</sup>
	+2CAL	-0.066	0.291	-0.133	-0.219 <sup>a</sup>	0.053 <sup>a</sup>	0.266	143.0 <sup>a</sup>
Baltic Supramax Index								
Jump diffusion	All options	-0.182 <sup>a</sup>	0.231 <sup>a</sup>	-0.086 <sup>a</sup>	-0.084 <sup>a</sup>	0.030 <sup>a</sup>	0.040	49.3 <sup>a</sup>
	+Q	-0.051	0.262	-0.258	-0.013	0.005	0.020	8.4 <sup>a</sup>
	+1CAL	-0.183 <sup>b</sup>	0.380 <sup>a</sup>	-0.178 <sup>a</sup>	-0.026 <sup>c</sup>	0.006	0.076	33.5 <sup>a</sup>
	+2CAL	0.077	-0.030	-0.002	0.010 <sup>c</sup>	-0.002	0.071	31.2 <sup>a</sup>
Lognormal	All options	1.264 <sup>a</sup>	-1.504 <sup>a</sup>	0.348 <sup>a</sup>	0.127 <sup>b</sup>	-0.047 <sup>c</sup>	0.272	428.3 <sup>a</sup>
	+Q	-0.400	-1.445 <sup>c</sup>	0.844	0.084	0.342 <sup>a</sup>	0.103	42.2 <sup>a</sup>
	+1CAL	1.223 <sup>b</sup>	-2.117 <sup>b</sup>	0.462	-0.301 <sup>a</sup>	0.266 <sup>a</sup>	0.393	255.7 <sup>a</sup>
	+2CAL	0.336	0.010	-0.083	-0.128 <sup>a</sup>	0.016	0.299	168.3 <sup>a</sup>

The regression specification is  $PE = \beta_0 + \beta_1 T + \beta_2 T^2 + \beta_3 \hat{\sigma}_S + \beta_4 TS + \varepsilon$ , where  $PE = (P^{\theta^*} - P^M)/P^M$  obtained from the Merton jump diffusion and lognormal models,  $T$  is the option time to maturity,  $T^2$  is the square of  $T$ ,  $\hat{\sigma}_S$  is the annualized 6-month rolling standard deviation of the spot freight rate  $S$ , and  $TS$  is the yield differential between the 30-year and 3-month US T-bill rates. Standard errors have been corrected for serial correlation and heteroscedasticity.  $F$  statistics are used to test the joint null hypothesis  $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ . Superscripts  $a, b, c$  indicate significance at 1%, 5%, 10% levels.

