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Optimal Fiscal and Monetary Rules in Normal and Abnormal Times *

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Abstract

We examine fiscal-monetary interactions in a New-Keynesian model with deep habits, distortionary taxes and a sovereign risk premium for government debt. Deep habits crucially affect the fiscal transmission mechanism in that these lead to a counter-cyclical mark-up, boosting the size of a demand-driven output expansion with important consequences for monetary and fiscal policy. We employ Bayesian estimates of the model to compute optimal monetary and fiscal policy first in 'normal times' with debt starting at its steady state and then in a crisis period with a much higher initial debt-GDP ratio. Policy is conducted in terms of optimal commitment, time consistent and simple Taylor-type rules. Welfare calculations and impulse responses indicate that the ability of the simple rules to closely mimic the Ramsey optimal policy, observed in the literature with optimal monetary policy alone, is still a feature of optimal policy with fiscal instruments, but only with 'passive' fiscal policy. For crisis management we find some support for slow consolidation with a more active role for tax increases rather than a decrease in government spending.

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Contents

1 Introduction

Both the efficacy of a fiscal stimulus and the appropriate speed of fiscal consolidation are controversial issues in applied macroeconomics. Of course they are closely related. On the former, the range of empirical government spending multipliers is wide – Ramey (2011a) surveys the literature and argues that this is between 0.8 and 1.5 – and the sign of the effect on private consumption is controversial. In fact, one strand of the empirical literature, using methods along the lines of Ramey and Shapiro (1998) and more recently Ramey (2011b), finds evidence for a *crowding-out* of consumption, while Structural Vector-Autoregressions (SVARs) in the spirit of Blanchard and Perotti (2002) and more recently Monacelli et al. (2010) provide evidence for a *crowding-in* effect. In addition, fiscal multipliers are found to be significantly higher in a recession regime (see e.g. Auerbach and Gorodnichenko, 2012; Batini et al., 2012, among others).

Canonical Dynamic Stochastic General Equilibrium (DSGE) models typically predict fiscal multipliers well below the empirical range and a crowding-out effect on private consumption. The main reason for this is to be found in the negative wealth effect triggered by the increase in government purchases. This, in fact, crowds out private consumption and investment and makes output respond in a less than proportional way. Woodford (2011), through rather simple algebraic manipulations, shows that the government spending multiplier is (i) necessarily below one in a neoclassical Real Business Cycle (RBC) model and exactly the same both in an RBC with monopolistic competition and in a sticky-price New-Keynesian (NK) model with strict inflation targeting; (ii) exactly one in an NK model with fixed real interest rate; (iii) somewhere between the two values in a model featuring a Taylor rule. In general, the more accommodative the monetary policy, the higher the fiscal multiplier. On the last point Canova and Pappa (2011) also provide empirical support. Moreover, substantially larger-than-one multipliers can be obtained in standard NK models if the zero lower bound on the nominal interest rate (ZLB) binds. Christiano et al. (2011) find that the spending multiplier may also reach 10 at the ZLB if the fiscal stimulus lasts for exactly the quarters when the ZLB is binding.

A modelling device that has been used to obtain the consumption crowding-in and higher fiscal multipliers in Real Business Cycle (RBC) models is the assumption that external 'deep habits' *à la* Ravn et al. (2006) are formed in private and public consumption, i.e. habits on the average consumption level of each variety of goods. Jacob (2011) shows that in a New-Keynesian (NK) model with deep habits, increasing degrees of price stickiness soften the expansionary effects of a fiscal stimulus and may overturn the results obtainable in a RBC model. However, Cantore et al. (2012) show that with an empirically plausible degree of price stickiness and either under an 'empirical' or an 'optimized' interest-rate rule the main results still hold.

This paper investigates these issues paying particular attention to the subtle interactions between fiscal and monetary policy that determine the outcome of fiscal stimuli and consolidations. We examine fiscal-monetary interactions in a NK DSGE model with deep habits, distortionary taxes and a sovereign risk premium for government debt. A number of possible interest rate and fiscal policies are compared: first, the welfare-optimal (Ramsey) policy; second, a time-consistent policy; third optimized simple Taylor type rules (of which a price-level rule is a special case of the interest rate rule). For the simple rule both passive and active fiscal policy stances are examined. We study policy rules responding both to continuous future stochastic shocks (policy in 'normal times') and to a one-off large shock to government debt ('crisis management'). This results in what we believe to be the first assessment of what is the *optimal timing and optimal* combination of instruments to achieve a fiscal consolidation using rules that are also suitable for future normal times.

Welfare calculations and impulse responses indicate that the ability of the simple rules to closely mimic the Ramsey optimal policy, observed in the literature with optimal monetary policy alone, is still a feature of optimal policy with fiscal instruments, but only with 'passive' fiscal policy. For crisis management we find some support for slow consolidation with a more active role for tax increases rather than a decrease in government spending.

The implications of these results agree with the findings of a number of recent studies. Batini et al. (2012) show, in the context of regime-switching vector-autoregressions, that smooth and gradual consolidations are to be preferred to frontloaded or aggressive consolidations, especially for economies in recession facing high risk premia on public debt. In addition, they find that tax hikes are less contractionary than spending cuts. Erceg and Linde (2013) obtain similar findings in a DSGE model of a currency union. Denes et al. (2013) highlight limitations of austerity measures, while Bi et al. (2013) show in a DSGE setting that, in the current economic environment, consolidation efforts are more likely to be contractionary rather than expansionary.

There are a few recent papers that address some of the issues in our paper: using a standard NK model with government sovereign risk, but without habit of a deep or 'superficial' kind, Corsetti et al. (2013) carry out a comparison of different fiscal consolidation scenarios. Apart from the model with deep habits, our study differs in that we consider optimal or optimized simple commitment rules whereas their paper studies ad-hoc policies. Leith et al. (2012) do examine optimal and optimized simple rules in a calibrated model with deep habits, but only for normal times. Perhaps the closest paper to ours is Kirsanova and Wren-Lewis (2012). In a simple core calibrated NK model without habits or a government sovereign risk their paper examines different ad hoc degrees of fiscal feedback alongside optimal monetary policy. As in our paper they allow fiscal policy to become 'active' and monetary policy 'passive' (as defined by Leeper (1991)) leaving the price level to jump to satisfy the government budget constraint. In contrast to all three studies we compare commitment and discretion, thus drawing conclusions regarding the importance of the former. Another novel feature of our paper is the consideration of the zero lower bound constraint for the interest rate in the design of optimal interest rate rules, and we impose an analogous upper bound constraint on the government debt/GDP ratio.

The rest of the paper is organized as follows. Section 2 sets out the model. Section 3 briefly summarizes the Bayesian estimation of the model drawing upon Cantore et al. (2013a). The main Section 4 carries out the policy experiments and Section 5 concludes. More technical details and proofs are appended to the paper.

2 The Model

Building on Cantore et al. (2012) we conduct the analysis within a NK model with Rotemberg price stickiness and convex investment adjustment costs augmented with deep habit formation. We refine the fiscal sector in that the government finances its expenditures by raising a mix of lump-sum and distortionary taxes and by issuing government bonds. In addition, we allow for a sovereign risk to generate a premium in the interest payments paid by the government.

2.1 Households

A continuum of identical households $j \in [0,1]$ has preferences over differentiated consumption varieties $i \in [0,1]$. Following Ravn et al. (2006), preferences feature habit formation at the level of individual goods, or deep habits (see also Jacob, 2011; Di Pace and Faccini, 2012; Zubairy, 2012; Cantore et al., 2013b). Similar to the more common *superficial habits*, i.e. habits on the overall level of consumption, deep habits may be internal or external, although it is common practice to use the latter version as this is analytically more tractable. In fact, internal deep habits lead to a time inconsistency problem (see Ravn et al., 2006), so we adopt external deep habits, i.e., *keeping up with the Joneses good by good*. In the microeconometric literature there is recent evidence of deep habit formation. For instance Verhelst and Van den Poel (2012) estimate a spatial panel model using scanner data from a large European retailer and test for both internal and external deep habit formation. While they find some categories with internal habit formation, this effect is generally small. On the contrary, the external habit effect is always positive and significant. In the macro-econometric literature there are also estimates of deep habits for the US. For instance, Ravn et al. (2006) use a Generalized Method of Moments estimator applied to the consumption Euler equation and use the additional restrictions that deep habits imply for the supply side of the economy. Zubairy (2013) estimates the deep habit parameters within the broader setting of a Bayesian estimation of a medium-scale NK model. Cantore et al. (2013a) compare superficial and deep habit formation within an estimated NK model for the US and provide empirical support in favour of the latter. Household j's optimization problem is

$$\max_{\left\{(X_t^c)^j, K_{t+1}^j, B_{t+1}^j, I_t^j, h_t^j\right\}} E_t \sum_{s=0}^{\infty} e_{t+s}^B \beta^{t+s} U((X_{t+s})^j, 1 - H_{t+s}^j),$$

subject to constraints

$$\begin{aligned}\left(1+\tau_t^C\right)(X_t^c)^j + \Omega_t + I_t^j + \tau_t^L + \frac{B_t^j}{P_t} + \frac{(B_t^g)^j}{P_t} &= \left(1-\tau_t^W\right)\frac{W_t}{P_t}H_t^j + \left(1-\tau_t^K\right)R_t^K K_t^j \\ &+ \frac{R_{t-1}B_{t-1}^j}{P_t} + \frac{R_{t-1}^g \Psi_{t-1}^{-1}\left(B_{t-1}^g\right)^j}{P_t} + \int_0^1 J_{it}di, \qquad (1)\end{aligned}$$

$$K_{t+1}^j = (1-\delta)K_t^j + e_t^I I_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right], \qquad (2)$$

where $\beta \in (0,1)$ is the discount factor, e_t^B is a preference shock, $(X_t)^j = X\left((X_t^c)^j, X_t^g\right)$ is a composite of habit-adjusted differentiated private and public consumption goods and H_t^j are hours of work. This assumption implies that government consumption delivers some utility to private agents (see e.g. Pappa, 2009; Cantore et al., 2012). Many studies, on the contrary, assume that public consumption goods are not utility-enhancing, i.e. they are simply a waste of resources. The private component of $(X_t)^j$ is

$$(X_t^c)^j = \left[\int_0^1 \left(C_{it}^j - \theta^c S_{it-1}^c\right)^{1-\frac{1}{e_t^P \zeta}} di\right]^{\frac{1}{1-\frac{1}{e_t^P \zeta}}}, \qquad (3)$$

where $\theta^c \in (0,1)$ is the degree of deep habit formation on each variety, ζ is the intratemporal elasticity of substitution, e^P is a price mark-up shock, and S_{it-1}^c denotes the stock of habit in the consumption of good i, which evolves over time according to

$$S_{it}^c = \varrho^c S_{it-1}^c + (1-\varrho^c)C_{it}, \qquad (4)$$

where $\varrho^c \in (0,1)$ implies persistence. The optimal level of demand for each variety, C_{it}^j, for a given composite is obtained by minimizing total expenditure $\int_0^1 P_{it} C_{it}^j di$ over C_{it}^j, subject to (3). This leads to

$$C_{it}^j = \left(\frac{P_{it}}{P_t}\right)^{-e_t^P \zeta} (X_t^c)^j + \theta^c S_{it-1}^c, \tag{5}$$

where P_{it} is the price of variety i, and $P_t \equiv \left[\int_0^1 P_{it}^{1-e_t^P\zeta} di\right]^{\frac{1}{1-e_t^P\zeta}}$ is the nominal price index. Multiplying (5) by P_{it} and integrating, real consumption expenditure C_t^j can be written as a function of the consumption composite and the stock of habit: $C_t^j = (X_t^c)^j + \Omega_t$, where $\Omega_t \equiv \theta^c \int_0^1 \frac{P_{it}}{P_t} S_{it-1}^c di$. Households hold K_t^j capital holdings, evolving according to (2) where δ is the capital depreciation rate, I_t^j is investment, $S(\cdot)$ represents an investment adjustment cost satisfying $S(1) = S'(1) = 0$ and $S''(1) > 0$, and e_t^I is an investment-specific shock. Investment is also a composite of goods, i.e. $I_t^j = \left[\int_0^1 \left(I_{it}^j\right)^{1-\frac{1}{e_t^P\zeta}} di\right]^{\frac{1}{1-\frac{1}{e_t^P\zeta}}}$, but does not feature habit formation. Expenditure minimisation leads to the optimal level of demand of private investment goods for each variety i:

$$I_{it}^j = \left(\frac{P_{it}}{P_t}\right)^{-e_t^P\zeta} I_t^j. \tag{6}$$

In the budget constraint, τ_t^C, τ_t^W and τ_t^K are tax rates on consumption, labour income and the return on capital, respectively and τ_t^L is a lump-sum tax. Households buy consumption goods, C_t^j; invest in investment goods, I_t^j, nominal private bond holdings, B_t^j, and nominal government bond holdings, $(B_t^g)^j$; receive the hourly wage, W_t, the rental rate of capital, R_t^K, the return on nominal private bond holdings, R_t, the return on nominal government bond holdings, R_t^g, augmented by the sovereign risk premium, Ψ_t, and firms' profits,$\int_0^1 J_{it} di$; and pay a mixture of lump-sum and distortionary taxes.

The first-order condition (FOC) with respect to (w.r.t.) the private consumption composite $(X_t^c)^j$ implies that the Lagrange multiplier on the household's budget constraint (1) is equal to $\Lambda_t^j = (1+\tau_t^C) U_{X^c,t}^j$, where $U_{X^c,t}^j$ is the marginal utility of the private consumption composite. Let $\Lambda_t^j Q_t^j$ be the multiplier on the capital accumulation equation (2), and Q_t^j represent Tobin's Q. Then, the FOC w.r.t. capital, K_{t+1}^j, implies $Q_t^j = E_t \left\{ D_{t,t+1}^j \left[R_{t+1}^K + (1-\delta) Q_{t+1}^j \right] \right\}$, where $D_{t,t+1}^j \equiv \beta E_t \left[\frac{e_{t+1}^B U_{X^c,t+1}^j}{e_t^B U_{X^c,t}^j} \frac{1+\tau_t^C}{1+\tau_{t+1}^C} \right]$ is the stochastic discount factor. The FOC w.r.t. investment, I_t^j, yields

$$e_t^I Q_t^j \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right) - S'\left(\frac{I_t^j}{I_{t-1}^j}\right)\frac{I_t^j}{I_{t-1}^j}\right) + E_t \left(e_{t+1}^I D_{t,t+1}^j Q_{t+1}^j S'\left(\frac{I_{t+1}^j}{I_t^j}\right)\left(\frac{I_{t+1}^j}{I_t^j}\right)^2 \right) = 1,$$

while the FOCs w.r.t. the private and government bond holdings delivers the following non-arbitrage condition for the two interest rates

$$1 = E_t \left[D^j_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right] = E_t \left[D^j_{t,t+1} \Psi_t^{-1} \frac{R^g_t}{\Pi_{t+1}} \right], \tag{7}$$

where $\Pi_t \equiv \frac{P_t}{P_{t+1}}$ is the gross inflation rate. Finally the FOC w.r.t hours implies: $-U^j_{H,t} = U^j_{X^c,t} \left(1 - \tau^W_t\right) (1+ \tau^C_t) \frac{W_t}{P_t}$.

Equation (7) implies $R^g_t = \Psi_t R_t$, i.e. that the government has to pay a premium, Ψ_t, on its interest payments. Such sovereign risk premium is modelled as an exponential function of government indebtedness,$\Psi_t = \exp\left(\phi \frac{b_t}{Y_t}\right)$, where $\phi \geq 0$ is a structural parameter.[1]

2.2 Government

As in Ravn et al. (2006) deep habits are present also in government consumption. This can be justified by assuming that households form habits also on consumption of government-provided goods. Alternatively, as in Leith et al. (2012) and Ravn et al. (2012), one can also argue that public goods are local in nature and households care about the provision of individual public goods in their constituency relative to other constituencies. For example, controversies over "post-code lotteries" in health care and other local services (Cummins et al., 2007) and comparisons of regional per capita government spending levels (MacKay, 2001) suggest that households care about their local government spending levels relative to those in other constituencies. Ravn et al. (2012) also propose the idea of procurement relationships that create a tendency for the government to favour transactions with sellers that supplied public goods in the past. In each period t, the government allocates spending $P_t G_t$ over differentiated goods sold by firms in a monopolistic market to maximize the quantity of a habit-adjusted composite good:

$$X^g_t = \left[\int_0^1 \left(G_{it} - \theta^g S^g_{it-1}\right)^{1-\frac{1}{e^P_t \zeta}} di \right]^{\frac{1}{1-\frac{1}{e^P_t \zeta}}},$$

[1] Corsetti et al. (2013) introduce the sovereign default by assuming a fiscal limit to the government-debt-to-GDP ratio. Whenever this ratio exceeds this limit a default in the form of a haircut will occur. However, the uncertainty surrounding the political process of a sovereign default is captured by granting the possibility of extracting such a limit each period from a probability distribution. In particular, each period, at a given level of indebtness, the ex-ante probability of default is given by the cumulative distribution function of a generalised beta distribution. With an appropriate calibration, such a mechanism generates a sovereign risk premium the quantitative effects of which are close to the simpler specification above. For a data-driven procedure to compute debt limits for advanced economies see Ghosh et al. (2013).

subject to the budget constraint $\int_0^1 P_{it}G_{it}di \leq P_t G_t$, where θ^g is the degree of deep habit formation in government spending and S^g_{it-1} denotes the stock of habits for this expenditure, which evolves as:

$$S^g_{it} = \varrho^g S^g_{it-1} + (1 - \varrho^g)G_{it}, \tag{8}$$

and exhibits persistence ρ^g. At the optimum

$$G_{it} = \left(\frac{P_{it}}{P_t}\right)^{-e^P_t \zeta} X^g_t + \theta^g S^g_{it-1}. \tag{9}$$

For simulations without a spending rule, aggregate real government consumption, G_t, is an autoregressive process

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \epsilon^G_t, \tag{10}$$

where ρ_G is an autoregressive parameter and ϵ^g_t is a mean zero, i.i.d. random shock with standard deviation σ^G.

The government budget constraint in real terms will read as follows:

$$b^g_t = \frac{R^g_{t-1}b^g_{t-1}}{\Pi_t(1+g_t)} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}, \tag{11}$$

where $b^g_t \equiv \frac{B^g_t}{P_t Y_t}$ is the debt-GDP ratio, $g_t \equiv \frac{Y_t - Y_{t-1}}{Y_t}$ and T_t represents total government revenue:

$$T_t = \tau^C_t C_t + \tau^W_t w_t h + \tau^K_t R^k_t K^p_t + \tau^L_t.$$

In order to reduce the number of tax instruments to one, we impose that τ^C_t, τ^W_t, τ^K_t and τ^L_t deviate from their steady state[2] in at by the same proportion (i.e. $\tau^C_t = \tau\bar{\tau}^C$, $\tau^W_t = \tau\bar{\tau}^W$, $\tau^K_t = \tau\bar{\tau}^K$,$\tau^L_t = \tau\bar{\tau}^L$) and that the proportional uniform tax change, τ, becomes one of our fiscal policy instruments. The other instrument we consider is government spending G_t. We allow the instruments to be adjusted according to

[2]The choices of steady-state tax rates and debt are discussed when we come to the policy exercises in Section 4.

the following Taylor-type rules:

$$\begin{aligned}\log\left(\frac{\tau_t}{\bar{\tau}}\right) &= \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right) + \rho_{\tau B}\log\left(\frac{b^g_{t-1}}{b^g}\right) + \rho_{\tau Y}\log\left(\frac{Y_t}{\bar{Y}}\right) + \epsilon^\tau_t, \text{ if } b^g \neq 0 \\ &= \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right) + \rho_{\tau B}\left(b^g_{t-1} - b^g\right) + \rho_{\tau Y}\log\left(\frac{Y_t}{\bar{Y}}\right) + \epsilon^\tau_t, \text{ if } b^g = 0 \qquad (12) \\ \log\left(\frac{G_t}{\bar{G}}\right) &= \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) - \rho_{GB}\log\left(\frac{b^g_{t-1}}{b^g}\right) - \rho_{GY}\log\left(\frac{Y_t}{\bar{Y}}\right) + \epsilon^G_t, \text{ if } b^g \neq 0 \\ &= \rho_G \log\left(\frac{\tau_t}{\bar{G}}\right) - \rho_{GB}\left(b^g_{t-1} - b^g\right) - \rho_{GY}\log\left(\frac{Y_t}{\bar{Y}}\right) + \epsilon^G_t, \text{ if } b^g = 0 \qquad (13)\end{aligned}$$

where ρ_τ implies persistence in the tax instrument, $\rho_{\tau B}$ is the responsiveness of the tax instrument to the deviation of government debt from its steady state, and $\rho_{\tau Y}$ is the responsiveness to the percentage deviation of the output gap. Parameters ρ_G, ρ_{GB}, and ρ_{GY} are the analogues in the expenditure rule, while ϵ^τ_t and ϵ^G_t are mean zero, i.i.d. fiscal shocks with standard deviations σ^τ and σ^G, respectively. Notice that these are Taylor-type rules as in Taylor (1993) that respond to deviations of output and debt from their deterministic steady state values and not from their flexi-price outcomes. Such rules have the advantage that they can be implemented using readily available macro-data series rather than from model-based theoretical constructs (see Schmitt-Grohe and Uribe (2007)). As reported below, a similar modelling choice is made for the monetary policy interest rate rule.[3]

2.3 Firms

A continuum of monopolistically competitive firms indexed by $i \in [0,1]$ rents capital, K_{it}, and hires labour, H_{it} to produce differentiated goods Y_{it} with convex technology $F\left(A_t H_{it}, K_{it}\right)$, where A_t is a labour-augmenting technology shock, which are sold at price P_{it}. Firms face quadratic price adjustment costs $\frac{\xi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t$, as in Rotemberg (1982) – where parameter ξ measures the degree of price stickiness – and maximize the following flow of discounted profits:

$$J_{it} = E_t\left\{\sum_{s=0}^{\infty} D_{t,t+s}\left[\begin{array}{c}\frac{P_{it+s}}{P_{t+s}}\left(C_{it+s} + G_{it+s} + I_{it+s}\right) \\ -\frac{W_{it+s}}{P_{t+s}} H_{it+s} - R^K_{t+s} K_{it+s} - \frac{\xi}{2}\left(\frac{P_{it+s}}{P_{it+s-1}} - 1\right)^2 Y_t\end{array}\right]\right\},$$

[3] In the context of a NK model with deep habits, Cantore et al. (2012) compare simple interest-rate rules embedding the model-based definition of the output gap to rules employing deviations of output from the steady state. They find that when the two types of rule are designed to be optimal, they result in almost identical real and inflation outcomes, though by means of different interest-rate paths.

with respect to K^p_{it+s}, H_{it+s}, C_{it+s}, S^c_{it+s}, G_{it+s}, S^g_{it+s} and P_{it+s} subject to (4), (5), (6), (8), (9), and the firm's resource contraint

$$C_{it+s} + G_{it+s} + I_{it+s} = F(H_{it}, K_{it}) - FC = Y_{it}, \tag{14}$$

where FC are fixed production costs, set to ensure that the free entry condition of long-run zero profits is satisfied. The corresponding first-order conditions for this problem are:

$$\begin{aligned}
R^K_t &= MC_t F_{K,it}, \\
\frac{W_t}{P_t} &= MC_t F_{H,it}, \\
\nu^c_t, &= \frac{P_{it}}{P_t} - MC_t + (1-\varrho^c)\lambda^c_t, \\
\lambda^c_t &= E_t D_{t,t+1}(\theta^c \nu^c_{t+1} + \varrho^c \lambda^c_{t+1}), \\
\nu^g_t &= \frac{P_{it}}{P_t} - MC_t + (1-\varrho^g)\lambda^g_t, \\
\lambda^g_t &= E_t D_{t,t+1}(\theta^g \nu^g_{t+1} + \varrho^g \lambda^g_{t+1}),
\end{aligned}$$

$$\begin{aligned}
&\frac{P_{it}}{P_t}(C_{it} + G_{it}) - \xi\left(\frac{P_{it}}{P_{it-1}} - 1\right)\frac{P_{it}}{P_{it-1}} Y_t + (1 - e^P_t \zeta)\left(\frac{P_{it}}{P_t}\right)^{1-e^P_t \zeta} I_t + e^P_t \zeta MC_t \left(\frac{P_{it}}{P_t}\right)^{-e^P_t \zeta} I_t \\
&- e^P_t \zeta \nu^c_t \left(\frac{P_{it}}{P_t}\right)^{-e^P_t \zeta} X^c_t - e^P_t \zeta \nu^g_t \left(\frac{P_{it}}{P_t}\right)^{-e^P_t \zeta} X^g_t + \xi E_t D_{t,t+1}\left[\left(\frac{P_{it+1}}{P_{it}} - 1\right)\frac{P_{it+1}}{P_{it}}\right] Y_{t+1} = 0.
\end{aligned}$$

Variables MC_t, ν^c_t, λ^c_t, ν^g_t, λ^g_t are the Lagrange multipliers associated with constraints (14), (5), (4), (9) and (8) respectively. In particular, MC_t is the shadow value of output and represents the firm's real marginal cost. Let MC^n_t denote the nominal marginal cost. The gross mark-up charged by final good firm i can be defined as $\mu_{it} \equiv P_{it}/MC^n_t = \frac{P_{it}}{P_t}/\frac{MC^n_t}{P_t} = p_{it}/MC_t$, where $p_{it} = \frac{P_{it}}{P_t}$. In the symmetric equilibrium all final good firms charge the same price, $P_{it} = P_t$, hence the relative price is unity, $p_{it} = 1$. It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost.

2.4 Monetary policy

Monetary policy is set according to a Taylor-type interest-rate rule:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \rho_y \log\left(\frac{Y_t}{\bar{Y}}\right) + \epsilon^M_t, \tag{15}$$

where ρ_r is the interest rate smoothing parameter and ρ_π and ρ_y are the monetary responses to inflation and output relative to its steady state, and ϵ_t^M is a mean zero, i.i.d. monetary policy shock with standard deviation σ^M.

2.5 Equilibrium

In equilibrium all markets clear. The model is completed by the resource constraint

$$Y_t = C_t + I_t + G_t + \frac{\xi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t + \frac{\chi}{2}\tau_t^2 Y_t, \tag{16}$$

and the following autoregressive processes for exogenous shocks:

$$\log\left(\frac{e_t^{\varkappa}}{\bar{e}^{\varkappa}}\right) = \rho_{\varkappa}\log\left(\frac{e_t^{\varkappa}}{\bar{e}^{\varkappa}}\right) + \epsilon_t^{\varkappa},$$

where (16) includes both price change costs and the cost of tax collection, $\varkappa = \{B, P, I, A\}$, $\rho_{\varkappa}$ are autoregressive parameter and $\epsilon_t^{\varkappa}$ are mean zero, i.i.d. random shock with standard deviation $\sigma^{\varkappa}$.

2.6 Functional forms

The utility function specializes as $U(X_t, 1-H_t) = \frac{\left[X_t^{(1-\varrho)}(1-H_t)^{\varrho}\right]^{1-\sigma_c}-1}{1-\sigma_c}$, where $\sigma_c > 0$ is the coefficient of relative risk aversion, and ω is a preference parameter that determines the relative weight of leisure and the consumption composite in utility. The consumption composite is a CES aggregate of private and public consumption, $X_t = \left\{\nu_x^{\frac{1}{\sigma_x}}\left(X_t^c\right)^{\frac{\sigma_x-1}{\sigma_x}} + (1-\nu_x)^{\frac{1}{\sigma_x}}\left(X_t^g\right)^{\frac{\sigma_x-1}{\sigma_x}}\right\}^{\frac{\sigma_x}{\sigma_x-1}}$, with ν_x representing the share of the private component in the aggregate and σ_x being the elasticity of substitution between the private and the public component. Investment adjustment costs are quadratic: $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, $\gamma > 0$, while the production function is Cobb-Douglas: $F(H_t, K_t) = (A_t H_t)^{\alpha} K_t^{1-\alpha}$, where α represents the labour share of income.

3 Bayesian Estimation

The model was estimated by Bayesian methods using US quarterly data for 6 observables (output, consumption, investment, government spending, nominal interest rate and inflation) over the period 1984:Q1-2008:Q3.

A number of structural parameters are kept fixed in the estimation procedure, in accordance with the usual practice in the literature (see Table 1). This is done so that the calibrated parameters reflect steady state values of the observed variables. An important parameter for the policy exercises is ϕ which determines the sovereign risk premium. Online Appendix A sets the calibration of ϕ. In the policy assessment we do not wish to underestimate the importance of the constraint imposed on fiscal authorities by financial markets. We therefore choose a value of ϕ at the upper end of the possible range. Another important parameter for welfare analysis is ν_x in the household utility. This is set so that the calibration of the government spending ratio, $\frac{G}{Y} = 0.2$, is optimal from the viewpoint of atomistic households. This is discussed further in Online Appendix B.

Estimation results from posteriors maximization are presented in Tables 2-3. We used the same priors as Smets and Wouters (2007) for common parameters whereas we used the estimates of Ravn et al. (2006) for the Deep habits parameters.