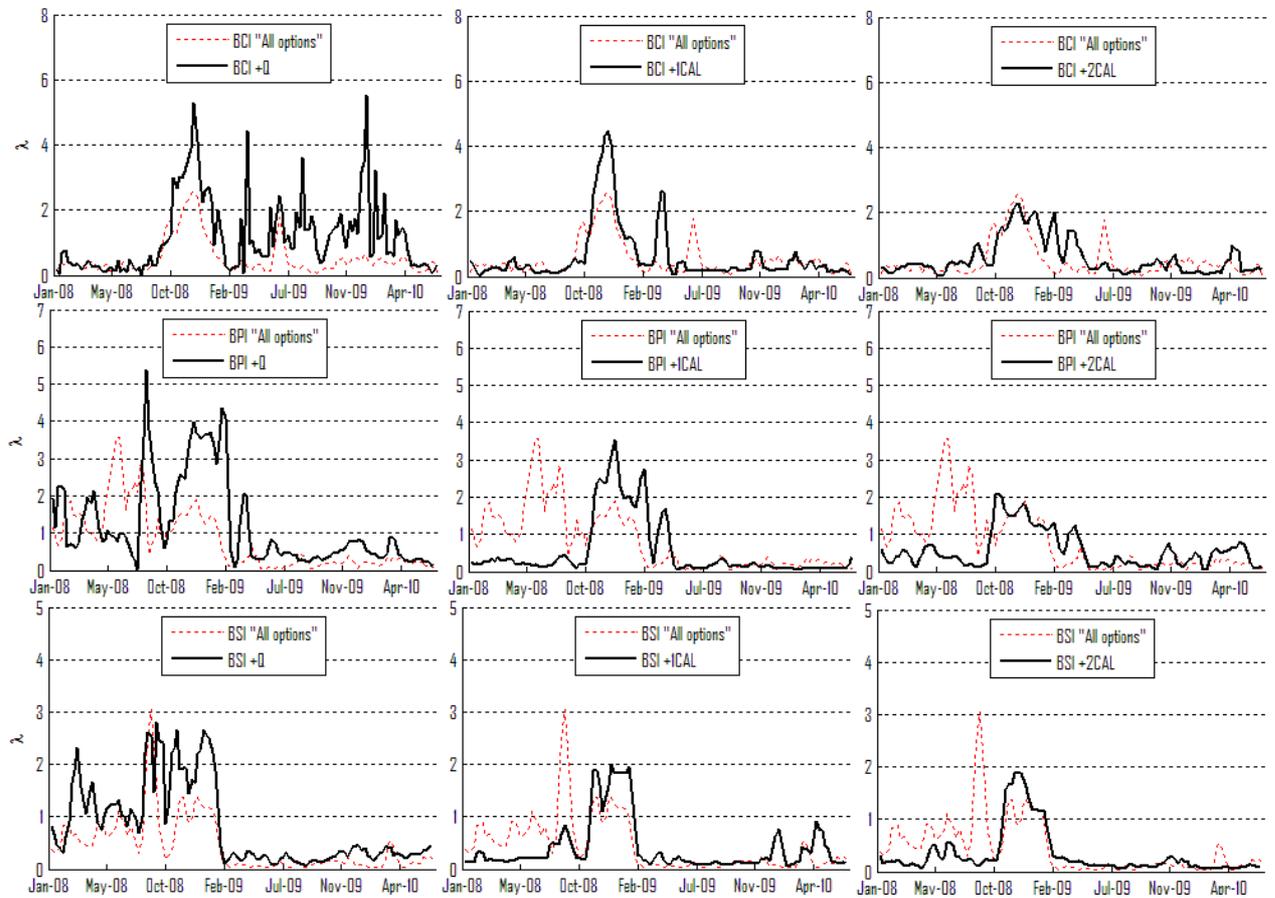


Fig. 1: Time plots of the estimates for the annual risk neutral jump arrival rate  $\lambda$  (131 estimates for each week over the period January 04, 2008 to July 02, 2010) obtained from the whole (“All options”) and maturity-partitioned (“+Q”, “+1CAL”, “+2CAL”) datasets, for each of the Baltic Capesize, Panamax and Supramax Indices.