Calibrated parameter	Symbol	Value
Discount factor	β	0.9902
Depreciation rate	δ	0.025
Labour share	α	0.70
Substitution elasticity of goods	ζ	5.3
Fixed cost	FC	0.13095
ES between leisure and consumption	ϱ	0.8640
Share of private consumption over total consumption	ν_x	0.7662
Tax collection parameter	χ	0.05
Risk premium parameter	ϕ	0.01
Implied steady state relationship		
Hours	H	0.33
Government expenditure-output ratio	g_y	0.2
Consumption-output ratio	c_y	0.6203
Investment-output ratio	i_y	$1 - g_y - i_y$
Tax collection costs - output ratio	$\frac{\chi\tau^2}{2Y}$	0.01

Table 1: Calibrated Parameters

The estimation results are presented below[4]

[4]Full details are presented in Cantore et al. (2013a).

parameter	prior mean	post. mean	5% CI	95% CI	Prior	prior stdev	
ρ_A	0.5	0.9812	0.9682	0.9957	beta	0.2	
ρ_G	0.5	0.9311	0.8929	0.9685	beta	0.2	
ρ_{ZI}	0.5	0.3640	0.1452	0.5671	beta	0.2	
ρ_B	0.5	0.7972	0.6670	0.9378	beta	0.2	
ρ_P	0.5	0.4997	0.1652	0.8285	beta	0.2	
ε_A	0.1	1.3487	1.0097	1.6951	invg	2.0	
ε_G	0.5	0.7793	0.6808	0.8732	invg	2.0	
ε_{ZI}	0.1	2.7415	1.7539	3.9920	invg	2.0	
ε_P	0.1	0.1015	0.0215	0.2241	invg	2.0	
ε_M	0.1	0.0759	0.0507	0.0995	invg	2.0	
ε_B	0.1	1.2952	0.9563	1.6334	invg	2.0	

Table 2: Posterior results for the exogenous shocks

parameter	prior mean	post. mean	5% CI	95% CI	Prior	prior stdev
γ	2	1.9802	1.0632	2.8989	norm	1.5
σ_c	1.5	1.3734	0.8193	1.9131	norm	0.3750
ϱ^C	0.8	0.8380	0.7090	0.9530	beta	0.10
θ^C	0.8	0.7047	0.6127	0.7981	beta	0.10
ϱ^G	0.8	0.9129	0.7914	0.9949	beta	0.10
θ^G	0.8	0.6760	0.5085	0.8388	beta	0.10
σ_x	0.999	0.7034	0.4620	0.9437	gamma	1.00
ξ	25.300	25.2331	23.5999	26.8421	norm	1.00
ρ_π	1.5	1.8337	1.5104	2.1494	norm	0.25
ρ_r	0.75	0.8529	0.8049	0.9023	beta	0.1
ρ_y	0.25	0.0338	0.0015	0.0657	norm	0.05

Table 3: Posteriors results for model parameters

4 Optimal Monetary and Fiscal Stabilization Policy

We consider two aspects of monetary and fiscal optimal stabilization policy. The first is stabilization policy for '*normal times*'. Rules are then designed to minimize an expected conditional welfare loss starting at some steady state. In this case the optimal policy problem is purely stochastic: optimal policy is in response to all future stochastic shocks hitting the economy. By contrast, '*crisis management*' starts with the economy far from the steady state (for whatever reason) so that policy is then required both for the economy to return to the steady state (a deterministic problem) and for it to deal with future stochastic shocks (the stochastic problem).

For both problems we adopt a linear-quadratic (LQ) set-up which, for a given set of observed policy instruments w_t, considers model linearized around a steady state in a general state-space form:

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{w}_t + \begin{bmatrix} \mathsf{u}_{t+1} \\ 0 \end{bmatrix} \tag{17}$$

where $\mathsf{z}_t, \mathsf{x}_t$ are vectors of backward and forward-looking variables, respectively, w_t is a vector of policy variables, and u_t is an i.i.d. zero mean shock variable with covariance matrix Σ_u.[5]

Let $\mathsf{y}_t^T \equiv [\mathsf{z}_t\, \mathsf{x}_t\, \mathsf{w}_t]$. Our balanced-growth steady state is that of the non-linear deterministic Ramsey problem. Then following the general procedure set out in Online Appendix C, a welfare-based quadratic large-distortions approximation to expected welfare loss at time t, $E_t[\Omega_t]$, where

$$\Omega_t = \frac{1}{2}(1-\beta)\sum_{i=0}^{\infty}\beta^t[\mathsf{y}_{t+\tau}^T Q \mathsf{y}_{t+\tau}] \tag{18}$$

where Q is a matrix. With the LQ approximation the normal and crises aspects of policy conveniently components decompose, but one optimal policy emerges conditional on the initial point.

In the absence of a further constraints, the policymaker's optimization problem at time $t = 0$ is to minimize Ω_0 given by (18) subject to (17) and given z_0. If the variances of shocks are sufficiently large, there are two problems with the solution to this LQ problem. The first is that the variance the debt/GDP ratio b_t/Y_t may be very high, even with a sovereign risk premium. The second is that this will lead to a large nominal interest rate variability and the possibility of the nominal interest rate becoming negative. We defer considerations of the latter zero-lower bound problem until section 4.2. Here we consider the former problem.

4.1 Debt-GDP Upper Bound Considerations

We pose the problem as in terms of a high probability of violating an upper bound constraint on the debt-GDP ratio (for example 100%). Using discounted averaging, recalling $b_t^g \equiv \frac{B_t^g}{P_t Y_t}$, define $\bar{b}^g \equiv E_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t b_t^g\right]$ to be the discounted future average of the debt-GDP path $\{b_t^g\}$ at $t = 0$. Our 'approximate form' of the upper bound constraint is a requirement that $\bar{b}^g$ is at least k_b standard deviations below an upper bound bound for b_t^g given by b_{ub}^g. The constraint is then $b_{ub}^g - \bar{b}^g \geq k_b sd(b_t^g)$ which

[5]Lower case variables are defined as deviations about the balanced growth steady state; for a typical variable X_t, $x_t \equiv \log X_t/X_t$ where X_t is the balanced growth steady state.

squaring becomes

$$(b_{ub}^g - \bar{b}^g)^2 \geq k_b^2 \left[E_0(1-\beta) \sum_{t=0}^{\infty} \beta^t (b_t^g - \bar{b}^g)^2 \right] \tag{19}$$

Lemma

A sufficient condition for this constraint is that the following two constraints are satisfied

$$\begin{aligned} E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t (b_t^g)^2 \right] &\leq m \\ \frac{b_{ub}^g}{1+k_b^2} - \bar{b}^g &\geq K_b \\ \text{where } K_b &= \max \left[0, \frac{k_b}{1+k_b^2} \sqrt{m(1+k_b^2) - b_{ub}^2} \right] \end{aligned}$$

Proof. See Appendix D.

Now write the second constraint as

$$\bar{b}^g \equiv E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t b_t^g \right] \leq \frac{b_{ub}^g}{1+k_b^2} - K_b$$

This means we must add two terms $E_0(1-\beta)[w_b \sum_{t=0}^{\infty} \beta^t (b_t^g)^2 + \mu_b \sum_{t=0}^{\infty} \beta^t b_t^g] = w_b E_0(1-\beta) \sum_{t=0}^{\infty} \beta^t (b_t^g + \frac{\mu_b}{2w_b})^2 -$ a constant, where $w_b, \mu_b > 0$ are Lagrange multipliers, to the Lagrangian of the optimization problem. Defining $b_t = b_t^g - b^g$, it follows now that the effect of the two extra constraints is to replace the single period loss function with

$$\mathsf{y}_t^T Q \mathsf{y}_t + w_b (b_t^g + b^*)^2 \tag{20}$$

where the role of the Lagrange multiplier is taken over by $-b^*$, an *asset accumulation* requirement for the fiscal authority (relative to the initial steady state). The upper bound constraint can then be achieved by a combination of raising w_b and lowering the variance of the debt-income ratio, and lowering its steady-state thereby making room for a higher variance without violating the constraint.

There are two possible ways of treating the steady state. The first is normative and seeks a quadratic approximation about the steady state of the Ramsey problem without upper bound considerations. Then with our choice of functional form for the risk premium, the steady state debt-income ratio $b^g = 0$ and ruling out the availability lump-sum taxes (which would be optimal to use exclusively) the tax rates τ^C, τ^W and τ^K are computed in the Ramsey problem. However we adopt a second approach which is to choose empirical values for these tax rates and a pre-crisis debt-to-GDP ratio $b^g = 0.60$. This enables us to examine policy in the current fiscal environment that doesn't call for radical changes in the tax structure and accumulation of government assets. Then following Christiano et al. (2010): $\tau^C = 0.05$, $\tau^W = 0.24$, $\tau^K = 0.32$. The lump-sum tax τ^L is then set equal to 0.0641 in order to target $b^g = 0.60$ and b^* in (20) becomes a debt-GDP target found computationally so that the Ramsey problem with the fiscal instrument as the uniform change to all tax rates, τ_t, and b_t^g gives the required steady state. With this steady state and an upper bound debt-income ratio of $b_{ub} = 1$, a choice of $w_b = 0.001$ in (20) results in an extremely low probability of violating the upper bound constraint, certainly far lower than the threshold we set later for the zero lower bound constraint on the nominal interest rate.

4.2 Policy for Normal Times

In this section we examine optimal policy using both monetary and fiscal instruments. As in Cantore et al. (2012) 'optimality' can mean the welfare-optimal (Ramsey) policy, or time-consistent policy or optimized Taylor-type interest rate and fiscal rules.|[6] For the latter we compare the use of either our taxation instrument τ_t alone or in conjunction with government spending G_t according to rules (12) alone or together with (13). Monetary policy is conducted according to (15).

One can think of this choice of rules as assigning responsibility for stabilizing inflation and debt to the monetary authority and fiscal authorities respectively.[7] With both the interest rate and the fiscal instruments responding to fluctuations of output the two authorities are sharing responsibility for output fluctuations.

The assignment issue arises in a different form in Leeper (1991) who provides the original characterisation of policy rules as being 'active' or 'passive'. An active monetary policy rule is one in which the monetary authority satisfies the Taylor principle in that they adjust nominal interest rates such that real interest rates rise in response to excess inflation. Conversely, a passive monetary rule is one which fails to satisfy this principle. In Leeper's terminology a passive fiscal policy is one in which the fiscal instrument is adjusted to stabilize the government's debt stock, while an active fiscal policy fails to do this. Our simple rules allow for both these possibilities.[8]

[6]The LQ solutions for these three policy regimes are now standard - see, for example, Levine et al. (2007) for details. Regarding discretionary policy, recent important contributions by Blake and Kirsanova (2012) and Dennis and Kirsanova (2013) raise the possibility of multiple discretionary equilibria. These are of two types: "point-in-time", which give multiple responses of the private sector to a given policy rule and those arising from more than one discretionary policy. The iterative algorithm we use rules out the former. The latter can in principle be found by experimenting with different initializations; however for the model and loss function employed in this paper we have not been able to find more than one equilibrium.

[7]For a recent discussion of the assignment issue see Kirsanova et al. (2009).

[8]Cochrane (2011) proposes passive fiscal rules to avoid the arbitrary assumption of a non-explosive path for the price level needed in the standard Blanchard-Kahn RE solution. But Sims (2013) points out that introducing a very small feedback from inflation to the tax-rate, together with ZLB constraint on the nominal interest rate and an upper bound on government

For simple rules we impose two 'feasibility' constraints (Schmitt-Grohe and Uribe (2007)): $\rho_{r\pi} \leq 5$ and $\rho_{\tau B}, \rho_{GB} \leq 0.25$ to avoid threat of excessive changes in the interest rate, tax rate and government spending.[9] Table 4 sets out the welfare outcomes under our three policy regimes with the optimized feedback parameters for the simple rule. For the latter we allow for both passive and active fiscal policy. These are implemented by constraining the parameter $\rho_{r\pi}$ to be greater (the Taylor principle) or less than unity respectively. The welfare loss is reported in brackets as a consumption equivalent percentage increase below the optimal policy, c_e.[10]

To allow us to assess their separate contributions to stabilization, four possible combinations of policy instruments are considered. First we consider all instruments together. Then focusing only on simple commitment rules, we switch off the use of government spending and taxation changes separately keeping these fiscal instruments at their steady state values. Finally we consider 'monetary policy alone'. Then for the case of active fiscal policy the model is saddlepath-stable and government debt is stabilized with all instruments held fixed at their steady state. But with passive policy a tax instrument is still required to stabilize government debt; we use taxes with minimal feedback on the debt-income ratio.

A number of features from these results stand out. First, with all three instruments the gains from commitment amount to $c_e = 0.03\%$ and almost all such gains can be achieved by an optimized simple rule with passive fiscal policy. An optimized rule with active fiscal policy by contrast is hardly better than discretion in welfare terms and consists of a constant tax rate. The optimized monetary rule involves no response to output changes and with a very high degree of persistence is close to a price-level rule.[11] Second, the results from switching off the fiscal instruments one at a time indicate that government spending is the more effective fiscal instrument. Indeed the optimized active fiscal rule without the use of government spending sees a substantial welfare loss compared with the fully optimal policy with all instruments of over a 1% consumption equivalent. Why are tax changes less effective for stabilization purposes? The

asset accumulation, are sufficient to rule out such explosive paths.

[9]In fact $\rho_{\tau B}, \rho_{GB} \leq 0.25$ is the minimal feedback for either instrument separately to stabilize the government debt-income ratio.

[10]To derive the welfare in terms of a consumption equivalent percentage increase. ($c_e \equiv \frac{\Delta C}{C} \times 10^2$), expanding $U(X_t, 1-N_t)$ as a Taylor series, $\Delta U = U_C \Delta C = CMU^C c_e \times 10^{-2}$. Losses X reported in the Table are of the order of variances expressed as percentages and have been scaled by $1-\beta$. Thus $X \times 10^{-4} = \Delta U$ and hence $c_e = \frac{X \times 10-2}{CMU^C}$. For the steady state of this model, $CMU^C = 0.503$. It follow that a welfare loss difference of $X = 1$ gives a consumption equivalent percentage difference of $c_e \approx 0.02\%$.

[11]There has been a recent interest in the case for price-level rather than inflation stability. Gaspar et al. (2010) provide an excellent review of this literature. The basic difference between the two regimes in that under an inflation targeting mark-up shock leads to a commitment to use the interest rate to accommodate an increase in the inflation rate falling back to its steady state. By contrast a price-level rule commits to a inflation rate below its steady state after the same initial rise. Under inflation targeting one lets bygones be bygones allowing the price level to drift to a permanently different price-level path whereas price-level targeting restores the price level to its steady state path. The latter can lower inflation variance and be welfare enhancing because forward-looking price-setters anticipates that a current increase in the general price level will be undone giving them an incentive to moderate the current adjustment of its own price.

reason must be the existence of tax distortions in the model and the more direct demand channel offered by government spending in our NK model. Third, with monetary policy alone and active fiscal policy all three tax instruments are held fixed at their steady states. Then it is left entirely to the price level to stabilize the economy and government debt in particular in the face of shocks. As a consequence the volatility of inflation is very high as seen in the impulse responses, discussed below. This regime leads to the highest possible variances and welfare costs of $c_e = 1.11\%$ so we can conclude that this is a measure of the maximum cost of business cycle fluctuations. Finally, apart from the case of simple rules with active fiscal policy, the standard deviation of the nominal interest rates are high indicating a zero lower bound problem. This we return to in a later sub-section. However even with a steady-state debt-GDP ratio at 60%, the upper end required in the Euro-zone, and an upper bound of 100%, the standard-deviations reported in Table 5 with w_b set at a low value $w_b = 0.001$ implies a very low probability of exceeding this upper bound.

Figures 1-8 show the impulse responses to a 1% increase in the technology, mark-up, investment and preference shocks respectively. For each shock we first display the impulse responses for all policy regimes. For these, the fully optimal commitment rule and the optimized simple rule with passive fiscal policy are very close, so in a second figure we focus only on these two.

We see the familiar impulse responses in a NK model across all three monetary policy regimes. For a technology shock output immediately rises and, inflation falls. The optimal policy is to commit to a sharp monetary relaxation before gradually returning to the steady state. Both consumption and leisure rise (the latter a familiar result in the NK literature) and hours fall. The productivity shock results in a fall in the mark -up, a rise in the real wage, the real marginal cost and inflation rises under optimal and time consistent policy. Consumption and investment rise, the latter in response to a fall in the real interest rate. Real variables - output, hours and consumption differ little between optimal and time consistent policy for all shocks, which explains the small welfare differences in Table 4 for all shocks combined. For a (negative) mark-up shock (a shock to the elasticity parameter ζ) output, consumption, investment, hours rise. Inflation and the nominal interest change by very little and in a fashion consistent with the Taylor rule. The investment shock causes output and hours to rise but crowds out consumption. In all these responses the optimized simple rule with passive fiscal policy closely mimics the fully optimal policy, confirming the welfare outcomes in Table 4.

If government spending does not react to public debt and real output deviations, in Figures 9 and 10 we can explore a fiscal stimulus through an exogenous impulse to government spending of size one

Policy Mix	Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	Loss (c_e)
All Instruments	Optimal	not applicable	not applicable	not applicable	1.98 (0)
All Instruments	Time Cons	not applicable	not applicable	not applicable	3.43 (0.03)
All Instruments	Simple (PF)	[0.91, 5.00, 0.00]	[0.15, 0.25, 0.36]	[0.39, 0.25, 0.39]	2.19 (0.004)
All Instruments	Simple (AF)	[0.00, 0.00 0.00]	[0.00, 0.00, 0.00]	[0.44, 0.00, 0.50]	2.80 (0.02)
R_t, T_t; $G_t = G$	Simple (PF)	[1.00, 5.00, 0.00]	[0.62, 0.25, 0.00]	[0.00, 0.00, 0.00]	4.16(1.04)
R_t, T_t; $G_t = G$	Simple (AF)	[0.00, 0.00 0.00]	[0.85, 0.00, 0.02]	[0.00, 0.00, 0.00]	53.3 (1.02)
R_t, G_t; $T_t = T$	Simple (PF)	[0.89, 5.00, 0.00]	[0.00, 0.00, 0.00]	[0.57, 0.25, 0.25]	2.26 (0.006)
R_t, G_t; $T_t = T$	Simple (AF)	[0.05, 0.00 0.00]	[0.00, 0.00, 0.00]	[0.44, 0.00, 0.50]	2.81 (0.02)
R_t; min T_t, $G_t = G$	Simple (PF)	[1.00, 5.00, 0.00]	[0.00, 0.25, 0.00]	[0.00, 0.00, 0.00]	4.43(0.05)
R_t; min T_t, $G_t = G$	Simple (AF)	[0.00, 0.00 0.00]	[0.00, 0.00, 0.00]	[0.00, 0.00, 0.00]	57.7 (1.11)

Table 4: Optimal Interest Rate, Taxation and Government Spending Rules: Welfare Outcomes. The welfare loss is reported as a consumption equivalent percentage increase above the optimal policy.

percent. With the tax rule in place, this increase in government spending is financed by a combination of distortionary and non-distortionary tax.

An increase in aggregate demand as such acts as a fiscal stimulus - in fact with $\frac{G}{Y} = 0.2$ in the steady state the impact multiplier is well over unity in our estimated model and almost identical across all policy regimes.[12] Inflation falls initially because the estimated degree of deep habits makes aggregate supply initially shift more than aggregate demand, but then rises, which elicits an interest rate initial rise but then a fall, again for all regimes. In our model with deep habits we see the familiar result in the literature highlighted in our introduction that a fiscal stimulus causes the mark-up to decrease, the real wage to rise and a crowding in of consumption. For a (negative) mark-up shock (a shock to the elasticity parameter ζ) output, consumption, investment, hours rise. Inflation and the nominal interest change by very little and in a fashion consistent with the Taylor rule. The investment shock causes output and hours to rise but crowds out consumption.

4.3 Interest Rate Zero Lower Bound Considerations

Table 5 indicates that the aggressive nature of these rules leads to high interest rate variances resulting in a ZLB problem for all the rules. From the table with our zero-inflation steady state and nominal interest rate of 1% per quarter, optimal policy variances between 1.00 and 1.49 of a normally distributed variable imply a probability per quarter of hitting the ZLB in the range $[0.14, 0.22]$. At the upper end of these ranges the ZLB would be hit almost every year. In this subsection we address this issue.

As for the upper bound on the debt-GDP ratio, we can impose a lower bound effect on the nominal

[12]Note that Figures 9 and 10 depict impulse response functions (irf) to a shock to government spending of size one percent and the fiscal multiplier is given by $\frac{\Delta Y_t}{\Delta G_t} = \frac{Y_t}{G_t} \times$ irf.

Policy Mix	Rule	sd(Y_t)	sd(Π_t)	sd(h_t)	sd(C_t)	sd(R_t)	sd(τ_t)	sd(G_t)	sd($\frac{B}{4Y}$)
All Instruments	Optimal	9.14	0.13	0.81	7.89	1.49	2.67	9.46	6.40
All Instruments	Time Cons	9.20	0.28	1.27	7.84	1.00	13.3	9.14	14.3
All Instruments	Simple (PF)	8.88	0.13	1.34	7.69	0.47	2.31	9.07	9.84
All Instruments	Simple (AF)	8.47	0.29	1.92	7.66	0	0	7.50	0.77
R_t, T_t; $G_t = G$	Simple (PF)	7.02	0.12	2.46	3.68	0.42	8,80	0	14.0
R_t, T_t; $G_t = G$	Simple (AF)	6.47	3.99	7.94	6.00	0	0	0	11.9
R_t, G_t; $T_t = T$	Simple (PF)	8.65	0.13	1.46	7.86	0.46	0	7.69	5.03
R_t, G_t; $T_t = T$	Simple (AF)	8.47	0.29	1.92	7.66	0	0	7.50	0.77
R_t; min T_t, $G_t = G$	Simple (PF)	8.73	0.12	4.91	16.1	0.43	96.6	0	445
R_t; min T_t, $G_t = G$	Simple (AF)	6.47	3.97	7.94	6.01	0	0	0	12.0

Table 5: Optimal Interest Rate, Taxation and Government Spending Rules: Volatility Outcomes

interest rate by modifying the discounted quadratic loss criterion as follows.[13] Consider first the ZLB constraint on the nominal on the nominal interest rate. Rather than requiring that the gross rate $R_t \geq 1$ for *any* realization of shocks, we impose the constraint that the mean gross rate should at least k standard deviation above the ZLB. Again, for analytical convenience we use discounted averages.

Define $\bar{R} \equiv E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_t\right]$ to be the discounted future average of the nominal interest rate path $\{R_t\}$. Our 'approximate form' of the ZLB constraint is a requirement that $\bar{R}$ is at least k_r standard deviations above the zero lower bound; i.e., using discounted averages that

$$\bar{R} \geq k_r sd(R_t) = k_r \sqrt{\overline{R^2} - (\bar{R})^2} \tag{21}$$

Squaring both sides of (21) we arrive at

$$E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_t^2\right] \quad \leq \quad K_r \left[E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_t\right]\right]^2 \tag{22}$$

where $K_r = 1 + k_r^{-2} > 1$

Again, as in upper bound debt-income ratio considerations, we can write this as two sufficient con-

[13]This follow the treatment of the ZLB in Woodford (2003) and Levine et al. (2008a)

straints

$$
\begin{aligned}
E_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t R_t^2\right] &\leq m \\
E_0\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t R_t\right] &\geq \sqrt{\frac{m}{K_r}}
\end{aligned}
$$

which is equivalent to adding $E_0(1-\beta)[w_r\sum_{t=0}^{\infty}\beta^t R_t^2+\mu_r\sum_{t=0}^{\infty}\beta^t R_t] = w_r E_0(1-\beta)\sum_{t=0}^{\infty}\beta^t(R_t-\frac{\mu_r}{2w_r})^2-$ a constant, where $w_r, \mu_r > 0$ are Lagrange multipliers, to the Lagrangian of the optimization problem.w_r It follows that the effect of the extra constraint is to follow the same optimization as before, except that the single period loss function of log-linearized variables is replaced with

$$
L_t = \mathrm{y}_t^T Q \mathrm{y}_t + w_r(r_t - r^*)^2 \tag{23}
$$

where $r_t \equiv \log\frac{R_r}{R}$ and r^* is a nominal interest rate target for the constrained problem relative to the steady state.

In our LQ approximation of the non-linear optimization problem we have linearized around the Ramsey steady state which has zero inflation. With a ZLB constraint, the policymaker's optimization problem is now to choose an unconditional distribution for r_t, shifted to the right by an amount r^*, about a new *positive* steady-state inflation rate, such that the probability of the interest rate hitting the lower bound is extremely low. This is implemented by choosing the weight w_r *for each of our policy rules* so that $z_0(p)\sigma_r < R(\Pi) - 1$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $R(\Pi) = \Pi R(1)$ is the shifted nominal gross interest rate corresponding to a gross inflation rate Π (all in the steady state). Then given σ_r the steady state positive gross inflation rate that will ensure $R_t \geq 1$ with probability $1-p$ is given by

$$
\Pi^* = \max\left[\frac{z_0(p)\sigma_r + 1}{R(1)}, 1\right] \tag{24}
$$

In our linear-quadratic framework we can write the intertemporal expected welfare loss at time $t=0$ as the sum of stochastic and deterministic components, $\Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0$. By increasing w_r we can lower σ_r thereby decreasing $\pi^* \equiv \Pi^* - 1$ (the net shifted inflation rate) and reducing the deterministic component, but at the expense of increasing the stochastic component of the welfare loss. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes a ZLB constraint,

$R_t \geq 1$ with probability $1-p$. Figures 11 – 13 and Table 6 show this solution to the problem for all three policy regimes with $p = 0.0025$; ie., a very stringent ZLB requirement that the probability of hitting the zero lower bound is only once every 400 quarters or 100 years.

Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	Adjusted Loss (c_e)	w_r	π^*	sd(R_t)
Optimal	not applicable	not applicable	not applicable	2.09 (0.00)	0.006	0	0.36
Time Cons	not applicable	not applicable	not applicable	6.08 (0.08)	0.005	0.1	0.37
Simple (PF)	[1.00, 1.76, 0.00]	[0.51, 0.25, 0.07]	[0.63, 0.00, 0.23]	2.28 (0.002)	0.007	0	0.33
Simple (AF)	[0.00, 0.00 0.00]	[0.00, 0.00, 0.00]	[0.39, 0.00, 0.50]	3.40 (0.03)	0	0	0

Table 6: Imposing the Zero Lower Bound with All Instruments.

In this analysis it is important to stress that the extra term in the welfare criterion for the nominal interest rate only exists to impose the relevant constraints. After computing optimal policy when we come to reporting the welfare loss this extra contribution is removed in the numbers reported to give an 'adjusted' loss.

From Table 6 we observe first, that the imposition of the ZLB constraint increases the gains from commitment, in fact it doubles from $c_e = 0.04\%$ to $c_e = 0.08\%$.[14] Second the aggressive response of the nominal interest rate in the optimized simple rule with passive fiscal policy seen previously with no ZLB considerations now gives way to a far more restrained stance. The interest rate regime now becomes a pure price level rule on which we have commented. Finally, alongside the stochastic stabilization bias of discretion we now see a deterministic steady-state inflationary bias of 0.1% per quarter.

Note that in our LQ framework, the zero interest rate bound is very occasionally hit; then the interest rate is allowed to become negative, possibly using a scheme proposed by Gesell (1934) and Keynes (1936). Our approach to the ZLB constraint (following Woodford (2003) and Schmitt-Grohe and Uribe (2004))[15] in effect replaces it with a nominal interest rate variability constraint which ensures the ZLB is hardly ever hit. By contrast the work of a number of authors including Adam and Billi (2007), Coenen and Wieland (2003), Eggertsson and Woodford (2003) and Eggertsson (2006) study optimal monetary policy with commitment in the face of a non-linear constraint $R_t \geq 1$ which allows for frequent episodes of liquidity traps in the form of $R_t = 1$. But it is open to question whether the solution methods in these papers are adequate for models as large as that of this paper. For deterministic and stochastic simulations of linearized DSGE models for a given policy rule, Holden and Paetz (2012) provide a particularly efficient and

[14] See Levine et al. (2008a) for further discussion of this result.

[15] As in Levine et al. (2008a), we generalize the treatment of these authors however by allowing the steady-state inflation rate to rise. Our policy prescription has recently been described as a "dual mandate" in which a central bank committed to a long-run inflation objective sufficiently high to avoid the ZLB constraint as well as a Taylor-type policy stabilization rule about such a rate - see Blanchard et al. (2010) and Gavin and Keen (2012).

implementable algorithm for general inequality constraints and a very useful assessment of this literature.

4.4 Crisis Management of High Debt: How Fast, How Deep?

Finally we examine the question of how fast should a fiscal consolidation proceed. To examine this we subject the model to a further initial unanticipated debt shock. First we decompose the state vector z_t in deviation form about the deterministic steady state into deterministic and stochastic components:

$$\mathsf{z}_t = \underbrace{\bar{\mathsf{z}}_t}_{\text{deterministic}} + \underbrace{\tilde{\mathsf{z}}_t}_{\text{stochastic}}$$

and similarly for x_t and the instruments $\mathsf{w}_t = \bar{\mathsf{w}}_t + \tilde{\mathsf{w}}_t$ Then exploiting the LQ strucure of the problem the expected quadratic welfare loss can be expressed as $E_t[\Omega_t] = \bar{\Omega}_t + E_t[\tilde{\Omega}_t]$ and the optimal policy design **decomposes** into

A: Min $\bar{\Omega}_t$ wrt $\bar{w}_t \rightarrow$ deterministic expected path

and

B: Min $E_t[\tilde{\Omega}_t]$ wrt $\tilde{w}_t \rightarrow$ stochastic state-contingent path or rule

We have already considered problem B (policy for normal times). Now we turn to problem A where the policymaker is faced with an initial increase in the debt-GDP ratio. We examine a 20% increase which is still sufficiently small for the linearization to be valid, but large enough to be of interest. We use the rules designed to avoid hitting the interest rate ZLB set out in Table 6. Figures 14 and 15 show the simulation results.

These four sets of trajectories provide the expected responses of output, consumption etc to the unanticipated debt shock. If the policymaker chooses to continue with the state-contingent simple rules designed for normal times in response to future technology, mark-up, investment and preference shocks she would announce a consolidation programme that follows one of the simple rules. But in this deterministic exercise there is no reason why she should not instead follow the trajectory of the optimal policy, as long as commitment is credible (policy A) and use the rule B for any unexpected deviation about this path (policy B).

Three features of our results then stand out: first, there appears to be *some support for slow consolidation*, in response to high initial debt. Along the optimal trajectory the debt to income ratio falls at the rate of around 1% per year. Second, this consolidation is achieved using tax increases rather than a decrease in government spending. Third, if the government lacks commitment or must stick with the active fiscal rules the optimal speed of fiscal consolidation is much faster. The simple rules with active fiscal policy are particularly striking. A debt shock brings about a substantial increase in the price level to stabilize the debt-GDP ratio. This acts as a large supply-side shock with output, consumption, the real wage and hours working rising sharply. As a consequence the debt falls quickly. Since this particular rule dominates the graph, Fig 15 removes it and focuses on the simple passive fiscal rule alongside optimal policy. These responses are more plausible. Optimal policy promotes some initial output growth which gives way to austerity. Taxes rise substantially along with some more modest fall in government spending. Debt falls very slowly. Unlike normal times, the optimized simple rules falls short of mimicking the performance of the optimal policy for debt reduction, but the optimal rate of decline of debt remains slow.