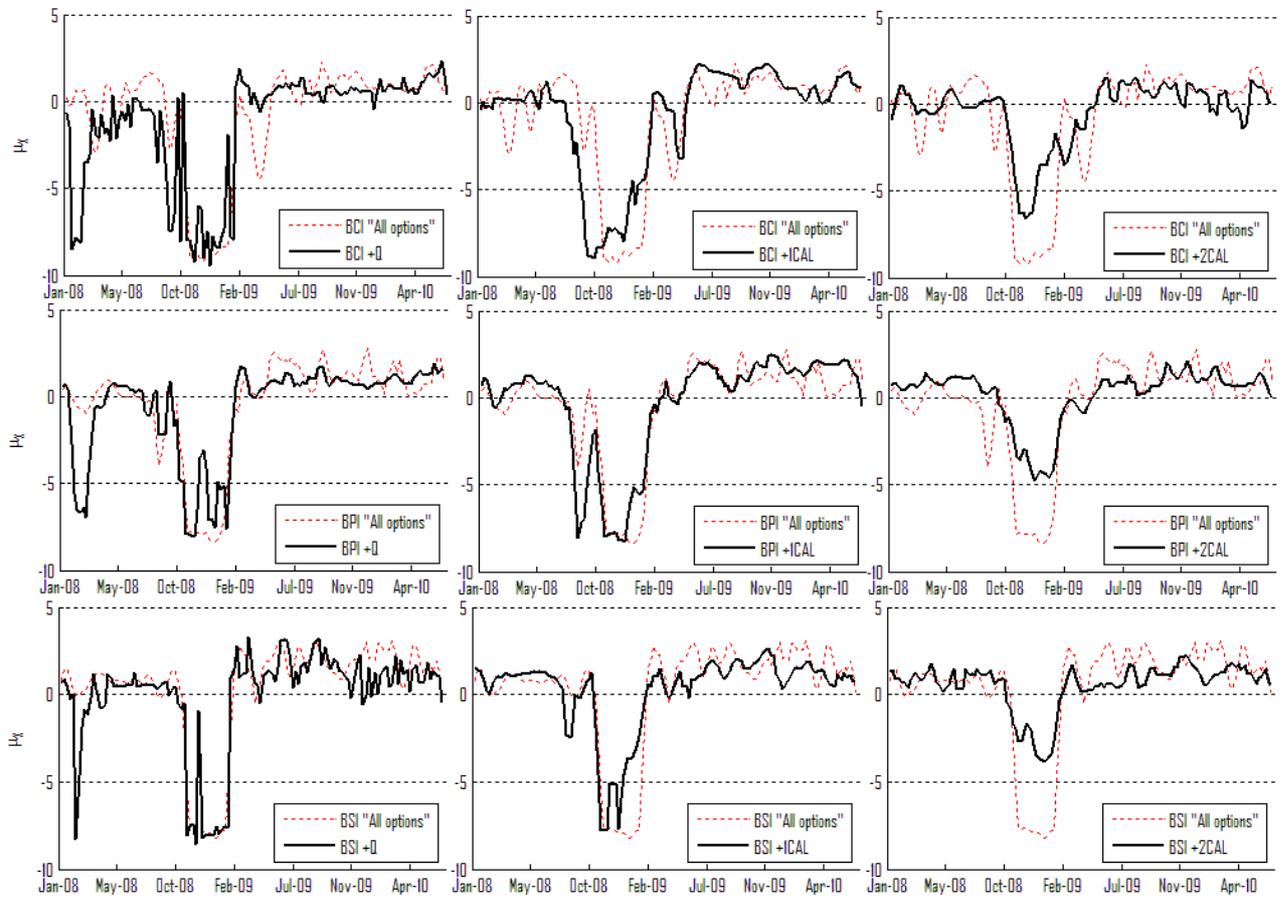


Fig. 2: Time plots of the estimates for the annual risk neutral mean jump size  $\mu_X$  (131 estimates for each week over the period January 04, 2008 to July 02, 2010) obtained from the whole (“All options”) and maturity-partitioned (“+Q”, “+1CAL”, “+2CAL”) datasets, for each of the Baltic Capesize, Panamax and Supramax Indices.