5 Conclusions

This paper has examined fiscal-monetary interactions in a NK DSGE model with deep habits, distortionary taxes and a sovereign risk premium for government debt. As shown in Cantore et al. (2012) deep habits crucially affect the fiscal transmission mechanism in that it leads to a counter-cyclical mark-up even when prices are flexible. This feature boosts the size of a output expansion or contraction with important consequences for optimal monetary and fiscal policy.

We proceed to use the model in conjuction with the Bayesian estimates of Cantore et al. (2013a) to compute optimal monetary and fiscal policy first in 'normal times' with debt at its steady state and then in a crisis period with a much higher initial debt-GDP ratio. For the former, we find that both taxation and government spending fiscal instruments alongside monetary policy, the gains from commitment amount to a consumption equivalent of $c_e = 0.03\%$ and almost all such gains can be achieved by an optimized simple rule with passive fiscal policy. An optimized rule with active fiscal policy by contrast is hardly better than discretion in welfare terms and consists of a constant tax rate. The optimized monetary rule involves no response to output changes and, with a very high degree of persistence, is close to a price-level rule. By switching off the fiscal instruments one at a time we find that government spending is the more effective fiscal instrument in welfare terms. With monetary policy alone and active fiscal policy all three tax instruments are held fixed at their steady states. This provides a measure of the maximum cost of business cycle fluctuations which turns out to be over 1% in consumption equivalent terms. Apart from the case of simple rules with active fiscal policy, the standard deviation of the nominal interest rates are high indicating a zero lower bound problem which we address by modifying the optimization problem and the subsequent monetary rule.

For crisis management, fiscal consolidation should be slow unless the fiscal authority cannot commit, or must stick with an active fiscal simple rule. For the former case optimal consolidation is best achieved using tax increases and should proceed slowly at a rate of approximately 1% of debt-GDP per year. Thus an economy that sets out with an initial debt-income ratio of 100% to achieve a requirement of 60% should allow 40 years, clearly much slower than envisaged in current austerity programmes in Europe and elsewhere.

Two priorities for future research seem apparent. First in this paper we have adopted the standard information assumptions - perfect information on the part of the private sector, but a limited use of data by the econometrician. Elsewhere we have highlighted these inconsistent and implausible information assumptions (Levine et al. (2012)). It would be of interest to see if our results remain intact under informational consistency. Second, our model assumes full employment so if anything we are underestimating the cost of consolidation. We plan to revisit all the issues and experiments in this paper using a model with search-match labour market frictions as set out in Cantore et al. (2013b).

References

Adam, K. and Billi, R. M. (2007). Discretionary monetary policy and the zero lower bound on nominal interest rates. *Journal of Monetary Economics*, 54(3):728–752.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Batini, N., Callegari, G., and Melina, G. (2012). Successful austerity in the united states, europe and japan. IMF Working Papers 12/190, International Monetary Fund.

Bi, H., Leeper, E. M., and Leith, C. (2013). Uncertain fiscal consolidations. *The Economic Journal*, 123(566):F31–F63.

Blake, A. and Kirsanova, T. (2012). Discretionary policy and multiple equilibria in lq re models. *Review of Economic Studies*, 79(4):1309–1339.

Blanchard, O., Dell'Ariccia, G., and Mauro, P. (2010). Rethinking macroeconomic policy. *Journal of Money, Credit and Banking*, 42:199–215.

Blanchard, O. and Perotti, R. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *The Quarterly Journal of Economics*, 117(4):1329–1368.

Canova, F. and Pappa, E. (2011). Fiscal policy, pricing frictions and monetary accommodation. *Economic Policy*, 26(68):555–598.

Cantore, C., Levine, P., and Melina, G. (2013a). Deep versus superficial habits and the fiscal stimulus. *School of Economics Discussion Paper, University of Surrey*. Forthcoming.

Cantore, C., Levine, P., and Melina, G. (2013b). A fiscal stimulus and jobless recovery. *The Scandinavian Journal of Economics*. Forthcoming.

Cantore, C., Levine, P., Melina, G., and Yang, B. (2012). A fiscal stimulus with deep habits and optimal monetary policy. *Economics Letters*, 117(1):348–353.

Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1):pp. 78–121.

Christiano, L., Motto, R., and Rostagno, M. (2010). Financial factors in economic fluctuations. *ECB Working Paper*, 1192.

Cochrane, J. H. (2011). Determinacy and Identification with Taylor Rules. *Journal of Political Economy*, 119:565–615.

Coenen, G. and Wieland, V. (2003). The zero-interest rate bound and the role of the exchange rate for monetary policy in japan. *Journal of Monetary Economics*, 50:1071–1101.

Corsetti, G., Kuester, K., Meier, A., and Müller, G. J. (2013). Sovereign risk, fiscal policy and macroeconomic stability. *Economic Journal*. Forthcoming.

Cummins, J., Francis, R., and Coffey, R. (2007). Local solutions or postcode lotteries. *Office of Public Management*, May.

Currie, D. and Levine, P. (1993). *Rules, Reputation and Macroeconomic Policy Coordination*. CUP.

Denes, M., Eggertsson, G. B., and Gilbukh, S. (2013). Deficits, public debt dynamics and tax and spending multipliers. *The Economic Journal*, 123(566):F133–F163.

Dennis, R. and Kirsanova, T. (2013). Expectations traps and coordination failures with discretionary policymaking. Working Papers 2013-02, Business School - Economics, University of Glasgow.

Di Pace, F. and Faccini, R. (2012). Deep habits and the cyclical behaviour of equilibrium unemployment and vacancies. *Journal of Economic Dynamics and Control*, 36(2):183–200.

Eggertsson, G. (2006). The deflation bias and committing to being irresponsible. *Journal of Money, Credit and Banking*, 36(2):283–322.

Eggertsson, G. and Woodford, M. (2003). The zero interest-rate bound and optimal monetary policy. *Brooking Papers on Economic Activity*, 1:139–211.

Erceg, C. J. and Linde, J. (2013). Fiscal consolidation in a currency union: Spending cuts vs. tax hikes. *Journal of Economic Dynamics and Control*, 37(2):422–445.

Gaspar, V., Smets, F., and Vestin, D. (2010). Is Time Ripe for Price Level Path Stability? In Siklos, P. L., Bohl, M. T., and Wohar, M. E., editors, *Challenges in central banking: the current institutional environment and forces affecting monetary policy*. Cambridge University Press.

Gavin, W. T. and Keen, B. D. (2012). The zero lower bound and the dual mandate. *Federal Reserve Bank of St. Louis Working Papers*, 2012-026.

Gesell, S. (1934). *The Natural Economic Order*. Free-Economy Publishing Co., Phlip Pye, San Antonio.

Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D., and Qureshi, M. S. (2013). Fiscal fatigue, fiscal space and debt sustainability in advanced economies. *The Economic Journal*, 123(566):F4–F30.

Holden, T. and Paetz, M. (2012). Efficient Simulation of DSGE Models with Inequality Constraints. Quantitative Macroeconomics Working Papers 21207b, Hamburg University, Department of Economics.

Jacob, P. (2011). Deep Habits, Price Rigidities and the Consumption Response to Government Spending. *SSRN eLibrary*.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, New York.

Kirsanova, T., Leith, C., and Wren-Lewis, S. (2009). Monetary and Fiscal Policy Interaction: The Current Consensus Assignment in the Light of Recent Developments. *The Economic Journal*, 119(541):F482–F496.

Kirsanova, T. and Wren-Lewis, S. (2012). Optimal fiscal feedback on debt in an economy with nominal rigidities. *The Economic Journal*, 122(559):238–264.

Leeper, E. M. (1991). Equilibria under Active and Passive Monetary and Fiscal Policies. *Journal of Monetary Economics*, 27:129 – 147.

Leith, C., Moldovan, I., and Rossi, R. (2012). Optimal monetary policy in a New Keynesian model with habits in consumption. *Review of Economic Dynamics*, 15(3):416–435.

Levine, P., McAdam, P., and Pearlman, J. (2007). Quantifying and Sustaining Welfare Gains from Monetary Commitment. ECB Working Paper No. 709, presented at the 12th International Conference on Computing in Economics and Finance, Cyprus, June, 2006.

Levine, P., McAdam, P., and Pearlman, J. (2008a). Quantifying and sustaining welfare gains from monetary commitment. *Journal of Monetary Economics*, 55(7):1253–1276.

Levine, P., Pearlman, J., Perendia, G., and Yang, B. (2012). Endogenous persistence in an estimated DSGE model under imperfect information. *The Economic Journal*, 122(565):1287–1312.

Levine, P., Pearlman, J., and Pierse, R. (2008b). Linear-Quadratic Approximation, Efficiency and Target-Implementability. *Journal of Economic Dynamics and Control*, 32:3315–3349.

MacKay, R. R. (2001). Regional taxing and spending: The search for balance. *Regional Studies*, 35(6):563–575.

Magill, M. (1977a). A Local Analysis of N-sector Capital Accumulation under Uncertainty. *Journal of Economic Theory*, 15(2):211–219.

Magill, M. (1977b). Some New Results on the Local Stability of the Process of Capital Accumulation. *Journal of Economic Theory*, 15(2):174–210.

Monacelli, T., Perotti, R., and Trigari, A. (2010). Unemployment fiscal multipliers. *Journal of Monetary Economics*, 57(5).

Pappa, E. (2009). The effects of fiscal shocks on employment and the real wage. *International Economic Review*, 50(1):217–244.

Ramey, V. A. (2011a). Can governments purchases stimulate the economy? *Journal of Economic Literature*, 49(3):673–685.

Ramey, V. A. (2011b). Identifying government spending shocks. It's all in the timing. *The Quarterly Journal of Economics*, 126(1):1–50.

Ramey, V. A. and Shapiro, M. D. (1998). Costly capital reallocation and the effects of government spending. *Carnegie-Rochester Conference Series on Public Policy*, 48(1):145–194.

Ravn, M., Schmitt-Grohé, S., and Uribe, M. (2006). Deep habits. *Review of Economic Studies*, 73(1):195–218.

Ravn, M., Schmitt-Grohé, S., and Uribe, M. (2012). Consumption, government spending, and the real exchange rate. *Journal of Monetary Economics*, 59:215–234.

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4):517–31.

Schmitt-Grohe, S. and Uribe, M. (2004). Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle. NBER Working Papers 10724.

Schmitt-Grohe, S. and Uribe, M. (2007). Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*, 54(6):1702–1725.

Sims, C. A. (2013). Paper money. *American Economic Review*, 103(2):563–584.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Verhelst, B. and Van den Poel, D. (2012). Deep habits in consumption: A spatial panel analysis using scanner data. *Ghent University*. Mimeo.

Woodford, M. (2003). *Foundations of a Theory of Monetary Policy*. Princeton University Press.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.

Zubairy, S. (2012). Interest rate rules and equilibrium stability under deep habits. *Macroeconomic Dynamics*. Forthcoming.

Zubairy, S. (2013). On fiscal multipliers: Estimates from a medium scale DSGE model. *International Economic Review*. Forthcoming.

ONLINE APPENDICES

A Calibration of the Sovereign Risk Premium Parameter ϕ

In order to calibrate the sovereign risk premium, we assume the debt-to-GDP being one at the steady state such that the gross steady-state sovereign spread is simply $\Psi = \exp(\phi)$. Hence given a yearly net spread, *spread*, the associated ϕ for our quarterly model is $\phi = \log\left(1 + \frac{spread}{4}\right)$. In the table below we report the parameter values ϕ takes at different levels of the sovereign spread.

Yearly spread (basis points)	Quarterly net spread ($spread/4$)	ϕ
0	0.00000	0.00000
50	0.00125	0.00054
100	0.00250	0.00108
200	0.00500	0.00217
300	0.00750	0.00325
400	0.01000	0.00432
500	0.01250	0.00540

Table 7: Calibration of the sovereign risk premium

B Optimal Choice of Government Spending and Calibration of ν_x

Consider the RBC core of the model without a nominal dimension. Then the social planner's deterministic problem at time $t = 0$ is to allocate consumption, hours, output, investment, capital stock and government spending over time so as to maximize $\sum_{t=0}^{\infty} \beta^t U_t(X_t, H_t)$ subject to a resource constraint

$$Y_t = C_t + I_t + G_t \tag{B.1}$$

where

$$
\begin{aligned}
X_t &= \left\{\nu_x^{\frac{1}{\sigma_x}}\left[X_t^c\right]^{\frac{\sigma_x-1}{\sigma_x}}+(1-\nu_x)^{\frac{1}{\sigma_x}}\left[X_t^g\right]^{\frac{\sigma_x-1}{\sigma_x}}\right\}^{\frac{\sigma_x}{\sigma_x-1}} && \text{(B.2)}\\
X_t^c &= C_t-\theta^c S_{t-1}^c && \text{(B.3)}\\
S_t^c &= \varrho^c S_{t-1}^c+(1-\varrho^c)C_t && \text{(B.4)}\\
X_t^g &= G_t-\theta^g S_{t-1}^g && \text{(B.5)}\\
S_t^g &= \varrho^g S_{t-1}^g+(1-\varrho^g)G_t && \text{(B.6)}\\
Y_t &= F(H_t,K_t)-FC && \text{(B.7)}\\
K_t &= (1-\delta)K_{t-1}+I_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right] && \text{(B.8)}
\end{aligned}
$$

To perform this optimization set up the Lagrangian

$$
\begin{aligned}
L_0 &= \sum_{t=0}^{\infty}\beta^t\Big[U_t(X_t,H_t)+\mu_{1,t}[Y_t-C_t-I_t-G_t] && \text{(B.9)}\\
&+ \mu_{2,t}^c[X_t^c-C_t+\theta^c S_{t-1}^c]+\mu_{3,t}^c[S_t^c-\varrho^c S_{t-1}^c-(1-\varrho^c)C_t] && \text{(B.10)}\\
&+ \mu_{2,t}^g[X_t^g-G_t+\theta^g S_{t-1}^g]+\mu_{3,t}^g[S_t^g-\varrho^g S_{t-1}^g-(1-\varrho^g)G_t] && \text{(B.11)}\\
&+ \mu_{4,t}\left[K_t-(1-\delta)K_{t-1}-I_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]\right] && \text{(B.12)}\\
&+ \mu_{5,t}[Y_t-F(H_t,K_t)+FC]\Big] && \text{(B.13)}
\end{aligned}
$$

We are interested only in the allocation between private and public consumption given hours, investment, capital stock and therefore output. The first-order conditions relevant for this problem are:

$$
\begin{aligned}
X_t^c &: U_{X^c,t}+\mu_{2,t}^c=0 && \text{(B.14)}\\
C_t &: -\mu_{1,t}-\mu_{2,t}^c-\mu_{3,t}^c(1-\varrho^c)=0 && \text{(B.15)}\\
S_{t-1}^c &: \beta^{-1}\mu_{3,t-1}^c-\varrho^c\mu_{3,t}^c+\theta^c\mu_{2,t}^c=0 && \text{(B.16)}\\
X_t^g &: U_{X^g,t}+\mu_{2,t}^g=0 && \text{(B.17)}\\
G_t &: -\mu_{1,t}-\mu_{2,t}^g+\mu_{3,t}^g(1-\varrho^g)=0 && \text{(B.18)}\\
S_{t-1}^g &: \beta^{-1}\mu_{3,t-1}^g-\varrho^g\mu_{3,t}^g+\theta^g\mu_{2,t}^g=0 && \text{(B.19)}
\end{aligned}
$$

It follows that the difference between the composite private and public marginal consumption is given by

$$U_{X^c,t} - U_{X^g,t} = \mu^g_{2,g} - \mu^c_{2,t} = -\mu^g_{3,t}(1-\varrho^g) + \mu^c_{3,t}(1-\varrho^c) \tag{B.20}$$

This result contrasts with the choice of the *atomistic* household who takes the stocks of habit as exogenous and therefore ignores the two constraints in habit putting $\mu^g_{3,t} = \mu^c_{3,t} = 0$. Thus $U_{X^c,t} = U_{C,t} = U_{X^g,t} = U_{G,t}$ holds and is individually optimal for this case giving an allocation for which the household would vote. We use this condition to calibrate the preference parameter ν_x. *However it is not the correct condition for the social optimum in the presence of external habit.*

The steady state of (B.14)-(B.19) is given by

$$\begin{aligned}
U_{X^c} + \mu^c_2 &= 0 && \text{(B.21)}\\
-\mu_1 - \mu^c_2 - \mu^c_3(1-\varrho^c) &= 0 && \text{(B.22)}\\
\mu^c_3 - \beta\varrho^c\mu^c_3 + \beta\theta^c\mu^c_2 &= 0 && \text{(B.23)}\\
U_{X^g} + \mu^g_2 &= 0 && \text{(B.24)}\\
-\mu_1 - \mu^g_2 - \mu^g_3(1-\varrho^g) &= 0 && \text{(B.25)}\\
\mu^g_3 - \beta\varrho^g\mu^g_3 + \beta\theta^g\mu^g_2 &= 0 && \text{(B.26)}\\
& && \text{(B.27)}
\end{aligned}$$

Some algebraic manipulation then leads to

$$U_{X^c} - U_{X^g} = \mu^c_2 - \mu^g_2 = \frac{\mu^c_2(1-\varrho^c)\beta\theta^c}{(1-\beta\varrho^c)} - \frac{\mu^g_2(1-\varrho^g)\beta\theta^g}{(1-\beta\varrho^g)} \tag{B.28}$$

so the social planner's allocation coincides with that chosen by the atomistic household ($\mu^c_2 = \mu^g_2$) iff

$$\frac{(1-\varrho^c)\theta^c}{(1-\beta\varrho^c)} = \frac{(1-\varrho^g)\theta^g}{(1-\beta\varrho^g)} \tag{B.29}$$

This condition holds if deep habit parameters are the same for private and public consumption (our priors), but otherwise will only hold by extreme coincidence.

Finally note that *the steady-state allocation of the social planner's inter-temporal problem is not the same as the optimum of the steady-state inter-temporal utility.* The latter is found by maximizing $U(X, H)$

subject to

$$X = \left\{ \nu_x^{\frac{1}{\sigma_x}} \left[X^c\right]^{\frac{\sigma_x - 1}{\sigma_x}} + (1-\nu_x)^{\frac{1}{\sigma_x}} \left[X^g\right]^{\frac{\sigma_x - 1}{\sigma_x}} \right\}^{\frac{\sigma_x}{\sigma_x - 1}} \tag{B.30}$$

$$X^c = C - \theta^c S^c \tag{B.31}$$

$$S^c = C \tag{B.32}$$

$$X^g = G - \theta^g S^g \tag{B.33}$$

$$S^g = G \tag{B.34}$$

$$Y = F(H, K) - FC \tag{B.35}$$

$$K = \delta^{-1} I \tag{B.36}$$

The two problems only coincide if $\beta = 1$ in which case (B.29) becomes simply $\theta^c = \theta^g$.

The *Ramsey problem* for the NK model adds the nominal side determining prices, given the monetary instrument, and inflation costs to the resource constraint. This however does not change the public-private consumption problem which remains as before.

An interesting implication of the non-optimality of the steady state of the inter-temporal problem is that once it is reached it is not optimal to stay there. In general, the solution to the social planner's problem starting from some arbitrary initial configuration of the economy is only *ex ante* optimal – anywhere along the trajectory (including the final steady state) there exists an incentive to re-optimize. This is just another way of saying that the solution to the social planner's problem is *time-inconsistent* and the same applies the the Ramsey problem. An implication of all this is that even if we calibrate preferences such that the observed G/Y is consistent with the steady state of the social optimum then a spending shock can immediately increase or decrease the inter-temporal utility as observed in our simulations. In fact our calibration imposes $U_{C,t} = U_{G,t}$ which is only individually optimal and then it turns out that an AR1 negative spending shock increases welfare despite lowering output. But all these depends on the relative strength of deep habit for private and public consumption.

C The Hamiltonian Quadratic Approximation of Welfare

Suppose we have a deterministic dynamic optimization problem expressed in the form[16]

$$\max \sum_{t=0}^{\infty} \beta^t U(X_{t-1}, W_t) \;\; s.t. \;\; X_t = f(X_{t-1}, W_t) \tag{C.1}$$

given initial and possibly tranversality conditions, which has a steady state solution $\bar{X}, \bar{W}$ for the states X_t and the policies W_t. Define $x_t = X_t - \bar{X}$ and $w_t = W_t - \bar{W}$ as representing the first-order approximation to deviations of states and policies from their steady states.

The Lagrangian L for the problem is

$$L = \sum_{t=0}^{\infty} \beta^t [U(X_{t-1}, W_t) - \lambda_t^T (X_t - f(X_{t-1}, W_t))] \tag{C.2}$$

so that a *necessary* condition for the solution to (C.1) is that the Lagrangian is stationary at all $\{X_s\}$, $\{W_s\}$ i.e.

$$U_W + \lambda_t^T f_W = 0 \qquad U_X - \frac{1}{\beta}\lambda_{t-1}^T + \lambda_t^T f_X = 0 \tag{C.3}$$

These necessary conditions for an optimum do not imply that the there is an asymptotic steady state to (C.3). However for the purposes of this paper, let us assume that this is the case, so that a steady state $\bar{\lambda}$ for the Lagrange multipliers exists as well. Now define the Hamiltonian $H_t = U(X_{t-1}, W_t) + \bar{\lambda}^T f(X_{t-1}, W_t)$. The following is the discrete time version of Magill (1977a):

Theorem 1: If a steady state solution $(\bar{X}, \bar{W}, \bar{\lambda})$ to the optimization problem (C.1) exists, then for any small initial perturbation x_0 about $\bar{X}$, the solution to the problem

$$\max \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} x_{t-1} & w_t \end{bmatrix} \begin{bmatrix} H_{XX} & H_{XW} \\ H_{WX} & H_{WW} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ w_t \end{bmatrix} \;\; s.t. \;\; x_t = f_X x_{t-1} + f_W w_t$$

where H_{XX}, etc denote second-order derivatives evaluated at $(\bar{X}, \bar{W})$, has the same stability properties

[16]This Appendix closely follows Levine et al. (2008b). An alternative representation of the problem is $U(X_t, W_t)$ and $E_t[X_{t+1}] = f(X_t, W_t)$ where X_t includes forward-looking non-predetermined variables and $E_t[X_{t+1}] = X_{t+1}$ for the deterministic problem where perfect foresight applies. Whichever one uses, it is easy to switch from one to the other by a simple re-definition. Magill (1977a) adopted a continuous-time model without forward-looking variables. We present a discrete-time version with forward-looking variables. As we demonstrate in the paper, although the inclusion of forward-looking variables significantly alters the nature of the optimization problem, these changes only affect the boundary conditions and not the steady state of the optimum which is all we require for LQ approximation.

as the solution to (C.1).

Judd (1998), (page 506) thus identifies this as the LQ approximation to the problem (C.1). The reason why this result holds is because the derivatives of the Lagrangian with respect to X_t and W_t are zero when evaluated at $(\bar{X}, \bar{W}, \bar{\lambda})$. By definition, $\sum_{t=0}^{\infty} \beta^t U(X_{t-1}, W_t) = \sum_{t=0}^{\infty} \beta^t [U(X_{t-1}, W_t) - \bar{\lambda}^T (X_t - f(X_{t-1}, W_t))]$, and the first-order term of the Taylor series expansion of the latter expression is zero. Although we deal directly with forward-looking systems in Theorem 3(b) below, we note that the above theorem applies both to backward-looking engineering-type systems and to rational expectations (RE) systems, in that approximation is about the long run of the optimum. However in the case of RE, the conventional optimum is obtained as a time-inconsistent solution, but the LQ approximation can also be used to obtain the timeless perspective optimum.

For the result of theorem to hold $(\bar{X}, \bar{W}, \bar{\lambda})$ must satisfy (C.3). These, it should be stressed, are necessary but not sufficient conditions for a local maximum. A standard sufficient condition for optimality is that the functions $f(X, W)$ and $U(X, W)$ are concave, but this is rarely satisfied in examples from economics. A more useful sufficient condition is the following:

Theorem 2: A sufficient condition for for the steady state of (C.3) to be a local maximum is that the matrix of second derivatives of H in (C) is negative semi-definite[17].

This condition is easy to check, but in the event that it does not hold, the following discrete time version of the sufficient conditions for an optimum in Magill (1977b) is applicable when the constraints and/or the welfare function are non-concave. It is based on iterating on the quadratic approximation to the value function. Part (a) below is a standard result, and relates to the fact that one requires a second-order condition to be met for the policy variables. Part (b), which is the main theoretical result of this paper, extends part (a) to the case when there are forward-looking variables.

Theorem 3:

(a) Case with no forward-looking variables: A necessary and sufficient condition for the solution (C.3) to the dynamic optimization problem (C.1) to be a local maximum is that $\beta f_W^T P_t f_W + H_{WW}$ is negative definite for all t, where the matrices $f_X, f_W, H_{XX}, H_{XW}, H_{WW}$ are all evaluated along the solution

[17] A simple example of a problem for which a maximum exists, but for which this sufficient condition does not hold is: max $x^2 - y^2$ such that $y = ax + b$. It is easy to see that the stationary point is a maximum when $|a| > 1$.

path and P_t satisfies the backwards Riccati equation given by:

$$P_{t-1} = \beta f_X^T P_t f_X - (\beta f_X^T P_t f_W + H_{XW})(\beta f_W^T P_t f_W + H_{WW})^{-1}(\beta f_W^T P_t f_X + H_{WX}) + H_{XX} \tag{C.4}$$

and the value function of small perturbations x_t about the path of the optimal solution dynamic optimization problem is given by $\frac{1}{2}x_t^T P_t x_t$.

(b) Case with forward and backward-looking variables: Consider a rational expectations system, where we order X_t as predetermined followed by non-predetermined variables, so that the latter dynamic constraints involve forward-looking expectations. Suppose that there is a long-run steady state solution to the first-order conditions. Then a further necessary and sufficient condition for this to be a maximum is that the bottom right-hand corner P_{22} of the the steady-state Riccati matrix P is negative definite.

Proof of Theorem

The basic idea is that the the optimal policy depends on the initial condition and the instruments and, in the case of an RE system, the jumps in the non-predetermined variables. Given the latter, one can take a dynamic programming approach to the problem to prove (a): taking variations about the optimal path, one may write the value function V_t at time t as a constant plus $\frac{1}{2}x_t^T P_t x_t$. Using (C), one can write the value function V_{t-1} (ignoring constants) as

$$V_{t-1} = \frac{1}{2}\max\left\{\beta(f_X x_{t-1} + f_W w_t)^T P_t (f_X x_{t-1} + f_W w_t) + \begin{bmatrix} x_{t-1} & w_t \end{bmatrix} \begin{bmatrix} H_{XX} & H_{XW} \\ H_{WX} & H_{WW} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ w_t \end{bmatrix}\right\} \tag{C.5}$$

with respect to w_t. The stated conditions for a maximum, and the update of P_t are straightforward to derive from this.

To prove (b), recall that from Currie and Levine (1993), we have the result under RE that V_0 is given by $\frac{1}{2}(x_0^{pT}(P_{11} - P_{12}P_{22}^{-1}P_{21})x_0^p + p_0^T P_{22}^{-1} p_0)$ where x_t^p are the deviations in the predetermined variables, p_0 is the initial value of the Lagrange multipliers associated with the non-predetermined variables (and is the source of the time inconsistency problem), and $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ is written conformably with predetermined and non-predetermined variables respectively. Clearly if P_{22}^{-1} is not non-negative definite, then the value of V_0 can be set arbitrarily large by appropriate choice of p_0; in such a case, a solution to the problem which tends to a steady state optimum does not exist.

As mentioned above we assume the existence of a steady state solution to (C.3) given by $[\bar{X}, \bar{W}, \bar{\lambda}]$, since we are interested in approximations about the latter. Hence the matrices in (C.4) (apart from P_t) are constant. Thus this theorem provides a means of checking whether a candidate solution to (C.3) actually is optimal. Note that the perturbed system is in standard linear-quadratic format, which is the basis for this result.[18]

Using these results our general procedure for approximating the non-linear optimization problem by a LQ one is as follows:

1. Set out the deterministic non-linear problem for the Ramsey Problem, to maximize the representative agents' utility subject to non-linear dynamic constraints.

2. Write down the Lagrangian for the problem.

3. Calculate the first order conditions. We do not require the initial conditions for an optimum since we ultimately only need the steady-state of the Ramsey problem.

4. Calculate the steady state of the first-order conditions. The terminal condition implied by this procedure is such that the system converges to this steady state.

5. Calculate a second-order Taylor series approximation, about the steady state, of the Hamiltonian associated with the Lagrangian in 2.

6. Calculate a first-order Taylor series approximation, about the steady state, of the first-order conditions and the original constraints.

7. Use 4. to eliminate the steady-state Lagrangian multipliers in 5. By appropriate elimination both the Hamiltonian and the constraints can be expressed in minimal form. This then gives us the accurate LQ approximation of the original non-linear optimization problem in the form of a minimal linear state-space representation of the constraints and a quadratic form of the utility expressed in terms of the states.

8. In an LQ procedure for computing ex ante optimal policy in stochastic setting compute the following two necessary and sufficient conditions for a particular steady state of the first order conditions to be a local maximum:

    (a) Condition 1: $\beta f_W^T P_t f_W + H_{WW}$ is negative definite

    (b) Condition 2: P_{22} is negative definite.

    If these conditions are satisfied then proceed to time consistent and optimized simple rules

[18] Levine et al. (2008b) show that Magill (1977a)'s result easily extends to the stochastic case as well.

APPENDIX FOR PUBLICATION

D Proof of Lemma

The constraint is then $b^g_{ub} - \overline{b^g} \geq k_b sd(b^g_t)$ which squaring becomes

$$(b^g_{ub} - \overline{b^g})^2 \geq k_b^2 \left[E_0(1-\beta) \sum_{t=0}^{\infty} \beta^t (b^g_t - \overline{b^g})^2 \right] = k_b^2 (\overline{(b^g)^2} - (\overline{b^g})^2) \tag{D.1}$$

defining

$$\overline{b^g} \equiv E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t b^g_t \right] \tag{D.2}$$

$$\overline{(b^g)^2} \equiv E_0 \left[(1-\beta) \sum_{t=0}^{\infty} \beta^t (b^g_t)^2 \right] \tag{D.3}$$

Completing the square, (D.1) can be written as

$$\left(\frac{b_{ub}}{\sqrt{1+k_b^2}} - \sqrt{1+k_b^2}\, \overline{b^g} \right)^2 + \frac{b_{ub}^2 k_b^2}{1+k_b^2} \geq k_b^2 \overline{(b^g)^2} \tag{D.4}$$

Then a little algebra shows that the two constraints in the lemma are sufficient to satisfy (D.4).