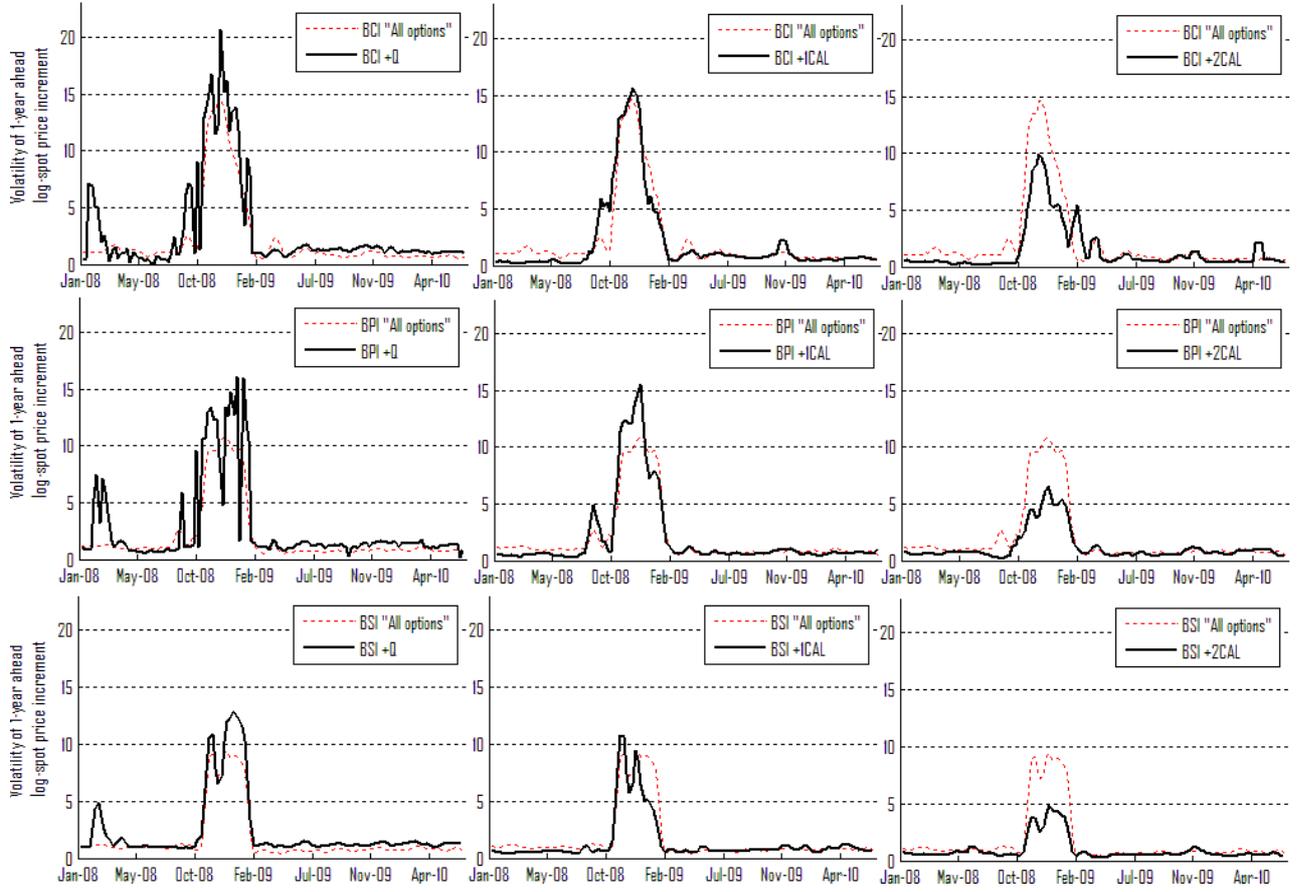


Fig. 3: Time plots of the computed annual risk neutral volatility  $c_2(\ln(S_1/S_0))^{1/2}$ , see Eq. (4), of the 1-year ahead log-increments of the fitted BCI, BPI and BSI spot price jump diffusion models. Computation is based on the weekly estimates for  $\lambda, \mu_X, \sigma_X, \sigma$  obtained from the whole (“All options”) and maturity-partitioned (“+Q”, “+1CAL”, “+2CAL”) datasets.

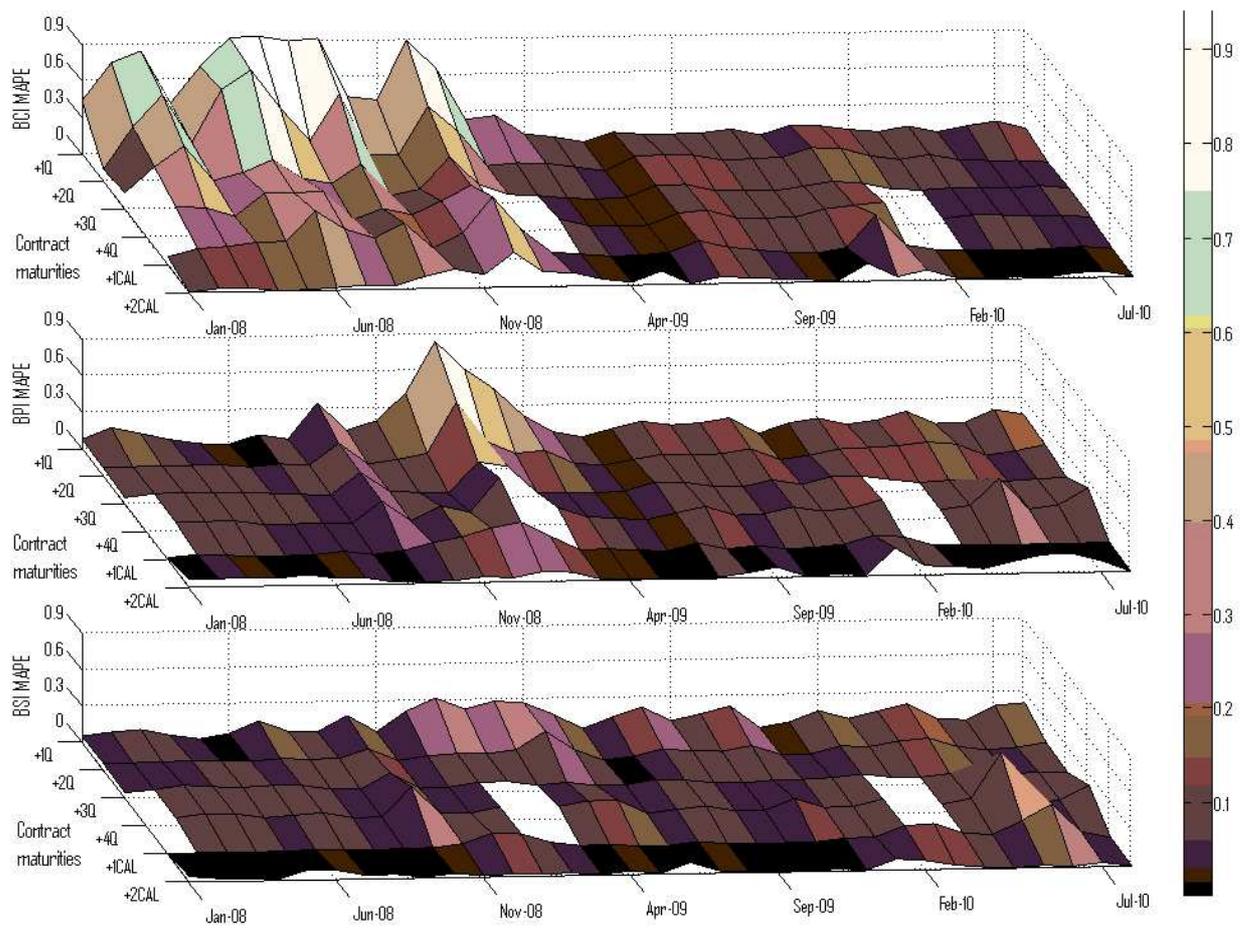


Fig. 4: Surfaces of the time series of the jump diffusion model MAPE for all traded contracts (+1Q, +2Q, +3Q, +4Q, +1CAL, +2CAL) in the capsize, panamax and supramax sectors.