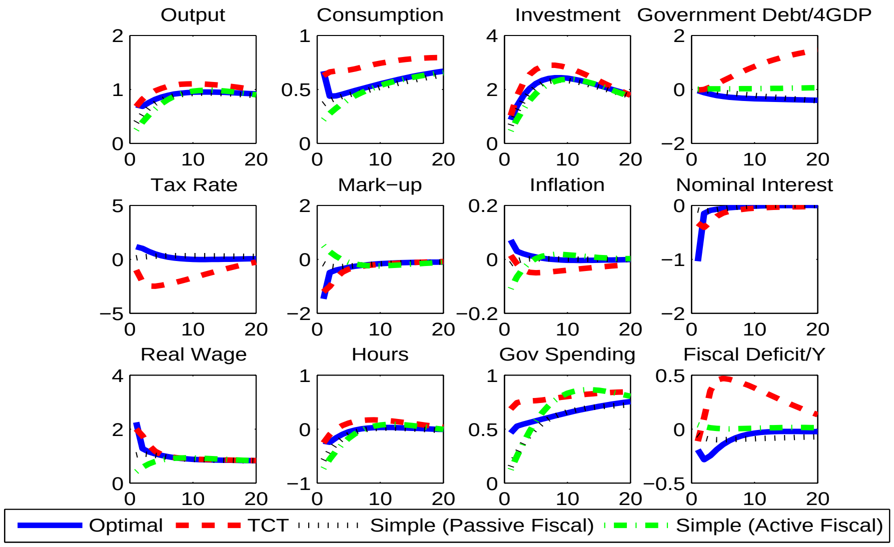

Figure 1: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Technology Shock

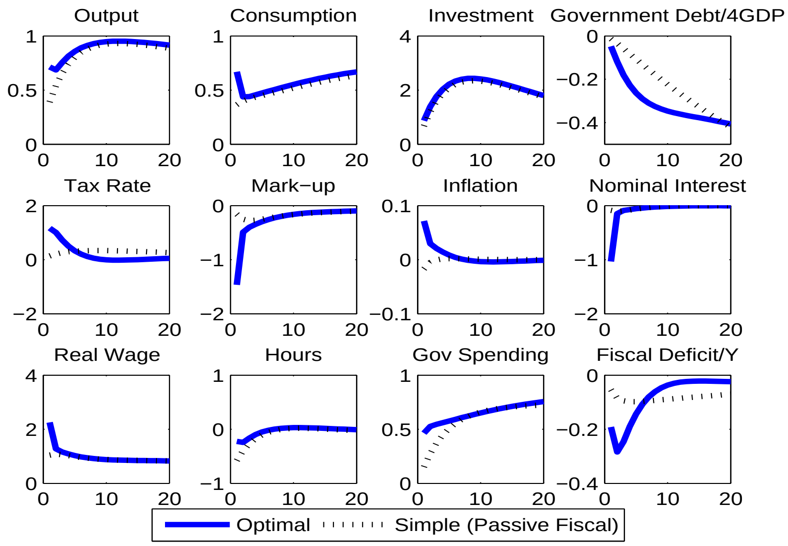

Figure 2: IRFs for Optimal Monetary Taxation and Government Spending Rules. Technology Shock

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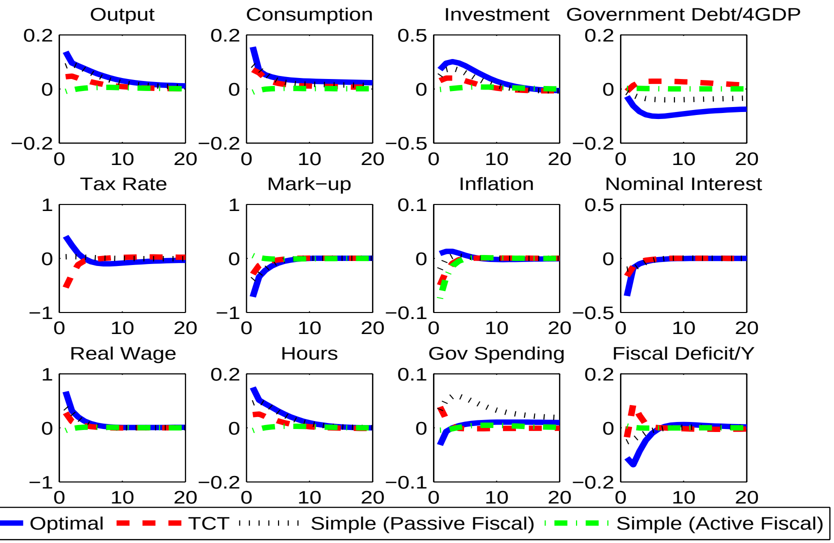


Figure 3: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Mark-up Shock

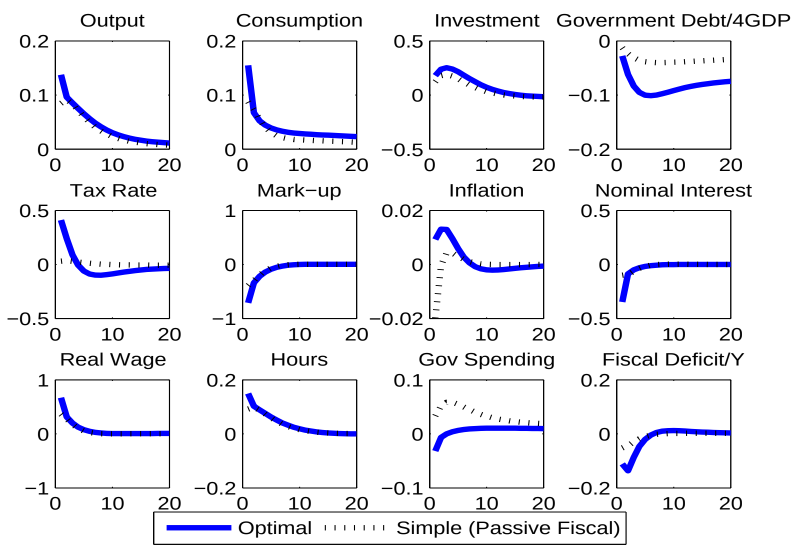


Figure 4: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Mark-up Shock

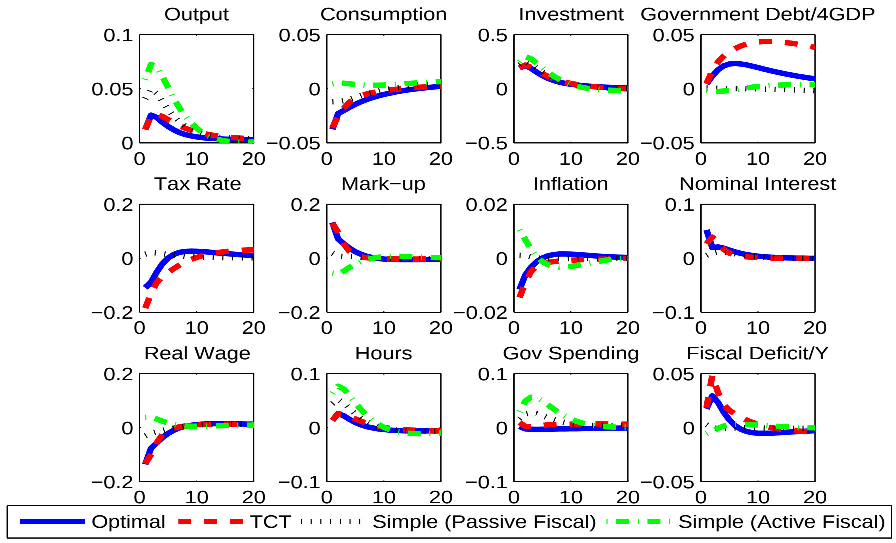


Figure 5: IRFs for Optimal Monetary, Taxation Rules and Government Spending Rules. Investment Shock

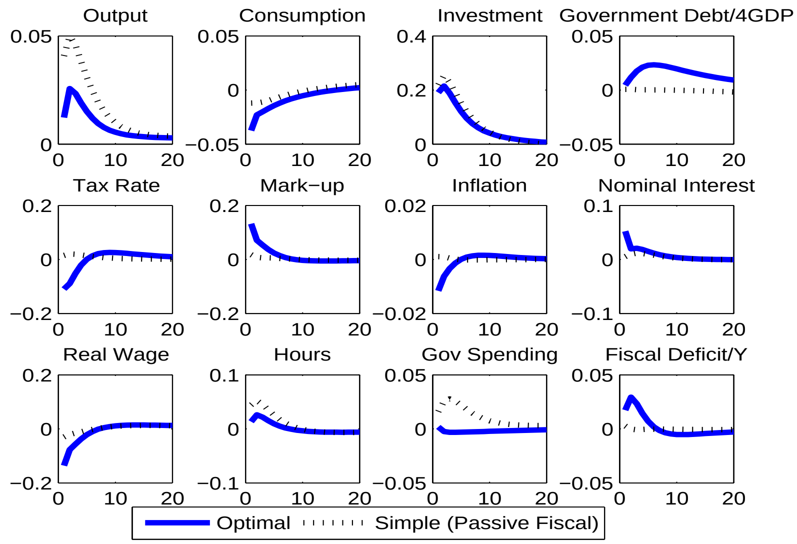


Figure 6: IRFs for Optimal Monetary, Taxation Rules and Government Spending Rules. Investment Shock

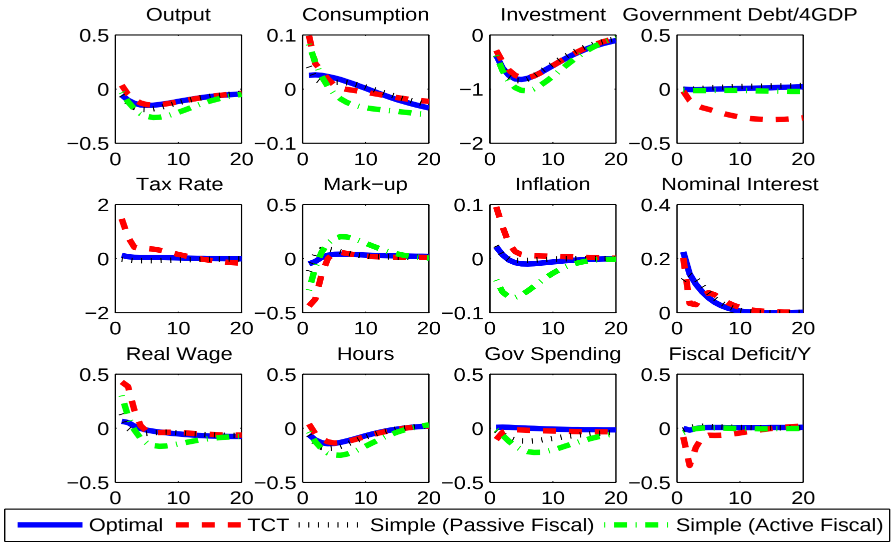


Figure 7: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Preference Shock

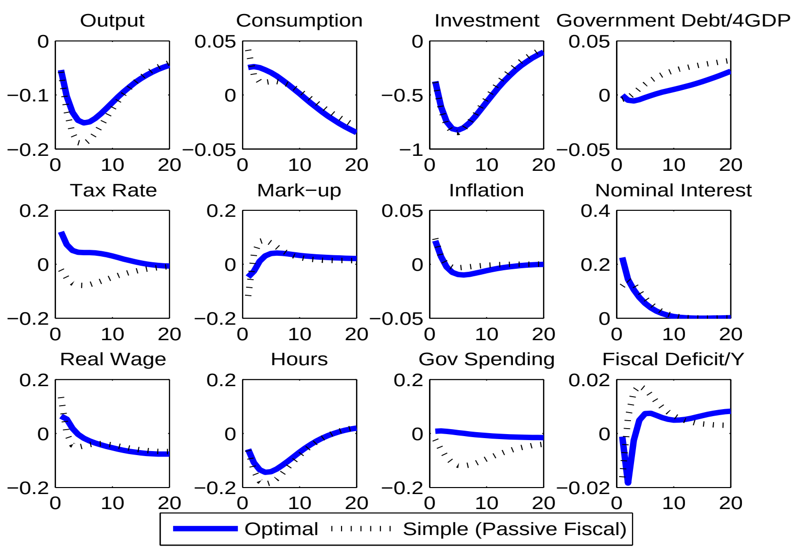


Figure 8: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Preference Shock

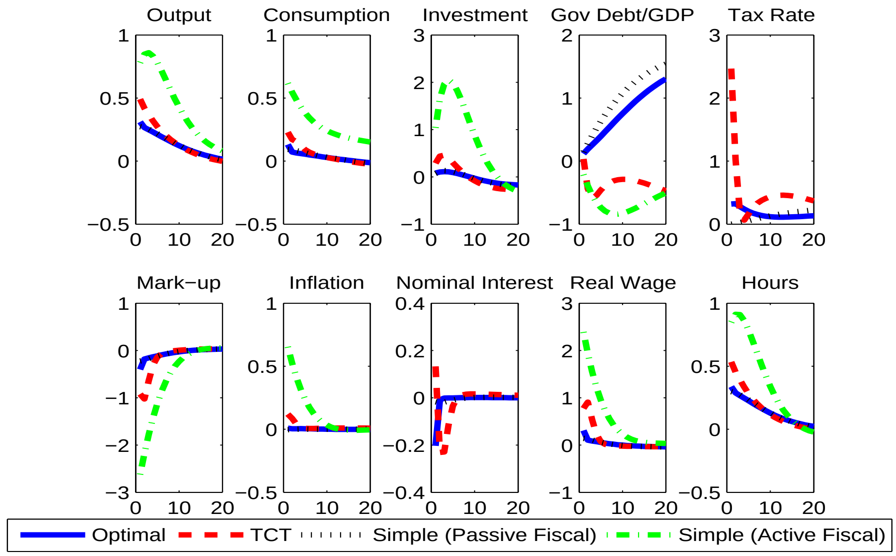


Figure 9: IRFs for Optimal Monetary and Taxation Rules. Fiscal Stimulus

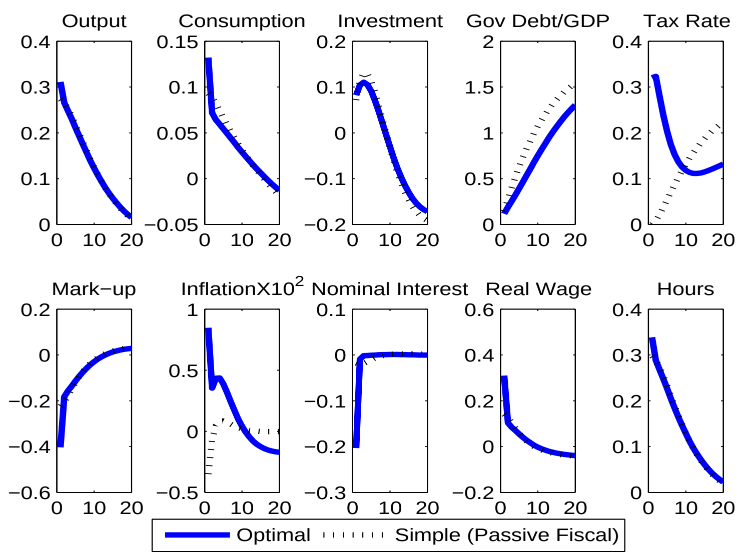


Figure 10: IRFs for Optimal Monetary and Taxation Rules. Fiscal Stimulus

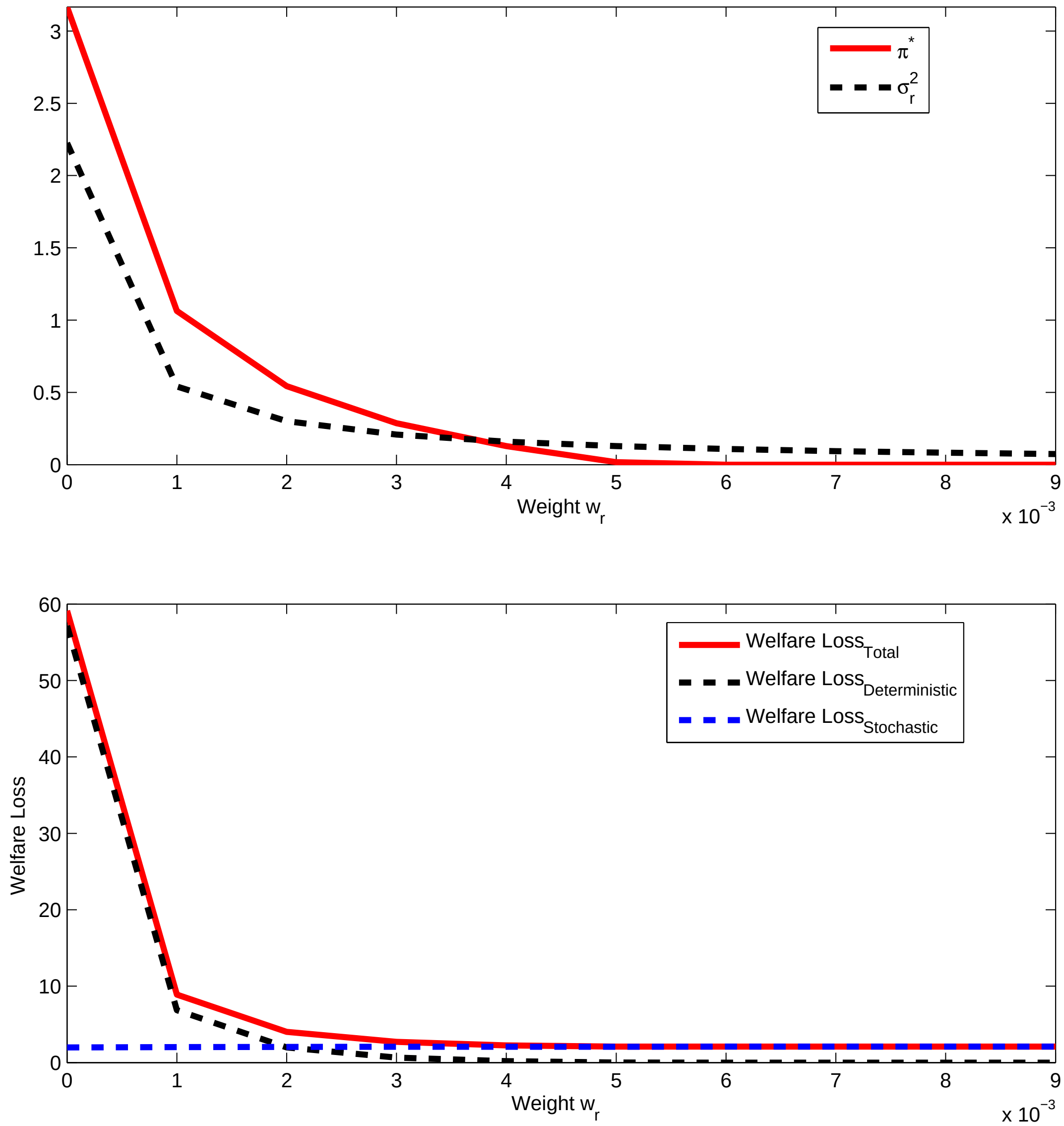
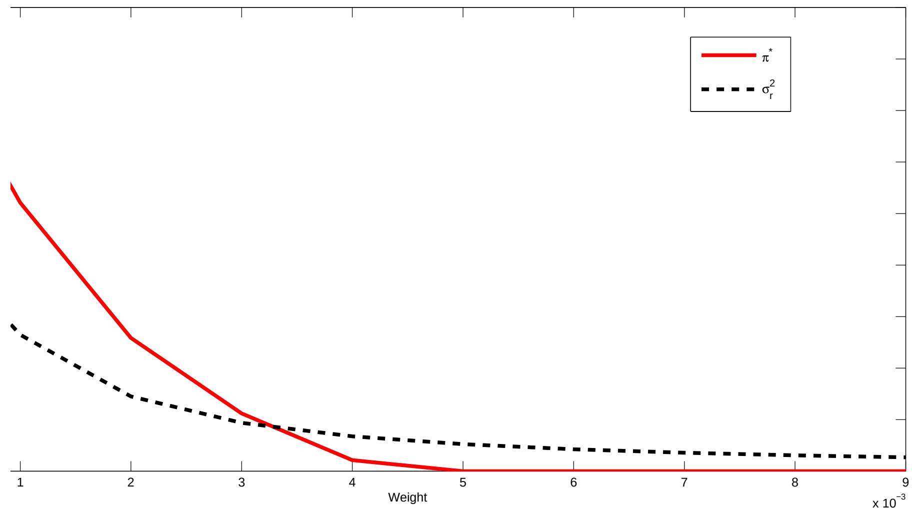


Figure 11: Imposition of ZLB for Optimal Monetary, Taxation and Government Spending Rules Rules



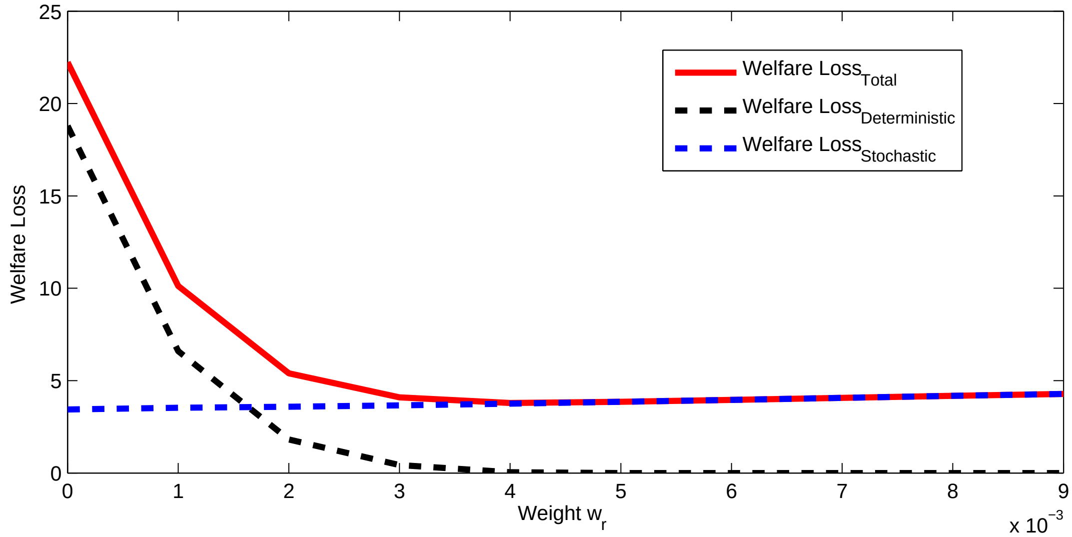


Figure 12: Imposition of ZLB for Time Consistent Monetary, Taxation and Government Spending Rules

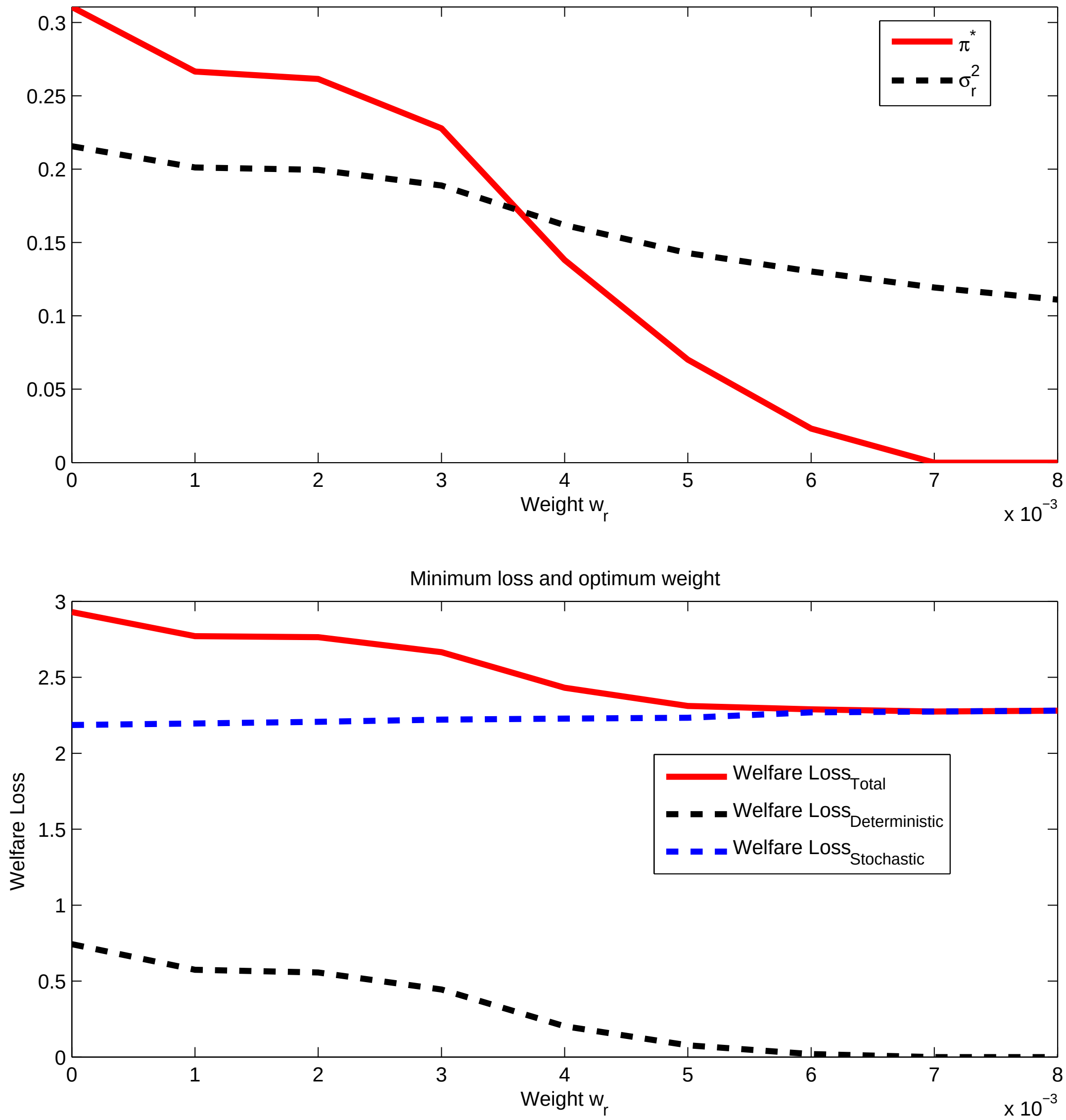


Figure 13: Imposition of ZLB for Simple Monetary, Taxation and Government Spending Rules with Passive Fiscal Policy

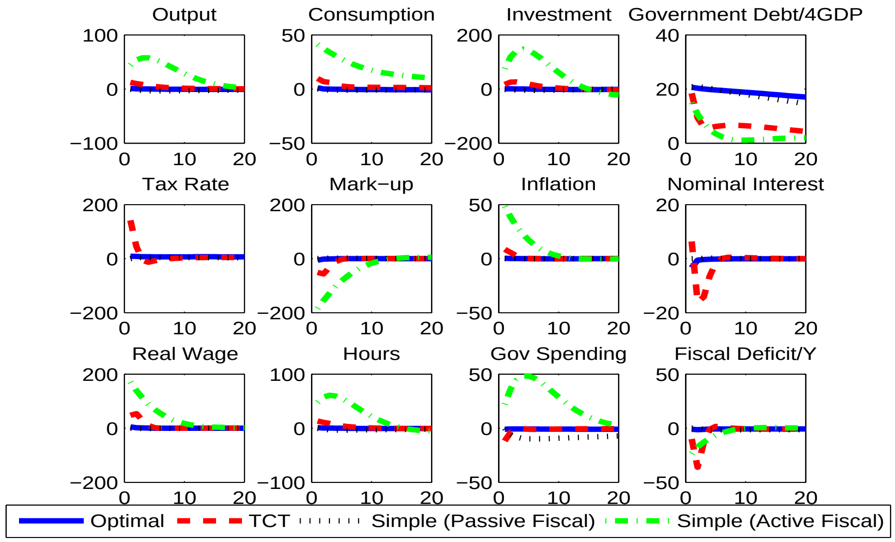


Figure 14: IRFs for Optimal Monetary, Taxation and Government Spending Rules. Debt Shock

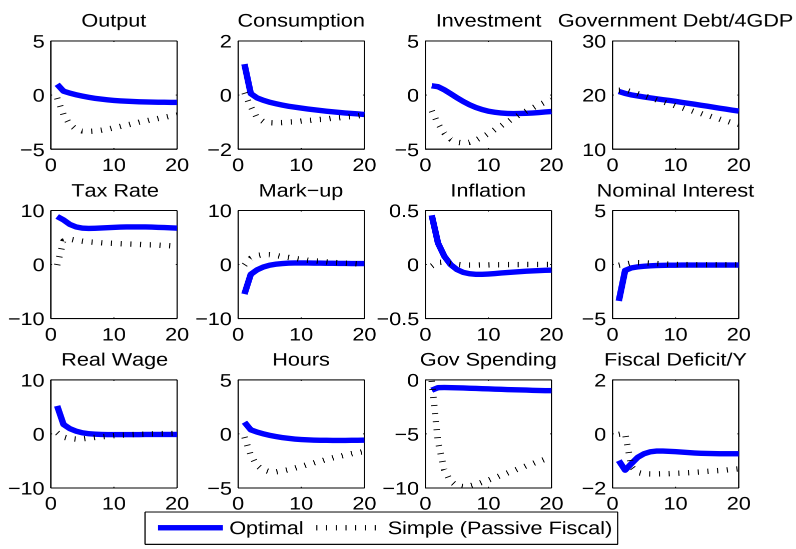


Figure 15: IRFs for for Optimal Monetary, Taxation and Government Spending Rules. Debt Shock

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